ECON 4101 Econometrics
CM23 Homework

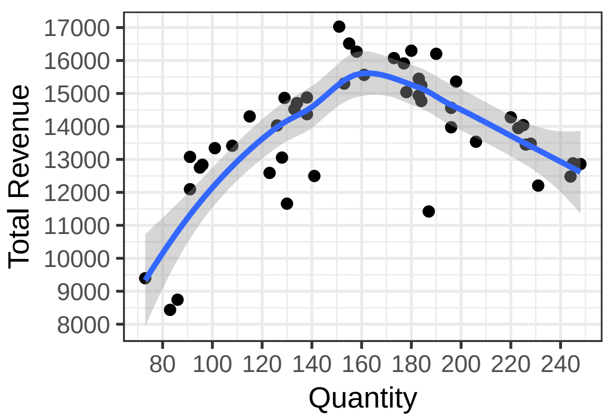
Pranav Singh

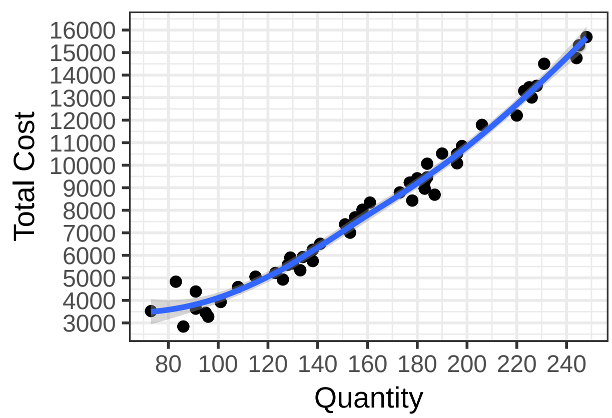
April 12, 2017

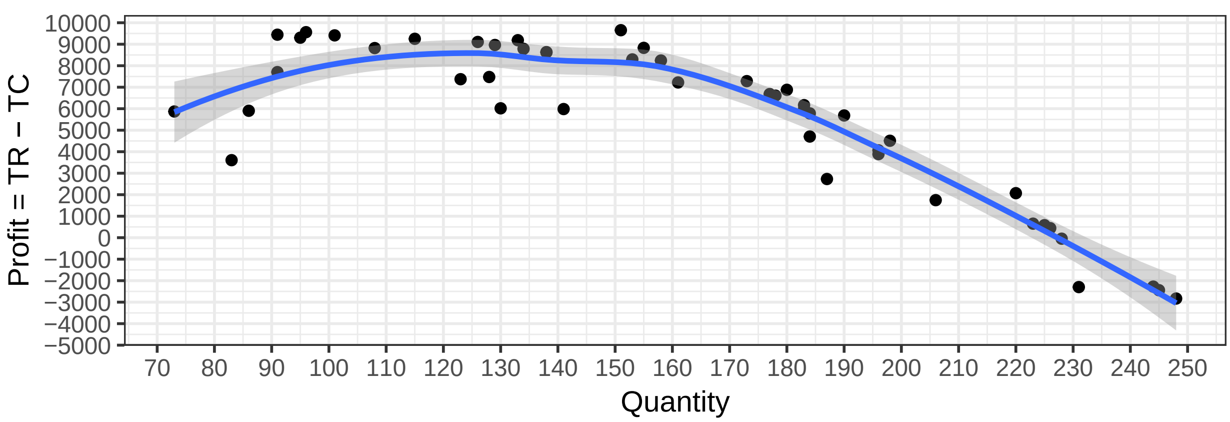
```
df <- read.csv("../../Data/monop.csv")
str(df)
```

```
## 'data.frame':    48 obs. of  3 variables:
##  $ tr: int  13536 12205 12881 12837 13477 14524 15041 14938 14278 12480 ...
##  $ tc: int  11790 14503 15323 3276 13523 5337 8431 8960 12207 14756 ...
##  $ q : int  206 231 245 96 228 133 178 183 220 244 ...
```

```
lay <- rbind(c(1, 2), c(3))
g1 <- ggplot(df, aes(x = q, y = tr)) + geom_point() + geom_smooth(method = "loess") +
    scale_x_continuous(breaks = pretty(df$q, n = 10)) + scale_y_continuous(breaks = pretty(df$tr,
    n = 10)) + ylab("Total Revenue") + xlab("Quantity")
g2 <- ggplot(df, aes(x = q, y = tc)) + geom_point() + geom_smooth(method = "loess") +
    scale_x_continuous(breaks = pretty(df$q, n = 10)) + scale_y_continuous(breaks = pretty(df$tc,
    n = 10)) + ylab("Total Cost") + xlab("Quantity")
g3 <- ggplot(df, aes(x = q, y = tr - tc)) + geom_point() + geom_smooth(method = "loess") +
    scale_x_continuous(breaks = pretty(df$q, n = 20)) + scale_y_continuous(breaks = pretty(-5000:10000,
    n = 20)) + ylab("Profit = TR - TC") + xlab("Quantity")
grid.arrange(grobs = list(g1, g2, g3), layout_matrix = lay)
```







Monopolist's Economic Model:

$$tr = \beta_1 q + \beta_2 q^2$$
$$tc = \alpha_1 + \alpha_2 q + \alpha_3 q^2$$

Part A

$$mr = \frac{d}{dq}\{tr\} = \beta_1 + 2\beta_2 q$$
$$mc = \frac{d}{dq}\{tc\} = \alpha_2 + 2\alpha_3 q$$

Part B

$$\begin{aligned}
mr &= mc \\
\implies \beta_1 + 2\beta_2 q^* &= \alpha_2 + 2\alpha_3 q^* \\
\implies (2\beta_2 - 2\alpha_3)q^* &= \alpha_2 - \beta_1 \\
\implies q^* &= \frac{\alpha_2 - \beta_1}{2(\beta_2 - \alpha_3)}
\end{aligned}$$

Part C

```
m.tr <- lm(tr ~ 0 + q + I(q^2), df)
summary(m.tr)
```

```
##
## Call:
## lm(formula = tr ~ 0 + q + I(q^2), data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3599.9  -287.4   258.7   732.2  2167.6
##
## Coefficients:
##         Estimate Std. Error t value            Pr(>|t|)
## q      174.28028    4.53989   38.39 <0.0000000000000002 ***
## I(q^2)  -0.50243    0.02354  -21.34 <0.0000000000000002 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1179 on 46 degrees of freedom
## Multiple R-squared:  0.9932, Adjusted R-squared:  0.9929
## F-statistic:  3346 on 2 and 46 DF,  p-value: < 0.00000000000000022
```

```
m.tc <- lm(tc ~ q + I(q^2), df)
summary(m.tc)
```

```
##
## Call:
## lm(formula = tc ~ q + I(q^2), data = df)
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1040.69  -352.62    -8.34   272.99  1325.43
##
## Coefficients:
##               Estimate Std. Error t value       Pr(>|t|)
## (Intercept) 2066.08294  727.21801   2.841        0.00673 **
## q             -1.57841    9.45245  -0.167        0.86813
## I(q^2)         0.22768    0.02889   7.881 0.000000000515 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 462.5 on 45 degrees of freedom
## Multiple R-squared:  0.9841, Adjusted R-squared:  0.9834
## F-statistic:  1396 on 2 and 45 DF,  p-value: < 0.00000000000000022
```

These estimates are appropriate (unbiased) for models that satisfy the first four Gauss-Markov assumptions: (1) the model is linear in parameters, (2) the observations are randomly sampled, (3) the model has zero conditional mean, and (4) there is no perfect collinearity between the model's terms.

```r
profitMaximizingQuantity <- function(alpha, beta) {
    (alpha[2] - beta[1])/(2 * (beta[2] - alpha[3]))
}
qstar <- profitMaximizingQuantity(m.tc$coefficients, m.tr$coefficients)
```

From part (b), we see that the OLS estimates suggest the profit maximizing level of output is $q^* = 120.4327681$.

Part D

```r
qstar <- round(qstar)
pred.tr <- predict(m.tr, data.frame(q = qstar))
pred.tc <- predict(m.tc, data.frame(q = qstar))
pred.profit <- pred.tr - pred.tc
```

The predicted profit is $8523.35.

Part E

```r
for (i in 1:5) {
    bg <- bgtest(m.tr, order = i)
    s <- sprintf("Breusch-Godfrey Test: AR(%d) P=%0.4f \n", i, bg$p.value)
    cat(s)
}
```

```
## Breusch-Godfrey Test: AR(1) P=0.0000
## Breusch-Godfrey Test: AR(2) P=0.0000
## Breusch-Godfrey Test: AR(3) P=0.0000
## Breusch-Godfrey Test: AR(4) P=0.0000
## Breusch-Godfrey Test: AR(5) P=0.0000
```

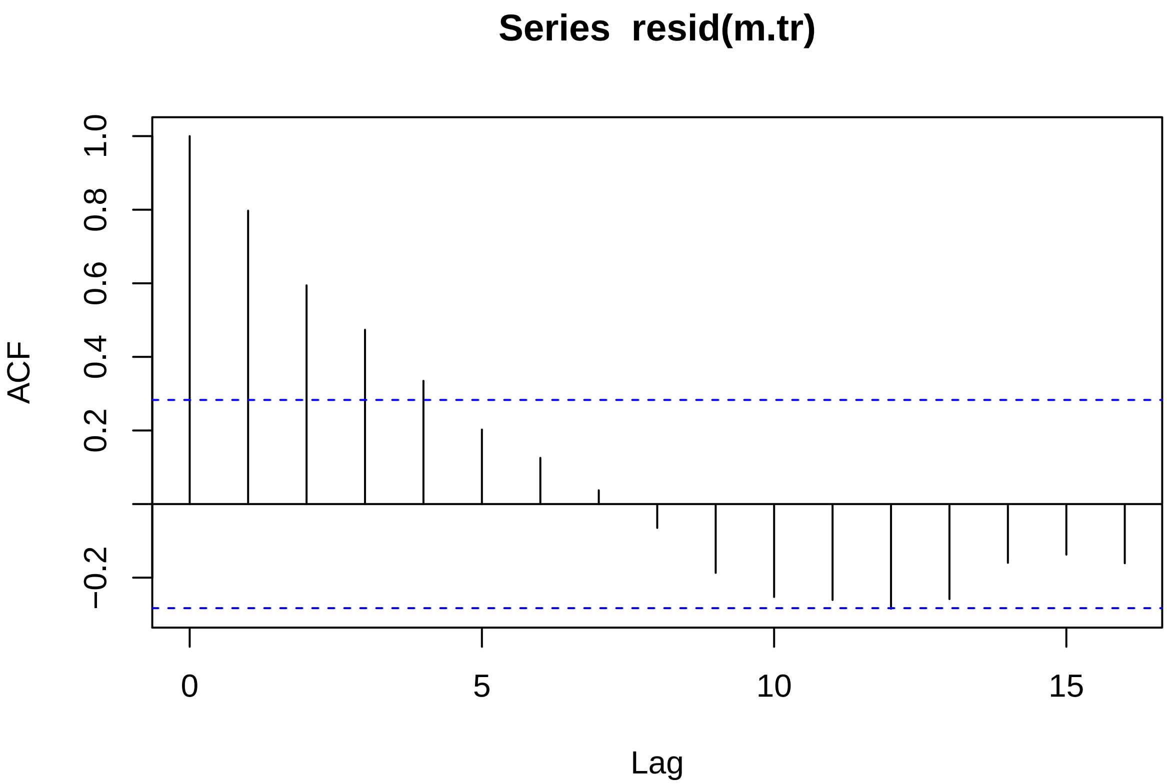
```r
for (i in 1:5) {
    bg <- bgtest(m.tc, order = i)
```

```
    s <- sprintf("Breusch-Godfrey Test: AR(%d) P=%0.4f \n", i, bg$p.value)
    cat(s)
}
```

```
## Breusch-Godfrey Test: AR(1) P=0.0052
## Breusch-Godfrey Test: AR(2) P=0.0163
## Breusch-Godfrey Test: AR(3) P=0.0306
## Breusch-Godfrey Test: AR(4) P=0.0630
## Breusch-Godfrey Test: AR(5) P=0.0653
```

Part F

```
acf(resid(m.tr))
```




```
m.tr.ols <- gls(tr ~ 0 + q + I(q^2), df)
m.tr.gls <- gls(tr ~ 0 + q + I(q^2), df, correlation = corARMA(p = 1, q = 0))
anova(m.tr.ols, m.tr.gls)
```

```
##          Model df      AIC      BIC    logLik   Test L.Ratio p-value
## m.tr.ols     1  3 822.9862 828.4721 -408.4931
## m.tr.gls     2  4 750.6851 757.9997 -371.3425 1 vs 2 74.3011  <.0001
```

```
summary(m.tr.gls)
```

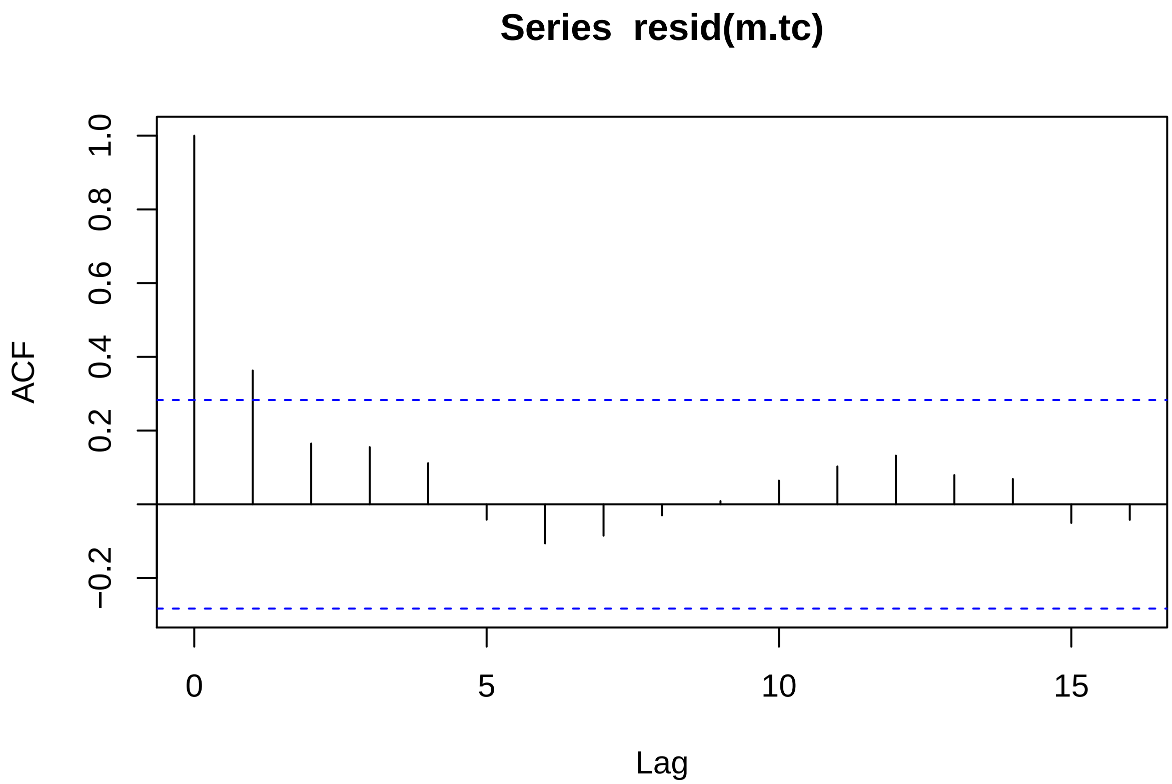
```
## Generalized least squares fit by REML
##   Model: tr ~ 0 + q + I(q^2)
##   Data: df
##        AIC      BIC    logLik
```

```
##   750.6851 757.9997 -371.3425
##
## Correlation Structure: AR(1)
##  Formula: ~1
##  Parameter estimate(s):
##       Phi
## 0.9340029
##
## Coefficients:
##            Value Std.Error   t-value p-value
## q      171.57669  6.849675  25.04888       0
## I(q^2)  -0.50845  0.021401 -23.75849       0
##
##  Correlation:
##        q
## I(q^2) -0.987
##
## Standardized residuals:
##        Min         Q1        Med         Q3        Max
## -1.9355124  0.2725241  0.5938623  0.8213691  1.8208907
##
## Residual standard error: 1489.975
## Degrees of freedom: 48 total; 46 residual
```

```r
confint(m.tr.gls)
```

```
##              2.5 %      97.5 %
## q      158.1515705 185.0018020
## I(q^2)  -0.5503944  -0.4665049
```

```r
acf(resid(m.tc))
```




```r
m.tc.ols <- gls(tc ~ q + I(q^2), df)
m.tc.gls <- gls(tc ~ q + I(q^2), df, correlation = corARMA(p = 1, q = 0))
anova(m.tc.ols, m.tc.gls)
```

```
##          Model df      AIC      BIC    logLik   Test  L.Ratio p-value
## m.tc.ols     1  4 722.8550 730.0816 -357.4275
## m.tc.gls     2  5 715.7076 724.7409 -352.8538 1 vs 2 9.147424  0.0025
```

```r
summary(m.tc.gls)
```

```
## Generalized least squares fit by REML
##   Model: tc ~ q + I(q^2)
##   Data: df
##        AIC      BIC    logLik
##   715.7076 724.7409 -352.8538
##
## Correlation Structure: AR(1)
##  Formula: ~1
##  Parameter estimate(s):
##       Phi
## 0.4826399
##
## Coefficients:
##                 Value Std.Error   t-value p-value
## (Intercept) 2358.6567  601.1383  3.923650  0.0003
## q             -5.6283    7.8102 -0.720631  0.4749
## I(q^2)         0.2415    0.0241 10.028540  0.0000
##
##  Correlation:
##        (Intr) q
```

```
## q      -0.968
## I(q^2)  0.936 -0.990
##
## Standardized residuals:
##         Min          Q1         Med          Q3         Max
## -2.20671077 -0.87478896 -0.05233391  0.53945058  2.65305988
##
## Residual standard error: 480.1036
## Degrees of freedom: 48 total; 45 residual
```

```r
confint(m.tc.gls)
```

```
##                   2.5 %       97.5 %
## (Intercept) 1180.4471798 3536.8662085
## q            -20.9359657    9.6794297
## I(q^2)         0.1943077    0.2887075
```

Part G

```r
qstar <- profitMaximizingQuantity(m.tc.gls$coefficients, m.tr.gls$coefficients)
```

The profi-maximizing level of output suggested by the GLS model is $q^* = 118.1433698$.

Part D

```r
qstar <- round(qstar)
pred.tr.gls <- predict(m.tr.gls, data.frame(q = qstar))
pred.tc.gls <- predict(m.tc.gls, data.frame(q = qstar))
pred.profit.gls <- pred.tr.gls - pred.tc.gls
```

The predicted profit is \$8109.12.