ECON 4101 Econometrics
CM15 Homework

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March 5, 2017

```
data <- fread("cobb_douglas_data.txt")
data[, `:=`(Year, as.Date(paste0(Year, "-01-01")))]
```

```
##           Year Output Labor Capital
##  1: 1899-01-01    100   100     100
##  2: 1900-01-01    101   105     107
##  3: 1901-01-01    112   110     114
##  4: 1902-01-01    122   118     122
##  5: 1903-01-01    124   123     131
##  6: 1904-01-01    122   116     138
##  7: 1905-01-01    143   125     149
##  8: 1906-01-01    152   133     163
##  9: 1907-01-01    151   138     176
## 10: 1908-01-01    126   121     185
## 11: 1909-01-01    155   140     198
## 12: 1910-01-01    159   144     208
## 13: 1911-01-01    153   145     216
## 14: 1912-01-01    177   152     226
## 15: 1913-01-01    184   154     236
## 16: 1914-01-01    169   149     244
## 17: 1915-01-01    189   154     266
## 18: 1916-01-01    225   182     298
## 19: 1917-01-01    227   196     335
## 20: 1918-01-01    223   200     366
## 21: 1919-01-01    218   193     387
## 22: 1920-01-01    231   193     407
## 23: 1921-01-01    179   147     417
## 24: 1922-01-01    240   161     431
##           Year Output Labor Capital
```

```
attach(data)
```

We model the data using a transformation of the Cobb-Douglas production function:

$$\ln Q = \ln A + \alpha \ln L + \beta \ln K$$

where Q = output, L = labor, K = capital, and A = constant.

```
log.Output <- log(Output)
log.Labor <- log(Labor)
log.Capital <- log(Capital)
mod1 <- lm(log.Output ~ log.Labor + log.Capital)
summary(mod1)
```

```
##
## Call:
## lm(formula = log.Output ~ log.Labor + log.Capital)
```

```
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.075282 -0.035234 -0.006439  0.038782  0.142114
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.17731    0.43429  -0.408  0.68721
## log.Labor    0.80728    0.14508   5.565 0.000016 ***
## log.Capital  0.23305    0.06353   3.668  0.00143 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.05814 on 21 degrees of freedom
## Multiple R-squared:  0.9574, Adjusted R-squared:  0.9534
## F-statistic: 236.1 on 2 and 21 DF,  p-value: 0.000000000000004038
```

```r
confint(mod1)
```

```
##                  2.5 %    97.5 %
## (Intercept) -1.0804715 0.7258521
## log.Labor    0.5055758 1.1089806
## log.Capital  0.1009361 0.3651708
```

One way to test the hypotheses

$$H_0 \ : \ \alpha + \beta = 1 H_1 \ : \ \alpha + \beta \neq 1$$

would be to use the t-statistic:

$$t_{\hat{\alpha}+\hat{\beta}} = \frac{(\hat{\alpha} + \hat{\beta}) - 1}{\text{se}(\hat{\alpha} + \hat{\beta})}$$

We can get the standard error from the covariance matrix of the estimators of the unrestricted fitted model as follows:

$$\text{se}(\hat{\alpha} + \hat{\beta}) = \sqrt{\widehat{\text{VAR}}(\hat{\alpha} + \hat{\beta})} = \sqrt{\widehat{\text{VAR}}(\hat{\alpha}) + \widehat{\text{VAR}}(\hat{\beta}) + 2\widehat{\text{COV}}(\hat{\alpha}, \hat{\beta})}$$

```r
vcov.params <- vcov(mod1)
vcov.params
```

```
##             (Intercept)    log.Labor  log.Capital
## (Intercept)  0.18861045 -0.059546854  0.019984179
## log.Labor   -0.05954685  0.021047093 -0.008383119
## log.Capital  0.01998418 -0.008383119  0.004036028
```

```r
se <- sqrt(vcov.params["log.Labor", "log.Labor"] + vcov.params["log.Capital", "log.Capital"] +
    2 * vcov.params["log.Labor", "log.Capital"])
t.stat <- unname((mod1$coefficients["log.Labor"] + mod1$coefficients["log.Capital"] -
    1)/se)
cat("t statistic: ", t.stat)
```

```
## t statistic:  0.4422483
```

```r
n <- nrow(data)
k <- 3
t.crit <- qt(0.975, n - k)
cat("t critical value: ", t.crit)
```

```
## t critical value:  2.079614
```

```r
pval <- pt(t.stat, df = n - k, lower.tail = F)
cat("p-value: ", pval)
```

```
## p-value:  0.3314154
```

The above test indicates we cannot reject the assumption that $\alpha + \beta = 1$, at least at the 95% confidence level.

Another way to test the assumption of constant returns to scale would be to use a restricted least squares framework as follows:

$$\begin{aligned}
\text{Original Model: } & \ln Q = \ln A + \alpha \ln L + \beta \ln K \\
\text{Restriction: } & \alpha + \beta = 1 \\
\implies & \ln Q = \ln A + (1 - \beta) \ln L + \beta \ln K \\
\implies & \ln Q - \ln L = \ln A + \beta(\ln K - \ln L)
\end{aligned}$$

```r
log.Output <- log.Output - log.Labor
log.Capital <- log.Capital - log.Labor
mod2 <- lm(log.Output ~ log.Capital)
summary(mod2)
```

```
##
## Call:
## lm(formula = log.Output ~ log.Capital)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.082565 -0.032869 -0.006925  0.040529  0.134443
##
## Coefficients:
##             Estimate Std. Error t value   Pr(>|t|)
## (Intercept)  0.01454    0.01998   0.728      0.474
## log.Capital  0.25413    0.04122   6.165 0.00000332 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.05707 on 22 degrees of freedom
## Multiple R-squared:  0.6334, Adjusted R-squared:  0.6167
## F-statistic:    38 on 1 and 22 DF,  p-value: 0.000003324
```

We use the F-statistic from the Ramsey RESET Test to compare our restricted and unrestricted models:

$$F = \frac{(SSE_R - SSE_U)/q}{SSE_U/(n-k-1)}$$

$$F \sim F_{\alpha,\, q,\, n-k-1}$$

where SSE_R is for restricted model and SSE_U is for unrestricted model, and $q =$ number of restrictions, and k is the number of terms in the unrestricted model. An `anova` summary of the two models generates our F-statistic value:

```r
anova(mod2, mod1)
```

```
## Analysis of Variance Table
##
## Model 1: log.Output ~ log.Capital
## Model 2: log.Output ~ log.Labor + log.Capital
##   Res.Df     RSS Df  Sum of Sq      F Pr(>F)
```

```
## 1     22 0.071643
## 2     21 0.070982  1 0.00066109 0.1956 0.6628
```

The insignificant p-value here corroborates our earlier finding that we cannot reject the assumption of constant returns to scale. Our findings here match those of Felipe [2005]. It is important to note, as the authors go to great lengths to explain, that these findings should not be interpreted as support for the inferential capabilities of the Cobb-Douglas production function. Rather, the authors offer a convincing argument that "all estimations of aggregate production functions do is to reproduce the distribution income accounting identity."

```
beta <- mod2$coefficients["log.Capital"]
alpha <- 1 - beta
ratio <- Output/(Labor^alpha * Capital^beta)
h <- exp(mod2$coefficients["(Intercept)"])
ggplot() + geom_line(aes(x = Year, y = ratio)) + geom_hline(aes(yintercept = h),
    linetype = "dashed")
```