Collateralized Commodity Obligations: *Modeling and Risk Assessment*

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Commodities as an asset class have been in the spotlight of investors' attention for the past decade. Trading volumes in commodity futures have grown exponentially and various commodity investment vehicles, such as commodity indexes and Exchange-Traded Funds (ETFs), have emerged. In addition, from mid-2000, commodity-linked notes and structured products have appeared and gained popularity.

These products have the form of a fixed-income instrument. A commodity-linked note pays a return linked to the performance of a basket of commodities over a defined period. On the maturity date, the note pays the initial principal amount plus return, if any, based on the percentage change in the underlying commodity basket. The underlying commodity basket can be focused on energy, agricultural commodities, or metals, for example, or it can be based on a commodity index, such as the Goldman Sachs or Dow Jones Commodity Index.

Sales of commodity-linked notes in 2008 were $15.8 billion, double the $7.8 billion in 2007, and the products continue to draw more investors, even as overall sales of structured notes declined in the aftermath of the financial crisis. The Scandinavian Structured Products Association recently reported that commodity-linked structured products saw a 277.72% surge year-on-year (which can be partially explained by poorly performing equity markets).

Although popular, most of the currently offered commodity-linked notes suffer from a significant drawback: They are "uniform" in their structure, in that they cannot differentiate between investors with different risk appetites. A more sophisticated commodity-linked structured product, the so-called *Collateralized Commodity Obligation* (CCO), overcomes this drawback by offering a variety of risk-return profiles. CCO transforms commodity price exposure to a *ratable* product, while still providing investors with a bond-like return that is linked to the performance of an underlying portfolio of commodities.

In December 2004, Barclays Capital launched the world's first Collateralized Commodity Obligation. Since then, the OTC market for these structured products has been growing steadily. Financial institutions, such as Barclays and Goldman Sachs, continue offering CCOs, and the biggest investors are insurance companies, commercial banks, and hedge funds in Europe and the United States.

Nowadays, CCOs are described in most investment handbooks and textbooks (see, e.g., Schofield [2007], Fabozzi, Fuss, and Kaiser [2008], Kothari [2009], Anson [2009], Kolb and Overdahl [2010]). However, there is virtually no literature (academic or professional) on how to price, hedge, or assess risks associated with these products.

Commodity-based structured products are very different from other products based on different asset classes, a fact that becomes quickly apparent when investigating the pricing and risks of these instruments. With this article, we hope to address these issues and to fill (at least partially) this gap in the literature.

STRUCTURE OF A TYPICAL CCO

CCOs are structured much like the well-known Collateralized Debt Obligations (CDOs). At a simple level, both CDOs and CCOs are transactions that transfer the risk of a reference portfolio of assets. CDOs transfer credit risk while CCOs transfer commodity price risk.

Where a CDO is created from a basket of underlying bonds, loans, or credit default swaps, the underlying securities in a CCO comprise a number (e.g., 100) of the so-called *commodity trigger swaps* (CTS). These are based on up to 16 precious metal, base metal, agricultural, and energy prices.

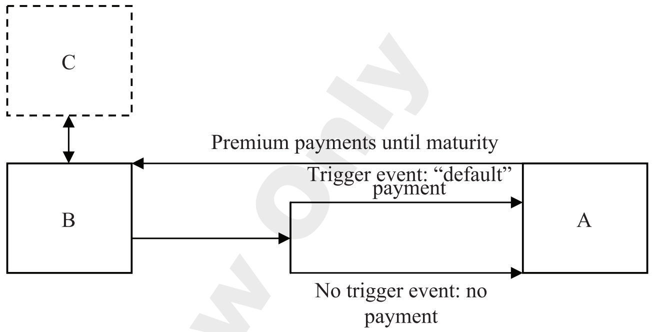
The structure of a commodity trigger swap is similar to that of a credit default swap (CDS), but there are two main differences. First, the *reference entity* of a CTS is a certain commodity price (rather than a company that might default). Second, the *credit event* of a CDS is replaced by the so-called *trigger event*, which occurs when the corresponding commodity price falls under (or rises above) some pre-set level at time T, which is the contract's maturity date.

Note that in the case of a CTS, the *protection buyer* pays premia until maturity T, while in the case of a CDS the payments are made until maturity or until the default of a reference entity, whichever comes first. Another difference between a CDS and CTS is that the *recovery rate* of a CTS is 0%, which implies 100% *loss in case of a trigger event*. A CTS can be seen as a European binary option (usually deep in- or out-of-money) on an underlying commodity (see Exhibit 1).

The asset side of a CCO is formed by a portfolio of, for example, 100 commodity trigger swaps (CTS), based on as many as 20 commodities. This portfolio is sold to a special-purpose vehicle (SPV). Just as in a CDO, the liability side of the SPV is formed by issuing notes belonging to tranches of increasing seniority. Losses encountered by CTSs will first affect the "equity" tranche, then the "mezzanine" tranches, and finally the "senior" tranches.

EXHIBIT 1

Basic Structure of a CTS. C is the Reference Commodity, Firm A is the Protection Buyer, and Firm B is the Protection Seller



The bond-style payoff of the CCO tranches consists of periodic interest payments of a floating rate (e.g., LIBOR) plus an additional premium, depending on the tranche. Besides the periodic interest payments, at the time of maturity the CCO also pays the principal less the losses generated by trigger events.

The cumulative loss of the portfolio, as well as the notional of a CCO tranche, depends on the total number of trigger events of the underlying CTSs that occur at the time of maturity T. Let L be the total number of trigger events in the CCO portfolio at maturity. Consider a CCO with k tranches indexed by $\kappa \in \{1,\ldots,k\}$ and *attachment points* $0 = K_0 < K_1 < \ldots < K_k$. The notional of a tranche κ at maturity depends exclusively on the total number of trigger events at maturity L and equals to

$$N_\kappa(T) = N_\kappa(0) f_\kappa(L) \quad \text{with}$$

$$f_\kappa(L) = \begin{cases} 1, & \text{if} \quad L < K_{\kappa-1} \\ \dfrac{K_\kappa - L}{K_\kappa - K_{\kappa-1}}, & \text{if} \quad L \in [K_{\kappa-1}, K_\kappa] \\ 0, & \text{if} \quad L > K_\kappa \end{cases}$$

In contrast to CDOs, where losses are generated by defaults (so investors are exposed to credit risk), in CCOs losses are due to movements in the underlying commodity prices (hence, the exposure to market risk). One very important difference between CDOs and CCOs, especially relevant in view of the recent financial crisis, is a complete transparency of the CCO structure. While in a typical loan-based (e.g., mortgage-based) CDO it is unknown which assets comprise a given CDO (who are the loan holders, what is their credit history, and so on), it is absolutely clear what kinds of (standardized) commodities underlie a given CCO.

The current economic situation had dire consequences for the performance of all kinds of loans, be it corporate bonds, mortgages, credit card, or car loans—typical assets underlying CDOs. This has led to downgrades and loss in value of CDO investments. Commodity markets suffered much less from the financial crisis and recovered sooner. This arguably leads to a potentially strong performance of CCOs, making them more attractive fixed-income investment instruments than CDOs, while also providing diversification benefits.

Another important feature contributes to the attractiveness of CCOs. Contrary to assets underlying a CDO, on whose performance there might be no historical data, there is an abundance of historical data on commodity prices. We shall exploit this fact by means of a data-driven historical simulation method for assessing CCO risks and performance. The approaches we describe here are universal in the sense that they can be applied not only to CCOs, but to any commodity-linked note or other structured product whose performance is linked to a portfolio of commodities.

ANALYSIS OF COMMODITY-LINKED STRUCTURED PRODUCTS AND CCOs

Recall that, at maturity, commodity-linked notes pay the principal amount plus the return (if any) related to the performance of some reference commodity portfolio. So the principal payment is guaranteed, while the return, or "interest," is uncertain. In this case, forecasting the return distribution of such a note is the same as forecasting the return distribution of the reference commodity portfolio.

This might seem a simple task. There is, after all, an abundance of historical commodity prices, so the historical return distribution of the reference portfolio can serve as such a forecast. In risk management, this is referred to as the *historical simulation approach*.

Alternatively, one can employ the so-called *variance-covariance method*: assume multivariate normal distribution for the commodity returns (in which case the portfolio's return is also normally distributed), estimate the variance-covariance matrix, and obtain the mean return and its variance via the well-known formulae (see, e.g., McNeal et al. [2005]). To deal with heavy tails in returns distribution, the normal distribution can be replaced by the Student-*t* with a low number of degrees of freedom.

These methods are well established in risk management for estimating loss distribution of a portfolio at relatively short horizons, but they may be completely unsuitable for the task at hand. Commodity-linked notes and other structured products have maturities that stretch three to five years into the future; hence, we are interested in the distribution of the reference portfolio return at these long horizons.

Commodity prices are characterized by long trends that can last for months or years. On the other hand, a common "wisdom" or "folklore" in commodity-related literature is that they always revert to some "long-term mean." This justifies the fact that commodity prices are often modeled by means of a mean-reversion process.

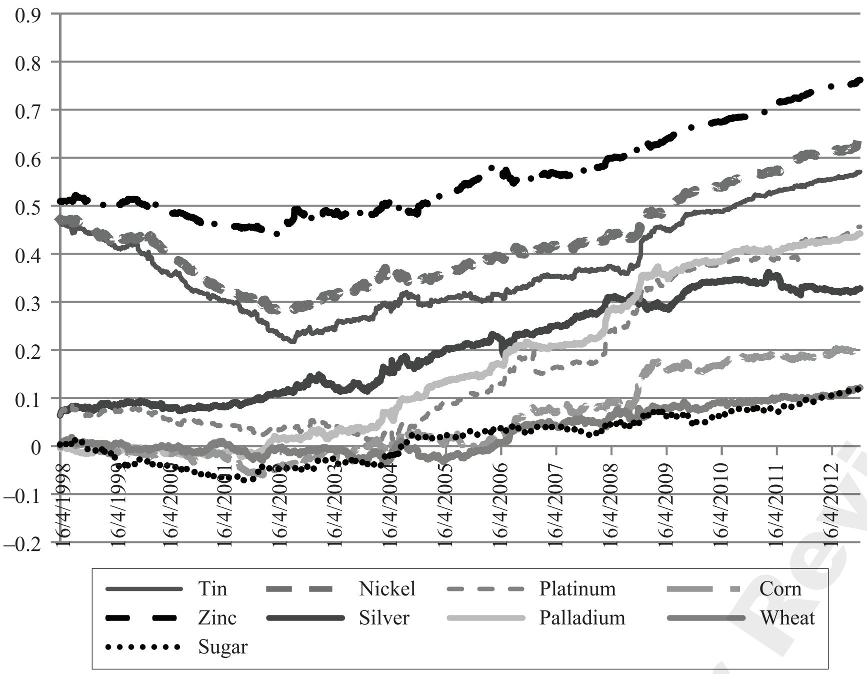
Price developments in oil and metals markets fuel the ongoing discussion on whether mean-reversion is still a valid assumption for energy and other commodities prices (see e.g., Geman [2005]). For example, the recent developments in the palladium and gold prices reject the assumption that they can be modeled by a mean-reversion model, since the prices have risen considerably for a long period. Other commodity prices, such as those of aluminum, lead, or copper, may well be modeled by a mean-reversion, but with higher long-term means than in the previous decades. This non-stationarity in mean should be taken into account when assessing the performance of any commodity-linked product at long time horizons.

Since the mid-2000s, commodity markets have attracted investors that were previously not active in these markets: hedge funds, mutual and pension funds, and other institutional investors. This has led to the so-called *financialization* of commodities (see, e.g., Tang and Xiong [2010]) and resulted in a significant increase in correlations between various commodities (as well as between commodities and other asset classes). Exhibit 2 demonstrates this effect on the example of lead and copper.

Finally, the recent financial crisis has led to significant increases in commodity price volatilities, as demonstrated in Exhibit 3. These effects can be referred to

E X H I B I T 2

Five-Year Rolling Correlations Between Lead and other Commodities



as the *second-order non-stationarity*, and are also a matter of concern when analyzing long-maturity commodity-linked products.

There are three main challenges when developing a methodology for analyzing commodity-linked structured products:

- Devising a flexible and realistic model for commodity price evolution, which persists for relatively long time horizons.
- Modeling complex inter-dependencies among commodities in the reference portfolio.
- Dealing with non-stationarities, primarily in mean, but also in volatilities and correlations.

In all, under the current changing market conditions, it is questionable whether any parametric model, no matter how realistic, can describe the commodity prices for the next three to five years.

In this article, we devise and estimate a highly flexible parametric model for joint commodity price evolution, one that incorporates time changing and uncertain commodity price trends. We show that such a model, although excellent at forecasting commodity price characteristics at one-year or shorter horizons, is less able to reliably forecast relevant aspects of commodity prices at longer horizons. Moreover, it will become apparent that the outcomes of CCO analysis are very sensitive to model assumptions and parameter estimates, implying a high degree of model risk.

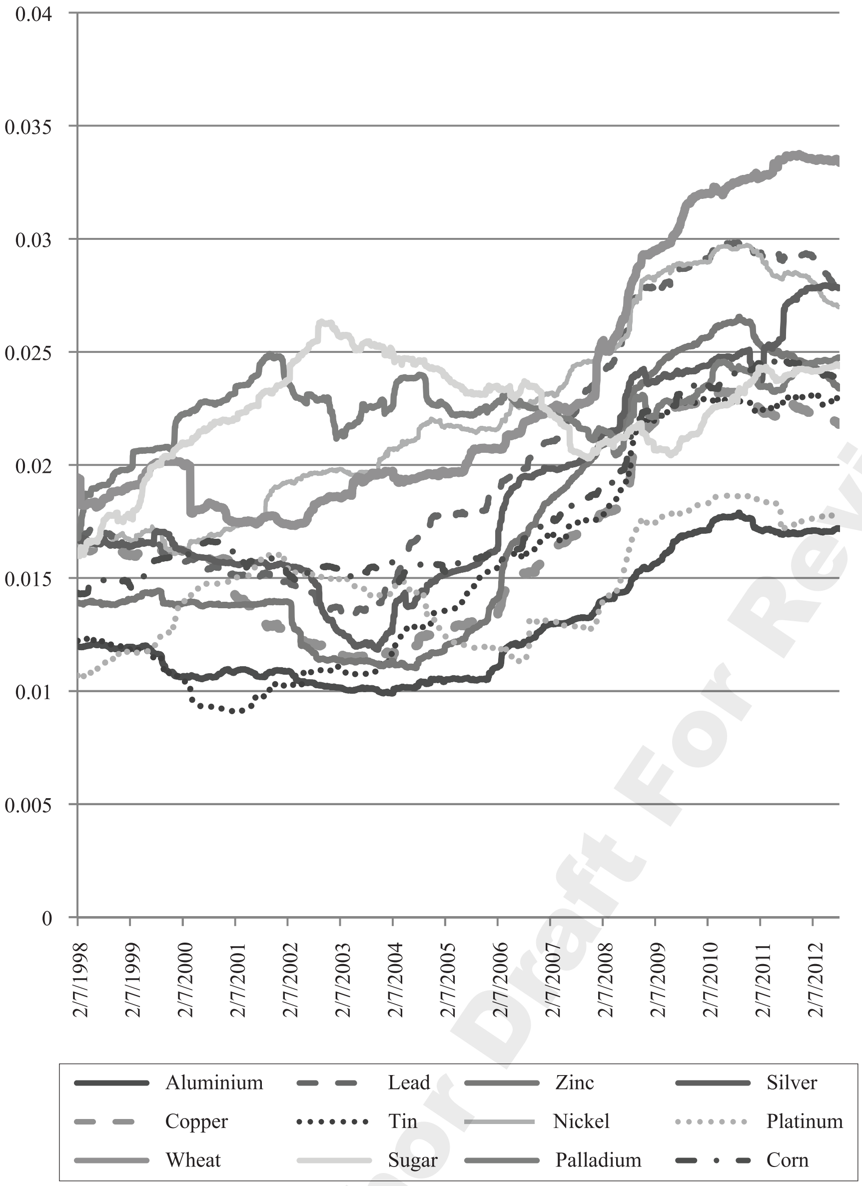
Analysis of CCOs represents another degree of complexity. They are tranched instruments, so their profit and loss distribution is much more complicated than that of the "regular" commodity-linked notes. While there is an abundant literature on pricing and rating of CDOs (see, e.g., Hull and White [2004, 2006, 2008]), there is practically nothing in the academic literature about the same issues for CCOs. The only exception is the paper by Chang et al. [2009], where structured products combining credit and commodity price risk (CDCO) are considered. Standard & Poor's (S&P) and Fitch rating agencies describe CCOs and their rating methodologies in their reports (see Standard & Poor's [2006], Fitch [2006]), as the tranches of CCOs issued by major banks were rated by these agencies.

S&P used standard uncorrelated mean-reversions for modeling commodity prices. The probabilities of trigger events of CTSs were then determined analytically. Subsequently, these probabilities (which replace the default probabilities in CDOs) were combined using the standard CDO evaluator, based on the Gaussian copula model (Standard & Poor's [2005]). The mean-reverting processes for commodities were assumed to be uncorrelated, but the trigger events were made correlated by using the historical returns correlations between commodities in the Gaussian copula.

Similarly, Fitch used its CDO evaluator (Fitch [2004]) with the input being the probabilities of trigger events, determined by Monte Carlo simulations. For commodity prices, the factor model was used, combining the Principal Component Analysis, a nonlinear GARCH process, a constant mean, and jumps. In this approach, the correlations between commodities were already taken into account at the first step. Both

Exhibit 3

Five-Year Rolling Volatilities



approaches, although quite different, were obviously designed with the existing CDO evaluators in mind.

In contrast to credit risk events (defaults of loans or CDSs), there is plenty of historical data on commodity prices, which can be explored here. In this article, we will simulate the default probabilities of CCO tranches *directly* by an appropriate commodity price model, and also by a nonparametric data-driven method of block bootstrap simulation. The essence of the latter method is resampling blocks of historical commodity returns, *simultaneously for all commodities in the portfolio*, which preserves their inter-dependencies.

In this method, we can use a long history of commodity returns, which are amply available (for example, in our simulation study we use 20 years of data). This ensures that we resample from a wide variety of economic environments represented by the data, and get a wide range of simulated price movements. Alternatively, we can resample only from the recent history, to better reflect the current state of the commodity market.

Our methodology is rather universal, in that it is applicable to any commodity-linked structured product. Here we apply it to CCOs, as these are more complex derivatives than most other commodity-linked products. But first we examine the historical performance of a typical CCO.

HISTORICAL PERFORMANCE: THE GOLDMAN SACHS CCO

In this section we investigate the historical performance of one typical example of a CCO, the one issued by Goldman Sachs International (GSI) in 2005. The asset side of this CCO consists of the portfolio of 100 CTSs in 12 different commodities with maturity of five years. (Note that there are no energy commodities among the reference commodities of this CCO.)

For each commodity, upper and lower trigger event strikes are defined as percentages of the corresponding commodity price at the time of issue. The trigger event strikes are evenly distributed between the upper and the lower strike. Exhibit 4 shows the lower and upper strike for each commodity, as well as the distribution of CTSs among the different commodities. Note that trigger events occur if commodity prices fall, as all strikes are below 100%.

The liability side of the CCO consists of notes belonging to five tranches of increasing seniority:

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EXHIBIT 4

Asset Side of the GSI CCO: Underlying Commodities, Upper and Lower Trigger Strikes and Number of CTSs per Commodity

No.	Commodity	Upper Trigger Event Strike	Lower Trigger Event Strike	# of Trigger Events (CTSs)
1	Aluminum	43%	35%	9
2	Tin	49%	42%	8
3	Zinc	25%	17%	9
4	Nickel	22%	14%	9
5	Silver	31%	24%	8
6	Platinum	30%	23%	8
7	Palladium	34%	27%	8
8	Corn	80%	74%	7
9	Wheat	59%	51%	9
10	Sugar	79%	72%	8
11	Copper	20%	12%	9
12	Lead	35%	28%	8

EXHIBIT 5

Attachment and Detachment Points for the Tranches of the GSI CCO

Tranche	Attachment Point	Detachment Point	Indicative Yield
BBB	12	18	LIBOR + 350 bp
A	18	21	LIBOR + 250 bp
AA	21	27	LIBOR + 190 bp
AAA	27	34	LIBOR + 115 bp
Super Senior	34	41	LIBOR + 60 bp

BBB, A, AA, AAA, and Super Senior. Each tranche has attachment and detachment points (Exhibit 5). If the total number of trigger events at maturity is lower than the attachment point, then the notional of the corresponding tranche remains intact; if it is between the attachment and detachment point, then part of the notional is lost; and if the number of trigger events is larger than the detachment point, then the whole notional is lost.

For example, suppose that in five years' time, the prices of lead, copper, and wheat fall to, respectively, 30%, 13%, and 55% of their value at time zero, while the prices of all other commodities stay above their upper strikes, and the hypothetical total number of trigger events is 19, which follows from Exhibit 4. This will result in a loss of 100% of the notional of the BBB tranche and 33.3% of the notional of the A tranche.

To investigate the historical performance of this CCO, were obtained the historical daily closing commodity prices from July 1, 1993, from Thomson Datastream. Exhibit 6 presents the list of commodities, their grade, and quality specifications used in the GSI CCO example.

The historical performance analysis is based on a hypothetical daily CCO issue from July 1, 1993, to January 31, 2005. The maturity of the contract is five years. The notional of tranches at maturity depends on the total number of trigger events generated by the portfolio of CTSs. For each hypothetical CCO, we calculate the number of trigger events and the realized present value of all payments for every CCO tranche.

Until 2003, most of the commodity prices underlying the CCO fluctuated around their long-term means. From 2003 on, the prices gradually started to show a strong upward trend, which persisted until the first quarter of 2008. Thus, no trigger events occurred after 2002.

However, by the end of 2008, all reference commodity prices except for sugar dropped significantly, while the CCOs issued after 2004 (for which the period from issue to maturity includes some of the crisis period) still experienced no trigger events. This can be explained by the fact that, despite the major drop in commodity markets, the prices decreased to levels higher than the average historical levels up to 2003.

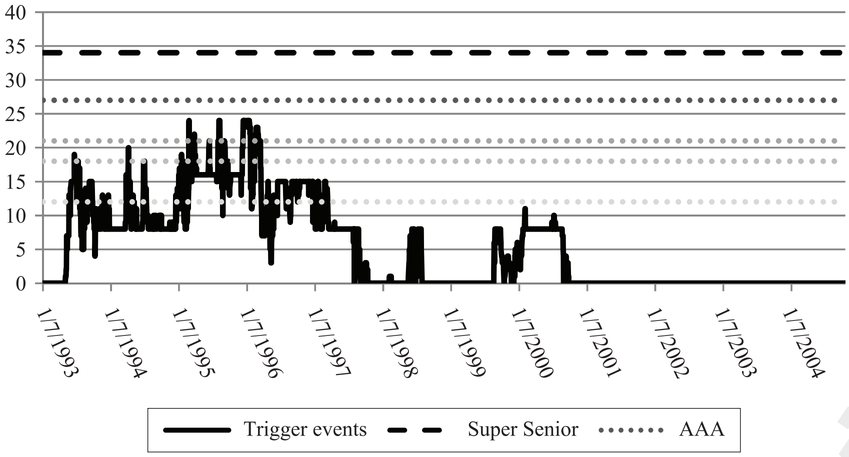
The total number of trigger events is calculated for the CCO issued every day from July 1, 1993, until January 31, 2005, and shown in Exhibit 7.

EXHIBIT 6

Technical Description of the Commodities

Commodity	Code	Type/Grade
Aluminium	LAHCASH	LME-Aluminium 99.7% Cash $/MT—A.M. OFFICIAL
Copper	LCPCASH	LME-Copper, Grade A Cash $/MT—A.M. OFFICIAL
Lead	LEDCASH	LME-Lead Cash $/MT—A.M. OFFICIAL
Tin	LTICASH	LME-Tin 99.85% Cash $/MT—A.M. OFFICIAL
Zinc	LZZCASH	LME-SHG Zinc 99.995% Cash $/MT—A.M. OFFICIAL
Nickel	LNICASH	LME-Nickel Cash $/MT—A.M. OFFICIAL
Silver	SLVCASH	Silver Fix LBM Cash cents/Troy ounce
Platinum	PLATFRE	London Platinum Free Market $/Troy ounce
Palladium	PALLADM	Palladium $/Troy ounce
Corn	CORNUS2	Corn No.2 Yellow cents/Bushel
Wheat	WHEATSF	Wheat No.2, Soft Red cents/Bushel
Sugar	WSUGDLY	Raw Sugar-ISO Daily Price cents/pound

Exhibit 7
The Number of Trigger Events and the Attachment Points of the Tranches



The historical analysis shows that the investors in the BBB tranche lose some of their notional in 16.8% of the hypothetically issued CCOs. The investors in the A and AA tranches lose some of their notional in, respectively, 2.49% and 1.42% of the issued CCOs. The investors in AAA and Super Senior tranches experience no losses.

We also calculated the realized present value for each tranche *i* of each hypothetical issue of the CCO as[1]:

$$PV_i = Disc.\ BondPayments + Disc.\ principal - Disc.\ Losses$$

The descriptive statistics for the whole historical period are shown in Exhibit 8. Exhibit 9 shows the realized present value of tranches, and a pattern very similar to that in Exhibit 7 can be seen. Before mid-1997, the present value of the tranches was quite volatile. After that, due to a very low number of trigger events, the tranche values remain almost constant.

The statistical summary of the realized present values shows that, from a historical perspective, the BBB and A tranches are the least desirable, combining a lower average present value with a higher volatility. Note that,

Exhibit 8
Descriptive Statistics for CCO Tranches' Realized Present Value (Issue 1993–2005), Where 1 Indicates Par

1993–2005	BBB	A	AA	AAA	SS
Mean	1.073	1.091	1.078	1.050	1.027
Median	1.148	1.107	1.081	1.049	1.026
Standard Deviation	0.187	0.104	0.037	0.001	0.001
Minimum	0.373	0.332	0.694	1.048	1.026
Maximum	1.159	1.114	1.087	1.053	1.027

Exhibit 9
Realized Present Values of Tranches at a Given Issue Date

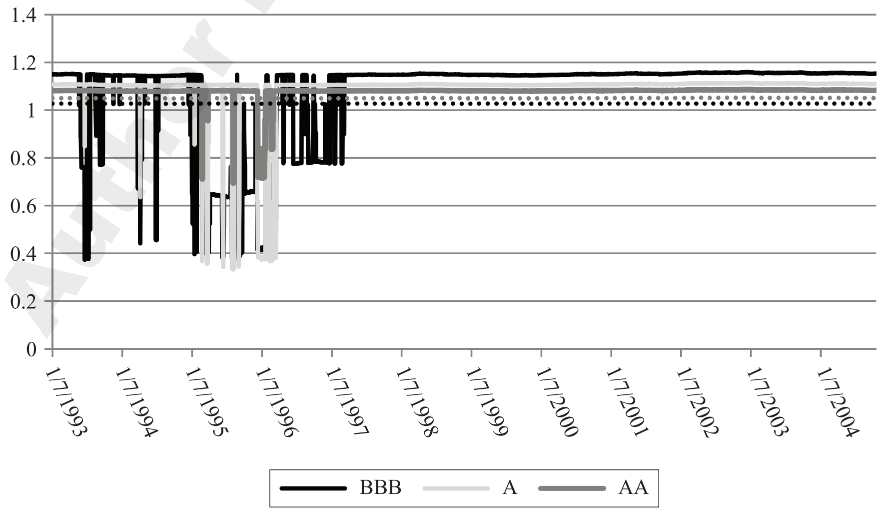


EXHIBIT 10

Descriptive Statistics for CCO Tranches Realized Present Value (Issue 1998–2005), where 1 Indicates Par

1998–2005	BBB	A	AA	AAA	SS
Mean	1.152	1.109	1.083	1.051	1.027
Median	1.151	1.108	1.083	1.050	1.027
Standard Deviation	0.004	0.003	0.002	0.001	0.001
Minimum	1.144	1.104	1.0793	1.049	1.026
Maximum	1.159	1.114	1.087	1.053	1.028

because triggers can only be hit after five years, the discounted interest rate payments result in minimum total values that are well above 0.

After mid-1997, the CCO performed significantly better: the returns of all tranches outperform the risk-free rate of return (e.g., five-year U.K. swap rate). The descriptive statistics for the present value of the CCOs issued from 1998 until 2005 (Exhibit 10) show significantly higher average realized present values and lower risk for tranches BBB, A, and AA.

Based on this analysis, we can conclude that all tranches would perform well (especially in the period of rising commodity prices from 1998–2008), and even the most risk-taking investors (BBB-investors) would have made a profit. All these results are completely backward-looking, however, so such a historical performance analysis can fail to assess future risks associated with CCOs. So in the next two sections we present a flexible parametric model for assessing CCOs performance and describe a model-free historical simulation approach to this problem.

METHODOLOGY

Before we describe the parametric and non-parametric methods for commodity price simulations and CCO risk assessment, we would like to mention an important link between CCOs and commodity options, which, at least in theory, can be exploited for the risk-neutral valuation of CCOs.

First of all, note that a Commodity Trigger Swap, illustrated in Exhibit 1, is essentially a *European digital put option* on a reference commodity. Consequently, the set of CTSs in the CCO portfolio for a particular reference commodity can be valued as a set of five-year European digital put options with a ladder of strikes.

These digital options also can be approximated by a combination of long and short put options (selling puts with the strike equal to the trigger level + ε and buying puts with the strike equal to the trigger level – ε, in appropriate amounts). If, on the exercise date, the corresponding commodity price is below trigger level – ε, the loss on the put options will be 2 ε* notional amount of commodities on the puts, because the loss is generated on the sold put options.

Valuation of these put options (either regular or digital) requires a forward curve and implied volatility smile/skew for all reference commodities. In particular, we would need the implied volatilities of (deep) out-of-the-money options with a long time to maturity. Unfortunately, commodity option markets are either illiquid or completely non-existent, so such volatilities would be very hard to obtain.

Even if we are able to calculate the prices of all the underlying put options (which, when combined, give us the total premium on the CCO as a whole), we are then facing the question of how to distribute this premium over the CCO tranches, i.e., how to determine the tranche coupons. Answering this question requires the risk neutral probabilities for all possible numbers of hit triggers, which, in turn, requires a (risk-neutral) correlation model.

Calibrating such a model is possible via prices of multi-commodity options, such as basket or spread options—again, instruments not readily available in the market. In all, while this risk-neutral valuation approach certainly seems appealing and elegant in theory, it is rather impossible to implement in practice. So in what follows, we will not be looking at the risk-neutral valuation of CCOs, but will concentrate on assessing CCO risk under the real-world measure, i.e., using historical data.

A Parametric Model: Mean-Reversion with Trend

In commodity literature as well as in practice, there are many parametric models for commodity prices. The most popular (and the most frequently used) model for a single commodity price is the mean reversion model (see, e.g., Schwartz [1997], Geman [2006]), possibly augmented with jumps and/or stochastic volatility. For an ensemble of commodity prices, correlated mean-reversion processes or factor models are often used.

The approach we introduce here can be thought of as a multivariate mean reversion with non-constant means. It allows for long-term trends such as those recently observed in many commodity markets. The model directly incorporates the dependencies between the commodities by means of the covariance matrixes of trends and returns. To account for the second-order non-stationarity, these matrixes can be estimated using only the recent data, i.e., the prices from the past five to seven years.

In the standard mean reversion model, the logarithm of the commodity price

$X_t = ln\ S_t$ is modeled as an Ornstein-Uhlenbeck process, given by the following stochastic differential equation:

$$dX_t = \kappa(\alpha - X_t)dt + \sigma\, dW_t \tag{1}$$

where κ is the mean reversion rate, α is the long run mean, σ is the volatility, and W_t is the standard Brownian motion. For energy and other commodity prices, there is an ongoing discussion on whether prices still follow mean reversion (Geman [2005]). For many commodities (energy, metals), prices have shown large upward trends since 2003, often attributed to a growing demand from China, then significant downward moves in the immediate aftermath of the financial crisis, followed by a substantial price recovery in 2010.

Mean reversion with a constant mean obviously cannot cope with such situations. An alternative would be to adjust Equation (1) by allowing the mean value α to vary (stochastically) over time, which is the essence of our approach outlined below.

Recall that, when studying multiple commodity-linked structured products such as CCOs, we need to model several correlated commodity prices simultaneously. Generalizing (1) to multiple commodities, we have

$$dX_{it} = \kappa_i\left(\alpha_{it} - X_{it}\right)dt + \sum_{j=1}^{g} c_{ij}\, dW_{jt} \tag{2}$$

where c_{ij} are the elements of the Cholesky decomposition of the variance-covariance matrix Σ and $W_{1t}, \ldots, W_{gt}$ are g independent Brownian motions. To get around the unrealistic assumption of constant mean price, we can use the so-called *mean reversion with trend*.

We assume that the long-run mean log-price α_{it} is not constant but represents a so-called *trend*, which is slowly (and stochastically) varying over time. The trend is filtered out from the historical price series by a state-space method using the Kalman filter. We use the so-called *local linear trend model*, or *a smooth trend model*, by Durbin and Koopman [2001].

For a univariate time series (y_t), this model has the following form:

$$\begin{aligned} y_t &= \mu_t + \beta_t + \omega_t, \quad && \omega_t \sim N\left(0, \sigma_\omega^2\right) \\ \mu_{t+1} &= \mu_t + \beta_t, && \\ \beta_{t+1} &= \beta_t + \eta_t, && \eta_t \sim N\left(0, q\sigma_\omega^2\right) \end{aligned}$$

where μ_t is the level and β_t is the slope. The addition of normally distributed random variables with non-zero variance allows the trend level and slope to vary over time. The parameter q is the *signal-to-noise ratio*. It determines how sensitive the estimated trend is to short-term fluctuations in the time series. The lower the value of q, the less sensitive the estimated trend is to these fluctuations.

We illustrate the trend estimation and forecasting on the example of the aluminium price. Exhibit 11 shows two graphs of the aluminium log-price, trends filtered out using the signal-to-noise ratios q of $3 \cdot 10^{-9}$, $6 \cdot 10^{-9}$ and $9 \cdot 10^{-9}$, together with the forecasted trends in December 2009 (upper graph) and December 2012 (lower graph), both with their 95% confidence bounds. Note that the trend forecasts are essentially projections of the recent (i.e., the past year) price developments.

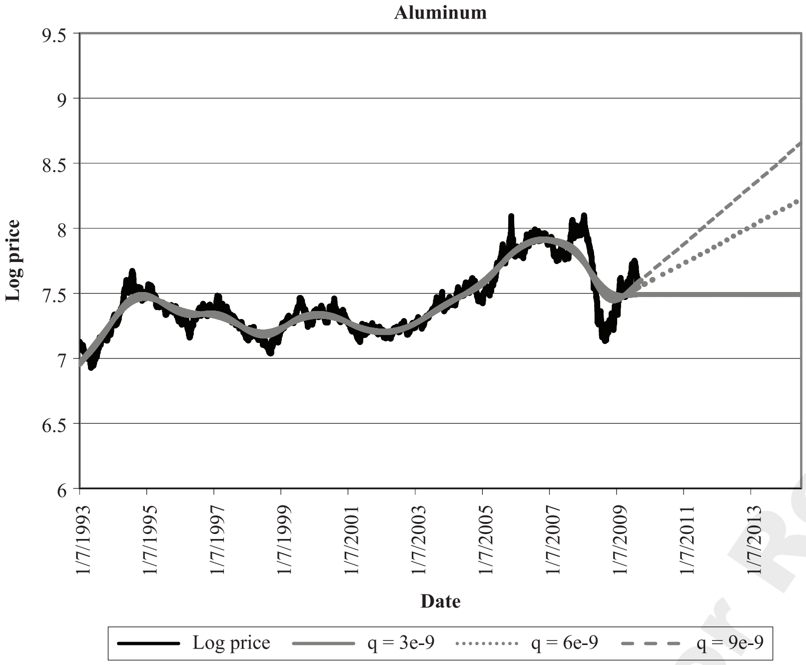
In December 2009, the forecasted trend is up (except for the lowest signal-to-noise ratio), as it picks up the recent recovery of the aluminium price. The upward trend is forecasted correctly for the following year, until the trend reverses in the summer of 2011. In December 2012, the situation is the opposite: for all signal-to-noise ratios, the forecasted trend is down, due to the persistent price fall in the past year. While not reported in this study, these observed features (in particular, correct forecasts of trend one year ahead) are observed for most commodities.

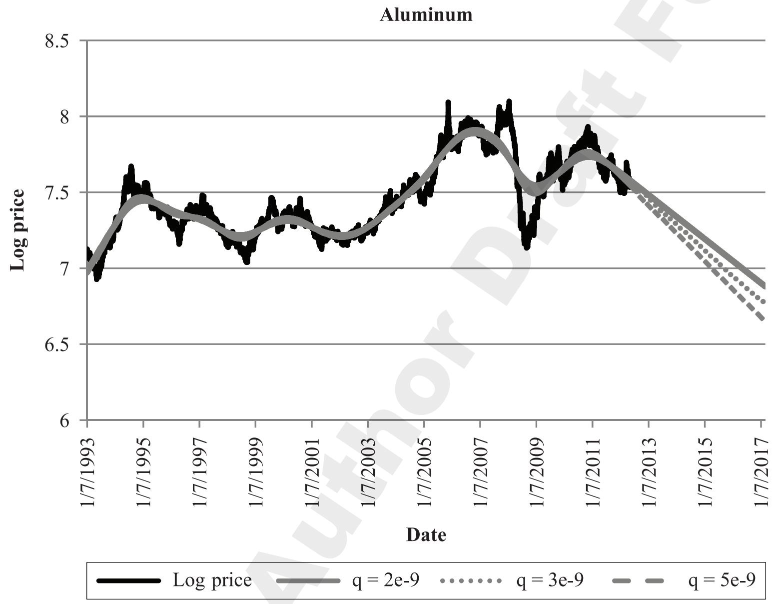
However, global commodity price trends historically have reversed every one to two years. So clearly it is extremely difficult to accurately forecast (by any method) where the prices will be at long horizons such as three to five years, which are the typical times to maturity of commodity-linked structured products.

The values of the signal-to-noise ratio used here seem very small. This is partially due to the fact that it is

Exhibit 11

Aluminum Price with Filtered Trend for Different Signal-to-Noise Ratios and Trend Forecasts N December 2009 (Upper) and December 2012 (Lower)





the multiplication factor for the daily variance, so converting it into the corresponding factor for annualized volatilities, we obtain the range of 0.008÷0.015. Furthermore, the considered time horizon (the CCO's time to maturity) is five years, further increasing the range of q to 0.02÷0.03. Finally, the signal-to-noise ratio scales the variance of the trend's *slope*, in relation to the variance of the log-price deviations from the trend.

The variance of the trend is quite low, especially compared to the variance of the log-price. So the five-year volatility of the trend's slope being in the range of 2%–3% of the total volatility is a very reasonable assumption.

To summarize, the absolute magnitude of q depends on measurement units, volatilities, and other model parameters and, hence, is quite irrelevant. What matters is the relative difference between various signal-to-noise ratios used for trend estimation and forecasting. Generally, the choice of q is quite subjective. However, as the aluminium example shows, this choice can be critical for forecasting the price trends and hence, for assessing the CCO performance.

We generalize the smooth trend model of Durbin and Koopman [2001] to the case of a multivariate time series. In this case, it is important to realize that the trends for different commodity prices are correlated. These trend correlations are summarized in the matrix R, which is estimated from the filtered trends. Given the long-term nature of the trends, we prefer to estimate this correlation matrix from a long history of data rather than from the most recent few years of data.

For the purpose of simulations, the trend is forecasted at the time horizon T. (For details on how to forecast trends, we refer to Durbin and Koopman [2001]). Let the mean forecasted trends for all commodities at time T (minus its value at time $t = 0$) be summarized in the vector m, and let the standard errors of the forecasts be in the diagonal matrix D. The (estimated) covariance matrix Ω of the future trends is

$$\Omega = D\,R\,D \tag{3}$$

Let B be the Cholesky decomposition of Ω. Then the trend values at a future date T (again, relative to its value at time zero) are simulated as a normal random vector with mean m and variance-covariance matrix Ω, by taking:

$$r = m + BZ \tag{4}$$

where Z is a drawing from the multivariate standard normal distribution.

Having filtered out the trends from all commodity prices, we set $\alpha_{it} = 0$ in (2), obtaining the zero-mean Ornstein-Uhlenbeck process for the de-trended log price $\tilde{X}_{it}$

$$d\tilde{X}_{it} = -\kappa_i \tilde{X}_{it} dt + \sum_{j=1}^{g} c_{ij} dW_{jt} \tag{5}$$

The mean reversion speeds κ_i (and the variance-covariance matrix Σ) are estimated by discretizing (5) by, for example, Euler scheme, and using a computationally efficient way to compute the Generalized Least Squares estimator of κ_i's and Σ, given by Davidson and MacKinnon [2004]. On the contrary to R, the matrix Σ can be estimated from the recent data, to reflect growing return correlations and volatilities.

The next step in the simulation procedure is generating drawings from the zero-mean mean reversion model (5), at time T. Recall that the solution to (5) is given by

$$\tilde{X}_{it} = e^{-\kappa_i t} \tilde{X}_{i0} + e^{-\kappa_i t} \int_0^t e^{\kappa_i s} \sum_{j=1}^{g} c_{ij} dW_{js} \tag{6}$$

From (5) it follows that the distribution of $\tilde{X}_{iT} - \tilde{X}_{i0}$ is normal with

$$\begin{aligned}
E\left[\tilde{X}_{iT} - \tilde{X}_{i0}\right] &= \left(e^{-T\kappa_i} - 1\right)\tilde{X}_{i0} \\
Var\left[\tilde{X}_{iT} - \tilde{X}_{i0}\right] &= e^{-2T\kappa_i} \int_0^T e^{2\kappa_i s} \sigma_i^2 \, ds = \frac{\sigma_{ii}^2}{2\kappa_i}\left(1 - e^{-2T\kappa_i}\right) \\
Cov\left[\tilde{X}_{iT} - \tilde{X}_{i0}, \tilde{X}_{jT} - \tilde{X}_{j0}\right] &= \frac{\sigma_{ij}^2}{\kappa_i + \kappa_j}\left(1 - e^{-T\left(\kappa_i + \kappa_j\right)}\right)
\end{aligned} \tag{7}$$

where σ_{ij}^2's are the elements of the variance-covariance matrix Σ.

The simulated commodity log-prices at the time horizon T are obtained by combining the simulated trend values at T, given in (4), with the values of simulated mean reversion models (5), whose distributional characteristics are given in (7). After that, for the CCO analysis, the total number of trigger events is calculated for each simulation. The loss distribution of CCO tranches is obtained by the Monte Carlo simulation procedure, repeating the above steps many times. For other commodity-linked structured products, the performance measure is also calculated for each simulation (providing one realization of such a performance measure), and the overall performance measure distribution is obtained by this Monte Carlo procedure.

For CCOs as well as for many other products whose payoff depends only on the "state of the world" at maturity, there is a significant computational gain: Due to the European style of commodity trigger swaps, we do not need to simulate entire paths (or scenarios) of commodity prices from $t = 0$ until maturity T, but only the joint distribution of these prices at maturity T. However, American exercise feature or other path-dependent payoffs can be easily incorporated into our model by simulating the price paths from $t = 0$ until T.

We can reduce the dimensionality of the mean-reversion part of model (5) by reducing the number of fundamental factors (sources of uncertainty) driving the commodity prices. In model (5), there are as many sources of uncertainty as there are commodities in the reference portfolio. However, due to high correlations between some commodities (e.g., between industrial metals), it is possible to formulate the multivariate model in terms of a smaller number of fundamental factors, obtained by, for example, the Principal Component Analysis. Then the estimated multifactor MR model can be used to simulate evolutions of the factors and, via the Principal Component loading, also obtain simulated paths of the individual commodity residuals and, hence, prices. However, this extension is beyond the scope of this article.

The multivariate smooth trend model for commodity prices is quite flexible; it deals with non-stationarities (in mean) of the price series, while modeling dependencies between commodity price trends (long-term fluctuations) and between daily returns (short-term fluctuations). The model can be extended to also incorporate non-stationary variances and correlations; however, this would require complex filtering techniques and a much greater estimation effort.

A simple way of dealing with this non-stationarity (adopted in this article) is to estimate the variance-covariance matrix of the multivariate mean reversion from the recent data only. Moreover, as the commodity price trends are the main drivers of the CCOs and other commodity-linked notes performance, the smooth trend model seems sufficient in this respect.

In the next section we test the performance of our model on the basis of simulations and show that its performance is not very robust to the choice of the signal-to-noise ratio or to the starting date of the simulation process. The alternative is offered by a non-parametric simulation approach, the moving block bootstrap.

Moving Block Bootstrap Approach

Bootstrapping is a statistical simulation technique introduced by Efron [1979] for assessing properties of estimators, such as their variance, by resampling the data from the empirical distribution. In practice, it amounts to repeatedly drawing (with replacement) the data from the original sample and re-calculating the quantity of interest on the basis of a new data sample. For this method to work well, the data must represent independent drawings from a distribution. If there is a serial dependence in the data (such as in case of a time series), the resampling would destroy this dependence.

The solution was provided by Carlstein [1986] and Kunsch [1989] (and further improved by Lall and Sharma [1996]), who suggested resampling not the individual observations, but blocks of them (hence the name "moving block bootstrap"). The serial dependence within blocks is preserved in this way, but the dependence between blocks is destroyed, as those are resampled independently. However, if long-enough blocks are chosen and the serial dependence in the data series is not too strong, arguably this has a minimal effect.

More formally, the moving block bootstrap amounts to randomly selecting blocks of consecutive observations with replacement from a set of overlapping blocks. Given the data series $\{X_i, i = 1,\ldots,n\}$ from a (possibly multivariate) stationary time series, the method uses a set of blocks $B_1,\ldots,B_b$ of length l, where $B_i = \{X_i,\ldots, X_{i+l-1}\}$, $b = n-l+1$. Blocks are drawn randomly (and independently) from the set $B_1,\ldots,B_b$ and joined together to form a simulated time series.

In our study, the moving block bootstrap is applied to the *multivariate series* of commodity returns, and the simulated series are used for approximating the loss distribution of CCO tranches. In this way, the cross-correlation structure of the commodity returns is preserved; we resample blocks of returns *simultaneously* for different commodities, i.e., the blocks with equal start and end date.

As mentioned before, the drawback of the block bootstrap is the failure to fully preserve the serial dependence in the data. However, one of the stylized facts of financial returns is that they exhibit (nearly) no autocorrelation (see e.g., McNeal et al. [2005]). The absolute or squared returns can be autocorrelated, indicating volatility clustering. This autocorrelation structure is to a large extent preserved by the moving block bootstrap, if the block length is sufficient to reflect this serial dependence. The choice of the block length is left to the model builder. However, in the simulation study we show that the method is robust to the block length.

The block bootstrap method is not directly applicable to non-stationary time series. As we argued above, commodity markets underwent significant changes in the past decade. In particular, commodity cross-correlations and volatilities have increased significantly since 2005, which is a sign of the second order non-stationarity (heteroskedasticity). However, if the underlying stochastic structure changes *slowly* in time (which is indeed the case with commodity cross-correlations and volatilities, as witnessed by Exhibits 2 and 3), the so-called *Local Block Bootstrap* (LBB) can be employed.

This bootstrap method was introduced by Paparoditis and Politis [2002]. It amounts to resampling blocks that are close to each other in time, i.e., a block that starts at time t can be replaced only by blocks whose starting point is in some sense "close" to t. In this way, the slowly changing stochastic structure of the process is preserved in simulations. Paparoditis and Politis [2002] have shown that the Local Block Bootstrap works well in capturing distributions of constant as well as non-stationary moments of the process for heteroskedastic models.

In our application, we are interested in simulating commodity returns for the next several years (usually three to five years, depending on the product's maturity), so in this case it is not so much the past evolution of the process but its current state (reflected in the recent history) that is relevant. Applying the Local Block Bootstrap in this case results in resampling only relatively recent blocks of data, while completely disregarding older observations. This is equivalent to the regular Moving Block Bootstrap applied to the more recent part of the historical dataset.

So in our bootstrap simulations, instead of Local Block Bootstrap we present the results of the regular Moving Block Bootstrap applied to the past seven years of data—the period characterized by higher correlations and volatilities, and hence, better reflecting the current state of commodity markets—and compare them to those obtained from a longer historical dataset.

SIMULATION STUDY FOR CCO TRANCHES

We run an extensive simulation study to assess default probabilities of CCO tranches for the example CCO (GS CCO with five-year maturity). For this study we use historical data from July 1, 1993, until December 1, 2012. We assume that the new CCO is issued either on February 16, 2010, or on December 1, 2012, and we simulate the loss distribution of these CCO tranches after five years, using half a million simulation runs. One of our main goals is to analyze the sensitivity of the results to the model parameters and the issue date, thus assessing the model risk.

We compare the default probabilities of the simulated tranches with those implied by the agencies' ratings. Exhibit 12 shows the default probabilities for different ratings, by S&P and Fitch, as of 2005 (upper table) and 2011 (lower table). It is not always clear exactly how the agencies define the "default" of a tranche, i.e., whether it is an event when the entire notional or just some part of the notional is lost at maturity T. So in our simulation study, we simulate probabilities for both types of events, where the entire notional of a tranche is lost ($N_T = 0$) and a part of the notional is lost ($0 \leq N_T < N_0$).

There are other ways (than default probability) to quantify risks of CCO tranches, such as calculating expected losses (the approach used by Moody's) or Value at Risk (VaR). Our calculations can be extended into both these directions. For example, the expected loss can be calculated by determining the loss per tranche per scenario and then averaging the losses per tranche over all scenarios. The VaR per tranche can be calculated similarly.

Mean-Reversion with Trend

We applied the multivariate smooth trend model, together with the multivariate mean reversion (5), to all the commodities in our example, and calibrated the parameters on the basis of the two long datasets of commodity prices (from 1993 to 2010 and from 1993 to 2012) and on the past five years of data (2007–2012).

For the entire historical dataset, yearly historical return volatilities range between 21% (aluminium) to 38% (wheat). The historical return correlations range from −0.02 (between sugar and lead) to 0.65 (between zinc and other industrial metals, e.g., zinc and lead). The average correlation between all pairs of commodities' returns is 0.25.

For the recent data (2005–2012), all volatilities and correlations are significantly higher (as we have already observed); the volatilities range from 26% p/a for aluminium and platinum to 50% p/a for wheat. The correlations are in the range between 0.02 (between sugar and wheat) and 0.75 (also between zinc and other industrial metals), with the average correlation being 0.35.

One remarkable difference is the growing correlation between agricultural commodities, such as sugar or wheat, with other commodities such as industrial metals. While for the longer dataset these correlations are all nearly zero, for the data 2007–2012, these correlations are on average 0.1.

The model parameters calibrated on the entire dataset of 1993–2012 are reported in Appendix A. Estimated standard deviations σ_ω of the log-price residuals (obtained by subtracting the level and the slope from the log-price), are similar to (but smaller than) the above volatility estimates. (If the parameters are calibrated on the more recent data, these standard deviations increase in the same way as the returns' volatilities.)

Calibrated values of the mean-reversion parameters do not change much for different signal-to-noise ratios or time periods. The mean reversion speed κ ranges from 0.010 for corn and aluminium to 0.023 for silver.

EXHIBIT 12

Credit Curve for CDO Tranches, by S&P and Fitch, for Five Years to Maturity; 2005 (Upper Table) and 2011 (Lower Table)

Credit Rating	AAA	AA	A	BBB
P(default) in % by S&P	0.12	0.36	0.71	2.81
P(default) in % by Fitch	0.03	0.20	0.56	1.58

Credit Rating	AAA	AA	A	BBB
P(default) in % by S&P	0.37	0.37	0.66	2.30
P(default) in % by Fitch	0.16	0.32	0.64	3.33

The average value of κ is 0.014. The calibrated mean-reversion volatilities are again similar to (but smaller than) the historical return volatilities.

The estimated mean-reversion correlations also closely reflect the behavior of the historical returns' correlations, although they are smaller than the historical returns correlations. This is because some of the overall returns correlations are due to the correlations of the trends, summarized in R.

Next, we analyze the sensitivity of the trend forecasts and simulated default probabilities to the date of CCO issue. As in the aluminium example in Exhibit 11, we consider two cases: a new CCO is issued on December 1, 2009, and on December 1, 2012. The model is estimated with the data available up to that date and simulations are run with the obtained estimates.

The results are staggeringly different. Appendix A shows the average simulated trend values and average forecasted five-year returns for all commodities, for the signal-to-noise ratio q of $2 \cdot 10^{-9}$. While most trends forecasted in the end of 2009 are going up, the opposite is true for 2012, when almost all forecasted trends are down. This is reflected in the default probabilities, simulated with 500, 000 Monte Carlo runs and shown (in %) for the signal-to-noise ratio of $2 \cdot 10^{-9}$ in Exhibits 13 and 14 (the standard errors are in parentheses).

These default probabilities are much higher for the CCO issued in 2012 than in 2009, due to the forecasted downward trends. The exception is the BBB tranche, for which it is predicted (in 2009) that investors almost certainly lose part (but not all) of their notional. This effect is caused by an exceptionally large decline in corn and wheat prices over the period mid-2008 to mid-2009, which is reflected in the forecasted trend.

In the composition of our example CCO, corn has high trigger event levels (74%–80% of the starting value). This means that, even for a slightly decreasing trend, these trigger levels will definitely be hit five years from now, implying a very high probability that the BBB tranche will be hit.

As we argued above, the trend forecasts are quite accurate for one to two years ahead, but it is questionable whether we can trust these (or, for that matter, any) forecasts that stretch five years into the future. Many commodities experienced large price movements over the past few years. In mid-2000, there were huge increases in commodity prices, which ended in mid-2008 (onset of the financial crisis), when a sharp decline set in.

In the period starting mid-2009, many commodity prices have recovered, and in 2012, commodity prices either stabilized or exhibited downward trends, possibly due to the double-dip recession. This rollercoaster makes forecasting the direction of commodity prices with a mean reversion model rather hard, if not impossible.

The sensitivity of the forecasts and the simulated default probabilities to the signal-to-noise ratio is also quite high, though not as dramatic as to the simulations' starting point). Exhibits 15 to 18 show the default probabilities for both issue dates and the signal-to-noise ratios q of $3 \cdot 10^{-9}$ and $5 \cdot 10^{-9}$. (The standard errors are similar to those in Exhibits 13 and 14 and are omitted.)

For larger values of q, the differences between 2009 and 2012 issues are more pronounced: 2009 issue gets even smaller and 2012 issue even larger default probabilities. Recall that higher q gives more weight to the more recent observations in the sample period, so the simulated trend is more pronounced, as it is largely influenced by the most recent observations.

Generally, the choice of the signal-to-noise ratio should be dictated by the time to maturity of the structured product. If we need to simulate the commodity price distributions for a long time (five years) into the future, a relatively small signal-to-noise ratio is called for. In this case, we get a smoother trend, picking up only large price moves in the past few years, but not the new short-term fluctuations.

Exhibit 13

Simulated Probabilities of Default for CCO Tranches (Issue December 2009, $q = 2 \cdot 10^{-9}$)

Tranche	Super Senior	AAA	AA	A	BBB
$P(N_T = 0)$	0	0.01	0.21	1.14	2.50
	(0)	(0.002)	(0.01)	(0.02)	(0.20)
$P(0 \leq N_T < N_0)$	0.01	0.16	0.89	1.93	99.45
	(0.001)	(0.01)	(0.01)	(0.02)	(0.01)

Exhibit 14

Simulated Probabilities of Default for CCO Tranches (Issue December 2012, $q = 2 \cdot 10^{-9}$)

Tranche	Super Senior	AAA	AA	A	BBB
$P(N_T = 0)$	0.32	1.47	4.37	10.14	15.86
	(0.02)	(0.03)	(0.04)	(0.06)	(0.06)
$P(0 \leq N_T < N_0)$	1.24	3.80	8.78	13.65	46.48
	(0.02)	(0.03)	(0.05)	(0.05)	(0.10)

Exhibit 15

Simulated Probabilities of Default for CCO Tranches (Issue December 2009, $q = 3 \cdot 10^{-9}$)

Tranche	Super Senior	AAA	AA	A	BBB
$P(N_T = 0)$	0	0	0	0.18	0.41
$P(0 \leq N_T < N_0)$	0	0.01	0.14	0.32	99.35

Exhibit 16

Simulated Probabilities of Default for CCO Tranches (Issue December 2009, $q = 5 \cdot 10^{-9}$)

Tranche	Super Senior	AAA	AA	A	BBB
$P(N_T = 0)$	0	0	0	0.006	0.018
$P(0 \leq N_T < N_0)$	0	0	0.005	0.014	97.77

Exhibit 17

Simulated Probabilities of Default for CCO Tranches (Issue December 2012, $q = 3 \cdot 10^{-9}$)

Tranche	Super Senior	AAA	AA	A	BBB
$P(N_T = 0)$	1.83	5.76	14.47	27.94	37.78
$P(0 \leq N_T < N_0)$	4.92	12.91	25.16	34.25	72.33

Exhibit 18

Simulated Probabilities of Default for CCO Tranches (Issue December 2012, $q = 5 \cdot 10^{-9}$)

Tranche	Super Senior	AAA	AA	A	BBB
$P(N_T = 0)$	4.47	11.72	26.36	45.57	55.89
$P(0 \leq N_T < N_0)$	10.14	23.97	42.28	52.46	82.93

In all, we find that the parametric modeling for the application at hand carries a lot of model risk. Sensitivities to the model parameters as well as to the historical data period are quite high. So, next we show the performance of the non-parametric method of bootstrap, which offers a viable alternative to the above parametric approach.

Moving Block Bootstrap

The moving block bootstrap is used to simulate the loss distributions for the tranches of the same CCO example. We simulate five-year multivariate series of commodity returns and, for each simulated path, calculate CCO tranche losses. The simulations are carried out 500 000 times, for several block lengths: 2, 5, 10, 20, 30, and 60 trading days.

To investigate the effects of the financial crisis and observed structural changes in commodity markets, four sets of historical data (from which we resample blocks) are considered separately: July 1993–December 2007 (relatively quiet period, characterized by low cross-commodity correlations), July 1993–December 2009 (longer dataset that includes the volatile crisis period as well as large declines and subsequent recoveries in commodity prices), July 2005–December 2012 (the most recent history, which includes the volatile crisis period, subsequent price recovery, and higher cross-commodity correlations), and the entire historical dataset of July 1993–December 2012. Note that, in the moving block bootstrap method, the future trends are simulated not directly but implicitly, by resampling blocks of returns from periods with different persistent price trends.

Exhibits 19 to 23 show the simulated probabilities of default (in %), defined as $PD = P(0 \leq N_T \leq N_0)$, for each historical dataset, tranche and block length. The standard deviations are omitted for the sake of clarity; their values are comparable to those obtained by the parametric method.

The first observation is that the results are quite robust to the block length. The differences in default probabilities for different block lengths are just fractions of a percent (or 5%–10% of default probability values, with a couple of exceptions for senior tranches and higher block lengths).

The second observation is that, when sampling from the set of historical returns during 1993–2009, the block bootstrap provides higher probabilities of default in comparison to those obtained from 1993–2007,

Exhibit 19

Simulated Probabilities of Default for CCO Tranches (Block Bootstrap Method); 1993–2007, $PD = P(0 \leq N_T \leq N_0)$

Block Length	BBB	A	AA	AAA	SS
2	12.024	2.066	1.574	0.099	0.010
5	12.038	2.207	1.710	0.104	0.008
10	12.115	2.412	1.859	0.137	0.009
20	11.597	2.154	1.645	0.130	0.014
30	11.936	2.201	1.686	0.137	0.010
60	12.158	2.113	1.622	0.103	0.004

Exhibit 20

Simulated Probabilities of Default for CCO Tranches (Block Bootstrap Method); 1993–2009, $PD = P(0 \le N_T \le N_0)$

Block Length	BBB	A	AA	AAA	SS
2	18.766	4.809	3.818	0.687	0.180
5	18.699	4.995	4.009	0.705	0.176
10	18.293	5.056	4.068	0.836	0.226
20	17.478	4.892	3.918	0.957	0.304
30	17.126	4.764	3.820	0.973	0.332
60	18.554	5.822	4.783	1.750	0.806

Exhibit 21

Simulated Probabilities of Default for CCO Tranches (Block Bootstrap Method); 2005–2012, $PD = P(0 \le N_T \le N_0)$

Block Length	BBB	A	AA	AAA	SS
2	0.110	0.063	0.053	0.032	0.017
5	0.113	0.067	0.057	0.035	0.019
10	0.098	0.058	0.048	0.030	0.016
20	0.075	0.043	0.035	0.022	0.011
30	0.049	0.025	0.020	0.012	0.006
60	0.045	0.025	0.021	0.013	0.007

Exhibit 22

Simulated Probabilities of Default for CCO Tranches (Block Bootstrap Method); 1993–2012, $PD = P(0 \le N_T \le N_0)$

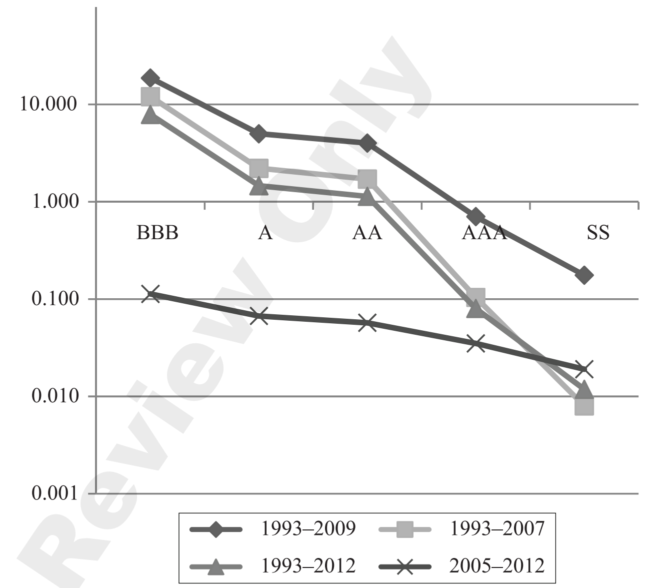
Block Length	BBB	A	AA	AAA	SS
2	7.880	1.369	1.045	0.076	0.012
5	7.890	1.463	1.135	0.080	0.012
10	7.935	1.593	1.229	0.100	0.011
20	7.589	1.420	1.085	0.092	0.013
30	7.801	1.444	1.107	0.094	0.009
60	7.945	1.387	1.065	0.072	0.005

2005–2012, or 1993–2012 data, as Exhibit 19 clearly shows. These results correctly suggest the inevitable downgrading of the highly rated CCO tranches in the immediate aftermath of the crisis.

On the other hand, resampling from 2005–2012 data leads to much lower probabilities of default, due to the remarkable recovery in commodity prices in 2010. Note that these lower default probabilities arise *despite* higher correlations and volatilities observed during this period, which, on their own, would lead to poorer performance of CCO tranches.

Exhibit 23

Default Probabilities $PD = P(0 \le N_T \le N_0)$ on Log Scale for all Tranches, Simulated by the Block Bootstrap with Block Length 5



A similar picture emerges if another definition of the default probability is considered: $PD = P(N_T = 0)$. Exhibits 24 to 27 show the corresponding simulated probabilities of default. Again, for the historical dataset 1993–2009, the simulated default probabilities are higher, and for 2005–2012 lower, for nearly all tranches.

Exhibit 28 shows the default probabilities calculated with the historical data from 1993 to 2009, as functions of the block length. Although the results are quite robust to the block length, we see that using very long blocks (of 60 trading days) results in slightly higher default probabilities for all tranches. Using even longer blocks amplifies this effect further. However, this effect diminishes if we use the data from 2005–2012 and dis-

Exhibit 24

Simulated Probabilities of Default for CCO Tranches (Block Bootstrap Method); 1993–2007, $PD = P(N_T = 0)$

Block Length	BBB	A	AA	AAA	SS
2	2.254	1.734	0.115	0.012	0.002
5	2.392	1.871	0.121	0.009	0.001
10	2.614	2.039	0.156	0.012	0.002
20	2.340	1.812	0.152	0.016	0.003
30	2.391	1.838	0.160	0.013	0.002
60	2.291	1.784	0.120	0.006	0.000

EXHIBIT 25

Simulated Probabilities of Default for CCO Tranches (Block Bootstrap Method); 1993–2009, $PD = P(N_T = 0)$

Block Length	BBB	A	AA	AAA	SS
2	5.188	4.131	0.770	0.209	0.072
5	5.376	4.321	0.792	0.201	0.065
10	5.435	4.382	0.934	0.258	0.096
20	5.258	4.220	1.057	0.342	0.147
30	5.116	4.118	1.070	0.372	0.167
60	6.221	5.117	1.886	0.877	0.482

EXHIBIT 26

Simulated Probabilities of Default for CCO Tranches (Block Bootstrap Method); 2005–2012, $PD = P(N_T = 0)$

Block Length	BBB	A	AA	AAA	SS
2	0.067	0.056	0.034	0.020	0.010
5	0.071	0.060	0.037	0.023	0.012
10	0.061	0.051	0.032	0.019	0.010
20	0.046	0.038	0.023	0.014	0.007
30	0.027	0.022	0.013	0.007	0.004
60	0.027	0.022	0.014	0.008	0.004

EXHIBIT 27

Simulated Probabilities of Default for CCO Tranches (Block Bootstrap Method); 1993–2012, $PD = P(N_T = 0)$

Block Length	BBB	A	AA	AAA	SS
2	1.493	1.150	0.088	0.015	0.0048
5	1.585	1.241	0.092	0.014	0.0048
10	1.726	1.348	0.113	0.014	0.0048
20	1.542	1.1950	0.107	0.015	0.0044
30	1.569	1.206	0.109	0.011	0.0027
60	1.504	1.171	0.083	0.007	0.0014

appears completely when using only pre-crisis data of 1993–2007.

This is due to the fact that the commodity prices have shown strong negative trends in 2008–2009, which is only captured in larger blocks. This ensures that all tranches (and, in particular, AAA and SS) get hit more often for simulations with a large block size. In other words, simulating with longer blocks enables us to capture longer clusters of simultaneous decrease of the prices, observed during the crisis period. For the data from 2005–2012, this effect is compensated by the strong increase in prices in 2010.

The default probabilities simulated using the longest dataset (1993–2012) come closest to those used by the rating agencies for the corresponding tranches. This might be a coincidence, but it might also point toward the conclusion that the rating agencies probably use long historical datasets for their default probabilities calculations.

In all, we find that the moving block bootstrap provides a viable and robust approach for simulating CCO tranches' loss distributions. It is robust to the block length and quite robust to the historical dataset from which the blocks are resampled. The differences in results for different historical data periods indicate that an appropriate part of the historical data, from which the returns are resampled, can be chosen to reflect the risk aversion of the investor and the current outlook for commodity prices.

CONCLUSIONS

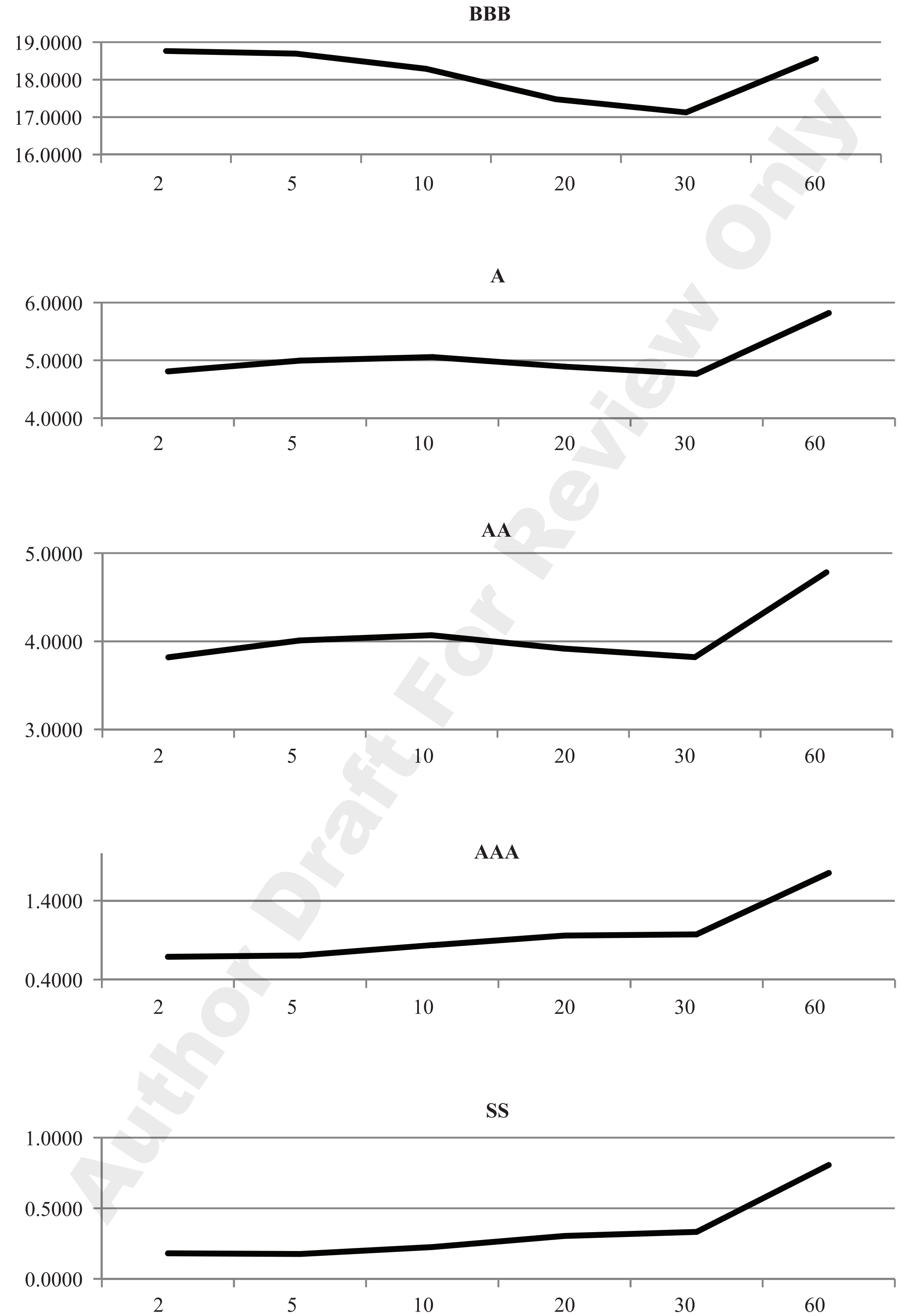
We have investigated methods for the risk assessment of commodity linked structured products on the example of Collaterized Commodity Obligations (CCOs), relatively new fixed-income instruments providing commodity exposure. We examined two methods for obtaining CCO tranche default probabilities: a parametric model of multivariate mean reversion with stochastic trend, and a non-parametric data-driven algorithm of moving block bootstrap. Both methods have the advantage of directly modeling the correlations among commodities, which are the essential characteristics of the underlying multivariate behavior.

Our parametric method is quite good at modeling an ensemble of commodity prices and capturing short- to medium-term trends. But it is perhaps less suited to the application at hand, namely CCO analysis, due to the long horizons at which the returns distribution must be forecast. The parametric approach appears to be quite sensitive to the choice of model parameters, such as the signal-to-noise ratio, and to the historical data period. Hence, it carries a significant model risk.

On the contrary, the block bootstrap method is less sensitive to the historical data period and is robust to the block length choice. It produces default probabilities that most closely match those used by the rating agencies to rate tranches of structured products. Moreover, the nonparametric data-driven approach is well suited to the analysis of CCOs and other commodity linked products, due to the abundance of historical commodity prices.

Exhibit 28

Probability of Default (in %) vs. Block Length (in trading days) for Different Tranches



APPENDIX

ESTIMATION AND SIMULATION RESULTS FOR THE PARAMETRIC MODEL

EXHIBIT A1

Calibrated Residual Standard Deviations and Mean Reversion Speeds, 1993–2012

Commodity	σ_ω	κ	s.e.(κ)
Aluminum	0.106	0.010	0.0016
Copper	0.143	0.011	0.0014
Lead	0.152	0.014	0.0017
Tin	0.130	0.011	0.0019
Zinc	0.143	0.014	0.0016
Nickel	0.189	0.013	0.0017
Silver	0.112	0.023	0.0026
Platinum	0.114	0.014	0.0017
Palladium	0.182	0.014	0.0018
Corn	0.150	0.010	0.0018
Wheat	0.145	0.022	0.0025
Sugar	0.148	0.015	0.0025

EXHIBIT A3

The Simulation Results for the Signal-to-Noise Ratio *q = 2e-9* and Dec. 2009: Trend Forecast (As A Fraction of Current Trend) with 95% Confidence Bounds (Trend, UB and LB), the Average Five-Year Returns with their Standard Deviation

Commodity	Trend	LB	UB	5-year log Returns	(se)
Aluminum	0.744	0.542	1.021	–0.455	0.1914
Copper	1.673	1.081	2.588	0.333	0.2568
Lead	1.879	1.177	3.000	0.513	0.2762
Tin	0.884	0.611	1.278	–0.244	0.2223
Zinc	2.280	1.449	3.588	0.633	0.2585
Nickel	0.877	0.489	1.573	–0.401	0.3355
Silver	2.109	1.575	2.823	0.776	0.1724
Platinum	1.284	0.909	1.815	0.088	0.1987
Palladium	2.302	1.332	3.980	0.517	0.3136
Corn	0.405	0.263	0.622	–0.917	0.2577
Wheat	0.185	0.124	0.276	–1.915	0.2360
Sugar	12.545	8.165	19.273	2.462	0.2545

EXHIBIT A2

Calibrated Residual Standard Deviation σ_ω for Different Signal-to-Noise Ratios

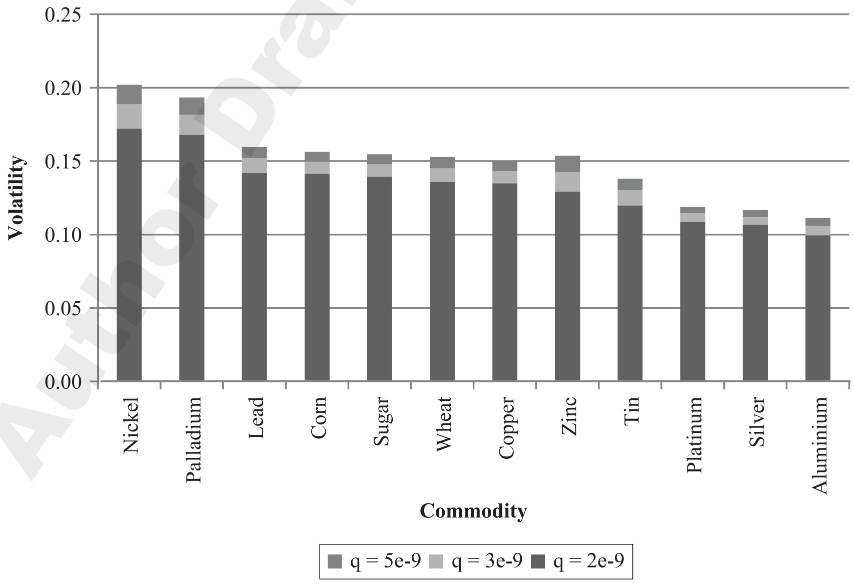


EXHIBIT A4

The Simulation Results for the Signal-to-Noise Ratio *q = 2e-9* and Dec. 2012: Trend Forecast (As a Fraction of Current Trend) with 95% Confidence Bounds (Trend, UB and LB), the Average Five-Year Returns with their Standard Deviation

Commodity	Trend	LB	UB	5-year log Returns	(se)
Aluminum	0.499	0.364	0.684	–0.671	0.189
Copper	0.706	0.461	1.079	–0.305	0.248
Lead	0.602	0.384	0.946	–0.594	0.267
Tin	0.463	0.313	0.684	–0.775	0.233
Zinc	0.572	0.371	0.884	–0.551	0.249
Nickel	0.314	0.177	0.555	–1.131	0.327
Silver	0.980	0.705	1.363	0.004	0.195
Platinum	0.694	0.496	0.971	–0.385	0.193
Palladium	0.662	0.383	1.143	–0.355	0.311
Corn	2.433	1.564	3.785	0.943	0.264
Wheat	2.290	1.486	3.528	0.750	0.252
Sugar	0.464	0.300	0.718	–0.655	0.261

ENDNOTES

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[1]We used the daily five-year U.K. swap rate for coupons and discounting.

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