CPD - HW assignment #1

1. *One-sided finite-difference approximation*
   Use the Taylor series expansion method discussed in the lecture notes to show that a second order accuracy finite difference approximation for $\left(\frac{\partial u}{\partial x}\right)_{x=x_i}$ can be written as
   $$\left(\frac{\partial u}{\partial x}\right)_{x=x_i} \approx \frac{-3u_i + 4u_{i+1} - u_{i+2}}{2\Delta x}$$
   Show that the leading term of the truncation error in the above approximation is $\mathcal{O}(\Delta x^2)$

2. *1-D Linear Advection Code*
   a) Run the linear_adv_lec_2.m code with different sigma values (sigma = 0.15, 0.05, 0.01), what happened to the solution at t=1? Can you describe the changes to the simulation results in a quantitative way (extra credit)?
   b) Run the linear_adv_lec_2.m code with a square wave as the initial profile: Q=1.0 for 0.4<x<0.6 and Q = 0.0 otherwise. Compare the final profile of Q with the initial condition and describe your result.
   c) Change the dt from 0.005 to 0.01. What happened to your simulation? Why?
   d) Change the u0 from 1 to -0.1 and re-run the Gaussian initial profile for Q(t=0), What happened to your simulation?
   e) Implement the central difference method for the spatial derivative in the linear_adv_lec_2.m code. Did you get a better solution due to the fact that it's second-order accurate in space?