ECON 4101 Econometrics
CM15 Homework

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```
data <- fread("cobb_douglas_data.txt")
data[, `:=`(Year, as.Date(paste0(Year, "-01-01")))]
```

```
##           Year Output Labor Capital
##  1: 1899-01-01    100   100     100
##  2: 1900-01-01    101   105     107
##  3: 1901-01-01    112   110     114
##  4: 1902-01-01    122   118     122
##  5: 1903-01-01    124   123     131
##  6: 1904-01-01    122   116     138
##  7: 1905-01-01    143   125     149
##  8: 1906-01-01    152   133     163
##  9: 1907-01-01    151   138     176
## 10: 1908-01-01    126   121     185
## 11: 1909-01-01    155   140     198
## 12: 1910-01-01    159   144     208
## 13: 1911-01-01    153   145     216
## 14: 1912-01-01    177   152     226
## 15: 1913-01-01    184   154     236
## 16: 1914-01-01    169   149     244
## 17: 1915-01-01    189   154     266
## 18: 1916-01-01    225   182     298
## 19: 1917-01-01    227   196     335
## 20: 1918-01-01    223   200     366
## 21: 1919-01-01    218   193     387
## 22: 1920-01-01    231   193     407
## 23: 1921-01-01    179   147     417
## 24: 1922-01-01    240   161     431
##           Year Output Labor Capital
```

```
attach(data)
```

We model the data using a transformation of the Cobb-Douglas production function:

$$\ln Q = \ln A + \alpha \ln L + \beta \ln K$$

where Q = output, L = labor, K = capital, and A = constant.

```
log.Output <- log(Output)
log.Labor <- log(Labor)
log.Capital <- log(Capital)
mod1 <- lm(log.Output ~ log.Labor + log.Capital)
summary(mod1)
```

```
##
## Call:
## lm(formula = log.Output ~ log.Labor + log.Capital)
```

```
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.075282 -0.035234 -0.006439  0.038782  0.142114
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.17731    0.43429  -0.408  0.68721
## log.Labor    0.80728    0.14508   5.565 0.000016 ***
## log.Capital  0.23305    0.06353   3.668  0.00143 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.05814 on 21 degrees of freedom
## Multiple R-squared:  0.9574, Adjusted R-squared:  0.9534
## F-statistic: 236.1 on 2 and 21 DF,  p-value: 0.000000000000004038
```

```
confint(mod1)
```

```
##                  2.5 %    97.5 %
## (Intercept) -1.0804715 0.7258521
## log.Labor    0.5055758 1.1089806
## log.Capital  0.1009361 0.3651708
```

The assumption of constant returns to scale requires that $\alpha + \beta = 1$. We test this assumption using the following restricted least squares framework:

$$\begin{aligned} \text{Original Model: } & \ln Q = \ln A + \alpha \ln L + \beta \ln K \\ \text{Restriction: } & \alpha + \beta = 1 \\ \implies & \ln Q = \ln A + (1-\beta) \ln L + \beta \ln K \\ \implies & \ln Q - \ln L = \ln A + \beta(\ln K - \ln L) \end{aligned}$$

```
log.Output <- log.Output - log.Labor
log.Capital <- log.Capital - log.Labor
mod2 <- lm(log.Output ~ log.Capital)
summary(mod2)
```

```
##
## Call:
## lm(formula = log.Output ~ log.Capital)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.082565 -0.032869 -0.006925  0.040529  0.134443
##
## Coefficients:
##             Estimate Std. Error t value   Pr(>|t|)
## (Intercept)  0.01454    0.01998   0.728      0.474
## log.Capital  0.25413    0.04122   6.165 0.00000332 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.05707 on 22 degrees of freedom
## Multiple R-squared:  0.6334, Adjusted R-squared:  0.6167
## F-statistic:    38 on 1 and 22 DF,  p-value: 0.000003324
```

```
beta <- mod2$coefficients["log.Capital"]
alpha <- 1 - beta
ratio <- Output/(Labor^alpha * Capital^beta)
h <- exp(mod2$coefficients["(Intercept)"])
ggplot() + geom_line(aes(x = Year, y = ratio)) + geom_hline(aes(yintercept = h),
    linetype = "dashed")
```