ECON 4101 Econometrics CM05 Homework

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Goal:

Use simple linear regression and analysis of variance to test the hypothesis that reported hectares of corn (soybeans) are explained by the number of pixels of corn (soybeans) in sample segment within county, from satellite data.

Data:

Survey and satellite data for 37 observations of corn and soy beans in 12 Iowa counties, obtained from the 1978 June Enumerative Survey of the U.S. Department of Agriculture and from land observatory satellites (LANDSAT) during the 1978 growing season.

Problem 1

```
colnames(data) <- c("county", "cornhec", "soyhec", "cornpix", "soypix")</pre>
n <- nrow(data)
print(paste0("Number of observations: ", n))
## [1] "Number of observations: 36"
sapply(data[, !"county"], function(x) c(summary(x), `Standard Deviation` = sd(x),
    Coefficient of Variance = sd(x)/mean(x))
##
                                cornhec
                                             soyhec
                                                        cornpix
                                                                     soypix
## Min.
                            64.7500000
                                         6.4700000 145.0000000 77.0000000
## 1st Qu.
                            95.6100000 76.5500000 243.8000000 170.8000000
## Median
                           115.3000000 103.1000000 294.5000000 211.5000000
                           119.2000000 98.8000000 295.3000000 207.4000000
## Mean
```

```
## 3rd Qu. 135.3000000 124.5000000 350.8000000 249.2000000 ## Max. 206.4000000 174.3000000 459.0000000 345.0000000 ## Standard Deviation 32.1964214 36.7839390 70.1254432 63.4847313 ## Coefficient of Variance 0.2701646 0.3723165 0.2374897 0.3060324
```

Problem 2

```
g1 <- ggplot(data = data, aes(x = cornpix, y = cornhec)) + geom_point() + labs(y = "Hectares of Corn",
    x = "Pixels of Corn")
g2 <- ggplot(data = data, aes(x = soypix, y = soyhec)) + geom_point() + labs(y = "Hectares of Soybeans"
    x = "Pixels of Soybeans")
grid.arrange(g1, g2, ncol = 2)
   200
                                                       150
                                                   Hectares of Soybeans
Hectares of Corn
    150
                                                       100
                                                        50
    100
                                                               100
                                                                     150
                                                                            200
                                                                                   250
                                                                                          300
                          300
                                                                                                 350
               200
                                      400
                                                                     Pixels of Soybeans
                     Pixels of Corn
```

Problem 3

```
lm.corn <- lm(data = data, cornhec ~ cornpix)
lm.soy <- lm(data = data, soyhec ~ soypix)

cornhec.hat <- predict(lm.corn)
soyhec.hat <- predict(lm.soy)

cornhec.bar <- mean(data$cornhec)
soyhec.bar <- mean(data$soyhec)
sse.corn <- sum((data$cornhec - cornhec.hat)^2)
sse.soy <- sum((data$soyhec - soyhec.hat)^2)
ssr.corn <- sum((cornhec.bar - cornhec.hat)^2)
ssr.soy <- sum((soyhec.bar - soyhec.hat)^2)
tss.corn <- ssr.corn + sse.corn
tss.soy <- ssr.soy + sse.soy
r2.corn <- ssr.corn/tss.corn
r2.soy <- ssr.soy/tss.soy</pre>
```

```
df.regression <- 1
df.error <- n - df.regression - 1</pre>
msr.corn <- ssr.corn/df.regression
msr.soy <- ssr.soy/df.regression</pre>
mse.corn <- sse.corn/df.error</pre>
mse.soy <- sse.soy/df.error</pre>
fstat.corn <- msr.corn/mse.corn</pre>
fstat.soy <- msr.soy/mse.soy</pre>
options(knitr.kable.NA = "")
anova.rownames <- c("Regression", "Error", "Total")</pre>
anova.corn <- data.frame(`Degrees of Freedom` = c(df.regression, df.error, df.regression +
    df.error), `Sum of Squares` = c(ssr.corn, sse.corn, tss.corn), `Mean Sum of Squares` = c(msr.corn,
    mse.corn, NA), `F Statistic` = c(fstat.corn, NA, NA))
rownames(anova.corn) <- anova.rownames</pre>
anova.soy <- data.frame(`Degrees of Freedom` = c(df.regression, df.error, df.regression +</pre>
    df.error), `Sum of Squares` = c(ssr.soy, sse.soy, tss.soy), `Mean Sum of Squares` = c(msr.soy,
    mse.soy, NA), `F Statistic` = c(fstat.soy, NA, NA))
rownames(anova.soy) <- anova.rownames</pre>
kable(anova.corn, digits = 4, caption = "ANOVA: CornHec ~ CornPix")
```

Table 1: ANOVA: CornHec ~ CornPix

	Degrees.of.Freedom	Sum.of.Squares	Mean.Sum.of.Squares	F.Statistic
Regression	1	24270.05	24270.0473	68.7005
Error	34	12011.29	353.2731	
Total	35	36281.33		

```
kable(anova.soy, digits = 4, caption = "ANOVA: SoyHec ~ SoyPix")
```

Table 2: ANOVA: SoyHec ~ SoyPix

	Degrees.of.Freedom	Sum.of.Squares	Mean.Sum.of.Squares	F.Statistic
Regression	1	30592.41	30592.4124	62.0439
Error	34	16764.62	493.0772	
Total	35	47357.04		

```
see.corn <- sqrt(mse.corn)
see.soy <- sqrt(mse.soy)

fcrit <- qf(0.95, df.regression, df.error)
pf.corn <- pf(fstat.corn, df.regression, df.error, lower.tail = F)
pf.soy <- pf(fstat.soy, df.regression, df.error, lower.tail = F)

print(paste0("Corn: Standard Error of Estimate = ", see.corn))</pre>
```

[1] "Corn: Standard Error of Estimate = 18.7955618381817"

```
print(paste0("Soybeans: Standard Error of Estimate = ", see.soy))
## [1] "Soybeans: Standard Error of Estimate = 22.2053408535331"
print(paste0("Critical value of F at .05 significance level: ", fcrit))
## [1] "Critical value of F at .05 significance level: 4.13001774565201"
print(paste0("Corn: (F-statistic, p-value) = (", fstat.corn, ", ", pf.corn, ")"))
## [1] "Corn: (F-statistic, p-value) = (68.7005158266815, 0.00000000112961941756451)"
print(paste0("Soybeans: (F-statistic, p-value) = (", fstat.soy, ", ", pf.soy, ")"))
## [1] "Soybeans: (F-statistic, p-value) = (62.0438638902307, 0.00000000358642239029152)"
Since the F-test statistic for both models is greater than the critical F-value, we conclude that, at the 5%
significance level, the predictors do help explain more of the variance of their corresponding responses than
does the null (intercept-only) model. That is, when modeling hectares of corn/soybeans, the simple linear
regression model that takes into account satellite pixels of corn/soybeans has a better fit than the null model
that only includes an intercept term.
aes.near.topleft \leftarrow aes(x = -Inf, y = Inf, hjust = -0.1, vjust = 2)
g1 <- g1 + geom_line(aes(y = cornhec.hat)) + geom_text(aes.near.topleft, label = lm_eqn(lm.corn),
    parse = T, size = 4)
g2 <- g2 + geom_line(aes(y = soyhec.hat)) + geom_text(aes.near.topleft, label = lm_eqn(lm.soy),
    parse = T, size = 4)
grid.arrange(g1, g2, nrow = 2)
   200
Hectares of Corn
           y = 8.3 + 0.38 \cdot x, R^2 = 0.669
   150
   100
                                                   300
                         200
                                                                             400
                                              Pixels of Corn
Hectares of Soybeans
           y = 2.2 + 0.47 \cdot x, R^2 = 0.646
    150
    100
     50
                  100
                                 150
                                                200
                                                               250
                                                                              300
                                                                                              350
                                           Pixels of Soybeans
```