Chapter 2 Exercise Solutions

Several exercises in this chapter differ from those in the 4th edition. An "*" following the exercise number indicates that the description has changed (e.g., new values). A second exercise number in parentheses indicates that the exercise number has changed. For example, "2-16* (2-9)" means that exercise 2-16 was 2-9 in the 4th edition, and that the description also differs from the 4th edition (in this case, asking for a time series plot instead of a digidot plot). New exercises are denoted with an "☺".

2-1*.

(a)

$$\bar{x} = \sum_{i=1}^{n} x_i \Big/ n = (16.05 + 16.03 + \cdots + 16.07)/12 = 16.029 \text{ oz}$$

(b)

$$s = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2 \Big/ n}{n-1}} = \sqrt{\frac{(16.05^2 + \cdots + 16.07^2) - (16.05 + \cdots + 16.07)^2 / 12}{12-1}} = 0.0202 \text{ oz}$$

MTB > Stat > Basic Statistics > Display Descriptive Statistics

Descriptive Statistics: Ex2-1

```
Variable   N  N*    Mean  SE Mean   StDev  Minimum      Q1  Median      Q3
Ex2-1     12   0  16.029  0.00583  0.0202   16.000  16.013  16.025  16.048
Variable  Maximum
Ex2-1      16.070
```

2-2.

(a)

$$\bar{x} = \sum_{i=1}^{n} x_i \Big/ n = (50.001 + 49.998 + \cdots + 50.004)/8 = 50.002 \text{ mm}$$

(b)

$$s = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2 \Big/ n}{n-1}} = \sqrt{\frac{(50.001^2 + \cdots + 50.004^2) - (50.001 + \cdots + 50.004)^2 / 8}{8-1}} = 0.003 \text{ mm}$$

MTB > Stat > Basic Statistics > Display Descriptive Statistics

Descriptive Statistics: Ex2-2

```
Variable  N  N*    Mean  SE Mean    StDev  Minimum      Q1  Median      Q3
Ex2-2     8   0  50.002  0.00122  0.00344   49.996  49.999  50.003  50.005
Variable  Maximum
Ex2-2      50.006
```

Chapter 2 Exercise Solutions

2-3.

(a)

$$\bar{x} = \sum_{i=1}^{n} x_i \Big/ n = (953 + 955 + \cdots + 959) \big/ 9 = 952.9 \text{ °F}$$

(b)

$$s = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2 \Big/ n}{n-1}} = \sqrt{\frac{(953^2 + \cdots + 959^2) - (953 + \cdots + 959)^2 / 9}{9-1}} = 3.7 \text{ °F}$$

MTB > Stat > Basic Statistics > Display Descriptive Statistics

Descriptive Statistics: Ex2-3

```
Variable  N  N*    Mean  SE Mean  StDev  Minimum      Q1  Median      Q3
Ex2-3     9   0  952.89     1.24   3.72   948.00  949.50  953.00  956.00
Variable  Maximum
Ex2-3      959.00
```

2-4.

(a)

In ranked order, the data are {948, 949, 950, 951, **<u>953</u>**, 954, 955, 957, 959}. The sample median is the middle value.

(b)

Since the median is the value dividing the ranked sample observations in half, it remains the same regardless of the size of the largest measurement.

2-5.

MTB > Stat > Basic Statistics > Display Descriptive Statistics

Descriptive Statistics: Ex2-5

```
Variable  N  N*    Mean  SE Mean  StDev  Minimum      Q1  Median      Q3
Ex2-5     8   0  121.25     8.00  22.63    96.00  102.50  117.00  144.50
Variable  Maximum
Ex2-5      156.00
```

2-6.

(a), (d)

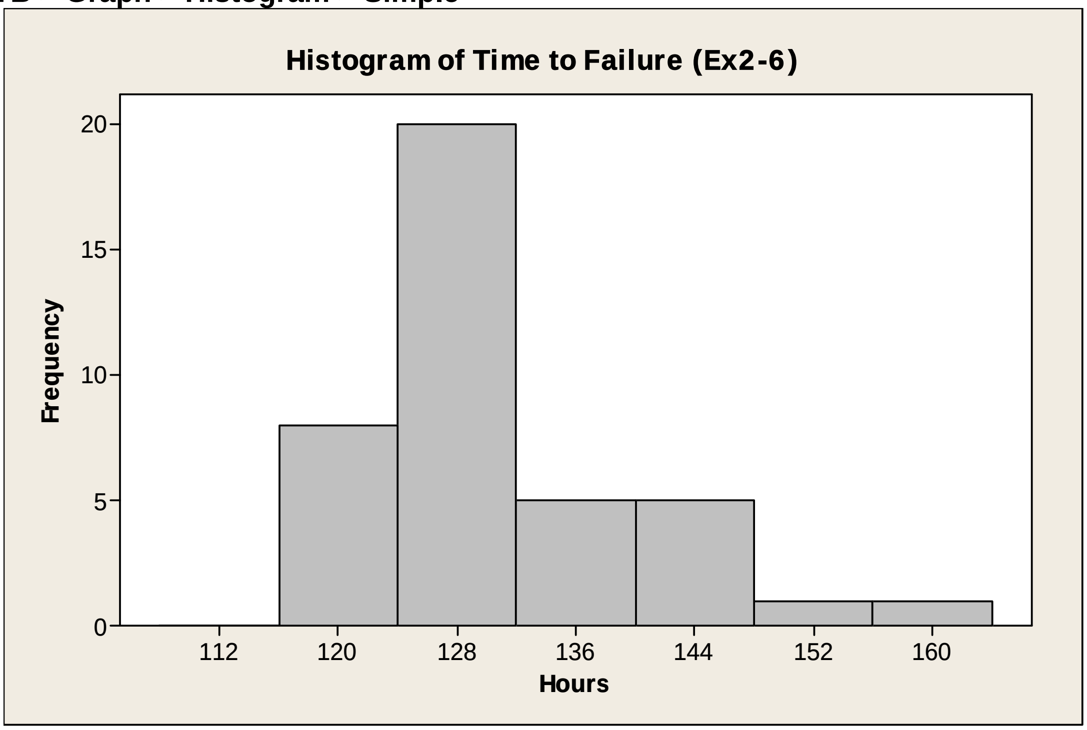
MTB > Stat > Basic Statistics > Display Descriptive Statistics

```
Descriptive Statistics: Ex2-6
Variable   N  N*    Mean  SE Mean  StDev  Minimum      Q1  Median      Q3
Ex2-6     40   0  129.98     1.41   8.91   118.00  124.00  128.00  135.25
Variable  Maximum
Ex2-6      160.00
```

(b)

Use $\sqrt{n} = \sqrt{40} \cong 7$ bins

MTB > Graph > Histogram > Simple



(c)

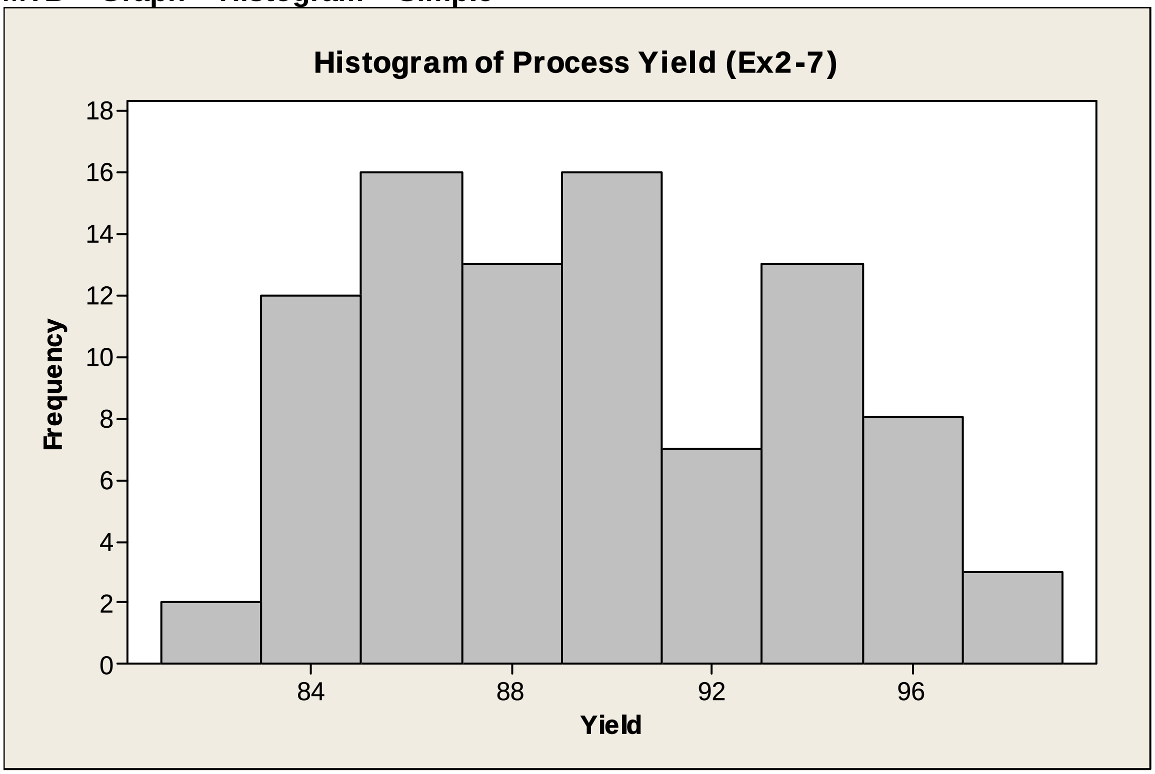
MTB > Graph > Stem-and-Leaf

```
Stem-and-Leaf Display: Ex2-6
Stem-and-leaf of Ex2-6  N  = 40
Leaf Unit = 1.0
 2   11  89
 5   12  011
 8   12  233
 17  12  444455555
 19  12  67
(5)  12  88999
 16  13  0111
 12  13  33
 10  13
 10  13  677
 7   13
 7   14  001
 4   14  22
HI 151, 160
```

2-7.

Use $\sqrt{n} = \sqrt{90} \cong 9$ bins

MTB > Graph > Histogram > Simple

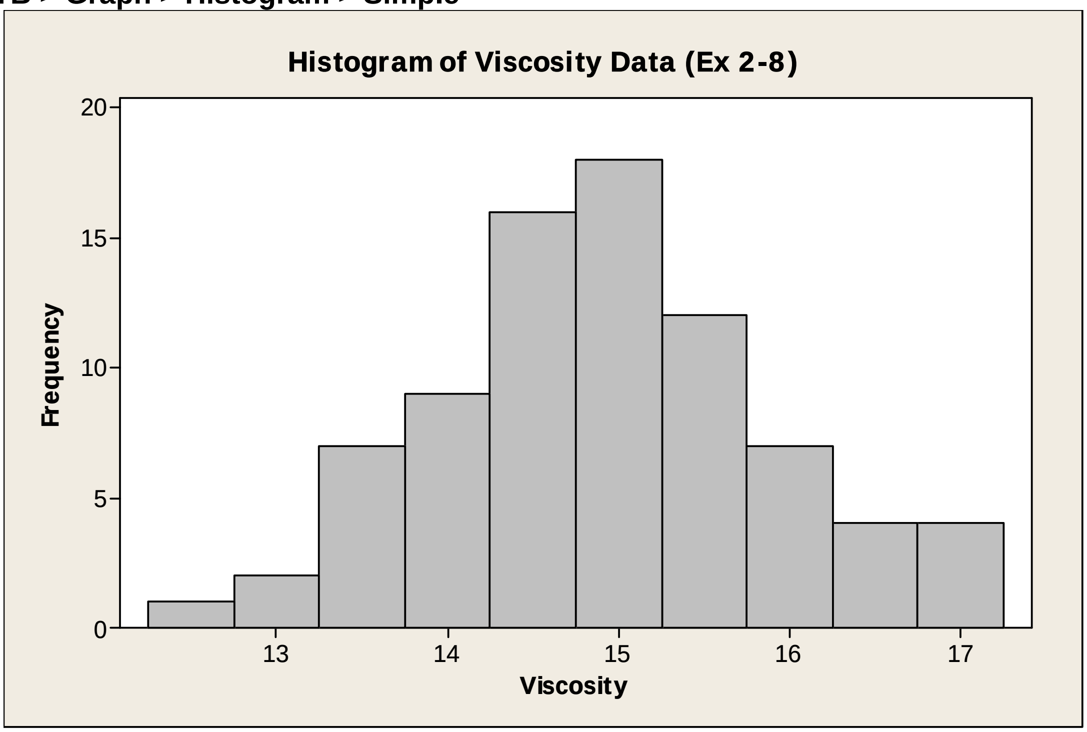


2-8.
(a)

```
    Stem-and-Leaf Plot
  2    12o|68
  6    13*|3134
 12    13o|776978
 28    14*|3133101332423404
(15)   14o|585669589889695
 37    15*|3324223422112232
 21    15o|568987666
 12    16*|144011
  6    16o|85996
  1    17*|0
Stem  Freq|Leaf
```

(b)
Use $\sqrt{n} = \sqrt{80} \cong 9$ bins

MTB > Graph > Histogram > Simple



Note that the histogram has 10 bins. The number of bins can be changed by editing the X scale. However, if 9 bins are specified, MINITAB generates an 8-bin histogram. Constructing a 9-bin histogram requires manual specification of the bin cut points. Recall that this formula is an approximation, and therefore either 8 or 10 bins should suffice for assessing the distribution of the data.

2-8(c) continued

MTB > %hbins 12.5 17 .5 c7

Row	Intervals	Frequencies	Percents
1	12.25 to 12.75	1	1.25
2	12.75 to 13.25	2	2.50
3	13.25 to 13.75	7	8.75
4	13.75 to 14.25	9	11.25
5	14.25 to 14.75	16	20.00
6	14.75 to 15.25	18	22.50
7	15.25 to 15.75	12	15.00
8	15.75 to 16.25	7	8.75
9	16.25 to 16.75	4	5.00
10	16.75 to 17.25	4	5.00
11	Totals	80	100.00

(d)

MTB > Graph > Stem-and-Leaf

Stem-and-Leaf Display: Ex2-8

```
Stem-and-leaf of Ex2-8  N  = 80
Leaf Unit = 0.10
 2    12  68
 6    13  1334
 12   13  677789
 28   14  0011122333333444
(15)  14  555566688889999
 37   15  1122222222333344
 21   15  566667889
 12   16  011144
 6    16  56899
 1    17  0
```

median observation rank is (0.5)(80) + 0.5 = 40.5
$x_{0.50}$ = (14.9 + 14.9)/2 = 14.9

Q1 observation rank is (0.25)(80) + 0.5 = 20.5
Q1 = (14.3 + 14.3)/2 = 14.3

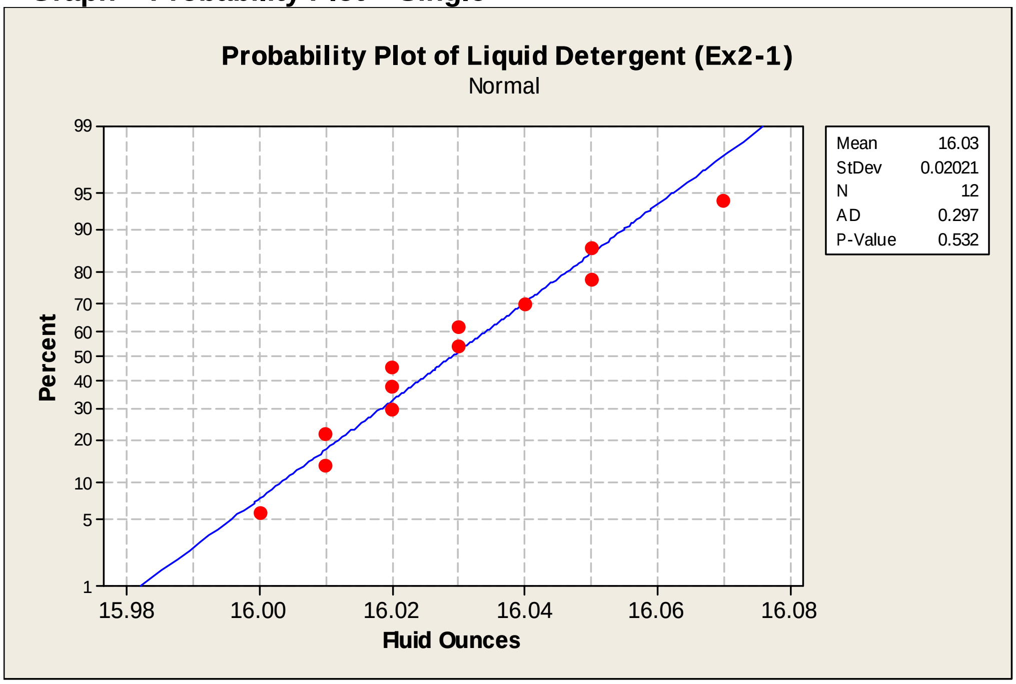
Q3 observation rank is (0.75)(80) + 0.5 = 60.5
Q3 = (15.6 + 15.5)/2 = 15.55

(d)
10^{th} percentile observation rank = (0.10)(80) + 0.5 = 8.5
$x_{0.10}$ = (13.7 + 13.7)/2 = 13.7

90^{th} percentile observation rank is (0.90)(80) + 0.5 = 72.5
$x_{0.90}$ = (16.4 + 16.1)/2 = 16.25

2-9 ☺.

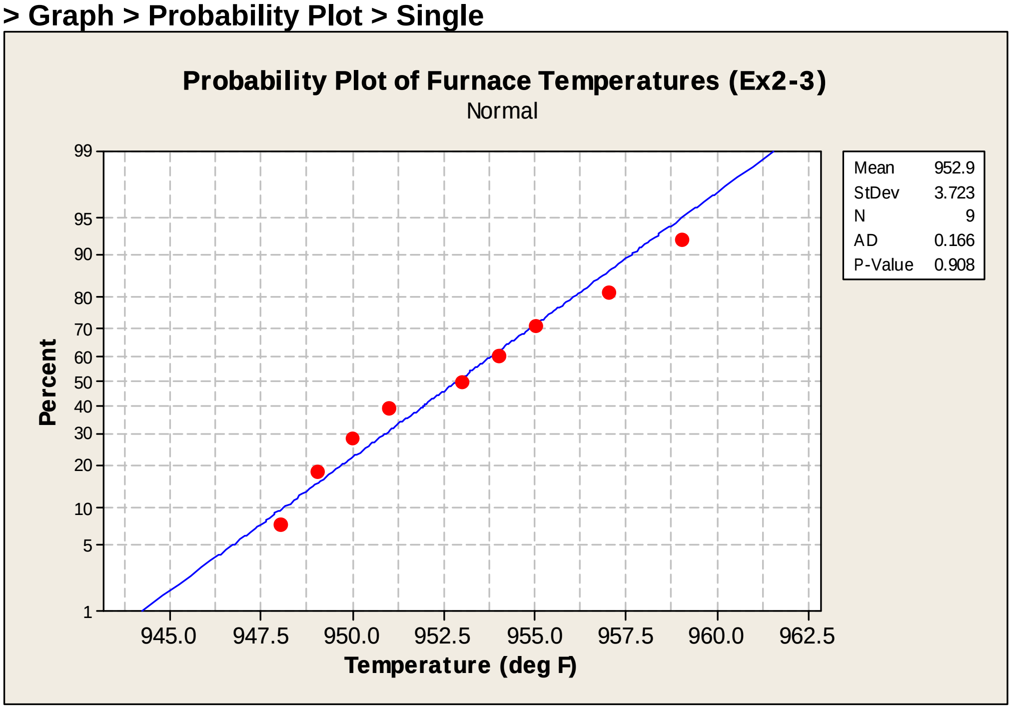
MTB > Graph > Probability Plot > Single



When plotted on a normal probability plot, the data points tend to fall along a straight line, indicating that a normal distribution adequately describes the volume of detergent.

2-10 ☺.

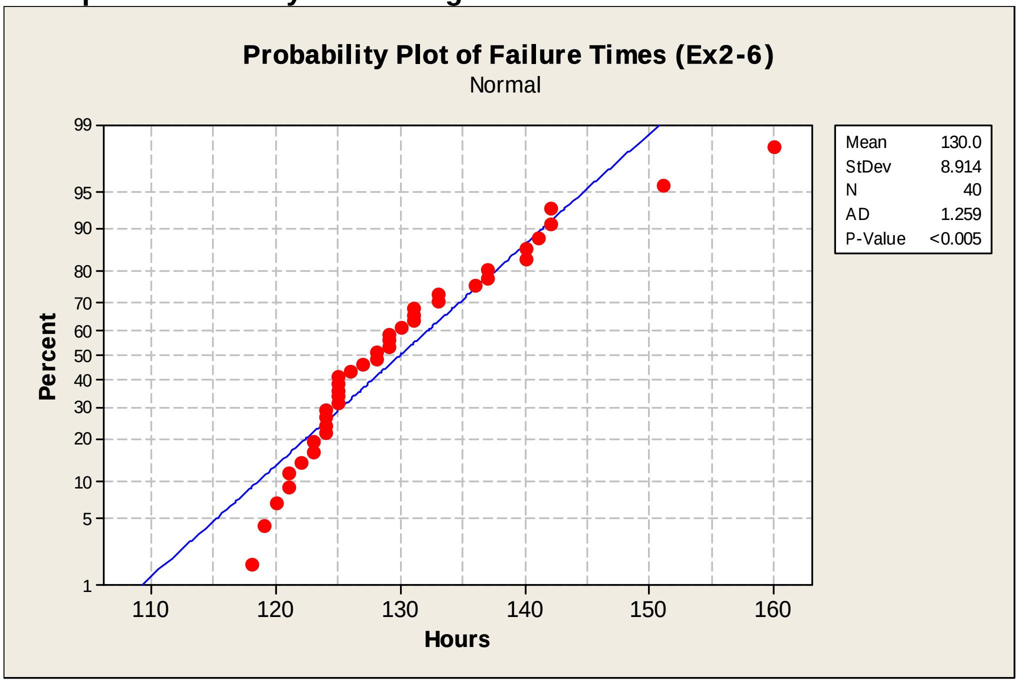
MTB > Graph > Probability Plot > Single



When plotted on a normal probability plot, the data points tend to fall along a straight line, indicating that a normal distribution adequately describes the furnace temperatures.

2-11 ☺.

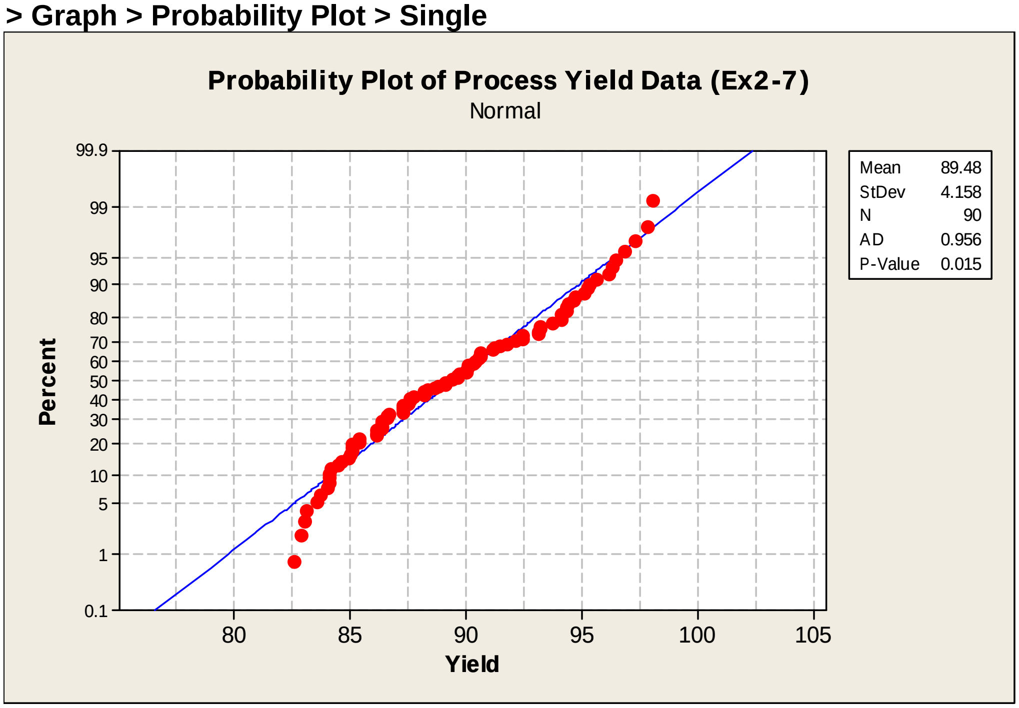
MTB > Graph > Probability Plot > Single



When plotted on a normal probability plot, the data points do not fall along a straight line, indicating that the normal distribution does not reasonably describe the failure times.

2-12 ☺.

MTB > Graph > Probability Plot > Single

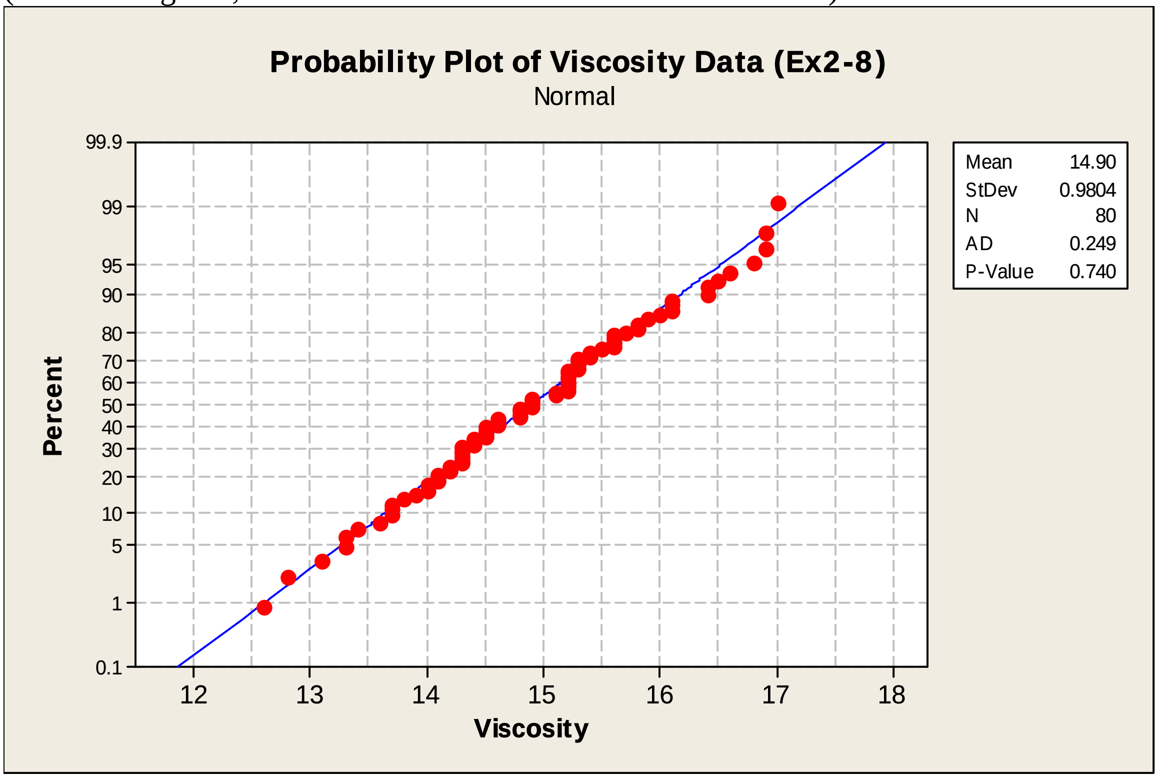


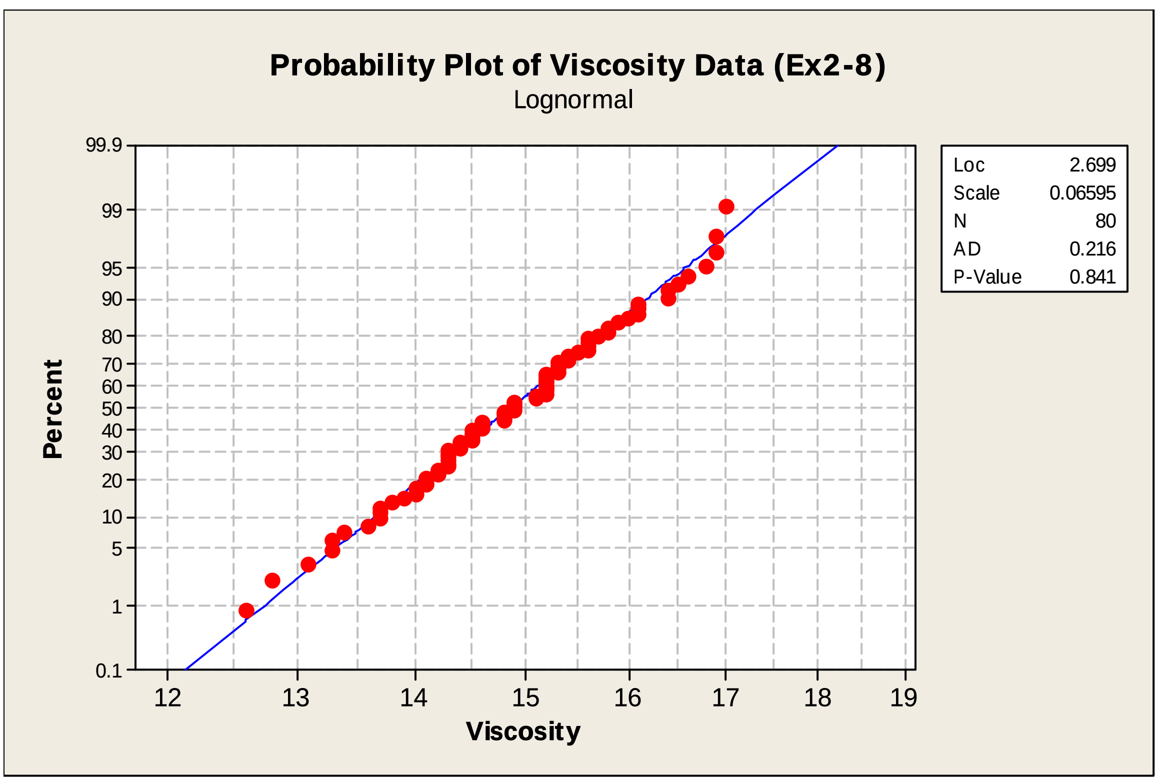
When plotted on a normal probability plot, the data points do not fall along a straight line, indicating that the normal distribution does not reasonably describe process yield.

2-13 ☺.

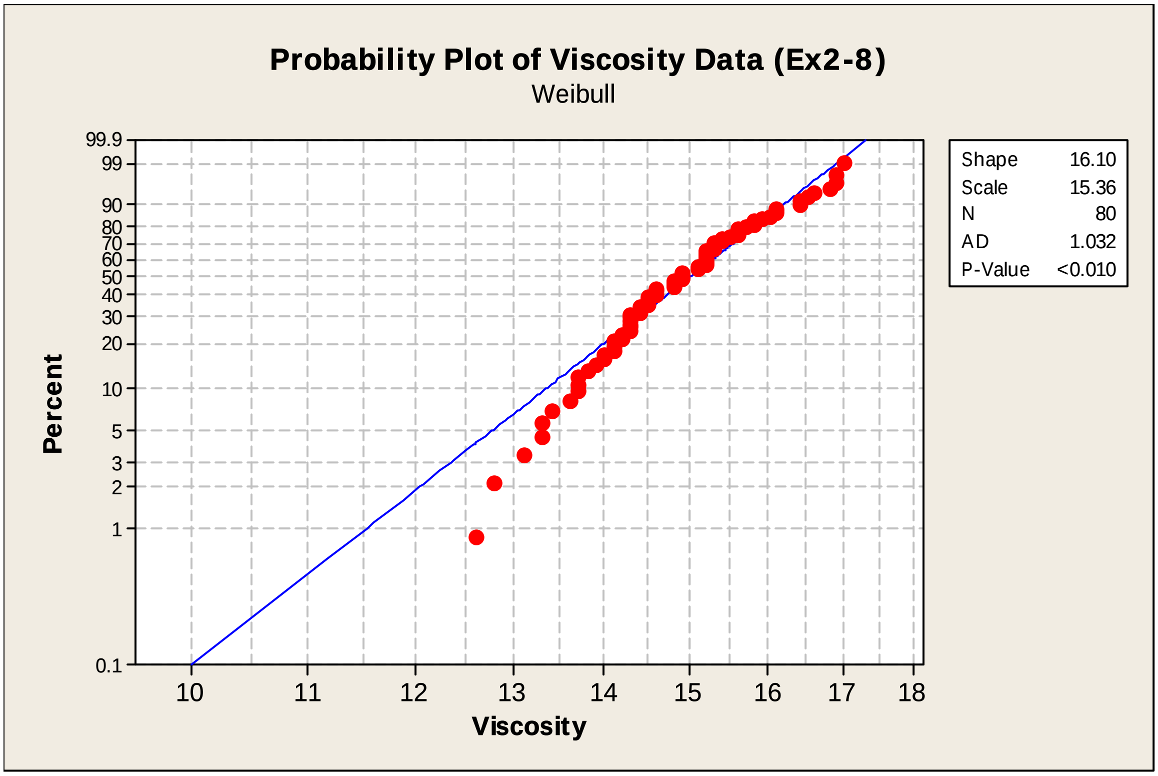
MTB > Graph > Probability Plot > Single

(In the dialog box, select Distribution to choose the distributions)





2-13 continued

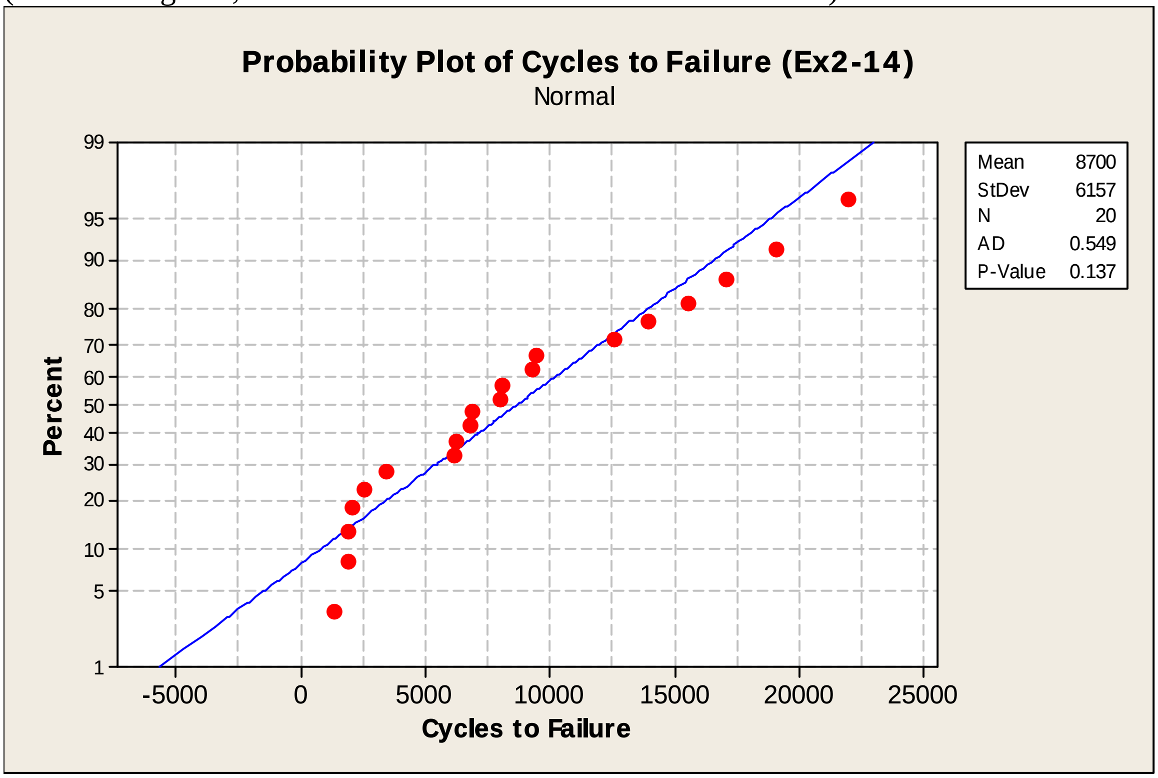


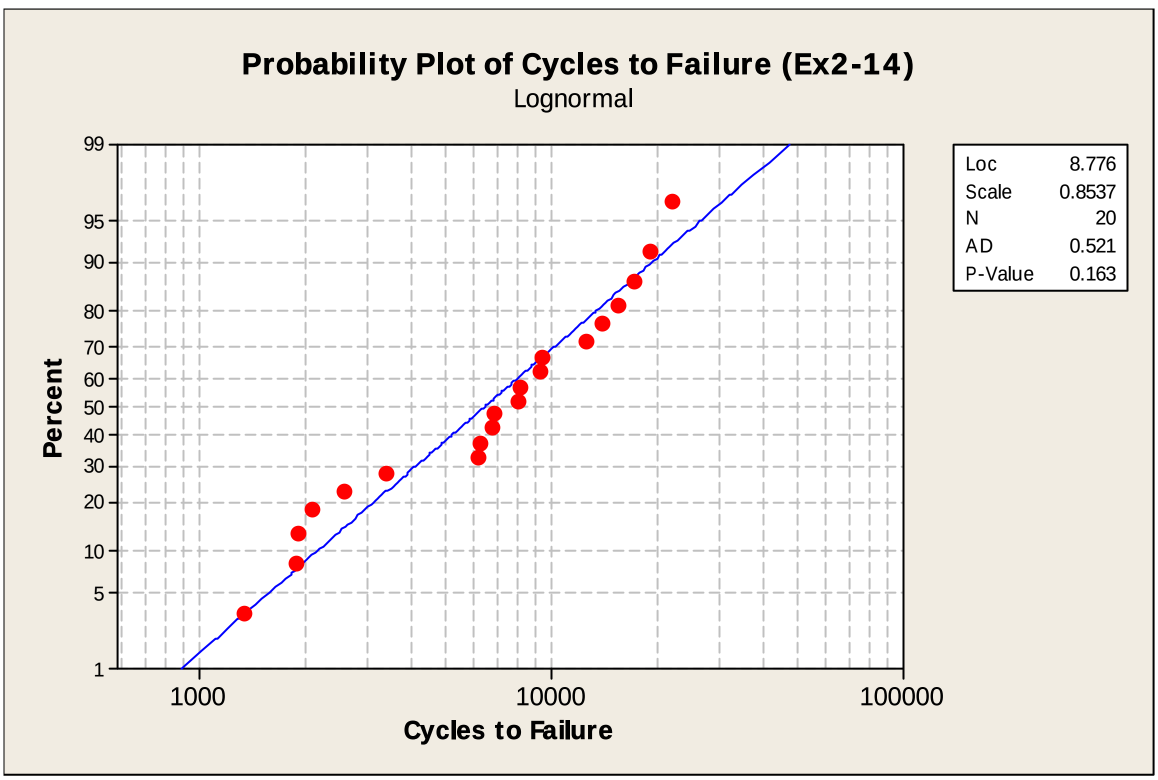
Both the normal and lognormal distributions appear to be reasonable models for the data; the plot points tend to fall along a straight line, with no bends or curves. However, the plot points on the Weibull probability plot are not straight—particularly in the tails—indicating it is not a reasonable model.

2-14 ☺.

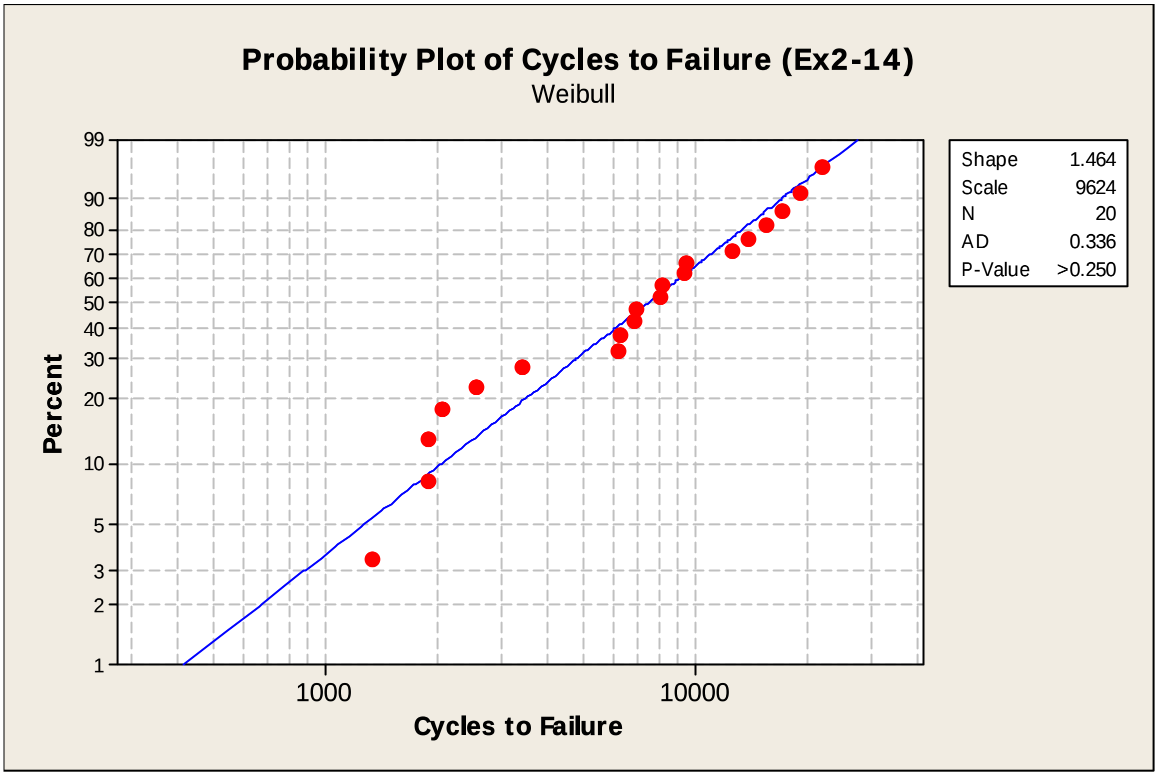
MTB > Graph > Probability Plot > Single

(In the dialog box, select Distribution to choose the distributions)





2-14 continued

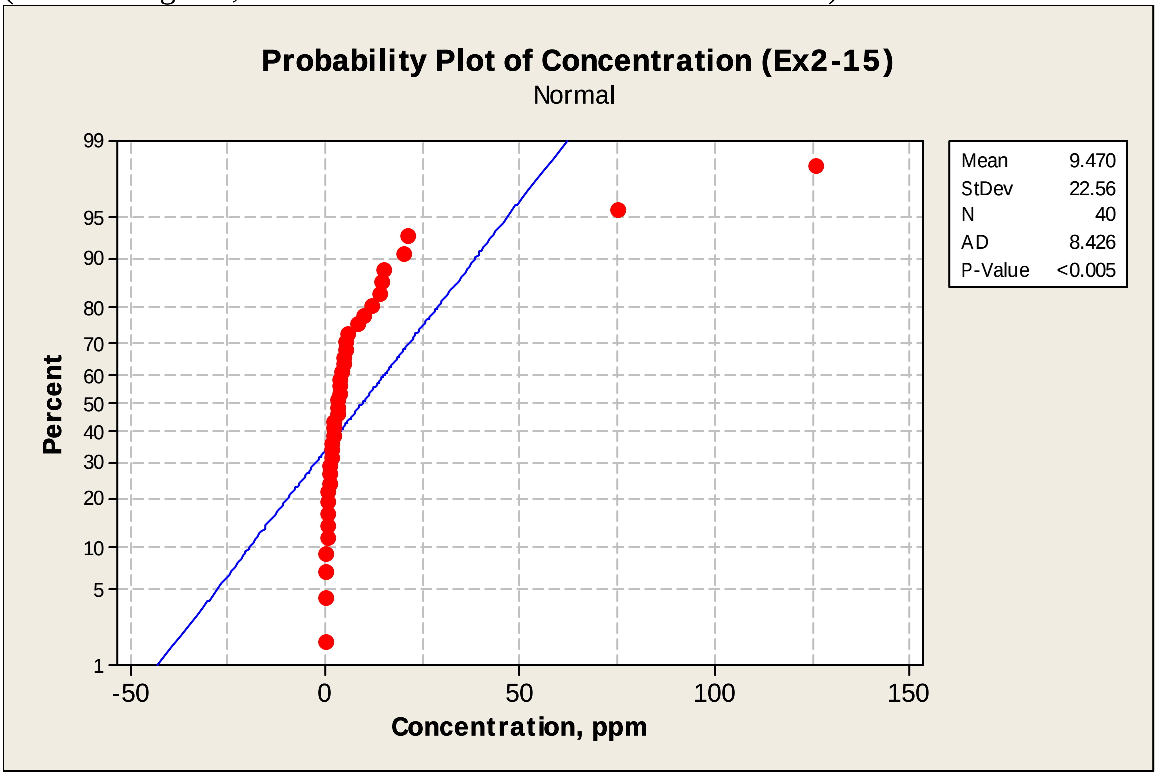


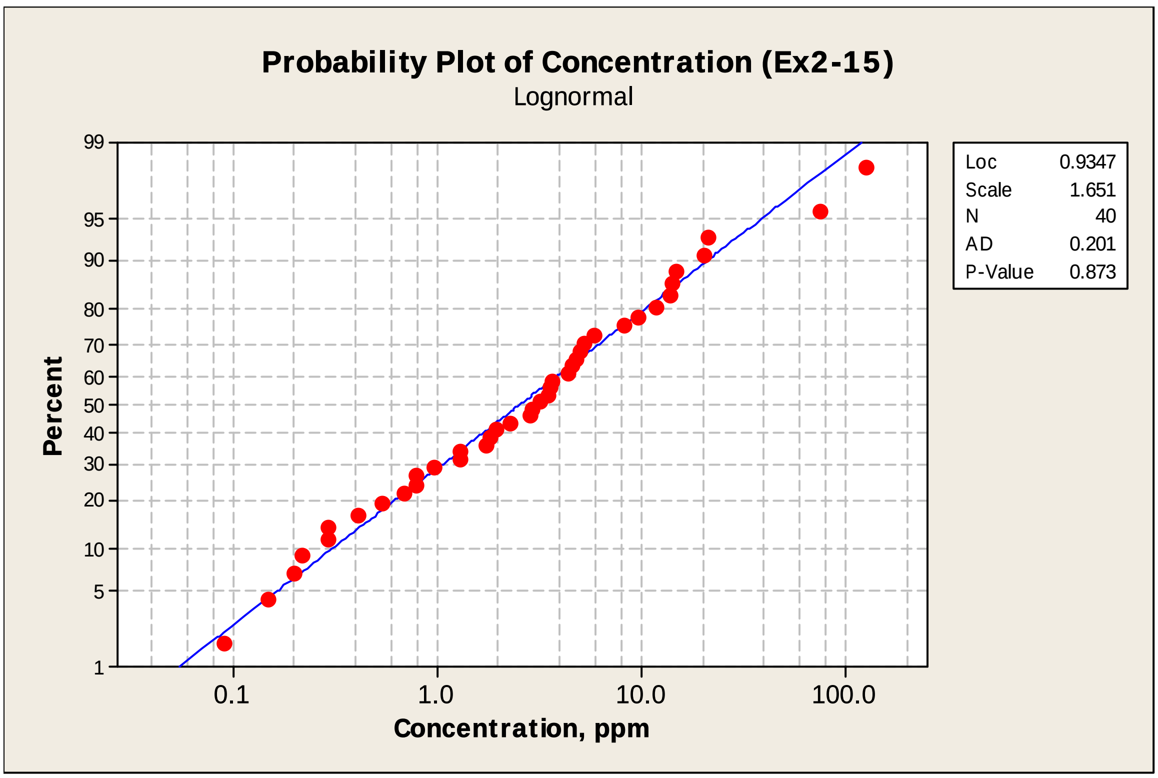
Plotted points do not tend to fall on a straight line on any of the probability plots, though the Weibull distribution appears to best fit the data in the tails.

2-15 ☺.

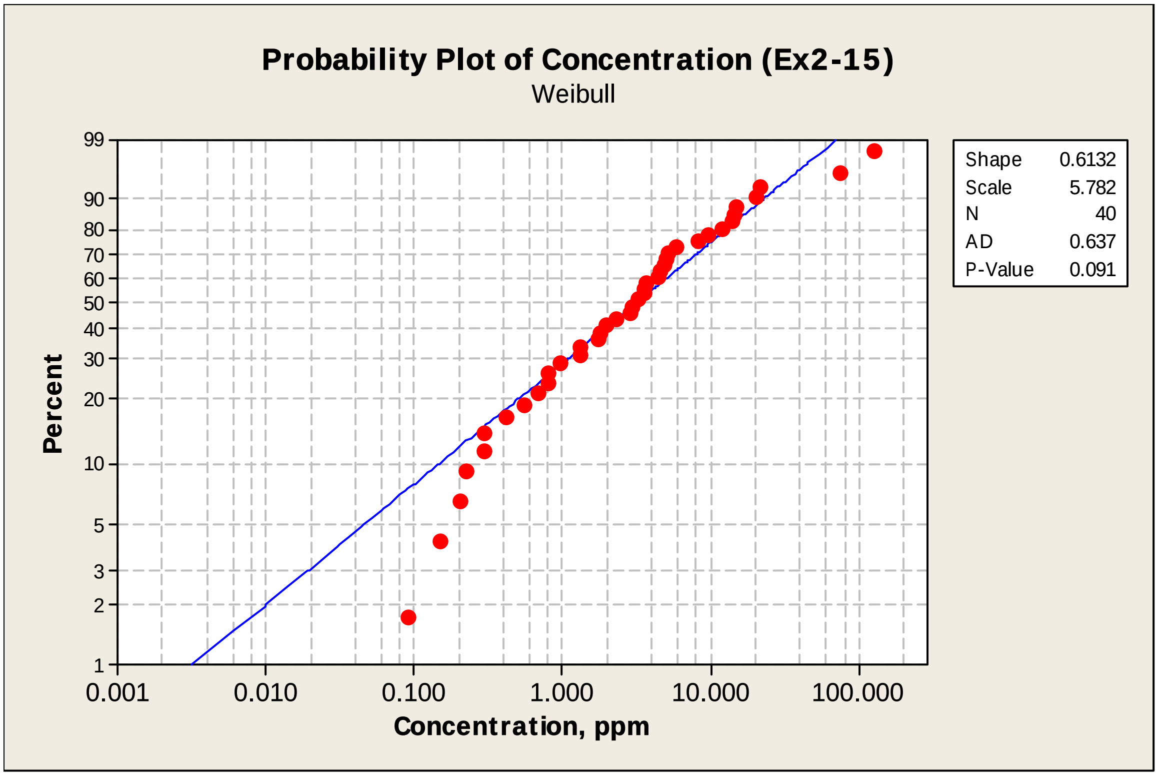
MTB > Graph > Probability Plot > Single

(In the dialog box, select Distribution to choose the distributions)





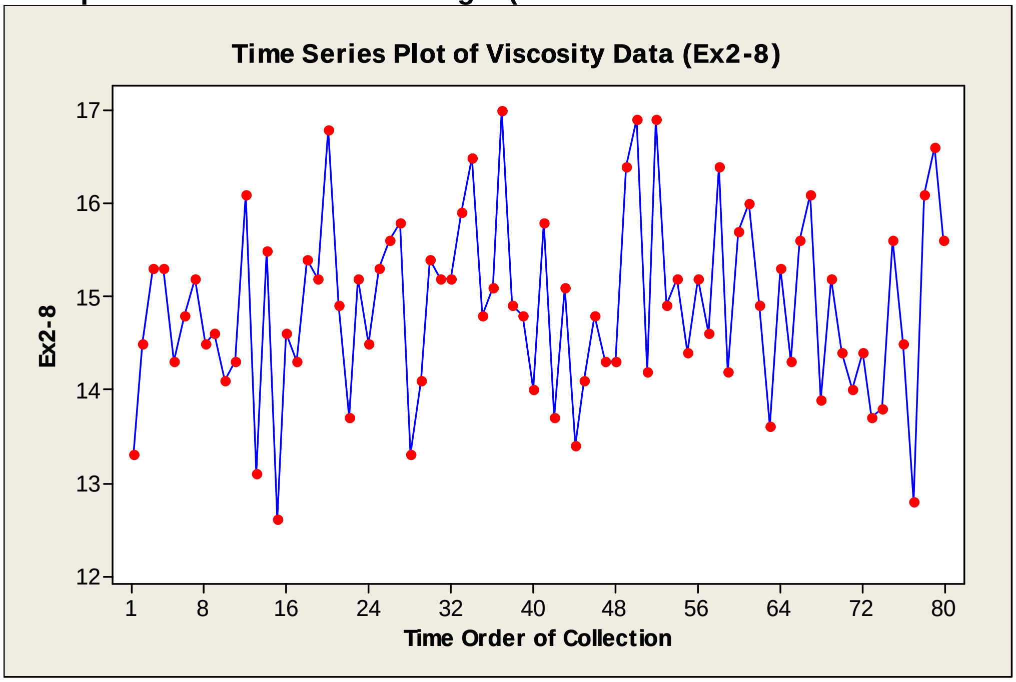
2-15 continued



The lognormal distribution appears to be a reasonable model for the concentration data. Plotted points on the normal and Weibull probability plots tend to fall off a straight line.

2-16* (2-9).

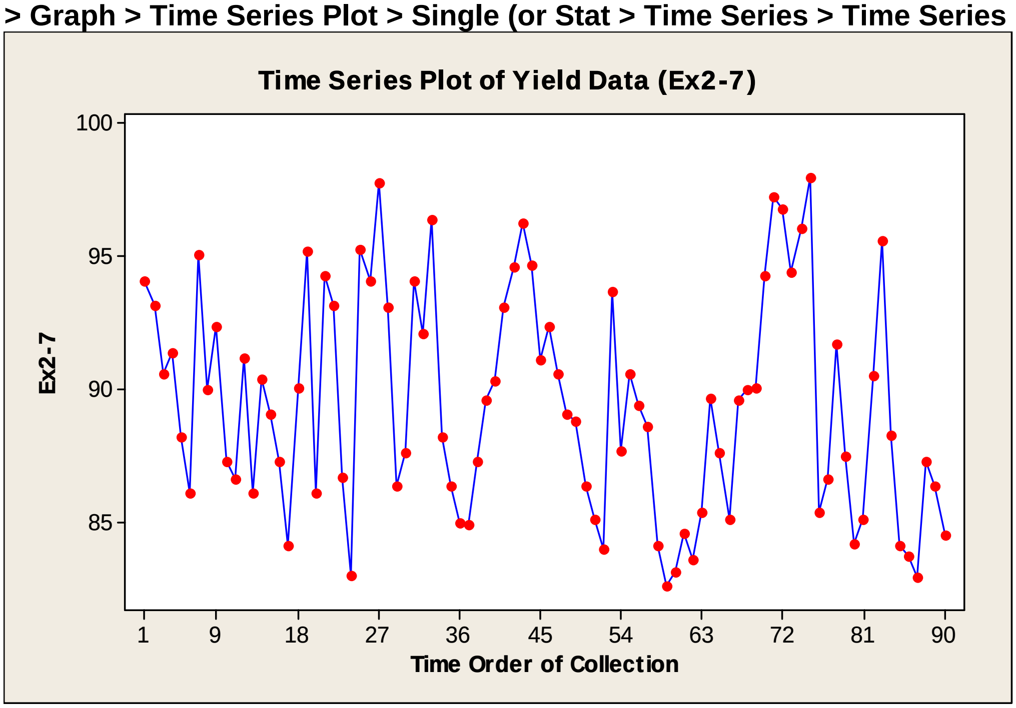
MTB > Graph > Time Series Plot > Single (or Stat > Time Series > Time Series Plot)



From visual examination, there are no trends, shifts or obvious patterns in the data, indicating that time is not an important source of variability.

2-17* (2-10).

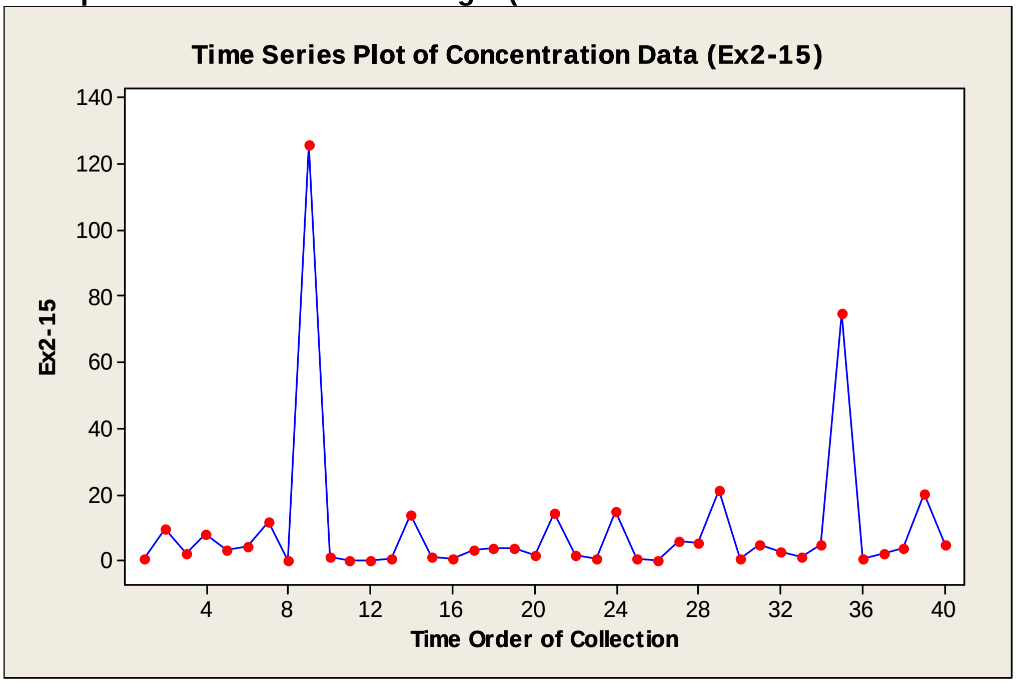
MTB > Graph > Time Series Plot > Single (or Stat > Time Series > Time Series Plot)



Time may be an important source of variability, as evidenced by potentially cyclic behavior.

2-18 ☺.

MTB > Graph > Time Series Plot > Single (or Stat > Time Series > Time Series Plot)



Although most of the readings are between 0 and 20, there are two unusually large readings (9, 35), as well as occasional "spikes" around 20. The order in which the data were collected may be an important source of variability.

2-19 (2-11).

MTB > Stat > Basic Statistics > Display Descriptive Statistics

Descriptive Statistics: Ex2-7

```
Variable   N  N*    Mean  SE Mean  StDev  Minimum      Q1  Median      Q3
Ex2-7     90   0  89.476    0.438  4.158   82.600  86.100  89.250  93.125
Variable  Maximum
Ex2-7      98.000
```

2-20 (2-12).

MTB > Graph > Stem-and-Leaf

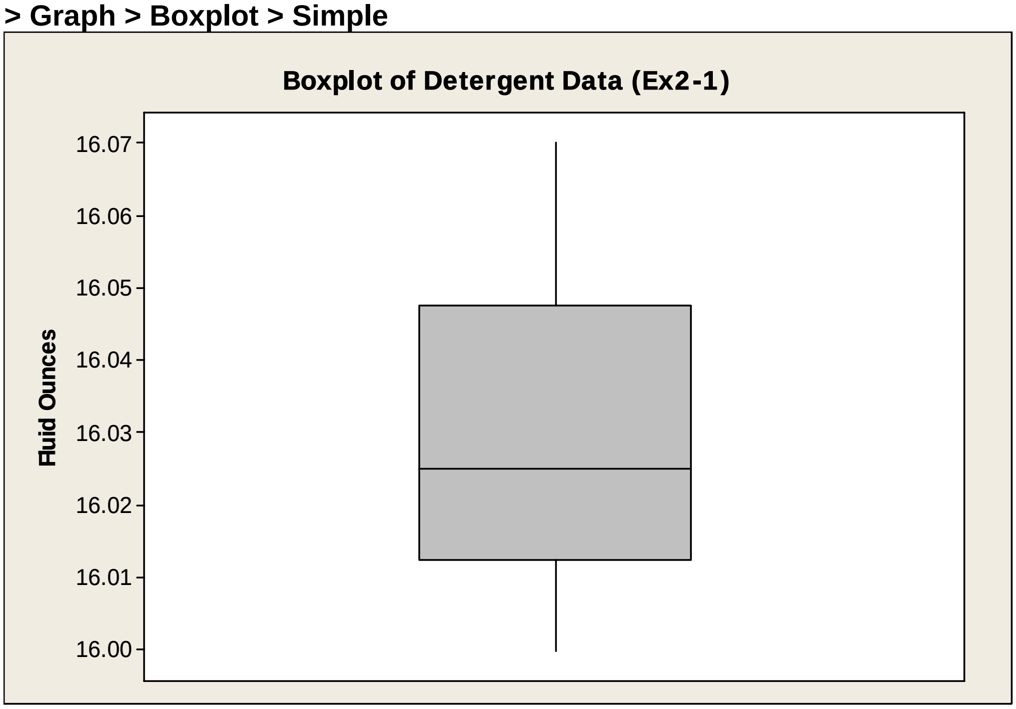
Stem-and-Leaf Display: Ex2-7

```
Stem-and-leaf of Ex2-7  N  = 90
Leaf Unit = 0.10
 2   82  69
 6   83  0167
 14  84  01112569
 20  85  011144
 30  86  1114444667
 38  87  33335667
 43  88  22368
(6)  89  114667
 41  90  0011345666
 31  91  1247
 27  92  144
 24  93  11227
 19  94  11133467
 11  95  1236
 7   96  1348
 3   97  38
 1   98  0
```

Neither the stem-and-leaf plot nor the frequency histogram reveals much about an underlying distribution or a central tendency in the data. The data appear to be fairly well scattered. The stem-and-leaf plot suggests that certain values may occur more frequently than others; for example, those ending in 1, 4, 6, and 7.

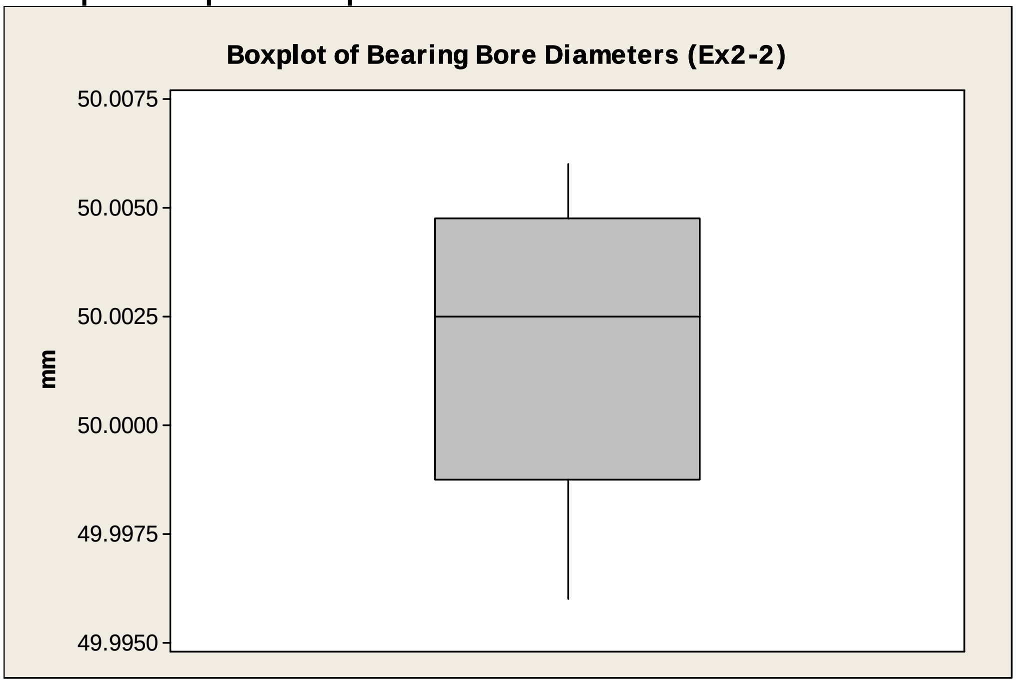
2-21 (2-13).

MTB > Graph > Boxplot > Simple



2-22 (2-14).

MTB > Graph > Boxplot > Simple



2-23 (2-15).

x: {the sum of two up dice faces}

sample space: {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

$$\Pr\{x=2\}=\Pr\{1,1\}=\tfrac{1}{6}\times\tfrac{1}{6}=\tfrac{1}{36}$$

$$\Pr\{x=3\}=\Pr\{1,2\}+\Pr\{2,1\}=\left(\tfrac{1}{6}\times\tfrac{1}{6}\right)+\left(\tfrac{1}{6}\times\tfrac{1}{6}\right)=\tfrac{2}{36}$$

$$\Pr\{x=4\}=\Pr\{1,3\}+\Pr\{2,2\}+\Pr\{3,1\}=\left(\tfrac{1}{6}\times\tfrac{1}{6}\right)+\left(\tfrac{1}{6}\times\tfrac{1}{6}\right)+\left(\tfrac{1}{6}\times\tfrac{1}{6}\right)=\tfrac{3}{36}$$

. . .

$$p(x)=\begin{cases}1/36;\ x=2 \quad 2/36;\ x=3 \quad 3/36;\ x=4 \quad 4/36;\ x=5 \quad 5/36;\ x=6 \quad 6/36;\ x=7\\ 5/36;\ x=8 \quad 4/36;\ x=9 \quad 3/36;\ x=10 \quad 2/36;\ x=11 \quad 1/36;\ x=12 \quad 0;\ \text{otherwise}\end{cases}$$

2-24 (2-16).

$$\bar{x}=\sum_{i=1}^{11}x_i\,p(x_i)=2\left(1/36\right)+3\left(2/36\right)+\cdots+12\left(1/36\right)=7$$

$$S=\sqrt{\frac{\sum_{i=1}^{n}x_i\,p(x_i)-\left[\sum_{i=1}^{n}x_i\,p(x_i)\right]^2\Big/n}{n-1}}=\sqrt{\frac{5.92-7^2/11}{10}}=0.38$$

2-25 (2-17).
This is a Poisson distribution with parameter λ = 0.02, x ~ POI(0.02).

(a)

$$\Pr\{x=1\} = p(1) = \frac{e^{-0.02}(0.02)^1}{1!} = 0.0196$$

(b)

$$\Pr\{x \ge 1\} = 1 - \Pr\{x=0\} = 1 - p(0) = 1 - \frac{e^{-0.02}(0.02)^0}{0!} = 1 - 0.9802 = 0.0198$$

(c)
This is a Poisson distribution with parameter λ = 0.01, x ~ POI(0.01).

$$\Pr\{x \ge 1\} = 1 - \Pr\{x=0\} = 1 - p(0) = 1 - \frac{e^{-0.01}(0.01)^0}{0!} = 1 - 0.9900 = 0.0100$$

Cutting the rate at which defects occur reduces the probability of one or more defects by approximately one-half, from 0.0198 to 0.0100.

2-26 (2-18).

For $f(x)$ to be a probability distribution, $\int_{-\infty}^{+\infty} f(x)dx$ must equal unity.

$$\int_0^\infty ke^{-x}dx = [-ke^{-x}]_0^\infty = -k[0-1] = k \Rightarrow 1$$

This is an exponential distribution with parameter λ=1.
μ = 1/λ = 1 (Eqn. 2-32)
σ^2 = 1/λ^2 = 1 (Eqn. 2-33)

2-27 (2-19).

$$p(x) = \begin{cases} (1+3k)/3; \quad x=1 \quad (1+2k)/3; \quad x=2 \\ (0.5+5k)/3; \quad x=3 \quad 0; \quad \text{otherwise} \end{cases}$$

(a)

To solve for k, use $F(x) = \sum_{i=1}^{\infty} p(x_i) = 1$

$$\frac{(1+3k)+(1+2k)+(0.5+5k)}{3} = 1$$

$$10k = 0.5$$

$$k = 0.05$$

2-27 continued
(b)

$$\mu = \sum_{i=1}^{3} x_i\, p(x_i) = 1\times\left[\frac{1+3(0.05)}{3}\right]+2\times\left[\frac{1+2(0.05)}{3}\right]+3\times\left[\frac{0.5+5(0.05)}{3}\right]=1.867$$

$$\sigma^2 = \sum_{i=1}^{3} x_i^2\, p(x_i) - \mu^2 = 1^2(0.383)+2^2(0.367)+3^2(0.250)-1.867^2 = 0.615$$

(c)

$$F(x)=\begin{cases}\dfrac{1.15}{3}=0.383;\, x=1\\ \dfrac{1.15+1.1}{3}=0.750;\, x=2\\ \dfrac{1.15+1.1+0.75}{3}=1.000;\, x=3\end{cases}$$

2-28 (2-20).

$$p(x)=kr^x;\quad 0<r<1;\quad x=0,1,2,\ldots$$

$$F(x)=\sum_{i=0}^{\infty} kr^x = 1 \text{ by definition}$$

$$k\left[1/(1-r)\right]=1$$

$$k=1-r$$

2-29 (2-21).
(a)
This is an exponential distribution with parameter λ = 0.125:
$\Pr\{x\le 1\}=F(1)=1-e^{-0.125(1)}=0.118$
Approximately 11.8% will fail during the first year.

(b)
Mfg. cost = \$50/calculator
Sale profit = \$25/calculator
Net profit = \$[-50(1 + 0.118) + 75]/calculator = \$19.10/calculator.
The effect of warranty replacements is to decrease profit by \$5.90/calculator.

2-30 (2-22).

$$\Pr\{x<12\}=F(12)=\int_{-\infty}^{12} f(x)dx=\int_{11.75}^{12} 4(x-11.75)dx=\left.\frac{4x^2}{2}\right|_{11.75}^{12}-47x\Big|_{11.75}^{12}=11.875-11.75=0.125$$

2-31* (2-23).

This is a binomial distribution with parameter p = 0.01 and n = 25. The process is stopped if $x \geq 1$.

$$\Pr\{x\geq 1\}=1-\Pr\{x<1\}=1-\Pr\{x=0\}=1-\binom{25}{0}(0.01)^0(1-0.01)^{25}=1-0.78=0.22$$

This decision rule means that 22% of the samples will have one or more nonconforming units, and the process will be stopped to look for a cause. This is a somewhat difficult operating situation.

This exercise may also be solved using Excel or MINITAB:
(1) **Excel Function BINOMDIST(x, n, p, TRUE)**
(2) **MTB > Calc > Probability Distributions > Binomial**

Cumulative Distribution Function

```
Binomial with n = 25 and p = 0.01
x  P( X <= x )
0     0.777821
```

2-32* (2-24).

x ~ BIN(25, 0.04) Stop process if $x \geq 1$.

$$\Pr\{x\geq 1\}=1-\Pr\{x<1\}=1-\Pr\{x=0\}=1-\binom{25}{0}(0.04)^0(1-0.04)^{25}=1-0.36=0.64$$

2-33* (2-25).

This is a binomial distribution with parameter p = 0.02 and n = 50.

$$\Pr\{\hat{p}\leq 0.04\}=\Pr\{x\leq 2\}=\sum_{x=0}^{4}\binom{50}{x}(0.02)^x(1-0.02)^{(50-x)}$$

$$=\binom{50}{0}(0.02)^0(1-0.02)^{50}+\binom{50}{1}(0.02)^1(1-0.02)^{49}+\cdots+\binom{50}{4}(0.02)^4(1-0.02)^{46}=0.921$$

2-34* (2-26).
This is a binomial distribution with parameter p = 0.01 and n = 100.
$\sigma = \sqrt{0.01(1-0.01)/100} = 0.0100$

$$\Pr\{\hat{p} > k\sigma + p\} = 1 - \Pr\{\hat{p} \le k\sigma + p\} = 1 - \Pr\{x \le n(k\sigma + p)\}$$

k = 1

$$\begin{aligned}1-\Pr\{x \le n(k\sigma+p)\} &= 1-\Pr\{x \le 100(1(0.0100)+0.01)\} = 1-\Pr\{x \le 2\}\\ &= 1-\sum_{x=0}^{2}\binom{100}{x}(0.01)^x(1-0.01)^{100-x}\\ &= 1-\left[\binom{100}{0}(0.01)^0(0.99)^{100}+\binom{100}{1}(0.01)^1(0.99)^{99}+\binom{100}{2}(0.01)^2(0.99)^{98}\right]\\ &= 1-[0.921] = 0.079\end{aligned}$$

k = 2

$$\begin{aligned}1-\Pr\{x \le n(k\sigma+p)\} &= 1-\Pr\{x \le 100(2(0.0100)+0.01)\} = 1-\Pr\{x \le 3\}\\ &= 1-\sum_{x=0}^{3}\binom{100}{x}(0.01)^x(0.99)^{100-x} = 1-\left[0.921+\binom{100}{3}(0.01)^3(0.99)^{97}\right]\\ &= 1-[0.982] = 0.018\end{aligned}$$

k = 3

$$\begin{aligned}1-\Pr\{x \le n(k\sigma+p)\} &= 1-\Pr\{x \le 100(3(0.0100)+0.01)\} = 1-\Pr\{x \le 4\}\\ &= 1-\sum_{x=0}^{4}\binom{100}{x}(0.01)^x(0.99)^{100-x} = 1-\left[0.982+\binom{100}{4}(0.01)^4(0.99)^{96}\right]\\ &= 1-[0.992] = 0.003\end{aligned}$$

2-35* (2-27).
This is a hypergeometric distribution with N = 25 and n = 5, without replacement.

(a)
Given D = 2 and x = 0:

$$\Pr\{\text{Acceptance}\} = p(0) = \frac{\binom{2}{0}\binom{25-2}{5-0}}{\binom{25}{5}} = \frac{(1)(33,649)}{(53,130)} = 0.633$$

This exercise may also be solved using Excel or MINITAB:
(1) **Excel Function HYPGEOMDIST(x, n, D, N)**
(2) **MTB > Calc > Probability Distributions > Hypergeometric**

Cumulative Distribution Function
```
Hypergeometric with N = 25, M = 2, and n = 5
x  P( X <= x )
0     0.633333
```

(b)
For the binomial approximation to the hypergeometric, $p = D/N$ = 2/25 = 0.08 and n = 5.

$$\Pr\{\text{acceptance}\} = p(0) = \binom{5}{0}(0.08)^0(1-0.08)^5 = 0.659$$

This approximation, though close to the exact solution for x = 0, violates the rule-of-thumb that n/N = 5/25 = 0.20 be less than the suggested 0.1. The binomial approximation is not satisfactory in this case.

(c)
For N = 150, n/N = 5/150 = 0.033 ≤ 0.1, so the binomial approximation would be a satisfactory approximation the hypergeometric in this case.

2-35 continued
(d)
Find n to satisfy $\Pr\{x \ge 1 \mid D \ge 5\} \ge 0.95$, or equivalently $\Pr\{x = 0 \mid D = 5\} < 0.05$.

$$p(0) = \frac{\binom{5}{0}\binom{25-5}{n-0}}{\binom{25}{n}} = \frac{\binom{5}{0}\binom{20}{n}}{\binom{25}{n}}$$

try $n = 10$

$$p(0) = \frac{\binom{5}{0}\binom{20}{10}}{\binom{25}{10}} = \frac{(1)(184,756)}{(3,268,760)} = 0.057$$

try $n = 11$

$$p(0) = \frac{\binom{5}{0}\binom{20}{11}}{\binom{25}{11}} = \frac{(1)(167,960)}{(4,457,400)} = 0.038$$

Let sample size $n = 11$.

2-36 (2-28).
This is a hypergeometric distribution with $N = 30$, $n = 5$, and $D = 3$.

$$\Pr\{x = 1\} = p(1) = \frac{\binom{3}{1}\binom{30-3}{5-1}}{\binom{30}{5}} = \frac{(3)(17,550)}{(142,506)} = 0.369$$

$$\Pr\{x \ge 1\} = 1 - \Pr\{x = 0\} = 1 - p(0) = 1 - \frac{\binom{3}{0}\binom{27}{5}}{\binom{30}{5}} = 1 - 0.567 = 0.433$$

2-37 (2-29).
This is a hypergeometric distribution with N = 500 pages, n = 50 pages, and D = 10 errors. Checking n/N = 50/500 = 0.1 ≤ 0.1, the binomial distribution can be used to approximate the hypergeometric, with $p = D/N$ = 10/500 = 0.020.

$$\Pr\{x=0\} = p(0) = \binom{50}{0}(0.020)^0(1-0.020)^{50-0} = (1)(1)(0.364) = 0.364$$

$$\Pr\{x \geq 2\} = 1-\Pr\{x \leq 1\} = 1-[\Pr\{x=0\}+\Pr\{x=1\}] = 1-p(0)-p(1)$$

$$= 1-0.364-\binom{50}{1}(0.020)^1(1-0.020)^{50-1} = 1-0.364-0.372 = 0.264$$

2-38 (2-30).
This is a Poisson distribution with λ = 0.1 defects/unit.

$$\Pr\{x \geq 1\} = 1-\Pr\{x=0\} = 1-p(0) = 1-\frac{e^{-0.1}(0.1)^0}{0!} = 1-0.905 = 0.095$$

This exercise may also be solved using Excel or MINITAB:
(1) **Excel Function POISSON(λ, x, TRUE)**
(2) **MTB > Calc > Probability Distributions > Poisson**

Cumulative Distribution Function
```
Poisson with mean = 0.1
x  P( X <= x )
0     0.904837
```

2-39 (2-31).
This is a Poisson distribution with λ = 0.00001 stones/bottle.

$$\Pr\{x \geq 1\} = 1-\Pr\{x=0\} = 1-\frac{e^{-0.00001}(0.00001)^0}{0!} = 1-0.99999 = 0.00001$$

2-40 (2-32).
This is a Poisson distribution with λ = 0.01 errors/bill.

$$\Pr\{x=1\} = p(1) = \frac{e^{-0.01}(0.01)^1}{1} = 0.0099$$

2-41 (2-33).

$$\Pr(t) = p(1-p)^{t-1};\quad t = 1, 2, 3, \ldots$$

$$\mu = \sum_{t=1}^{\infty} t\left[p(1-p)^{t-1} \right] = p\frac{d}{dq}\left[\sum_{t=1}^{\infty} q^t \right] = \frac{1}{p}$$

2-42 (2-34).

This is a Pascal distribution with Pr{defective weld} = p = 0.01, r = 3 welds, and x = 1 + (5000/100) = 51.

$$\Pr\{x = 51\} = p(51) = \binom{51-1}{3-1}(0.01)^3(1-0.01)^{51-3} = (1225)(0.000001)(0.617290) = 0.0008$$

$$\Pr\{x > 51\} = \Pr\{r = 0\} + \Pr\{r = 1\} + \Pr\{r = 2\}$$

$$= \binom{50}{0}0.01^0 0.99^{50} + \binom{50}{1}0.01^1 0.99^{49}\binom{50}{2}0.01^2 0.99^{48} = 0.9862$$

2-43* (2-35).

$x \sim N(40, 5^2)$; n = 50,000

How many fail the minimum specification, LSL = 35 lb.?

$$\Pr\{x \le 35\} = \Pr\left\{ z \le \frac{35-40}{5} \right\} = \Pr\{z \le -1\} = \Phi(-1) = 0.159$$

So, the number that fail the minimum specification are (50,000) × (0.159) = 7950.

This exercise may also be solved using Excel or MINITAB:

(1) **Excel Function NORMDIST(X, μ, σ, TRUE)**

(2) **MTB > Calc > Probability Distributions > Normal**

Cumulative Distribution Function
```
Normal with mean = 40 and standard deviation = 5
 x  P( X <= x )
35     0.158655
```

How many exceed 48 lb.?

$$\Pr\{x > 48\} = 1 - \Pr\{x \le 48\} = 1 - \Pr\left\{ z \le \frac{48-40}{5} \right\} = 1 - \Pr\{z \le 1.6\}$$

$$= 1 - \Phi(1.6) = 1 - 0.945 = 0.055$$

So, the number that exceed 48 lb. is (50,000) × (0.055) = 2750.

2-44* (2-36).
x ~ $N(5, 0.02^2)$; LSL = 4.95 V; USL = 5.05 V

$$\Pr\{\text{Conformance}\} = \Pr\{\text{LSL} \le x \le \text{USL}\} = \Pr\{x \le \text{USL}\} - \Pr\{x \le \text{LSL}\}$$
$$= \Phi\left(\frac{5.05-5}{0.02}\right) - \Phi\left(\frac{4.95-5}{0.02}\right) = \Phi(2.5) - \Phi(-2.5) = 0.99379 - 0.00621 = 0.98758$$

2-45* (2-37).
The process, with mean 5 V, is currently centered between the specification limits (target = 5 V). Shifting the process mean in either direction would increase the number of nonconformities produced.

Desire Pr{Conformance} = 1 / 1000 = 0.001. Assume that the process remains centered between the specification limits at 5 V. Need Pr{$x \le$ LSL} = 0.001 / 2 = 0.0005.

$$\Phi(z) = 0.0005$$
$$z = \Phi^{-1}(0.0005) = -3.29$$

$$z = \frac{\text{LSL}-\mu}{\sigma}, \quad \text{so } \sigma = \frac{\text{LSL}-\mu}{z} = \frac{4.95-5}{-3.29} = 0.015$$

Process variance must be reduced to 0.015^2 to have at least 999 of 1000 conform to specification.

2-46 (2-38).
$x \sim N(\mu, 4^2)$. Find μ such that $\Pr\{x < 32\} = 0.0228$.

$$\Phi^{-1}(0.0228) = -1.9991$$
$$\frac{32-\mu}{4} = -1.9991$$
$$\mu = -4(-1.9991) + 32 = 40.0$$

2-47 (2-39).
$x \sim N(900, 35^2)$

$$\Pr\{x > 1000\} = 1 - \Pr\{x \le 1000\}$$
$$= 1 - \Pr\left\{x \le \frac{1000-900}{35}\right\}$$
$$= 1 - \Phi(2.8571)$$
$$= 1 - 0.9979$$
$$= 0.0021$$

2-48 (2-40).

$x \sim N(5000, 50^2)$. Find LSL such that $\Pr\{x < \text{LSL}\} = 0.005$

$$\Phi^{-1}(0.005) = -2.5758$$

$$\frac{\text{LSL} - 5000}{50} = -2.5758$$

$$\text{LSL} = 50(-2.5758) + 5000 = 4871$$

2-49 (2-41).

$x_1 \sim N(7500, \sigma_1^2 = 1000^2)$; $x_2 \sim N(7500, \sigma_2^2 = 500^2)$; LSL = 5,000 h; USL = 10,000 h

sales = \$10/unit, defect = \$5/unit, profit = \$10 × Pr{good} + \$5 × Pr{bad} – c

<u>For Process 1</u>

$$\begin{aligned}\text{proportion defective} = p_1 &= 1 - \Pr\{\text{LSL} \le x_1 \le \text{USL}\} = 1 - \Pr\{x_1 \le \text{USL}\} + \Pr\{x_1 \le \text{LSL}\} \\ &= 1 - \Pr\left\{z_1 \le \frac{10{,}000 - 7{,}500}{1{,}000}\right\} + \Pr\left\{z_1 \le \frac{5{,}000 - 7{,}500}{1{,}000}\right\} \\ &= 1 - \Phi(2.5) + \Phi(-2.5) = 1 - 0.9938 + 0.0062 = 0.0124\end{aligned}$$

profit for process 1 = 10 (1 – 0.0124) + 5 (0.0124) – c_1 = 9.9380 – c_1

<u>For Process 2</u>

$$\begin{aligned}\text{proportion defective} = p_2 &= 1 - \Pr\{\text{LSL} \le x_2 \le \text{USL}\} = 1 - \Pr\{x_2 \le \text{USL}\} + \Pr\{x_2 \le \text{LSL}\} \\ &= 1 - \Pr\left\{z_2 \le \frac{10{,}000 - 7{,}500}{500}\right\} + \Pr\left\{z_2 \le \frac{5{,}000 - 7{,}500}{500}\right\} \\ &= 1 - \Phi(5) + \Phi(-5) = 1 - 1.0000 + 0.0000 = 0.0000\end{aligned}$$

profit for process 2 = 10 (1 – 0.0000) + 5 (0.0000) – c_2 = 10 – c_2

If $c_2 > c_1 + 0.0620$, then choose process 1

2-50 (2-42).
Proportion less than lower specification:

$$p_l = \Pr\{x < 6\} = \Pr\left\{z \le \frac{6-\mu}{1}\right\} = \Phi(6-\mu)$$

Proportion greater than upper specification:

$$p_u = \Pr\{x > 8\} = 1 - \Pr\{x \le 8\} = 1 - \Pr\left\{z \le \frac{8-\mu}{1}\right\} = 1 - \Phi(8-\mu)$$

$$\begin{aligned}
\text{Profit} &= +C_0 p_{\text{within}} - C_1 p_l - C_2 p_u \\
&= C_0[\Phi(8-\mu) - \Phi(6-\mu)] - C_1[\Phi(6-\mu)] - C_2[1-\Phi(8-\mu)] \\
&= (C_0 + C_2)[\Phi(8-\mu)] - (C_0 + C_1)[\Phi(6-\mu)] - C_2
\end{aligned}$$

$$\frac{d}{d\mu}[\Phi(8-\mu)] = \frac{d}{d\mu}\left[\int_{-\infty}^{8-\mu} \frac{1}{\sqrt{2\pi}} \exp(-t^2/2)dt\right]$$

Set $s = 8 - \mu$ and use chain rule

$$\frac{d}{d\mu}[\Phi(8-\mu)] = \frac{d}{ds}\left[\int_{-\infty}^{s} \frac{1}{\sqrt{2\pi}} \exp(-t^2/2)dt\right]\frac{ds}{d\mu} = -\frac{1}{\sqrt{2\pi}} \exp\left(-1/2\times(8-\mu)^2\right)$$

$$\frac{d(\text{Profit})}{d\mu} = -(C_0 + C_2)\left[\frac{1}{\sqrt{2\pi}} \exp\left(-1/2\times(8-\mu)^2\right)\right] + (C_0 + C_1)\left[\frac{1}{\sqrt{2\pi}} \exp\left(-1/2\times(6-\mu)^2\right)\right]$$

Setting equal to zero

$$\frac{C_0 + C_1}{C_0 + C_2} = \frac{\exp\left(-1/2\times(8-\mu)^2\right)}{\exp\left(-1/2\times(8-\mu)^2\right)} = \exp(2\mu - 14)$$

So $\mu = \frac{1}{2}\left[\ln\left(\frac{C_0 + C_1}{C_0 + C_2}\right) + 14\right]$ maximizes the expected profit.

2-51 (2-43).

For the binomial distribution, $p(x) = \binom{n}{x} p^x (1-p)^{n-x}; \quad x = 0,1,\ldots,n$

$$\mu = E(x) = \sum_{i=1}^{\infty} x_i\, p(x_i) = \sum_{x=0}^{n} x\left[\binom{n}{x} p^x (1-p)^{n-x}\right] = n\left[p + (1-p)\right]^{n-1} p = np$$

$$\sigma^2 = E[(x-\mu)^2] = E(x^2) - [E(x)]^2$$

$$E(x^2) = \sum_{i=1}^{\infty} x_i^2\, p(x_i) = \sum_{x=0}^{n} x^2\left[\binom{n}{x} p^x (1-p)^{n-x}\right] = np + (np)^2 - np^2$$

$$\sigma^2 = \left[np + (np)^2 - np^2\right] - \left[np\right]^2 = np(1-p)$$

2-52 (2-44).

For the Poisson distribution, $p(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}; x = 0,1,\ldots$

$$\mu = E[x] = \sum_{i=1}^{\infty} x_i\, p(x_i) = \sum_{x=0}^{\infty} x\left(\frac{e^{-\lambda}\lambda^x}{x!}\right) = e^{-\lambda}\lambda \sum_{x=0}^{\infty} \frac{\lambda^{(x-1)}}{(x-1)!} = e^{-\lambda}\lambda\left(e^{\lambda}\right) = \lambda$$

$$\sigma^2 = E[(x-\mu)^2] = E(x^2) - [E(x)]^2$$

$$E(x^2) = \sum_{i=1}^{\infty} x_i^2\, p(x_i) = \sum_{x=0}^{\infty} x^2\left(\frac{e^{-\lambda}\lambda^x}{x!}\right) = \lambda^2 + \lambda$$

$$\sigma^2 = (\lambda^2 + \lambda) - \left[\lambda\right]^2 = \lambda$$

2-53 (2-45).
For the exponential distribution, $f(x) = \lambda e^{-\lambda x}; x \ge 0$

For the mean:

$$\mu = \int_0^{+\infty} x f(x) dx = \int_0^{+\infty} x\left(\lambda e^{-\lambda x}\right) dx$$

Integrate by parts, setting $u = x$ and $dv = \lambda \exp(-\lambda x)$

$$uv - \int v du = \left[-x \exp\left(-\lambda x\right)\right]_0^{+\infty} + \int_0^{+\infty} \exp\left(-\lambda x\right) dx = 0 + \frac{1}{\lambda} = \frac{1}{\lambda}$$

For the variance:

$$\sigma^2 = E[(x-\mu)^2] = E(x^2) - [E(x)^2] = E(x^2) - \left(\frac{1}{\lambda}\right)^2$$

$$E(x^2) = \int_{-\infty}^{+\infty} x^2 f(x) dx = \int_0^{+\infty} x^2 \lambda \exp(-\lambda x) dx$$

Integrate by parts, setting $u = x^2$ and $dv = \lambda \exp(-\lambda x)$

$$uv - \int v du = \left[x^2 \exp(-\lambda x)\right]_0^{+\infty} + 2\int_0^{+\infty} x \exp(-\lambda x) dx = (0-0) + \frac{2}{\lambda^2}$$

$$\sigma^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

Chapter 3 Exercise Solutions

3-1.
$n = 15$; $\bar{x} = 8.2535$ cm; $\sigma = 0.002$ cm

(a)
$\mu_0 = 8.25$, $\alpha = 0.05$
Test H_0: $\mu = 8.25$ vs. H_1: $\mu \neq 8.25$. Reject H_0 if $|Z_0| > Z_{\alpha/2}$.

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{8.2535 - 8.25}{0.002/\sqrt{15}} = 6.78$$

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$
Reject H_0: $\mu = 8.25$, and conclude that the mean bearing ID is not equal to 8.25 cm.

(b)
P-value $= 2[1 - \Phi(Z_0)] = 2[1 - \Phi(6.78)] = 2[1 - 1.00000] = 0$

(c)

$$\bar{x} - Z_{\alpha/2}\left(\sigma/\sqrt{n}\right) \le \mu \le \bar{x} + Z_{\alpha/2}\left(\sigma/\sqrt{n}\right)$$

$$8.25 - 1.96\left(0.002/\sqrt{15}\right) \le \mu \le 8.25 + 1.96\left(0.002/\sqrt{15}\right)$$

$$8.249 \le \mu \le 8.251$$

MTB > Stat > Basic Statistics > 1-Sample Z > Summarized data

One-Sample Z

```
Test of mu = 8.2535 vs not = 8.2535
The assumed standard deviation = 0.002
 N     Mean  SE Mean        95% CI             Z      P
15  8.25000  0.00052  (8.24899, 8.25101)  -6.78  0.000
```

3-2.
$n = 8$; $\bar{x} = 127$ psi; $\sigma = 2$ psi

(a)
$\mu_0 = 125$; $\alpha = 0.05$
Test H_0: $\mu = 125$ vs. H_1: $\mu > 125$. Reject H_0 if $Z_0 > Z_\alpha$.

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{127 - 125}{2/\sqrt{8}} = 2.828$$

$Z_\alpha = Z_{0.05} = 1.645$
Reject H_0: $\mu = 125$, and conclude that the mean tensile strength exceeds 125 psi.

3-2 continued

(b)

P-value = $1 - \Phi(Z_0) = 1 - \Phi(2.828) = 1 - 0.99766 = 0.00234$

(c)

In strength tests, we usually are interested in whether some minimum requirement is met, not simply that the mean does not equal the hypothesized value. A one-sided hypothesis test lets us do this.

(d)

$$\bar{x} - Z_\alpha\left(\sigma/\sqrt{n}\right) \le \mu$$
$$127 - 1.645\left(2/\sqrt{8}\right) \le \mu$$
$$125.8 \le \mu$$

MTB > Stat > Basic Statistics > 1-Sample Z > Summarized data

```
One-Sample Z
Test of mu = 125 vs > 125
The assumed standard deviation = 2
                          95%
                        Lower
N     Mean  SE Mean     Bound     Z      P
8  127.000    0.707   125.837  2.83  0.002
```

3-3.

$x \sim N(\mu, \sigma)$; $n = 10$

(a)

$\bar{x} = 26.0$; $s = 1.62$; $\mu_0 = 25$; $\alpha = 0.05$

Test H_0: $\mu = 25$ vs. H_1: $\mu > 25$. Reject H_0 if $t_0 > t_\alpha$.

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{26.0 - 25}{1.62/\sqrt{10}} = 1.952$$

$t_{\alpha, n-1} = t_{0.05, 10-1} = 1.833$

Reject H_0: $\mu = 25$, and conclude that the mean life exceeds 25 h.

MTB > Stat > Basic Statistics > 1-Sample t > Samples in columns

```
One-Sample T: Ex3-3
Test of mu = 25 vs > 25
                                                95%
                                              Lower
Variable   N     Mean   StDev  SE Mean        Bound     T      P
Ex3-3     10  26.0000  1.6248   0.5138      25.0581  1.95  0.042
```

3-3 continued

(b)

$\alpha = 0.10$

$$\bar{x} - t_{\alpha/2,n-1} S/\sqrt{n} \le \mu \le \bar{x} + t_{\alpha/2,n-1} S/\sqrt{n}$$

$$26.0 - 1.833\left(1.62/\sqrt{10}\right) \le \mu \le 26.0 + 1.833\left(1.62/\sqrt{10}\right)$$

$$25.06 \le \mu \le 26.94$$

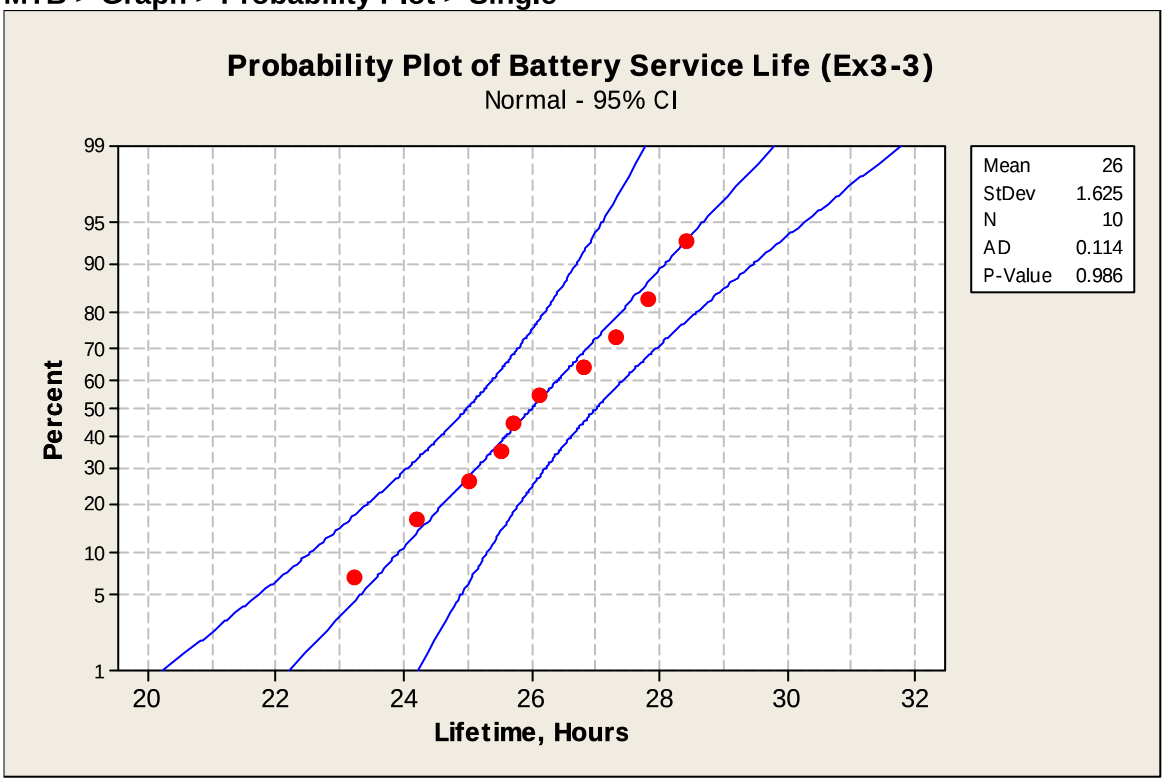
MTB > Stat > Basic Statistics > 1-Sample t > Samples in columns

One-Sample T: Ex3-3

```
Test of mu = 25 vs not = 25
Variable    N     Mean    StDev  SE Mean         90% CI             T       P
Ex3-3      10  26.0000  1.6248   0.5138  (25.0581, 26.9419)  1.95  0.083
```

(c)

MTB > Graph > Probability Plot > Single



The plotted points fall approximately along a straight line, so the assumption that battery life is normally distributed is appropriate.

3-4.

$x \sim N(\mu, \sigma)$; $n = 10$; $\bar{x} = 26.0$ h; $s = 1.62$ h; $\alpha = 0.05$; $t_{\alpha, n-1} = t_{0.05,9} = 1.833$

$$\bar{x} - t_{\alpha,n-1}\left(S/\sqrt{n}\right) \le \mu$$

$$26.0 - 1.833\left(1.62/\sqrt{10}\right) \le \mu$$

$$25.06 \le \mu$$

The manufacturer might be interested in a lower confidence interval on mean battery life when establishing a warranty policy.

3-5.

(a)

$x \sim N(\mu, \sigma)$, $n = 10$, $\bar{x} = 13.39618 \times 1000$ Å, $s = 0.00391$

$\mu_0 = 13.4 \times 1000$ Å, $\alpha = 0.05$

Test H_0: $\mu = 13.4$ vs. H_1: $\mu \ne 13.4$. Reject H_0 if $|t_0| > t_{\alpha/2}$.

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{13.39618 - 13.4}{0.00391/\sqrt{10}} = -3.089$$

$t_{\alpha/2,\, n-1} = t_{0.025,\, 9} = 2.262$

Reject H_0: $\mu = 13.4$, and conclude that the mean thickness differs from 13.4×1000 Å.

MTB > Stat > Basic Statistics > 1-Sample t > Samples in columns

```
One-Sample T: Ex3-5
Test of mu = 13.4 vs not = 13.4
Variable   N     Mean   StDev  SE Mean        95% CI              T      P
Ex3-5     10  13.3962  0.0039   0.0012  (13.3934, 13.3990)  -3.09  0.013
```

(b)

$\alpha = 0.01$

$$\bar{x} - t_{\alpha/2,n-1}\left(S/\sqrt{n}\right) \le \mu \le \bar{x} + t_{\alpha/2,n-1}\left(S/\sqrt{n}\right)$$

$$13.39618 - 3.2498\left(0.00391/\sqrt{10}\right) \le \mu \le 13.39618 + 3.2498\left(0.00391/\sqrt{10}\right)$$

$$13.39216 \le \mu \le 13.40020$$

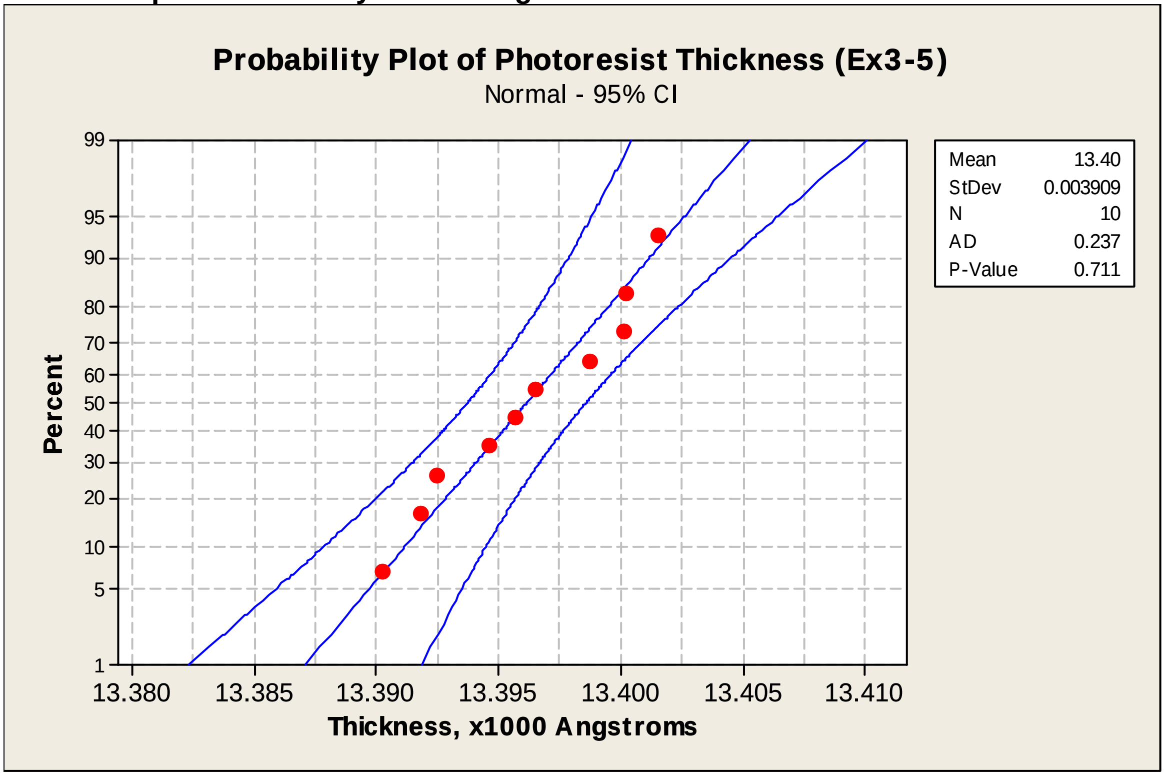
MTB > Stat > Basic Statistics > 1-Sample t > Samples in columns

```
One-Sample T: Ex3-5
Test of mu = 13.4 vs not = 13.4
Variable   N     Mean   StDev  SE Mean        99% CI              T      P
Ex3-5     10  13.3962  0.0039   0.0012  (13.3922, 13.4002)  -3.09  0.013
```

3-5 continued

(c)

MTB > Graph > Probability Plot > Single



The plotted points form a reverse-“S” shape, instead of a straight line, so the assumption that battery life is normally distributed is not appropriate.

3-6.

(a)

$x \sim N(\mu, \sigma)$, $\mu_0 = 12$, $\alpha = 0.01$

$n = 10$, $\bar{x} = 12.015$, $s = 0.030$

Test H_0: $\mu = 12$ vs. H_1: $\mu > 12$. Reject H_0 if $t_0 > t_\alpha$.

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{12.015 - 12}{0.0303/\sqrt{10}} = 1.5655$$

$t_{\alpha/2,\, n-1} = t_{0.005,\, 9} = 3.250$

Do not reject H_0: $\mu = 12$, and conclude that there is not enough evidence that the mean fill volume exceeds 12 oz.

MTB > Stat > Basic Statistics > 1-Sample t > Samples in columns

One-Sample T: Ex3-6

```
Test of mu = 12 vs > 12
                                                   99%
                                                 Lower
Variable   N     Mean   StDev  SE Mean      Bound     T      P
Ex3-6     10  12.0150  0.0303   0.0096    11.9880  1.57  0.076
```

3-6 continued

(b)

$\alpha = 0.05$

$t_{\alpha/2,\, n-1} = t_{0.025,\, 9} = 2.262$

$$\bar{x} - t_{\alpha/2,n-1}\left(S/\sqrt{n}\right) \le \mu \le \bar{x} + t_{\alpha/2,n-1}\left(S/\sqrt{n}\right)$$

$$12.015 - 2.262\left(S/\sqrt{10}\right) \le \mu \le 12.015 + 2.62\left(S/\sqrt{10}\right)$$

$$11.993 \le \mu \le 12.037$$

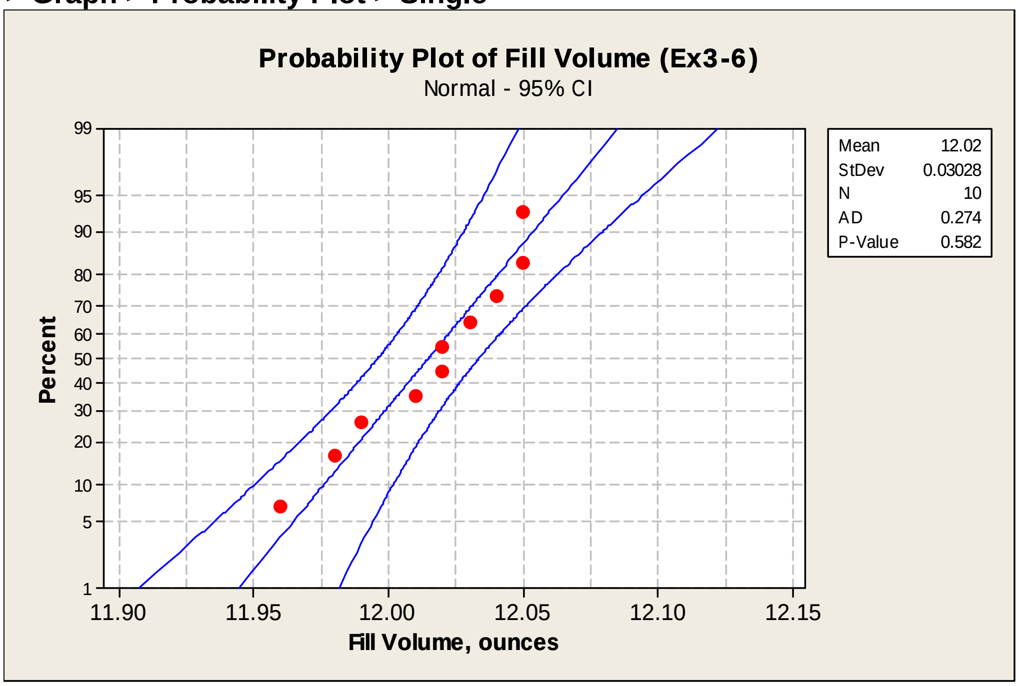
MTB > Stat > Basic Statistics > 1-Sample t > Samples in columns

One-Sample T: Ex3-6

```
Test of mu = 12 vs not = 12
Variable    N     Mean   StDev  SE Mean         95% CI             T      P
Ex3-6      10  12.0150  0.0303   0.0096  (11.9933, 12.0367)  1.57  0.152
```

(c)

MTB > Graph > Probability Plot > Single



The plotted points fall approximately along a straight line, so the assumption that fill volume is normally distributed is appropriate.

3-7.

$\sigma = 4$ lb, $\alpha = 0.05$, $Z_{\alpha/2} = Z_{0.025} = 1.9600$, total confidence interval width = 1 lb, find n

$$2\left[Z_{\alpha/2}\left(\sigma/\sqrt{n}\right)\right] = \text{total width}$$

$$2\left[1.9600\left(4/\sqrt{n}\right)\right] = 1$$

$$n = 246$$

3-8.

(a)

$x \sim N(\mu, \sigma)$, $\mu_0 = 0.5025$, $\alpha = 0.05$

$n = 25$, $\bar{x} = 0.5046$ in, $\sigma = 0.0001$ in

Test H_0: $\mu = 0.5025$ vs. H_1: $\mu \neq 0.5025$. Reject H_0 if $|Z_0| > Z_{\alpha/2}$.

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{0.5046 - 0.5025}{0.0001/\sqrt{25}} = 105$$

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$

Reject H_0: $\mu = 0.5025$, and conclude that the mean rod diameter differs from 0.5025.

MTB > Stat > Basic Statistics > 1-Sample Z > Summarized data

One-Sample Z

```
Test of mu = 0.5025 vs not = 0.5025
The assumed standard deviation = 0.0001
 N      Mean   SE Mean         95% CI              Z      P
25  0.504600  0.000020  (0.504561, 0.504639)  105.00  0.000
```

(b)

P-value $= 2[1 - \Phi(Z_0)] = 2[1 - \Phi(105)] = 2[1 - 1] = 0$

(c)

$$\bar{x} - Z_{\alpha/2}\left(\sigma/\sqrt{n}\right) \leq \mu \leq \bar{x} + Z_{\alpha/2}\left(\sigma/\sqrt{n}\right)$$

$$0.5046 - 1.960\left(0.0001/\sqrt{25}\right) \leq \mu \leq 0.5046 + 1.960\left(0.0001/\sqrt{25}\right)$$

$$0.50456 \leq \mu \leq 0.50464$$

3-9.

$x \sim N(\mu, \sigma)$, $n = 16$, $\bar{x} = 10.259$ V, $s = 0.999$ V

(a)

$\mu_0 = 12$, $\alpha = 0.05$

Test H_0: $\mu = 12$ vs. H_1: $\mu \neq 12$. Reject H_0 if $|t_0| > t_{\alpha/2}$.

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{10.259 - 12}{0.999/\sqrt{16}} = -6.971$$

$t_{\alpha/2, n-1} = t_{0.025, 15} = 2.131$

Reject H_0: $\mu = 12$, and conclude that the mean output voltage differs from 12V.

MTB > Stat > Basic Statistics > 1-Sample t > Samples in columns

One-Sample T: Ex3-9

```
Test of mu = 12 vs not = 12
Variable   N     Mean   StDev  SE Mean        95% CI            T      P
Ex3-9     16  10.2594  0.9990   0.2498  (9.7270, 10.7917)  -6.97  0.000
```

3-9 continued

(b)

$$\bar{x} - t_{\alpha/2,n-1}\left(S/\sqrt{n}\right) \le \mu \le \bar{x} + t_{\alpha/2,n-1}\left(S/\sqrt{n}\right)$$

$$10.259 - 2.131\left(0.999/\sqrt{16}\right) \le \mu \le 10.259 + 2.131\left(0.999/\sqrt{16}\right)$$

$$9.727 \le \mu \le 10.792$$

(c)

$\sigma_0^2 = 1$, $\alpha = 0.05$

Test H_0: $\sigma^2 = 1$ vs. H_1: $\sigma^2 \ne 1$. Reject H_0 if $\chi^2_0 > \chi^2_{\alpha/2,\, n-1}$ or $\chi^2_0 < \chi^2_{1-\alpha/2,\, n-1}$.

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{(16-1)0.999^2}{1} = 14.970$$

$\chi^2_{\alpha/2,\, n-1} = \chi^2_{0.025,16-1} = 27.488$

$\chi^2_{1-\alpha/2,\, n-1} = \chi^2_{0.975,16-1} = 6.262$

Do not reject H_0: $\sigma^2 = 1$, and conclude that there is insufficient evidence that the variance differs from 1.

(d)

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}$$

$$\frac{(16-1)0.999^2}{27.488} \le \sigma^2 \le \frac{(16-1)0.999^2}{6.262}$$

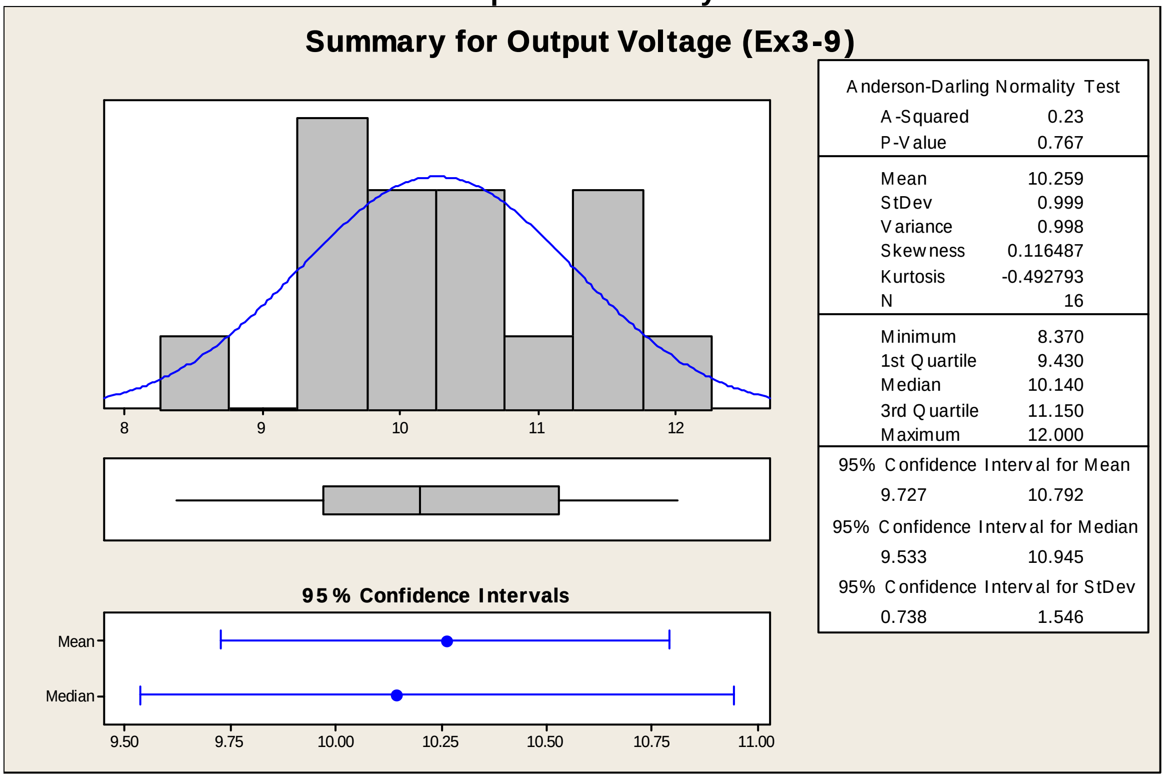
$$0.545 \le \sigma^2 \le 2.391$$

$$0.738 \le \sigma \le 1.546$$

Since the 95% confidence interval on σ contains the hypothesized value, $\sigma_0^2 = 1$, the null hypothesis, H_0: $\sigma^2 = 1$, cannot be rejected.

3-9 (d) continued

MTB > Stat > Basic Statistics > Graphical Summary



(e)

$$\alpha = 0.05;\ \ \chi^2_{1-\alpha,n-1} = \chi^2_{0.95,15} = 7.2609$$

$$\sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha,n-1}}$$

$$\sigma^2 \le \frac{(16-1)0.999^2}{7.2609}$$

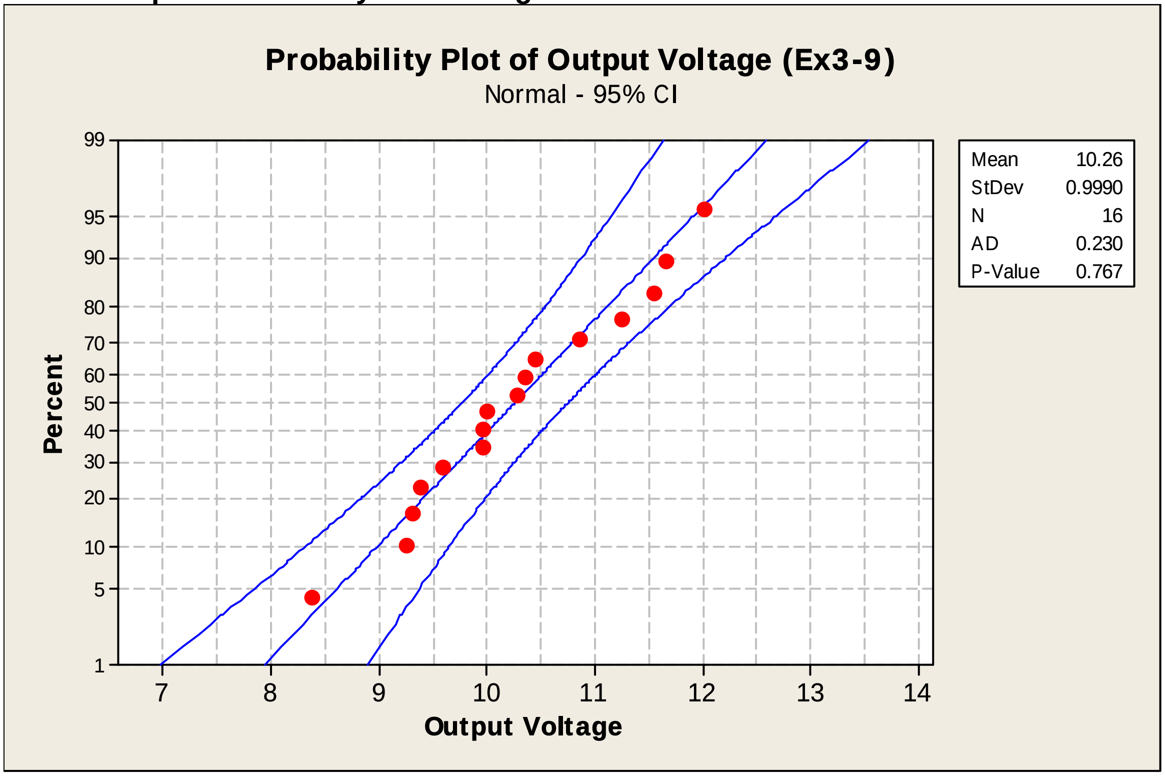
$$\sigma^2 \le 2.062$$

$$\sigma \le 1.436$$

3-9 continued

(f)

MTB > Graph > Probability Plot > Single



From visual examination of the plot, the assumption of a normal distribution for output voltage seems appropriate.

3-10.

$n_1 = 25$, $\bar{x}_1 = 2.04\ l$, $\sigma_1 = 0.010\ l$; $n_2 = 20$, $\bar{x}_2 = 2.07\ l$, $\sigma_2 = 0.015\ l$;

(a)

$\alpha = 0.05$, $\Delta_0 = 0$

Test H_0: $\mu_1 - \mu_2 = 0$ versus H_0: $\mu_1 - \mu_2 \neq 0$. Reject H_0 if $Z_0 > Z_{\alpha/2}$ or $Z_0 < -Z_{\alpha/2}$.

$$Z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} = \frac{(2.04 - 2.07) - 0}{\sqrt{0.010^2/25 + 0.015^2/20}} = -7.682$$

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96 \qquad -Z_{\alpha/2} = -1.96$

Reject H_0: $\mu_1 - \mu_2 = 0$, and conclude that there is a difference in mean net contents between machine 1 and machine 2.

(b)

P-value $= 2[1 - \Phi(Z_0)] = 2[1 - \Phi(-7.682)] = 2[1 - 1.00000] = 0$

3-10 continued
(c)

$$(\bar{x}_1 - \bar{x}_2) - Z_{\alpha/2}\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2} \le (\mu_1 - \mu_2) \le (\bar{x}_1 - \bar{x}_2) + Z_{\alpha/2}\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}$$

$$(2.04 - 2.07) - 1.9600\sqrt{0.010^2/25 + 0.015^2/20} \le (\mu_1 - \mu_2) \le (2.04 - 2.07) + 1.9600\sqrt{0.010^2/25 + 0.015^2/20}$$

$$-0.038 \le (\mu_1 - \mu_2) \le -0.022$$

The confidence interval for the difference does not contain zero. We can conclude that the machines do not fill to the same volume.

3-11.
(a)
MTB > Stat > Basic Statistics > 2-Sample t > Samples in different columns

Two-Sample T-Test and CI: Ex3-11T1, Ex3-11T2

```
Two-sample T for Ex3-11T1 vs Ex3-11T2
          N   Mean  StDev  SE Mean
Ex3-11T1  7  1.383  0.115    0.043
Ex3-11T2  8  1.376  0.125    0.044
Difference = mu (Ex3-11T1) - mu (Ex3-11T2)
Estimate for difference:  0.006607
95% CI for difference:  (-0.127969, 0.141183)
T-Test of difference = 0 (vs not =): T-Value = 0.11  P-Value = 0.917  DF = 13
Both use Pooled StDev = 0.1204
```

Do not reject H_0: $\mu_1 - \mu_2 = 0$, and conclude that there is not sufficient evidence of a difference between measurements obtained by the two technicians.

(b)
The practical implication of this test is that it does not matter which technician measures parts; the readings will be the same. If the null hypothesis had been rejected, we would have been concerned that the technicians obtained different measurements, and an investigation should be undertaken to understand why.

(c)
$n_1 = 7$, $\bar{x}_1 = 1.383$, $S_1 = 0.115$; $n_2 = 8$, $\bar{x}_2 = 1.376$, $S_2 = 0.125$

$\alpha = 0.05$, $t_{\alpha/2,\, n1+n2-2} = t_{0.025,\, 13} = 2.1604$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(7-1)0.115^2 + (8-1)0.125^2}{7+8-2}} = 0.120$$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,n_1+n_2-2}S_p\sqrt{1/n_1 + 1/n_2} \le (\mu_1 - \mu_2) \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,n_1+n_2-2}S_p\sqrt{1/n_1 + 1/n_2}$$

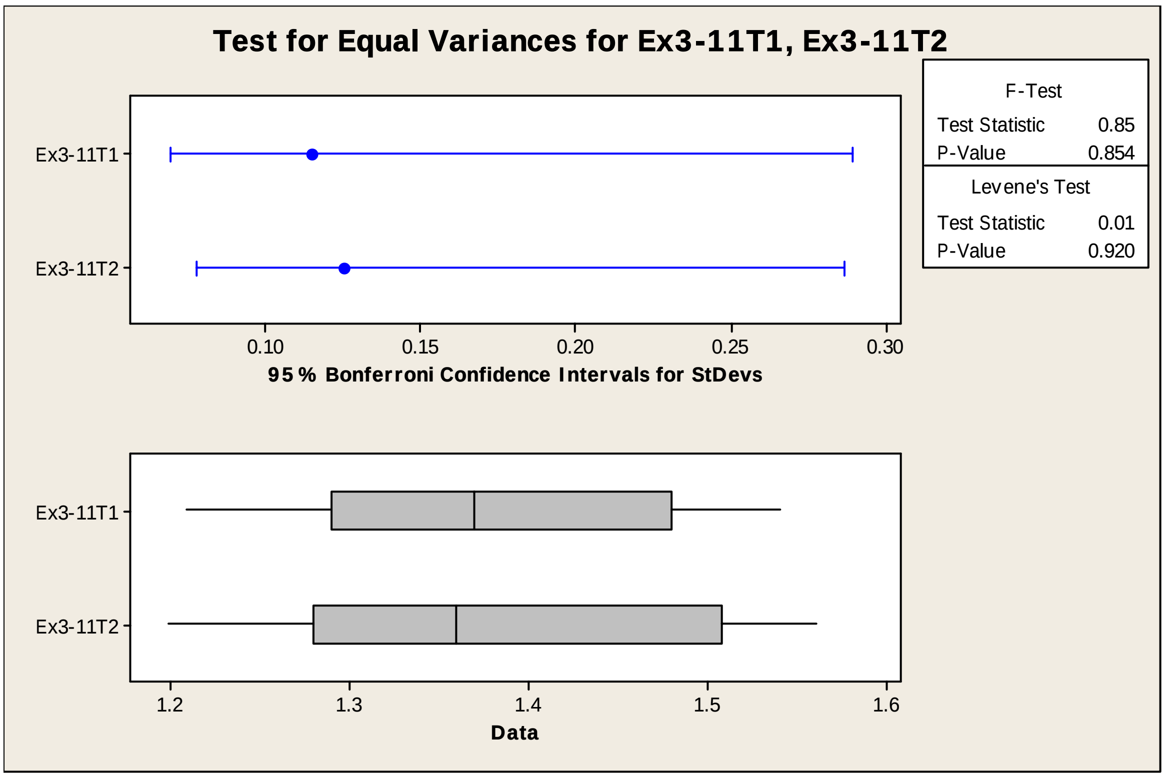
$$(1.383 - 1.376) - 2.1604(0.120)\sqrt{1/7 + 1/8} \le (\mu_1 - \mu_2) \le (1.383 - 1.376) + 2.1604(0.120)\sqrt{1/7 + 1/8}$$

$$-0.127 \le (\mu_1 - \mu_2) \le 0.141$$

The confidence interval for the difference contains zero. We can conclude that there is no difference in measurements obtained by the two technicians.

3-11 continued
(d)
$\alpha = 0.05$
Test $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_1 : \sigma_1^2 \neq \sigma_2^2$.
Reject H_0 if $F_0 > F_{\alpha/2, n_1-1, n_2-1}$ or $F_0 < F_{1-\alpha/2, n_1-1, n_2-1}$.
$F_0 = S_1^2 / S_2^2 = 0.115^2 / 0.125^2 = 0.8464$
$F_{\alpha/2, n_1-1, n_2-1} = F_{0.05/2, 7-1, 8-1} = F_{0.025, 6, 7} = 5.119$
$F_{1-\alpha/2, n_1-1, n_2-1} = F_{1-0.05/2, 7-1, 8-1} = F_{0.975, 6, 7} = 0.176$

MTB > Stat > Basic Statistics > 2 Variances > Summarized data



Do not reject H_0, and conclude that there is no difference in variability of measurements obtained by the two technicians.

If the null hypothesis is rejected, we would have been concerned about the difference in measurement variability between the technicians, and an investigation should be undertaken to understand why.

3-11 continued

(e)

$\alpha = 0.05 \qquad F_{1-\alpha/2,n_2-1,n_1-1} = F_{0.975,7,6} = 0.1954; \quad F_{\alpha/2,n_2-1,n_1-1} = F_{0.025,7,6} = 5.6955$

$$\frac{S_1^2}{S_2^2} F_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{S_1^2}{S_2^2} F_{\alpha/2,n_2-1,n_1-1}$$

$$\frac{0.115^2}{0.125^2}(0.1954) \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{0.115^2}{0.125^2}(5.6955)$$

$$0.165 \le \frac{\sigma_1^2}{\sigma_2^2} \le 4.821$$

(f)

$n_2 = 8; \ \bar{x}_2 = 1.376; \ S_2 = 0.125$

$\alpha = 0.05; \ \chi^2_{\alpha/2,n_2-1} = \chi^2_{0.025,7} = 16.0128; \ \chi^2_{1-\alpha/2,n_2-1} = \chi^2_{0.975,7} = 1.6899$

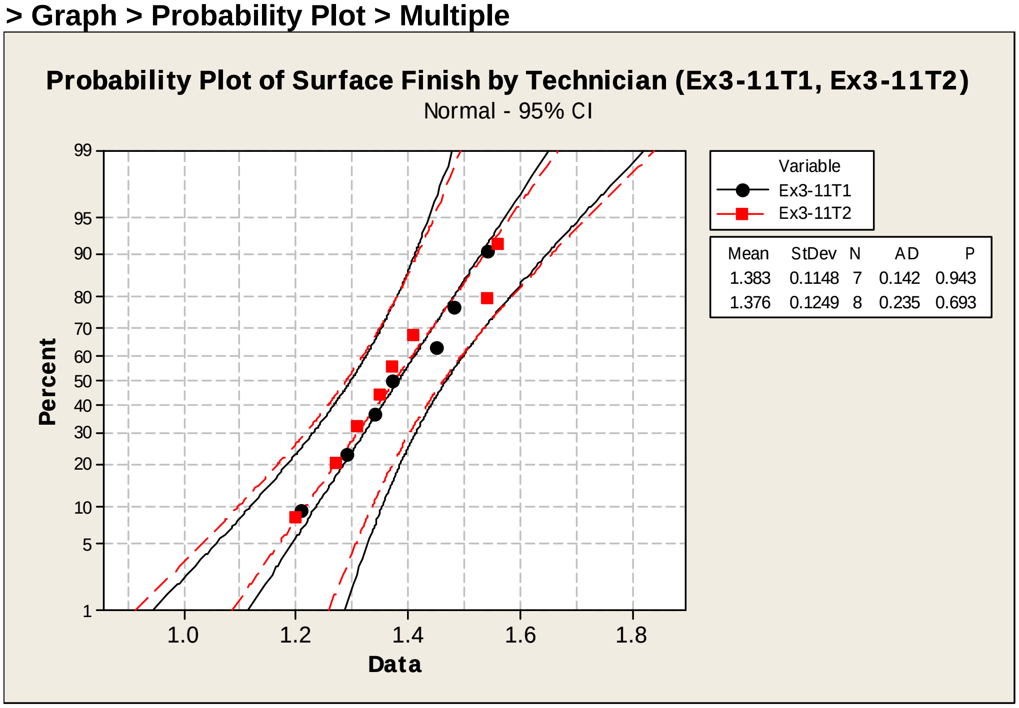
$$\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}$$

$$\frac{(8-1)0.125^2}{16.0128} \le \sigma^2 \le \frac{(8-1)0.125^2}{1.6899}$$

$$0.007 \le \sigma^2 \le 0.065$$

(g)

MTB > Graph > Probability Plot > Multiple



The normality assumption seems reasonable for these readings.

3-12.
From Eqn. 3-54 and 3-55, for $\sigma_1^2 \neq \sigma_2^2$ and both unknown, the test statistic is

$$t_0^* = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S_1^2/n_1 + S_2^2/n_2}} \text{ with degrees of freedom } \nu = \frac{\left(S_1^2/n_1 + S_2^2/n_2\right)^2}{\frac{\left(S_1^2/n_1\right)^2}{(n_1+1)} + \frac{\left(S_2^2/n_2\right)^2}{(n_2+1)}} - 2$$

A 100(1-α)% confidence interval on the difference in means would be:

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,\nu}\sqrt{S_1^2/n_1 + S_2^2/n_2} \le (\mu_1 - \mu_2) \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,\nu}\sqrt{S_1^2/n_1 + S_2^2/n_2}$$

3-13.
Saltwater quench: $n_1 = 10$, $\bar{x}_1 = 147.6$, $S_1 = 4.97$
Oil quench: $n_2 = 10$, $\bar{x}_2 = 149.4$, $S_2 = 5.46$

(a)
Assume $\sigma_1^2 = \sigma_2^2$
MTB > Stat > Basic Statistics > 2-Sample t > Samples in different columns

Two-Sample T-Test and CI: Ex3-13SQ, Ex3-13OQ
```
Two-sample T for Ex3-13SQ vs Ex3-13OQ
           N    Mean  StDev  SE Mean
Ex3-13SQ  10  147.60   4.97      1.6
Ex3-13OQ  10  149.40   5.46      1.7
Difference = mu (Ex3-13SQ) - mu (Ex3-13OQ)
Estimate for difference:  -1.80000
95% CI for difference:  (-6.70615, 3.10615)
T-Test of difference = 0 (vs not =): T-Value = -0.77  P-Value = 0.451  DF = 18
Both use Pooled StDev = 5.2217
```
Do not reject H_0, and conclude that there is no difference between the quenching processes.

(b)
$\alpha = 0.05$, $t_{\alpha/2,\, n1+n2-2} = t_{0.025,\, 18} = 2.1009$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(10-1)4.97^2 + (10-1)5.46^2}{10+10-2}} = 5.22$$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,n_1+n_2-2} S_p\sqrt{1/n_1 + 1/n_2} \le (\mu_1 - \mu_2) \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,n_1+n_2-2} S_p\sqrt{1/n_1 + 1/n_2}$$

$$(147.6 - 149.4) - 2.1009(5.22)\sqrt{1/10 + 1/10} \le (\mu_1 - \mu_2) \le (147.6 - 149.4) + 2.1009(5.22)\sqrt{1/10 + 1/10}$$

$$-6.7 \le (\mu_1 - \mu_2) \le 3.1$$

3-13 continued

(c)

$\alpha = 0.05 \qquad F_{1-\alpha/2,n_2-1,n_1-1} = F_{0.975,9,9} = 0.2484; \;\; F_{\alpha/2,n_2-1,n_1-1} = F_{0.025,9,9} = 4.0260$

$$\frac{S_1^2}{S_2^2} F_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{S_1^2}{S_2^2} F_{\alpha/2,n_2-1,n_1-1}$$

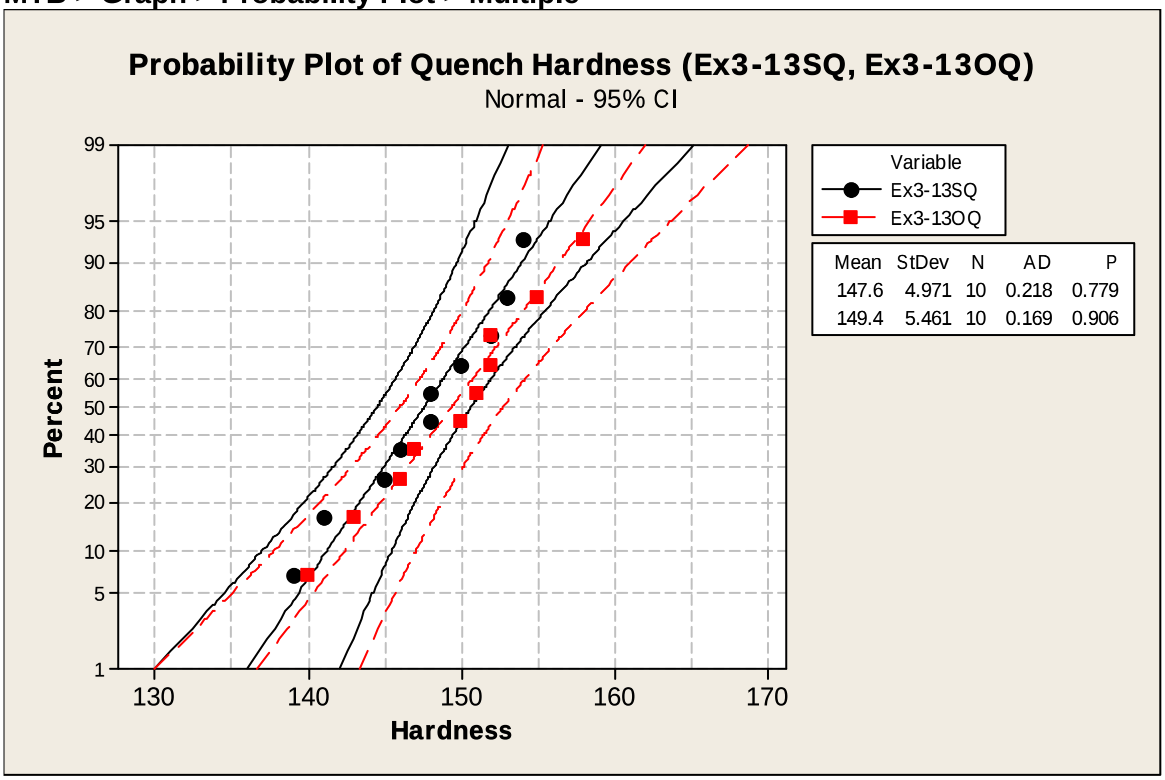
$$\frac{4.97^2}{5.46^2}(0.2484) \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{4.97^2}{5.46^2}(4.0260)$$

$$0.21 \le \frac{\sigma_1^2}{\sigma_2^2} \le 3.34$$

Since the confidence interval includes the ratio of 1, the assumption of equal variances seems reasonable.

(d)

MTB > Graph > Probability Plot > Multiple



The normal distribution assumptions for both the saltwater and oil quench methods seem reasonable.

3-14.
$n = 200$, $x = 18$, $\hat{p} = x/n = 18/200 = 0.09$

(a)
$p_0 = 0.10$, $\alpha = 0.05$. Test H_0: $p = 0.10$ versus H_1: $p \neq 0.10$. Reject H_0 if $|Z_0| > Z_{\alpha/2}$.
$np_0 = 200(0.10) = 20$

Since $(x = 18) < (np_0 = 20)$, use the normal approximation to the binomial for $x < np_0$.

$$Z_0 = \frac{(x+0.5)-np_0}{\sqrt{np_0(1-p_0)}} = \frac{(18+0.5)-20}{\sqrt{20(1-0.10)}} = -0.3536$$

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$

Do not reject H_0, and conclude that the sample process fraction nonconforming does not differ from 0.10.

P-value $= 2[1 - \Phi|Z_0|] = 2[1 - \Phi|-0.3536|] = 2[1 - 0.6382] = 0.7236$

MTB > Stat > Basic Statistics > 1 Proportion > Summarized data

Test and CI for One Proportion

```
Test of p = 0.1 vs p not = 0.1
Sample   X    N  Sample p         95% CI          Z-Value  P-Value
1       18  200  0.090000  (0.050338, 0.129662)    -0.47    0.637
```

Note that MINITAB uses an exact method, not an approximation.

(b)
$\alpha = 0.10$, $Z_{\alpha/2} = Z_{0.10/2} = Z_{0.05} = 1.645$

$$\hat{p} - Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n} \le p \le \hat{p} + Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$$
$$0.09 - 1.645\sqrt{0.09(1-0.09)/200} \le p \le 0.09 + 1.645\sqrt{0.09(1-0.09)/200}$$
$$0.057 \le p \le 0.123$$

3-15.
$n = 500$, $x = 65$, $\hat{p} = x/n = 65/500 = 0.130$

(a)
$p_0 = 0.08$, $\alpha = 0.05$. Test H_0: $p = 0.08$ versus H_1: $p \neq 0.08$. Reject H_0 if $|Z_0| > Z_{\alpha/2}$.
$np_0 = 500(0.08) = 40$
Since $(x = 65) > (np_0 = 40)$, use the normal approximation to the binomial for $x > np_0$.

$$Z_0 = \frac{(x - 0.5) - np_0}{\sqrt{np_0(1 - p_0)}} = \frac{(65 - 0.5) - 40}{\sqrt{40(1 - 0.08)}} = 4.0387$$

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$
Reject H_0, and conclude the sample process fraction nonconforming differs from 0.08.

MTB > Stat > Basic Statistics > 1 Proportion > Summarized data

Test and CI for One Proportion
```
Test of p = 0.08 vs p not = 0.08
Sample   X    N  Sample p          95% CI          Z-Value  P-Value
1       65  500  0.130000  (0.100522, 0.159478)     4.12    0.000
```
Note that MINITAB uses an exact method, not an approximation.

(b)
P-value $= 2[1 - \Phi|Z_0|] = 2[1 - \Phi|4.0387|] = 2[1 - 0.99997] = 0.00006$

(c)
$\alpha = 0.05$, $Z_\alpha = Z_{0.05} = 1.645$

$$p \leq \hat{p} + Z_\alpha \sqrt{\hat{p}(1 - \hat{p})/n}$$

$$p \leq 0.13 + 1.645\sqrt{0.13(1 - 0.13)/500}$$

$$p \leq 0.155$$

3-16.

(a)

$n_1 = 200$, $x_1 = 10$, $\hat{p}_1 = x_1/n_1 = 10/200 = 0.05$

$n_2 = 300$, $x_2 = 20$, $\hat{p}_2 = x_2/n_2 = 20/300 = 0.067$

(b)

Use $\alpha = 0.05$.

Test H_0: $p_1 = p_2$ versus H_1: $p_1 \neq p_2$. Reject H_0 if $Z_0 > Z_{\alpha/2}$ or $Z_0 < -Z_{\alpha/2}$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{10+20}{200+300} = 0.06$$

$$Z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(1/n_1 + 1/n_2\right)}} = \frac{0.05 - 0.067}{\sqrt{0.06(1-0.06)\left(1/200 + 1/300\right)}} = -0.7842$$

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96 \quad -Z_{\alpha/2} = -1.96$

Do not reject H_0. Conclude there is no strong evidence to indicate a difference between the fraction nonconforming for the two processes.

MTB > Stat > Basic Statistics > 2 Proportions > Summarized data

Test and CI for Two Proportions

```
Sample   X    N  Sample p
1       10  200  0.050000
2       20  300  0.066667
Difference = p (1) - p (2)
Estimate for difference:  -0.0166667
95% CI for difference:  (-0.0580079, 0.0246745)
Test for difference = 0 (vs not = 0):  Z = -0.77  P-Value = 0.442
```

(c)

$$(\hat{p}_1 - \hat{p}_2) - Z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \le (p_1 - p_2) \le (\hat{p}_1 - \hat{p}_2) + Z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$(0.050 - 0.067) - 1.645\sqrt{\frac{0.05(1-0.05)}{200} + \frac{0.067(1-0.067)}{300}} \le (p_1 - p_2) \le (0.05 - 0.067) + 1.645\sqrt{\frac{0.05(1-0.05)}{200} + \frac{0.067(1-0.067)}{300}}$$

$$-0.052 \le (p_1 - p_2) \le 0.018$$

3-17.*

before: $n_1 = 10$, $x_1 = 9.85$, $S_1^2 = 6.79$

after: $n_2 = 8$, $x_2 = 8.08$, $S_2^2 = 6.18$

(a)

Test $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_1 : \sigma_1^2 \neq \sigma_2^2$, at $\alpha = 0.05$

Reject H_0 if $F_0 > F_{\alpha/2, n_1-1, n_2-2}$ or $F_0 < F_{1-\alpha/2, n_1-1, n_2-1}$

$F_{\alpha/2, n_1-1, n_2-2} = F_{0.025,9,7} = 4.8232$; $F_{1-\alpha/2, n_1-1, n_2-1} = F_{0.975,9,7} = 0.2383$

$F_0 = S_1^2 / S_2^2 = 6.79/6.18 = 1.0987$

$F_0 = 1.0987 < 4.8232$ and > 0.2383, so do not reject H_0

MTB > Stat > Basic Statistics > 2 Variances > Summarized data

```
Test for Equal Variances
95% Bonferroni confidence intervals for standard deviations
Sample   N    Lower    StDev    Upper
     1  10  1.70449  2.60576  5.24710
     2   8  1.55525  2.48596  5.69405
F-Test (normal distribution)
Test statistic = 1.10, p-value = 0.922
```

The impurity variances before and after installation are the same.

(b)

Test H_0: $\mu_1 = \mu_2$ versus H_1: $\mu_1 > \mu_2$, $\alpha = 0.05$.

Reject H_0 if $t_0 > t_{\alpha, n1+n2-2}$.

$t_{\alpha, n1+n2-2} = t_{0.05,\ 10+8-2} = 1.746$

$$S_P = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(10-1)6.79 + (8-1)6.18}{10+8-2}} = 2.554$$

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{S_P\sqrt{1/n_1 + 1/n_2}} = \frac{9.85-8.08}{2.554\sqrt{1/10+1/8}} = 1.461$$

MTB > Stat > Basic Statistics > 2-Sample t > Summarized data

```
Two-Sample T-Test and CI
Sample   N  Mean  StDev  SE Mean
1       10  9.85   2.61     0.83
2        8  8.08   2.49     0.88
Difference = mu (1) - mu (2)
Estimate for difference:  1.77000
95% lower bound for difference:  -0.34856
T-Test of difference = 0 (vs >): T-Value = 1.46  P-Value = 0.082  DF = 16
Both use Pooled StDev = 2.5582
```

The mean impurity after installation of the new purification unit is not less than before.

3-18.
$n_1 = 16$, $\bar{x}_1 = 175.8$ psi, $n_2 = 16$, $\bar{x}_2 = 181.3$ psi, $\sigma_1 = \sigma_2 = 3.0$ psi

Want to demonstrate that μ_2 is greater than μ_1 by at least 5 psi, so H_1: $\mu_1 + 5 < \mu_2$. So test a difference $\Delta_0 = -5$, test H_0: $\mu_1 - \mu_2 = -5$ versus H_1: $\mu_1 - \mu_2 < -5$.

Reject H_0 if $Z_0 < -Z_\alpha$.

$\Delta_0 = -5$ $\qquad -Z_\alpha = -Z_{0.05} = -1.645$

$$Z_0 = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \Delta_0}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} = \frac{(175.8 - 181.3) - (-5)}{\sqrt{3^2/16 + 3^2/16}} = -0.4714$$

$(Z_0 = -0.4714) > -1.645$, so do not reject H_0.

The mean strength of Design 2 does not exceed Design 1 by 5 psi.

P-value $= \Phi(Z_0) = \Phi(-0.4714) = 0.3187$

MTB > Stat > Basic Statistics > 2-Sample t > Summarized data

Two-Sample T-Test and CI

```
Sample   N    Mean  StDev  SE Mean
1       16  175.80   3.00     0.75
2       16  181.30   3.00     0.75
Difference = mu (1) - mu (2)
Estimate for difference:  -5.50000
95% upper bound for difference:  -3.69978
T-Test of difference = -5 (vs <): T-Value = -0.47  P-Value = 0.320  DF = 30
Both use Pooled StDev = 3.0000
```

Note: For equal variances and sample sizes, the Z-value is the same as the t-value. The P-values are close due to the sample sizes.

3-19.
Test H_0: $\mu_d = 0$ versus H_1: $\mu_d \neq 0$. Reject H_0 if $|t_0| > t_{\alpha/2,\, n1 + n2 - 2}$.

$t_{\alpha/2,\, n1 + n2 - 2} = t_{0.005,22} = 2.8188$

$$\bar{d} = 1/n \sum_{j=1}^{n} \left(x_{\text{Micrometer}, j} - x_{\text{Vernier}, j} \right) = 1/12 \left[(0.150 - 0.151) + \cdots + (0.151 - 0.152) \right] = -0.000417$$

$$S_d^2 = \frac{\sum_{j=1}^{n} d_j^2 - \left(\sum_{j=1}^{n} d_j \right)^2 \Big/ n}{(n-1)} = 0.001311^2$$

$$t_0 = \bar{d} \Big/ \left(S_d / \sqrt{n} \right) = -0.000417 \Big/ \left(0.001311 / \sqrt{12} \right) = -1.10$$

($|t_0| = 1.10$) < 2.8188, so do not reject H_0. There is no strong evidence to indicate that the two calipers differ in their mean measurements.

MTB > Stat > Basic Statistics > Paired t > Samples in Columns

Paired T-Test and CI: Ex3-19MC, Ex3-19VC

```
Paired T for Ex3-19MC - Ex3-19VC
             N       Mean     StDev   SE Mean
Ex3-19MC    12   0.151167  0.000835  0.000241
Ex3-19VC    12   0.151583  0.001621  0.000468
Difference  12  -0.000417  0.001311  0.000379
95% CI for mean difference: (-0.001250, 0.000417)
T-Test of mean difference = 0 (vs not = 0): T-Value = -1.10  P-Value = 0.295
```

3-20.

(a)

The alternative hypothesis H_1: μ > 150 is preferable to H_1: μ < 150 we desire a true mean weld strength greater than 150 psi. In order to achieve this result, H_0 must be rejected in favor of the alternative H_1, μ > 150.

(b)

$n = 20$, $\bar{x} = 153.7$, $s = 11.5$, $\alpha = 0.05$

Test H_0: $\mu = 150$ versus H_1: $\mu > 150$. Reject H_0 if $t_0 > t_{\alpha,\, n-1}$. $t_{\alpha,\, n-1} = t_{0.05,19} = 1.7291$.

$$t_0 = \left(\bar{x} - \mu\right) \Big/ \left(S/\sqrt{n}\right) = \left(153.7 - 150\right) \Big/ \left(11.5/\sqrt{20}\right) = 1.4389$$

($t_0 = 1.4389$) < 1.7291, so do not reject H_0. There is insufficient evidence to indicate that the mean strength is greater than 150 psi.

MTB > Stat > Basic Statistics > 1-Sample t > Summarized data

One-Sample T

```
Test of mu = 150 vs > 150
                                 95%
                               Lower
 N     Mean   StDev  SE Mean    Bound     T      P
20  153.700  11.500    2.571  149.254  1.44  0.083
```

3-21.

$n = 20$, $\bar{x} = 752.6$ ml, $s = 1.5$, $\alpha = 0.05$

(a)

Test H_0: $\sigma^2 = 1$ versus H_1: $\sigma^2 < 1$. Reject H_0 if $\chi^2_0 < \chi^2_{1-\alpha,\, n-1}$.

$\chi^2_{1-\alpha,\, n-1} = \chi^2_{0.95,19} = 10.1170$

$$\chi_0^2 = \left[(n-1)S^2\right] \Big/ \sigma_0^2 = \left[(20-1)1.5^2\right] \Big/ 1 = 42.75$$

$\chi^2_0 = 42.75 > 10.1170$, so do not reject H_0. The standard deviation of the fill volume is not less than 1ml.

(b)

$\chi^2_{\alpha/2,\, n-1} = \chi^2_{0.025,19} = 32.85$. $\chi^2_{1-\alpha/2,\, n-1} = \chi^2_{0.975,19} = 8.91$.

$$(n-1)S^2 \Big/ \chi^2_{\alpha/2,n-1} \le \sigma^2 \le (n-1)S^2 \Big/ \chi^2_{1-\alpha/2,n-1}$$

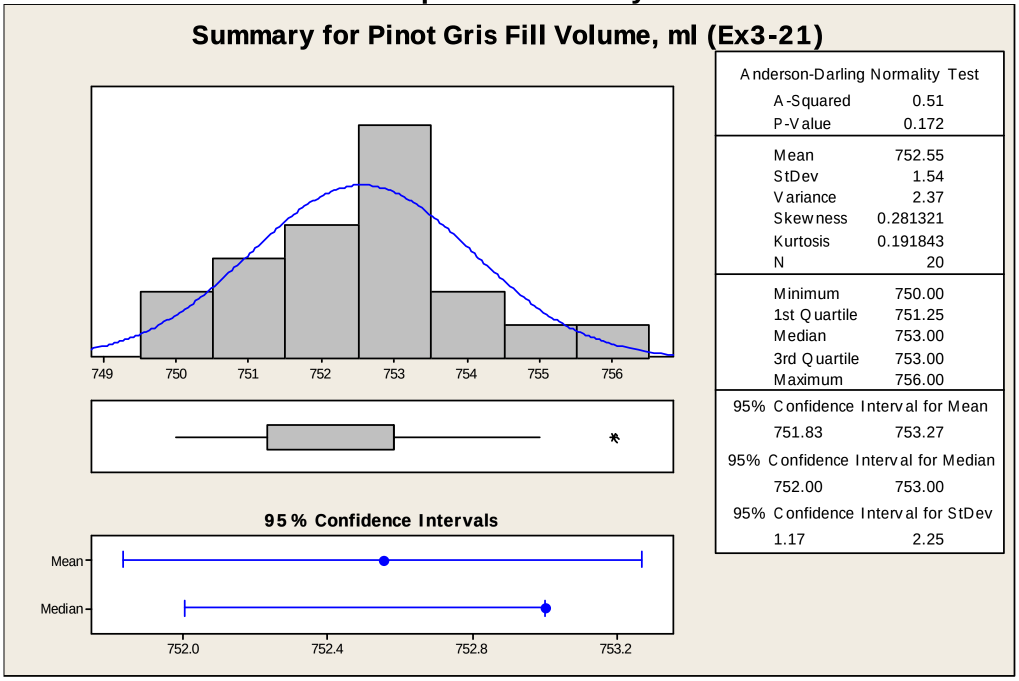
$$(20-1)1.5^2 / 32.85 \le \sigma^2 \le (20-1)1.5^2 / 8.91$$

$$1.30 \le \sigma^2 \le 4.80$$

$$1.14 \le \sigma \le 2.19$$

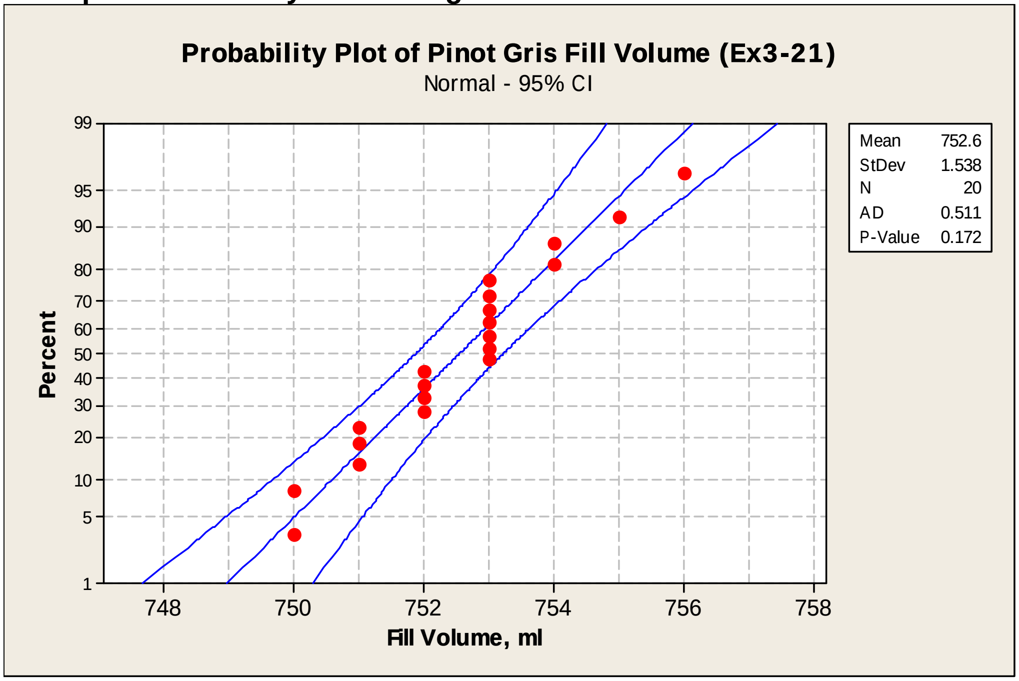
3-21 (b) continued

MTB > Stat > Basic Statistics > Graphical Summary



(c)

MTB > Graph > Probability Plot > SIngle



The plotted points do not fall approximately along a straight line, so the assumption that battery life is normally distributed is not appropriate.

3-22.

$\mu_0 = 15$, $\sigma^2 = 9.0$, $\mu_1 = 20$, $\alpha = 0.05$. Test H_0: $\mu = 15$ versus H_1: $\mu \neq 15$.
What n is needed such that the Type II error, β, is less than or equal to 0.10?

$$\delta = \mu_1 - \mu_2 = 20 - 15 = 5 \qquad d = |\delta|/\sigma = 5/\sqrt{9} = 1.6667$$

From Figure 3-7, the operating characteristic curve for two-sided at $\alpha = 0.05$, $n = 4$.
Check:

$$\beta = \Phi\left(Z_{\alpha/2} - \delta\sqrt{n}/\sigma\right) - \Phi\left(-Z_{\alpha/2} - \delta\sqrt{n}/\sigma\right) = \Phi\left(1.96 - 5\sqrt{4}/3\right) - \Phi\left(-1.96 - 5\sqrt{4}/3\right)$$

$$= \Phi(-1.3733) - \Phi(-5.2933) = 0.0848 - 0.0000 = 0.0848$$

MTB > Stat > Power and Sample Size > 1-Sample Z

Power and Sample Size

```
1-Sample Z Test
Testing mean = null (versus not = null)
Calculating power for mean = null + difference
Alpha = 0.05  Assumed standard deviation = 3
             Sample  Target
Difference     Size   Power  Actual Power
         5        4     0.9      0.915181
```

3-23.

Let $\mu_1 = \mu_0 + \delta$. From Eqn. 3-46, $\beta = \Phi\left(Z_{\alpha/2} - \delta\sqrt{n}/\sigma\right) - \Phi\left(-Z_{\alpha/2} - \delta\sqrt{n}/\sigma\right)$

If $\delta > 0$, then $\Phi\left(-Z_{\alpha/2} - \delta\sqrt{n}/\sigma\right)$ is likely to be small compared with β. So,

$$\beta \approx \Phi\left(Z_{\alpha/2} - \delta\sqrt{n}/\sigma\right)$$

$$\Phi(\beta) \approx \Phi^{-1}\left(Z_{\alpha/2} - \delta\sqrt{n}/\sigma\right)$$

$$-Z_\beta \approx Z_{\alpha/2} - \delta\sqrt{n}/\sigma$$

$$n \approx \left[(Z_{\alpha/2} + Z_\beta)\sigma/\delta\right]^2$$

3-24.

Maximize: $Z_0 = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$ Subject to: $n_1 + n_2 = N$.

Since $(\bar{x}_1 - \bar{x}_2)$ is fixed, an equivalent statement is

Minimize: $L = \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2} = \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{N - n_1}$

$$\frac{dL}{dn_1}\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{N - n_1}\right) = \frac{dL}{dn_1}\left[n_1^{-1}\sigma_1^2 + \left(N - n_1\right)^{-1}\sigma_2^2\right]$$

$$= -1n_1^{-2}\sigma_1^2 + (-1)(-1)\left(N - n_1\right)^{-2}\sigma_2^2 = 0$$

$$= -\frac{\sigma_1^2}{n_1^2} + \frac{\sigma_2^2}{\left(N - n_1\right)^2} = 0$$

$$\frac{n_1}{n_2} = \frac{\sigma_1}{\sigma_2}$$

Allocate N between n_1 and n_2 according to the ratio of the standard deviations.

3-25.

Given $x \sim N,\ n_1,\ \bar{x}_1,\ n_2,\ \bar{x}_2,\ x_1$ independent of x_2 .

Assume $\mu_1 = 2\mu_2$ and let $Q = (\bar{x}_1 - \bar{x}_2)$.

$$E(Q) = E(\bar{x}_1 - 2\bar{x}_2) = \mu_1 - 2\mu_2 = 0$$

$$\text{var}(Q) = \text{var}(\bar{x}_1 - 2\bar{x}_2) = \text{var}(\bar{x}_1) + \text{var}(2\bar{x}_2) = \text{var}(\bar{x}_1) + 2^2\ \text{var}(\bar{x}_2) = \frac{\text{var}(x_1)}{n_1} + 4\frac{\text{var}(x_2)}{n_2}$$

$$Z_0 = \frac{Q - 0}{SD(Q)} = \frac{\bar{x}_1 - 2\bar{x}_2}{\sqrt{\sigma_1^2/n_1 + 4\sigma_2^2/n_2}}$$

And, reject H_0 if $\left|Z_0\right| > Z_{\alpha/2}$

3-26.
(a)
Wish to test H_0: $\lambda = \lambda_0$ versus H_1: $\lambda \neq \lambda_0$.

Select random sample of n observations $x_1, x_2, \ldots, x_n$. Each $x_i \sim$ POI(λ). $\sum_{i=1}^{n} x_i \sim \text{POI}(n\lambda)$.

Using the normal approximation to the Poisson, if n is large, $\bar{x} = x/n = \sim$ N(λ, λ/n).
$Z_0 = (\bar{x} - \lambda) \big/ \sqrt{\lambda_0 / n}$. Reject H_0: $\lambda = \lambda_0$ if $|Z_0| > Z_{\alpha/2}$

(b)
$x \sim$ Poi(λ), $n = 100$, $x = 11$, $\bar{x} = x/N = 11/100 = 0.110$
Test H_0: $\lambda = 0.15$ versus H_1: $\lambda \neq 0.15$, at $\alpha = 0.01$. Reject H_0 if $|Z_0| > Z_{\alpha/2}$.
$Z_{\alpha/2} = Z_{0.005} = 2.5758$
$Z_0 = (\bar{x} - \lambda_0) \big/ \sqrt{\lambda_0/n} = (0.110 - 0.15) \big/ \sqrt{0.15/100} = -1.0328$
($|Z_0| = 1.0328$) < 2.5758, so do not reject H_0.

3-27.
$x \sim$ Poi(λ), $n = 5$, $x = 3$, $\bar{x} = x/N = 3/5 = 0.6$
Test H_0: $\lambda = 0.5$ versus H_1: $\lambda > 0.5$, at $\alpha = 0.05$. Reject H_0 if $Z_0 > Z_\alpha$.
$Z_\alpha = Z_{0.05} = 1.645$
$Z_0 = (\bar{x} - \lambda_0) \big/ \sqrt{\lambda_0/n} = (0.6 - 0.5) \big/ \sqrt{0.5/5} = 0.3162$
($Z_0 = 0.3162$) < 1.645, so do not reject H_0.

3-28.
$x \sim$ Poi(λ), $n = 1000$, $x = 688$, $\bar{x} = x/N = 688/1000 = 0.688$
Test H_0: $\lambda = 1$ versus H_1: $\lambda \neq 1$, at $\alpha = 0.05$. Reject H_0 if $|Z_0| > Z_\alpha$.
$Z_{\alpha/2} = Z_{0.025} = 1.96$
$Z_0 = (\bar{x} - \lambda_0) \big/ \sqrt{\lambda_0/n} = (0.688 - 1) \big/ \sqrt{1/1000} = -9.8663$
($|Z_0| = 9.8663$) > 1.96, so reject H_0.

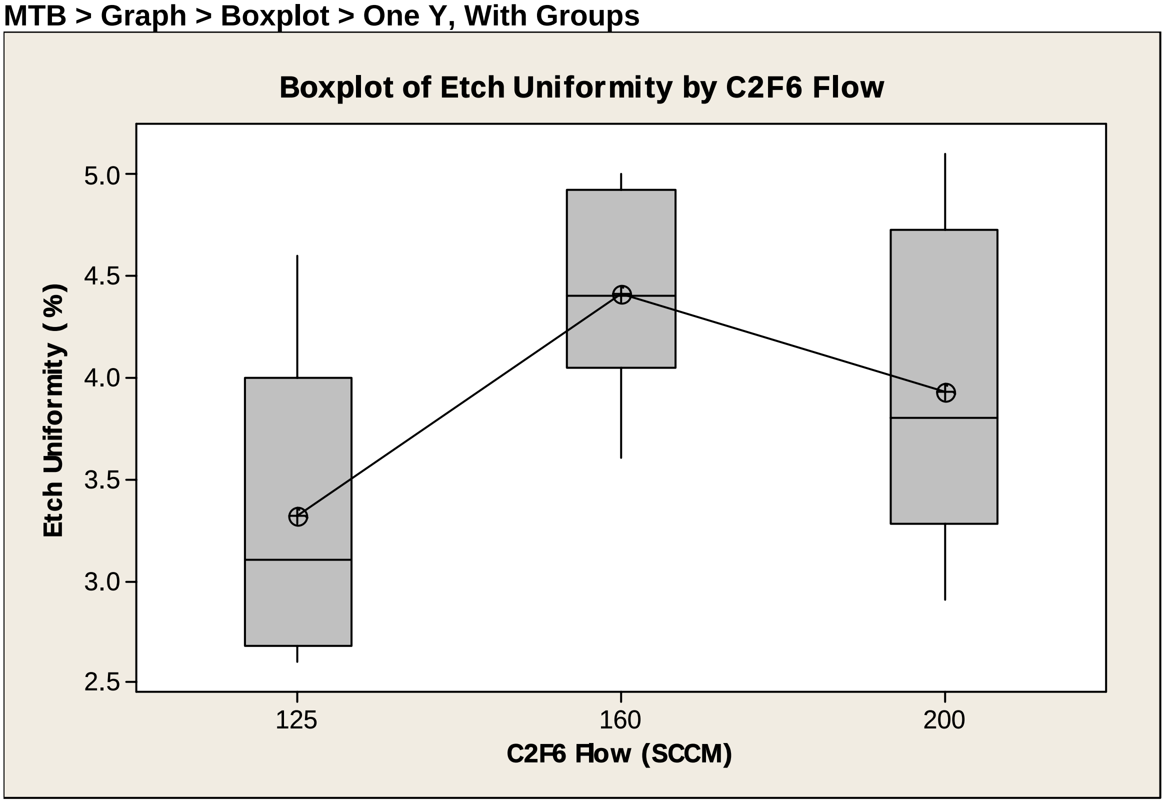
3-29.
(a)
MTB > Stat > ANOVA > One-Way

One-way ANOVA: Ex3-29Obs versus Ex3-29Flow

```
Source      DF      SS     MS     F      P
Ex3-29Flow   2   3.648  1.824  3.59  0.053
Error       15   7.630  0.509
Total       17  11.278
S = 0.7132   R-Sq = 32.34%   R-Sq(adj) = 23.32%
                          Individual 95% CIs For Mean Based on
                          Pooled StDev
Level  N    Mean   StDev  -----+---------+---------+---------+----
125    6  3.3167  0.7600  (---------*----------)
160    6  4.4167  0.5231                    (----------*---------)
200    6  3.9333  0.8214            (----------*---------)
                          -----+---------+---------+---------+----
                             3.00      3.60      4.20      4.80
Pooled StDev = 0.7132
```

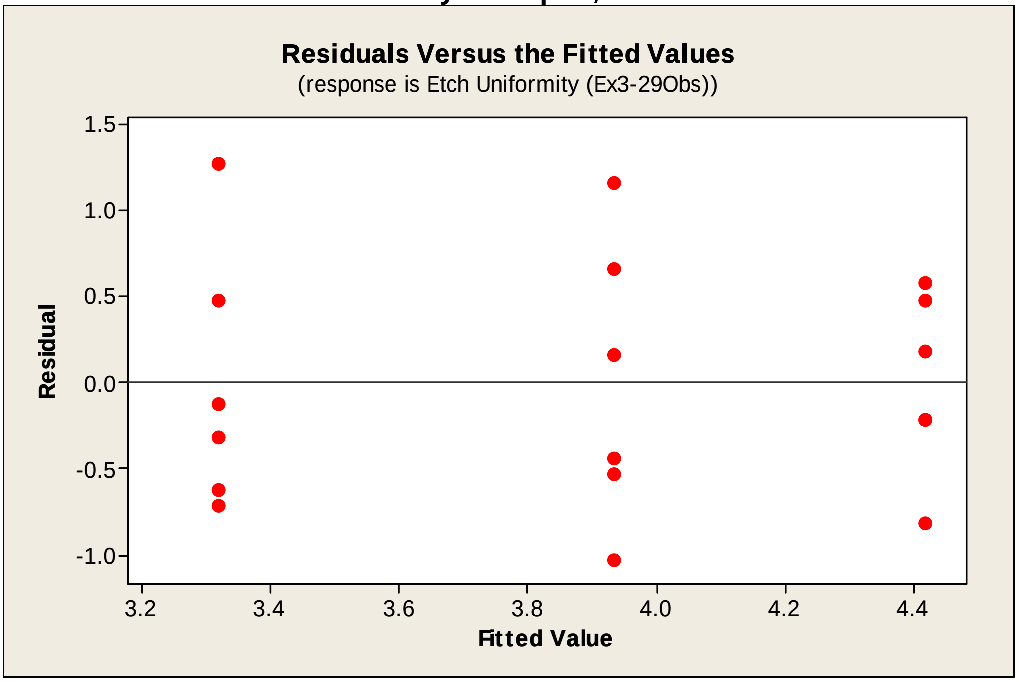
($F_{0.05,2,15}$ = 3.6823) > (F_0 = 3.59), so flow rate does not affect etch uniformity at a significance level α = 0.05. However, the *P*-value is just slightly greater than 0.05, so there is some evidence that gas flow rate affects the etch uniformity.

(b)
MTB > Stat > ANOVA > One-Way > Graphs, Boxplots of data
MTB > Graph > Boxplot > One Y, With Groups



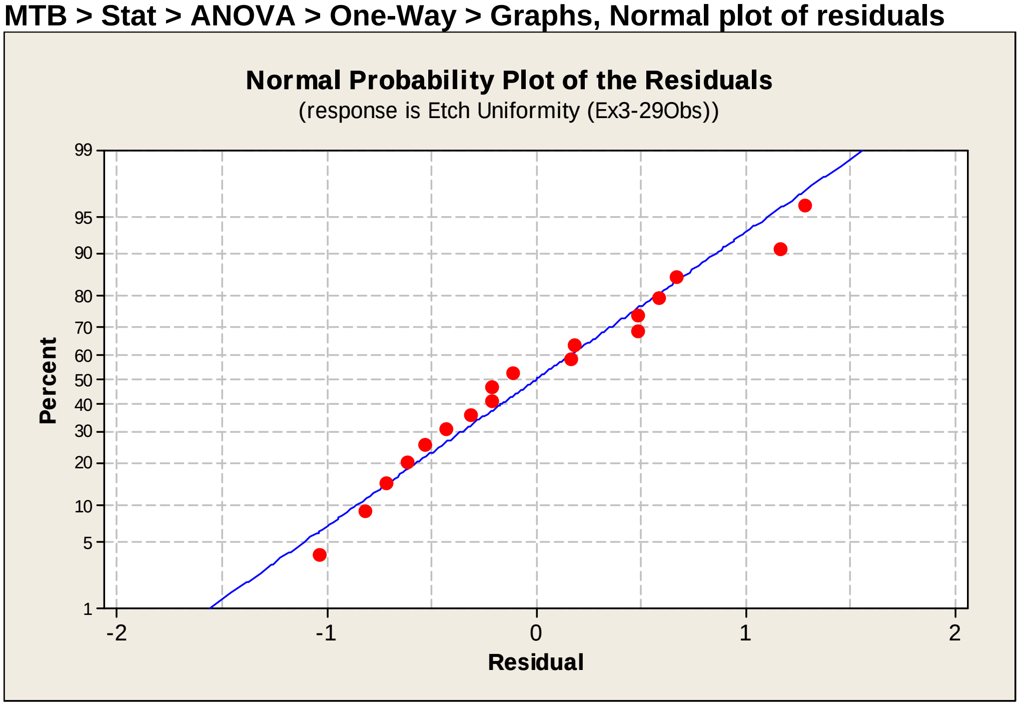
Gas flow rate of 125 SCCM gives smallest mean percentage uniformity.

3-29 continued
(c)
MTB > Stat > ANOVA > One-Way > Graphs, Residuals versus fits



Residuals are satisfactory.

(d)
MTB > Stat > ANOVA > One-Way > Graphs, Normal plot of residuals

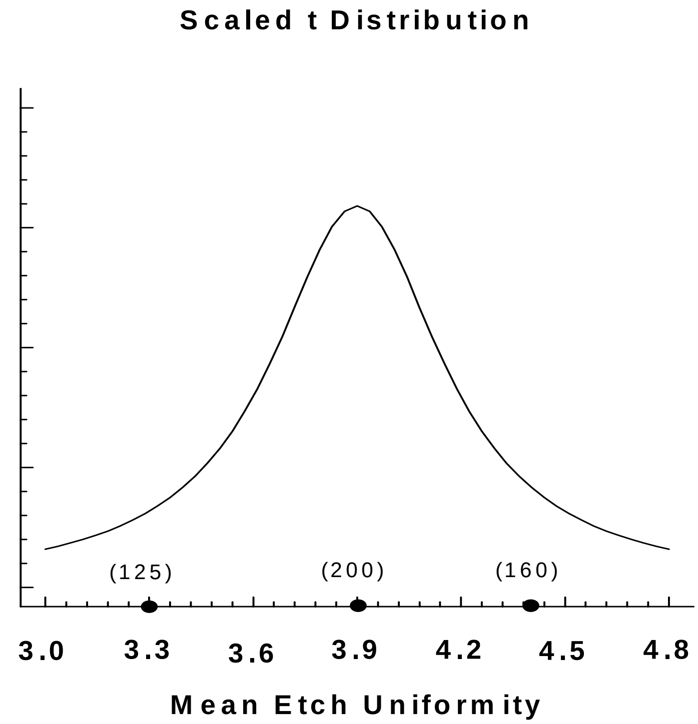


The normality assumption is reasonable.

3-30.

Flow Rate	Mean Etch Uniformity
125	3.3%
160	4.4%
200	3.9%

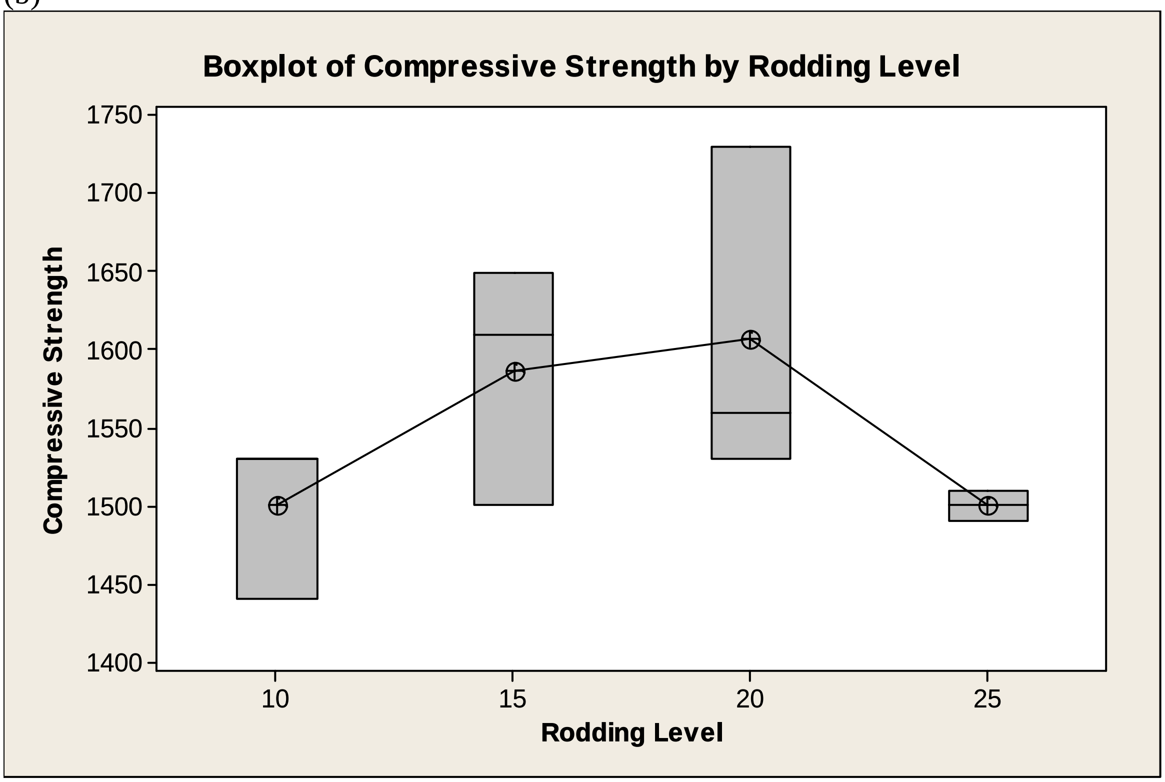
scale factor $= \sqrt{MS_E / n} = \sqrt{0.5087/6} = 0.3$



The graph does not indicate a large difference between the mean etch uniformity of the three different flow rates. The statistically significant difference between the mean uniformities can be seen by centering the t distribution between, say, 125 and 200, and noting that 160 would fall beyond the tail of the curve.

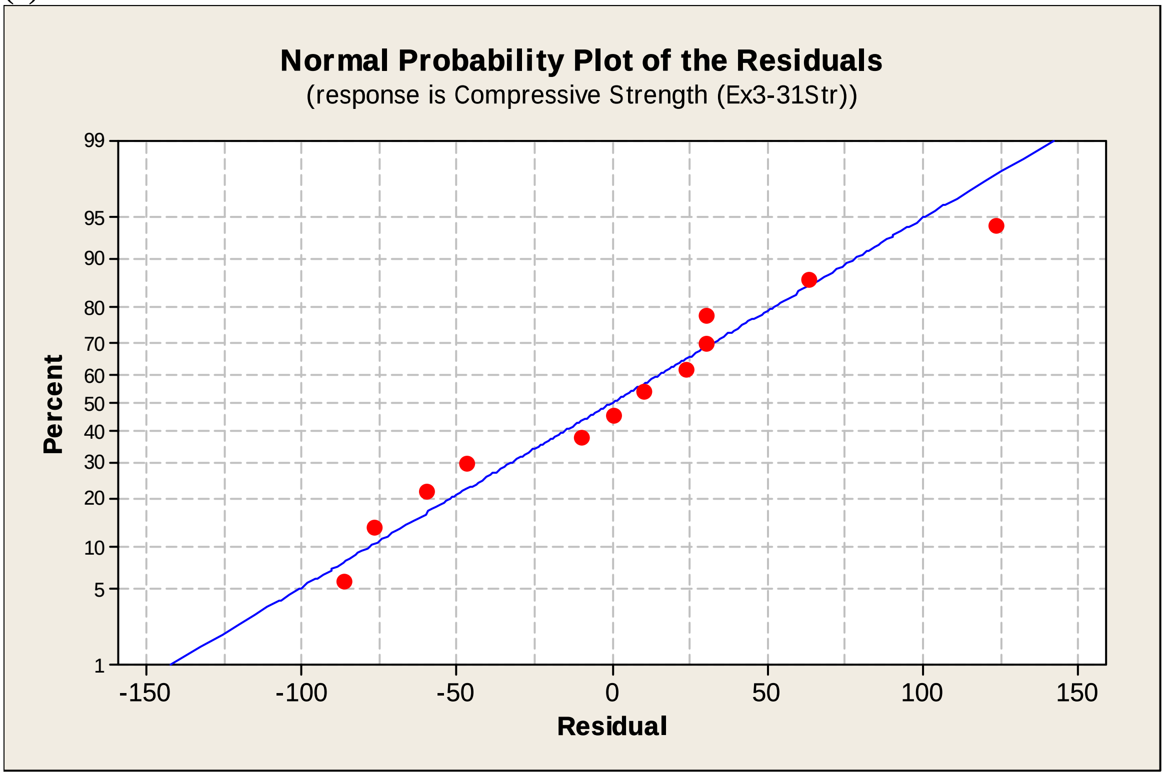
3-31.
(a)
MTB > Stat > ANOVA > One-Way > Graphs> Boxplots of data, Normal plot of residuals

One-way ANOVA: Ex3-31Str versus Ex3-31Rod
```
Source     DF     SS    MS     F      P
Ex3-31Rod   3  28633  9544  1.87  0.214
Error       8  40933  5117
Total      11  69567
S = 71.53   R-Sq = 41.16%   R-Sq(adj) = 19.09%
                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level  N    Mean  StDev  ----+---------+---------+---------+-----
10     3  1500.0   52.0  (-----------*----------)
15     3  1586.7   77.7            (-----------*-----------)
20     3  1606.7  107.9               (-----------*-----------)
25     3  1500.0   10.0  (-----------*----------)
                         ----+---------+---------+---------+-----
                          1440      1520      1600      1680
Pooled StDev = 71.5
```
No difference due to rodding level at $\alpha = 0.05$.

(b)


Level 25 exhibits considerably less variability than the other three levels.

3-31 continued
(c)

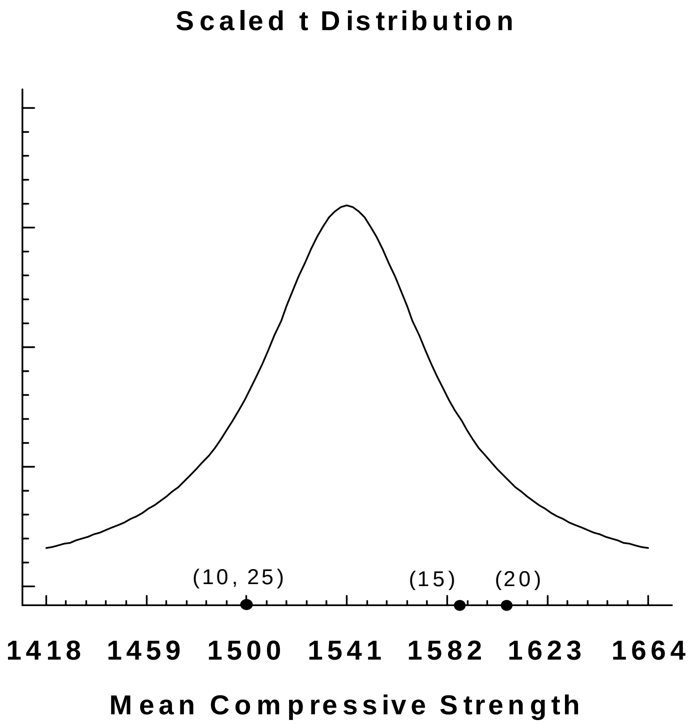


The normal distribution assumption for compressive strength is reasonable.

3-32.

Rodding Level	Mean Compressive Strength
10	1500
15	1587
20	1607
25	1500

scale factor = $\sqrt{MS_E/n} = \sqrt{5117/3} = 41$



There is no difference due to rodding level.

3-33.

(a)

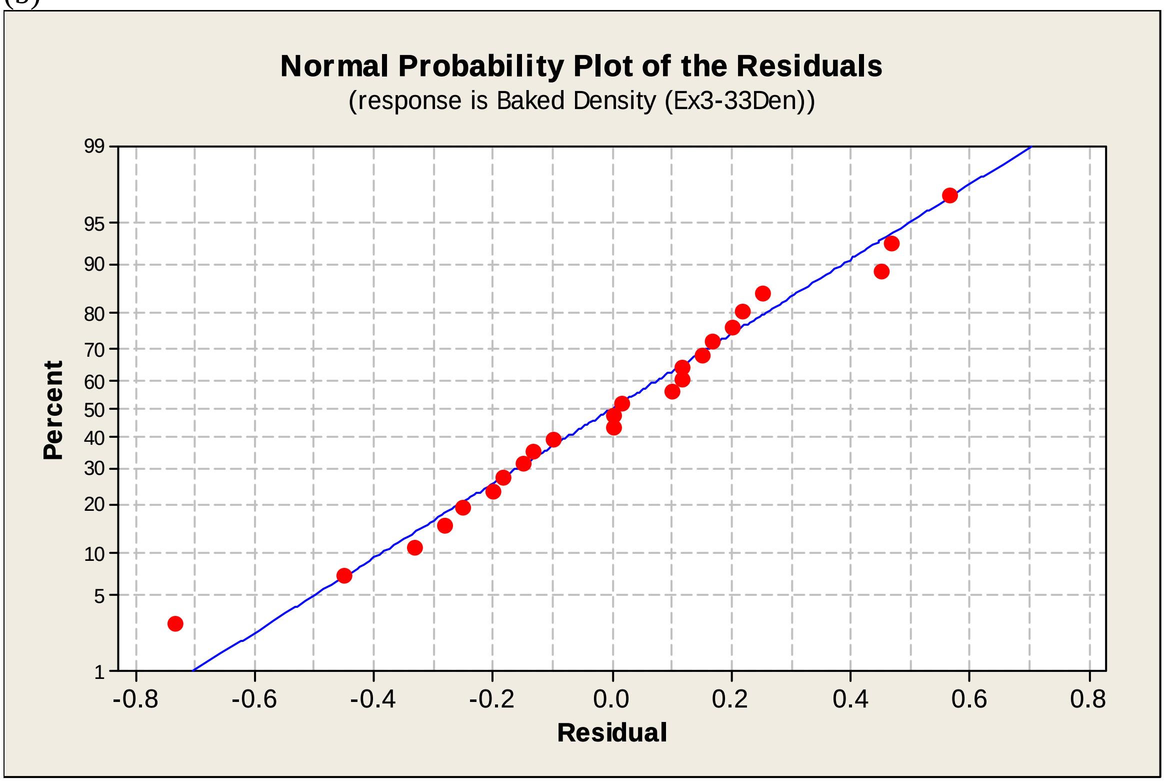
MTB > Stat > ANOVA > One-Way > Graphs> Boxplots of data, Normal plot of residuals

One-way ANOVA: Ex3-33Den versus Ex3-33T

```
Source   DF     SS     MS     F      P
Ex3-33T   3  0.457  0.152  1.45  0.258
Error    20  2.097  0.105
Total    23  2.553
S = 0.3238   R-Sq = 17.89%   R-Sq(adj) = 5.57%
                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level  N    Mean  StDev  --------+---------+---------+---------+-
500    6  41.700  0.141                 (----------*----------)
525    6  41.583  0.194            (----------*----------)
550    6  41.450  0.339       (----------*----------)
575    6  41.333  0.497  (----------*----------)
                         --------+---------+---------+---------+-
                              41.25     41.50     41.75     42.00
Pooled StDev = 0.324
```

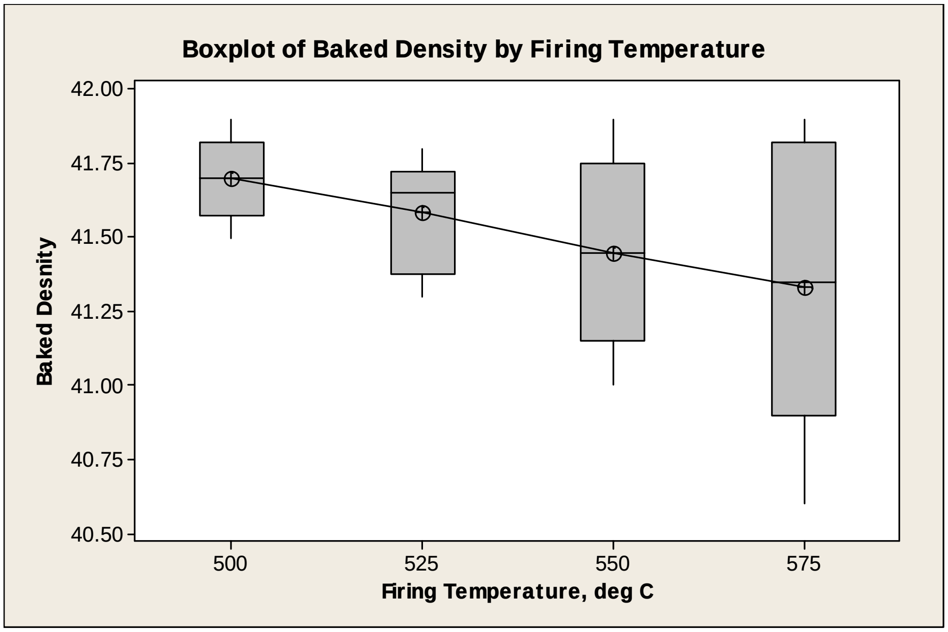
Temperature level does not significantly affect mean baked anode density.

(b)



Normality assumption is reasonable.

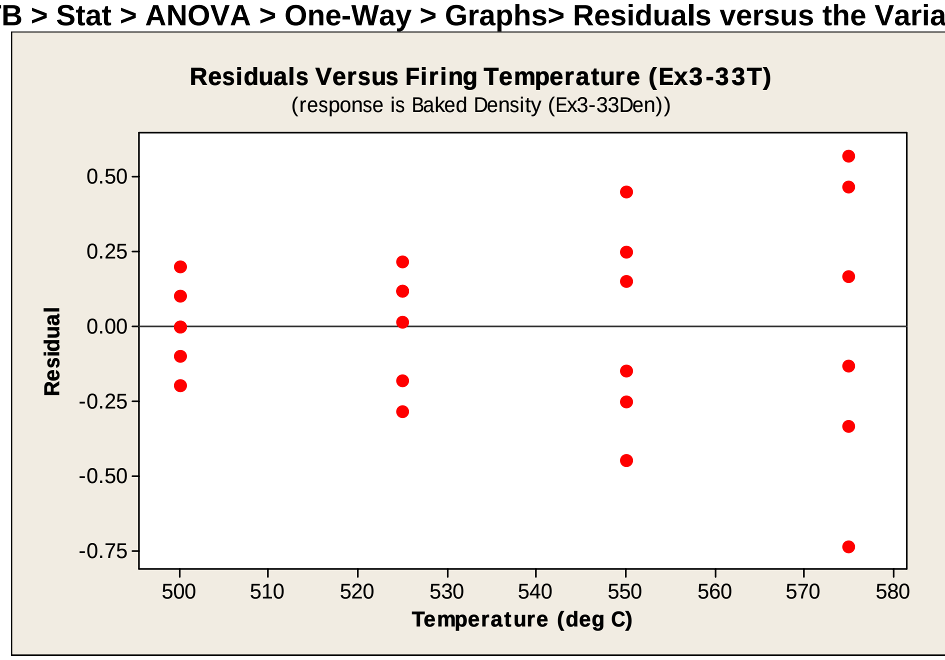
3-33 continued
(c)



Since statistically there is no evidence to indicate that the means are different, select the temperature with the smallest variance, 500°C (see Boxplot), which probably also incurs the smallest cost (lowest temperature).

3-34.

MTB > Stat > ANOVA > One-Way > Graphs> Residuals versus the Variables



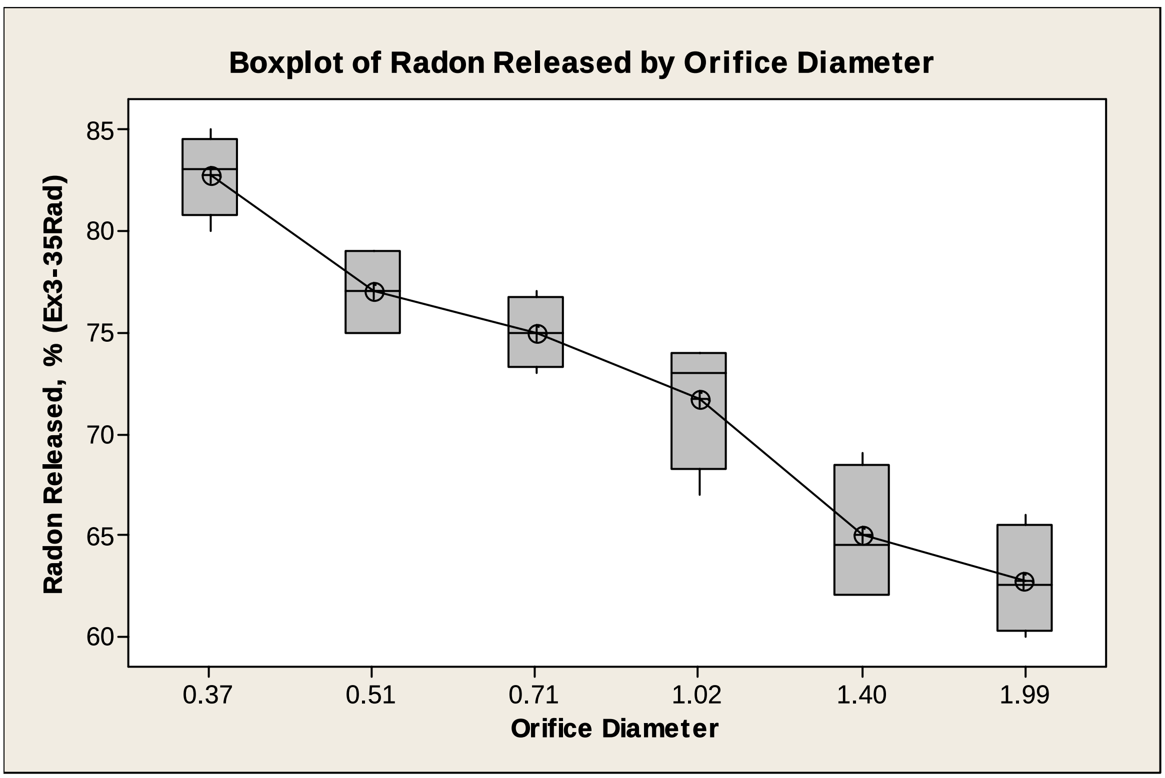
As firing temperature increases, so does variability. More uniform anodes are produced at lower temperatures. Recommend 500°C for smallest variability.

3-35.
(a)
MTB > Stat > ANOVA > One-Way > Graphs> Boxplots of data

One-way ANOVA: Ex3-35Rad versus Ex3-35Dia

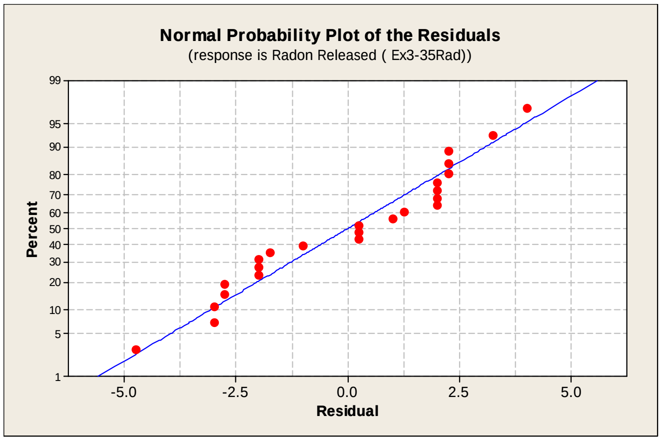
```
Source     DF       SS      MS      F      P
Ex3-35Dia   5  1133.38  226.68  30.85  0.000
Error      18   132.25    7.35
Total      23  1265.63
S = 2.711   R-Sq = 89.55%   R-Sq(adj) = 86.65%
                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level  N    Mean  StDev  ----+---------+---------+---------+-----
0.37   4  82.750  2.062                              (---*---)
0.51   4  77.000  2.309                      (---*---)
0.71   4  75.000  1.826                   (---*---)
1.02   4  71.750  3.304              (----*---)
1.40   4  65.000  3.559     (---*---)
1.99   4  62.750  2.754  (---*---)
                         ----+---------+---------+---------+-----
                          63.0      70.0      77.0      84.0
Pooled StDev = 2.711
```

Orifice size does affect mean % radon release, at $\alpha = 0.05$.

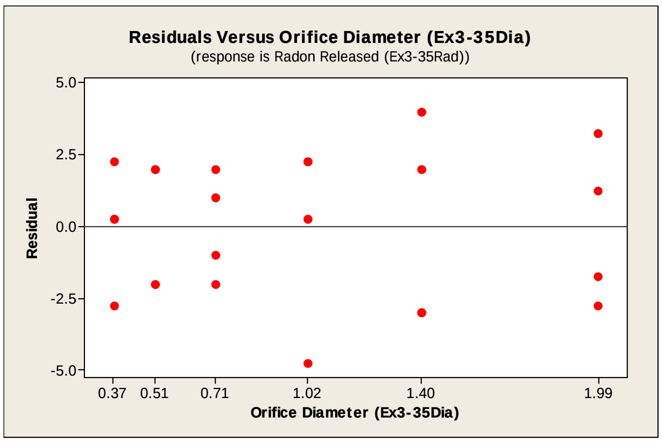


Smallest % radon released at 1.99 and 1.4 orifice diameters.

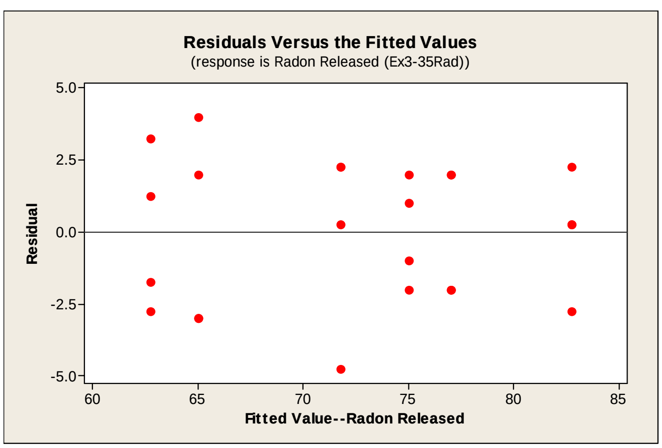
3-35 continued
(b)
MTB > Stat > ANOVA > One-Way > Graphs> Normal plot of residuals, Residuals versus fits, Residuals versus the Variables



Residuals violate the normality distribution.



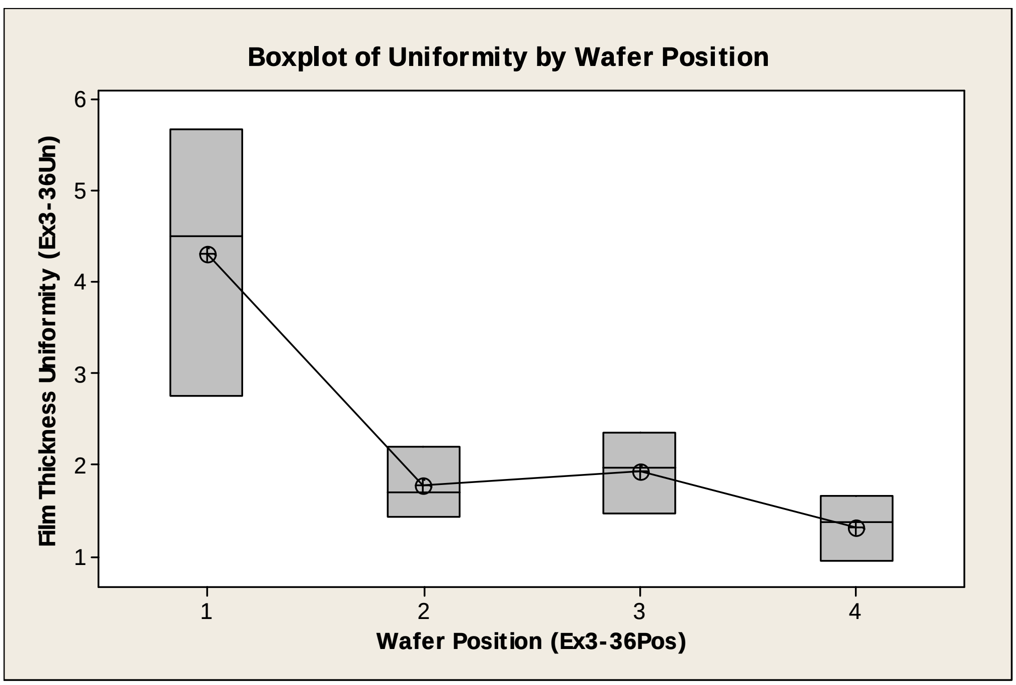
The assumption of equal variance at each factor level appears to be violated, with larger variances at the larger diameters (1.02, 1.40, 1.99).



Variability in residuals does not appear to depend on the magnitude of predicted (or fitted) values.

3-36.
(a)
MTB > Stat > ANOVA > One-Way > Graphs, Boxplots of data

One-way ANOVA: Ex3-36Un versus Ex3-36Pos
```
Source     DF      SS     MS     F      P
Ex3-36Pos   3  16.220  5.407  8.29  0.008
Error       8   5.217  0.652
Total      11  21.437
S = 0.8076   R-Sq = 75.66%   R-Sq(adj) = 66.53%
                          Individual 95% CIs For Mean Based on
                          Pooled StDev
Level  N    Mean   StDev  --------+---------+---------+---------+-
1      3  4.3067  1.4636                      (------*------)
2      3  1.7733  0.3853     (------*------)
3      3  1.9267  0.4366      (------*------)
4      3  1.3167  0.3570  (------*------)
                          --------+---------+---------+---------+-
                                1.5       3.0       4.5       6.0
Pooled StDev = 0.8076
```
There is a statistically significant difference in wafer position, 1 is different from 2, 3, and 4.

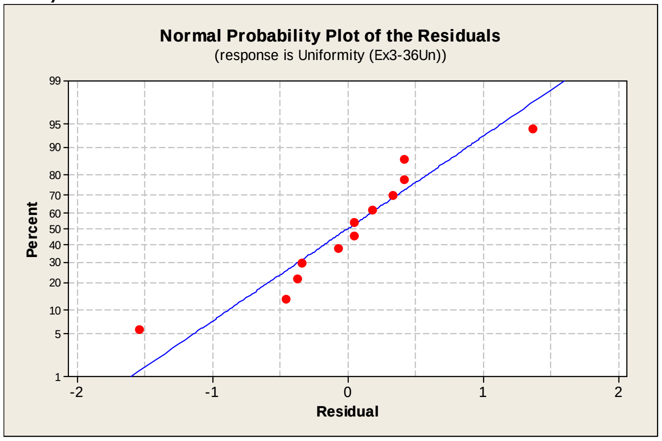


(b)
$$\hat{\sigma}_\tau^2 = \frac{\text{MS}_{\text{factor}} - \text{MS}_E}{n} = \frac{5.4066 - 0.6522}{12} = 0.3962$$

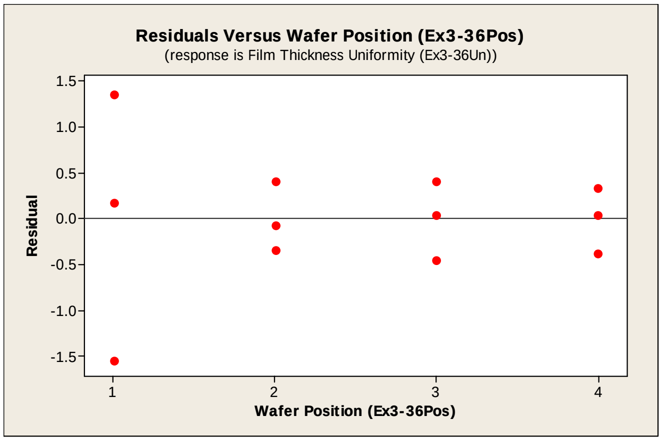
(c)
$$\hat{\sigma}^2 = \text{MS}_E = 0.6522$$
$$\hat{\sigma}_{\text{uniformity}}^2 = \hat{\sigma}_\tau^2 + \hat{\sigma}^2 = 0.3962 + 0.6522 = 1.0484$$

3-36 continued

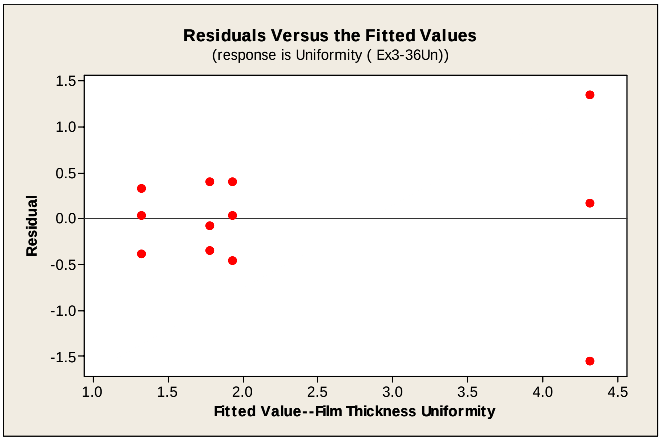
(d) **MTB > Stat > ANOVA > One-Way > Graphs> Normal plot of residuals, Residuals versus fits, Residuals versus the Variables**



Normality assumption is probably not unreasonable, but there are two very unusual observations – the outliers at either end of the plot – therefore model adequacy is questionable.



Both outlier residuals are from wafer position 1.



The variability in residuals does appear to depend on the magnitude of predicted (or fitted) values.

Chapter 4 Exercise Solutions

Several exercises in this chapter differ from those in the 4th edition. An "*" following the exercise number indicates that the description has changed. New exercises are denoted with an "☺". A second exercise number in parentheses indicates that the exercise number has changed.

4-1.
"Chance" or "common" causes of variability represent the inherent, natural variability of a process - its background noise. Variation resulting from "assignable" or "special" causes represents generally large, unsatisfactory disturbances to the usual process performance. Assignable cause variation can usually be traced, perhaps to a change in material, equipment, or operator method.

A Shewhart control chart can be used to monitor a process and to identify occurrences of assignable causes. There is a high probability that an assignable cause has occurred when a plot point is outside the chart's control limits. By promptly identifying these occurrences and acting to permanently remove their causes from the process, we can reduce process variability in the long run.

4-2.
The control chart is mathematically equivalent to a series of statistical hypothesis tests. If a plot point is within control limits, say for the average $\bar{X}$, the null hypothesis that the mean is some value is not rejected. However, if the plot point is outside the control limits, then the hypothesis that the process mean is at some level is rejected. A control chart shows, graphically, the results of many sequential hypothesis tests.

NOTE TO INSTRUCTOR FROM THE AUTHOR (D.C. Montgomery):
There has been some debate as to whether a control chart is really equivalent to hypothesis testing. Deming (see *Out of the Crisis*, MIT Center for Advanced Engineering Study, Cambridge, MA, pp. 369) writes that:

> "Some books teach that use of a control chart is test of hypothesis: the process is in control, or it is not. Such errors may derail self-study".

Deming also warns against using statistical theory to study control chart behavior (false-alarm probability, OC-curves, average run lengths, and normal curve probabilities. Wheeler (see "Shewhart's Charts: Myths, Facts, and Competitors", *ASQC Quality Congress Transactions* (1992), Milwaukee, WI, pp. 533–538) also shares some of these concerns:

> "While one may mathematically model the control chart, and while such a model may be useful in comparing different statistical procedures on a theoretical basis, these models do not justify any procedure in practice, and their exact probabilities, risks, and power curves do not actually apply in practice."

4-2 continued

On the other hand, Shewhart, the inventor of the control chart, did not share these views in total. From Shewhart (*Statistical Method from the Viewpoint of Quality Control* (1939), U.S. Department of Agriculture Graduate School, Washington DC, p. 40, 46):

> "As a background for the development of the operation of statistical control, the formal mathematical theory of testing a statistical hypothesis is of outstanding importance, but it would seem that we must continually keep in mind the fundamental difference between the formal theory of testing a statistical hypothesis and the empirical theory of testing a hypothesis employed in the operation of statistical control. In the latter, one must also test the hypothesis that the sample of data was obtained under conditions that may be considered random. …
> The mathematical theory of distribution characterizing the formal and mathematical concept of a state of statistical control constitutes an unlimited storehouse of helpful suggestions from which practical criteria of control must be chosen, and the general theory of testing statistical hypotheses must serve as a background to guide the choice of methods of making a running quality report that will give the maximum service as time goes on."

Thus Shewhart does not discount the role of hypothesis testing and other aspects of statistical theory. However, as we have noted in the text, the purposes of the control chart are more general than those of hypothesis tests. The real value of a control chart is monitoring stability over time. Also, from Shewhart's 1939 book, (p. 36):

> "The control limits as most often used in my own work have been set so that after a state of statistical control has been reached, one will look for assignable causes when they are not present not more than approximately three times in 1000 samples, when the distribution of the statistic used in the criterion is normal."

Clearly, Shewhart understood the value of statistical theory in assessing control chart performance.

My view is that the proper application of statistical theory to control charts can provide useful information about how the charts will perform. This, in turn, will guide decisions about what methods to use in practice. If you are going to apply a control chart procedure to a process with unknown characteristics, it is prudent to know how it will work in a more idealized setting. In general, before recommending a procedure for use in practice, it should be demonstrated that there is some underlying model for which it performs well. The study by Champ and Woodall (1987), cited in the text, that shows the ARL performance of various sensitizing rules for control charts is a good example. This is the basis of the recommendation against the routine use of these rules to enhance the ability of the Shewhart chart to detect small process shifts.

4-3.
Relative to the control chart, the type I error represents the probability of concluding the process is out of control when it isn't, meaning a plot point is outside the control limits when in fact the process is still in control. In process operation, high frequencies of false alarms could lead could to excessive investigation costs, unnecessary process adjustment (and increased variability), and lack of credibility for SPC methods.

The type II error represents the probability of concluding the process is in control, when actually it is not; this results from a plot point within the control limits even though the process mean has shifted out of control. The effect on process operations of failing to detect an out-of-control shift would be an increase in non-conforming product and associated costs.

4-4.
The statement that a process is in a state of statistical control means that assignable or special causes of variation have been removed; characteristic parameters like the mean, standard deviation, and probability distribution are constant; and process behavior is predictable. One implication is that any improvement in process capability (i.e., in terms of non-conforming product) will require a change in material, equipment, method, etc.

4-5.
No. The fact that a process operates in a state of statistical control does not mean that nearly all product meets specifications. It simply means that process behavior (mean and variation) is statistically predictable. We may very well predict that, say, 50% of the product will not meet specification limits! *Capability* is the term, which refers to the ability to meet product specifications, and a process must be in control in order to calculate capability.

4-6.
The logic behind the use of 3-sigma limits on Shewhart control charts is that they give good results in practice. Narrower limits will result in more investigations for assignable causes, and perhaps more false alarms. Wider limits will result in fewer investigations, but perhaps fewer process shifts will be promptly identified.

Sometimes probability limits are used - particularly when the underlying distribution of the plotted statistic is known. If the underlying distribution is unknown, care should be exercised in selecting the width of the control limits. Historically, however, 3-sigma limits have been very successful in practice.

4-7.
Warning limits on control charts are limits that are inside the control limits. When warning limits are used, control limits are referred to as action limits. Warning limits, say at 2-sigma, can be used to increase chart sensitivity and to signal process changes more quickly than the 3-sigma action limits. The Western Electric rule, which addresses this type of shift is to consider a process to be out of control if 2 of 3 plot points are between 2 sigma and 3 sigma of the chart centerline.

4-8.
The concept of a rational subgroup is used to maximize the chance for detecting variation between subgroups. Subgroup samples can be structured to identify process shifts. If it is expected that a process will shift and stay at the new level until a corrective action, then sampling consecutive (or nearly) units maximizes the variability between subgroups and minimizes the variability within a subgroup. This maximizes the probability of detecting a shift.

4-9.
I would want assignable causes to occur between subgroups and would prefer to select samples as close to consecutive as possible. In most SPC applications, process changes will not be self-correcting, but will require action to return the process to its usual performance level. The probability of detecting a change (and therefore initiating a corrective action) will be maximized by taking observations in a sample as close together as possible.

4-10.
This sampling strategy will very likely underestimate the size of the true process variability. Similar raw materials and operating conditions will tend to make any five-piece sample alike, while variability caused by changes in batches or equipment may remain undetected. An out-of-control signal on the R chart will be interpreted to be the result of differences between cavities. Because true process variability will be underestimated, there will likely be more false alarms on the $\bar{x}$ chart than there should be.

4-11.
(a)
No.

(b)
The problem is that the process may shift to an out-of-control state and back to an in-control state in less than one-half hour. Each subgroup should be a random sample of all parts produced in the last 2½ hours.

4-12.
No. The problem is that with a slow, prolonged trend upwards, the sample average will tend to be the value of the 3rd sample --- the highs and lows will average out. Assume that the trend must last 2½ hours in order for a shift of detectable size to occur. Then a better sampling scheme would be to simply select 5 consecutive parts every 2½ hours.

4-13.
No. If time order of the data is not preserved, it will be impossible to separate the presence of assignable causes from underlying process variability.
4-14.
An operating characteristic curve for a control chart illustrates the tradeoffs between sample size n and the process shift that is to be detected. Generally, larger sample sizes are needed to increase the probability of detecting small changes to the process. If a large shift is to be detected, then smaller sample sizes can be used.

4-15.
The costs of sampling, excessive defective units, and searches for assignable causes impact selection of the control chart parameters of sample size n, sampling frequency h, and control limit width. The larger n and h, the larger will be the cost of sampling. This sampling cost must be weighed against the cost of producing non-conforming product.

4-16.
Type I and II error probabilities contain information on statistical performance; an ARL results from their selection. ARL is more meaningful in the sense of the operations information that is conveyed and could be considered a measure of the process performance of the sampling plan.

4-17.
Evidence of runs, trends or cycles? NO. There are no runs of 5 points or cycles. So, we can say that the plot point pattern appears to be random.

4-18.
Evidence of runs, trends or cycles? YES, there is one "low - high - low - high" pattern (Samples 13 – 17), which might be part of a cycle. So, we can say that the pattern does not appear random.

4-19.
Evidence of runs, trends or cycles? YES, there is a "low - high - low - high - low" wave (all samples), which might be a cycle. So, we can say that the pattern does not appear random.

4-20.
Three points exceed the 2-sigma warning limits - points #3, 11, and 20.

4-21.
Check:

- Any point outside the 3-sigma control limits? NO.
- 2 of 3 beyond 2 sigma of centerline? NO.
- 4 of 5 at 1 sigma or beyond of centerline? YES. Points #17, 18, 19, and 20 are outside the lower 1-sigma area.
- 8 consecutive points on one side of centerline? NO.

One out-of-control criteria is satisfied.

4-22.
Four points exceed the 2-sigma warning limits - points #6, 12, 16, and 18.

4-23.
Check:

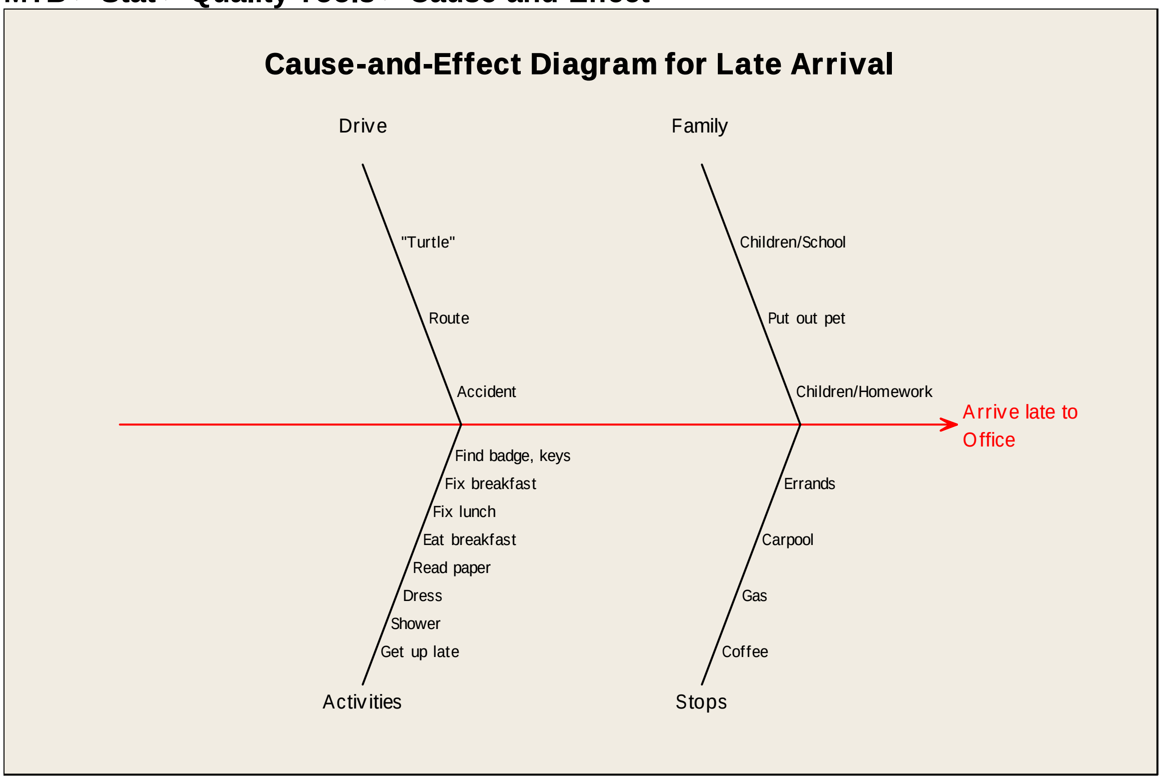
- Any point outside the 3-sigma control limits? NO. (Point #12 is within the lower 3-sigma control limit.)
- 2 of 3 beyond 2 sigma of centerline? YES, points #16, 17, and 18.
- 4 of 5 at 1 sigma or beyond of centerline? YES, points #5, 6, 7, 8, and 9.
- 8 consecutive points on one side of centerline? NO.

Two out-of-control criteria are satisfied.

4-24.
The pattern in Figure (a) matches the control chart in Figure (2).
The pattern in Figure (b) matches the control chart in Figure (4).
The pattern in Figure (c) matches the control chart in Figure (5).
The pattern in Figure (d) matches the control chart in Figure (1).
The pattern in Figure (e) matches the control chart in Figure (3).

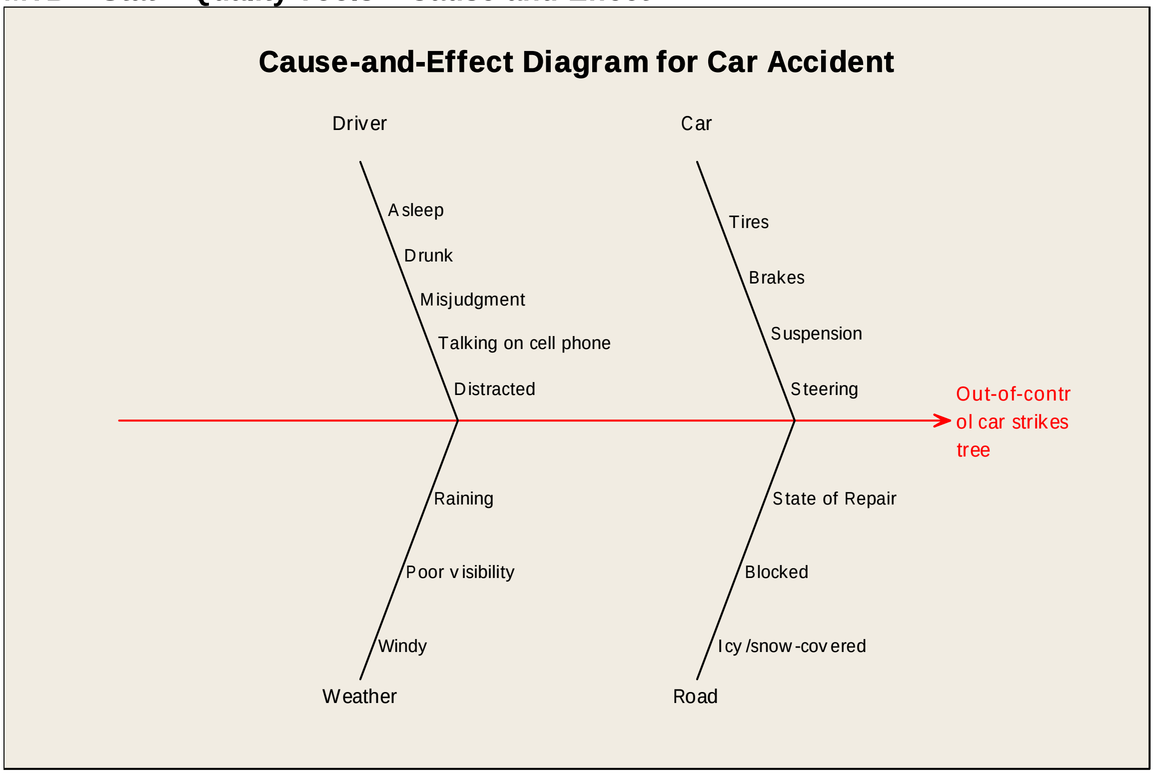
4-25 (4-30).
Many possible solutions.

MTB > Stat > Quality Tools > Cause-and-Effect



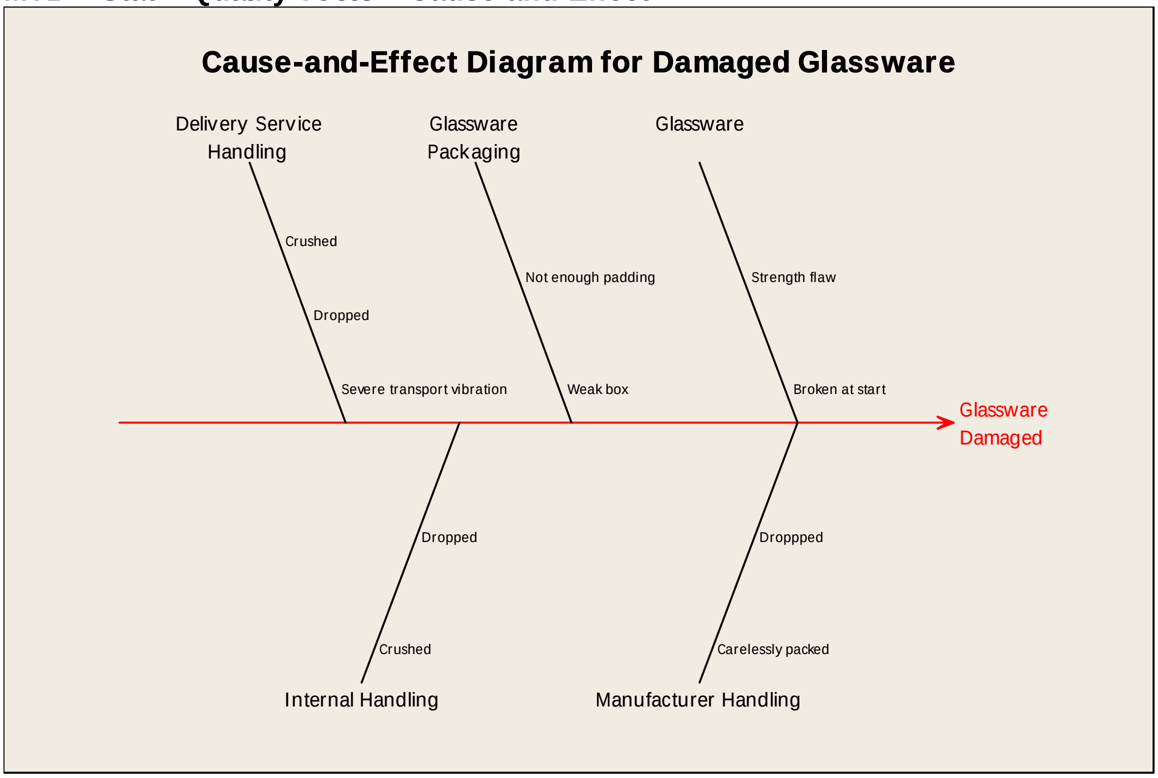
4-26 (4-31).
Many possible solutions.

MTB > Stat > Quality Tools > Cause-and-Effect



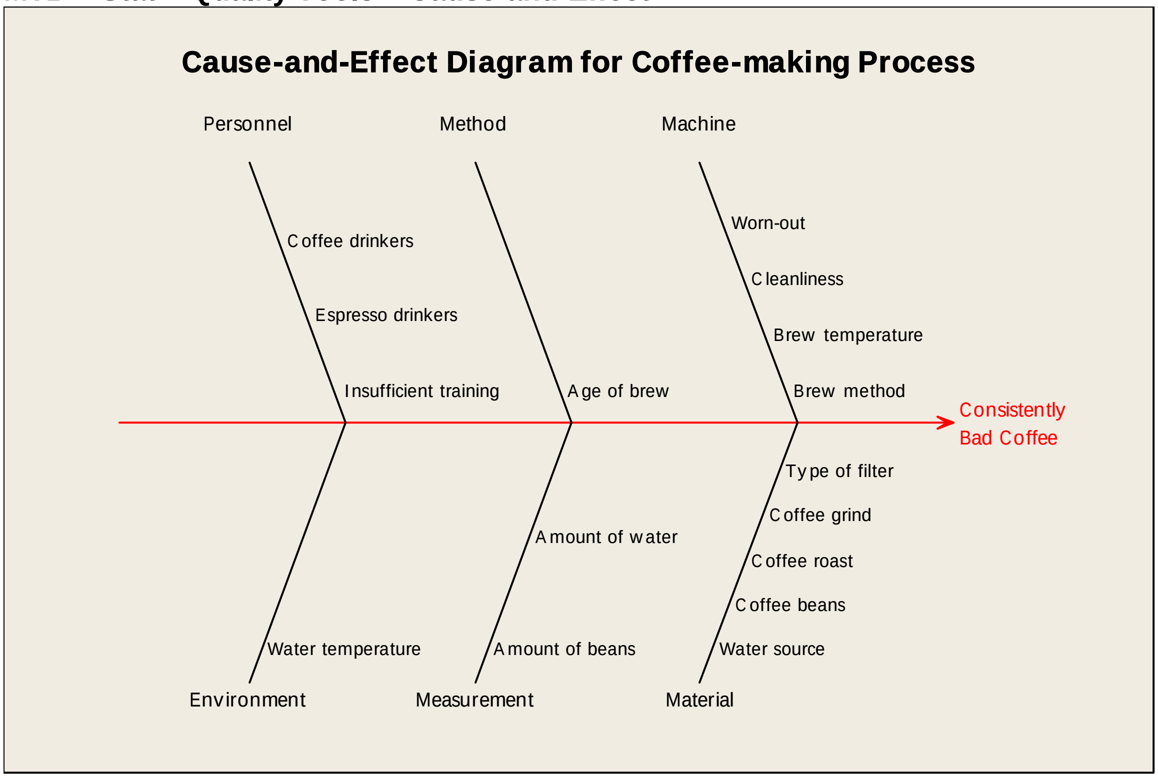
4-27 (4-32).
Many possible solutions.

MTB > Stat > Quality Tools > Cause-and-Effect

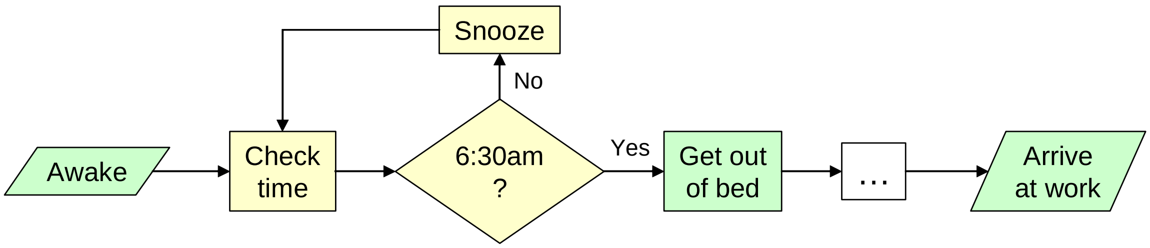


4-28☺.
Many possible solutions.

MTB > Stat > Quality Tools > Cause-and-Effect

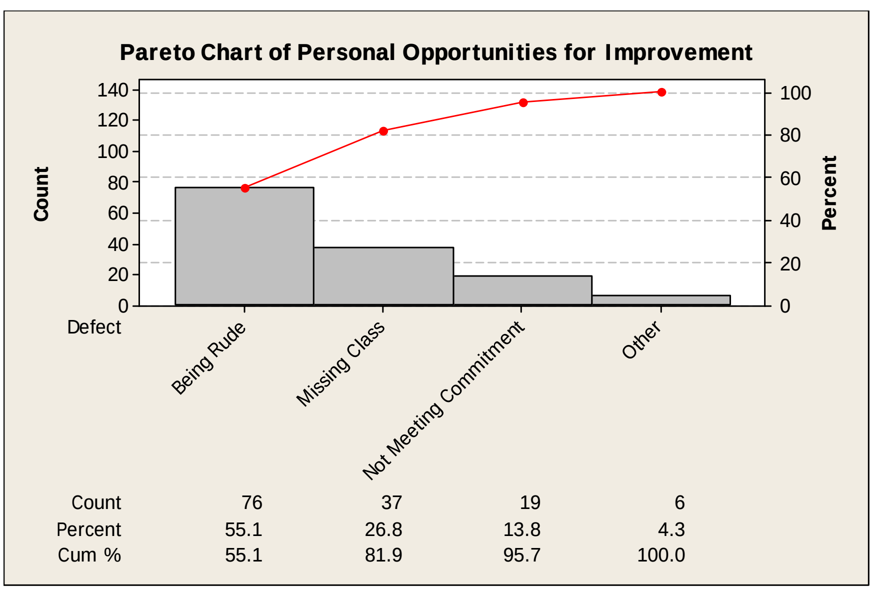


4-29☺.
Many possible solutions, beginning and end of process are shown below. Yellow is non-value-added activity; green is value-added activity.



4-31☺.
Example of a check sheet to collect data on personal opportunities for improvement. Many possible solutions, including defect categories and counts.

	Month/Day									
Defect	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**…**	**31**	**TOTAL**
Overeating	0	2	1	0	1	0	1	…	1	6
Being Rude	10	11	9	9	7	10	11	…	9	76
Not meeting commitments	4	2	2	2	1	0	1	…	7	19
Missing class	4	6	3	2	7	9	4	…	2	37
Etc.										
TOTAL	18	21	15	13	16	19	17		19	138



To reduce total count of defects, “Being Rude” represents the greatest opportunity to make an improvement. The next step would be to determine the causes of “Being Rude” and to work on eliminating those causes.

4-32☺.

$m = 5$

$$\alpha_1 = \Pr\{\text{at least 1 out-of-control}\} = \Pr\{\text{1 of 5 beyond}\} + \Pr\{\text{2 of 5 beyond}\} + \cdots + \Pr\{\text{5 of 5 beyond}\}$$

$$= 1 - \Pr\{\text{0 of 5 beyond}\} = 1 - \binom{5}{0}(0.0027)^0(1-0.0027)^5 = 1 - 0.9866 = 0.0134$$

MTB > Calc > Probability Distributions > Binomial, Cumulative Probability

Cumulative Distribution Function

```
Binomial with n = 5 and p = 0.0027
x  P( X <= x )
0     0.986573
```

$m = 10$

$$\alpha_1 = 1 - \Pr\{\text{0 of 10 beyond}\} = 1 - \binom{10}{0}(0.0027)^0(1-0.0027)^{10} = 1 - 0.9733 = 0.0267$$

Cumulative Distribution Function

```
Binomial with n = 10 and p = 0.0027
x  P( X <= x )
0     0.973326
```

$m = 20$

$$\alpha_1 = 1 - \Pr\{\text{0 of 20 beyond}\} = 1 - \binom{20}{0}(0.0027)^0(1-0.0027)^{20} = 0.0526$$

Cumulative Distribution Function

```
Binomial with n = 20 and p = 0.0027
x  P( X <= x )
0     0.947363
```

$m = 30$

$$\alpha_1 = 1 - \Pr\{\text{0 of 30 beyond}\} = 1 - \binom{30}{0}(0.0027)^0(1-0.0027)^{30} = 0.0779$$

Cumulative Distribution Function

```
Binomial with n = 30 and p = 0.0027
x  P( X <= x )
0     0.922093
```

$m = 50$

$$\alpha_1 = 1 - \Pr\{\text{0 of 50 beyond}\} = 1 - \binom{50}{0}(0.0027)^0(1-0.0027)^{50} = 0.1025$$

Cumulative Distribution Function

```
Binomial with n = 50 and p = 0.0027
x  P( X <= x )
0     0.873556
```

Although the probability that a single point plots beyond the control limits is 0.0027, as the number of samples increases (m), the probability that at least one of the points is beyond the limits also increases.

4-33☺.

When the process mean μ and variance σ^2 are unknown, they must be estimated by sample means $\bar{x}$ and standard deviations s. However, the points used to estimate these sample statistics are not independent—they do not reflect a random sample from a population. In fact, sampling frequencies are often designed to increase the likelihood of detecting a special or assignable cause. The lack of independence in the sample statistics will affect the estimates of the process population parameters.

Chapter 5 Exercise Solutions

Notes:

1. Several exercises in this chapter differ from those in the 4th edition. An "*" indicates that the description has changed. A second exercise number in parentheses indicates that the exercise number has changed. New exercises are denoted with an "☺".
2. The MINITAB convention for determining whether a point is out of control is: (1) if a plot point is within the control limits, it is in control, or (2) if a plot point is on or beyond the limits, it is out of control.
3. MINITAB uses pooled standard deviation to estimate standard deviation for control chart limits and capability estimates. This can be changed in dialog boxes or under Tools>Options>Control Charts and Quality Tools>Estimating Standard Deviation.
4. MINITAB defines some sensitizing rules for control charts differently than the standard rules. In particular, a run of *n* consecutive points on one side of the center line is defined as 9 points, not 8. This can be changed under Tools > Options > Control Charts and Quality Tools > Define Tests.

5-1.

(a) for $n = 5$, $A_2 = 0.577$, $D_4 = 2.114$, $D_3 = 0$

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_m}{m} = \frac{34.5 + 34.2 + \cdots + 34.2}{24} = 34.00$$

$$\bar{R} = \frac{R_1 + R_2 + \cdots + R_m}{m} = \frac{3 + 4 + \cdots + 2}{24} = 4.71$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 34.00 + 0.577(4.71) = 36.72$$

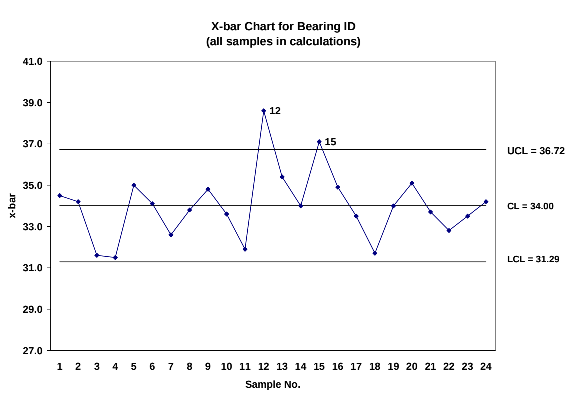
$$\text{CL}_{\bar{x}} = \bar{\bar{x}} = 34.00$$

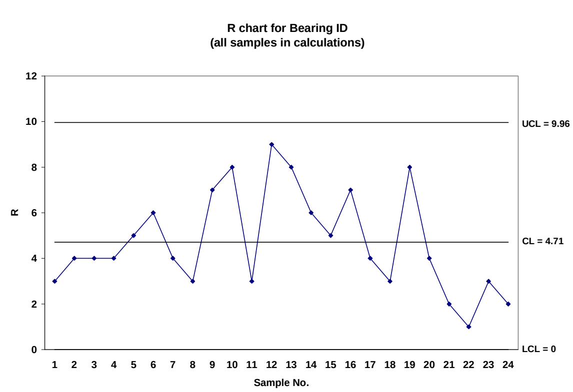
$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 34.00 - 0.577(4.71) = 31.29$$

$$\text{UCL}_{\text{R}} = D_4\bar{R} = 2.115(4.71) = 9.96$$

$$\text{CL}_{\text{R}} = \bar{R} = 4.71$$

$$\text{LCL}_{\text{R}} = D_3\bar{R} = 0(4.71) = 0.00$$





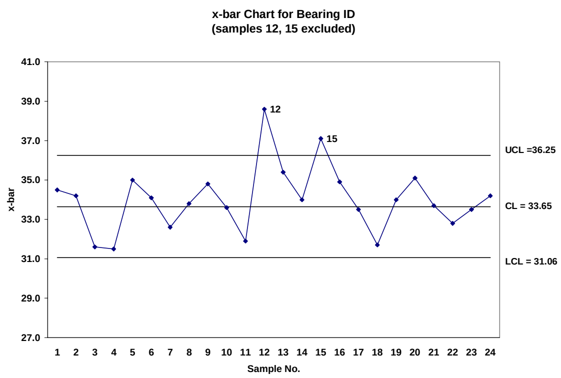
5-1 (a) continued

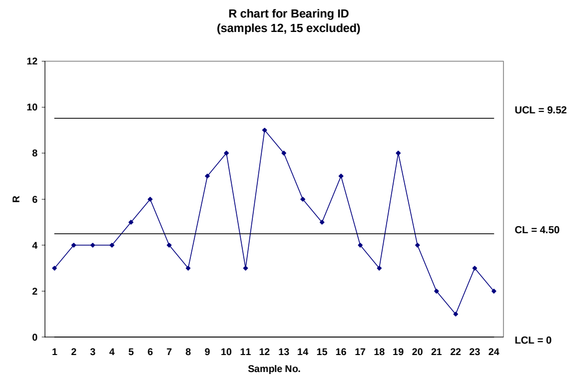
The process is not in statistical control; $\bar{x}$ is beyond the upper control limit for both Sample No. 12 and Sample No. 15. Assuming an assignable cause is found for these two out-of-control points, the two samples can be excluded from the control limit calculations. The new process parameter estimates are:

$$\bar{\bar{x}} = 33.65; \quad \bar{R} = 4.5; \quad \hat{\sigma}_x = \bar{R}/d_2 = 4.5/2.326 = 1.93$$

$$\mathrm{UCL}_{\bar{x}} = 36.25; \mathrm{CL}_{\bar{x}} = 33.65; \mathrm{LCL}_{\bar{x}} = 31.06$$

$$\mathrm{UCL}_{\mathrm{R}} = 9.52; \mathrm{CL}_{\mathrm{R}} = 4.5; \mathrm{LCL}_{\mathrm{R}} = 0.00$$





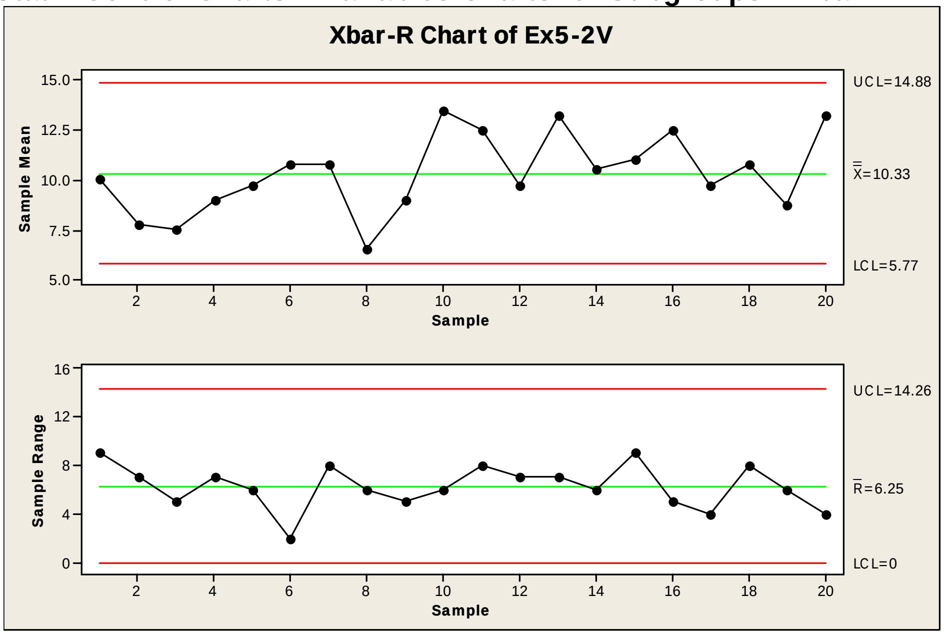
(b)

$$\hat{p} = \Pr\{x < \mathrm{LSL}\} + \Pr\{x > \mathrm{USL}\} = \Pr\{x < 20\} + \Pr\{x > 40\} = \Pr\{x < 20\} + \left[1 - \Pr\{x < 40\}\right]$$

$$= \Phi\left(\frac{20 - 33.65}{1.93}\right) + \left[1 - \Phi\left(\frac{40 - 33.65}{1.93}\right)\right]$$

$$= \Phi(-7.07) + 1 - \Phi(3.29) = 0 + 1 - 0.99950 = 0.00050$$

5-2.
(a)
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R

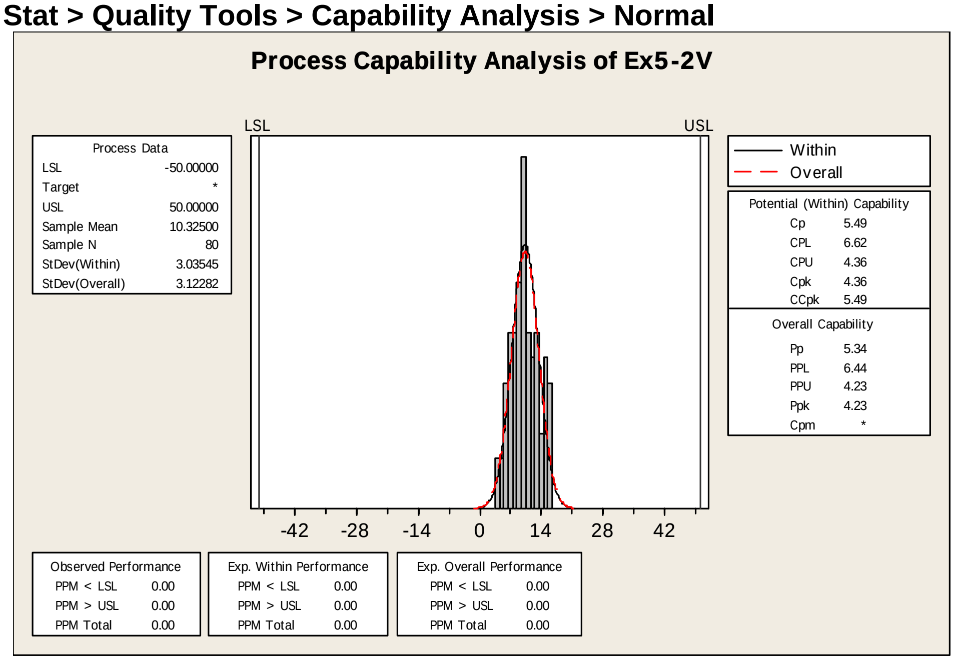


The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

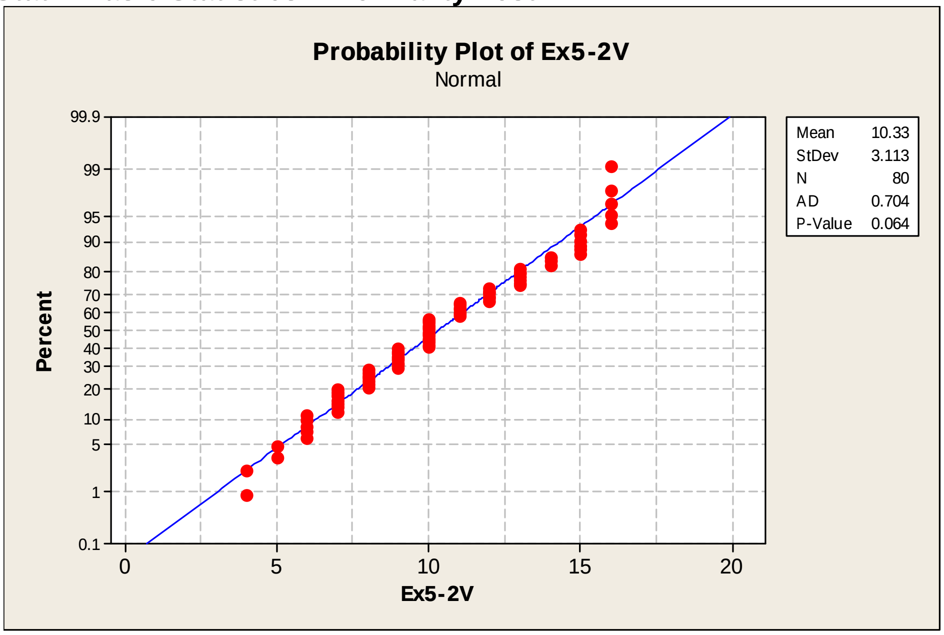
(b)
$n = 4$, $\bar{\bar{x}} = 10.33$, $\bar{R} = 6.25$, $\hat{\sigma}_X = \bar{R}/d_2 = 6.25/2.059 = 3.035$. Actual specs are 350 ± 5 V.
With x_i = (observed voltage on unit i – 350) × 10: $\text{USL}_\text{T} = +50$, $\text{LSL}_\text{T} = -50$

$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{+50-(-50)}{6(3.035)} = 5.49$$, so the process is capable.

MTB > Stat > Quality Tools > Capability Analysis > Normal

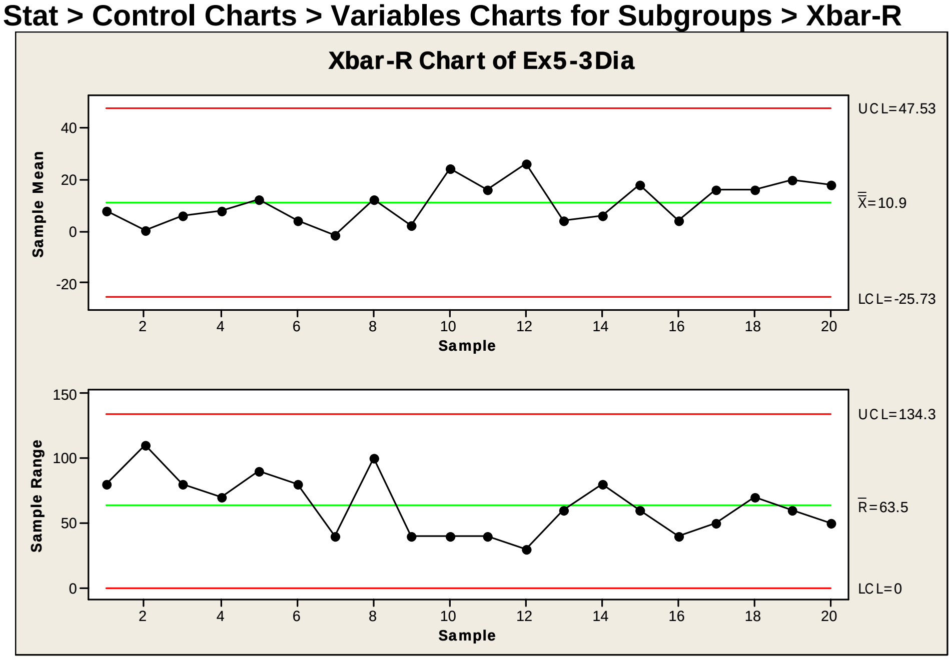


5-2 continued
(c)
MTB > Stat > Basic Statistics > Normality Test



A normal probability plot of the transformed output voltage shows the distribution is close to normal.

5-3.
(a)
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R



The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

5-3 continued

(b)

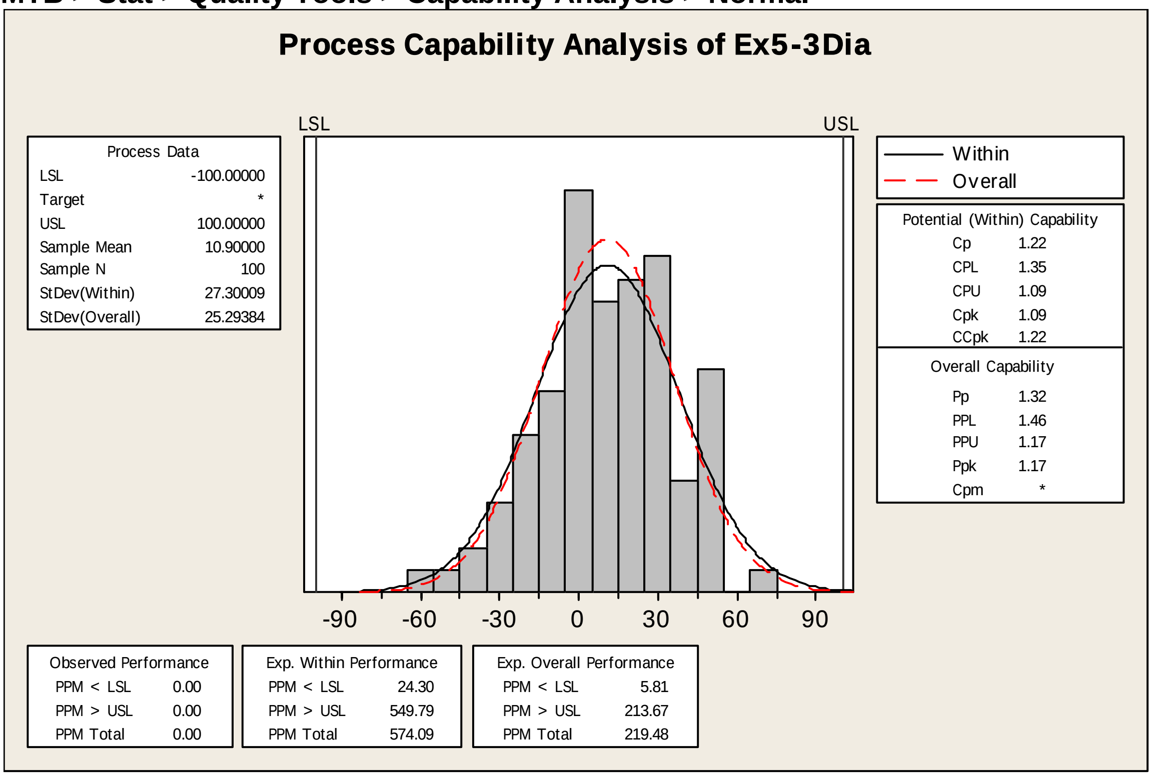
$\hat{\sigma}_x = \bar{R}/d_2 = 63.5/2.326 = 27.3$

(c)

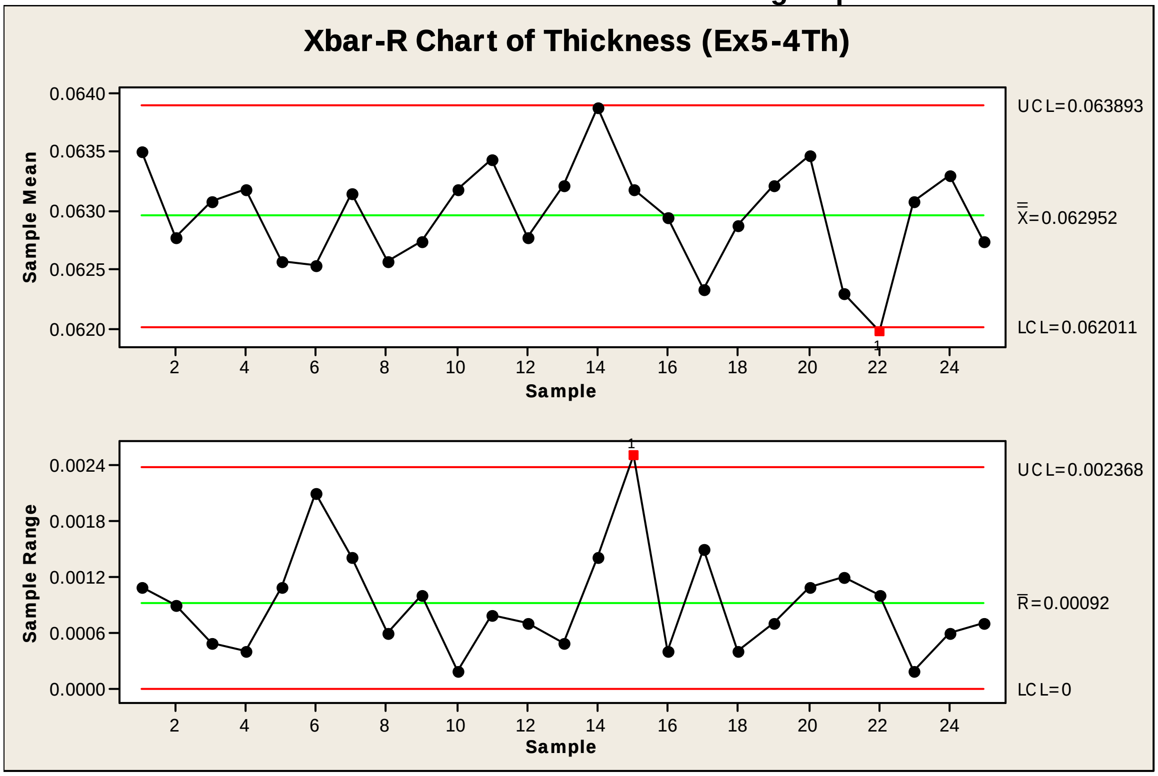
USL = +100, LSL = –100

$\hat{C}_P = \dfrac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \dfrac{+100-(-100)}{6(27.3)} = 1.22$, so the process is capable.

MTB > Stat > Quality Tools > Capability Analysis > Normal



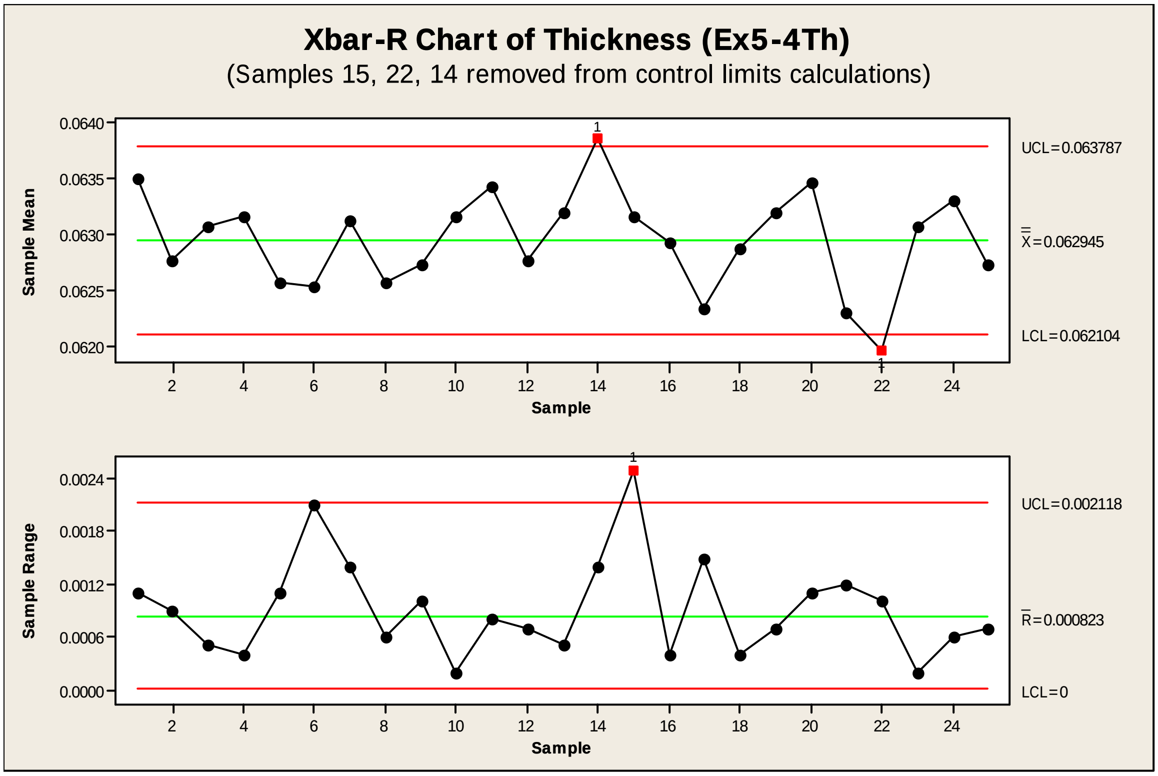
5-4.
(a)
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R




```
Test Results for Xbar Chart of Ex5-4Th
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  22
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  22

Test Results for R Chart of Ex5-4Th
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  15
* WARNING * If graph is updated with new data, the results above may no
          * longer be correct.
```

5-4 continued

The process is out-of-control, failing tests on both the $\overline{x}$ and the R charts. Assuming assignable causes are found, remove the out-of-control points (samples 15, 22) and re-calculate control limits. With the revised limits, sample 14 is also out-of-control on the $\overline{x}$ chart. Removing all three samples from calculation, the new control limits are:



(b)

$\hat{\sigma}_x = \overline{R} / d_2 = 0.000823 / 1.693 = 0.000486$

(c)

Natural tolerance limits are: $\overline{\overline{x}} \pm 3\hat{\sigma}_x = 0.06295 \pm 3(0.000486) = [0.061492, 0.064408]$

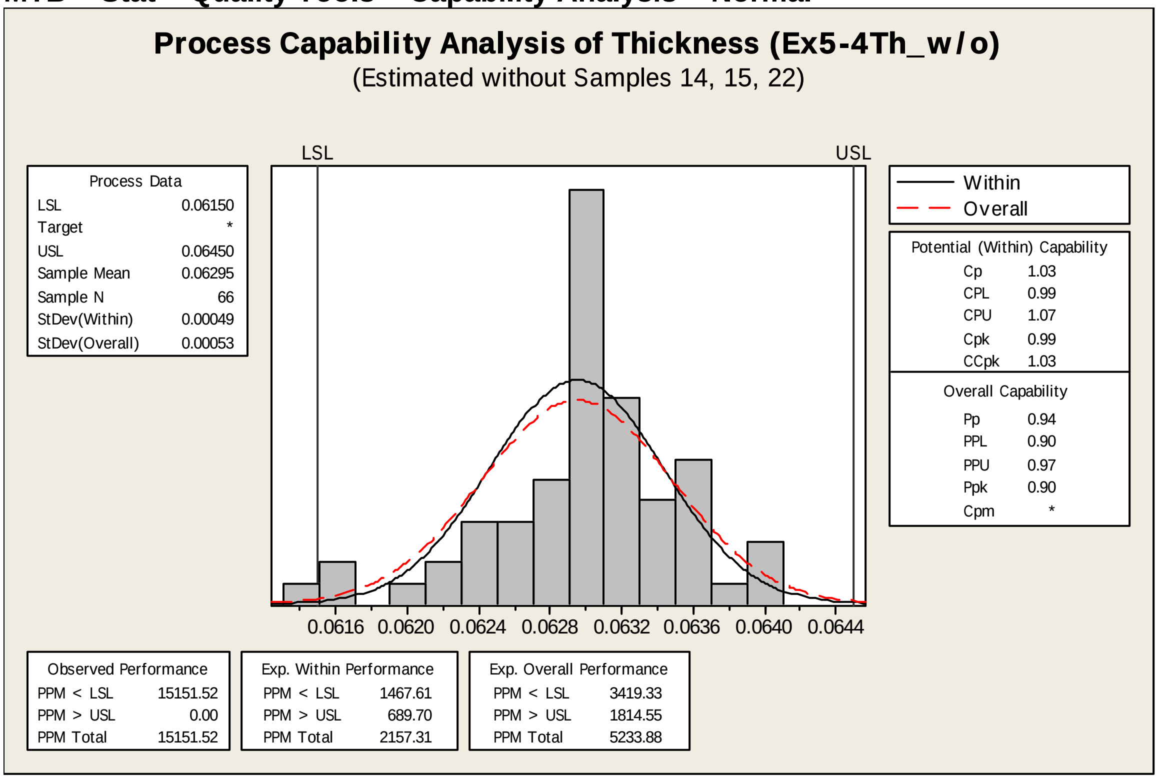
5-4 continued

(d)

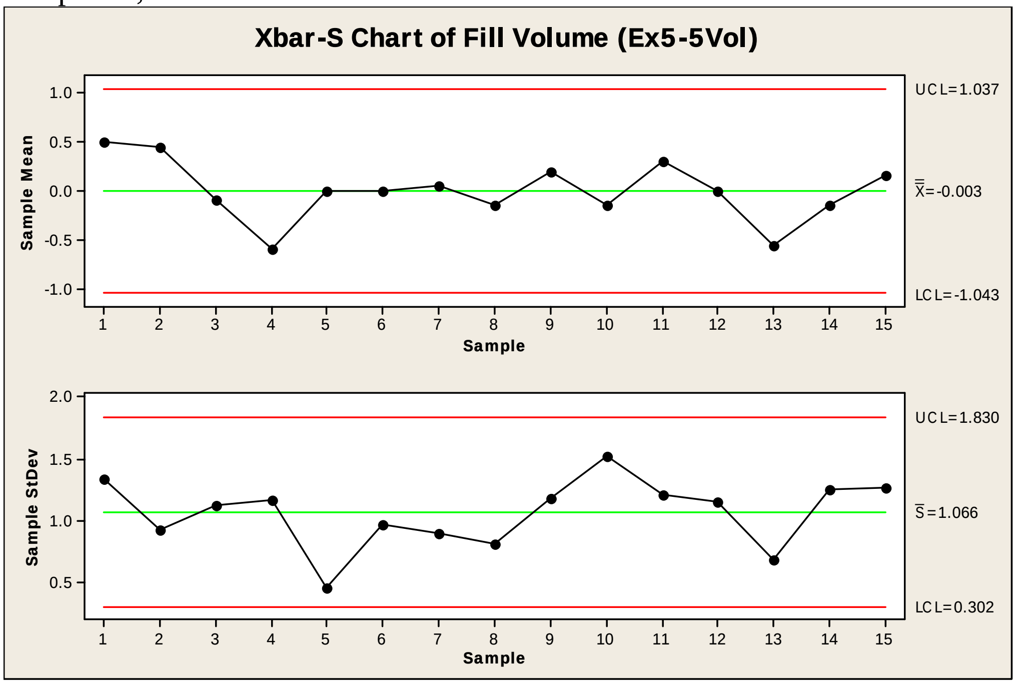
Assuming that printed circuit board thickness is normally distributed, and excluding samples 14, 15, and 22 from the process capability estimation:

$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{+0.0015 - (-0.0015)}{6(0.000486)} = 1.028$$

MTB > Stat > Quality Tools > Capability Analysis > Normal

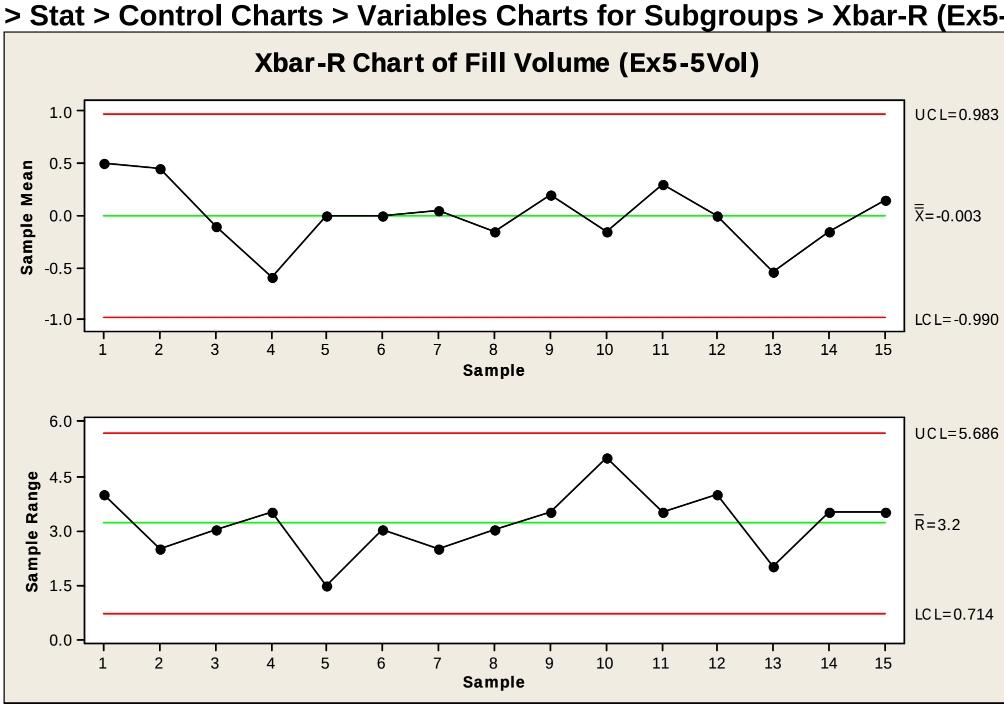


5-5.
(a)
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S (Ex5-5Vol)
Under “Options, Estimate” select Sbar as method to estimate standard deviation.



The process is in statistical control, with no out-of-control signals, runs, trends, or cycles.

(b)
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R (Ex5-5Vol)



The process is in statistical control, with no out-of-control signals, runs, trends, or cycles. There is no difference in interpretation from the $\bar{x}-s$ chart.

5-5 continued

(c)

Let $\alpha = 0.010$. $n = 15$, $\bar{s} = 1.066$.

$$\text{CL} = \bar{s}^2 = 1.066^2 = 1.136$$

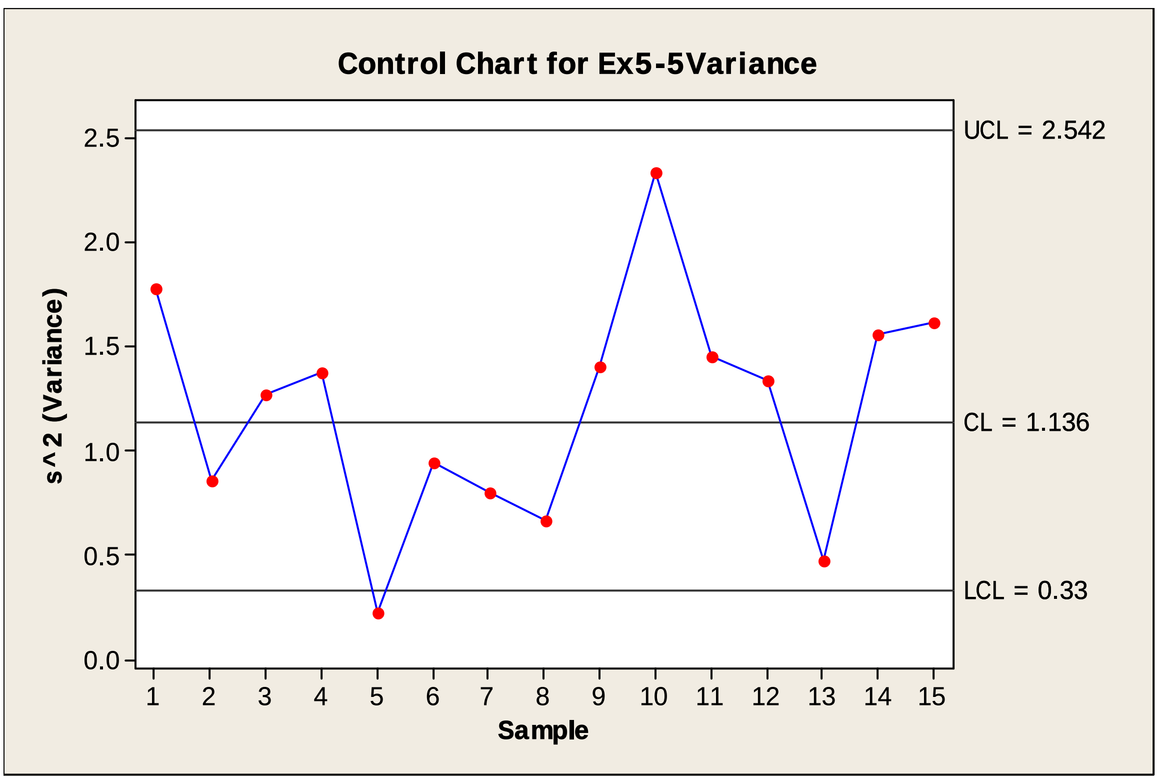
$$\text{UCL} = \bar{s}^2/(n-1)\,\chi^2_{\alpha/2,n-1} = 1.066^2/(15-1)\left(\chi^2_{0.010/2,15-1}\right) = 1.066^2/(15-1)(31.32) = 2.542$$

$$\text{LCL} = \bar{s}^2/(n-1)\,\chi^2_{1-(\alpha/2),n-1} = 1.066^2/(15-1)\left(\chi^2_{1-(0.010/2),15-1}\right) = 1.066^2/(15-1)(4.07) = 0.330$$

MINITAB's control chart options do not include an s^2 or variance chart. To construct an s^2 control chart, first calculate the sample standard deviations and then create a time series plot. To obtain sample standard deviations: **Stat > Basic Statistics > Store Descriptive Statistics**. "Variables" is column with sample data (Ex5-5Vol), and "By Variables" is the sample ID column (Ex5-5Sample). In "Statistics" select "Variance". Results are displayed in the session window. Copy results from the session window by holding down the keyboard "Alt" key, selecting only the variance column, and then copying & pasting to an empty worksheet column (results in Ex5-5Variance).

Graph > Time Series Plot > Simple

Control limits can be added using: **Time/Scale > Reference Lines > Y positions**

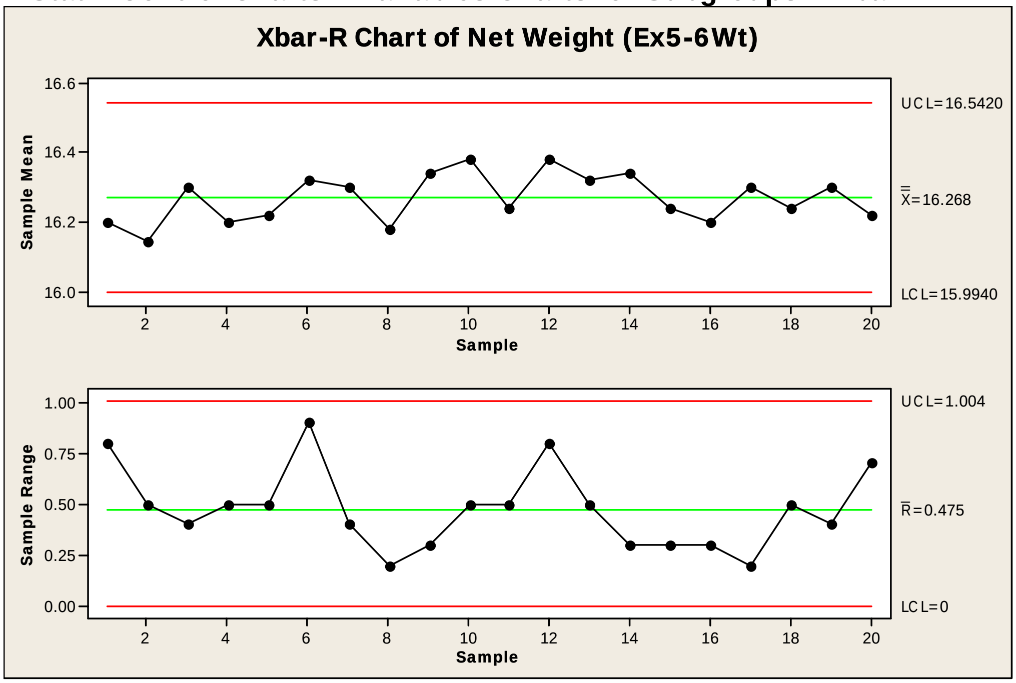


Sample 5 signals out of control below the lower control limit. Otherwise there are no runs, trends, or cycles. If the limits had been calculated using $\alpha = 0.0027$ (not tabulated in textbook), sample 5 would be within the limits, and there would be no difference in interpretation from either the $\bar{x} - s$ or the x–R chart.

5-6.

(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R



The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

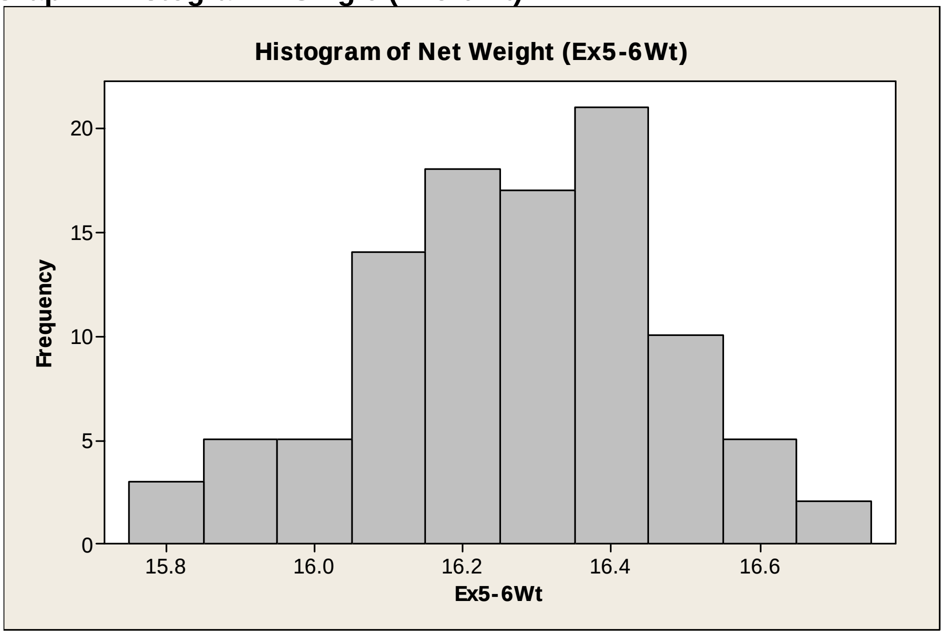
(b)

$n = 5;\ \ \bar{\bar{x}} = 16.268;\ \ \bar{R} = 0.475;\ \ \hat{\sigma}_x = \bar{R}/d_2 = 0.475/2.326 = 0.204$

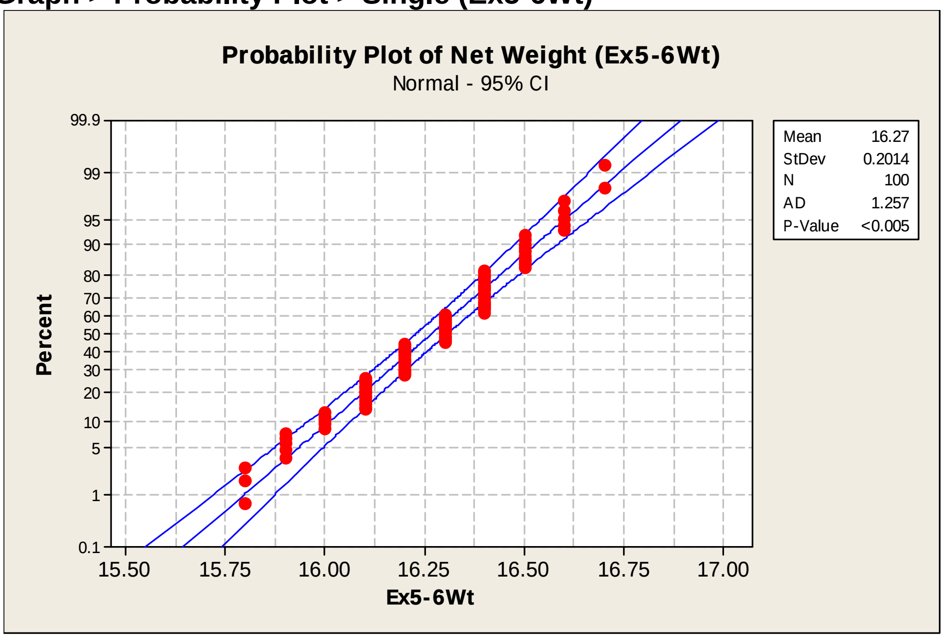
5-6 continued

(c)

MTB > Graph > Histogram > Single (Ex5-6Wt)



MTB > Graph > Probability Plot > Single (Ex5-6Wt)



Visual examination indicates that fill weights approximate a normal distribution - the histogram has one mode, and is approximately symmetrical with a bell shape. Points on the normal probability plot generally fall along a straight line.

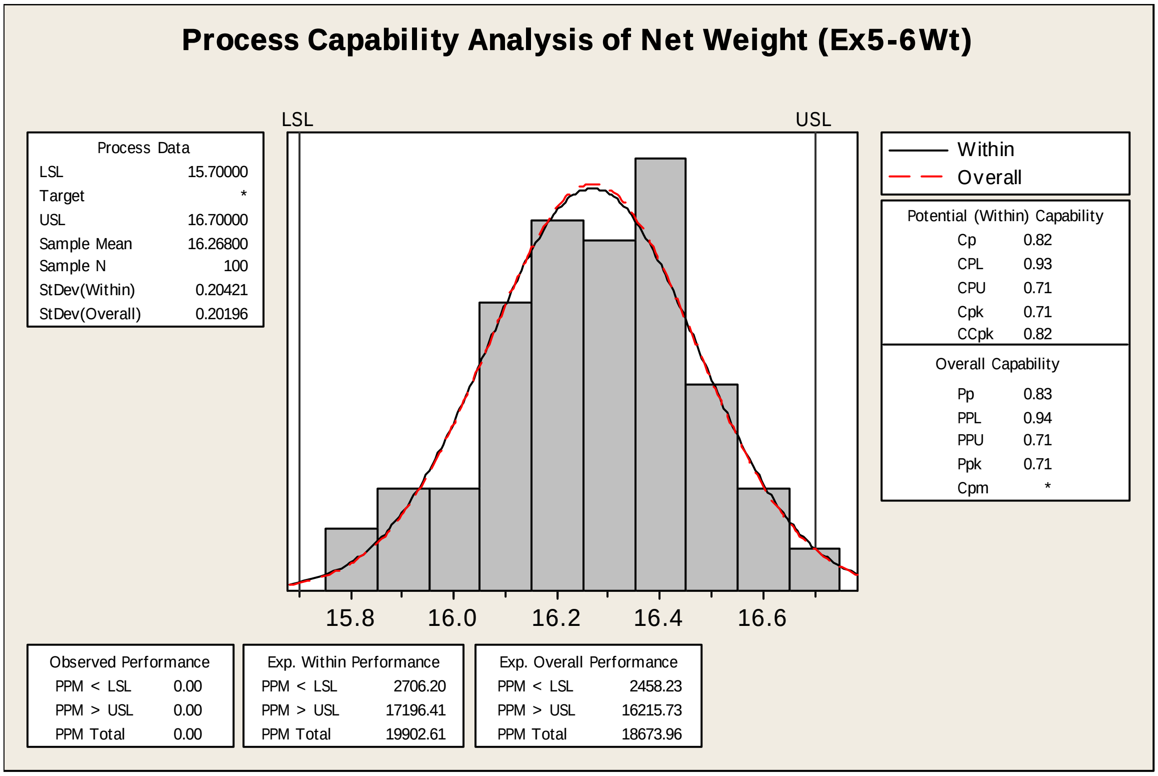
5-6 continued

(d)

$$\hat{C}_P \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{+0.5 - (-0.5)}{6(0.204)} = 0.82$$, so the process is not capable of meeting specifications.

MTB > Stat > Quality Tools > Capability Analysis > Normal

Under "Estimate" select Rbar as method to estimate standard deviation.



(e)

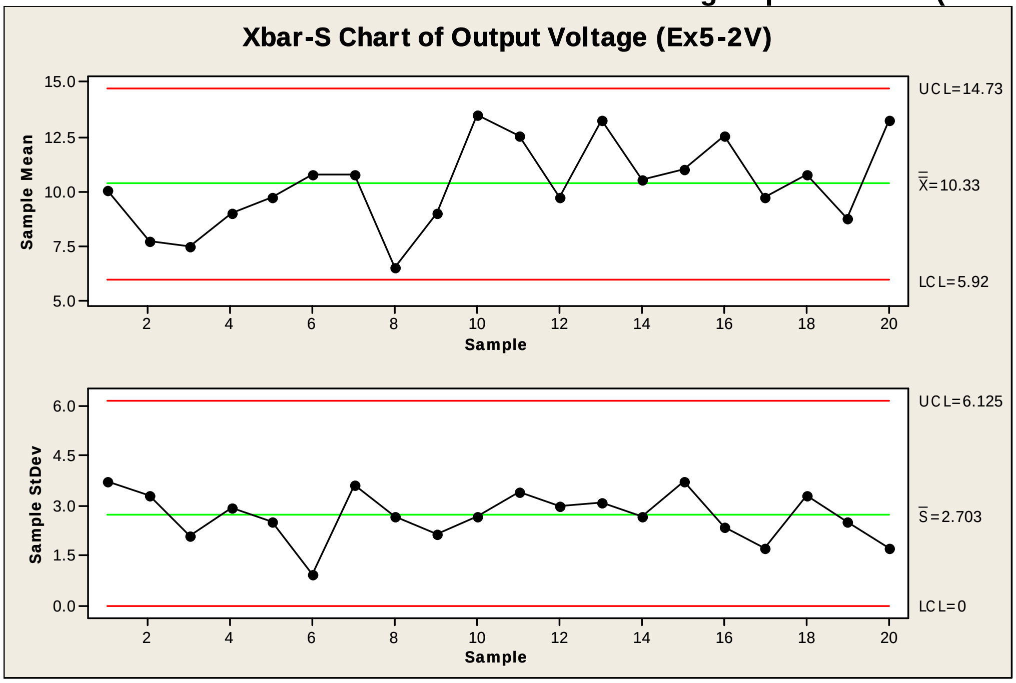
$$\hat{p}_{\text{lower}} = \Pr\{x < \text{LSL}\} = \Pr\{x < 15.7\} = \Phi\left(\frac{15.7 - 16.268}{0.204}\right) = \Phi(-2.78) = 0.0027$$

The MINITAB process capability analysis also reports

```
          Exp. "Overall" Performance
          PPM < LSL         2458.23
          PPM > USL        16215.73
          PPM Total        18673.96
```

5-7.

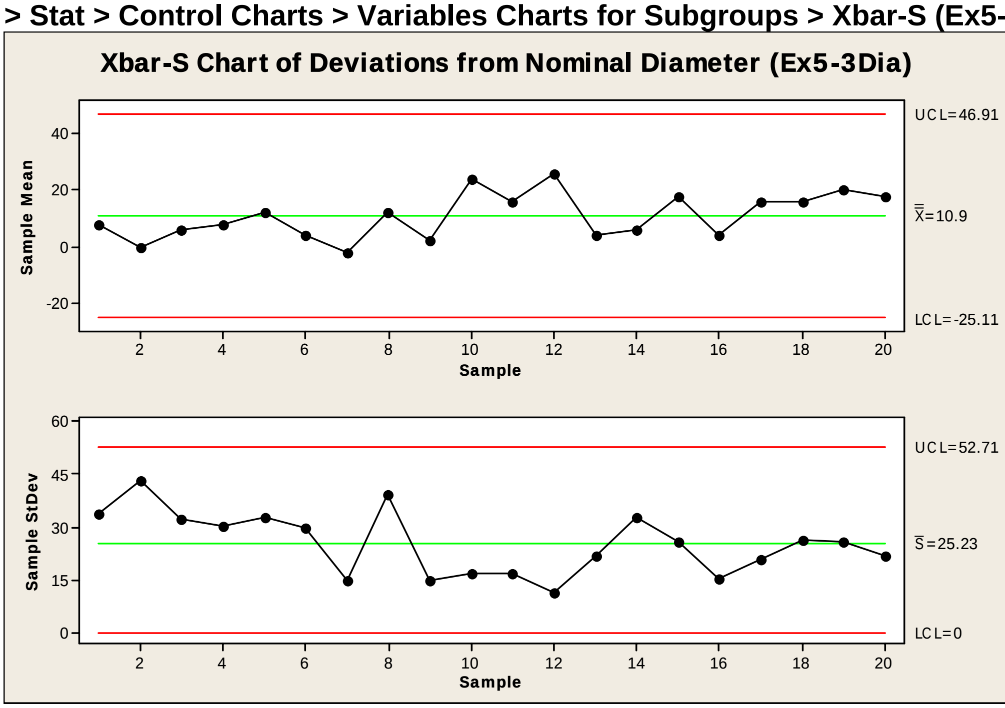
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S (Ex5-2Vl)



The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

5-8.

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S (Ex5-3Dia)

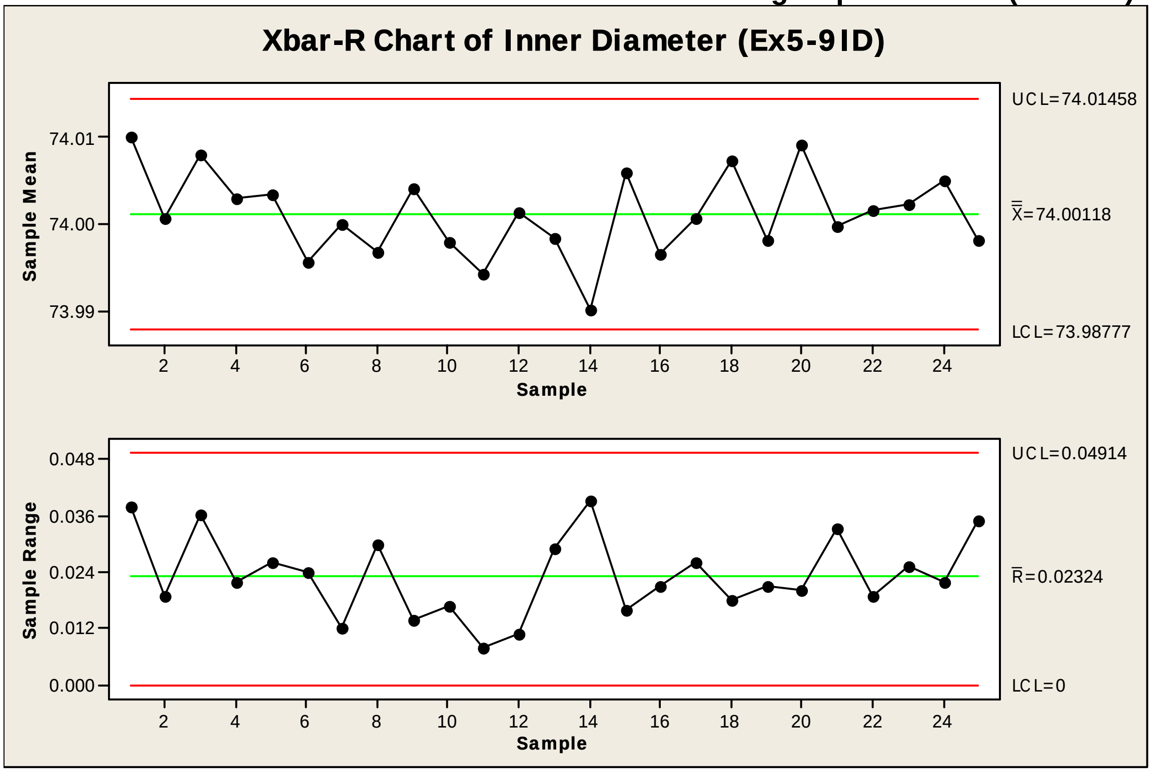


The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

5-9☺.

(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R (Ex5-9ID)



The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

(b)

The control limits on the $\bar{x}$ charts in Example 5-3 were calculated using $\bar{S}$ to estimate σ, in this exercise $\bar{R}$ was used to estimate σ. They will not always be the same, and in general, the $\bar{x}$ control limits based on $\bar{S}$ will be slightly different than limits based on $\bar{R}$.

5-9 continued

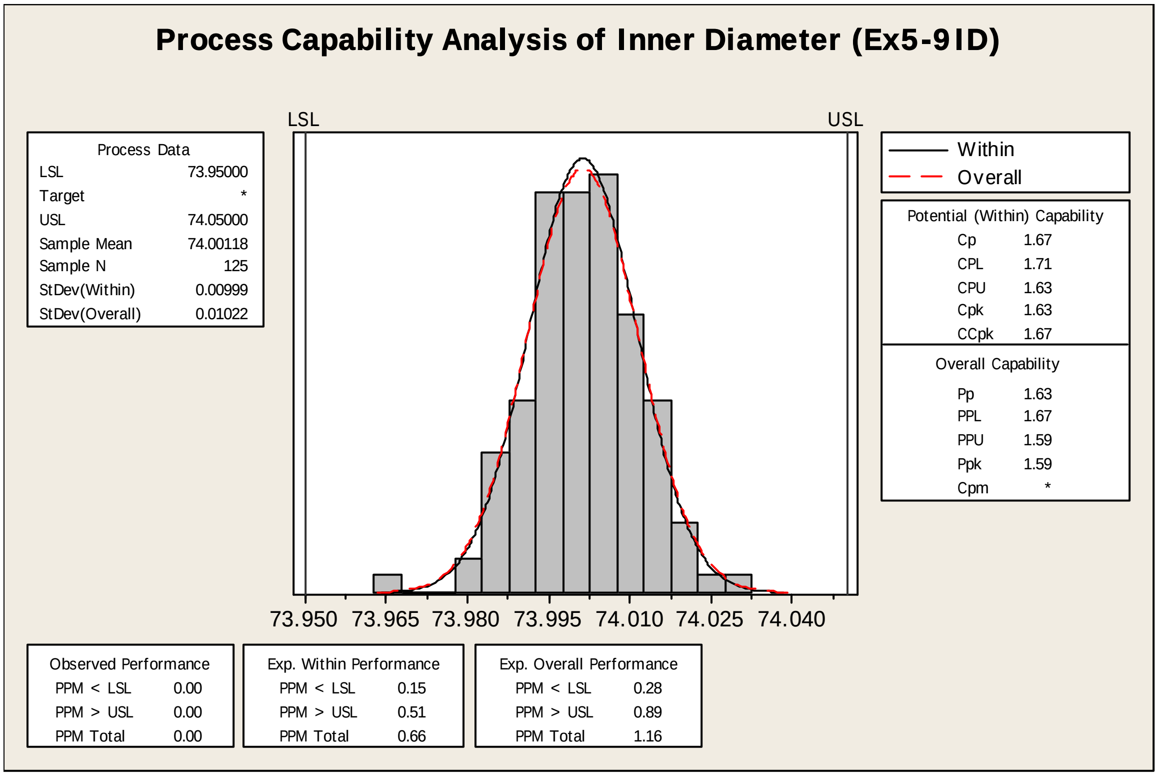
(c)

$$\hat{\sigma}_x = \bar{R}/d_2 = 0.02324/2.326 = 0.009991$$

$\hat{C}_P = \dfrac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \dfrac{74.05 - 73.95}{6(0.009991)} = 1.668$, so the process is not capable of meeting specifications.

MTB > Stat > Quality Tools > Capability Analysis > Normal

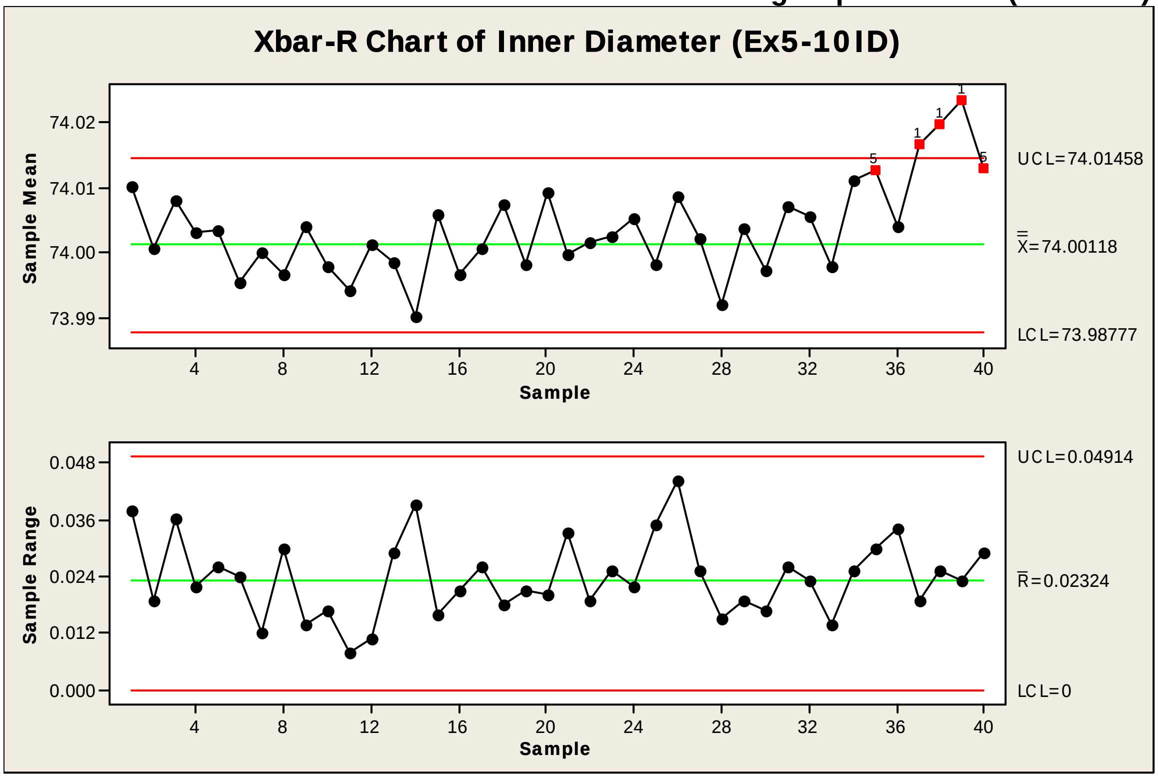
Under "Estimate" select Rbar as method to estimate standard deviation.



$$\begin{aligned}
\hat{p} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\
&= \Pr\{x < 73.95\} + \Pr\{x > 74.05\} \\
&= \Pr\{x < 73.95\} + \left[1 - \Pr\{x < 74.05\}\right] \\
&= \Phi\left(\frac{73.95 - 74.00118}{0.009991}\right) + \left[1 - \Phi\left(\frac{74.05 - 74.00118}{0.009991}\right)\right] \\
&= \Phi(-5.123) + 1 - \Phi(4.886) \\
&= 0 + 1 - 1 \\
&= 0
\end{aligned}$$

5-10☺.

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R (Ex5-10ID)




```
Test Results for Xbar Chart of Ex5-10ID
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  37, 38, 39
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  35, 37, 38, 39, 40
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  38, 39, 40
```

The control charts indicate that the process is in control, until the $\bar{x}$ -value from the 37th sample is plotted. Since this point and the three subsequent points plot above the upper control limit, an assignable cause has likely occurred, increasing the process mean.

5-11 (5-9).

$n=10;\ \mu=80$ in-lb; $\sigma_x=10$ in-lb; and $A=0.949;\ B_6=1.669;\ B_5=0.276$

$$\text{centerline}_{\bar{x}}=\mu=80$$
$$\text{UCL}_{\bar{x}}=\mu+A\sigma_x=80+0.949(10)=89.49$$
$$\text{LCL}_{\bar{x}}=\mu-A\sigma_x=80-0.949(10)=70.51$$
$$\text{centerline}_S=c_4\sigma_x=0.9727(10)=9.727$$
$$\text{UCL}_S=B_6\sigma_x=1.669(10)=16.69$$
$$\text{LCL}_S=B_5\sigma_x=0.276(10)=2.76$$

5-12* (5-10).

$n=6$ items/sample; $\sum_{i=1}^{50}\bar{x}_i=2000;\ \sum_{i=1}^{50}R_i=200;\ m=50$ samples

(a)

$$\bar{\bar{x}}=\frac{\sum_{i=1}^{50}\bar{x}_i}{m}=\frac{2000}{50}=40;\quad \bar{R}=\frac{\sum_{i=1}^{50}R_i}{m}=\frac{200}{50}=4$$
$$\text{UCL}_{\bar{x}}=\bar{\bar{x}}+A_2\bar{R}=40+0.483(4)=41.932$$
$$\text{LCL}_{\bar{x}}=\bar{\bar{x}}-A_2\bar{R}=40-0.483(4)=38.068$$
$$\text{UCL}_R=D_4\bar{R}=2.004(4)=8.016$$
$$\text{LCL}_R=D_3\bar{R}=0(4)=0$$

(b)

natural tolerance limits: $\bar{\bar{x}}\pm3\hat{\sigma}_x=\bar{\bar{x}}\pm3\left(\bar{R}/d_2\right)=40\pm3(4/2.534)=[35.264,44.736]$

(c)

$\hat{C}_P=\dfrac{\text{USL - LSL}}{6\hat{\sigma}_x}=\dfrac{+5.0-(-5.0)}{6(1.579)}=1.056$, so the process is not capable.

(d)

$\hat{p}_{\text{scrap}}=\Pr\{x<\text{LSL}\}=\Pr\{x<36\}=\Phi\left(\dfrac{36-40}{1.579}\right)=\Phi(-2.533)=0.0057$, or 0.57%.

$\hat{p}_{\text{rework}}=\Pr\{x>\text{USL}\}=1-\Pr\{x<\text{USL}\}=1-\Phi\left(\dfrac{47-40}{1.579}\right)=1-\Phi(4.433)=1-0.999995=0.000005$ or 0.0005%.

(e)

First, center the process at 41, not 40, to reduce scrap and rework costs. Second, reduce variability such that the natural process tolerance limits are closer to, say, $\hat{\sigma}_x\approx1.253$.

5-13* (5-11).

$n = 4$ items/subgroup; $\sum_{i=1}^{50} \bar{x}_i = 1000$; $\sum_{i=1}^{50} S_i = 72$; $m = 50$ subgroups

(a)

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{50} \bar{x}_i}{m} = \frac{1000}{50} = 20$$

$$\bar{S} = \frac{\sum_{i=1}^{50} S_i}{m} = \frac{72}{50} = 1.44$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_3\bar{S} = 20 + 1.628(1.44) = 22.34$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_3\bar{S} = 20 - 1.628(1.44) = 17.66$$

$$\text{UCL}_S = B_4\bar{S} = 2.266(1.44) = 3.26$$

$$\text{LCL}_S = B_3\bar{S} = 0(1.44) = 0$$

(b)

natural process tolerance limits: $\bar{\bar{x}} \pm 3\hat{\sigma}_x = \bar{\bar{x}} \pm 3\left(\frac{\bar{S}}{c_4}\right) = 20 \pm 3\left(\frac{1.44}{0.9213}\right) = [15.3, 24.7]$

(c)

$\hat{C}_P = \frac{\text{USL - LSL}}{6\hat{\sigma}_x} = \frac{+4.0-(-4.0)}{6(1.44/0.9213)} = 0.85$, so the process is not capable.

(d)

$\hat{p}_{\text{rework}} = \Pr\{x > \text{USL}\} = 1 - \Pr\{x \le \text{USL}\} = 1 - \Phi\left(\frac{23-20}{1.44/0.9213}\right) = 1 - \Phi(1.919) = 1 - 0.9725 = 0.0275$

or 2.75%.

$\hat{p}_{\text{scrap}} = \Pr\{x < \text{LSL}\} = \Phi\left(\frac{15-20}{1.44/0.9213}\right) = \Phi(-3.199) = 0.00069$, or 0.069%

Total = 2.88% + 0.069% = 2.949%

(e)

$\hat{p}_{\text{rework}} = 1 - \Phi\left(\frac{23-19}{1.44/0.9213}\right) = 1 - \Phi(2.56) = 1 - 0.99477 = 0.00523$, or 0.523%

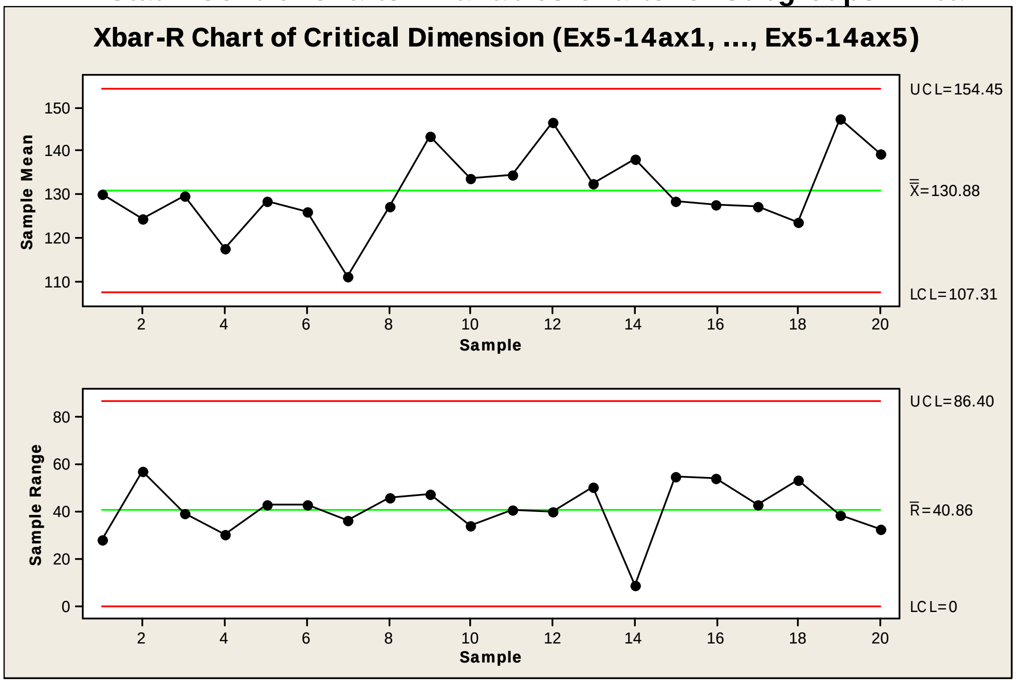
$\hat{p}_{\text{scrap}} = \Phi\left(\frac{15-19}{1.44/0.9213}\right) = \Phi(-2.56) = 0.00523$, or 0.523%

Total = 0.523% + 0.523% = 1.046%

Centering the process would reduce rework, but increase scrap. A cost analysis is needed to make the final decision. An alternative would be to work to improve the process by reducing variability.

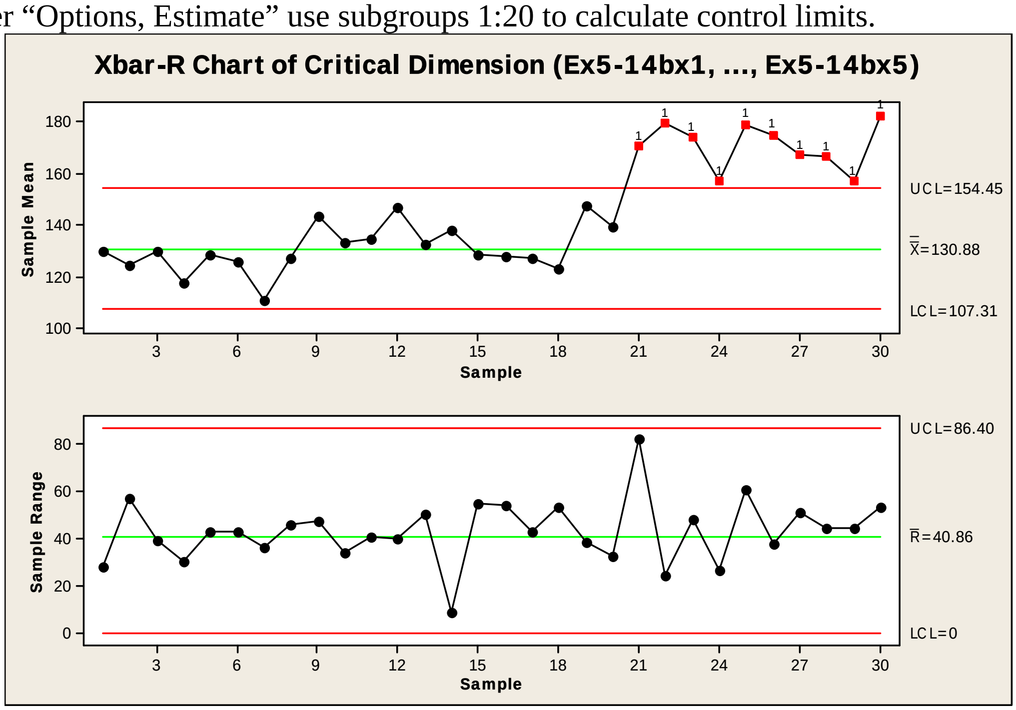
5-14 (5-12).
(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R



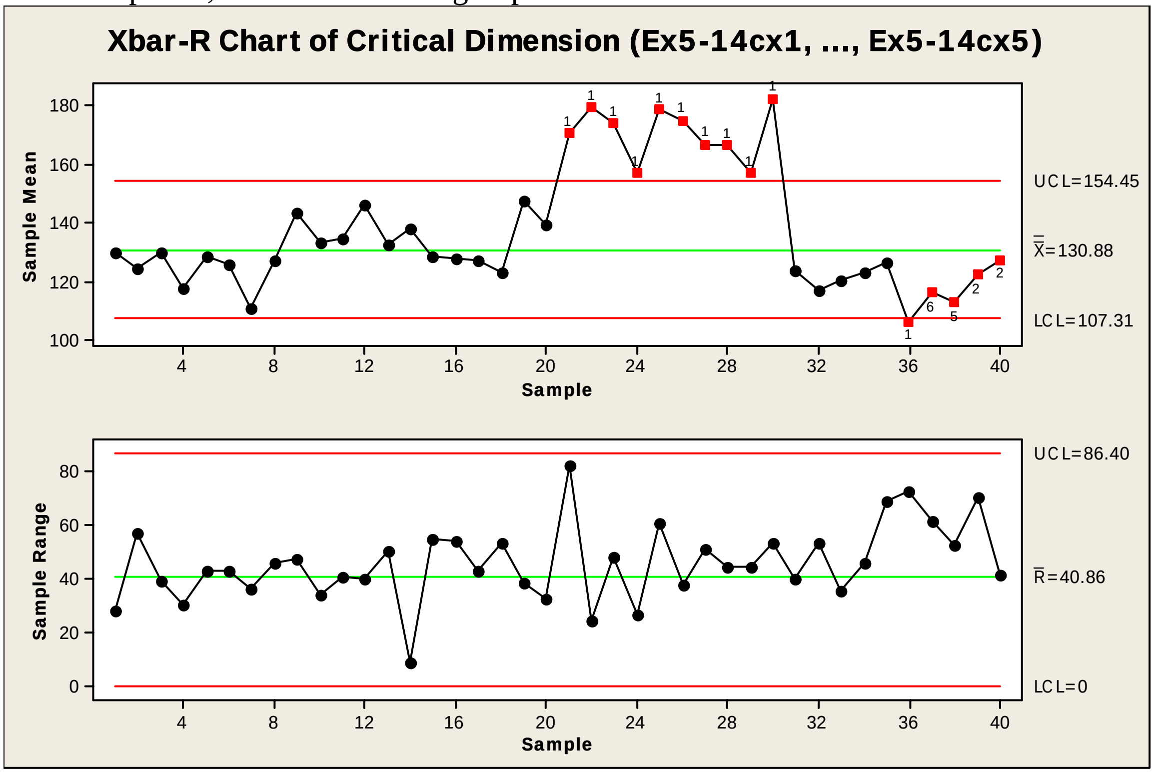
The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

(b)
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R
Under "Options, Estimate" use subgroups 1:20 to calculate control limits.



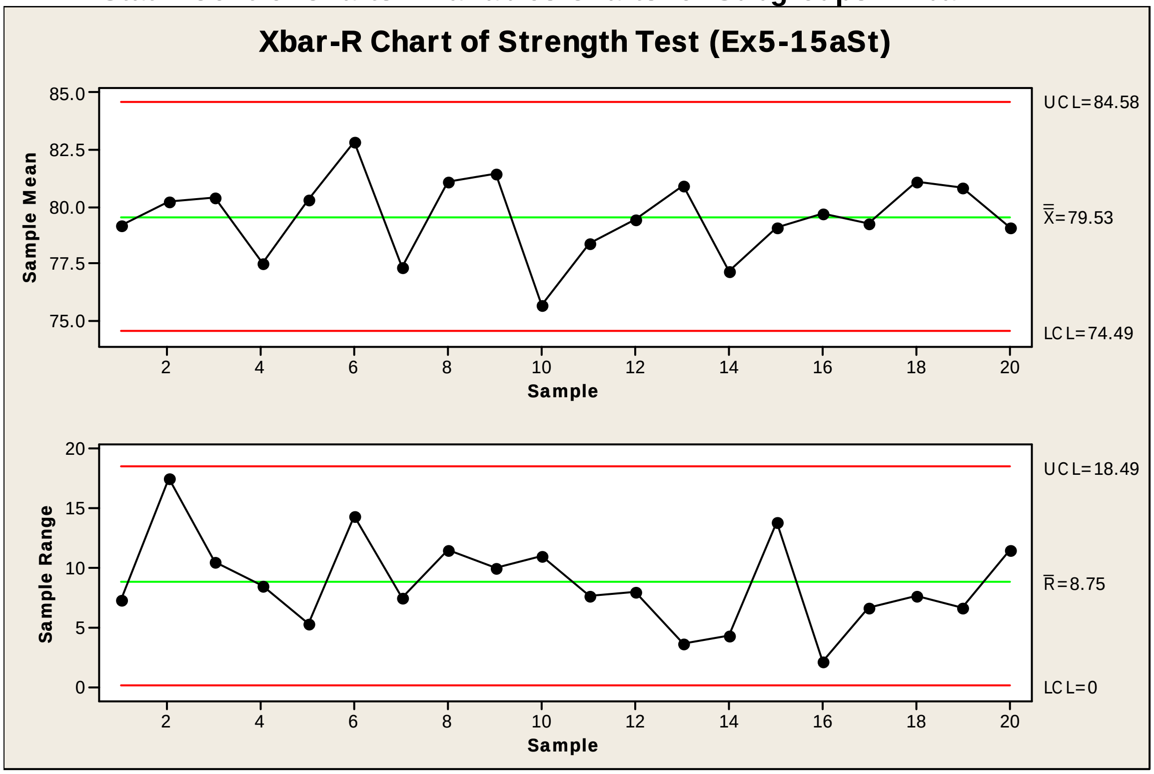
Starting at Sample #21, the process average has shifted to above the UCL = 154.45.

5-14 continued
(c)
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R
Under "Options, Estimate" use subgroups 1:20 to calculate control limits.



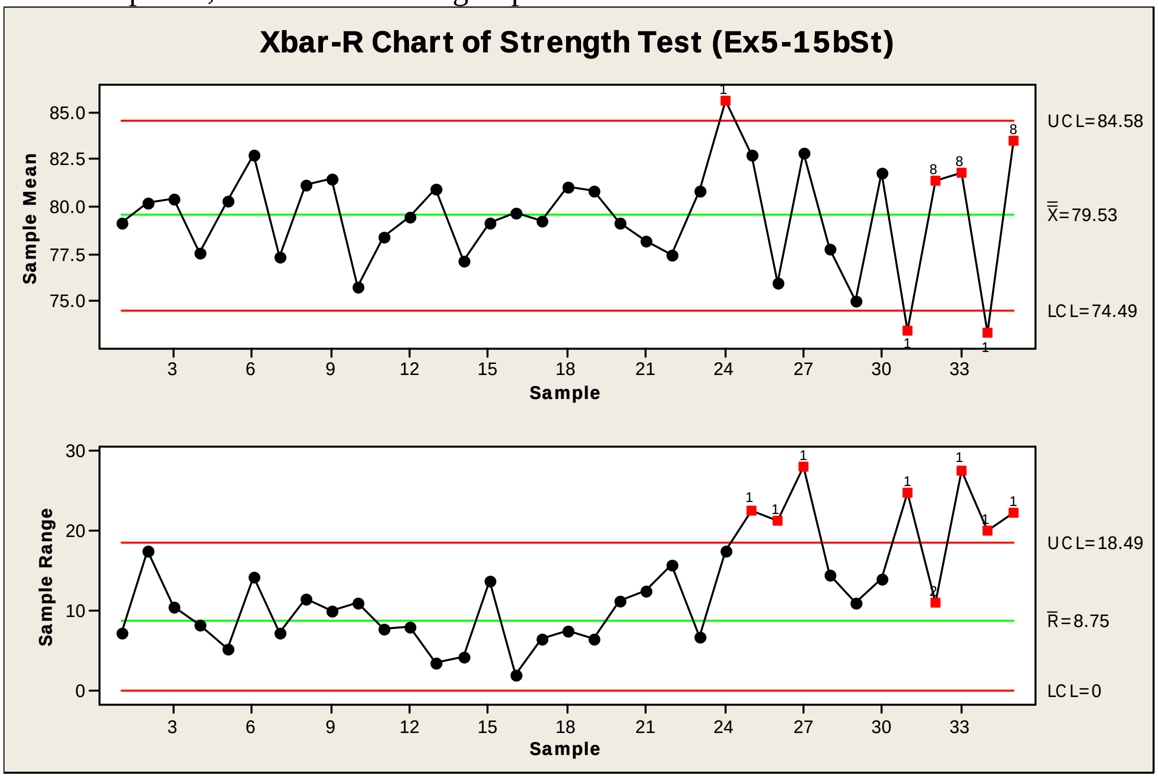
The adjustment overcompensated for the upward shift. The process average is now between $\overline{\overline{x}}$ and the LCL, with a run of ten points below the centerline, and one sample (#36) below the LCL.

5-15* (5-13).
(a)
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R



Yes, the process is in control—though we should watch for a possible cyclic pattern in the averages.

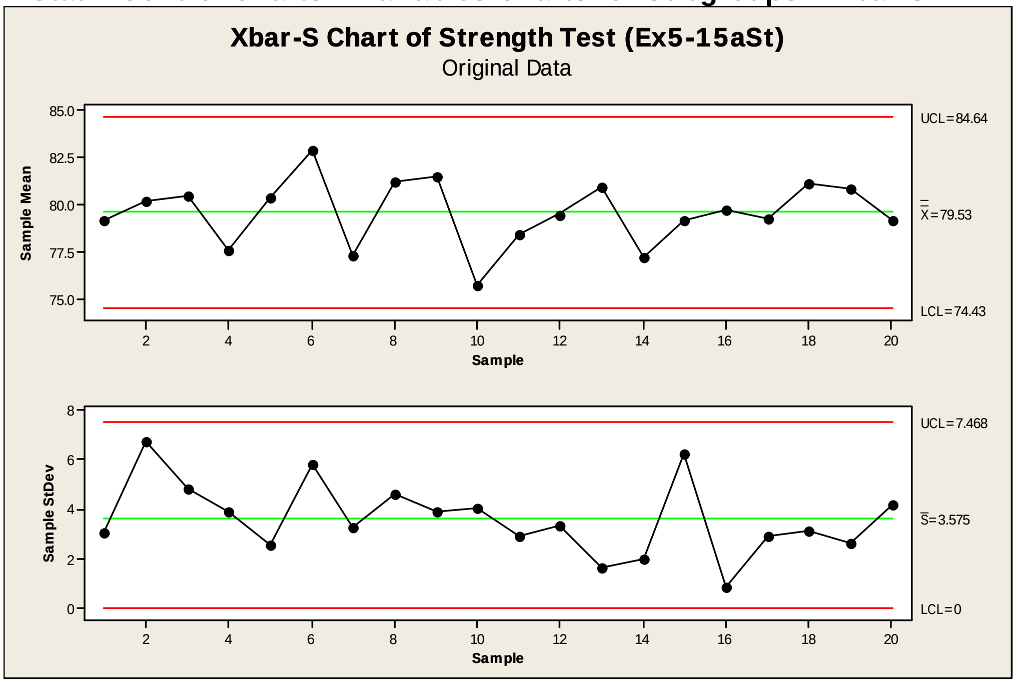
5-15 continued
(b)
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R
Under "Options, Estimate" use subgroups 1:20 to calculate control limits.



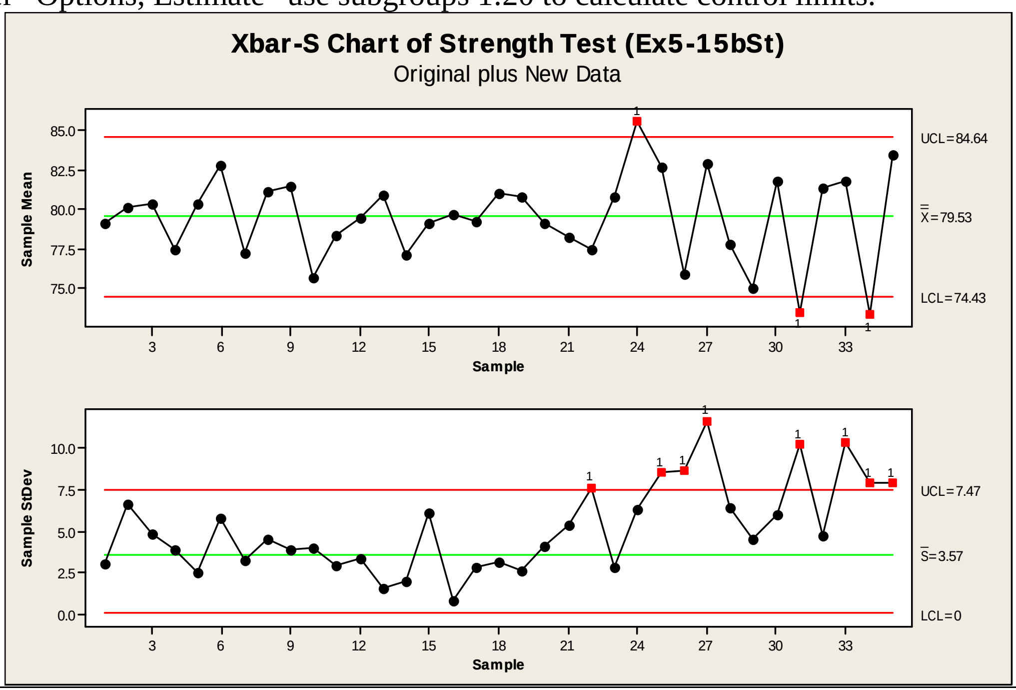
Test Results for R Chart of Ex5-15bSt
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  25, 26, 27, 31, 33, 34, 35
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  32, 33, 34, 35
```

A strongly cyclic pattern in the averages is now evident, but more importantly, there are several out-of-control points on the range chart.

5-16 (5-14).
(a)
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S



Under "Options, Estimate" use subgroups 1:20 to calculate control limits.



Test Results for Xbar Chart of Ex5-15bSt

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  24, 31, 34
```

Test Results for S Chart of Ex5-15bSt

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  22, 25, 26, 27, 31, 33, 34, 35
```

5-16 continued
(b)
Yes, the *s* chart detects the change in process variability more quickly than the *R* chart did, at sample #22 versus sample #24.

5-17 (5-15).
$n_{\text{old}} = 5;\ \bar{\bar{x}}_{\text{old}} = 34.00;\ \bar{R}_{\text{old}} = 4.7$

(a)
for $n_{\text{new}} = 3$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} + A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 34 + 1.023\left[\frac{1.693}{2.326}\right](4.7) = 37.50$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} - A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 34 - 1.023\left[\frac{1.693}{2.326}\right](4.7) = 30.50$$

$$\text{UCL}_{R} = D_{4(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 2.574\left[\frac{1.693}{2.326}\right](4.7) = 8.81$$

$$\text{CL}_{R} = \bar{R}_{\text{new}} = \left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = \left[\frac{1.693}{2.326}\right](4.7) = 3.42$$

$$\text{LCL}_{R} = D_{3(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 0\left[\frac{1.693}{2.326}\right](4.7) = 0$$

(b)
The $\bar{x}$ control limits for $n = 5$ are "tighter" (31.29, 36.72) than those for $n = 3$ (30.50, 37.50). This means a 2σ shift in the mean would be detected more quickly with a sample size of $n = 5$.

5-17 continued
(c)
for n = 8

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} + A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 34 + 0.373\left[\frac{2.847}{2.326}\right](4.7) = 36.15$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} - A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 34 - 0.373\left[\frac{2.847}{2.326}\right](4.7) = 31.85$$

$$\text{UCL}_{R} = D_{4(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 1.864\left[\frac{2.847}{2.326}\right](4.7) = 10.72$$

$$\text{CL}_{R} = \bar{R}_{\text{new}} = \left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = \left[\frac{2.847}{2.326}\right](4.7) = 5.75$$

$$\text{LCL}_{R} = D_{3(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 0.136\left[\frac{2.847}{2.326}\right](4.7) = 0.78$$

(d)
The $\bar{x}$ control limits for n = 8 are even "tighter" (31.85, 36.15), increasing the ability of the chart to quickly detect the 2σ shift in process mean.

5-18☺.
n_{old} = 5, $\bar{\bar{x}}_{\text{old}}$ = 74.001, $\bar{R}_{\text{old}}$ = 0.023, n_{new} = 3

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} + A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 74.001 + 1.023\left[\frac{1.693}{2.326}\right](0.023) = 74.018$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} - A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 74.001 - 1.023\left[\frac{1.693}{2.326}\right](0.023) = 73.984$$

$$\text{UCL}_{R} = D_{4(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 2.574\left[\frac{1.693}{2.326}\right](0.023) = 0.043$$

$$\text{CL}_{R} = \bar{R}_{\text{new}} = \left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = \left[\frac{1.693}{2.326}\right](0.023) = 0.017$$

$$\text{LCL}_{R} = D_{3(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 0\left[\frac{1.693}{2.326}\right](0.023) = 0$$

5-19 (5-16).

$n = 7; \quad \sum_{i=1}^{35} \bar{x}_i = 7805; \quad \sum_{i=1}^{35} R_i = 1200; \quad m = 35$ samples

(a)

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{35} \bar{x}_i}{m} = \frac{7805}{35} = 223$$

$$\bar{R} = \frac{\sum_{i=1}^{35} R_i}{m} = \frac{1200}{35} = 34.29$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2 \bar{R} = 223 + 0.419(34.29) = 237.37$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2 \bar{R} = 223 - 0.419(34.29) = 208.63$$

$$\text{UCL}_R = D_4 \bar{R} = 1.924(34.29) = 65.97$$

$$\text{LCL}_R = D_3 \bar{R} = 0.076(34.29) = 2.61$$

(b)

$$\hat{\mu} = \bar{\bar{x}} = 223; \quad \hat{\sigma}_x = \bar{R} / d_2 = 34.29 / 2.704 = 12.68$$

(c)

$\hat{C}_P = \dfrac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \dfrac{+35 - (-35)}{6(12.68)} = 0.92$, the process is not capable of meeting specifications.

$$\hat{p} = \Pr\{x > \text{USL}\} + \Pr\{x < \text{LSL}\} = 1 - \Pr\{x < \text{USL}\} + \Pr\{x < \text{LSL}\} = 1 - \Pr\{x \le 255\} + \Pr\{x \le 185\}$$

$$= 1 - \Phi\left(\frac{255 - 223}{12.68}\right) + \Phi\left(\frac{185 - 223}{12.68}\right) = 1 - \Phi(2.52) + \Phi(-3.00) = 1 - 0.99413 + 0.00135 = 0.0072$$

(d)

The process mean should be located at the nominal dimension, 220, to minimize non-conforming units.

$$\hat{p} = 1 - \Phi\left(\frac{255 - 220}{12.68}\right) + \Phi\left(\frac{185 - 220}{12.68}\right) = 1 - \Phi(2.76) + \Phi(-2.76) = 1 - 0.99711 + 0.00289 = 0.00578$$

5-20 (5-17).

$$n=5; \quad \sum_{i=1}^{25}\bar{x}_i = 662.50; \quad \sum_{i=1}^{25} R_i = 9.00; \quad m = 25 \text{ samples}$$

(a)

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{25}\bar{x}_i}{m} = \frac{662.50}{25} = 26.50$$

$$\bar{R} = \frac{\sum_{i=1}^{25} R_i}{m} = \frac{9.00}{25} = 0.36$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 26.50 + 0.577(0.36) = 26.71$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 26.50 - 0.577(0.36) = 26.29$$

$$\text{UCL}_R = D_4\bar{R} = 2.114(0.36) = 0.76$$

$$\text{LCL}_R = D_3\bar{R} = 0(0.36) = 0$$

(b)

$$\hat{\sigma}_x = \bar{R}/d_2 = 0.36/2.326 = 0.155$$

$$\hat{p} = \Pr\{x > \text{USL}\} + \Pr\{x < \text{LSL}\} = 1 - \Pr\{x \le \text{USL}\} + \Pr\{x < \text{LSL}\}$$

$$= 1 - \Phi\left(\frac{26.90 - 26.50}{0.155}\right) + \Phi\left(\frac{25.90 - 26.50}{0.155}\right) = 1 - \Phi(2.58) + \Phi(-3.87) = 1 - 0.99506 + 0.00005$$

$$= 0.00499$$

(c)

$$\hat{p} = 1 - \Phi\left(\frac{26.90 - 26.40}{0.155}\right) + \Phi\left(\frac{25.90 - 26.40}{0.155}\right) = 1 - \Phi(3.23) + \Phi(-3.23)$$

$$= 1 - 0.99938 + 0.00062 = 0.00124$$

5-21 (5-18).
$n = 5;\ \bar{\bar{x}} = 20.0;\ \bar{S} = 1.5;\ m = 50$ samples

(a)
$\hat{\sigma}_x = \bar{S} / c_4 = 1.5 / 0.9400 = 1.60$

(b)
$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_3\bar{S} = 20.0 + 1.427(1.5) = 22.14$$
$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_3\bar{S} = 20.0 - 1.427(1.5) = 17.86$$
$$\text{UCL}_S = B_4\bar{S} = 2.089(1.5) = 3.13$$
$$\text{LCL}_S = B_3\bar{S} = 0(1.5) = 0$$

(c)
$$\Pr\{\text{in control}\} = \Pr\{\text{LCL} \le \bar{x} \le \text{UCL}\} = \Pr\{\bar{x} \le \text{UCL}\} - \Pr\{\bar{x} \le \text{LCL}\}$$
$$= \Phi\left(\frac{22.14 - 22}{1.6/\sqrt{5}}\right) - \Phi\left(\frac{17.86 - 22}{1.6/\sqrt{5}}\right) = \Phi(0.20) - \Phi(-5.79)$$
$$= 0.57926 - 0 = 0.57926$$

5-22 (5-19).
$$\Pr\{\text{detect}\} = 1 - \Pr\{\text{not detect}\} = 1 - [\Pr\{\text{LCL} \le \bar{x} \le \text{UCL}\}] = 1 - [\Pr\{\bar{x} \le \text{UCL}\} - \Pr\{\bar{x} \le \text{LCL}\}]$$
$$= 1 - \left[\Phi\left(\frac{\text{UCL}_{\bar{x}} - \mu_{\text{new}}}{\sigma_x/\sqrt{n}}\right) - \Phi\left(\frac{\text{LCL}_{\bar{x}} - \mu_{\text{new}}}{\sigma_x/\sqrt{n}}\right)\right] = 1 - \left[\Phi\left(\frac{209 - 188}{6/\sqrt{4}}\right) - \Phi\left(\frac{191 - 188}{6/\sqrt{4}}\right)\right]$$
$$= 1 - \Phi(7) + \Phi(1) = 1 - 1 + 0.84134 = 0.84134$$

5-23 (5-20).
$X \sim N;\quad n = 5;\quad \bar{\bar{x}} = 104;\quad \bar{R} = 9.30;\quad \text{USL=110};\quad \text{LSL=90}$
$\hat{\sigma}_x = \bar{R} / d_2 = 9.30 / 2.326 = 3.998$ and $6\hat{\sigma}_x = 6(3.998) = 23.99$ is larger than the width of the tolerance band, 2(10) = 20. So, even if the mean is located at the nominal dimension, 100, not all of the output will meet specification.
$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{+10 - (-10)}{6(3.998)} = 0.8338$$

5-24* (5-21).

$n = 2; \quad \mu = 10; \quad \sigma_x = 2.5$. These are standard values.

(a)

$\text{centerline}_{\bar{x}} = \mu = 10$

$\text{UCL}_{\bar{x}} = \mu + A\sigma_x = 10 + 2.121(2.5) = 15.30$

$\text{LCL}_{\bar{x}} = \mu - A\sigma_x = 10 - 2.121(2.5) = 4.70$

(b)

$\text{centerline}_R = d_2\sigma_x = 1.128(2.5) = 2.82$

$\text{UCL}_R = D_2\sigma = 3.686(2.5) = 9.22$

$\text{LCL}_R = D_1\sigma = 0(2.5) = 0$

(c)

$\text{centerline}_S = c_4\sigma_x = 0.7979(2.5) = 1.99$

$\text{UCL}_S = B_6\sigma = 2.606(2.5) = 6.52$

$\text{LCL}_S = B_5\sigma = 0(2.5) = 0$

5-25 (5-22).

$n = 5; \quad \overline{\overline{x}} = 20; \quad \overline{R} = 4.56; \quad m = 25$ samples

(a)

$$\text{UCL}_{\overline{x}} = \overline{\overline{x}} + A_2\overline{R} = 20 + 0.577(4.56) = 22.63$$
$$\text{LCL}_{\overline{x}} = \overline{\overline{x}} - A_2\overline{R} = 20 - 0.577(4.56) = 17.37$$
$$\text{UCL}_R = D_4\overline{R} = 2.114(4.56) = 9.64$$
$$\text{LCL}_R = D_3\overline{R} = 0(4.56) = 0$$

(b)

$\hat{\sigma}_x = \overline{R}/d_2 = 4.56/2.326 = 1.96$

(c)

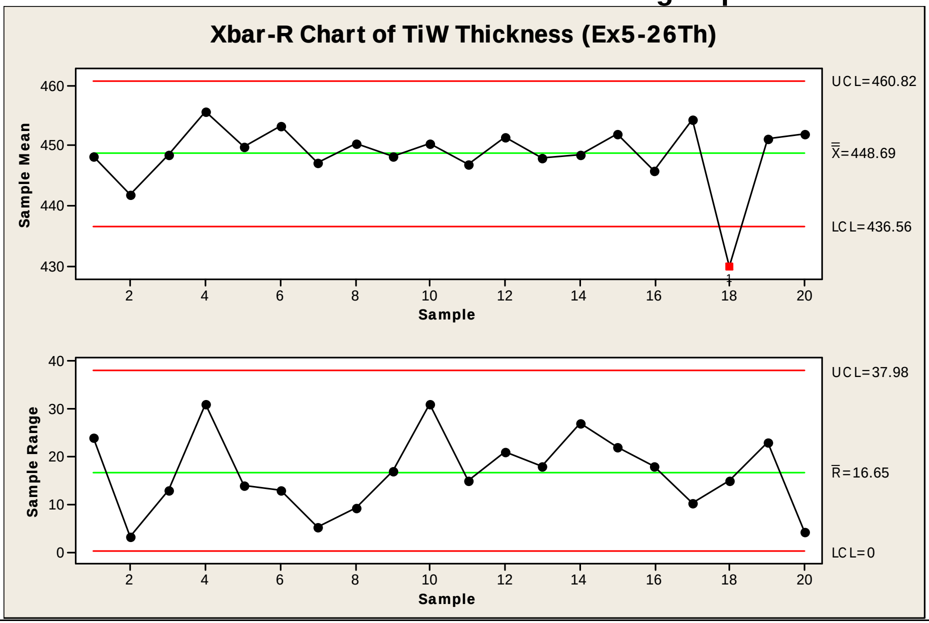
$\hat{C}_P = \dfrac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \dfrac{+5-(-5)}{6(1.96)} = 0.85$, so the process is not capable of meeting specifications.

(d)

$$\Pr\{\text{not detect}\} = \Pr\{\text{LCL} \le \overline{x} \le \text{UCL}\} = \Pr\{\overline{x} \le \text{UCL}\} - \Pr\{\overline{x} \le \text{LCL}\}$$
$$= \Phi\left(\frac{\text{UCL}_{\overline{x}} - \mu_{\text{new}}}{\hat{\sigma}_x/\sqrt{n}}\right) - \Phi\left(\frac{\text{LCL}_{\overline{x}} - \mu_{\text{new}}}{\hat{\sigma}_x/\sqrt{n}}\right) = \Phi\left(\frac{22.63-24}{1.96/\sqrt{5}}\right) - \Phi\left(\frac{17.37-24}{1.96/\sqrt{5}}\right)$$
$$= \Phi(-1.56) + \Phi(-7.56) = 0.05938 - 0 = 0.05938$$

5-26☺.

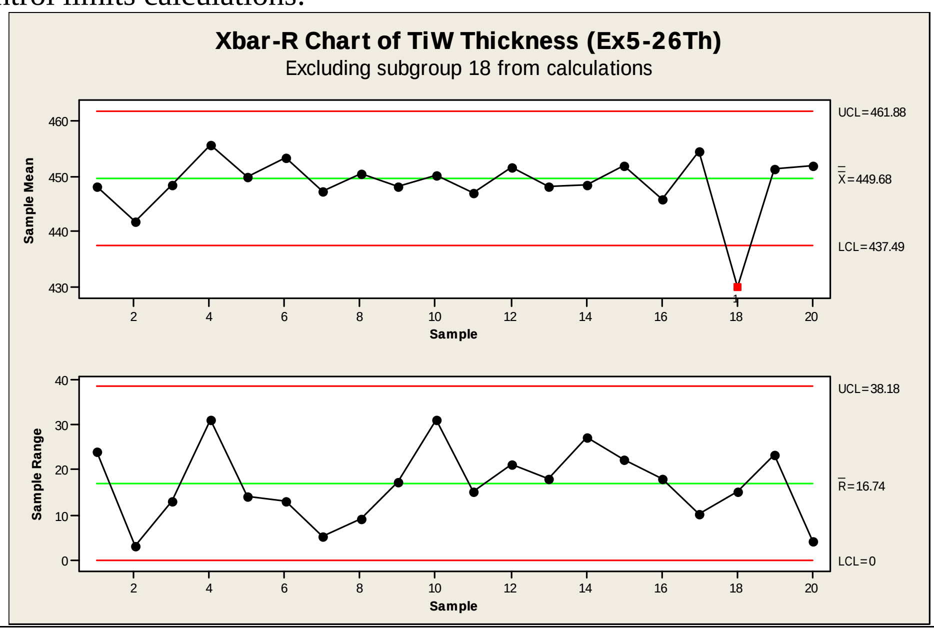
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R



Test Results for Xbar Chart of Ex5-26Th

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  18
```

The process is out of control on the $\bar{x}$ chart at subgroup 18. Excluding subgroup 18 from control limits calculations:



Test Results for Xbar Chart of Ex5-26Th

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  18
```

No additional subgroups are beyond the control limits, so these limits can be used for future production.

5-26 continued

(b)

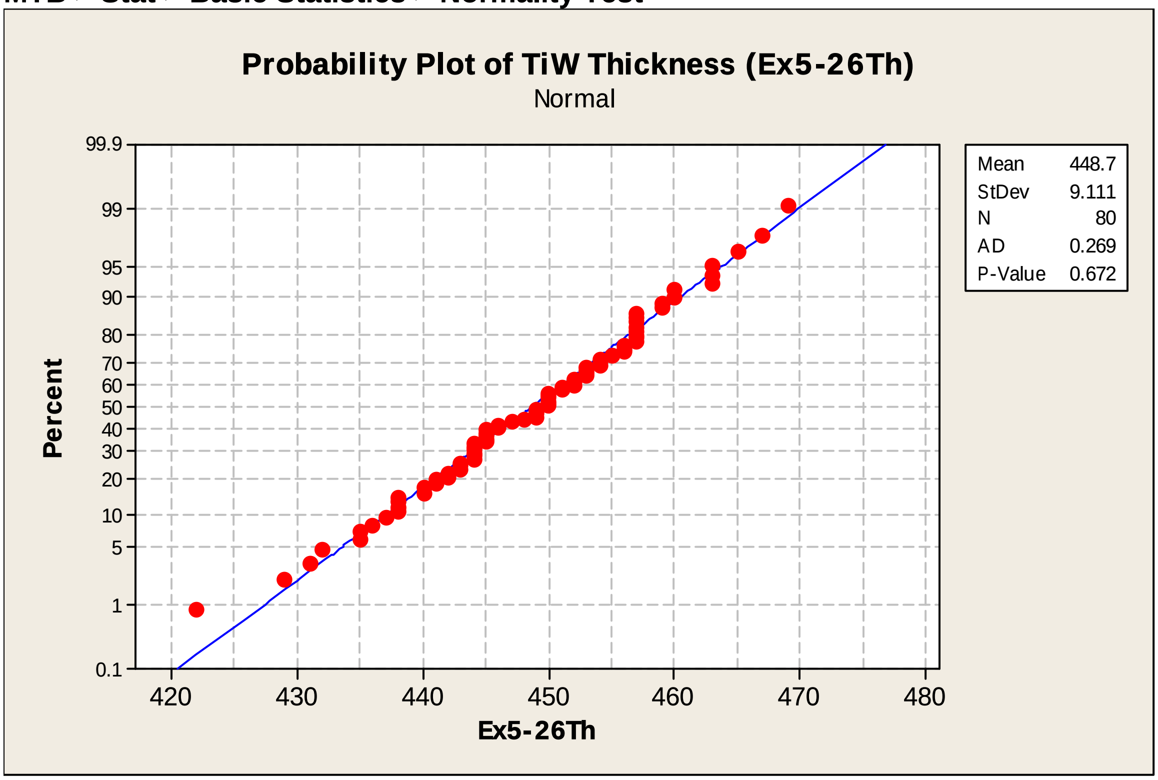
Excluding subgroup 18:

$\overline{x} = 449.68$

$\hat{\sigma}_x = \overline{R} / d_2 = 16.74 / 2.059 = 8.13$

(c)

MTB > Stat > Basic Statistics > Normality Test

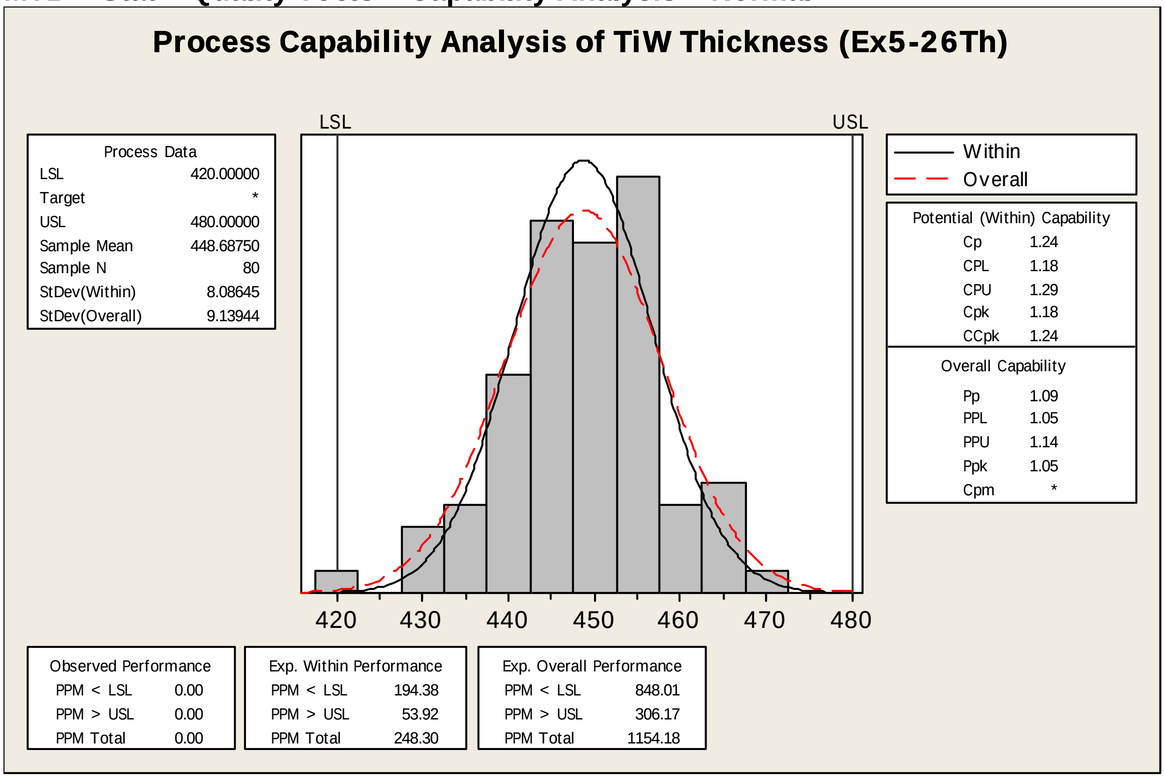


A normal probability plot of the TiW thickness measurements shows the distribution is close to normal.

5-26 continued
(d)
USL = +30, LSL = –30

$\hat{C}_P = \dfrac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \dfrac{+30-(-30)}{6(8.13)} = 1.23$, so the process is capable.

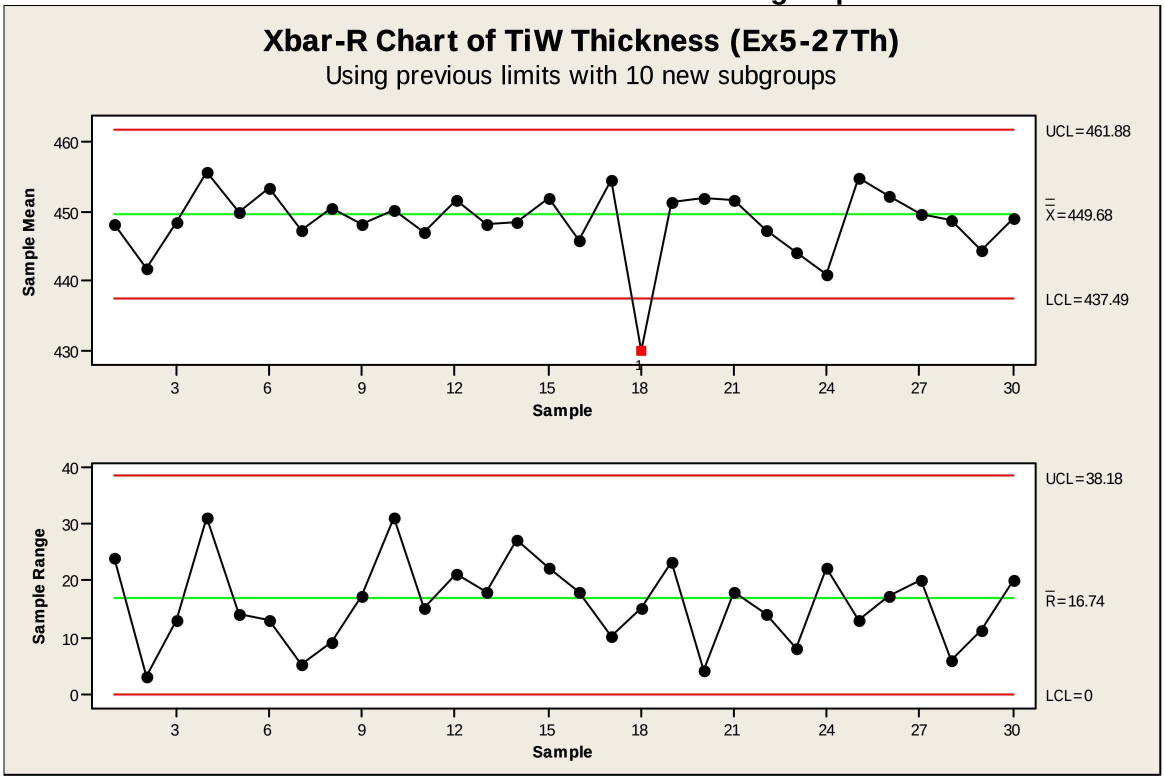
MTB > Stat > Quality Tools > Capability Analysis > Normal



The Potential (Within) Capability, Cp = 1.24, is estimated from the within-subgroup variation, or in other words, σ_x is estimated using $\bar{R}$. This is the same result as the manual calculation.

5-27☺.

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R



Test Results for Xbar Chart of Ex5-27Th
`TEST 1. One point more than 3.00 standard deviations from center line.`
`Test Failed at points:  18`

The process continues to be in a state of statistical control.

5-28☺.

$n_{\text{old}} = 4; \quad \bar{\bar{x}}_{\text{old}}\ 449.68; \quad \bar{R}_{\text{ old}} = 16.74; \quad n_{\text{new}} = 2$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} + A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 449.68 + 1.880\left[\frac{1.128}{2.059}\right](16.74) = 466.92$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}}_{\text{old}} - A_{2(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 449.68 - 1.880\left[\frac{1.128}{2.059}\right](16.74) = 432.44$$

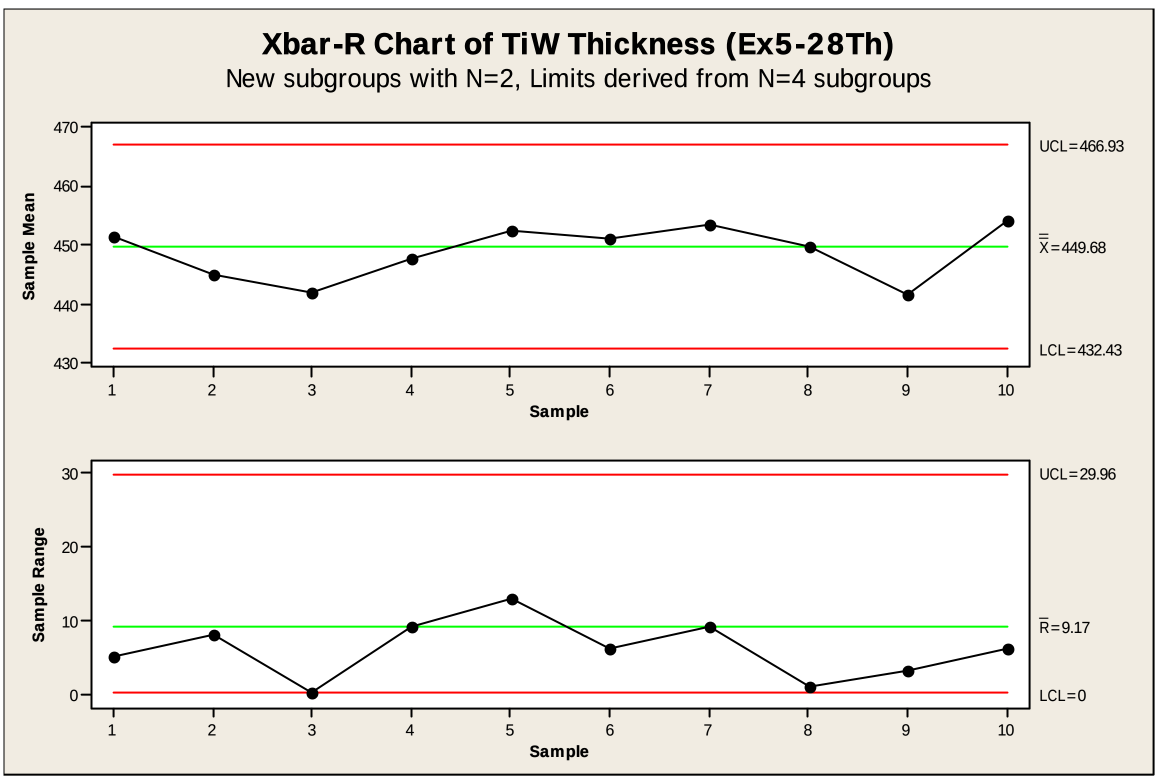
$$\text{UCL}_{R} = D_{4(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 3.267\left[\frac{1.128}{2.059}\right](16.74) = 29.96$$

$$\text{CL}_{R} = \bar{R}_{\text{new}} = \left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = \left[\frac{1.128}{2.059}\right](16.74) = 9.17$$

$$\text{LCL}_{R} = D_{3(\text{new})}\left[\frac{d_{2(\text{new})}}{d_{2(\text{old})}}\right]\bar{R}_{\text{old}} = 0\left[\frac{1.128}{2.059}\right](16.74) = 0$$

$$\hat{\sigma}_{\text{new}} = \bar{R}_{\text{new}} \big/ d_{2(\text{new})} = 9.17/1.128 = 8.13$$

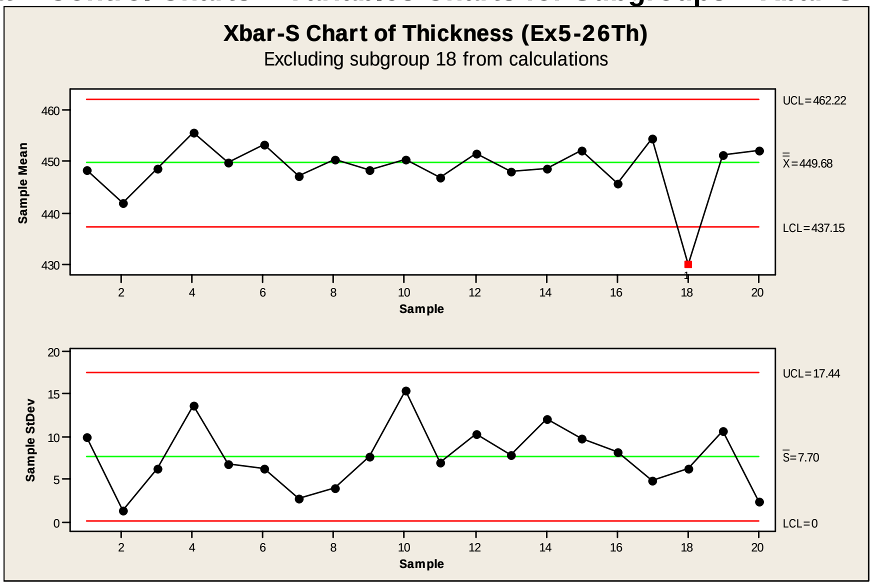
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R
Select Xbar-R options, Parameters, and enter new parameter values.



The process remains in statistical control.

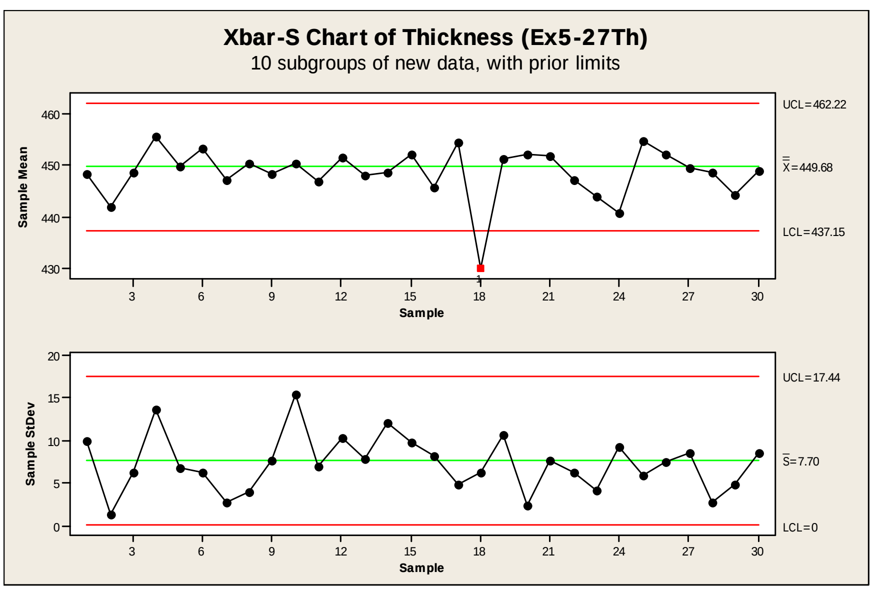
5-29☺.
The process is out of control on the $\bar{x}$ chart at subgroup 18. After finding assignable cause, exclude subgroup 18 from control limits calculations:

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S




```
Xbar-S Chart of Ex5-26Th
Test Results for Xbar Chart of Ex5-26Th
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  18
```

No additional subgroups are beyond the control limits, so these limits can be used for future production.



The process remains in statistical control.

5-30 (5-23).

$$n = 6; \quad \sum_{i=1}^{30} \bar{x}_i = 6000; \quad \sum_{i=1}^{30} R_i = 150; \quad m = 30 \text{ samples}$$

(a)

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{30} \bar{x}_i}{m} = \frac{6000}{30} = 200$$

$$\bar{R} = \frac{\sum_{i=1}^{30} R_i}{m} = \frac{150}{30} = 5$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2 \bar{R} = 200 + 0.483(5) = 202.42$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2 \bar{R} = 200 - 0.483(5) = 197.59$$

$$\text{UCL}_R = D_4 \bar{R} = 2.004(5) = 10.02$$

$$\text{LCL}_R = D_3 \bar{R} = 0(5) = 0$$

(b)

$$\hat{\sigma}_x = \bar{R} / d_2 = 5 / 2.534 = 1.97$$

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{+5 - (-5)}{6(1.97)} = 0.85$$

The process is not capable of meeting specification. Even though the process is centered at nominal, the variation is large relative to the tolerance.

(c)

$$\beta - \text{risk} = \Pr\{\text{not detect}\} = \Phi\left(\frac{202.42 - 199}{1.97/\sqrt{6}}\right) - \Phi\left(\frac{197.59 - 199}{1.97/\sqrt{6}}\right)$$

$$= \Phi(4.25) - \Phi(-1.75) = 1 - 0.04006 = 0.95994$$

5-31 (5-24).

$\mu_0 = 100; \quad L = 3; \quad n = 4; \quad \sigma = 6; \quad \mu_1 = 92$

$k = (\mu_1 - \mu_0)/\sigma = (92 - 100)/6 = -1.33$

$$\begin{aligned}
\Pr\{\text{detecting shift on 1st sample}\} &= 1 - \Pr\{\text{not detecting shift on 1st sample}\} \\
&= 1 - \beta \\
&= 1 - \left[\Phi\left(L - k\sqrt{n}\right) - \Phi\left(-L - k\sqrt{n}\right)\right] \\
&= 1 - \left[\Phi\left(3 - (-1.33)\sqrt{4}\right) - \Phi\left(-3 - (-1.33)\sqrt{4}\right)\right] \\
&= 1 - \left[\Phi(5.66) - \Phi(-0.34)\right] \\
&= 1 - \left[1 - 0.37\right] \\
&= 0.37
\end{aligned}$$

5-32 (5-25).

(a)

$\bar{\bar{x}} = 104.05; \quad \bar{R} = 3.95$

$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 104.05 + 0.577(3.95) = 106.329$

$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 104.05 - 0.577(3.95) = 101.771$

$\text{UCL}_R = D_4\bar{R} = 2.114(3.95) = 8.350$

$\text{LCL}_R = D_3\bar{R} = 0(3.95) = 0$

Sample #4 is out of control on the Range chart. So, excluding #4 and recalculating:

$\bar{\bar{x}} = 104; \quad \bar{R} = 3.579$

$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 104 + 0.577(3.579) = 106.065$

$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 104 - 0.577(3.579) = 101.935$

$\text{UCL}_R = D_4\bar{R} = 2.114(3.579) = 7.566$

$\text{LCL}_R = D_3\bar{R} = 0(3.579) = 0$

(b)

Without sample #4, $\hat{\sigma}_x = \bar{R}/d_2 = 3.579/2.326 = 1.539$

(c)

$\text{UNTL} = \bar{\bar{x}} + 3\hat{\sigma}_x = 104 + 3(1.539) = 108.62$

$\text{LNTL} = \bar{\bar{x}} - 3\hat{\sigma}_x = 104 - 3(1.539) = 99.38$

5-32 continued

(d)

$$\hat{p} = 1 - \Phi\left(\frac{107-104}{1.539}\right) + \Phi\left(\frac{99-104}{1.539}\right) = 1 - \Phi(1.95) + \Phi(-3.25) = 1 - 0.9744 + 0.0006 = 0.0262$$

(e)

To reduce the fraction nonconforming, first center the process at nominal.

$$\hat{p} = 1 - \Phi\left(\frac{107-103}{1.539}\right) + \Phi\left(\frac{99-103}{1.539}\right) = 1 - \Phi(2.60) + \Phi(-2.60) = 1 - 0.9953 + 0.0047 = 0.0094$$

Next work on reducing the variability; if $\hat{\sigma}_x = 0.667$, then almost 100% of parts will be within specification.

$$\hat{p} = 1 - \Phi\left(\frac{107-103}{0.667}\right) + \Phi\left(\frac{99-103}{0.667}\right) = 1 - \Phi(5.997) + \Phi(-5.997) = 1 - 1.0000 + 0.0000 = 0.0000$$

5-33 (5-26).

$$n = 5; \quad \sum_{i=1}^{30} \bar{x}_i = 607.8; \quad \sum_{i=1}^{30} R_i = 144; \quad m = 30$$

(a)

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{m} \bar{x}_i}{m} = \frac{607.8}{30} = 20.26$$

$$\bar{R} = \frac{\sum_{i=1}^{m} R_i}{m} = \frac{144}{30} = 4.8$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 20.26 + 0.577(4.8) = 23.03$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 20.26 - 0.577(4.8) = 17.49$$

$$\text{UCL}_R = D_4\bar{R} = 2.114(4.8) = 10.147$$

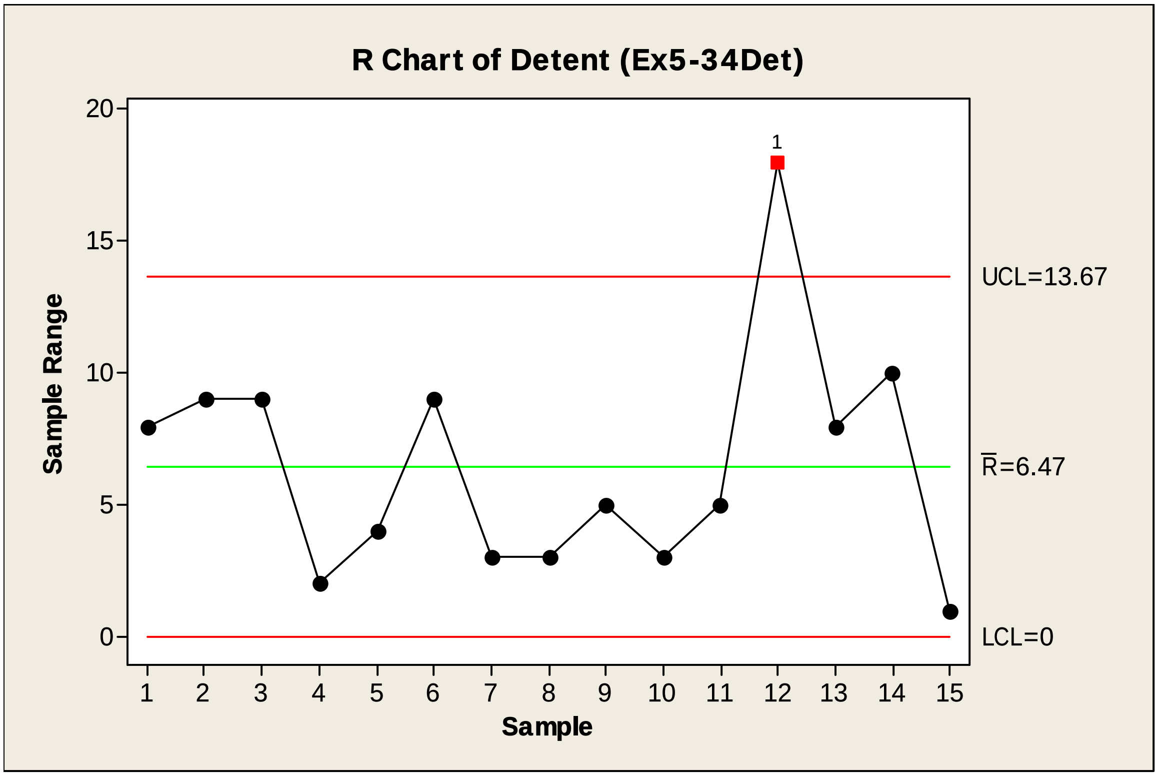
$$\text{LCL}_R = D_3\bar{R} = 0(4.8) = 0$$

(b)

$$\hat{\sigma}_x = \bar{R}/d_2 = 4.8/2.326 = 2.064$$

$$\hat{p} = \Pr\{x < \text{LSL}\} = \Phi\left(\frac{16-20.26}{2.064}\right) = \Phi(-2.064) = 0.0195$$

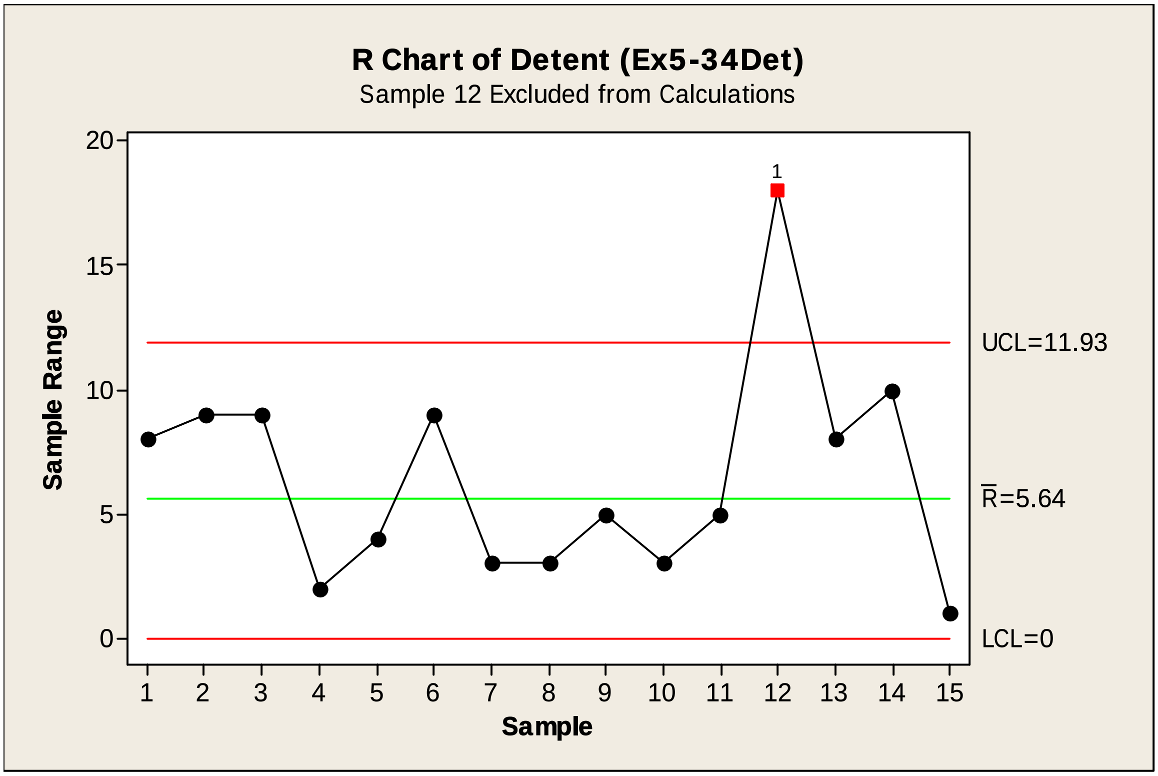
5-34 (5-27).
(a)
MTB > Stat > Control Charts > Variables Charts for Subgroups > R
Under "Options, Estimate" select Rbar as method to estimate standard deviation.



Test Results for R Chart of Ex5-34Det
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

Process is not in statistical control -- sample #12 exceeds the upper control limit on the Range chart.

5-34 continued
(b)
Excluding Sample Number 12:
MTB > Stat > Control Charts > Variables Charts for Subgroups > R
Under “Options, Estimate” omit subgroup 12 and select Rbar.



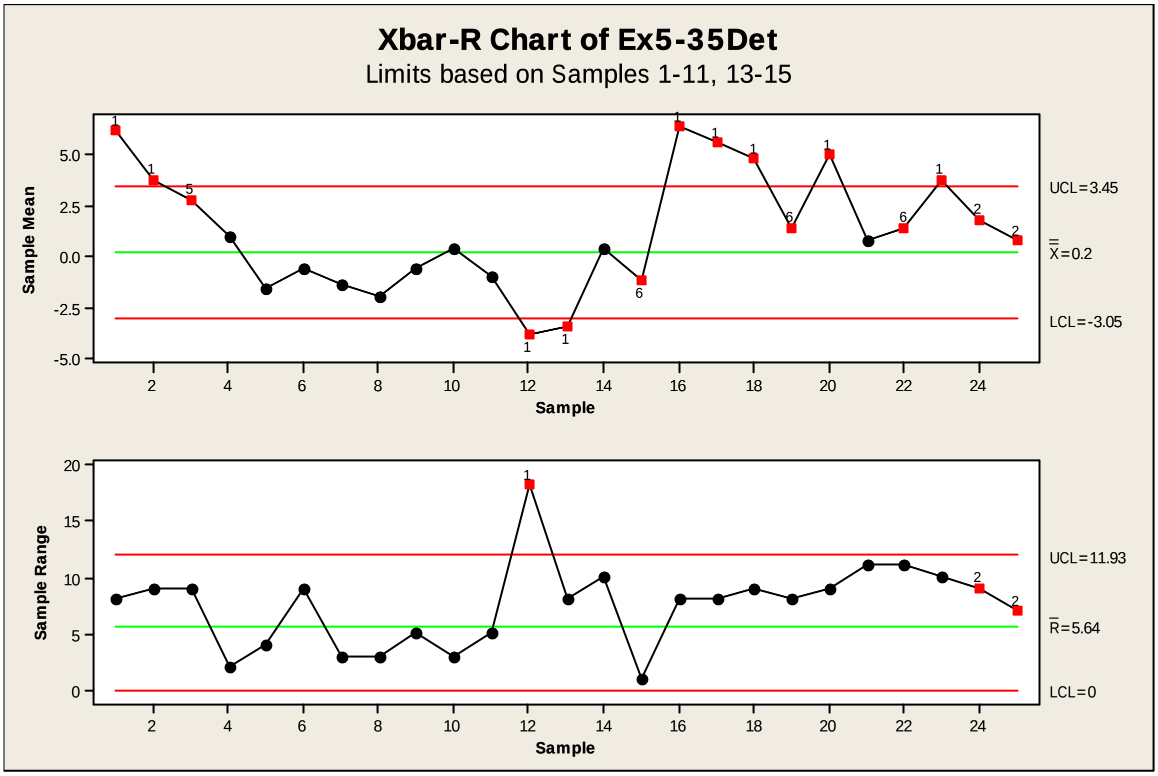
Test Results for R Chart of Ex5-34Det
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

(c)
Without sample #12: $\hat{\sigma}_x = \bar{R} / d_2 = 5.64 / 2.326 = 2.42$

(d)
Assume the cigar lighter detent is normally distributed. Without sample #12:

$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{0.3220 - 0.3200}{6(2.42 \times 0.0001)} = 1.38$$

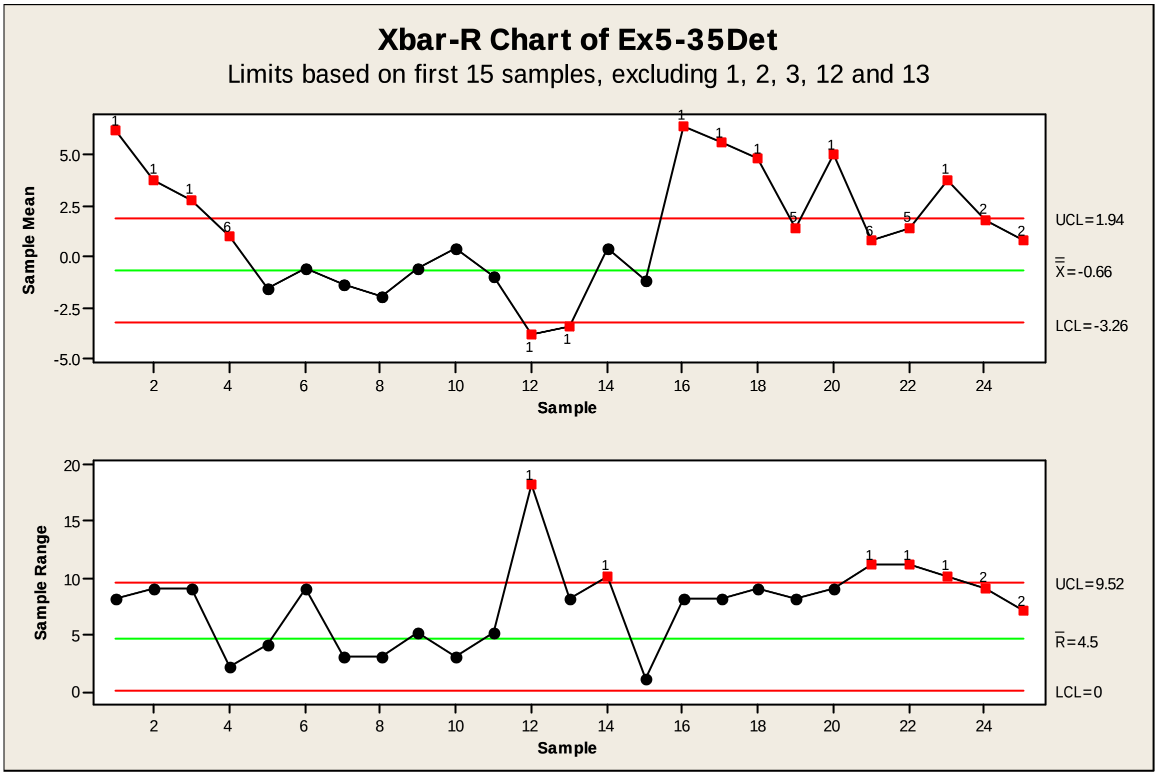
5-35 (5-28).
MTB > Stat > Control Charts > Variables Charts for Subgroups > R
Under "Options, Estimate" use subgroups 1:11 and 13:15, and select Rbar.




```
Test Results for Xbar Chart of Ex5-35Det
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 2, 12, 13, 16, 17, 18, 20, 23
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  24, 25
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  2, 3, 13, 17, 18, 20
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  15, 19, 20, 22, 23, 24

Test Results for R Chart of Ex5-35Det
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  24, 25
```

5-35 continued

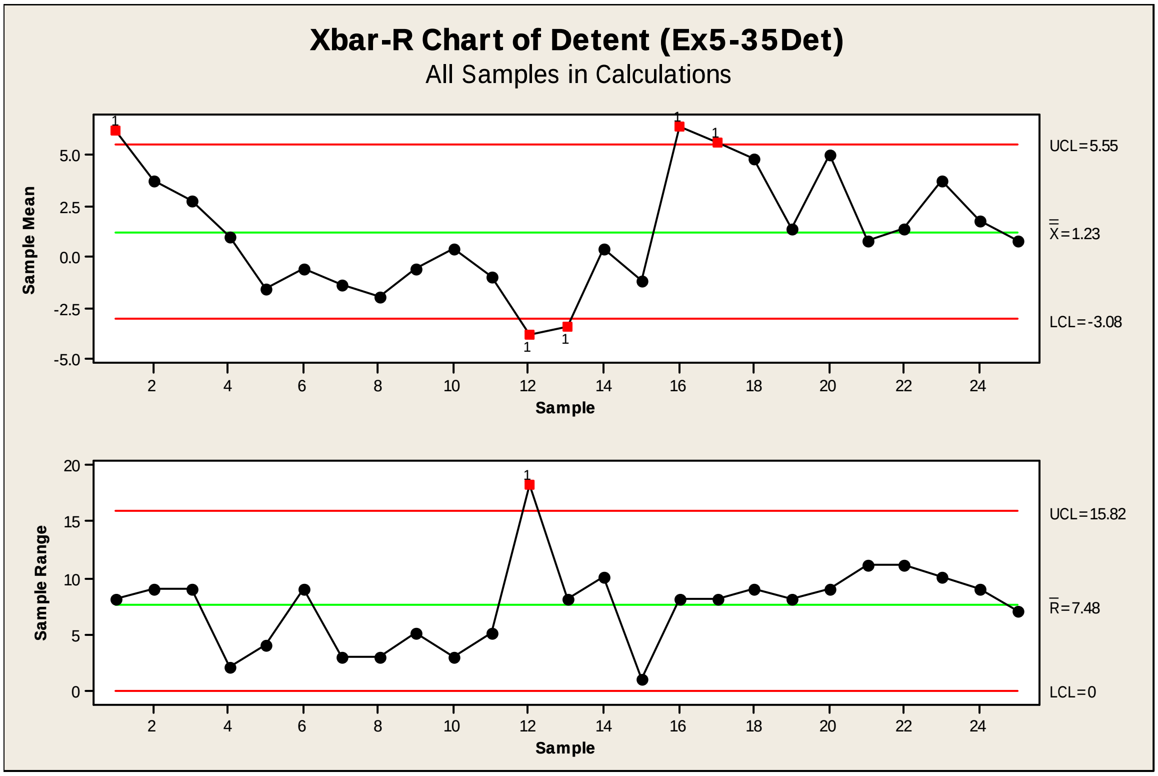
We are trying to establish trial control limits from the first 15 samples to monitor future production. Note that samples 1, 2, 12, and 13 are out of control on the $\bar{x}$ chart. If these samples are removed and the limits recalculated, sample 3 is also out of control on the $\bar{x}$ chart. Removing sample 3 gives



Sample 14 is now out of control on the R chart. No additional samples are out of control on the $\bar{x}$ chart. While the limits on the above charts may be used to monitor future production, the fact that 6 of 15 samples were out of control and eliminated from calculations is an early indication of process instability.

(a) Given the large number of points after sample 15 beyond both the $\bar{x}$ and R control limits on the charts above, the process appears to be unstable.

5-35 continued
(b)



With Test 1 only:

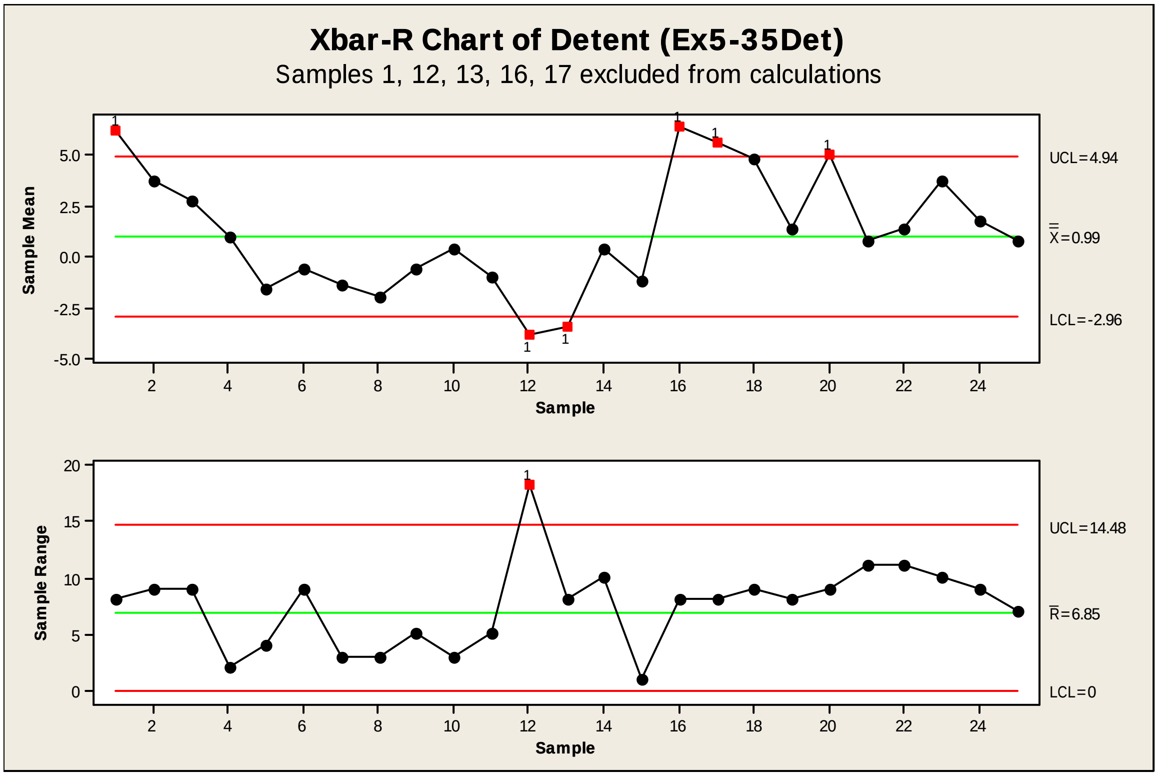
Test Results for Xbar Chart of Ex5-35Det

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 12, 13, 16, 17
```

Test Results for R Chart of Ex5-35Det

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

5-35 (b) continued
Removing samples 1, 12, 13, 16, and 17 from calculations:

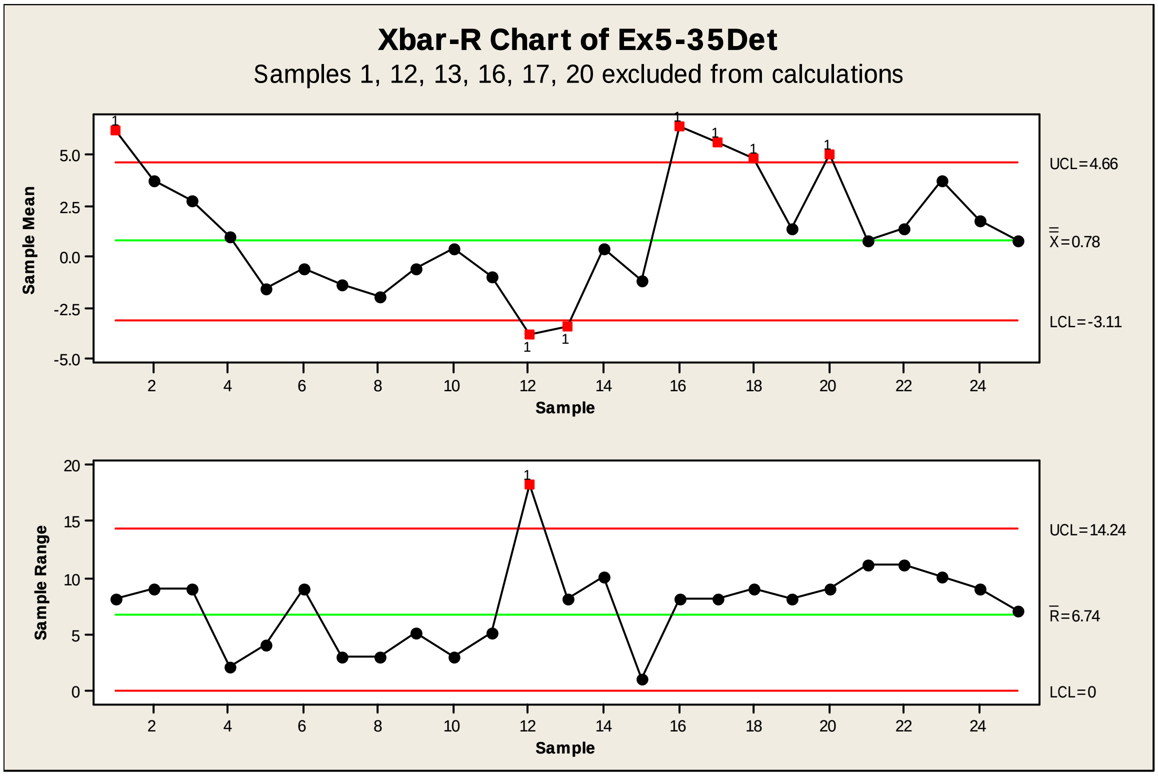


With Test 1 only:

Test Results for Xbar Chart of Ex5-35Det
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 12, 13, 16, 17, 20
```
Test Results for R Chart of Ex5-35Det
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

5-35 continued

Sample 20 is now also out of control. Removing sample 20 from calculations,



With Test 1 only:

```
Test Results for Xbar Chart of Ex5-35Det
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 12, 13, 16, 17, 18, 20
Test Results for R Chart of Ex5-35Det
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

Sample 18 is now out-of-control, for a total 7 of the 25 samples, with runs of points both above and below the centerline. This suggests that the process is inherently unstable, and that the sources of variation need to be identified and removed.

5-36 (5-29).

(a)

$$n=5;\quad m_x=20;\quad m_y=10;\quad \sum_{i=1}^{20}R_{x,i}=18.608;\quad \sum_{i=1}^{10}R_{y,i}=6.978$$

$$\hat{\sigma}_x=\bar{R}_x/d_2=\left(\sum_{i=1}^{20}R_{x,i}/m_x\right)\Big/d_2=(18.608/20)/2.326=0.400$$

$$\hat{\sigma}_y=\bar{R}_y/d_2=\left(\sum_{i=1}^{10}R_{y,i}/m_y\right)\Big/d_2=(6.978/10)/2.326=0.300$$

(b)

Want Pr{(x – y) < 0.09} = 0.006. Let $z = x - y$. Then

$$\hat{\sigma}_z=\sqrt{\hat{\sigma}_x^2+\hat{\sigma}_y^2}=\sqrt{0.4^2+0.3^2}=0.500$$

$$\Phi\left(\frac{0.09-z}{\hat{\sigma}_z}\right)=0.006$$

$$\Phi^{-1}\left(\frac{0.09-z}{0.500}\right)=\Phi(0.006)$$

$$\left(\frac{0.09-z}{0.500}\right)=-2.5121$$

$$z=+2.5121(0.500)+0.09=1.346$$

5-37 (5-30).

$$n=6;\quad \sum_{i=1}^{30}\bar{x}_i=12,870;\quad \sum_{i=1}^{30}R_i=1350;\quad m=30$$

(a)

$$\bar{R}=\frac{\sum_{i=1}^{m}R_i}{m}=\frac{1350}{30}=45.0$$

$$\text{UCL}_R=D_4\bar{R}=2.004(45.0)=90.18$$

$$\text{LCL}_R=D_3\bar{R}=0(45.0)=0$$

(b)

$$\hat{\mu}=\bar{\bar{x}}=\frac{\sum_{i=1}^{m}\bar{x}_i}{m}=\frac{12,870}{30}=429.0$$

$$\hat{\sigma}_x=\bar{R}/d_2=45.0/2.534=17.758$$

5-37 continued

(c)

USL = 440 + 40 = 480; LSL = 440 - 40 = 400

$$\hat{C}_p \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{480 - 400}{6(17.758)} = 0.751$$

$$\hat{p} = 1 - \Phi\left(\frac{480 - 429}{17.758}\right) + \Phi\left(\frac{400 - 429}{17.758}\right) = 1 - \Phi(2.87) + \Phi(-1.63) = 1 - 0.9979 + 0.0516 = 0.0537$$

(d)

To minimize fraction nonconforming the mean should be located at the nominal dimension (440) for a constant variance.

5-38 (5-31).

$$n = 4; \quad \sum_{i=1}^{30} \bar{x}_i = 12{,}870; \quad \sum_{i=1}^{30} S_i = 410; \quad m = 30$$

(a)

$$\bar{S} = \frac{\sum_{i=1}^{m} S_i}{m} = \frac{410}{30} = 13.667$$

$$\text{UCL}_S = B_4 \bar{S} = 2.266(13.667) = 30.969$$

$$\text{LCL}_S = B_3 \bar{S} = 0(13.667) = 0$$

(b)

$$\hat{\mu} = \bar{\bar{x}} = \frac{\sum_{i=1}^{m} \bar{x}_i}{m} = \frac{12{,}870}{30} = 429.0$$

$$\hat{\sigma}_x = \bar{S} / c_4 = 13.667 / 0.9213 = 14.834$$

5-39 (5-32).
(a)
$n = 4; \quad \mu = 100; \quad \sigma_x = 8$
$\text{UCL}_{\bar{x}} = \mu + 2\sigma_{\bar{x}} = \mu + 2\left(\sigma_x / \sqrt{n}\right) = 100 + 2\left(8/\sqrt{4}\right) = 108$
$\text{LCL}_{\bar{x}} = \mu - 2\sigma_{\bar{x}} = \mu - 2\left(\sigma_x / \sqrt{n}\right) = 100 - 2\left(8/\sqrt{4}\right) = 92$

(b)
$k = Z_{\alpha/2} = Z_{0.005/2} = Z_{0.0025} = 2.807$
$\text{UCL}_{\bar{x}} = \mu + k\sigma_{\bar{x}} = \mu + k\left(\sigma_x / \sqrt{n}\right) = 100 + 2.807\left(8/\sqrt{4}\right) = 111.228$
$\text{LCL}_{\bar{x}} = \mu - k\sigma_{\bar{x}} = \mu - k\left(\sigma_x / \sqrt{n}\right) = 100 - 2.807\left(8/\sqrt{4}\right) = 88.772$

5-40 (5-33).
$n = 5; \quad \text{UCL}_{\bar{x}} = 104; \quad \text{centerline}_{\bar{x}} = 100; \quad \text{LCL}_{\bar{x}} = 96; \quad k = 3; \quad \mu = 98; \quad \sigma_x = 8$
Pr{out-of-control signal by at least 3rd plot point}
$= 1 - \Pr\{\text{not detected by 3rd sample}\} = 1 - [\Pr\{\text{not detected}\}]^3$
$\Pr\{\text{not detected}\} = \Pr\{\text{LCL}_{\bar{x}} \le \bar{x} \le \text{UCL}_{\bar{x}}\} = \Pr\{\bar{x} \le \text{UCL}_{\bar{x}}\} - \Pr\{\bar{x} \le \text{LCL}_{\bar{x}}\}$

$$= \Phi\left(\frac{\text{UCL}_{\bar{x}} - \mu}{\sigma_{\bar{x}}}\right) - \Phi\left(\frac{\text{LCL}_{\bar{x}} - \mu}{\sigma_{\bar{x}}}\right) = \Phi\left(\frac{104-98}{8/\sqrt{5}}\right) - \Phi\left(\frac{96-98}{8/\sqrt{5}}\right) = \Phi(1.68) - \Phi(-0.56)$$

$= 0.9535 - 0.2877 = 0.6658$
$1 - [\Pr\{\text{not detected}\}]^3 = 1 - (0.6658)^3 = 0.7049$

5-41 (5-34).

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1 - \Pr\{\text{not detect}\}} = \frac{1}{1 - 0.6658} = 2.992$$

5-42 (5-35).

$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{\text{USL} - \text{LSL}}{6\left(\bar{S}/c_4\right)} = \frac{202.50\text{-}197.50}{6\left(1.000/0.9213\right)} = 0.7678$$

The process is not capable of meeting specifications.

5-43 (5-36).

$n = 4;\quad \mu = 200;\quad \sigma_x = 10$

(a)

$$\text{centerline}_S = c_4\sigma = 0.9213(10) = 9.213$$
$$\text{UCL}_S = B_6\sigma_x = 2.088(10) = 20.88$$
$$\text{LCL}_S = B_5\sigma_x = 0(10) = 0$$

(b)

$$k = Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$$
$$\text{UCL}_{\bar{x}} = \mu + k\sigma_{\bar{x}} = \mu + k\left(\sigma_x/\sqrt{n}\right) = 200 + 1.96\left(10/\sqrt{4}\right) = 209.8$$
$$\text{LCL}_{\bar{x}} = \mu - k\sigma_{\bar{x}} = \mu - k\left(\sigma_x/\sqrt{n}\right) = 200 - 1.96\left(10/\sqrt{4}\right) = 190.2$$

5-44 (5-37).

n = 9; USL = 600 + 20 = 620; LSL = 600 – 20 = 580

(a)

$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{\text{USL} - \text{LSL}}{6\left(\bar{R}/d_2\right)} = \frac{620 - 580}{6(17.82/2.970)} = 1.111$$

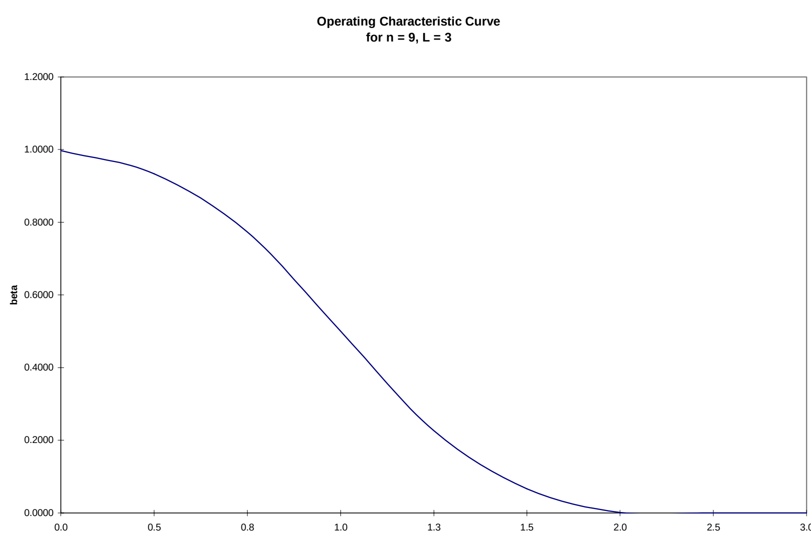
Process is capable of meeting specifications.

(b)

$$n = 9;\quad L = 3;\quad \beta = \Phi\left(L - k\sqrt{n}\right) - \Phi\left(-L - k\sqrt{n}\right)$$

for k = {0, 0.5, 0.75, 1.0, 1.25, 1.5, 2.0, 2.5, 3.0},

β = {0.9974, 0.9332, 0.7734, 0.5, 0.2266, 0.0668, 0.0013, 0.0000, 0.0000}



5-45 (5-38).

$n = 7; \quad \sum_{i=1}^{30} \bar{x}_i = 2700; \quad \sum_{i=1}^{30} R_i = 120; \quad m = 30$

(a)

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{m} \bar{x}_i}{m} = \frac{2700}{30} = 90; \quad \bar{R} = \frac{\sum_{i=1}^{m} R_i}{m} = \frac{120}{30} = 4$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 90 + 0.419(4) = 91.676$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 90 - 0.419(4) = 88.324$$

$$\text{UCL}_R = D_4\bar{R} = 1.924(4) = 7.696$$

$$\text{LCL}_R = D_3\bar{R} = 0.076(4) = 0.304$$

(b)

$$\hat{\sigma}_x = \bar{R}/d_2 = 4/2.704 = 1.479$$

(c)

$$\bar{S} = c_4\hat{\sigma}_x = 0.9594(1.479) = 1.419$$

$$\text{UCL}_S = 1.882(1.419) = 2.671$$

$$\text{LCL}_S = 0.118(1.419) = 0.167$$

5-46 (5-39).

$n = 9; \quad \mu = 600; \quad \sigma_x = 12; \quad \alpha = 0.01$

$$k = Z_{\alpha/2} = Z_{0.01/2} = Z_{0.005} = 2.576$$

$$\text{UCL}_{\bar{x}} = \mu + k\sigma_{\bar{x}} = \mu + k\left(\sigma_x/\sqrt{n}\right) = 600 + 2.576\left(12/\sqrt{9}\right) = 610.3$$

$$\text{LCL}_{\bar{x}} = \mu - k\sigma_{\bar{x}} = \mu - k\left(\sigma_x/\sqrt{n}\right) = 600 - 2.576\left(12/\sqrt{9}\right) = 589.7$$

5-47 (5-40).

$$\hat{\sigma}_x = \bar{R}/d_2 = 20.59/2.059 = 10$$

$$\Pr\{\text{detect shift on 1st sample}\} = \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} = \Pr\{\bar{x} < \text{LCL}\} + 1 - \Pr\{\bar{x} \le \text{UCL}\}$$

$$= \Phi\left(\frac{\text{LCL} - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) = \Phi\left(\frac{785 - 790}{10/\sqrt{4}}\right) + 1 - \Phi\left(\frac{815 - 790}{10/\sqrt{4}}\right)$$

$$= \Phi(-1) + 1 - \Phi(5) = 0.1587 + 1 - 1.0000 = 0.1587$$

5-48 (5-41).

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1-\Pr\{\text{not detect}\}} = \frac{1}{\Pr\{\text{detect}\}} = \frac{1}{0.1587} = 6.30$$

5-49 (5-42).

(a)

$$\hat{\sigma}_x = \bar{R}/d_2 = 8.91/2.970 = 3.000$$

$$\alpha = \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} = \Phi\left(\frac{\text{LCL}-\bar{\bar{x}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL}-\bar{\bar{x}}}{\sigma_{\bar{x}}}\right)$$

$$= \Phi\left(\frac{357-360}{3/\sqrt{9}}\right) + 1 - \Phi\left(\frac{363-360}{3/\sqrt{9}}\right) = \Phi(-3) + 1 - \Phi(3) = 0.0013 + 1 - 0.9987 = 0.0026$$

(b)

$$\hat{C}_P = \frac{\text{USL}-\text{LSL}}{6\hat{\sigma}_x} = \frac{+6-(-6)}{6(3)} = 0.667$$

The process is not capable of producing all items within specification.

(c)

$\mu_{\text{new}} = 357$

$$\Pr\{\text{not detect on 1st sample}\} = \Pr\{\text{LCL} \le \bar{x} \le \text{UCL}\} = \Phi\left(\frac{\text{UCL}-\mu_{\text{new}}}{\hat{\sigma}_x/\sqrt{n}}\right) - \Phi\left(\frac{\text{LCL}-\mu_{\text{new}}}{\hat{\sigma}_x/\sqrt{n}}\right)$$

$$= \Phi\left(\frac{363-357}{3/\sqrt{9}}\right) - \Phi\left(\frac{357-357}{3/\sqrt{9}}\right) = \Phi(6) - \Phi(0) = 1.0000 - 0.5000 = 0.5000$$

(d)

$$\alpha = 0.01; \quad k = Z_{\alpha/2} = Z_{0.01/2} = Z_{0.005} = 2.576$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + k\sigma_{\bar{x}} = \bar{\bar{x}} + k\left(\hat{\sigma}_x/\sqrt{n}\right) = 360 + 2.576\left(3/\sqrt{9}\right) = 362.576$$

$$\text{LCL}_{\bar{x}} = 360 - 2.576\left(3/\sqrt{9}\right) = 357.424$$

5-50 (5-43).

(a)

$$\hat{\sigma}_x = \overline{R}/d_2 = 8.236/2.059 = 4.000$$

(b)

$$\overline{S} = c_4\hat{\sigma}_x = 0.9213(4) = 3.865$$
$$\text{UCL}_S = B_4\overline{S} = 2.266(3.685) = 8.351$$
$$\text{LCL}_S = B_3\overline{S} = 0(3.685) = 0$$

(c)

$$\hat{p} = \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} = \Phi\left(\frac{\text{LSL} - \overline{\overline{x}}}{\hat{\sigma}_x}\right) + 1 - \Phi\left(\frac{\text{USL} - \overline{\overline{x}}}{\hat{\sigma}_x}\right)$$
$$= \Phi\left(\frac{595 - 620}{4}\right) + 1 - \Phi\left(\frac{625 - 620}{4}\right)$$
$$= \Phi(-6.25) + 1 - \Phi(1.25) = 0.0000 + 1 - 0.8944 = 0.1056$$

(d)

To reduce the fraction nonconforming, try moving the center of the process from its current mean of 620 closer to the nominal dimension of 610. Also consider reducing the process variability.

(e)

$$\Pr\{\text{detect on 1st sample}\} = \Pr\{\overline{x} < \text{LCL}\} + \Pr\{\overline{x} > \text{UCL}\}$$
$$= \Phi\left(\frac{\text{LCL} - \mu_{\text{new}}}{\sigma_{\overline{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \mu_{\text{new}}}{\sigma_{\overline{x}}}\right)$$
$$= \Phi\left(\frac{614 - 610}{4/\sqrt{4}}\right) + 1 - \Phi\left(\frac{626 - 610}{4/\sqrt{4}}\right)$$
$$= \Phi(2) + 1 - \Phi(8) = 0.9772 + 1 - 1.0000 = 0.9772$$

(f)

$$\Pr\{\text{detect by 3rd sample}\} = 1 - \Pr\{\text{not detect by 3rd sample}\}$$
$$= 1 - (\Pr\{\text{not detect}\})^3 = 1 - (1 - 0.9772)^3 = 1.0000$$

5-51 (5-44).

(a)

$\hat{\mu} = \bar{\bar{x}} = 706.00; \quad \hat{\sigma}_x = \bar{S} / c_4 = 1.738 / 0.9515 = 1.827$

(b)

$$\text{UNTL} = \bar{\bar{x}} + 3\hat{\sigma}_x = 706 + 3(1.827) = 711.48$$
$$\text{LNTL} = 706 - 3(1.827) = 700.52$$

(c)

$$\begin{aligned}
\hat{p} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\
&= \Phi\left(\frac{\text{LSL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) + 1 - \Phi\left(\frac{\text{USL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) \\
&= \Phi\left(\frac{703 - 706}{1.827}\right) + 1 - \Phi\left(\frac{709 - 706}{1.827}\right) \\
&= \Phi(-1.642) + 1 - \Phi(1.642) = 0.0503 + 1 - 0.9497 = 0.1006
\end{aligned}$$

(d)

$$\begin{aligned}
\Pr\{\text{detect on 1st sample}\} &= \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} \\
&= \Phi\left(\frac{\text{LCL} - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) \\
&= \Phi\left(\frac{703.8 - 702}{1.827/\sqrt{6}}\right) + 1 - \Phi\left(\frac{708.2 - 702}{1.827/\sqrt{6}}\right) \\
&= \Phi(2.41) + 1 - \Phi(8.31) = 0.9920 + 1 - 1.0000 = 0.9920
\end{aligned}$$

(e)

$$\begin{aligned}
\Pr\{\text{detect by 3rd sample}\} &= 1 - \Pr\{\text{not detect by 3rd sample}\} \\
&= 1 - (\Pr\{\text{not detect}\})^3 = 1 - (1 - 0.9920)^3 = 1.0000
\end{aligned}$$

5-52 (5-45).

(a)

$\hat{\mu} = \bar{\bar{x}} = 700; \quad \hat{\sigma}_x = \bar{S} / c_4 = 7.979 / 0.9213 = 8.661$

(b)

$$\begin{aligned}\hat{p} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\ &= \Phi\left(\frac{\text{LSL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) + 1 - \Phi\left(\frac{\text{USL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) \\ &= \Phi\left(\frac{690 - 700}{8.661}\right) + 1 - \Phi\left(\frac{720 - 700}{8.661}\right) \\ &= \Phi(-1.15) + 1 - \Phi(2.31) = 0.1251 + 1 - 0.9896 = 0.1355\end{aligned}$$

(c)

$$\begin{aligned}\alpha &= \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} \\ &= \Phi\left(\frac{\text{LCL} - \bar{\bar{x}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \bar{\bar{x}}}{\sigma_{\bar{x}}}\right) \\ &= \Phi\left(\frac{690 - 700}{8.661/\sqrt{4}}\right) + 1 - \Phi\left(\frac{710 - 700}{8.661/\sqrt{4}}\right) \\ &= \Phi(-2.31) + 1 - \Phi(2.31) = 0.0104 + 1 - 0.9896 = 0.0208\end{aligned}$$

(d)

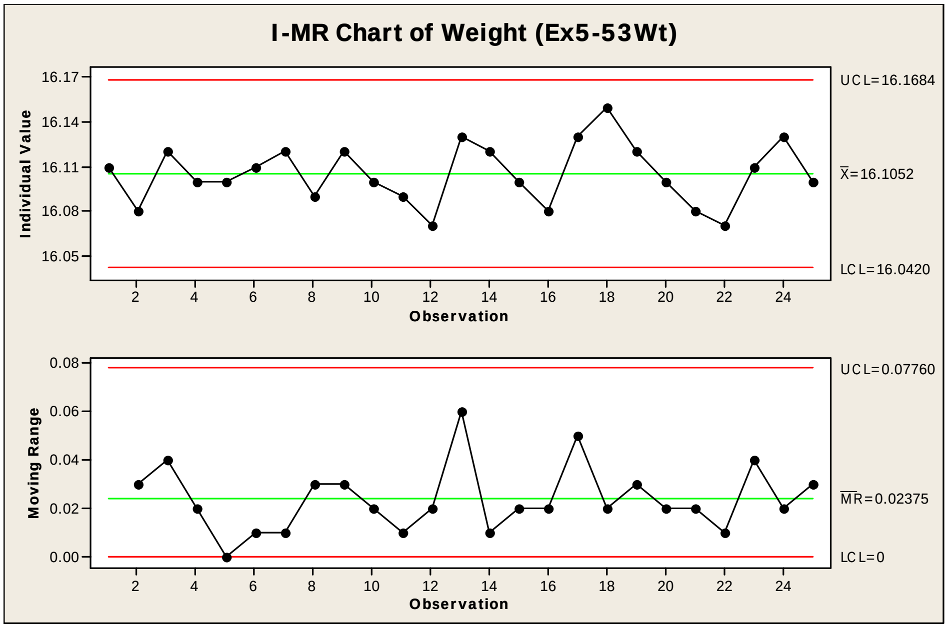
$$\begin{aligned}\Pr\{\text{detect on 1st sample}\} &= \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} \\ &= \Phi\left(\frac{\text{LCL} - \mu_{\text{new}}}{\sigma_{\bar{x},\text{new}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \mu_{\text{new}}}{\sigma_{\bar{x},\text{new}}}\right) \\ &= \Phi\left(\frac{690 - 693}{12/\sqrt{4}}\right) + 1 - \Phi\left(\frac{710 - 693}{12/\sqrt{4}}\right) \\ &= \Phi(-0.5) + 1 - \Phi(2.83) = 0.3085 + 1 - 0.9977 = 0.3108\end{aligned}$$

(e)

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1 - \Pr\{\text{not detect}\}} = \frac{1}{\Pr\{\text{detect}\}} = \frac{1}{0.3108} = 3.22$$

5-53 (5-46).

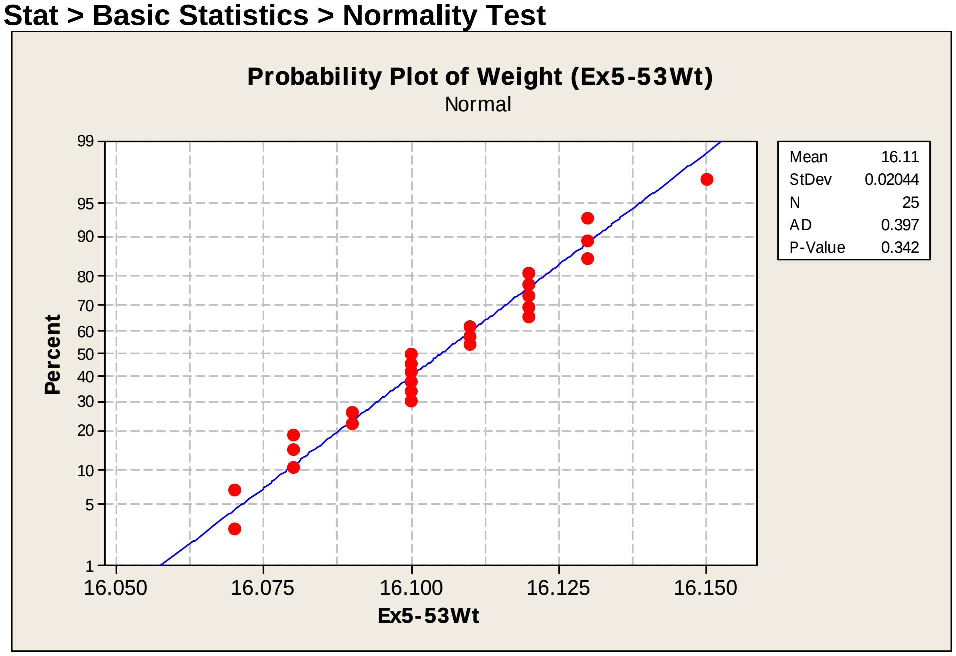
MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



There may be a "sawtooth" pattern developing on the Individuals chart.

$\overline{\overline{x}} = 16.1052$; $\hat{\sigma}_x = 0.021055$; $\overline{\text{MR2}} = 0.02375$

MTB > Stat > Basic Statistics > Normality Test



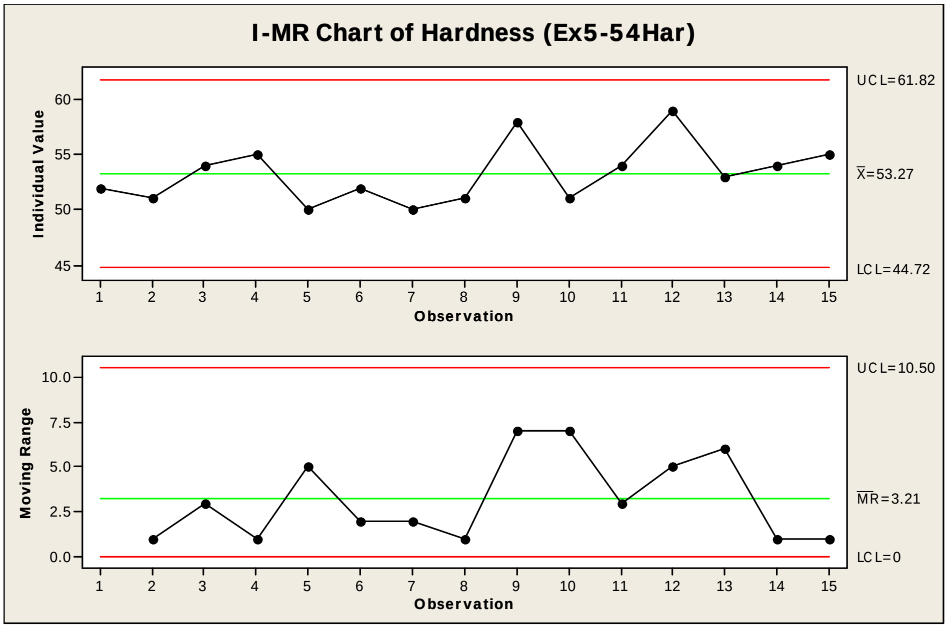
Visual examination of the normal probability indicates that the assumption of normally distributed coffee can weights is valid.

$$\%\text{underfilled} = 100\% \times \Pr\{x < 16 \text{ oz}\}$$

$$= 100\% \times \Phi\left(\frac{16 - 16.1052}{0.021055}\right) = 100\% \times \Phi(-4.9964) = 0.00003\%$$

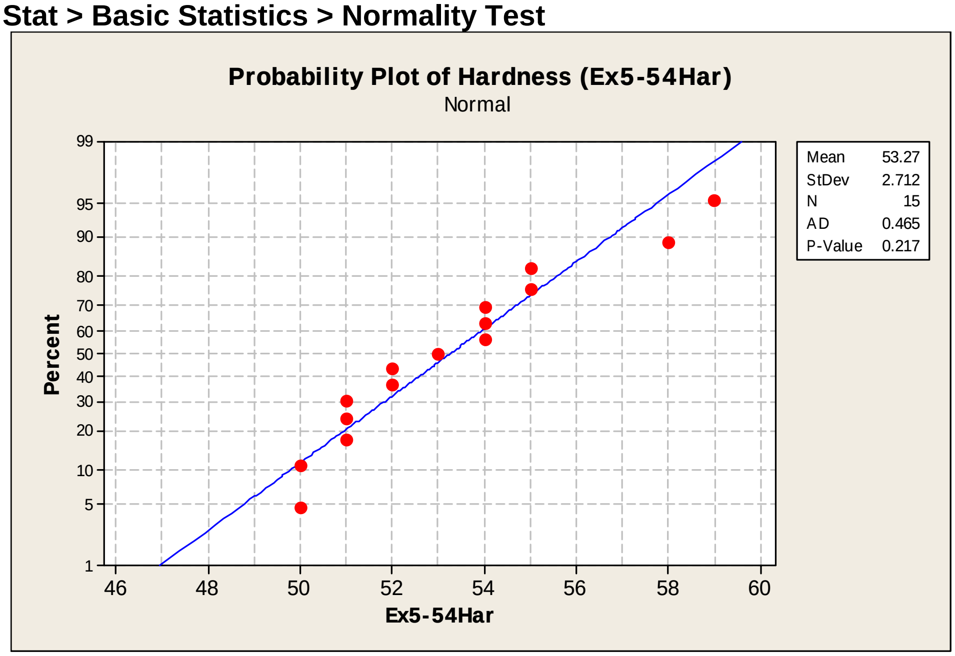
5-54(5-47).

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



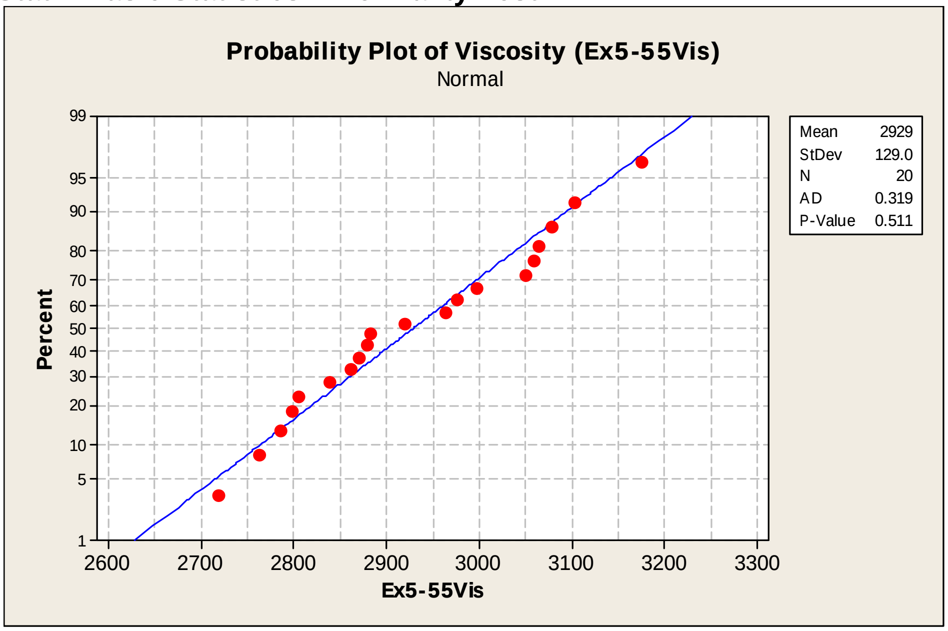
$\overline{\overline{x}} = 53.2667;\quad \hat{\sigma}_x = 2.84954;\quad \overline{\text{MR2}} = 3.21429$

MTB > Stat > Basic Statistics > Normality Test



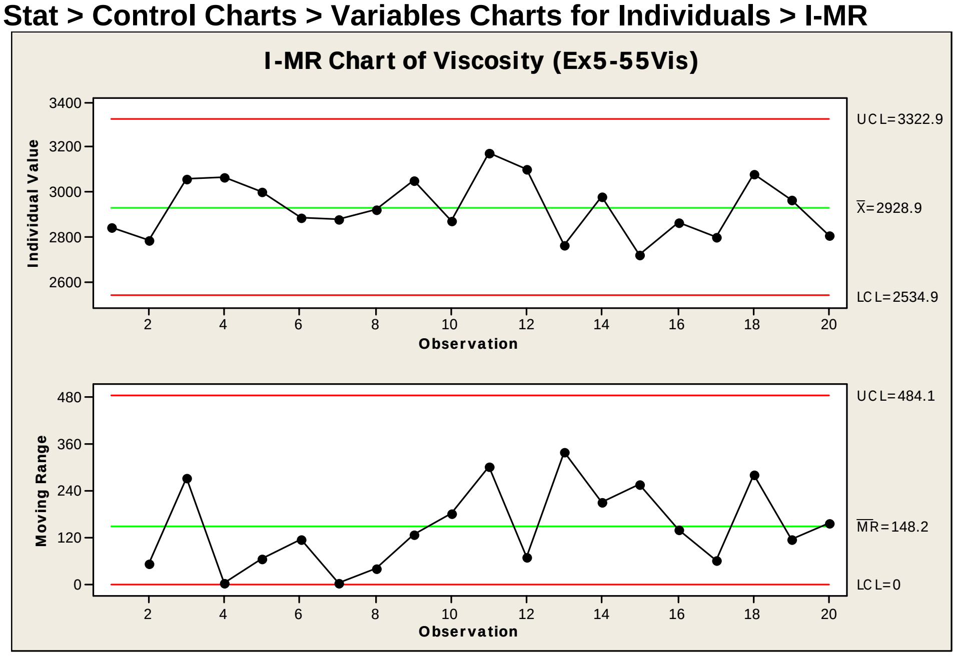
Although the observations at the tails are not very close to the straight line, the *p*-value is greater than 0.05, indicating that it may be reasonable to assume that hardness is normally distributed.

5-55 (5-48).
(a)
MTB > Stat > Basic Statistics > Normality Test



Viscosity measurements do appear to follow a normal distribution.

(b)
MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR

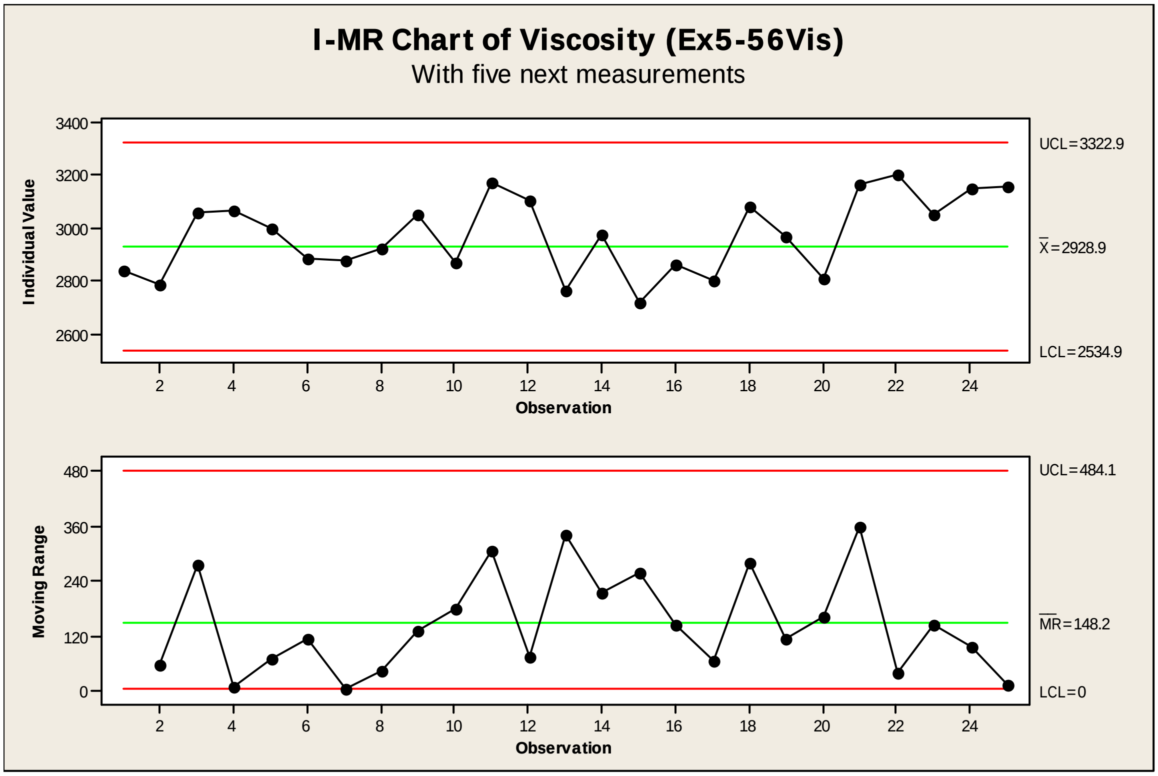


The process appears to be in statistical control, with no out-of-control points, runs, trends, or other patterns.

(c)
$\hat{\mu} = \overline{\overline{x}} = 2928.9; \quad \hat{\sigma}_x = 131.346; \quad \overline{\text{MR2}} = 148.158$

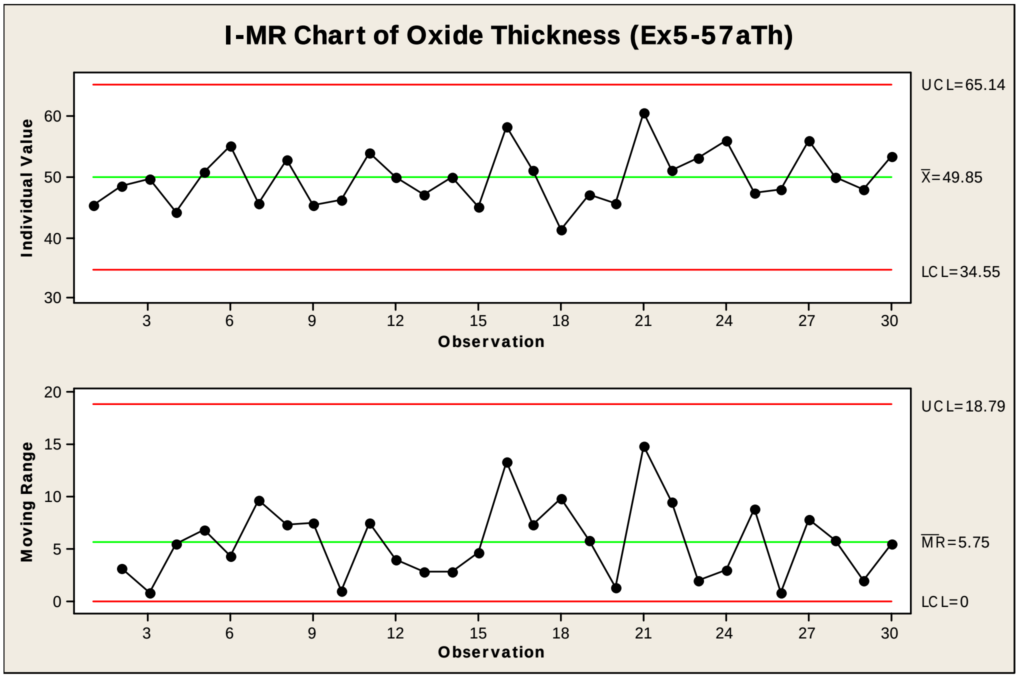
5-56 (5-49).

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



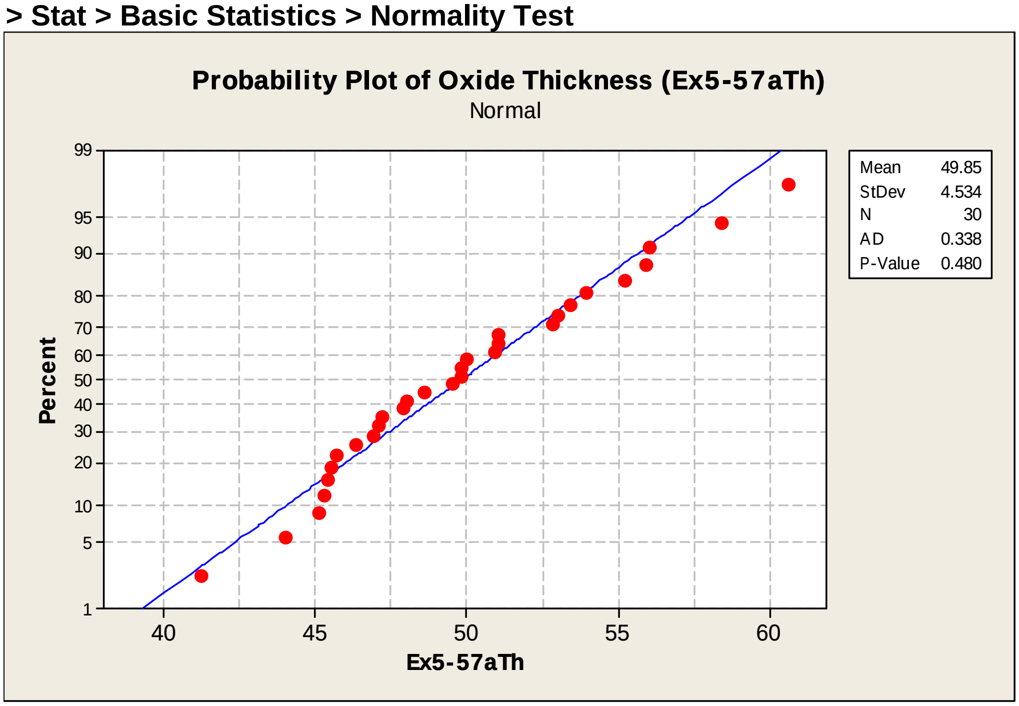
All points are inside the control limits. However all of the new points on the *I* chart are above the center line, indicating that a shift in the mean may have occurred.

5-57 (5-50).
(a)
MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



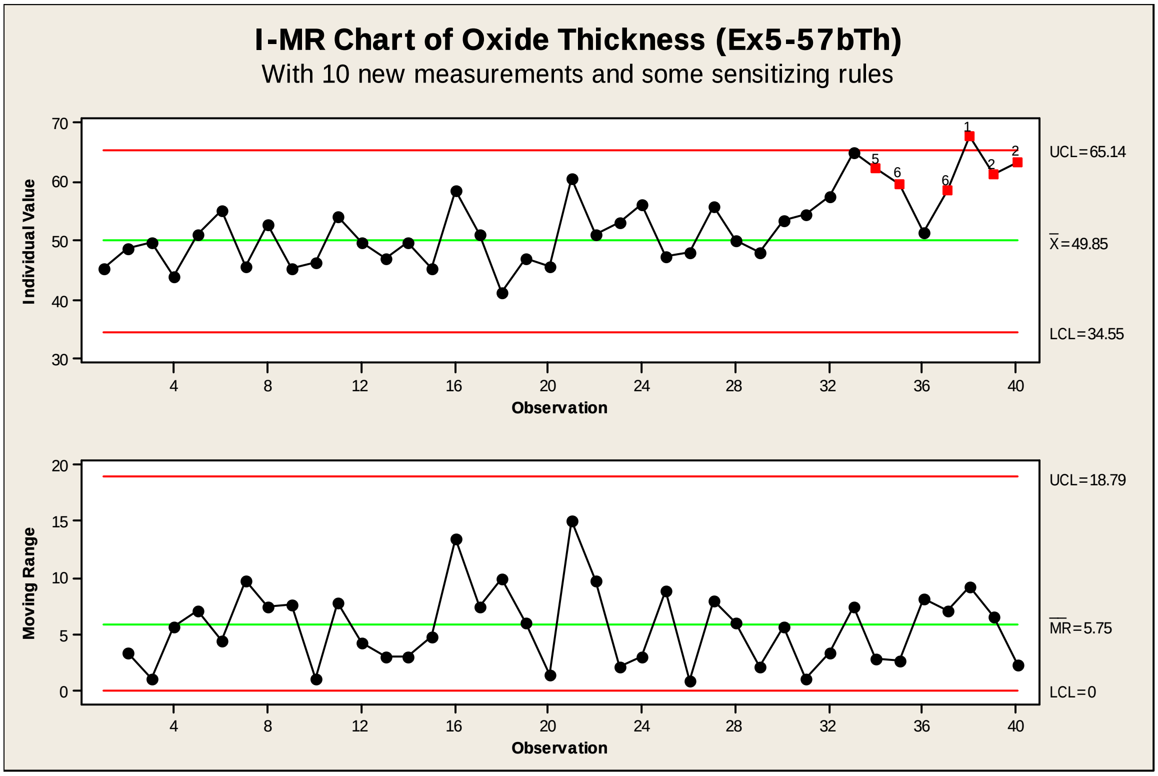
The process is in statistical control.

MTB > Stat > Basic Statistics > Normality Test



The normality assumption is reasonable.

5-57 continued
(b)
MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



Test Results for I Chart of Ex5-57bTh

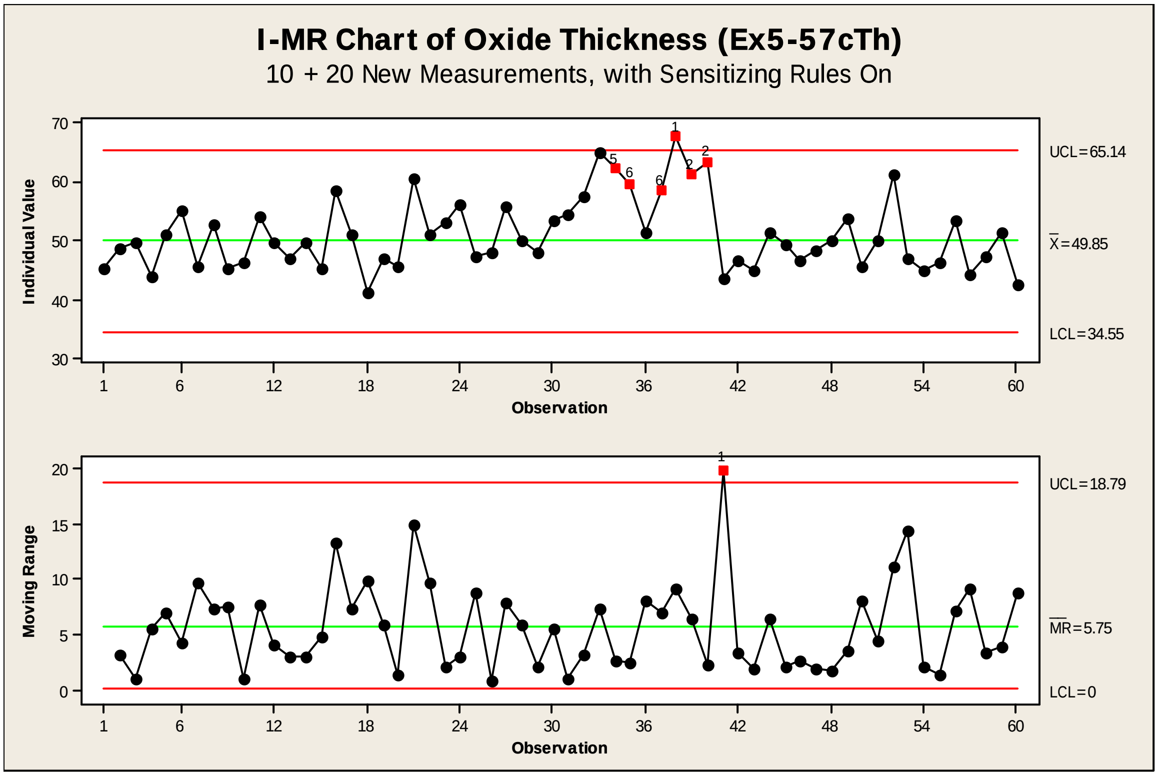
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  38
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  38, 39, 40
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  34, 39, 40
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  35, 37, 38, 39, 40
```

We have turned on some of the sensitizing rules in MINITAB to illustrate their use. There is a run above the centerline, several 4 of 5 beyond 1 sigma, and several 2 of 3 beyond 2 sigma on the *x* chart. However, even without use of the sensitizing rules, it is clear that the process is out of control during this period of operation.

5-57 continued

(c)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



The process has been returned to a state of statistical control.

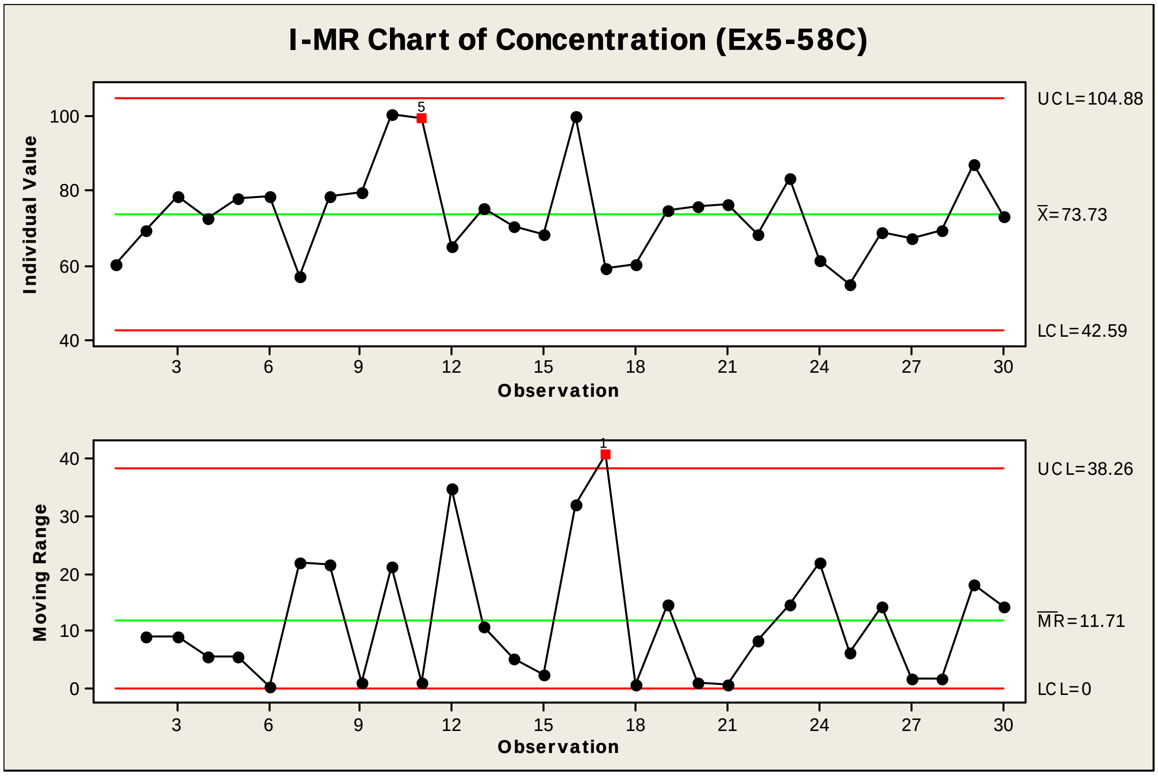
5-58 (5-51).

(a)

The normality assumption is a little bothersome for the concentration data, in particular due to the curve of the larger values and three distant values.

(b)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



Test Results for I Chart of Ex5-58C

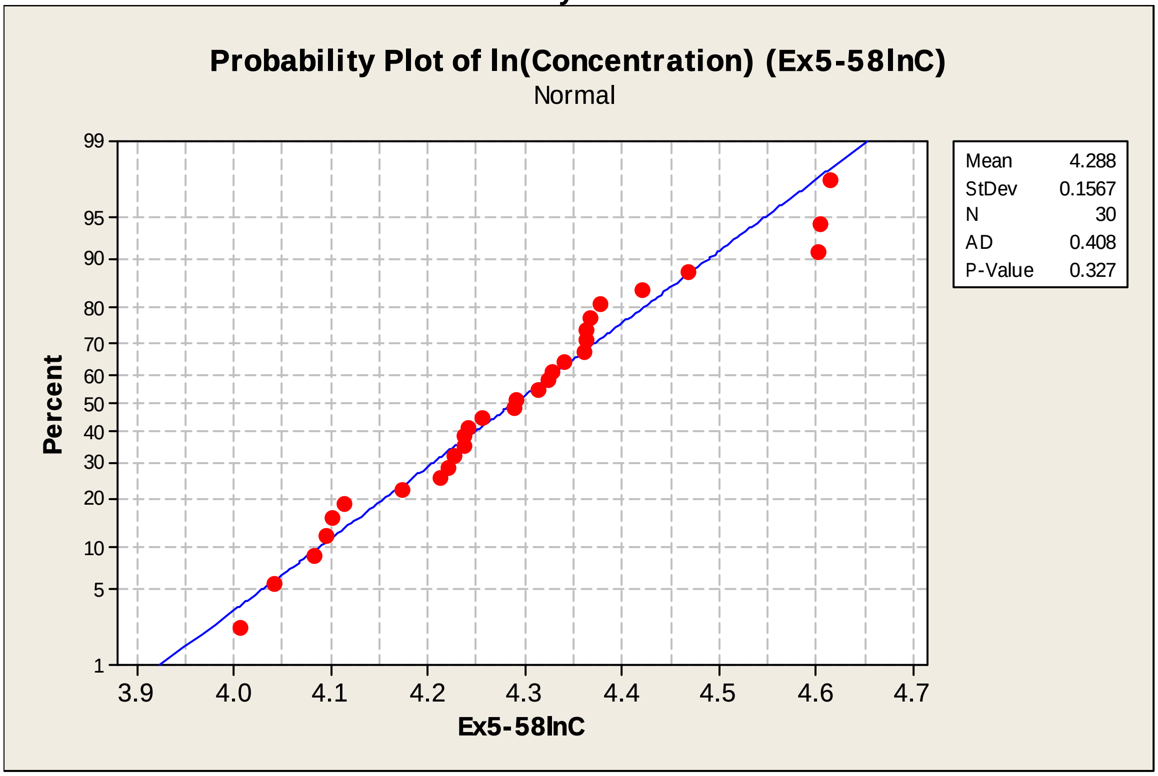
```
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  11
```

Test Results for MR Chart of Ex5-58C

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  17
```

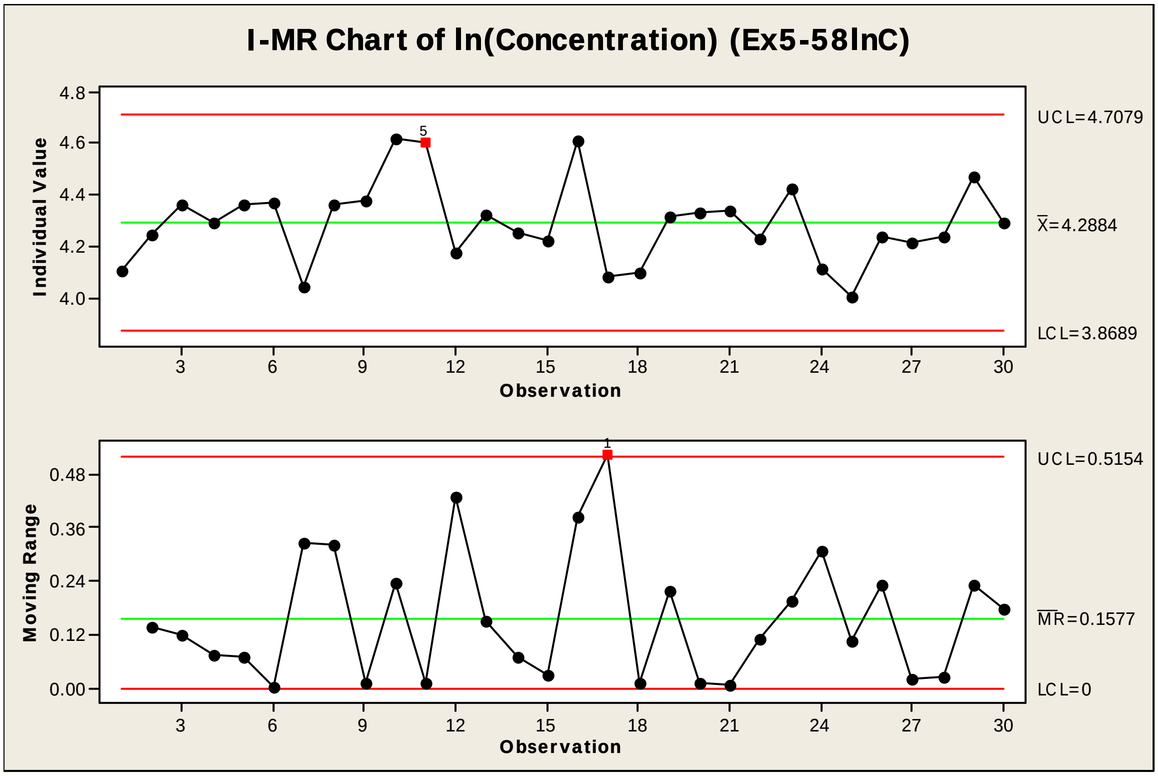
The process is not in control, with two Western Electric rule violations.

5-58 continued
(c)
MTB > Stat > Basic Statistics > Normality Test



The normality assumption is still troubling for the natural log of concentration, again due to the curve of the larger values and three distant values.

5-58 continued
(d)
MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



Test Results for I Chart of Ex5-58lnC

```
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  11
```

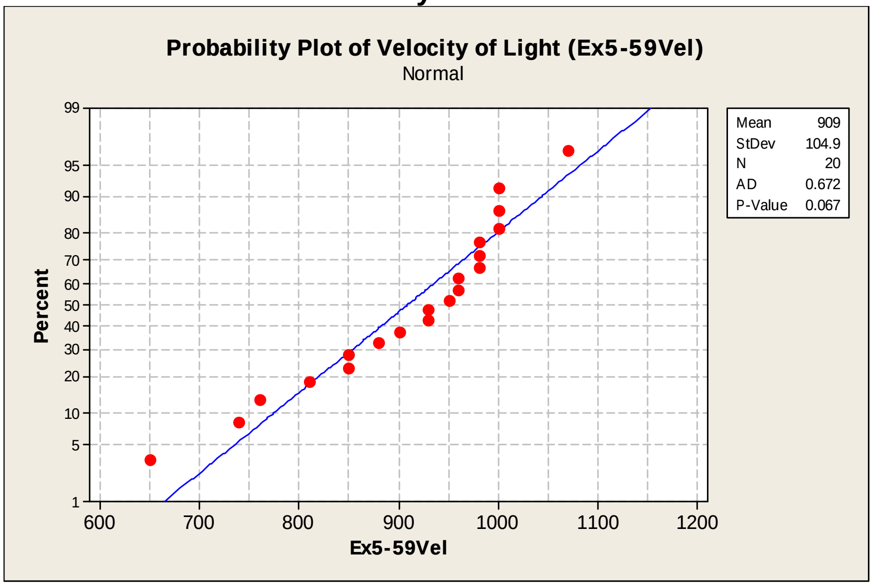
Test Results for MR Chart of Ex5-58lnC

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  17
```

The process is still not in control, with the same to Western Electric Rules violations. There does not appear to be much difference between the two control charts (actual and natural log).

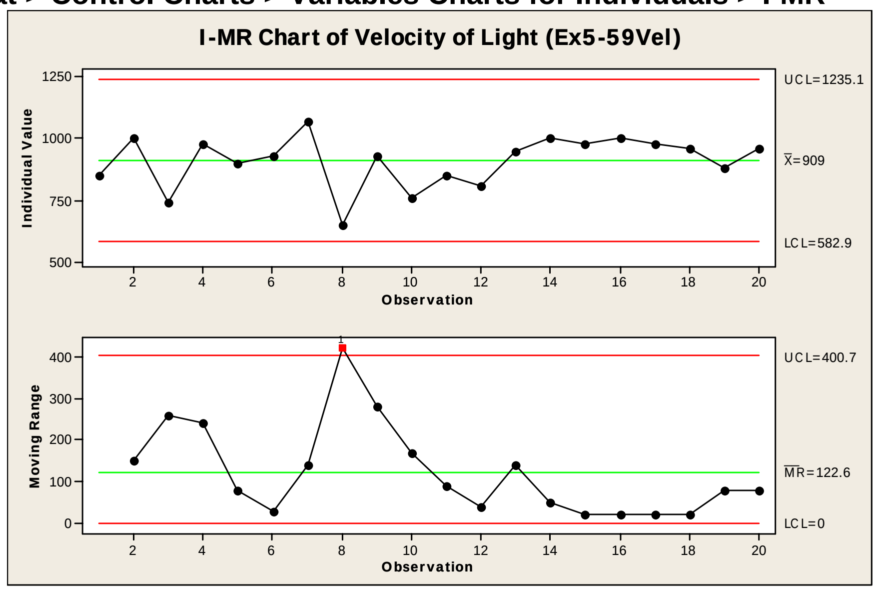
5-59☺.

MTB > Stat > Basic Statistics > Normality Test



Velocity of light measurements are approximately normally distributed.

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



I-MR Chart of Ex5-59Vel

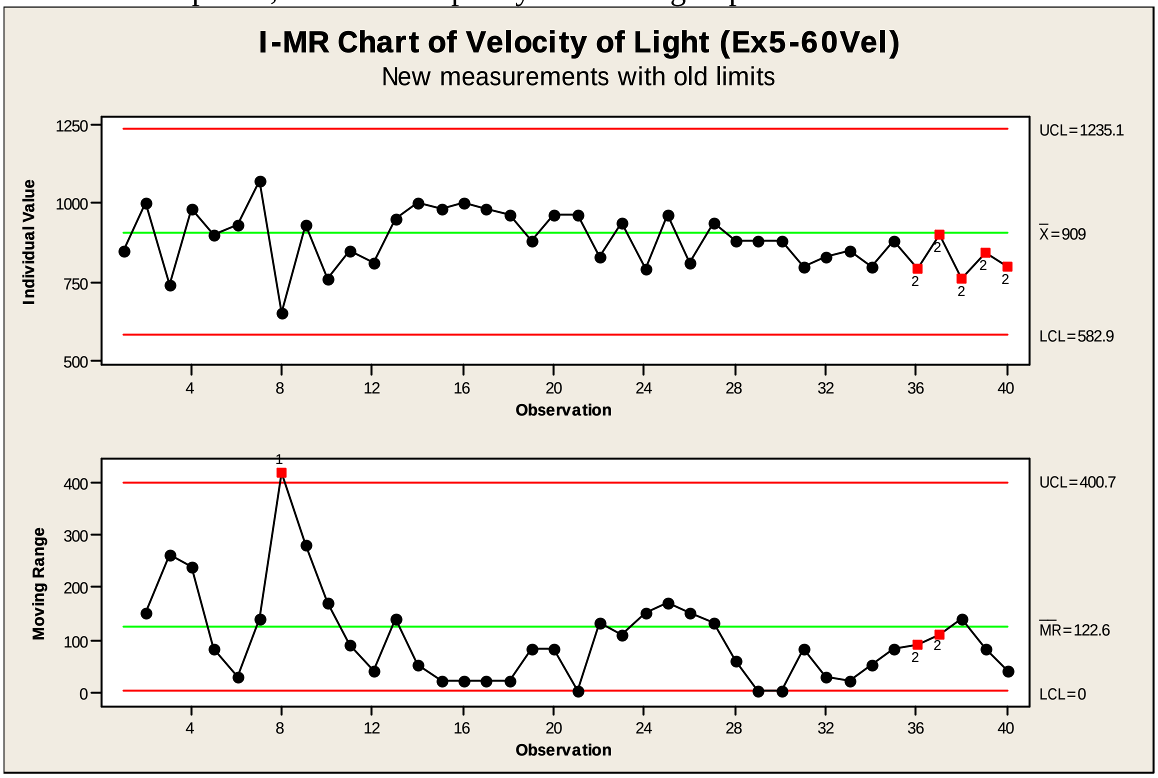
Test Results for MR Chart of Ex5-59Vel

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  8
```

The out-of-control signal on the moving range chart indicates a significantly large difference between successive measurements (7 and 8). Since neither of these measurements seems unusual, use all data for control limits calculations.

There may also be an early indication of less variability in the later measurements. For now, consider the process to be in a state of statistical process control.

5-60☺.
(a)
MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR
Select I-MR Options, Estimate to specify which subgroups to use in calculations



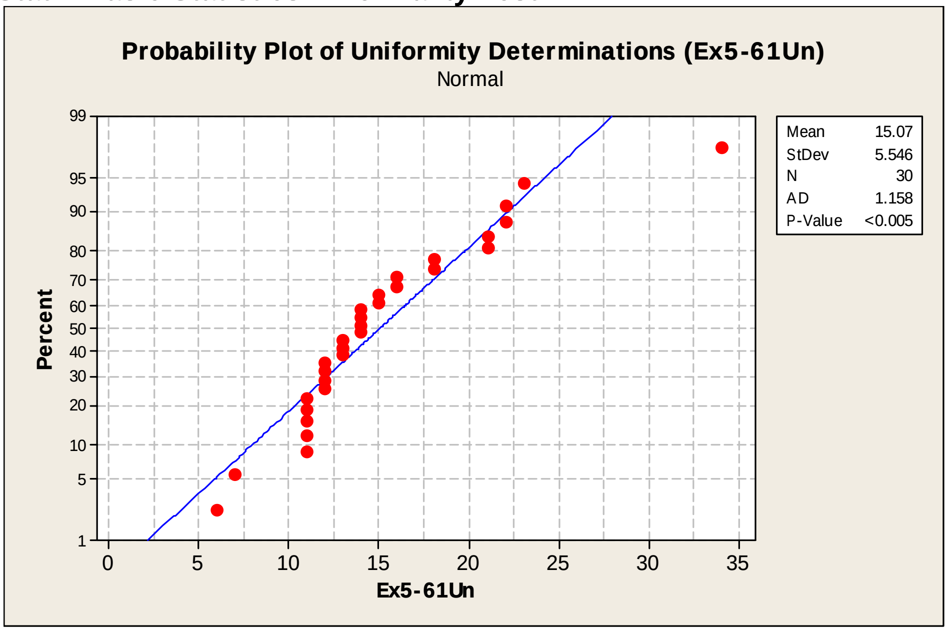
I-MR Chart of Ex5-60Vel
Test Results for I Chart of Ex5-60Vel
```
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  36, 37, 38, 39, 40
```

Test Results for MR Chart of Ex5-60Vel
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  8
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  36, 37
```

The velocity of light in air is not changing, however the method of measuring is producing varying results—this is a chart of the measurement process. There is a distinct downward trend in measurements, meaning the method is producing gradually smaller measurements.

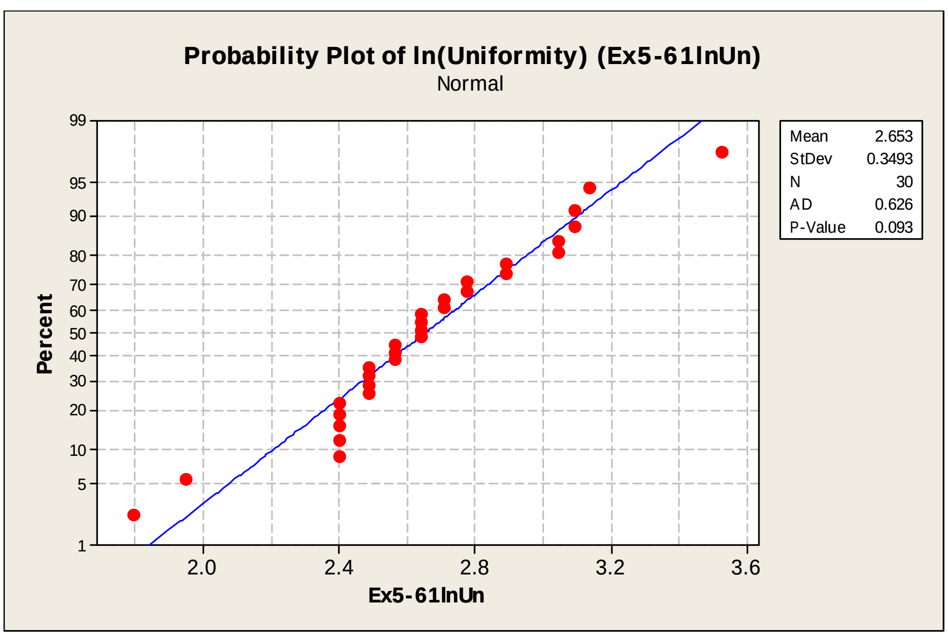
(b)
Early measurements exhibit more variability than the later measurements, which is reflected in the number of observations below the centerline of the moving range chart.

5-61☺.
(a)
MTB > Stat > Basic Statistics > Normality Test



The data are not normally distributed, as evidenced by the "S"- shaped curve to the plot points on a normal probability plot, as well as the Anderson-Darling test p-value.

The data are skewed right, so a compressive transform such as natural log or square-root may be appropriate.

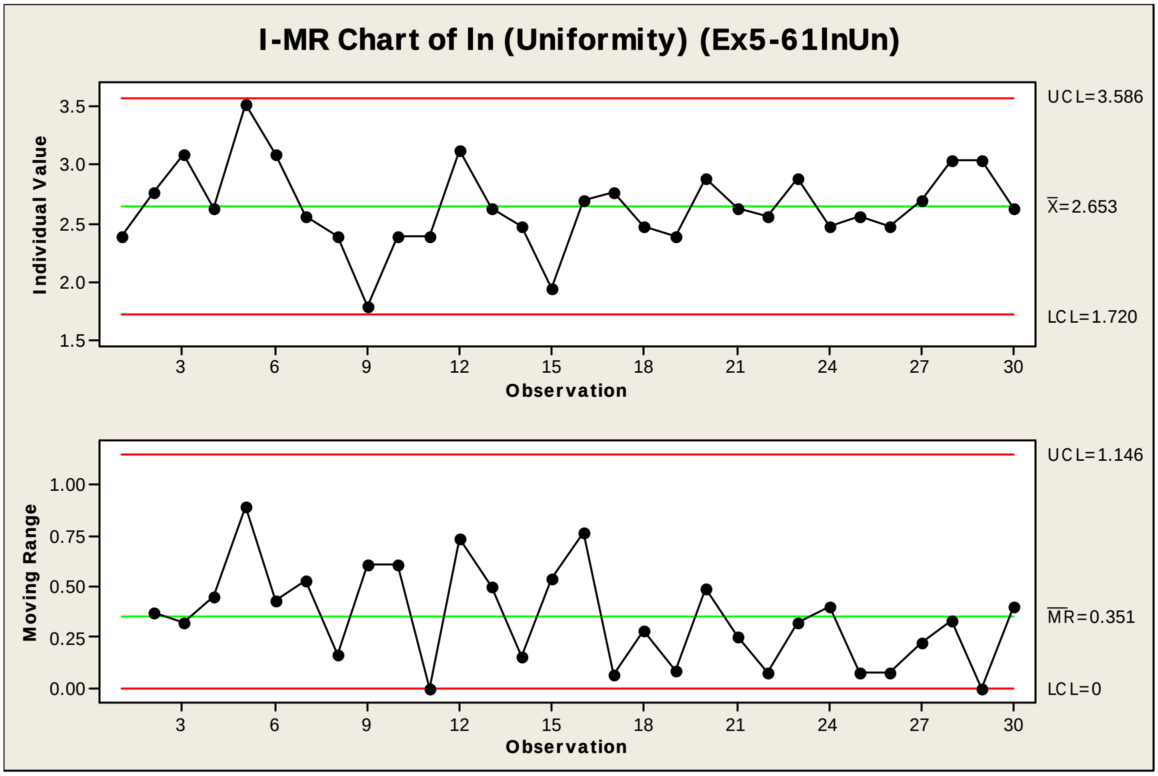


The distribution of the natural-log transformed uniformity measurements is approximately normally distributed.

5-61 continued

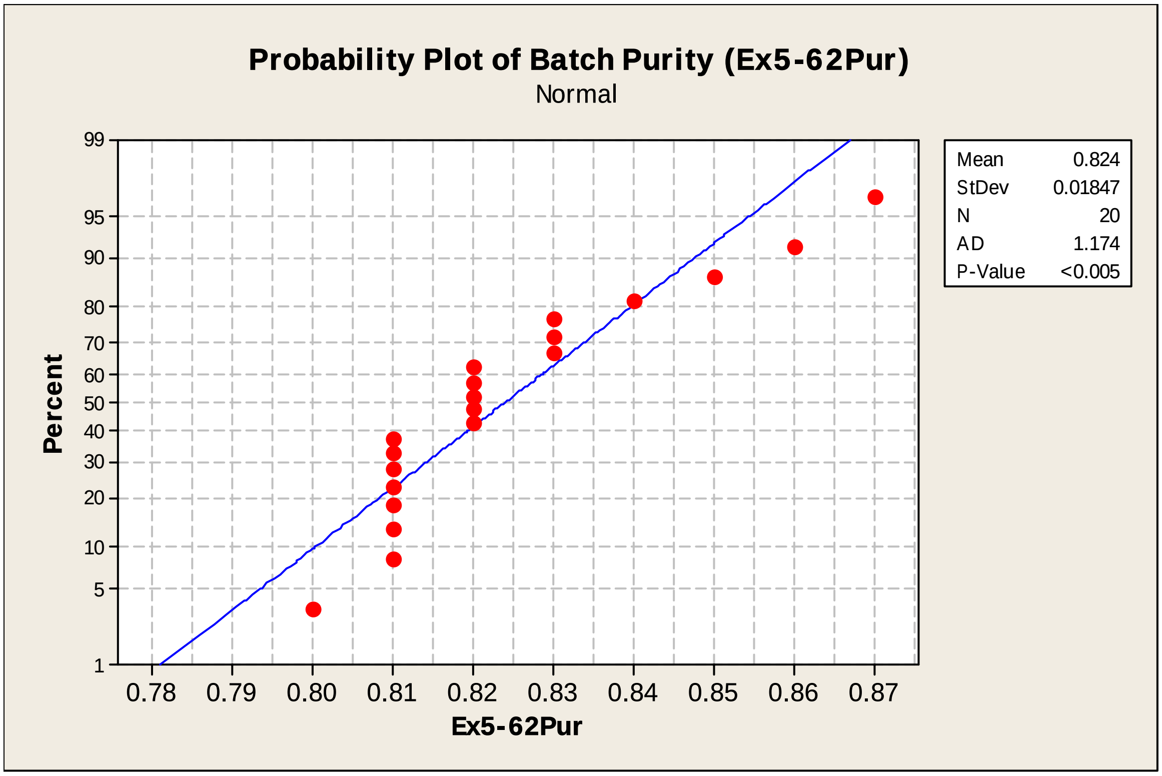
(b)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



The etching process appears to be in statistical control.

5-62 (5-52).
(a)
MTB > Stat > Basic Statistics > Normality Test

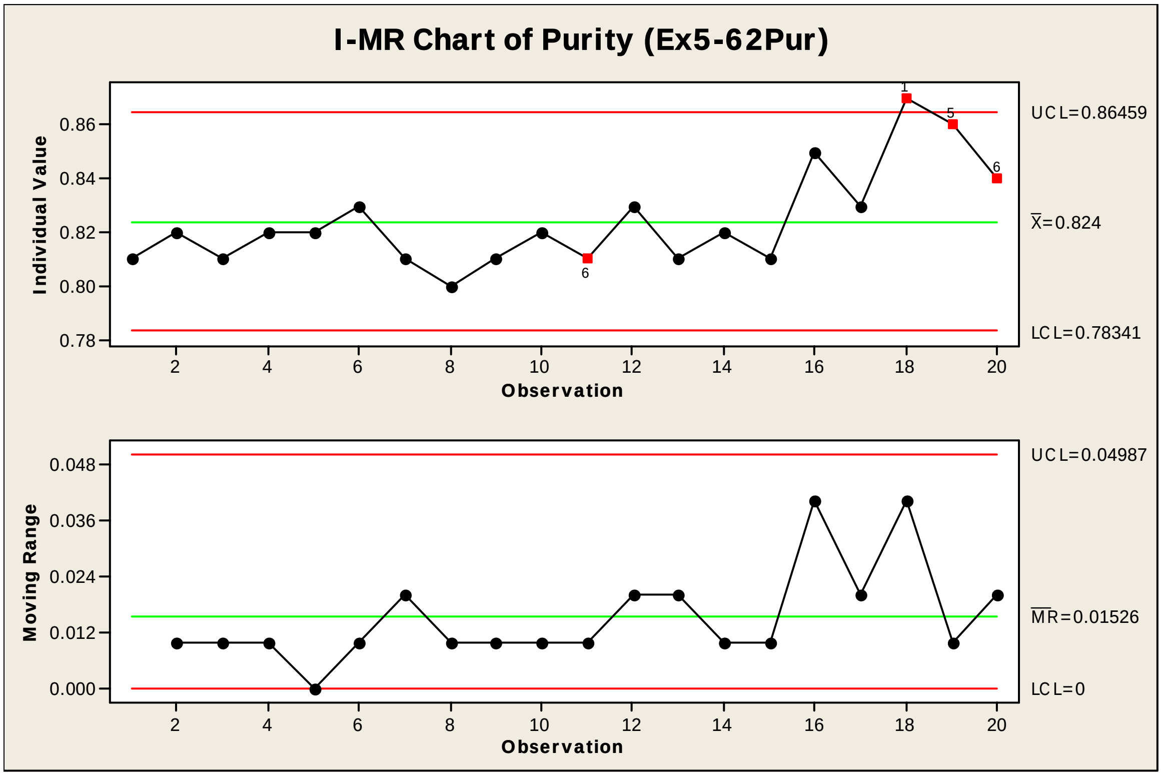


Purity is not normally distributed.

5-62 continued

(b)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



Test Results for I Chart of Ex5-62Pur

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  18
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  19
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  11, 20
```

The process is not in statistical control.

(c)

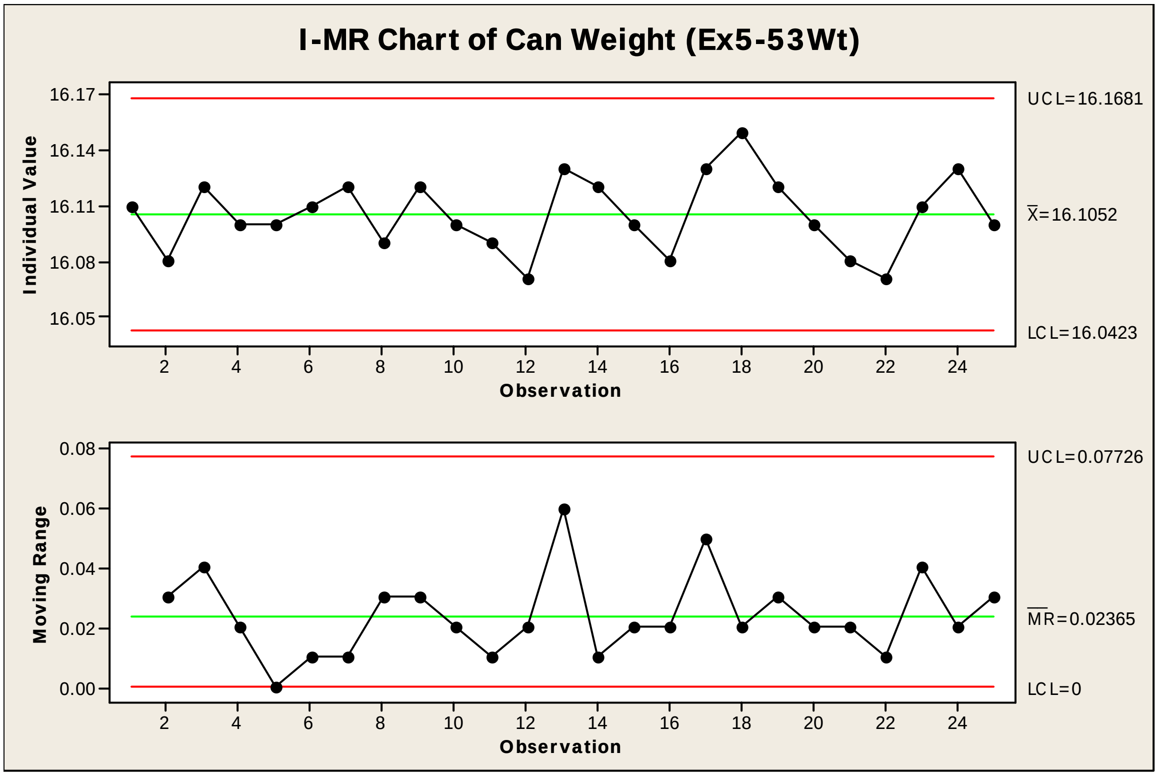
all data: $\hat{\mu} = 0.824$, $\hat{\sigma}_x = 0.0135$

without sample 18: $\hat{\mu} = 0.8216$, $\hat{\sigma}_x = 0.0133$

5-63 (5-53).

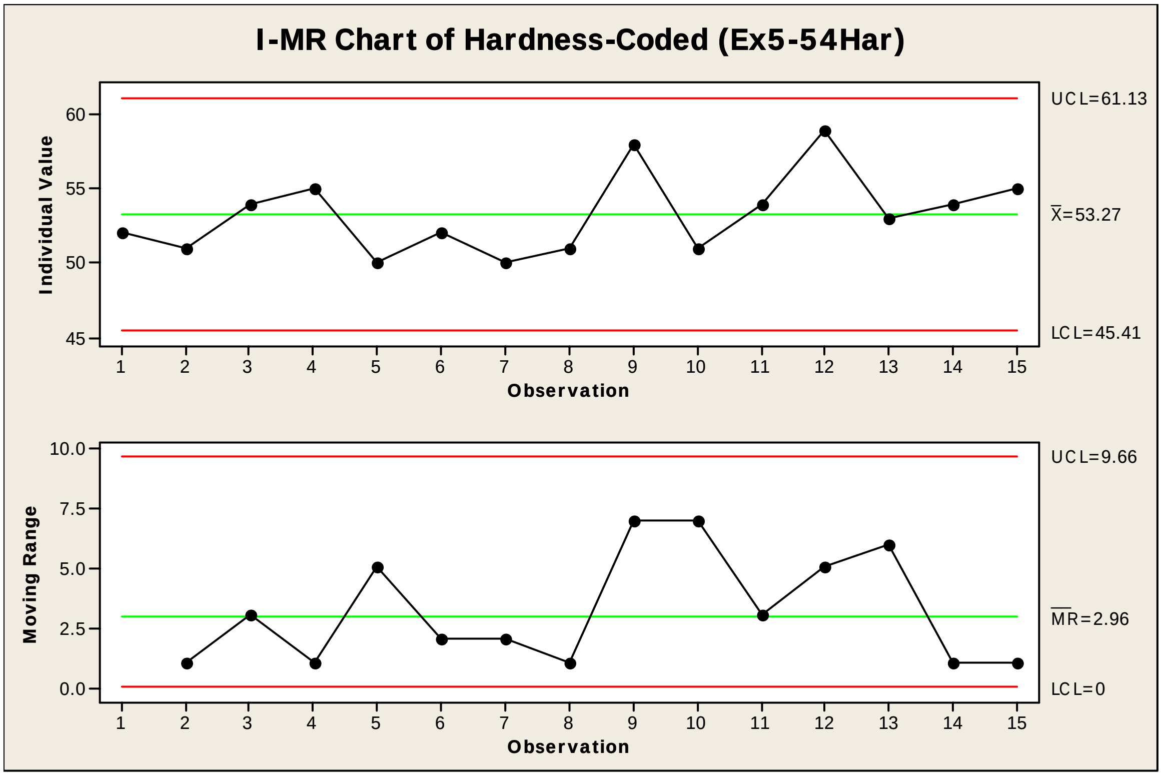
MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR

Select "Estimate" to change the method of estimating sigma



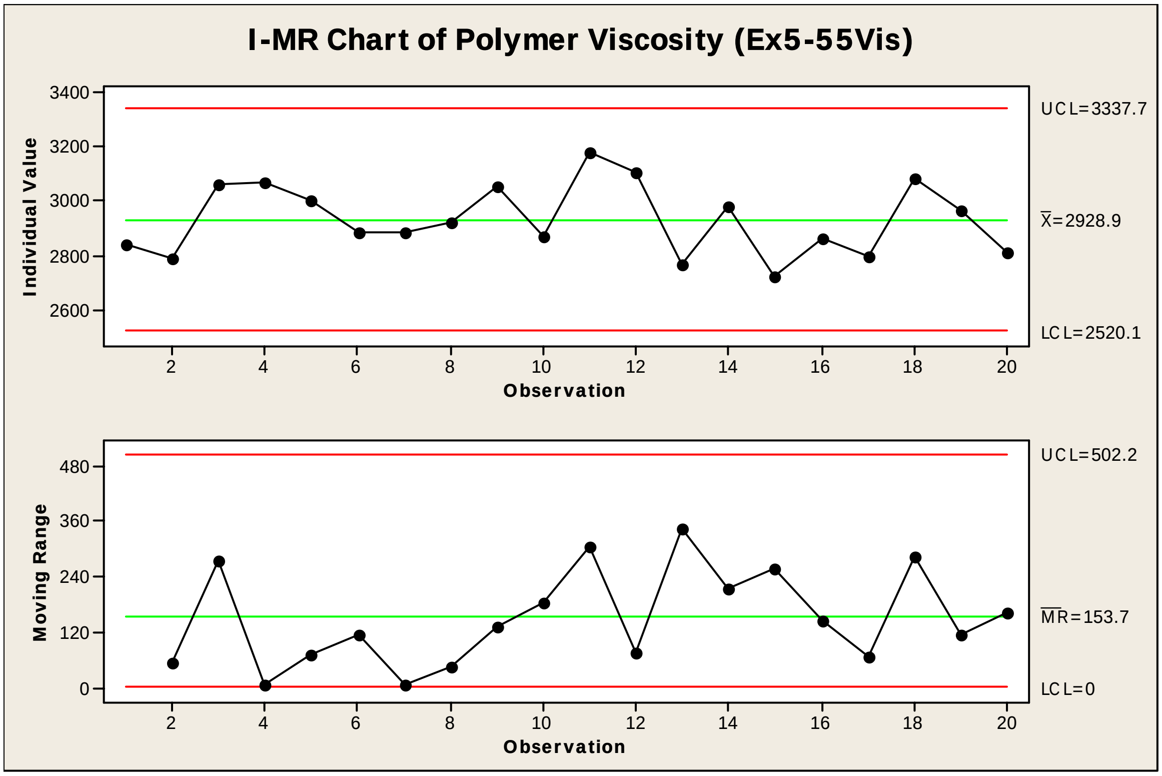
There is no difference between this chart and the one in Exercise 5-53; control limits for both are essentially the same.

5-64 (5-54).
MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR
Select "Estimate" to change the method of estimating sigma



The median moving range method gives slightly tighter control limits for both the Individual and Moving Range charts, with no practical difference for this set of observations.

5-65 (5-55).
MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR
Select "Estimate" to change the method of estimating sigma

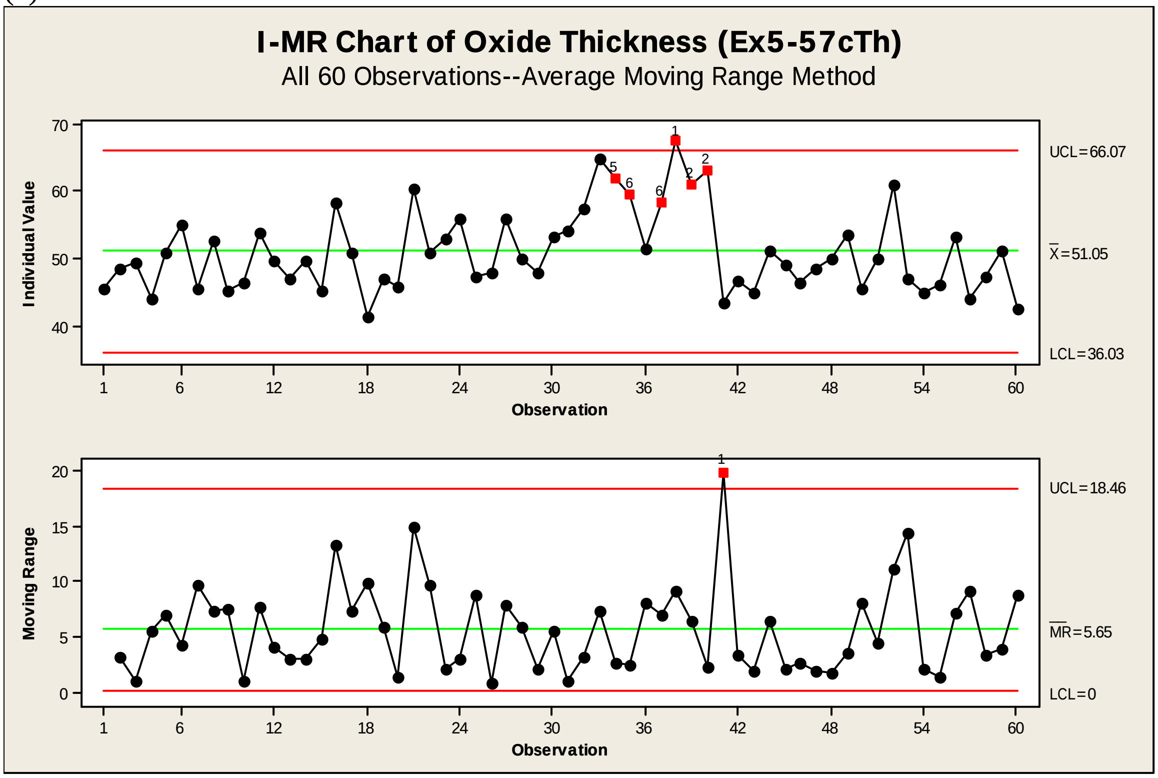


The median moving range method gives slightly wider control limits for both the Individual and Moving Range charts, with no practical meaning for this set of observations.

5-66 (5-56).

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR

(a)

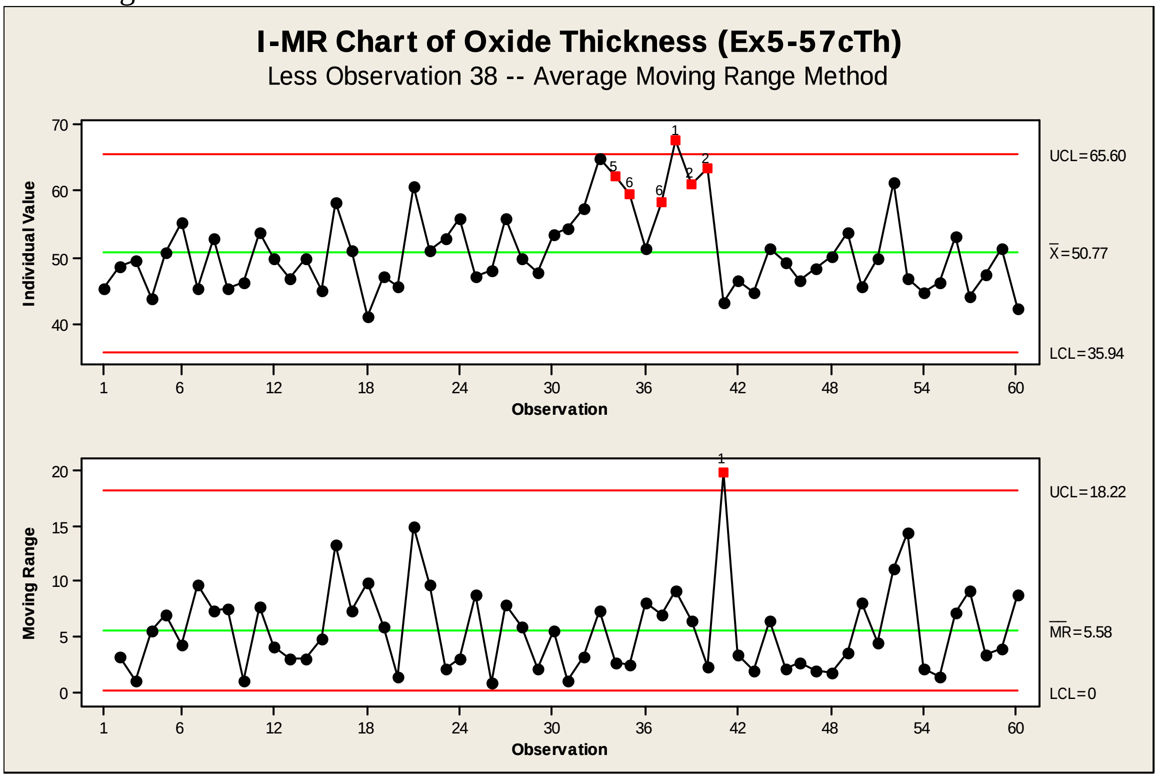



```
Test Results for I Chart of Ex5-57cTh
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  38
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  38, 39, 40
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  34, 39, 40
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  35, 37, 38, 39, 40

Test Results for MR Chart of Ex5-57cTh
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  41
```

Recall that observations on the Moving Range chart are correlated with those on the Individuals chart—that is, the out-of-control signal on the MR chart for observation 41 is reflected by the shift between observations 40 and 41 on the Individuals chart.

Remove observation 38 and recalculate control limits.

5-66 (a) continued
Excluding observation 38 from calculations:



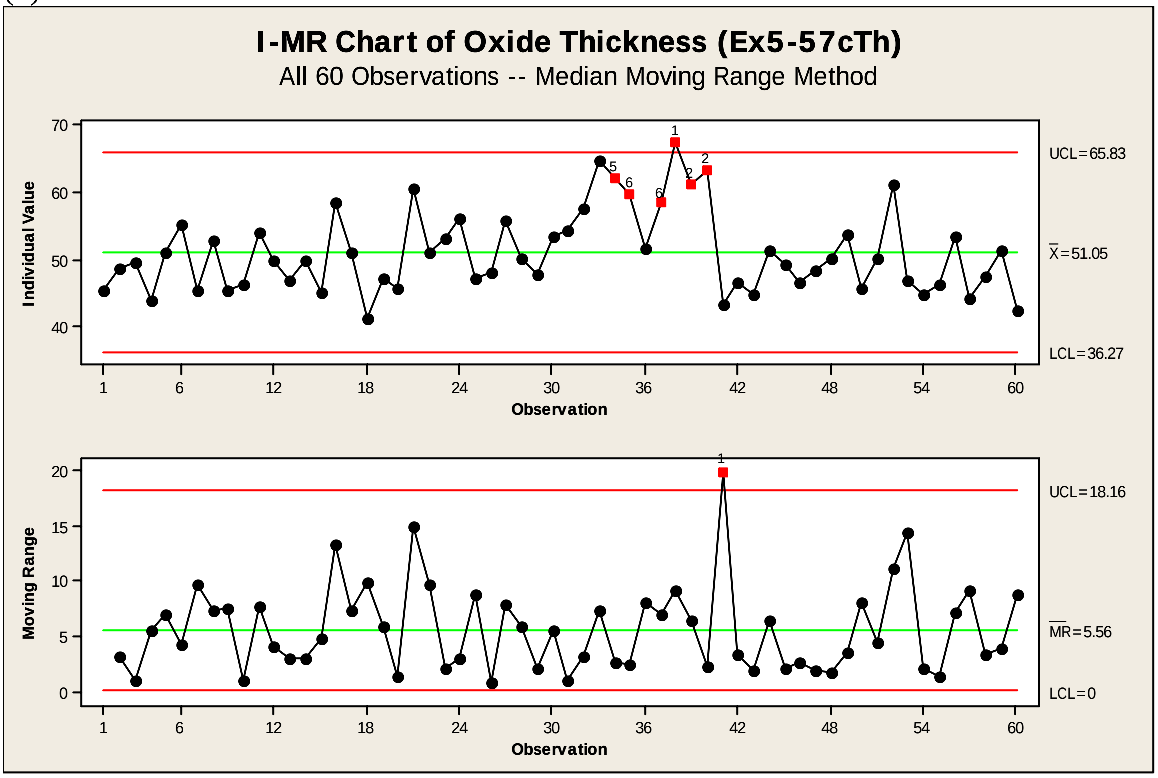
Test Results for I Chart of Ex5-57cTh

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  38
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  38, 39, 40
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  34, 39, 40
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  35, 37, 38, 39, 40
```

Test Results for MR Chart of Ex5-57cTh

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  41
```

5-66 continued
(b)



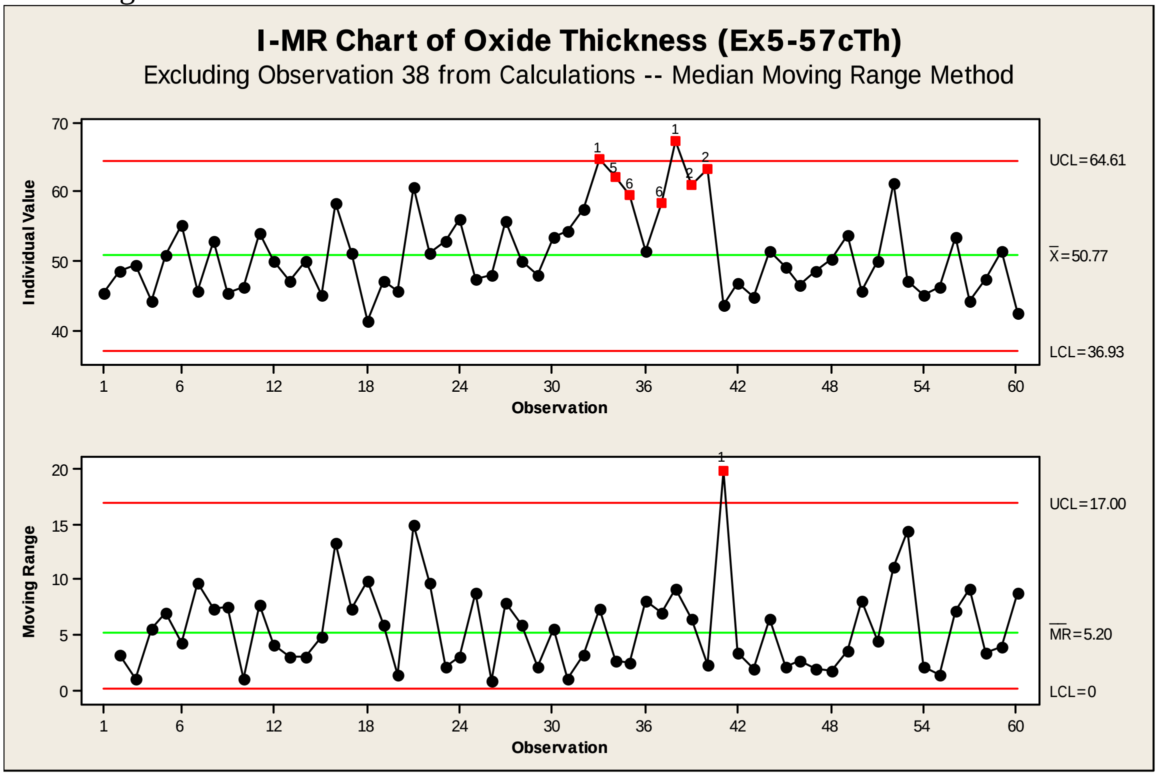
Test Results for I Chart of Ex5-57cTh

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  38
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  38, 39, 40
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  34, 39, 40
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  35, 37, 38, 39, 40
```

Test Results for MR Chart of Ex5-57cTh

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  41
```

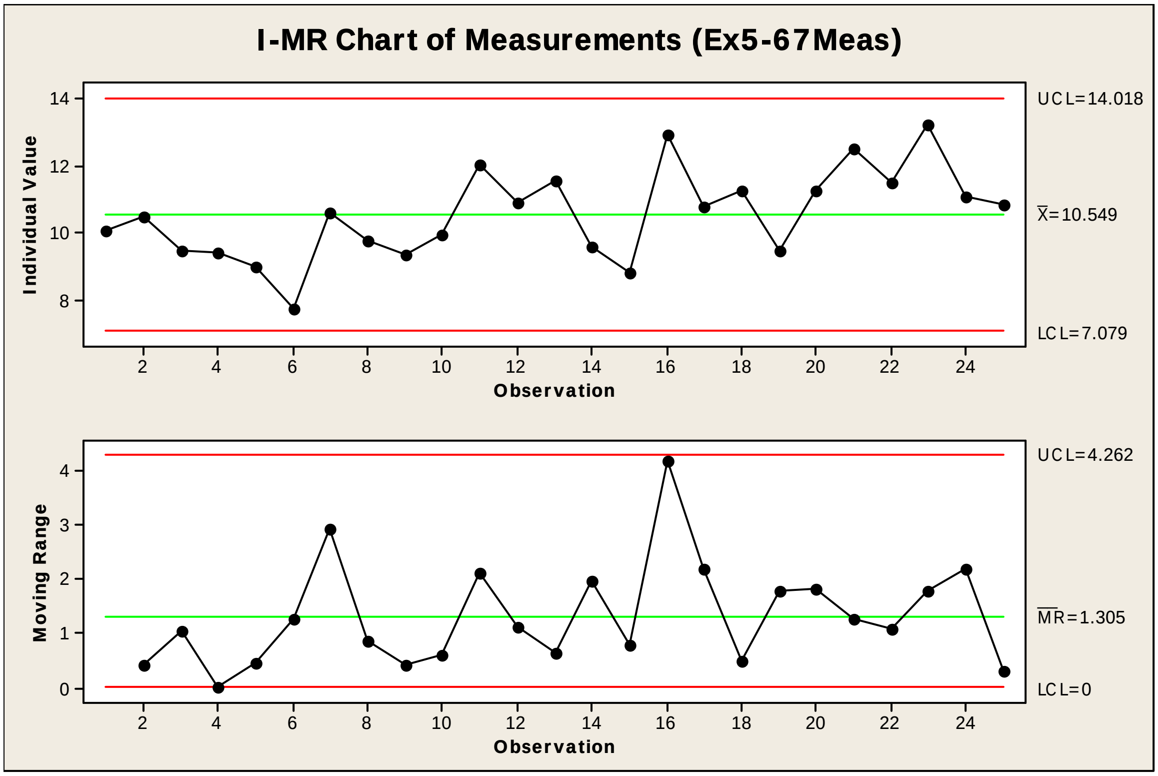
5-66 (b) continued
Excluding observation 38 from calculations:




```
Test Results for I Chart of Ex5-57cTh
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  33, 38
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  38, 39, 40
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  34, 39, 40
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  35, 37, 38, 39, 40

Test Results for MR Chart of Ex5-57cTh
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  41
```

(c)
The control limits estimated by the median moving range are tighter and detect the shift in process level at an earlier sample, 33.

5-67 (5-57).
(a)
MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



$\hat{\sigma}_x = \overline{R} / d_2 = 1.305/1.128 = 1.157$

(b)
MTB > Stat > Basic Statistics > Descriptive Statistics

Descriptive Statistics: Ex5-67Meas

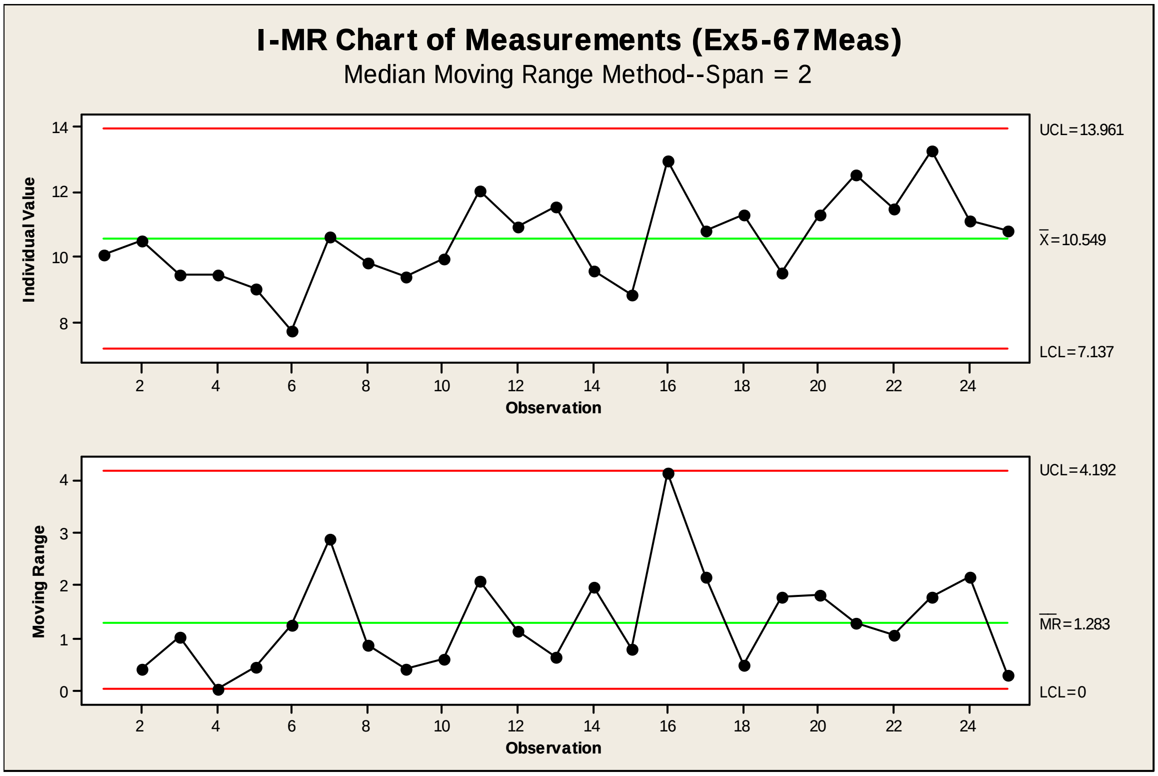
Variable	Total Count	Mean	StDev	Median
Ex5-67Meas	25	10.549	1.342	10.630

$\hat{\sigma}_x = S / c_4 = 1.342 / 0.7979 = 1.682$

5-67 continued

(c)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



$\hat{\sigma}_x = \bar{R} / d_2 = 1.283/1.128 = 1.137$

(d)

Average MR3 Chart: $\hat{\sigma}_x = \bar{R} / d_2 = 2.049/1.693 = 1.210$

Average MR4 Chart: $\hat{\sigma}_x = \bar{R} / d_2 = 2.598/2.059 = 1.262$

Average MR19 Chart: $\hat{\sigma}_x = \bar{R} / d_2 = 5.186/3.689 = 1.406$

Average MR20 Chart: $\hat{\sigma}_x = \bar{R} / d_2 = 5.36/3.735 = 1.435$

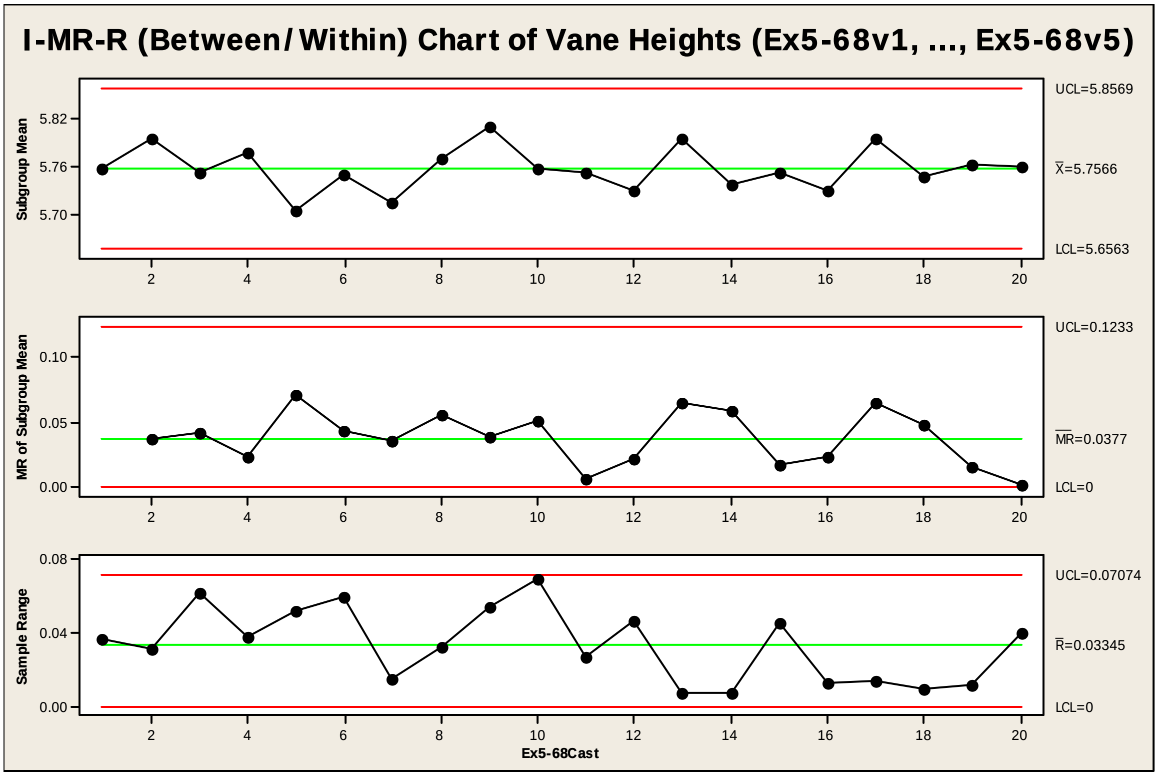
(e)

As the span of the moving range is increased, there are fewer observations to estimate the standard deviation, and the estimate becomes less reliable. For this example, σ gets larger as the span increases. This tends to be true for unstable processes.

5-68 (5-58).

MTB > Stat > Control Charts > Variables Charts for Subgroups > I-MR-R/S (Between/Within)

Select "**I-MR-R/S Options, Estimate**" and choose R-bar method to estimate standard deviation



I-MR-R/S Standard Deviations of Ex5-68v1, ..., Ex5-68v5

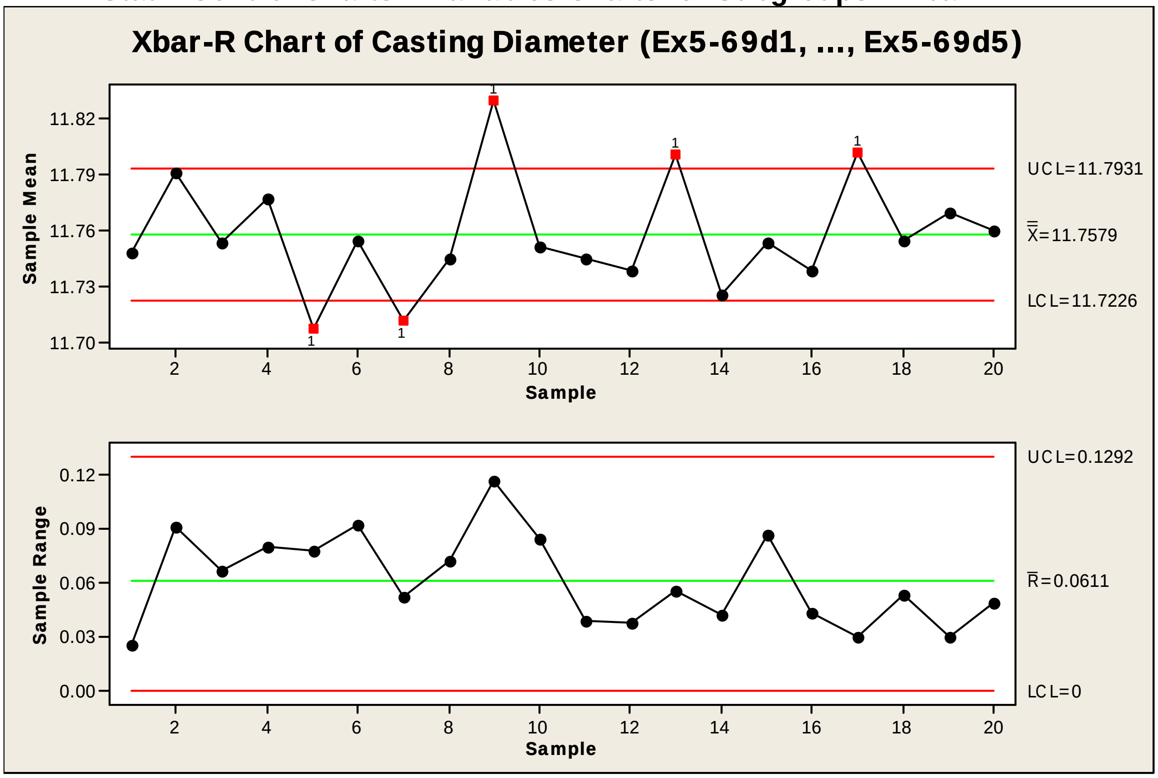
```
Standard Deviations
Between         0.0328230
Within          0.0143831
Between/Within  0.0358361
```

The Individuals and Moving Range charts for the subgroup means are identical. When compared to the *s* chart for all data, the *R* chart tells the same story—same data pattern and no out-of-control points. For this example, the control schemes are identical.

5-69 (5-59).

(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R




```
Xbar-R Chart of Ex5-69d1, ..., Ex5-69d5
Test Results for Xbar Chart of Ex5-69d1, ..., Ex5-69d5
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  5, 7, 9, 13, 17
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  7
```

(b)

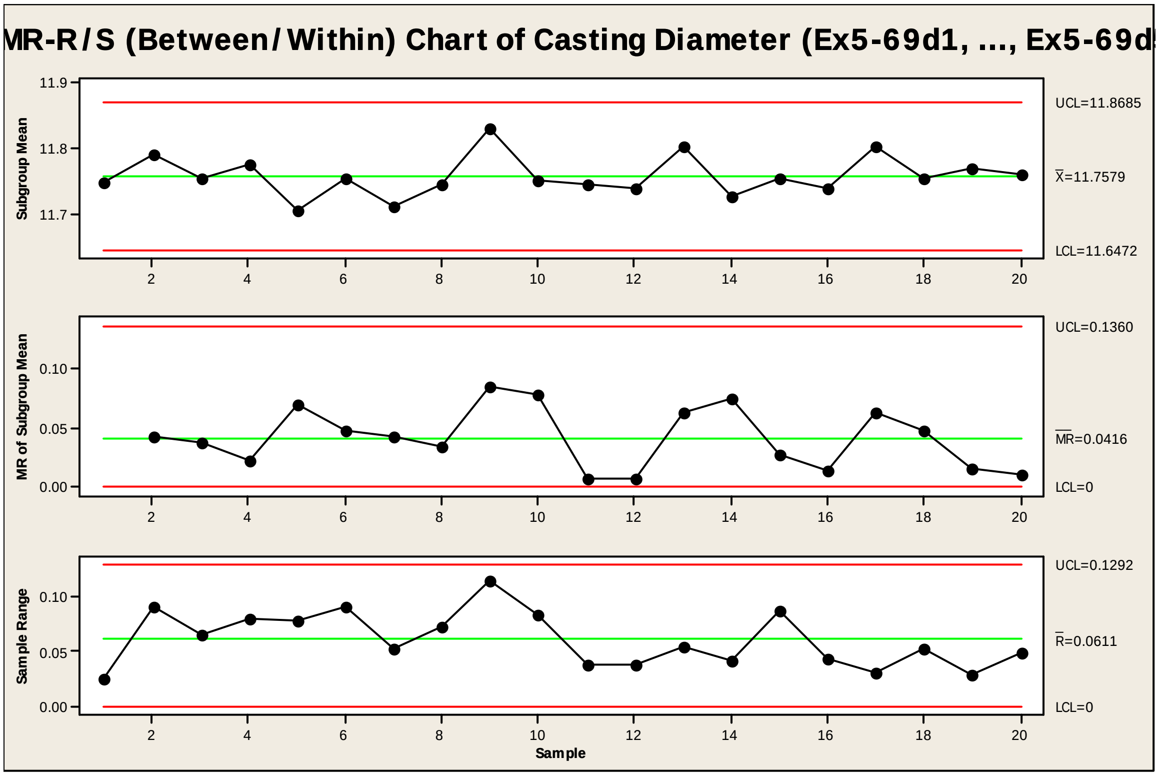
Though the *R* chart is in control, plot points on the $\bar{x}$ chart bounce below and above the control limits. Since these are high precision castings, we might expect that the diameter of a single casting will not change much with location. If no assignable cause can be found for these out-of-control points, we may want to consider treating the averages as an Individual value and graphing "between/within" range charts. This will lead to a understanding of the greatest source of variability, between castings or within a casting.

5-69 continued

(c)

MTB > Stat > Control Charts > Variables Charts for Subgroups > I-MR-R/S (Between/Within)

Select "**I-MR-R/S Options, Estimate**" and choose R-bar method to estimate standard deviation



I-MR-R/S (Between/Within) Chart of Ex5-69d1, ..., Ex5-69d5

I-MR-R/S Standard Deviations of Ex5-69d1, ..., Ex5-69d5

```
Standard Deviations
Between         0.0349679
Within          0.0262640
Between/Within  0.0437327
```

(d)

We are taking several diameter measurements on a single precision casting.

(e)

The "within" chart is the usual *R* chart ($n > 1$). It describes the measurement variability within a sample (variability in diameter of a single casting). Though the nature of this process leads us to believe that the diameter at any location on a single casting does not change much, we should continue to monitor "within" to look for wear, damage, etc., in the wax mold.

5-70 (5-60).
(a)
Both total process variability and the overall process average could be estimated from a single measurement on one wafer from each lot. Individuals X and Moving Range charts should be used for process monitoring.

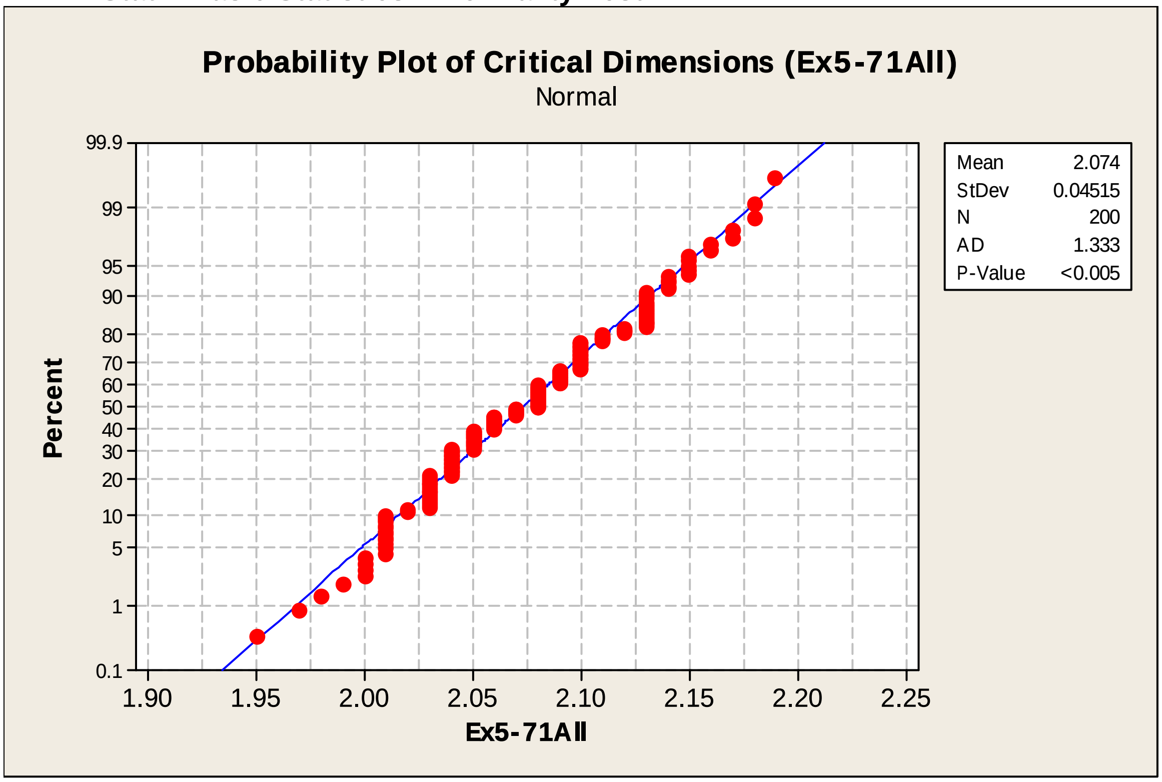
(b)
Assuming that each wafer is processed separately, within-wafer variability could be monitored with a standard $\bar{X} - R$ control chart. The data from each wafer could also be used to monitor between-wafer variability by maintaining an individuals X and moving range chart for each of the five fixed positions. The Minitab "between/within" control charts do this in three graphs: (1) wafer mean $(\bar{x}_{ww})$ is an "individual value", (2) moving range is the difference between successive wafers, and (3) sample range is the difference within a wafer (R_{ww}). Alternatively, a multivariate process control technique could be used.

(c)
Both between-wafer and total process variability could be estimated from measurements at one point on five consecutive wafers. If it is necessary to separately monitor the variation at each location, then either five $\bar{X} - R$ charts or some multivariate technique is needed. If the positions are essentially identical, then only one location, with one $\bar{X} - R$ chart, needs to be monitored.

(d)
Within-wafer variability can still be monitored with randomly selected test sites. However, no information will be obtained about the pattern of variability within a wafer.

(e)
The simplest scheme would be to randomly select one wafer from each lot and treat the average of all measurements on that wafer as one observation. Then a chart for individual x and moving range would provide information on lot-to-lot variability.

5-71 (5-61).
(a)
MTB > Stat > Basic Statistics > Normality Test



Although the p-value is very small, the plot points do fall along a straight line, with many repeated values. The wafer critical dimension is approximately normally distributed.

The natural tolerance limits (± 3 sigma above and below mean) are:

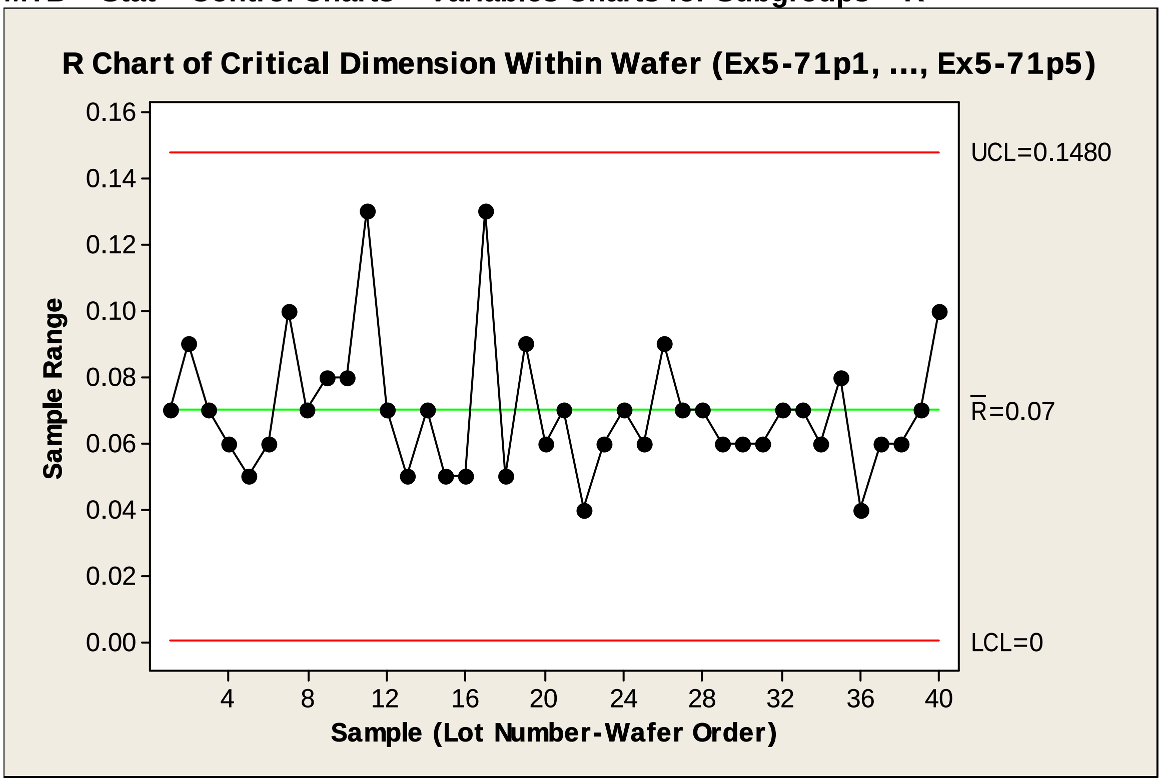
$\bar{x} = 2.074, s = 0.04515$

$$\text{UNTL} = \bar{x} + 3s = 2.074 + 3(0.04515) = 2.209$$

$$\text{LNTL} = \bar{x} - 3s = 2.074 - 3(0.04515) = 1.939$$

5-71 continued
(b)
To evaluate within-wafer variability, construct an *R* chart for each sample of 5 wafer positions (two wafers per lot number), for a total of 40 subgroups.

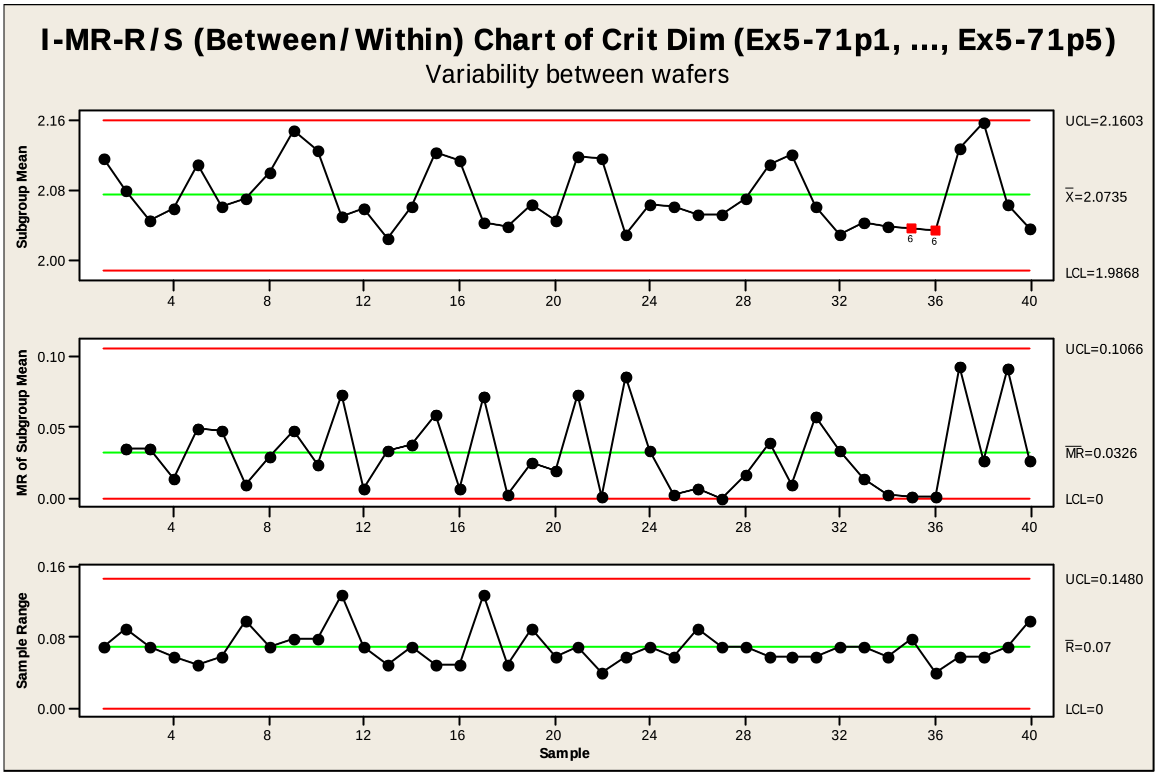
MTB > Stat > Control Charts > Variables Charts for Subgroups > R



The Range chart is in control, indicating that within-wafer variability is also in control.

5-71 continued
(c)
To evaluate variability between wafers, set up Individuals and Moving Range charts where the *x* statistic is the average wafer measurement and the moving range is calculated between two wafer averages.

MTB > Stat > Control Charts > Variables Charts for Subgroups > I-MR-R/S (Between/Within)
Select "**I-MR-R/S Options, Estimate**" and choose R-bar method to estimate standard deviation



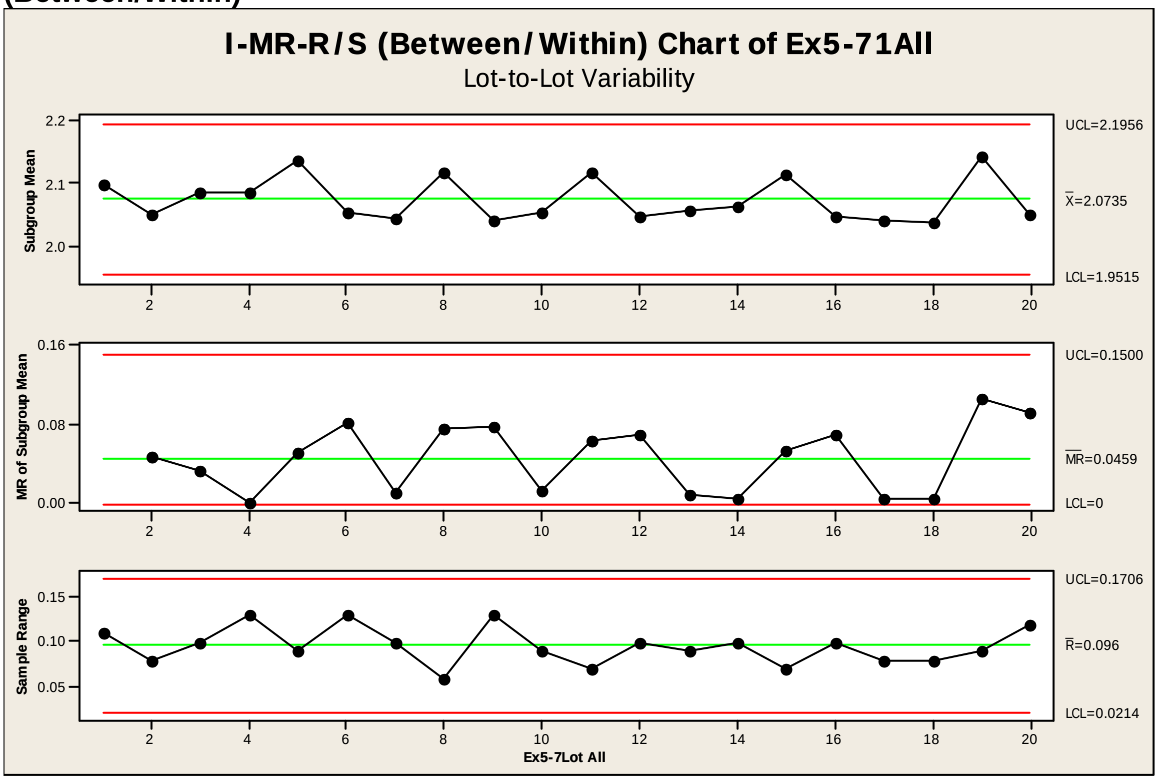
I-MR-R/S Standard Deviations of Ex5-71p1, ..., Ex5-71p5

Standard Deviations	
Between	0.0255911
Within	0.0300946
Between/Within	0.0395043

Both "between" control charts (Individuals and Moving Range) are in control, indicating that between-wafer variability is also in-control. The "within" chart (Range) is not required to evaluate variability between wafers.

5-71 continued
(d)
To evaluate lot-to-lot variability, three charts are needed: (1) lot average, (2) moving range between lot averages, and (3) range within a lot—the Minitab “between/within” control charts.

MTB > Stat > Control Charts > Variables Charts for Subgroups > I-MR-R/S (Between/Within)



I-MR-R/S Standard Deviations of Ex5-71All

```
Standard Deviations
Between         0.0394733
Within          0.0311891
Between/Within  0.0503081
```

All three control charts are in control, indicating that the lot-to-lot variability is also in-control.

Chapter 6 Exercise Solutions

Notes:

1. New exercises are denoted with an "☺".
2. For these solutions, we follow the MINITAB convention for determining whether a point is out of control. If a plot point is *within* the control limits, it is considered to be in control. If a plot point is *on* or *beyond* the control limits, it is considered to be out of control.
3. MINITAB defines some sensitizing rules for control charts differently than the standard rules. In particular, a run of *n* consecutive points on one side of the center line is defined as 9 points, not 8. This can be changed under Tools > Options > Control Charts and Quality Tools > Define Tests. Also fewer special cause tests are available for attributes control charts.

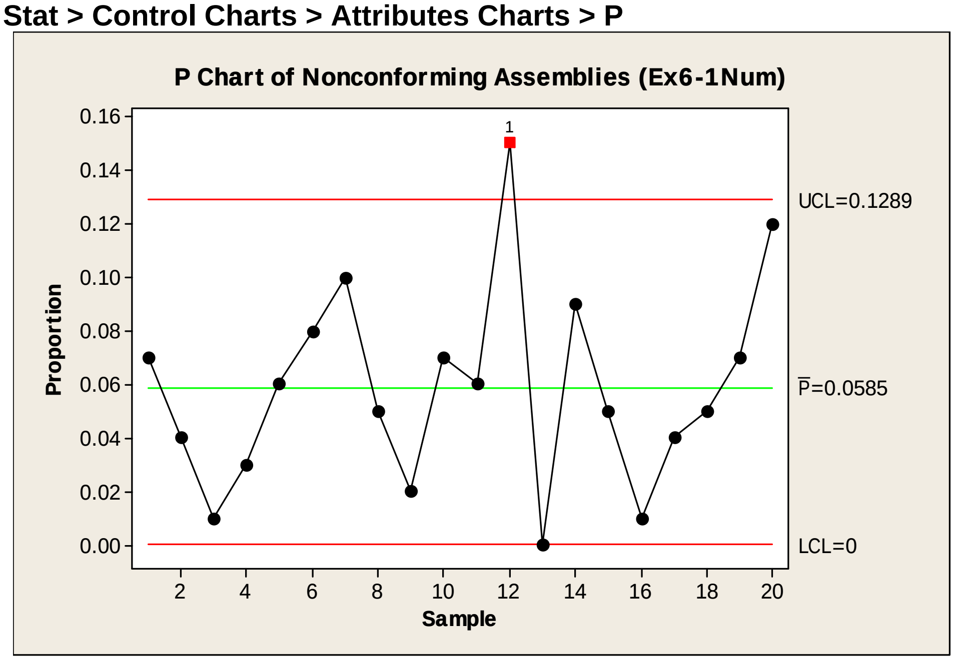
6-1.

$$n = 100;\quad m = 20;\quad \sum_{i=1}^{m} D_i = 117;\quad \bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{117}{20(100)} = 0.0585$$

$$\text{UCL}_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0585 + 3\sqrt{\frac{0.0585(1-0.0585)}{100}} = 0.1289$$

$$\text{LCL}_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0585 - 3\sqrt{\frac{0.0585(1-0.0585)}{100}} = 0.0585 - 0.0704 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P



Test Results for P Chart of Ex6-1Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

6-1 continued

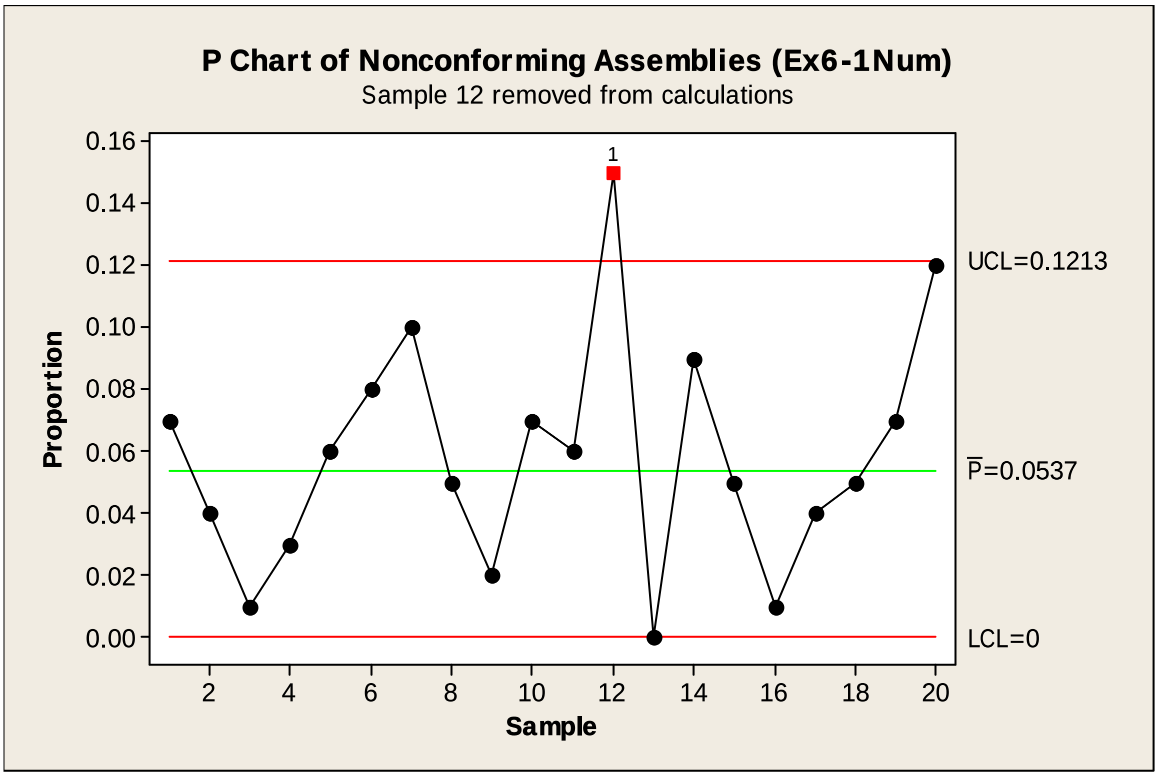
Sample 12 is out-of-control, so remove from control limit calculation:

$$n = 100; \quad m = 19; \quad \sum_{i=1}^{m} D_i = 102; \quad \overline{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{102}{19(100)} = 0.0537$$

$$\text{UCL}_p = 0.0537 + 3\sqrt{\frac{0.0537(1-0.0537)}{100}} = 0.1213$$

$$\text{LCL}_p = 0.0537 - 3\sqrt{\frac{0.0537(1-0.0537)}{100}} = 0.0537 - 0.0676 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P



Test Results for P Chart of Ex6-1Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

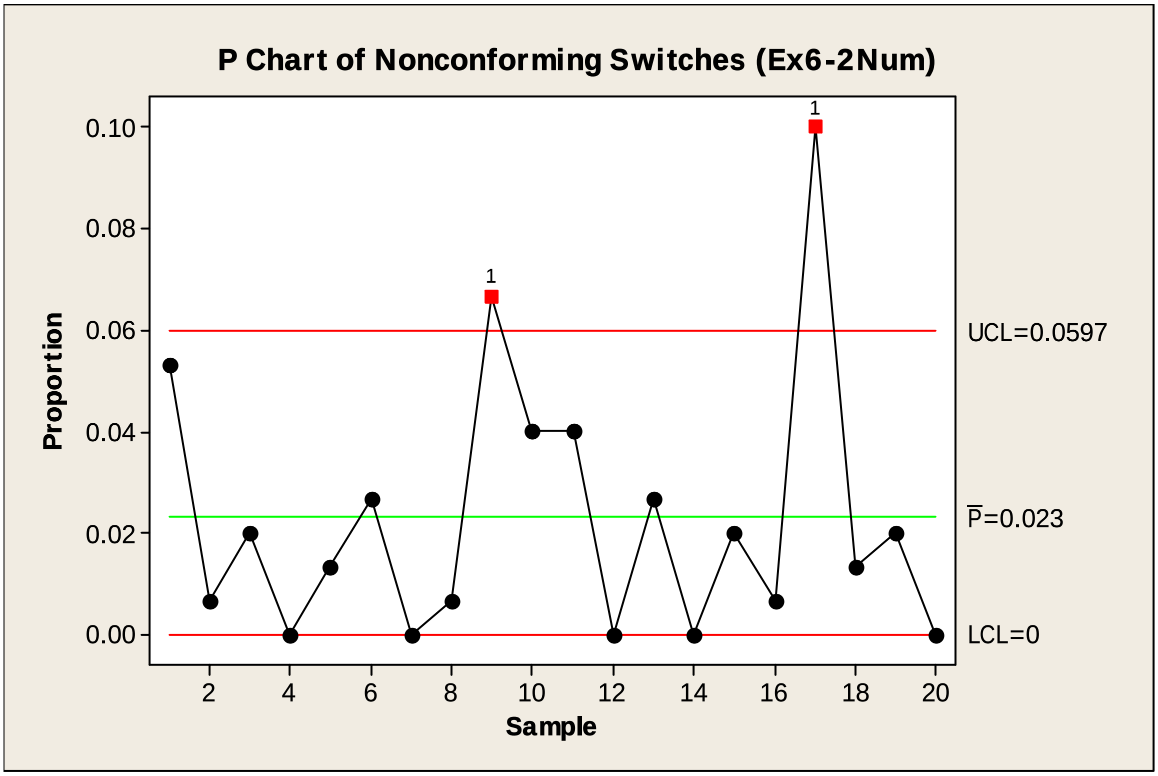
6-2.

$$n = 150; \quad m = 20; \quad \sum_{i=1}^{m} D_i = 69; \quad \overline{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{69}{20(150)} = 0.0230$$

$$\text{UCL}_p = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.0230 + 3\sqrt{\frac{0.0230(1-0.0230)}{150}} = 0.0597$$

$$\text{LCL}_p = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.0230 - 3\sqrt{\frac{0.0230(1-0.0230)}{150}} = 0.0230 - 0.0367 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P

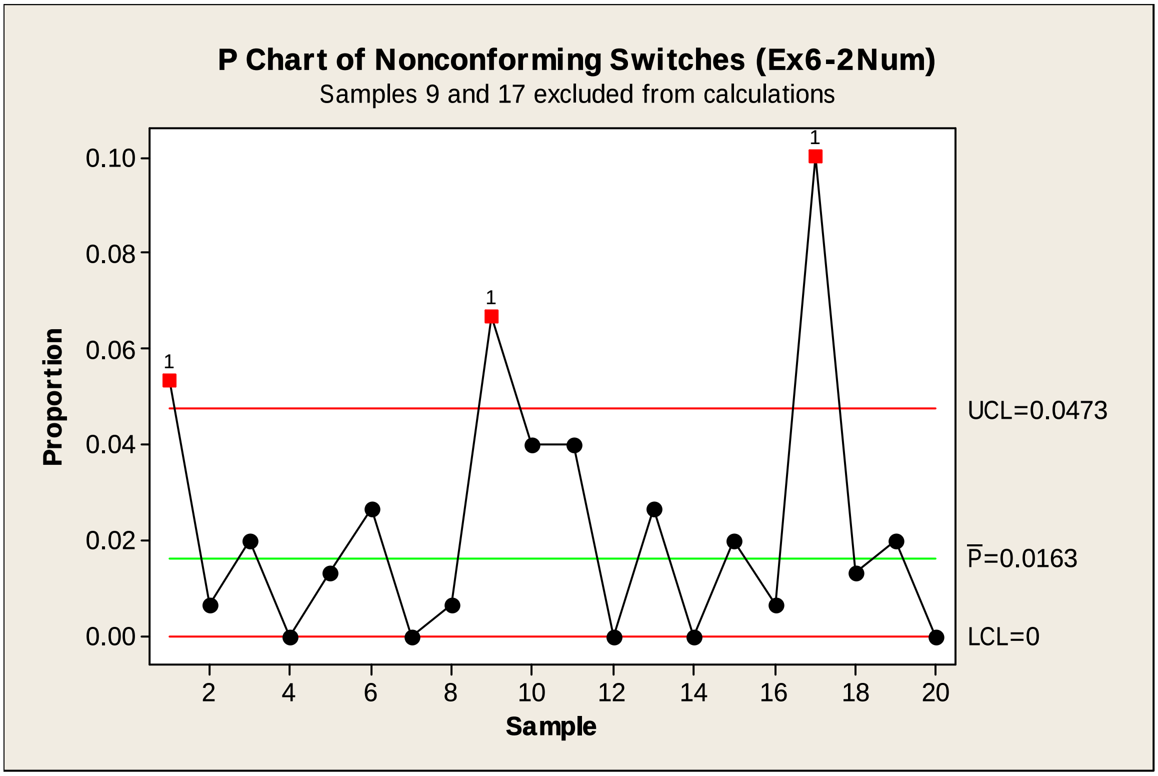


Test Results for P Chart of Ex6-2Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  9, 17
```

6-2 continued
Re-calculate control limits without samples 9 and 17:

MTB > Stat > Control Charts > Attributes Charts > P



Test Results for P Chart of Ex6-2Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 9, 17
```

6-2 continued

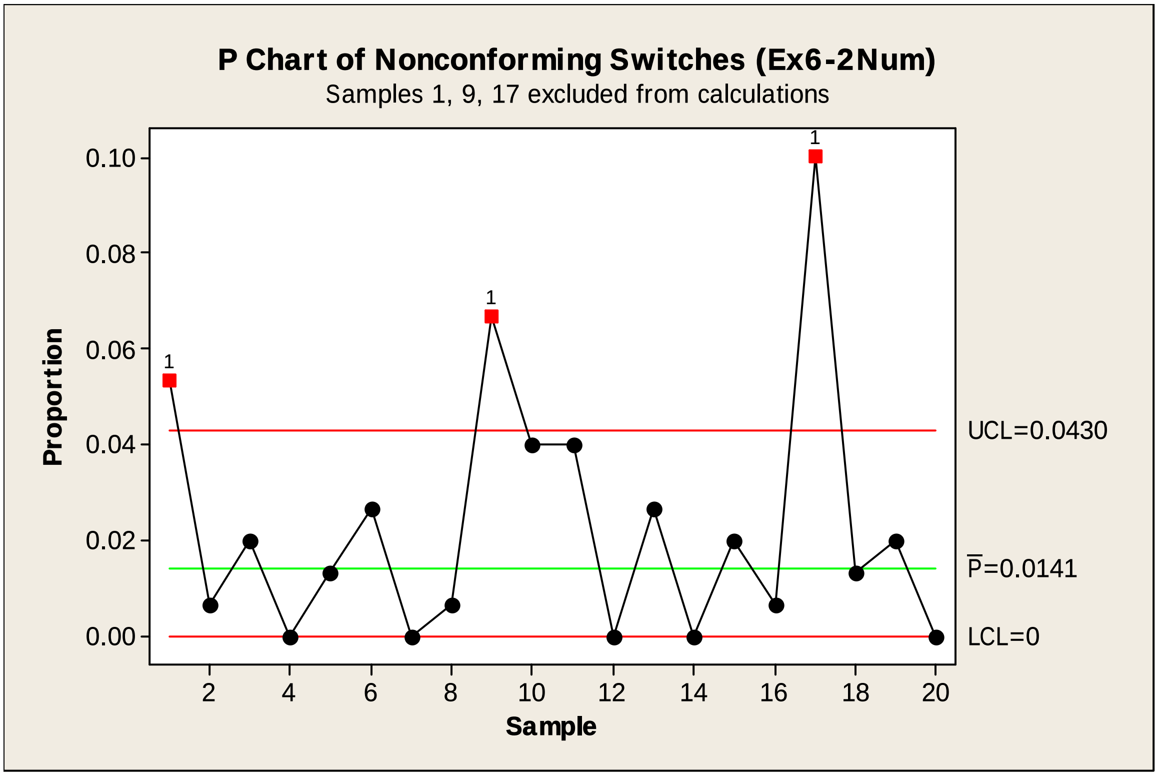
Also remove sample 1 from control limits calculation:

$$n = 150;\quad m = 17;\quad \sum_{i=1}^{m} D_i = 36;\quad \bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{36}{17(150)} = 0.0141$$

$$\text{UCL}_p = 0.0141 + 3\sqrt{\frac{0.0141(1-0.0141)}{150}} = 0.0430$$

$$\text{LCL}_p = 0.0141 - 3\sqrt{\frac{0.0141(1-0.0141)}{150}} = 0.0141 - 0.0289 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P



Test Results for P Chart of Ex6-2Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 9, 17
```

6-3.
NOTE: There is an error in the table in the textbook. The Fraction Nonconforming for Day 5 should be 0.046.

$$m = 10; \quad \sum_{i=1}^{m} n_i = 1000; \quad \sum_{i=1}^{m} D_i = 60; \quad \overline{p} = \sum_{i=1}^{m} D_i \Big/ \sum_{i=1}^{m} n_i = 60/1000 = 0.06$$

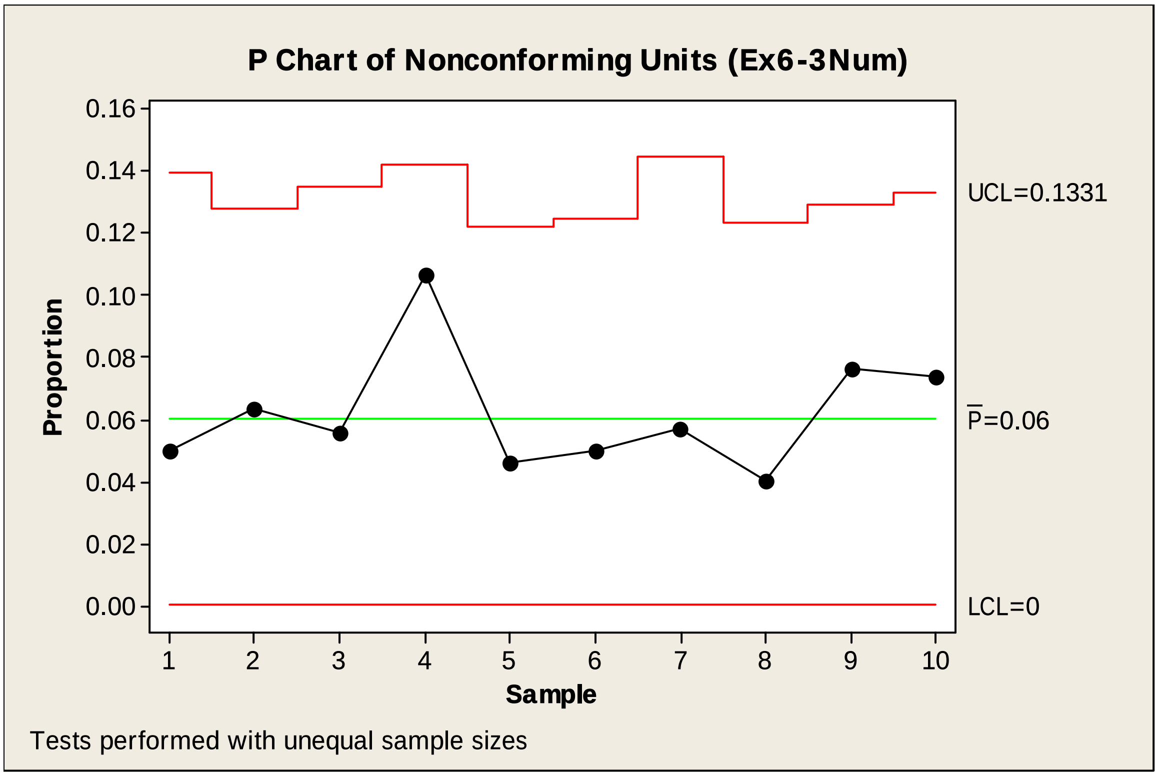
$$\text{UCL}_i = \overline{p} + 3\sqrt{\overline{p}(1-\overline{p})/n_i} \quad \text{and} \quad \text{LCL}_i = \max\{0, \overline{p} - 3\sqrt{\overline{p}(1-\overline{p})/n_i}\}$$

As an example, for $n = 80$:

$$\text{UCL}_1 = \overline{p} + 3\sqrt{\overline{p}(1-\overline{p})/n_1} = 0.06 + 3\sqrt{0.06(1-0.06)/80} = 0.1397$$

$$\text{LCL}_1 = \overline{p} - 3\sqrt{\overline{p}(1-\overline{p})/n_1} = 0.06 - 3\sqrt{0.06(1-0.06)/80} = 0.06 - 0.0797 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P



The process appears to be in statistical control.

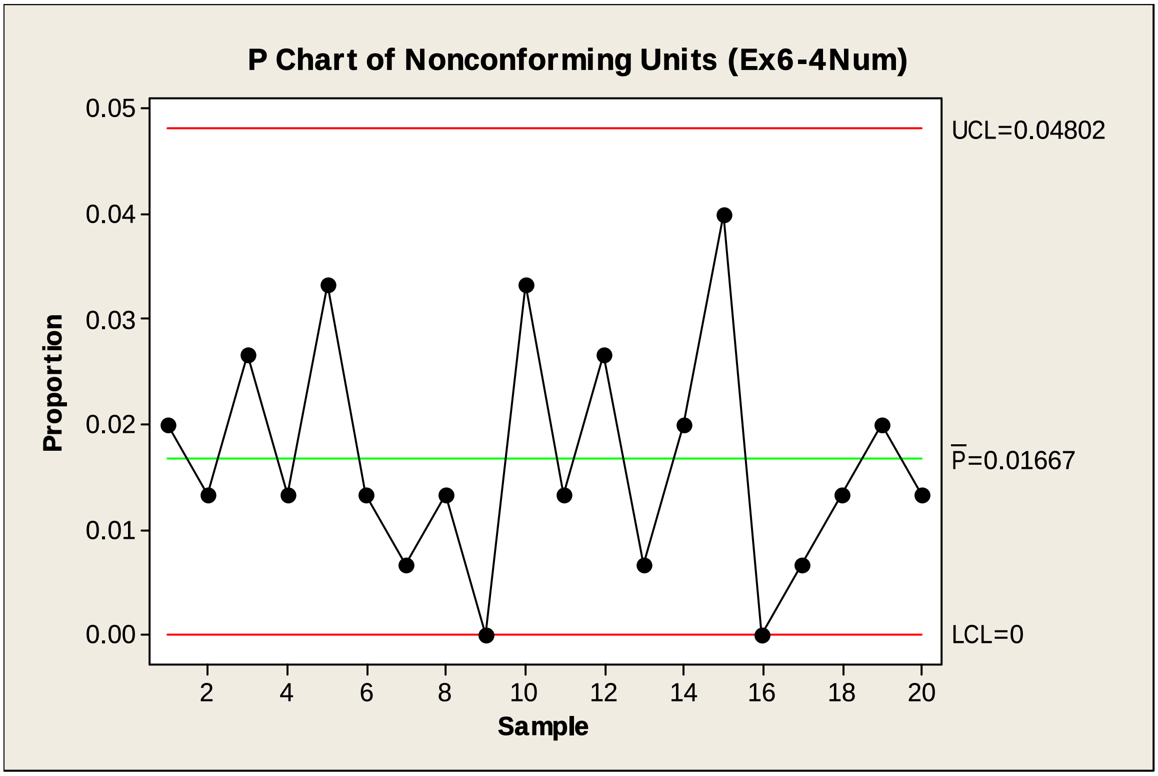
6-4.
(a)

$$n = 150; \quad m = 20; \quad \sum_{i=1}^{m} D_i = 50; \quad \overline{p} = \sum_{i=1}^{m} D_i \Big/ mn = 50/20(150) = 0.0167$$

$$\text{UCL} = \overline{p} + 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.0167 + 3\sqrt{0.0167(1-0.0167)/150} = 0.0480$$

$$\text{LCL} = \overline{p} - 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.0167 - 3\sqrt{0.0167(1-0.0167)/150} = 0.0167 - 0.0314 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P



The process appears to be in statistical control.

(b)
Using Equation 6-12,

$$n > \frac{(1-p)}{p} L^2$$

$$> \frac{(1-0.0167)}{0.0167} (3)^2$$

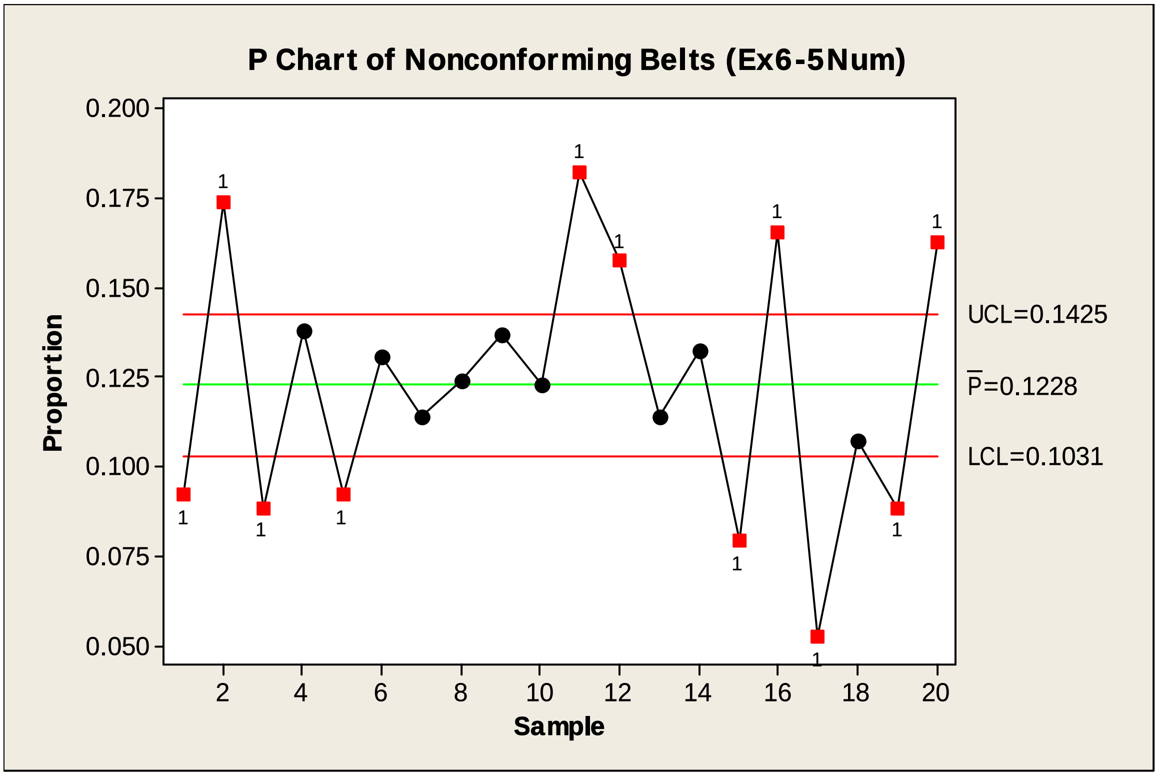
$$> 529.9 \qquad \text{Select } n = 530.$$

6-5.
(a)

$$\text{UCL} = \overline{p} + 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.1228 + 3\sqrt{0.1228(1-0.1228)/2500} = 0.1425$$

$$\text{LCL} = \overline{p} - 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.1228 - 3\sqrt{0.1228(1-0.1228)/2500} = 0.1031$$

MTB > Stat > Control Charts > Attributes Charts > P




```
Test Results for P Chart of Ex6-5Num
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 2, 3, 5, 11, 12, 15, 16, 17, 19, 20
```

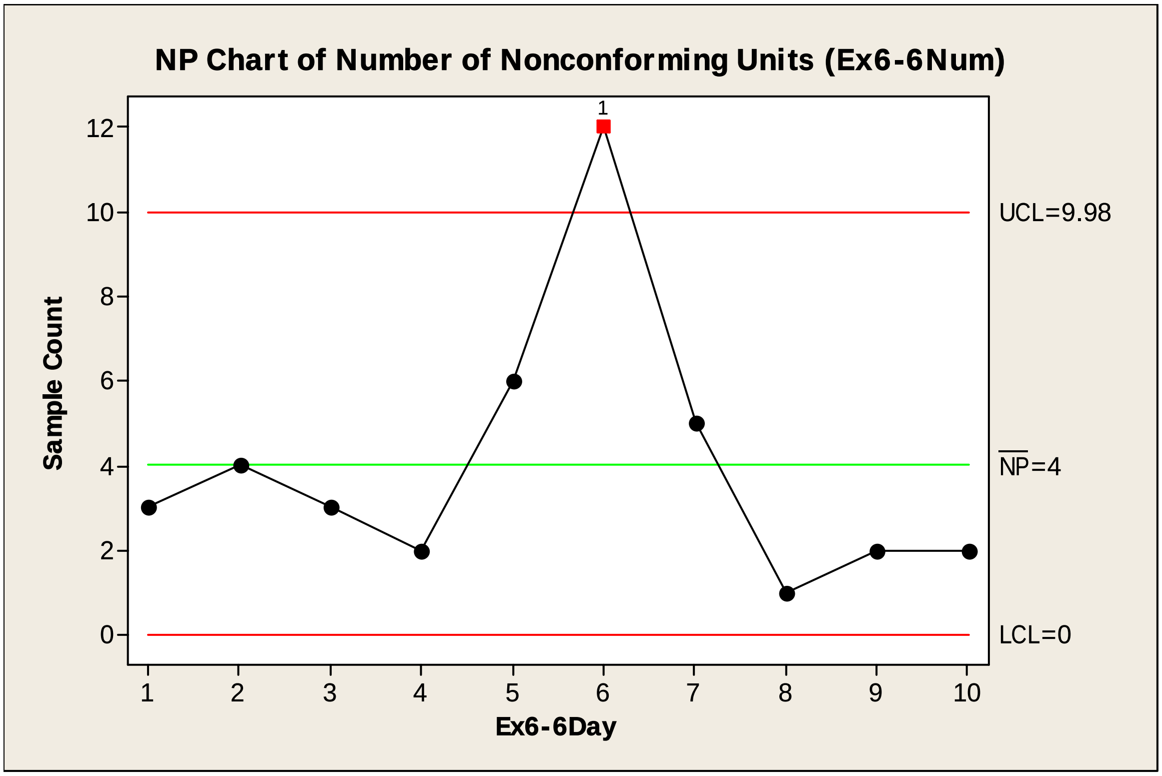
(b)
So many subgroups are out of control (11 of 20) that the data should not be used to establish control limits for future production. Instead, the process should be investigated for causes of the wild swings in *p*.

6-6.

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 4 + 3\sqrt{4(1-0.008)} = 9.976$$

$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 4 - 3\sqrt{4(1-0.008)} = 4 - 5.976 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > NP

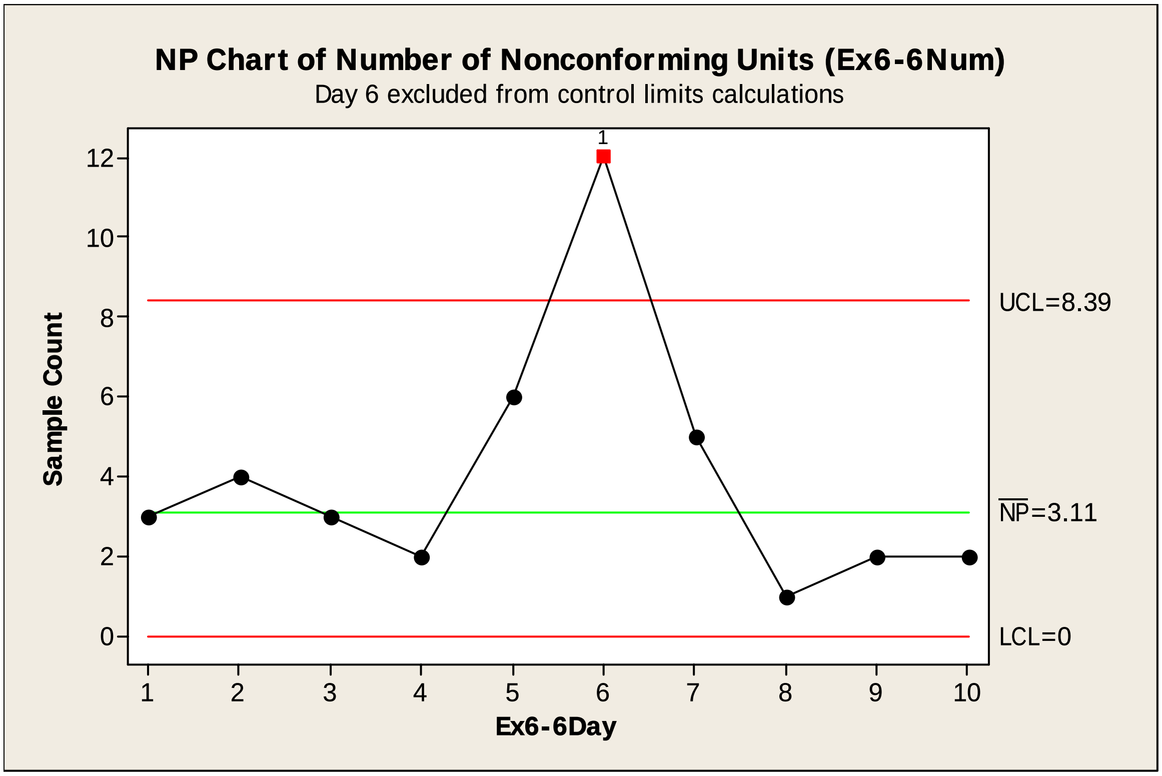


Test Results for NP Chart of Ex6-6Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  6
```

6.6 continued

Recalculate control limits without sample 6:




```
Test Results for NP Chart of Ex6-6Num
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  6
```

Recommend using control limits from second chart (calculated less sample 6).

6-7.

$\overline{p} = 0.02; n = 50$

$$\text{UCL} = \overline{p} + 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.02 + 3\sqrt{0.02(1-0.02)/50} = 0.0794$$

$$\text{LCL} = \overline{p} - 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.02 - 3\sqrt{0.02(1-0.02)/50} = 0.02 - 0.0594 \Rightarrow 0$$

Since $p_{\text{new}} = 0.04 < 0.1$ and $n = 50$ is "large", use the Poisson approximation to the binomial with $\lambda = np_{\text{new}} = 50(0.04) = 2.00$.

Pr{detect|shift}

= 1 – Pr{not detect|shift}

= 1 – β

= 1 – [Pr{$D < n$UCL | λ} – Pr{$D \le n$LCL | λ}]

= 1 – Pr{$D < 50(0.0794)$ | 2} + Pr{$D \le 50(0)$ | 2}

= 1 – POI(3,2) + POI(0,2) = 1 – 0.857 + 0.135 = 0.278

where POI(·) is the cumulative Poisson distribution.

Pr{detected by 3rd sample} = 1 – Pr{detected after 3rd} = 1 – $(1 – 0.278)^3$ = 0.624

6-8.

$$m = 10;\quad n = 250;\quad \sum_{i=1}^{10} \hat{p}_i = 0.0440;\quad \overline{p} = \frac{0.0440}{10} = 0.0044$$

$$\text{UCL} = \overline{p} + 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.0044 + 3\sqrt{0.0044(1-0.0044)/250} = 0.0170$$

$$\text{UCL} = \overline{p} - 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.0044 - 3\sqrt{0.0044(1-0.0044)/250} = 0.0044 - 0.0126 \Rightarrow 0$$

No. The data from the shipment do not indicate statistical control. From the 6th sample, $(\hat{p}_6 = 0.020) > 0.0170$, the UCL.

6-9.

$\bar{p} = 0.10; n = 64$

$\text{UCL} = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.10 + 3\sqrt{0.10(1-0.10)/64} = 0.2125$

$\text{LCL} = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.10 - 3\sqrt{0.10(1-0.10)/64} = 0.10 - 0.1125 \Rightarrow 0$

$$\begin{aligned}\beta &= \Pr\{D < n\text{UCL} \mid p\} - \Pr\{D \le n\text{LCL} \mid p\} \\ &= \Pr\{D < 64(0.2125) \mid p\} - \Pr\{D \le 64(0) \mid p\} \\ &= \Pr\{D < 13.6) \mid p\} - \Pr\{D \le 0 \mid p\}\end{aligned}$$

p	$\Pr\{D \le 13 \mid p\}$	$\Pr\{D \le 0 \mid p\}$	β
0.05	0.999999	0.037524	0.962475
0.10	0.996172	0.001179	0.994993
0.20	0.598077	0.000000	0.598077
0.21	**0.519279**	**0.000000**	**0.519279**
0.22	0.44154	0.000000	0.44154
0.215	0.480098	0.000000	0.480098
0.212	0.503553	0.000000	0.503553

Assuming L = 3 sigma control limits,

$$\begin{aligned}n &> \frac{(1-p)}{p}L^2 \\ &> \frac{(1-0.10)}{0.10}(3)^2 \\ &> 81\end{aligned}$$

6-10.

$np = 16.0;\quad n = 100;\quad \bar{p} = 16/100 = 0.16$

$\text{UCL} = np + 3\sqrt{np(1-\bar{p})} = 16 + 3\sqrt{16(1-0.16)} = 27.00$

$\text{LCL} = np - 3\sqrt{np(1-\bar{p})} = 16 - 3\sqrt{16(1-0.16)} = 5.00$

(a)

np_{new} = 20.0 > 15, so use normal approximation to binomial distribution.

$$
\begin{aligned}
\Pr\{\text{detect shift on 1st sample}\} &= 1-\beta \\
&= 1 - [\Pr\{D < \text{UCL} \mid p\} - \Pr\{D \le \text{LCL} \mid p\}] \\
&= 1 - \Phi\left(\frac{\text{UCL} + 1/2 - np}{\sqrt{np(1-p)}}\right) + \Phi\left(\frac{\text{LCL} - 1/2 - np}{\sqrt{np(1-p)}}\right) \\
&= 1 - \Phi\left(\frac{27 + 0.5 - 20}{\sqrt{20(1-0.2)}}\right) + \Phi\left(\frac{5 - 0.5 - 20}{\sqrt{20(1-0.2)}}\right) \\
&= 1 - \Phi(1.875) + \Phi(-3.875) \\
&= 1 - 0.970 + 0.000 \\
&= 0.030
\end{aligned}
$$

Pr{detect by at least 3rd}

= 1 – Pr{detected after 3rd}

= 1 – $(1 – 0.030)^3$

= 0.0873

(b)

Assuming L = 3 sigma control limits,

$$
\begin{aligned}
n &> \frac{(1-p)}{p} L^2 \\
&> \frac{(1-0.16)}{0.16} (3)^2 \\
&> 47.25
\end{aligned}
$$

So, n = 48 is the minimum sample size for a positive LCL.

6-11.

$p = 0.10;\quad p_{\text{new}} = 0.20;\quad$ desire $\Pr\{\text{detect}\} = 0.50;\quad$ assume $k = 3$ sigma control limits

$\delta = p_{\text{new}} - p = 0.20 - 0.10 = 0.10$

$$n = \left(\frac{k}{\delta}\right)^2 p(1-p) = \left(\frac{3}{0.10}\right)^2 (0.10)(1-0.10) = 81$$

6-12.
n = 100, p = 0.08, UCL = 0.161, LCL = 0

(a)
$np = 100(0.080) = 8$
$\text{UCL} = np + 3\sqrt{np(1-p)} = 8 + 3\sqrt{8(1-0.080)} = 16.14$
$\text{LCL} = np - 3\sqrt{np(1-p)} = 8 - 3\sqrt{8(1-0.080)} = 8 - 8.1388 \Rightarrow 0$

(b)
p = 0.080 < 0.1 and n =100 is large, so use Poisson approximation to the binomial.

Pr{type I error} = α
= Pr{D < LCL | p} + Pr{D > UCL | p}
= Pr{D < LCL | p} + [1 – Pr{$D \leq$ UCL | p}]
= Pr{D < 0 | 8} + [1 – Pr{$D \leq$ 16 | 8}]
= 0 + [1 – POI(16,8)]
= 0 + [1 – 0.996]
= 0.004
where POI(·) is the cumulative Poisson distribution.

(c)
np_{new} = 100(0.20) = 20 > 15, so use the normal approximation to the binomial.

$$\begin{aligned}
\Pr\{\text{type II error}\} &= \beta \\
&= \Pr\{\hat{p} < \text{UCL} \mid p_{\text{new}}\} - \Pr\{\hat{p} \leq \text{LCL} \mid p_{\text{new}}\} \\
&= \Phi\left(\frac{\text{UCL} - p_{\text{new}}}{\sqrt{p(1-p)/n}}\right) - \Phi\left(\frac{\text{LCL} - p_{\text{new}}}{\sqrt{p(1-p)/n}}\right) \\
&= \Phi\left(\frac{0.161 - 0.20}{\sqrt{0.08(1-0.08)/100}}\right) - \Phi\left(\frac{0 - 0.20}{\sqrt{0.08(1-0.08)/100}}\right) \\
&= \Phi(-1.44) - \Phi(-7.37) \\
&= 0.07494 - 0 \\
&= 0.07494
\end{aligned}$$

(d)
Pr{detect shift by at most 4th sample}
= 1 – Pr{not detect by 4th}
= 1 – $(0.07494)^4$
= 0.99997

6-13.
(a)
$\bar{p} = 0.07$; $k = 3$ sigma control limits; $n = 400$

$$\text{UCL} = \bar{p} + 3\sqrt{p(1-p)/n} = 0.07 + 3\sqrt{0.07(1-0.07)/400} = 0.108$$
$$\text{LCL} = \bar{p} - 3\sqrt{p(1-p)/n} = 0.07 - 3\sqrt{0.07(1-0.07)/400} = 0.032$$

(b)
np_{new} = 400(0.10) = > 40, so use the normal approximation to the binomial.

$$\begin{aligned}
\Pr\{\text{detect on 1st sample}\} &= 1 - \Pr\{\text{not detect on 1st sample}\} \\
&= 1 - \beta \\
&= 1 - [\Pr\{\hat{p} < \text{UCL} \mid p\} - \Pr\{\hat{p} \le \text{LCL} \mid p\}] \\
&= 1 - \Phi\left(\frac{\text{UCL} - p}{\sqrt{p(1-p)/n}}\right) + \Phi\left(\frac{\text{LCL} - p}{\sqrt{p(1-p)/n}}\right) \\
&= 1 - \Phi\left(\frac{0.108 - 0.1}{\sqrt{0.1(1-0.1)/400}}\right) + \Phi\left(\frac{0.032 - 0.1}{\sqrt{0.1(1-0.1)/400}}\right) \\
&= 1 - \Phi(0.533) + \Phi(-4.533) \\
&= 1 - 0.703 + 0.000 \\
&= 0.297
\end{aligned}$$

(c)
Pr{detect on 1st or 2nd sample}
= Pr{detect on 1st} + Pr{not on 1st}×Pr{detect on 2nd}
= 0.297 + (1 – 0.297)(0.297)
= 0.506

6-14.
p = 0.20 and L = 3 sigma control limits

$$\begin{aligned}
n &> \frac{(1-p)}{p}L^2 \\
&> \frac{(1-0.20)}{0.20}(3)^2 \\
&> 36
\end{aligned}$$

For Pr{detect} = 0.50 after a shift to p_{new} = 0.26,

$$\delta = p_{\text{new}} - p = 0.26 - 0.20 = 0.06$$
$$n = \left(\frac{k}{\delta}\right)^2 p(1-p) = \left(\frac{3}{0.06}\right)^2 (0.20)(1-0.20) = 400$$

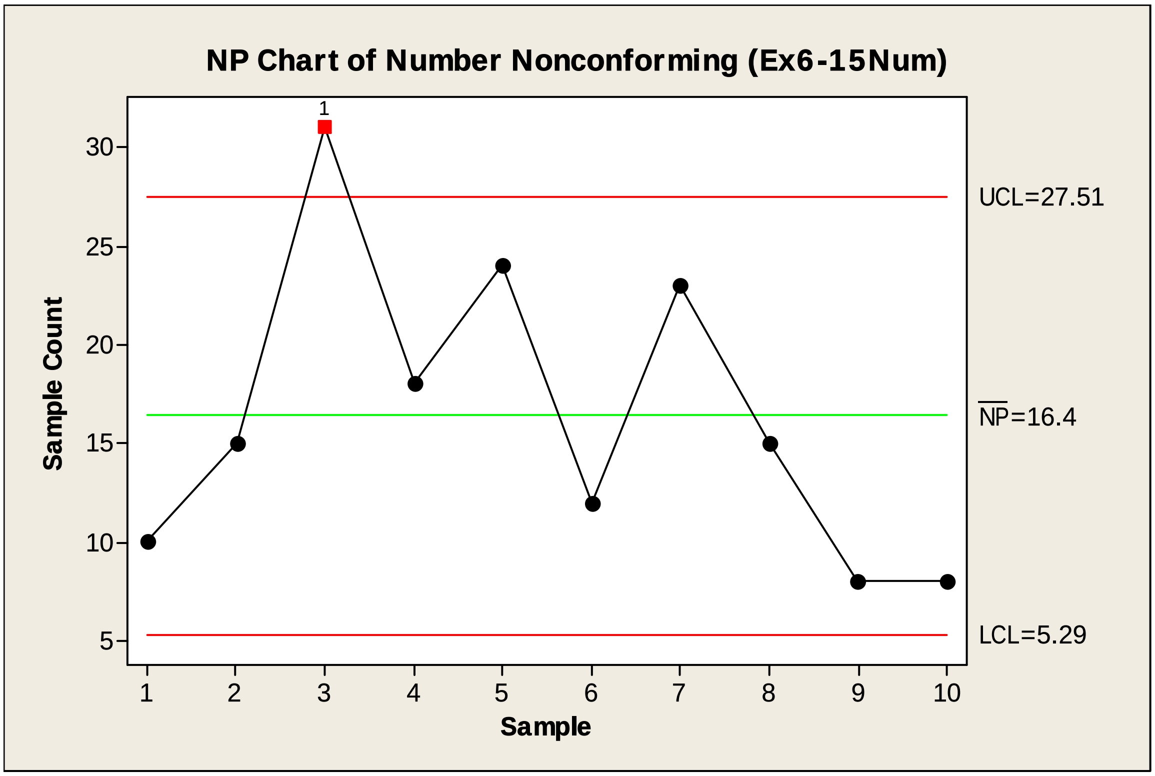
6-15.
(a)

$$m=10; \quad n=100; \quad \sum_{i=1}^{10} D_i = 164; \quad \bar{p} = \sum_{i=1}^{10} D_i \Big/ (mn) = 164 \big/ [10(100)] = 0.164; \quad n\bar{p} = 16.4$$

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 16.4 + 3\sqrt{16.4(1-0.164)} = 27.51$$

$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 16.4 - 3\sqrt{16.4(1-0.164)} = 5.292$$

MTB > Stat > Control Charts > Attributes Charts > NP

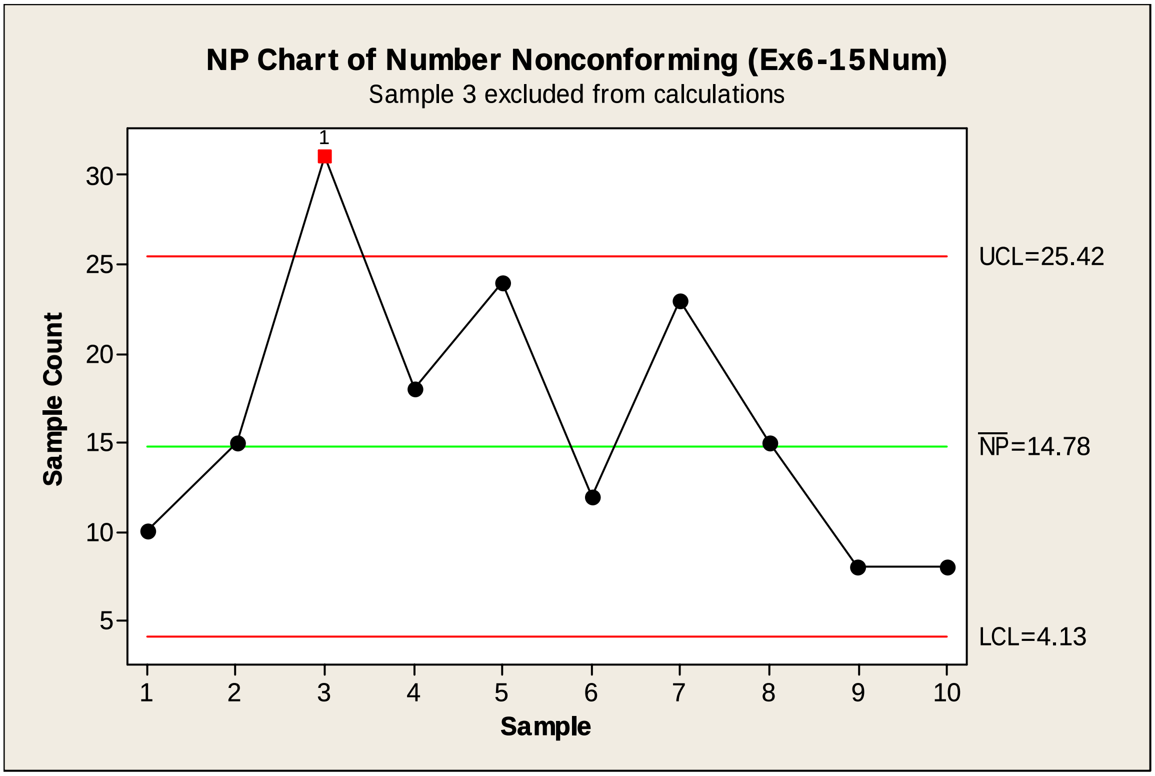


Test Results for NP Chart of Ex6-15Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  3
```

6-15 continued

Recalculate control limits less sample 3:



Test Results for NP Chart of Ex6-15Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  3
```

6-15 continued
(b)
$p_{\text{new}} = 0.30$. Since $p = 0.30$ is not too far from 0.50, and $n = 100 > 10$, the normal approximation to the binomial can be used.

$$\begin{aligned}
\Pr\{\text{detect on 1st}\} &= 1 - \Pr\{\text{not detect on 1st}\} \\
&= 1 - \beta \\
&= 1 - [\Pr\{D < \text{UCL} \mid p\} - \Pr\{D \le \text{LCL} \mid p\}] \\
&= 1 - \Phi\left(\frac{\text{UCL} + 1/2 - np}{\sqrt{np(1-p)}}\right) + \Phi\left(\frac{\text{LCL} - 1/2 - np}{\sqrt{np(1-p)}}\right) \\
&= 1 - \Phi\left(\frac{25.42 + 0.5 - 30}{\sqrt{30(1-0.3)}}\right) + \Phi\left(\frac{4.13 - 0.5 - 30}{\sqrt{30(1-0.3)}}\right) \\
&= 1 - \Phi(-0.8903) + \Phi(-5.7544) \\
&= 1 - (0.187) + (0.000) \\
&= 0.813
\end{aligned}$$

6-16.
(a)

$$\text{UCL}_p = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.03 + 3\sqrt{0.03(1-0.03)/200} = 0.0662$$

$$\text{LCL}_p = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.03 - 3\sqrt{0.03(1-0.03)/200} = 0.03 - 0.0362 \Rightarrow 0$$

(b)
$p_{\text{new}} = 0.08$. Since ($p_{\text{new}} = 0.08$) < 0.10 and n is large, use the Poisson approximation to the binomial.

$$\begin{aligned}
\Pr\{\text{detect on 1st sample} \mid p\} &= 1 - \Pr\{\text{not detect} \mid p\} \\
&= 1 - \beta \\
&= 1 - [\Pr\{\hat{p} < \text{UCL} \mid p\} - \Pr\{\hat{p} \le \text{LCL} \mid p\}] \\
&= 1 - \Pr\{D < n\text{UCL} \mid np\} + \Pr\{D \le n\text{LCL} \mid np\} \\
&= 1 - \Pr\{D < 200(0.0662) \mid 200(0.08)\} + \Pr\{D \le 200(0) \mid 200(0.08)\} \\
&= 1 - \text{POI}(13,16) + \text{POI}(0,16) \\
&= 1 - 0.2745 + 0.000 \\
&= 0.7255
\end{aligned}$$

where POI($\cdot$) is the cumulative Poisson distribution.
Pr{detect by at least 4th} = 1 – Pr{detect after 4th} = 1 – $(1 - 0.7255)^4$ = 0.9943

6-17.

(a)

$$\bar{p} = \sum_{i=1}^{m} D_i \Big/ (mn) = 1200 \big/ [30(400)] = 0.10; \quad n\bar{p} = 400(0.10) = 40$$

$$\text{UCL}_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 40 + 3\sqrt{40(1-0.10)} = 58$$

$$\text{LCL}_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 40 - 3\sqrt{40(1-0.10)} = 22$$

(b)

np_{new} = 400 (0.15) = 60 > 15, so use the normal approximation to the binomial.

$$\begin{aligned}
\Pr\{\text{detect on 1st sample} \mid p\} &= 1 - \Pr\{\text{not detect on 1st sample} \mid p\} \\
&= 1 - \beta \\
&= 1 - [\Pr\{D < \text{UCL} \mid np\} - \Pr\{D \le \text{LCL} \mid np\}] \\
&= 1 - \Phi\left(\frac{\text{UCL} + 1/2 - np}{\sqrt{np(1-p)}}\right) + \Phi\left(\frac{\text{LCL} - 1/2 - np}{\sqrt{np(1-p)}}\right) \\
&= 1 - \Phi\left(\frac{58 + 0.5 - 60}{\sqrt{60(1-0.15)}}\right) + \Phi\left(\frac{22 - 0.5 - 60}{\sqrt{60(1-0.15)}}\right) \\
&= 1 - \Phi(-0.210) + \Phi(-5.39) \\
&= 1 - 0.417 + 0.000 \\
&= 0.583
\end{aligned}$$

6-18.
(a)
$$\text{UCL} = p + 3\sqrt{p(1-p)/n}$$
$$n = p(1-p)\left(\frac{3}{\text{UCL}-p}\right)^2 = 0.1(1-0.1)\left(\frac{3}{0.19-0.1}\right)^2 = 100$$

(b)
Using the Poisson approximation to the binomial, $\lambda = np = 100(0.10) = 10$.
$$\begin{aligned}\Pr\{\text{type I error}\} &= \Pr\{\hat{p} < \text{LCL} \mid p\} + \Pr\{\hat{p} > \text{UCL} \mid p\} \\ &= \Pr\{D < n\text{LCL} \mid \lambda\} + 1 - \Pr\{D \le n\text{UCL} \mid \lambda\} \\ &= \Pr\{D < 100(0.01) \mid 10\} + 1 - \Pr\{D \le 100(0.19) \mid 10\} \\ &= \text{POI}(0,10) + 1 - \text{POI}(19,10) \\ &= 0.000 + 1 - 0.996 \\ &= 0.004\end{aligned}$$
where POI(·) is the cumulative Poisson distribution.

(c)
$p_{\text{new}} = 0.20$.
Using the Poisson approximation to the binomial, $\lambda = np_{\text{new}} = 100(0.20) = 20$.
$$\begin{aligned}\Pr\{\text{type II error}\} &= \beta \\ &= \Pr\{D < n\text{UCL} \mid \lambda\} - \Pr\{D \le n\text{LCL} \mid \lambda\} \\ &= \Pr\{D < 100(0.19) \mid 20\} - \Pr\{D \le 100(0.01) \mid 20\} \\ &= \text{POI}(18,20) - \text{POI}(1,20) \\ &= 0.381 - 0.000 \\ &= 0.381\end{aligned}$$
where POI(·) is the cumulative Poisson distribution.

6-19.
NOTE: There is an error in the textbook. This is a continuation of Exercise 6-17, not 6-18.

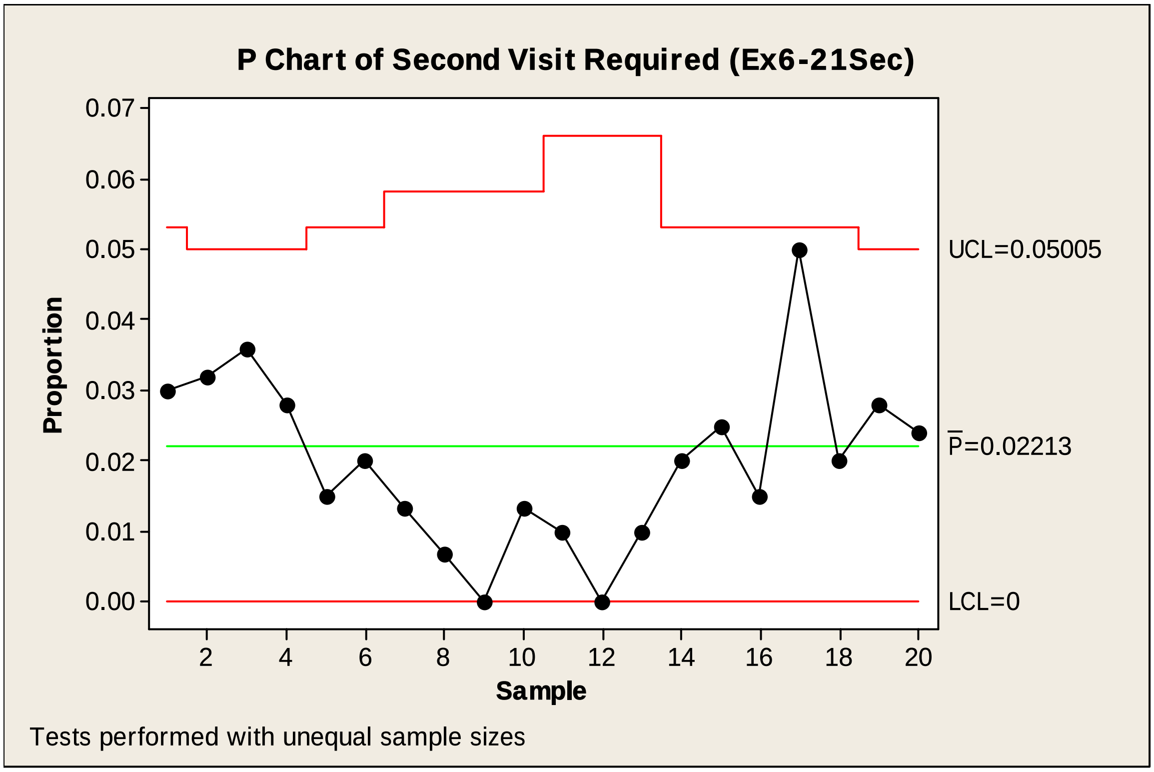
from 6-17(b), $1 - \beta = 0.583$
$\text{ARL}_1 = 1/(1-\beta) = 1/(0.583) = 1.715 \cong 2$

6-20.
from 6-18(c), $\beta = 0.381$
$\text{ARL}_1 = 1/(1-\beta) = 1/(1-0.381) = 1.616 \cong 2$

6-21.
(a)
For a p chart with variable sample size: $\overline{p} = \sum_i D_i / \sum_i n_i = 83/3750 = 0.0221$ and control limits are at $\overline{p} \pm 3\sqrt{\overline{p}(1-\overline{p})/n_i}$

n_i	[LCL_i, UCL_i]
100	[0, 0.0662]
150	[0, 0.0581]
200	[0, 0.0533]
250	[0, 0.0500]

MTB > Stat > Control Charts > Attributes Charts > P



Process is in statistical control.

(b)
There are two approaches for controlling future production. The first approach would be to plot $\hat{p}_i$ and use constant limits unless there is a different size sample or a plot point near a control limit. In those cases, calculate the exact control limits by $\overline{p} \pm 3\sqrt{\overline{p}(1-\overline{p})/n_i} = 0.0221 \pm 3\sqrt{0.0216/n_i}$. The second approach, preferred in many cases, would be to construct standardized control limits with control limits at ± 3, and to plot $Z_i = (\hat{p}_i - 0.0221) \Big/ \sqrt{0.0221(1-0.0221)/n_i}$.

6-22.

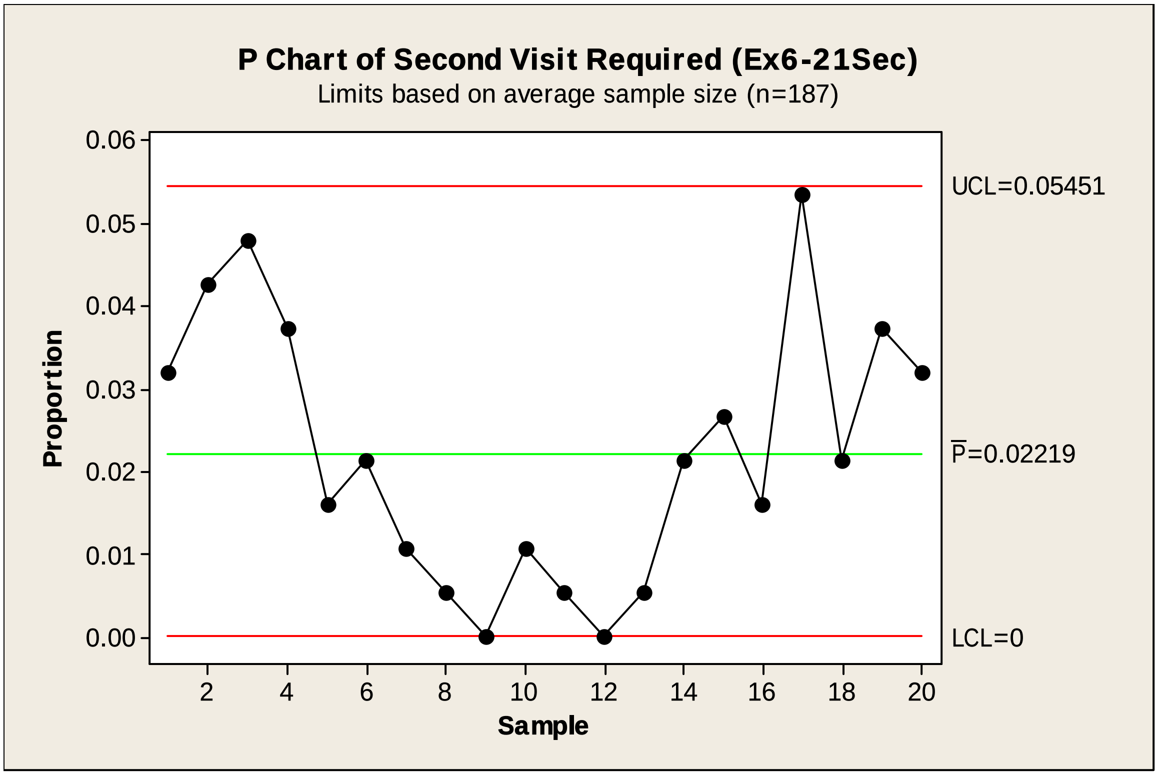
MTB > Stat > Basic Statistics > Display Descriptive Statistics

Descriptive Statistics: Ex6-21Req

```
Variable    N   Mean
Ex6-21Req  20  187.5
```

Average sample size is 187.5, however MINITAB accepts only integer values for *n*. Use a sample size of *n* = 187, and carefully examine points near the control limits.

MTB > Stat > Control Charts > Attributes Charts > P

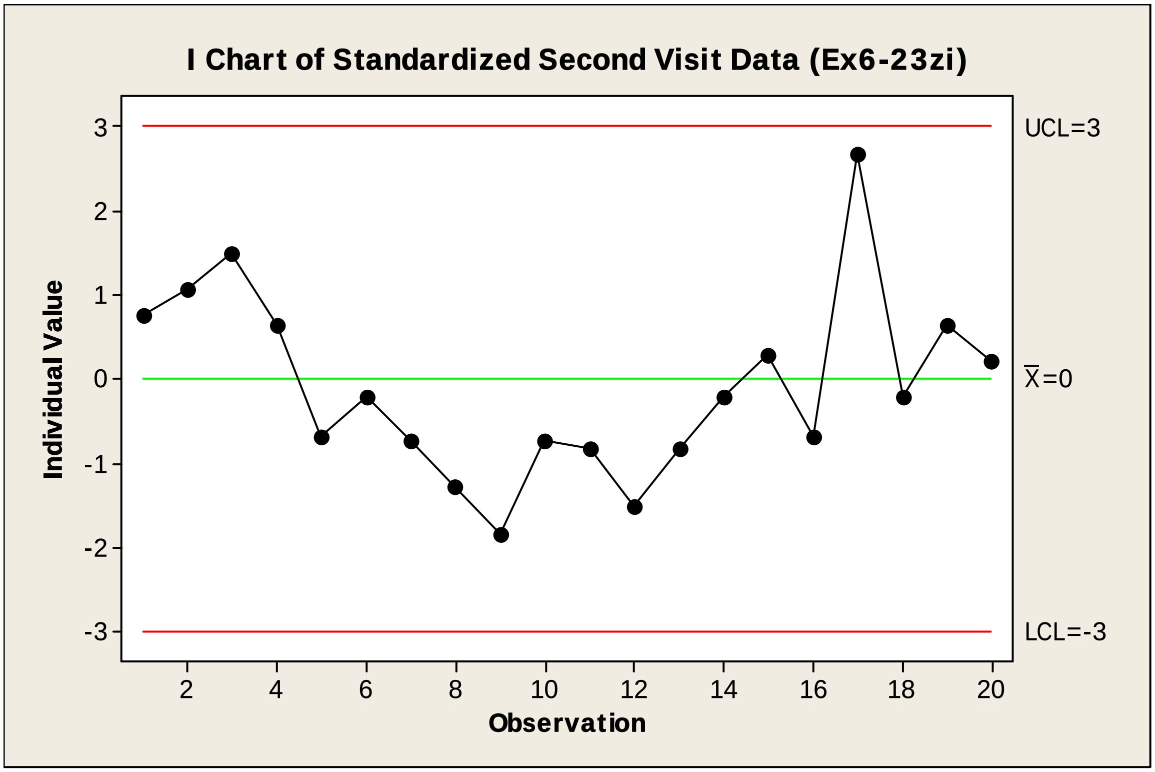


Process is in statistical control.

6-23.

$z_i = (\hat{p}_i - \overline{p}) \Big/ \sqrt{\overline{p}(1-\overline{p})/n_i} = (\hat{p}_i - 0.0221) \Big/ \sqrt{0.0216/n_i}$

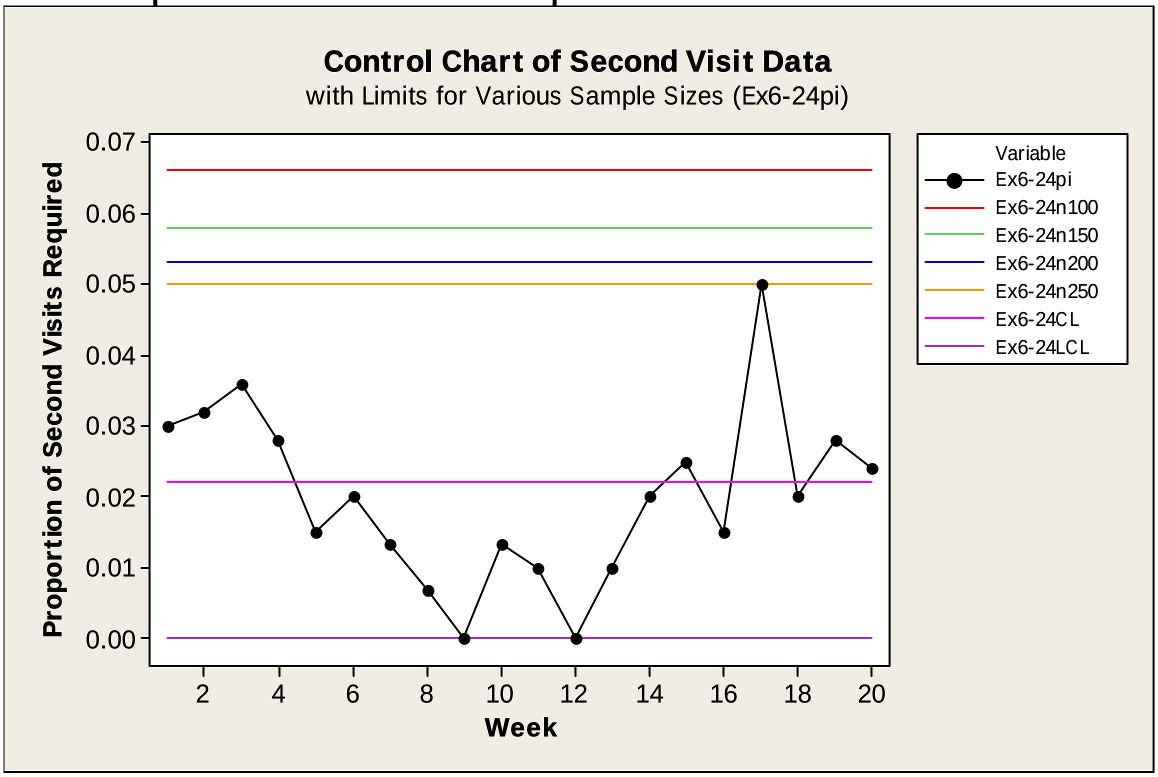
MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals



Process is in statistical control.

6-24.
CL = 0.0221, LCL = 0
UCL_{100} = 0.0662, UCL_{150} = 0.0581, UCL_{200} = 0.0533, UCL_{250} = 0.0500

MTB > Graph > Time Series Plot > Multiple



6-25.
UCL = 0.0399; $\bar{p}$ = CL = 0.01; LCL = 0; n = 100

$$\begin{aligned} n &> \left(\frac{1-p}{p}\right)L^2 \\ &> \left(\frac{1-0.01}{0.01}\right)3^2 \\ &> 891 \\ &\geq 892 \end{aligned}$$

6-26.
The np chart is inappropriate for varying sample sizes because the centerline (process center) would change with each n_i.

6-27.
n = 400; UCL = 0.0809; p = CL = 0.0500; LCL = 0.0191

(a)

$$\begin{aligned} 0.0809 = 0.05 + L\sqrt{0.05(1-0.05)/400} &= 0.05 + L(0.0109) \\ L &= 2.8349 \end{aligned}$$

(b)

$$\text{CL} = np = 400(0.05) = 20$$
$$\text{UCL} = np + 2.8349\sqrt{np(1-p)} = 20 + 2.8349\sqrt{20(1-0.05)} = 32.36$$
$$\text{LCL} = np - 2.8349\sqrt{np(1-p)} = 20 - 2.8349\sqrt{20(1-0.05)} = 7.64$$

(c)
n = 400 is large and p = 0.05 < 0.1, use Poisson approximation to binomial.

$$\begin{aligned} &\Pr\{\text{detect shift to 0.03 on 1st sample}\} \\ &= 1 - \Pr\{\text{not detect}\} \\ &= 1 - \beta \\ &= 1 - [\Pr\{D < \text{UCL} \mid \lambda\} - \Pr\{D \leq \text{LCL} \mid \lambda\}] \\ &= 1 - \Pr\{D < 32.36 \mid 12\} + \Pr\{D \leq 7.64 \mid 12\} \\ &= 1 - \text{POI}(32,12) + \text{POI}(7,12) \\ &= 1 - 1.0000 + 0.0895 \\ &= 0.0895 \end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

6-28.
(a)

$$\begin{aligned}
\text{UCL} &= p + L\sqrt{p(1-p)/n} \\
0.0962 &= 0.0500 + L\sqrt{0.05(1-0.05)/400} \\
L &= 4.24
\end{aligned}$$

(b)
p = 15, λ = np = 400(0.15) = 60 > 15, use normal approximation to binomial.

$$\begin{aligned}
&\Pr\{\text{detect on 1st sample after shift}\} \\
&= 1 - \Pr\{\text{not detect}\} \\
&= 1 - \beta \\
&= 1 - [\Pr\{\hat{p} < \text{UCL} \mid p\} - \Pr\{\hat{p} \le \text{LCL} \mid p\}] \\
&= 1 - \Phi\left(\frac{\text{UCL} - p}{\sqrt{p(1-p)/n}}\right) + \Phi\left(\frac{\text{LCL} - p}{\sqrt{p(1-p)/n}}\right) \\
&= 1 - \Phi\left(\frac{0.0962 - 0.15}{\sqrt{0.15(1-0.15)/400}}\right) + \Phi\left(\frac{0.0038 - 0.15}{\sqrt{0.15(1-0.15)/400}}\right) \\
&= 1 - \Phi(-3.00) + \Phi(-8.19) \\
&= 1 - 0.00135 + 0.000 \\
&= 0.99865
\end{aligned}$$

6-29.
p = 0.01; L = 2

(a)

$$\begin{aligned}
n &> \left(\frac{1-p}{p}\right)L^2 \\
&> \left(\frac{1-0.01}{0.01}\right)2^2 \\
&> 396 \\
&\ge 397
\end{aligned}$$

(b)
δ = 0.04 – 0.01 = 0.03

$$n = \left(\frac{L}{\delta}\right)^2 p(1-p) = \left(\frac{2}{0.03}\right)^2 (0.01)(1-0.01) = 44$$

6-30.

(a)

$$
\begin{aligned}
&\Pr\{\text{type I error}\}\\
&= \Pr\{\hat{p} < \text{LCL} \mid p\} + \Pr\{\hat{p} > \text{UCL} \mid p\}\\
&= \Pr\{D < n\text{LCL} \mid np\} + 1 - \Pr\{D \le n\text{UCL} \mid np\}\\
&= \Pr\{D < 100(0.0050) \mid 100(0.04)\} + 1 - \Pr\{D \le 100(0.075) \mid 100(0.04)\}\\
&= \text{POI}(0,4) + 1 - \text{POI}(7,4)\\
&= 0.018 + 1 - 0.948\\
&= 0.070
\end{aligned}
$$

where POI(·) is the cumulative Poisson distribution.

(b)

$$
\begin{aligned}
&\Pr\{\text{type II error}\}\\
&= \beta\\
&= \Pr\{D < n\text{UCL} \mid np\} - \Pr\{D \le n\text{LCL} \mid np\}\\
&= \Pr\{D < 100(0.075) \mid 100(0.06)\} - \Pr\{D \le 100(0.005) \mid 100(0.06)\\
&= \text{POI}(7,6) - \text{POI}(0,6)\\
&= 0.744 - 0.002\\
&= 0.742
\end{aligned}
$$

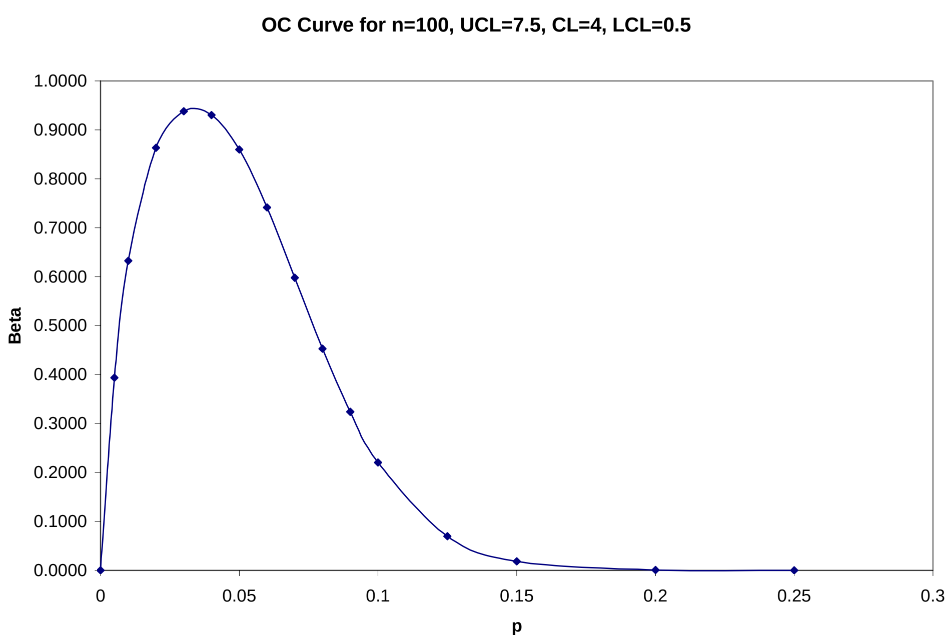
where POI(·) is the cumulative Poisson distribution.

6-30 continued
(c)

$$\begin{aligned}\beta &= \Pr\{D < n\text{UCL} \mid np\} - \Pr\{D \le n\text{LCL} \mid np\} \\ &= \Pr\{D < 100(0.0750) \mid 100p\} - \Pr\{D \le 100(0.0050) \mid 100p\} \\ &= \Pr\{D < 7.5 \mid 100p\} - \Pr\{D \le 0.5 \mid 100p\}\end{aligned}$$

Excel : workbook Chap06.xls : worksheet Ex6-30

p	np	Pr{D<7.5|np}	Pr{D<=0.5|np}	beta
0	0	1.0000	1.0000	0.0000
0.005	0.5	1.0000	0.6065	0.3935
0.01	1	1.0000	0.3679	0.6321
0.02	2	0.9989	0.1353	0.8636
0.03	3	0.9881	0.0498	0.9383
0.04	4	0.9489	0.0183	0.9306
0.05	5	0.8666	0.0067	0.8599
0.06	6	0.7440	0.0025	0.7415
0.07	7	0.5987	0.0009	0.5978
0.08	8	0.4530	0.0003	0.4526
0.09	9	0.3239	0.0001	0.3238
0.1	10	0.2202	0.0000	0.2202
0.125	12.5	0.0698	0.0000	0.0698
0.15	15	0.0180	0.0000	0.0180
0.2	20	0.0008	0.0000	0.0008
0.25	25	0.0000	0.0000	0.0000



(d)
from part (a), α = 0.070: $\text{ARL}_0 = 1/\alpha = 1/0.070 = 14.29 \cong 15$
from part (b), $\beta = 0.0742$: $\text{ARL}_1 = 1/(1-\beta) = 1/(1-0.742) = 3.861 \cong 4$

6-31.
$n = 100$; $\bar{p} = 0.02$

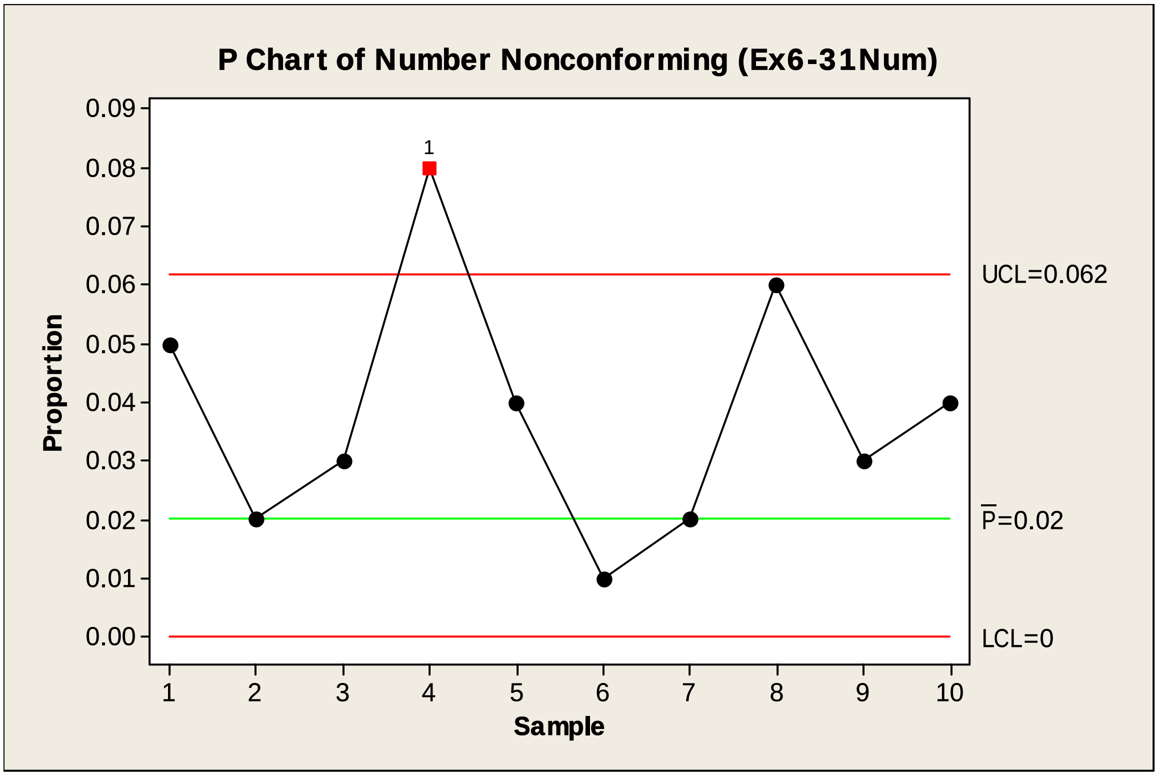
(a)

$$\text{UCL} = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.02 + 3\sqrt{0.02(1-0.02)/100} = 0.062$$

$$\text{LCL} = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.02 - 3\sqrt{0.02(1-0.02)/100} \Rightarrow 0$$

(b)

MTB > Stat > Control Charts > Attributes Charts > P




```
Test Results for P Chart of Ex6-31Num
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  4
```

Sample 4 exceeds the upper control limit.
$\bar{p} = 0.038$ and $\hat{\sigma}_p = 0.0191$

6-32.

$$\text{LCL} = n\bar{p} - k\sqrt{n\bar{p}(1-\bar{p})} > 0$$

$$n\bar{p} > k\sqrt{n\bar{p}(1-\bar{p})}$$

$$n > k^2\left(\frac{1-\bar{p}}{\bar{p}}\right)$$

6-33.

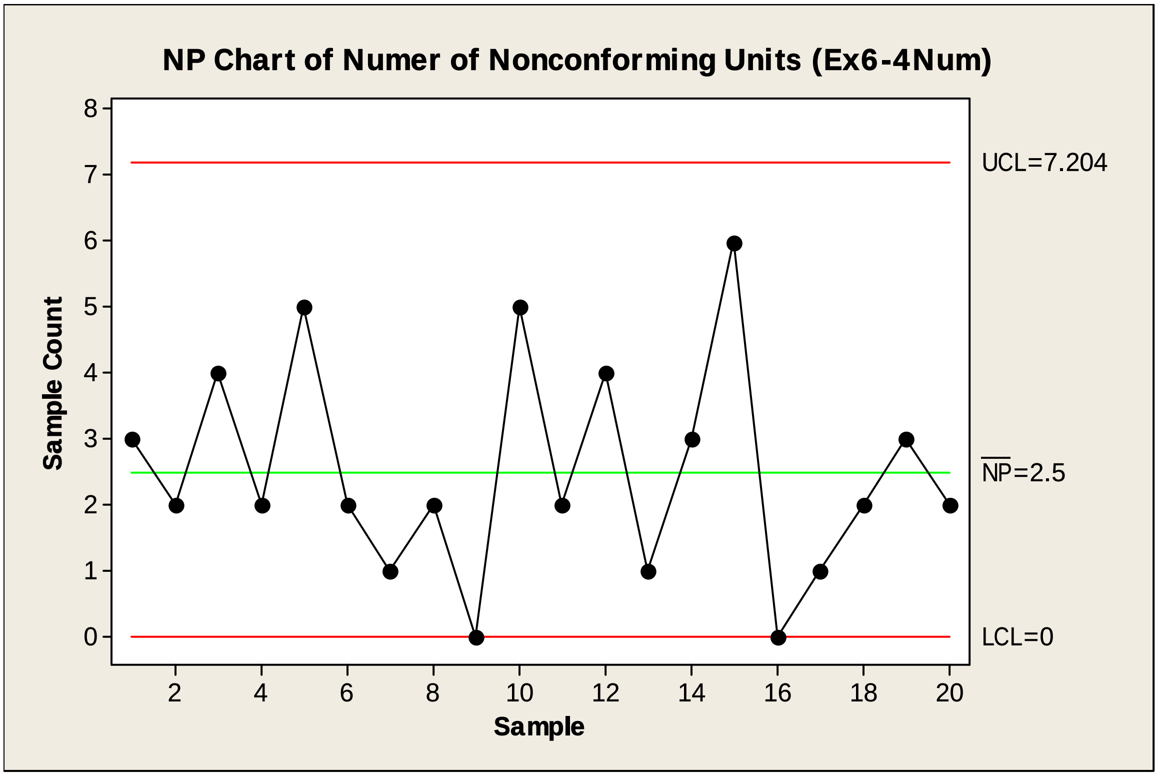
$n = 150; \quad m = 20; \quad \sum D = 50; \quad \overline{p} = 0.0167$

$\text{CL} = n\overline{p} = 150(0.0167) = 2.505$

$\text{UCL} = n\overline{p} + 3\sqrt{n\overline{p}(1-\overline{p})} = 2.505 + 3\sqrt{2.505(1-0.0167)} = 7.213$

$\text{LCL} = n\overline{p} - 3\sqrt{n\overline{p}(1-\overline{p})} = 2.505 - 4.708 \Rightarrow 0$

MTB > Stat > Control Charts > Attributes Charts > NP



The process is in control; results are the same as for the *p* chart.

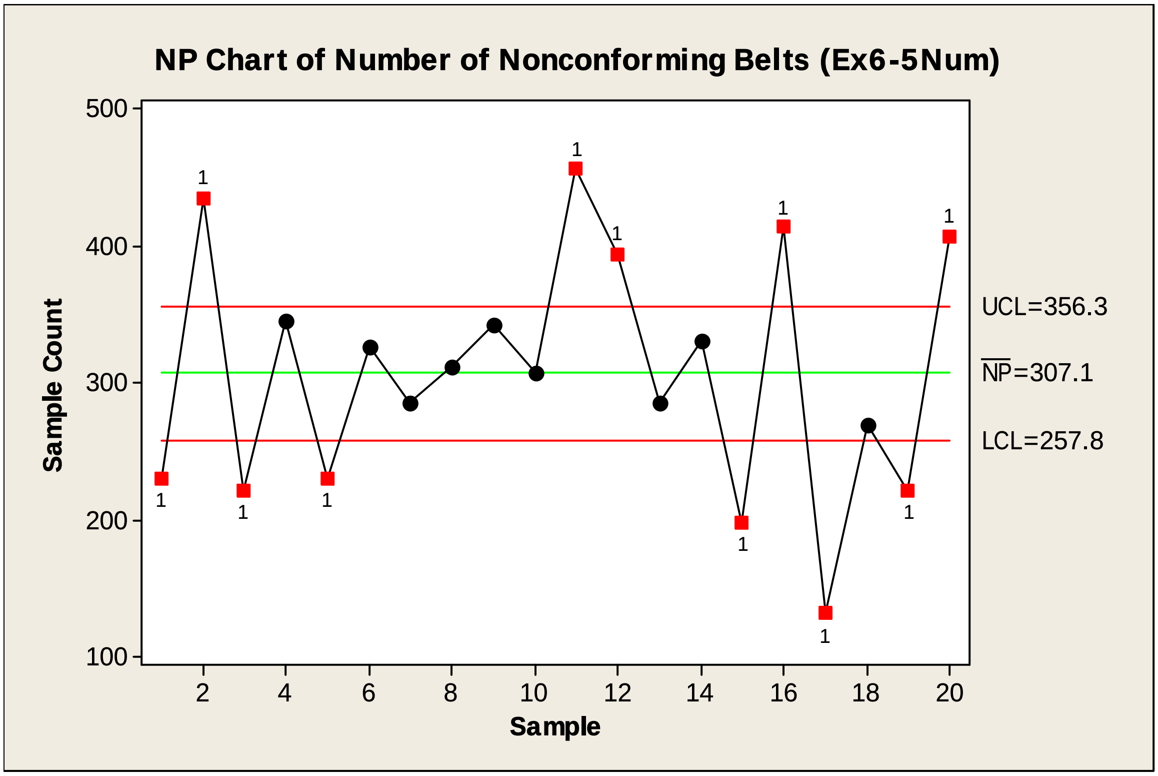
6-34.

$$\text{CL} = n\bar{p} = 2500(0.1228) = 307$$

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 307 + 3\sqrt{307(1-0.1228)} = 356.23$$

$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 307 - 3\sqrt{307(1-0.1228)} = 257.77$$

MTB > Stat > Control Charts > Attributes Charts > NP




```
Test Results for NP Chart of Ex6-5Num
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 2, 3, 5, 11, 12, 15, 16, 17, 19, 20
```

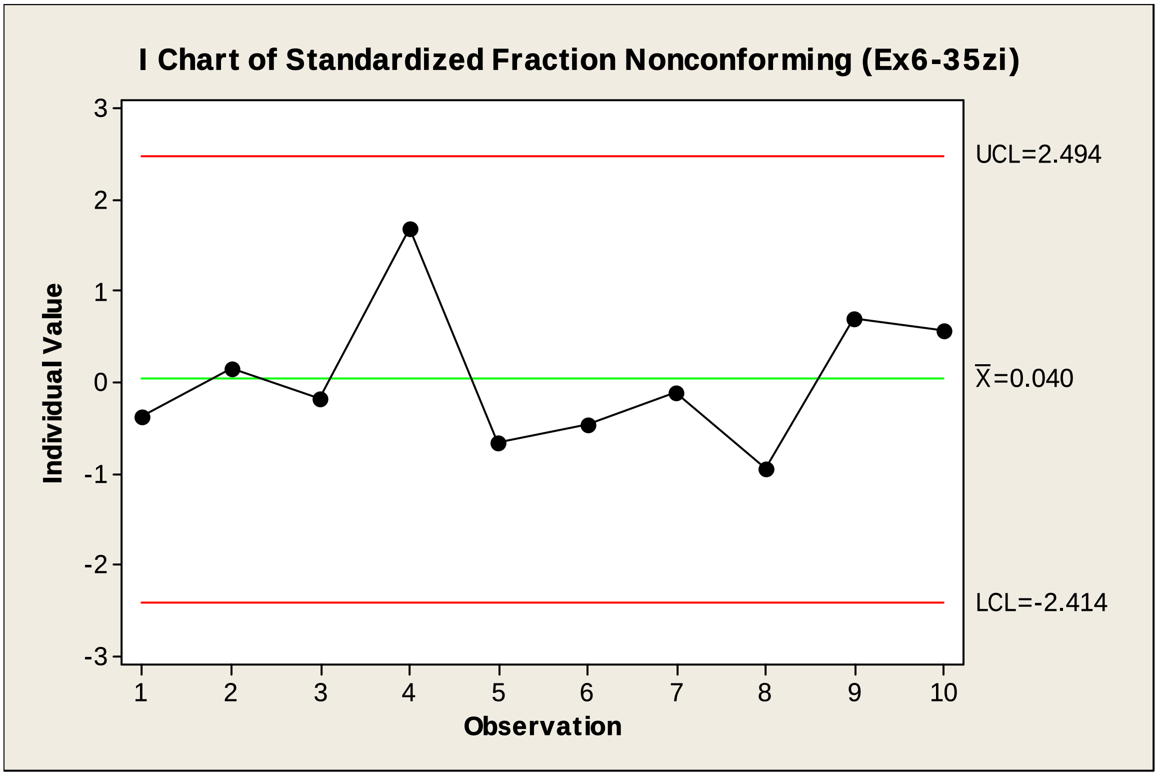
Like the *p* control chart, many subgroups are out of control (11 of 20), indicating that this data should not be used to establish control limits for future production.

6-35.

$\bar{p} = 0.06$

$z_i = (\hat{p}_i - 0.06) \Big/ \sqrt{0.06(1-0.06)/n_i} = (\hat{p}_i - 0.06) \Big/ \sqrt{0.0564/n_i}$

MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals



The process is in control; results are the same as for the *p* chart.

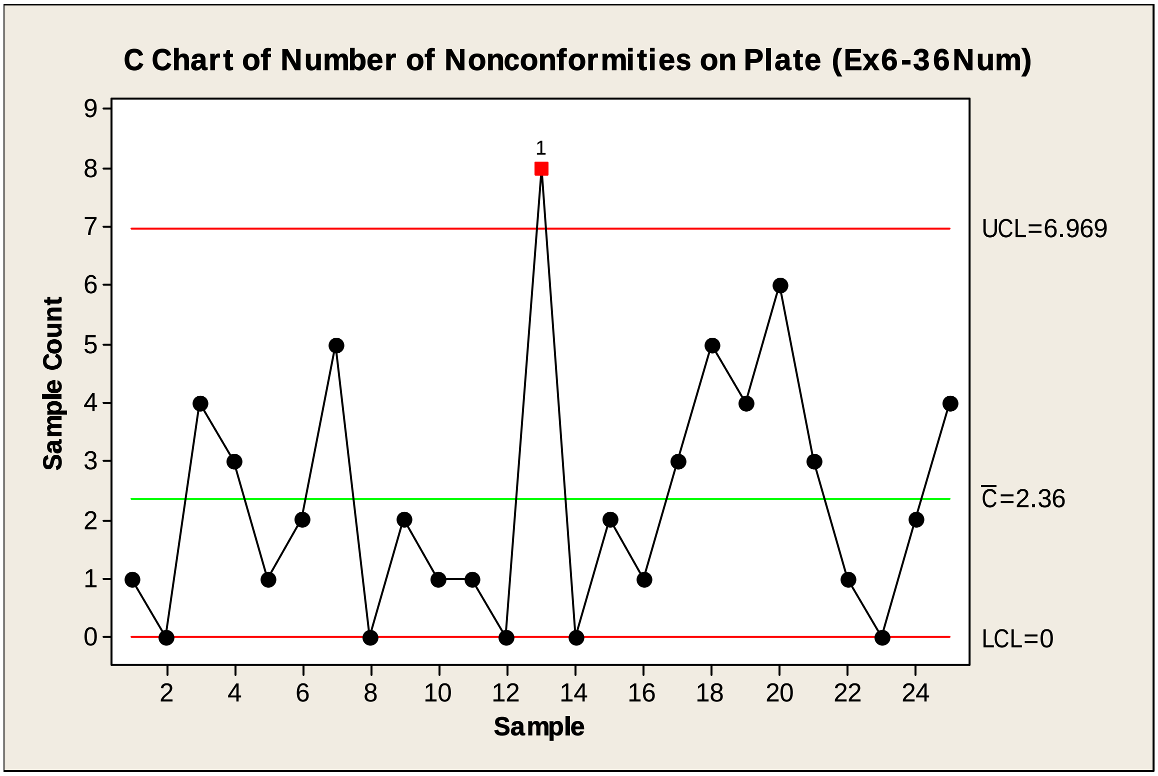
6-36.

$$\text{CL} = \bar{c} = 2.36$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 2.36 + 3\sqrt{2.36} = 6.97$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 2.36 - 3\sqrt{2.36} \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > C



Test Results for C Chart of Ex6-36Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  13
```

No. The plate process does not seem to be in statistical control.

6-37.

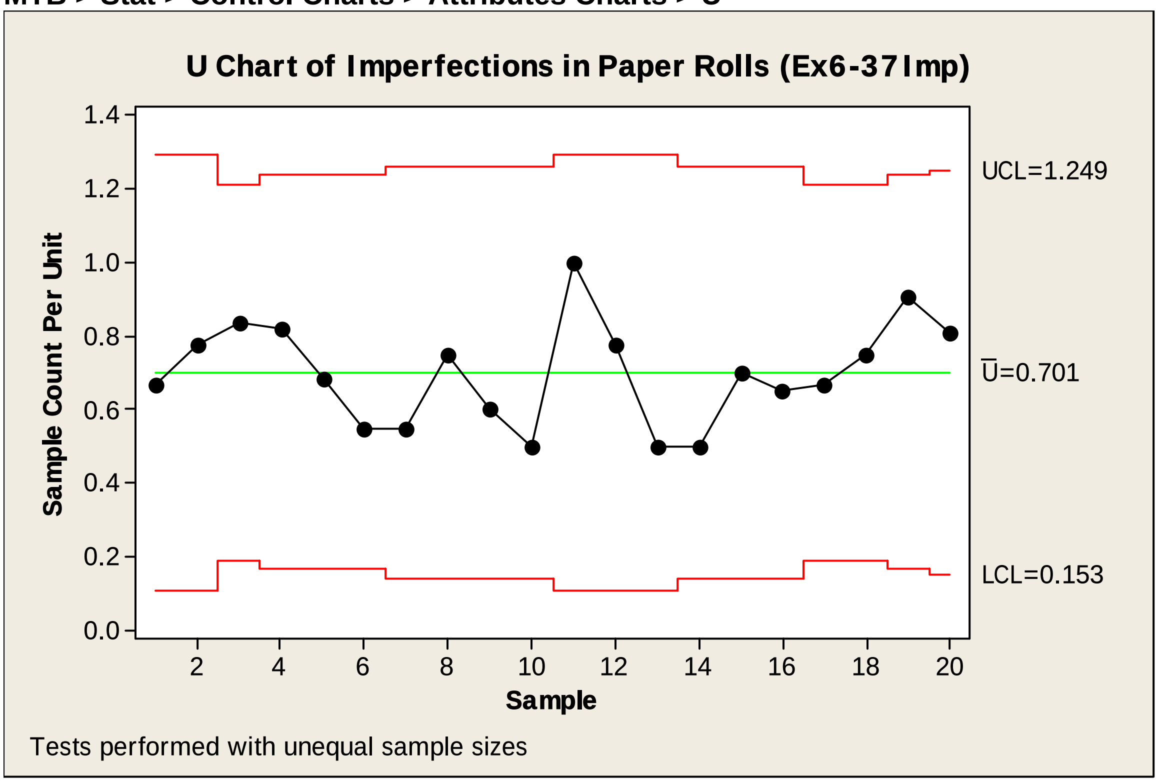
$$\text{CL} = \bar{u} = 0.7007$$

$$\text{UCL}_i = \bar{u} + 3\sqrt{\bar{u}/n_i} = 0.7007 + 3\sqrt{0.7007 / n_i}$$

$$\text{LCL}_i = \bar{u} - 3\sqrt{\bar{u}/n_i} = 0.7007 - 3\sqrt{0.7007 / n_i}$$

n_i	[LCL$_i$, UCL$_i$]
18	[0.1088, 1.2926]
20	[0.1392, 1.2622]
21	[0.1527, 1.2487]
22	[0.1653, 1.2361]
24	[0.1881, 1.2133]

MTB > Stat > Control Charts > Attributes Charts > U



6-38.

$$\text{CL} = \bar{u} = 0.7007; \quad \bar{n} = 20.55$$

$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/\bar{n}} = 0.7007 + 3\sqrt{0.7007/20.55} = 1.2547$$

$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/\bar{n}} = 0.7007 - 3\sqrt{0.7007/20.55} = 0.1467$$

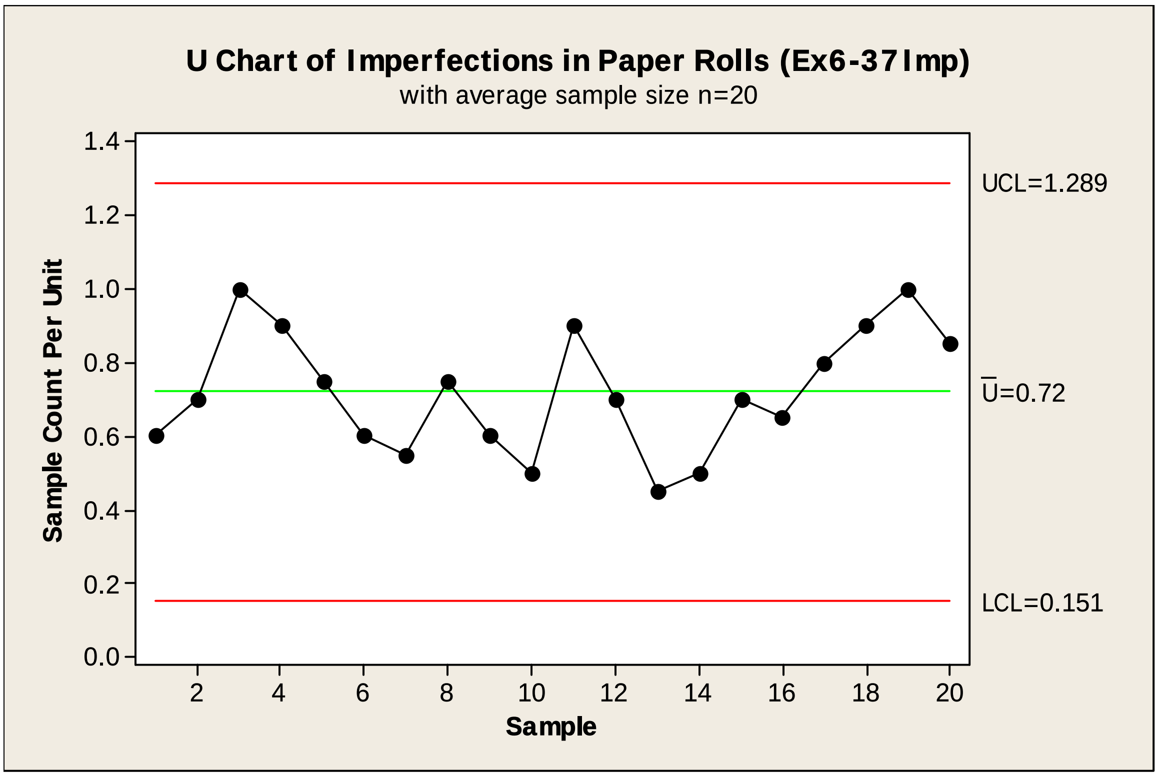
MTB > Stat > Basic Statistics > Display Descriptive Statistics

Descriptive Statistics: Ex6-37Rol

```
Variable    N    Mean
Ex6-37Rol  20  20.550
```

Average sample size is 20.55, however MINITAB accepts only integer values for *n*. Use a sample size of *n* = 20, and carefully examine points near the control limits.

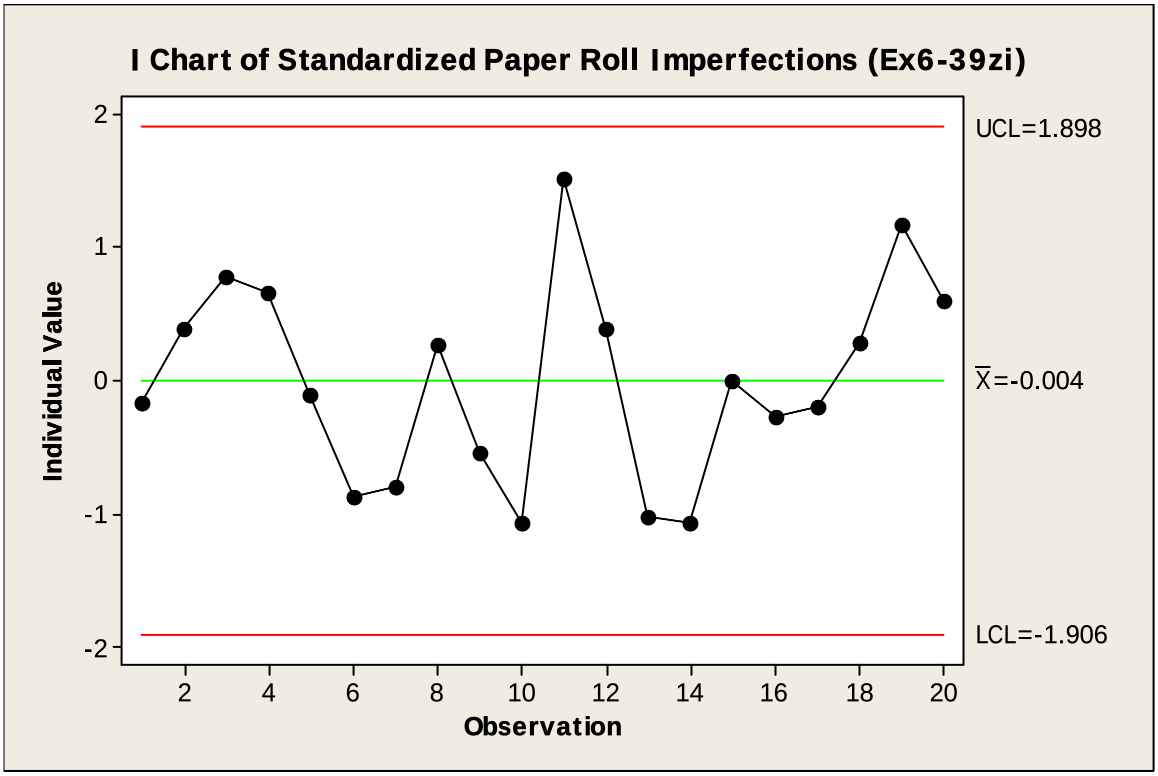
MTB > Stat > Control Charts > Attributes Charts > U



6-39.

$$z_i = (u_i - \bar{u}) \Big/ \sqrt{\bar{u}/n_i} = (u_i - 0.7007) \Big/ \sqrt{0.7007 / n_i}$$

MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals



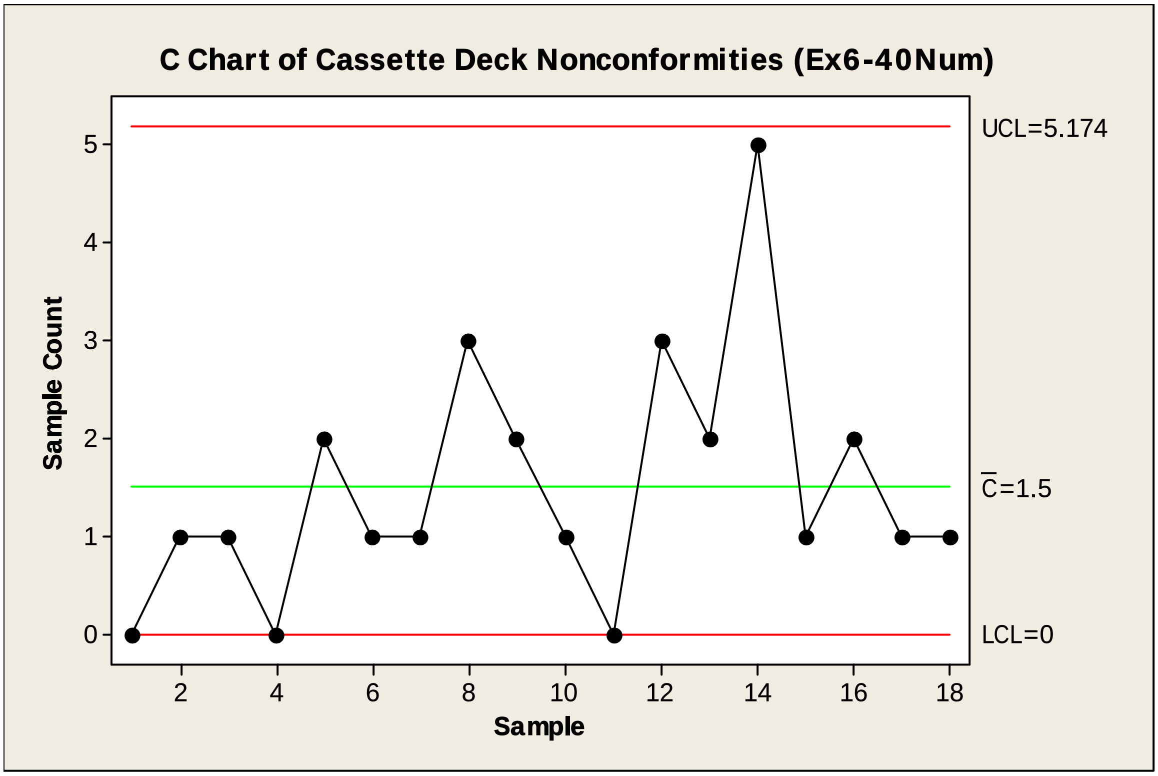
6-40.
c chart based on # of nonconformities per cassette deck

$\text{CL} = \bar{c} = 1.5$

$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 1.5 + 3\sqrt{1.5} = 5.17$

$\text{LCL} \Rightarrow 0$

MTB > Stat > Control Charts > Attributes Charts > C

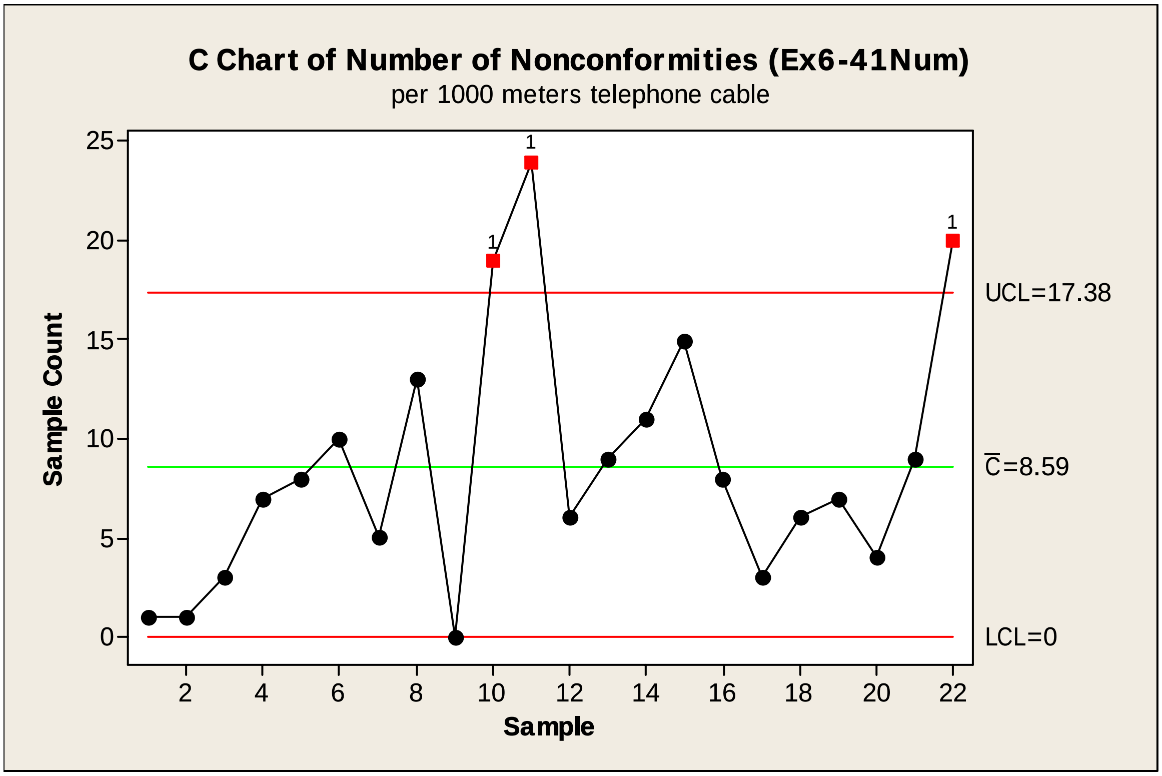


Process is in statistical control. Use these limits to control future production.

6-41.

$$\text{CL} = \bar{c} = 8.59; \quad \text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 8.59 + 3\sqrt{8.59} = 17.384; \quad \text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 8.59 - 3\sqrt{8.59} \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > C



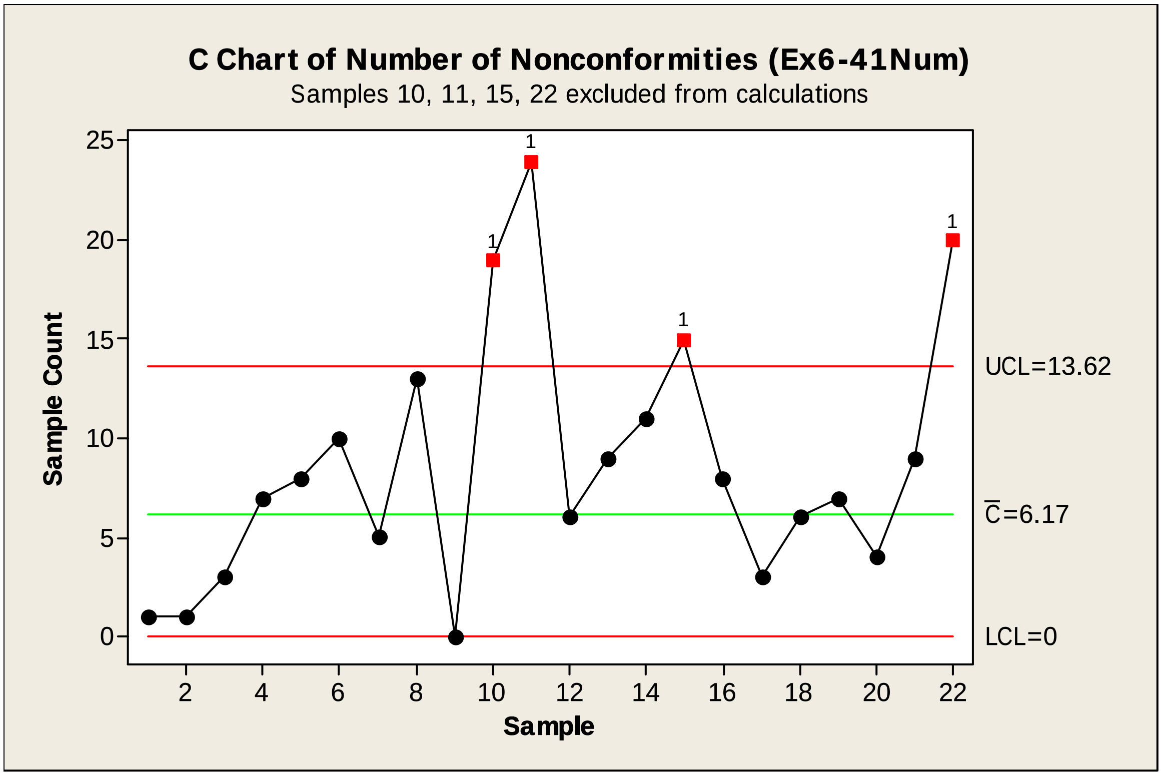
Test Results for C Chart of Ex6-41Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  10, 11, 22
```

6-41 continued

Process is not in statistical control; three subgroups exceed the UCL. Exclude subgroups 10, 11 and 22, then re-calculate the control limits. Subgroup 15 will then be out of control and should also be excluded.

$$\text{CL} = \bar{c} = 6.17; \quad \text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 6.17 + 3\sqrt{6.17} = 13.62; \quad \text{LCL} \Rightarrow 0$$




```
Test Results for C Chart of Ex6-41Num
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  10, 11, 15, 22
```

6-42.
(a)
The new inspection unit is n = 4 cassette decks. A c chart of the total number of nonconformities per inspection unit is appropriate.

$$\text{CL} = n\bar{c} = 4(1.5) = 6$$

$$\text{UCL} = n\bar{c} + 3\sqrt{n\bar{c}} = 6 + 3\sqrt{6} = 13.35$$

$$\text{LCL} = n\bar{c} - 3\sqrt{n\bar{c}} = 6 - 3\sqrt{6} \Rightarrow 0$$

(b)
The sample is n =1 new inspection units. A u chart of average nonconformities per inspection unit is appropriate.

$$\text{CL} = \bar{u} = \frac{\text{total nonconformities}}{\text{total inspection units}} = \frac{27}{(18/4)} = 6.00$$

$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 6 + 3\sqrt{6/1} = 13.35$$

$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 6 - 3\sqrt{6/1} \Rightarrow 0$$

6-43.
(a)
The new inspection unit is n = 2500/1000 = 2.5 of the old unit. A c chart of the total number of nonconformities per inspection unit is appropriate.

$$\text{CL} = n\bar{c} = 2.5(6.17) = 15.43$$

$$\text{UCL} = n\bar{c} + 3\sqrt{n\bar{c}} = 15.43 + 3\sqrt{15.43} = 27.21$$

$$\text{LCL} = n\bar{c} - 3\sqrt{n\bar{c}} = 15.43 - 3\sqrt{15.43} = 3.65$$

The plot point, $\hat{c}$, is the total number of nonconformities found while inspecting a sample 2500m in length.

(b)
The sample is n =1 new inspection units. A u chart of average nonconformities per inspection unit is appropriate.

$$\text{CL} = \bar{u} = \frac{\text{total nonconformities}}{\text{total inspection units}} = \frac{111}{(18 \times 1000)/2500} = 15.42$$

$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 15.42 + 3\sqrt{15.42/1} = 27.20$$

$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 15.42 - 3\sqrt{15.42/1} = 3.64$$

The plot point, $\hat{u}$, is the average number of nonconformities found in 2500m, and since n = 1, this is the same as the total number of nonconformities.

6-44.

(a)

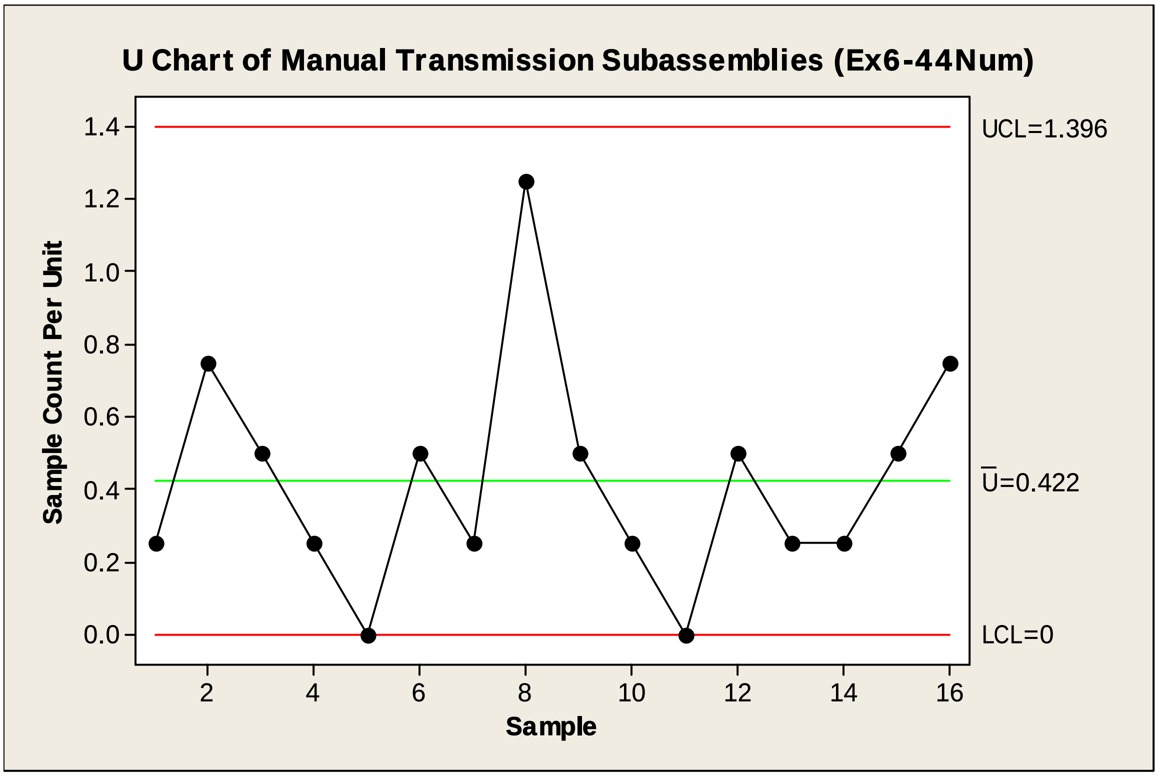
A u chart of average number of nonconformities per unit is appropriate, with n = 4 transmissions in each inspection.

$$\text{CL} = \bar{u} = \sum u_i / m = \left(\sum x_i / n\right) / m = (27/4)/16 = 6.75/16 = 0.422$$

$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 0.422 + 3\sqrt{0.422/4} = 1.396$$

$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 0.422 - 3\sqrt{0.422/4} = -0.211 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > U



(b)

The process is in statistical control.

(c)

The new sample is n = 8/4 = 2 inspection units. However, since this chart was established for ***average*** nonconformities per unit, the same control limits may be used for future production with the new sample size. (If this was a c chart for ***total*** nonconformities in the sample, the control limits would need revision.)

6-45.
(a)
$\text{CL} = \bar{c} = 4$

$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 4 + 3\sqrt{4} = 10$

$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 4 - 3\sqrt{4} \Rightarrow 0$

(b)
$c = 4; \quad n = 4$

$\text{CL} = \bar{u} = c/n = 4/4 = 1$

$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 1 + 3\sqrt{1/4} = 2.5$

$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 1 - 3\sqrt{1/4} \Rightarrow 0$

6-46.
Use the cumulative Poisson tables.
$\bar{c} = 16$

$\Pr\{x \le 21 \mid c = 16\} = 0.9108; \text{ UCL} = 21$

$\Pr\{x \le 10 \mid c = 16\} = 0.0774; \text{ LCL} = 10$

6-47.
(a)
$\text{CL} = \bar{c} = 9$

$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 9 + 3\sqrt{9} = 18$

$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 9 - 3\sqrt{9} = 0$

(b)
$c = 16; \quad n = 4$

$\text{CL} = \bar{u} = c/n = 16/4 = 4$

$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 4 + 3\sqrt{4/4} = 7$

$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 4 - 3\sqrt{4/4} = 1$

6-48.
u chart with $u = 6.0$ and $n = 3$. $c = u \times n = 18$. Find limits such that Pr{$D \leq$ UCL} = 0.980 and Pr{$D <$ LCL} = 0.020. From the cumulative Poisson tables:

x	Pr{$D \leq x \mid c = 18$}
9	0.015
10	0.030
26	0.972
27	0.983

UCL = x/n = 27/3 = 9, and LCL = x/n = 9/3 = 3. As a comparison, the normal distribution gives:

$$\text{UCL} = \bar{u} + z_{0.980}\sqrt{\bar{u}/n} = 6 + 2.054\sqrt{6/3} = 8.905$$
$$\text{LCL} = \bar{u} + z_{0.020}\sqrt{\bar{u}/n} = 6 - 2.054\sqrt{6/3} = 3.095$$

6-49.
Using the cumulative Poisson distribution:

x	Pr{$D \leq x \mid c = 7.6$}
2	0.019
3	0.055
12	0.954
13	0.976

for the c chart, UCL = 13 and LCL = 2. As a comparison, the normal distribution gives

$$\text{UCL} = \bar{c} + z_{0.975}\sqrt{\bar{c}} = 7.6 + 1.96\sqrt{7.6} = 13.00$$
$$\text{LCL} = \bar{c} - z_{0.025}\sqrt{\bar{c}} = 7.6 - 1.96\sqrt{7.6} = 2.20$$

6-50.
Using the cumulative Poisson distribution with $c = u\,n = 1.4(10) = 14$:

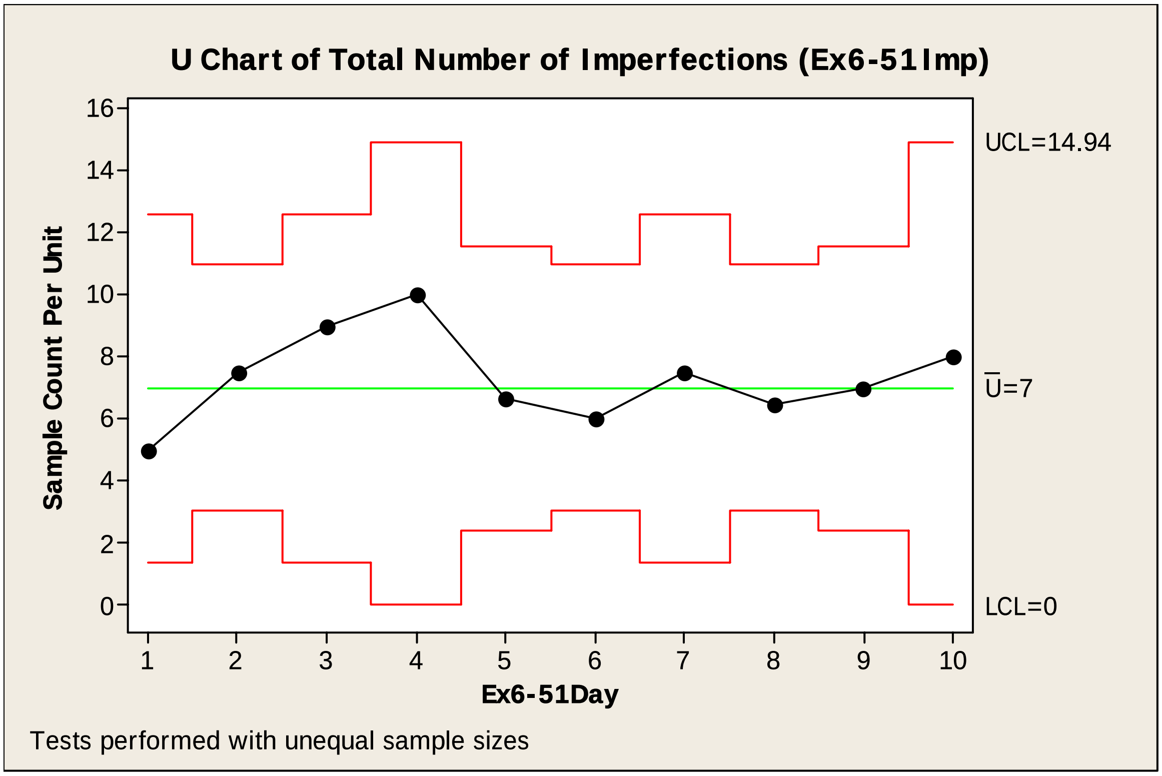
x	Pr{$D \leq x \mid c = 14$}
7	0.032
8	0.062
19	0.923
20	0.952

UCL = x/n = 20/10 = 2.00, and LCL = x/n = 7/10 = 0.70. As a comparison, the normal distribution gives:

$$\text{UCL} = \bar{u} + z_{0.95}\sqrt{\bar{u}/n} = 1.4 + 1.645\sqrt{1.4/10} = 2.016$$
$$\text{LCL} = \bar{u} + z_{0.05}\sqrt{\bar{u}/n} = 1.4 - 1.645\sqrt{1.4/10} = 0.784$$

6-51.
u chart with control limits based on each sample size:
$\bar{u}=7;\quad \text{UCL}_i=7+3\sqrt{7/n_i};\quad \text{LCL}_i=7-3\sqrt{7/n_i}$

MTB > Stat > Control Charts > Attributes Charts > U



The process is in statistical control.

6-52.
(a)
From the cumulative Poisson table, Pr{$x \le 6 \mid c = 2.0$} = 0.995. So set UCL = 6.0.

(b)
Pr{two consecutive out-of-control points} = (0.005)(0.005) = 0.00003

6-53.
A c chart with one inspection unit equal to 50 manufacturing units is appropriate.
$\bar{c} = 850/100 = 8.5$. From the cumulative Poisson distribution:

x	$\Pr\{D \le x \mid c = 8.5\}$
3	0.030
13	0.949
14	0.973

LCL = 3 and UCL = 13. For comparison, the normal distribution gives

$$\text{UCL} = \bar{c} + z_{0.97}\sqrt{\bar{c}} = 8.5 + 1.88\sqrt{8.5} = 13.98$$

$$\text{LCL} = \bar{c} + z_{0.03}\sqrt{\bar{c}} = 8.5 - 1.88\sqrt{8.5} = 3.02$$

6-54.
(a)
Plot the number of nonconformities per water heater on a c chart.

$$\text{CL} = \bar{c} = \sum D/m = 924/176 = 5.25$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 5.25 + 3\sqrt{5.25} = 12.12$$

$$\text{LCL} \Rightarrow 0$$

Plot the results after inspection of each water heater, approximately 8/day.

(b)
Let new inspection unit n = 2 water heaters

$$\text{CL} = n\bar{c} = 2(5.25) = 10.5$$

$$\text{UCL} = n\bar{c} + 3\sqrt{n\bar{c}} = 10.5 + 3\sqrt{10.5} = 20.22$$

$$\text{LCL} = n\bar{c} - 3\sqrt{n\bar{c}} = 10.5 - 3\sqrt{10.5} = 0.78$$

(c)

$$\begin{aligned}
\Pr\{\text{type I error}\} &= \Pr\{D < \text{LCL} \mid c\} + \Pr\{D > \text{UCL} \mid c\} \\
&= \Pr\{D < 0.78 \mid 10.5\} + \left[1 - \Pr\{D \le 20.22 \mid 10.5\}\right] \\
&= \text{POI}(0,10.5) + \left[1 - \text{POI}(20,10.5)\right] \\
&= 0.000 + \left[1 - 0.997\right] \\
&= 0.003
\end{aligned}$$

6-55.
$\bar{u} = 4.0$ average number of nonconformities/unit. Desire α = 0.99. Use the cumulative Poisson distribution to determine the UCL:

MTB : worksheet Chap06.mtw

Ex6-55X	Ex6-55alpha
0	0.02
1	0.09
2	0.24
3	0.43
4	0.63
5	0.79
6	0.89
7	0.95
8	0.98
9	0.99
10	1.00
11	1.00

An UCL = 9 will give a probability of 0.99 of concluding the process is in control, when in fact it is.

6-56.
Use a c chart for nonconformities with an inspection unit n = 1 refrigerator.
$\sum D_i = 16$ in 30 refrigerators; $\bar{c} = 16/30 = 0.533$

(a)
3-sigma limits are $\bar{c} \pm 3\sqrt{\bar{c}} = 0.533 \pm 3\sqrt{0.533} = [0, 2.723]$

(b)
$$\begin{aligned}
\alpha &= \Pr\{D < \text{LCL} \mid c\} + \Pr\{D > \text{UCL} \mid c\} \\
&= \Pr\{D < 0 \mid 0.533\} + \left[1 - \Pr\{D \le 2.72 \mid 0.533\}\right] \\
&= 0 + \left[1 - \text{POI}(2, 0.533)\right] \\
&= 1 - 0.983 \\
&= 0.017
\end{aligned}$$
where POI(·) is the cumulative Poisson distribution.

6-56 continued

(c)

$$\begin{aligned}\beta &= \Pr\{\text{not detecting shift}\}\\ &= \Pr\{D < \text{UCL} \mid c\} - \Pr\{D \le \text{LCL} \mid c\}\\ &= \Pr\{D < 2.72 \mid 2.0\} - \Pr\{D \le 0 \mid 2.0\}\\ &= \text{POI}(2,2) - \text{POI}(0,2)\\ &= 0.6767 - 0.1353\\ &= 0.5414\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

(d)

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1-0.541} = 2.18 \approx 2$$

6-57.

$\bar{c} = 0.533$

(a)

$$\bar{c} \pm 2\sqrt{\bar{c}} = 0.533 + 2\sqrt{0.533} = [0, 1.993]$$

(b)

$$\begin{aligned}\alpha &= \Pr\{D < \text{LCL} \mid \bar{c}\} + \Pr\{D > \text{UCL} \mid \bar{c}\}\\ &= \Pr\{D < 0 \mid 0.533\} + \left[1 - \Pr\{D \le 1.993 \mid 0.533\}\right]\\ &= 0 + \left[1 - \text{POI}(1, 0.533)\right]\\ &= 1 - 0.8996\\ &= 0.1004\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

(c)

$$\begin{aligned}\beta &= \Pr\{D < \text{UCL} \mid c\} - \Pr\{D \le \text{LCL} \mid c\}\\ &= \Pr\{D < 1.993 \mid 2\} - \Pr\{D \le 0 \mid 2\}\\ &= \text{POI}(1,2) - \text{POI}(0,2)\\ &= 0.406 - 0.135\\ &= 0.271\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

(d)

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1-0.271} = 1.372 \approx 2$$

6-58.
1 inspection unit = 10 radios, $\bar{u} = 0.5$ average nonconformities/radio

$\text{CL} = \bar{c} = \bar{u} \times n = 0.5(10) = 5$

$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 5 + 3\sqrt{5} = 11.708$

$\text{LCL} \Rightarrow 0$

6-59.
$\bar{u} =$ average # nonconformities/calculator = 2

(a)
c chart with $\bar{c} = \bar{u} \times n = 2(2) = 4$ nonconformities/inspection unit

$\text{CL} = \bar{c} = 4$

$\text{UCL} = \bar{c} + k\sqrt{\bar{c}} = 4 + 3\sqrt{4} = 10$

$\text{LCL} = \bar{c} - k\sqrt{\bar{c}} = 4 - 3\sqrt{4} \Rightarrow 0$

(b)
Type I error =

$$
\begin{aligned}
\alpha &= \Pr\{D < \text{LCL} \mid \bar{c}\} + \Pr\{D > \text{UCL} \mid \bar{c}\} \\
&= \Pr\{D < 0 \mid 4\} + \left[1 - \Pr\{D \le 10 \mid 4\}\right] \\
&= 0 + \left[1 - \text{POI}(10, 4)\right] \\
&= 1 - 0.997 \\
&= 0.003
\end{aligned}
$$

where POI(·) is the cumulative Poisson distribution.

6-60.
1 inspection unit = 6 clocks, $\bar{u} = 0.75$ nonconformities/clock

$\text{CL} = \bar{c} = \bar{u} \times n = 0.75(6) = 4.5$

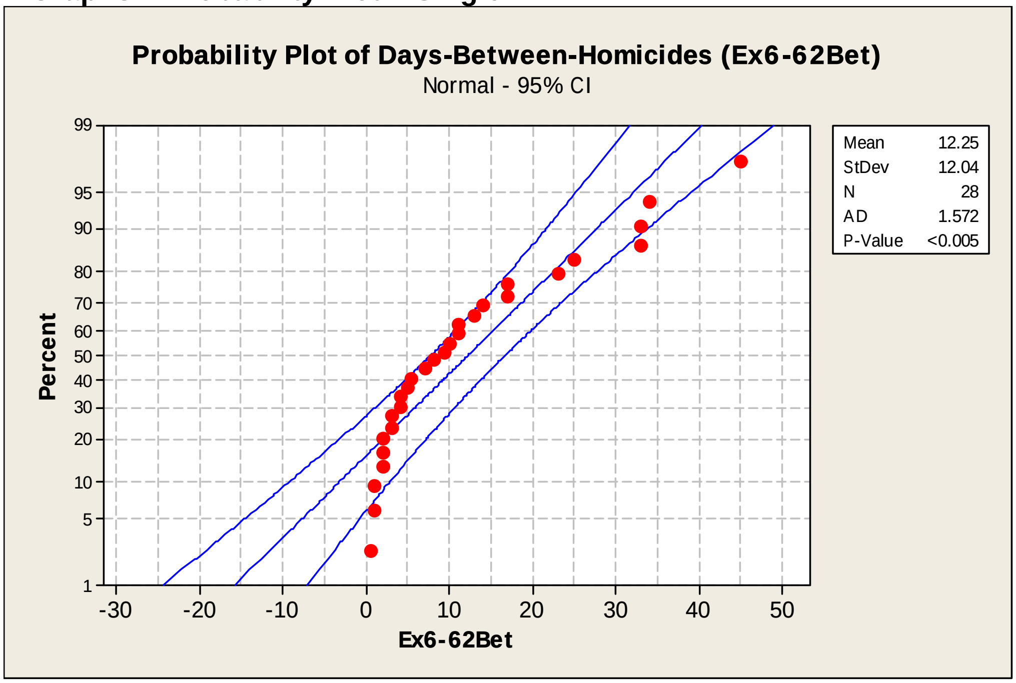
$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 4.5 + 3\sqrt{4.5} = 10.86$

$\text{LCL} \Rightarrow 0$

6-61.
c: nonconformities per unit; L: sigma control limits

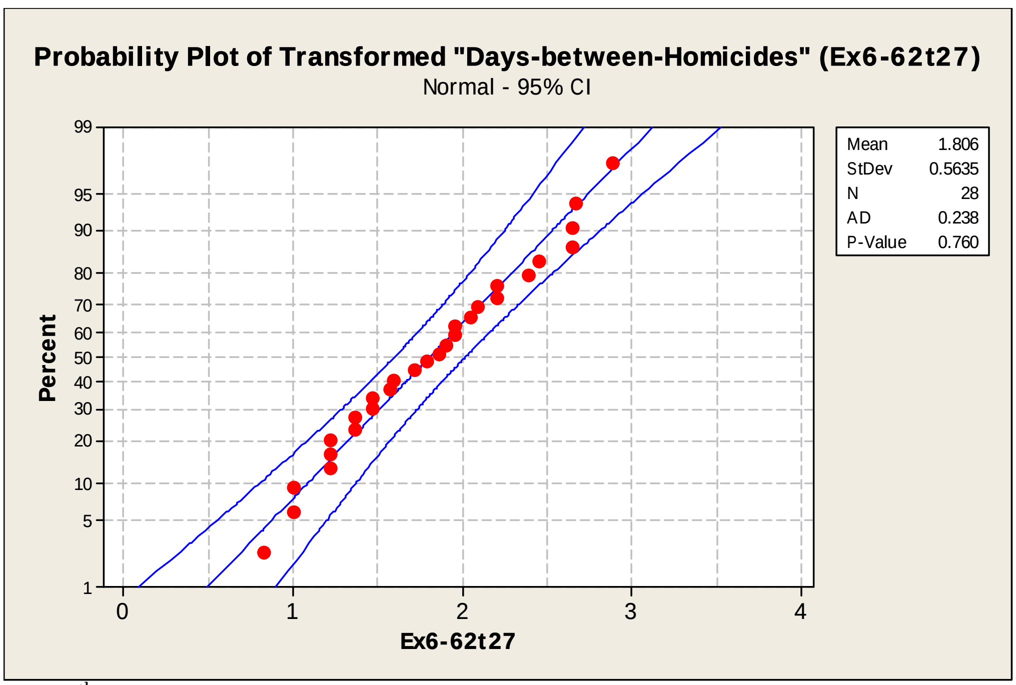
$$
\begin{aligned}
n\bar{c} - L\sqrt{n\bar{c}} &> 0 \\
n\bar{c} &> L\sqrt{n\bar{c}} \\
n &> L^2 / \bar{c}
\end{aligned}
$$

6-62.
(a)
MTB > Graphs > Probability Plot > Single



There is a huge curve in the plot points, indicating that the normal distribution assumption is not reasonable.

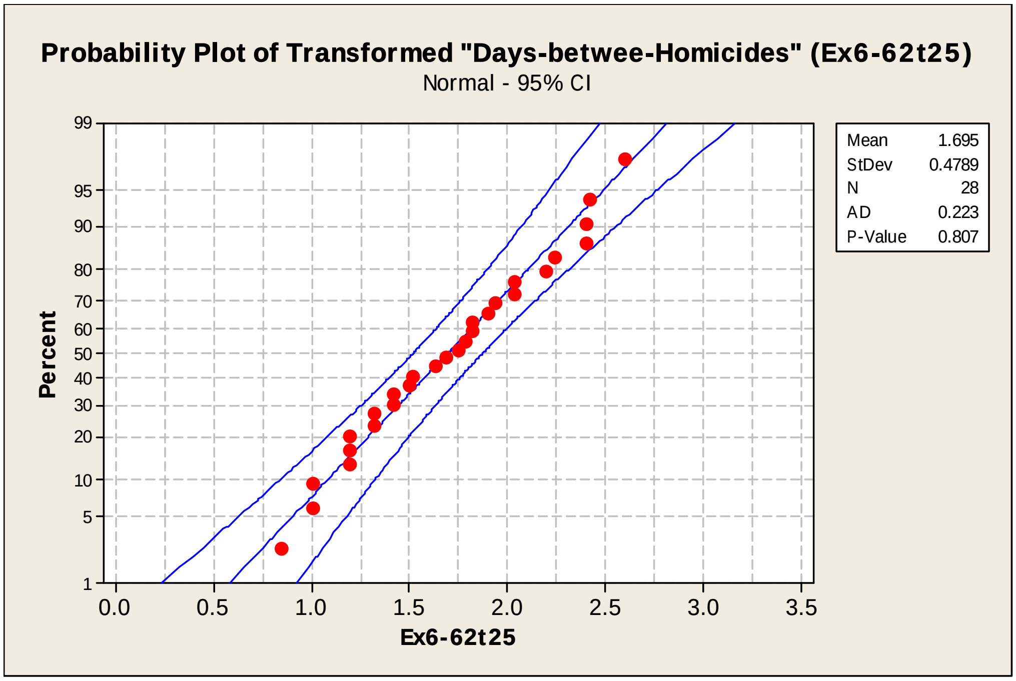
(b)



The 0.2777th root transformation makes the data more closely resemble a sample from a normal distribution.

6-62 continued

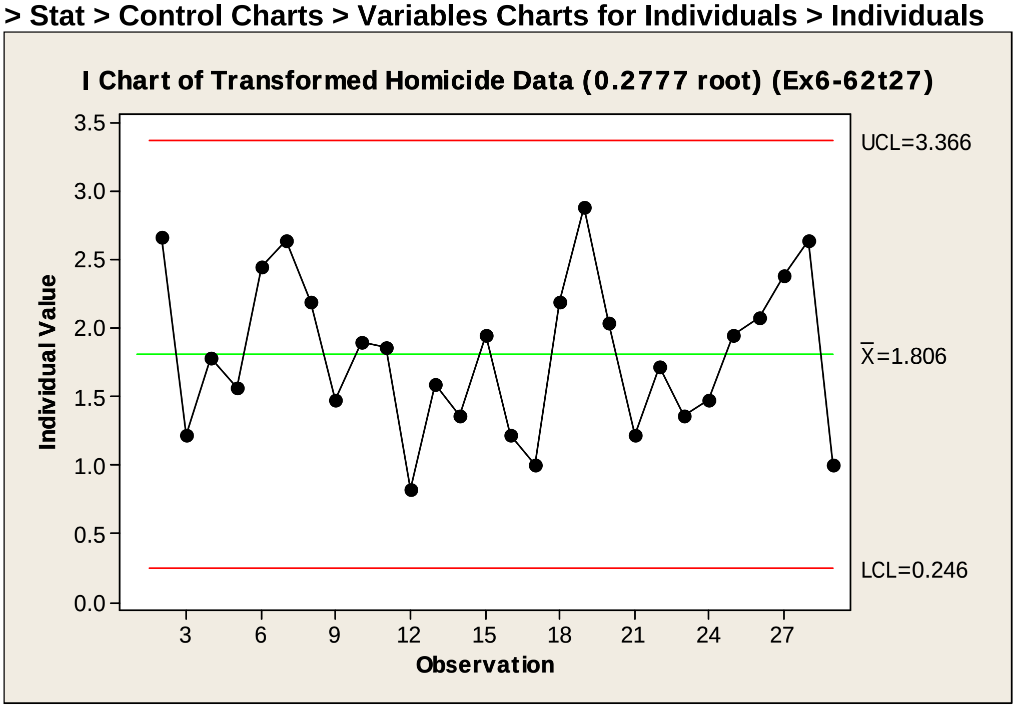
(c)



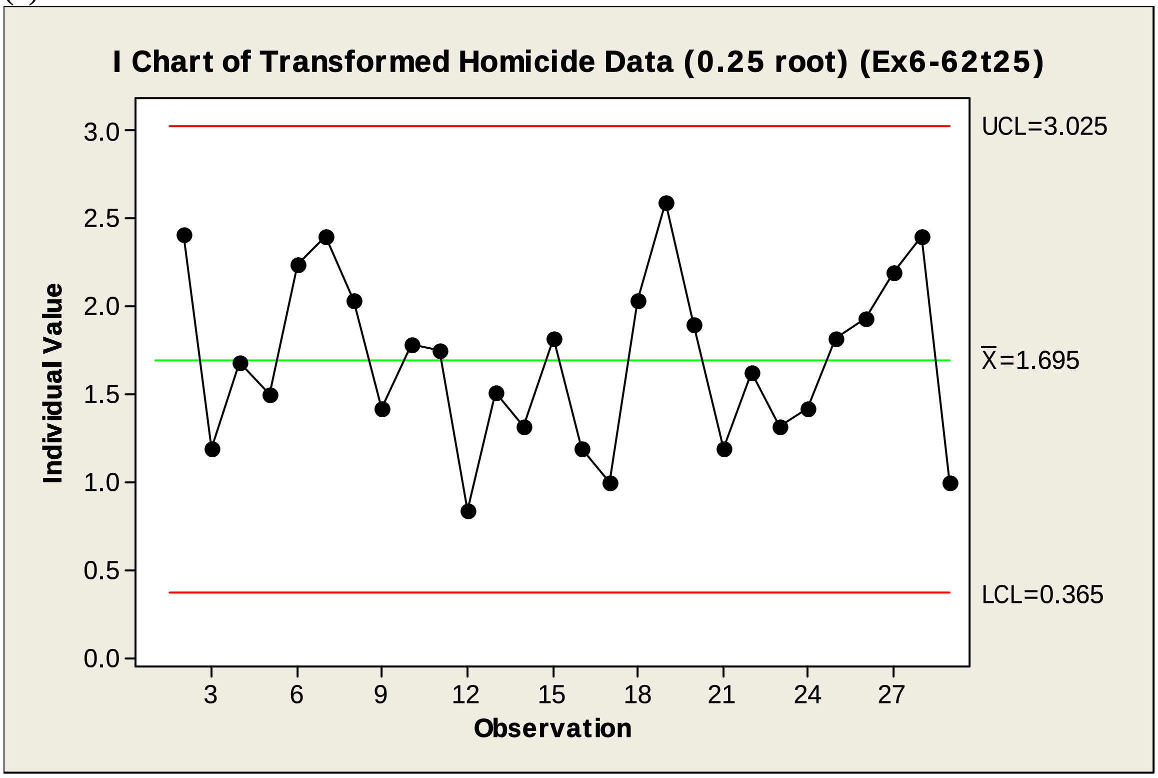
The 0.25th root transformation makes the data more closely resemble a sample from a normal distribution. It is not very different from the transformed data in (b).

(d)

MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals



6-62 continued
(e)



Both Individuals charts are similar, with an identical pattern of points relative to the UCL, mean and LCL. There is no difference in interpretation.

(f)
The "process" is stable, meaning that the days-between-homicides is approximately constant. If a change is made, say in population, law, policy, workforce, etc., which affects the rate at which homicides occur, the mean time between may get longer (or shorter) with plot points above the upper (or below the lower) control limit.

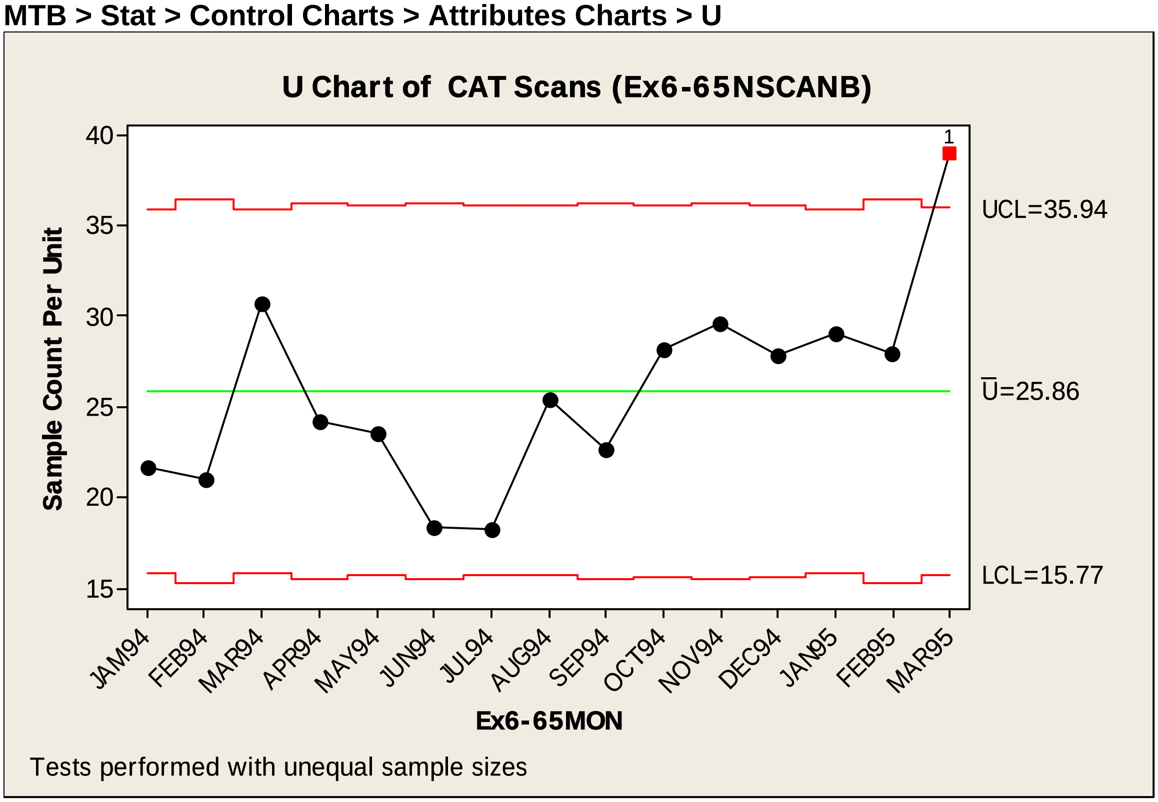
6-63.
There are endless possibilities for collection of attributes data from nonmanufacturing processes. Consider a product distribution center (or any warehouse) with processes for filling and shipping orders. One could track the number of orders filled incorrectly (wrong parts, too few/many parts, wrong part labeling,), packaged incorrectly (wrong material, wrong package labeling), invoiced incorrectly, etc. Or consider an accounting firm—errors in statements, errors in tax preparation, etc. (hopefully caught internally with a verification step).

6-64.
If time-between-events data (say failure time) is being sought for internally generated data, it can usually be obtained reliably and consistently. However, if you're looking for data on time-between-events that must be obtained from external sources (for example, time-to-field failures), it may be hard to determine with sufficient accuracy—both the "start" and the "end". Also, the conditions of use and the definition of "failure" may not be consistently applied.

There are ways to address these difficulties. Collection of "start" time data may be facilitated by serializing or date coding product.

6-65☺.
The variable NYRSB can be thought of as an "inspection unit", representing an identical "area of opportunity" for each "sample". The "process characteristic" to be controlled is the rate of CAT scans. A *u* chart which monitors the average number of CAT scans per NYRSB is appropriate.

MTB > Stat > Control Charts > Attributes Charts > U



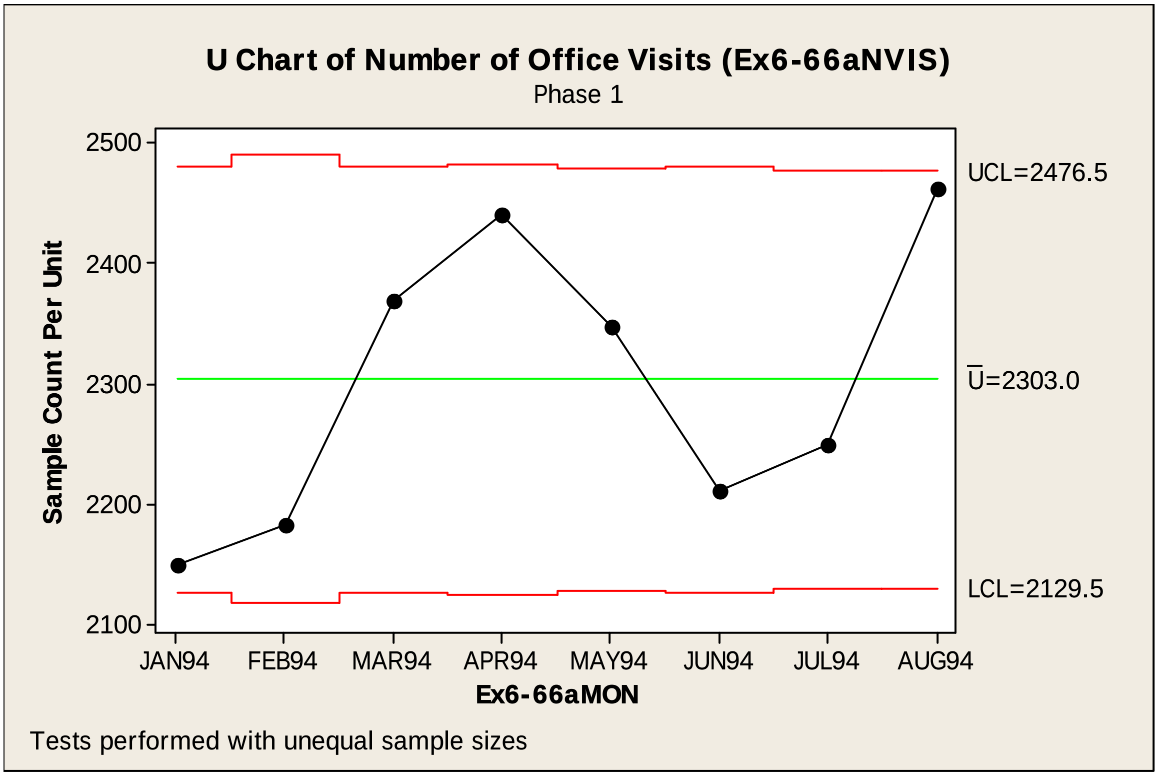
Test Results for U Chart of Ex6-65NSCANB

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  15
```

The rate of monthly CAT scans is out of control.

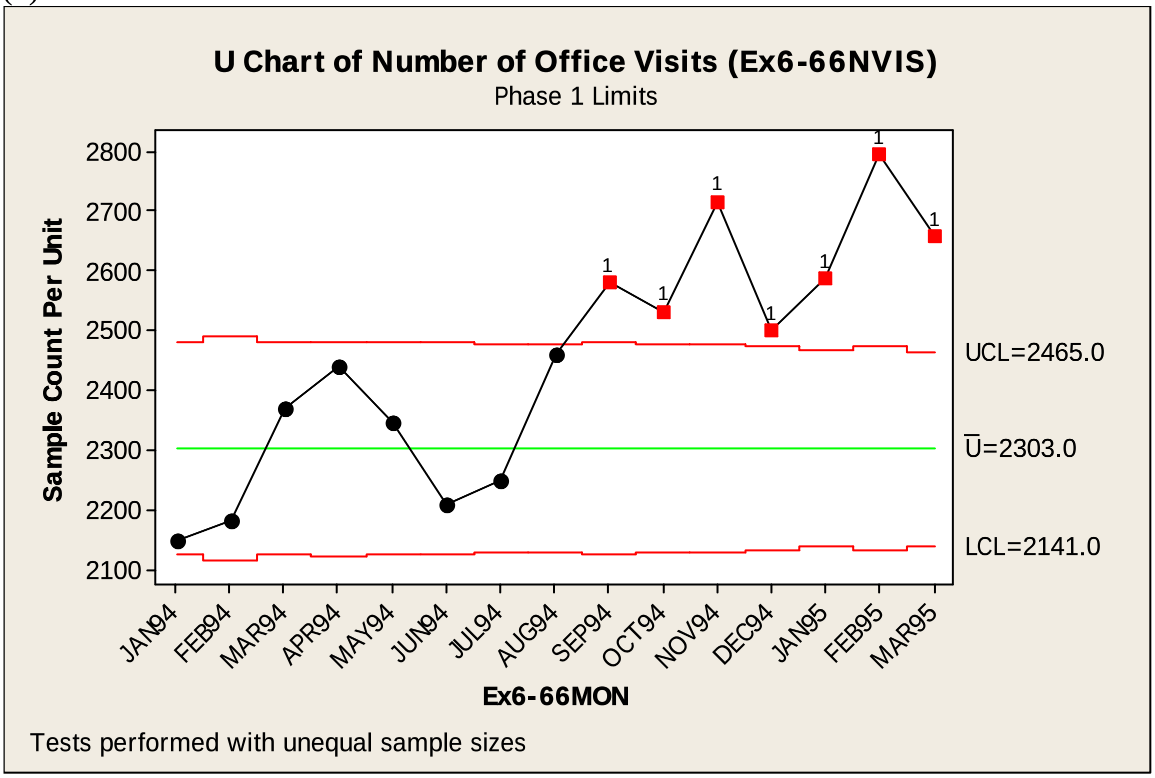
6-66☺.
The variable NYRSE can be thought of as an "inspection unit", representing an identical "area of opportunity" for each "sample". The "process characteristic" to be controlled is the rate of office visits. A *u* chart which monitors the average number of office visits per NYRSB is appropriate.

(a)
MTB > Stat > Control Charts > Attributes Charts > U



The chart is in statistical control

6-66 continued
(b)

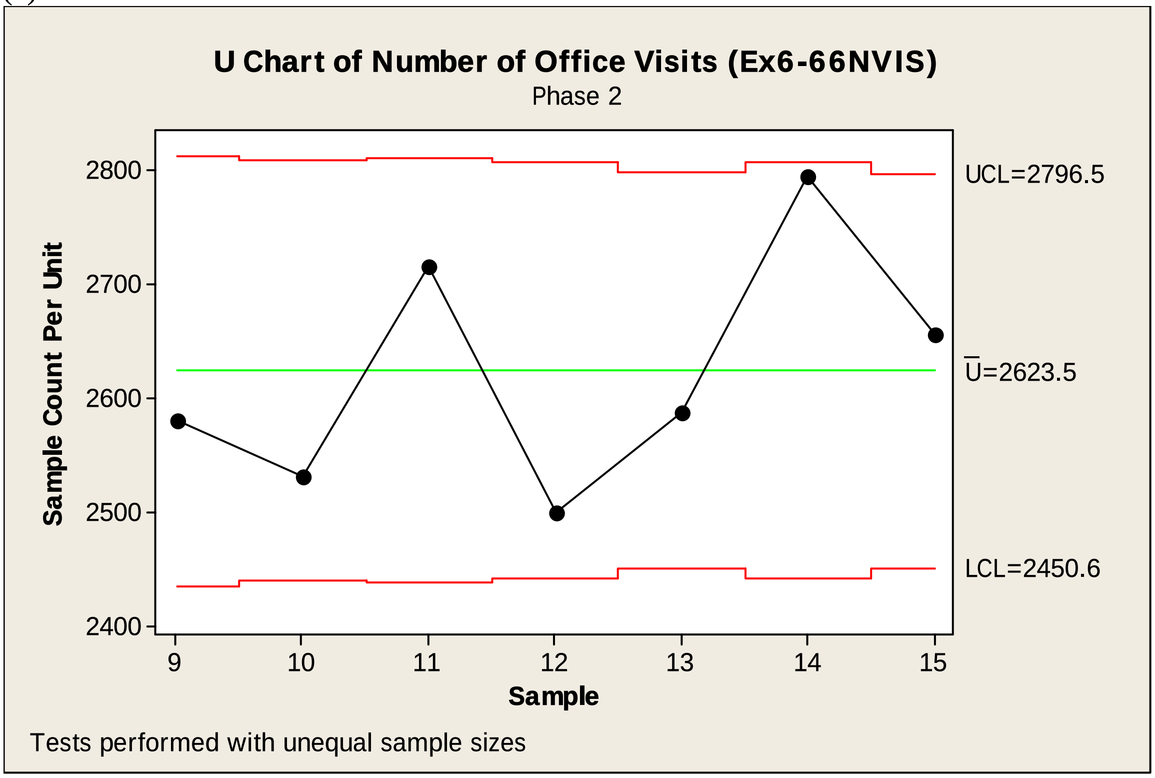


Test Results for U Chart of Ex6-66NVIS

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  9, 10, 11, 12, 13, 14, 15
```

The phase 2 data appears to have shifted up from phase 1. The 2^{nd} phase is not in statistical control relative to the 1^{st} phase.

6-66 continued
(c)



The Phase 2 data, separated from the Phase 1 data, are in statistical control.

Chapter 7 Exercise Solutions

Note: Several exercises in this chapter differ from those in the 4th edition. An "*" indicates that the description has changed. A second exercise number in parentheses indicates that the exercise number has changed. New exercises are denoted with an "☺".

7-1.

$$\hat{\mu} = \bar{\bar{x}} = 74.001; \quad \bar{R} = 0.023; \quad \hat{\sigma} = \bar{R}/d_2 = 0.023/2.326 = 0.010$$

$$\text{SL} = 74.000 \pm 0.035 = [73.965, 74.035]$$

$$\hat{C}_p \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{74.035 - 73.965}{6(0.010)} = 1.17$$

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \frac{74.001 - 73.965}{3(0.010)} = 1.20$$

$$\hat{C}_{pu} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} = \frac{74.035 - 74.001}{3(0.010)} = 1.13$$

$$\hat{C}_{pk} = \min\left(\hat{C}_{pl}, \hat{C}_{pu}\right) = 1.13$$

7-2.

In Exercise 5-1, samples 12 and 15 are out of control, and the new process parameters are used in the process capability analysis.

$$n = 5; \quad \hat{\mu} = \bar{\bar{x}} = 33.65; \quad \bar{R} = 4.5; \quad \hat{\sigma} = \bar{R}/d_2 = 1.93$$

$$\text{USL} = 40; \quad \text{LSL} = 20$$

$$\hat{C}_p \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{40 - 20}{6(1.93)} = 1.73$$

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \frac{33.65 - 20}{3(1.93)} = 2.36$$

$$\hat{C}_{pu} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} = \frac{40 - 33.65}{3(1.93)} = 1.10$$

$$\hat{C}_{pk} = \min\left(\hat{C}_{pl}, \hat{C}_{pu}\right) = 1.10$$

7-3.

$\hat{\mu} = \bar{\bar{x}} = 10.375; \bar{R}_x = 6.25; \hat{\sigma}_x = \bar{R}/d_2 = 6.25/2.059 = 3.04$

$\text{USL}_x = [(350+5)-350]\times 10 = 50; \text{LSL}_x = [(350-5)-350]\times 10 = -50$

$x_i = (\text{obs}_i - 350)\times 10$

$$\hat{C}_p = \frac{\text{USL}_x - \text{LSL}_x}{6\hat{\sigma}_x} = \frac{50-(-50)}{6(3.04)} = 5.48$$

The process produces product that uses approximately 18% of the total specification band.

$$\hat{C}_{pu} = \frac{\text{USL}_x - \hat{\mu}}{3\hat{\sigma}_x} = \frac{50-10.375}{3(3.04)} = 4.34$$

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{10.375-(-50)}{3(3.04)} = 6.62$$

$$\hat{C}_{pk} = \min(\hat{C}_{pu}, \hat{C}_{pl}) = 4.34$$

This is an extremely capable process, with an estimated percent defective much less than 1 ppb. Note that the C_{pk} is less than C_p, indicating that the process is not centered and is not achieving potential capability. However, this PCR does not tell *where* the mean is located within the specification band.

$$V = \frac{T - \bar{\bar{x}}}{S} = \frac{0-10.375}{3.04} = -3.4128$$

$$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1+V^2}} = \frac{5.48}{\sqrt{1+(-3.4128)^2}} = 1.54$$

Since C_{pm} is greater than 4/3, the mean μ lies within approximately the middle fourth of the specification band.

$$\hat{\xi} = \frac{\hat{\mu} - T}{\hat{\sigma}} = \frac{10.375-0}{3.04} = 3.41$$

$$\hat{C}_{pkm} = \frac{\hat{C}_{pk}}{\sqrt{1+\hat{\xi}^2}} = \frac{1.54}{\sqrt{1+3.41^2}} = 0.43$$

7-4.

$n = 5$; $\bar{\bar{x}} = 0.00109$; $\bar{R} = 0.00635$; $\hat{\sigma}_x = 0.00273$; tolerances: 0 ± 0.01

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{0.01 + 0.01}{6(0.00273)} = 1.22$$

The process produces product that uses approximately 82% of the total specification band.

$$\hat{C}_{pu} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} = \frac{0.01 - 0.00109}{3(0.00273)} = 1.09$$

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \frac{0.00109 - (-0.01)}{3(0.00273)} = 1.35$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 1.09$$

This process is not considered capable, failing to meet the minimally acceptable definition of capable $C_{pk} \geq 1.33$

$$V = \frac{T - \bar{\bar{x}}}{S} = \frac{0 - 0.00109}{0.00273} = -0.399$$

$$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1 + V^2}} = \frac{1.22}{\sqrt{1 + (-0.399)^2}} = 1.13$$

Since C_{pm} is greater than 1, the mean μ lies within approximately the middle third of the specification band.

$$\hat{\xi} = \frac{\hat{\mu} - T}{\hat{\sigma}} = \frac{0.00109 - 0}{0.00273} = 0.399$$

$$\hat{C}_{pkm} = \frac{\hat{C}_{pk}}{\sqrt{1 + \hat{\xi}^2}} = \frac{1.09}{\sqrt{1 + 0.399^2}} = 1.01$$

7-5.

$\hat{\mu} = \overline{\overline{x}} = 100; \overline{s} = 1.05; \hat{\sigma}_x = \overline{s}/c_4 = 1.05/0.9400 = 1.117$

(a)

Potential: $\hat{C}_p = \dfrac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \dfrac{(95+10)-(95-10)}{6(1.117)} = 2.98$

(b)

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{100-(95-10)}{3(1.117)} = 4.48$$

Actual: $\hat{C}_{pu} = \dfrac{\text{USL}_x - \hat{\mu}}{3\hat{\sigma}_x} = \dfrac{(95+10)-100}{3(1.117)} = 1.49$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 1.49$$

(c)

$$\begin{aligned}
\hat{p}_{\text{Actual}} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\
&= \Pr\{x < \text{LSL}\} + \left[1 - \Pr\{x \le \text{USL}\}\right] \\
&= \Pr\left\{z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right\} + \left[1 - \Pr\left\{z \le \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\}\right] \\
&= \Pr\left\{z < \frac{85-100}{1.117}\right\} + \left[1 - \Pr\left\{z \le \frac{105-100}{1.117}\right\}\right] \\
&= \Phi(-13.429) + \left[1 - \Phi(4.476)\right] \\
&= 0.0000 + \left[1 - 0.999996\right] \\
&= 0.000004
\end{aligned}$$

$$\begin{aligned}
\hat{p}_{\text{Potential}} &= \Pr\left\{z < \frac{85-95}{1.117}\right\} + \left[1 - \Pr\left\{z \le \frac{105-95}{1.117}\right\}\right] \\
&= \Phi(-8.953) + \left[1 - \Phi(8.953)\right] \\
&= 0.000000 + \left[1 - 1.000000\right] \\
&= 0.000000
\end{aligned}$$

7-6☺.

$n = 4; \quad \hat{\mu} = \bar{\bar{x}} = 199; \quad \bar{R} = 3.5; \quad \hat{\sigma}_x = \bar{R}/d_2 = 3.5/2.059 = 1.70$

USL = 200 + 8 = 208; LSL = 200 – 8 = 192

(a)

Potential: $\hat{C}_p = \dfrac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \dfrac{208 - 192}{6(1.70)} = 1.57$

The process produces product that uses approximately 64% of the total specification band.

(b)

$$\hat{C}_{pu} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} = \frac{208 - 199}{3(1.70)} = 1.76$$

Actual: $\hat{C}_{pl} = \dfrac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \dfrac{199 - 192}{3(1.70)} = 1.37$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 1.37$$

(c)

The current fraction nonconforming is:

$$\begin{aligned}
\hat{p}_{\text{Actual}} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\
&= \Pr\{x < \text{LSL}\} + \left[1 - \Pr\{x \le \text{USL}\}\right] \\
&= \Pr\left\{z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right\} + \left[1 - \Pr\left\{z \le \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\}\right] \\
&= \Pr\left\{z < \frac{192 - 199}{1.70}\right\} + \left[1 - \Pr\left\{z \le \frac{208 - 199}{1.70}\right\}\right] \\
&= \Phi(-4.1176) + \left[1 - \Phi(5.2941)\right] \\
&= 0.0000191 + \left[1 - 1\right] \\
&= 0.0000191
\end{aligned}$$

If the process mean could be centered at the specification target, the fraction nonconforming would be:

$$\begin{aligned}
\hat{p}_{\text{Potential}} &= 2 \times \Pr\left\{z < \frac{192 - 200}{1.70}\right\} \\
&= 2 \times 0.0000013 \\
&= 0.0000026
\end{aligned}$$

7-7☺.

$n = 2; \quad \hat{\mu} = \bar{\bar{x}} = 39.7; \quad \bar{R} = 2.5; \quad \hat{\sigma}_x = \bar{R}/d_2 = 2.5/1.128 = 2.216$

USL = 40 + 5 = 45; LSL = 40 – 5 = 35

(a)

Potential: $\hat{C}_p = \dfrac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \dfrac{45-35}{6(2.216)} = 0.75$

(b)

$$\hat{C}_{pu} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} = \frac{45 - 39.7}{3(2.216)} = 0.80$$

Actual: $\hat{C}_{pl} = \dfrac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \dfrac{39.7 - 35}{3(2.216)} = 0.71$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 0.71$$

(c)

$$V = \frac{\bar{x} - T}{s} = \frac{39.7 - 40}{2.216} = -0.135$$

$$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1+V^2}} = \frac{0.75}{\sqrt{1+(-0.135)^2}} = 0.74$$

$$\hat{C}_{pkm} = \frac{\hat{C}_{pk}}{\sqrt{1+V^2}} = \frac{0.71}{\sqrt{1+(-0.135)^2}} = 0.70$$

The closeness of estimates for C_p, C_{pk}, C_{pm}, and C_{pkm} indicate that the process mean is very close to the specification target.

(d)

The current fraction nonconforming is:

$$\begin{aligned}
\hat{p}_{\text{Actual}} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\
&= \Pr\{x < \text{LSL}\} + \left[1 - \Pr\{x \le \text{USL}\}\right] \\
&= \Pr\left\{z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right\} + \left[1 - \Pr\left\{z \le \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\}\right] \\
&= \Pr\left\{z < \frac{35 - 39.7}{2.216}\right\} + \left[1 - \Pr\left\{z \le \frac{45 - 39.7}{2.216}\right\}\right] \\
&= \Phi(-2.12094) + \left[1 - \Phi(2.39170)\right] \\
&= 0.0169634 + \left[1 - 0.991615\right] \\
&= 0.025348
\end{aligned}$$

7-7 (d) continued
If the process mean could be centered at the specification target, the fraction nonconforming would be:

$$\begin{aligned}\hat{p}_{\text{Potential}} &= 2\times\Pr\left\{z<\frac{35-40}{2.216}\right\}\\ &= 2\times\Pr\{z<-2.26\}\\ &= 2\times 0.01191\\ &= 0.02382\end{aligned}$$

7-8 (7-6).
$\hat{\mu}=75;\ \overline{S}=2;\ \hat{\sigma}=\hat{S}\big/c_4=2\big/0.9400=2.13$

(a)
Potential: $\hat{C}_p=\dfrac{\text{USL}-\text{LSL}}{6\hat{\sigma}}=\dfrac{2(8)}{6(2.13)}=1.25$

(b)
$$\hat{C}_{pl}=\frac{\hat{\mu}-\text{LSL}}{3\hat{\sigma}}=\frac{75-(80-8)}{3(2.13)}=0.47$$

Actual: $\hat{C}_{pu}=\dfrac{\text{USL}-\hat{\mu}}{3\hat{\sigma}}=\dfrac{80+8-75}{3(2.13)}=2.03$

$$\hat{C}_{pk}=\min(\hat{C}_{pl},\hat{C}_{pu})=0.47$$

(c) Let $\hat{\mu}=80$

$$\begin{aligned}\hat{p}_{\text{Potential}} &= \Pr\{x<\text{LSL}\}+\Pr\{x>\text{USL}\}\\ &= \Pr\left\{z<\frac{\text{LSL}-\hat{\mu}}{\hat{\sigma}}\right\}+1-\Pr\left\{z\le\frac{\text{USL}-\hat{\mu}}{\hat{\sigma}}\right\}\\ &= \Pr\left\{z<\frac{72-80}{2.13}\right\}+1-\Pr\left\{z\le\frac{88-80}{2.13}\right\}\\ &= \Phi(-3.756)+1-\Phi(3.756)\\ &= 0.000086+1-0.999914\\ &= 0.000172\end{aligned}$$

7-9 (7-7).
Assume n = 5

<u>Process A</u>

$$\hat{\mu} = \bar{\bar{x}}_A = 100;\ \bar{s}_A = 3;\ \hat{\sigma}_A = \bar{s}_A / c_4 = 3/0.9400 = 3.191$$

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{(100+10)-(100-10)}{6(3.191)} = 1.045$$

$$\hat{C}_{pu} = \frac{\text{USL}_x - \hat{\mu}}{3\hat{\sigma}_x} = \frac{(100+10)-100}{3(3.191)} = 1.045$$

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{100-(100-10)}{3(3.191)} = 1.045$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 1.045$$

$$V = \frac{\bar{x} - T}{s} = \frac{100-100}{3.191} = 0$$

$$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1+V^2}} = \frac{1.045}{\sqrt{1+(0)^2}} = 1.045$$

$$\begin{aligned}
\hat{p} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\
&= \Pr\{x < \text{LSL}\} + 1 - \Pr\{x \le \text{USL}\} \\
&= \Pr\left\{z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right\} + 1 - \Pr\left\{z \le \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\} \\
&= \Pr\left\{z < \frac{90-100}{3.191}\right\} + 1 - \Pr\left\{z \le \frac{110-100}{3.191}\right\} \\
&= \Phi(-3.13) + 1 - \Phi(3.13) \\
&= 0.00087 + 1 - 0.99913 \\
&= 0.00174
\end{aligned}$$

<u>Process B</u>

$$\hat{\mu} = \bar{\bar{x}}_B = 105;\ \bar{s}_B = 1;\ \hat{\sigma}_B = \bar{s}_B / c_4 = 1/0.9400 = 1.064$$

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{(100+10)-(100-10)}{6(1.064)} = 3.133$$

$$\hat{C}_{pl} = \frac{\hat{\mu}_x - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{105-(100-10)}{3(1.064)} = 4.699$$

$$\hat{C}_{pu} = \frac{\text{USL}_x - \hat{\mu}_x}{3\hat{\sigma}_x} = \frac{(100+10)-105}{3(1.064)} = 1.566$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 1.566$$

7-9 continued

$$V = \frac{\bar{x} - T}{s} = \frac{100 - 105}{1.064} = -4.699$$

$$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1+V^2}} = \frac{3.133}{\sqrt{1+(-4.699)^2}} = 0.652$$

$$\begin{aligned}
\hat{p} &= \Pr\left\{z < \frac{90-105}{1.064}\right\} + 1 - \Pr\left\{z \le \frac{110-105}{1.064}\right\} \\
&= \Phi(-14.098) + 1 - \Phi(4.699) \\
&= 0.000000 + 1 - 0.999999 \\
&= 0.000001
\end{aligned}$$

Prefer to use Process B with estimated process fallout of 0.000001 instead of Process A with estimated fallout 0.001726.

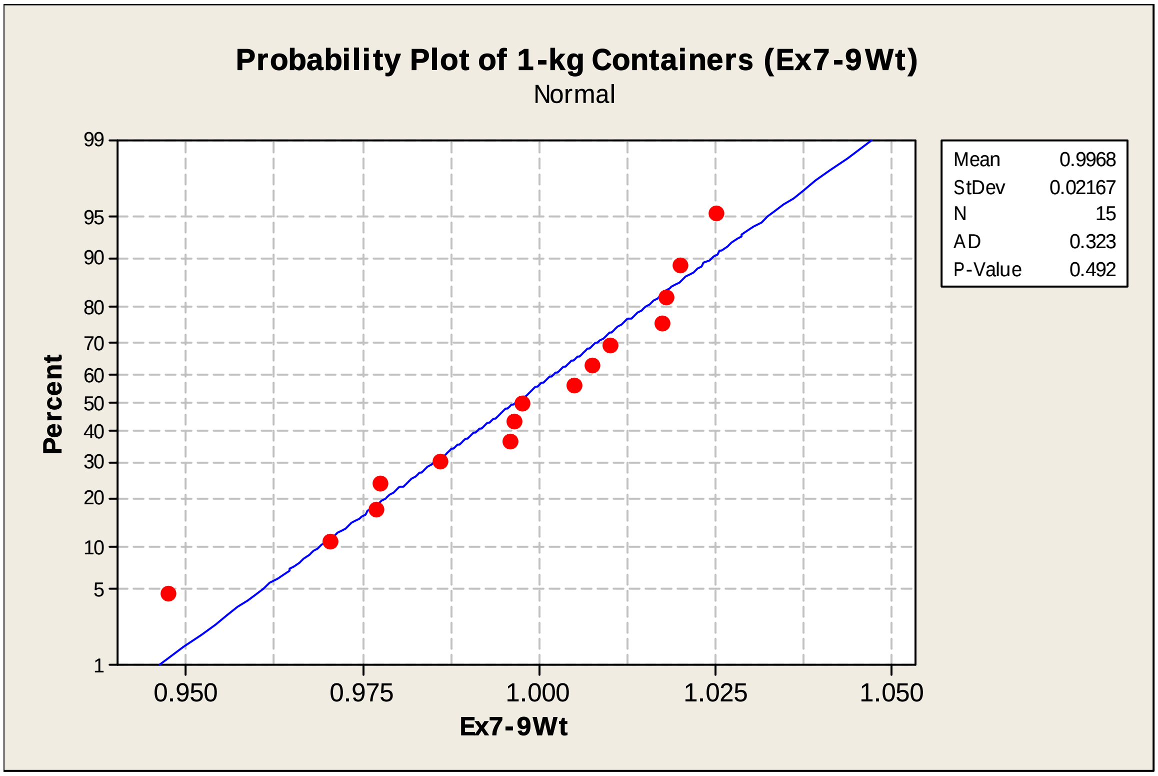
7-10 (7-8).

Process A: $\hat{\mu}_A = 20(100) = 2000; \hat{\sigma}_A = \sqrt{20\hat{\sigma}^2} = \sqrt{20(3.191)^2} = 14.271$

Process B: $\hat{\mu}_B = 20(105) = 2100; \hat{\sigma}_B = \sqrt{20\hat{\sigma}^2} = \sqrt{20(1.064)^2} = 4.758$

Process B will result in fewer defective assemblies. For the parts $\left(\hat{C}_{pk,A} = 1.045\right) < \left(1.566 = \hat{C}_{pk,B}\right)$ indicates that more parts from Process B are within specification than from Process A.

7-11 (7-9).
MTB > Stat > Basic Statistics > Normality Test



A normal probability plot of the 1-kg container weights shows the distribution is close to normal.

$$\bar{x} \approx p_{50} = 0.9975; \quad p_{84} = 1.0200$$
$$\hat{\sigma} = p_{84} - p_{50} = 1.0200 - 0.9975 = 0.0225$$
$$6\hat{\sigma} = 6(0.0225) = 0.1350$$

7-12☺.
LSL = 0.985 kg

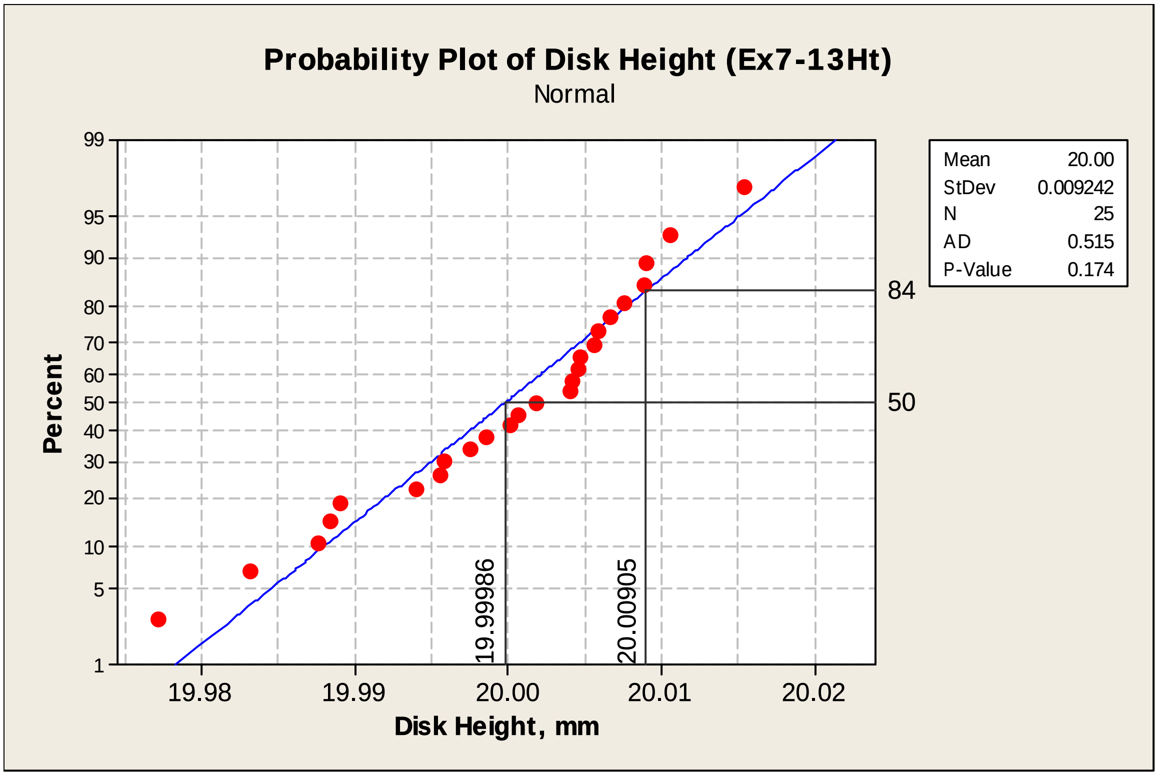
$$C_{pl} = \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \frac{0.9975 - 0.985}{3(0.0225)} = 0.19$$

$$\hat{p} = \Pr\left\{ z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}} \right\} = \Pr\left\{ z < \frac{0.985 - 0.9975}{0.0225} \right\} = \Phi(-0.556) = 0.289105$$

7-13☺.

MTB > Stat > Basic Statistics > Normality Test

(Add percentile lines at Y values 50 and 84 to estimate μ and σ.)



A normal probability plot of computer disk heights shows the distribution is close to normal.

$$\bar{x} \approx p_{50} = 19.99986$$

$$p_{84} = 20.00905$$

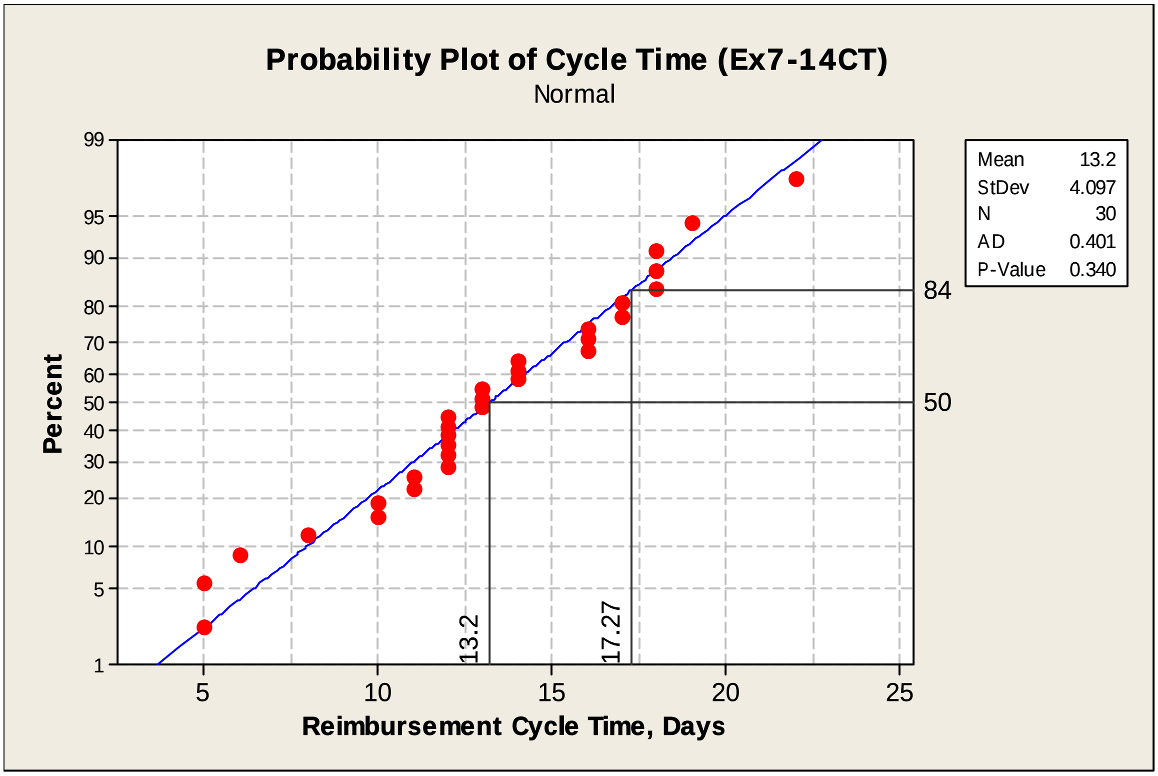
$$\hat{\sigma} = p_{84} - p_{50} = 20.00905 - 19.99986 = 0.00919$$

$$6\hat{\sigma} = 6(0.00919) = 0.05514$$

7-14☺.

MTB > Stat > Basic Statistics > Normality Test

(Add percentile lines at Y values 50 and 84 to estimate μ and σ.)



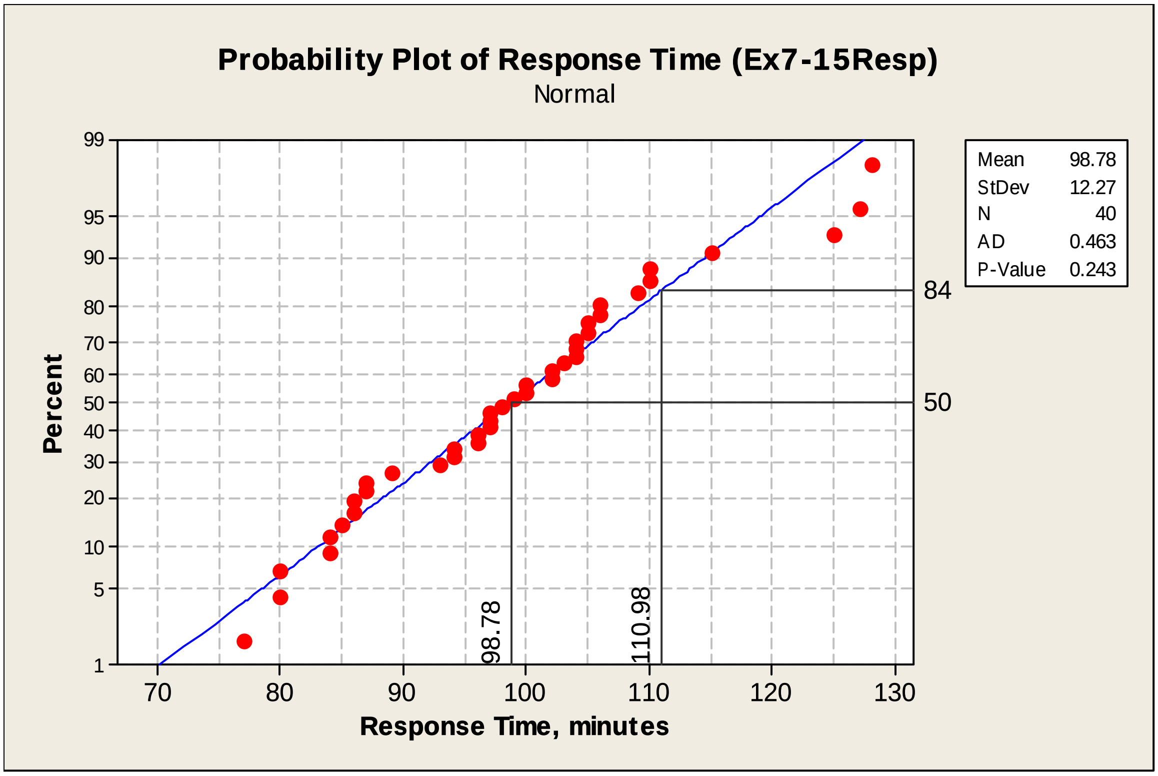
A normal probability plot of reimbursement cycle times shows the distribution is close to normal.

$$\bar{x} \approx p_{50} = 13.2$$
$$p_{84} = 17.27$$
$$\hat{\sigma} = p_{84} - p_{50} = 17.27 - 13.2 = 4.07$$
$$6\hat{\sigma} = 6(4.07) = 24.42$$

7-15☺.

MTB > Stat > Basic Statistics > Normality Test

(Add percentile lines at Y values 50 and 84 to estimate μ and σ.)



A normal probability plot of response times shows the distribution is close to normal.

(a)

$$\bar{x} \approx p_{50} = 98.78$$

$$p_{84} = 110.98$$

$$\hat{\sigma} = p_{84} - p_{50} = 110.98 - 98.78 = 12.2$$

$$6\hat{\sigma} = 6(12.2) = 73.2$$

(b)

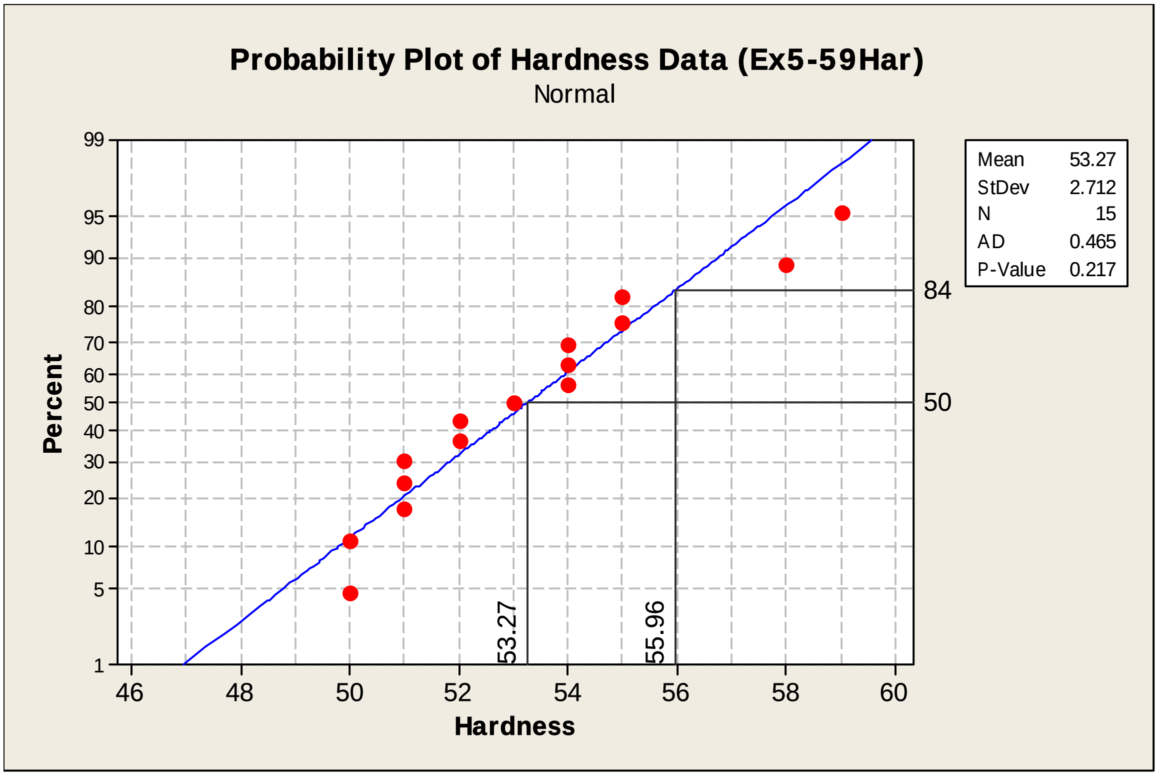
USL = 2 hrs = 120 mins

$$C_{pu} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} = \frac{120 - 98.78}{3(12.2)} = 0.58$$

$$\hat{p} = \Pr\left\{z > \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\} = 1 - \Pr\left\{z < \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\} = 1 - \Pr\left\{z < \frac{120 - 98.78}{12.2}\right\}$$

$$= 1 - \Phi(1.739) = 1 - 0.958983 = 0.041017$$

7-16 (7-10).
MTB > Stat > Basic Statistics > Normality Test
(Add percentile lines at Y values 50 and 84 to estimate μ and σ.)



A normal probability plot of hardness data shows the distribution is close to normal.

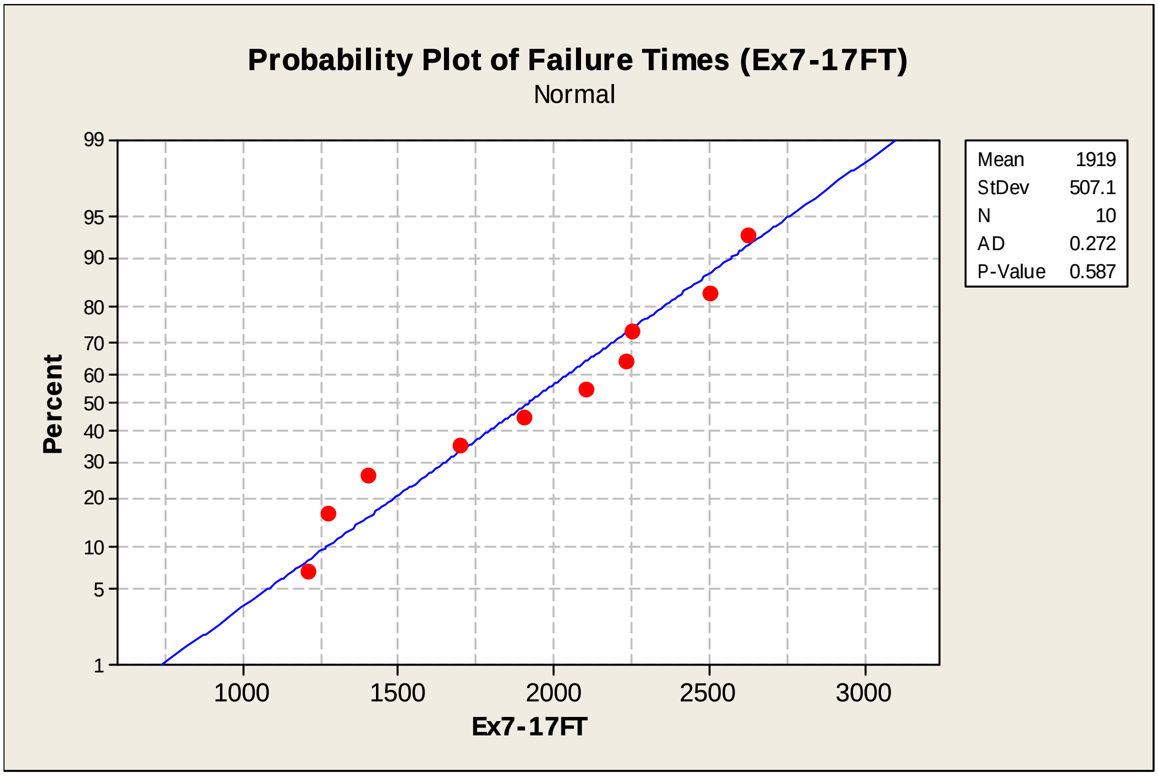
$\bar{x} \approx p_{50} = 53.27$

$p_{84} = 55.96$

$\hat{\sigma} = p_{84} - p_{50} = 55.96 - 53.27 = 2.69$

$6\hat{\sigma} = 6(2.69) = 16.14$

7-17 (7-11).
MTB > Stat > Basic Statistics > Normality Test



The plot shows that the data is not normally distributed; so it is not appropriate to estimate capability.

7-18 (7-12).
LSL = 75; USL = 85; n = 25; S = 1.5

(a)
$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{85 - 75}{6(1.5)} = 1.11$$

(b)
$$\alpha = 0.05$$
$$\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,24} = 12.40$$
$$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,24} = 39.36$$
$$\hat{C}_p\sqrt{\frac{\chi^2_{1-\alpha/2,n-1}}{n-1}} \le C_p \le \hat{C}_p\sqrt{\frac{\chi^2_{\alpha/2,n-1}}{n-1}}$$
$$1.11\sqrt{\frac{12.40}{25-1}} \le C_p \le 1.11\sqrt{\frac{39.36}{25-1}}$$
$$0.80 \le C_p \le 1.42$$
This confidence interval is wide enough that the process may either be capable (ppm = 27) or far from it (ppm ≈ 16,395).

7-19 (7-13).
$$n = 50$$
$$\hat{C}_p = 1.52$$
$$1-\alpha = 0.95$$
$$\chi^2_{1-\alpha,n-1} = \chi^2_{0.95,49} = 33.9303$$
$$\hat{C}_p\sqrt{\frac{\chi^2_{1-\alpha,n-1}}{n-1}} \le C_p$$
$$1.52\sqrt{\frac{33.9303}{49}} = 1.26 \le C_p$$

The company cannot demonstrate that the PCR exceeds 1.33 at a 95% confidence level.
$$1.52\sqrt{\frac{\chi^2_{1-\alpha,49}}{49}} = 1.33$$
$$\chi^2_{1-\alpha,49} = 49\left(\frac{1.33}{1.52}\right)^2 = 37.52$$
$$1-\alpha = 0.88$$
$$\alpha = 0.12$$

7-20 (7-14).

$n = 30;\ \bar{x} = 97;\ S = 1.6;\ \text{USL} = 100;\ \text{LSL} = 90$

(a)

$$\hat{C}_{pu} = \frac{\text{USL}_x - \hat{\mu}_x}{3\hat{\sigma}_x} = \frac{100 - 97}{3(1.6)} = 0.63$$

$$\hat{C}_{pl} = \frac{\hat{\mu}_x - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{97 - 90}{3(1.6)} = 1.46$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 0.63$$

(b)

$\alpha = 0.05$

$z_{\alpha/2} = z_{0.025} = 1.960$

$$\hat{C}_{pk}\left[1 - z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}}\right] \le C_{pk} \le \hat{C}_{pk}\left[1 + z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}}\right]$$

$$0.63\left[1 - 1.96\sqrt{\frac{1}{9(30)(0.63)^2} + \frac{1}{2(30-1)}}\right] \le C_{pk} \le 0.63\left[1 + 1.96\sqrt{\frac{1}{9(30)(0.63)^2} + \frac{1}{2(30-1)}}\right]$$

$$0.4287 \le C_{pk} \le 0.8313$$

7-21 (7-15).
USL = 2350; LSL = 2100; nominal = 2225; $\bar{x} = 2275$; $s = 60$; $n = 50$

(a)

$$\hat{C}_{pu} = \frac{\text{USL}_x - \hat{\mu}_x}{3\hat{\sigma}_x} = \frac{2350 - 2275}{3(60)} = 0.42$$

$$\hat{C}_{pl} = \frac{\hat{\mu}_x - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{2275 - 2100}{3(60)} = 0.97$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 0.42$$

(b)

$\alpha = 0.05$; $z_{\alpha/2} = z_{0.025} = 1.960$

$$\hat{C}_{pk}\left[1 - z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}}\right] \le C_{pk} \le \hat{C}_{pk}\left[1 + z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}}\right]$$

$$0.42\left[1 - 1.96\sqrt{\frac{1}{9(50)(0.42)^2} + \frac{1}{2(50-1)}}\right] \le C_{pk} \le 0.42\left[1 + 1.96\sqrt{\frac{1}{9(50)(0.42)^2} + \frac{1}{2(50-1)}}\right]$$

$$0.2957 \le C_{pk} \le 0.5443$$

7-22 (7-16).
from Ex. 7-20, $\hat{C}_{pk} = 0.63$; $z_{\alpha/2} = 1.96$; $n = 30$

$$\hat{C}_{pk}\left[1 - z_{\alpha/2}\sqrt{\frac{1}{2(n-1)}}\right] \le C_{pk} \le \hat{C}_{pk}\left[1 + z_{\alpha/2}\sqrt{\frac{1}{2(n-1)}}\right]$$

$$0.63\left[1 - 1.96\sqrt{\frac{1}{2(30-1)}}\right] \le C_{pk} \le 0.63\left[1 + 1.96\sqrt{\frac{1}{2(30-1)}}\right]$$

$$0.47 \le C_{pk} \le 0.79$$

The approximation yields a narrower confidence interval, but it is not too far off.

7-23 (7-17).

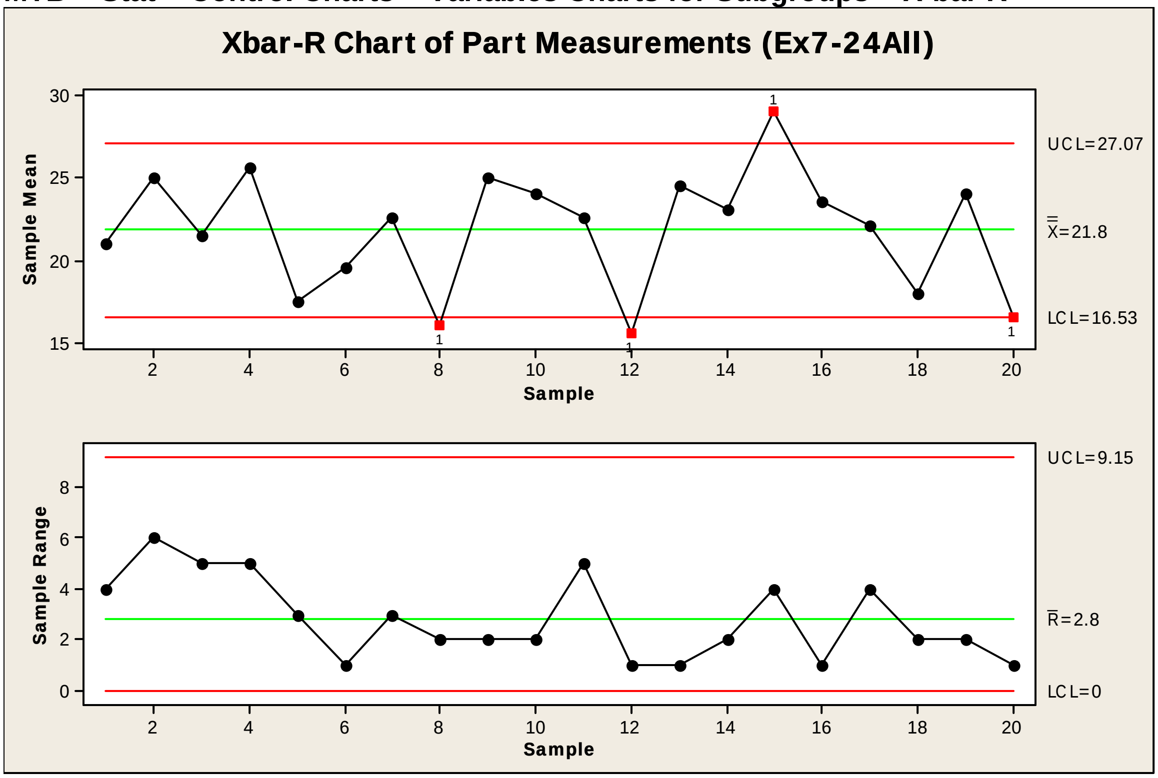
$\sigma_{OI} = 0$; $\hat{\sigma}_I = 3$; $\hat{\sigma}_{\text{Total}} = 5$

$$\hat{\sigma}_{\text{Total}}^2 = \hat{\sigma}_{\text{Meas}}^2 + \hat{\sigma}_{\text{Process}}^2$$

$$\hat{\sigma}_{\text{Process}} = \sqrt{\hat{\sigma}_{\text{Total}}^2 - \hat{\sigma}_{\text{Meas}}^2} = \sqrt{5^2 - 3^2} = 4$$

7-24 (7-18).
(a)
$n = 2; \bar{\bar{x}} = 21.8; \bar{R} = 2.8; \hat{\sigma}_{\text{Gauge}} = 2.482$

MTB > Stat > Control Charts > Variables Charts for Subgroups > X-bar R



Test Results for Xbar Chart of Ex7-24All

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  8, 12, 15, 20
```

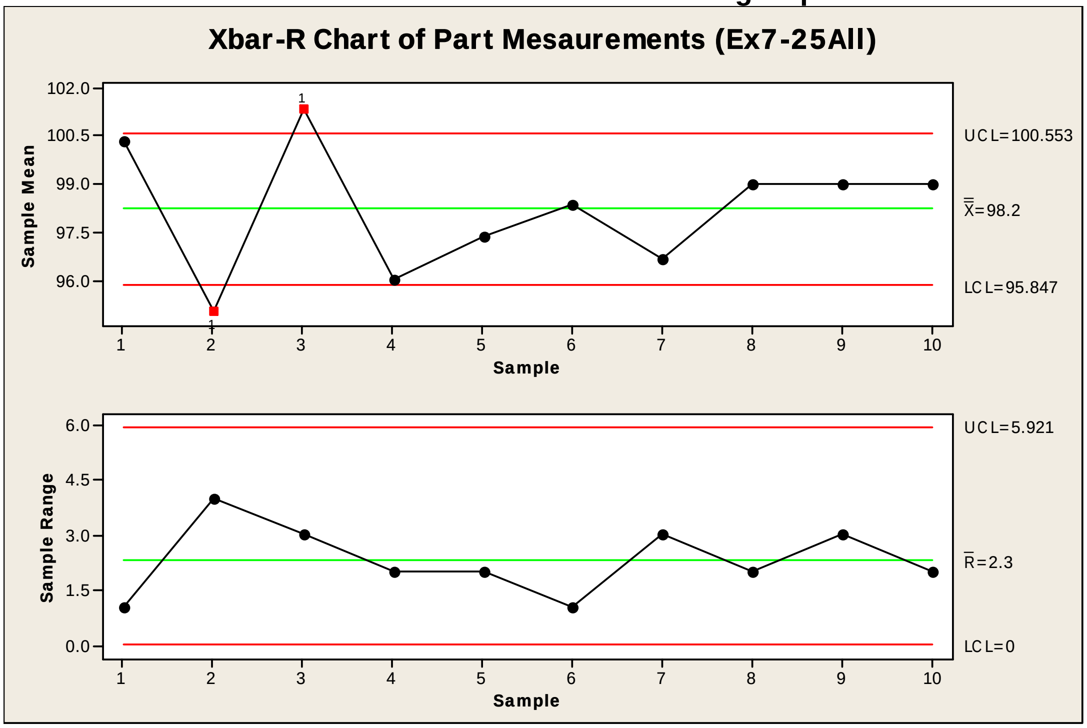
The R chart is in control, and the $\bar{x}$ chart has a few out-of-control parts. The new gauge is more repeatable than the old one.

(b) specs: 25 ± 15

$$\frac{P}{T} = \frac{6\hat{\sigma}_{\text{Gauge}}}{\text{USL} - \text{LSL}} \times 100 = \frac{6(2.482)}{2(15)} \times 100 = 49.6\%$$

7-25 (7-19).

MTB > Stat > Control Charts > Variables Charts for Subgroups > X-bar R



Test Results for Xbar Chart of Ex7-25All

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  2, 3
```

The $\bar{x}$ chart has a couple out-of-control points, and the R chart is in control. This indicates that the operator is not having difficulty making consistent measurements.

(b)

$$\bar{\bar{x}} = 98.2;\ \bar{R} = 2.3;\ \hat{\sigma}_{\text{Gauge}} = \bar{R}/d_2 = 2.3/1.693 = 1.359$$

$$\hat{\sigma}^2_{\text{Total}} = 4.717$$

$$\hat{\sigma}^2_{\text{Product}} = \hat{\sigma}^2_{\text{Total}} - \hat{\sigma}^2_{\text{Gauge}} = 4.717 - 1.359^2 = 2.872$$

$$\hat{\sigma}_{\text{Product}} = 1.695$$

(c)

$$\frac{\hat{\sigma}_{\text{Gauge}}}{\hat{\sigma}_{\text{Total}}} \times 100 = \frac{1.359}{\sqrt{4.717}} \times 100 = 62.5\%$$

(d)

USL = 100 + 15 = 115; LSL = 100 – 15 = 85

$$\frac{P}{T} = \frac{6\hat{\sigma}_{\text{Gauge}}}{\text{USL} - \text{LSL}} = \frac{6(1.359)}{115 - 85} = 0.272$$

7-26 (7-20).
(a)
Excel : workbook Chap07.xls : worksheet Ex7-26

$\bar{\bar{x}}_1 = 50.03;\ \bar{R}_1 = 1.70;\ \bar{\bar{x}}_2 = 49.87;\ \bar{R}_2 = 2.30$

$\bar{\bar{R}} = 2.00$

$n = 3$ repeat measurements

$d_2 = 1.693$

$\hat{\sigma}_{\text{Repeatability}} = \bar{\bar{R}} / d_2 = 2.00/1.693 = 1.181$

$R_{\bar{\bar{x}}} = 0.17$

$n = 2$ operators

$d_2 = 1.128$

$\hat{\sigma}_{\text{Reproducibility}} = R_{\bar{\bar{x}}} / d_2 = 0.17/1.128 = 0.151$

(b)

$\hat{\sigma}^2_{\text{Measurement Error}} = \hat{\sigma}^2_{\text{Repeatability}} + \hat{\sigma}^2_{\text{Reproducibility}} = 1.181^2 + 0.151^2 = 1.418$

$\hat{\sigma}_{\text{Measurement Error}} = 1.191$

(c) specs: 50 ± 10

$$\frac{P}{T} = \frac{6\hat{\sigma}_{\text{Gauge}}}{\text{USL} - \text{LSL}} \times 100 = \frac{6(1.191)}{60 - 40} \times 100 = 35.7\%$$

7-27 (7-21).

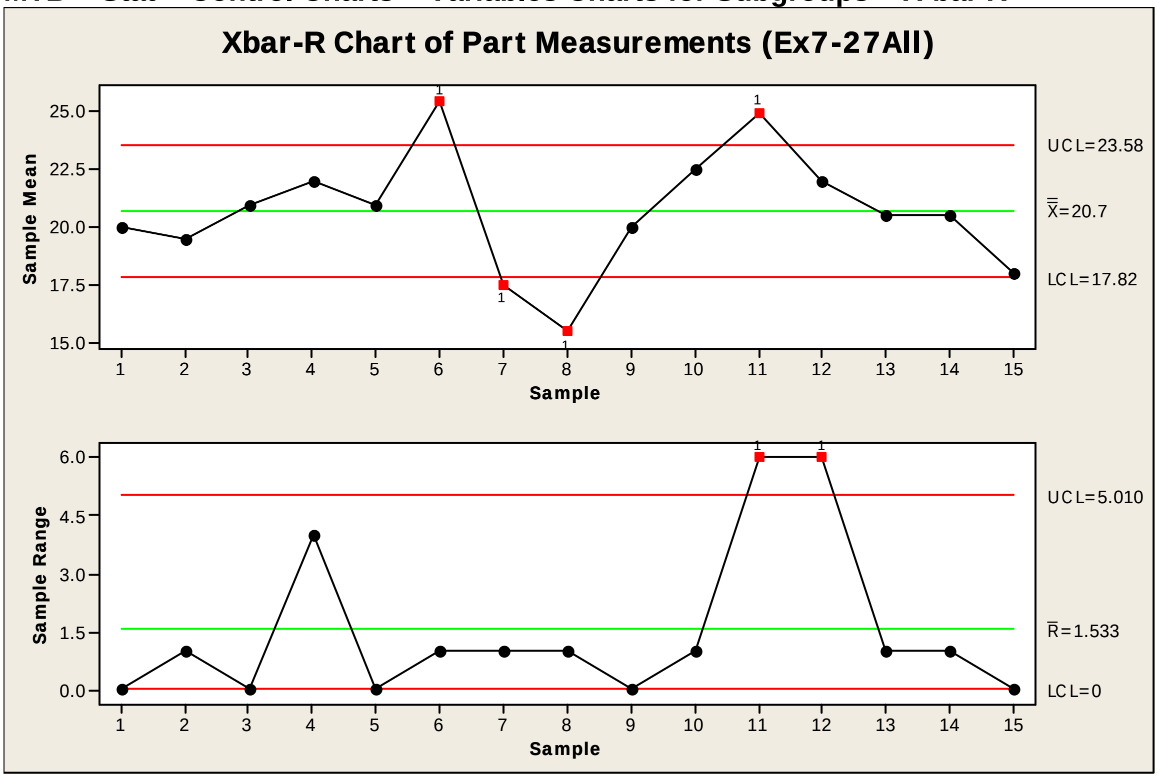
(a)

$\hat{\sigma}_{\text{Gauge}} = \bar{R}/d_2 = 1.533/1.128 = 1.359$

Gauge capability: $6\hat{\sigma} = 8.154$

(b)

MTB > Stat > Control Charts > Variables Charts for Subgroups > X-bar R




```
Test Results for R Chart of Ex7-27All
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  11, 12
```

Out-of-control points on R chart indicate operator difficulty with using gage.

7-28☺.

MTB > Stat > ANOVA > Balanced ANOVA
In Results, select "Display expected mean squares and variance components"

ANOVA: Ex7-28Reading versus Ex7-28Part, Ex7-28Op

```
Factor      Type    Levels
Ex7-28Part  random      20
Ex7-28Op    random       3

Factor      Values
Ex7-28Part   1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11, 12, 13, 14, 15, 16, 17,
18, 19, 20
Ex7-28Op    1, 2, 3

Analysis of Variance for Ex7-28Reading
Source                DF        SS      MS      F      P
Ex7-28Part            19  1185.425  62.391  87.65  0.000
Ex7-28Op               2     2.617   1.308   1.84  0.173
Ex7-28Part*Ex7-28Op   38    27.050   0.712   0.72  0.861
Error                 60    59.500   0.992
Total                119  1274.592

S = 0.995825   R-Sq = 95.33%   R-Sq(adj) = 90.74%

                                         Expected Mean Square
                         Variance  Error  for Each Term (using
   Source               component   term  unrestricted model)
1  Ex7-28Part             10.2798      3  (4) + 2 (3) + 6 (1)
2  Ex7-28Op                0.0149      3  (4) + 2 (3) + 40 (2)
3  Ex7-28Part*Ex7-28Op    -0.1399      4  (4) + 2 (3)
4  Error                   0.9917         (4)
```

$$\hat{\sigma}^2_{\text{Repeatability}} = MS_{\text{Error}} = 0.992$$

$$\hat{\sigma}^2_{\text{Part}\times\text{Operator}} = \frac{MS_{\text{P}\times\text{O}} - MS_{\text{E}}}{n} = \frac{0.712 - 0.992}{2} = -0.1400 \Rightarrow 0$$

$$\hat{\sigma}^2_{\text{Operator}} = \frac{MS_{\text{O}} - MS_{\text{P}\times\text{O}}}{pn} = \frac{1.308 - 0.712}{20(2)} = 0.0149$$

$$\hat{\sigma}^2_{\text{Part}} = \frac{MS_{\text{P}} - MS_{\text{P}\times\text{O}}}{on} = \frac{62.391 - 0.712}{3(2)} = 10.2798$$

The manual calculations match the MINITAB results. Note the Part × Operator variance component is negative. Since the Part × Operator term is not significant ($\alpha = 0.10$), we can fit a reduced model without that term. For the reduced model:

ANOVA: Ex7-28Reading versus Ex7-28Part, Ex7-28Op
…

```
                                  Expected
                                  Mean Square
                                  for Each
                                  Term (using
                Variance   Error  unrestricted
   Source      component    term  model)
1  Ex7-28Part    10.2513       3  (3) + 6 (1)
2  Ex7-28Op       0.0106       3  (3) + 40 (2)
3  Error          0.8832          (3)
```

(a)

$$\hat{\sigma}^2_{\text{Reproducibility}} = \hat{\sigma}^2_{\text{Operator}} = 0.0106$$

$$\hat{\sigma}^2_{\text{Repeatability}} = \hat{\sigma}^2_{\text{Error}} = 0.8832$$

(b)

$$\hat{\sigma}^2_{\text{Gauge}} = \hat{\sigma}^2_{\text{Reproducibility}} + \hat{\sigma}^2_{\text{Repeatability}} = 0.0106 + 0.8832 = 0.8938$$

$$\hat{\sigma}_{\text{Gauge}} = 0.9454$$

(c)

$$\widehat{P/T} = \frac{6 \times \hat{\sigma}_{\text{Gauge}}}{\text{USL-LSL}} = \frac{6 \times 0.9454}{60 - 6} = 0.1050$$

This gauge is borderline capable since the estimate of P/T ratio just exceeds 0.10.

Estimates of variance components, reproducibility, repeatability, and total gauge variability may also be found using:

MTB > Stat > Quality Tools > Gage Study > Gage R&R Study (Crossed)

Gage R&R Study - ANOVA Method

Two-Way ANOVA Table With Interaction

Source	DF	SS	MS	F	P
Ex7-28Part	19	1185.43	62.3908	87.6470	0.000
Ex7-28Op	2	2.62	1.3083	1.8380	0.173
Ex7-28Part * Ex7-28Op	38	27.05	0.7118	0.7178	0.861
Repeatability	60	59.50	0.9917		
Total	119	1274.59			

Two-Way ANOVA Table Without Interaction

Source	DF	SS	MS	F	P
Ex7-28Part	19	1185.43	62.3908	70.6447	0.000
Ex7-28Op	2	2.62	1.3083	1.4814	0.232
Repeatability	98	86.55	0.8832		
Total	119	1274.59			

Gage R&R

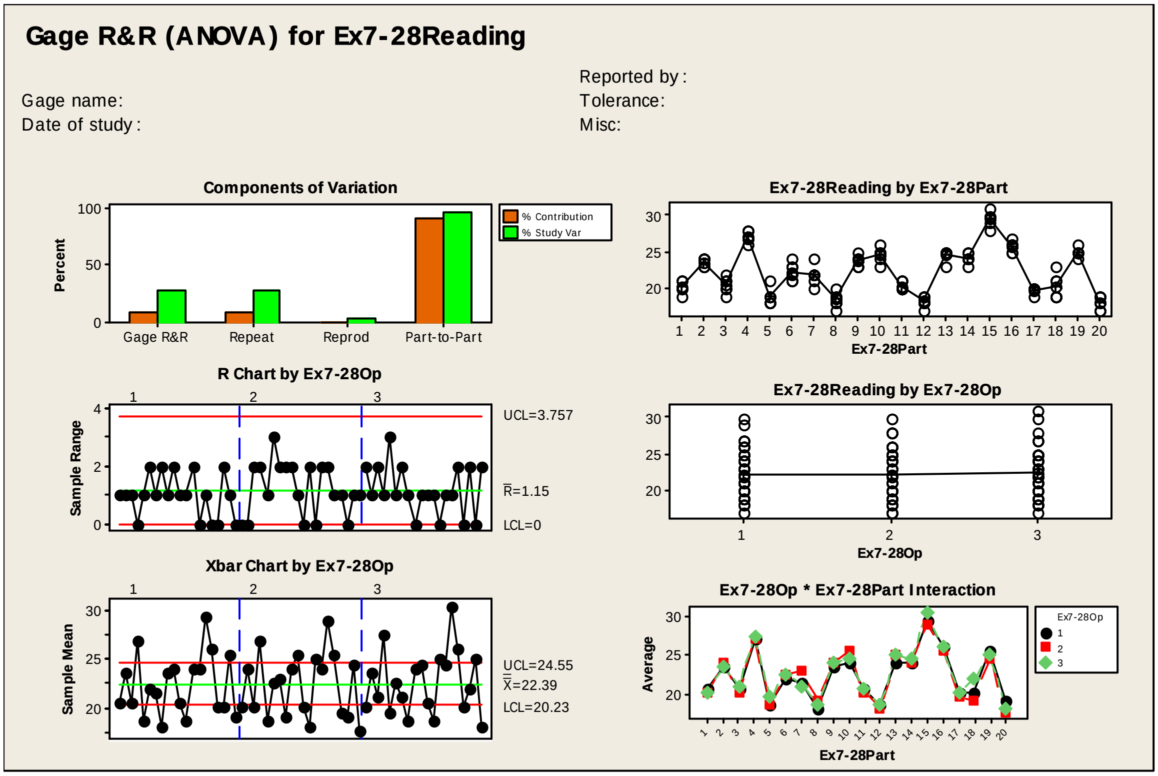
Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	0.8938	8.02
Repeatability	0.8832	7.92
Reproducibility	0.0106	0.10
Ex7-28Op	0.0106	0.10
Part-To-Part	10.2513	91.98
Total Variation	11.1451	100.00

Source	StdDev (SD)	Study Var (6 * SD)	%Study Var (%SV)
Total Gage R&R	0.94541	5.6724	28.32
Repeatability	0.93977	5.6386	28.15
Reproducibility	0.10310	0.6186	3.09
Ex7-28Op	0.10310	0.6186	3.09
Part-To-Part	3.20176	19.2106	95.91
Total Variation	3.33842	20.0305	100.00

Number of Distinct Categories = 4

7-28 continued

Visual representations of variability and stability are also provided:



7-29☺.

$\hat{\sigma}^2_{\text{Part}} = 10.2513; \hat{\sigma}^2_{\text{Total}} = 11.1451$

$\hat{\rho}_P = \dfrac{\hat{\sigma}^2_{\text{Part}}}{\hat{\sigma}^2_{\text{Total}}} = \dfrac{10.2513}{11.1451} = 0.9198$

$\widehat{SNR} = \sqrt{\dfrac{2\hat{\rho}_P}{1-\hat{\rho}_P}} = \sqrt{\dfrac{2(0.9198)}{1-0.9198}} = 4.79$

$\widehat{DR} = \dfrac{1+\hat{\rho}_P}{1-\hat{\rho}_P} = \dfrac{1+0.9198}{1-0.9198} = 23.94$

SNR = 4.79 indicates that fewer than five distinct levels can be reliably obtained from the measurements. This is near the AIAG-recommended value of five levels or more, but larger than a value of two (or less) that indicates inadequate gauge capability. (Also note that the MINITAB Gage R&R output indicates "`Number of Distinct Categories = 4`"; this is also the number of distinct categories of parts that the gauge is able to distinguish)

DR = 23.94, exceeding the minimum recommendation of four. By this measure, the gauge is capable.

7-30 (7-22).

$\mu = \mu_1 + \mu_2 + \mu_3 = 100 + 75 + 75 = 250$

$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^3} = \sqrt{4^2 + 4^2 + 2^2} = 6$

$$
\begin{aligned}
\Pr\{x > 262\} &= 1 - \Pr\{x \le 262\} \\
&= 1 - \Pr\left\{z \le \frac{262-\mu}{\sigma}\right\} \\
&= 1 - \Pr\left\{z \le \frac{262-250}{6}\right\} \\
&= 1 - \Phi(2.000) \\
&= 1 - 0.9772 \\
&= 0.0228
\end{aligned}
$$

7-31 (7-23).

$x_1 \sim N(20, 0.3^2);\ x_2 \sim N(19.6, 0.4^2)$

Nonconformities will occur if $y = x_1 - x_2 < 0.1$ or $y = x_1 - x_2 > 0.9$

$$\mu_y = \mu_1 - \mu_2 = 20 - 19.6 = 0.4$$

$$\sigma_y^2 = \sigma_1^2 + \sigma_2^2 = 0.3^2 + 0.4^2 = 0.25$$

$$\sigma_y = 0.50$$

$$\begin{aligned}
\Pr\{\text{Nonconformities}\} &= \Pr\{y < \text{LSL}\} + \Pr\{y > \text{USL}\} \\
&= \Pr\{y < 0.1\} + \Pr\{y > 0.9\} \\
&= \Pr\{y < 0.1\} + 1 - \Pr\{y \le 0.9\} \\
&= \Phi\left(\frac{0.1 - 0.4}{\sqrt{0.25}}\right) + 1 - \Phi\left(\frac{0.9 - 0.4}{\sqrt{0.25}}\right) \\
&= \Phi(-0.6) + 1 - \Phi(1.00) \\
&= 0.2743 + 1 - 0.8413 \\
&= 0.4330
\end{aligned}$$

7-32 (7-24).

$$\begin{aligned}
\text{Volume} &= L \times H \times W \\
&\cong \mu_L \mu_H \mu_W + (L - \mu_L)\mu_H \mu_W + (H - \mu_H)\mu_L \mu_W + (W - \mu_W)\mu_L \mu_H
\end{aligned}$$

$$\hat{\mu}_{\text{Volume}} \cong \mu_L \mu_H \mu_W = 6.0(3.0)(4.0) = 72.0$$

$$\begin{aligned}
\sigma^2_{\text{Volume}} &\cong \mu_L^2 \sigma_H^2 \sigma_W^2 + \mu_H^2 \sigma_L^2 \sigma_W^2 + \mu_W^2 \sigma_L^2 \sigma_H^2 \\
&= 6.0^2(0.01)(0.01) + 3.0^2(0.01)(0.01) + 4.0^2(0.01)(0.01) \\
&= 0.0061
\end{aligned}$$

7-33 (7-25).

$$\begin{aligned}
\text{Weight} &= d \times W \times L \times T \\
&\cong d\left[\mu_W \mu_L \mu_T + (W - \mu_W)\mu_L \mu_T + (L - \mu_L)\mu_W \mu_T + (T - \mu_T)\mu_W \mu_L\right]
\end{aligned}$$

$$\hat{\mu}_{\text{Weight}} \cong d[\mu_W \mu_L \mu_T] = 0.08(10)(20)(3) = 48$$

$$\begin{aligned}
\hat{\sigma}^2_{\text{Weight}} &\cong d^2\left[\hat{\mu}_W^2 \hat{\sigma}_L^2 \hat{\sigma}_T^2 + \hat{\mu}_L^2 \hat{\sigma}_W^2 \hat{\sigma}_T^2 + \hat{\mu}_T^2 \hat{\sigma}_W^2 \hat{\sigma}_L^2\right] \\
&= 0.08^2\left[10^2(0.3^2)(0.1^2) + 20^2(0.2^2)(0.1^2) + 3^2(0.2^2)(0.3^2)\right] = 0.00181
\end{aligned}$$

$$\hat{\sigma}_{\text{Weight}} \cong 0.04252$$

7-34 (7-26).

$$s = (3 + 0.05x)^2 \text{ and } f(x) = \frac{1}{26}(5x - 2);\ 2 \le x \le 4$$

$$E(x) = \mu_x = \int x f(x)dx = \int_2^4 x\left[\frac{1}{26}(5x-2)\right]dx = \frac{1}{26}\left(\frac{5}{3}x^3\Big|_2^4 - x^2\Big|_2^4\right) = 3.1282$$

$$E\left(x^2\right) = \int x^2 f(x)dx = \int_2^4 x^2\left[\frac{1}{26}(5x-2)\right]dx = \frac{1}{26}\left(\frac{5}{4}x^4\Big|_2^4 - \frac{2}{3}x^3\Big|_2^4\right) = 10.1026$$

$$\sigma_x^2 = E(x^2) - \left[E(x)\right]^2 = 10.1026 - (3.1282)^2 = 0.3170$$

$$\mu_s \cong g(x) = \left[3 + 0.05(\mu_x)\right]^2 = \left[3 + 0.05(3.1282)\right]^2 = 9.9629$$

$$\sigma_s^2 \cong \left[\frac{\partial g(x)}{\partial x}\right]^2\Bigg|_{\mu_x} \sigma_x^2$$

$$= \left[\frac{\partial(3+0.05x)^2}{\partial x}\right]^2\Bigg|_{\mu_x} \sigma_x^2$$

$$= 2(3 + 0.05\mu_x)(0.05)\sigma_x^2$$

$$= 2\left[3 + 0.05(3.1282)\right](0.05)(0.3170)$$

$$= 0.1001$$

7-35 (7-27).

$$I = E/(R_1 + R_2)$$

$$\mu_I \cong \mu_E/(\mu_{R_1} + \mu_{R_2})$$

$$\sigma_I^2 \cong \frac{\sigma_E^2}{(\mu_{R_1} + \mu_{R_2})} + \frac{\mu_E}{(\mu_{R_1} + \mu_{R_2})^2}\left(\sigma_{R_1}^2 + \sigma_{R_2}^2\right)$$

7-36 (7-28).

$$x_1 \sim N(\mu_1, 0.400^2);\ x_2 \sim N(\mu_2, 0.300^2)$$

$$\mu_y = \mu_1 - \mu_2$$

$$\sigma_y = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{0.400^2 + 0.300^2} = 0.5$$

$$\Pr\{y < 0.09\} = 0.006$$

$$\Pr\left\{z < \frac{0.09 - \mu_y}{\sigma_y}\right\} = \Phi^{-1}(0.006)$$

$$\frac{0.09 - \mu_y}{0.5} = -2.512$$

$$\mu_y = -[0.5(-2.512) - 0.09] = 1.346$$

7-37 (7-29).

$\text{ID} \sim N(2.010, 0.002^2)$ and $\text{OD} \sim N(2.004, 0.001^2)$

Interference occurs if $y = \text{ID} - \text{OD} < 0$

$$\mu_y = \mu_{\text{ID}} - \mu_{\text{OD}} = 2.010 - 2.004 = 0.006$$

$$\sigma_y^2 = \sigma_{\text{ID}}^2 + \sigma_{\text{OD}}^2 = 0.002^2 + 0.001^2 = 0.000005$$

$$\sigma_y = 0.002236$$

$$\begin{aligned}
\Pr\{\text{positive clearance}\} &= 1 - \Pr\{\text{interference}\} \\
&= 1 - \Pr\{y < 0\} \\
&= 1 - \Phi\left(\frac{0 - 0.006}{\sqrt{0.000005}}\right) \\
&= 1 - \Phi(-2.683) \\
&= 1 - 0.0036 \\
&= 0.9964
\end{aligned}$$

7-38 (7-30).

$$\alpha = 0.01$$

$$\gamma = 0.80$$

$$\chi^2_{1-\gamma,4} = \chi^2_{0.20,4} = 5.989$$

$$n \cong \frac{1}{2} + \left(\frac{2-\alpha}{\alpha}\right)\frac{\chi^2_{1-\gamma,4}}{4} = \frac{1}{2} + \left(\frac{2-0.01}{0.01}\right)\frac{5.989}{4} = 299$$

7-39 (7-31).
$n = 10$; $x \sim N(300, 10^2)$; $\alpha = 0.10$; $\gamma = 0.95$; one-sided
From Appendix VIII: $K = 2.355$
$\text{UTL} = \bar{x} + KS = 300 + 2.355(10) = 323.55$

7-40 (7-32).
$n = 25$; $x \sim N(85, 1^2)$; $\alpha = 0.10$; $\gamma = 0.95$; one-sided
From Appendix VIII: $K = 1.838$
$\bar{x} - KS = 85 - 1.838(1) = 83.162$

7-41 (7-33).
$n = 20$; $x \sim N(350, 10^2)$; $\alpha = 0.05$; $\gamma = 0.90$; one-sided
From Appendix VIII: $K = 2.208$
$\text{UTL} = \bar{x} + KS = 350 + 2.208(10) = 372.08$

7-42 (7-34).
$\alpha = 0.05$

$\gamma = 0.90$

$\chi^2_{1-\gamma,4} = \chi^2_{0.10,4} = 7.779$

$$n \cong \frac{1}{2} + \left(\frac{2-\alpha}{\alpha}\right)\frac{\chi^2_{1-\gamma,4}}{4} = \frac{1}{2} + \left(\frac{2-0.05}{0.05}\right)\frac{7.779}{4} = 77$$

After the data are collected, a natural tolerance interval would be the smallest to largest observations.

7-43 (7-35).
$x \sim N\left(0.1264, 0.0003^2\right)$

(a)
$\alpha = 0.05$; $\gamma = 0.95$; and two-sided
From Appendix VII: $K = 2.445$
TI on $x: \bar{x} \pm KS = 0.1264 \pm 2.445(0.0003) = [0.1257, 0.1271]$

(b)
$\alpha = 0.05; t_{\alpha/2,n-1} = t_{0.025,39} = 2.023$
CI on $\bar{x}: \bar{x} \pm t_{\alpha/2,n-1} S/\sqrt{n} = 0.1264 \pm 2.023\left(0.0003/\sqrt{40}\right) = [0.1263, 0.1265]$

Part (a) is a tolerance interval on individual thickness observations; part (b) is a confidence interval on mean thickness. In part (a), the interval relates to individual observations (random variables), while in part (b) the interval refers to a parameter of a distribution (an unknown constant).

7-44 (7-36).
$\alpha = 0.05; \gamma = 0.95$

$$n = \frac{\log(1-\gamma)}{\log(1-\alpha)} = \frac{\log(1-0.95)}{\log(1-0.05)} = 59$$

The largest observation would be the nonparametric upper tolerance limit.

Chapter 8 Exercise Solutions

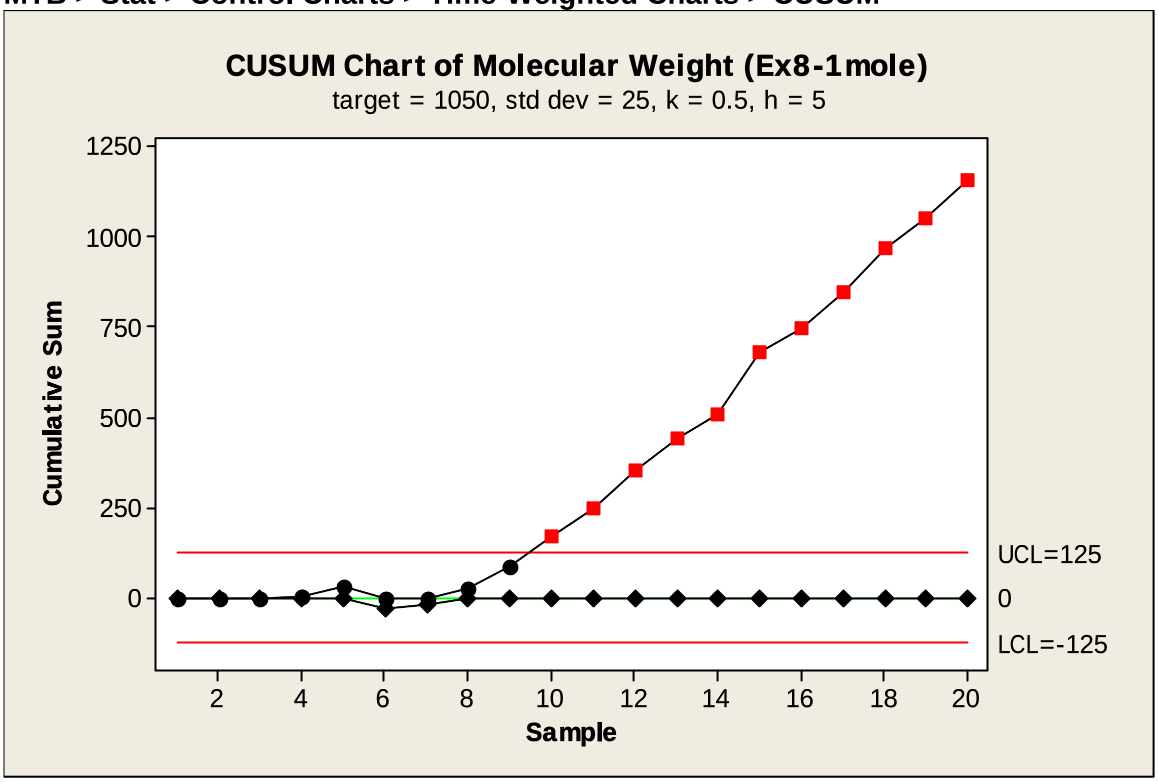
Several exercises in this chapter differ from those in the 4th edition. An "*" following the exercise number indicates that the description has changed. New exercises are denoted with an "☺". A number in parentheses gives the exercise number from the 4th edition.

8-1.

$\mu_0 = 1050$; $\sigma = 25$; $\delta = 1\sigma$; $K = (\delta/2)\sigma = (1/2)25 = 12.5$; $H = 5\sigma = 5(25) = 125$

(a)

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



The process signals out of control at observation 10. The point at which the assignable cause occurred can be determined by counting the number of increasing plot points. The assignable cause occurred after observation 10 – 3 = 7.

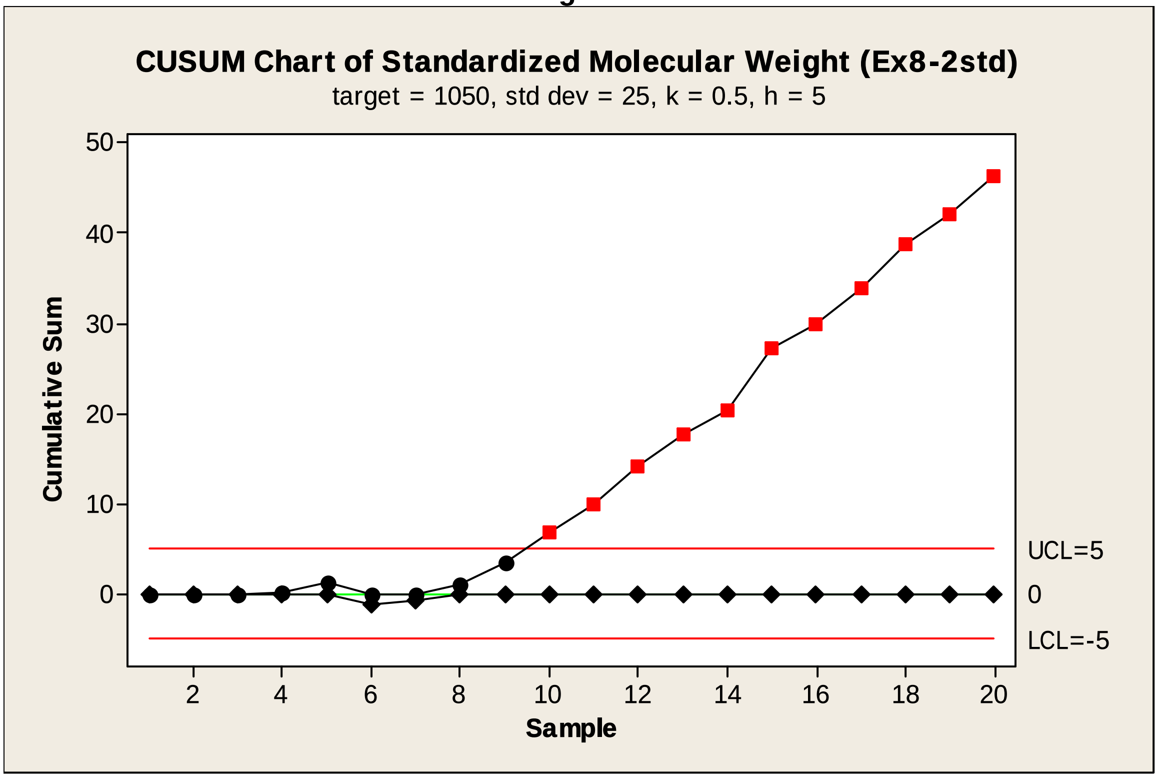
(b)

$\hat{\sigma} = \overline{\text{MR}2}/d_2 = 38.8421/1.128 = 34.4345$

No. The estimate used for σ is much smaller than that from the data.

8-2.

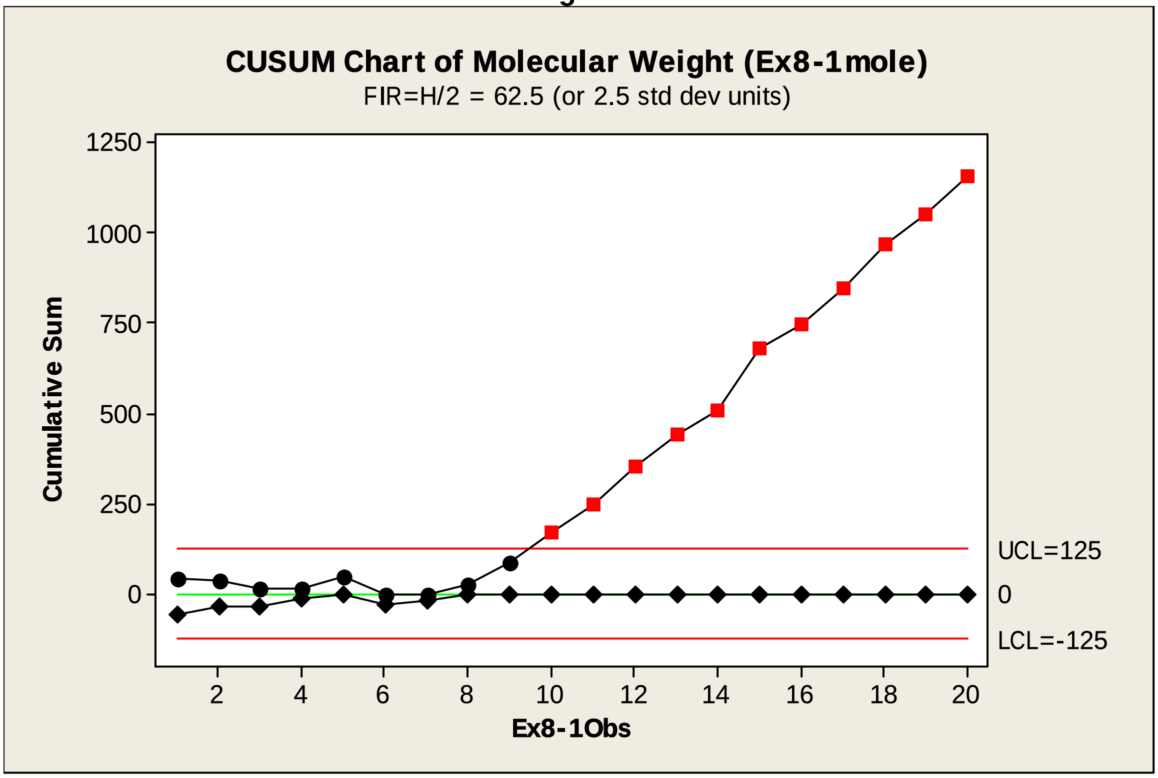
MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



The process signals out of control at observation 10. The assignable cause occurred after observation 10 – 3 = 7.

8-3.
(a)
$\mu_0 = 1050$, $\sigma = 25$, $k = 0.5$, $K = 12.5$, $h = 5$, $H/2 = 125/2 = 62.5$
FIR = $H/2 = 62.5$, in std dev units = $62.5/25 = 2.5$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



For example,

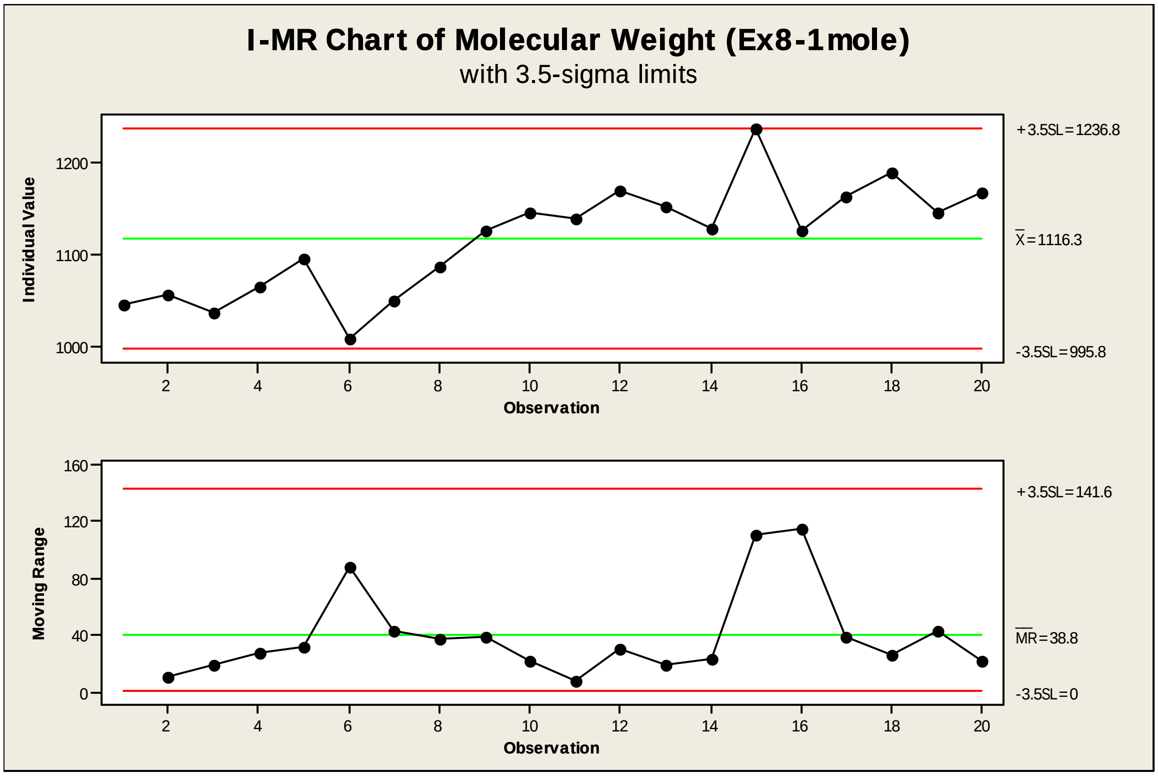
$$C_1^+ = \max\left[0, x_i - \left(\mu_0 - K\right) + C_0^+\right] = \max\left[0, 1045 - (1050 + 12.5) + 62.5\right] = 45$$

Using the tabular CUSUM, the process signals out of control at observation 10, the same as the CUSUM without a FIR feature.

8-3 continued

(b)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



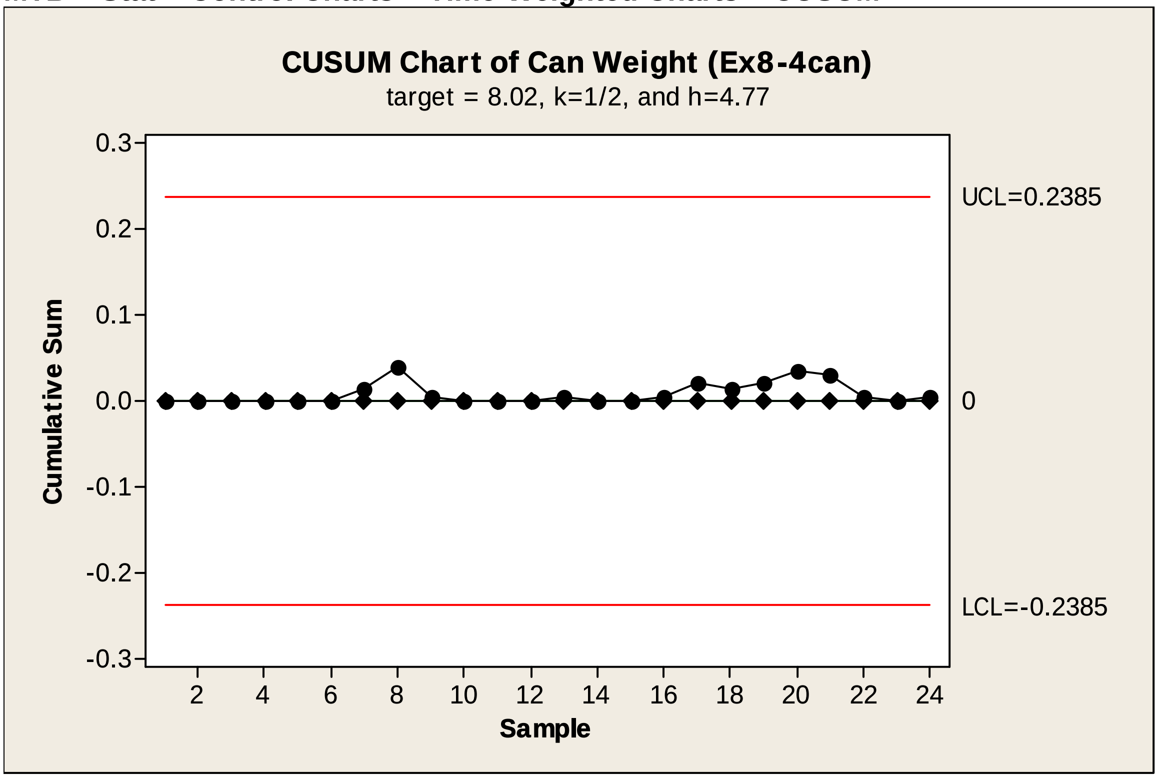
Using 3.5σ limits on the Individuals chart, there are no out-of-control signals. However there does appear to be a trend up from observations 6 through 12—this is the situation detected by the cumulative sum.

8-4.

$\mu_0 = 8.02$, $\sigma = 0.05$, $k = 0.5$, $h = 4.77$, $H = h\sigma = 4.77\ (0.05) = 0.2385$

(a)

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



There are no out-of-control signals.

(b)

$\hat{\sigma} = \overline{\text{MR2}}/1.128 = 0.0186957/1.128 = 0.0166$, so $\sigma = 0.05$ is probably not reasonable.

<u>In Exercise 8-4:</u>

$\mu_0 = 8.02;\ \sigma = 0.05;\ k = 1/2;\ h = 4.77;\ b = h + 1.166 = 4.77 + 1.166 = 5.936$

$\delta^* = 0;\quad \Delta^+ = \delta^* - k = 0 - 0.5 = -0.5;\quad \Delta^- = -\delta^* - k = -0 - 0.5 = -0.5$

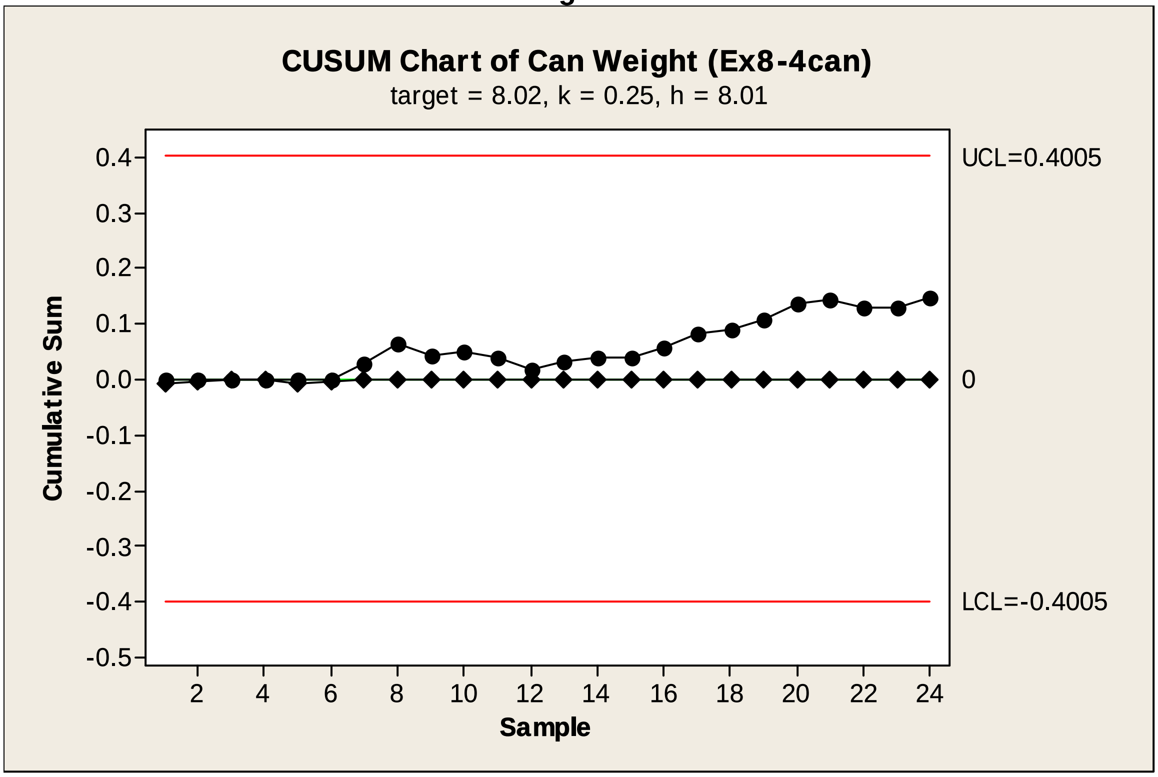
$$\text{ARL}_0^+ = \text{ARL}_0^- \cong \frac{\exp[-2(-0.5)(5.936)] + 2(-0.5)(5.936) - 1}{2(-0.5)^2} = 742.964$$

$$\frac{1}{\text{ARL}_0} = \frac{1}{\text{ARL}_0^+} + \frac{1}{\text{ARL}_0^-} = \frac{2}{742.964} = 0.0027$$

$\text{ARL}_0 = 1/0.0027 = 371.48$

8-5.
$\mu_0 = 8.02,\ \sigma = 0.05,\ k = 0.25,\ h = 8.01,\ H = h\sigma = 8.01\ (0.05) = 0.4005$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



There are no out-of-control signals.

<u>In Exercise 8-5:</u>

$$\mu_0 = 8.02;\ \sigma = 0.05;\ k = 0.25;\ h = 8.01;\ b = h + 1.166 = 8.01 + 1.166 = 9.176$$

$$\delta^* = 0; \quad \Delta^+ = \delta^* - k = 0 - 0.25 = -0.25; \quad \Delta^- = -\delta^* - k = -0 - 0.25 = -0.25$$

$$\text{ARL}_0^+ = \text{ARL}_0^- \cong \frac{\exp[-2(-0.25)(9.176)] + 2(-0.25)(9.176) - 1}{2(-0.25)^2} = 741.6771$$

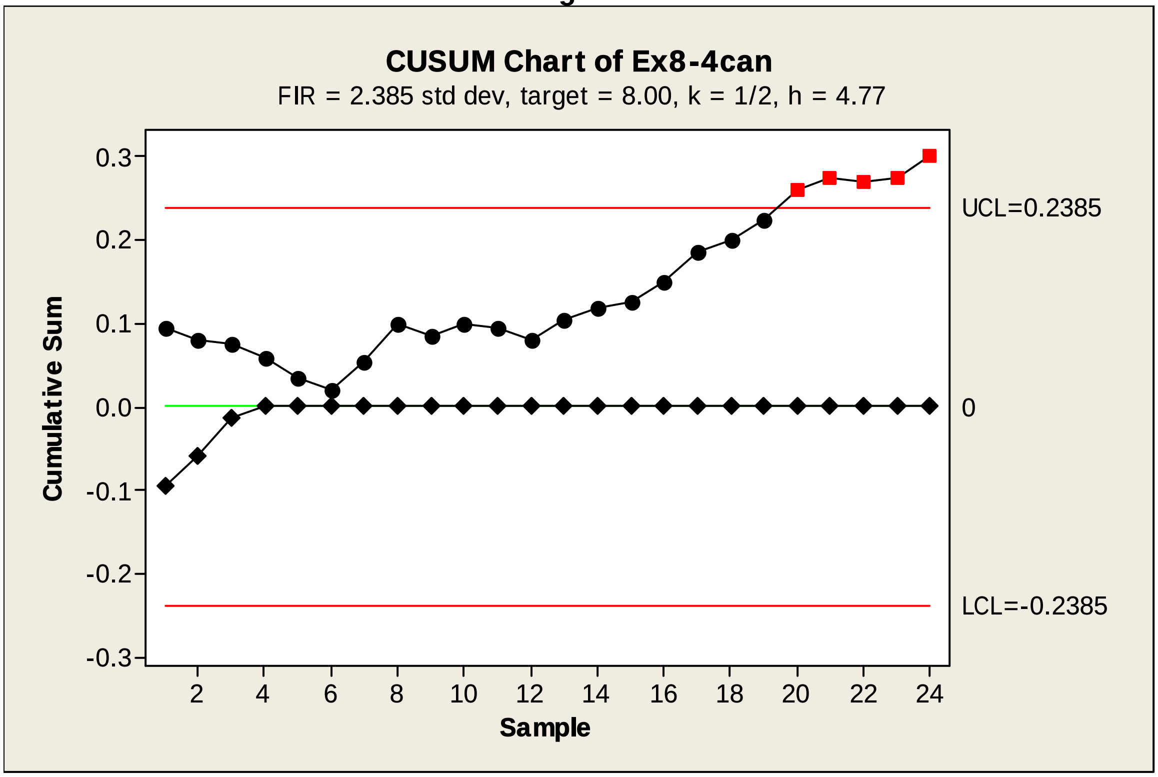
$$\frac{1}{\text{ARL}_0} = \frac{1}{\text{ARL}_0^+} + \frac{1}{\text{ARL}_0^-} = \frac{2}{741.6771} = 0.0027$$

$$\text{ARL}_0 = 1/0.0027 = 370.84$$

The theoretical performance of these two CUSUM schemes is the same for Exercises 8-4 and 8-5.

8-6.
$\mu_0 = 8.00$, $\sigma = 0.05$, $k = 0.5$, $h = 4.77$, $H = h\ \sigma = 4.77\ (0.05) = 0.2385$
FIR = $H/2$, FIR in # of standard deviations = $h/2 = 4.77/2 = 2.385$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



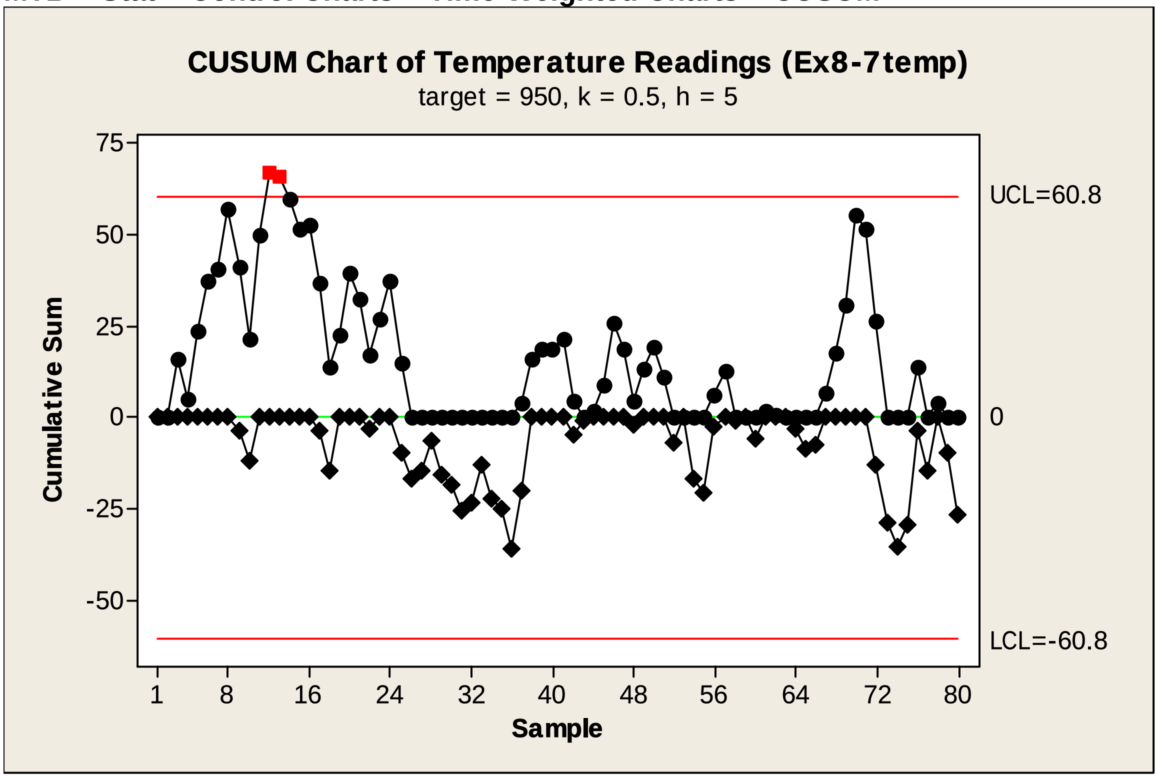
The process signals out of control at observation 20. Process was out of control at process start-up.

8-7.

(a) $\hat{\sigma} = \overline{\text{MR}2}/d_2 = 13.7215/1.128 = 12.16$

(b) $\mu_0 = 950; \hat{\sigma} = 12.16; k = 1/2; h = 5$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



Test Results for CUSUM Chart of Ex8-7temp

```
TEST. One point beyond control limits.
Test Failed at points:  12, 13
```

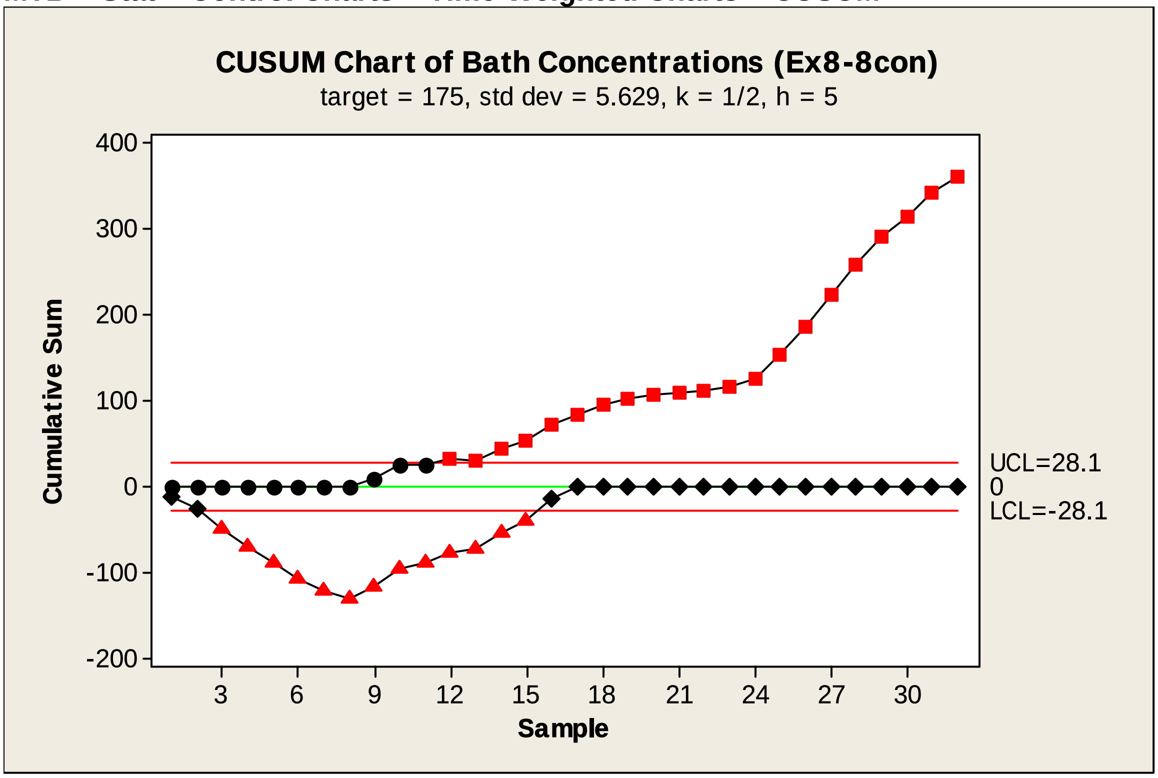
The process signals out of control at observation 12. The assignable cause occurred after observation 12 – 10 = 2.

8-8.

(a) $\hat{\sigma} = \overline{\text{MR2}}/d_2 = 6.35/1.128 = 5.629$ (from a Moving Range chart with CL = 6.35)

(b) $\mu_0 = 175; \hat{\sigma} = 5.629; k = 1/2; h = 5$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



Test Results for CUSUM Chart of Ex8-8con

```
TEST. One point beyond control limits.
Test Failed at points:  12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25,
                        26, 27, 28, 29, 30, 31, 32
```

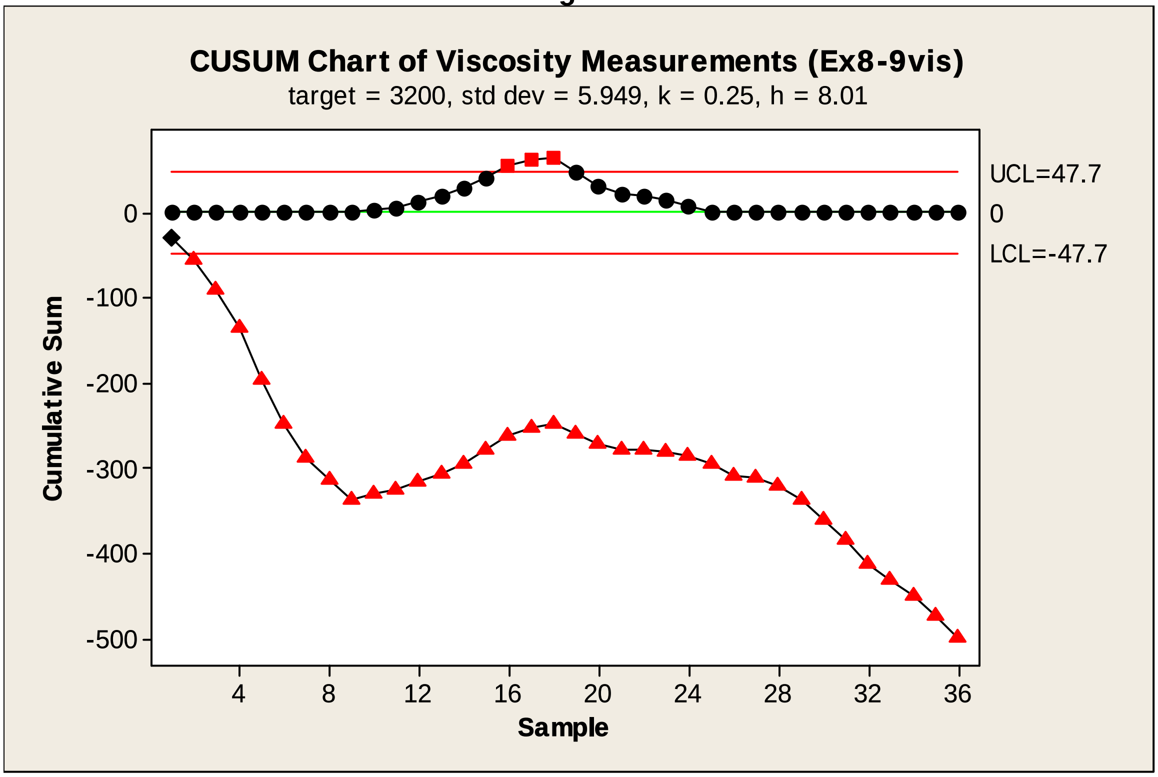
The process signals out of control on the lower side at sample 3 and on the upper side at sample 12. Assignable causes occurred after startup (sample 3 – 3 = 0) and after sample 8 (12 – 4).

8-9.

(a) $\hat{\sigma} = \overline{\text{MR}2}/d_2 = 6.71/1.128 = 5.949$ (from a Moving Range chart with CL = 6.71)

(b) $\mu_0 = 3200; \hat{\sigma} = 5.949; k = 0.25; h = 8.01$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM




```
Test Results for CUSUM Chart of Ex8-9vis
TEST. One point beyond control limits.
Test Failed at points:  16, 17, 18
```

The process signals out of control on the lower side at sample 2 and on the upper side at sample 16. Assignable causes occurred after startup (sample 2 – 2) and after sample 9 (16 – 7).

(c)

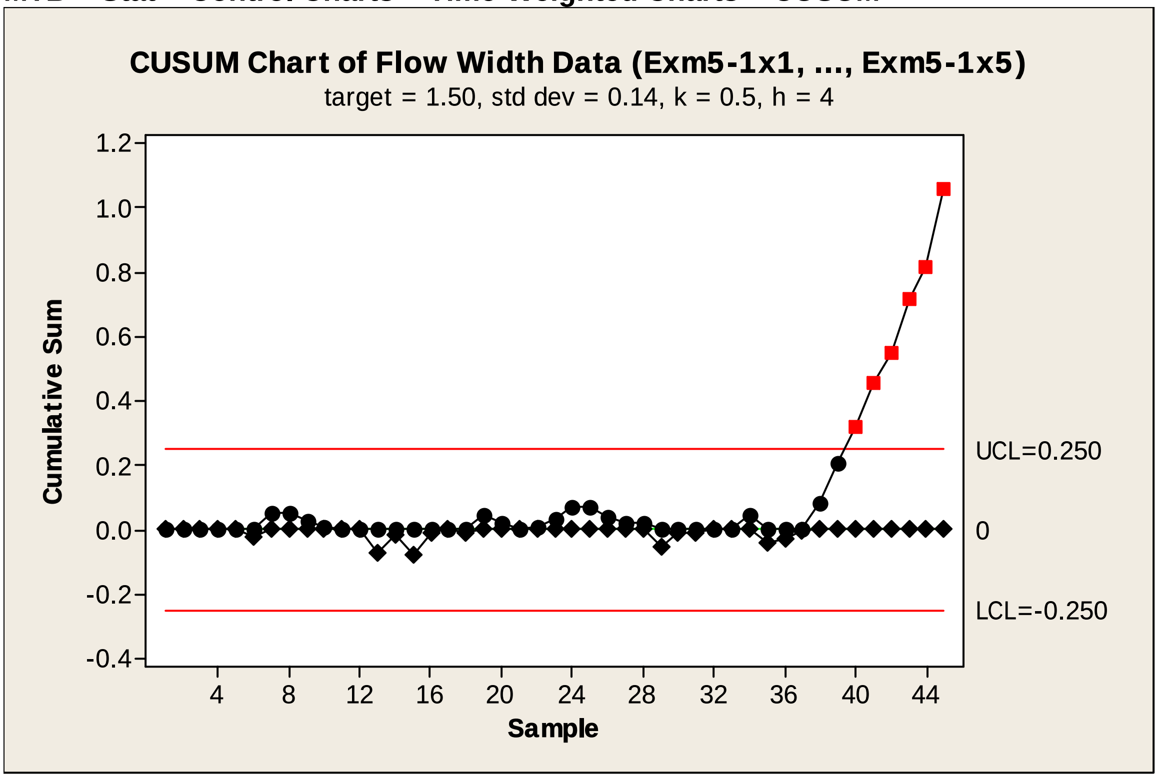
Selecting a smaller shift to detect, $k = 0.25$, should be balanced by a larger control limit, $h = 8.01$, to give longer in-control ARLs with shorter out-of-control ARLs.

8-10*.

$n = 5;\ \mu_0 = 1.50;\ \sigma = 0.14; \quad \sigma_{\bar{x}} = \sigma/\sqrt{n} = 0.14/\sqrt{5} = 0.0626$

$\delta = 1;\ k = \delta/2 = 0.5;\ h = 4; \quad K = k\sigma_{\bar{x}} = 0.0313;\ H = h\sigma_{\bar{x}} = 0.2504$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



Test Results for CUSUM Chart of Exm5-1x1, ..., Exm5-1x5

```
TEST. One point beyond control limits.
Test Failed at points:  40, 41, 42, 43, 44, 45
```

The CUSUM chart signals out of control at sample 40, and remains above the upper limit. The $\bar{x}$-R chart shown in Figure 5-4 signals out of control at sample 43. This CUSUM detects the shift in process mean earlier, at sample 40 versus sample 43.

8-11.

$$V_i = \left(\sqrt{|y_i|} - 0.822 \right) \Big/ 0.349$$

Excel file: workbook Chap08.xls : worksheet Ex8-11

mu0 =	1050											
sigma =	25											
delta =	1 sigma											
k =	0.5											
h =	5											
				one-sided upper cusum					**one-sided lower cusum**			
Obs, i	**xi**	**yi**	**vi**	**Si+**	**N+**	**OOC?**	**When?**		**Si-**	**N-**	**OOC?**	**When?**
No FIR				0					0			
1	1045	-0.2	-1.07	0	0				0.57	1		
2	1055	0.2	-1.07	0	0				1.15	2		
3	1037	-0.52	-0.29	0	0				0.94	3		
4	1064	0.56	-0.21	0	0				0.65	4		
5	1095	1.8	1.49	0.989	1				0	0		
6	1008	-1.68	1.36	1.848	2				0	0		
7	1050	0	-2.36	0	0				1.86	1		
8	1087	1.48	1.13	0.631	1				0.22	2		
9	1125	3	2.61	2.738	2				0	0		
10	1146	3.84	3.26	5.498	3	OOC	7		0	0		
11	1139	3.56	3.05	8.049	4	OOC	7		0	0		
12	1169	4.76	3.90	11.44	5	OOC	7		0	0		
13	1151	4.04	3.40	14.35	6	OOC	7		0	0		
14	1128	3.12	2.71	16.55	7	OOC	7		0	0		
15	1238	7.52	5.50	21.56	8	OOC	7		0	0		
16	1125	3	2.61	23.66	9	OOC	7		0	0		
17	1163	4.52	3.74	26.9	10	OOC	7		0	0		
18	1188	5.52	4.38	30.78	11	OOC	7		0	0		
19	1146	3.84	3.26	33.54	12	OOC	7		0	0		
20	1167	4.68	3.84	36.88	13	OOC	7		0	0		

The process is out of control after observation 10 – 3 = 7. Process variability is increasing.

8-12.

$$V_i = \left(\sqrt{|y_i|} - 0.822\right) \Big/ 0.349$$

Excel file : workbook Chap08.xls : worksheet Ex8-12

mu0 =	175											
sigma =	5.6294	(from Exercise 8-8)										
delta =	1 sigma											
k =	0.5											
h =	5											
				one-sided upper cusum					**one-sided lower cusum**			
i	**xi**	**yi**	**vi**	**Si+**	**N+**	**OOC?**	**When?**		**Si-**	**N-**	**OOC?**	**When?**
No FIR				0					0			
1	160	-2.6646	2.32	1.822	1				0	0		
2	158	-3.0199	2.62	**3.946**	**2**				0	0		
3	150	-4.4410	3.68	7.129	3	OOC	0		0	0		
4	151	-4.2633	3.56	10.19	4	OOC	0		0	0		
5	153	-3.9081	3.31	13	5	OOC	0		0	0		
6	154	-3.7304	3.18	15.68	6	OOC	0		0	0		
7	158	-3.0199	2.62	17.8	7	OOC	0		0	0		
8	162	-2.3093	2.00	19.3	8	OOC	0		0	0		
9	186	1.9540	1.65	20.45	9	OOC	0		0	0		
10	195	3.5528	3.05	23	10	OOC	0		0	0		
11	179	0.7106	0.06	22.56	11	OOC	0		0	0		
12	184	1.5987	1.27	23.32	12	OOC	0		0	1		
13	175	0.0000	-2.36	20.47	13	OOC	0		1.86	0		
14	192	3.0199	2.62	22.59	14	OOC	0		0	0		
15	186	1.9540	1.65	23.74	15	OOC	0		0	0		
16	197	3.9081	3.31	26.55	16	OOC	0		0	0		
17	190	2.6646	2.32	28.37	17	OOC	0		0	0		
18	189	2.4869	2.16	30.04	18	OOC	0		0	0		
19	185	1.7764	1.46	31	19	OOC	0		0	0		
20	182	1.2435	0.84	31.34	20	OOC	0		0	0		
…												

The process was last in control at period 2 – 2 = 0. Process variability has been increasing since start-up.

8-13.
Standardized, two-sided cusum with k = 0.2 and h = 8

In control ARL performance:

$$\delta^* = 0$$

$$\Delta^+ = \delta^* - k = 0 - 0.2 = -0.2$$

$$\Delta^- = -\delta^* - k = -0 - 0.2 = -0.2$$

$$b = h + 1.166 = 8 + 1.166 = 9.166$$

$$\text{ARL}_0^+ = \text{ARL}_0^- \cong \frac{\exp[-2(-0.2)(9.166)] + 2(-0.2)(9.166) - 1}{2(-0.2)^2} = 430.556$$

$$\frac{1}{\text{ARL}_0} = \frac{1}{\text{ARL}_0^+} + \frac{1}{\text{ARL}_0^-} = \frac{2}{430.556} = 0.005$$

$$\text{ARL}_0 = 1/0.005 = 215.23$$

Out of control ARL Performance:

$$\delta^* = 0.5$$

$$\Delta^+ = \delta^* - k = 0.5 - 0.2 = 0.3$$

$$\Delta^- = -\delta^* - k = -0.5 - 0.2 = -0.7$$

$$b = h + 1.166 = 8 + 1.166 = 9.166$$

$$\text{ARL}_1^+ = \frac{\exp[-2(0.3)(9.166)] + 2(0.3)(9.166) - 1}{2(0.3)^2} = 25.023$$

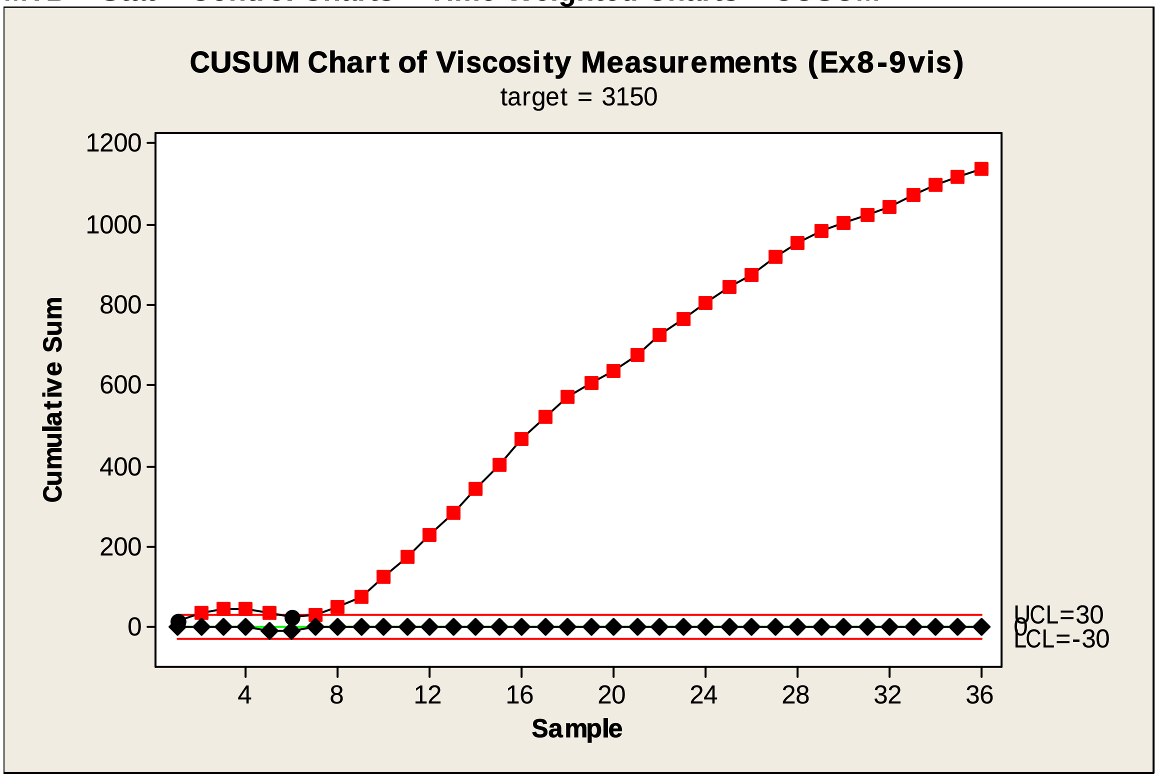
$$\text{ARL}_1^- = \frac{\exp[-2(-0.7)(9.166)] + 2(-0.7)(9.166) - 1}{2(-0.7)^2} = 381,767$$

$$\frac{1}{\text{ARL}_1} = \frac{1}{\text{ARL}_1^+} + \frac{1}{\text{ARL}_1^-} = \frac{1}{25.023} + \frac{1}{381,767} = 0.040$$

$$\text{ARL}_1 = 1/0.040 = 25.02$$

8-14.
$\mu_0 = 3150$, $s = 5.95238$, $k = 0.5$, $h = 5$
$K = ks = 0.5\ (5.95238) = 2.976$, $H = hs = 5(5.95238) = 29.762$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



MINITAB displays both the upper and lower sides of a CUSUM chart on the same graph; there is no option to display a single-sided chart. The upper CUSUM is used to detect upward shifts in the level of the process.

The process signals out of control on the upper side at sample 2. The assignable cause occurred at start-up (2 – 2).

8-15☺.

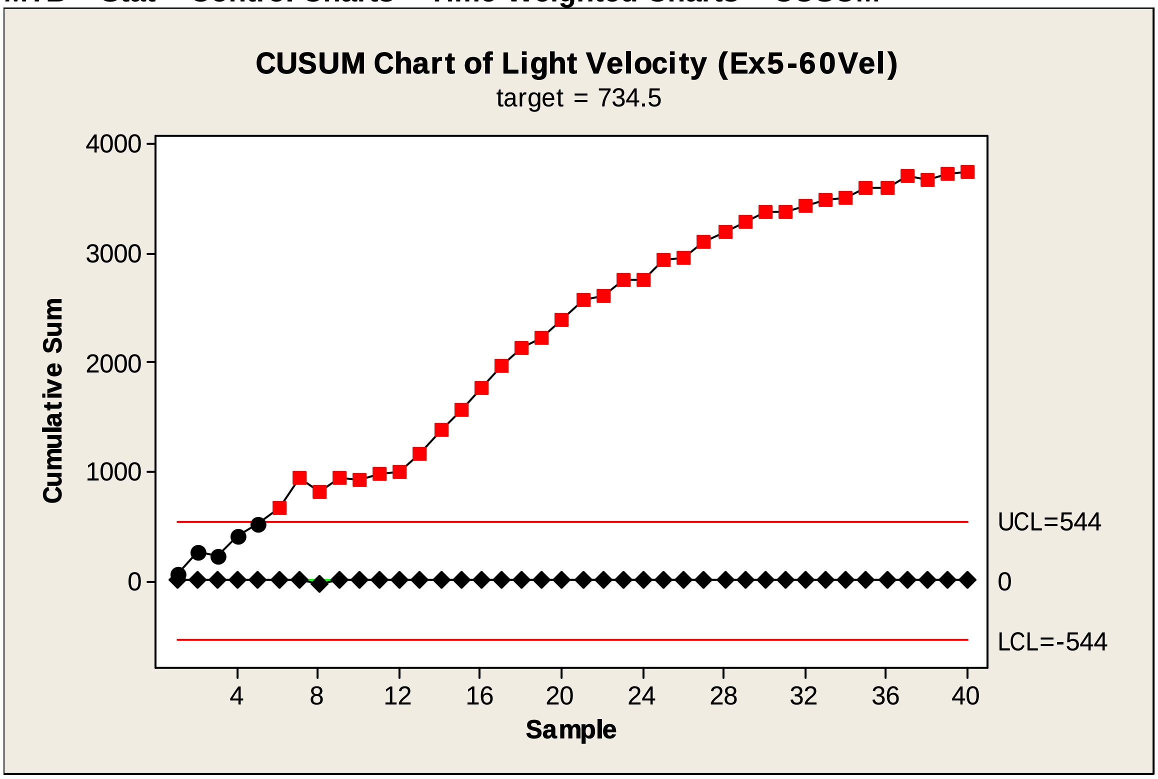
$\hat{\sigma} = \overline{\text{MR2}}/d_2 = 122.6/1.128 = 108.7$ (from a Moving Range chart with CL = 122.6)

$\mu_0 = 734.5; k = 0.5; h = 5$

$K = k\hat{\sigma} = 0.5(108.7) = 54.35$

$H = h\hat{\sigma} = 5(108.7) = 543.5$

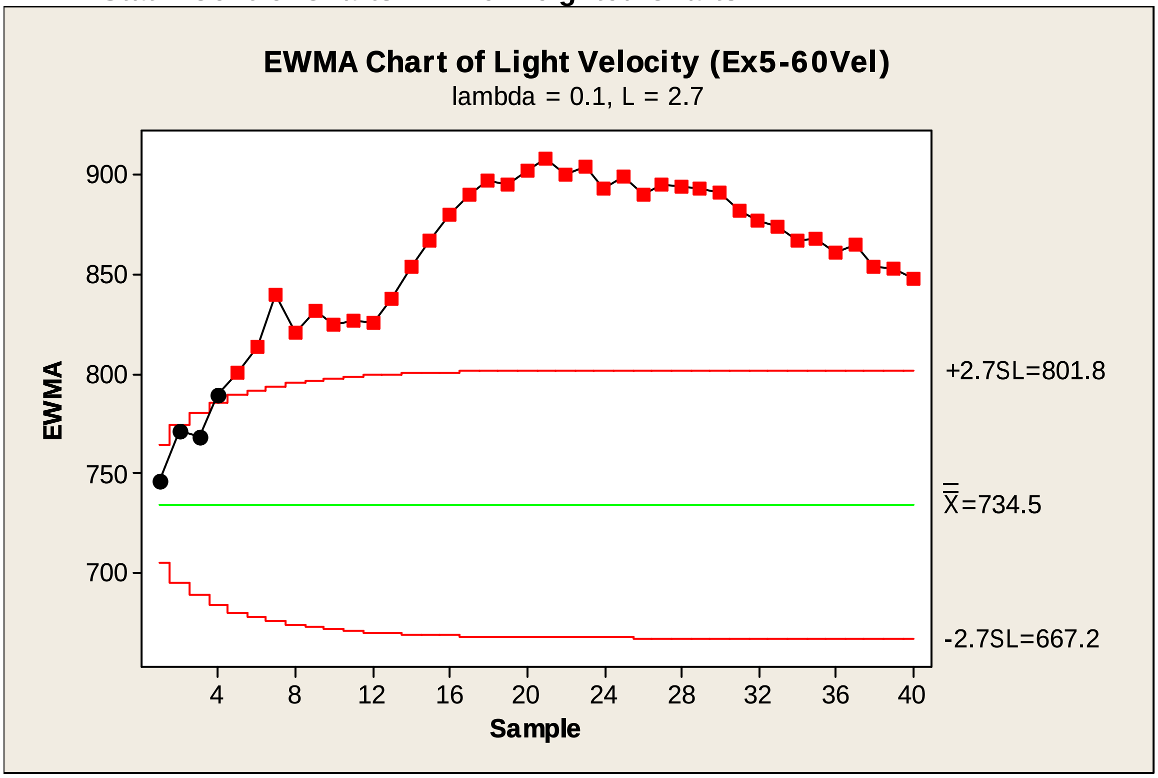
MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



The Individuals I-MR chart, with a centerline at $\bar{x} = 909$, displayed a distinct downward trend in measurements, starting at about sample 18. The CUSUM chart reflects a consistent run above the target value 734.5, from virtually the first sample. There is a distinct signal on both charts, of either a trend/drift or a shit in measurements. The out-of-control signals should lead us to investigate and determine the assignable cause.

8-16☺.
$\lambda = 0.1$; $L = 2.7$; CL = $\mu_0 = 734.5$; $\sigma = 108.7$

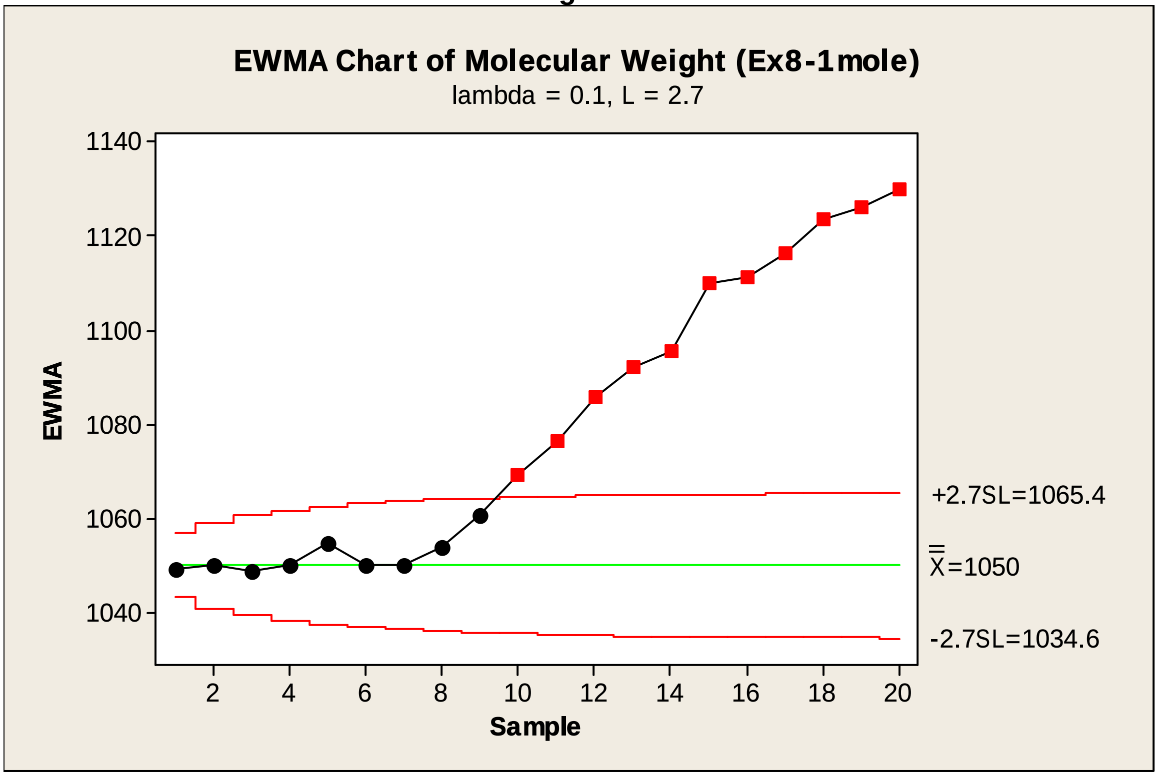
MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



The EWMA chart reflects the consistent trend above the target value, 734.5, and also indicates the slight downward trend starting at about sample 22.

8-17 (8-15).
$\lambda = 0.1$, $L = 2.7$, $\sigma = 25$, CL = μ_0 = 1050, UCL = 1065.49, LCL = 1034.51

MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



Process exceeds upper control limit at sample 10; the same as the CUSUM chart.

8-18 (8-16).
(a)
$\lambda = 0.1$, $L = 3$

$$\text{limits} = \mu_0 \pm L\sigma\sqrt{\lambda/(2-\lambda)} = 10 \pm 3(1)\sqrt{0.1/(2-0.1)} = [9.31, 10.69]$$

(b)
$\lambda = 0.2$, $L = 3$

$$\text{limits} = \mu_0 \pm L\sigma\sqrt{\lambda/(2-\lambda)} = 10 \pm 3(1)\sqrt{0.2/(2-0.2)} = [9, 11]$$

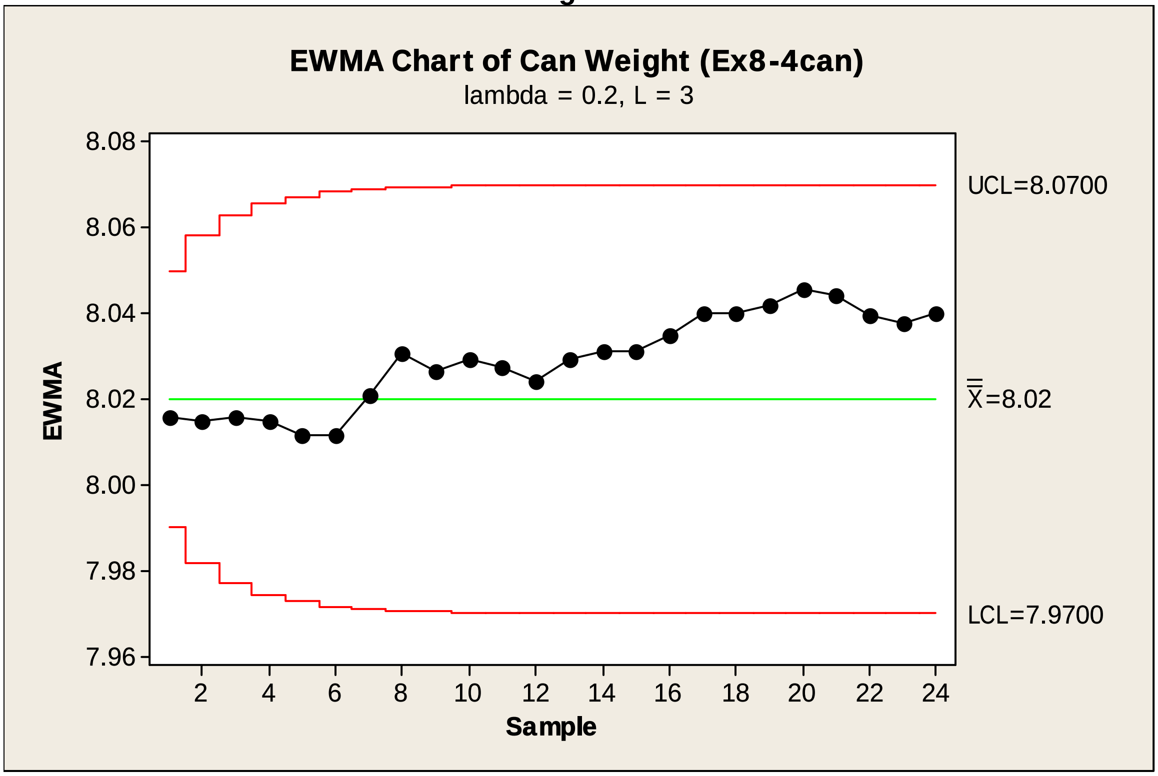
(c)
$\lambda = 0.4$, $L = 3$

$$\text{limits} = \mu_0 \pm L\sigma\sqrt{\lambda/(2-\lambda)} = 10 \pm 3(1)\sqrt{0.4/(2-0.4)} = [8.5, 11.5]$$

As λ increases, the width of the control limits also increases.

8-19 (8-17).
$\lambda = 0.2$, $L = 3$. Assume $\sigma = 0.05$. CL = $\mu_0 = 8.02$, UCL = 8.07, LCL = 7.97

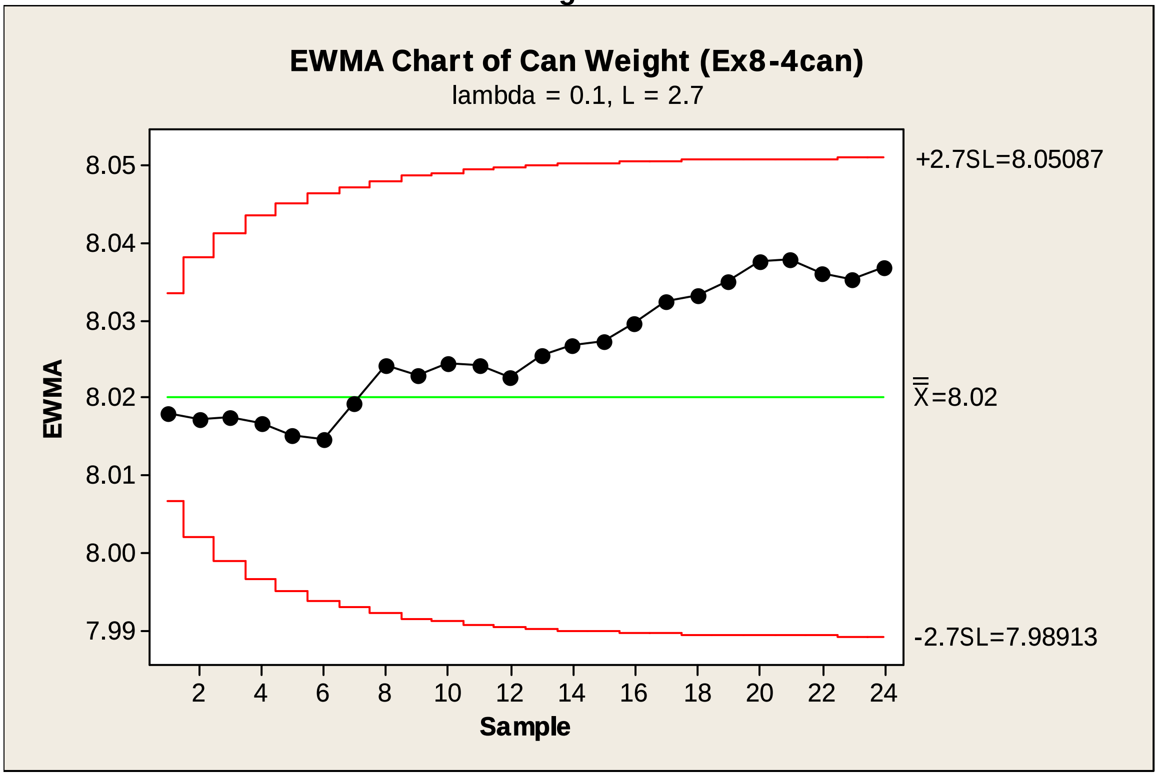
MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



The process is in control.

8-20 (8-18).
$\lambda = 0.1$, $L = 2.7$. Assume $\sigma = 0.05$. CL = $\mu_0 = 8.02$, UCL = 8.05, LCL = 7.99

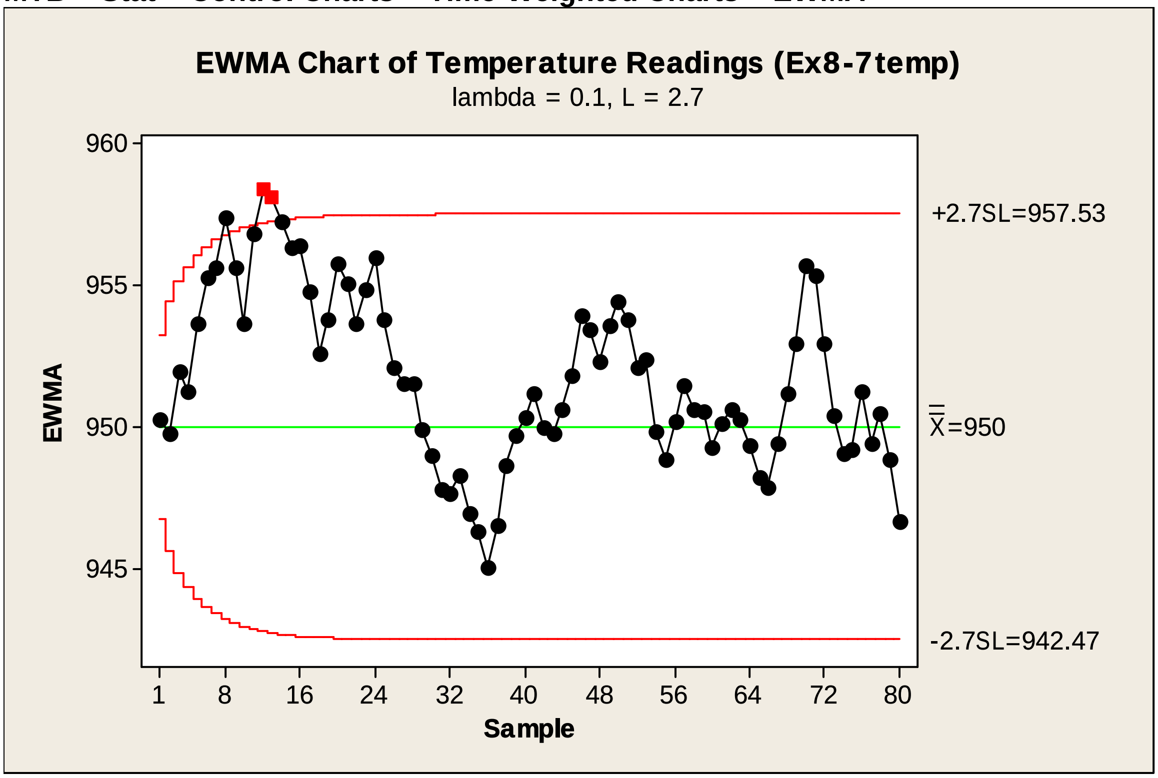
MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



The process is in control. There is not much difference between the control charts.

8-21 (8-19).
$\lambda = 0.1$, $L = 2.7$, $\hat{\sigma} = 12.16$, CL = μ_0 = 950, UCL = 957.53, LCL = 942.47.

MTB > Stat > Control Charts > Time-Weighted Charts > EWMA

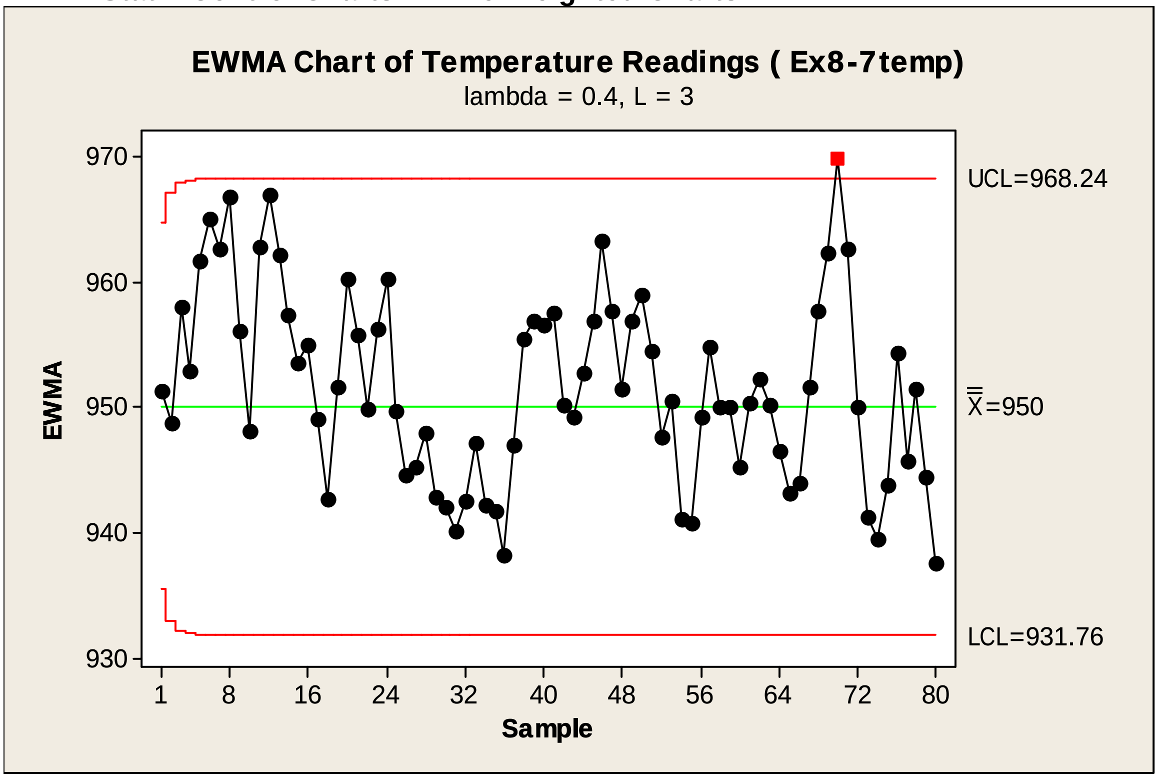


Test Results for EWMA Chart of Ex8-7temp
```
TEST. One point beyond control limits.
Test Failed at points:  12, 13
```

Process is out of control at samples 8 (beyond upper limit, but not flagged on chart), 12 and 13.

8-22 (8-20).
$\lambda = 0.4$, $L = 3$, $\hat{\sigma} = 12.16$, CL = μ_0 = 950, UCL = 968.24, LCL = 931.76.

MTB > Stat > Control Charts > Time-Weighted Charts > EWMA

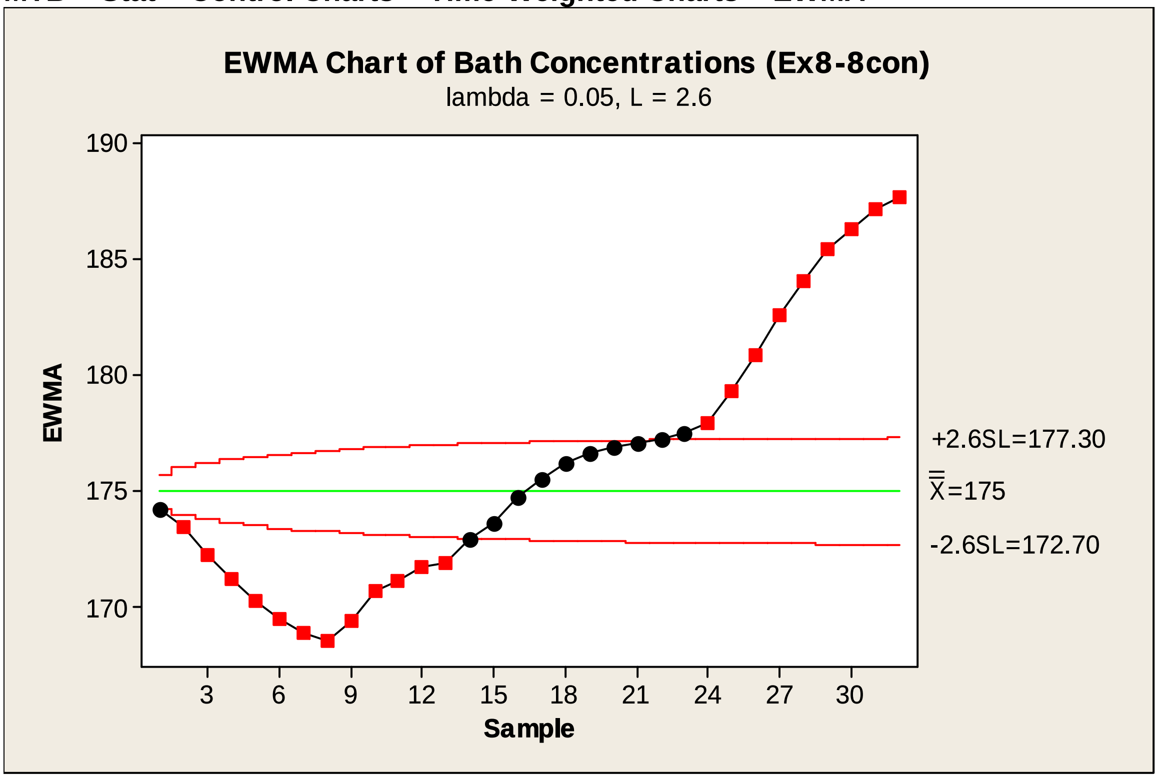



```
Test Results for EWMA Chart of Ex8-7temp
TEST. One point beyond control limits.
Test Failed at points:  70
```

With the larger λ, the process is out of control at observation 70, as compared to the chart in the Exercise 21 (with the smaller λ) which signaled out of control at earlier samples.

8-23 (8-21).
$\lambda = 0.05$, $L = 2.6$, $\hat{\sigma} = 5.634$, CL = μ_0 = 175, UCL = 177.30, LCL = 172.70.

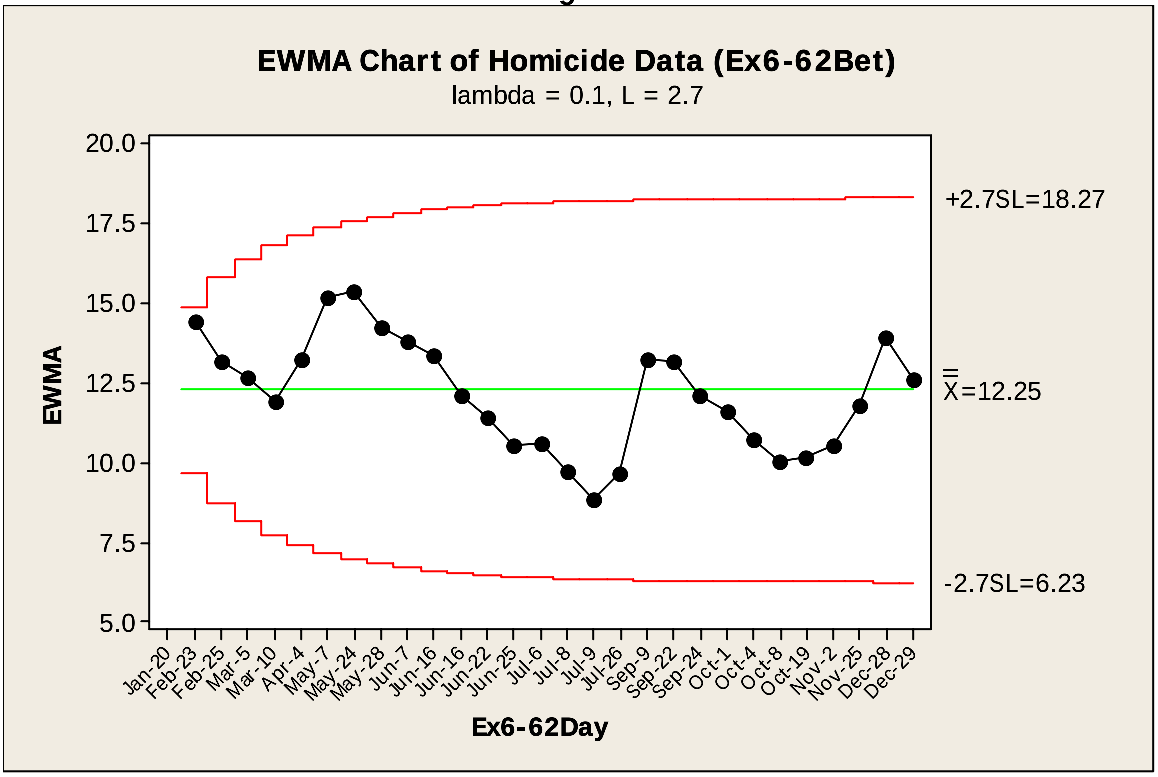
MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



Process is out of control. The process average of $\hat{\mu} = 183.594$ is too far from the process target of μ_0 = 175 for the process variability. The data is grouped into three increasing levels.

8-24☺.
$\lambda = 0.1$, $L = 2.7$

MTB > Stat > Control Charts > Time-Weighted Charts > EWMA

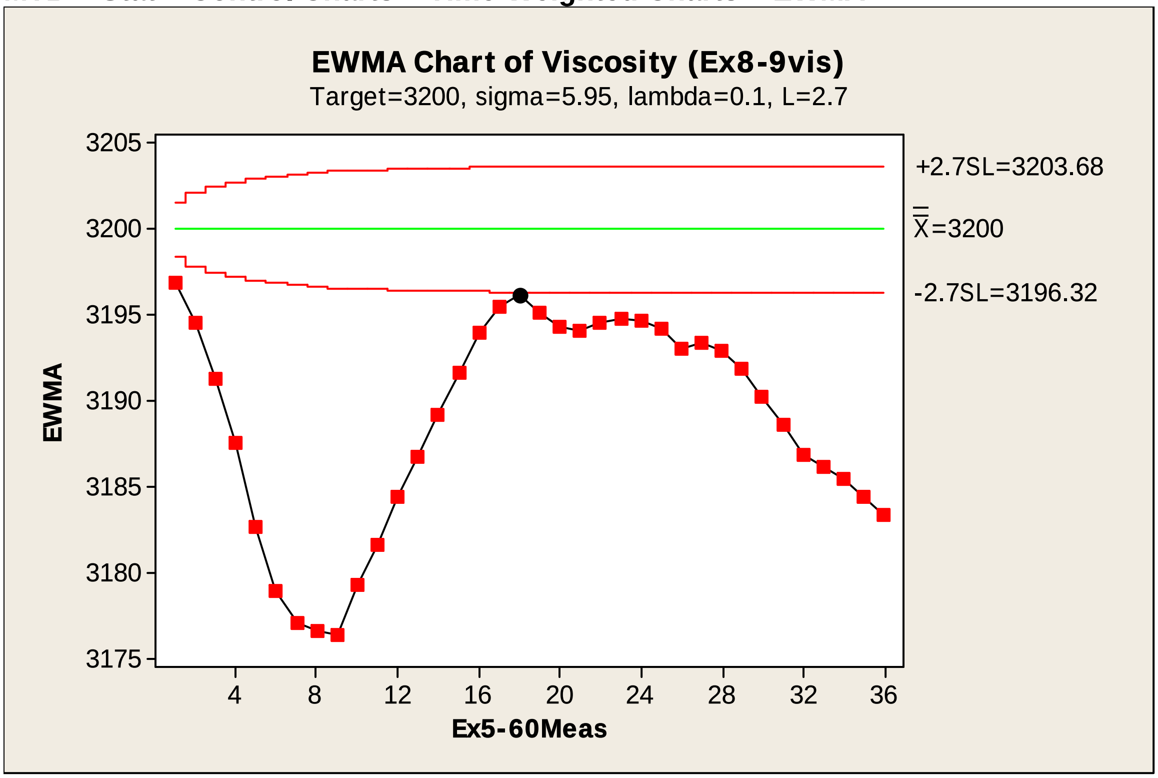


In Exercise 6-62, Individuals control charts of 0.2777th- and 0.25th-root transformed data showed no out-of-control signals. The EWMA chart also does not signal out of control. As mentioned in the text (Section 8.4-3), a properly designed EWMA chart is very robust to the assumption of normally distributed data.

8-25 (8-22).

$\mu_0 = 3200$, $\hat{\sigma} = 5.95$ (from Exercise 8-9), $\lambda = 0.1$, $L = 2.7$

MTB > Stat > Control Charts > Time-Weighted Charts > EWMA

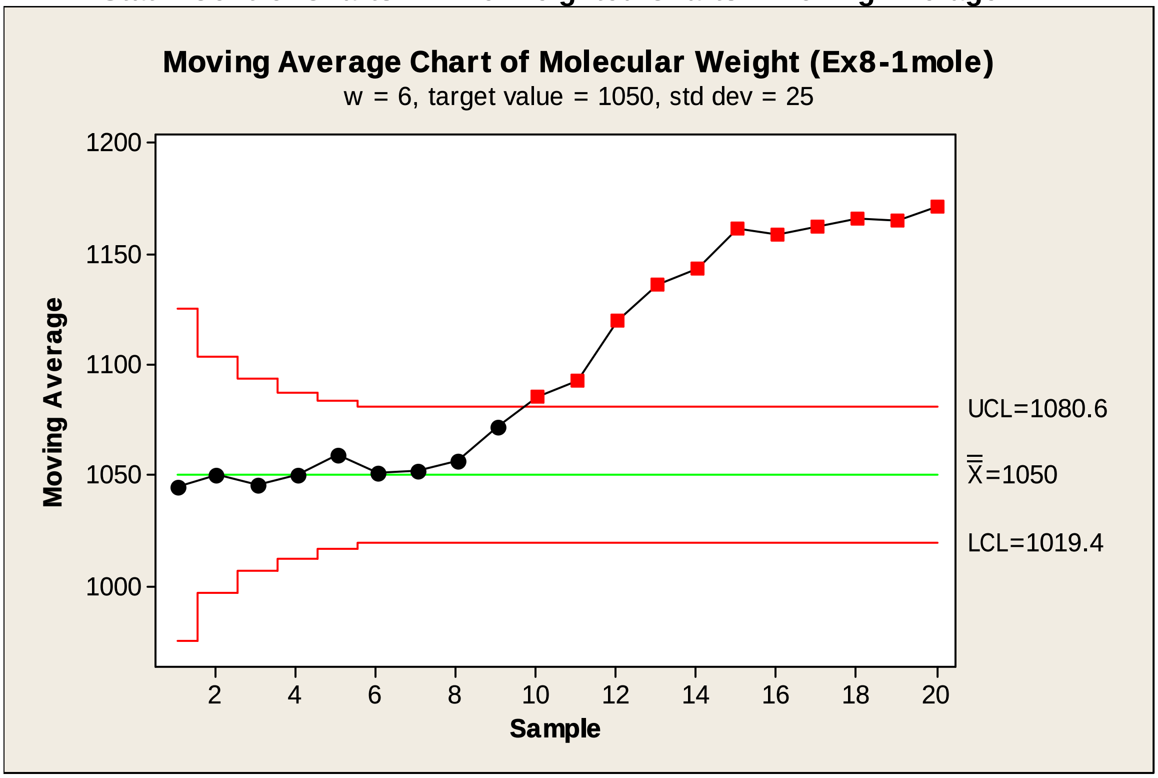


The process is out of control from the first sample.

8-26 (8-23).

$w = 6$, $\mu_0 = 1050$, $\sigma = 25$, CL = 1050, UCL = 1080.6, LCL = 1019.4

MTB > Stat > Control Charts > Time-Weighted Charts > Moving Average

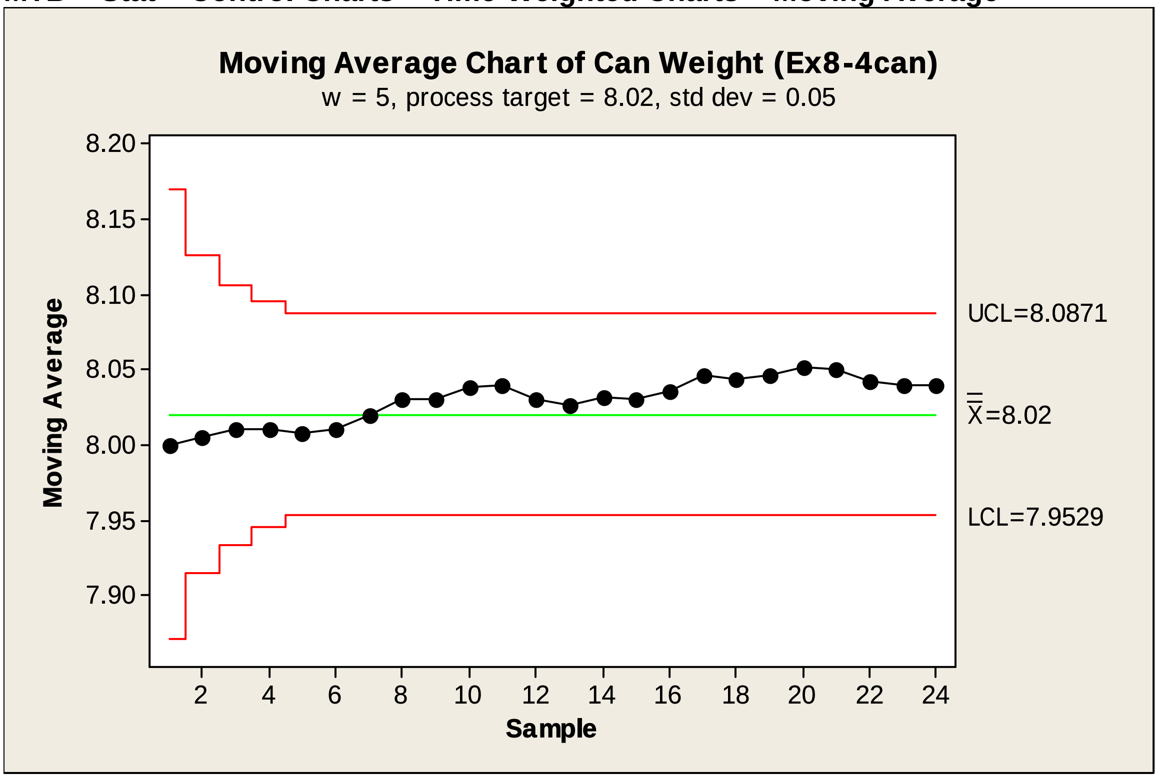



```
Test Results for Moving Average Chart of Ex8-1mole
TEST. One point beyond control limits.
Test Failed at points:  10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20
```

Process is out of control at observation 10, the same result as for Exercise 8-1.

8-27 (24).
$w = 5$, $\mu_0 = 8.02$, $\sigma = 0.05$, CL = 8.02, UCL = 8.087, LCL = 7.953

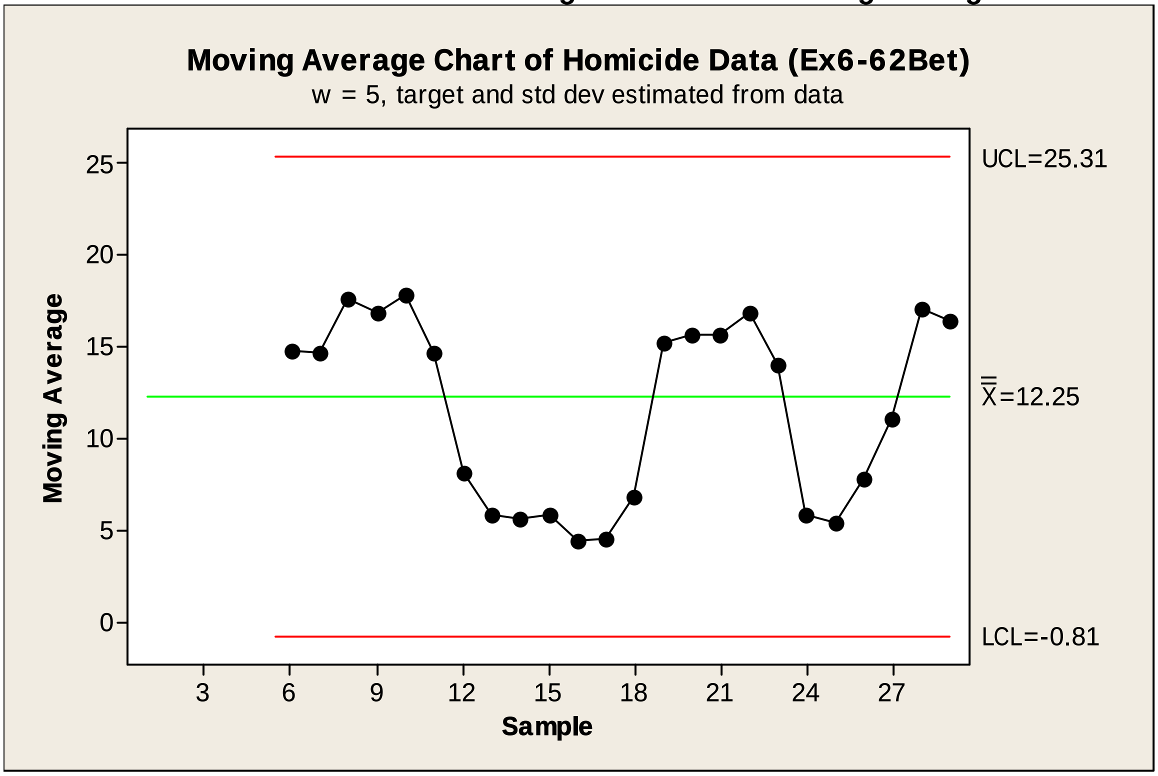
MTB > Stat > Control Charts > Time-Weighted Charts > Moving Average



The process is in control, the same result as for Exercise 8-4.

8-28☺.
$w = 5$

MTB > Stat > Control Charts > Time-Weighted Charts > Moving Average



Because these plot points are an average of five observations, the nonnormality of the individual observations should be of less concern. The approximate normality of the averages is a consequence of the Central Limit Theorem.

8-29 (8-25).

Assume that t is so large that the starting value $Z_0 = \overline{\overline{x}}$ has no effect.

$$E(Z_t) = E[\lambda \overline{x}_t + (1-\lambda)(Z_{t-1})] = E\left[\lambda \sum_{j=0}^{\infty} (1-\lambda)^j \overline{x}_{t-j}\right] = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E(\overline{x}_{t-j})$$

Since $E(\overline{x}_{t-j}) = \mu$ and $\lambda \sum_{j=0}^{\infty} (1\text{-}\lambda)^j = 1$, $E(Z_t) = \mu$

8-30 (8-26).

$$\begin{aligned}\text{var}(Z_t) &= \text{var}\left[\lambda \sum_{j=0}^{\infty} (1-\lambda)^j \overline{x}_{t-j}\right] \\ &= \left[\lambda^2 \sum_{j=0}^{\infty} (1-\lambda)^{2j}\right]\left[\text{var}(\overline{x}_{t-j})\right] \\ &= \frac{\lambda}{2-\lambda}\left(\frac{\sigma^2}{n}\right)\end{aligned}$$

8-31 (8-27).

For the EWMA chart, the steady-state control limits are $\overline{\overline{x}} \pm 3\sigma\sqrt{\dfrac{\lambda}{(2-\lambda)n}}$.

Substituting λ = 2/(w + 1), $\overline{\overline{x}} \pm 3\sigma\sqrt{\dfrac{\left(\dfrac{2}{w+1}\right)}{\left(2-\dfrac{2}{w+1}\right)n}} = \overline{\overline{x}} \pm 3\sigma\sqrt{\dfrac{1}{wn}} = \overline{\overline{x}} \pm \dfrac{3\sigma}{\sqrt{wn}}$,

which are the same as the limits for the MA chart.

8-32 (8-28).

The average age of the data in a w-period moving average is $\dfrac{1}{w}\sum_{j=0}^{w-1} j = \dfrac{w-1}{2}$. In the EWMA, the weight given to a sample mean j periods ago is $\lambda(1-\lambda)^j$, so the average age is $\lambda \sum_{j=0}^{\infty} (1-\lambda)^j j = \dfrac{1-\lambda}{\lambda}$. By equating average ages:

$$\frac{1-\lambda}{\lambda} = \frac{w-1}{2}$$

$$\lambda = \frac{2}{w+1}$$

8-33 (8-29).

For $n > 1$, Control limits $= \mu_0 \pm \dfrac{3}{\sqrt{w}}\left(\dfrac{\sigma}{\sqrt{n}}\right) = \mu_0 \pm \dfrac{3\sigma}{\sqrt{wn}}$

8-34 (8-30).

$\bar{x}$ chart: CL = 10, UCL = 16, LCL = 4

$$\begin{aligned}\text{UCL} &= \text{CL} + k\sigma_{\bar{x}} \\ 16 &= 10 - k\sigma_{\bar{x}} \\ k\sigma_{\bar{x}} &= 6\end{aligned}$$

EWMA chart:

$$\begin{aligned}\text{UCL} &= \text{CL} + l\sigma\sqrt{\lambda/[(2-\lambda)n]} \\ &= \text{CL} + l\,\sigma/\sqrt{n}\,\sqrt{0.1/(2-0.1)} = 10 + 6(0.2294) = 11.3765 \\ \text{LCL} &= 10 - 6(0.2294) = 8.6236\end{aligned}$$

8-35 (8-31).

$\lambda = 0.4$

For EWMA, steady-state limits are $\pm L\sigma\sqrt{\lambda/(2-\lambda)}$

For Shewhart, steady-state limits are $\pm k\sigma$

$$\begin{aligned}k\sigma &= L\sigma\sqrt{\lambda/(2-\lambda)} \\ k &= L\sqrt{0.4/(2-0.4)} \\ k &= 0.5L\end{aligned}$$

8-36 (8-32).

The two alternatives to plot a CUSUM chart with transformed data are:

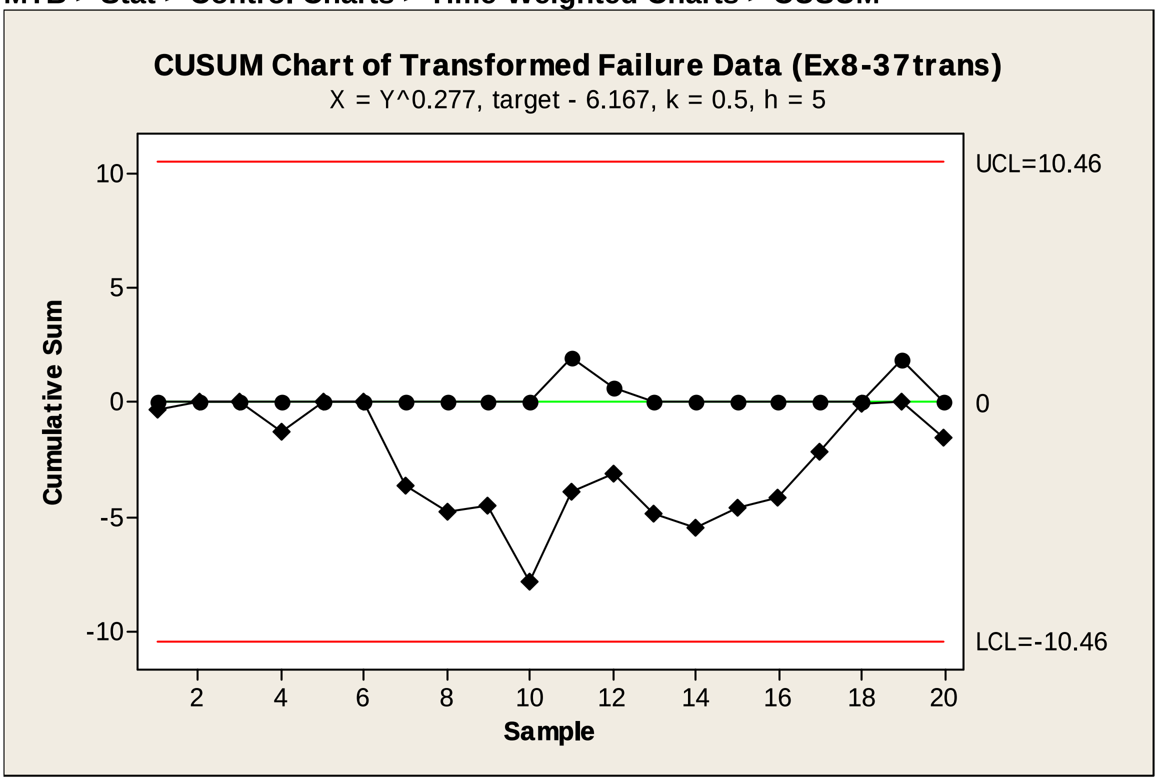
1. Transform the data, target (if given), and standard deviation (if given), then use these results in the CUSUM Chart dialog box, or
2. Transform the target (if given) and standard deviation (if given), then use the Box-Cox tab under CUSUM Options to transform the data.

The solution below uses alternative #2.

From Example 6-6, transform time-between-failures (Y) data to approximately normal distribution with $X = Y^{0.2777}$.

$T_Y = 700$, $T_X = 700^{0.2777} = 6.167$, $k = 0.5$, $h = 5$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



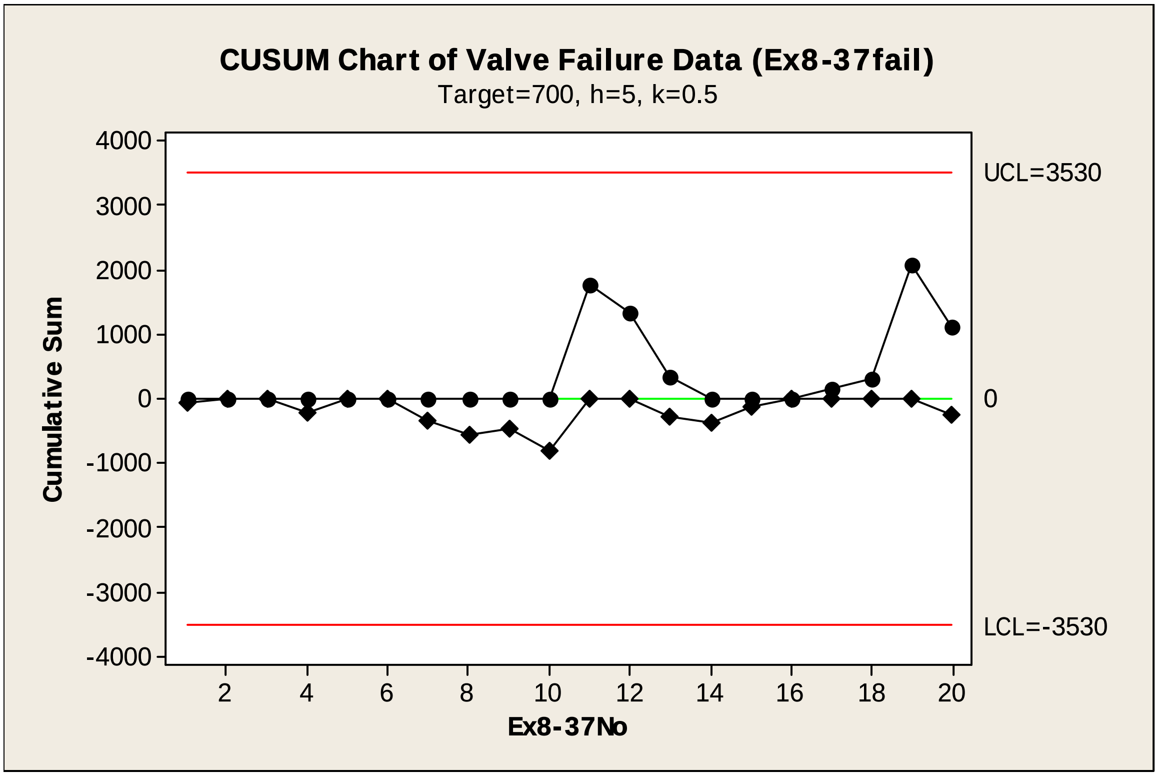
A one-sided <u>lower</u> CUSUM is needed to detect an increase in failure rate, or equivalently a decrease in the time-between-failures. Evaluate the lower CUSUM on the MINITAB chart to assess stability.

The process is in control.

8-37 (8-33).

$\mu_0 = 700$, $h = 5$, $k = 0.5$, estimate σ using the average moving range

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM,
also CUSUM options > Estimate > Average Moving Range



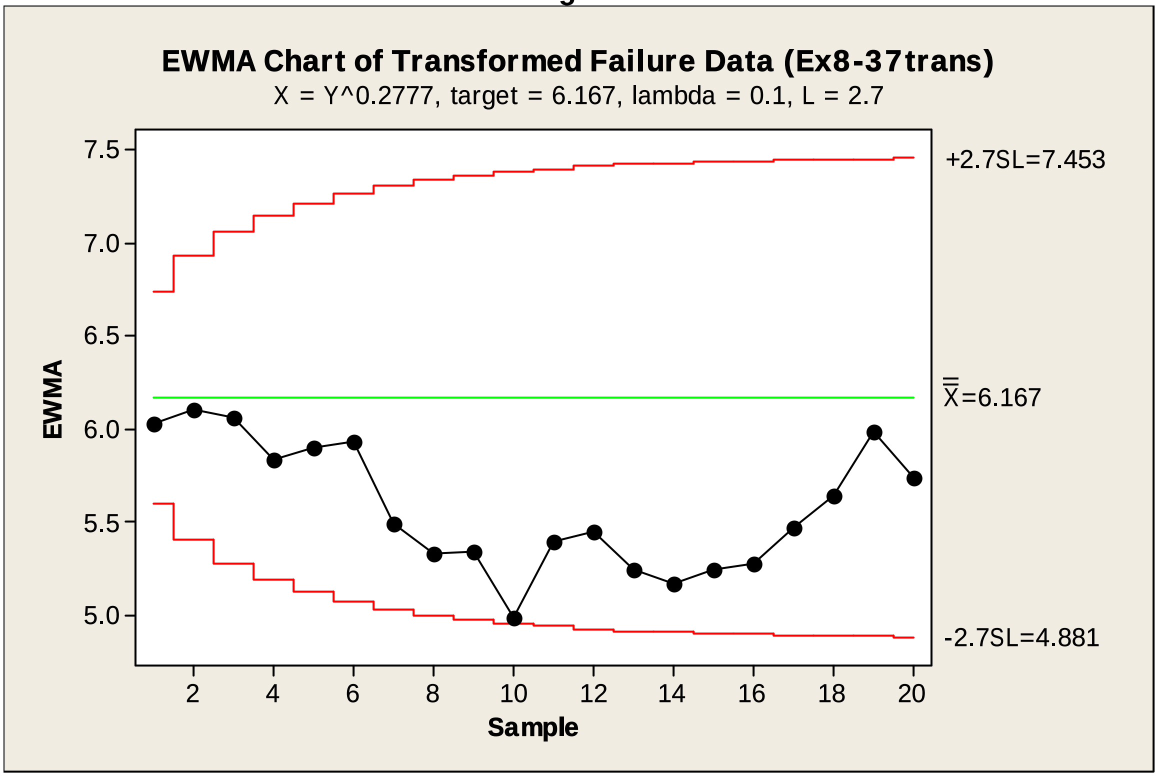
A one-sided lower CUSUM is needed to detect an increase in failure rate. Evaluate the lower CUSUM on the MINITAB chart to assess stability.

The process is in control.

Though the data are not normal, the CUSUM works fairly well for monitoring the process; this chart is very similar to the one constructed with the transformed data.

8-38 (8-34).
$\mu_0 = T_X = 700^{0.2777} = 6.167$, $\lambda = 0.1$, $L = 2.7$

MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



Valve failure times are in control.

8-39 (8-35).
The standard (two-sided) EWMA can be modified to form a one-sided statistic in much the same way a CUSUM can be made into a one-sided statistic. The standard (two-sided) EWMA is

$$z_i = \lambda x_i + (1-\lambda) z_{i-1}$$

Assume that the center line is at μ_0. Then a one-sided upper EWMA is

$$z_i^+ = \max\left[\mu_0, \lambda x_i + (1-\lambda) z_{i-1}\right],$$

and the one-sided lower EWMA is

$$z_i^- = \min\left[\mu_0, \lambda x_i + (1-\lambda) z_{i-1}\right].$$

Chapter 9 Exercise Solutions

Note: Many of the exercises in this chapter were solved using Microsoft Excel 2002, not MINITAB. The solutions, with formulas, charts, etc., are in **Chap09.xls.**

9-1.

$\hat{\sigma}_A = 2.530,\ n_A = 15,\ \hat{\mu}_A = 101.40$

$\hat{\sigma}_B = 2.297,\ n_B = 9,\ \hat{\mu}_B = 60.444$

$\hat{\sigma}_C = 1.815,\ n_C = 18,\ \hat{\mu}_C = 75.333$

$\hat{\sigma}_D = 1.875,\ n_D = 18,\ \hat{\mu}_D = 50.111$

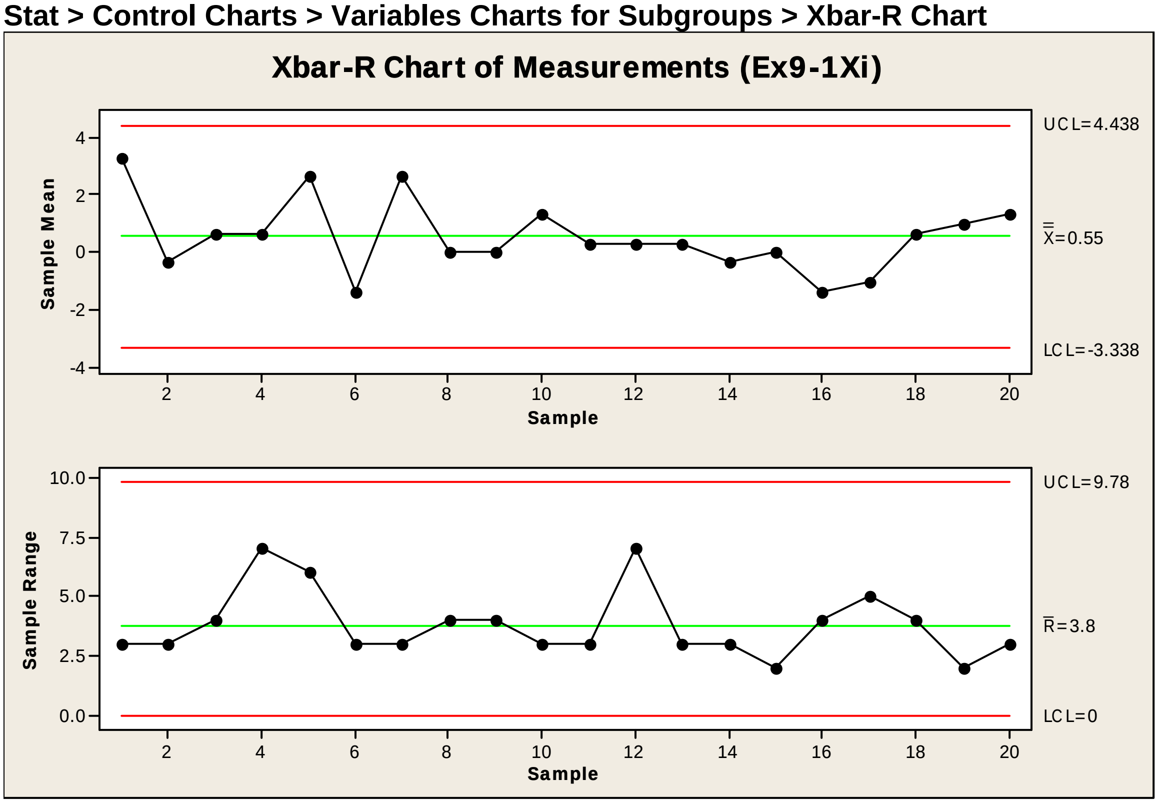
Standard deviations are approximately the same, so the DNOM chart can be used.

$\bar{R} = 3.8,\ \hat{\sigma} = 2.245,\ n = 3$

$\bar{x}$ chart: CL = 0.55, UCL = 4.44, LCL = –3.34

R chart: CL = 3.8, UCL = $D_4\bar{R}$ = 2.574 (3.8) = 9.78, LCL = 0

Stat > Control Charts > Variables Charts for Subgroups > Xbar-R Chart

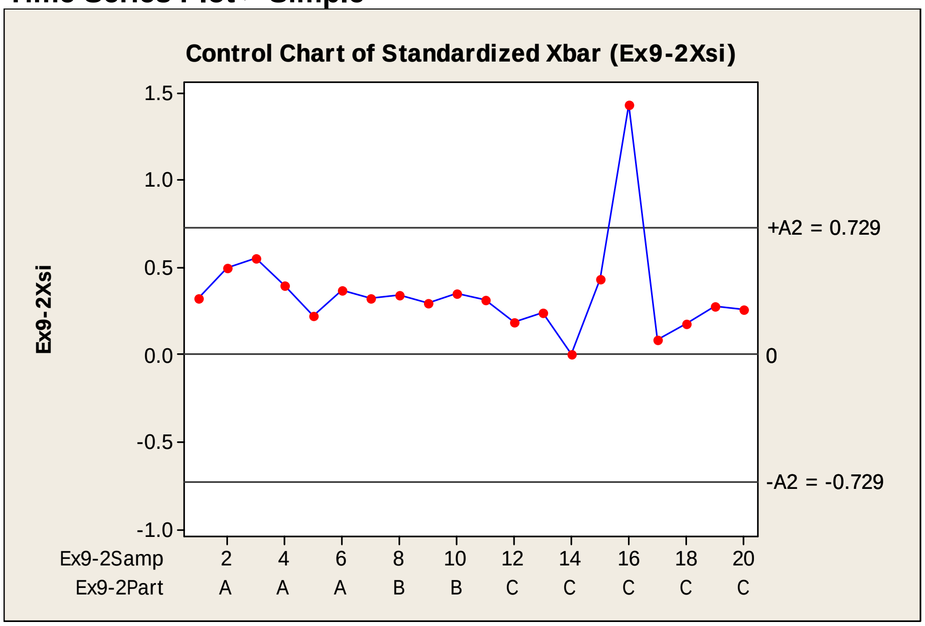


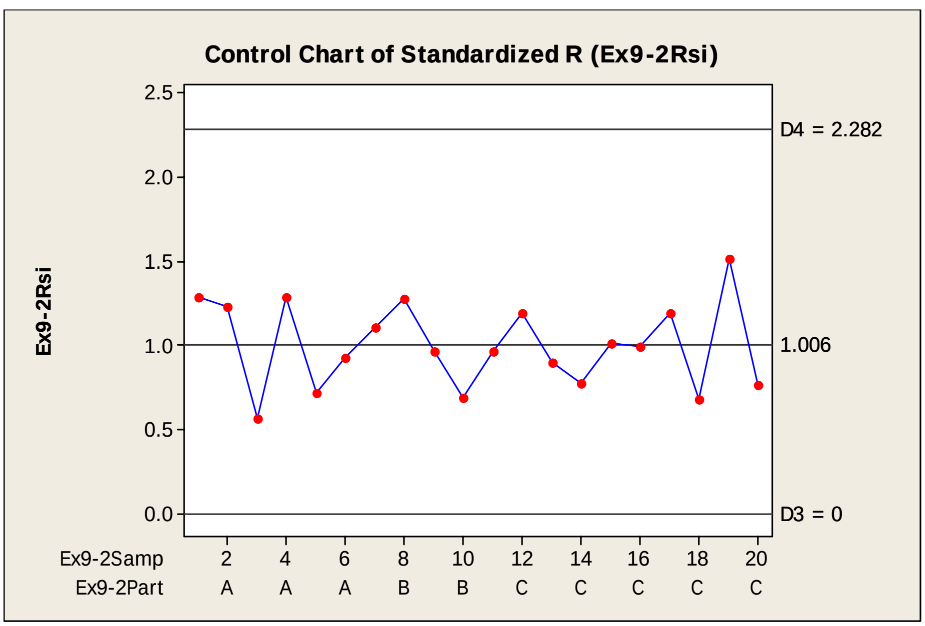
Process is in control, with no samples beyond the control limits or unusual plot patterns.

9-2.
Since the standard deviations are not the same, use a standardized $\bar{x}$ and R charts.
Calculations for standardized values are in:
Excel : workbook Chap09.xls : worksheet : Ex9-2.

$$n = 4, D_3 = 0, D_4 = 2.282, A_2 = 0.729; \quad \bar{R}_A = 19.3, \bar{R}_B = 44.8, \bar{R}_C = 278.2$$

Graph > Time Series Plot > Simple





Process is out of control at Sample 16 on the $\bar{x}$ chart.

9-3.
In a short production run situation, a standardized CUSUM could be used to detect smaller deviations from the target value. The chart would be designed so that δ, in standard deviation units, is the same for each part type. The standardized variable $(y_{i,j} - \mu_{0,j})/\sigma_j$ (where j represents the part type) would be used to calculate each plot statistic.

9-4.
Note: In the textbook, the 4th part on Day 246 should be "1385" not "1395".

Set up a standardized c chart for defect counts. The plot statistic is $Z_i = (c_i - \bar{c})/\sqrt{\bar{c}}$, with CL = 0, UCL = +3, LCL = –3.

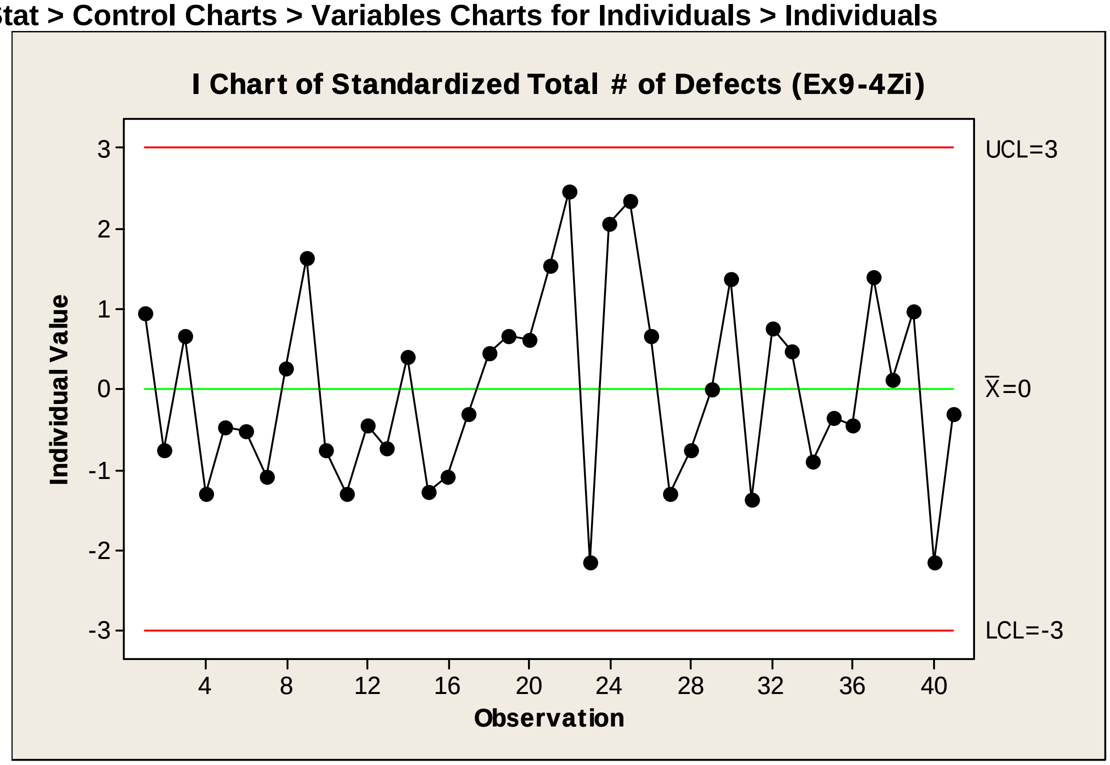
Stat > Basic Statistics > Display Descriptive Statistics

Descriptive Statistics: Rx9-4Def

Rx9-4Def	1055	13.25
	1130	64.00
	1261	12.67
	1385	26.63
	4610	4.67
	8611	50.13

$\bar{c}_{1055} = 13.25, \bar{c}_{1130} = 64.00, \bar{c}_{1261} = 12.67, \bar{c}_{1385} = 26.63, \bar{c}_{4610} = 4.67, \bar{c}_{8611} = 50.13$

Stat > Control Charts > Variables Charts for Individuals > Individuals

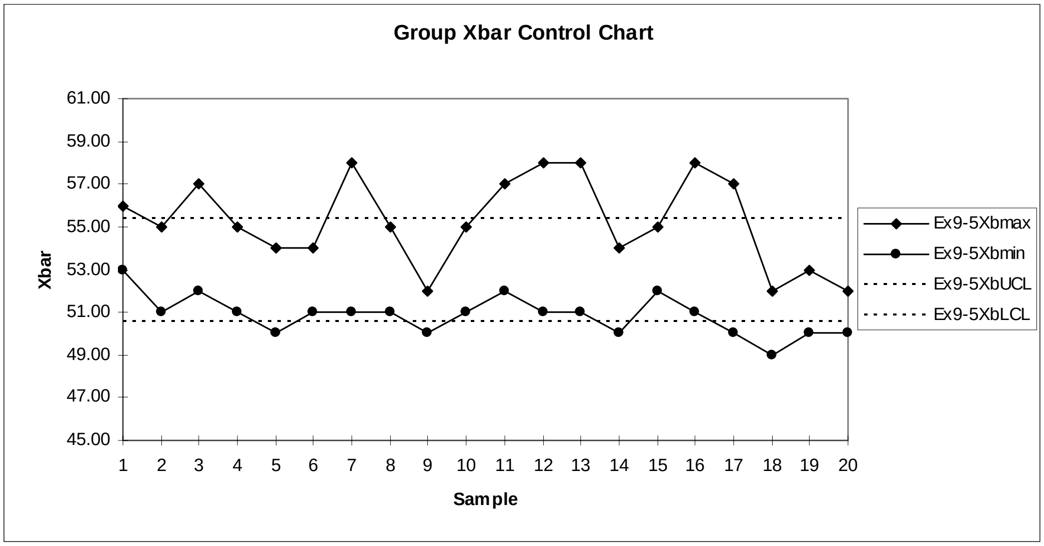


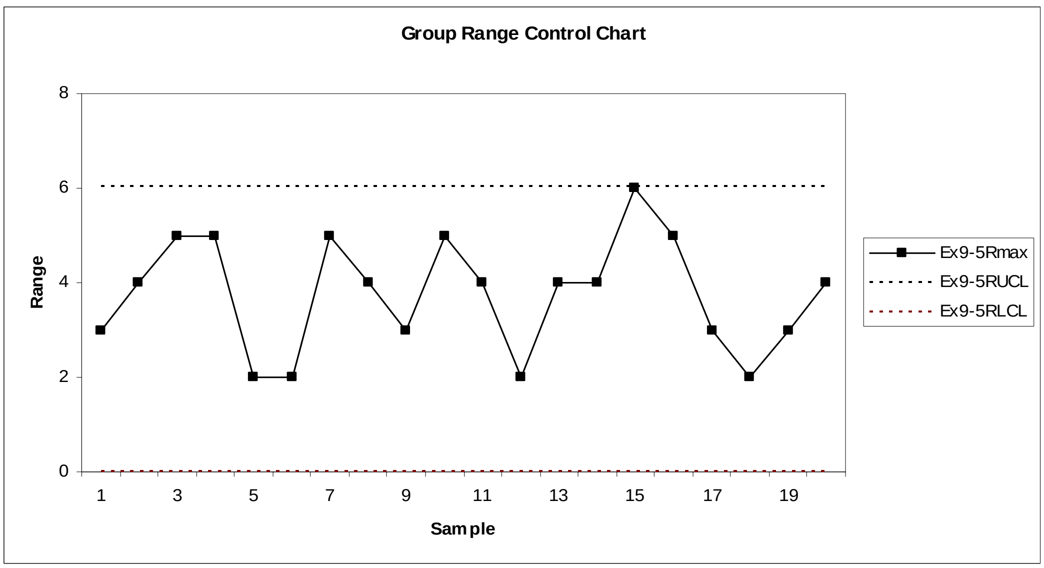
Process is in control.

9-5.

Excel : Workbook Chap09.xls : Worksheet Ex9-5

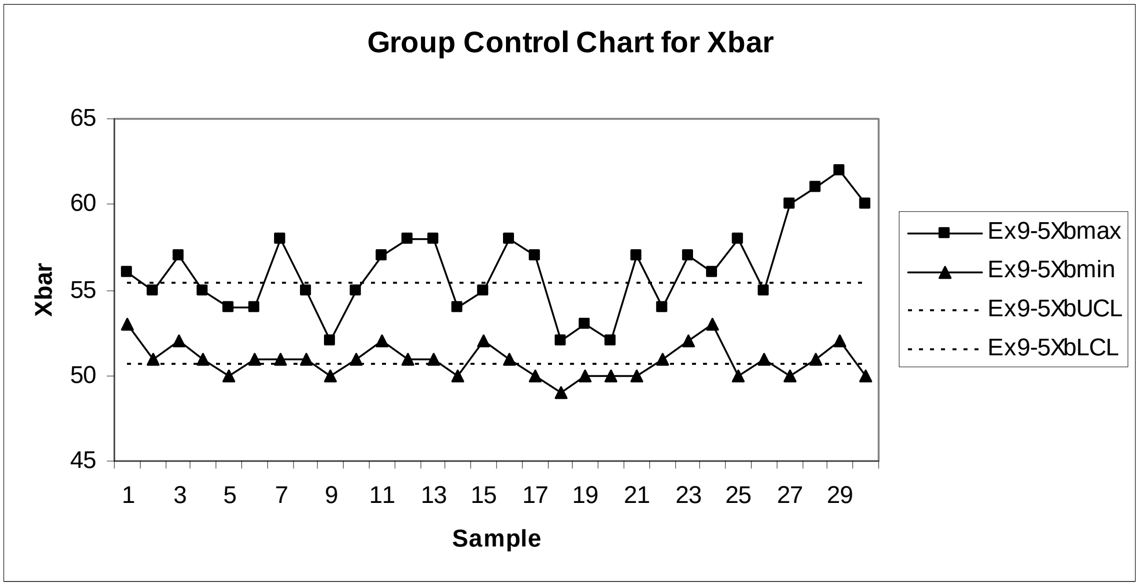
Grand Avg =	52.988	
Avg R =	2.338	
s =	4	heads
n =	3	units
A2 =	1.023	
D3 =	0	
D4 =	2.574	
Xbar UCL =	55.379	
Xbar LCL =	50.596	
R UCL =	6.017	
R LCL =	0.000	

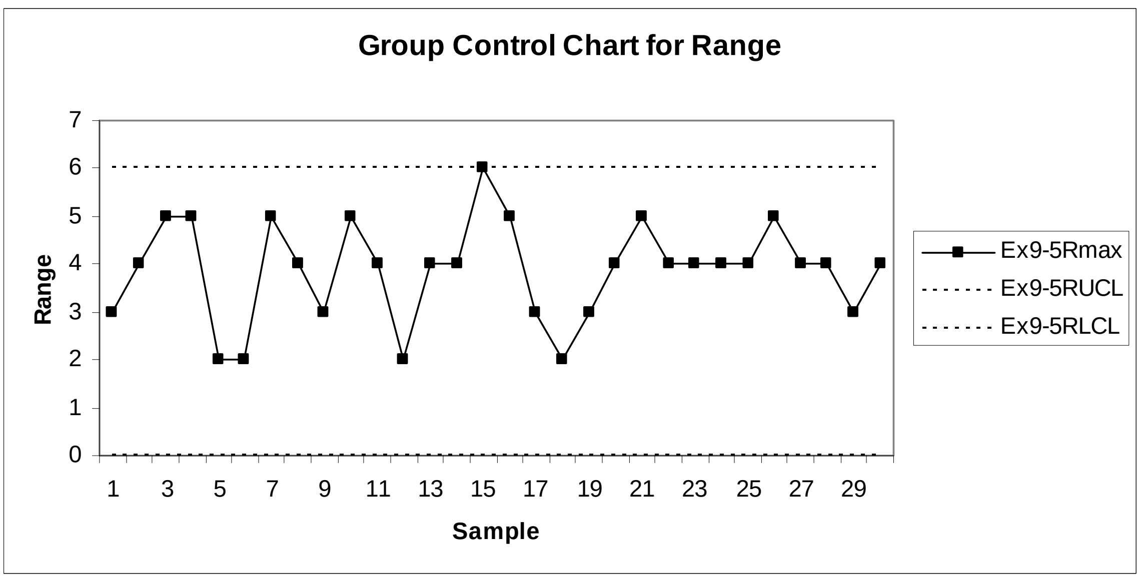




There is no situation where one single head gives the maximum or minimum value of $\overline{x}$ six times in a row. There are many values of $\overline{x}$ max and $\overline{x}$ min that are outside the control limits, so the process is out-of-control. The assignable cause affects all heads, not just a specific one.

9-6.
Excel : Workbook Chap09.xls : Worksheet Ex9-6



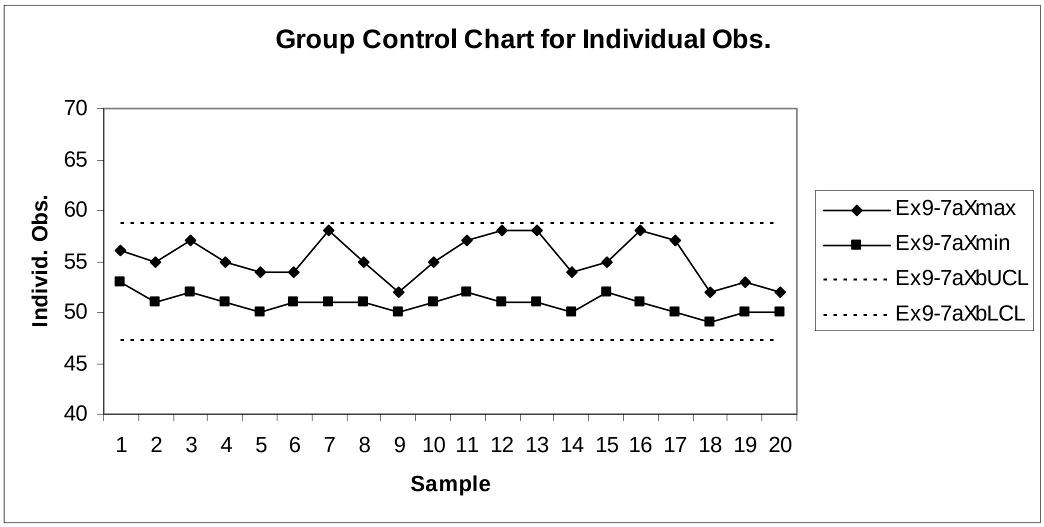


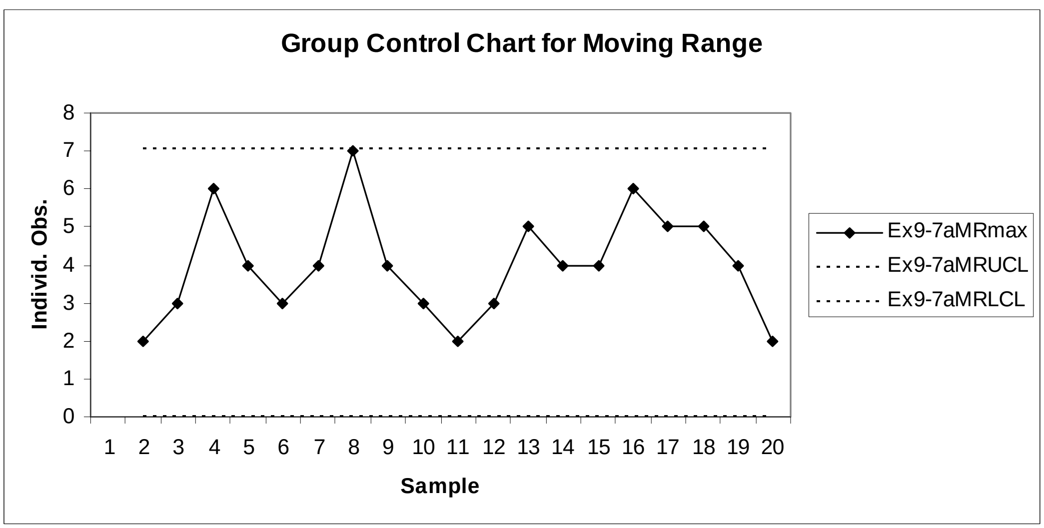
The last four samples from Head 4 are the maximum of all heads; a process change may have caused output of this head to be different from the others.

9-7.
(a)

Excel : Workbook Chap09.xls : Worksheet Ex9-7A

Grand Avg =	52.988	
Avg MR =	2.158	
s =	4	heads
n =	2	units
d2 =	1.128	
D3 =	0	
D4 =	3.267	
Xbar UCL =	58.727	
Xbar LCL =	47.248	
R UCL =	7.050	
R LCL =	0.000	

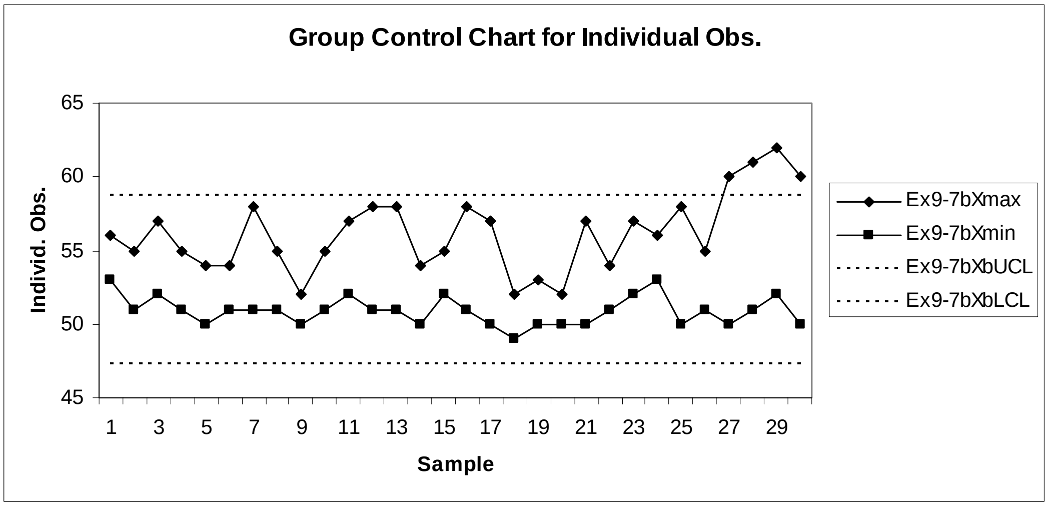


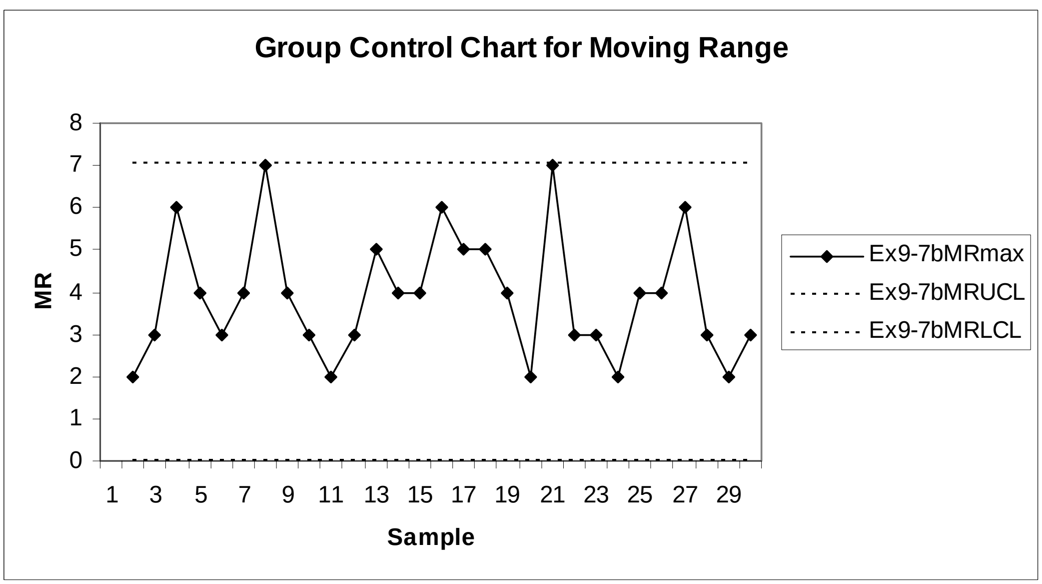


See the discussion in Exercise 9-5.

9-7 continued
(b)
Excel : Workbook Chap09.xls : Worksheet Ex9-7b

Grand Avg =	52.988	
Avg MR =	2.158	
s =	4	heads
n =	2	units
d2 =	1.128	
D3 =	0	
D4 =	3.267	
Xbar UCL =	58.727	
Xbar LCL =	47.248	
R UCL =	7.050	
R LCL =	0.000	

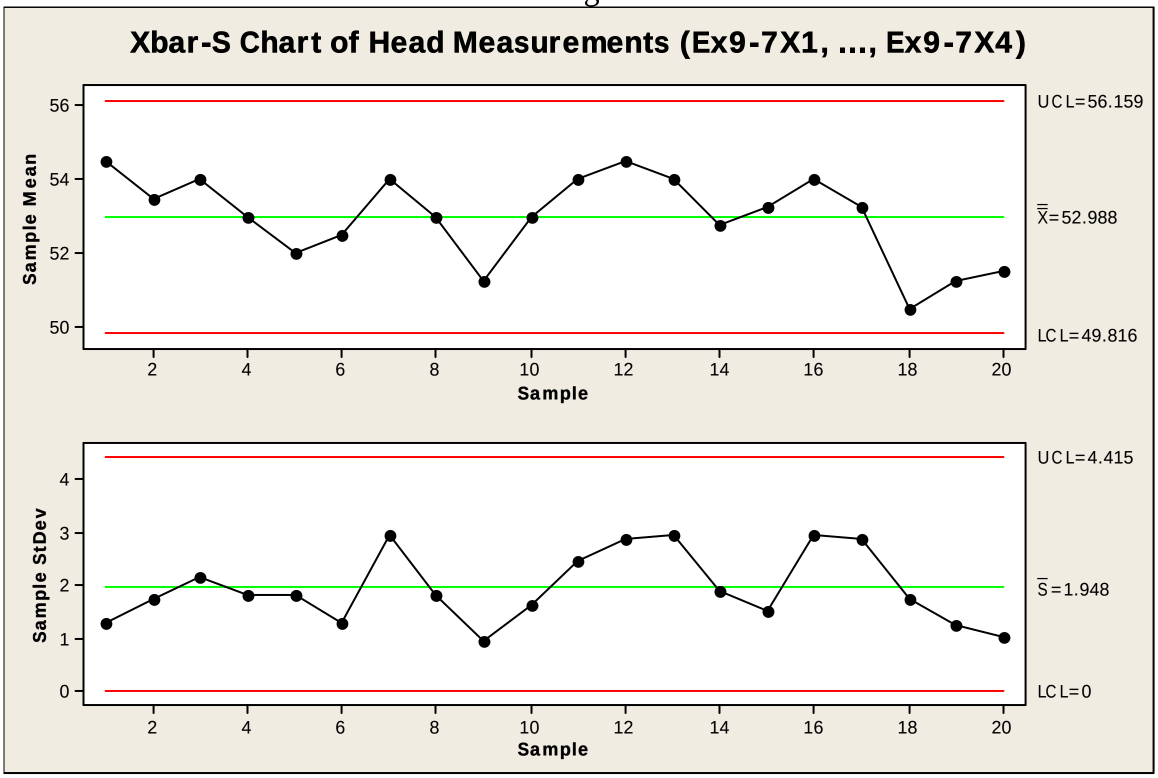




The last four samples from Head 4 remain the maximum of all heads; indicating a potential process change.

9-7 continued
(c)

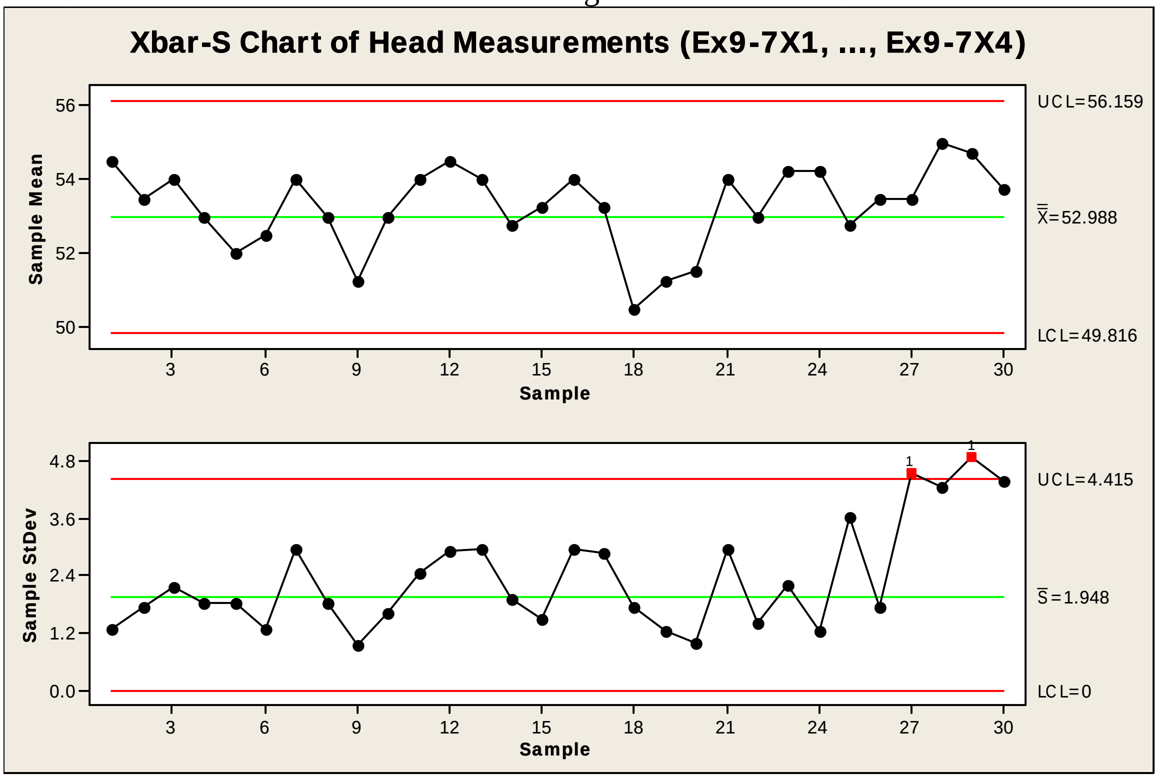
Stat > Control Charts > Variables Charts for Subgroups > Xbar-S Chart
Note: Use "Sbar" as the method for estimating standard deviation.



Failure to recognize the multiple stream nature of the process had led to control charts that fail to identify the out-of-control conditions in this process.

9-7 continued
(d)

Stat > Control Charts > Variables Charts for Subgroups > Xbar-S Chart
Note: Use "Sbar" as the method for estimating standard deviation.



Test Results for S Chart of Ex9-7X1, ..., Ex9-7X4
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  27, 29
```

Only the *S* chart gives any indication of out-of-control process.

9-8.

Stat > Basic Statistics > Display Descriptive Statistics

Descriptive Statistics: Ex9-8Xbar, Ex9-8R

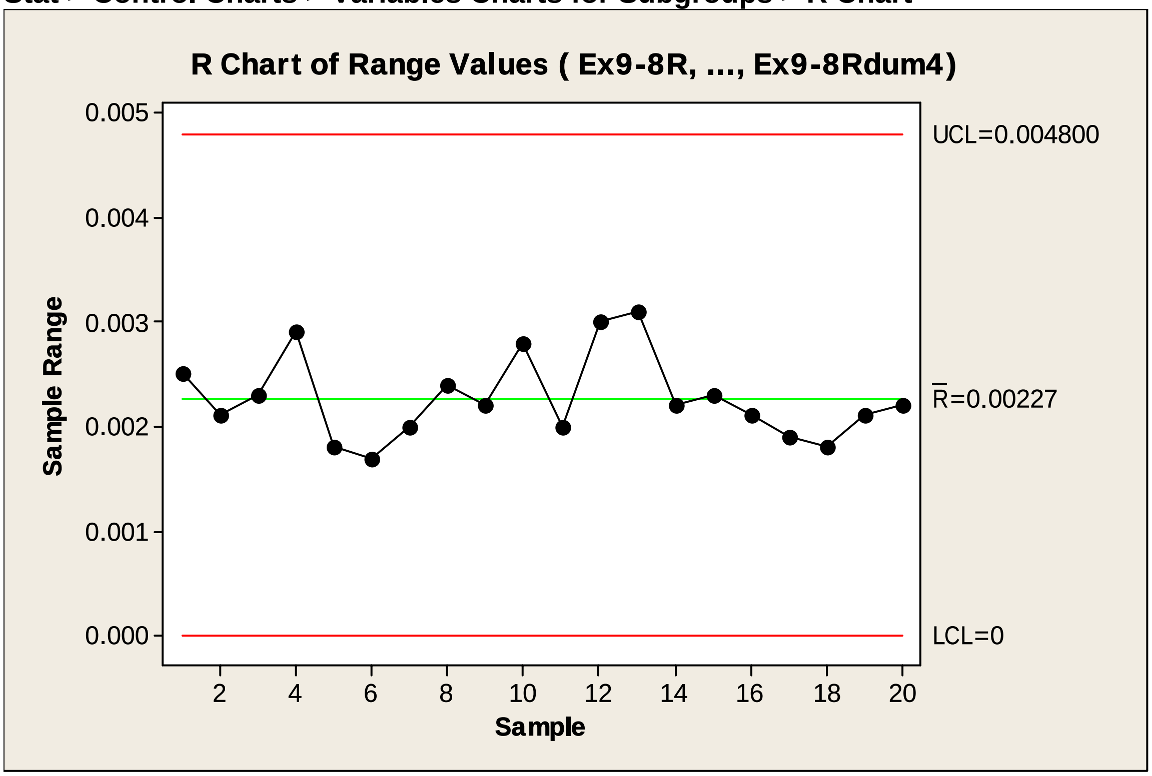
```
Variable       Mean
Ex9-8Xbar   0.55025
Ex9-8R     0.002270
```

$n = 5$

$\bar{\bar{x}} = 0.55025, \bar{R} = 0.00227, \hat{\sigma} = \bar{R} / d_2 = 0.00227 / 2.326 = 0.000976$

$\widehat{\text{PCR}} = (\text{USL-LSL}) / 6\hat{\sigma} = (0.552 - 0.548) / [6(0.000976)] = 6.83$

Stat > Control Charts > Variables Charts for Subgroups > R Chart



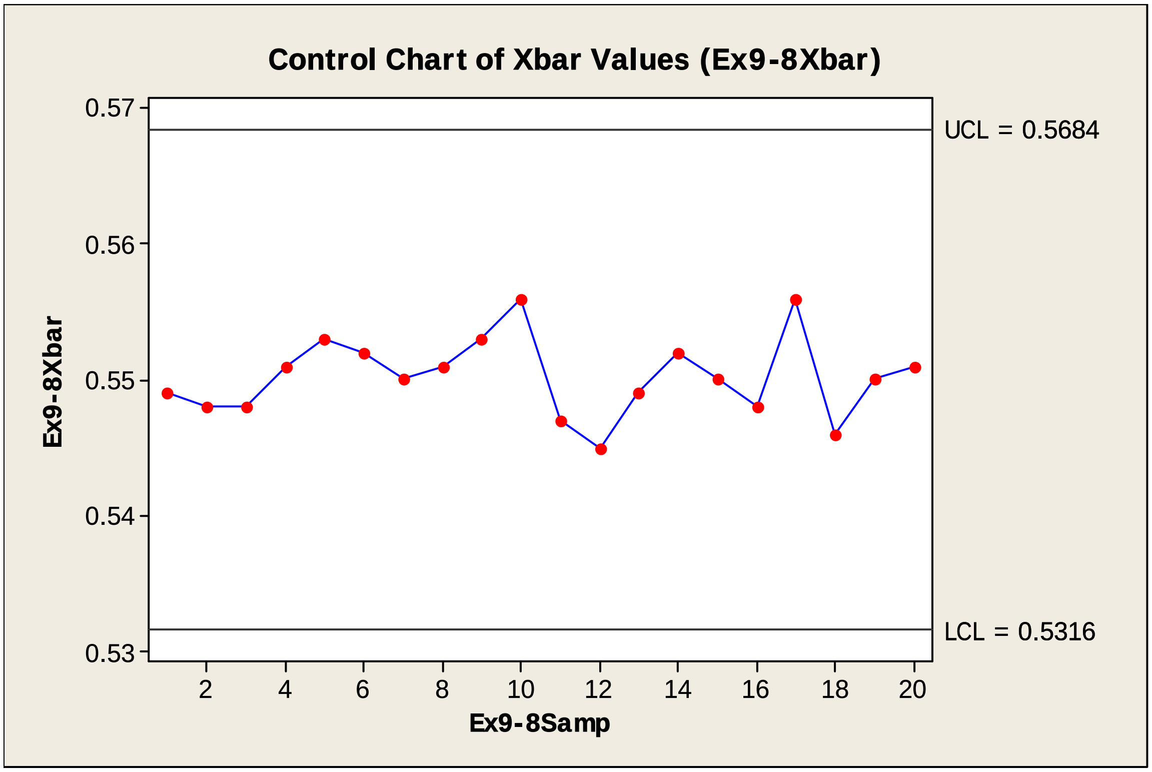
The process variability, as shown on the *R* chart is in control.

9-8 continued
(a)
3-sigma limits:
$\delta = 0.01, Z_\delta = Z_{0.01} = 2.33$

$$\text{UCL} = \text{USL} - \left(Z_\delta - 3\big/\sqrt{n}\right)\hat{\sigma} = (0.550 + 0.020) - \left(2.33 - 3\big/\sqrt{20}\right)(0.000976) = 0.5684$$

$$\text{LCL} = \text{LSL} + \left(Z_\delta - 3\big/\sqrt{n}\right)\hat{\sigma} = (0.550 - 0.020) + \left(2.33 - 3\big/\sqrt{20}\right)(0.000976) = 0.5316$$

Graph > Time Series Plot > Simple
Note: Reference lines have been used set to the control limit values.



The process mean falls within the limits that define 1% fraction nonconforming.

Notice that the control chart does not have a centerline. Since this type of control scheme allows the process mean to vary over the interval—with the assumption that the overall process performance is not appreciably affected—a centerline is not needed.

9-8 continued
(b)

$\gamma = 0.01,\ Z_\gamma = Z_{0.01} = 2.33$

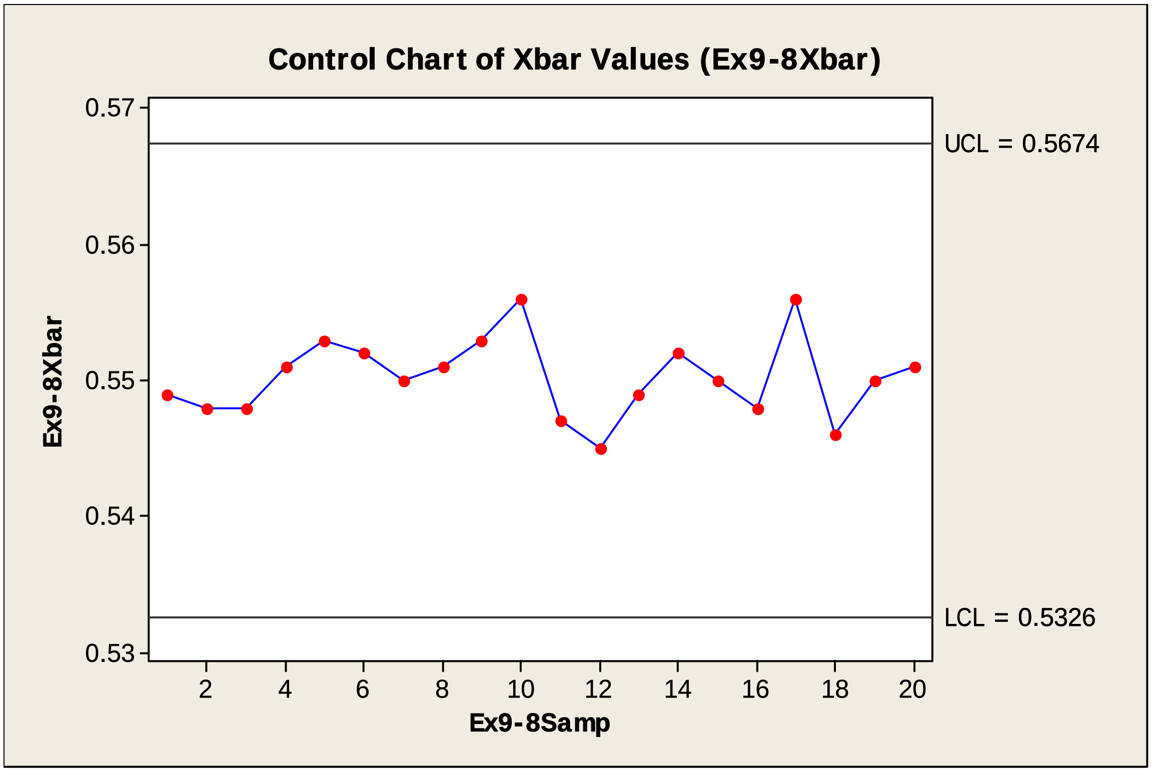
$1-\beta = 0.90,\ Z_\beta = z_{0.10} = 1.28$

$$\text{UCL} = \text{USL} - \left(Z_\gamma + Z_\beta \big/ \sqrt{n}\right)\hat{\sigma} = (0.550+0.020) - \left(2.33 + 1.28\big/\sqrt{20}\right)(0.000976) = 0.5674$$

$$\text{LCL} = \text{LSL} + \left(Z_\gamma + Z_\beta \big/ \sqrt{n}\right)\hat{\sigma} = (0.550-0.020) + \left(2.33 + 1.28\big/\sqrt{20}\right)(0.000976) = 0.5326$$

Chart control limits for part (b) are slightly narrower than for part (a).

Graph > Time Series Plot > Simple
Note: Reference lines have been used set to the control limit values.



The process mean falls within the limits defined by 0.90 probability of detecting a 1% fraction nonconforming.

9-9.
(a)
3-sigma limits:

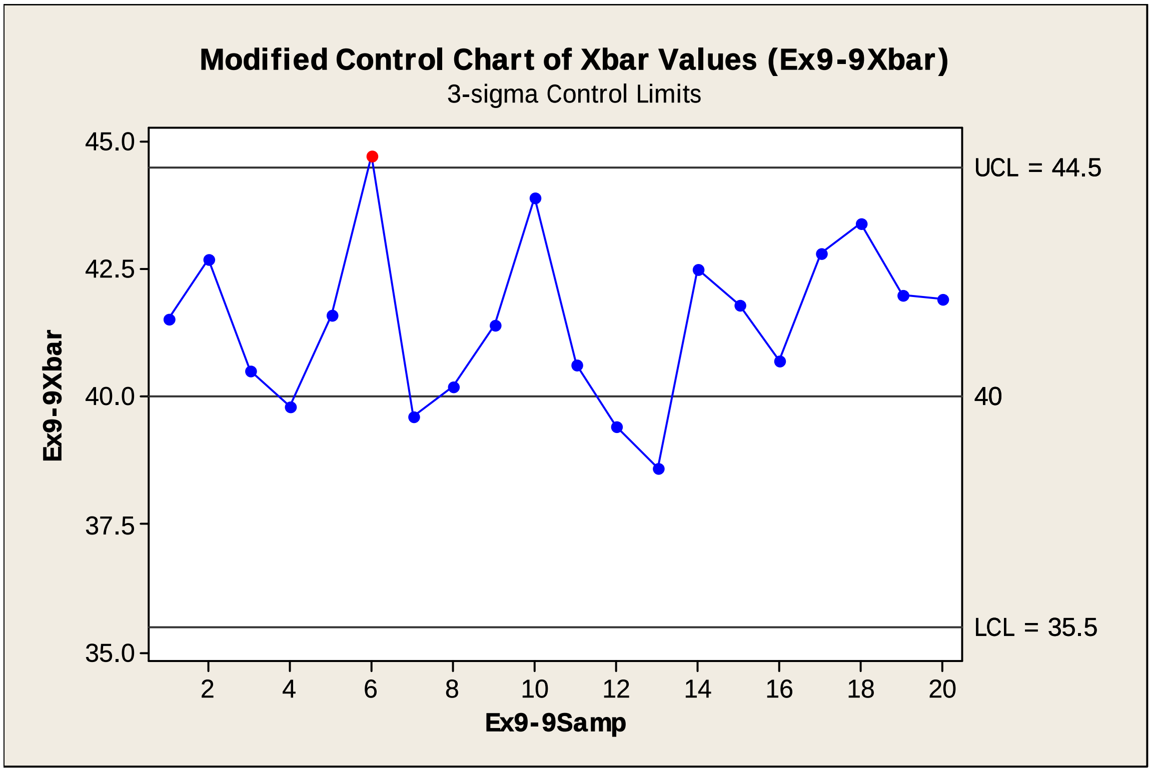
$$n = 5, \delta = 0.001, Z_\delta = Z_{0.001} = 3.090$$

$$\text{USL} = 40 + 8 = 48, \text{ LSL} = 40 - 8 = 32$$

$$\text{UCL} = \text{USL} - \left(Z_\delta - 3/\sqrt{n}\right)\sigma = 48 - \left(3.090 - 3/\sqrt{5}\right)(2.0) = 44.503$$

$$\text{LCL} = \text{LSL} + \left(Z_\delta - 3/\sqrt{n}\right)\sigma = 32 + \left(3.090 - 3/\sqrt{5}\right)(2.0) = 35.497$$

Graph > Time Series Plot > Simple
Note: Reference lines have been used set to the control limit values.



Process is out of control at sample #6.

9-9 continued

(b)

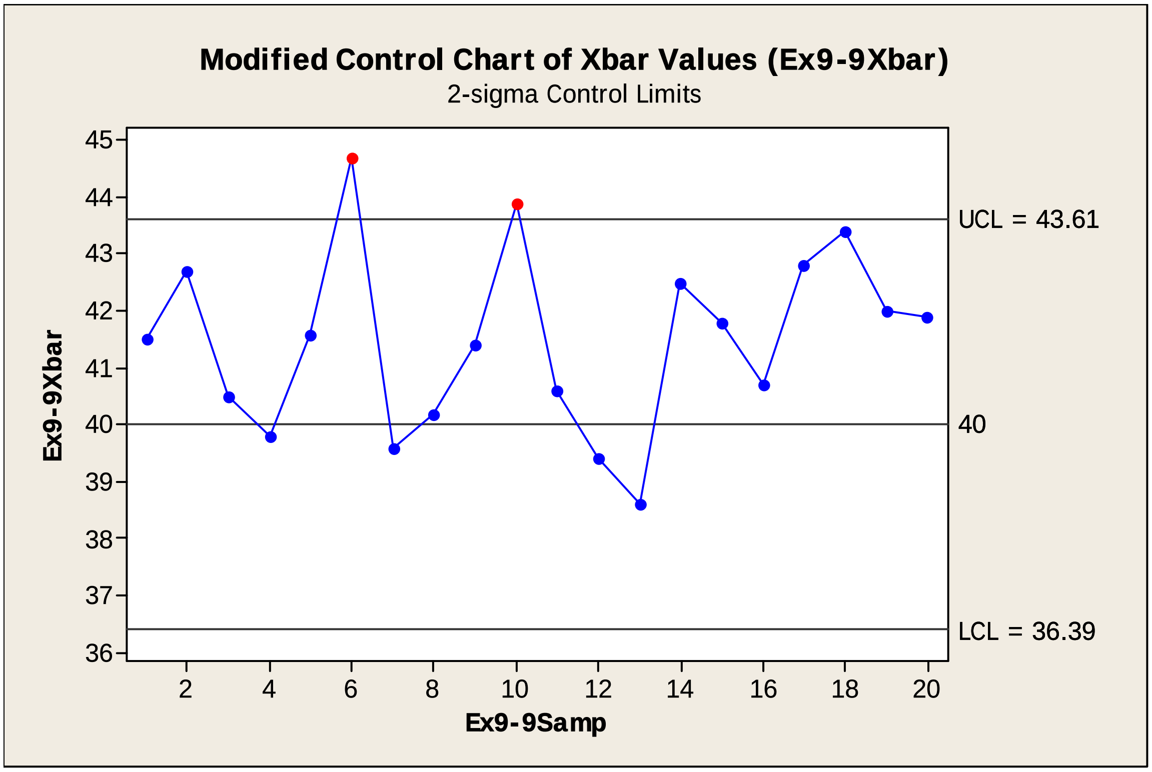
2-sigma limits:

$$\text{UCL} = \text{USL} - \left(Z_\delta - 2/\sqrt{n}\right)\sigma = 48 - \left(3.090 - 2/\sqrt{5}\right)(2.0) = 43.609$$

$$\text{LCL} = \text{LSL} + \left(Z_\delta - 2/\sqrt{n}\right)\sigma = 32 + \left(3.090 - 2/\sqrt{5}\right)(2.0) = 36.391$$

Graph > Time Series Plot > Simple

Note: Reference lines have been used set to the control limit values.



With 3-sigma limits, sample #6 exceeds the UCL, while with 2-sigma limits both samples #6 and #10 exceed the UCL.

9-9 continued

(c)

$\gamma = 0.05,\ Z_\gamma = Z_{0.05} = 1.645$

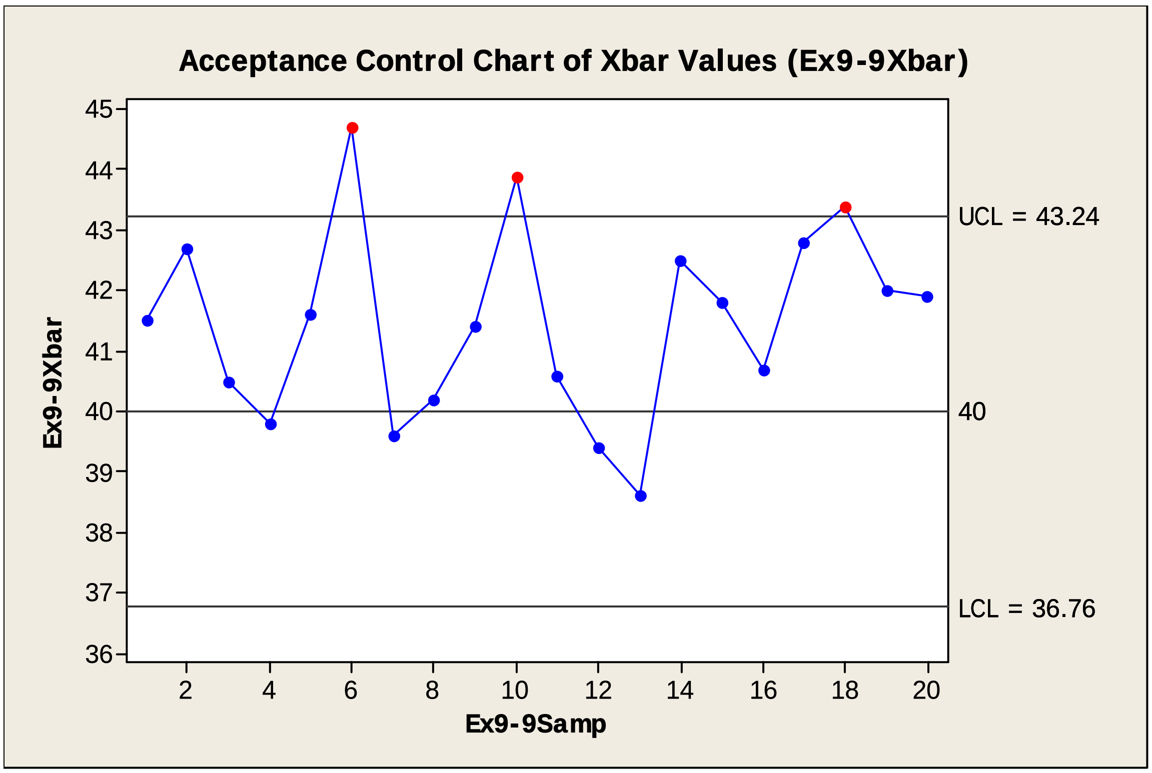
$1-\beta = 0.95,\ Z_\beta = Z_{0.05} = 1.645$

$$\text{UCL} = \text{USL} - \left(Z_\gamma + z_\beta \big/ \sqrt{n}\right)\sigma = 48 - \left(1.645 + 1.645 \big/ \sqrt{5}\right)(2.0) = 43.239$$

$$\text{LCL} = \text{LSL} + \left(Z_\gamma + z_\beta \big/ \sqrt{n}\right)\sigma = 32 + \left(1.645 + 1.645 \big/ \sqrt{5}\right)(2.0) = 36.761$$

Graph > Time Series Plot > Simple

Note: Reference lines have been used set to the control limit values.



Sample #18 also signals an out-of-control condition.

9-10.
Design an acceptance control chart.

Accept in-control fraction nonconforming $= 0.1\% \rightarrow \delta = 0.001,\ Z_\delta = Z_{0.001} = 3.090$
with probability $1-\alpha = 0.95 \rightarrow \alpha = 0.05,\ Z_\alpha = Z_{0.05} = 1.645$
Reject at fraction nonconforming $= 2\% \rightarrow \gamma = 0.02,\ Z_\gamma = Z_{0.02} = 2.054$
with probability $1-\beta = 0.90 \rightarrow \beta = 0.10,\ Z_\beta = Z_{0.10} = 1.282$

$$n = \left(\frac{Z_\alpha + Z_\beta}{Z_\delta - Z_\gamma}\right)^2 = \left(\frac{1.645+1.282}{3.090-2.054}\right)^2 = 7.98 \approx 8$$

$$\text{UCL} = \text{USL} - \left(Z_\gamma + Z_\beta \big/ \sqrt{n}\right)\sigma = \text{USL} - \left(2.054 + 1.282 \big/ \sqrt{8}\right)\sigma = \text{USL} - 2.507\sigma$$

$$\text{LCL} = \text{LSL} + \left(Z_\gamma + Z_\beta \big/ \sqrt{n}\right)\sigma = \text{LSL} + \left(2.054 + 1.282 \big/ \sqrt{8}\right)\sigma = \text{LSL} + 2.507\sigma$$

9-11.

$\mu = 0,\ \sigma = 1.0,\ n = 5,\ \delta = 0.00135,\ Z_\delta = Z_{0.00135} = 3.00$

For 3-sigma limits, $Z_\alpha = 3$

$$\text{UCL} = \text{USL} - \left(z_\delta - z_\alpha/\sqrt{n}\right)\sigma = \text{USL} - \left(3.000 - 3/\sqrt{5}\right)(1.0) = \text{USL} - 1.658$$

$$\Pr\{\text{Accept}\} = \Pr\{\bar{x} < \text{UCL}\} = \Phi\left(\frac{\text{UCL} - \mu_0}{\sigma/\sqrt{n}}\right) = \Phi\left(\frac{\text{USL} - 1.658 - \mu_0}{1.0/\sqrt{5}}\right) = \Phi\left((\Delta - 1.658)\sqrt{5}\right)$$

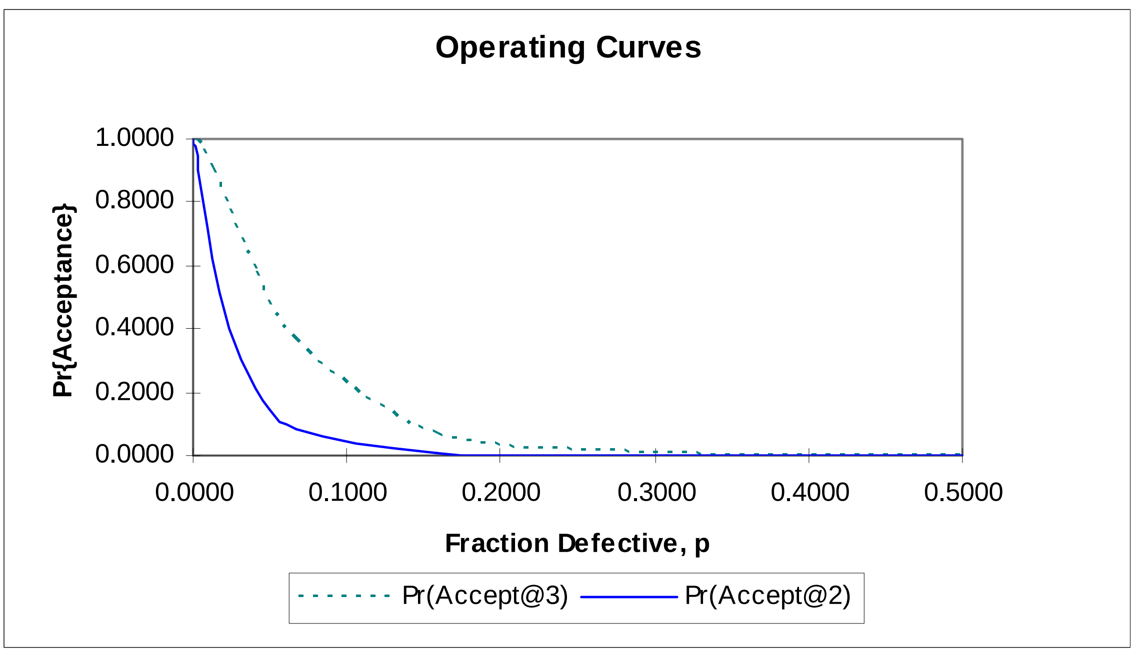
where $\Delta = \text{USL} - \mu_0$

For 2-sigma limits, $Z_\alpha = 2 \Rightarrow \Pr\{\text{Accept}\} = \Phi\left((\Delta - 2.106)\sqrt{5}\right)$

$$p = \Pr\{x > \text{USL}\} = 1 - \Pr\{x \le \text{USL}\} = 1 - \Phi\left(\frac{\text{USL} - \mu_0}{\sigma}\right) = 1 - \Phi(\Delta)$$

Excel : Workbook Chap09.xls : Worksheet Ex9-11

DELTA=USL-mu0	CumNorm(DELTA)	p	Pr(Accept@3)	Pr(Accept@2)
3.50	0.9998	0.0002	1.0000	0.9991
3.25	0.9994	0.0006	0.9998	0.9947
3.00	0.9987	0.0013	0.9987	0.9772
2.50	0.9938	0.0062	0.9701	0.8108
2.25	0.9878	0.0122	0.9072	0.6263
2.00	0.9772	0.0228	0.7778	0.4063
1.75	0.9599	0.0401	0.5815	0.2130
1.50	0.9332	0.0668	0.3619	0.0877
1.00	0.8413	0.1587	0.0706	0.0067
0.50	0.6915	0.3085	0.0048	0.0002
0.25	0.5987	0.4013	0.0008	0.0000
0.00	0.5000	0.5000	0.0001	0.0000



9-12.
Design a modified control chart.

n = 8, USL = 8.01, LSL = 7.99, S = 0.001
δ = 0.00135, $Z_\delta = Z_{0.00135}$ = 3.000
For 3-sigma control limits, $Z_\alpha = 3$

$$\text{UCL} = \text{USL} - \left(Z_\delta - Z_\alpha \big/ \sqrt{n}\right)\sigma = 8.01 - \left(3.000 - 3\big/\sqrt{8}\right)(0.001) = 8.008$$

$$\text{LCL} = \text{LSL} + \left(Z_\delta - Z_\alpha \big/ \sqrt{n}\right)\sigma = 7.99 + \left(3.000 - 3\big/\sqrt{8}\right)(0.001) = 7.992$$

9-13.
Design a modified control chart.

n = 4, USL = 70, LSL = 30, S = 4
δ = 0.01, Z_δ = 2.326
1 − α = 0.995, α = 0.005, Z_α = 2.576

$$\text{UCL} = \text{USL} - \left(Z_\delta - Z_\alpha \big/ \sqrt{n}\right)\sigma = (50 + 20) - \left(2.326 - 2.576\big/\sqrt{4}\right)(4) = 65.848$$

$$\text{LCL} = \text{LSL} + \left(Z_\delta - Z_\alpha \big/ \sqrt{n}\right)\sigma = (50 - 20) + \left(2.326 - 2.576\big/\sqrt{4}\right)(4) = 34.152$$

9-14.
Design a modified control chart.

n = 4, USL = 820, LSL = 780, S = 4
δ = 0.01, Z_δ = 2.326
1 − α = 0.90, α = 0.10, Z_α = 1.282

$$\text{UCL} = \text{USL} - \left(Z_\delta - Z_\alpha \big/ \sqrt{n}\right)\sigma = (800 + 20) - \left(2.326 - 1.282\big/\sqrt{4}\right)(4) = 813.26$$

$$\text{LCL} = \text{LSL} + \left(Z_\delta - Z_\alpha \big/ \sqrt{n}\right)\sigma = (800 - 20) + \left(2.326 - 1.282\big/\sqrt{4}\right)(4) = 786.74$$

9-15.

$n = 4,\ \overline{R} = 8.236,\ \overline{\overline{x}} = 620.00$

(a)

$\hat{\sigma}_x = \overline{R}/d_2 = 8.236/2.059 = 4.000$

(b)

$$\begin{aligned}
\hat{p} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\
&= \Pr\{x < 595\} + \left[1 - \Pr\{x \le 625\}\right] \\
&= \Phi\left(\frac{595-620}{4.000}\right) + \left[1 - \Phi\left(\frac{625-620}{4.000}\right)\right] \\
&= 0.0000 + \left[1 - 0.8944\right] \\
&= 0.1056
\end{aligned}$$

(c)

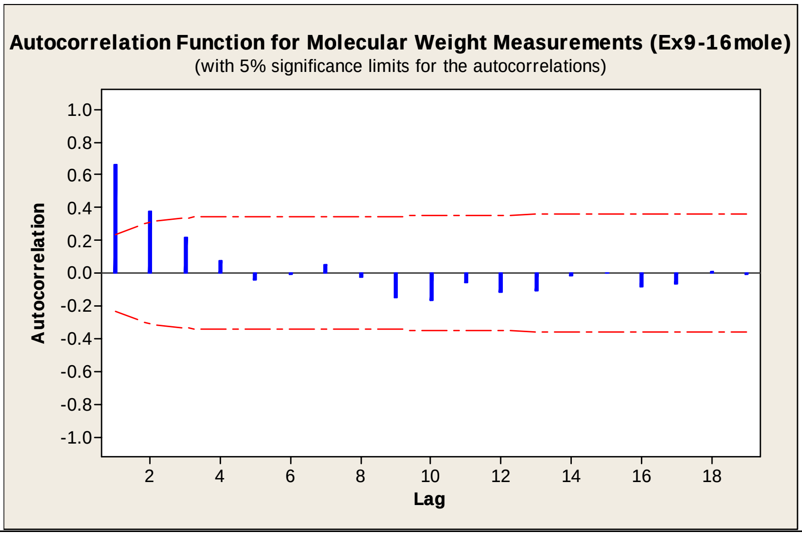
$\delta = 0.005,\ Z_\delta = Z_{0.005} = 2.576$

$\alpha = 0.01,\ Z_\alpha = Z_{0.01} = 2.326$

$$\text{UCL} = \text{USL} - \left(Z_\delta - Z_\alpha \big/ \sqrt{n}\right)\sigma = 625 - \left(2.576 - 2.326\big/\sqrt{4}\right)4 = 619.35$$

$$\text{LCL} = \text{LSL} + \left(Z_\delta - Z_\alpha \big/ \sqrt{n}\right)\sigma = 595 + \left(2.576 - 2.326\big/\sqrt{4}\right)4 = 600.65$$

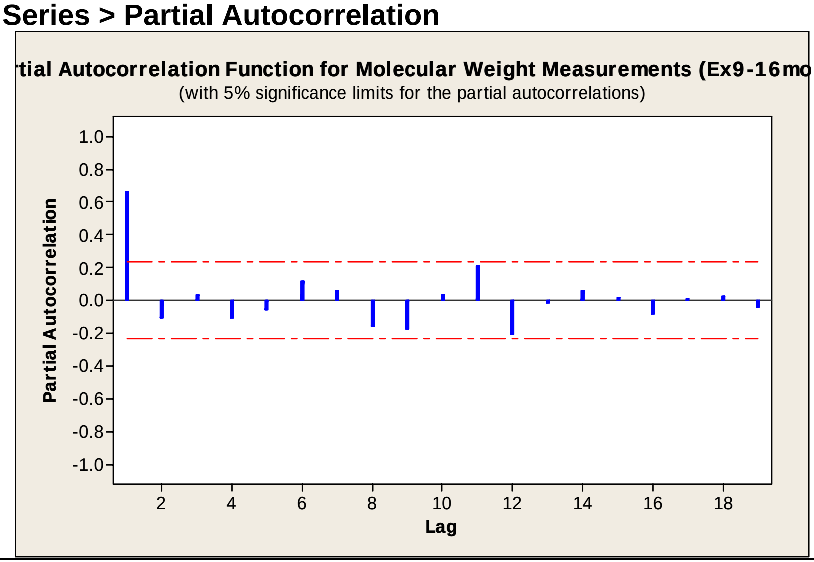
9-16. Note: In the textbook, the 5th column, the 5th row should be "2000" not "2006".
(a)
Stat > Time Series > Autocorrelation



Autocorrelation Function: Ex9-16mole

```
Lag        ACF      T    LBQ
  1   0.658253   5.70  33.81
  2   0.373245   2.37  44.84
  3   0.220536   1.30  48.74
  4   0.072562   0.42  49.16
  5  -0.039599  -0.23  49.29  …
```

Stat > Time Series > Partial Autocorrelation



Partial Autocorrelation Function: Ex9-16mole

```
Lag       PACF      T
  1   0.658253   5.70
  2  -0.105969  -0.92
  3   0.033132   0.29
  4  -0.110802  -0.96
  5  -0.055640  -0.48  …
```

The decaying sine wave of the ACFs combined with a spike at lag 1 for the PACFs suggests an autoregressive process of order 1, AR(1).

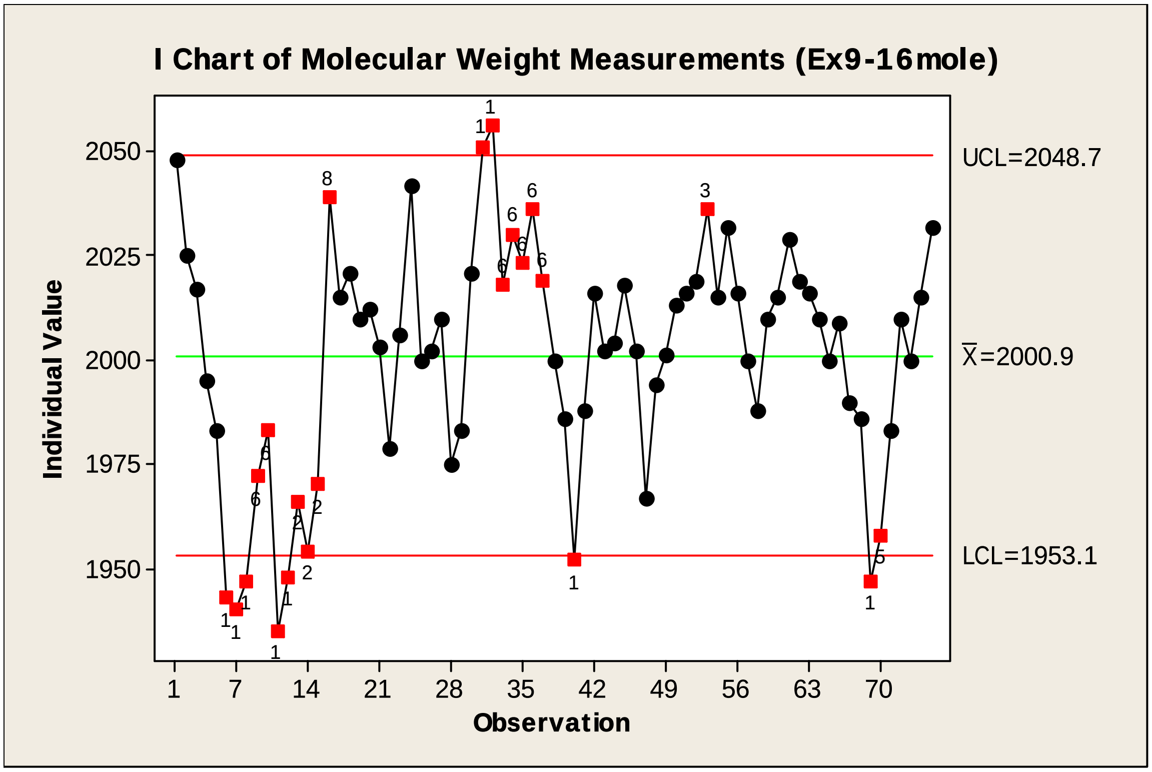
9-16 continued

(b)

x chart: CL = 2001, UCL = 2049, LCL = 1953

$\hat{\sigma} = \overline{\text{MR}}/d_2 = 17.97/1.128 = 15.93$

Stat > Control Charts > Variables Charts for Individuals > Individuals



Test Results for I Chart of Ex9-16mole

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  6, 7, 8, 11, 12, 31, 32, 40, 69
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  12, 13, 14, 15
TEST 3. 6 points in a row all increasing or all decreasing.
Test Failed at points:  7, 53
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  7, 8, 12, 13, 14, 32, 70
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  8, 9, 10, 11, 12, 13, 14, 15, 33, 34, 35, 36, 37
TEST 8. 8 points in a row more than 1 standard deviation from center line
     (above and below CL).
Test Failed at points:  12, 13, 14, 15, 16, 35, 36, 37
```

The process is out of control on the *x* chart, violating many runs tests, with big swings and very few observations actually near the mean.

9-16 continued

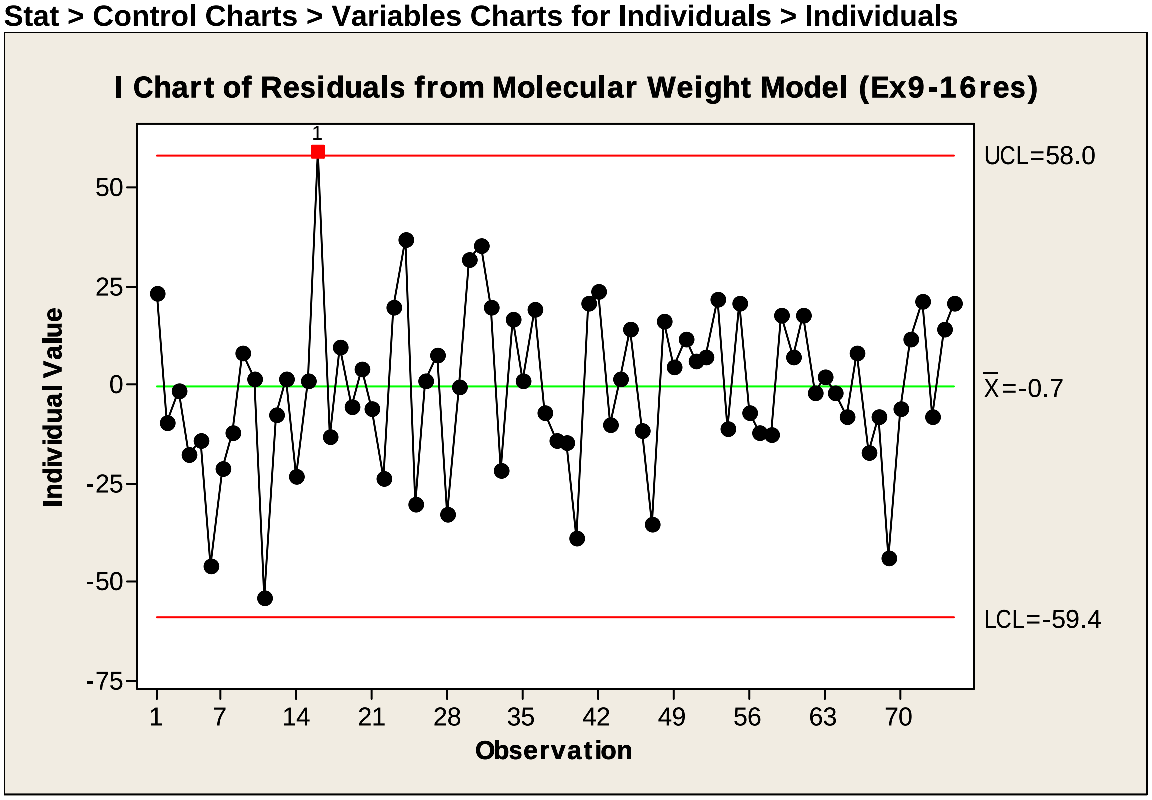
(c)

Stat > Time Series > ARIMA

ARIMA Model: Ex9-16mole

```
Estimates at each iteration
Iteration       SSE      Parameters
        0  50173.7  0.100  1800.942
        1  41717.0  0.250  1500.843
        2  35687.3  0.400  1200.756
        3  32083.6  0.550   900.693
        4  30929.9  0.675   650.197
        5  30898.4  0.693   613.998
        6  30897.1  0.697   606.956
        7  30897.1  0.698   605.494
        8  30897.1  0.698   605.196
Relative change in each estimate less than 0.0010

Final Estimates of Parameters
Type          Coef  SE Coef       T      P
AR   1      0.6979   0.0852    8.19  0.000
Constant   605.196    2.364  256.02  0.000
Mean       2003.21     7.82…
```

Stat > Control Charts > Variables Charts for Individuals > Individuals



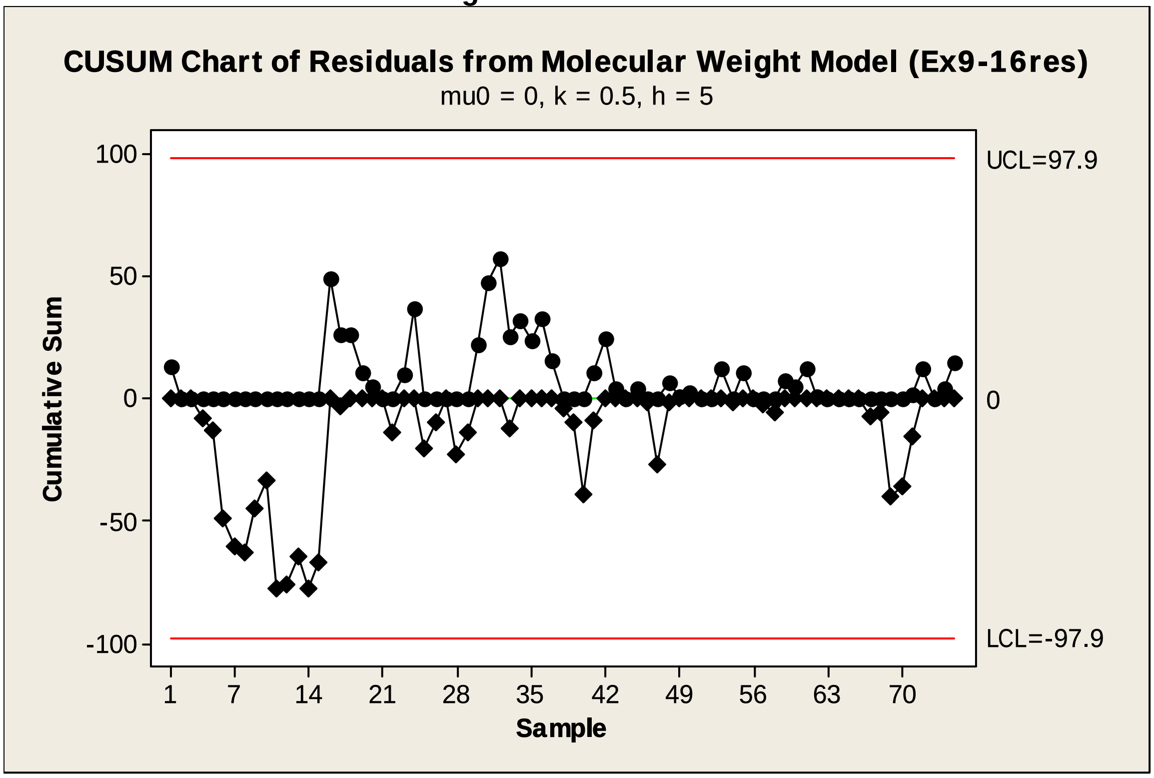
Test Results for I Chart of Ex9-16res

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  16
```

Observation 16 signals out of control above the upper limit. There are no other violations of special cause tests.

9-17.
Let $\mu_0 = 0$, $\delta = 1$ sigma, $k = 0.5$, $h = 5$.

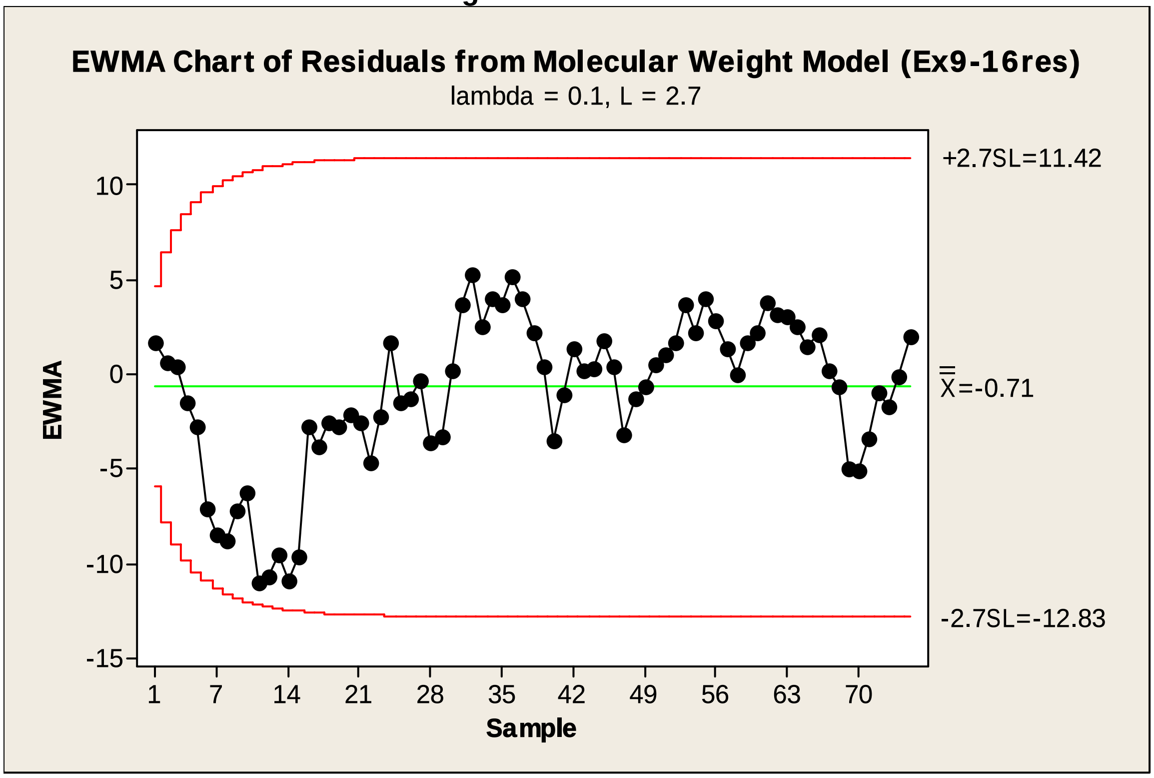
Stat > Control Charts > Time-Weighted Charts > CUSUM



No observations exceed the control limit. The residuals are in control.

9-18.
Let $\lambda = 0.1$ and $L = 2.7$ (approximately the same as a CUSUM with $k = 0.5$ and $h = 5$).

Stat > Control Charts > Time-Weighted Charts > EWMA



Process is in control.

9-19.

To find the optimal λ, fit an ARIMA (0,1,1) (= EWMA = IMA(1,1)).

Stat > Time Series > ARIMA

ARIMA Model: Ex9-16mole

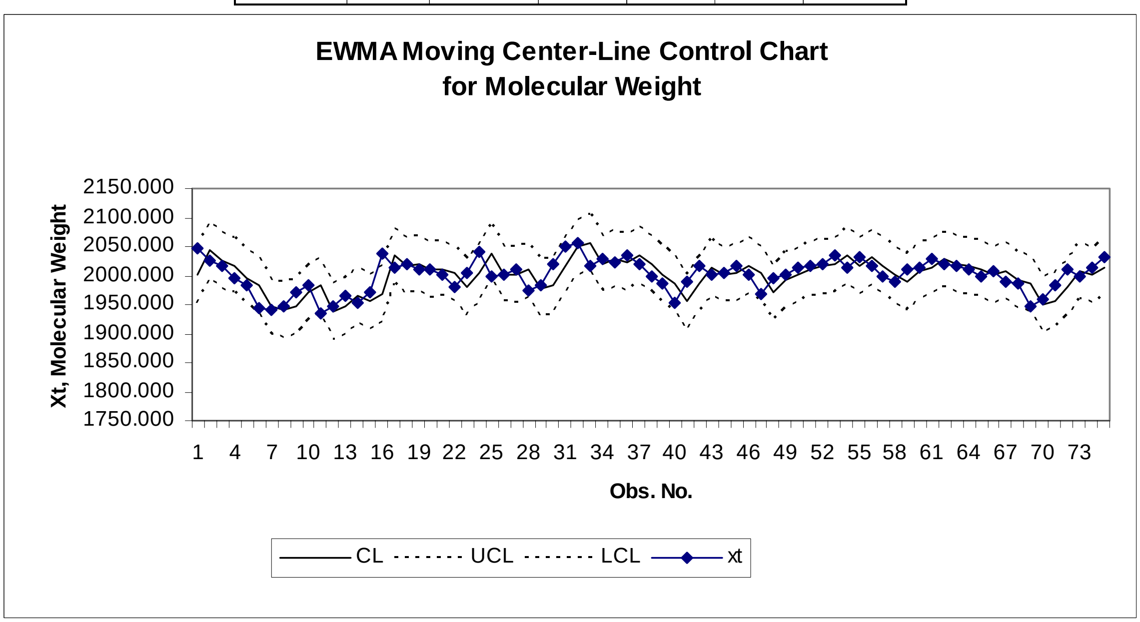
```
…
Final Estimates of Parameters
Type         Coef  SE Coef      T      P
MA   1     0.0762   0.1181   0.65  0.521
Constant   -0.211    2.393  -0.09  0.930
…
```

$\lambda = 1 - \text{MA1} = 1 - 0.0762 = 0.9238$

$\hat{\sigma} = \overline{\text{MR}}/d_2 = 17.97/1.128 = 15.93$

Excel : Workbook Chap09.xls : Worksheet Ex9-19

t	xt	zt	CL	UCL	LCL	OOC?
0		2000.947				
1	2048	2044.415	2000.947	2048.749	1953.145	No
2	2025	2026.479	2044.415	2092.217	1996.613	No
3	2017	2017.722	2026.479	2074.281	1978.677	No
4	1995	1996.731	2017.722	2065.524	1969.920	No
5	1983	1984.046	1996.731	2044.533	1948.929	No
6	1943	1946.128	1984.046	2031.848	1936.244	No
7	1940	1940.467	1946.128	1993.930	1898.326	No
8	1947	1946.502	1940.467	1988.269	1892.665	No
9	1972	1970.057	1946.502	1994.304	1898.700	No
10	1983	1982.014	1970.057	2017.859	1922.255	No
11	1935	1938.582	1982.014	2029.816	1934.212	No
12	1948	1947.282	1938.582	1986.384	1890.780	No
13	1966	1964.574	1947.282	1995.084	1899.480	No
14	1954	1954.806	1964.574	2012.376	1916.772	No
15	1970	1968.842	1954.806	2002.608	1907.004	No
16	2039	2033.654	1968.842	2016.644	1921.040	above UCL
…						

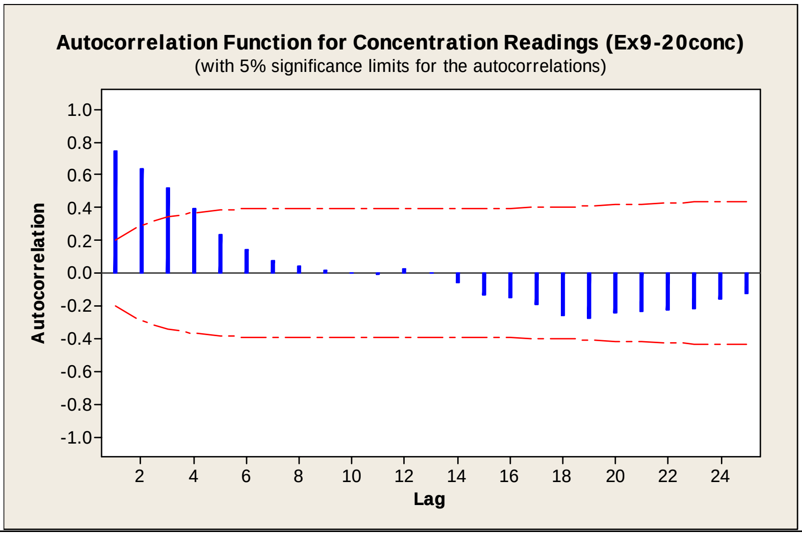


Observation 6 exceeds the upper control limit compared to one out-of-control signal at observation 16 on the Individuals control chart.

9-20

(a)

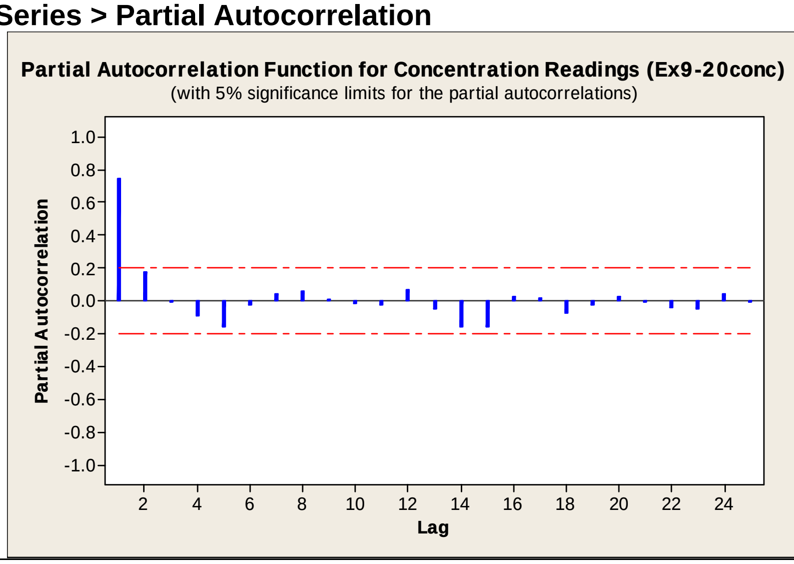
Stat > Time Series > Autocorrelation



Autocorrelation Function: Ex9-20conc

Lag	ACF	T	LBQ
1	0.746174	7.46	57.36
2	0.635375	4.37	99.38
3	0.520417	3.05	127.86
4	0.390108	2.10	144.03
5	0.238198	1.23	150.12 …

Stat > Time Series > Partial Autocorrelation



Partial Autocorrelation Function: Ex9-20conc

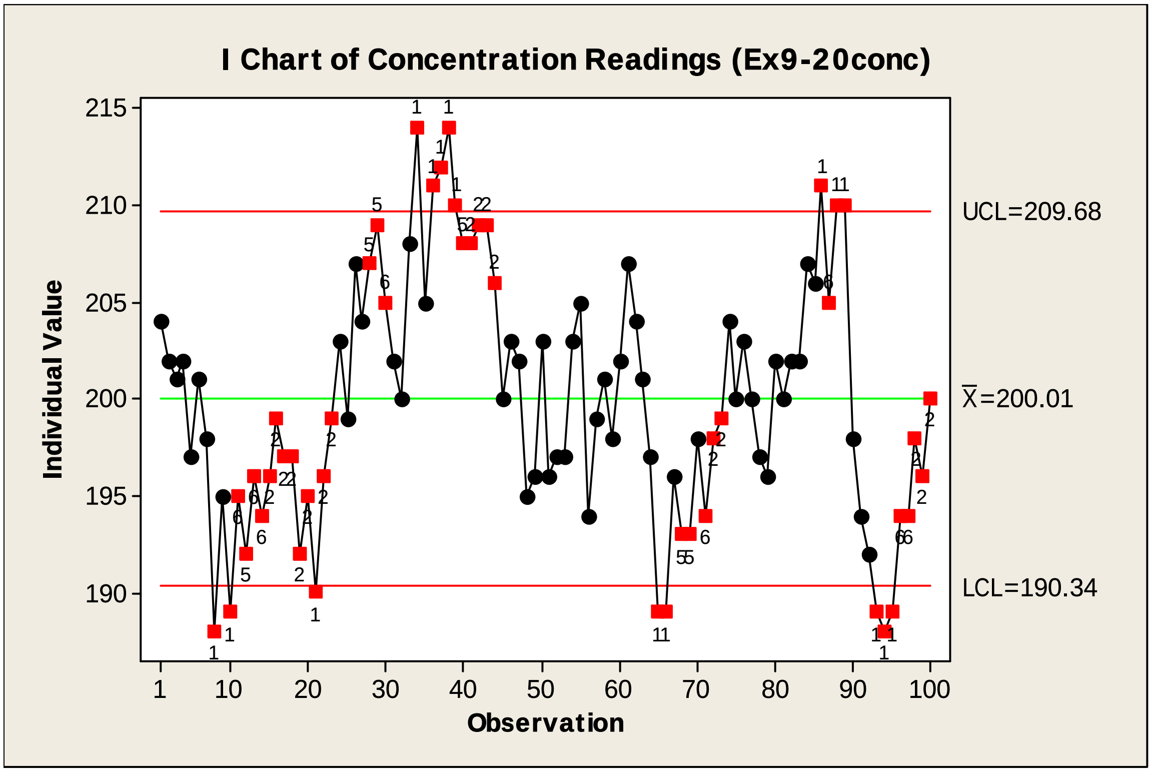
Lag	PACF	T
1	0.746174	7.46
2	0.177336	1.77
3	-0.004498	-0.04
4	-0.095134	-0.95
5	-0.158358	-1.58 …

The decaying sine wave of the ACFs combined with a spike at lag 1 for the PACFs suggests an autoregressive process of order 1, AR(1).

9-20 continued
(b)

$\hat{\sigma} = \overline{\text{MR}}/d_2 = 3.64/1.128 = 3.227$

Stat > Control Charts > Variables Charts for Individuals > Individuals




```
Test Results for I Chart of Ex9-20conc
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  8, 10, 21, 34, 36, 37, 38, 39, 65, 66, 86, 88, 89, 93,
                        94, 95
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  15, 16, 17, 18, 19, 20, 21, 22, 23, 41, 42, 43, 44, 72,
                        73, 98, 99, 100
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  10, 12, 21, 28, 29, 34, 36, 37, 38, 39, 40, 41, 42, 43,
                        66, 68, 69, 86, 88, 89, 93, 94, 95
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  11, 12, 13, 14, 15, 22, 29, 30, 36, 37, 38, 39, 40, 41,
                        42, 43, 44, 68, 69, 71, 87, 88, 89, 94, 95, 96, 97, 99
TEST 8. 8 points in a row more than 1 standard deviation from center line
     (above and below CL).
Test Failed at points:  15, 40, 41, 42, 43, 44
```

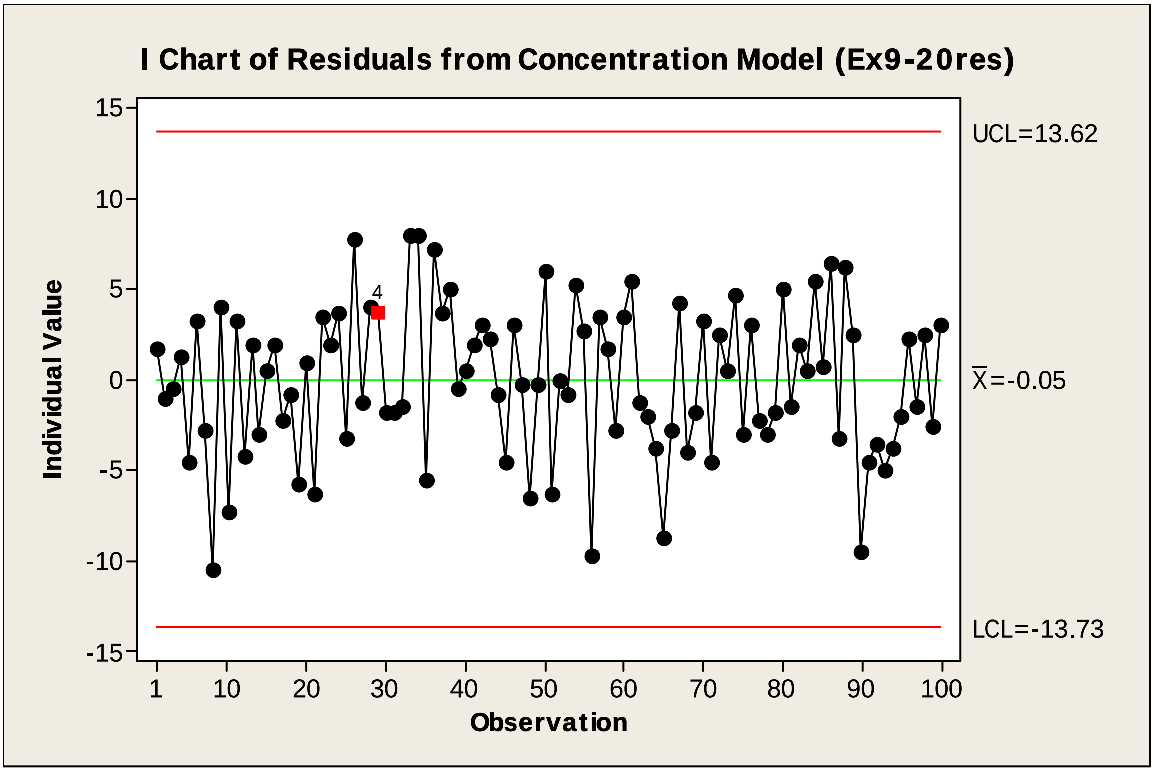
The process is out of control on the *x* chart, violating many runs tests, with big swings and very few observations actually near the mean.

9-20 continued
(c)
Stat > Time Series > ARIMA

ARIMA Model: Ex9-20conc

```
…
Final Estimates of Parameters
Type           Coef   SE Coef        T       P
AR    1      0.7493    0.0669    11.20   0.000
Constant    50.1734    0.4155   120.76   0.000
Mean        200.122     1.657
…
```

Stat > Control Charts > Variables Charts for Individuals > Individuals

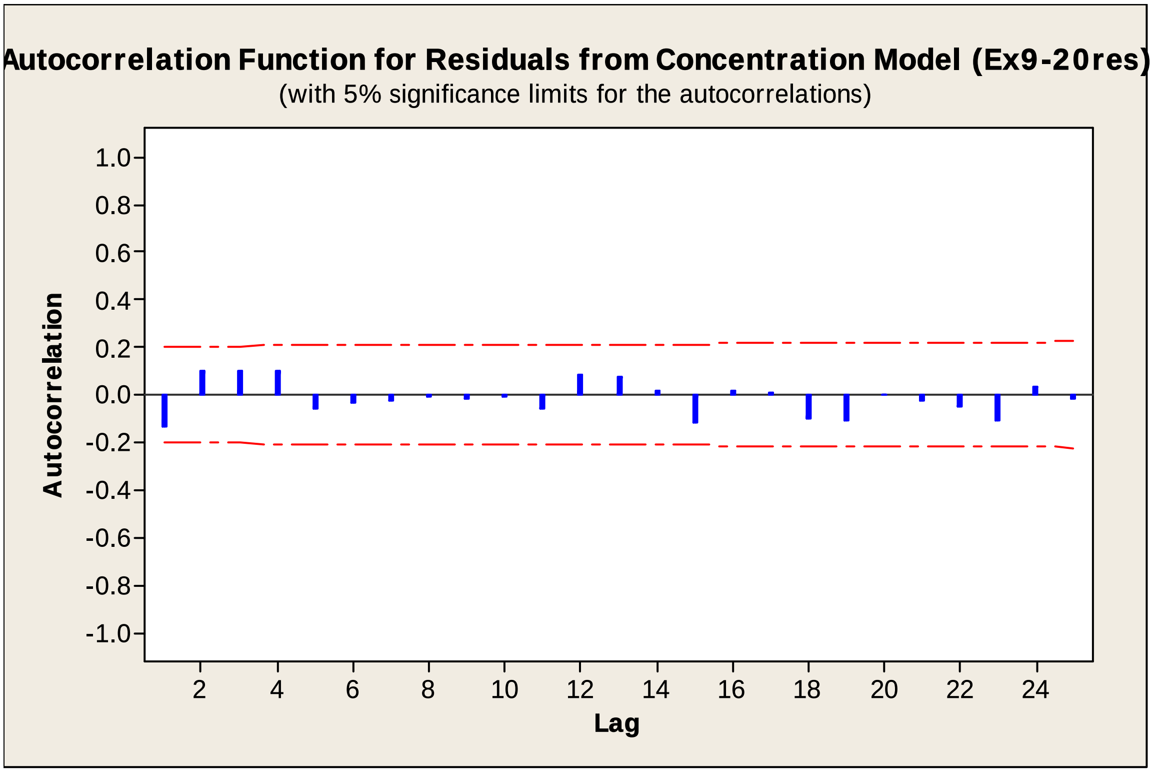


Test Results for I Chart of Ex9-20res

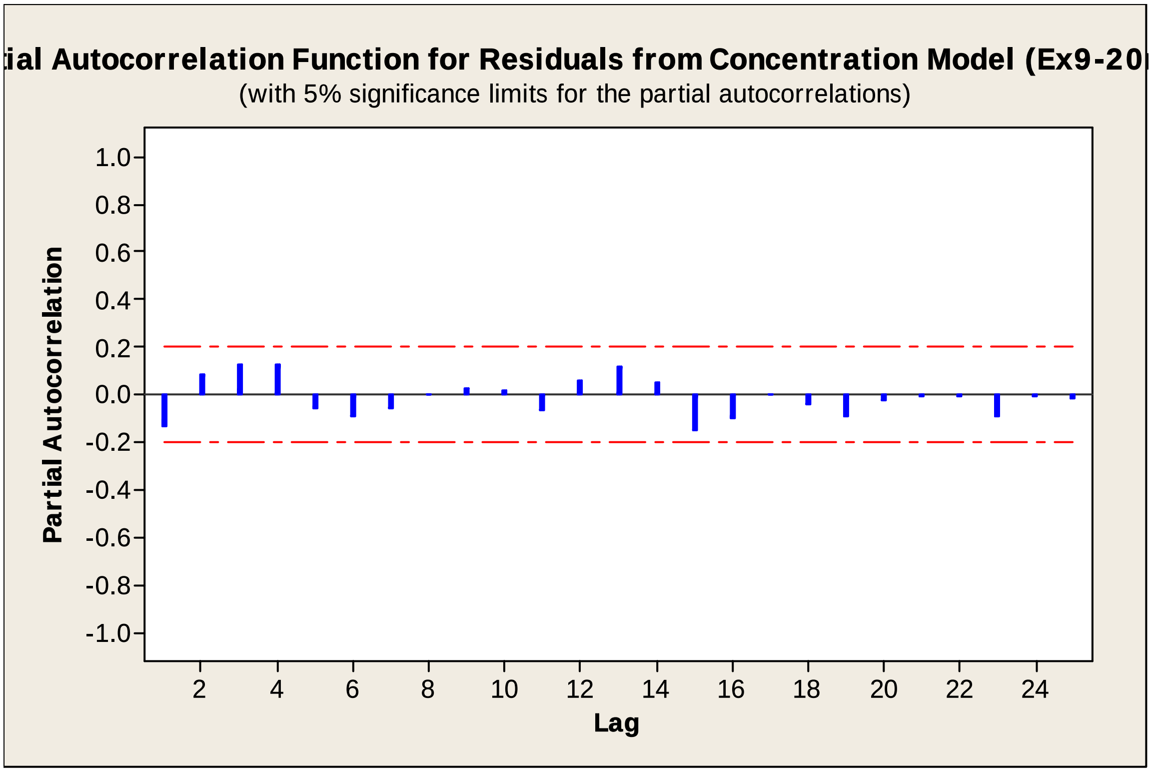
```
TEST 4. 14 points in a row alternating up and down.
Test Failed at points:  29
```

Observation 29 signals out of control for test 4, however this is not unlikely for a dataset of 100 observations. Consider the process to be in control.

9-20 continued
(d)
Stat > Time Series > Autocorrelation

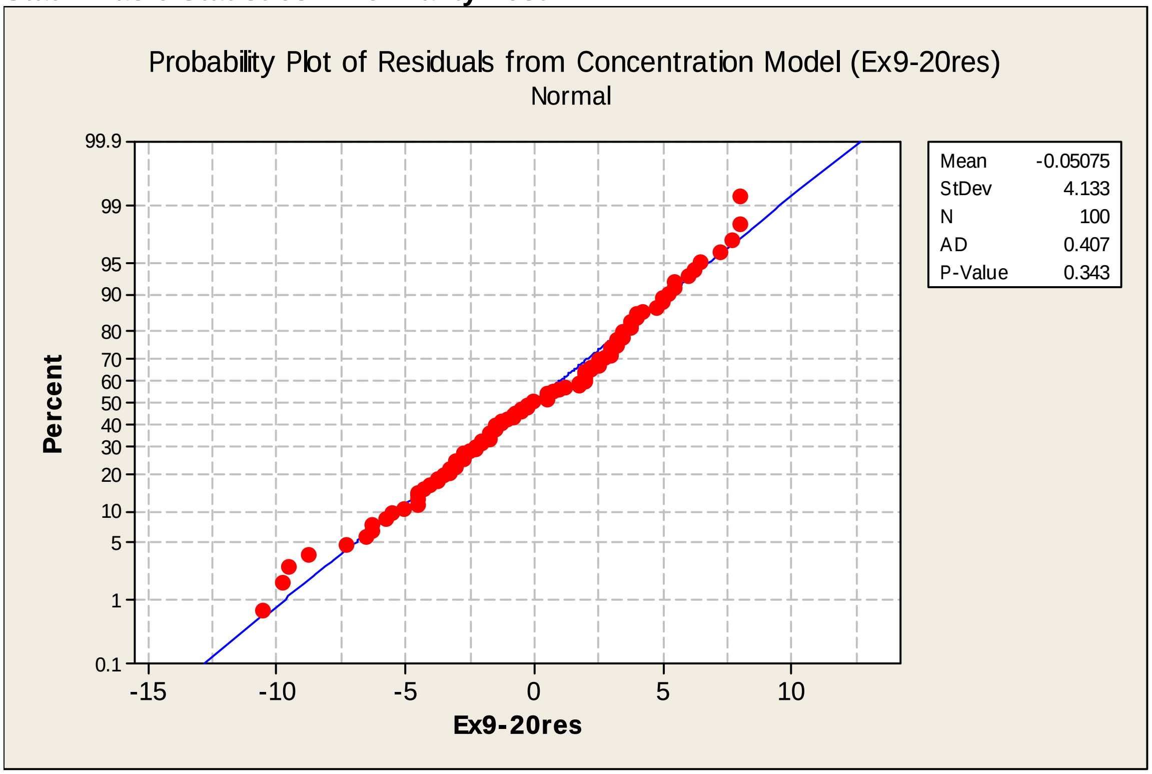


Stat > Time Series > Partial Autocorrelation



9-20 (d) continued

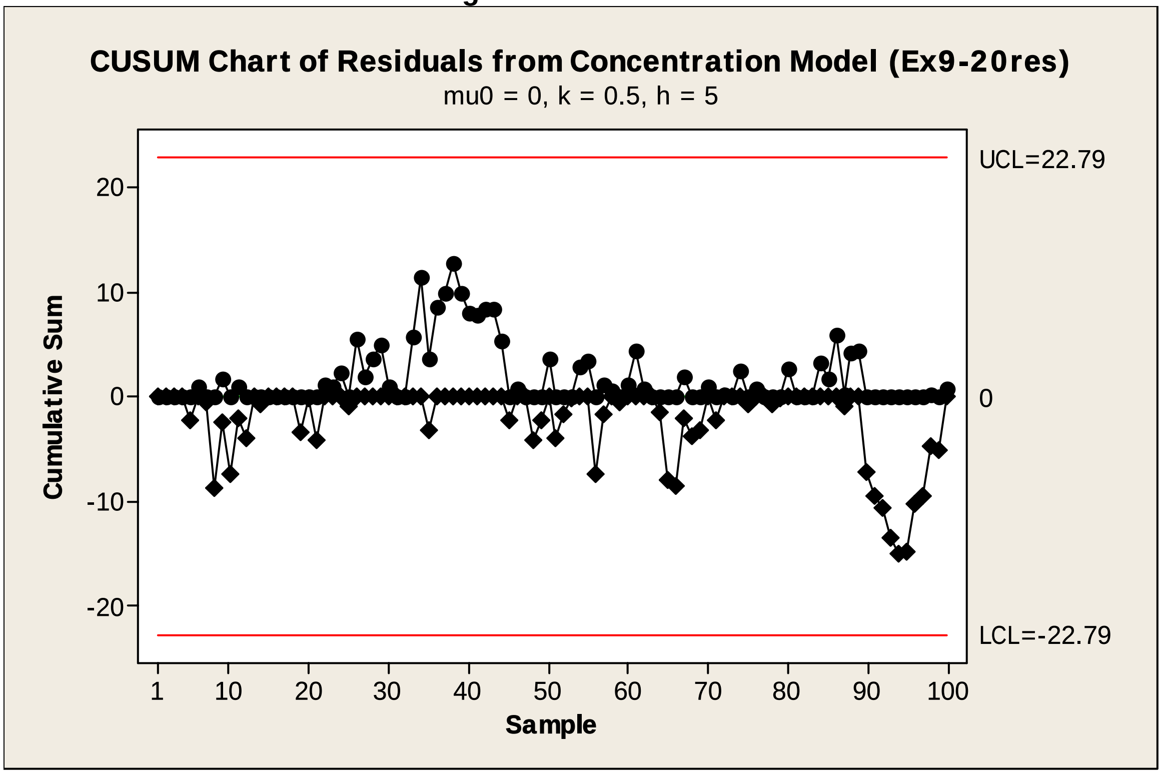
Stat > Basic Statistics > Normality Test



Visual examination of the ACF, PACF and normal probability plot indicates that the residuals are normal and uncorrelated.

9-21.
Let $\mu_0 = 0$, $\delta = 1$ sigma, $k = 0.5$, $h = 5$.

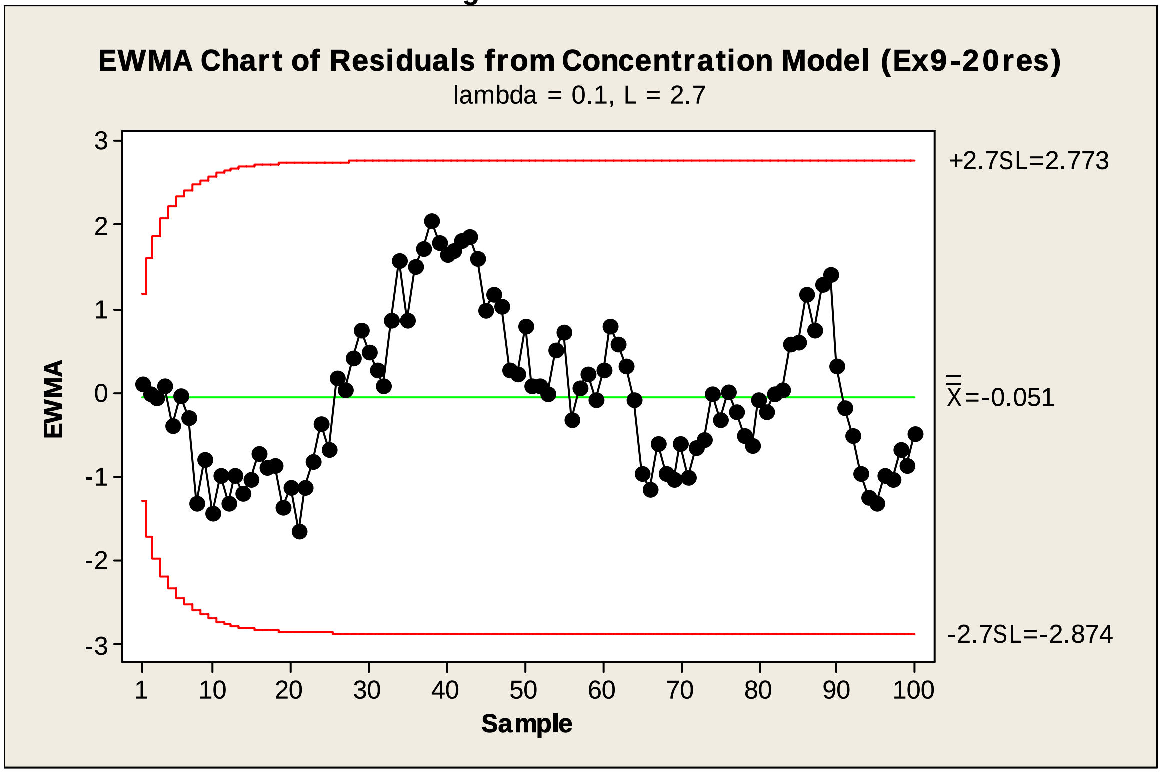
Stat > Control Charts > Time-Weighted Charts > CUSUM



No observations exceed the control limit. The residuals are in control, and the AR(1) model for concentration should be a good fit.

9-22.
Let $\lambda = 0.1$ and $L = 2.7$ (approximately the same as a CUSUM with $k = 0.5$ and $h = 5$).

Stat > Control Charts > Time-Weighted Charts > EWMA



No observations exceed the control limit. The residuals are in control.

9-23.
To find the optimal λ, fit an ARIMA (0,1,1) (= EWMA = IMA(1,1)).
Stat > Time Series > ARIMA

ARIMA Model: Ex9-20conc

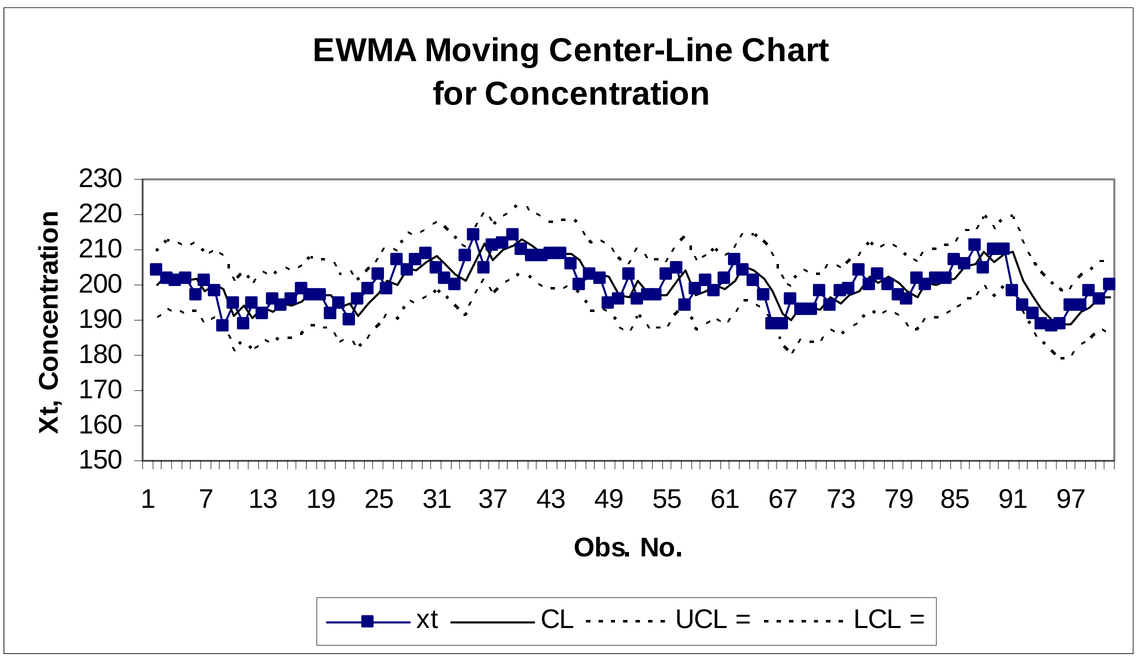
```
…
Final Estimates of Parameters
Type          Coef   SE Coef       T       P
MA   1      0.2945    0.0975    3.02   0.003
Constant   -0.0452    0.3034   -0.15   0.882
…
```

$\lambda = 1 - \text{MA1} = 1 - 0.2945 = 0.7055$

$\hat{\sigma} = \overline{\text{MR}}/d_2 = 3.64/1.128 = 3.227$

Excel : Workbook Chap09.xls : Worksheet Ex9-23

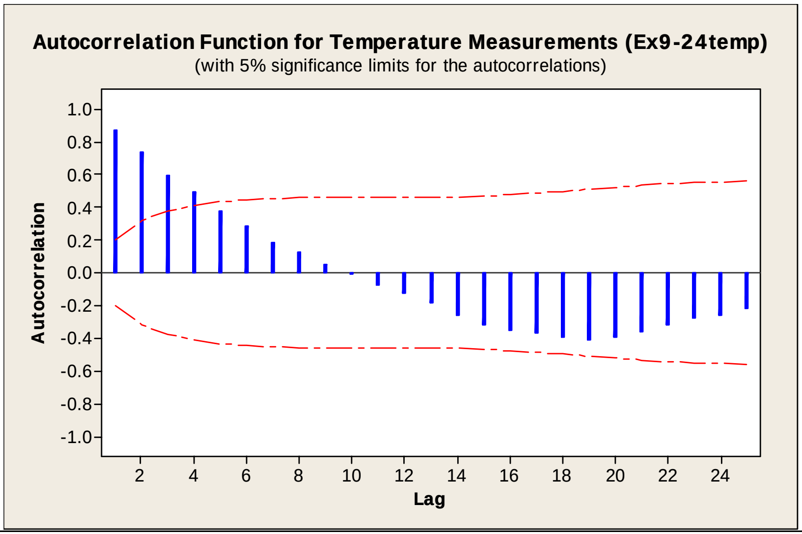
lamda =	0.706	sigma^ =	3.23			
t	xt	zt	CL	UCL =	LCL =	OOC?
0		200.010				
1	204	202.825	200.010	209.691	190.329	0
2	202	202.243	202.825	212.506	193.144	0
3	201	201.366	202.243	211.924	192.562	0
4	202	201.813	201.366	211.047	191.685	0
5	197	198.418	201.813	211.494	192.132	0
6	201	200.239	198.418	208.099	188.737	0
7	198	198.660	200.239	209.920	190.558	0
8	188	191.139	198.660	208.341	188.979	below LCL
9	195	193.863	191.139	200.820	181.458	0
10	189	190.432	193.863	203.544	184.182	0
…						



The control chart of concentration data signals out of control at three observations (8, 56, 90).

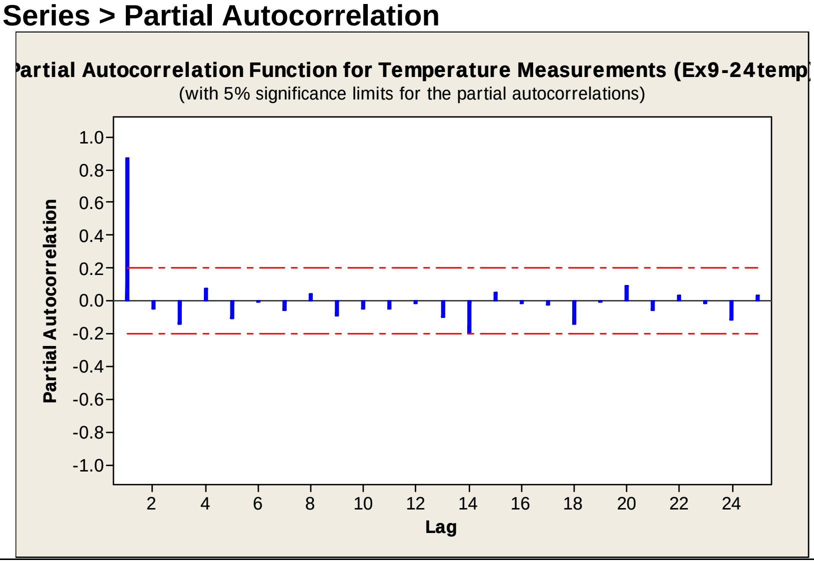
9-24.

(a) **Stat > Time Series > Autocorrelation**




```
Autocorrelation Function: Ex9-24temp
Lag        ACF      T     LBQ
  1   0.865899   8.66   77.25
  2   0.737994   4.67  133.94
  3   0.592580   3.13  170.86
  4   0.489422   2.36  196.31
  5   0.373763   1.71  211.31…
```

Stat > Time Series > Partial Autocorrelation



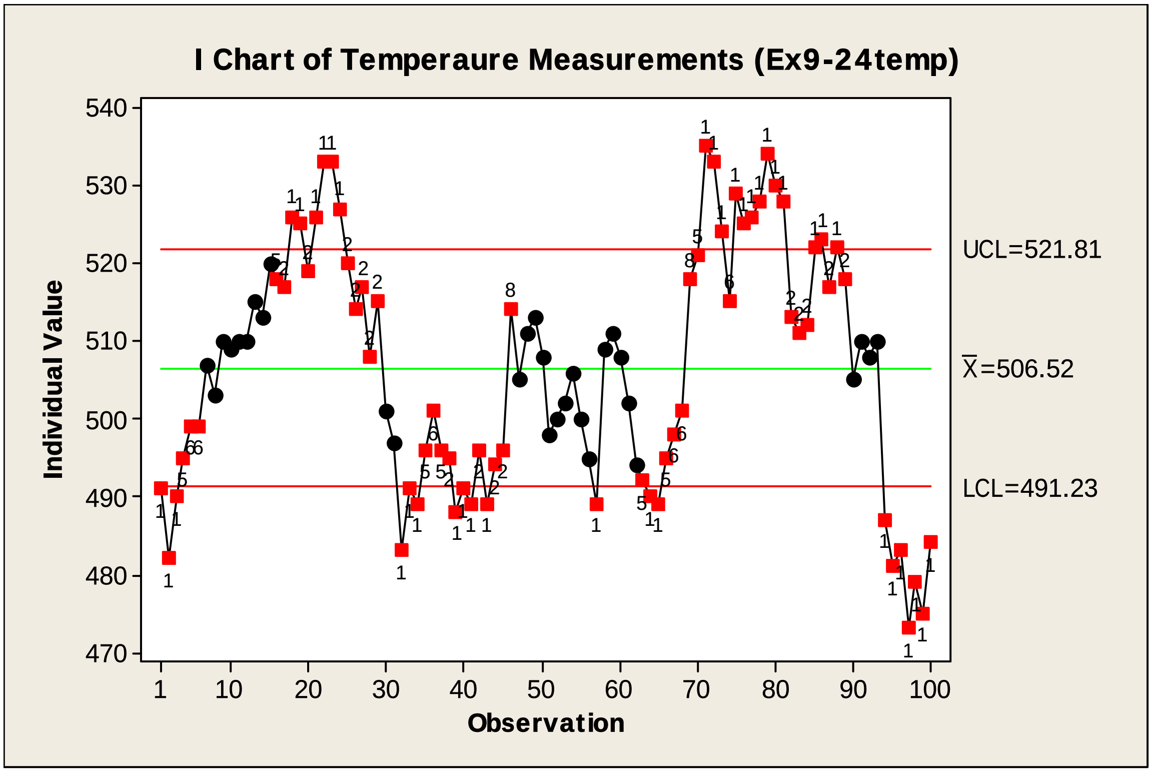

```
Partial Autocorrelation Function: Ex9-24temp
Lag       PACF      T
  1   0.865899   8.66
  2  -0.047106  -0.47
  3  -0.143236  -1.43
  4   0.078040   0.78
  5  -0.112785  -1.13…
```

Slow decay of ACFs with sinusoidal wave indicates autoregressive process. PACF graph suggest order 1.

9-24 continued

(b)

Stat > Control Charts > Variables Charts for Individuals > Individuals




```
Test Results for I Chart of Ex9-24temp
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 2, 3, 18, 19, 21, 22, 23, 24, 32, 33, 34, …

TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, …

TEST 3. 6 points in a row all increasing or all decreasing.
Test Failed at points:  65, 71

TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  2, 3, 4, 16, 17, 18, 19, 20, 21, 22, 23, 24, …

TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  4, 5, 6, 16, 17, 18, 19, 20, 21, 22, 23, 24, …

TEST 8. 8 points in a row more than 1 standard deviation from center line
     (above and below CL).
Test Failed at points:  20, 21, 22, 23, 24, 25, 26, 27, 36, 37, 38, 39, …
```

Process is out of control, violating many of the tests for special causes. The temperature measurements appear to wander over time.

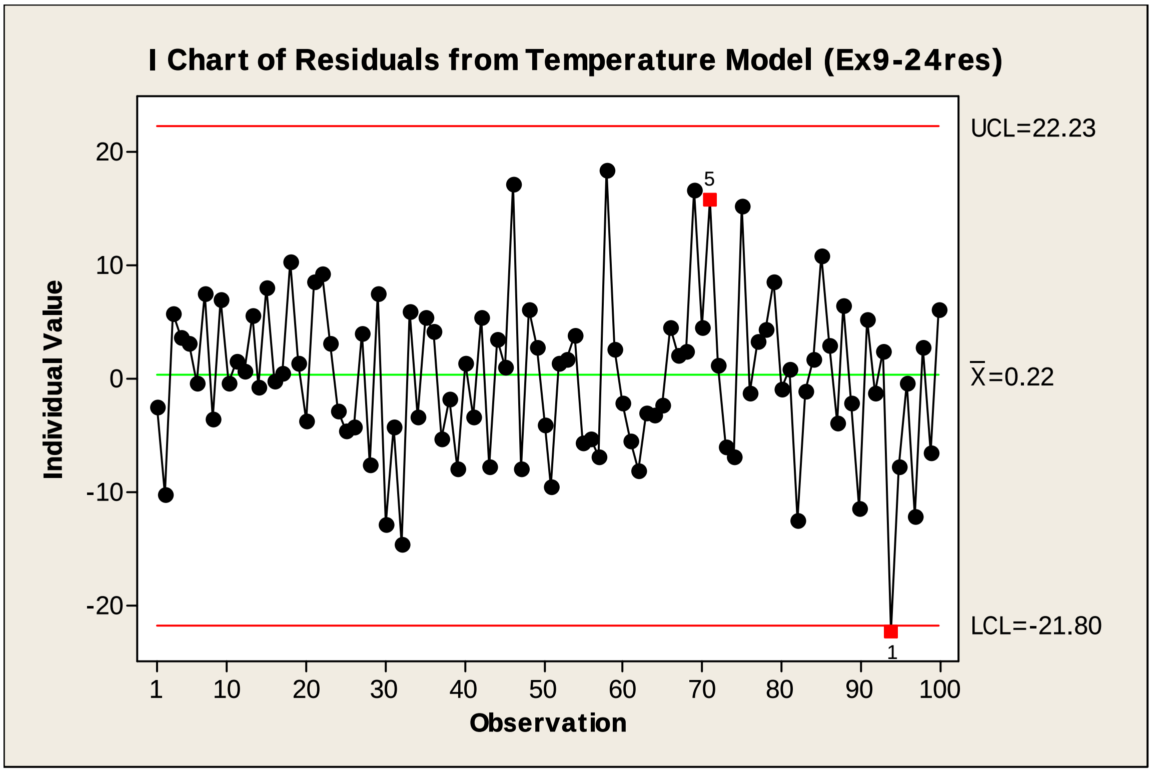
9-24 continued

(c) **Stat > Time Series > ARIMA**

ARIMA Model: Ex9-24temp

```
…
Final Estimates of Parameters
Type          Coef   SE Coef      T      P
AR    1     0.8960    0.0480  18.67  0.000
Constant   52.3794    0.7263  72.12  0.000
Mean       503.727     6.985
…
```

Stat > Control Charts > Variables Charts for Individuals > Individuals



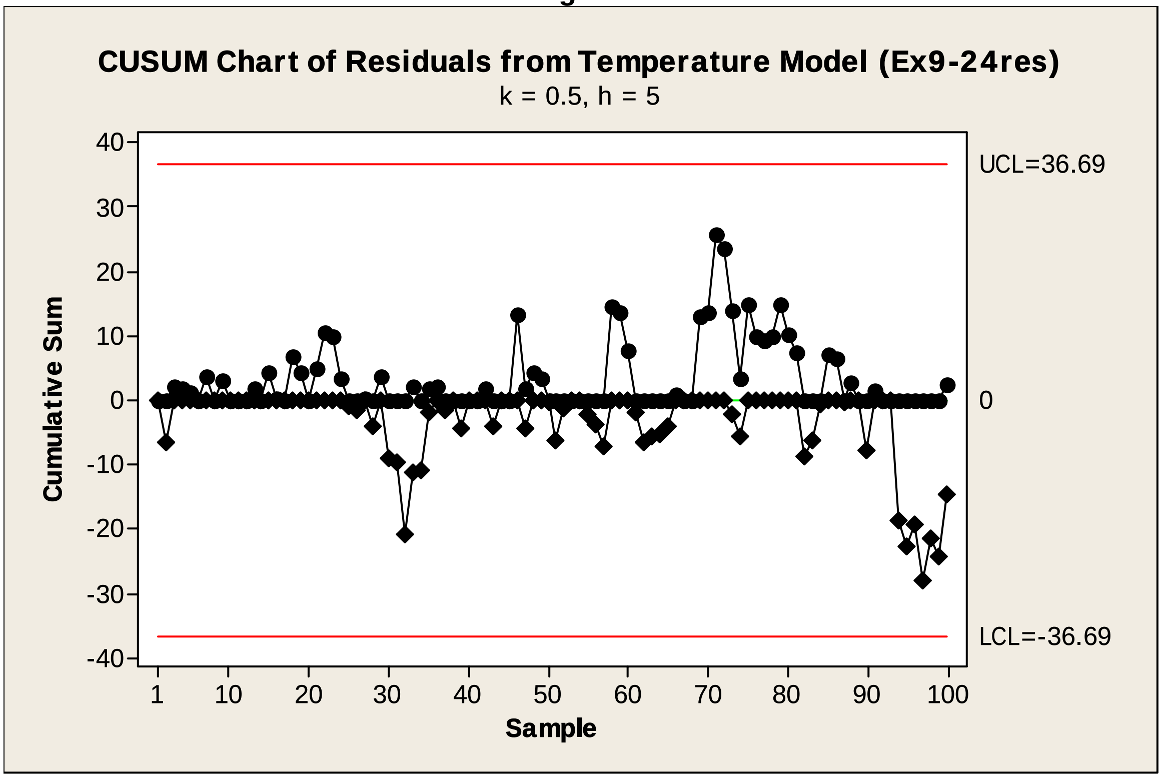
Test Results for I Chart of Ex9-24res

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  94
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  71
```

Observation 94 signals out of control above the upper limit, and observation 71 fails Test 5. The residuals do not exhibit cycles in the original temperature readings, and points are distributed between the control limits. The chemical process is in control.

9-25.

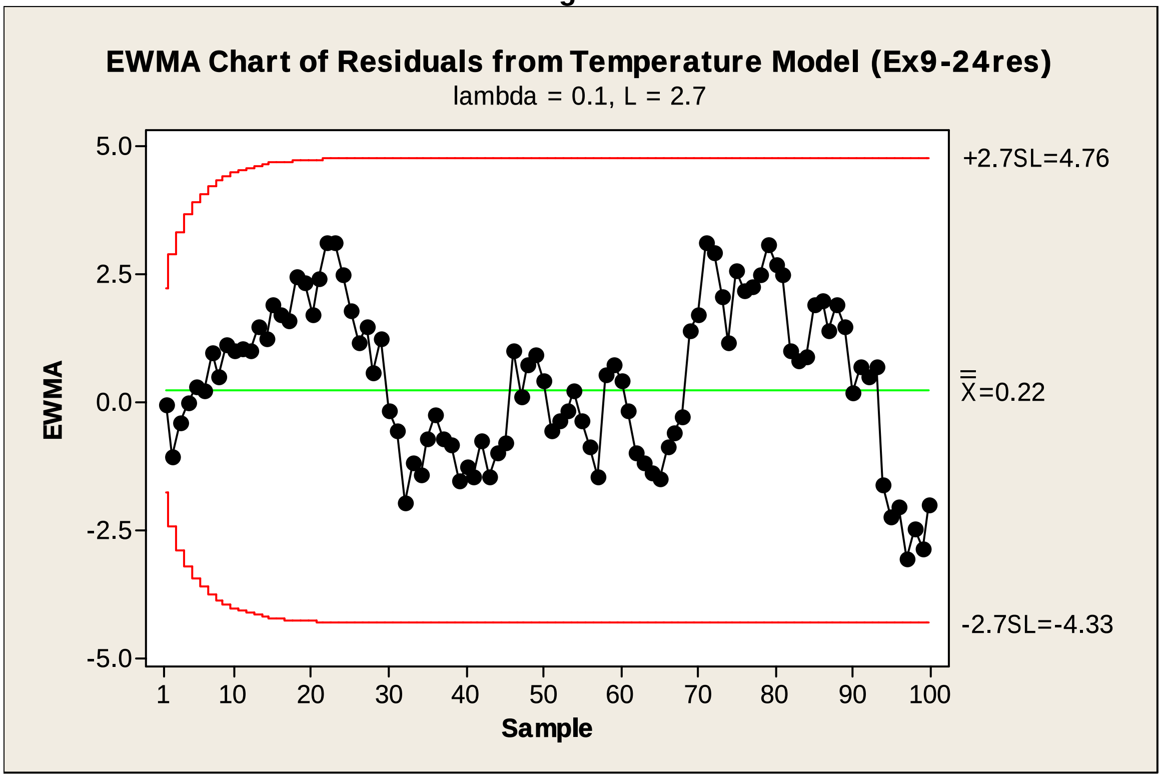
MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



No observations exceed the control limits. The residuals are in control, indicating the process is in control. This is the same conclusion as applying an Individuals control chart to the model residuals.

9-26.

MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



No observations exceed the control limits. The residuals are in control, indicating the process is in control. This is the same conclusion as applying the Individuals and CUSUM control charts to the model residuals.

9-27.
To find the optimal λ, fit an ARIMA (0,1,1) (= EWMA = IMA(1,1)).

Stat > Time Series > ARIMA

ARIMA Model: Ex9-24temp

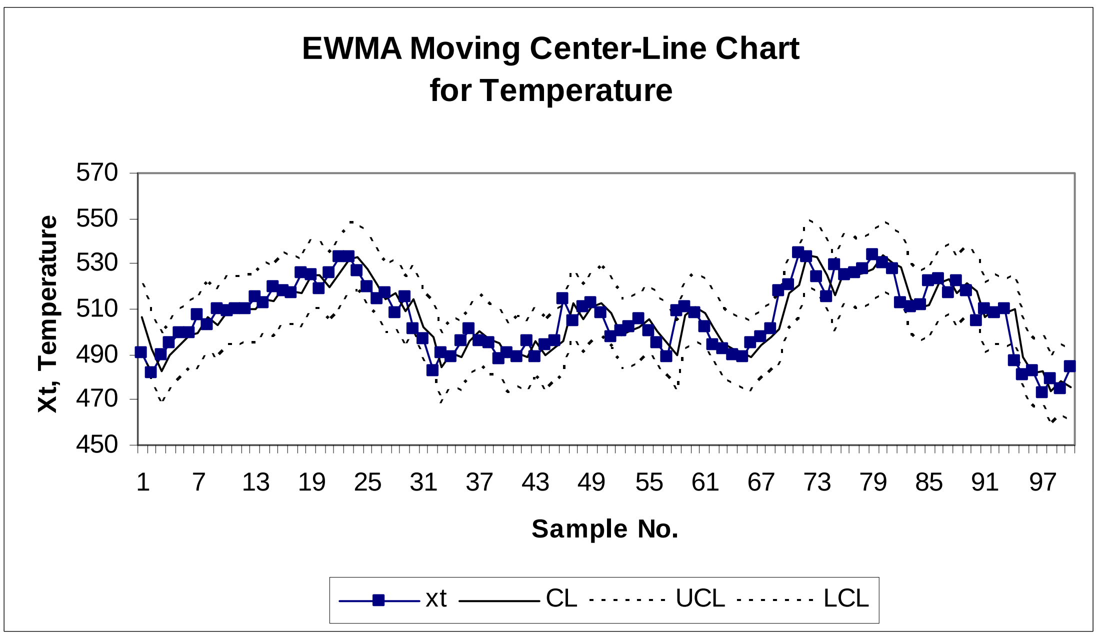
```
…
Final Estimates of Parameters
Type          Coef   SE Coef      T      P
MA   1      0.0794    0.1019   0.78  0.438
Constant   -0.0711    0.6784  -0.10  0.917
…
```

$\lambda = 1 - \text{MA1} = 1 - 0.0794 = 0.9206$

$\hat{\sigma} = \overline{\text{MR}}/d_2 = 5.75/1.128 = 5.0975$ (from a Moving Range chart with CL = 5.75)

Excel : Workbook Chap09.xls : Worksheet Ex9-27

		lambda =	0.921	sigma^ =	5.098	
t	xt	zt	CL	UCL	LCL	OOC?
0		506.520				
1	491	492.232	506.520	521.813	491.227	below LCL
2	482	482.812	492.232	507.525	476.940	0
3	490	489.429	482.812	498.105	467.520	0
4	495	494.558	489.429	504.722	474.137	0
5	499	498.647	494.558	509.850	479.265	0
6	499	498.972	498.647	513.940	483.355	0
7	507	506.363	498.972	514.265	483.679	0
8	503	503.267	506.363	521.655	491.070	0
9	510	509.465	503.267	518.560	487.974	0
10	509	509.037	509.465	524.758	494.173	0



A few observations exceed the upper limit (46, 58, 69) and the lower limit (1, 94), similar to the two out-of-control signals on the Individuals control chart (71, 94).

9-28.

(a)

When the data are positively autocorrelated, adjacent observations will tend to be similar, therefore making the moving ranges smaller. This would tend to produce an estimate of the process standard deviation that is too small.

(b)

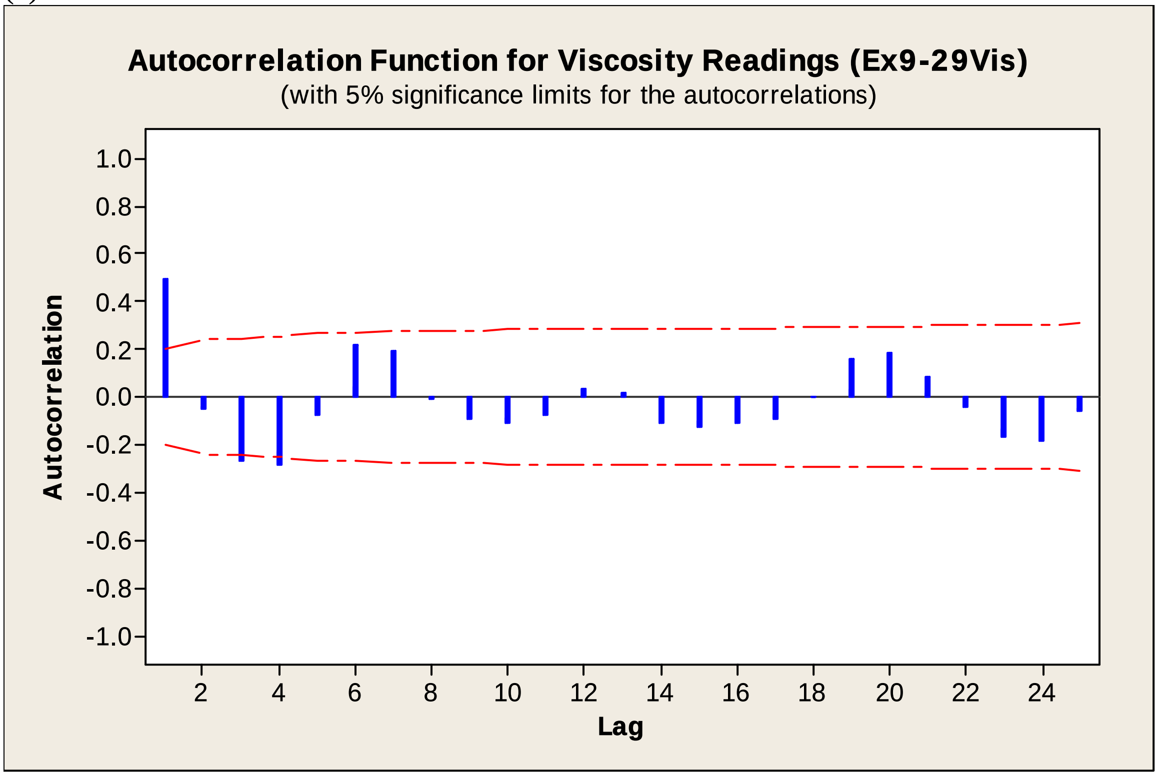
S^2 is still an unbiased estimator of σ^2 when the data are positively autocorrelated. There is nothing in the derivation of the expected value of $S^2 = \sigma^2$ that depends on an assumption of independence.

(c)

If assignable causes are present, it is not good practice to estimate σ^2 from S^2. Since it is difficult to determine whether a process generating autocorrelated data – or really any process – is in control, it is generally a bad practice to use S^2 to estimate σ^2.

9-29.

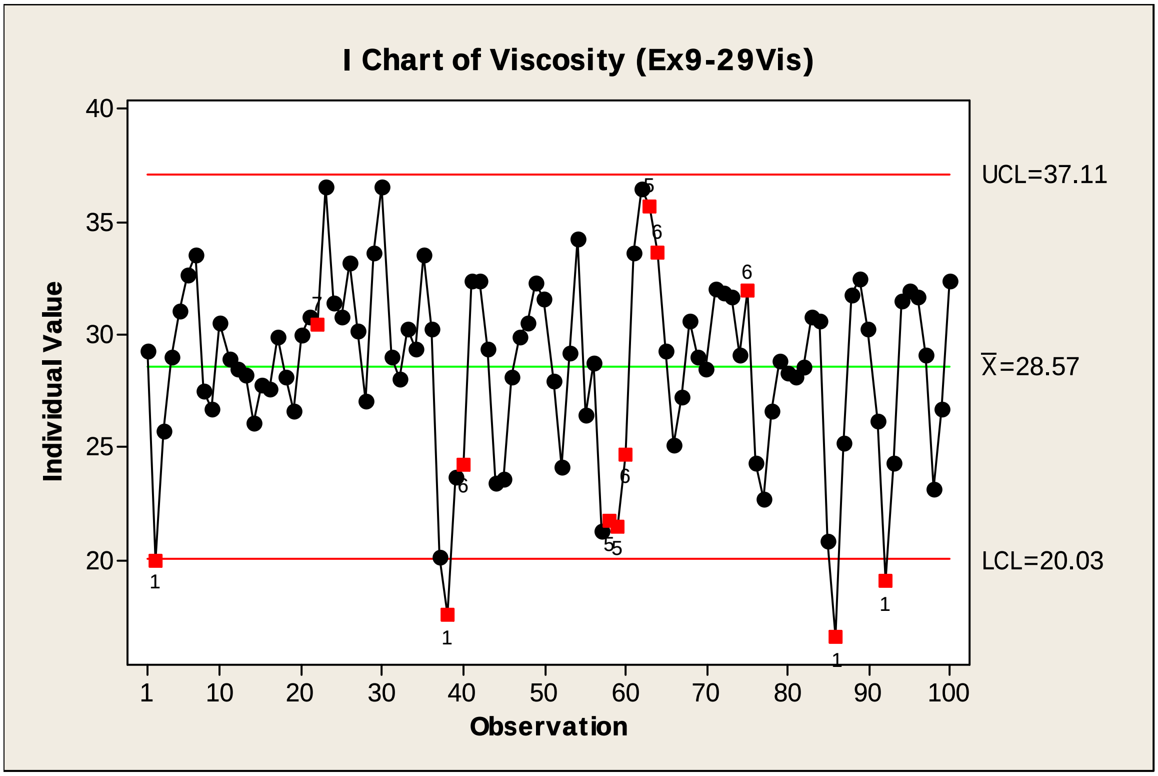
(a) **Stat > Time Series > Autocorrelation**




```
Autocorrelation Function: Ex9-29Vis
Lag         ACF      T    LBQ
  1    0.494137   4.94  25.16
  2   -0.049610  -0.41  25.41
  3   -0.264612  -2.17  32.78
  4   -0.283150  -2.22  41.29
  5   -0.071963  -0.54  41.85
…
```

$r_1 = 0.49$, indicating a strong positive correlation at lag 1. There is a serious problem with autocorrelation in viscosity readings.

9-29 continued
(b)
Stat > Control Charts > Variables Charts for Individuals > Individuals

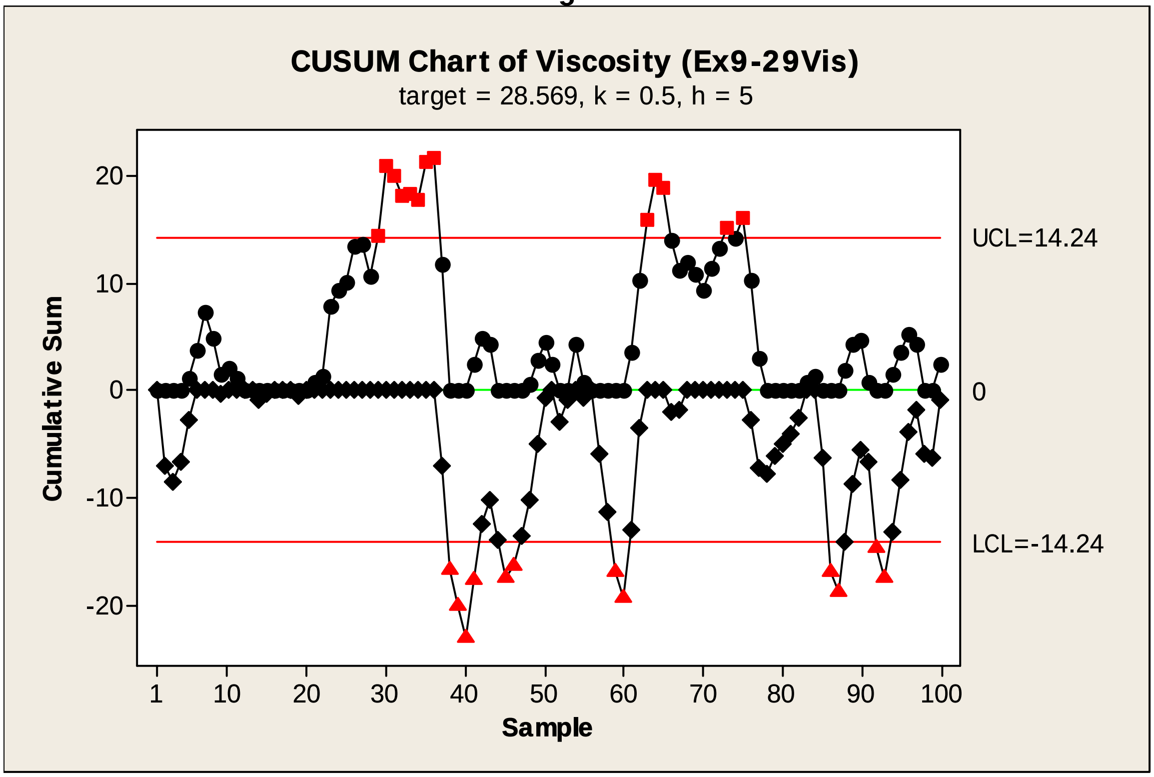



```
Test Results for I Chart of Ex9-29Vis
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  2, 38, 86, 92
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  38, 58, 59, 63, 86
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  40, 60, 64, 75
TEST 7. 15 points within 1 standard deviation of center line (above and below
     CL).
Test Failed at points:  22
TEST 8. 8 points in a row more than 1 standard deviation from center line
     (above and below CL).
Test Failed at points:  64
```

Process is out of control, violating many of the tests for special causes. The viscosity measurements appear to wander over time.

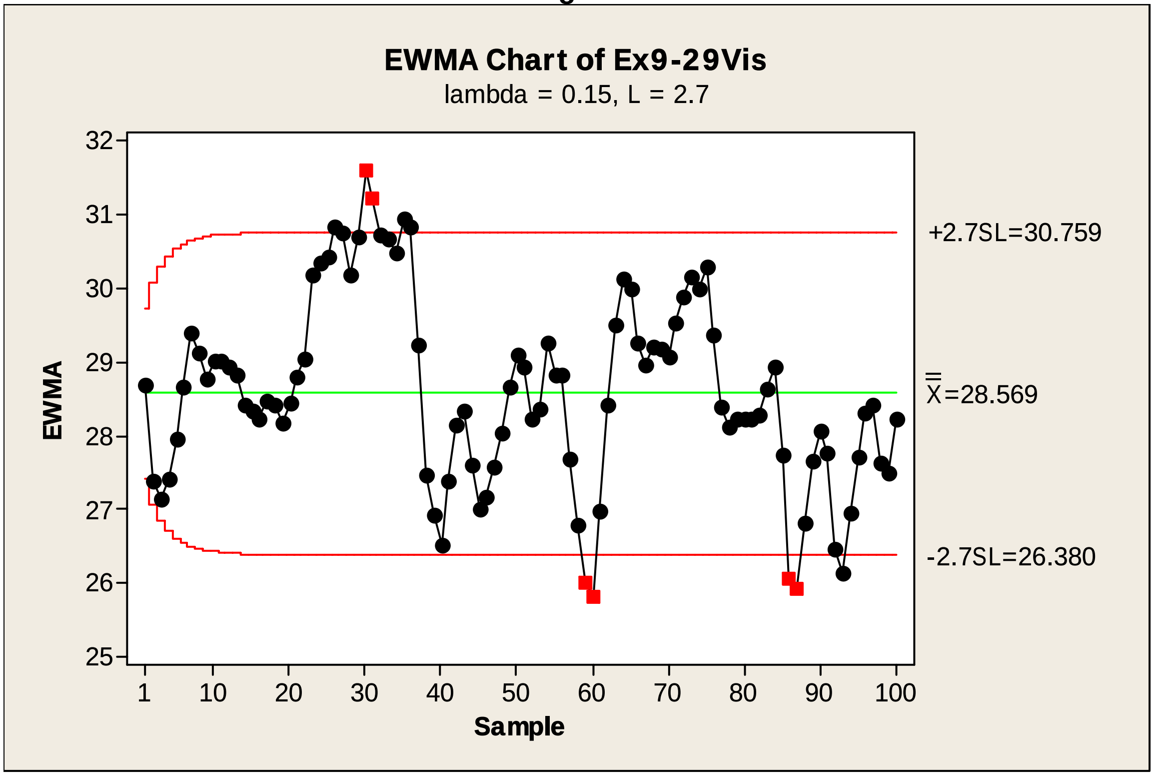
9-29 continued
(c)
Let target = $\mu_0 = 28.569$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



Several observations are out of control on both the lower and upper sides.

9-29 continued
(d)
MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



The process is not in control. There are wide swings in the plot points and several are beyond the control limits.

9-29 continued
(e)
To find the optimal λ, fit an ARIMA (0,1,1) (= EWMA = IMA(1,1)).

Stat > Time Series > ARIMA

ARIMA Model: Ex9-29Vis

```
...
Final Estimates of Parameters
Type         Coef  SE Coef      T      P
MA    1   -0.1579   0.1007  -1.57  0.120
Constant   0.0231   0.4839   0.05  0.962
```

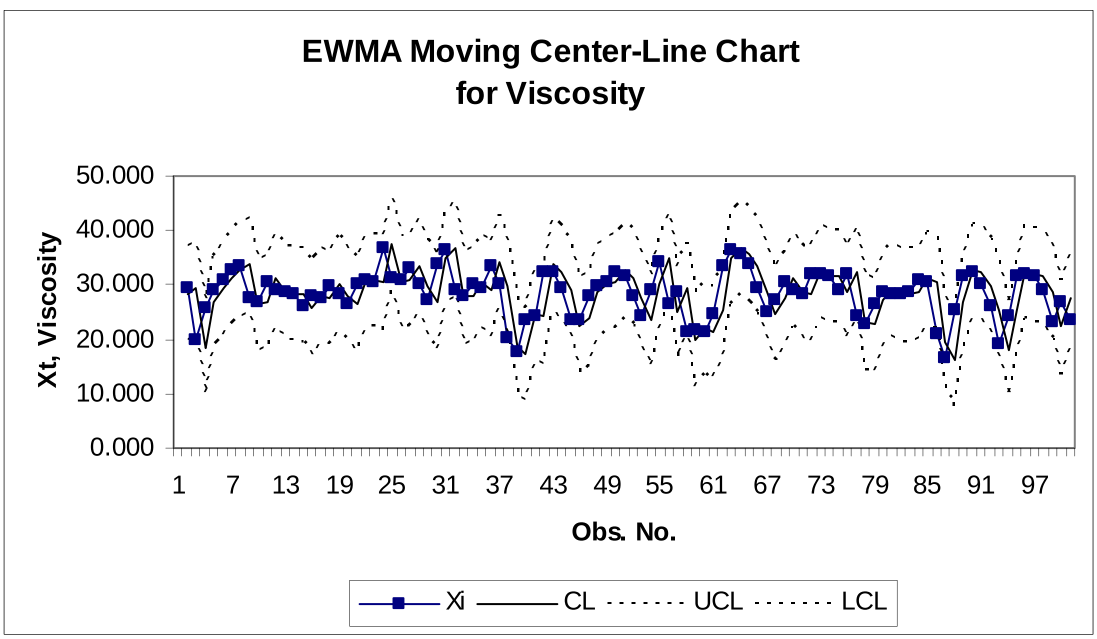
λ = 1 – MA1 = 1 – (– 0.1579) = 1.1579

$\hat{\sigma} = \overline{\text{MR}}/d_2 = 3.21/1.128 = 2.8457$ (from a Moving Range chart with CL = 5.75)

Excel : Workbook Chap09.xls : Worksheet Ex9-29

		lambda =	1.158	sigma^ =	2.85	
I	**Xi**	**Zi**	**CL**	**UCL**	**LCL**	**OOC?**
0		28.479				
1	29.330	29.464	28.479	37.022	19.937	0
2	**19.980**	**18.482**	**29.464**	**38.007**	**20.922**	**below LCL**
3	25.760	26.909	18.482	27.025	9.940	0
4	29.000	29.330	26.909	35.452	18.367	0
5	31.030	31.298	29.330	37.873	20.788	0
6	32.680	32.898	31.298	39.841	22.756	0
7	33.560	33.665	32.898	41.441	24.356	0
8	27.500	26.527	33.665	42.207	25.122	0
9	26.750	26.785	26.527	35.069	17.984	0
10	30.550	31.144	26.785	35.328	18.243	0

…



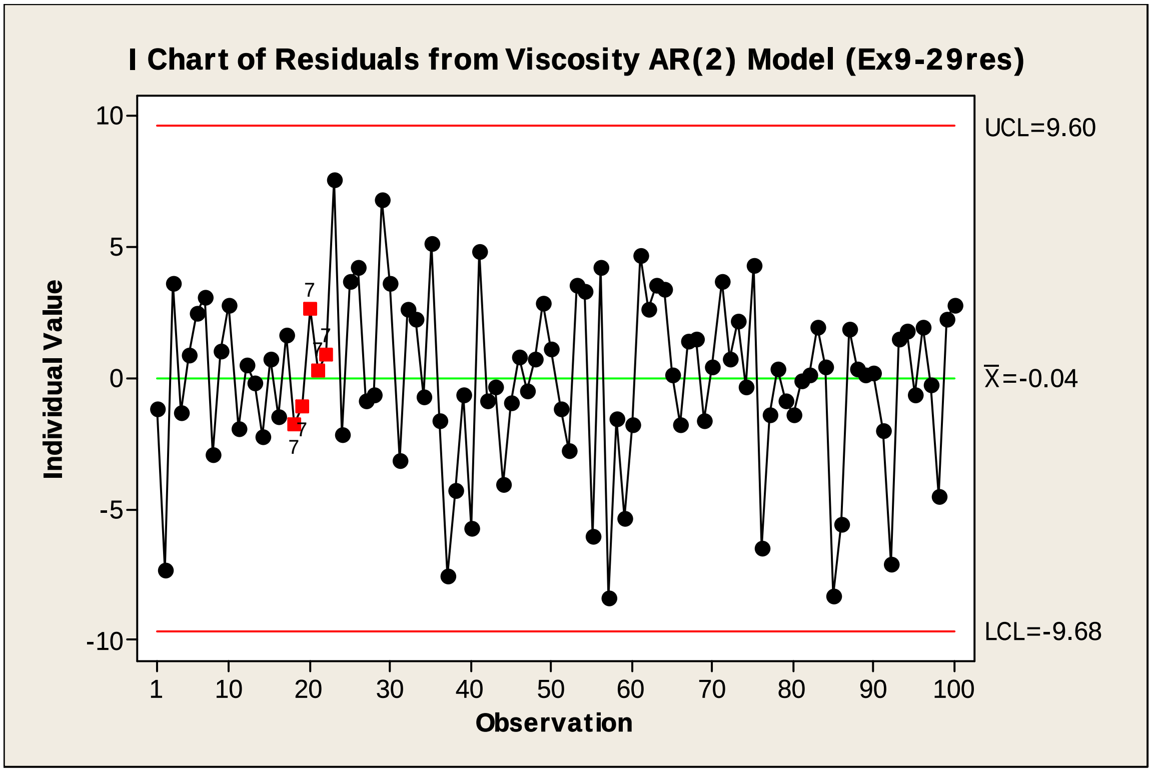
A few observations exceed the upper limit (87) and the lower limit (2, 37, 55, 85).

9-29 continued
(f)

Stat > Time Series > ARIMA
ARIMA Model: Ex9-29Vis

```
…
Final Estimates of Parameters
Type          Coef   SE Coef      T      P
AR    1     0.7193    0.0923   7.79  0.000
AR    2    -0.4349    0.0922  -4.72  0.000
Constant   20.5017    0.3278  62.54  0.000
Mean       28.6514    0.4581
…
```

Stat > Control Charts > Variables Charts for Individuals > Individuals



Test Results for I Chart of Ex9-29res

```
TEST 7. 15 points within 1 standard deviation of center line (above and below
     CL).
Test Failed at points:  18, 19, 20, 21, 22
```

The model residuals signal a potential issue with viscosity around observation 20. Otherwise the process appears to be in control, with a good distribution of points between the control limits and no patterns.

9-30.
λ = 0.01/hr or 1/λ = 100hr; δ = 2.0
a_1 = \$0.50/sample; a_2 = \$0.10/unit; a'_3 = \$5.00; a_3 = \$2.50; a_4 = \$100/hr
g = 0.05hr/sample; D = 2hr

(a)
Excel : workbook Chap09.xls : worksheet Ex9-30a
$n = 5$, $k = 3$, $h = 1$, $\alpha = 0.0027$

$$\beta = \Phi\left(\frac{(\mu_0 + k\,\sigma/\sqrt{n}) - (\mu_0 + 2\sigma)}{\sigma/\sqrt{n}}\right) - \Phi\left(\frac{(\mu_0 - k\,\sigma/\sqrt{n}) - (\mu_0 + 2\sigma)}{\sigma/\sqrt{n}}\right)$$
$$= \Phi\left(3 - 2\sqrt{5}\right) - \Phi\left(-3 - 2\sqrt{5}\right)$$
$$= \Phi(-1.472) - \Phi(-7.472)$$
$$= 0.0705 - 0.0000$$
$$= 0.0705$$
$$\tau \cong \frac{h}{2} - \frac{\lambda h^2}{12} = 0.4992$$
$$\frac{\alpha e^{-\lambda h}}{\left(1 - e^{-\lambda h}\right)} \cong \frac{\alpha}{\lambda h} = 0.27$$
$E(L)$ = \$3.79/hr

(b)
$n = 3$, $k_{\text{opt}} = 2.210$, $h_{\text{opt}} = 1.231$, $\alpha = 0.027$, $1 - \beta = 0.895$
$E(L)$ = \$3.6098/hr

9-31.
$\lambda = 0.01$/hr or $1/\lambda = 100$hr; $\delta = 2.0$
$a_1 = \$0.50$/sample; $a_2 = \$0.10$/unit; $a'_3 = \$50$; $a_3 = \$25$; $a_4 = \$100$/hr
$g = 0.05$hr/sample; $D = 2$hr

(a)
Excel : workbook Chap09.xls : worksheet Ex9-31
$n = 5$, $k = 3$, $h = 1$, $\alpha = 0.0027$

$$\begin{aligned}\beta &= \Phi\left(\frac{(\mu_0 + k\,\sigma/\sqrt{n}) - (\mu_0 + \delta\sigma)}{\sigma/\sqrt{n}}\right) - \Phi\left(\frac{(\mu_0 - k\,\sigma/\sqrt{n}) - (\mu_0 + \delta\sigma)}{\sigma/\sqrt{n}}\right)\\ &= \Phi\left(k - \delta\sqrt{n}\right) - \Phi\left(-k - \delta\sqrt{n}\right)\\ &= \Phi\left(3 - 2\sqrt{5}\right) - \Phi\left(-3 - 2\sqrt{5}\right)\\ &= \Phi(-1.472) - \Phi(-7.472)\\ &= 0.0705 - 0.0000\\ &= 0.0705\end{aligned}$$

$$\tau \cong \frac{h}{2} - \frac{\lambda h^2}{12} = \frac{1}{2} - \frac{0.01(1^2)}{12} = 0.4992$$

$$\frac{\alpha e^{-\lambda h}}{\left(1 - e^{-\lambda h}\right)} \cong \frac{\alpha}{\lambda h} = \frac{0.0027}{0.01(1)} = 0.27$$

$E(L) = \$4.12$/hr

(b)
$n = 5$, $k = 3$, $h = 0.5$, $\alpha = 0.0027$, $\beta = 0.0705$

$$\tau \cong \frac{h}{2} - \frac{\lambda h^2}{12} = \frac{0.5}{2} - \frac{0.01(0.5^2)}{12} = 0.2498$$

$$\frac{\alpha e^{-\lambda h}}{\left(1 - e^{-\lambda h}\right)} \cong \frac{\alpha}{\lambda h} = \frac{0.0027}{0.01(0.5)} = 0.54$$

$E(L) = \$4.98$/hr

(c)
$n = 5$, $k_{\text{opt}} = 3.080$, $h_{\text{opt}} = 1.368$, $\alpha = 0.00207$, $1 - \beta = 0.918$
$E(L) = \$4.01392$/hr

9-32.
Excel : workbook Chap09.xls : worksheet Ex9-32
D_0 = 2hr, D_1 = 2hr
V_0 = \$500, Δ = \$25
$n = 5$, $k = 3$, $h = 1$, $\alpha = 0.0027$, $\beta = 0.0705$
$E(L)$ = \$13.16/hr

9-33.
Excel : workbook Chap09.xls : worksheet Ex9-33
λ = 0.01/hr or $1/\lambda$ = 100hr
$\delta = 2.0$
a_1 = \$2/sample
a_2 = \$0.50/unit
a'_3 = \$75
a_3 = \$50
a_4 = \$200/hr
g = 0.05 hr/sample
D = 1 hr

(a)
$n = 5$, $k = 3$, $h = 0.5$, $\alpha = 0.0027$

$$\begin{aligned}\beta &= \Phi\left(k-\delta\sqrt{n}\right)-\Phi\left(-k-\delta\sqrt{n}\right)\\ &= \Phi\left(3-1\sqrt{5}\right)-\Phi\left(-3-1\sqrt{5}\right)\\ &= \Phi(-1.472)-\Phi(-7.472)\\ &= 0.775-0.0000\\ &= 0.775\end{aligned}$$

$$\tau \cong \frac{h}{2}-\frac{\lambda h^2}{12}=\frac{0.5}{2}-\frac{0.01(0.5^2)}{12}=0.2498$$

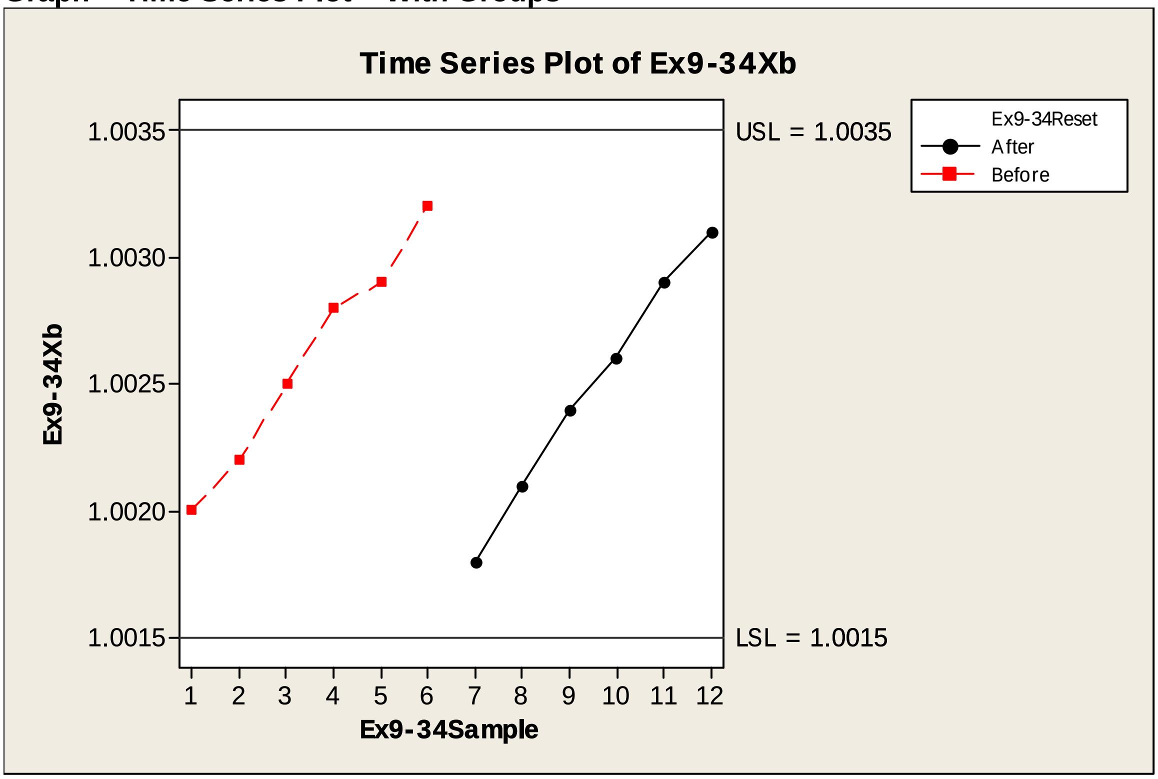
$$\frac{\alpha e^{-\lambda h}}{\left(1-e^{-\lambda h}\right)} \cong \frac{\alpha}{\lambda h}=\frac{0.0027}{0.01(0.5)}=0.54$$

$E(L)$ = \$16.17/hr

(b)
$n = 10$, $k_{\text{opt}} = 2.240$, $h_{\text{opt}} = 2.489018$, $\alpha = 0.025091$, $1-\beta = 0.8218083$
$E(L)$ = \$10.39762/hr

9-34.
It is good practice visually examine data in order to understand the type of tool wear occurring. The plot below shows that the tool has been reset to approximately the same level as initially and the rate of tool wear is approximately the same after reset.

Graph > Time Series Plot > With Groups



$n = 5;\ \bar{R} = 0.00064;\ \hat{\sigma} = \bar{R}/d_2 = 0.00064/2.326 = 0.00028$

CL = $\bar{R}$ = 0.00064, UCL = $D_4\bar{R}$ = 2.114(0.00064) = 0.00135, LCL = 0

$\bar{x}$ chart initial settings:
CL = LSL + 3σ = 1.0015 + 3(0.00028) = 1.00234
$\text{UCL} = \text{CL} + 3\sigma_{\bar{x}} = 1.00234 + 3\left(0.00028/\sqrt{5}\right) = 1.00272$

$\text{LCL} = \text{CL} - 3\sigma_{\bar{x}} = 1.00234 - 3\left(0.00028/\sqrt{5}\right) = 1.00196$

$\bar{x}$ chart at tool reset:
CL = USL – 3σ = 1.0035 – 3(0.00028) = 1.00266 (maximum permissible average)
$\text{UCL} = \text{CL} + 3\sigma_{\bar{x}} = 1.00266 + 3\left(0.00028/\sqrt{5}\right) = 1.00304$

$\text{LCL} = \text{CL} - 3\sigma_{\bar{x}} = 1.00266 - 3\left(0.00028/\sqrt{5}\right) = 1.00228$

Chapter 10 Exercise Solutions

Note: MINITAB's **Tsquared** functionality does not use summary statistics, so many of these exercises have been solved in Excel.

10-1.

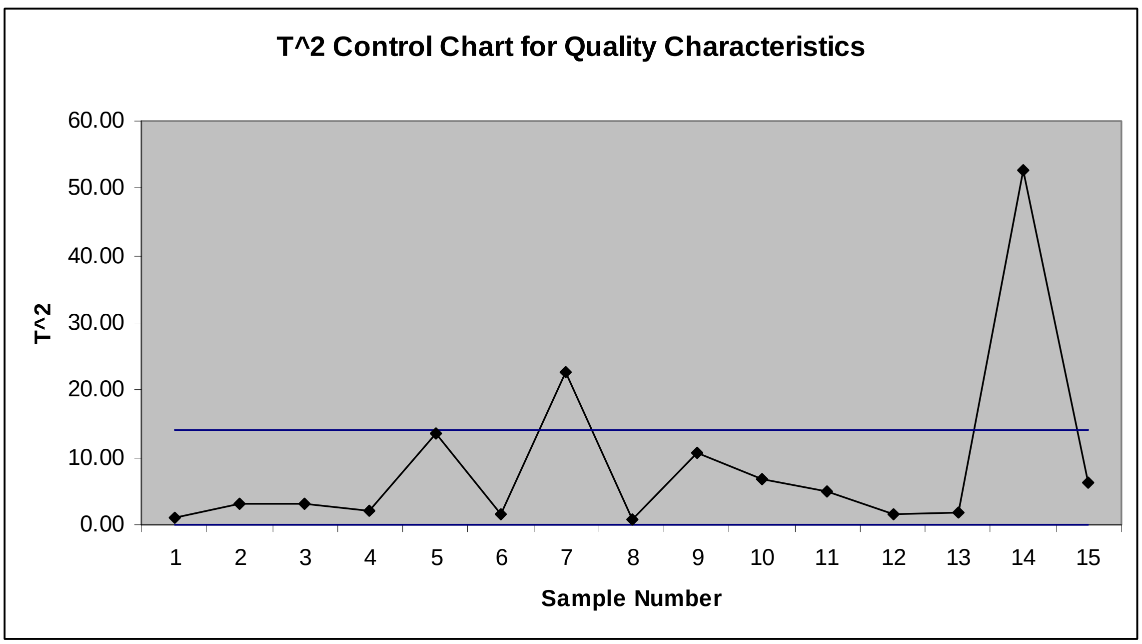
Phase 2 T^2 control charts with m = 50 preliminary samples, n = 25 sample size, p = 2 characteristics. Let α = 0.001.

$$\begin{aligned}\text{UCL} &= \frac{p(m+1)(n-1)}{mn-m-p+1}F_{\alpha,p,mn-m-p+1}\\ &= \frac{2(50+1)(25-1)}{50(25)-50-2+1}F_{0.001,2,1199}\\ &= (2448/1199)(6.948) = 14.186\end{aligned}$$

LCL = 0

Excel : workbook Chap10.xls : worksheet Ex10-1

Sample No.	1	2	3	4	5	6	7	8	9	
xbar1	58	60	50	54	63	53	42	55	46	
xbar2	32	33	27	31	38	30	20	31	25	
diff1	3	5	-5	-1	8	-2	-13	0	-9	
diff2	2	3	-3	1	8	0	-10	1	-5	
matrix calc	0.0451	0.1268	0.1268	0.0817	0.5408	0.0676	0.9127	0.0282	0.4254	
t2 = n * calc	1.1268	3.1690	3.1690	2.0423	13.5211	1.6901	22.8169	0.7042	10.6338	
UCL =	14.1850	14.1850	14.1850	14.1850	14.1850	14.1850	14.1850	14.1850	14.1850	
LCL =	0	0	0	0	0	0	0	0	0	
OOC?	In control	In control	In control	In control	In control	In control	Above UCL	In control	In control	…



Process is out of control at samples 7 and 14.

10-2.

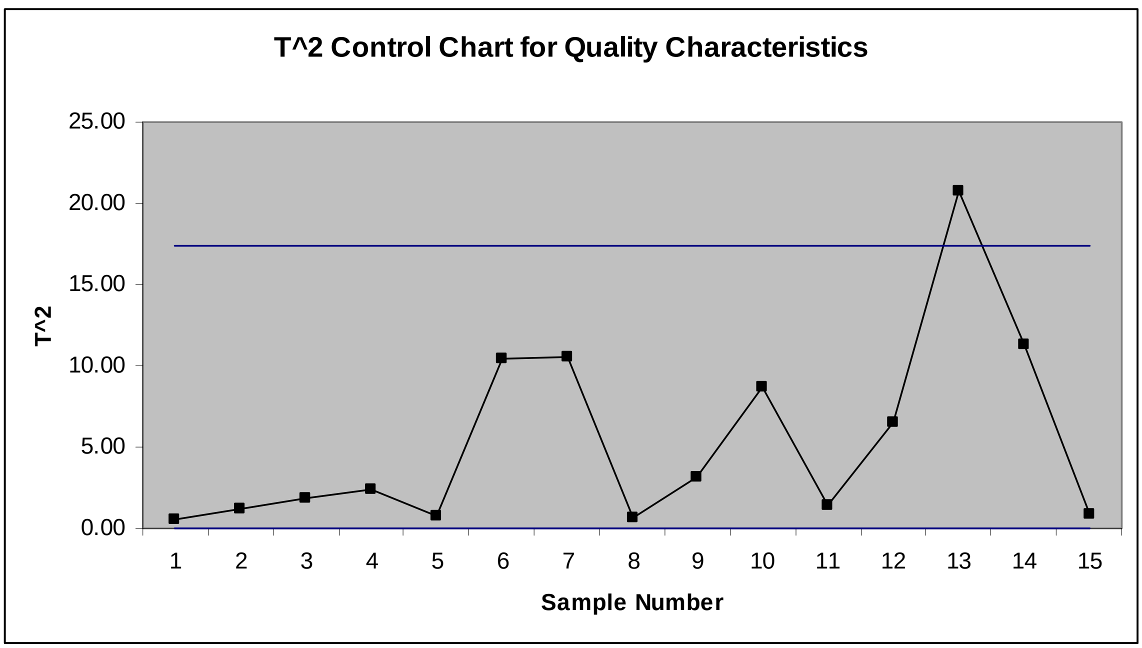
Phase 2 T^2 control limits with m = 30 preliminary samples, n = 10 sample size, p = 3 characteristics. Let α = 0.001.

$$\begin{aligned}\text{UCL} &= \frac{p(m+1)(n-1)}{mn-m-p+1}F_{\alpha,p,mn-m-p+1} \\ &= \frac{3(30+1)(10-1)}{30(10)-30-3+1}F_{0.001,3,268} \\ &= \left(\frac{837}{268}\right)(5.579) \\ &= 17.425\end{aligned}$$

LCL = 0

Excel : workbook Chap10.xls : worksheet Ex10-2

Sample No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
xbar1	3.1	3.3	2.6	2.8	3	4	3.8	3	2.4	2	3.2	3.7	4.1	3.8	3.2
xbar2	3.7	3.9	3	3	3.3	4.6	4.2	3.3	3	2.6	3.9	4	4.7	4	3.6
xbar3	3	3.1	2.4	2.5	2.8	3.5	3	2.7	2.2	1.8	3	3	3.2	2.9	2.8
diff1	0.1	0.3	-0.4	-0.2	0	1	0.8	0	-0.6	-1	0.2	0.7	1.1	0.8	0.2
diff2	0.2	0.4	-0.5	-0.5	-0.2	1.1	0.7	-0.2	-0.5	-0.9	0.4	0.5	1.2	0.5	0.1
diff3	0.2	0.3	-0.4	-0.3	0	0.7	0.2	-0.1	-0.6	-1	0.2	0.2	0.4	0.1	0
matrix calc	0.0528	0.1189	0.1880	0.2372	0.0808	1.0397	1.0593	0.0684	0.3122	0.8692	0.1399	0.6574	2.0793	1.1271	0.0852
t2 = n * calc	0.5279	1.1887	1.8800	2.3719	0.8084	10.3966	10.5932	0.6844	3.1216	8.6922	1.3990	6.5741	20.7927	11.2706	0.8525
UCL =	17.4249	17.4249	17.4249	17.4249	17.4249	17.4249	17.4249	17.4249	17.4249	17.4249	17.4249	17.4249	17.4249	17.4249	17.4249
LCL =	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
OOC?	In control	In control	In control	In control	In control	In control	In control	In control	In control	In control	In control	In control	Above UCL	In control	In control

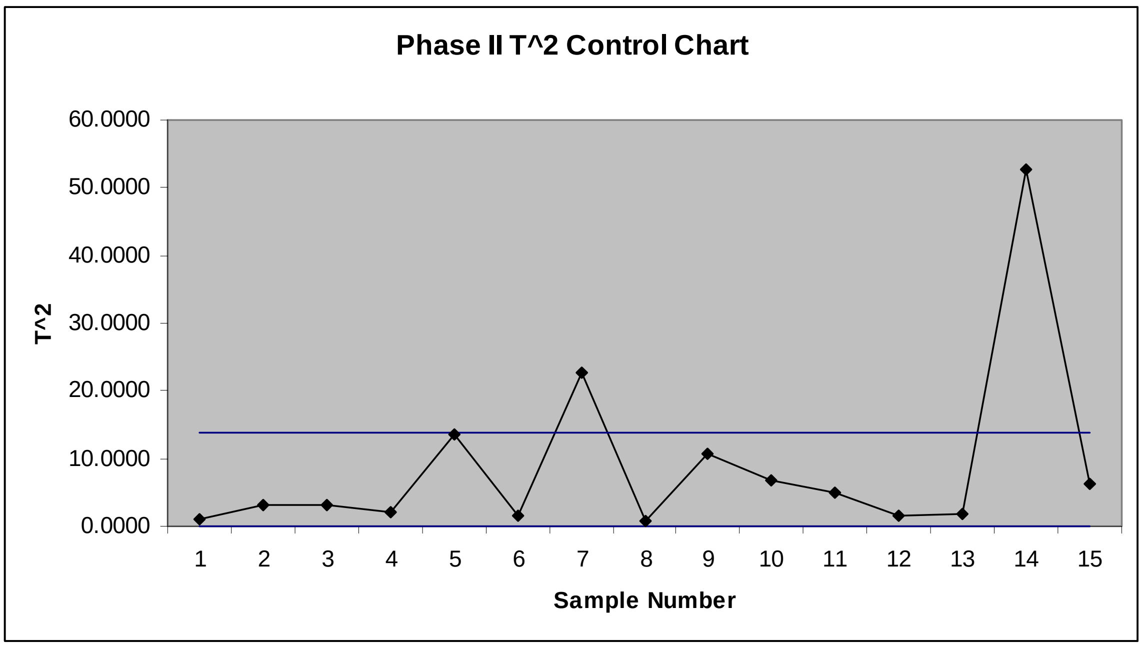


Process is out of control at sample 13.

10-3.
Phase 2 T^2 control limits with $p = 2$ characteristics. Let $\alpha = 0.001$.
Since population parameters are known, the chi-square formula will be used for the upper control limit: $\text{UCL} = \chi^2_{\alpha,p} = \chi^2_{0.001,2} = 13.816$

Excel : workbook Chap10.xls : worksheet Ex10-3

Sample No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
xbar1	58	60	50	54	63	53	42	55	46	50	49	57	58	75	55
xbar2	32	33	27	31	38	30	20	31	25	29	27	30	33	45	27
diff1	3	5	-5	-1	8	-2	-13	0	-9	-5	-6	2	3	20	0
diff2	2	3	-3	1	8	0	-10	1	-5	-1	-3	0	3	15	-3
matrix calc	0.0451	0.1268	0.1268	0.0817	0.5408	0.0676	0.9127	0.0282	0.4254	0.2676	0.2028	0.0676	0.0761	2.1127	0.2535
t2 = n * calc	1.1268	3.1690	3.1690	2.0423	13.5211	1.6901	22.8169	0.7042	10.6338	6.6901	5.0704	1.6901	1.9014	52.8169	6.3380
UCL =	13.8150	13.8150	13.8150	13.8150	13.8150	13.8150	13.8150	13.8150	13.8150	13.8150	13.8150	13.8150	13.8150	13.8150	13.8150
LCL =	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
OOC?	In control	In control	In control	In control	In control	In control	Above UCL	In control	In control	In control	In control	In control	In control	Above UCL	In control



Process is out of control at samples 7 and 14. Same results as for parameters estimated from samples.

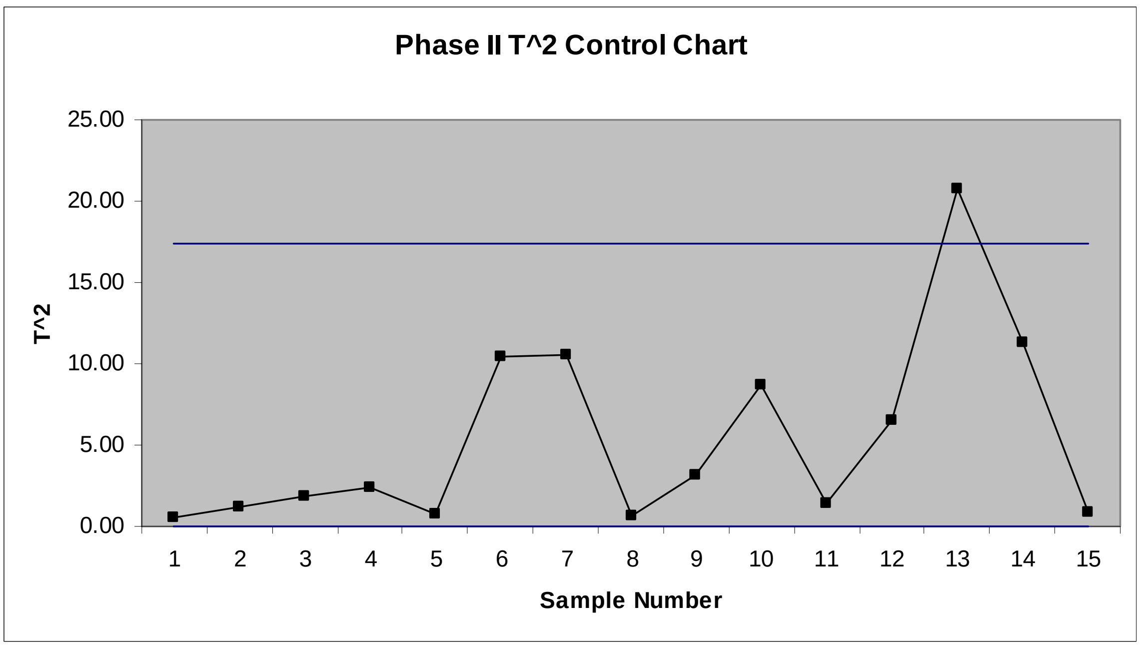
10-4.

Phase 2 T^2 control limits with $p = 3$ characteristics. Let $\alpha = 0.001$.

Since population parameters are known, the chi-square formula will be used for the upper control limit: $\text{UCL} = \chi^2_{\alpha,p} = \chi^2_{0.001,3} = 16.266$

Excel : workbook Chap10.xls : worksheet Ex10-4

Sample No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
xbar1	3.1	3.3	2.6	2.8	3	4	3.8	3	2.4	2	3.2	3.7	4.1	3.8	3.2
xbar2	3.7	3.9	3	3	3.3	4.6	4.2	3.3	3	2.6	3.9	4	4.7	4	3.6
xbar3	3	3.1	2.4	2.5	2.8	3.5	3	2.7	2.2	1.8	3	3	3.2	2.9	2.8
diff1	0.1	0.3	-0.4	-0.2	0	1	0.8	0	-0.6	-1	0.2	0.7	1.1	0.8	0.2
diff2	0.2	0.4	-0.5	-0.5	-0.2	1.1	0.7	-0.2	-0.5	-0.9	0.4	0.5	1.2	0.5	0.1
diff3	0.2	0.3	-0.4	-0.3	0	0.7	0.2	-0.1	-0.6	-1	0.2	0.2	0.4	0.1	0
matrix calc	0.0528	0.1189	0.1880	0.2372	0.0808	1.0397	1.0593	0.0684	0.3122	0.8692	0.1399	0.6574	2.0793	1.1271	0.0852
t2 = n * calc	0.5279	1.1887	1.8800	2.3719	0.8084	10.3966	10.5932	0.6844	3.1216	8.6922	1.3990	6.5741	20.7927	11.2706	0.8525
UCL =	16.2660	16.2660	16.2660	16.2660	16.2660	16.2660	16.2660	16.2660	16.2660	16.2660	16.2660	16.2660	16.2660	16.2660	16.2660
LCL =	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
OOC?	In control	In control	In control	In control	In control	In control	In control	In control	In control	In control	In control	In control	**Above UCL**	In control	In control



Process is out of control at sample 13. Same as results for parameters estimated from samples.

10-5.
m = 30 preliminary samples, n = 3 sample size, p = 6 characteristics, $\alpha = 0.005$

(a)
Phase II limits:

$$\begin{aligned}\text{UCL} &= \frac{p(m+1)(n-1)}{mn-m-p+1}F_{\alpha,p,mn-m-p+1}\\ &= \frac{6(30+1)(3-1)}{30(3)-30-6+1}F_{0.005,6,55}\\ &= \left(\frac{372}{55}\right)(3.531)\\ &= 23.882\end{aligned}$$

LCL = 0

(b)
chi-square limit: $\text{UCL} = \chi^2_{\alpha,p} = \chi^2_{0.005,6} = 18.548$

The Phase II UCL is almost 30% larger than the chi-square limit.

(c)
Quality characteristics, p = 6. Samples size, n = 3. α = 0.005. Find "m" such that exact Phase II limit is within 1% of chi-square limit, 1.01(18.548) = 18.733.

Excel : workbook Chap10.xls : worksheet Ex10-5

m	num	denom	F	UCL
30	372	55	3.531	23.8820
40	492	75	3.407	22.3527
50	612	95	3.338	21.5042
60	732	115	3.294	20.9650
70	852	135	3.263	20.5920
80	972	155	3.240	20.3184
90	1092	175	3.223	20.1095
100	1212	195	3.209	19.9447
…				
717	8616	1429	3.107	18.7337
718	8628	1431	3.107	18.7332
719	8640	1433	3.107	18.7331
720	8652	1435	3.107	18.7328
721	8664	1437	3.107	18.7325
722	8676	1439	3.107	18.7324

720 preliminary samples must be taken to ensure that the exact Phase II limit is within 1% of the chi-square limit.

10-6.
m = 30 preliminary samples, n = 5 sample size, p = 6 characteristics, $\alpha = 0.005$

(a)
Phase II UCL:

$$\begin{aligned}\text{UCL} &= \frac{p(m+1)(n-1)}{mn-m-p+1}F_{\alpha,p,mn-m-p+1} \\ &= \frac{6(30+1)(5-1)}{30(5)-30-6+1}F_{0.005,6,115} \\ &= \left(\frac{744}{115}\right)(3.294) \\ &= 21.309\end{aligned}$$

(b)
chi-square UCL: $\text{UCL} = \chi^2_{\alpha,p} = \chi^2_{0.005,6} = 18.548$
The Phase II UCL is almost 15% larger than the chi-square limit.

(c)
Quality characteristics, p = 6. Samples size, n = 5. $\alpha = 0.005$. Find "m" such that exact Phase II limit is within 1% of chi-square limit, 1.01(18.548) = 18.733.

Excel : workbook Chap10.xls : worksheet Ex10-6

m	num	denom	F	UCL
30	744	115	3.294	21.3087
40	984	155	3.240	20.5692
50	1224	195	3.209	20.1422
60	1464	235	3.189	19.8641
70	1704	275	3.174	19.6685
80	1944	315	3.164	19.5237
90	2184	355	3.155	19.4119
100	2424	395	3.149	19.3232
…				
390	9384	1555	3.106	18.7424
400	9624	1595	3.105	18.7376
410	9864	1635	3.105	18.7330
411	9888	1639	3.105	18.7324
412	9912	1643	3.105	18.7318

411 preliminary samples must be taken to ensure that the exact Phase II limit is within 1% of the chi-square limit.

10-7.
m = 25 preliminary samples, n = 3 sample size, p = 10 characteristics, $\alpha = 0.005$

(a)
Phase II UCL:

$$\begin{aligned}\text{UCL} &= \frac{p(m+1)(n-1)}{mn-m-p+1}F_{\alpha,p,mn-m-p+1}\\ &= \frac{10(25+1)(3-1)}{25(3)-25-10+1}F_{0.005,10,41}\\ &= \left(\frac{520}{41}\right)(3.101)\\ &= 39.326\end{aligned}$$

(b)
chi-square UCL: UCL $= \chi^2_{\alpha,p} = \chi^2_{0.005,10} = 25.188$

The Phase II UCL is more than 55% larger than the chi-square limit.

(c)
Quality characteristics, p = 10. Samples size, n = 3. $\alpha = 0.005$. Find "m" such that exact Phase II limit is within 1% of chi-square limit, 1.01(25.188) = 25.440.

Excel : workbook Chap10.xls : worksheet Ex10-7

m	num	denom	F	UCL
25	520	41	3.101	39.3259
35	720	61	2.897	34.1991
45	920	81	2.799	31.7953
55	1120	101	2.742	30.4024
65	1320	121	2.704	29.4940
75	1520	141	2.677	28.8549
85	1720	161	2.657	28.3808
95	1920	181	2.641	28.0154
105	2120	201	2.629	27.7246
…				
986	19740	1963	2.530	25.4405
987	19760	1965	2.530	25.4401
988	19780	1967	2.530	25.4399
989	19800	1969	2.530	25.4398
990	19820	1971	2.530	25.4394

988 preliminary samples must be taken to ensure that the exact Phase II limit is within 1% of the chi-square limit.

10-8.
m = 25 preliminary samples, n = 5 sample size, p = 10 characteristics, α = 0.005

(a)
Phase II UCL:

$$\begin{aligned}\text{UCL} &= \frac{p(m+1)(n-1)}{mn-m-p+1}F_{\alpha,p,mn-m-p+1}\\ &= \frac{10(25+1)(5-1)}{25(5)-25-10+1}F_{0.005,10,91}\\ &= \left(\frac{1040}{91}\right)(2.767)\\ &= 31.625\end{aligned}$$

(b)
chi-square UCL: $\text{UCL} = \chi^2_{\alpha,p} = \chi^2_{0.005,10} = 25.188$
The Phase II UCL is more than 25% larger than the chi-square limit.

(c)
Quality characteristics, p = 10. Samples size, n = 5. α = 0.005. Find "m" such that exact Phase II limit is within 1% of chi-square limit, 1.01(25.188) = 25.440.

Excel : workbook Chap10.xls : worksheet Ex10-8

m	num	denom	F	UCL
25	1040	91	2.767	31.6251
35	1440	131	2.689	29.5595
45	1840	171	2.648	28.4967
55	2240	211	2.623	27.8495
65	2640	251	2.606	27.4141
75	3040	291	2.594	27.1011
85	3440	331	2.585	26.8651
95	3840	371	2.578	26.6812
105	4240	411	2.572	26.5335
540	21640	2151	2.529	25.4419
541	21680	2155	2.529	25.4413
542	21720	2159	2.529	25.4408
543	21760	2163	2.529	25.4405
544	21800	2167	2.529	25.4399
545	21840	2171	2.529	25.4394

544 preliminary samples must be taken to ensure that the exact Phase II limit is within 1% of the chi-square limit.

10-9.
p = 10 quality characteristics, n = 3 sample size, m = 25 preliminary samples. Assume α = 0.01.

Phase I UCL:

$$\begin{aligned}\text{UCL} &= \frac{p(m-1)(n-1)}{mn-m-p+1}F_{\alpha,p,mn-m-p+1} \\ &= \frac{10(25-1)(3-1)}{25(3)-25-10+1}F_{0.01,10,41} \\ &= \left(\frac{480}{41}\right)(2.788) \\ &= 32.638\end{aligned}$$

Phase II UCL:

$$\begin{aligned}\text{UCL} &= \frac{p(m+1)(n-1)}{mn-m-p+1}F_{\alpha,p,mn-m-p+1} \\ &= \frac{10(25+1)(3-1)}{25(3)-25-10+1}F_{0.01,10,41} \\ &= \left(\frac{520}{41}\right)(2.788) \\ &= 35.360\end{aligned}$$

10-10.
Excel : workbook Chap10.xls : worksheet Ex10-10

(a)

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & 0.7 & 0.7 & 0.7 \\ 0.7 & 1 & 0.7 & 0.7 \\ 0.7 & 0.7 & 1 & 0.7 \\ 0.7 & 0.7 & 0.7 & 1 \end{bmatrix}$$

(b)

$$\text{UCL} = \chi^2_{\alpha,p} = \chi^2_{0.01,4} = 13.277$$

10-10 continued

(c)

$$T^2 = n(\mathbf{y}-\boldsymbol{\mu})'\Sigma^{-1}(\mathbf{y}-\boldsymbol{\mu})$$

$$=1\left(\begin{bmatrix}3.5\\3.5\\3.5\\3.5\end{bmatrix}-\begin{bmatrix}0\\0\\0\\0\end{bmatrix}\right)'\begin{bmatrix}1&0.7&0.7&0.7\\0.7&1&0.7&0.7\\0.7&0.7&1&0.7\\0.7&0.7&0.7&1\end{bmatrix}^{-1}\left(\begin{bmatrix}3.5\\3.5\\3.5\\3.5\end{bmatrix}-\begin{bmatrix}0\\0\\0\\0\end{bmatrix}\right)$$

$$=15.806$$

Yes. Since $\left(T^2=15.806\right)>\left(\text{UCL}=13.277\right)$, an out-of-control signal is generated.

(d)

$$T^2_{(1)} = n\left(\mathbf{y}_{(1)}-\boldsymbol{\mu}_{(1)}\right)'\Sigma^{-1}_{(1)}\left(\mathbf{y}_{(1)}-\boldsymbol{\mu}_{(1)}\right)$$

$$=1\left(\begin{bmatrix}3.5\\3.5\\3.5\end{bmatrix}-\begin{bmatrix}0\\0\\0\end{bmatrix}\right)'\begin{bmatrix}1&0.7&0.7\\0.7&1&0.7\\0.7&0.7&1\end{bmatrix}^{-1}\left(\begin{bmatrix}3.5\\3.5\\3.5\end{bmatrix}-\begin{bmatrix}0\\0\\0\end{bmatrix}\right)$$

$$=15.313$$

$$T^2_{(1)}=T^2_{(2)}=T^2_{(3)}=T^2_{(4)}=15.313$$

$$d_i = T^2 - T^2_{(i)}$$

$$d_1=d_2=d_3=d_4=15.806-15.313=0.493$$

$$\chi^2_{0.01,1}=6.635$$

No. First, since all d_i are smaller than $\chi^2_{0.01,1}$, no variable is identified as a relatively large contributor. Second, since the standardized observations are equal (that is, all variables had the same shift), this information does not assist in identifying which a process variable shifted.

(e)

Since $(T^2=28.280)>(\text{UCL}=13.277)$, an out-of-control signal is generated.

(f)

$$\chi^2_{0.01,1}=6.635$$

$$T^2_{(1)}=15.694;\quad d_1=12.585$$

$$T^2_{(2)}=21.979;\quad d_2=6.300$$

$$T^2_{(3)}=14.479;\quad d_3=13.800$$

$$T^2_{(4)}=25.590;\quad d_4=2.689$$

Investigate variables 1 and 3.

10-11.
Excel : workbook Chap10.xls : worksheet Ex10-11

(a)

$$\Sigma = \begin{bmatrix} 1 & 0.8 & 0.8 \\ 0.8 & 1 & 0.8 \\ 0.8 & 0.8 & 1 \end{bmatrix}$$

(b)
$\text{UCL} = \chi^2_{\alpha,p} = \chi^2_{0.05,3} = 7.815$

(c)
$T^2 = 11.154$
Yes. Since $\left(T^2 = 11.154\right) > \left(\text{UCL} = 7.815\right)$, an out-of-control signal is generated.

(d)
$\chi^2_{0.05,1} = 3.841$
$T^2_{(1)} = 11.111; \quad d_1 = 0.043$
$T^2_{(2)} = 2.778; \quad d_2 = 8.376$
$T^2_{(3)} = 5.000; \quad d_3 = 6.154$
Variables 2 and 3 should be investigated.

(e)
Since $(T^2 = 6.538) > (\text{UCL} = 7.815)$, an out-of-control signal is not generated.

(f)
$\chi^2_{0.05,1} = 3.841$
$T^2_{(1)} = 5.000; \quad d_1 = 1.538$
$T^2_{(2)} = 5.000; \quad d_2 = 1.538$
$T^2_{(3)} = 4.444; \quad d_3 = 2.094$
Since an out-of-control signal was not generated in (e), it is not necessary to calculate the diagnostic quantities. This is confirmed since none of the d_i's exceeds the UCL.

10-12.

Excel : workbook Chap10.xls : worksheet Ex10-12

$m = 40$

$$\bar{\mathbf{x}}' = \begin{bmatrix} 15.339 & 0.104 \end{bmatrix}; \quad \mathbf{S}_1 = \begin{bmatrix} 4.440 & -0.016 \\ -0.016 & 0.001 \end{bmatrix}$$

$$\mathbf{V}'\mathbf{V} = \begin{bmatrix} 121.101 & -0.256 \\ -0.256 & 0.071 \end{bmatrix}; \quad \mathbf{S}_2 = \begin{bmatrix} 1.553 & -0.003 \\ -0.003 & 0.001 \end{bmatrix}$$

10-13.

Excel : workbook Chap10.xls : worksheet Ex10-13

$m = 40$

$$\bar{\mathbf{x}}' = \begin{bmatrix} 15.339 & 0.104 & 88.125 \end{bmatrix}; \quad \mathbf{S}_1 = \begin{bmatrix} 4.440 & -0.016 & 5.395 \\ -0.016 & 0.001 & -0.014 \\ 5.395 & -0.014 & 27.599 \end{bmatrix}$$

$$\mathbf{V}'\mathbf{V} = \begin{bmatrix} 121.101 & -0.256 & 43.720 \\ -0.256 & 0.071 & 0.950 \\ 43.720 & 0.950 & 587.000 \end{bmatrix}; \quad \mathbf{S}_2 = \begin{bmatrix} 1.553 & -0.003 & -0.561 \\ -0.003 & 0.001 & 0.012 \\ -0.561 & 0.012 & 7.526 \end{bmatrix}$$

10-14.

Excel : workbook Chap10.xls : worksheet Ex10-14

xbar	
xbar1	10.607
xbar2	21.207

S1	
3.282	3.305
3.305	5.641

V'V	
133.780	80.740
80.740	67.150

S2	
2.307	1.392
1.392	1.158

10-15.

Excel : workbook Chap10.xls : worksheet Ex10-15

p =	4			
mu' =	0	0	0	0

Sigma =	1	0.75	0.75	0.75
	0.75	1	0.75	0.75
	0.75	0.75	1	0.75
	0.75	0.75	0.75	1

Sigma-1 =	3.0769	-0.9231	-0.9231	-0.9231
	-0.9231	3.0769	-0.9231	-0.9231
	-0.9231	-0.9231	3.0769	-0.9231
	-0.9231	-0.9231	-0.9231	3.0769

y' =	1	1	1	1

y =	1
	1
	1
	1

y' Sigma-1 =	0.308	0.308	0.308	0.308

y' Sigma-1 y =	1.231

delta = 1.109

ARL0 = 200

From Table 10-3, select (lambda, H) pair that closely minimizes ARL1

delta =	1	1.5
lambda =	0.1	0.2
UCL = H =	12.73	13.87
ARL1 =	12.17	6.53

Select $\lambda = 0.1$ with an UCL = $H = 12.73$. This gives an ARL_1 between 7.22 and 12.17.

10-16.

Excel : workbook Chap10.xls : worksheet Ex10-16

p =	4			
mu' =	0	0	0	0

Sigma =	1	0.9	0.9	0.9
	0.9	1	0.9	0.9
	0.9	0.9	1	0.9
	0.9	0.9	0.9	1

Sigma-1 =	7.568	-2.432	-2.432	-2.432
	-2.432	7.568	-2.432	-2.432
	-2.432	-2.432	7.568	-2.432
	-2.432	-2.432	-2.432	7.568

y' =	1	1	1	1

y =	1
	1
	1
	1

y' Sigma-1 =	0.270	0.270	0.270	0.270

y' Sigma-1 y =	1.081
delta =	1.040
ARL0 =	500

From Table 10-4, select (lambda, H) pair

delta =	1	1.5
lambda =	0.105	0.18
UCL = H =	15.26	16.03
ARLmin =	14.60	7.65

Select $\lambda = 0.105$ with an UCL = $H = 15.26$. This gives an ARL_{min} near 14.60.

10-17.

Excel : workbook Chap10.xls : worksheet Ex10-17

p =	2	
mu' =	0	0

Sigma =	1	0.8
	0.8	1

Sigma-1 =	2.7778	-2.2222
	-2.2222	2.7778

y' =	1	1

y =	1
	1

y' Sigma-1 =	0.556	0.556
y' Sigma-1 y =	1.111	
delta =	1.054	
ARL0 =	200	

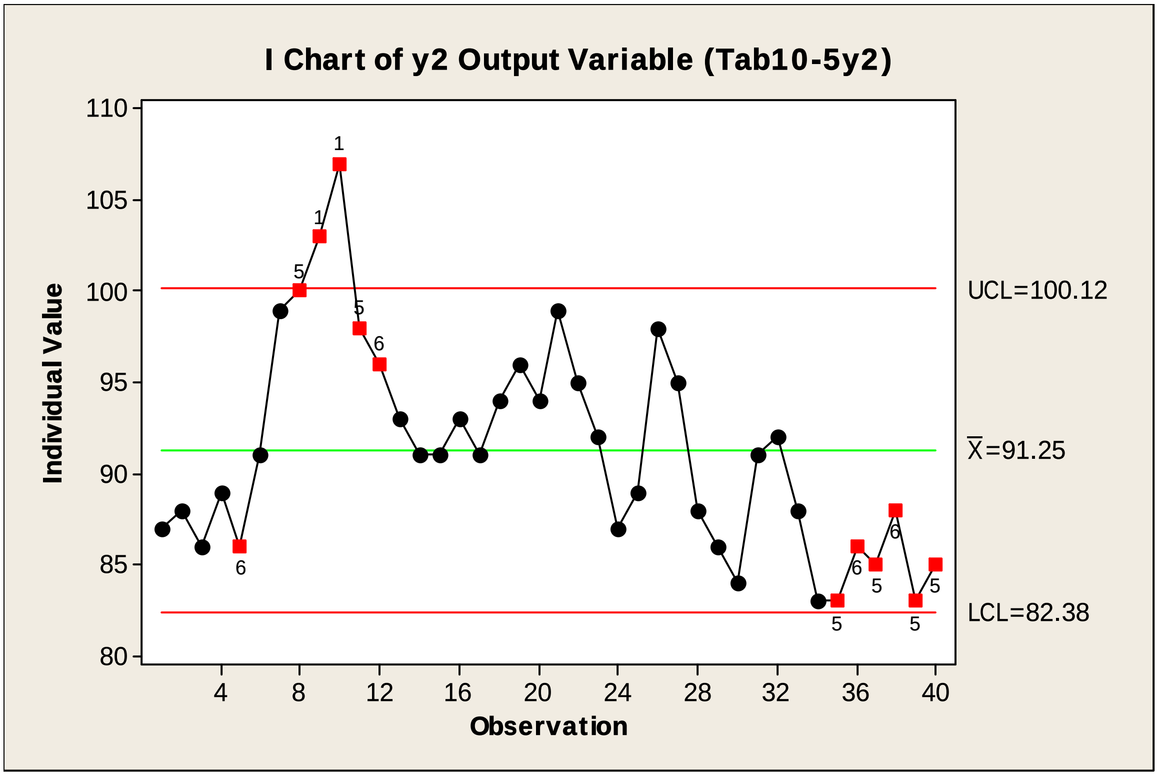
From Table 10-3, select (lambda, H) pair that closely minimizes ARL1

delta =	1	1	1.5	1.5
lambda =	0.1	0.2	0.2	0.3
UCL = H =	8.64	9.65	9.65	10.08
ARL1 =	10.15	10.20	5.49	5.48

Select $\lambda = 0.2$ with an UCL = H = 9.65. This gives an ARL_1 between 5.49 and 10.20.

10-18.
(a)
Note: In the textbook Table 10-5, the y_2 values for Observations 8, 9, and 10 should be 100, 103, and 107.

Stat > Control Charts > Variables Charts for Individuals > Individuals



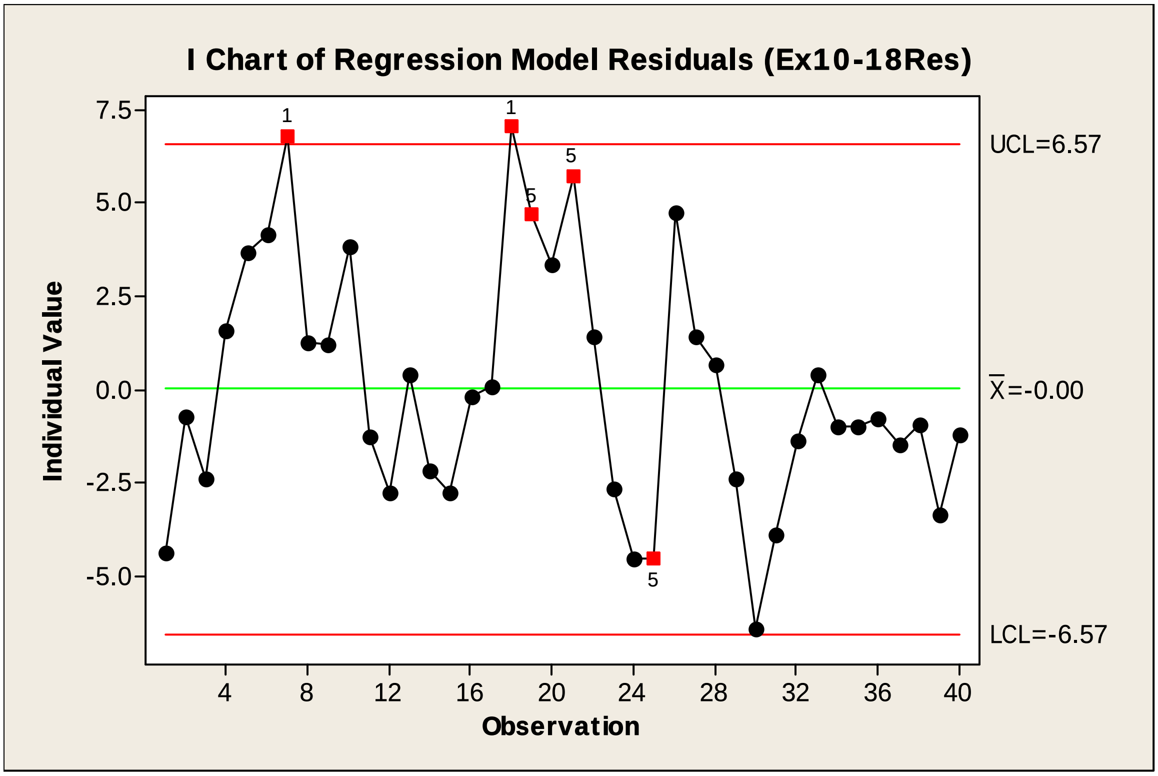
Test Results for I Chart of Tab10-5y2

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  9, 10
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  8, 9, 10, 11, 35, 37, 39, 40
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  5, 10, 11, 12, 36, 37, 38, 39, 40
TEST 8. 8 points in a row more than 1 standard deviation from center line
     (above and below CL).
Test Failed at points:  40
```

10-18 continued
(b)
Stat > Regression > Regression

Regression Analysis: Tab10-5y2 versus Tab10-5x1, Tab10-5x2, ...
```
The regression equation is
Tab10-5y2 = 215 - 0.666 Tab10-5x1 - 11.6 Tab10-5x2 + 0.435 Tab10-5x3
            + 0.192 Tab10-5x4 - 3.2 Tab10-5x5 + 0.73 Tab10-5x6 + 6.1 Tab10-5x7
            + 10.9 Tab10-5x8 - 215 Tab10-5x9
```

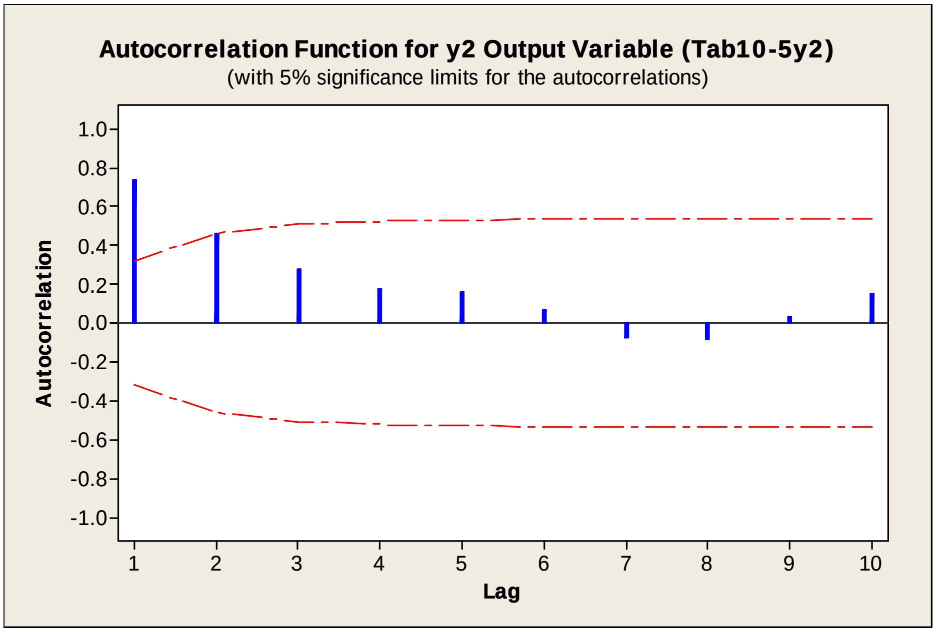
Stat > Control Charts > Variables Charts for Individuals > Individuals

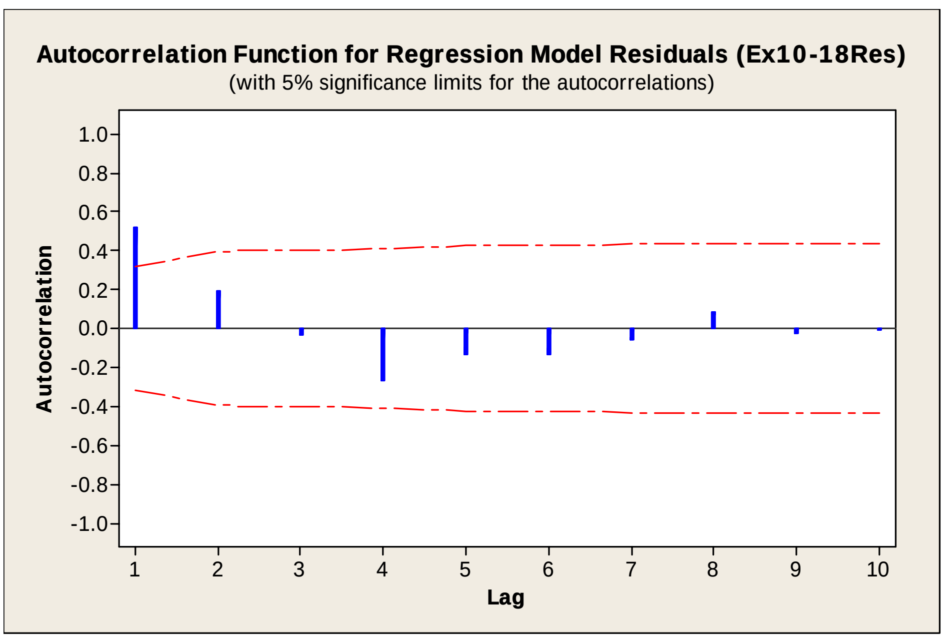


Test Results for I Chart of Ex10-18Res
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  7, 18
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  19, 21, 25
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  21
```

Plot points on the residuals control chart are spread between the control limits and do not exhibit the downward trend of the response y_2 control chart.

10-18 continued
(c)
Stat > Time Series > Autocorrelation





The decaying sine wave of ACFs for Response y_2 suggests an autoregressive process, while the ACF for the residuals suggests a random process.

10-19.

Different approaches can be used to identify insignificant variables and reduce the number of variables in a regression model. This solution uses MINITAB's "Best Subsets" functionality to identify the best-fitting model with as few variables as possible.

Stat > Regression > Best Subsets

Best Subsets Regression: Tab10-5y1 versus Tab10-5x1, Tab10-5x2, ...

```
Response is Tab10-5y1
                                         T T T T T T T T T
                                         a a a a a a a a a
                                         b b b b b b b b b
                                         1 1 1 1 1 1 1 1 1
                                         0 0 0 0 0 0 0 0 0
                                         - - - - - - - - -
                                         5 5 5 5 5 5 5 5 5
                       Mallows           x x x x x x x x x
Vars  R-Sq  R-Sq(adj)      C-p        S  1 2 3 4 5 6 7 8 9
   1  43.1       41.6     52.9   1.3087                  X
   1  31.3       29.5     71.3   1.4378        X
   2  62.6       60.5     24.5   1.0760  X               X
   2  55.0       52.5     36.4   1.1799        X         X
   3  67.5       64.7     18.9   1.0171  X     X         X
   3  66.8       64.0     19.9   1.0273  X             X X
   4  72.3       69.1     13.3  0.95201  X   X X         X
   4  72.1       68.9     13.6  0.95522  X     X       X X
   5  79.5       76.5      4.0  0.83020  X   X X       X X       ******
   5  73.8       69.9     13.0  0.93966  X   X     X   X X
   6  79.9       76.2      5.5  0.83550  X   X X   X   X X
   6  79.8       76.1      5.6  0.83693  X X X X       X X
   7  80.3       76.0      6.8  0.83914  X   X X X   X X X
   7  80.1       75.8      7.1  0.84292  X   X X X X   X X
…
```

Best Subsets Regression: Tab10-5y2 versus Tab10-5x1, Tab10-5x2, ...

```
Response is Tab10-5y2
                                         T T T T T T T T T
                                         a a a a a a a a a
                                         b b b b b b b b b
                                         1 1 1 1 1 1 1 1 1
                                         0 0 0 0 0 0 0 0 0
                                         - - - - - - - - -
                                         5 5 5 5 5 5 5 5 5
                       Mallows           x x x x x x x x x
Vars  R-Sq  R-Sq(adj)      C-p        S  1 2 3 4 5 6 7 8 9
   1  36.1       34.4     24.0   4.6816      X
   1  35.8       34.1     24.2   4.6921                  X
   2  55.1       52.7      8.1   3.9751                X X
   2  50.7       48.1     12.2   4.1665      X         X
   3  61.6       58.4      4.0   3.7288      X X       X
   3  59.8       56.4      5.7   3.8160      X         X X
   4  64.9       60.9      2.9   3.6147  X   X         X X
   4  64.4       60.4      3.4   3.6387      X X       X X
   5  67.7       62.9      2.3   3.5208  X   X X       X X       ******
   5  65.2       60.1      4.7   3.6526  X   X     X   X X
   6  67.8       62.0      4.2   3.5660  X   X X     X X X
   6  67.8       61.9      4.3   3.5684  X   X X X     X X
   7  67.9       60.9      6.1   3.6149  X X X X     X X X
   7  67.8       60.8      6.2   3.6200  X   X X X   X X X
…
```

For output variables *y*1 and *y*2, a regression model of input variables *x*1, *x*3, *x*4, *x*8, and *x*9 maximize adjusted R^2 (minimize S) and minimize Mallow's C-p.

10-19 continued

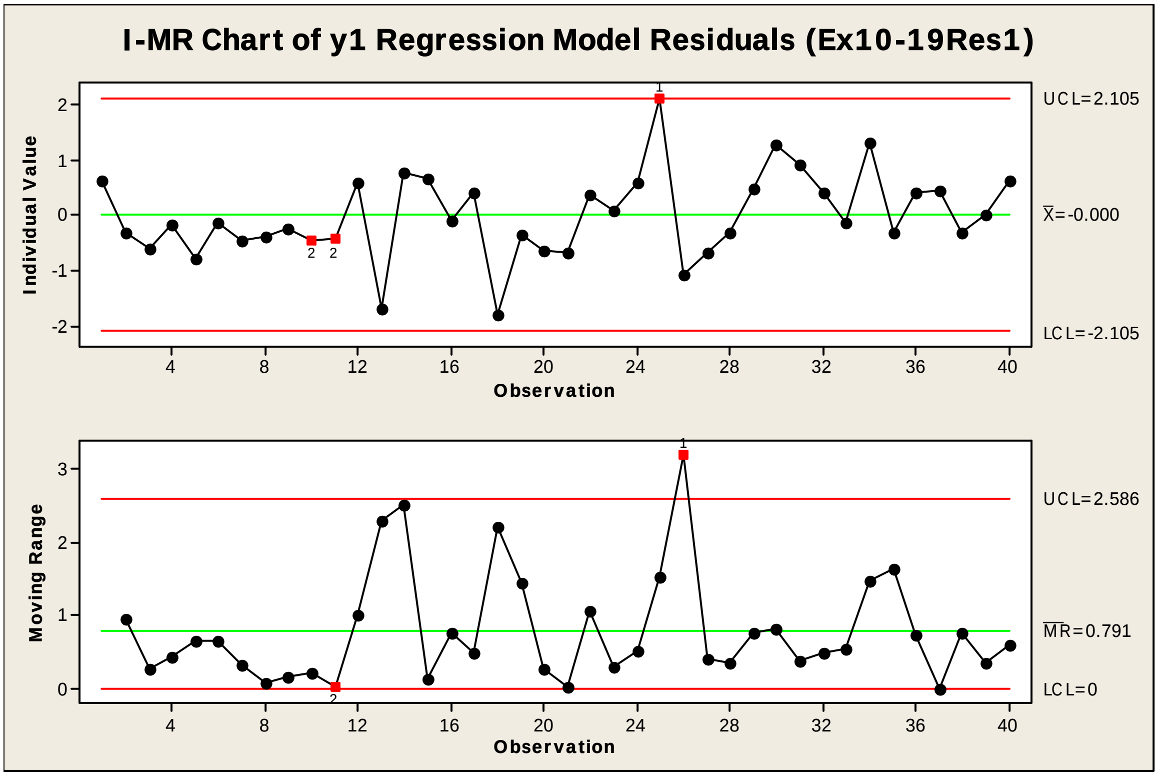
Stat > Regression > Regression

```
Regression Analysis: Tab10-5y1 versus Tab10-5x1, Tab10-5x3, ...
The regression equation is
Tab10-5y1 = 819 + 0.431 Tab10-5x1 - 0.124 Tab10-5x3 - 0.0915 Tab10-5x4
            + 2.64 Tab10-5x8 + 115 Tab10-5x9

Predictor      Coef  SE Coef      T      P
Constant     818.80    29.14  28.10  0.000
Tab10-5x1   0.43080  0.08113   5.31  0.000
Tab10-5x3  -0.12396  0.03530  -3.51  0.001
Tab10-5x4  -0.09146  0.02438  -3.75  0.001
Tab10-5x8    2.6367   0.7604   3.47  0.001
Tab10-5x9    114.81    23.65   4.85  0.000

S = 0.830201   R-Sq = 79.5%   R-Sq(adj) = 76.5%

Analysis of Variance
Source          DF       SS      MS      F      P
Regression       5   90.990  18.198  26.40  0.000
Residual Error  34   23.434   0.689
Total           39  114.424
```

```
Regression Analysis: Tab10-5y2 versus Tab10-5x1, Tab10-5x3, ...
The regression equation is
Tab10-5y2 = 244 - 0.633 Tab10-5x1 + 0.454 Tab10-5x3 + 0.176 Tab10-5x4
            + 11.2 Tab10-5x8 - 236 Tab10-5x9

Predictor     Coef  SE Coef      T      P
Constant     244.4    123.6   1.98  0.056
Tab10-5x1  -0.6329   0.3441  -1.84  0.075
Tab10-5x3   0.4540   0.1497   3.03  0.005
Tab10-5x4   0.1758   0.1034   1.70  0.098
Tab10-5x8   11.175    3.225   3.47  0.001
Tab10-5x9   -235.7    100.3  -2.35  0.025

S = 3.52081   R-Sq = 67.7%   R-Sq(adj) = 62.9%

Analysis of Variance
Source          DF       SS      MS      F      P
Regression       5   882.03  176.41  14.23  0.000
Residual Error  34   421.47   12.40
Total           39  1303.50
```

10-19 continued

Stat > Control Charts > Variables Charts for Individuals > Individuals



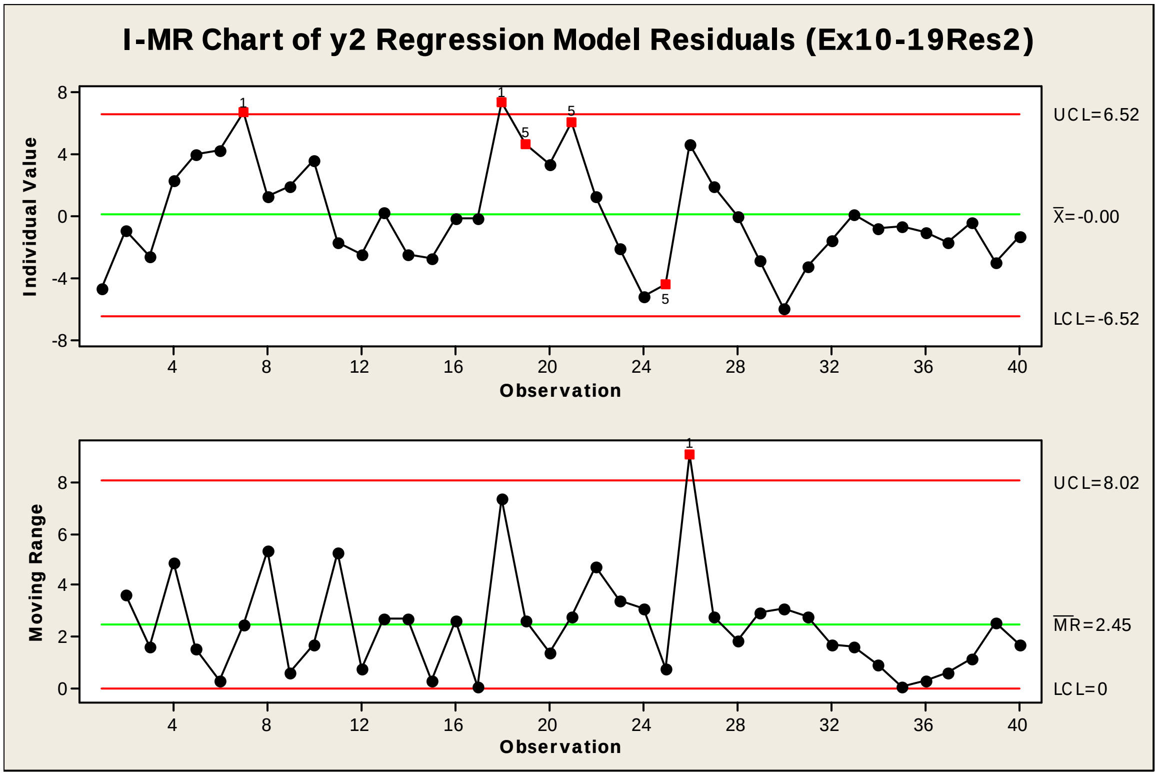
Test Results for I Chart of Ex10-19Res1

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  25
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  10, 11
```

Test Results for MR Chart of Ex10-19Res1

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  26
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  11
```

10-19 continued



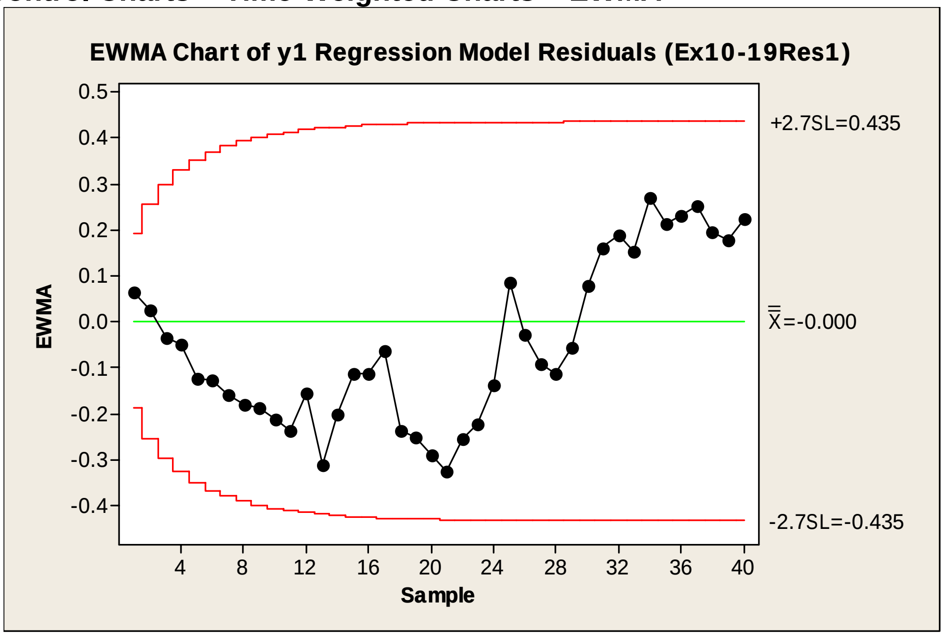

```
Test Results for I Chart of Ex10-19Res2
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  7, 18
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  19, 21, 25
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  7, 21

Test Results for MR Chart of Ex10-19Res2
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  26
```

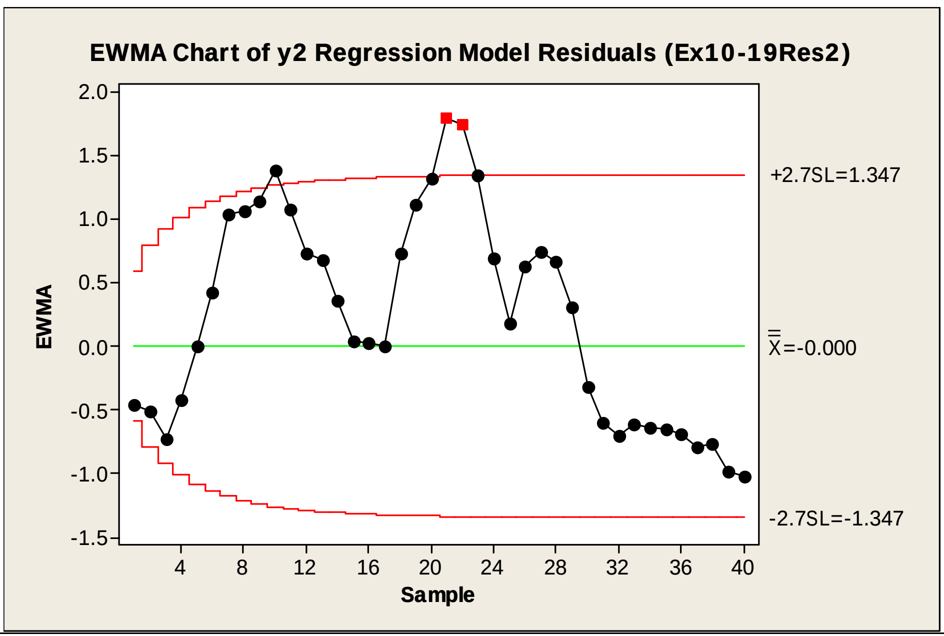
For response *y*1, there is not a significant difference between control charts for residuals from either the full regression model (Figure 10-10, no out-of-control observations) and the subset regression model (observation 25 is OOC).

For response *y*2, there is not a significant difference between control charts for residuals from either the full regression model (Exercise 10-18, observations 7 and 18 are OOC) and the subset regression model (observations 7 and 18 are OOC).

10-20.
Use λ = 0.1 and L = 2.7.

Stat > Control Charts > Time-Weighted Charts > EWMA





Test Results for EWMA Chart of Ex10-19Res2
```
TEST. One point beyond control limits.
Test Failed at points:  21, 22
```

The EWMA control chart for residuals from the response $y1$ subset model has no out-of-control signals. However the chart for $y2$ residuals still indicates a problem beginning near observation 20. A potential advantage to using the EWMA control chart for residuals from a regression model is the quicker detection of small shifts in the process.

10-21.

(a)

Stat > Multivariate > Principal Components

Note: To work in standardized variables in MINITAB, select Correlation Matrix.

Note: To obtain principal component scores, select Storage and enter columns for Scores.

Principal Component Analysis: Ex10-21X1, Ex10-21X2, Ex10-21X3, Ex10-21X4

```
Eigenanalysis of the Correlation Matrix
Eigenvalue  2.3181  1.0118  0.6088  0.0613
Proportion   0.580   0.253   0.152   0.015
Cumulative   0.580   0.832   0.985   1.000

Variable      PC1     PC2     PC3     PC4
Ex10-21X1  0.594  -0.334   0.257   0.685
Ex10-21X2  0.607  -0.330   0.083  -0.718
Ex10-21X3  0.286   0.794   0.534  -0.061
Ex10-21X4  0.444   0.387  -0.801   0.104
```

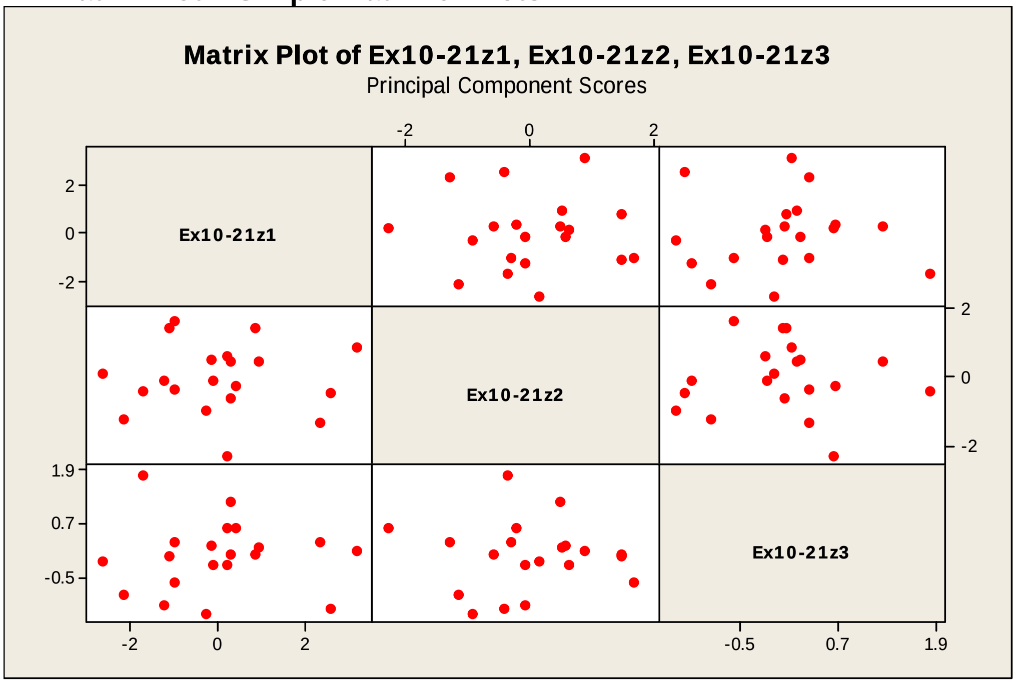
Principal Component Scores

Ex10-21z1	Ex10-21z2	Ex10-21z3
0.29168	-0.60340	0.02496
0.29428	0.49153	1.23823
0.19734	0.64094	-0.20787
0.83902	1.46958	0.03929
3.20488	0.87917	0.12420
0.20327	-2.29514	0.62545
-0.99211	1.67046	-0.58815
-1.70241	-0.36089	1.82157
-0.14246	0.56081	0.23100
-0.99498	-0.31493	0.33164
0.94470	0.50471	0.17976
-1.21950	-0.09129	-1.11787
2.60867	-0.42176	-1.19166
-0.12378	-0.08767	-0.19592
-1.10423	1.47259	0.01299
-0.27825	-0.94763	-1.31445
-2.65608	0.13529	-0.11243
2.36528	-1.30494	0.32286
0.41131	-0.21893	0.64480
-2.14662	-1.17849	-0.86838

10-21 continued

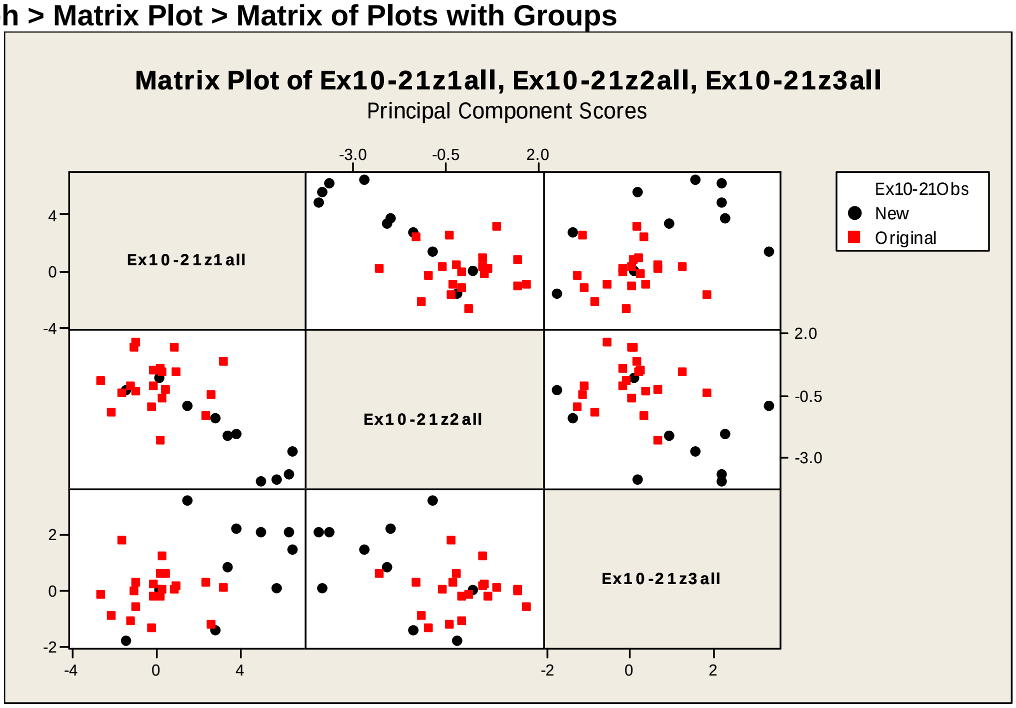
(b)

Graph > Matrix Plot > Simple Matrix of Plots



(c) Note: Principal component scores for new observations were calculated in Excel. See **Excel : workbook Chap10.xls : worksheet Ex10-21.**

Graph > Matrix Plot > Matrix of Plots with Groups



Although a few new points are within area defined by the original points, the majority of new observations are clearly different from the original observations.

10-22.

(a)

Stat > Multivariate > Principal Components

Note: To work in standardized variables in MINITAB, select Correlation Matrix.

Note: To obtain principal component scores, select Storage and enter columns for Scores.

Principal Component Analysis: Ex10-22x1, Ex10-22x2, Ex10-22x3, …, Ex10-22x9

Eigenanalysis of the Correlation Matrix

Eigenvalue	3.1407	2.0730	1.3292	1.0520	0.6129	0.3121	0.2542	0.1973	0.0287
Proportion	0.349	0.230	0.148	0.117	0.068	0.035	0.028	0.022	0.003
Cumulative	0.349	0.579	0.727	0.844	0.912	0.947	0.975	0.997	1.000

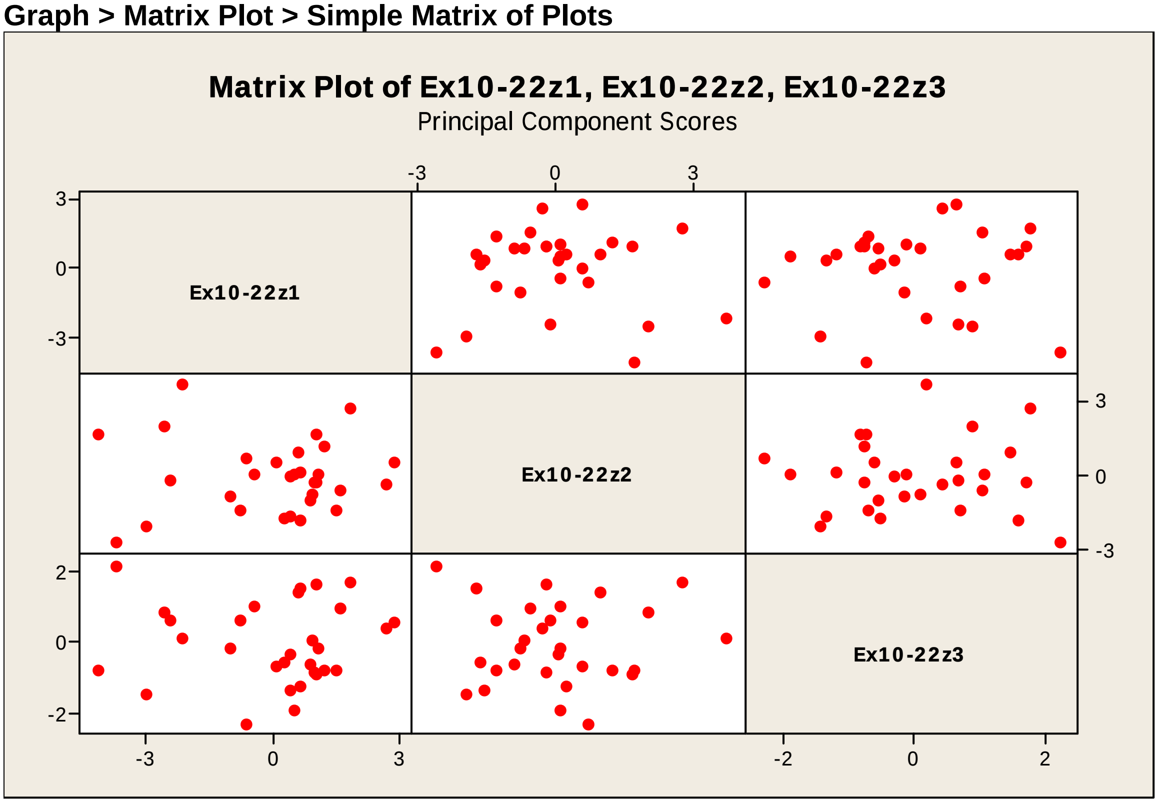
Variable	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9
Ex10-22x1	-0.406	0.204	-0.357	-0.261	0.068	-0.513	0.322	0.467	0.090
Ex10-22x2	0.074	-0.267	0.662	-0.199	0.508	-0.380	0.166	-0.006	-0.124
Ex10-22x3	-0.465	0.050	-0.000	0.156	0.525	0.232	-0.602	0.256	-0.018
Ex10-22x4	0.022	0.409	0.575	-0.200	-0.431	0.135	-0.162	0.471	0.099
Ex10-22x5	-0.436	-0.372	0.089	0.048	-0.277	0.262	0.262	0.152	-0.651
Ex10-22x6	-0.163	0.579	0.108	0.032	0.332	0.419	0.529	-0.244	-0.022
Ex10-22x7	-0.425	-0.407	0.175	-0.014	-0.127	0.193	0.188	-0.105	0.723
Ex10-22x8	-0.120	0.145	0.202	0.874	-0.123	-0.368	0.089	0.021	0.035
Ex10-22x9	0.448	-0.238	-0.115	0.247	0.240	0.323	0.297	0.632	0.133

(b)

72.7% of the variability is explained by the first 3 principal components.

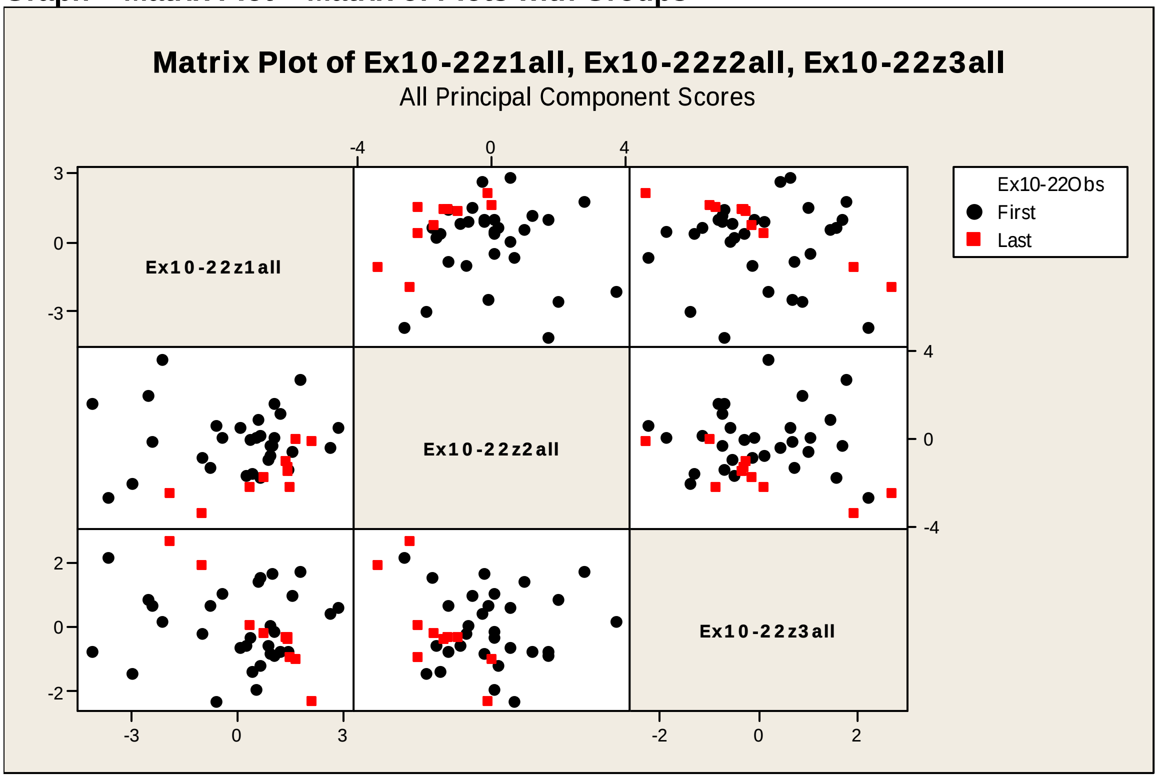
(c)

Graph > Matrix Plot > Simple Matrix of Plots



10-22 continued
(d) Note: Principal component scores for new observations were calculated in Excel. See **Excel : workbook Chap10.xls : worksheet Ex10-22.**

Graph > Matrix Plot > Matrix of Plots with Groups



Several points lie outside the area defined by the first 30 observations, indicating that the process is not in control.

Chapter 11 Exercise Solutions

11-1.
y_t : observation
z_t: EWMA

(a)

$$
\begin{aligned}
z_t &= \lambda y_t + (1-\lambda) z_{t-1} \\
z_t &= \lambda y_t + z_{t-1} - \lambda z_{t-1} \\
z_t - z_{t-1} &= \lambda y_t + z_{t-1} - z_{t-1} - \lambda z_{t-1} \\
z_t - z_{t-1} &= \lambda y_t - \lambda z_{t-1} \\
z_t - z_{t-1} &= \lambda (y_t - z_{t-1})
\end{aligned}
$$

(b)

$$
\begin{aligned}
z_{t-1} - z_{t-2} &= \lambda e_{t-1} \quad \text{(as a result of part (a))} \\
z_{t-1} - z_{t-2} + (e_t - e_{t-1}) &= \lambda e_{t-1} + (e_t - e_{t-1}) \\
z_{t-1} + e_t - z_{t-2} - e_{t-1} &= e_t - (1-\lambda) e_{t-1} \\
y_t - y_{t-1} &= e_t - (1-\lambda) e_{t-1}
\end{aligned}
$$

11-2.

Excel : workbook Chap11.xls : worksheet Ex 11-2

T =	0
lambda =	0.3
L =	10
g =	0.8

Obs	Orig_out	Orig_Nt	Adj_out_t	EWMA_t	|EWMA_t|>L?	Adj_Obs_t+1	Cum_Adj
1	0	0					
2	16	16	16	4.800	no	0.0	0.0
3	24	8	24	10.560	yes	-9.0	-9.0
4	29	5	20.000	6.000	no	0.0	-9.0
5	34	5	25.000	11.700	yes	-9.375	-18.375
6	24	-10	5.625	1.688	no	0.000	-18.375
7	31	7	12.625	4.969	no	0.000	-18.375
8	26	-5	7.625	5.766	no	0.000	-18.375
9	38	12	19.625	9.923	no	0.000	-18.375
10	29	-9	10.625	10.134	yes	-3.984	-22.359
…							
45	22	9	8.025	-0.127	no	0.000	-13.975
46	-9	-31	-22.975	-6.982	no	0.000	-13.975
47	3	12	-10.975	-8.179	no	0.000	-13.975
48	12	9	-1.975	-6.318	no	0.000	-13.975
49	3	-9	-10.975	-7.715	no	0.000	-13.975
50	12	9	-1.975	-5.993	no	0.000	-13.975
SS =	21468		6526.854				
Average =	17.24		0.690				

Bounded Adjustment Chart for Ex 11-2

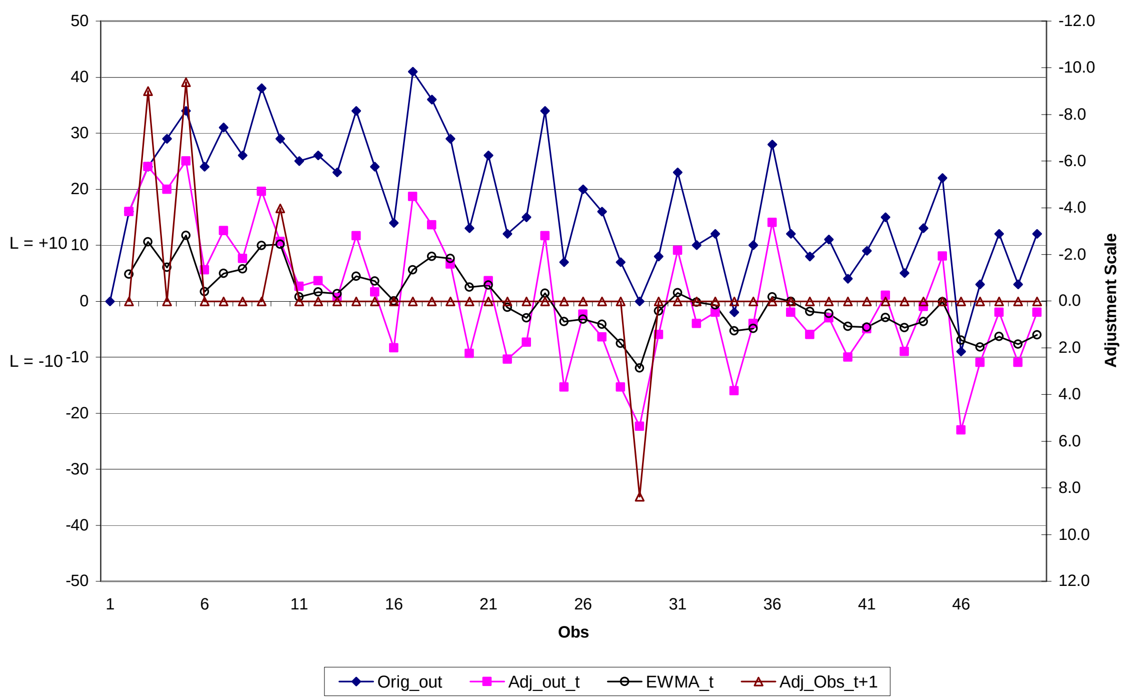


Chart with $\lambda = 0.2$ gives SS = 9780 and average deviation from target = 1.76. The chart with $\lambda = 0.3$ exhibits less variability and is closer to target on average.

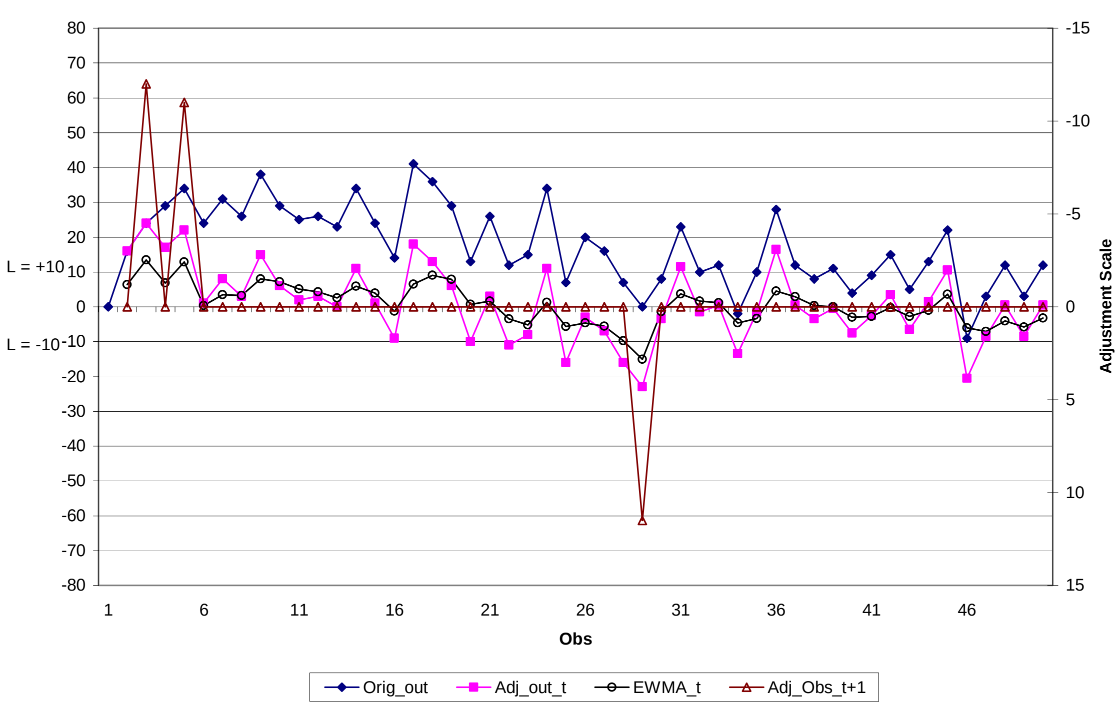
11-3.

Excel : workbook Chap11.xls : worksheet Ex 11-3

Target yt =	0
lambda =	0.4
L =	10
g =	0.8

Obs	**Orig_out**	**Orig_Nt**	**Adj_out_t**	**EWMA_t**	**|EWMA_t|>L?**	**Adj_Obs_t+1**	**Cum_Adj**
1	0	0					
2	16	16	16	6.400	no	0	0
3	**24**	**8**	**24**	**13.440**	**yes**	**-12**	**-12**
4	29	5	17	6.800	no	0	-12
5	**34**	**5**	**22**	**12.880**	**yes**	**-11**	**-23**
6	24	-10	1	0.400	no	0	-23
7	31	7	8	3.440	no	0	-23
8	26	-5	3	3.264	no	0	-23
9	38	12	15	7.958	no	0	-23
10	29	-9	6	7.175	no	0	-23
…							
46	-9	-31	-20.5	-6.061	no	0	-11.5
47	3	12	-8.5	-7.037	no	0	-11.5
48	12	9	0.5	-4.022	no	0	-11.5
49	3	-9	-8.5	-5.813	no	0	-11.5
50	12	9	0.5	-3.288	no	0	-11.5
SS =	21468		5610.25				
Average =	17.24		0.91				

Bounded Adjustment Chart for Ex 11-3



The chart with $\lambda = 0.4$ exhibits less variability, but is further from target on average than for the chart with $\lambda = 0.3$.

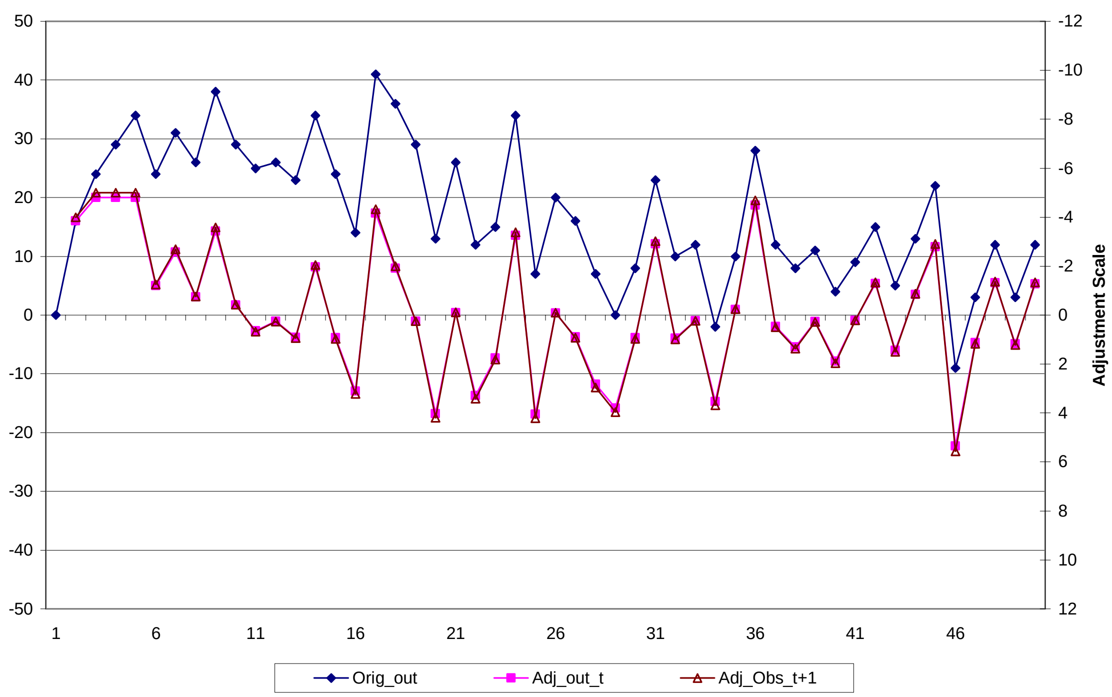
11-4.

Excel : workbook Chap11.xls : worksheet Ex 11-4

T =	0
lambda =	0.2
g =	0.8

Obs	**Orig_out**	**Orig_Nt**	**Adj_out_t**	**Adj_Obs_t+1**	**Cum_Adj**
1	0	0			
2	16	16	16.0	-4.0	-4.0
3	24	8	20.0	-5.0	-9.0
4	29	5	20.0	-5.0	-14.0
5	34	5	20.0	-5.0	-19.0
6	24	-10	5.0	-1.3	-20.3
7	31	7	10.8	-2.7	-22.9
8	26	-5	3.1	-0.8	-23.7
9	38	12	14.3	-3.6	-27.3
10	29	-9	1.7	-0.4	-27.7
...					
45	22	9	11.6	-2.9	-13.3
46	-9	-31	-22.3	5.6	-7.7
47	3	12	-4.7	1.2	-6.5
48	12	9	5.5	-1.4	-7.9
49	3	-9	-4.9	1.2	-6.7
50	12	9	5.3	-1.3	-8.0
SS =	21468		5495.9		
Average =	17.24		0.7		

Integral Control for Ex 11-4

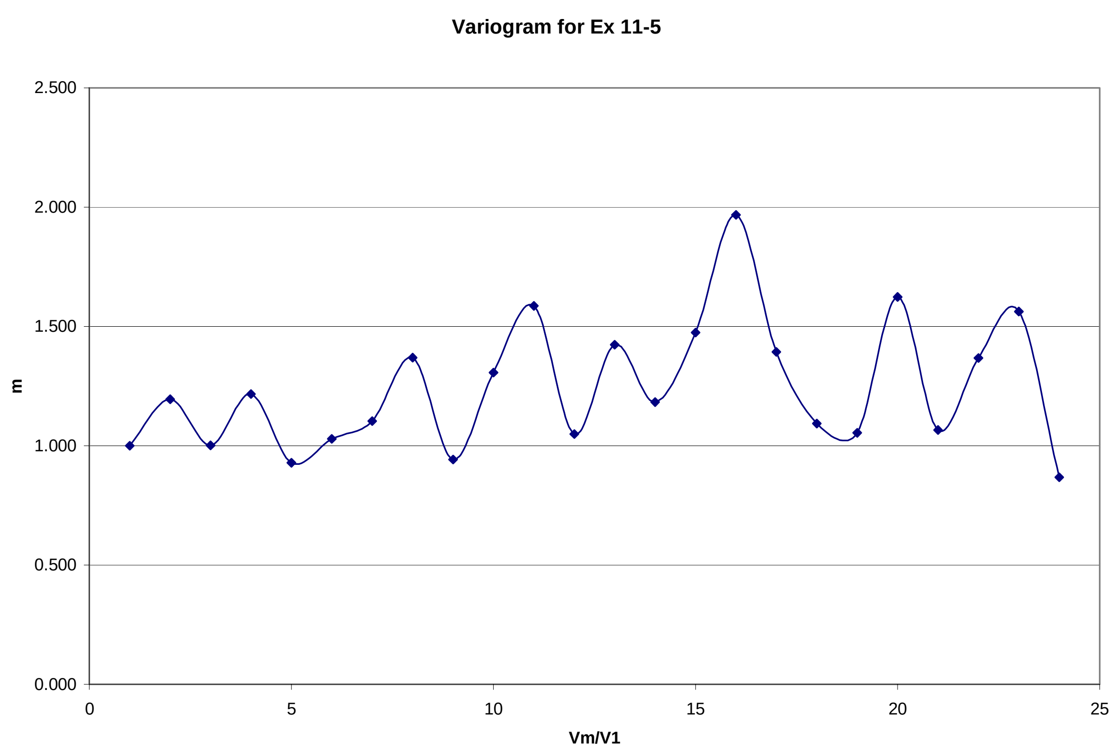


The chart with process adjustment after every observation exhibits approximately the same variability and deviation from target as the chart with $\lambda = 0.4$.

11-5.

Excel : workbook Chap11.xls : worksheet Ex 11-5

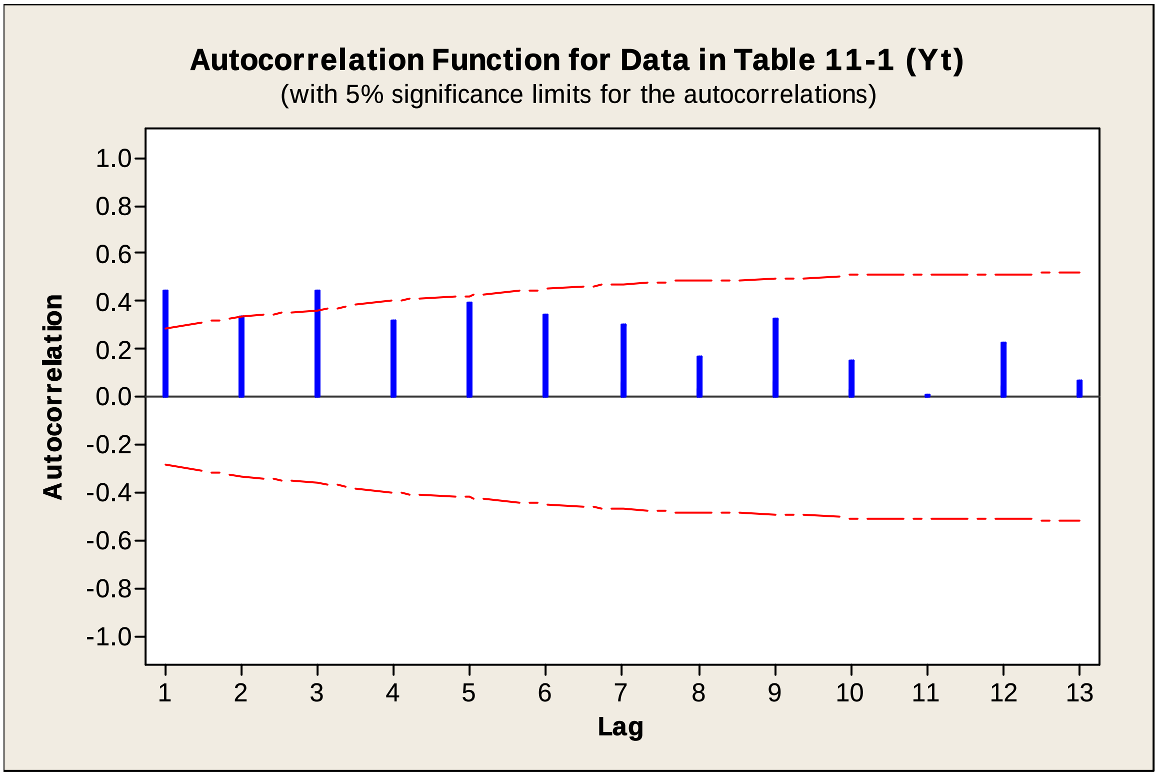
t	**Yt / m =>**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	0									
2	16	16								
3	24	8	24							
4	29	5	13	29						
5	34	5	10	18	34					
6	24	-10	-5	0	8	24				
7	31	7	-3	2	7	15	31			
8	26	-5	2	-8	-3	2	10	26		
9	38	12	7	14	4	9	14	22	38	
10	29	-9	3	-2	5	-5	0	5	13	29
…										
Var_m =		147.11	175.72	147.47	179.02	136.60	151.39	162.43	201.53	138.70
Var_m/Var_1 =		1.000	1.195	1.002	1.217	0.929	1.029	1.104	1.370	0.943



11-5 continued

MTB : Chap11.mtw : Yt

Stat > Time Series > Autocorrelation Function



Autocorrelation Function: Yt

Lag	ACF	T	LBQ
1	0.440855	3.12	10.31
2	0.334961	2.01	16.39
3	0.440819	2.45	27.14
4	0.316478	1.58	32.80
5	0.389094	1.85	41.55
6	0.345327	1.54	48.59
7	0.299822	1.28	54.03
8	0.164698	0.68	55.71
9	0.325056	1.33	62.41
10	0.149321	0.59	63.86
11	0.012158	0.05	63.87
12	0.228540	0.90	67.44
13	0.066173	0.26	67.75

Variogram appears to be increasing, so the observations are correlated and there may be some mild indication of nonstationary behavior. The slow decline in the sample ACF also indicates the data are correlated and potentially nonstationary.

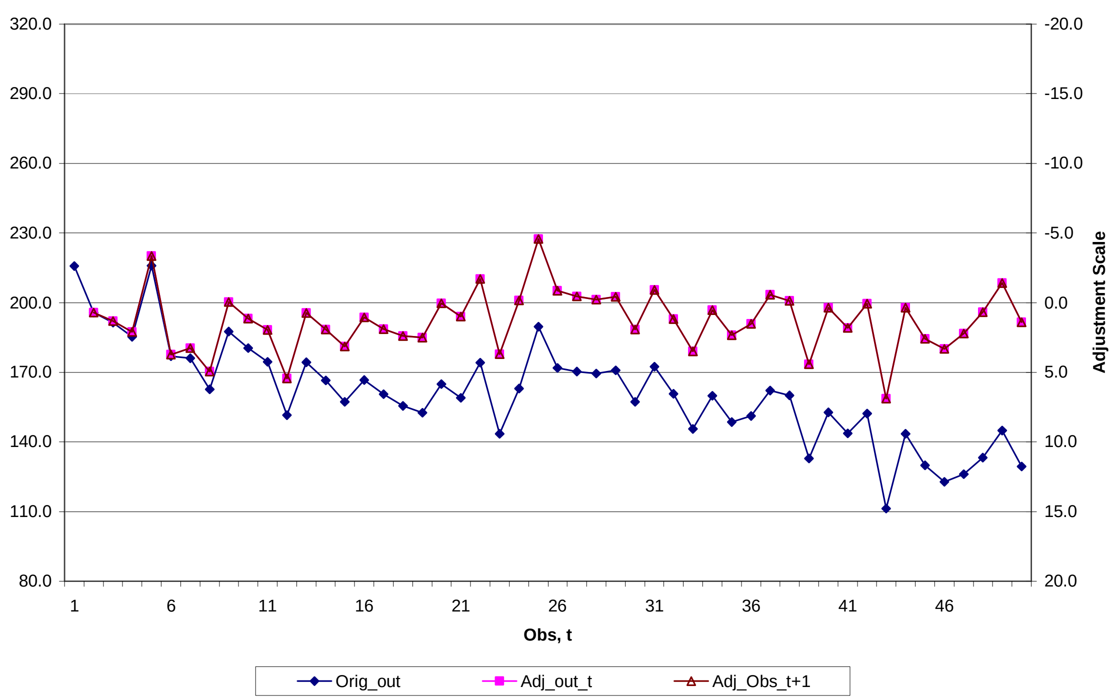
11-6.
(a) and (b)
Excel : workbook Chap11.xls : worksheet Ex 11-6a

T =	200
lambda =	0.2
g =	1.2

Obs, t	Orig_out	Orig_Nt	Adj_out_t	Adj_Obs_t+1	Cum_Adj
1	215.8	0.0			
2	195.8	-20.0	195.8	0.7	0.7
3	191.3	-4.5	192.0	1.3	2.0
4	185.3	-6.0	187.3	2.1	4.1
5	216.0	30.7	220.1	-3.4	0.8
6	176.9	-39.1	177.7	3.7	4.5
7	176.0	-0.9	180.5	3.2	7.8
8	162.6	-13.4	170.4	4.9	12.7
9	187.5	24.9	200.2	0.0	12.7
10	180.5	-7.0	193.2	1.1	13.8
…					
49	145.0	11.8	208.4	-1.4	62.0
50	129.5	-15.5	191.5	1.4	63.4

	Unadjusted	Adjusted
SS =	1,323,871.8	1,818,510.3
Average =	161.3	192.2
Variance =	467.8	160.9

Integral Control for Ex 11-6(a)



Significant reduction in variability with use of integral control scheme.

11-6 continued

(c)

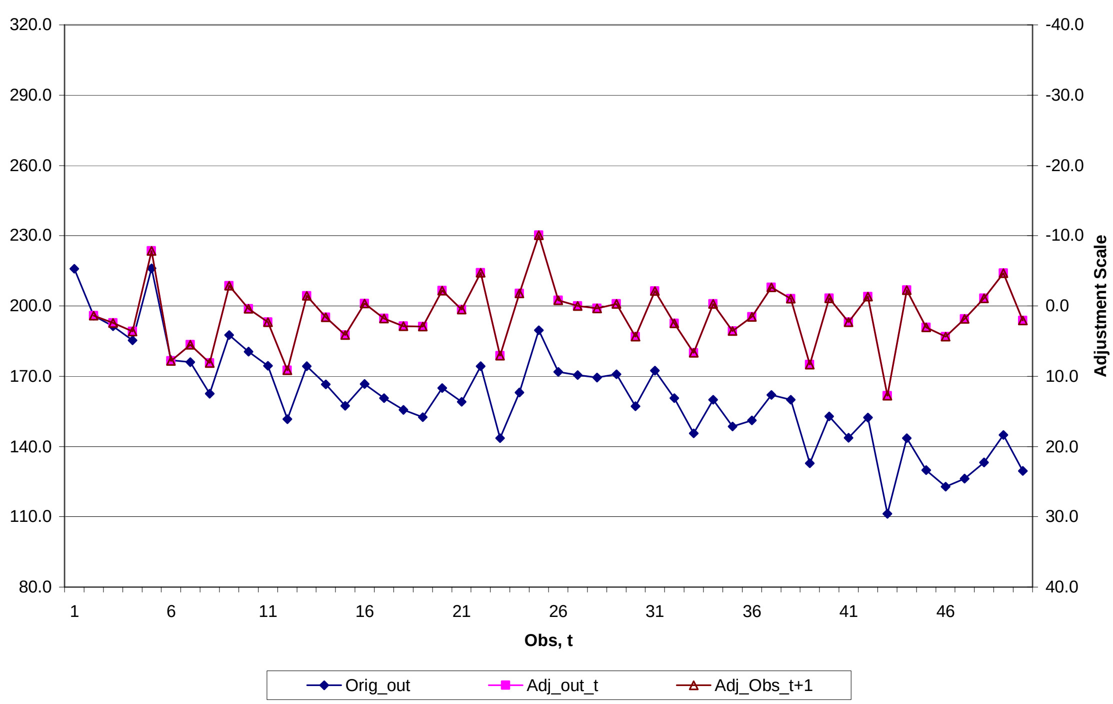
Excel : workbook Chap11.xls : worksheet Ex 11-6c

T =	200
lambda =	0.4
g =	1.2

Obs, t	**Orig_out**	**Orig_Nt**	**Adj_out_t**	**Adj_Obs_t+1**	**Cum_Adj**
1.0	215.8	0.0			
2.0	195.8	-20.0	195.8	1.4	1.4
3.0	191.3	-4.5	192.7	2.4	3.8
4.0	185.3	-6.0	189.1	3.6	7.5
5.0	216.0	30.7	223.5	-7.8	-0.4
6.0	176.9	-39.1	176.5	7.8	7.5
7.0	176.0	-0.9	183.5	5.5	13.0
8.0	162.6	-13.4	175.6	8.1	21.1
9.0	187.5	24.9	208.6	-2.9	18.2
10.0	180.5	-7.0	198.7	0.4	18.7
…					
50.0	129.5	-15.5	193.9	2.0	66.4

	Unadjusted	**Adjusted**
SS =	1,323,871.8	1,888,995.0
Average =	161.3	195.9
Variance =	467.8	164.0

Integral Control for Ex 11-6(c)



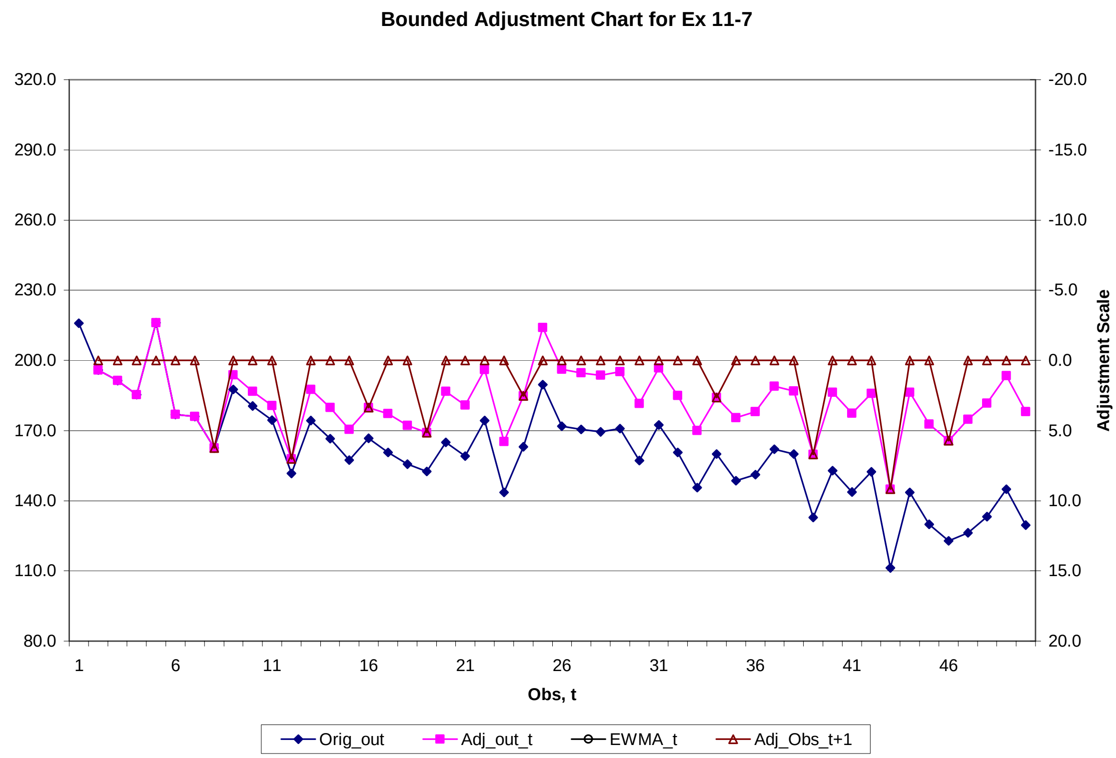
Variances are similar for both integral adjustment control schemes ($\lambda = 0.2$ and $\lambda = 0.4$).

11-7.

Excel : workbook Chap11.xls : worksheet Ex 11-7

Target yt =	200
lambda =	0.2
L =	12
g =	1.2

Obs, t	**Orig_out**	**Orig_Nt**	**Adj_out_t**	**EWMA_t**	**|EWMA_t|>L?**	**Adj_Obs_t+1**	**Cum_Adj**
1	215.8	0					
2	195.8	-20	196	-0.840	no	0.0	0.0
3	191.3	-4.5	191.300	-2.412	no	0.0	0.000
4	185.3	-6	185.300	-4.870	no	0.000	0.000
5	216.0	30.7	216.000	-0.696	no	0.000	0.000
6	176.9	-39.1	176.900	-5.177	no	0.000	0.000
7	176.0	-0.9	176.000	-8.941	no	0.000	0.000
8	162.6	-13.4	162.600	-14.633	yes	6.233	6.233
9	187.5	24.9	193.733	-1.253	no	0.000	6.233
10	180.5	-7	186.733	-3.656	no	0.000	6.233
…							
46	122.9	-7	165.699	-12.969	yes	5.717	48.516
47	126.2	3.3	174.716	-5.057	no	0.000	48.516
48	133.2	7	181.716	-7.702	no	0.000	48.516
49	145.0	11.8	193.516	-7.459	no	0.000	48.516
50	129.5	-15.5	178.016	-10.364	no	0.000	48.516
SS =	1,323,872		1,632,265				
Average =	161.304		182.051				
Variance =	467.8		172.7				



Behavior of the bounded adjustment control scheme is similar to both integral control schemes ($\lambda = 0.2$ and $\lambda = 0.4$).

11-8.

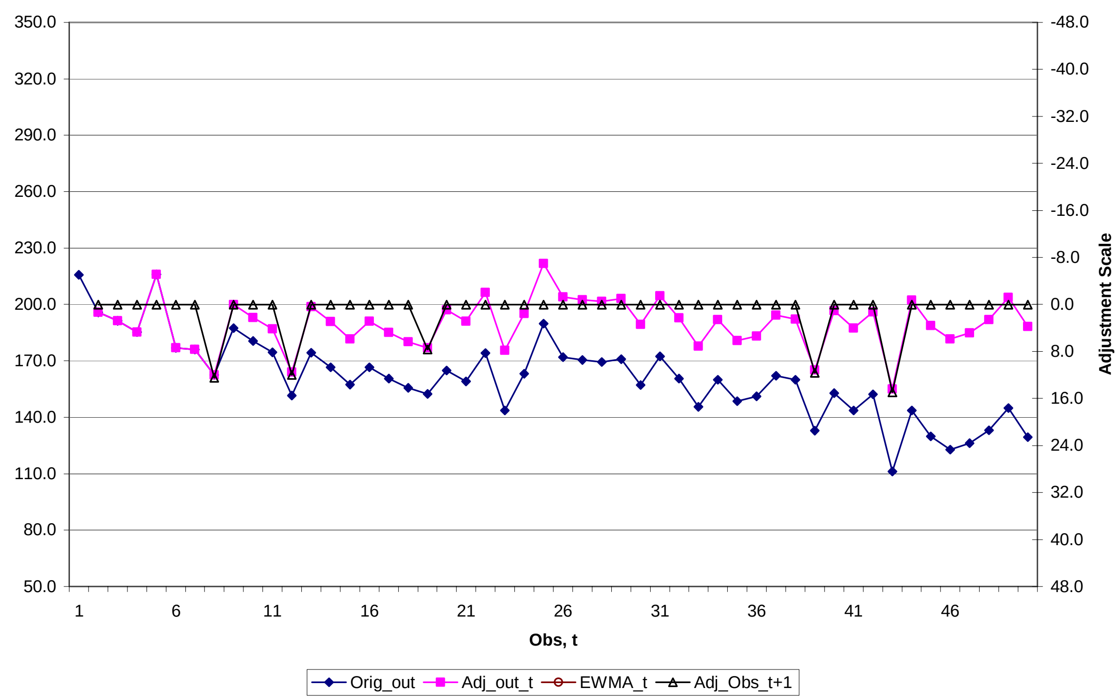
Excel : workbook Chap11.xls : worksheet Ex 11-8

T =	200
lambda =	0.4
L =	15
g =	1.2

Obs, t	Orig_out	Orig_Nt	Adj_out_t	EWMA_t	|EWMA_t|>L?	Adj_Obs_t+1	Cum_Adj
1	215.8	0					
2	195.8	-20	196	-1.680	no	0.0	0.0
3	191.3	-4.5	191.300	-4.488	no	0.0	0.000
4	185.3	-6	185.300	-8.573	no	0.000	0.000
5	216.0	30.7	216.000	1.256	no	0.000	0.000
6	176.9	-39.1	176.900	-8.486	no	0.000	0.000
7	176.0	-0.9	176.000	-14.692	no	0.000	0.000
8	162.6	-13.4	162.600	-23.775	yes	12.467	12.467
9	187.5	24.9	199.967	-0.013	no	0.000	12.467
10	180.5	-7	192.967	-2.821	no	0.000	12.467
…							
46	122.9	-7	181.658	-9.720	no	0.000	58.758
47	126.2	3.3	184.958	-11.849	no	0.000	58.758
48	133.2	7	191.958	-10.326	no	0.000	58.758
49	145.0	11.8	203.758	-4.693	no	0.000	58.758
50	129.5	-15.5	188.258	-7.513	no	0.000	58.758

	Orig_out		Adj_out_t
SS =	1,323,872		1,773,083
Average =	161.304		189.784
Variance =	467.81		170.86

Bounded Adjustment Chart for Ex 11-8



Behavior of both bounded adjustment control schemes are similar to each other and simlar to the integral control schemes.

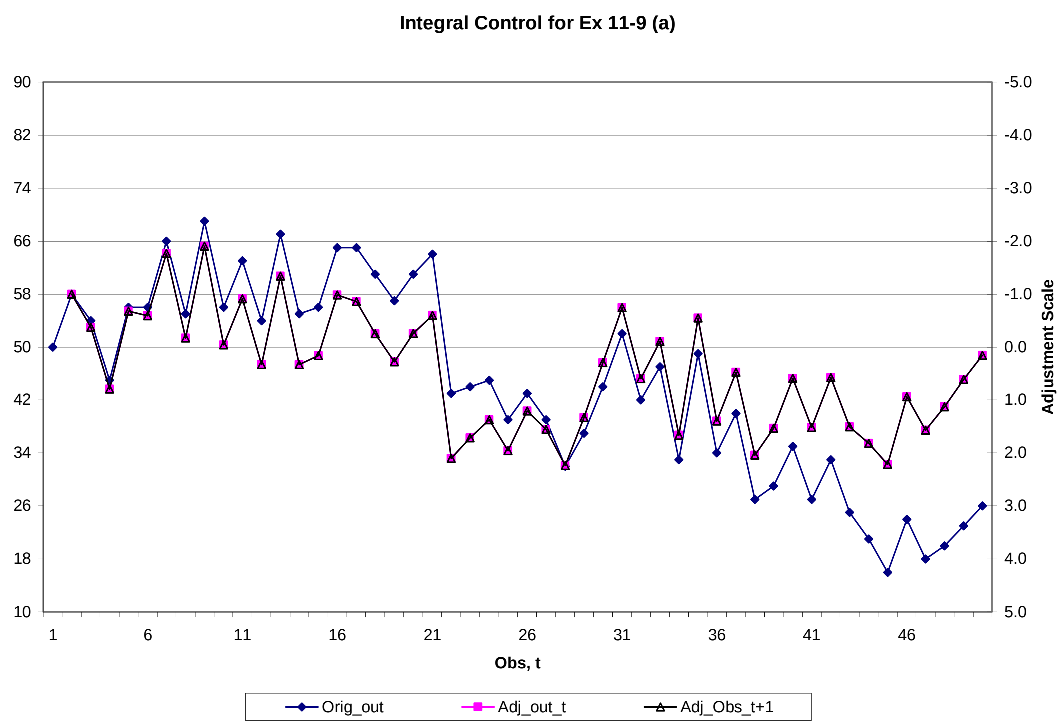
11-9.
(a) and (b)
Excel : workbook Chap11.xls : worksheet Ex 11-9a

T =	50
lambda =	0.2
g =	1.6

Obs	Orig_out	Orig_Nt	Adj_out_t	Adj_Obs_t+1	Cum_Adj
1	50				
2	58	8.0	58.0	-1.0	-1.0
3	54	-4.0	53.0	-0.4	-1.4
4	45	-9.0	43.6	0.8	-0.6
5	56	11.0	55.4	-0.7	-1.3
6	56	0.0	54.7	-0.6	-1.8
7	66	10.0	64.2	-1.8	-3.6
8	55	-11.0	51.4	-0.2	-3.8
9	69	14.0	65.2	-1.9	-5.7
10	56	-13.0	50.3	0.0	-5.7
…					
49	23	3.0	45.1	0.6	22.7
50	26	3.0	48.7	0.2	22.9

	Unadjusted	Adjusted
SS =	109,520	108,629
Average =	44.4	46.262
Variance =	223.51	78.32

Significant reduction in variability with use of an integral control scheme.



11-9 continued

(c)

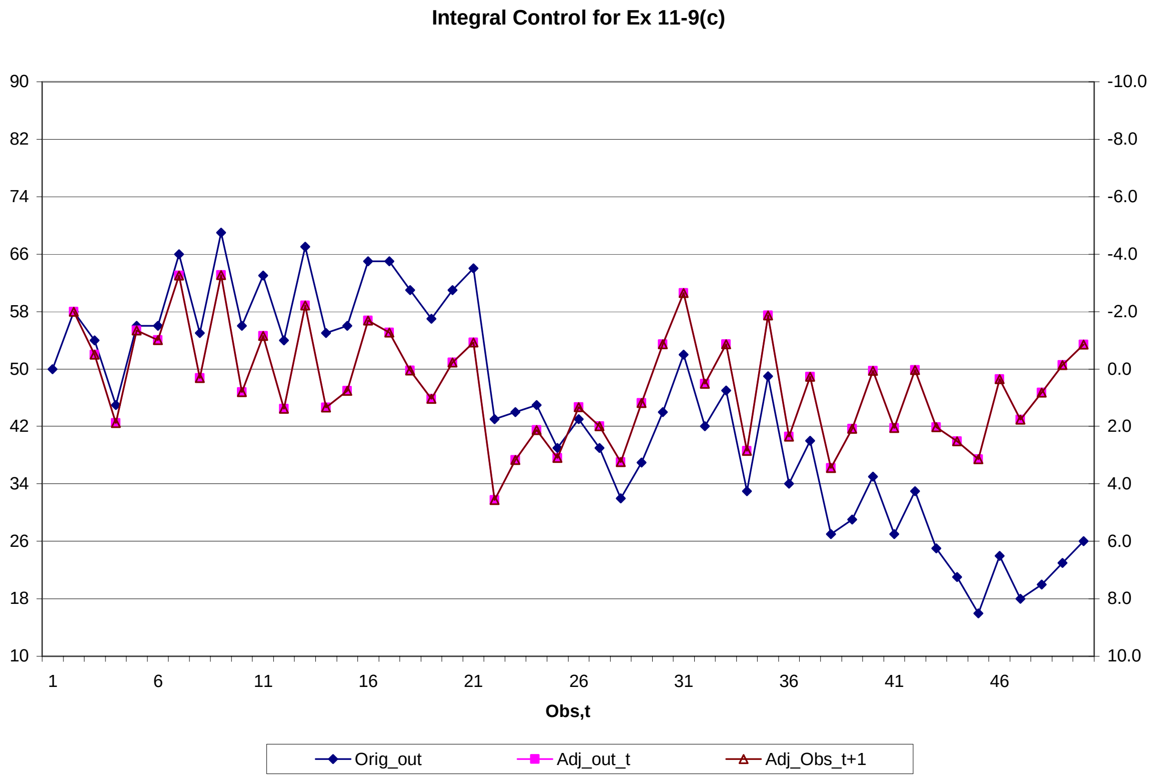
Excel : workbook Chap11.xls : worksheet Ex 11-9c

T =	50
lambda =	0.4
g =	1.6

Obs	**Orig_out**	**Orig_Nt**	**Adj_out_t**	**Adj_Obs_t+1**	**Cum_Adj**
1	50				
2	58	8.0	58.0	-2.0	-2.0
3	54	-4.0	52.0	-0.5	-2.5
4	45	-9.0	42.5	1.9	-0.6
5	56	11.0	55.4	-1.3	-2.0
6	56	0.0	54.0	-1.0	-3.0
7	66	10.0	63.0	-3.3	-6.2
8	55	-11.0	48.8	0.3	-5.9
9	69	14.0	63.1	-3.3	-9.2
10	56	-13.0	46.8	0.8	-8.4
…					
49	23	3.0	50.5	-0.1	27.4
50	26	3.0	53.4	-0.8	26.5

SS =	109,520		114,819
Average =	44.4		47.833
Variance =	223.51		56.40

There is a slight reduction in variability with use of $\lambda = 0.4$, as compared to $\lambda = 0.2$, with a process average slightly closer to the target of 50.



Integral Control for Ex 11-9(c)
90
82
74
66
58
50
42
34
26
18
10
-10.0
-8.0
-6.0
-4.0
-2.0
0.0
2.0
4.0
6.0
8.0
10.0
1
6
11
16
21
26
31
36
41
46
Obs,t
Orig_out
Adj_out_t
Adj_Obs_t+1

11-10.

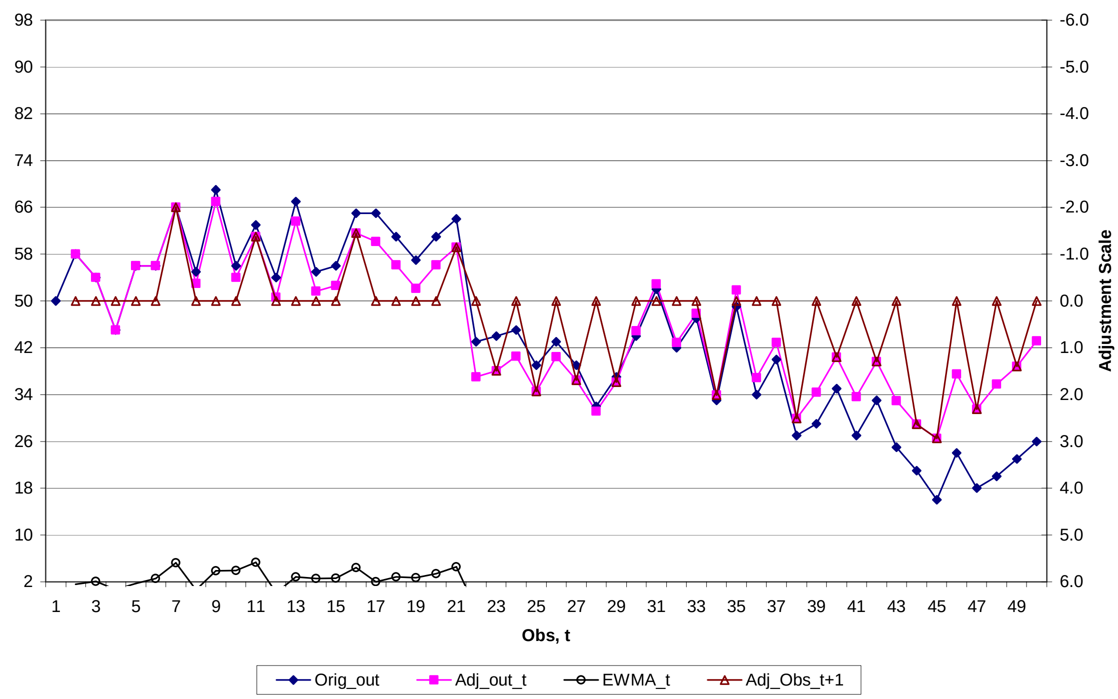
Excel : workbook Chap11.xls : worksheet Ex 11-10

Target yt =	50
lambda =	0.2
L =	4
g =	1.6

Obs, t	**Orig_out**	**Orig_Nt**	**Adj_out_t**	**EWMA_t**	**|EWMA_t|>L?**	**Adj_Obs_t+1**	**Cum_Adj**
1	50	0					
2	58	8	58	1.600	no	0.0	0.0
3	54	-4	54.00	2.080	no	0.00	0.00
4	45	-9	45.00	0.664	no	0.00	0.00
5	56	11	56.00	1.731	no	0.00	0.00
6	56	0	56.00	2.585	no	0.00	0.00
7	66	10	66.00	5.268	yes	-2.00	-2.00
8	55	-11	53.00	0.600	no	0.00	-2.00
9	69	14	67.00	3.880	no	0.00	-2.00
10	56	-13	54.00	3.904	no	0.00	-2.00
...							
46	24	8	37.48	-2.505	no	0.00	13.48
47	18	-6	31.48	-5.709	yes	2.32	15.79
48	20	2	35.79	-2.842	no	0.00	15.79
49	23	3	38.79	-4.515	yes	1.40	17.19
50	26	3	43.19	-1.362	no	0.00	17.19

SS =	109,520	107,822
Average =	44.4	45.620
Variance =	223.51	121.72

Bounded Adjustment Chart for Ex 11-10



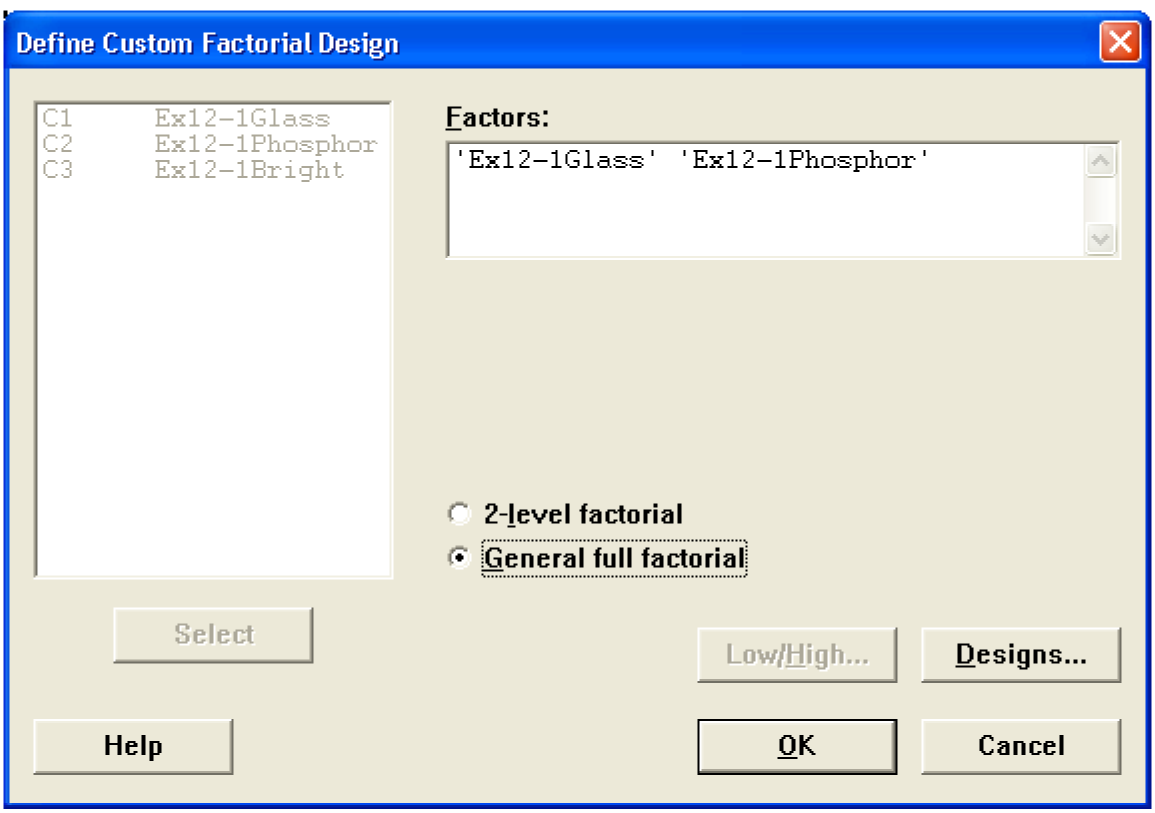
Nearly the same performance as the integral control scheme, with similar means and sums of squares, but different variances (bounded adjustment variance is larger).

Chapter 12 Exercise Solutions

Note: To analyze an experiment in MINITAB, the initial experimental layout must be created in MINITAB or defined by the user. The Excel data sets contain only the data given in the textbook; therefore some information required by MINITAB is not included. Detailed MINITAB instructions are provided for Exercises 12-1 and 12-2 to define and create designs. The remaining exercises are worked in a similar manner, and only the solutions are provided.

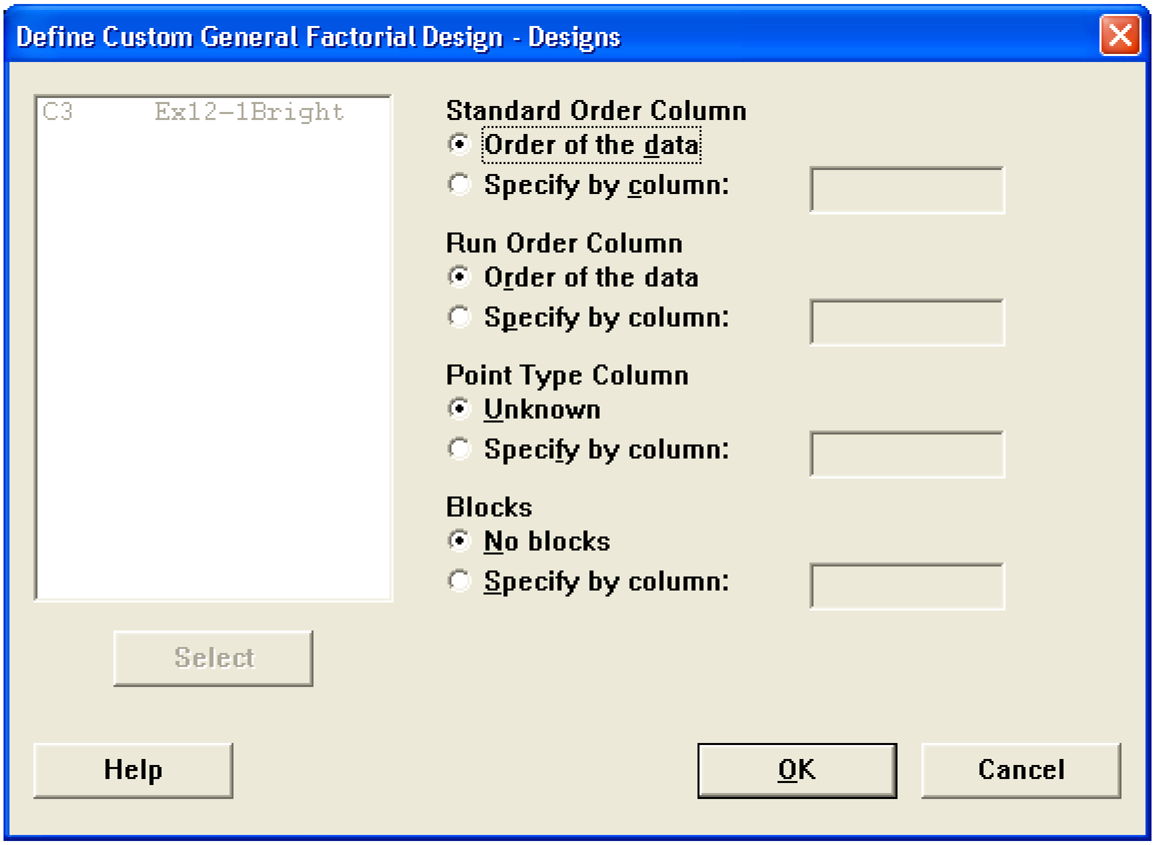
12-1.
This experiment is three replicates of a factorial design in two factors—two levels of glass type and three levels of phosphor type—to investigate brightness. Enter the data into the MINITAB worksheet using the first three columns: one column for glass type, one column for phosphor type, and one column for brightness. This is how the Excel file is structured (**Chap12.xls**). Since the experiment layout was not created in MINITAB, the design must be defined before the results can be analyzed.

After entering the data in MINITAB, select **Stat > DOE > Factorial > Define Custom Factorial Design**. Select the two factors (Glass Type and Phosphor Type), then for this exercise, check "**General full factorial**". The dialog box should look:



12-1 continued

Next, select “**Designs**”. For this exercise, no information is provided on standard order, run order, point type, or blocks, so leave the selections as below, and click “**OK**” twice.



Note that MINITAB added four new columns (4 through 7) to the worksheet. DO NOT insert or delete columns between columns 1 through 7. MINITAB recognizes these contiguous seven columns as a designed experiment; inserting or deleting columns will cause the design layout to become corrupt.

The design and data are in the MINITAB worksheet **Ex12-1.MTW**.

12-1 continued

Select **Stat > DOE > Factorial > Analyze Factorial Design**. Select the response (Brightness), then click on "**Terms**", verify that the selected terms are Glass Type, Phosphor Type, and their interaction, click "**OK**". Click on "**Graphs**", select "**Residuals Plots : Four in one**". The option to plot residuals versus variables is for continuous factor levels; since the factor levels in this experiment are categorical, do not select this option. Click "**OK**". Click on "**Storage**", select "**Fits**" and "**Residuals**", and click "**OK**" twice.

General Linear Model: Ex12-1Bright versus Ex12-1Glass, Ex12-1Phosphor

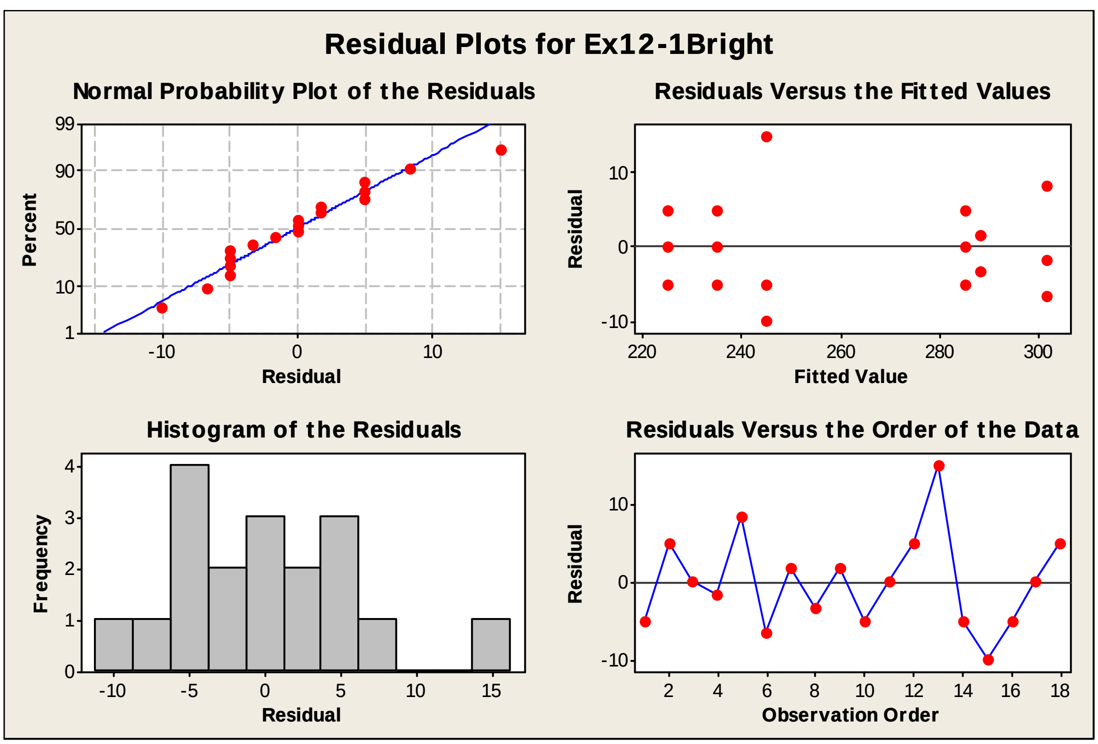
Factor	Type	Levels	Values
Ex12-1Glass	fixed	2	1, 2
Ex12-1Phosphor	fixed	3	1, 2, 3

Analysis of Variance for Ex12-1Bright, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Ex12-1Glass	1	14450.0	14450.0	14450.0	273.79	0.000
Ex12-1Phosphor	2	933.3	933.3	466.7	8.84	0.004
Ex12-1Glass*Ex12-1Phosphor	2	133.3	133.3	66.7	1.26	0.318
Error	12	633.3	633.3	52.8		
Total	17	16150.0				

S = 7.26483   R-Sq = 96.08%   R-Sq(adj) = 94.44%

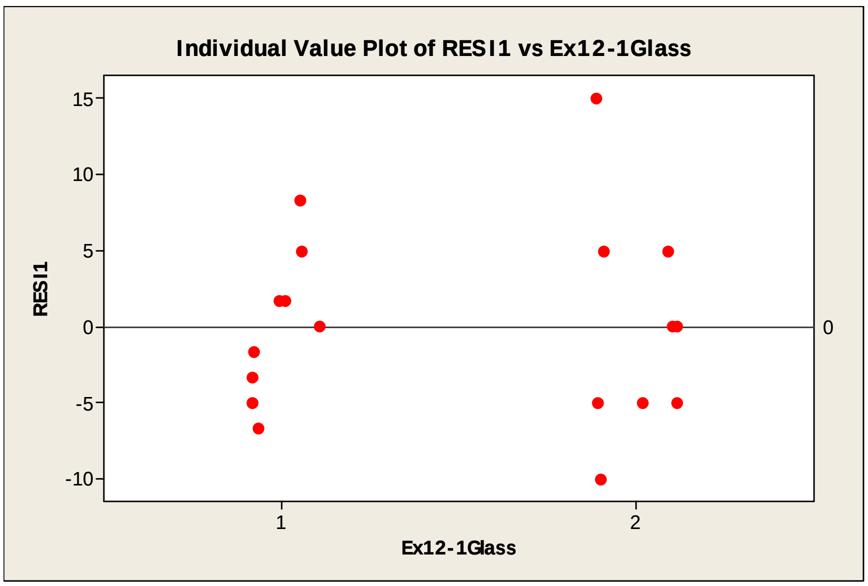
No indication of significant interaction (P-value is greater than 0.10). Glass type (A) and phosphor type (B) significantly affect television tube brightness (P-values are less than 0.10).

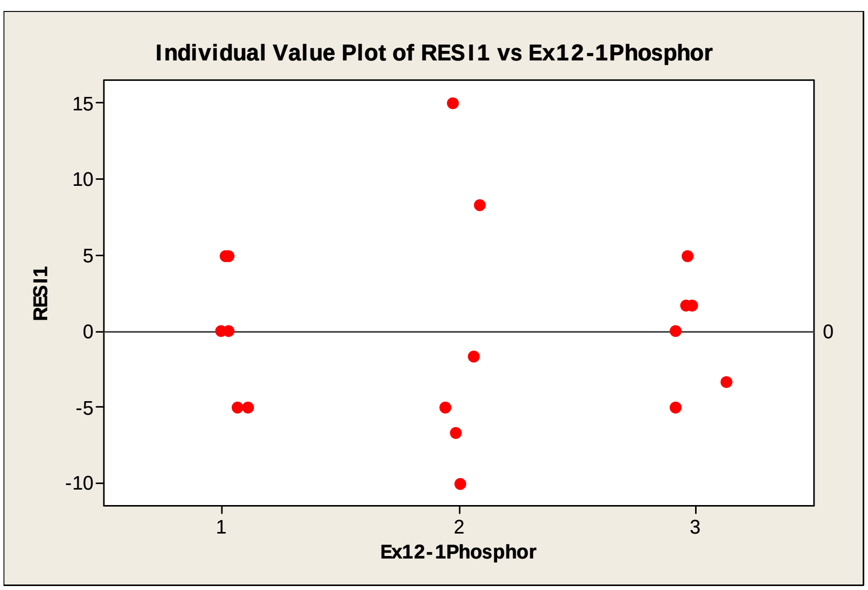


12-1 continued

Visual examination of residuals on the normal probability plot, histogram, and versus fitted values reveals no problems. The plot of residuals versus observation order is not meaningful since no order was provided with the data. If the model were re-fit with only Glass Type and Phosphor Type, the residuals should be re-examined.

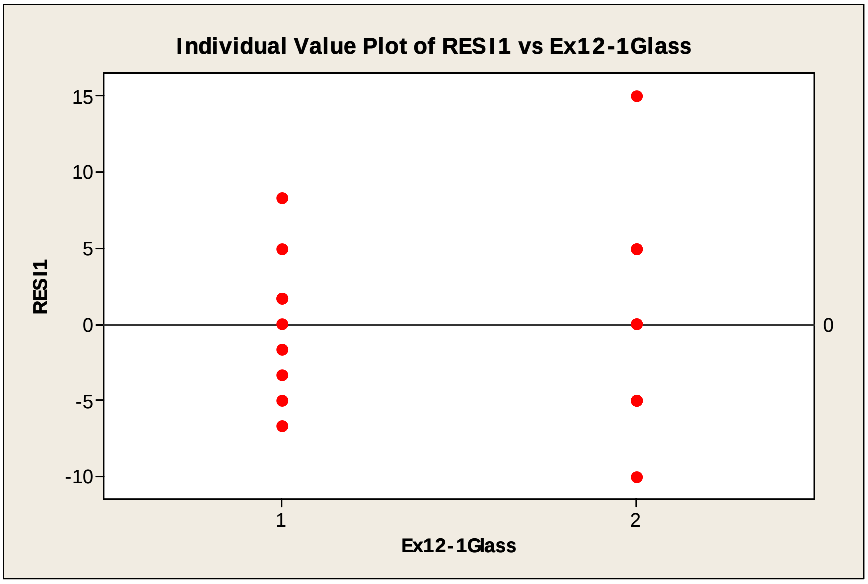
To plot residuals versus the two factors, select **Graph > Individual Value Plot > One Y with Groups**. Select the column with stored residuals (**RESI1**) as the **Graph variable** and select one of the factors (Glass Type or Phosphor Type) as the **Categorical variable for grouping**. Click on "**Scale**", select the "**Reference Lines**" tab, and enter "**0**" for the Y axis, then click "**OK**" twice.

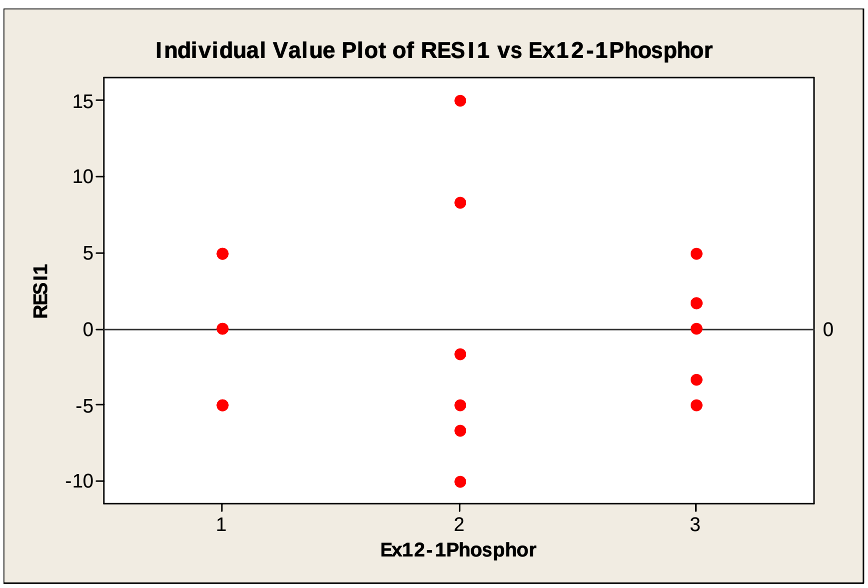




12-1 continued

Note that the plot points are "jittered" about the factor levels. To remove the jitter, select the graph to make it active then: **Editor > Select Item > Individual Symbols** and then **Editor > Edit Individual Symbols > Jitter** and de-select **Add jitter to direction**.

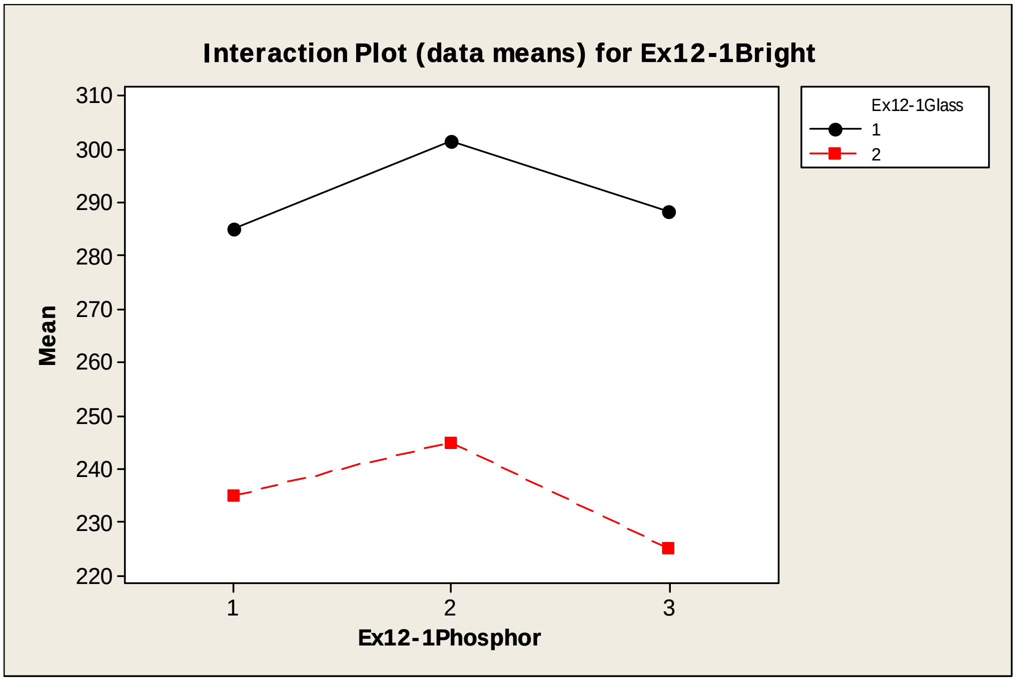




Variability appears to be the same for both glass types; however, there appears to be more variability in results with phosphor type 2.

12-1 continued

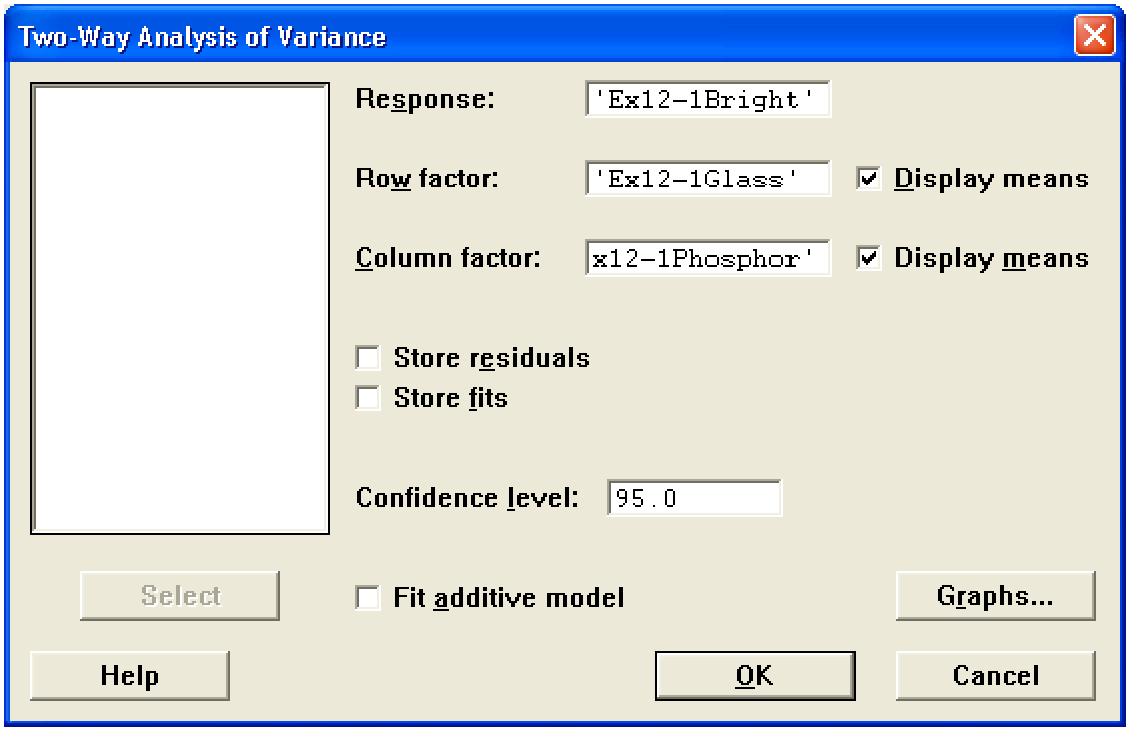
Select **Stat > DOE > Factorial > Factorial Plots**. Select "**Interaction Plot**" and click on "**Setup**", select the response (Brightness) and both factors (Glass Type and Phosphor Type), and click "**OK**" twice.



The absence of a significant interaction is evident in the parallelism of the two lines. Final selected combination of glass type and phosphor type depends on the desired brightness level.

12-1 continued

<u>Alternate Solution:</u> This exercise may also be solved using MINITAB's ANOVA functionality instead of its DOE functionality. The DOE functionality was selected to illustrate the approach that will be used for most of the remaining exercises. To obtain results which match the output in the textbook's Table 12.5, select **Stat > ANOVA > Two-Way**, and complete the dialog box as below.



Two-way ANOVA: Ex12-1Bright versus Ex12-1Glass, Ex12-1Phosphor

```
Source          DF       SS       MS       F      P
Ex12-1Glass      1  14450.0  14450.0  273.79  0.000
Ex12-1Phosphor   2    933.3    466.7    8.84  0.004
Interaction      2    133.3     66.7    1.26  0.318
Error           12    633.3     52.8
Total           17  16150.0
S = 7.265   R-Sq = 96.08%   R-Sq(adj) = 94.44%

                      Individual 95% CIs For Mean Based on
                      Pooled StDev
Ex12-1Glass     Mean  -----+---------+---------+---------+----
1            291.667                              (--*-)
2            235.000  (--*-)
                      -----+---------+---------+---------+----
                         240       260       280       300

                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Ex12-1Phosphor     Mean  -------+---------+---------+---------+--
1               260.000      (-------*-------)
2               273.333                       (-------*-------)
3               256.667  (-------*-------)
                         -------+---------+---------+---------+--
                            256.0     264.0     272.0     280.0
```

12-2.

Since the standard order (Run) is provided, one approach to solving this exercise is to create a 2^3 factorial design in MINITAB, then enter the data. Another approach would be to create a worksheet containing the data, then define a customer factorial design. Both approaches would achieve the same result. This solution uses the first approach.

Select **Stat > DOE > Factorial > Create Factorial Design**. Leave the design type as a 2-level factorial with default generators, and change the Number of factors to "**3**". Select "**Designs**", highlight **full factorial**, change number of replicates to "**2**", and click "**OK**". Select "**Factors**", enter the factor names, leave factor types as "**Numeric**" and factor levels as -1 and +1, and click "**OK**" twice. The worksheet is in run order, to change to standard order (and ease data entry) select **Stat > DOE > Display Design** and choose standard order. The design and data are in the MINITAB worksheet **Ex12-2.MTW**.

(a)
To analyze the experiment, select **Stat > DOE > Factorial > Analyze Factorial Design**. Select "**Terms**" and verify that all terms (A, B, C, AB, AC, BC, ABC) are included.

Factorial Fit: Life versus Cutting Speed, Metal Hardness, Cutting Angle

```
Estimated Effects and Coefficients for Life (coded units)
Term                              Effect    Coef  SE Coef      T      P
Constant                                  413.13    12.41  33.30  0.000
Cutting Speed                      18.25    9.13    12.41   0.74  0.483
Metal Hardness                     84.25   42.12    12.41   3.40  0.009 **
Cutting Angle                      71.75   35.88    12.41   2.89  0.020 **
Cutting Speed*Metal Hardness      -11.25   -5.62    12.41  -0.45  0.662
Cutting Speed*Cutting Angle      -119.25  -59.62    12.41  -4.81  0.001 **
Metal Hardness*Cutting Angle      -24.25  -12.12    12.41  -0.98  0.357
Cutting Speed*Metal Hardness*     -34.75  -17.37    12.41  -1.40  0.199
  Cutting Angle

S = 49.6236   R-Sq = 85.36%   R-Sq(adj) = 72.56%

Analysis of Variance for Life (coded units)
Source              DF  Seq SS  Adj SS  Adj MS     F      P
Main Effects         3   50317   50317   16772  6.81  0.014
2-Way Interactions   3   59741   59741   19914  8.09  0.008
3-Way Interactions   1    4830    4830    4830  1.96  0.199
Residual Error       8   19700   19700    2462
  Pure Error         8   19700   19700    2463
Total               15  134588
…
```

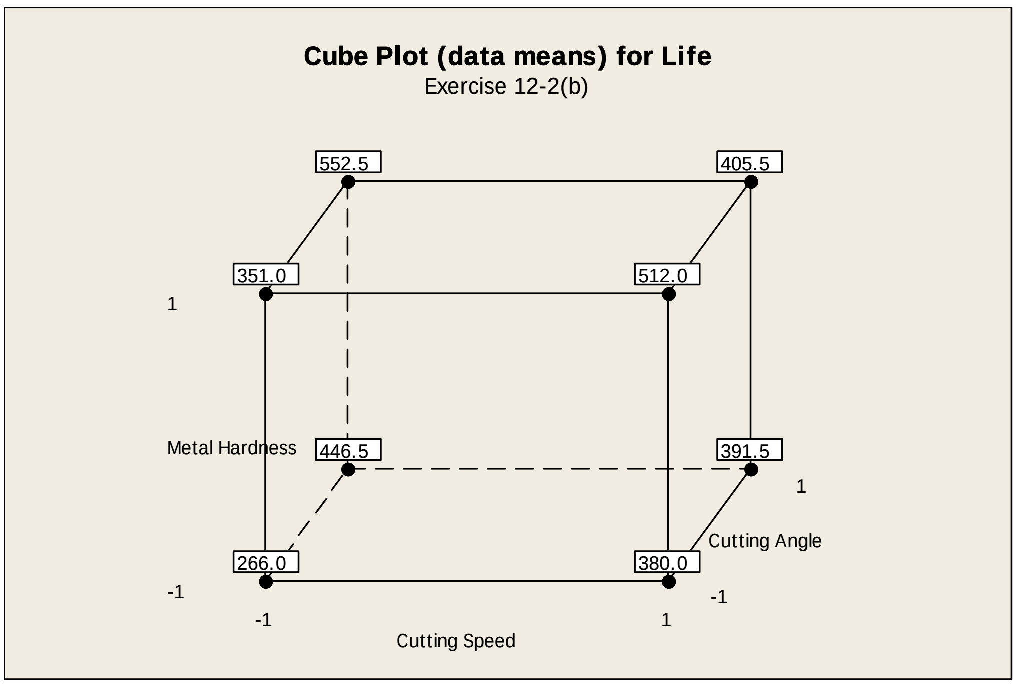
Based on ANOVA results, a full factorial model is not necessary. Based on *P*-values less than 0.10, a reduced model in Metal Hardness, Cutting Angle, and Cutting Speed*Cutting Angle is more appropriate. Cutting Speed will also be retained to maintain a hierarchical model.

12-2(a) continued

Factorial Fit: Life versus Cutting Speed, Metal Hardness, Cutting Angle

```
Estimated Effects and Coefficients for Life (coded units)
Term                           Effect    Coef  SE Coef      T      P
Constant                                413.13    12.47  33.12  0.000
Cutting Speed                   18.25     9.13    12.47   0.73  0.480
Metal Hardness                  84.25    42.12    12.47   3.38  0.006
Cutting Angle                   71.75    35.88    12.47   2.88  0.015
Cutting Speed*Cutting Angle   -119.25   -59.62    12.47  -4.78  0.001

S = 49.8988   R-Sq = 79.65%   R-Sq(adj) = 72.25%

Analysis of Variance for Life (coded units)
Source              DF   Seq SS   Adj SS   Adj MS      F      P
Main Effects         3    50317    50317    16772   6.74  0.008
2-Way Interactions   1    56882    56882    56882  22.85  0.001
Residual Error      11    27389    27389     2490
  Lack of Fit        3     7689     7689     2563   1.04  0.425
  Pure Error         8    19700    19700     2463
Total               15   134588
```

(b)
The combination that maximizes tool life is easily seen from a cube plot. Select **Stat > DOE > Factorial > Factorial Plots**. Choose and set-up a "**Cube Plot**".

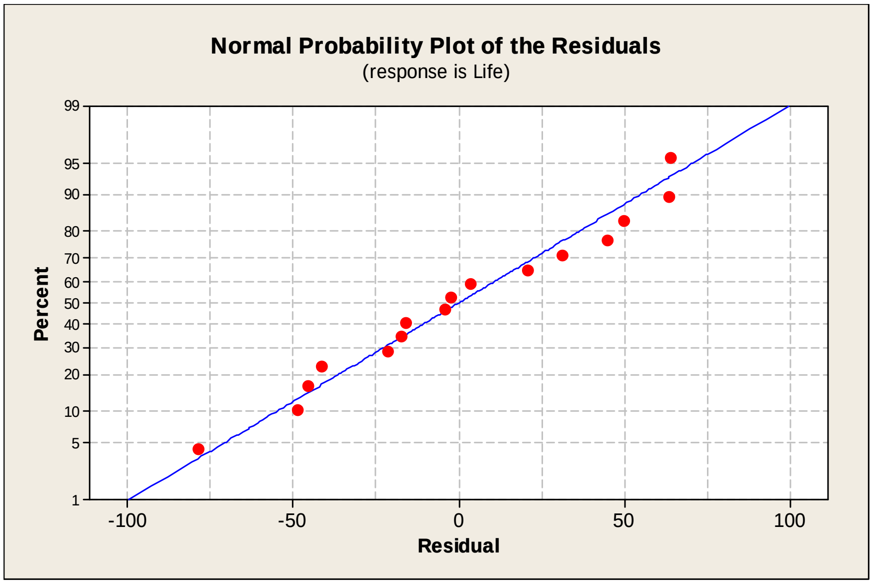


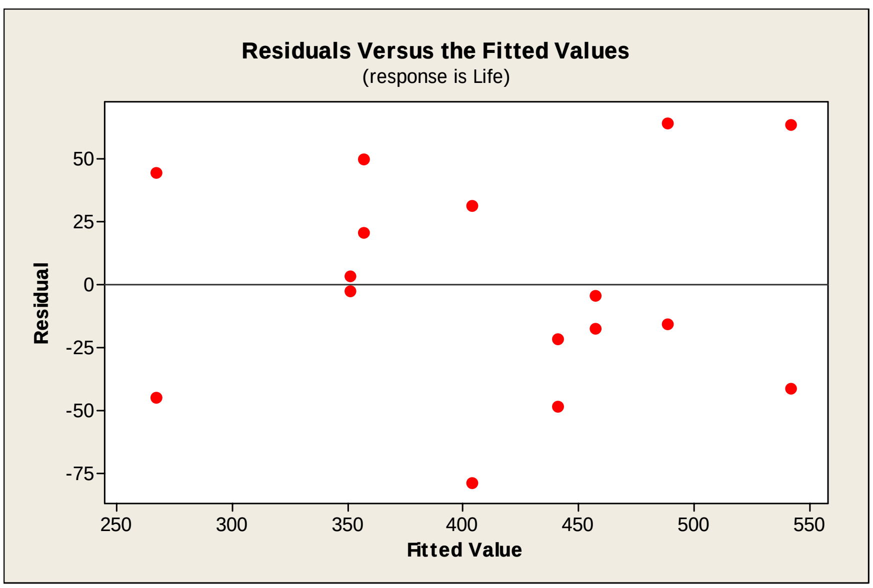
Longest tool life is at A-, B+ and C+, for an average predicted life of 552.5.

(c)
From examination of the cube plot, we see that the low level of cutting speed and the high level of cutting angle gives good results regardless of metal hardness.

12-3.

To find the residuals, select **Stat > DOE > Factorial > Analyze Factorial Design**. Select "**Terms**" and verify that all terms for the reduced model (A, B, C, AC) are included. Select "**Graphs**", and for residuals plots choose "**Normal plot**" and "**Residuals versus fits**". To save residuals to the worksheet, select "**Storage**" and choose "**Residuals**".





Normal probability plot of residuals indicates that the normality assumption is reasonable. Residuals versus fitted values plot shows that the equal variance assumption across the prediction range is reasonable.

12-4.

Create a 2^4 factorial design in MINITAB, and then enter the data. The design and data are in the MINITAB worksheet **Ex12-4.MTW**.

Select **Stat > DOE > Factorial > Analyze Factorial Design**. Since there are two replicates of the experiment, select "**Terms**" and verify that all terms are selected.

Factorial Fit: Total Score versus Sweetener, Syrup to Water, ...

```
Estimated Effects and Coefficients for Total Score (coded units)
Term                                 Effect     Coef  SE Coef       T      P
Constant                                     182.781   0.9504  192.31  0.000
Sweetener                            -9.062   -4.531   0.9504   -4.77  0.000 *
Syrup to Water                       -1.313   -0.656   0.9504   -0.69  0.500
Carbonation                          -2.688   -1.344   0.9504   -1.41  0.177
Temperature                           3.938    1.969   0.9504    2.07  0.055 *
Sweetener*Syrup to Water              4.062    2.031   0.9504    2.14  0.048 *
Sweetener*Carbonation                 0.687    0.344   0.9504    0.36  0.722
Sweetener*Temperature                -2.188   -1.094   0.9504   -1.15  0.267
Syrup to Water*Carbonation           -0.563   -0.281   0.9504   -0.30  0.771
Syrup to Water*Temperature           -0.188   -0.094   0.9504   -0.10  0.923
Carbonation*Temperature               1.688    0.844   0.9504    0.89  0.388
Sweetener*Syrup to Water*Carbonation -5.187   -2.594   0.9504   -2.73  0.015 *
Sweetener*Syrup to Water*Temperature  4.688    2.344   0.9504    2.47  0.025 *
Sweetener*Carbonation*Temperature    -0.938   -0.469   0.9504   -0.49  0.629
Syrup to Water*Carbonation*          -0.938   -0.469   0.9504   -0.49  0.629
  Temperature
Sweetener*Syrup to Water*             2.438    1.219   0.9504    1.28  0.218
  Carbonation*Temperature

Analysis of Variance for Total Score (coded units)
Source              DF   Seq SS   Adj SS  Adj MS     F      P
Main Effects         4   852.63  852.625  213.16  7.37  0.001
2-Way Interactions   6   199.69  199.688   33.28  1.15  0.379
3-Way Interactions   4   405.13  405.125  101.28  3.50  0.031
4-Way Interactions   1    47.53   47.531   47.53  1.64  0.218
Residual Error      16   462.50  462.500   28.91
  Pure Error        16   462.50  462.500   28.91
Total               31  1967.47
```

From magnitude of effects, type of sweetener is dominant, along with interactions involving both sweetener and the ratio of syrup to water. Use an $\alpha = 0.10$ and select terms with P-value less than 0.10. To preserve model hierarchy, the reduced model will contain the significant terms (sweetener, temperature, sweetener*syrup to water, sweetener*syrup to water*carbonation, sweetener*syrup to water*temperature), as well as lower-order terms included in the significant terms (main effects: syrup to water, carbonation; two-factor interactions: sweetener*carbonation, sweetener*temperature, syrup to water*carbonation, syrup to water*temperature).

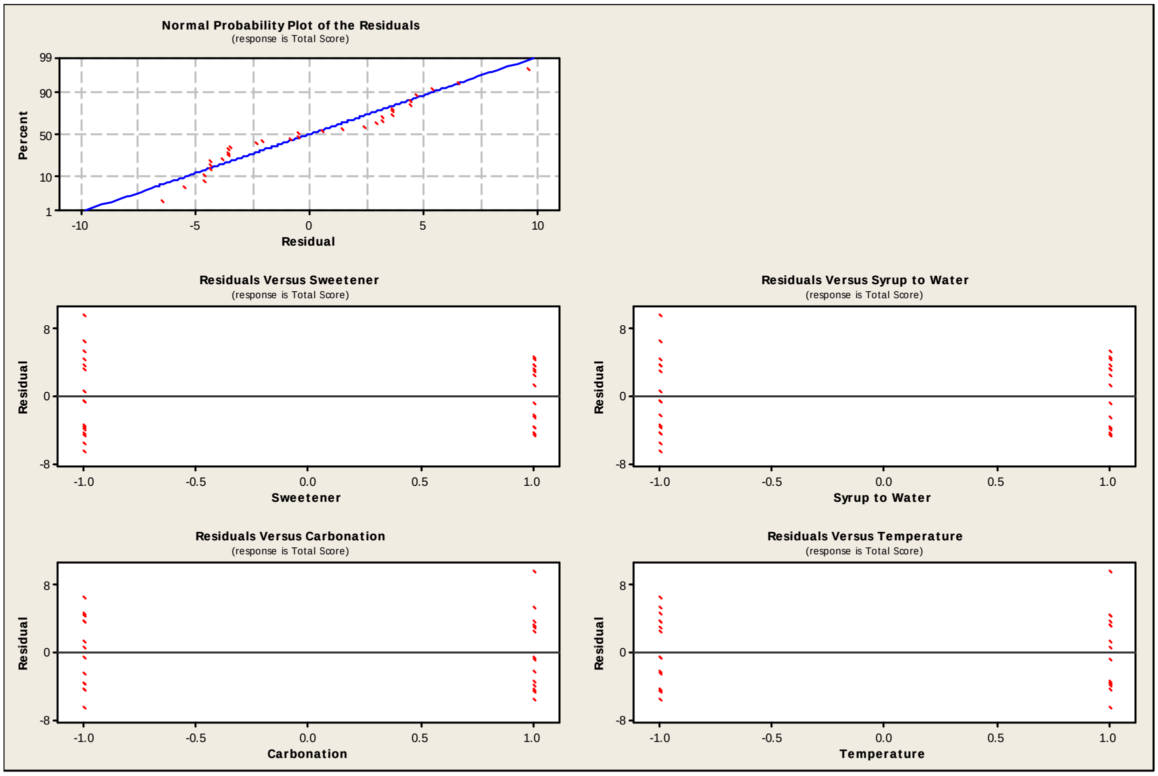
12-4 continued

Factorial Fit: Total Score versus Sweetener, Syrup to Water, ...

```
Estimated Effects and Coefficients for Total Score (coded units)
Term                                 Effect     Coef  SE Coef       T      P
Constant                                     182.781   0.9244  197.73  0.000
Sweetener                            -9.062   -4.531   0.9244   -4.90  0.000
Syrup to Water                       -1.313   -0.656   0.9244   -0.71  0.486
Carbonation                          -2.688   -1.344   0.9244   -1.45  0.162
Temperature                           3.938    1.969   0.9244    2.13  0.046
Sweetener*Syrup to Water              4.062    2.031   0.9244    2.20  0.040
Sweetener*Carbonation                 0.688    0.344   0.9244    0.37  0.714
Sweetener*Temperature                -2.188   -1.094   0.9244   -1.18  0.251
Syrup to Water*Carbonation           -0.563   -0.281   0.9244   -0.30  0.764
Syrup to Water*Temperature           -0.188   -0.094   0.9244   -0.10  0.920
Sweetener*Syrup to Water*Carbonation -5.188   -2.594   0.9244   -2.81  0.011
Sweetener*Syrup to Water*Temperature  4.688    2.344   0.9244    2.54  0.020

Analysis of Variance for Total Score (coded units)
Source              DF   Seq SS   Adj SS   Adj MS     F      P
Main Effects         4   852.63   852.63   213.16  7.80  0.001
2-Way Interactions   5   176.91   176.91    35.38  1.29  0.306
3-Way Interactions   2   391.06   391.06   195.53  7.15  0.005
Residual Error      20   546.88   546.88    27.34
  Lack of Fit        4    84.38    84.38    21.09  0.73  0.585
  Pure Error        16   462.50   462.50    28.91
Total               31  1967.47
```

12-5.

To find the residuals, select **Stat > DOE > Factorial > Analyze Factorial Design**. Select "**Terms**" and verify that all terms for the reduced model are included. Select "**Graphs**", choose "**Normal plot**" of residuals and "**Residuals versus variables**", and then select the variables.



There appears to be a slight indication of inequality of variance for sweetener and syrup ratio, as well as a slight indication of an outlier. This is not serious enough to warrant concern.

12-6.

Select **Stat > DOE > Factorial > Analyze Factorial Design**. Select "**Terms**" and verify that all terms for the reduced model are selected.

Factorial Fit: Total Score versus Sweetener, Syrup to Water, ...

Estimated Effects and Coefficients for Total Score (coded units)

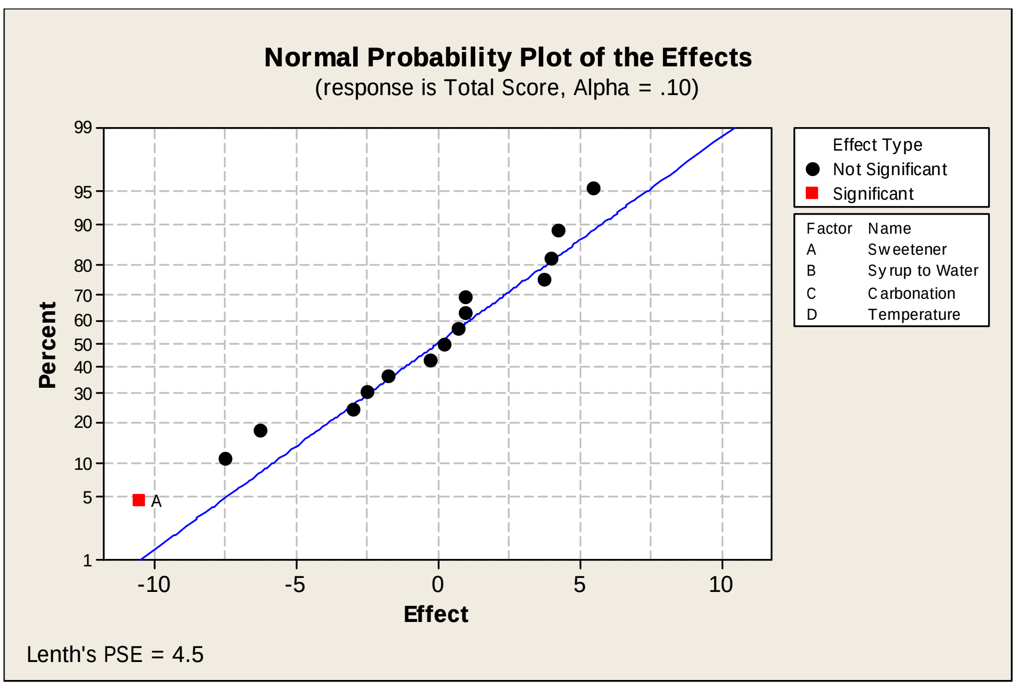
Term	Effect	Coef	SE Coef	T	P
Constant		182.781	0.9244	197.73	0.000
Sweetener	-9.062	-4.531	0.9244	-4.90	0.000
Syrup to Water	-1.313	-0.656	0.9244	-0.71	0.486
Carbonation	-2.688	-1.344	0.9244	-1.45	0.162
Temperature	3.938	1.969	0.9244	2.13	0.046
Sweetener*Syrup to Water	4.062	2.031	0.9244	2.20	0.040
Sweetener*Carbonation	0.688	0.344	0.9244	0.37	0.714
Sweetener*Temperature	-2.188	-1.094	0.9244	-1.18	0.251
Syrup to Water*Carbonation	-0.563	-0.281	0.9244	-0.30	0.764
Syrup to Water*Temperature	-0.188	-0.094	0.9244	-0.10	0.920
Sweetener*Syrup to Water*Carbonation	-5.188	-2.594	0.9244	-2.81	0.011
Sweetener*Syrup to Water*Temperature	4.688	2.344	0.9244	2.54	0.020

The ratio of the coefficient estimate to the standard error is distributed as *t* statistic, and a value greater than approximately |2| would be considered significant. Also, if the confidence interval includes zero, the factor is not significant. From examination of the above table, factors A, D, AB, ABC, and ABD appear to be significant.

12-7.
Create a 2^4 factorial design in MINITAB, and then enter the data. The design and data are in the MINITAB worksheet **Ex12-7.MTW**. Select **Stat > DOE > Factorial > Analyze Factorial Design**. Since there is only one replicate of the experiment, select "**Terms**" and verify that all terms are selected. Then select "**Graphs**", choose the normal effects plot, and set alpha to 0.10

Factorial Fit: Total Score versus Sweetener, Syrup to Water, ...

```
Estimated Effects and Coefficients for Total Score (coded units)
Term                                      Effect      Coef
Constant                                           183.625
Sweetener                                -10.500    -5.250
Syrup to Water                            -0.250    -0.125
Carbonation                                0.750     0.375
Temperature                                5.500     2.750
Sweetener*Syrup to Water                   4.000     2.000
Sweetener*Carbonation                      1.000     0.500
Sweetener*Temperature                     -6.250    -3.125
Syrup to Water*Carbonation                -1.750    -0.875
Syrup to Water*Temperature                -3.000    -1.500
Carbonation*Temperature                    1.000     0.500
Sweetener*Syrup to Water*Carbonation      -7.500    -3.750
Sweetener*Syrup to Water*Temperature       4.250     2.125
Sweetener*Carbonation*Temperature          0.250     0.125
Syrup to Water*Carbonation*               -2.500    -1.250
  Temperature
Sweetener*Syrup to Water*                  3.750     1.875
  Carbonation*Temperature
…
```

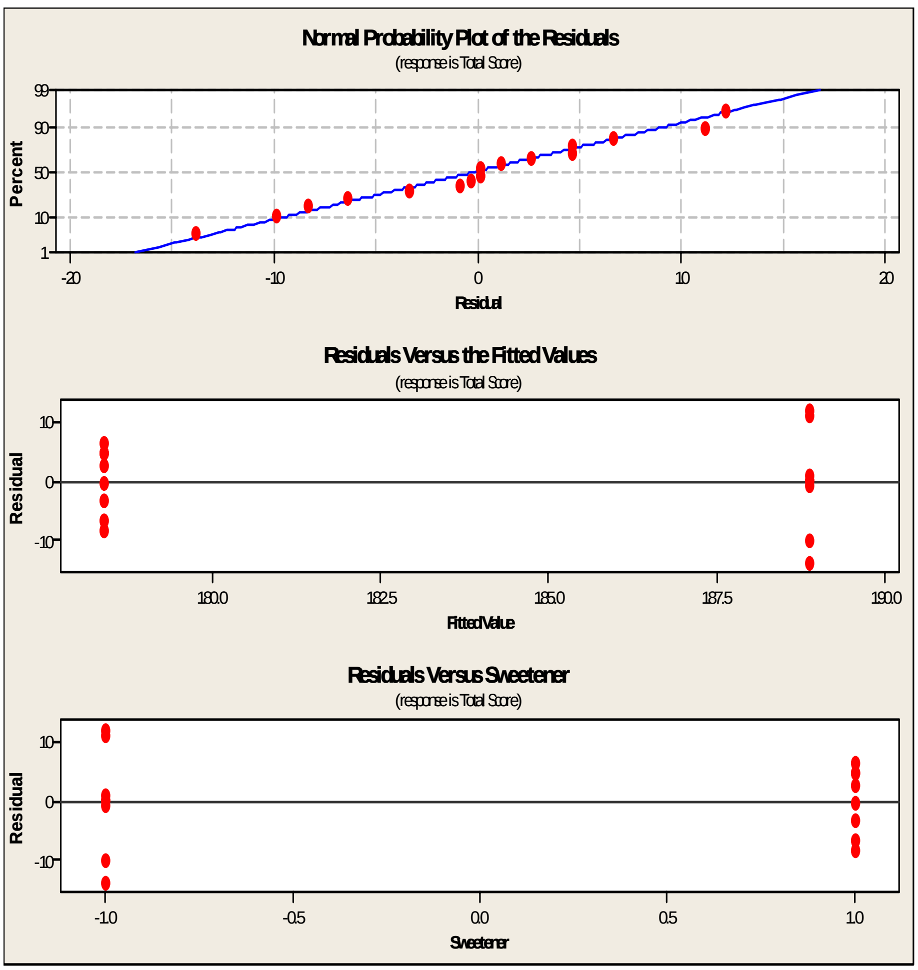


12-7 continued

From visual examination of the normal probability plot of effects, only factor A (sweetener) is significant. Re-fit and analyze the reduced model.

Factorial Fit: Total Score versus Sweetener

```
Estimated Effects and Coefficients for Total Score (coded units)
Term          Effect      Coef  SE Coef       T      P
Constant             183.625    1.865  98.48  0.000
Sweetener  -10.500    -5.250    1.865  -2.82  0.014

S = 7.45822   R-Sq = 36.15%   R-Sq(adj) = 31.59%

Analysis of Variance for Total Score (coded units)
Source          DF    Seq SS   Adj SS  Adj MS      F      P
Main Effects     1    441.00  441.000  441.00   7.93  0.014
Residual Error  14    778.75  778.750   55.63
  Pure Error    14    778.75  778.750   55.63
Total           15   1219.75
```



There appears to be a slight indication of inequality of variance for sweetener, as well as in the predicted values. This is not serious enough to warrant concern.

12-8.

The ABCD interaction is confounded with blocks, or days.

Day 1		Day 2	
a	*d*	(1)	*bc*
b	*abd*	*ab*	*bd*
c	*acd*	*ac*	*cd*
abc	*bcd*	*ad*	*abcd*

Treatment combinations within a day should be run in random order.

12-9.
A 2^5 design in two blocks will lose the ABCDE interaction to blocks.

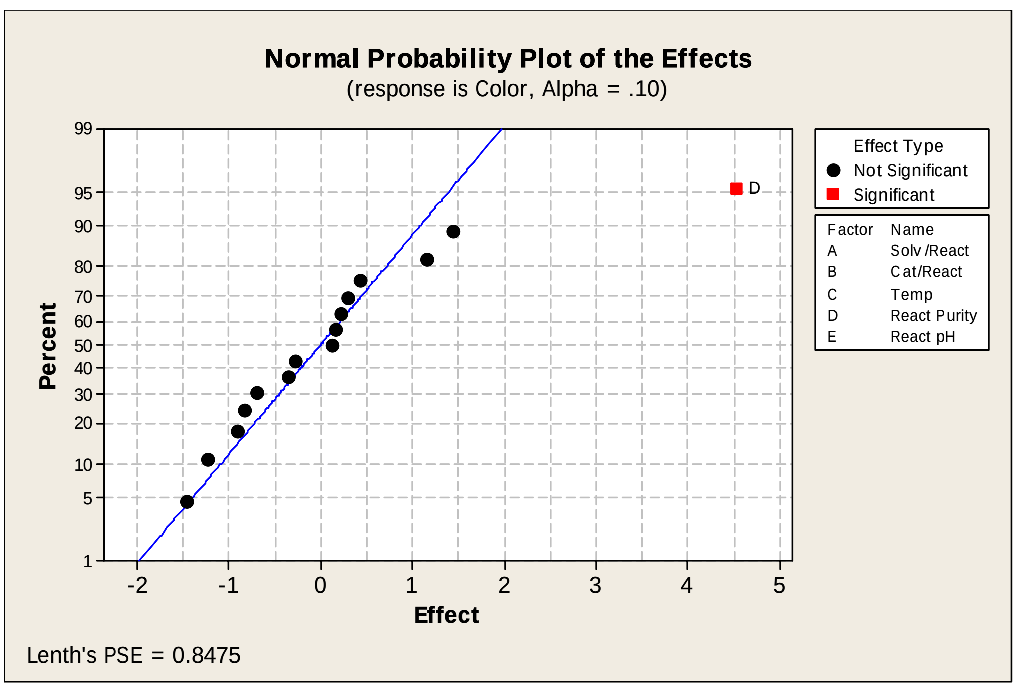
Block 1		Block 2	
(1)	*ae*	*a*	*e*
ab	*be*	*b*	*abe*
ac	*ce*	*c*	*ace*
bc	*abce*	*abc*	*bce*
ad	*de*	*d*	*ade*
bd	*abde*	*abd*	*bde*
cd	*acde*	*acd*	*cde*
abcd	*bcde*	*bcd*	*abcde*

12-10.
(a)

Create a 2^{5-1} factorial design in MINITAB, and then enter the data. The design and data are in the MINITAB worksheet **Ex12-10.MTW**. Select **Stat > DOE > Factorial > Analyze Factorial Design**. Since there is only one replicate of the experiment, select "**Terms**" and verify that all main effects and interaction effects are selected. Then select "**Graphs**", choose the normal effects plot, and set alpha to 0.10.

Factorial Fit: Color versus Solv/React, Cat/React, ...

```
Estimated Effects and Coefficients for Color (coded units)
Term                        Effect     Coef
Constant                             2.7700
Solv/React                  1.4350   0.7175
Cat/React                  -1.4650  -0.7325
Temp                       -0.2725  -0.1363
React Purity                4.5450   2.2725
React pH                   -0.7025  -0.3513
Solv/React*Cat/React        1.1500   0.5750
Solv/React*Temp            -0.9125  -0.4562
Solv/React*React Purity    -1.2300  -0.6150
Solv/React*React pH         0.4275   0.2138
Cat/React*Temp              0.2925   0.1462
Cat/React*React Purity      0.1200   0.0600
Cat/React*React pH          0.1625   0.0812
Temp*React Purity          -0.8375  -0.4187
Temp*React pH              -0.3650  -0.1825
React Purity*React pH       0.2125   0.1062
```

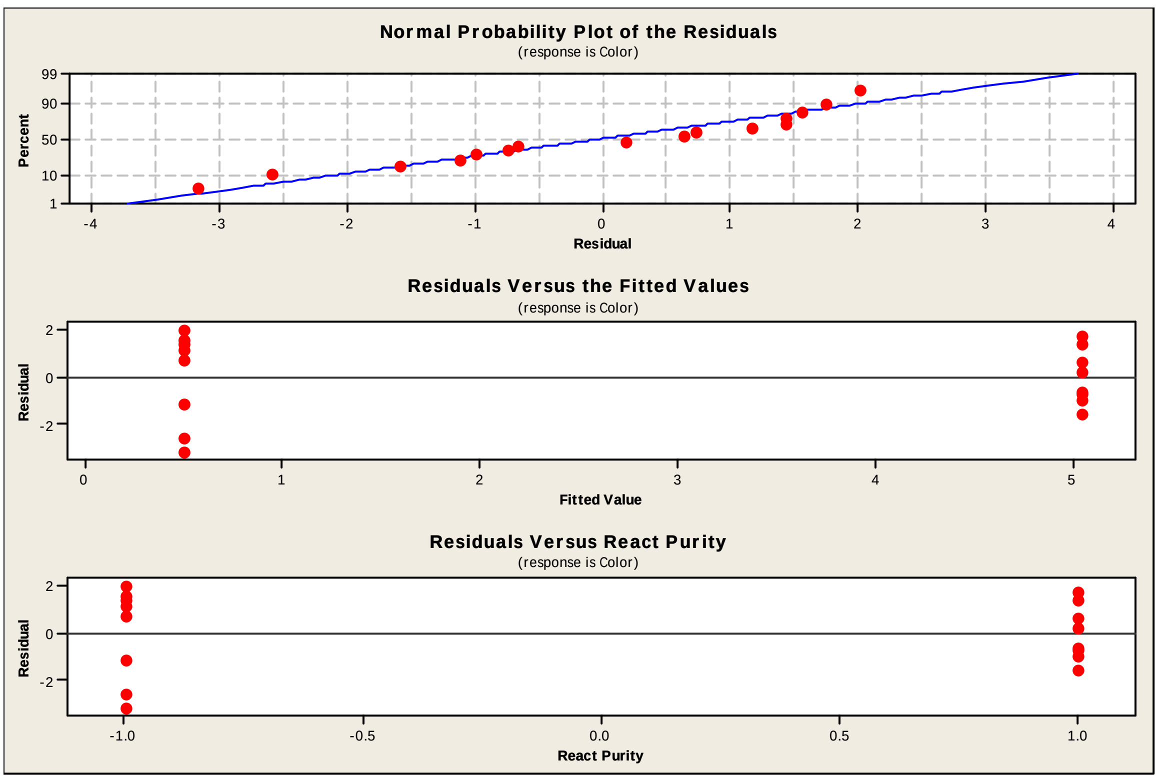


12-10 (a) continued

From visual examination of the normal probability plot of effects, only factor D (reactant purity) is significant. Re-fit and analyze the reduced model.

```
Factorial Fit: Color versus React Purity
Estimated Effects and Coefficients for Color (coded units)
Term           Effect    Coef  SE Coef      T      P
Constant                2.770   0.4147   6.68  0.000
React Purity    4.545   2.272   0.4147   5.48  0.000

S = 1.65876   R-Sq = 68.20%   R-Sq(adj) = 65.93%

Analysis of Variance for Color (coded units)
Source          DF   Seq SS  Adj SS  Adj MS      F      P
Main Effects     1    82.63   82.63  82.628  30.03  0.000
Residual Error  14    38.52   38.52   2.751
  Pure Error    14    38.52   38.52   2.751
Total           15   121.15
```

(b)



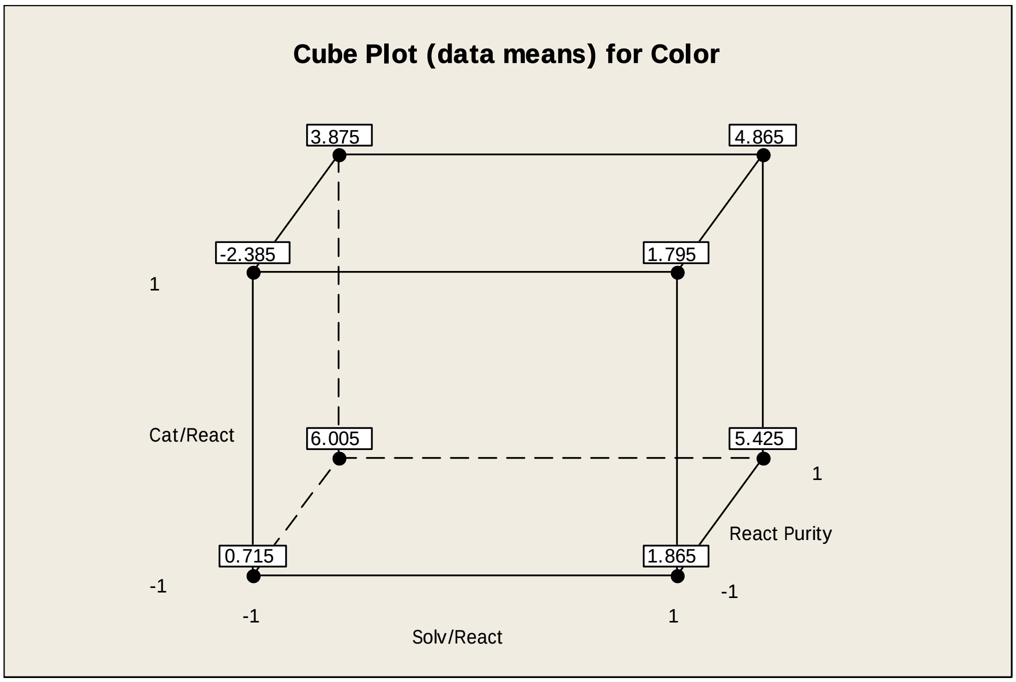
Residual plots indicate that there may be problems with both the normality and constant variance assumptions.

12-10 continued

(c)

There is only one significant factor, D (reactant purity), so this design collapses to a one-factor experiment, or simply a 2-sample *t*-test.

Looking at the original normal probability plot of effects and effect estimates, the 2nd and 3rd largest effects in absolute magnitude are A (solvent/reactant) and B (catalyst/reactant). A cube plot in these factors shows how the design can be collapsed into a replicated 2^3 design. The highest color scores are at high reactant purity; the lowest at low reactant purity.



12-11.

Enter the factor levels and yield data into a MINITAB worksheet, then define the experiment using **Stat > DOE > Factorial > Define Custom Factorial Design**. The design and data are in the MINITAB worksheet **Ex12-11.MTW**.

(a) and (b)

Select **Stat > DOE > Factorial > Analyze Factorial Design**. Since there is only one replicate of the experiment, select "**Terms**" and verify that all main effects and two-factor interaction effects are selected.

Factorial Fit: yield versus A:Temp, B:Matl1, C:Vol, D:Time, E:Matl2

```
Estimated Effects and Coefficients for yield (coded units)
Term             Effect    Coef
Constant                 19.238
A:Temp           -1.525  -0.762
B:Matl1          -5.175  -2.587
C:Vol             2.275   1.138
D:Time           -0.675  -0.337
E:Matl2           2.275   1.138
A:Temp*B:Matl1    1.825   0.913
A:Temp*D:Time    -1.275  -0.638
…
Alias Structure
I + A:Temp*C:Vol*E:Matl2 + B:Matl1*D:Time*E:Matl2 + A:Temp*B:Matl1*C:Vol*D:Time
A:Temp + C:Vol*E:Matl2 + B:Matl1*C:Vol*D:Time + A:Temp*B:Matl1*D:Time*E:Matl2
B:Matl1 + D:Time*E:Matl2 + A:Temp*C:Vol*D:Time + A:Temp*B:Matl1*C:Vol*E:Matl2
C:Vol + A:Temp*E:Matl2 + A:Temp*B:Matl1*D:Time + B:Matl1*C:Vol*D:Time*E:Matl2
D:Time + B:Matl1*E:Matl2 + A:Temp*B:Matl1*C:Vol + A:Temp*C:Vol*D:Time*E:Matl2
E:Matl2 + A:Temp*C:Vol + B:Matl1*D:Time + A:Temp*B:Matl1*C:Vol*D:Time*E:Matl2
A:Temp*B:Matl1 + C:Vol*D:Time + A:Temp*D:Time*E:Matl2 + B:Matl1*C:Vol*E:Matl2
A:Temp*D:Time + B:Matl1*C:Vol + A:Temp*B:Matl1*E:Matl2 + C:Vol*D:Time*E:Matl2
```

From the Alias Structure shown in the Session Window, the complete defining relation is: I = ACE = BDE = ABCD.

The aliases are:

$A*I = A*ACE = A*BDE = A*ABCD \Rightarrow A = CE = ABDE = BCD$

$B*I = B*ACE = B*BDE = B*ABCD \Rightarrow B = ABCE = DE = ACD$

$C*I = C*ACE = C*BDE = C*ABCD \Rightarrow C = AE = BCDE = ABD$

…

$AB*I = AB*ACE = AB*BDE = AB*ABCD \Rightarrow AB = BCE = ADE = CD$

The remaining aliases are calculated in a similar fashion.

12-11 continued

(c)

A	B	C	D	E	yield
-1	-1	-1	-1	1	23.2
1	1	-1	-1	-1	15.5
1	-1	-1	1	-1	16.9
-1	1	1	-1	-1	16.2
-1	-1	1	1	-1	23.8
1	-1	1	-1	1	23.4
-1	1	-1	1	1	16.8
1	1	1	1	1	18.1

$[A] = A + CE + BCD + ABDE$
$= ¼ (–23.2 + 15.5 + 16.9 – 16.2 – 23.8 + 23.4 – 16.8 + 18.1) = ¼ (–6.1) = –1.525$

$[AB] = AB + BCE + ADE + CD$
$= ¼ (+23.2 +15.5 – 16.9 -16.2 +23.8 – 23.4 – 16.8 + 18.1) = ¼ (7.3) = 1.825$

This are the same effect estimates provided in the MINITAB output above. The other main effects and interaction effects are calculated in the same way.

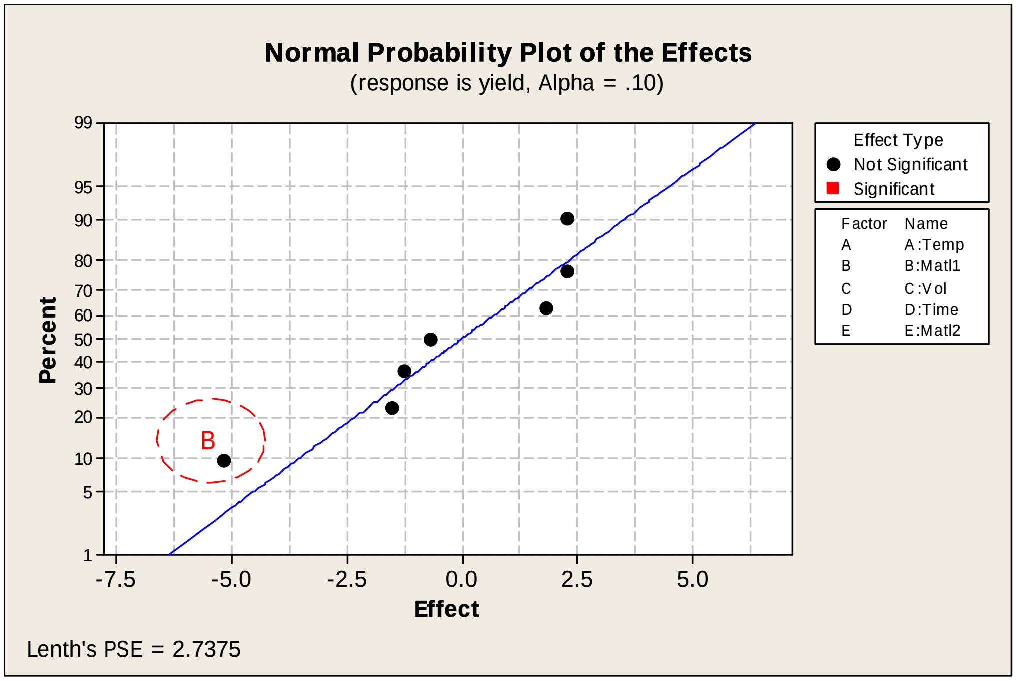
(d)

Select **Stat > DOE > Factorial > Analyze Factorial Design**. Since there is only one replicate of the experiment, select "**Terms**" and verify that all main effects and two-factor interaction effects are selected. Then select "**Graphs**", choose the normal effects plot, and set alpha to 0.10.

Factorial Fit: yield versus A:Temp, B:Matl1, C:Vol, D:Time, E:Matl2
…

```
…
Analysis of Variance for yield (coded units)
Source              DF  Seq SS  Adj SS  Adj MS  F  P
Main Effects         5  79.826  79.826  15.965  *  *
2-Way Interactions   2   9.913   9.913   4.956  *  *
Residual Error       0       *       *       *
Total                7  89.739
…
```

12-11 (d) continued



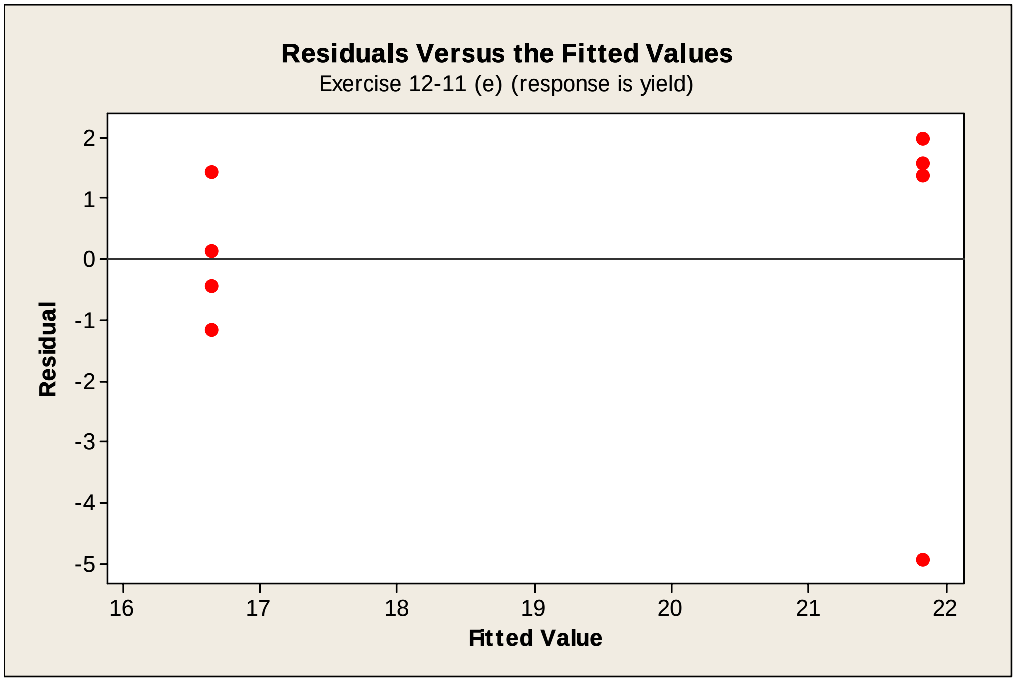
Although none of the effects is significant at 0.10, main effect B (amount of material 1) is more than twice as large as the 2nd largest effect (absolute values) and falls far from a line passing through the remaining points. Re-fit a reduced model containing only the B main effect, and pool the remaining terms to estimate error.

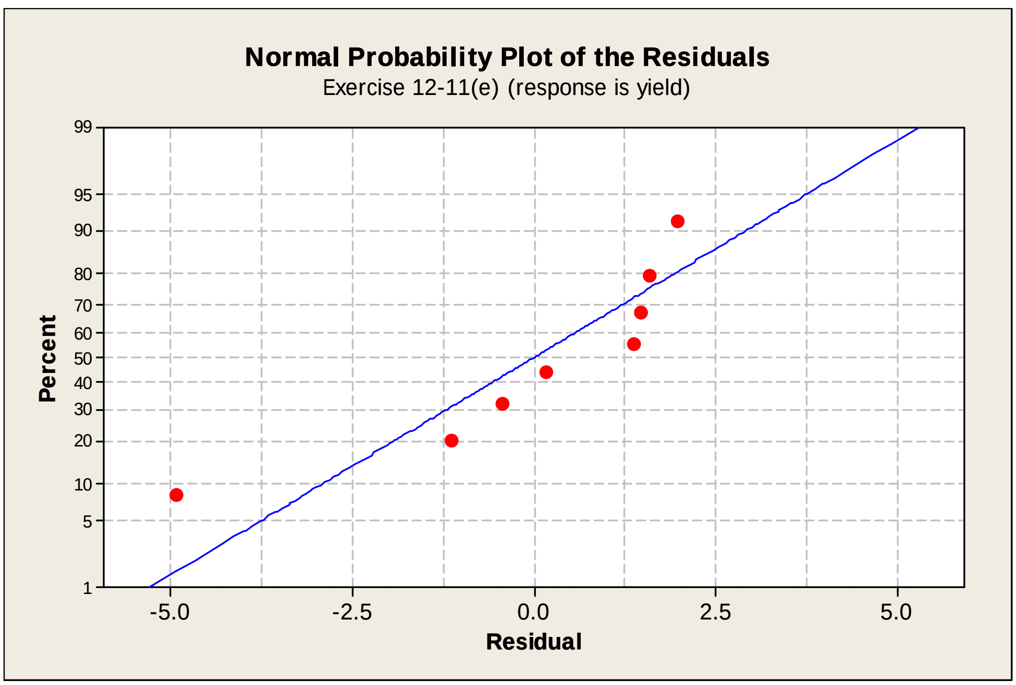
Select **Stat > DOE > Factorial > Analyze Factorial Design**. Select "**Terms**" and select "**B**". Then select "**Graphs**", and select the "**Normal plot**" and "**Residuals versus fits**" residual plots.

Factorial Fit: yield versus B:Matl1

```
Estimated Effects and Coefficients for yield (coded units)
Term       Effect     Coef  SE Coef      T      P
Constant            19.238   0.8682  22.16  0.000
B:Matl1    -5.175   -2.587   0.8682  -2.98  0.025
…
Analysis of Variance for yield (coded units)
Source          DF   Seq SS   Adj SS   Adj MS      F      P
Main Effects     1    53.56    53.56   53.561   8.88  0.025
Residual Error   6    36.18    36.18    6.030
  Pure Error     6    36.18    36.18    6.030
Total            7    89.74
…
```

12-11 continued
(e)





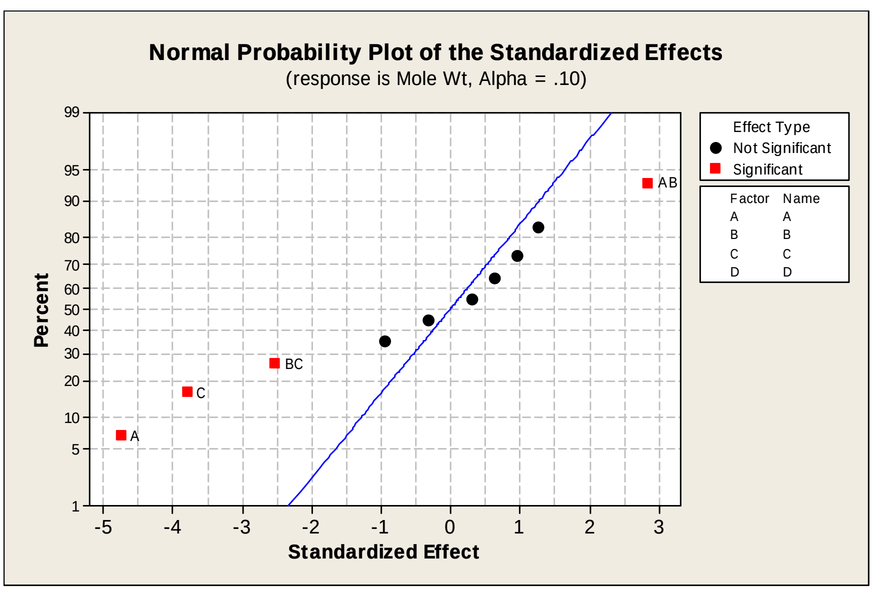
Residual plots indicate a potential outlier. The run should be investigated for any issues which occurred while running the experiment. If no issues can be identified, it may be necessary to make additional experimental runs

12-12.
Create a 2^4 factorial design in MINITAB, and then enter the data. The design and data are in the MINITAB worksheet **Ex12-12.MTW**.

(a)
Select **Stat > DOE > Factorial > Analyze Factorial Design**. Since this is a single replicate of the experiment, select "**Terms**" and verify that all main effects and two-factor interaction effects are selected. Then select "**Graphs**", choose the normal effects plot, and set alpha to 0.10.

Factorial Fit: Mole Wt versus A, B, C, D

```
Estimated Effects and Coefficients for Mole Wt (coded units)
Term        Effect     Coef  SE Coef       T      P
Constant             837.50    3.953  211.87  0.000
A           -37.50   -18.75    3.953   -4.74  0.005 *
B            10.00     5.00    3.953    1.26  0.262
C           -30.00   -15.00    3.953   -3.79  0.013 *
D            -7.50    -3.75    3.953   -0.95  0.386
A*B          22.50    11.25    3.953    2.85  0.036 *
A*C          -2.50    -1.25    3.953   -0.32  0.765
A*D           5.00     2.50    3.953    0.63  0.555
B*C         -20.00   -10.00    3.953   -2.53  0.053 *
B*D           2.50     1.25    3.953    0.32  0.765
C*D           7.50     3.75    3.953    0.95  0.386
…
Analysis of Variance for Mole Wt (coded units)
Source              DF  Seq SS  Adj SS  Adj MS     F      P
Main Effects         4    9850    9850  2462.5  9.85  0.014
2-Way Interactions   6    4000    4000   666.7  2.67  0.151
Residual Error       5    1250    1250   250.0
Total               15   15100
```



The main effects A and C and two two-factor interactions with B (AB, BC) are significant. The main effect B must be kept in the model to maintain hierarchy. Re-fit and analyze a reduced model containing A, B, C, AB, and BC.

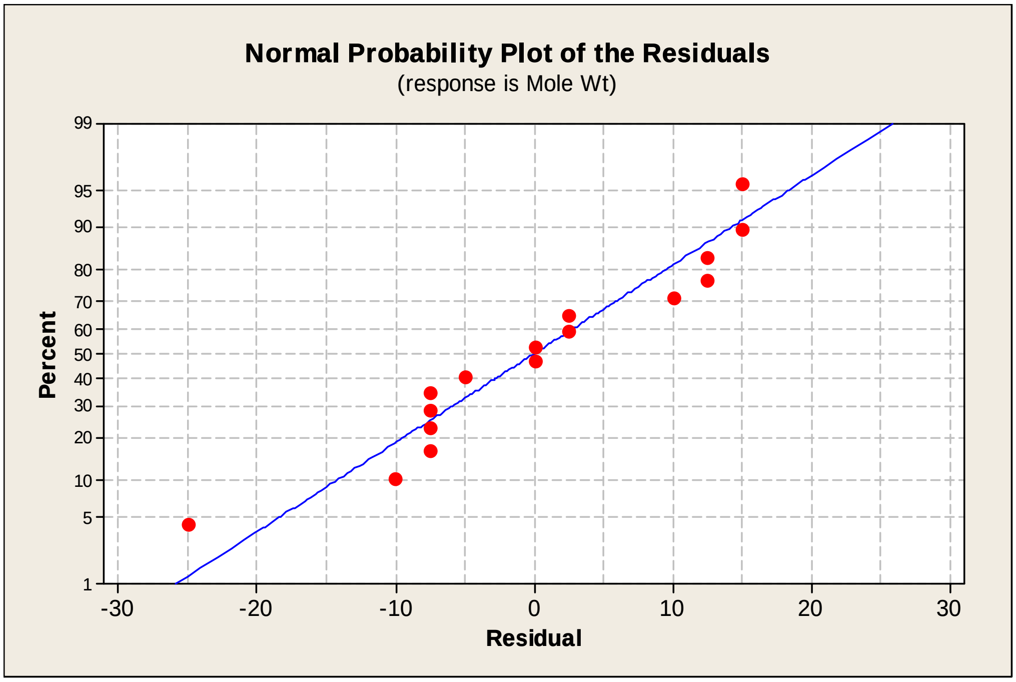
12-12 continued
(b)
Select **Stat > DOE > Factorial > Analyze Factorial Design**. Select "**Terms**" and select "**A, B, C, AB, BC**". Then select "**Graphs**", and select the "**Normal plot**" and "**Residuals versus fits**" residual plots.

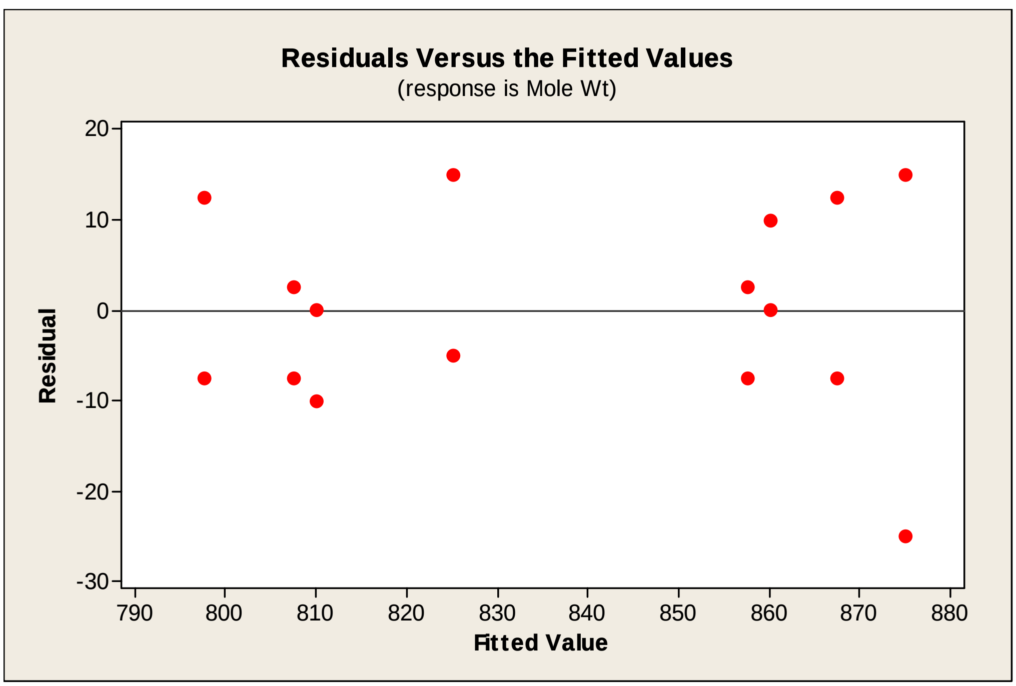
Factorial Fit: Mole Wt versus A, B, C

```
Estimated Effects and Coefficients for Mole Wt (coded units)
Term       Effect    Coef  SE Coef       T      P
Constant           837.50    3.400  246.30  0.000
A          -37.50  -18.75    3.400   -5.51  0.000 *
B           10.00    5.00    3.400    1.47  0.172
C          -30.00  -15.00    3.400   -4.41  0.001 *
A*B         22.50   11.25    3.400    3.31  0.008 *
B*C        -20.00  -10.00    3.400   -2.94  0.015 *
…
Analysis of Variance for Mole Wt (coded units)
Source              DF   Seq SS   Adj SS   Adj MS      F      P
Main Effects         3   9625.0   9625.0   3208.3  17.34  0.000
2-Way Interactions   2   3625.0   3625.0   1812.5   9.80  0.004
Residual Error      10   1850.0   1850.0    185.0
  Lack of Fit        2    250.0    250.0    125.0   0.63  0.559
  Pure Error         8   1600.0   1600.0    200.0
Total               15  15100.0
…
```

The same terms remain significant, A, C, AB, and BC.

12-12 continued
(c)

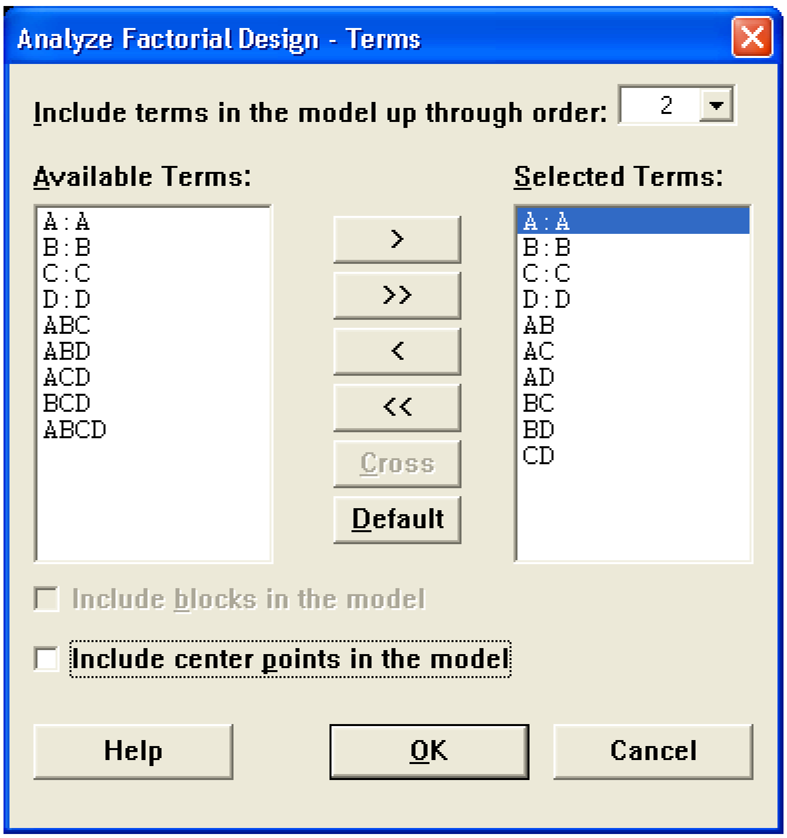




A “modest” outlier appears on both plots; however neither plot reveals a major problem with the normality and constant variance assumptions.

12-13.
Create a 2^4 factorial design with four center points in MINITAB, and then enter the data. The design and data are in the MINITAB worksheet **Ex12-13.MTW**.

(a)
Select **Stat > DOE > Factorial > Analyze Factorial Design**. Select "**Terms**" and verify that all main effects and two-factor interactions are selected. Also, DO NOT include the center points in the model (uncheck the default selection). This will ensure that if both lack of fit and curvature are not significant, the main and interaction effects are tested for significance against the correct residual error (lack of fit + curvature + pure error). See the dialog box below.



To summarize MINITAB's functionality, curvature is always tested against pure error and lack of fit (if available), regardless of whether center points are included in the model. The inclusion/exclusion of center points in the model affects the total residual error used to test significance of effects. Assuming that lack of fit and curvature tests are not significant, all three (curvature, lack of fit, and pure error) should be included in the residual mean square.

12-13 (a) continued

When looking at results in the ANOVA table, the first test to consider is the "lack of fit" test, which is a test of significance for terms not included in the model (in this exercise, the three-factor and four-factor interactions). If lack of fit is significant, the model is not correctly specified, and some terms need to be added to the model.

If lack of fit is not significant, the next test to consider is the "curvature" test, which is a test of significance for the pure quadratic terms. If this test is significant, no further statistical analysis should be performed because the model is inadequate.

If tests for both lack of fit and curvature are not significant, then it is reasonable to pool the curvature, pure error, and lack of fit (if available) and use this as the basis for testing for significant effects. (In MINITAB, this is accomplished by not including center points in the model.)

Factorial Fit: Mole Wt versus A, B, C, D

```
Estimated Effects and Coefficients for Mole Wt (coded units)
Term      Effect    Coef  SE Coef      T      P
Constant          848.00    8.521  99.52  0.000
A         -37.50  -18.75    9.527  -1.97  0.081
B          10.00    5.00    9.527   0.52  0.612
C         -30.00  -15.00    9.527  -1.57  0.150
D          -7.50   -3.75    9.527  -0.39  0.703
A*B        22.50   11.25    9.527   1.18  0.268
A*C        -2.50   -1.25    9.527  -0.13  0.898
A*D         5.00    2.50    9.527   0.26  0.799
B*C       -20.00  -10.00    9.527  -1.05  0.321
B*D         2.50    1.25    9.527   0.13  0.898
C*D         7.50    3.75    9.527   0.39  0.703
…
Analysis of Variance for Mole Wt (coded units)
Source              DF  Seq SS  Adj SS  Adj MS      F      P
Main Effects         4    9850    9850  2462.5   1.70  0.234
2-Way Interactions   6    4000    4000   666.7   0.46  0.822
Residual Error       9   13070   13070  1452.2
  Curvature          1    8820    8820  8820.0  16.60  0.004 *
  Lack of Fit        5    1250    1250   250.0   0.25  0.915
  Pure Error         3    3000    3000  1000.0
Total               19   26920
…
```

(b)
The test for curvature is significant (*P*-value = 0.004). Although one could pick a "winning combination" from the experimental runs, a better strategy is to add runs that would enable estimation of the quadratic effects. This approach to sequential experimentation is presented in Chapter 13.

12-14.
From Table 12-23 in the textbook, a 2^{8-4}_{IV} design has a complete defining relation of:

$$\begin{aligned} I &= BCDE = ACDF = ABCG = ABDH \\ &= ABEF = ADEG = ACEH = BDFG = BCFH = CDGH \\ &= CEFG = DEFH = AFGH = ABCDEFGH \end{aligned}$$

The runs would be:

Run	***A***	***B***	***C***	***D***	***E=BCD***	***F=ACD***	***G=ABC***	***H=ABD***
1	–	–	–	–	–	–	–	–
2	+	–	–	–	–	+	+	+
3	–	+	–	–	+	–	+	+
4	+	+	–	–	+	+	–	–
5	–	–	+	–	+	+	+	–
6	+	–	+	–	+	–	–	+
7	–	+	+	–	–	+	–	+
8	+	+	+	–	–	–	+	–
9	–	–	–	+	+	+	–	+
10	+	–	–	+	+	–	+	–
11	–	+	–	+	–	+	+	–
12	+	+	–	+	–	–	–	+
13	–	–	+	+	–	–	+	+
14	+	–	+	+	–	+	–	–
15	–	+	+	+	+	–	–	–
16	+	+	+	+	+	+	+	+

A=BCDE=CDF=BCG=BDH=BEF=DEG=CEH=ABDFG=ACDGH=ABCFH=ACEFG=ADEFH=FGH=BCDEFGH
B=CDE=ACDF=ACG=ADH=AEF=ABDEG=ABCEH=DFG=CFH=BCDGH=BCEFG=BDEFH=ABFGH=ACDEFGH
C=BDE=ADF=ABG=ABDH=ABCEF=ACDEF=AEH=BCDFG=BFH=DGH=EFG=CDEFH=ACFGH=ABDEFGH
D=BCE=ACF=ABCG=ABH=ABDEF=AEG=ACDEH=BFG=BCDFH=CGH=CDEFG=EFH=ADFGH=ABCEFGH
E=BCD=ACDEF=ABCEG=ABDEH=ABF=ADG=ACH=BDEFG=BCEFH=CDEGH=CFG=DFH=AEFGH=ABCDFGH
F=BCDEF=ACD=ABCFG=ABDFH=ABE=ADEFG=ACEFH=BDE=BCH=CDFGH=CEG=DEH=AGH=ABCDEGH
G=BCDEG=ACDFG=ABC=ABDGH=ABEFG=ADE=ACEGH=BDF=BFGH=CDH=CEF=DEFGH=AFH=ABCDEFH
H=BCDEH=ACDFH=ABCGH=ABD=ABEFH=ADEGH= ACE=BDFGH=BCF=CDG=CEFGH=DEF=AFG=ABCDEFG
AB=ACDE=BCDF=CG=DH=EF=BDEG=BCEH=ADFG=ACFH=ABCDFH=ABCEFG=ABDEFH=BFGH=CDEFGH
AC=ABDE=DF=BG=BCDH=BCEF=CDEG=EH=ABCDFG=ABFH=ADGH=AEFG=ACDEFH=CFGH=BDEFGH
etc.

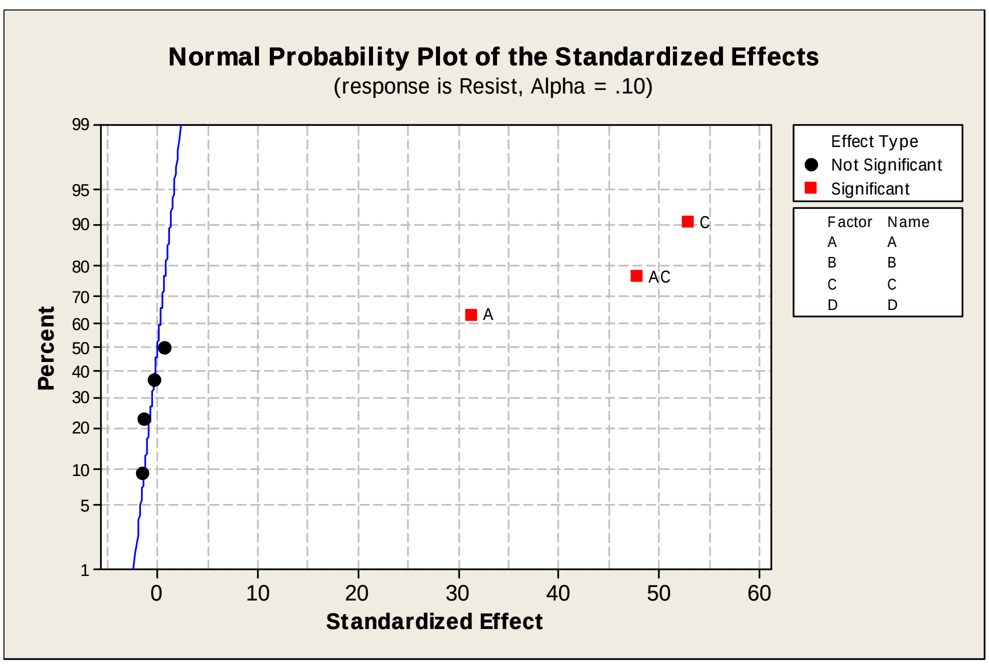
Main effects are clear of 2-factor interactions, and at least some 2-factor interactions are aliased with each other, so this is a resolution IV design. A lower resolution design would have some 2-factor interactions and main effects aliased together. The source of interest for any combined main and 2-factor interaction effect would be in question. Since significant 2-factor interactions often occur in practice, this problem is of concern.

12-15.
Enter the factor levels and resist data into a MINITAB worksheet, including a column indicating whether a run is a center point run (1 = not center point, 0 = center point). Then define the experiment using **Stat > DOE > Factorial > Define Custom Factorial Design**. The design and data are in the MINITAB worksheet **Ex12-15.MTW**.

(a)
Select **Stat > DOE > Factorial > Analyze Factorial Design**. Select "**Terms**" and verify that all main effects and two-factor interactions are selected. Also, DO NOT include the center points in the model (uncheck the default selection). Then select "**Graphs**", choose the normal effects plot, and set alpha to 0.10.

Factorial Fit: Resist versus A, B, C, D

```
Estimated Effects and Coefficients for Resist (coded units)
Term        Effect    Coef  SE Coef      T      P
Constant            60.433   0.6223  97.12  0.000
A           47.700  23.850   0.7621  31.29  0.000 *
B           -0.500  -0.250   0.7621  -0.33  0.759
C           80.600  40.300   0.7621  52.88  0.000 *
D           -2.400  -1.200   0.7621  -1.57  0.190
A*B          1.100   0.550   0.7621   0.72  0.510
A*C         72.800  36.400   0.7621  47.76  0.000 *
A*D         -2.000  -1.000   0.7621  -1.31  0.260
…
Analysis of Variance for Resist (coded units)
Source              DF   Seq SS   Adj SS   Adj MS       F      P
Main Effects         4  17555.3  17555.3  4388.83  944.51  0.000
2-Way Interactions   3  10610.1  10610.1  3536.70  761.13  0.000
Residual Error       4     18.6     18.6     4.65
  Curvature          1      5.6      5.6     5.61    1.30  0.338
  Pure Error         3     13.0     13.0     4.33
Total               11  28184.0
```



12-15 continued
Examining the normal probability plot of effects, the main effects A and C and their two-factor interaction (AC) are significant. Re-fit and analyze a reduced model containing A, C, and AC.

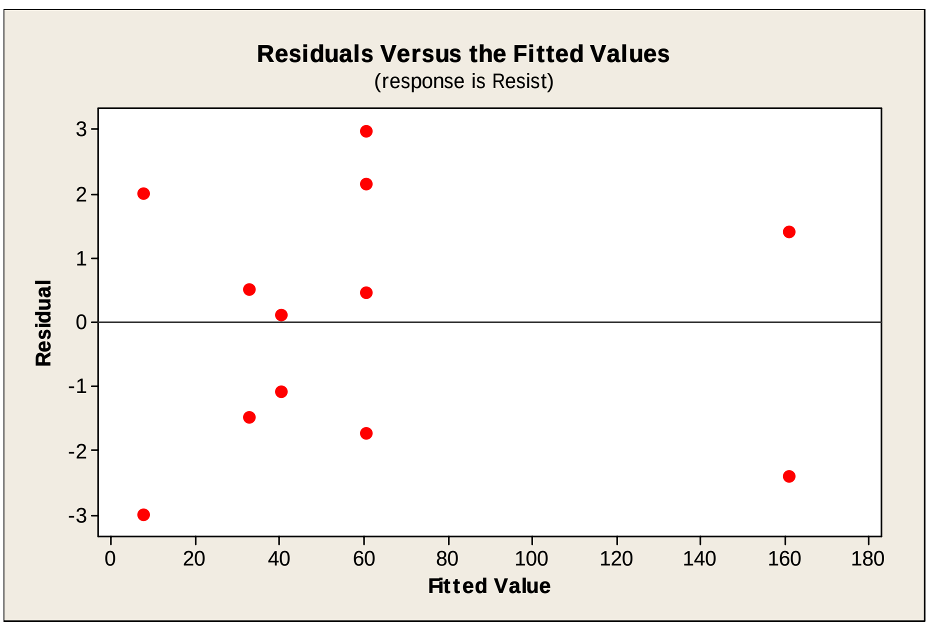
Select **Stat > DOE > Factorial > Analyze Factorial Design**. Select "**Terms**" and select "**A, C, AC**". Then select "**Graphs**", and select the "**Normal plot**" and "**Residuals versus fits**" residual plots.

(b)

```
Factorial Fit: Resist versus A, C
Estimated Effects and Coefficients for Resist (coded units)
Term      Effect   Coef  SE Coef      T      P
Constant          60.43   0.6537  92.44  0.000
A          47.70  23.85   0.8007  29.79  0.000 *
C          80.60  40.30   0.8007  50.33  0.000 *
A*C        72.80  36.40   0.8007  45.46  0.000 *
…
Analysis of Variance for Resist (coded units)
Source               DF   Seq SS   Adj SS   Adj MS        F      P
Main Effects          2  17543.3  17543.3   8771.6  1710.43  0.000
2-Way Interactions    1  10599.7  10599.7  10599.7  2066.89  0.000
Residual Error        8     41.0     41.0      5.1
  Curvature           1      5.6      5.6      5.6     1.11  0.327
  Pure Error          7     35.4     35.4      5.1
Total                11  28184.0
```

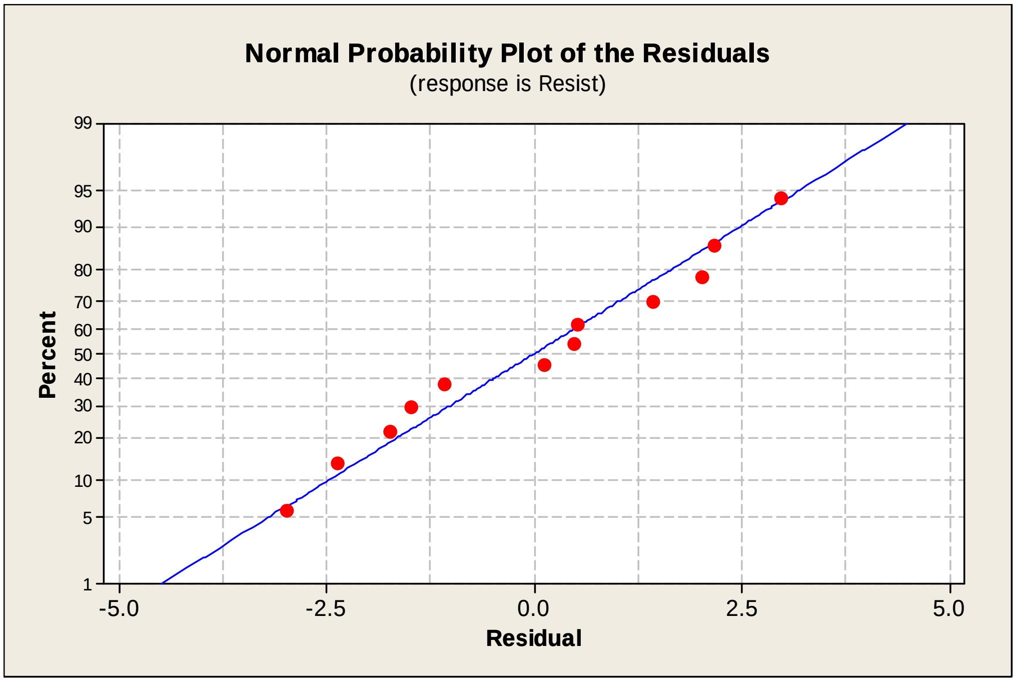
Curvature is not significant (*P*-value = 0.327), so continue with analysis.

(c)



A funnel pattern at the low value and an overall lack of consistent width suggest a problem with equal variance across the prediction range.

12-15 continued
(d)



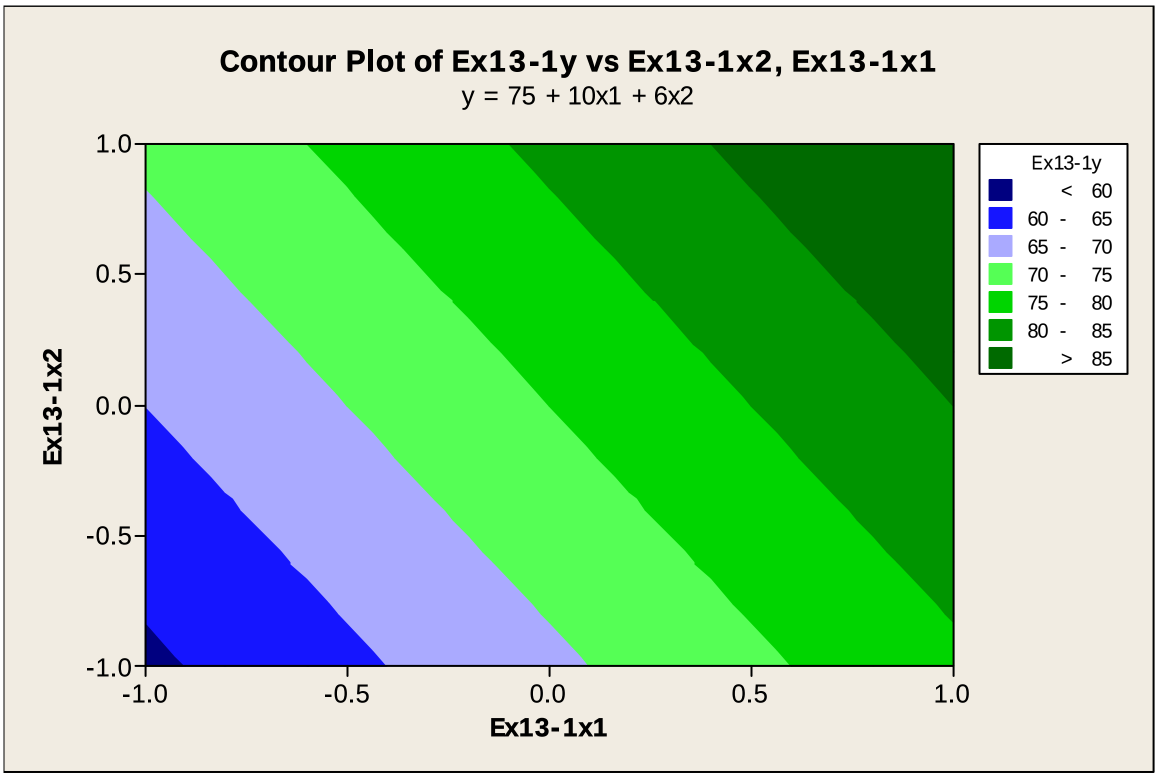
The normal probability plot of residuals is satisfactory.

The concern with variance in the predicted resistivity indicates that a data transformation may be needed.

Chapter 13 Exercise Solutions

Note: To analyze an experiment in MINITAB, the initial experimental layout must be created in MINITAB or defined by the user. The Excel data sets contain only the data given in the textbook; therefore some information required by MINITAB is not included. The MINITAB instructions provided for the factorial designs in Chapter 12 are similar to those for response surface designs in this Chapter.

13-1.
(a)
Graph > Contour Plot



(b)

$\hat{y} = 75 + 10x_1 + 6x_2 \quad -1 \le x_1 \le 1; 1 \le x_2 \le 1$

$\frac{x_2}{x_1} = \frac{6}{10} = 0.6$

$\Delta x_1 = 1$

$\Delta x_2 = 0.6$

13-2.

$\hat{y} = 50 + 2x_1 - 15x_2 + 3x_3 \quad -1 \le x_i \le +1; i = 1,2,3$

select x_2 with largest absolute coefficient, $\hat{\beta}_2 = -15,$ and set $\Delta x_2 = 1.0$

$$\Delta x_1 = \frac{\hat{\beta}_1}{\hat{\beta}_2 / \Delta x_2} = \frac{2}{-15/1.0} = -0.13$$

$$\Delta x_3 = \frac{\hat{\beta}_3}{\hat{\beta}_2 / \Delta x_2} = \frac{3}{-15/1.0} = -0.20$$

13-3.

(a)
This design is a CCD with k = 2 and α = 1.5. The design is not rotatable.

13-3 continued
(b)
Enter the factor levels and response data into a MINITAB worksheet, including a column indicating whether a run is a center point run (1 = not center point, 0 = center point). Then define the experiment using **Stat > DOE > Response Surface > Define Custom Response Surface Design**. The design and data are in the MINITAB worksheet **Ex13-3.MTW**.

Select **Stat > DOE > Response Surface > Analyze Response Surface Design**. Select "**Terms**" and verify that all main effects, two-factor interactions, and quadratic terms are selected.

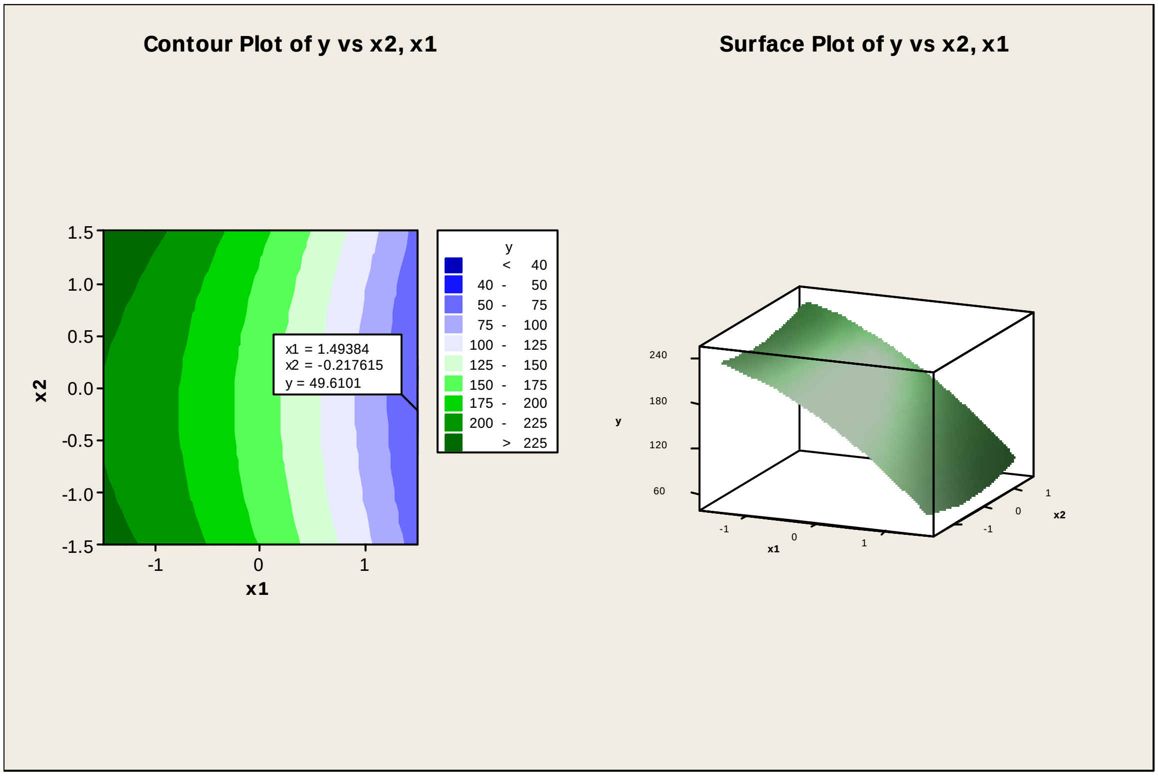
Response Surface Regression: y versus x1, x2

```
The analysis was done using coded units.
Estimated Regression Coefficients for y
Term         Coef  SE Coef        T      P
Constant  160.868    4.555   35.314  0.000
x1        -87.441    4.704  -18.590  0.000
x2          3.618    4.704    0.769  0.471
x1*x1     -24.423    7.461   -3.273  0.017
x2*x2      15.577    7.461    2.088  0.082
x1*x2      -1.688   10.285   -0.164  0.875
…
Analysis of Variance for y
Source          DF   Seq SS   Adj SS   Adj MS       F      P
Regression       5  30583.4  30583.4   6116.7   73.18  0.000
  Linear         2  28934.2  28934.2  14467.1  173.09  0.000
  Square         2   1647.0   1647.0    823.5    9.85  0.013
  Interaction    1      2.3      2.3      2.3    0.03  0.875
Residual Error   6    501.5    501.5     83.6
  Lack-of-Fit    3     15.5     15.5      5.2    0.03  0.991
  Pure Error     3    486.0    486.0    162.0
Total           11  31084.9
…
Estimated Regression Coefficients for y using data in uncoded units
Term          Coef
Constant  160.8682
x1        -58.2941
x2          2.4118
x1*x1     -10.8546
x2*x2       6.9231
x1*x2      -0.7500
```

13-3 continued

(c)

Stat > DOE > Response Surface > Contour/Surface Plots



From visual examination of the contour and surface plots, it appears that minimum purity can be achieved by setting x_1 (time) = +1.5 and letting x_2 (temperature) range from –1.5 to + 1.5. The range for x_2 agrees with the ANOVA results indicating that it is statistically insignificant (P-value = 0.471). The level for temperature could be established based on other considerations, such as cost. A flag is planted at one option on the contour plot above.

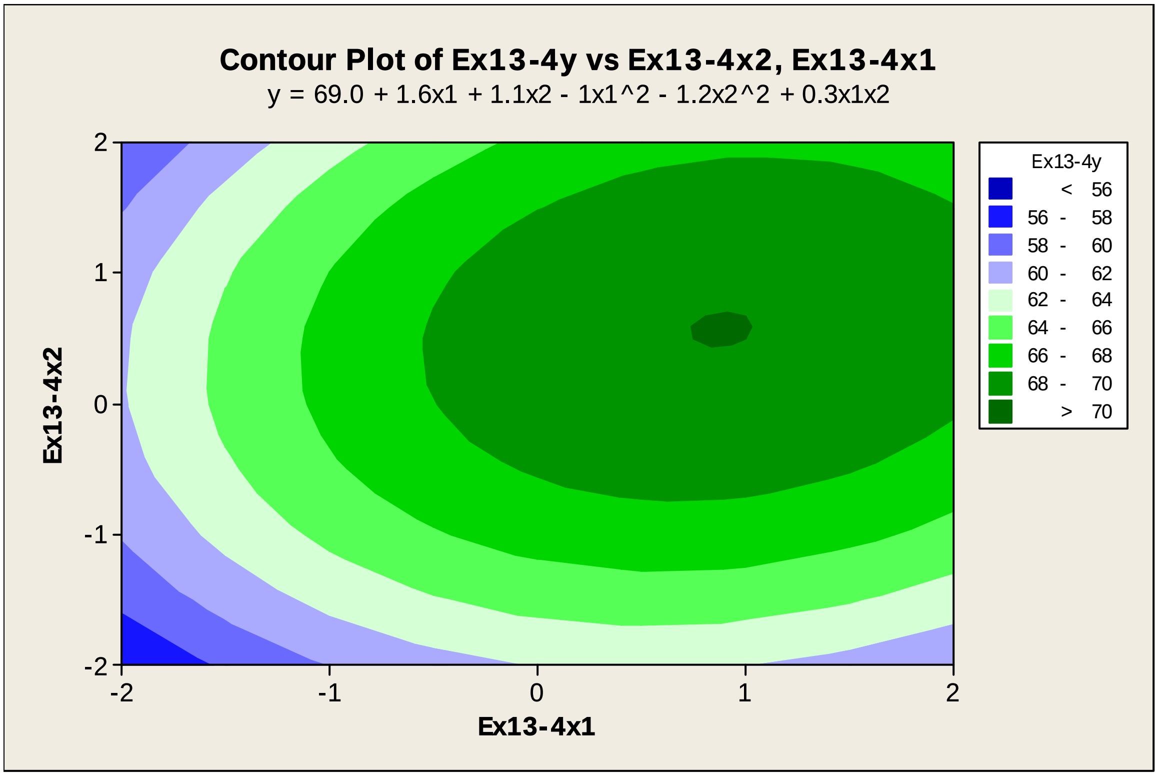
(d)

$$\text{Temp} = 50x_1 + 750 = 50(+1.50) + 750 = 825$$

$$\text{Time} = 15x_2 + 30 = 15(-0.22) + 30 = 26.7$$

13-4.

Graph > Contour Plot



$\hat{y}_{\max} = 70.012$ at $x_1 \approx +0.9, x_2 \approx +0.6$

(b)

$$\frac{\partial \hat{y}}{\partial x_1} = \frac{\partial(69.0 + 1.6x_1 + 1.1x_2 - 1x_1^2 - 1.2x_2^2 + 0.3x_1x_2)}{\partial x_1} \Rightarrow 0$$

$$= 1.6 - 2x_1 + 0.3x_2 = 0$$

$$\frac{\partial \hat{y}}{\partial x_2} = 1.1 - 2.4x_2 + 0.3x_1 = 0$$

$$x_1 = -13.9/(-15.7) = 0.885$$

$$x_2 = [-1.1 - 0.3(0.885)]/(-2.4) = 0.569$$

13-5.
(a)
The design is a CCD with k = 2 and α = 1.4. The design is rotatable.

(b)
Since the standard order is provided, one approach to solving this exercise is to create a two-factor response surface design in MINITAB, then enter the data.

Select **Stat > DOE > Response Surface > Create Response Surface Design**. Leave the design type as a 2-factor, central composite design. Select "**Designs**", highlight the design with five center points (13 runs), and enter a custom alpha value of exactly 1.4 (the rotatable design is α = 1.41421). The worksheet is in run order, to change to standard order (and ease data entry) select **Stat > DOE > Display Design** and choose standard order. The design and data are in the MINITAB worksheet **Ex13-5.MTW**.

To analyze the experiment, select **Stat > DOE > Response Surface > Analyze Response Surface Design**. Select "**Terms**" and verify that a full quadratic model (A, B, A^2, B^2, AB) is selected.

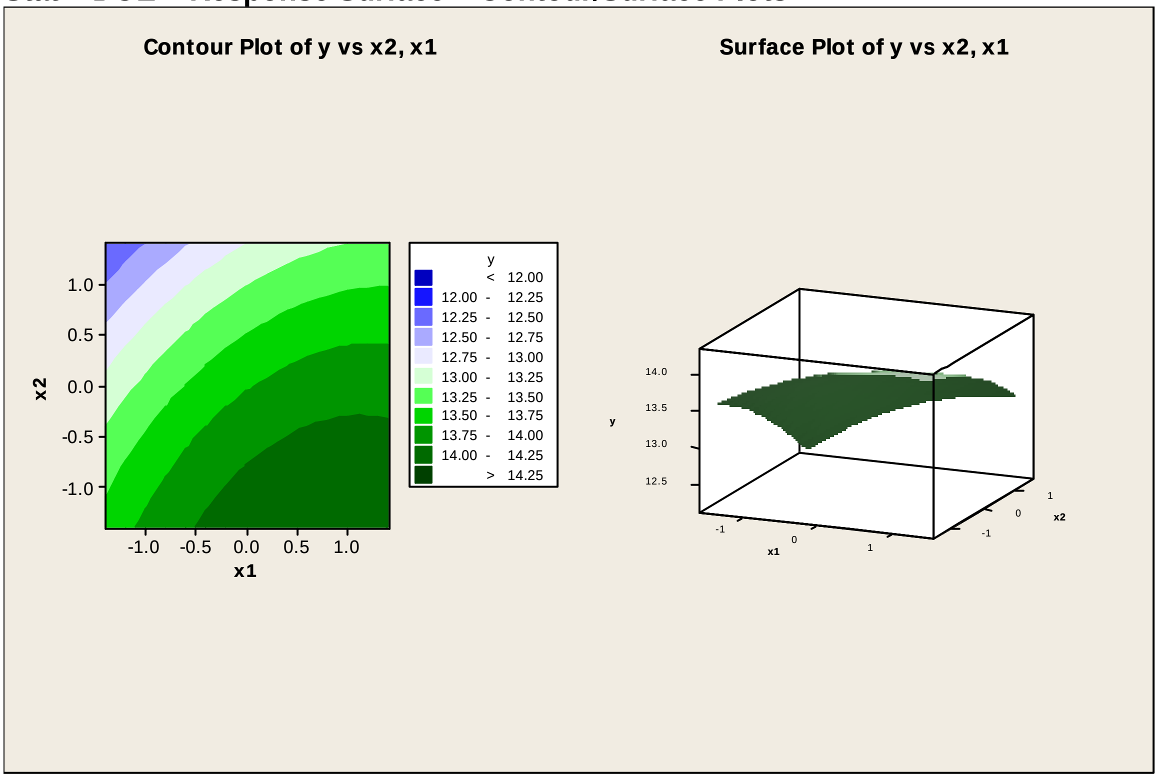
Response Surface Regression: y versus x1, x2

```
The analysis was done using coded units.
Estimated Regression Coefficients for y
Term         Coef  SE Coef        T      P
Constant  13.7273  0.04309  318.580  0.000
x1         0.2980  0.03424    8.703  0.000
x2        -0.4071  0.03424  -11.889  0.000
x1*x1     -0.1249  0.03706   -3.371  0.012
x2*x2     -0.0790  0.03706   -2.132  0.070
x1*x2      0.0550  0.04818    1.142  0.291
…
Analysis of Variance for y
Source          DF   Seq SS   Adj SS   Adj MS       F      P
Regression       5  2.16128  2.16128  0.43226   46.56  0.000
  Linear         2  2.01563  2.01563  1.00781  108.54  0.000
  Square         2  0.13355  0.13355  0.06678    7.19  0.020
  Interaction    1  0.01210  0.01210  0.01210    1.30  0.291
Residual Error   7  0.06499  0.06499  0.00928
  Lack-of-Fit    3  0.03271  0.03271  0.01090    1.35  0.377
  Pure Error     4  0.03228  0.03228  0.00807
Total           12  2.22628
…
Estimated Regression Coefficients for y using data in uncoded units
Term         Coef
Constant  13.7273
x1         0.2980
x2        -0.4071
x1*x1     -0.1249
x2*x2     -0.0790
x1*x2      0.0550
```

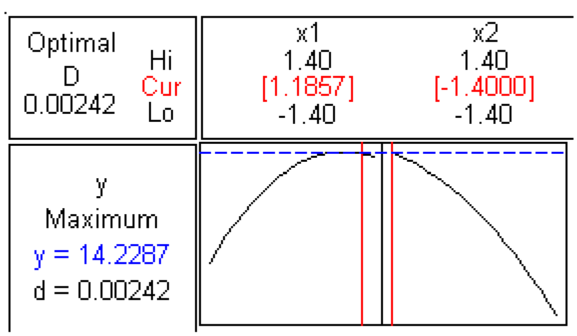
13-5 (b) continued

Values of x_1 and x_2 maximizing the Mooney viscosity can be found from visual examination of the contour and surface plots, or using MINITAB's Response Optimizer.

Stat > DOE > Response Surface > Contour/Surface Plots



Stat > DOE > Response Surface > Response Optimizer
In Setup, let Goal = maximize, Lower = 10, Target = 20, and Weight = 7.



From the plots and the optimizer, setting x_1 in a range from 0 to +1.4 and setting x_2 between -1 and -1.4 will maximize viscosity.

13-6.

The design is a full factorial of three factors at three levels. Since the runs are listed in a patterned (but not standard) order, one approach to solving this exercise is to create a general full factorial design in MINITAB, and then enter the data.

Select **Stat > DOE > Factoriall > Create Factorial Design**. Change the design type to a general full factorial design, and select the number of factors as "**3**". Select "**Designs**" to establish three levels for each factor, then select "**Factors**" to specify the actual level values. In order to analyze this experiment using the Response Surface functionality, it must also be defined using **Stat > DOE > Response Surface > Define Custom Response Surface Design**. The design and data are in the MINITAB worksheet **Ex13-6.MTW**.

(a)
To analyze the experiment, select **Stat > DOE > Response Surface > Analyze Response Surface Design**. Select "**Terms**" and verify that a full quadratic model is selected.

Response Surface Regression: y1 versus x1, x2, x3

```
The analysis was done using coded units.
Estimated Regression Coefficients for y1
Term        Coef  SE Coef       T      P
Constant  327.62    38.76   8.453  0.000
x1        177.00    17.94   9.866  0.000
x2        109.43    17.94   6.099  0.000
x3        131.47    17.94   7.328  0.000
x1*x1      32.01    31.08   1.030  0.317
x2*x2     -22.38    31.08  -0.720  0.481
x3*x3     -29.06    31.08  -0.935  0.363
x1*x2      66.03    21.97   3.005  0.008
x1*x3      75.47    21.97   3.435  0.003
x2*x3      43.58    21.97   1.983  0.064
…
Analysis of Variance for y1
Source          DF   Seq SS   Adj SS  Adj MS      F      P
Regression       9  1248237  1248237  138693  23.94  0.000
  Linear         3  1090558  1090558  363519  62.74  0.000
  Square         3    14219    14219    4740   0.82  0.502
  Interaction    3   143461   143461   47820   8.25  0.001
Residual Error  17    98498    98498    5794
Total           26  1346735
…
Estimated Regression Coefficients for y1 using data in uncoded units
Term          Coef
Constant  327.6237
x1        177.0011
x2        109.4256
x3        131.4656
x1*x1      32.0056
x2*x2     -22.3844
x3*x3     -29.0578
x1*x2      66.0283
x1*x3      75.4708
x2*x3      43.5833
```

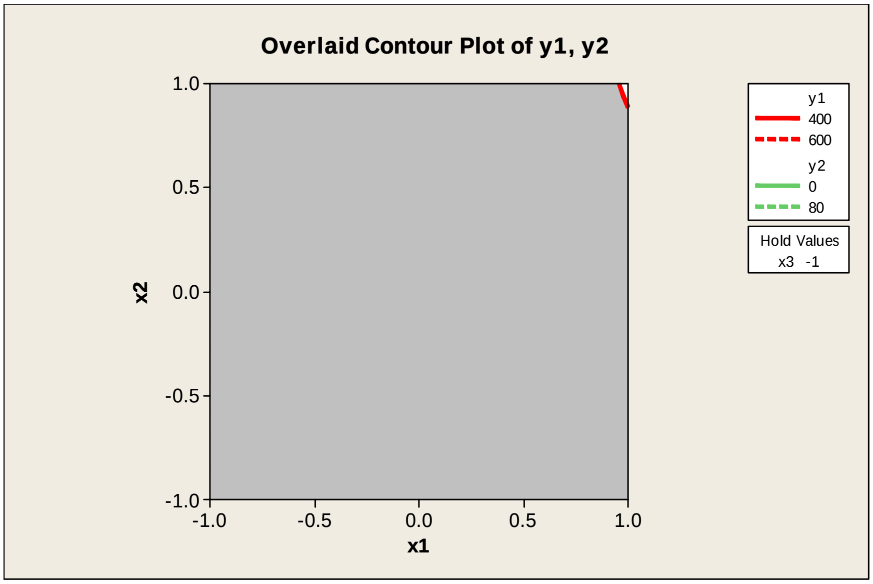
13-6 continued
(b)
To analyze the experiment, select **Stat > DOE > Response Surface > Analyze Response Surface Design**. Select "**Terms**" and verify that a full quadratic model is selected.

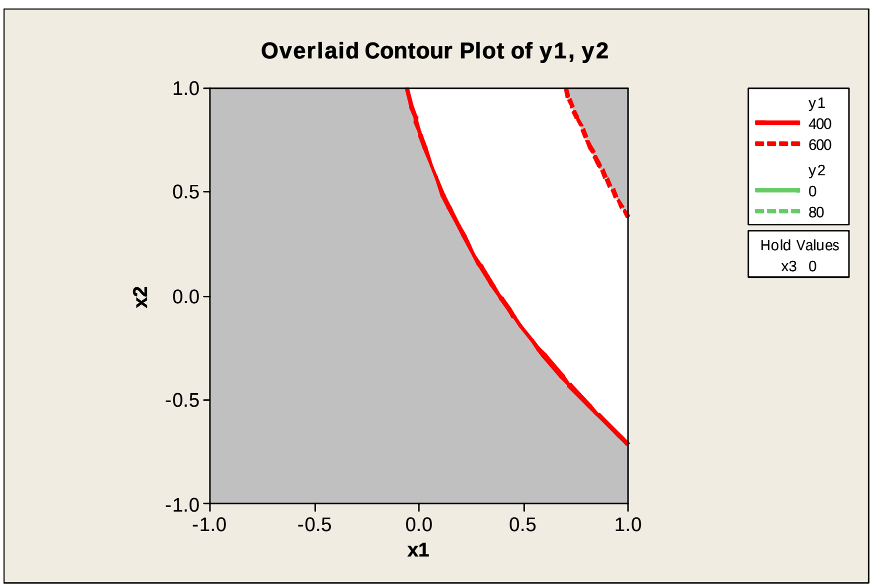
Response Surface Regression: y2 versus x1, x2, x3

```
The analysis was done using coded units.
Estimated Regression Coefficients for y2
Term         Coef  SE Coef       T      P
Constant  34.890    22.31   1.564  0.136
x1        11.528    10.33   1.116  0.280
x2        15.323    10.33   1.483  0.156
x3        29.192    10.33   2.826  0.012
x1*x1      4.198    17.89   0.235  0.817
x2*x2     -1.319    17.89  -0.074  0.942
x3*x3     16.779    17.89   0.938  0.361
x1*x2      7.719    12.65   0.610  0.550
x1*x3      5.108    12.65   0.404  0.691
x2*x3     14.082    12.65   1.113  0.281
…
Analysis of Variance for y2
Source          DF   Seq SS   Adj SS   Adj MS     F      P
Regression       9  27170.7  27170.7  3018.97  1.57  0.202
  Linear         3  21957.3  21957.3  7319.09  3.81  0.030
  Square         3   1805.5   1805.5   601.82  0.31  0.815
  Interaction    3   3408.0   3408.0  1135.99  0.59  0.629
Residual Error  17  32650.2  32650.2  1920.60
Total           26  59820.9
…
Estimated Regression Coefficients for y2 using data in uncoded units
Term         Coef
Constant  34.8896
x1        11.5278
x2        15.3233
x3        29.1917
x1*x1      4.1978
x2*x2     -1.3189
x3*x3     16.7794
x1*x2      7.7192
x1*x3      5.1083
x2*x3     14.0825
```

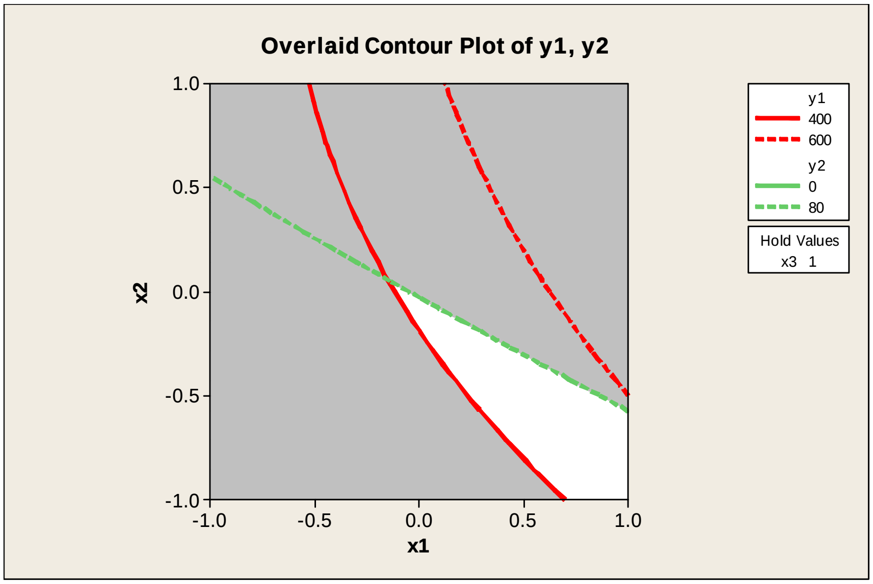
13-6 continued
(c)
Both overlaid contour plots and the response optimizer can be used to identify settings to achieve both objectives.

Stat > DOE > Response Surface > Overlaid Contour Plot
After selecting the responses, select the first two factors x_1 and x_2. Select "Contours" to establish the low and high contours for both y_1 and y_2. Since the goal is to hold y_1 (resistivity) at 500, set low = 400 and high = 600. The goal is to minimize y_2 (standard deviation) set low = 0 (the minimum of the observed results) and high = 80 (the 3rd quartile of the observed results).

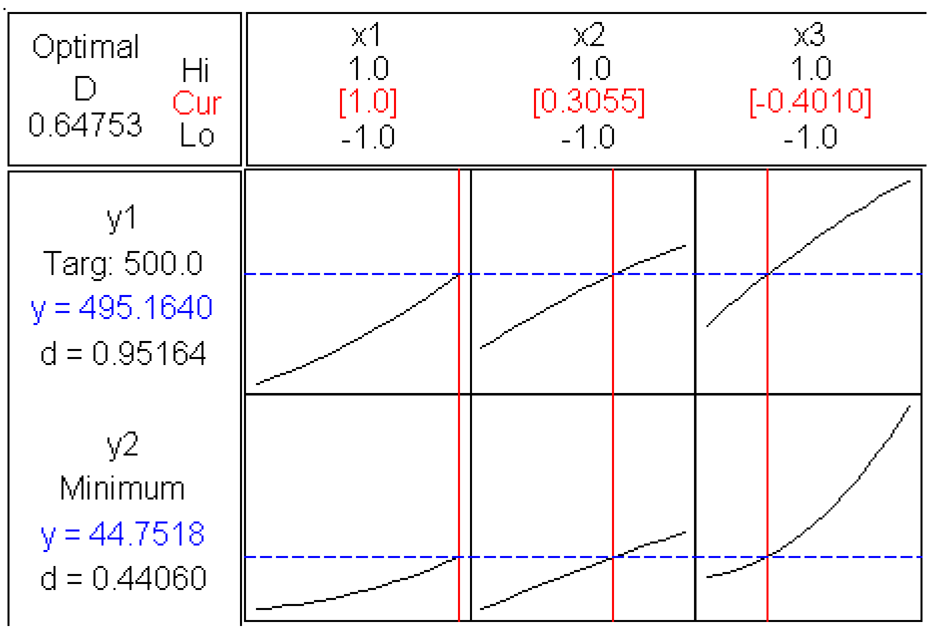




13-6 (c) continued



Stat > DOE > Response Surface > Response Optimizer
In Setup, for y_1 set Goal = Target, Lower = 400, Target = 500, Upper = 600. For y_2, set Goal = Minimize, Target = 0, and Upper = 80. Leave all Weight and Importance values at 1. The graph below represents one possible solution.



At $x_1 = 1.0$, $x_2 = 0.3$ and $x_3 = -0.4$, the predicted resistivity mean is 495.16 and standard deviation is 44.75.

13-7.
Enter the factor levels and response data into a MINITAB worksheet, and then define the experiment using **Stat > DOE > Factorial > Define Custom Factorial Design**. The design and data are in the MINITAB worksheet **Ex13-7.MTW**.

(a)
The defining relation for this half-fraction design is I = ABCD (from examination of the plus and minus signs).

$A+BCD$	$AB+CD$	$CE+ABDE$
$B+ACD$	$AC+BD$	$DE+ABCE$
$C+ABD$	$AD+BC$	$ABE+CDE$
$D+ABC$	$AE+BCDE$	$ACE+BDE$
E	$BE+ACDE$	$ADE+BCE$

This is a resolution IV design. All main effects are clear of 2-factor interactions, but some 2-factor interactions are aliased with each other.

Stat > DOE > Factorial > Analyze Factorial Design

Factorial Fit: Mean versus A, B, C, D, E

…

```
Alias Structure
I + A*B*C*D
A + B*C*D
B + A*C*D
C + A*B*D
D + A*B*C
E + A*B*C*D*E
A*B + C*D
A*C + B*D
A*D + B*C
A*E + B*C*D*E
B*E + A*C*D*E
C*E + A*B*D*E
D*E + A*B*C*E
```

13-7 continued
(b)
The full model for mean:

Stat > DOE > Factorial > Analyze Factorial Design

Factorial Fit: Height versus A, B, C, D, E

```
Estimated Effects and Coefficients for Height (coded units)
Term       Effect     Coef  SE Coef       T      P
Constant            7.6256  0.02021  377.41  0.000
A          0.2421   0.1210  0.02021    5.99  0.000
B         -0.1638  -0.0819  0.02021   -4.05  0.000
C         -0.0496  -0.0248  0.02021   -1.23  0.229
D          0.0912   0.0456  0.02021    2.26  0.031
E         -0.2387  -0.1194  0.02021   -5.91  0.000
A*B       -0.0296  -0.0148  0.02021   -0.73  0.469
A*C        0.0012   0.0006  0.02021    0.03  0.976
A*D       -0.0229  -0.0115  0.02021   -0.57  0.575
A*E        0.0637   0.0319  0.02021    1.58  0.124
B*E        0.1529   0.0765  0.02021    3.78  0.001
C*E       -0.0329  -0.0165  0.02021   -0.81  0.421
D*E        0.0396   0.0198  0.02021    0.98  0.335
A*B*E      0.0021   0.0010  0.02021    0.05  0.959
A*C*E      0.0196   0.0098  0.02021    0.48  0.631
A*D*E     -0.0596  -0.0298  0.02021   -1.47  0.150
…
Analysis of Variance for Height (coded units)
Source              DF   Seq SS   Adj SS   Adj MS      F      P
Main Effects         5  1.83846  1.83846  0.36769  18.76  0.000
2-Way Interactions   7  0.37800  0.37800  0.05400   2.76  0.023
3-Way Interactions   3  0.04726  0.04726  0.01575   0.80  0.501
Residual Error      32  0.62707  0.62707  0.01960
  Pure Error        32  0.62707  0.62707  0.01960
Total               47  2.89078
```

The reduced model for mean:

Factorial Fit: Height versus A, B, D, E

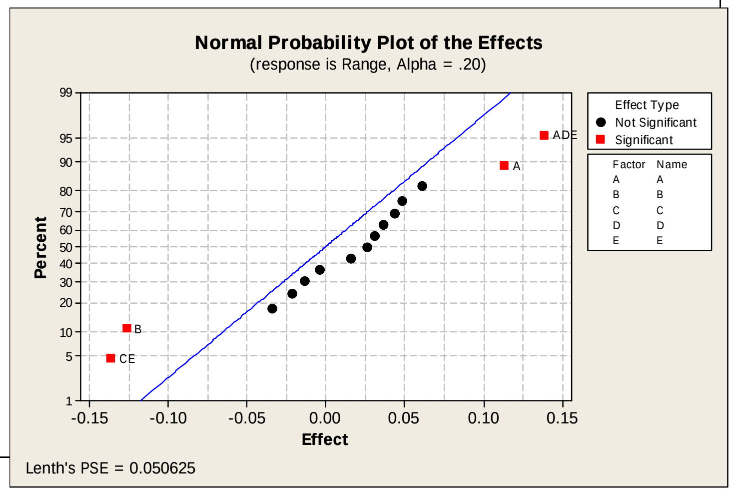
```
Estimated Effects and Coefficients for Height (coded units)
Term       Effect     Coef  SE Coef       T      P
Constant            7.6256  0.01994  382.51  0.000
A          0.2421   0.1210  0.01994    6.07  0.000
B         -0.1638  -0.0819  0.01994   -4.11  0.000
D          0.0913   0.0456  0.01994    2.29  0.027
E         -0.2387  -0.1194  0.01994   -5.99  0.000
B*E        0.1529   0.0765  0.01994    3.84  0.000
…
Analysis of Variance for Height (coded units)
Source              DF  Seq SS  Adj SS   Adj MS      F      P
Main Effects         4  1.8090  1.8090  0.45224  23.71  0.000
2-Way Interactions   1  0.2806  0.2806  0.28060  14.71  0.000
Residual Error      42  0.8012  0.8012  0.01908
  Lack of Fit       10  0.1742  0.1742  0.01742   0.89  0.554
  Pure Error        32  0.6271  0.6271  0.01960
Total               47  2.8908
```

13-7 continued
(c)
The full model for range:

Factorial Fit: Range versus A, B, C, D, E

Estimated Effects and Coefficients for Range (coded units)

Term	Effect	Coef
Constant		0.21937
A	0.11375	0.05688
B	-0.12625	-0.06312
C	0.02625	0.01313
D	0.06125	0.03062
E	-0.01375	-0.00687
A*B	0.04375	0.02188
A*C	-0.03375	-0.01688
A*D	0.03625	0.01812
A*E	-0.00375	-0.00188
B*E	0.01625	0.00812
C*E	-0.13625	-0.06812
D*E	-0.02125	-0.01063
A*B*E	0.03125	0.01562
A*C*E	0.04875	0.02437
A*D*E	0.13875	0.06937



The reduced model for range:

Factorial Fit: Range versus A, B, C, D, E

Estimated Effects and Coefficients for Range (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		0.21937	0.01625	13.50	0.000
A	0.11375	0.05688	0.01625	3.50	0.008
B	-0.12625	-0.06312	0.01625	-3.88	0.005
C	0.02625	0.01313	0.01625	0.81	0.443
D	0.06125	0.03062	0.01625	1.88	0.096
E	-0.01375	-0.00687	0.01625	-0.42	0.683
C*E	-0.13625	-0.06812	0.01625	-4.19	0.003
A*D*E	0.13875	0.06937	0.01625	4.27	0.003

…

Analysis of Variance for Range (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	0.13403	0.13403	0.026806	6.34	0.011
2-Way Interactions	1	0.07426	0.07426	0.074256	17.58	0.003
3-Way Interactions	1	0.07701	0.07701	0.077006	18.23	0.003
Residual Error	8	0.03380	0.03380	0.004225		
Total	15	0.31909				

…

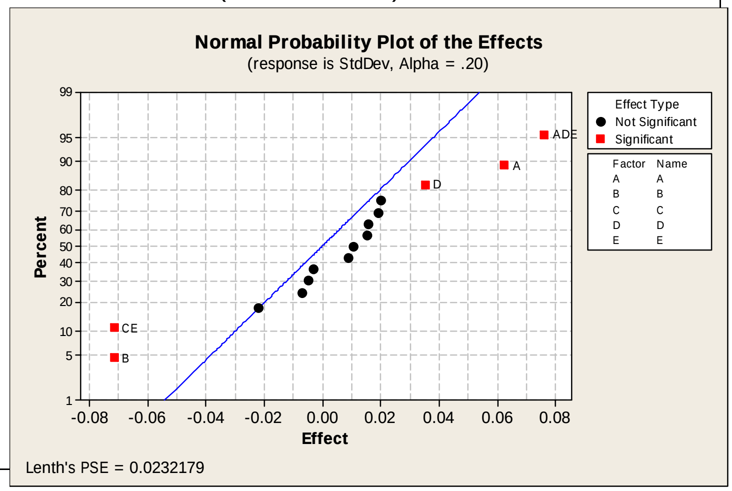
13-7 (c) continued

The full model for standard deviation:

Factorial Fit: StdDev versus A, B, C, D, E

Estimated Effects and Coefficients for StdDev (coded units)

Term	Effect	Coef
Constant		0.11744
A	0.06259	0.03129
B	-0.07149	-0.03574
C	0.01057	0.00528
D	0.03536	0.01768
E	-0.00684	-0.00342
A*B	0.01540	0.00770
A*C	-0.02185	-0.01093
A*D	0.01906	0.00953
A*E	-0.00329	-0.00165
B*E	0.00877	0.00438
C*E	-0.07148	-0.03574
D*E	-0.00468	-0.00234
A*B*E	0.01556	0.00778
A*C*E	0.01997	0.00999
A*D*E	0.07643	0.03822



The reduced model for standard deviation:

Factorial Fit: StdDev versus A, B, C, D, E

Estimated Effects and Coefficients for StdDev (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		0.11744	0.007559	15.54	0.000
A	0.06259	0.03129	0.007559	4.14	0.003
B	-0.07149	-0.03574	0.007559	-4.73	0.001
C	0.01057	0.00528	0.007559	0.70	0.504
D	0.03536	0.01768	0.007559	2.34	0.047
E	-0.00684	-0.00342	0.007559	-0.45	0.663
C*E	-0.07148	-0.03574	0.007559	-4.73	0.001
A*D*E	0.07643	0.03822	0.007559	5.06	0.001

…

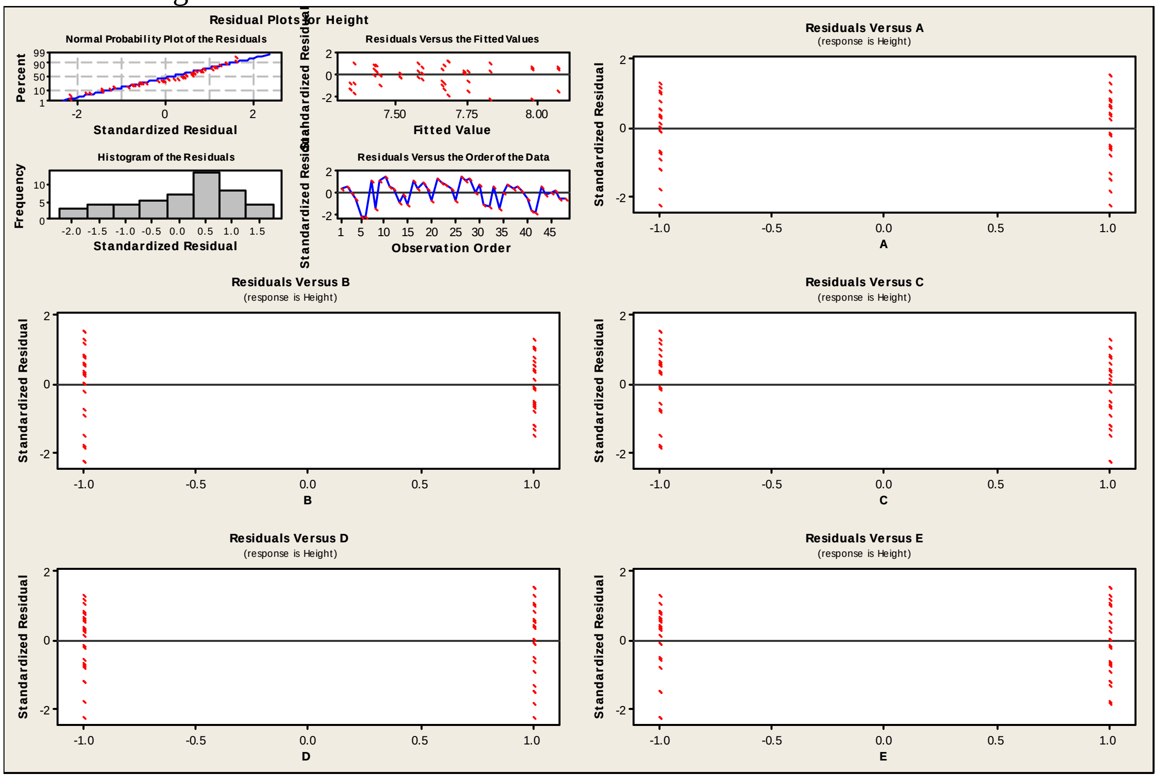
Analysis of Variance for StdDev (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	0.041748	0.041748	0.0083496	9.13	0.004
2-Way Interactions	1	0.020438	0.020438	0.0204385	22.36	0.001
3-Way Interactions	1	0.023369	0.023369	0.0233690	25.56	0.001
Residual Error	8	0.007314	0.007314	0.0009142		
Total	15	0.092869				

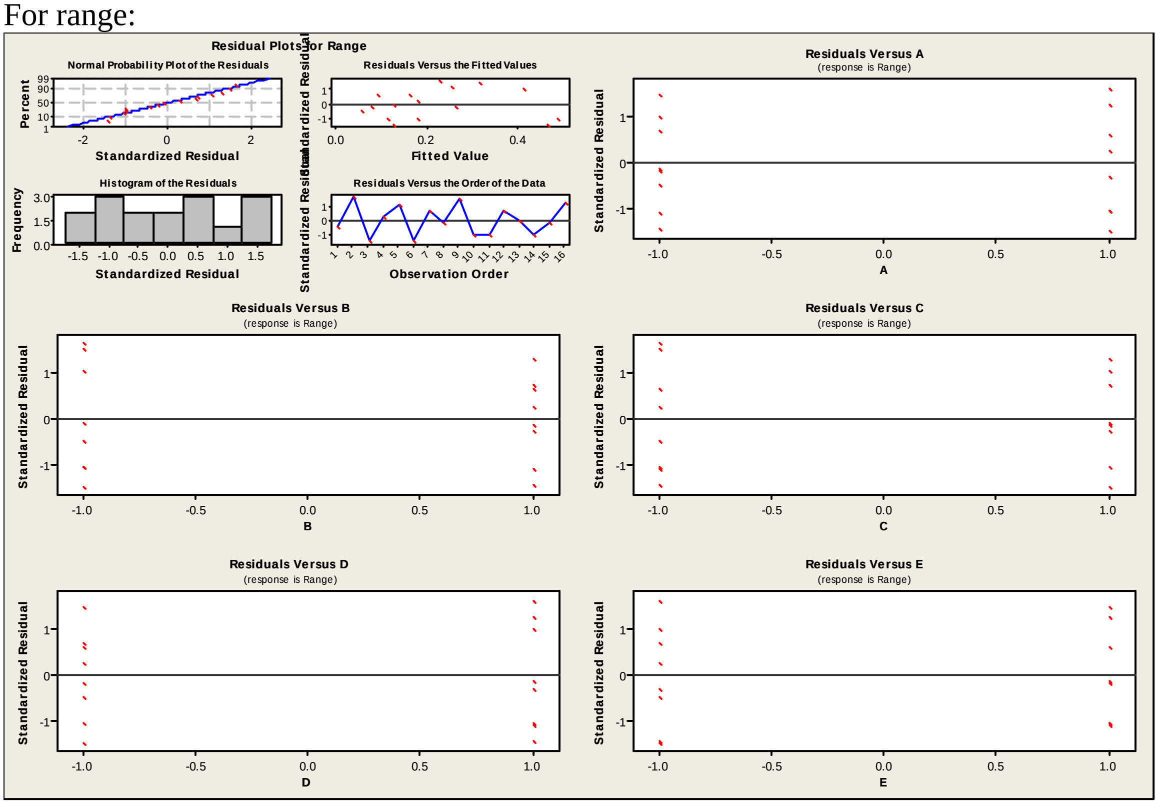
For both models of variability, interactions CE (transfer time × quench oil temperature) and ADE=BCE, along with factors B (heating time) and A (furnace temperature) are significant. Factors C and E are included to keep the models hierarchical.

(d)

For mean height:

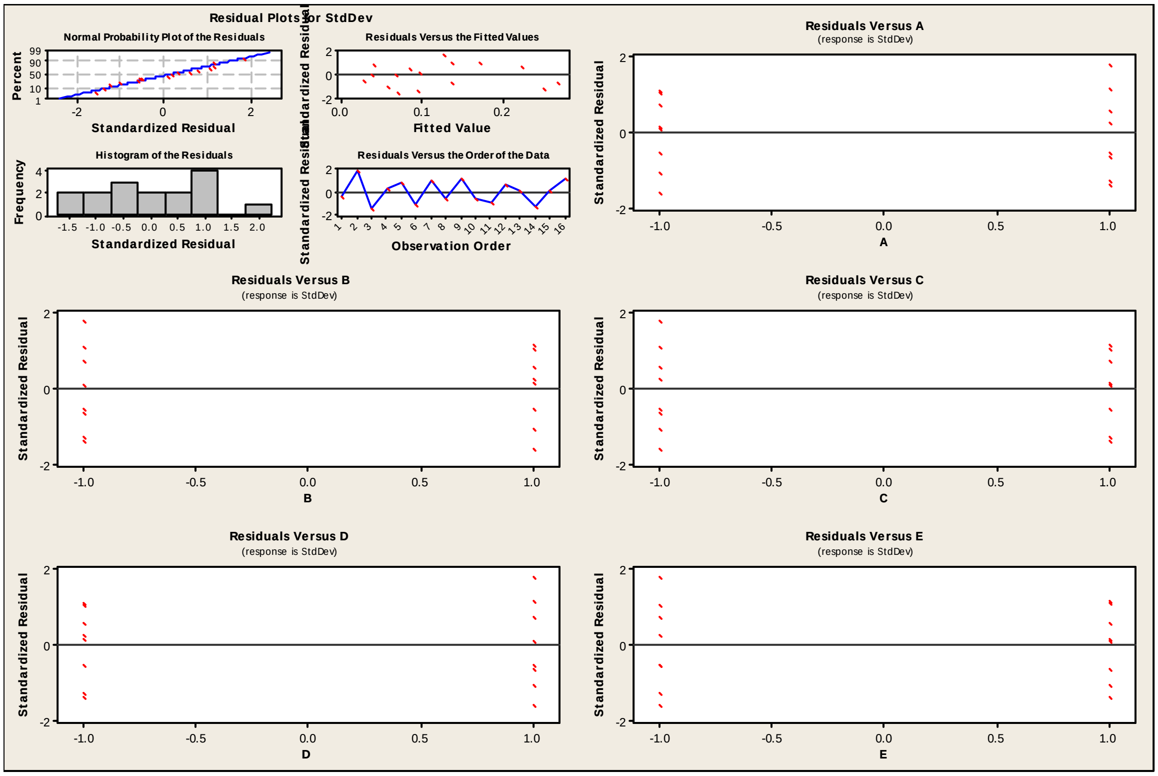


For range:



13-7 (d) continued

For standard deviation:



Mean Height
Plot of residuals versus predicted indicates constant variance assumption is reasonable. Normal probability plot of residuals support normality assumption. Plots of residuals versus each factor shows that variance is less at low level of factor E.

Range
Plot of residuals versus predicted shows that variance is approximately constant over range of predicted values. Residuals normal probability plot indicate normality assumption is reasonable Plots of residuals versus each factor indicate that the variance may be different at different levels of factor D.

Standard Deviation
Residuals versus predicted plot and residuals normal probability plot support constant variance and normality assumptions. Plots of residuals versus each factor indicate that the variance may be different at different levels of factor D.

(e)
This is not the best 16-run design for five factors. A resolution V design can be generated with E = ± ABCD, then none of the 2-factor interactions will be aliased with each other.

13-8.

Factor E is hard to control (a "noise" variable). Using equations (13-6) and (13-7) the mean and variance models are:

Mean Free Height = 7.63 + 0.12A – 0.081B + 0.046D

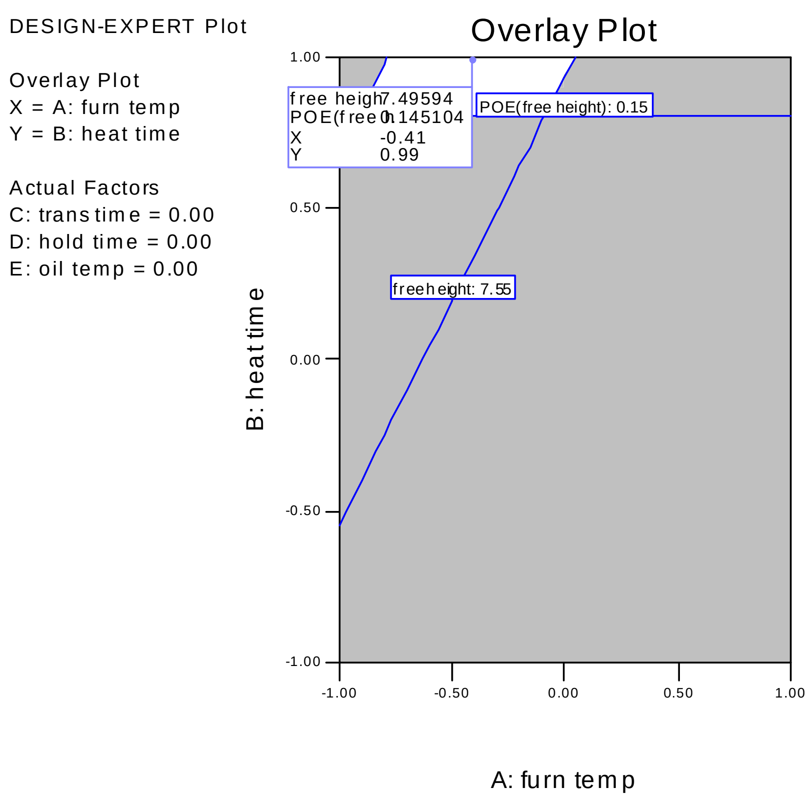
Variance of Free Height = $\sigma^2_E\,(-0.12 + 0.077B)^2 + \sigma^2$

Assume (following text) that $\sigma^2_E = 1$ and $\hat{\sigma}^2 = MS_E = 0.02$, so

Variance of Free Height = $(-0.12 + 0.077B)^2 + 0.02$

For the current factor levels, Free Height Variance could be calculated in the MINITAB worksheet, and then contour plots in factors A, B, and D could be constructed using the Graph > Contour Plot functionality. These contour plots could be compared with a contour plot of Mean Free Height, and optimal settings could be identified from visual examination of both plots. This approach is fully described in the solution to Exercise 13-12.

The overlaid contour plot below (constructed in Design-Expert) shows one solution with mean Free Height ≅ 7.49 and minimum standard deviation of 0.056 at A = –0.44 and B = 0.99.



13-9.

Factors D and E are noise variables. Assume $\sigma_D^2 = \sigma_E^2 = 1$. Using equations (13-6) and (13-7), the mean and variance are:

Mean Free Height = 7.63 + 0.12A – 0.081B

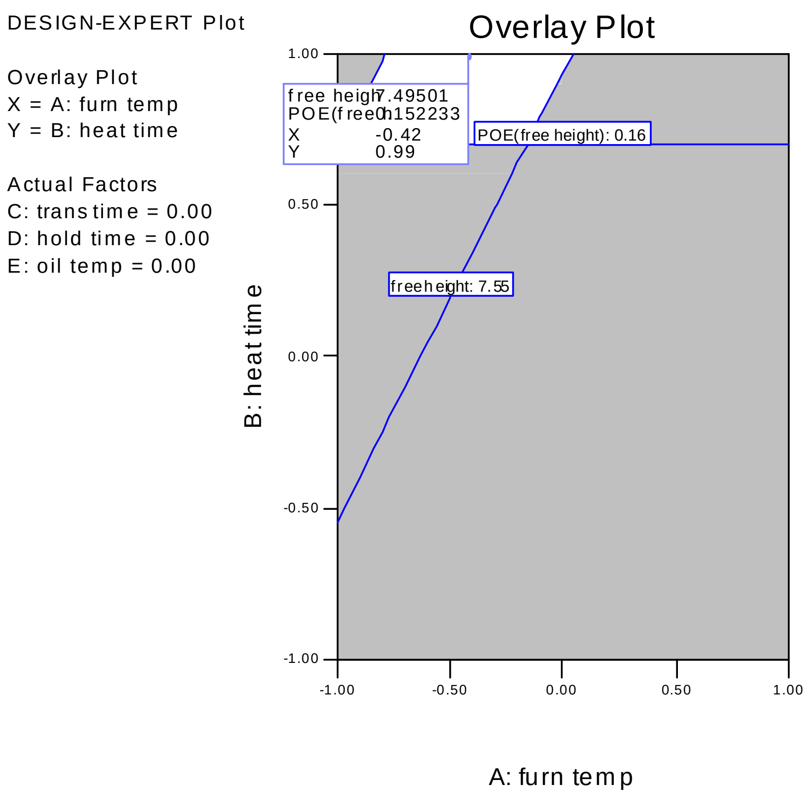
Variance of Free Height = $\sigma^2_D\,(+0.046)^2 + \sigma^2_E\,(-0.12 + 0.077B)^2 + \sigma^2$

Using $\hat{\sigma}^2 = MS_E = 0.02$:

Variance of Free Height = $(0.046)^2 + (-0.12 + 0.077B)^2 + 0.02$

For the current factor levels, Free Height Variance could be calculated in the MINITAB worksheet, and then contour plots in factors A, B, and D could be constructed using the Graph > Contour Plot functionality. These contour plots could be compared with a contour plot of Mean Free Height, and optimal settings could be identified from visual examination of both plots. This approach is fully described in the solution to Exercise 13-12.

The overlaid contour plot below (constructed in Design-Expert) shows one solution with mean Free Height ≅ 7.50 and minimum standard deviation of Free Height to be: A = –0.42 and B = 0.99.



13-10.

Note: Several *y* values are incorrectly listed in the textbook. The correct values are: 66, 70, 78, 60, 80, 70, 100, 75, 65, 82, 68, 63, 100, 80, 83, 90, 87, 88, 91, 85. These values are used in the Excel and MINITAB data files.

Since the runs are listed in a patterned (but not standard) order, one approach to solving this exercise is to create a general full factorial design in MINITAB, and then enter the data. The design and data are in the MINITAB worksheet **Ex13-10.MTW**.

Stat > DOE > Response Surface > Analyze Response Surface Design

Response Surface Regression: y versus x1, x2, x3

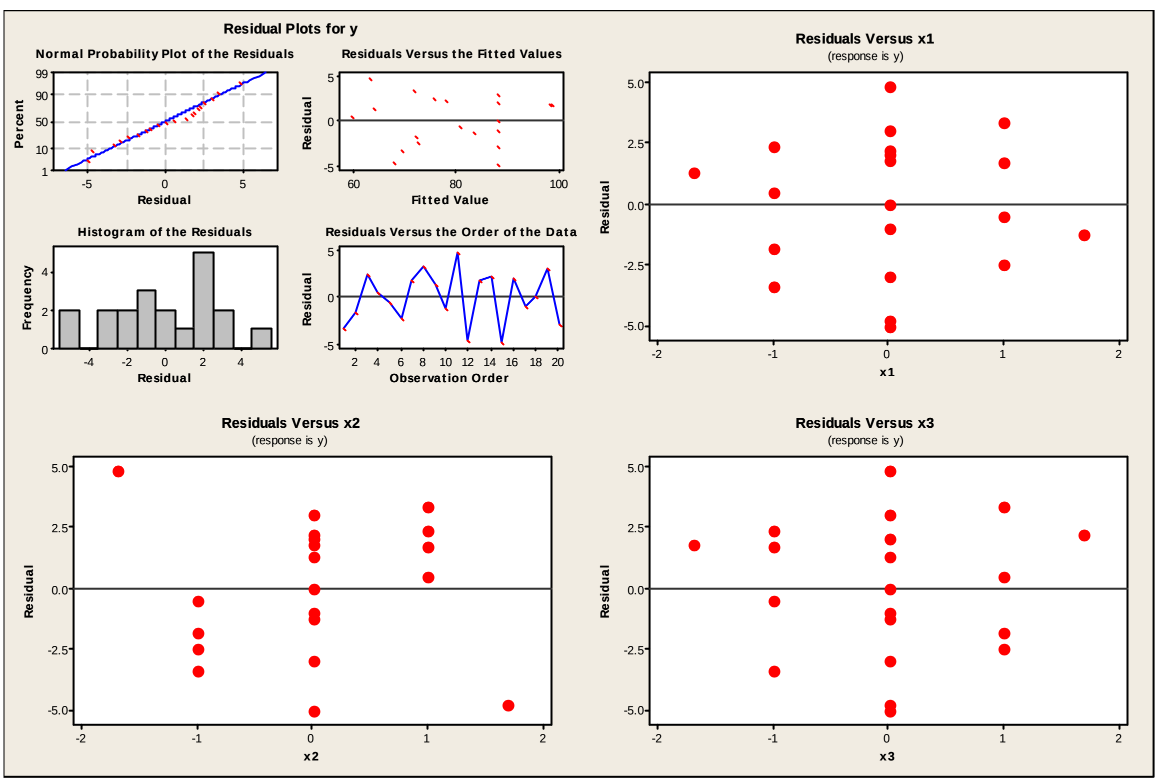
```
The analysis was done using coded units.
Estimated Regression Coefficients for y
Term          Coef  SE Coef       T      P
Constant   87.359    1.513  57.730  0.000
x1          9.801    1.689   5.805  0.000
x2          2.289    1.689   1.356  0.205
x3        -10.176    1.689  -6.027  0.000
x1*x1     -14.305    2.764  -5.175  0.000
x2*x2     -22.305    2.764  -8.069  0.000
x3*x3       2.195    2.764   0.794  0.446
x1*x2       8.132    3.710   2.192  0.053
x1*x3      -7.425    3.710  -2.001  0.073
x2*x3     -13.081    3.710  -3.526  0.005
…
Analysis of Variance for y
Source          DF   Seq SS   Adj SS   Adj MS      F      P
Regression       9  2499.29  2499.29  277.699  20.17  0.000
  Linear         3   989.17   989.17  329.723  23.95  0.000
  Square         3  1217.74  1217.74  405.914  29.49  0.000
  Interaction    3   292.38   292.38   97.458   7.08  0.008
Residual Error  10   137.66   137.66   13.766
  Lack-of-Fit    5    92.33    92.33   18.466   2.04  0.227
  Pure Error     5    45.33    45.33    9.067
Total           19  2636.95
…
Estimated Regression Coefficients for y using data in uncoded units
Term         Coef
Constant  87.3589
x1         5.8279
x2         1.3613
x3        -6.0509
x1*x1     -5.0578
x2*x2     -7.8862
x3*x3      0.7759
x1*x2      2.8750
x1*x3     -2.6250
x2*x3     -4.6250
```

13-10 continued

Reduced model:

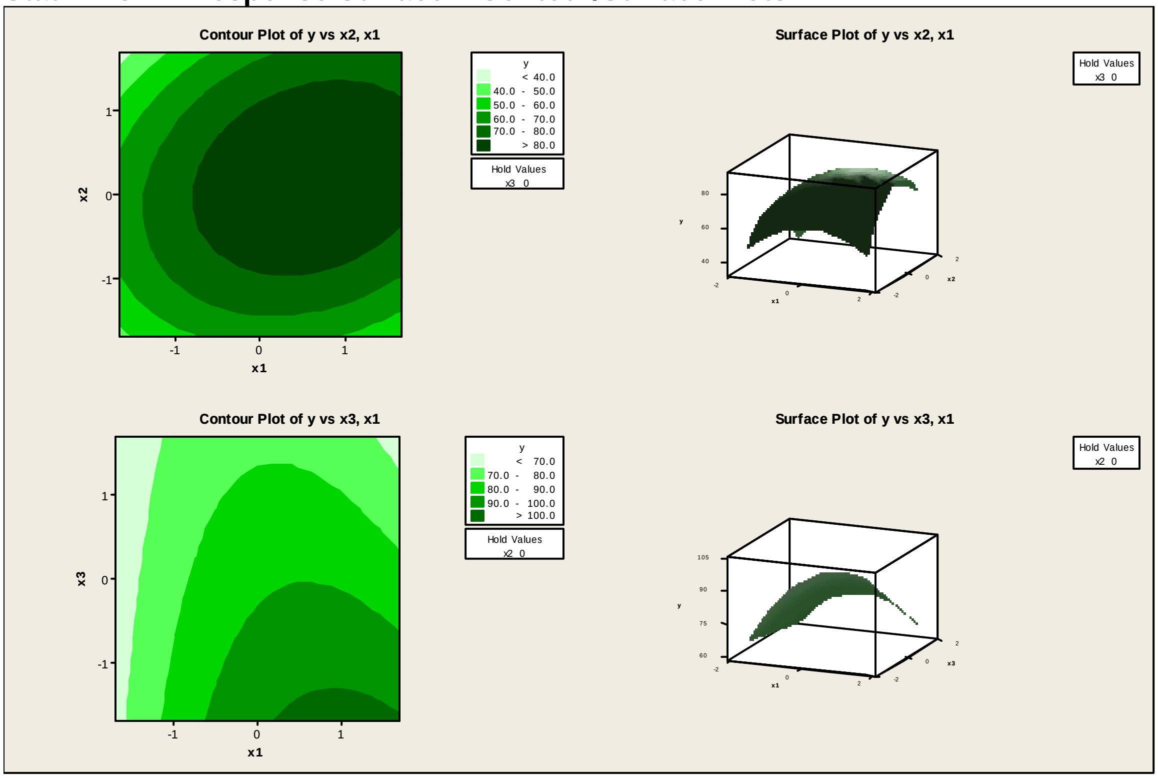
Response Surface Regression: y versus x1, x2, x3

```
The analysis was done using coded units.
Estimated Regression Coefficients for y
Term          Coef  SE Coef       T      P
Constant    87.994    1.263  69.685  0.000
x1           9.801    1.660   5.905  0.000
x2           2.289    1.660   1.379  0.195
x3         -10.176    1.660  -6.131  0.000
x1*x1      -14.523    2.704  -5.371  0.000
x2*x2      -22.523    2.704  -8.329  0.000
x1*x2        8.132    3.647   2.229  0.048
x1*x3       -7.425    3.647  -2.036  0.067
x2*x3      -13.081    3.647  -3.587  0.004
…
Analysis of Variance for y
Source          DF   Seq SS   Adj SS   Adj MS      F      P
Regression       8  2490.61  2490.61  311.327  23.40  0.000
  Linear         3   989.17   989.17  329.723  24.78  0.000
  Square         2  1209.07  1209.07  604.534  45.44  0.000
  Interaction    3   292.38   292.38   97.458   7.33  0.006
Residual Error  11   146.34   146.34   13.303
  Lack-of-Fit    6   101.00   101.00   16.834   1.86  0.257
  Pure Error     5    45.33    45.33    9.067
Total           19  2636.95
…
```



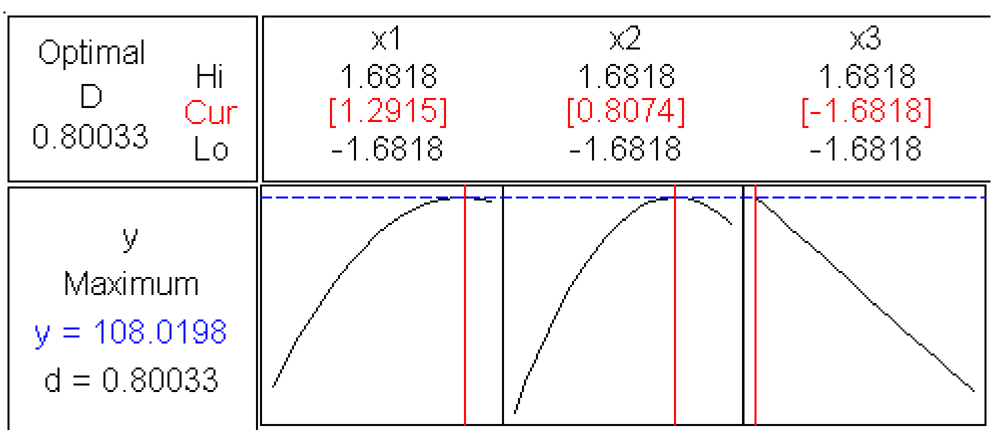
13-10 continued

Stat > DOE > Response Surface > Contour/Surface Plots



Stat > DOE > Response Surface > Response Optimizer

Goal = Maximize, Lower = 60, Upper = 120, Weight = 1, Importance = 1



One solution maximizing growth is $x_1 = 1.292$, $x_2 = 0.807$, and $x3 = -1.682$. Predicted yield is approximately 108 grams.

13-11.

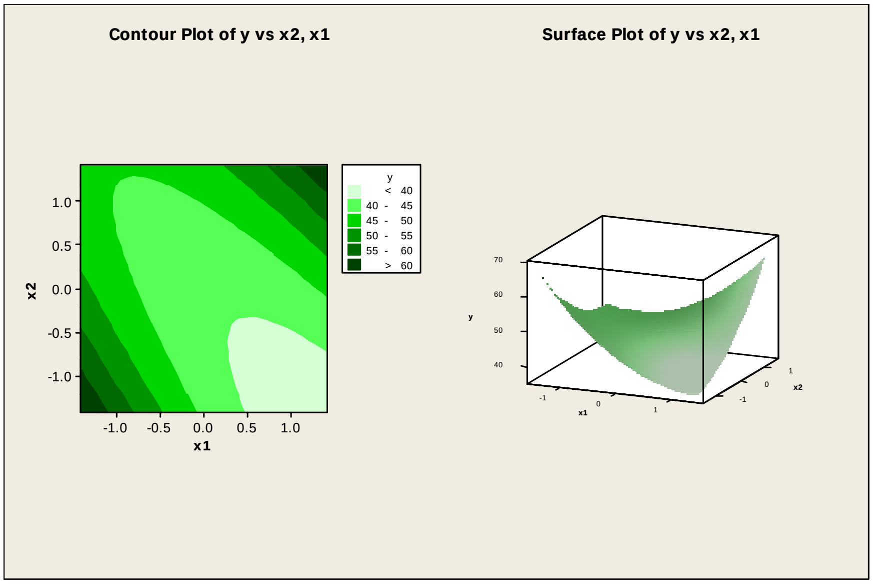
Since the runs are listed in a patterned (but not standard) order, one approach to solving this exercise is to create a general full factorial design in MINITAB, and then enter the data. The design and data are in the MINITAB worksheet **Ex13-11.MTW**.

Stat > DOE > Response Surface > Analyze Response Surface Design

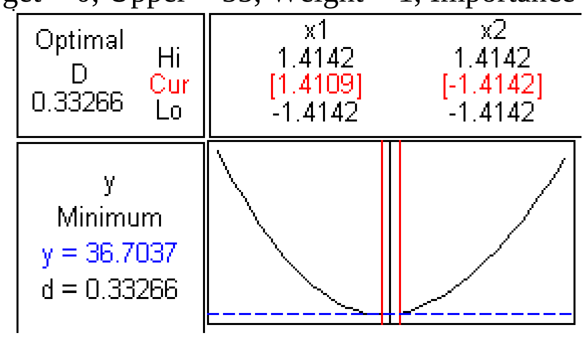
Response Surface Regression: y versus x1, x2

```
The analysis was done using coded units.
Estimated Regression Coefficients for y
Term         Coef  SE Coef       T      P
Constant  41.200    2.100  19.616  0.000
x1        -1.970    1.660  -1.186  0.274
x2         1.457    1.660   0.878  0.409
x1*x1      3.712    1.781   2.085  0.076
x2*x2      2.463    1.781   1.383  0.209
x1*x2      6.000    2.348   2.555  0.038
…
Analysis of Variance for y
Source          DF  Seq SS  Adj SS   Adj MS      F      P
Regression       5  315.60  315.60   63.119   2.86  0.102
  Linear         2   48.02   48.02   24.011   1.09  0.388
  Square         2  123.58  123.58   61.788   2.80  0.128
  Interaction    1  144.00  144.00  144.000   6.53  0.038
Residual Error   7  154.40  154.40   22.058
  Lack-of-Fit    3  139.60  139.60   46.534  12.58  0.017
  Pure Error     4   14.80   14.80    3.700
Total           12  470.00
```

13-11 continued

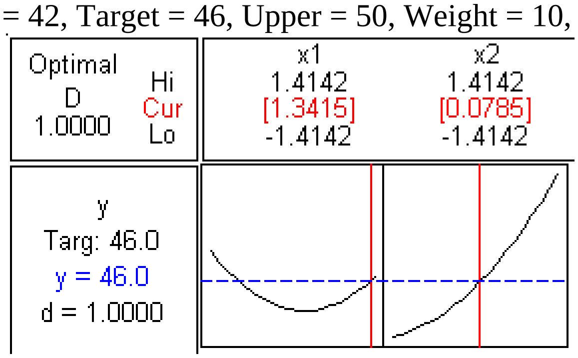


(a)
Goal = Minimize, Target = 0, Upper = 55, Weight = 1, Importance = 1



Recommended operating conditions are temperature = +1.4109 and pressure = -1.4142, to achieve predicted filtration time of 36.7.

(b)
Goal = Target, Lower = 42, Target = 46, Upper = 50, Weight = 10, Importance = 1



Recommended operating conditions are temperature = +1.3415 and pressure = -0.0785, to achieve predicted filtration time of 46.0.

13-12.

The design and data are in the MINITAB worksheet **Ex13-12.MTW**

Stat > DOE > Response Surface > Analyze Response Surface Design

Response Surface Regression: y versus x1, x2, z

```
The analysis was done using coded units.
Estimated Regression Coefficients for y
Term           Coef  SE Coef       T      P
Constant   87.3333    1.681  51.968  0.000
x1          9.8013    1.873   5.232  0.001
x2          2.2894    1.873   1.222  0.256
z          -6.1250    1.455  -4.209  0.003
x1*x1     -13.8333    3.361  -4.116  0.003
x2*x2     -21.8333    3.361  -6.496  0.000
z*z         0.1517    2.116   0.072  0.945
x1*x2       8.1317    4.116   1.975  0.084
x1*z       -4.4147    2.448  -1.804  0.109
x2*z       -7.7783    2.448  -3.178  0.013
…
Analysis of Variance for y
Source          DF   Seq SS   Adj SS   Adj MS      F      P
Regression       9  2034.94  2034.94  226.105  13.34  0.001
  Linear         3   789.28   789.28  263.092  15.53  0.001
  Square         3   953.29   953.29  317.764  18.75  0.001
  Interaction    3   292.38   292.38   97.458   5.75  0.021
Residual Error   8   135.56   135.56   16.945
  Lack-of-Fit    3    90.22    90.22   30.074   3.32  0.115
  Pure Error     5    45.33    45.33    9.067
Total           17  2170.50
…
Estimated Regression Coefficients for y using data in uncoded units
Term         Coef
Constant  87.3333
x1         5.8279
x2         1.3613
z         -6.1250
x1*x1     -4.8908
x2*x2     -7.7192
z*z        0.1517
x1*x2      2.8750
x1*z      -2.6250
x2*z      -4.6250
```

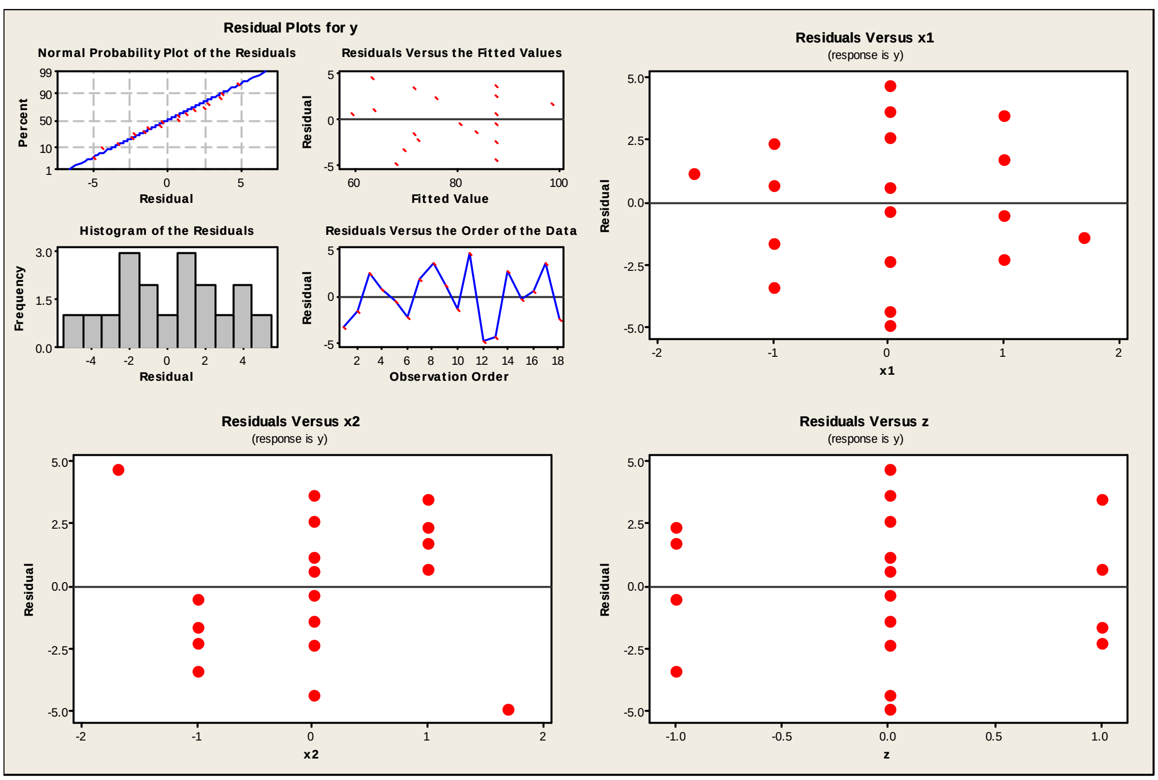
The coefficients for x_1z and x_2z (the two interactions involving the noise variable) are significant (P-values ≤ 0.10), so there is a robust design problem.

13-12 continued

Reduced model:

Response Surface Regression: y versus x1, x2, z

```
The analysis was done using coded units.
Estimated Regression Coefficients for y
Term          Coef  SE Coef       T      P
Constant    87.361    1.541  56.675  0.000
x1           9.801    1.767   5.548  0.000
x2           2.289    1.767   1.296  0.227
z           -6.125    1.373  -4.462  0.002
x1*x1      -13.760    3.019  -4.558  0.001
x2*x2      -21.760    3.019  -7.208  0.000
x1*x2        8.132    3.882   2.095  0.066
x1*z        -4.415    2.308  -1.912  0.088
x2*z        -7.778    2.308  -3.370  0.008
…
Analysis of Variance for y
Source          DF   Seq SS   Adj SS   Adj MS      F      P
Regression       8  2034.86  2034.86  254.357  16.88  0.000
  Linear         3   789.28   789.28  263.092  17.46  0.000
  Square         2   953.20   953.20  476.602  31.62  0.000
  Interaction    3   292.38   292.38   97.458   6.47  0.013
Residual Error   9   135.64   135.64   15.072
  Lack-of-Fit    4    90.31    90.31   22.578   2.49  0.172
  Pure Error     5    45.33    45.33    9.067
Total           17  2170.50
…
```



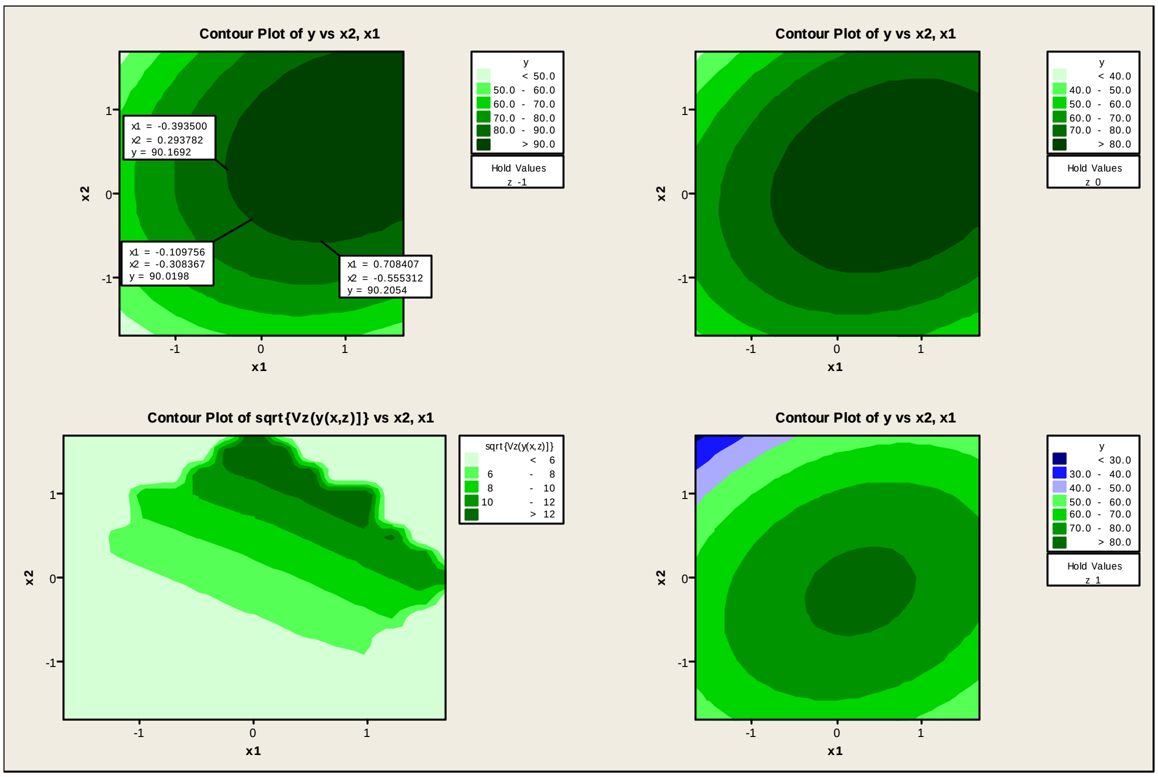
13-12 continued

$y_{\text{Pred}} = 87.36 + 5.83x_1 + 1.36x_2 - 4.86x_1^2 - 7.69x_2^2 + (-6.13 - 2.63x_1 - 4.63x_2)z$

For the mean yield model, set $z = 0$:
Mean Yield $= 87.36 + 5.83x_1 + 1.36x_2 - 4.86x_1^2 - 7.69x_2^2$

For the variance model, assume $\sigma_z^2 = 1$:
Variance of Yield $= \sigma_z^2\,(-6.13 - 2.63x_1 - 4.63x_2)^2 + \hat{\sigma}^2$
$= (-6.13 - 2.63x_1 - 4.63x_2)^2 + 15.072$

This equation can be added to the worksheet and used in a contour plot with x_1 and x_2. (Refer to MINITAB worksheet **Ex13-12.MTW**.)



Examination of contour plots for Free Height show that heights greater than 90 are achieved with $z = -1$. Comparison with the contour plot for variability shows that growth greater than 90 with minimum variability is achieved at approximately $x_1 = -0.11$ and $x_2 = -0.31$ (mean yield of about 90 with a standard deviation between 6 and 8). There are other combinations that would work.

13-13.

If $h(\mathbf{x},\mathbf{z}) = \sum_{i=1}^{r}\gamma_i z_i + \sum_{i=1}^{k}\sum_{j=1}^{r}\delta_{ij}x_i z_j$, then $\dfrac{\partial h(\mathbf{x},\mathbf{z})}{\partial z_i} = \gamma_i + \sum_{u=1}^{k}\delta_{ui}x_u$, and

$$V\left[y(\mathbf{x},\mathbf{z})\right] = \sigma_z^2\sum_{i=1}^{r}\left(\gamma_i + \sum_{u=1}^{k}\delta_{ui}x_u\right)^2 + \sigma^2$$

If $h(\mathbf{x},\mathbf{z}) = \sum_{i=1}^{r}\gamma_i z_i + \sum_{i=1}^{k}\sum_{j=1}^{r}\delta_{ij}x_i z_j + \sum\sum_{i<j=2}^{r}\lambda_{ij}z_i z_j$,

then $\sum_{i=1}^{r}\dfrac{\partial h(\mathbf{x},\mathbf{z})}{\partial z_i} = \sum_{i=1}^{r}\gamma_i + \sum_{i=1}^{r}\sum_{u=1}^{k}\delta_{ui}x_u + \sum\sum_{i<j=2}^{r}\lambda_{ij}(z_i + z_j)$, and

$$V\left[y(\mathbf{x},\mathbf{z})\right] = V\sum_{i=1}^{r}\left[\gamma_i + \sum_{u=1}^{k}\delta_{ui}x_u + \sum_{j>i}^{r}\lambda_{ij}(z_i + z_j)\right]z_i + \sigma^2$$

There will be additional terms in the variance expression arising from the third term inside the square brackets.

13-14.

If $h(\mathbf{x},\mathbf{z}) = \sum_{i=1}^{r}\gamma_i z_i + \sum_{i=1}^{k}\sum_{j=1}^{r}\delta_{ij}x_i z_j + \sum\sum_{i<j=2}^{r}\lambda_{ij}z_i z_j + \sum_{i=1}^{r}\theta_i z_i^2$, then

$$\sum_{i=1}^{r}\frac{\partial h(\mathbf{x},\mathbf{z})}{\partial z_i} = \sum_{i=1}^{r}\gamma_i + \sum_{i=1}^{r}\sum_{u=1}^{k}\delta_{ui}x_u + \sum\sum_{i<j=2}^{r}\lambda_{ij}(z_i + z_j) + 2\sum_{i=1}^{r}\theta_i z_i$$, and

$$V\left[y(\mathbf{x},\mathbf{z})\right] = V\sum_{i=1}^{r}\left[\gamma_i + \sum_{u=1}^{k}\delta_{ui}x_u + \sum_{j>i}^{r}\lambda_{ij}(z_i + z_j) + 2\theta_i z_i^2\right]z_i + \sigma^2$$

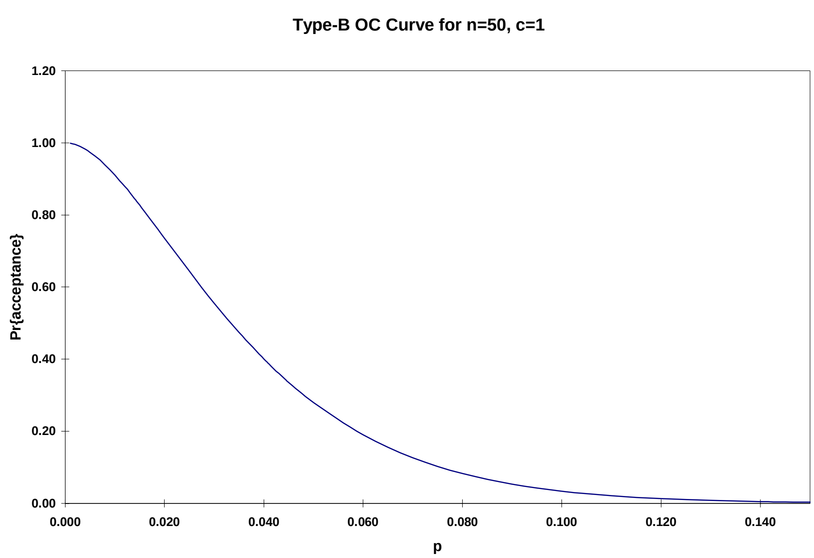
There will be additional terms in the variance expression arising from the last two terms inside the square brackets.

Chapter 14 Exercise Solutions

Note: Many of the exercises in this chapter are easily solved with spreadsheet application software. The BINOMDIST, HYPGEOMDIST, and graphing functions in Microsoft® Excel were used for these solutions. Solutions are in the Excel workbook **Chap14.xls**.

14-1.

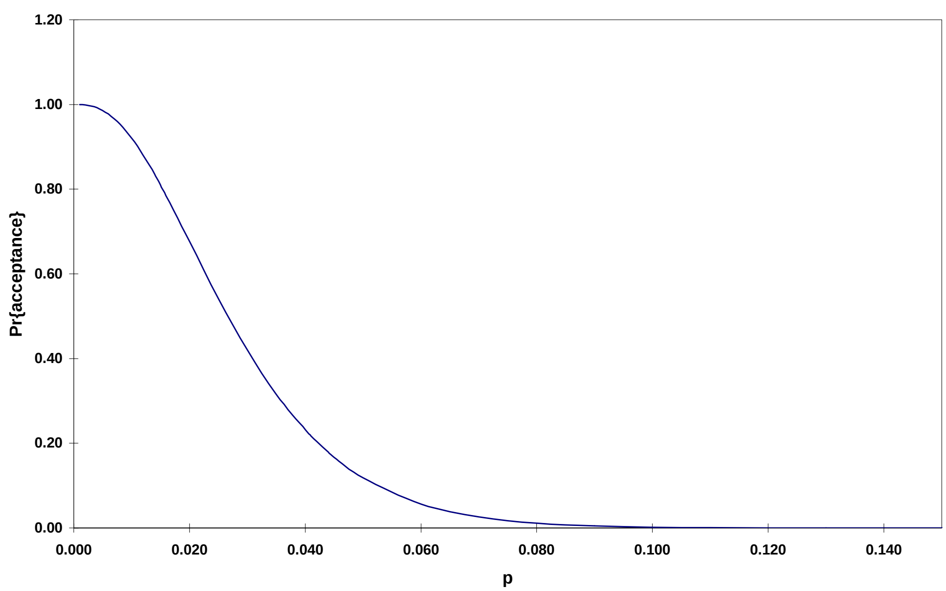
p	f(d=0)	f(d=1)	Pr{d<=c}
0.001	0.95121	0.04761	0.99881
0.002	0.90475	0.09066	0.99540
0.003	0.86051	0.12947	0.98998
0.004	0.81840	0.16434	0.98274
0.005	0.77831	0.19556	0.97387
0.006	0.74015	0.22339	0.96353
0.007	0.70382	0.24807	0.95190
0.008	0.66924	0.26986	0.93910
0.009	0.63633	0.28895	0.92528
0.010	0.60501	0.30556	0.91056
0.020	0.36417	0.37160	0.73577
0.030	0.21807	0.33721	0.55528
0.040	0.12989	0.27060	0.40048
0.050	0.07694	0.20249	0.27943
0.060	0.04533	0.14467	0.19000
0.070	0.02656	0.09994	0.12649
0.080	0.01547	0.06725	0.08271
0.090	0.00896	0.04428	0.05324
0.100	0.00515	0.02863	0.03379



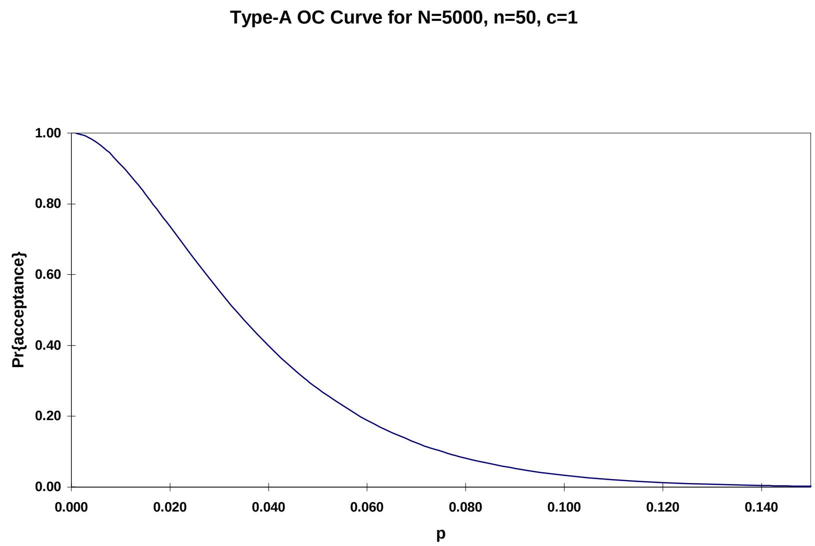
14-2.

p	f(d=0)	f(d=1)	f(d=2)	Pr{d<=c}
0.001	0.90479	0.09057	0.00449	0.99985
0.002	0.81857	0.16404	0.01627	0.99888
0.003	0.74048	0.22281	0.03319	0.99649
0.004	0.66978	0.26899	0.05347	0.99225
0.005	0.60577	0.30441	0.07572	0.98590
0.006	0.54782	0.33068	0.09880	0.97730
0.007	0.49536	0.34920	0.12185	0.96641
0.008	0.44789	0.36120	0.14419	0.95327
0.009	0.40492	0.36773	0.16531	0.93796
0.010	0.36603	0.36973	0.18486	0.92063
0.020	0.13262	0.27065	0.27341	0.67669
0.030	0.04755	0.14707	0.22515	0.41978
0.040	0.01687	0.07029	0.14498	0.23214
0.050	0.00592	0.03116	0.08118	0.11826
0.060	0.00205	0.01312	0.04144	0.05661
0.070	0.00071	0.00531	0.01978	0.02579
0.080	0.00024	0.00208	0.00895	0.01127
0.090	0.00008	0.00079	0.00388	0.00476
0.100	0.00003	0.00030	0.00162	0.00194
0.200	0.00000	0.00000	0.00000	0.00000

Type-B OC Curve for n=100, c=2

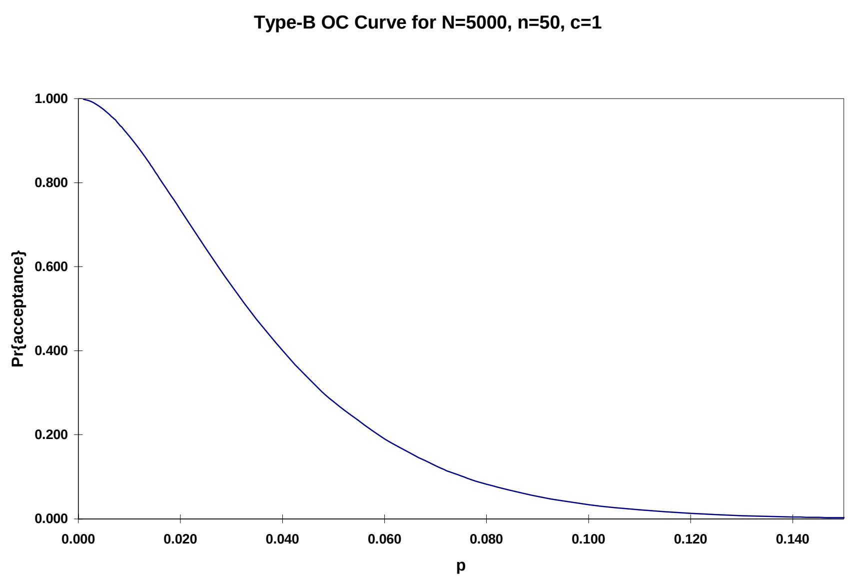


14-3.
(a)



P_a (d = 35) = 0.9521, or $\alpha \cong 0.05$
P_a (d = 375) = 0.10133, or $\beta \cong 0.10$

(b)



P_a (p = 0.007) = 0.9521, or $\alpha \cong 0.05$
P_a (p = 0.075) = 0.10133, or $\beta \cong 0.10$

(c)
Based on values for α and β, the difference between the two curves is small; either is appropriate.

Chapter 14 Exercise Solutions

14-4.

$p_1 = 0.01; 1-\alpha = 1-0.05 = 0.95; \; p_2 = 0.10; \; \beta = 0.10$

From the binomial nomograph, select n = 35 and c = 1, resulting in actual α = 0.04786 and β = 0.12238.

14-5.

$p_1 = 0.05; 1-\alpha = 1-0.05 = 0.95; \; p_2 = 0.15; \; \beta = 0.10$

From the binomial nomograph, the sampling plan is n = 80 and c = 7.

14-6.

$p_1 = 0.02; 1-\alpha = 1-0.01 = 0.99; \; p_2 = 0.06; \; \beta = 0.10$

From the binomial nomograph, select a sampling plan of n = 300 and c = 12.

14-7.

LTPD = 0.05

	N1 =	5000	N2 =	10000	
	n1 =	500	n1 =	1000	
	pmax =	0.0200	pmax =	0.0200	
	cmax =	10	cmax =	20	
	binomial		binomial		
p	Pr{d<=10}	Pr{reject}	Pr{d<=20}	Pr{reject}	difference
0.0010	1.00000	0.0000	1.00000	0.0000	0.00000
0.0020	1.00000	0.0000	1.00000	0.0000	0.00000
0.0030	1.00000	0.0000	1.00000	0.0000	0.00000
0.0040	0.99999	0.0000	1.00000	0.0000	-0.00001
0.0050	0.99994	0.0001	1.00000	0.0000	-0.00006
0.0060	0.99972	0.0003	1.00000	0.0000	-0.00027
0.0070	0.99903	0.0010	0.99999	0.0000	-0.00095
0.0080	0.99729	0.0027	0.99991	0.0001	-0.00263
0.0090	0.99359	0.0064	0.99959	0.0004	-0.00600
0.0100	0.98676	0.0132	0.99850	0.0015	-0.01175
0.0200	0.58304	0.4170	0.55910	0.4409	0.02395
0.0250	0.29404	0.7060	0.18221	0.8178	0.11183
0.0300	0.11479	0.8852	0.03328	0.9667	0.08151
0.0400	0.00967	0.9903	0.00030	0.9997	0.00938
0.0500	0.00046	0.9995	0.00000	1.0000	0.00046
0.0600	0.00001	1.0000	0.00000	1.0000	0.00001
0.0700	0.00000	1.0000	0.00000	1.0000	0.00000

Different sample sizes offer different levels of protection. For $N = 5{,}000$, $P_a(p = 0.025) = 0.294$; while for $N = 10{,}000$, $P_a(p = 0.025) = 0.182$. Also, the consumer is protected from a LTPD = 0.05 by $P_a(N = 5{,}000) = 0.00046$ and $P_a(N = 10{,}000) = 0.00000$, but pays for the high probability of rejecting acceptable lots like those with $p = 0.025$.

14-8.

	N1 =	1000	N2 =	5000
	n1 =	32	n1 =	71
	pmax =	0.01	pmax =	0.01
	cmax =	0	cmax =	1
	binomial		binomial	
p	Pr{d<=0}	Pr{reject}	Pr{d<=1}	Pr{reject}
0.0002	0.99382	0.0062	0.98610	0.0139
0.0004	0.98767	0.0123	0.97238	0.0276
0.0006	0.98157	0.0184	0.95886	0.0411
0.0008	0.97550	0.0245	0.94552	0.0545
0.0010	0.96946	0.0305	0.93236	0.0676
0.0020	0.93982	0.0602	0.86924	0.1308
0.0030	0.91107	0.0889	0.81033	0.1897
0.0040	0.88316	0.1168	0.75536	0.2446
0.0050	0.85608	0.1439	0.70407	0.2959
0.0060	0.82981	0.1702	0.65622	0.3438
0.0070	0.80432	0.1957	0.61157	0.3884
0.0080	0.77958	0.2204	0.56992	0.4301
0.0090	0.75558	0.2444	0.53107	0.4689
0.0100	**0.73230**	**0.2677**	**0.49484**	**0.5052**
0.0200	0.53457	0.4654	0.24312	0.7569
0.0300	0.38898	0.6110	0.11858	0.8814
0.0400	0.28210	0.7179	0.05741	0.9426
0.0500	0.20391	0.7961	0.02758	0.9724
0.0600	0.14688	0.8531	0.01315	0.9868
0.0700	0.10543	0.8946	0.00622	0.9938
0.0800	0.07541	0.9246	0.00292	0.9971
0.0900	0.05374	0.9463	0.00136	0.9986
0.1000	0.03815	0.9618	0.00063	0.9994
0.2000	0.00099	0.9990	0.00000	1.0000
0.3000	0.00002	1.0000	0.00000	1.0000
0.3500	0.00000	1.0000	0.00000	1.0000

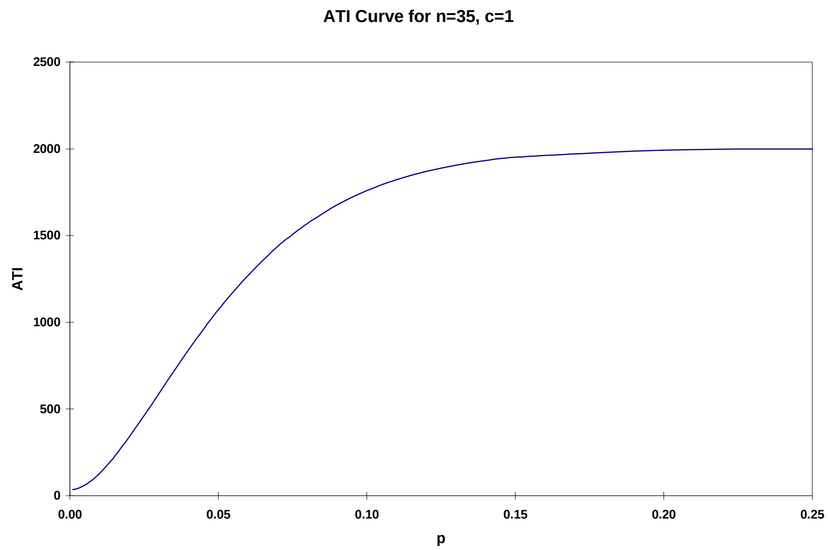
This plan offers vastly different protections at various levels of defectives, depending on the lot size. For example, at $p = 0.01$, $P_a(p = 0.01) = 0.7323$ for $N = 1000$, and $P_a(p = 0.01) = 0.4949$ for $N = 5000$.

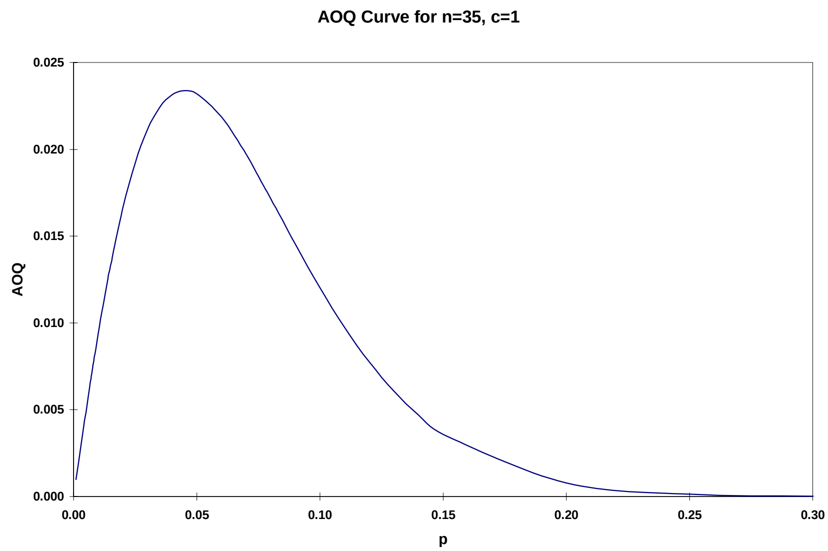
14-9.
$n = 35$; $c = 1$; $N = 2{,}000$

$$\begin{aligned}\text{ATI} &= n + (1 - P_a)(N - n)\\ &= 35 + (1 - P_a)(2000 - 35)\\ &= 2000 - 1965 P_a\end{aligned}$$

$$\begin{aligned}\text{AOQ} &= \frac{P_a p (N - n)}{N}\\ &= (1965/2000) P_a p\end{aligned}$$

$$\text{AOQL} = 0.0234$$

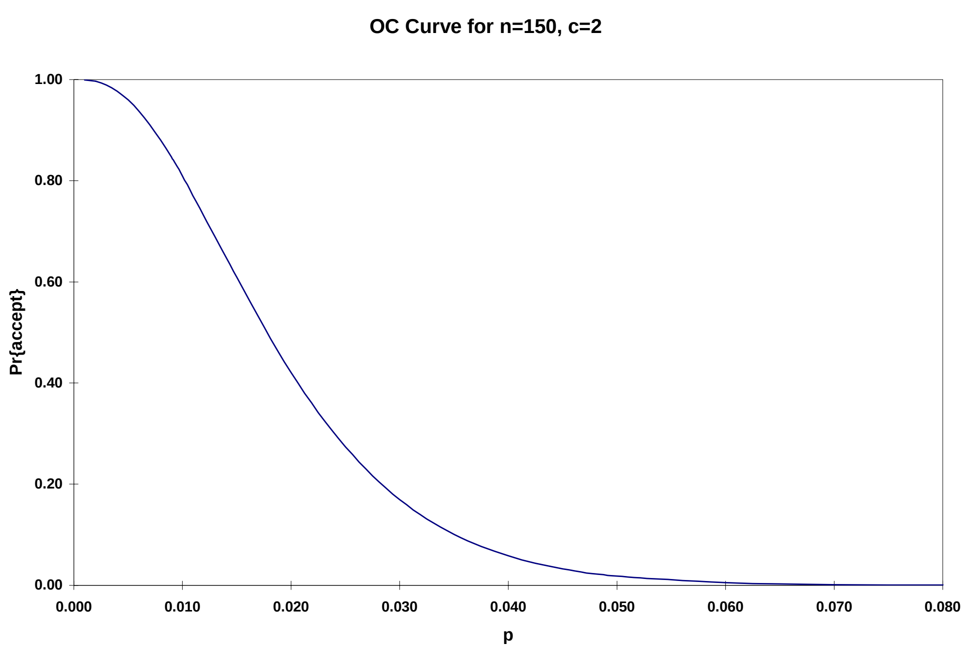




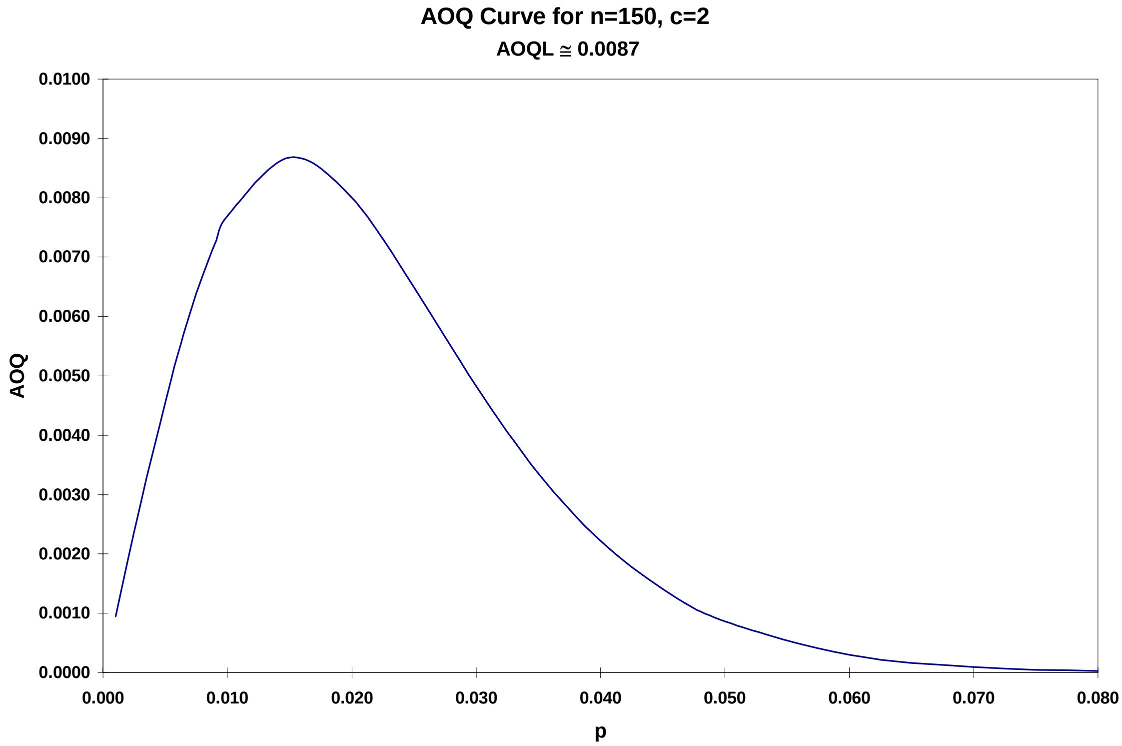
14-10.
$N = 3000$, $n = 150$, $c = 2$

p	Pa=Pr{d<=2}	AOQ		ATI
0.001	0.99951	0.0009		151
0.002	0.99646	0.0019		160
0.003	0.98927	0.0028		181
0.004	0.97716	0.0037		215
0.005	**0.95991**	0.0046		264
0.006	0.93769	0.0053		328
0.007	0.91092	0.0061		404
0.008	0.88019	0.0067		491
0.009	0.84615	0.0072		588
0.010	0.80948	0.0077		693
0.015	0.60884	**0.0087**	**AOQL**	1265
0.020	0.42093	0.0080		1800
0.025	0.27341	0.0065		2221
0.030	0.16932	0.0048		2517
0.035	**0.10098**	0.0034		2712
0.040	0.05840	0.0022		2834
0.045	0.03292	0.0014		2906
0.050	0.01815	0.0009		2948
0.060	0.00523	0.0003		2985
0.070	0.00142	0.0001		2996
0.080	0.00036	0.0000		2999
0.090	0.00009	0.0000		3000
0.100	0.00002	0.0000		3000

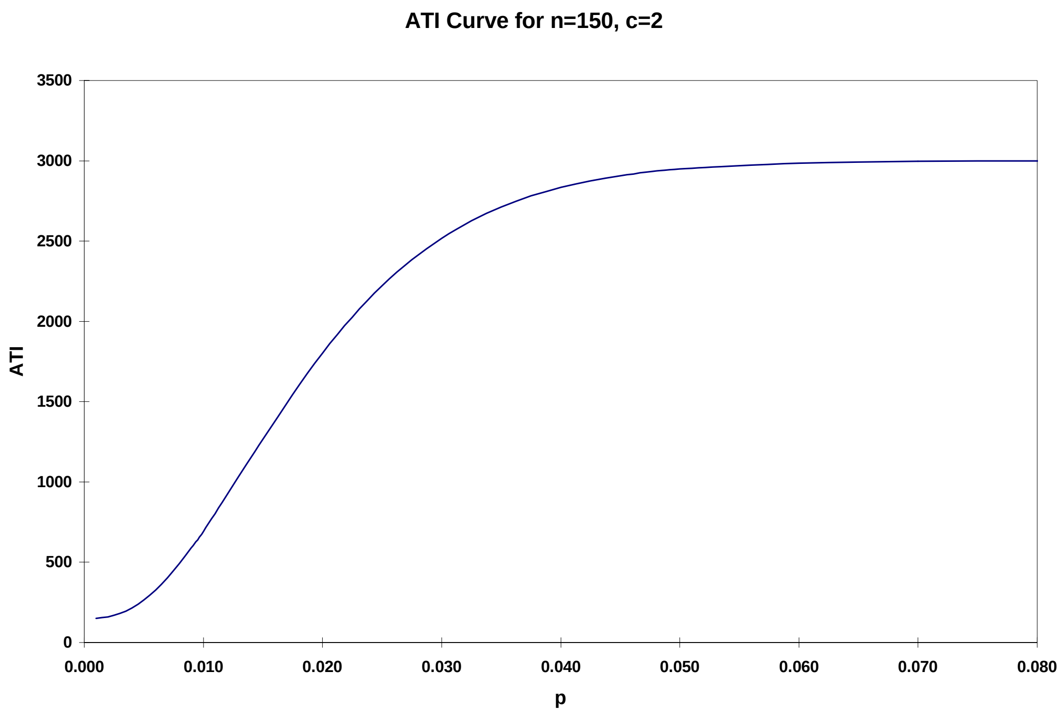
(a)



14-10 continued
(b)

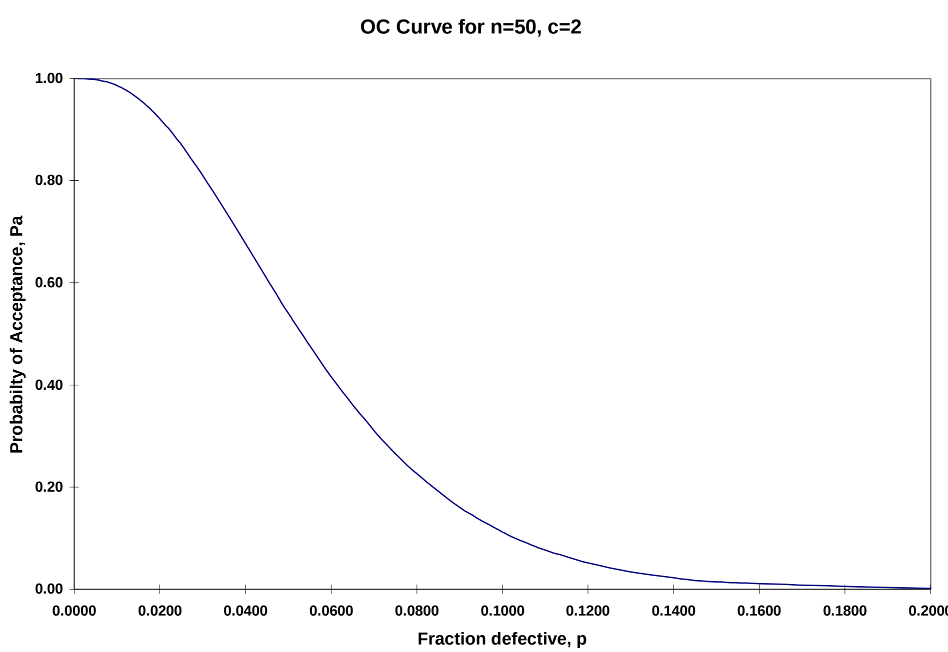


(c)



14-11.
(a)
$N = 5000$, $n = 50$, $c = 2$

p	Pa=Pr{d<=1}	Pr{reject}
0.0010	0.99998	0.00002
0.0020	0.99985	0.00015
0.0030	0.99952	0.00048
0.0040	0.99891	0.00109
0.0050	0.99794	0.00206
0.0060	0.99657	0.00343
0.0070	0.99474	0.00526
0.0080	0.99242	0.00758
0.0090	0.98957	0.01043
0.0100	0.98618	0.01382
0.0200	0.92157	0.07843
0.0300	0.81080	0.18920
0.0400	0.67671	0.32329
0.0500	0.54053	0.45947
0.0600	0.41625	0.58375
0.0700	0.31079	0.68921
0.0800	0.22597	0.77403
0.0900	0.16054	0.83946
0.1000	0.11173	0.88827
0.1010	0.10764	0.89236
0.1020	0.10368	0.89632
0.1030	0.09985	0.90015
0.1040	0.09614	0.90386
0.1050	0.09255	0.90745
0.2000	0.00129	0.99871
0.3000	0.00000	1.00000



14-11 continued

(b)

$p = 0.1030$ will be rejected about 90% of the time.

(c)

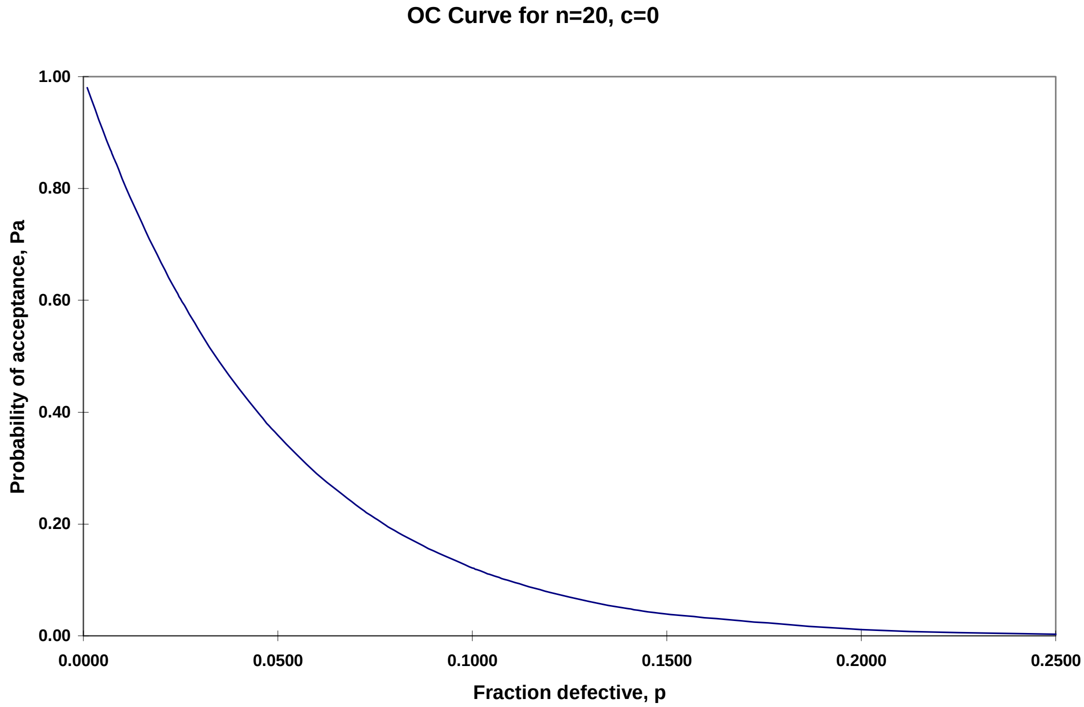
A zero-defects sampling plan, with acceptance number $c = 0$, will be extremely hard on the vendor because the P_a is low even if the lot fraction defective is low. Generally, quality improvement begins with the manufacturing process control, not the sampling plan.

(d)

From the nomograph, select $n = 20$, yielding $P_a = 1 - 0.11372 = 0.88638 \approx 0.90$. The OC curve for this zero-defects plan is much steeper.

p	Pa=Pr{d<=0}	Pr{reject}
0.0010	0.98019	0.01981
0.0020	0.96075	0.03925
0.0030	0.94168	0.05832
0.0040	0.92297	0.07703
0.0050	0.90461	0.09539
0.0060	0.88660	0.11340
0.0070	0.86893	0.13107
0.0080	0.85160	0.14840
0.0090	0.83459	0.16541
0.0100	0.81791	0.18209
0.0200	0.66761	0.33239
0.0300	0.54379	0.45621
0.0400	0.44200	0.55800
0.0500	0.35849	0.64151
0.0600	0.29011	0.70989
0.0700	0.23424	0.76576
0.0800	0.18869	0.81131
0.0900	0.15164	0.84836
0.1000	0.12158	0.87842
0.2000	0.01153	0.98847
0.3000	0.00080	0.99920
0.4000	0.00004	0.99996
0.5000	0.00000	1.00000

14-11 (d) continued



(e)

$\Pr\{\text{reject} \mid p = 0.005, c = 0\} = 0.09539$

$\Pr\{\text{reject} \mid p = 0.005, c = 2\} = 0.00206$

$\text{ATI}_{c=0} = n + (1 - P_a)(N - n) = 20 + (0.09539)(5000 - 20) = 495$

$\text{ATI}_{c=2} = 50 + (0.00206)(5000 - 50) = 60$

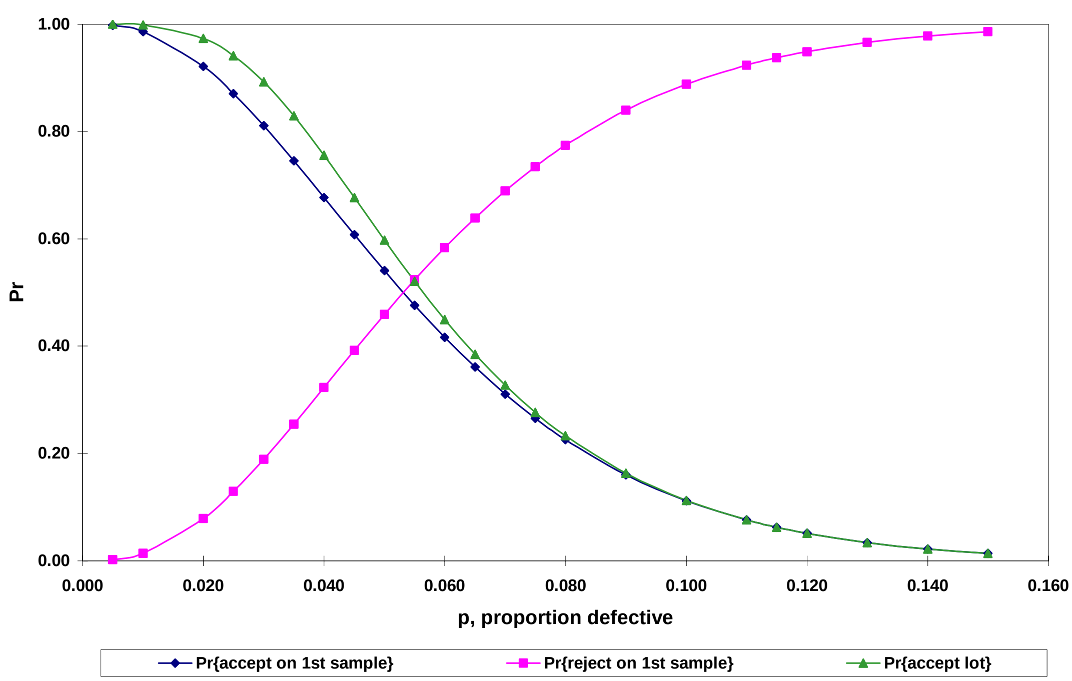
The c = 2 plan is preferred because the c = 0 plan will reject good lots 10% of the time.

14-12.

$n_1 = 50$, $c_1 = 2$, $n_2 = 100$, $c_2 = 6$

P	d1 = Pal	PrI	3 Pr{d1=3,d2<=3}	4 Pr{d1=4,d3<=2}	5 Pr{d1=5,d2<=1}	6 Pr{d1=6,d2=0}	Pall	Pa
0.005	0.9979	0.0021	0.0019	0.0001	0.0000	0.0000	0.0019	0.9999
0.010	0.9862	0.0138	0.0120	0.0013	0.0001	0.0000	0.0120	0.9982
0.020	0.9216	0.0784	0.0521	0.0098	0.0011	0.0001	0.0522	0.9737
0.025	0.8706	0.1294	0.0707	0.0152	0.0019	0.0001	0.0708	0.9414
0.030	0.8108	0.1892	0.0818	0.0193	0.0025	0.0001	0.0820	0.8928
0.035	0.7452	0.2548	0.0842	0.0212	0.0029	0.0002	0.0844	0.8296
0.040	0.6767	0.3233	0.0791	0.0209	0.0030	0.0002	0.0793	0.7560
0.045	0.6078	0.3922	0.0690	0.0190	0.0028	0.0002	0.0692	0.6770
0.050	**0.5405**	**0.4595**	**0.0567**	**0.0161**	**0.0024**	**0.0002**	**0.0568**	**0.5974**
0.055	0.4763	0.5237	0.0442	0.0129	0.0020	0.0001	0.0444	0.5207
0.060	0.4162	0.5838	0.0330	0.0098	0.0015	0.0001	0.0331	0.4494
0.065	0.3610	0.6390	0.0238	0.0072	0.0011	0.0001	0.0238	0.3848
0.070	0.3108	0.6892	0.0165	0.0051	0.0008	0.0001	0.0166	0.3274
0.075	0.2658	0.7342	0.0111	0.0035	0.0006	0.0000	0.0112	0.2770
0.080	0.2260	0.7740	0.0073	0.0023	0.0004	0.0000	0.0073	0.2333
0.090	0.1605	0.8395	0.0029	0.0009	0.0002	0.0000	0.0029	0.1635
0.100	0.1117	0.8883	0.0011	0.0004	0.0001	0.0000	0.0011	0.1128
0.110	0.0763	0.9237	0.0004	0.0001	0.0000	0.0000	0.0004	0.0767
0.115	0.0627	0.9373	0.0002	0.0001	0.0000	0.0000	0.0002	0.0629
0.120	0.0513	0.9487	0.0001	0.0000	0.0000	0.0000	0.0001	0.0514
0.130	0.0339	0.9661	0.0000	0.0000	0.0000	0.0000	0.0000	0.0339
0.140	0.0221	0.9779	0.0000	0.0000	0.0000	0.0000	0.0000	0.0221
0.150	0.0142	0.9858	0.0000	0.0000	0.0000	0.0000	0.0000	0.0142

Primary and Supplementary OC Curves for n1=50, c1=2, n2=100, c2=6



14-13.

(a)

$p_1 = 0.01; 1-\alpha = 1-0.05 = 0.95;\ p_2 = 0.10;\ \beta = 0.10$

$k = 1.0414;\ h_1 = 0.9389;\ h_2 = 1.2054;\ s = 0.0397$

$X_A = -0.9389 + 0.0397n;\ X_R = 1.2054 + 0.0397n$

n	XA	XR	Acc	Rej
1	-0.899	1.245	n/a	2
2	-0.859	1.285	n/a	2
3	-0.820	1.325	n/a	2
4	-0.780	1.364	n/a	2
5	-0.740	1.404	n/a	2
…	…	…	…	…
20	-0.144	2.000	n/a	2
21	-0.104	2.040	n/a	3
22	-0.064	2.080	n/a	3
23	-0.025	2.120	n/a	3
24	0.015	2.159	0	3
25	0.055	2.199	0	3
…	…	…	…	…
45	0.850	2.994	0	3
46	0.890	3.034	0	4
47	0.929	3.074	0	4
48	0.969	3.113	0	4
49	1.009	3.153	1	4
50	1.049	3.193	1	4

The sampling plan is n = 49; Acc = 1; Rej = 4.

(b)

Three points on the OC curve are:

$p_1 = 0.01;\ P_a = 1-\alpha = 0.95$

$$p = s = 0.0397;\ P_a = \frac{h_2}{h_1 + h_2} = \frac{1.2054}{0.9389 + 1.2054} = 0.5621$$

$p_2 = 0.10;\ P_a = \beta = 0.10$

14-14.

(a)

$p_1 = 0.02; 1-\alpha = 1-0.05 = 0.95;\ p_2 = 0.15; \beta = 0.10$

$k = 0.9369; h_1 = 1.0436; h_2 = 1.3399; s = 0.0660$

$X_A = -1.0436 + 0.0660n; X_R = 1.3399 + 0.0660n$

n	XA	XR	Acc	Rej
1	-0.978	1.406	n/a	2
2	-0.912	1.472	n/a	2
3	-0.846	1.538	n/a	2
4	-0.780	1.604	n/a	2
5	-0.714	1.670	n/a	2
…	…	…	…	…
20	0.276	2.659	n/a	2
21	0.342	2.725	n/a	3
22	0.408	2.791	n/a	3
23	0.474	2.857	n/a	3
24	0.540	2.923	0	3
25	0.606	2.989	0	3
…	…	…	…	…
45	1.925	4.309	0	3
46	1.991	4.375	0	4
47	2.057	4.441	0	4
48	2.123	4.507	0	4
49	2.189	4.572	1	4
50	2.255	4.638	1	4

The sampling plan is n = 49, Acc = 1 and Rej = 4.

(b)

$p_1 = 0.02; P_a = \alpha = 0.95$

$$p = s = 0.0660; P_a = \frac{h_2}{h_1 + h_2} = \frac{1.3399}{1.0436 + 1.3399} = 0.5622$$

$p_2 = 0.15; P_a = \beta = 0.10$

14-15.

$$\text{AOQ} = \left[P_a \times p \times (N-n)\right] \big/ \left[N - P_a \times (np) - (1-P_a) \times (Np)\right]$$

14-16.
N = 3000, AQL = 1%
General level II
Sample size code letter = K
Normal sampling plan: n = 125, Ac = 3, Re = 4
Tightened sampling plan: n = 125, Ac = 2, Re = 3
Reduced sampling plan: n = 50, Ac = 1, Re = 4

14-17.
N = 3000, AQL = 1%
General level I
Normal sampling plan: Sample size code letter = H, n = 50, Ac = 1, Re = 2
Tightened sampling plan: Sample size code letter = J, n = 80, Ac = 1, Re = 2
Reduced sampling plan: Sample size code letter = H, n = 20, Ac = 0, Re = 2

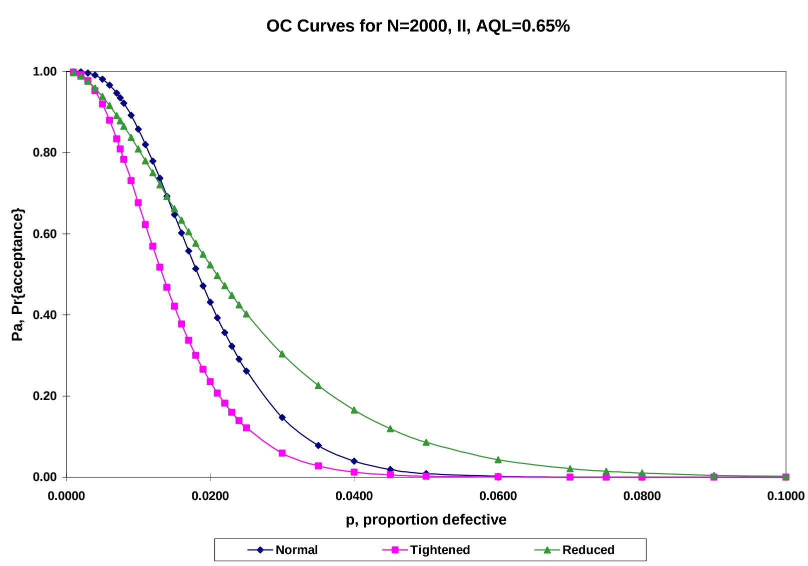
14-18.
N = 10,000; AQL = 0.10%; General inspection level II; Sample size code letter = L
Normal: up to letter K, n = 125, Ac = 0, Re = 1
Tightened: n = 200, Ac = 0, Re = 1
Reduced: up to letter K, n = 50, Ac = 0, Re = 1

14-19.

(a)
N = 5000, AQL = 0.65%; General level II; Sample size code letter = L
Normal sampling plan: n = 200, Ac = 3, Re = 4
Tightened sampling plan: n = 200, Ac = 2, Re = 3
Reduced sampling plan: n = 80, Ac = 1, Re = 4

(b)

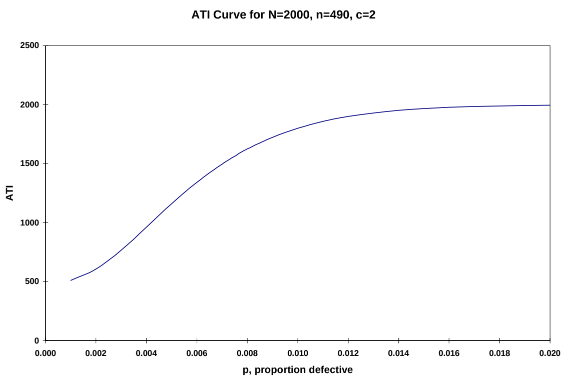
N = 5000	normal	tightened	reduced
n =	200	200	80
c =	3	2	1
p	Pa=Pr{d<=3}	Pa=Pr{d<=2}	Pa=Pr{d<=1}
0.0010	0.9999	0.9989	0.9970
0.0020	0.9992	0.9922	0.9886
0.0030	0.9967	0.9771	0.9756
0.0040	0.9911	0.9529	0.9588
0.0050	0.9813	0.9202	0.9389
0.0060	0.9667	0.8800	0.9163
0.0070	0.9469	0.8340	0.8916
0.0080	0.9220	0.7838	0.8653
0.0090	0.8922	0.7309	0.8377
0.0100	0.8580	0.6767	0.8092
0.0200	0.4315	0.2351	0.5230
0.0300	0.1472	0.0593	0.3038
0.0400	0.0395	0.0125	0.1654
0.0500	0.0090	0.0023	0.0861
0.0600	0.0018	0.0004	0.0433
0.0700	0.0003	0.0001	0.0211
0.0800	0.0001	0.0000	0.0101
0.0900	0.0000	0.0000	0.0047
0.1000	0.0000	0.0000	0.0022

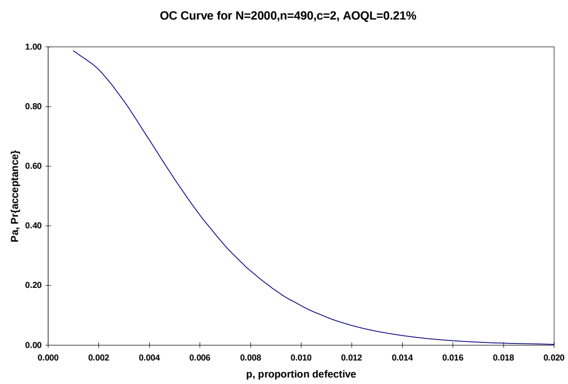


14-20.

N = 2000; LTPD = 1%; p = 0.25%
n = 490; c = 2; AOQL = 0.2%

p	D = N*p	Pa	ATI	AOQ	
0.001	2	0.9864	511	0.0007	
0.002	4	0.9235	605	0.0014	
0.003	6	0.8165	767	0.0018	
0.004	8	0.6875	962	0.0021	
0.005	10	0.5564	1160	**0.0021**	**AOQL**
0.006	12	0.4361	1341	0.0020	
0.007	14	0.3330	1497	0.0018	
0.008	15	0.2886	1564	0.0016	
0.008	16	0.2489	1624	0.0015	
0.009	18	0.1827	1724	0.0012	
0.010	20	0.1320	1801	0.0010	
0.011	22	0.0942	1858	0.0008	
0.012	24	0.0664	1900	0.0006	
0.013	26	0.0464	1930	0.0005	
0.014	28	0.0321	1952	0.0003	
0.015	30	0.0220	1967	0.0002	
0.016	32	0.0150	1977	0.0002	
0.017	34	0.0102	1985	0.0001	
0.018	36	0.0068	1990	0.0001	
0.019	38	0.0046	1993	0.0001	
0.020	40	0.0031	1995	0.0000	





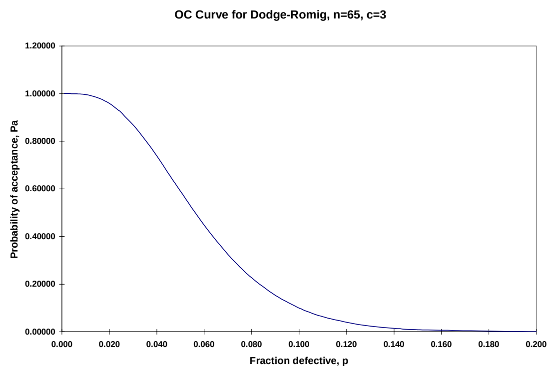
The AOQL is 0.21%.

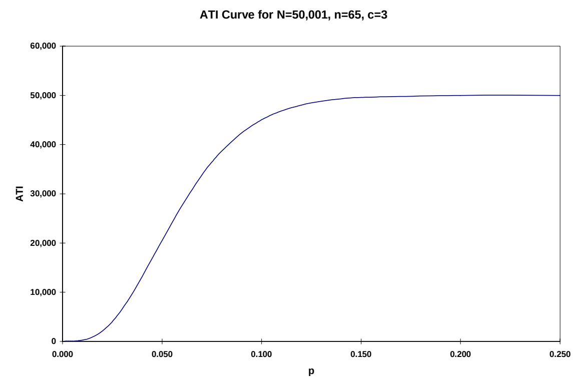
Note that this solution uses the cumulative binomial distribution in a spreadsheet formulation. A more precise solution would use the hypergeometric distribution to represent this sampling plan of n = 490 from N = 2000, without replacement.

14-21.
Dodge-Romig single sampling, AOQL = 3%, average process fallout = p = 0.50% defective

(a)
Minimum sampling plan that meets the quality requirements is 50,001 ≤ N ≤ 100,000; n = 65; c = 3.

(b)





$$\text{let } N = 50,001$$
$$P_a = Binom(3, 65, 0.005) = 0.99967$$
$$\text{ATI} = n + (1 - P_a)(N - n) = 65 + (1 - 0.99967)(50,001 - 65) = 82$$

On average, if the vendor's process operates close to process average, the average inspection required will be 82 units.

(c)
LTPD = 10.3%

14-22.

(a)
N = 8000; AOQL = 3%; $p \leq 1\%$
n = 65; c = 3; LTPD = 10.3%

(b)
$$P_a = \sum_{d=0}^{c} \text{binomial}(n, p) = \sum_{d=0}^{3} b(65, 0.01) = 0.9958$$
$$\text{ATI} = n + (1 - P_a)(N - n) = 65 + (1 - 0.9958)(8000 - 65) \approx 98$$

(c)
N = 8000; AOQL = 3%; $p \leq 0.25\%$
n = 46; c = 2; LTPD = 11.6%

$$P_a = \sum_{d=0}^{c} \text{binomial}(n, p) = \sum_{d=0}^{2} b(46, 0.0025) = 0.9998$$
$$\text{ATI} = n + (1 - P_a)(N - n) = 46 + (1 - 0.9998)(8000 - 46) \approx 48$$

Chapter 15 Exercise Solutions

15-1.
LSL = 0.70 g/cm^3, $p_1 = 0.02$; $1 - \alpha = 1 - 0.10 = 0.90$; $p_2 = 0.10$; $\beta = 0.05$

(a)
From the variables nomograph, the sampling plan is $n = 35$; $k = 1.7$.
Calculate $\bar{x}$ and S.
Accept the lot if $\left[Z_{\text{LSL}} = (\bar{x} - \text{LSL})/S\right] \geq 1.7$.

(b)
$\bar{x} = 0.73$; $S = 1.05 \times 10^{-2}$

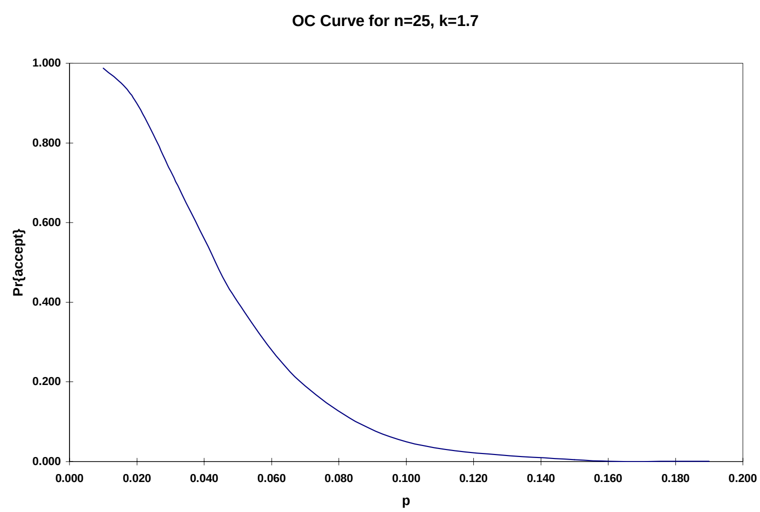
$$\left[Z_{\text{LSL}} = (0.73 - 0.70)/(1.05 \times 10^{-2}) = 2.8571\right] \geq 1.7$$

Accept the lot.

(c)
Excel workbook Chap15.xls : worksheet Ex15-1

From the variables nomograph at $n = 35$ and $k = 1.7$:

	p	Pr{accept}	
	0.010	0.988	
	0.016	0.945	
p1	**0.020**	**0.900**	**1-alpha**
	0.025	0.820	
	0.030	0.730	
	0.040	0.560	
	0.050	0.400	
	0.070	0.190	
p2	**0.100**	**0.050**	**beta**
	0.150	0.005	
	0.190	0.001	



$P_a\{p = 0.05\} \approx 0.38$ (from nomograph)

Chapter 15 Exercise Solution

15-2.

$\text{LSL} = 150; \sigma = 5$

$p_1 = 0.005; 1 - \alpha = 1 - 0.05 = 0.95; \; p_2 = 0.02; \beta = 0.10$

From variables nomograph, n = 120 and k = 2.3.
Calculate $\bar{x}$ and S.
Accept the lot if $\left[Z_{\text{LSL}} = (\bar{x} - 150)/S \right] \geq 2.3$

15-3.
The equations do not change: AOQ = $P_a p (N - n)$ / N and ATI = $n + (1 - P_a)(N - n)$.
The design of a variables plan in rectifying inspection is somewhat different from the attribute plan design, and generally involves some trial-and-error search.

For example, for a given AOQL = $P_a p_m (N - n) / N$ (where p_m is the value of p that maximizes AOQ), we know n and k are related, because both P_a and p_m are functions of n and k. Suppose n is arbitrarily specified. Then a k can be found to satisfy the AOQL equation. No convenient mathematical method exists to do this, and special Romig tables are usually employed. Now, for a specified process average, n and k will define P_a. Finally, ATI is found from the above equation. Repeat until the n and k that minimize ATI are found.

15-4.
AQL = 1.5%, N = 7000, standard deviation unknown
Assume single specification limit - Form 1, Inspection level IV
From Table 15-1 (A-2):
Sample size code letter = M
From Table 15-2 (B-1):
$n = 50$, $k_{\text{normal}} = 1.80$, $k_{\text{tightened}} = 1.93$

A reduced sampling ($n_{\text{reduced}} = 20$, $k_{\text{reduced}} = 1.51$) can be obtained from the full set of tables in MIL-STD-414 using Table B-3. The table required to do this is available on the Montgomery SQC website: www.wiley.com/college/montgomery

15-5.
Under MIL STD 105E, Inspection level II, Sample size code letter = L:

	Normal	**Tightened**	**Reduced**
n	200	200	80
Ac	7	5	3
Re	8	6	6

The MIL STD 414 sample sizes are considerably smaller than those for MIL STD 105E.

Chapter 15 Exercise Solution

15-6.
N = 500, inspection level II, AQL = 4%
Sample size code letter = E
Assume single specification limit
Normal sampling: n = 7, k = 1.15
Tightened sampling: n = 7, k = 1.33

15-7.
LSL = 225psi, AQL = 1%, N = 100,000
Assume inspection level IV, sample size code letter = O
Normal sampling: n = 100, k = 2.00
Tightened sampling: n = 100, k = 2.14
Assume normal sampling is in effect.
$\bar{x} = 255; S = 10$
$\left[Z_{\text{LSL}} = \left(\bar{x} - \text{LSL}\right)/\text{S} = \left(255-225\right)/10 = 3.000\right] > 2.00,$ so accept the lot.

15-8.
σ = 0.005 g/cm^3

$$\bar{x}_1 = 0.15; 1-\alpha = 1-0.95 = 0.05$$
$$\frac{\bar{x}_A - \bar{x}_1}{\sigma/\sqrt{n}} = \Phi(1-\alpha)$$
$$\frac{\bar{x}_A - 0.15}{0.005/\sqrt{n}} = +1.645$$

$$\bar{x}_2 = 0.145; \beta = 0.10$$
$$\frac{\bar{x}_A - \bar{x}_2}{\sigma/\sqrt{n}} = \Phi(\beta)$$
$$\frac{\bar{x}_A - 0.145}{0.005/\sqrt{n}} = -1.282$$

$n \approx 9$ and the target $\bar{x}_A$ = 0.1527

Chapter 15 Exercise Solution

15-9.
target = 3ppm; $\sigma = 0.10$ppm; $p_1 = 1\% = 0.01$; $p_2 = 8\% = 0.08$

(a)
$1 - \alpha = 0.95$; $\beta = 1 - 0.90 = 0.10$
From the nomograph, the sampling plan is $n = 30$ and $k = 1.8$.

(b)
Note: The tables from MIL-STD-414 required to complete this part of the exercise are available on the Montgomery SQC website: www.wiley.com/college/montgomery

AQL = 1%; $N = 5000$; σ unknown
Double specification limit, assume inspection level IV
From Table A-2:
sample size code letter = M
From Table A-3:
Normal: $n = 50$, M = 1.00 ($k = 1.93$)
Tightened: $n = 50$, M = 1.71 ($k = 2.08$)
Reduced: $n = 20$, M = 4.09 ($k = 1.69$)

σ known allows smaller sample sizes than σ unknown.

(c)
$1 - \alpha = 0.95$; $\beta = 0.10$; $p_1 = 0.01$; $p_2 = 0.08$
From nomograph (for attributes): $n = 60$, $c = 2$

The sample size is slightly larger than required for the variables plan (a). Variables sampling would be more efficient if σ were known.

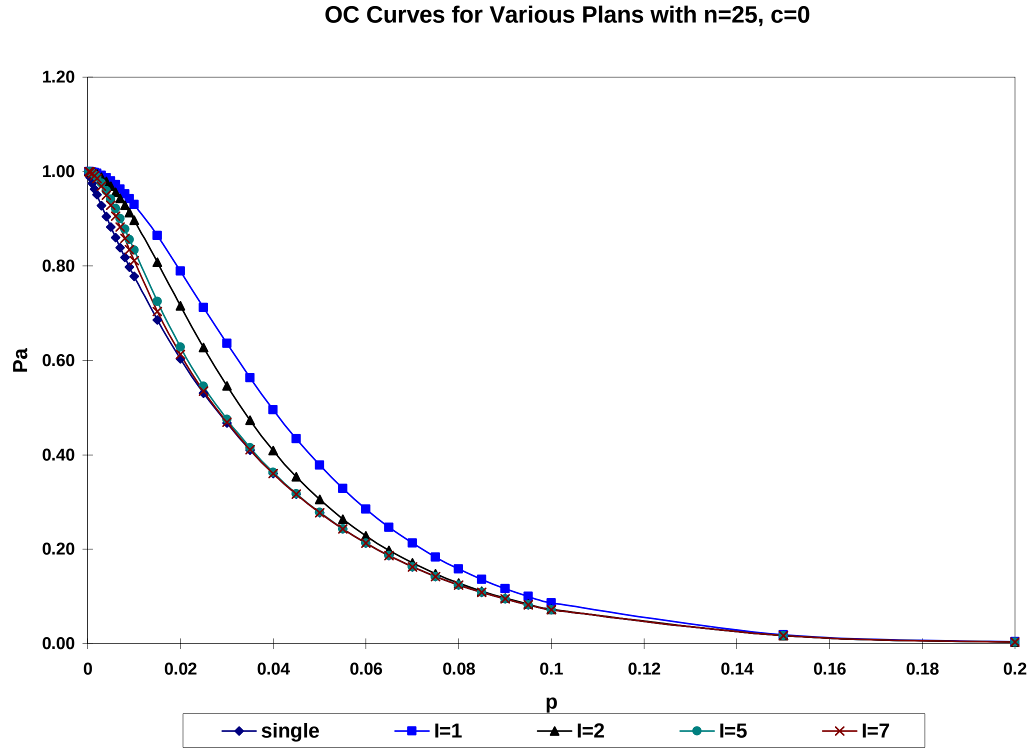
(d)
AQL = 1%; $N = 5{,}000$
Assume inspection level II: sample size code letter = L
Normal: $n = 200$, Ac = 5, Re = 6
Tightened: $n = 200$, Ac = 3, Re = 4
Reduced: $n = 80$, Ac = 2, Re = 5

The sample sizes required are much larger than for the other plans.

Chapter 15 Exercise Solution

15-10.

Excel workbook Chap15.xls : worksheet Ex15-10



Compared to single sampling with c = 0, chain sampling plans with c = 0 have slightly less steep OC curves.

Chapter 15 Exercise Solution

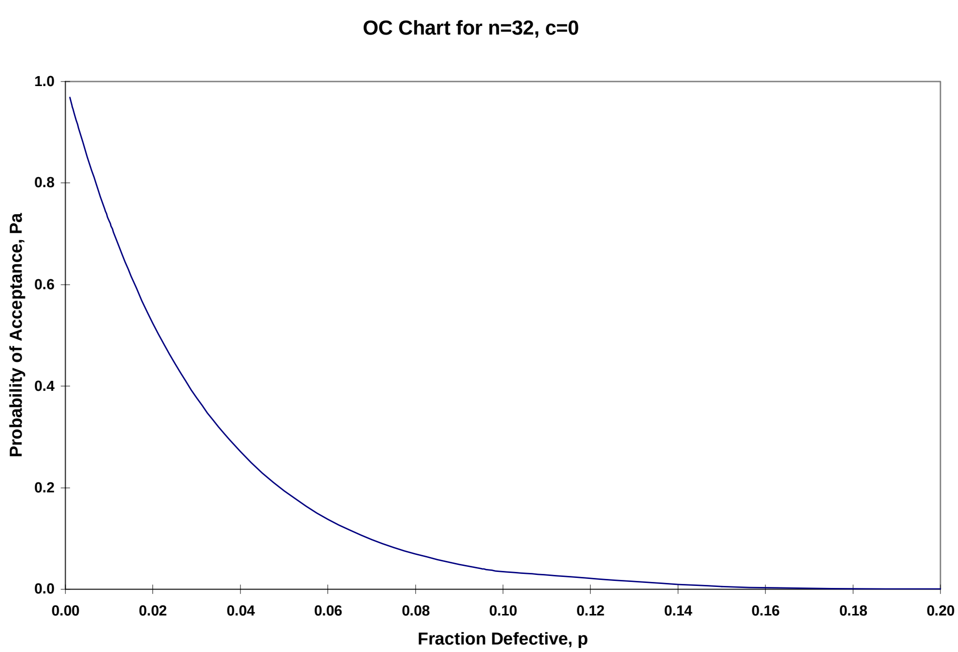
15-11.

N = 30,000; average process fallout = 0.10% = 0.001, n = 32, c = 0

Excel workbook Chap15.xls : worksheet Ex15-11

(a)

p	Pa	Pr{reject}
0.0010	0.9685	0.0315
0.0020	0.9379	0.0621
0.0030	0.9083	0.0917
0.0040	0.8796	0.1204
0.0050	0.8518	0.1482
0.0060	0.8248	0.1752
0.0070	0.7987	0.2013
0.0080	0.7733	0.2267
0.0090	0.7488	0.2512
0.0100	0.7250	0.2750
0.0200	0.5239	0.4761
0.0300	0.3773	0.6227
0.0400	0.2708	0.7292
0.0500	0.1937	0.8063
0.0600	0.1381	0.8619
0.0700	0.0981	0.9019
0.0800	0.0694	0.9306
0.0900	0.0489	0.9511
0.1000	0.0343	0.9657
0.2000	0.0008	0.9992
0.3000	0.0000	1.0000



Chapter 15 Exercise Solution

15-11 continued

(b)

$$\text{ATI} = n + (1 - P_a)(N - n)$$
$$= 32 + (1 - 0.9685)(30000 - 32)$$
$$= 976$$

(c)

Chain-sampling: $n = 32$, $c = 0$, $i = 3$, $p = 0.001$

$$P_a = P(0,n) + P(1,n)[P(0,n)]^i$$
$$P(0,n) = P(0,32) = 0.9685$$
$$P(1,n) = P(1,32) = 0.0310$$
$$P_a = 0.9685 + (0.0310)(0.9685)^3 = 0.9967$$

$$\text{ATI} = 32 + (1 - 0.9967)(30000 - 32) = 131$$

Compared to conventional sampling, the P_a for chain sampling is slightly larger, but the average number inspected is much smaller.

(d)

$P_a = 0.9958$, there is little change in performance by increasing i.

$$\text{ATI} = 32 + (1 - 0.9958)(30000 - 32) = 158$$

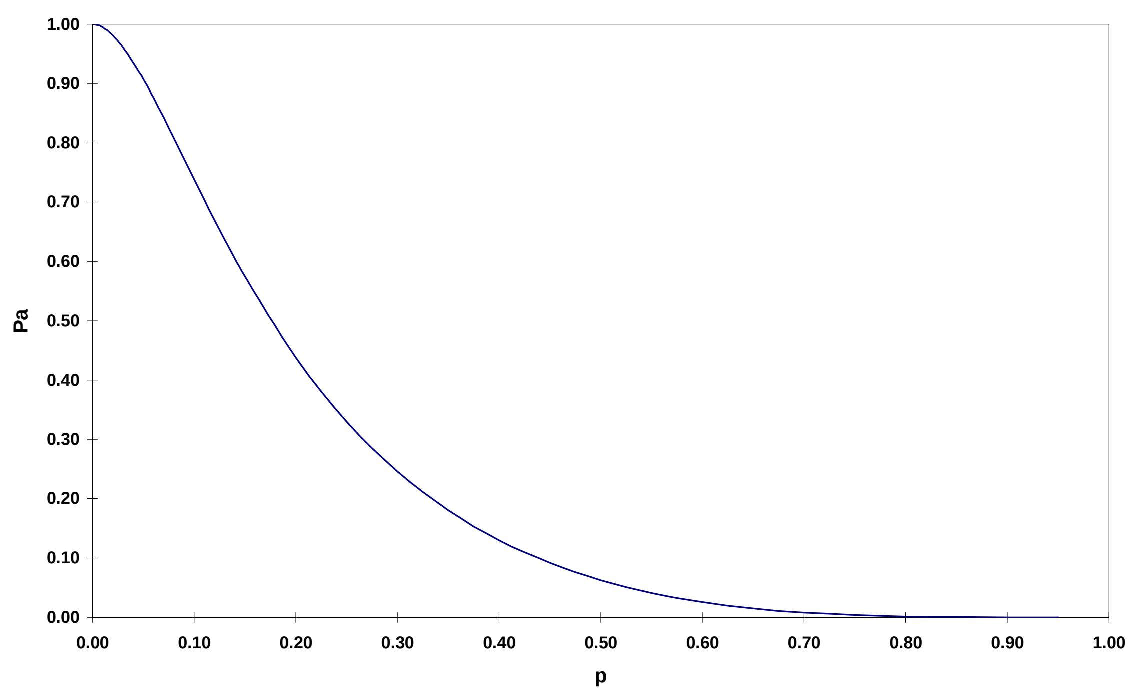
Chapter 15 Exercise Solution

15-12.
$n = 4$, $c = 0$, $i = 3$

Excel workbook Chap15.xls : worksheet Ex15-1

p	P(0,4)	P(1,4)	Pa
0.0010	0.9960	0.0040	0.9999
0.0100	0.9606	0.0388	0.9950
0.0200	0.9224	0.0753	0.9815
0.0300	0.8853	0.1095	0.9613
0.0500	0.8145	0.1715	0.9072
0.0600	0.7807	0.1993	0.8756
0.0700	0.7481	0.2252	0.8423
0.0800	0.7164	0.2492	0.8080
0.0900	0.6857	0.2713	0.7732
0.1000	0.6561	0.2916	0.7385
0.2000	0.4096	0.4096	0.4377
0.3000	0.2401	0.4116	0.2458
0.4000	0.1296	0.3456	0.1304
0.5000	0.0625	0.2500	0.0626
0.6000	0.0256	0.1536	0.0256
0.7000	0.0081	0.0756	0.0081
0.8000	0.0016	0.0256	0.0016
0.9000	0.0001	0.0036	0.0001
0.9500	0.0000	0.0005	0.0000

OC Curve for ChSP-1 n=4,c=0



Chapter 15 Exercise Solution

15-13.
$N = 500$, $n = 6$
If $c = 0$, accept. If $c = 1$, accept if $i = 4$. Need to find $P_a\{p = 0.02\}$

$$P_a = P(0,6) + P(1,6)[P(0,6)]^4 = 0.88584 + 0.10847(0.88584)^4 = 0.95264$$

15-14.
Three different CSP-1 plans with AOQL = 0.198% would be:

1. $f = ½$ and $i = 140$
2. $f = 1/10$ and $i = 550$
3. $f = 1/100$ and $i = 1302$

15-15.
Average process fallout, $p = 0.15\% = 0.0015$ and $q = 1 - p = 0.9985$

1. $f = ½$ and $i = 140$: $u = 155.915$, $v = 1333.3$, AFI = 0.5523, $P_a = 0.8953$
2. $f = 1/10$ and $i = 550$: $u = 855.530$, $v = 6666.7$, AFI = 0.2024, $P_a = 0.8863$
3. $f = 1/100$ and $i = 1302$: $u = 4040.000$, $v = 66,666.7$, AFI = 0.0666, $P_a = 0.9429$

	f = 1/2 and i = 140			**f = 1/10 and i = 550**			**f = 1/100 and i = 1302**		
p	**u**	**v**	**Pa**	**u**	**v**	**Pa**	**u**	**v**	**Pa**
0.0010	1.5035E+02	2000.0000	0.9301	7.3373E+02	10000.0000	0.9316	2.6790E+03	100000.0000	0.9739
0.0015	1.5592E+02	1333.3333	0.8953	8.5553E+02	6666.6667	0.8863	4.0401E+03	66666.6667	0.9429
0.0020	1.6175E+02	1000.0000	0.8608	1.0037E+03	5000.0000	0.8328	6.2765E+03	50000.0000	0.8885
0.0025	1.6788E+02	800.0000	0.8266	1.1848E+03	4000.0000	0.7715	1.0010E+04	40000.0000	0.7998
0.0030	1.7431E+02	666.6667	0.7927	1.4066E+03	3333.3333	0.7032	1.6331E+04	33333.3333	0.6712
0.0035	1.8106E+02	571.4286	0.7594	1.6795E+03	2857.1429	0.6298	2.7161E+04	28571.4286	0.5127
0.0040	1.8816E+02	500.0000	0.7266	2.0162E+03	2500.0000	0.5536	4.5912E+04	25000.0000	0.3526
0.0045	1.9562E+02	444.4444	0.6944	2.4329E+03	2222.2222	0.4774	7.8675E+04	22222.2222	0.2202
0.0050	2.0346E+02	400.0000	0.6628	2.9502E+03	2000.0000	0.4040	1.3638E+05	20000.0000	0.1279
0.0060	2.2037E+02	333.3333	0.6020	4.3972E+03	1666.6667	0.2749	4.2131E+05	16666.6667	0.0381
0.0070	2.3909E+02	285.7143	0.5444	6.6619E+03	1428.5714	0.1766	1.3395E+06	14285.7143	0.0106
0.0080	2.5984E+02	250.0000	0.4904	1.0238E+04	1250.0000	0.1088	4.3521E+06	12500.0000	0.0029
0.0090	2.8284E+02	222.2222	0.4400	1.5930E+04	1111.1111	0.0652	1.4383E+07	11111.1111	0.0008
0.0100	3.0839E+02	200.0000	0.3934	2.5056E+04	1000.0000	0.0384	4.8192E+07	10000.0000	0.0002
0.0150	4.8648E+02	133.3333	0.2151	2.7157E+05	666.6667	0.0024	2.3439E+10	6666.6667	0.0000
0.0200	7.9590E+02	100.0000	0.1116	3.3467E+06	500.0000	0.0001	1.3262E+13	5000.0000	0.0000
0.0250	1.3449E+03	80.0000	0.0561	4.4619E+07	400.0000	0.0000	8.2804E+15	4000.0000	0.0000
0.0300	2.3371E+03	66.6667	0.0277	6.2867E+08	333.3333	0.0000	5.5729E+18	3333.3333	0.0000
0.0350	4.1604E+03	57.1429	0.0135	9.2451E+09	285.7143	0.0000	3.9936E+21	2857.1429	0.0000
0.0400	7.5602E+03	50.0000	0.0066	1.4085E+11	250.0000	0.0000	3.0255E+24	2500.0000	0.0000
0.0450	1.3984E+04	44.4444	0.0032	2.2128E+12	222.2222	0.0000	2.4121E+27	2222.2222	0.0000
0.0500	2.6266E+04	40.0000	0.0015	3.5731E+13	200.0000	0.0000	2.0179E+30	2000.0000	0.0000
0.0600	9.6355E+04	33.3333	0.0003	1.0035E+16	166.6667	0.0000	1.6195E+36	1666.6667	0.0000
0.0700	3.6921E+05	28.5714	0.0001	3.0852E+18	142.8571	0.0000	1.5492E+42	1428.5714	0.0000
0.0800	1.4676E+06	25.0000	0.0000	1.0318E+21	125.0000	0.0000	1.7586E+48	1250.0000	0.0000
0.0900	6.0251E+06	22.2222	0.0000	3.7410E+23	111.1111	0.0000	2.3652E+54	1111.1111	0.0000
0.1000	2.5471E+07	20.0000	0.0000	1.4676E+26	100.0000	0.0000	3.7692E+60	1000.0000	0.0000

Chapter 15 Exercise Solution

15-16.
CSP-1 with AOQL = 1.90%
Plan A: $f = 1/5$ and $i = 38$
Plan B: $f = 1/25$ and $i = 86$

15-17.
Plan A: AFI = 0.5165 and $P_a\{p = 0.0375\} = 0.6043$
Plan B: AFI = 0.5272 and $P_a\{p = 0.0375\} = 0.4925$

Prefer Plan B over Plan A since it has a lower P_a at the unacceptable level of p.