

ECON 4101 Econometrics

CM23 Homework

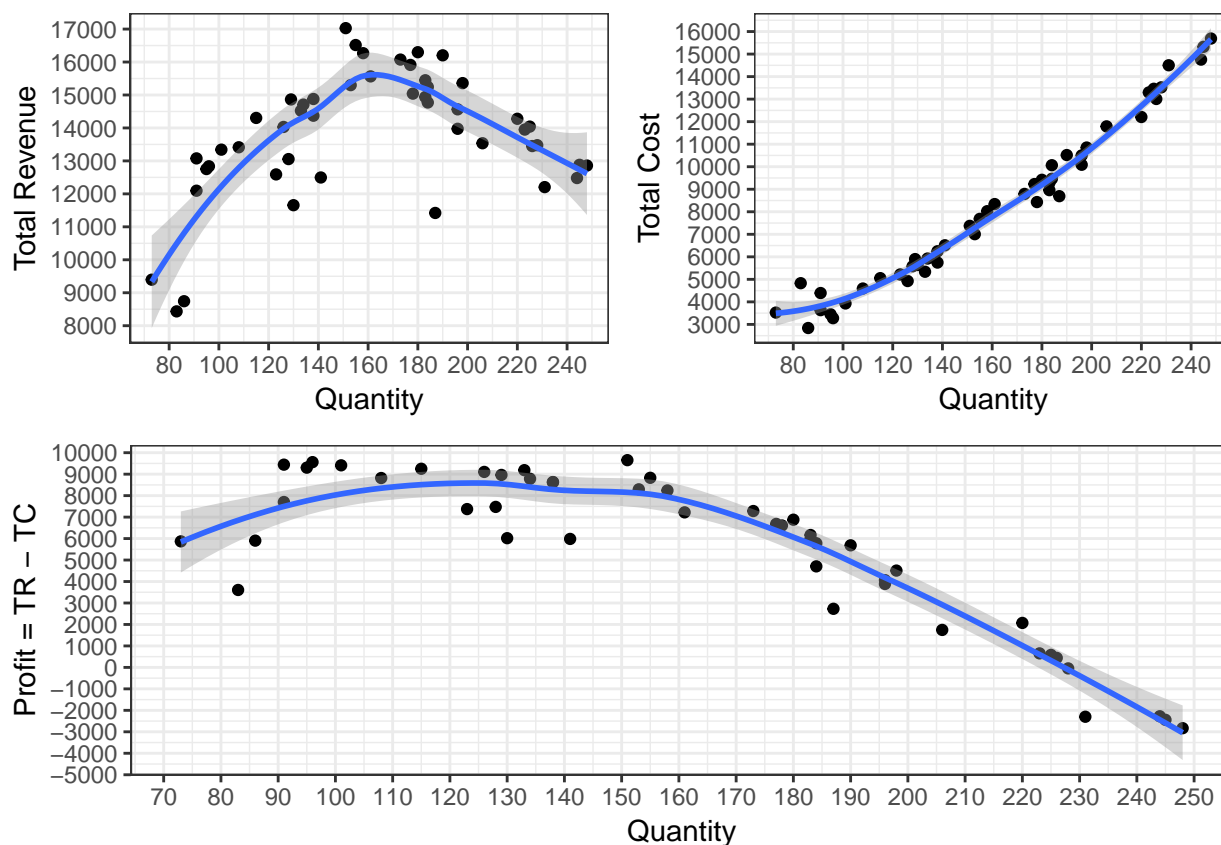
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```
df <- read.csv("../Data/monop.csv")
str(df)
```

```
## 'data.frame': 48 obs. of 3 variables:
## $ tr: int 13536 12205 12881 12837 13477 14524 15041 14938 14278 12480 ...
## $ tc: int 11790 14503 15323 3276 13523 5337 8431 8960 12207 14756 ...
## $ q : int 206 231 245 96 228 133 178 183 220 244 ...

lay <- rbind(c(1, 2), c(3))
g1 <- ggplot(df, aes(x = q, y = tr)) + geom_point() + geom_smooth(method = "loess") +
  scale_x_continuous(breaks = pretty(df$q, n = 10)) + scale_y_continuous(breaks = pretty(df$tr,
    n = 10)) + ylab("Total Revenue") + xlab("Quantity")
g2 <- ggplot(df, aes(x = q, y = tc)) + geom_point() + geom_smooth(method = "loess") +
  scale_x_continuous(breaks = pretty(df$q, n = 10)) + scale_y_continuous(breaks = pretty(df$tc,
    n = 10)) + ylab("Total Cost") + xlab("Quantity")
g3 <- ggplot(df, aes(x = q, y = tr - tc)) + geom_point() + geom_smooth(method = "loess") +
  scale_x_continuous(breaks = pretty(df$q, n = 20)) + scale_y_continuous(breaks = pretty(-5000:10000,
    n = 20)) + ylab("Profit = TR - TC") + xlab("Quantity")
grid.arrange(grobs = list(g1, g2, g3), layout_matrix = lay)
```



Monopolist's Economic Model:

$$tr = \beta_1 q + \beta_2 q^2$$
$$tc = \alpha_1 + \alpha_2 q + \alpha_3 q^2$$

Part A

$$mr = \frac{d}{dq}\{tr\} = \beta_1 + 2\beta_2 q$$
$$mc = \frac{d}{dq}\{tc\} = \alpha_2 + 2\alpha_3 q$$

Part B

$$\begin{aligned} mr &= mc \\ \Rightarrow \beta_1 + 2\beta_2 q^* &= \alpha_2 + 2\alpha_3 q^* \\ \Rightarrow (2\beta_2 - 2\alpha_3)q^* &= \alpha_2 - \beta_1 \\ \Rightarrow q^* &= \frac{\alpha_2 - \beta_1}{2(\beta_2 - \alpha_3)} \end{aligned}$$

Part C

```
m.tr <- lm(tr ~ 0 + q + I(q^2), df)
summary(m.tr)

##
## Call:
## lm(formula = tr ~ 0 + q + I(q^2), data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3599.9  -287.4   258.7   732.2  2167.6
##
## Coefficients:
##              Estimate Std. Error t value      Pr(>|t|)
## q             174.28028     4.53989   38.39 <0.0000000000000002 ***
## I(q^2)        -0.50243     0.02354  -21.34 <0.0000000000000002 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1179 on 46 degrees of freedom
## Multiple R-squared:  0.9932, Adjusted R-squared:  0.9929
## F-statistic: 3346 on 2 and 46 DF,  p-value: < 0.00000000000000022

m.tc <- lm(tc ~ q + I(q^2), df)
summary(m.tc)

##
## Call:
## lm(formula = tc ~ q + I(q^2), data = df)
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1040.69  -352.62    -8.34   272.99  1325.43
##
## Coefficients:
##              Estimate Std. Error t value      Pr(>|t|)
## (Intercept) 2066.08294   727.21801    2.841    0.00673 **
## q           -1.57841     9.45245   -0.167    0.86813
## I(q^2)       0.22768     0.02889    7.881 0.000000000515 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 462.5 on 45 degrees of freedom
## Multiple R-squared:  0.9841, Adjusted R-squared:  0.9834
## F-statistic: 1396 on 2 and 45 DF,  p-value: < 0.00000000000000022
```

These estimates are appropriate (unbiased) for models that satisfy the first four Gauss-Markov assumptions: (1) the model is linear in parameters, (2) the observations are randomly sampled, (3) the model has zero conditional mean, and (4) there is no perfect collinearity between the model's terms.

```
profitMaximizingQuantity <- function(alpha, beta) {
  (alpha[2] - beta[1])/(2 * (beta[2] - alpha[3]))
}
qstar <- profitMaximizingQuantity(m.tc$coefficients, m.tr$coefficients)
```

From part (b), we see that the OLS estimates suggest the profit maximizing level of output is $q^* = 120.4327681$.

Part D

```
qstar <- round(qstar)
pred.tr <- predict(m.tr, data.frame(q = qstar))
pred.tc <- predict(m.tc, data.frame(q = qstar))
pred.profit <- pred.tr - pred.tc
```

The predicted profit is \$8523.35.

Part E

```
for (i in 1:5) {
  bg <- bgtest(m.tr, order = i)
  s <- sprintf("Breusch-Godfrey Test: AR(%d) P=%0.4f \n", i, bg$p.value)
  cat(s)
}
```

```
## Breusch-Godfrey Test: AR(1) P=0.0000
## Breusch-Godfrey Test: AR(2) P=0.0000
## Breusch-Godfrey Test: AR(3) P=0.0000
## Breusch-Godfrey Test: AR(4) P=0.0000
## Breusch-Godfrey Test: AR(5) P=0.0000
```

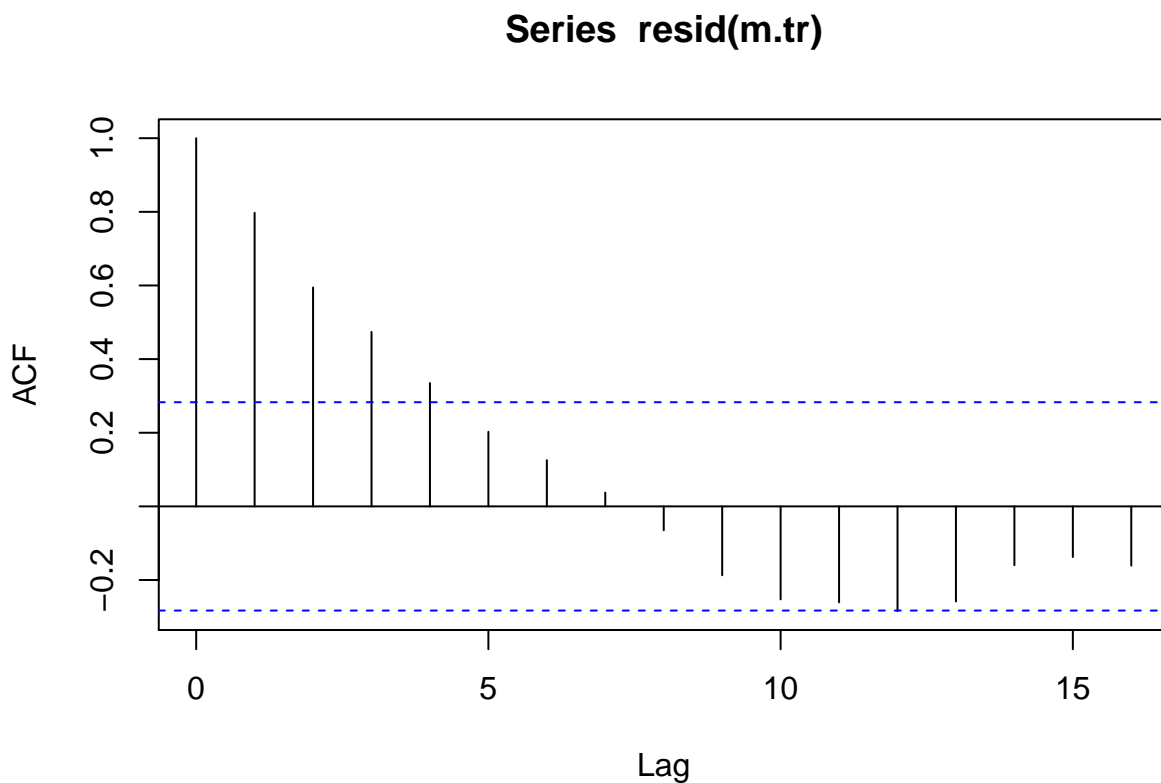
```
for (i in 1:5) {
  bg <- bgtest(m.tc, order = i)
```

```
s <- sprintf("Breusch-Godfrey Test: AR(%d) P=%0.4f \n", i, bg$p.value)
cat(s)
}
```

```
## Breusch-Godfrey Test: AR(1) P=0.0052
## Breusch-Godfrey Test: AR(2) P=0.0163
## Breusch-Godfrey Test: AR(3) P=0.0306
## Breusch-Godfrey Test: AR(4) P=0.0630
## Breusch-Godfrey Test: AR(5) P=0.0653
```

Part F

```
acf(resid(m.tr))
```



```
m.tr.ols <- gls(tr ~ 0 + q + I(q^2), df)
m.tr.gls <- gls(tr ~ 0 + q + I(q^2), df, correlation = corARMA(p = 1, q = 0))
anova(m.tr.ols, m.tr.gls)
```

```
##           Model df      AIC      BIC    logLik   Test L.Ratio p-value
## m.tr.ols      1   3 822.9862 828.4721 -408.4931
## m.tr.gls      2   4 750.6851 757.9997 -371.3425 1 vs 2 74.3011 <.0001
```

```
summary(m.tr.gls)
```

```
## Generalized least squares fit by REML
## Model: tr ~ 0 + q + I(q^2)
## Data: df
##           AIC      BIC    logLik
```

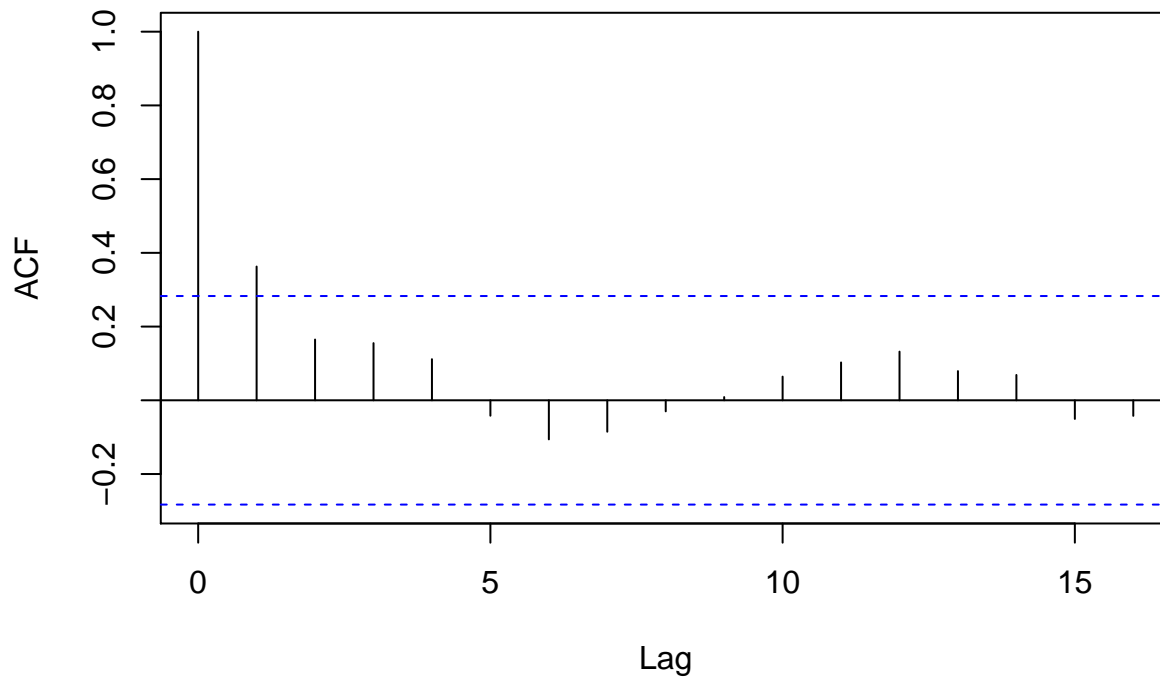
```

##    750.6851 757.9997 -371.3425
##
## Correlation Structure: AR(1)
## Formula: ~1
## Parameter estimate(s):
##      Phi
## 0.9340029
##
## Coefficients:
##              Value Std.Error   t-value p-value
## q          171.57669   6.849675   25.04888     0
## I(q^2)    -0.50845   0.021401  -23.75849     0
##
## Correlation:
##      q
## I(q^2) -0.987
##
## Standardized residuals:
##      Min      Q1      Med      Q3      Max
## -1.9355124  0.2725241  0.5938623  0.8213691  1.8208907
##
## Residual standard error: 1489.975
## Degrees of freedom: 48 total; 46 residual
confint(m.tr.gls)

##              2.5 %      97.5 %
## q          158.1515705 185.0018020
## I(q^2)    -0.5503944 -0.4665049
acf(resid(m.tc))

```

Series resid(m.tc)



```
m.tc.ols <- gls(tc ~ q + I(q^2), df)
m.tc.gls <- gls(tc ~ q + I(q^2), df, correlation = corARMA(p = 1, q = 0))
anova(m.tc.ols, m.tc.gls)
```

	Model	df	AIC	BIC	logLik	Test	L.Ratio	p-value
##	m.tc.ols	1	4	722.8550	730.0816	-357.4275		
##	m.tc.gls	2	5	715.7076	724.7409	-352.8538	1 vs 2	9.147424 0.0025

```
summary(m.tc.gls)
```

```
## Generalized least squares fit by REML
## Model: tc ~ q + I(q^2)
## Data: df
##      AIC      BIC    logLik
## 715.7076 724.7409 -352.8538
##
## Correlation Structure: AR(1)
## Formula: ~1
## Parameter estimate(s):
##      Phi
## 0.4826399
##
## Coefficients:
##              Value Std.Error   t-value p-value
## (Intercept) 2358.6567  601.1383   3.923650  0.0003
## q            -5.6283   7.8102  -0.720631  0.4749
## I(q^2)         0.2415   0.0241  10.028540  0.0000
##
## Correlation:
##      (Intr) q
```

```
## q          -0.968
## I(q^2)    0.936 -0.990
##
## Standardized residuals:
##          Min          Q1          Med          Q3          Max
## -2.20671077 -0.87478896 -0.05233391  0.53945058  2.65305988
##
## Residual standard error: 480.1036
## Degrees of freedom: 48 total; 45 residual
```

```
confint(m.tc.gls)
```

```
##                2.5 %          97.5 %
## (Intercept) 1180.4471798 3536.8662085
## q           -20.9359657   9.6794297
## I(q^2)       0.1943077    0.2887075
```

Part G

```
qstar <- profitMaximizingQuantity(m.tc.gls$coefficients, m.tr.gls$coefficients)
```

The profit-maximizing level of output suggested by the GLS model is $q^* = 118.1433698$.

Part H

```
qstar <- round(qstar)

tr.last <- df[48, ]$tr
tc.last <- df[48, ]$tc

beta1 <- coef(m.tr.gls)[1]
beta2 <- coef(m.tr.gls)[2]
phi.tr <- 0.9340029

alpha1 <- coef(m.tc.gls)[1]
alpha2 <- coef(m.tc.gls)[2]
alpha3 <- coef(m.tc.gls)[3]
phi.tc <- 0.4826399

pred.tr.gls.1 <- beta1 * qstar + beta2 * qstar^2 + phi.tr * tr.last
pred.tc.gls.1 <- alpha1 + alpha2 * qstar + alpha3 * qstar^2 + phi.tc * tc.last
pred.profit.gls.1 <- pred.tr.gls.1 - pred.tc.gls.1

tr.last <- pred.tr.gls.1
tc.last <- pred.tc.gls.1

pred.tr.gls.2 <- beta1 * qstar + beta2 * qstar^2 + phi.tr * tr.last
pred.tc.gls.2 <- alpha1 + alpha2 * qstar + alpha3 * qstar^2 + phi.tc * tc.last
pred.profit.gls.2 <- pred.tr.gls.2 - pred.tc.gls.2

tr.last <- pred.tr.gls.2
```

```
tc.last <- pred.tc.gls.2

pred.tr.gls.3 <- beta1 * qstar + beta2 * qstar^2 + phi.tr * tr.last
pred.tc.gls.3 <- alpha1 + alpha2 * qstar + alpha3 * qstar^2 + phi.tc * tc.last
pred.profit.gls.3 <- pred.tr.gls.3 - pred.tc.gls.3
```

The predicted profits for the next three months are (in dollars): 13656.77, 24199.23, 34603.37.