Reverse Stress Testing

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Abstract

This article outlines a framework for the analysis of extreme events based on forward-looking reverse stress testing. We carry out a portfolio simulation and identify stress scenarios which are critical for bank solvency as the ones contributing the most to cost of capital, as expressed by KVA scenario differentials.

Applications include model validation, trading limits, model risk management and hedging. A pricing model is invalid if it breaks on a path leading to stress conditions, causing alpha leaks that go undetected in market risk models such as value-at-risk (VaR), expected shortfall and stressed VaR. Trading limits are best predicated on incremental cost of capital and model risk capital can be assessed by computing cost of capital with Bayesian averages. Stress scenarios also have the potential to suggest risk-reducing hedges.

Keywords: model validation, stress testing, reverse stress testing, capital models, risk margins, trading limits, cost of capital, KVA, model risk, short rate models, PFE.

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Contents

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1 Introduction

Regulated entities such as broker-dealers and insurance firms don't have full freedom in their modeling choices.

In classical portfolio optimization theory, investors express their views and preferences by selecting a risk measure and a model for generating future scenarios (see de Finetti (2017) and Markowitz (1952)). If more than one model is selected, uncertainty is accounted for via Baysian averages (see Black and Litterman (1990)).

Market makers instead are subject to various regulations. The CCAR/ICAAP stress testing framework, the Stressed VaR metric (see Federal Reserve Board (2019)), the guidelines for model validation (see Federal Reserve Board (2011)) and the regulatory metrics for trade limits (see European Banking Authority (2011)) are all examples of the constraints to banks' portfolio optimization, which were introduced by regulators with the primary purpose of safeguarding banks' long term solvency.

Examples of backward looking or *direct* stress testing include the CCAR/ICAAP selection of historical stress scenarios and macro-scenario projections using rating transition probabilities as in Bangia et al. (2002). In direct stress testing, one infers stress scenarios from macro-economic statistics and then evaluates the bank's resilience under these scenarios.

Reverse stress testing instead is based on forward looking analytics. Stress scenarios are then data-mined out of a sample obtained by means of a forward-looking portfolio simulation. The scarce literature on the topic of reverse stress tasting is summarized in Grundke (2011). The interest on reverse stress testing has been recently highlighted by the BIS and the FSA, who advocate the practice as a way to share the responsibility of identifying potential future adverse states with the financial sector. Although preliminary approaches mimic the spirit of CCAR and ICAAP historical scenario selections, we believe there is merit to attempt a more systematic approach based on large scale portfolio simulations based on synthetic scenarios.

In this article, we set out a theoretical framework and a computational methodology for the analysis of extreme events in the form of forward-looking reverse stress testing, gathering the future scenarios under which bank solvency could be impaired. The simulation targets the bespoke characteristics of the portfolio held, and the scenarios are generated using the model of choice of the bank. We define our stress scenarios as the ones under which the capital demand and dividend accruals for a bank reach its peak, as assessed by an economic capital based KVA metrics. The more computationally intensive exercise creates the opportunity to identify both the systemic and the idiosyncratic weaknesses of the portfolio, without relying on historical events.

Although extreme stress scenarios could impair solvency, once identified, there is a chance they can be hedged. A typical risk transfer hedging strategy to ameliorate the risk under stress scenarios triggers a negative KVA change to the bank and may consequently give rise to a day-one profit within the KVA accounting framework in IFRS17 for insurance contracts, see IASB (2017). A discussion of this standard in the context of banking portfolios is in Albanese, Caenazzo, and Crépey (2016). Cost-of-capital reducing trades can either be transacted with external entities or be executed on an intra-dealer basis as cross-selling opportunities.

The possibility, at least in principle, to hedge stress scenarios, is essential from the viewpoint of model validation. The crucial caveat is that hedging stress scenarios is particularly exposed to model risk: the use of economically unrealistic models carries more risks than rewards. In our case study, the Hull-White model signals solvency risk in unrealistic deep negative rates (e.g., at -350 bps), which entice buying floors struck at unrealistically deep negative rates as a capital optimization strategy. As detailed in Albanese, Crépey, and Iabichino (2020), a misspecified model results in a systematic alpha-leakage for the hedged process which results from dynamic recalibrations and is typically autocorrelated as a function of time. Alpha leakage is mathematically a negative drift term in a Doob-Meyer decomposition and, by definition, will go undetected by market risk models such as value-at-risk (VaR), expected shortfall and stressed VaR. In our opinion, a pricing model should be declared invalid if systematic recalibrations are required on a path leading to identified stress conditions.

To further manage the uncertainty of model choice and assess capital requirements for model risk, we propose a variation of the Black and Litterman (1990) methodology for portfolio optimization. Taking a Bayesian average among valid models by generating a composite scenario set based on multiple model specifications typically increases capital requirements. The difference derives from model risk and can be used as a model risk capital buffer.

A solvency based reserve stress test could also be useful to strengthen trading limits. We identify a new credit limit metric that is sensitive to the idiosyncratic risks inherent to a counterparty and their correlation with all the other relevant risk factors.

This paper is organised as follows. Section 2 develops our economic capital based KVA reverse stress testing methodology, as well as its long-term hedging implications. Section 3 revolves around model validation and model risk. Section 4 is on credit and trading limits. Section 5 concludes.

2 The Reverse Stress Testing Framework

2.1 Economical Capital Based KVA Setup

In the line of Albanese, Caenazzo, and Crépey (2016, 2017), Crépey, Sabbagh, and Song (2019), Albanese, Crépey, Hoskinson, and Saadeddine (2019), and consistently with IFRS17, we define the KVA as a risk margin metric used for remunerating bank shareholders at some hurdle rate h for their capital at risk. Moreover, instead of implementing Pillar I regulatory capital models, we follow a more principled Pillar II economical based approach.

Indeed, capital models under Pillar I are consistent in intent with our assumptions, but they have numerous assumptions motivated by the desire to reduce the computational complexity. For instance, the definition of EEPE takes a single counterparty centric view; default clustering is neglected as are complex tail dependencies; Gaussian copulas in the default RWA are highly stylized assumptions and the possibility to create doom loops (e.g., between the CVA RWA and CDS hedges) are neglected. Pillar I approximations also unlink risk capital from the volatility of XVA metrics, a feature which however has been identified by regulators as one of the primary drivers of the 2008 financial crisis.

Instead, following a Pillar II approach, we anchor risk capital at the level of an economic capital (EC) defined as the 97.5% expected shortfall of core equity tier 1 capital (CET1) depletions over a one-year time period. CET1 is the highest quality of a bank's capital and is earmarked to absorb unexpected losses. The market and counterparty credit losses deplete CET1 and they are thus covered by capital inflows.

For notational simplicity, we use the risk-free asset growing at the OIS rate as a numéraire. We assume the physical measure equal to the prevailing pricing measure, for lack of faithful knowledge about the discrepancy between the two at the very large horizon of XVA computations (i.e. the final maturity of the portfolio assumed held on a run-off basis). Denoting by $\mathbb{E}_t$ the conditional expectation with respect to the pricing probability measure, we set

$$\mathrm{EC}_t = \mathbb{E}_t\big[\mathrm{CET1}_t - \mathrm{CET1}_{t+1} \ \mid\ \mathrm{CET1}_t - \mathrm{CET1}_{t+1y} \geq \mathrm{VaR}_t\big], \tag{1}$$

where

$$\mathrm{VaR}_t = \inf\{y; \mathbb{E}_t \mathbb{1}_{\{\mathrm{CET1}_t - \mathrm{CET1}_{t+1} \leq y\}} \geq 97.5\%\} \tag{2}$$

is the corresponding conditional value-at-risk with confidence level 97.5%.

Our KVA is defined as, similarly to a risk margin in Solvency II,

$$\mathrm{KVA}_t = \mathbb{E}_t \int_t^{\infty} h(\mathrm{EC}_s - \mathrm{KVA}_s)^+ ds. \tag{3}$$

Here $(\mathrm{EC}_s - \mathrm{KVA}_s)^+$ corresponds to shareholder capital at risk and h is an intertemporal hurdle rate representing a target for earnings per unit of capital invested. Hence

The EEPE is defined as (in the case of an unsecured portfolio) $\int_0^1 \max_{s\in[0,t]} \mathbb{E}(\mathrm{MtM}_s^+)dt$, where MtM is the counterparty-risk free valuation of the portfolio.

$h(\mathrm{EC}_s - \mathrm{KVA}_s)^+$ corresponds to the instantaneous dividend return rate paid by the bank and the KVA values dividend accruals. The choice of h (e.g. 12%) impacts stress scenarios, as a smaller h emphasizes longer term risk and vice-versa.

It holds that $\mathrm{KVA}_0 \geq 0$, with equality to 0 if and only if the CET1 process is constant over time. This property, added to the interpretation of KVA_0 as a weighted average of future economic capital, make it a very natural metric for the inter-temporal risk of the bank.

In practice, our (time 0) KVA is based on a simulation of CET1 depletions that is used for projecting out a term structure of EC obtained by replacing $\mathbb{E}_t$ by $\mathbb{E}_0$ in (1)-(2).

To illustrate our methodology, we discuss throughout the paper a case study concerning the counterparty credit risk embedded into a portfolio of about 2,500 counterparties, 100,000 derivative trades with exposure to rates, credit, and foreign exchange factors for G10 currencies. Using models calibrated to option prices and correlated with dynamic copulas, we use a simulation of both market and credit risk factors over 200 time points, covering 50 years, and 20,000 primary scenarios. We use nested simulations to account for portfolio CVA and FVA volatility. Nested simulations are computed by branching off an additional 1,000 secondary scenarios at yearly frequency, covering the residual life of the portfolio. These results, generated with a calculation time of around 3 hours on a single server, are then used to compute the distribution of CET1 requirement over time. Figure 1 depicts CET1 consumption over time for the OTC book considered in our case study, simulated as per (12), i.e. assuming market risk perfectly hedged and counterparty credit risk unhedged.

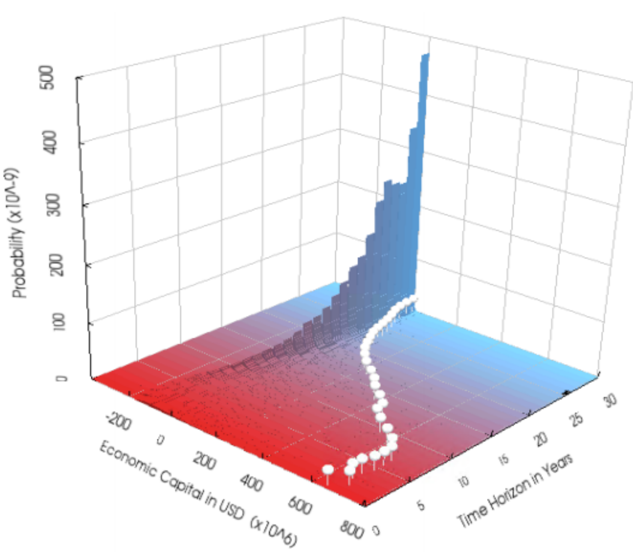

Figure 1: Distribution of CET1 demand over time. The white dots mark the economic capital profile at the 97.5% confidence level.

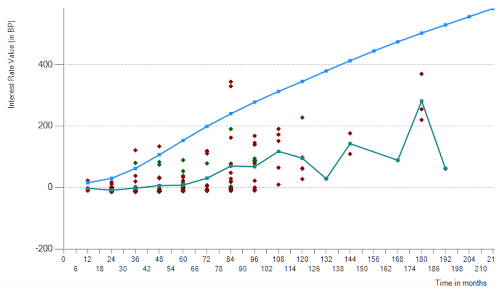

Figure 2: USD OIS overnight rate (in BPS) for stress scenarios using the SD model (9)). The blue and green lines represent the forward curve and the forward curve conditional to stress scenarios (average of the dots), respectively.

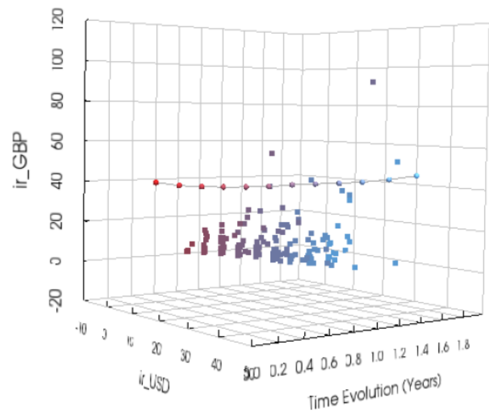

Figure 3: USD OIS vs. GBP SONIA overnight rates (in BPS) for stress scenarios using the SD model.

2.2 Scenario Differential KVA and Reverse Stress Testing

The KVA naturally probes stress scenarios where the whole portfolio suffers from extreme losses. The relative contribution of each scenario ω to the overall KVA, denoted

$$\delta_\omega \mathrm{KVA}, \tag{4}$$

is defined as the difference between the KVA and the KVA recomputed omitting scenario ω. In view of (3), δ_ωKVA represents the dividend accruals paid by the bank to rise capital, under scenario ω. The metric δ_ωKVA enables us to rank scenarios as a function of their total dividend liabilities, which are proportional to the risk incurred along the scenario ω.

Among the pool of scenarios, we select stress scenarios in the confidence interval $[99\%, 99.9\%]$, i.e. we select the most adverse 180 scenarios out of the 20,000 primary ones (where the 20 most extreme were neglected).

A drill-down analysis of stress scenarios identifies the points in time where the most substantial CET1 depletions occur. Through an interval-by-interval analysis, we gather the vector collecting the corresponding states of each economic factors. In the univariate projection of the analysis in Figure 2, as well as in the bivariate projection of Figure 3, each depicted point is sourced from one Monte Carlo scenario, signaling the risk factor state at the time where the largest CET1 depletion materializes.

Having identified the most adverse economic states, we then extend our analysis to identify the counterparties which exacerbate CET1 depletion in those states (see Figure 4).

2.3 Hedging Solvency Risk

Knowledge of stress scenarios helps identify candidate trades that have the potential of reducing cost of capital.

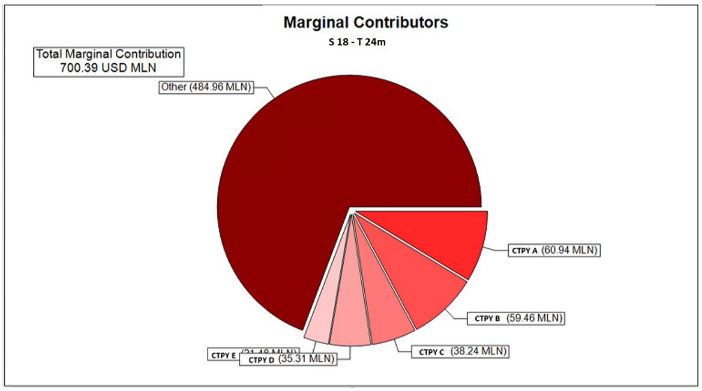

Figure 4: Top 5 marginal contributors to an extreme stress scenario.

More precisely, if EP is the entry price of a candidate trade and if $\Delta\text{CA} = \Delta\text{CVA} + \Delta\text{FVA}$ is the implied change in reserve capital requirements, then it is advantageous to execute the trade at the condition that

$$\Delta\text{KVA} + \Delta\text{CA} + \text{EP} < 0. \tag{5}$$

In a hedging trade the bank is a price taker and the counterparty a price maker. The entry price EP in general is not the fair valuation of the transaction since it includes the valuation adjustments of the counterparty and a profit margin (i.e. the counterparty KVA in case the counterparty is a broker dealer).

A precise accounting treatment for cost of capital is only available at present in the insurance industry and is part of the IFRS 17 standard. Most banks use KVA as a guideline to make economically sound decisions but do not reflect it into accounting statements, as they do for CVA and FVA. Nevertheless, if banks were to adopt a formal KVA accounting framework along the lines of IFRS 17, then a hedging transaction satisfying (5) yields a day one profit whose size is the negative of the left hand side, i.e.

$$\text{day-one-profit} = -(\Delta\text{KVA} + \Delta\text{CA} + \text{EP}) > 0. \tag{6}$$

Such a situation bears vague analogies with the Black-Scholes-Merton theory that delta-hedging an option does not change its valuation but can reduce the risk to the market maker to zero. We are in a very different context of market imperfections, lack of completeness, non-zero cost of capital, and cost of funding that go beyond the traditional Modigliani-Miller assumptions. In our case, the solvency risk analysis identifies long-run hedges, quite different in spirit from short-term delta hedges, that potentially reduce cost of capital.

Due to established monetary policies, low rates environments occur at the downturn of credit cycles. Consequently, default losses are more significant when interest rates are lower. For the chosen subset of risk factors in our portfolio, the correlation between stress scenario occurring during a downturn of the credit cycle is well visible from Figures 2 and 3. The analysis also reveals the presence of a second cluster which is idiosyncratic to the portfolio and reflects concentrations of cash-flows or convexity at certain points in time and under specific conditions in the future.

Due to the balance of offer and demand, entry prices for macro hedges typically embed a higher premium than entry prices for idiosyncratic risk. In view of the condition (5), hedges discovered by reverse stress testing are relatively more effective at evening out idiosyncratic risk as this attracts lower profit premia. In the case of macroeconomic hedges, (5) only allows one to place an upper bound on macro risk, depending on the premium embedded in the cost of hedging and provided capital optimization strategies are executed systematically.

The least onerous optimization trades in terms of liquidity premium on the entry price are the ones executed internally to a bank, i.e. cross-selling between trading desks. Optimizing trades with external counterparties, be it other broker dealers or buy-side funds, normally attract a profit margin.

Figure 5 provides an executive summary of our economic capital KVA based, long-run hedging oriented, reserve stress testing methodology.

3 Model Validation

In the two sections that follow, keeping fixed all the other model components, we investigate the reverse stress testing properties of different interest rates models. For the sake of simplicity, we focus on just two models, and we discuss individually their features in terms of extreme scenarios, XVA numbers, and the different actionable insight they suggest in terms of risk appetite and hedging.

3.1 Champion and Challenger

As "champion" model, we choose the Hull and White (1990) 2-factor model (HW), a Gaussian model which, thanks to its analytical solution, is highly used in the XVA industry to speed up computation times. As "challenger", we choose the 2-factor stochastic drift model (SD) of Albanese and Trovato (2008). Both models are based on a mean reverting process of the form

$$d\rho_t = \kappa(\theta_t - \rho_t)dt + \sigma(t)\rho_t^{\beta} dW^{(1)}, d\theta_t = k(a - \theta_t)dt + \nu dW^{(2)}, \tag{7}$$

with $d\langle W^{(1)}, W^{(2)}\rangle_t = \gamma dt$.

In the HW model, the short rate $r_t = \rho_t$ is an unbounded process, the parameter β equals 0, and the deterministic term structure for $\sigma(t)$ is calibrated to market observables. In the SD model, instead, the short rate process is defined by an equation of the form

$$r_t = \phi(t) + \lambda(t)\rho_t. \tag{8}$$

Here $\lambda(t)$ is a drift adjustment factor and

$$\phi(t) = \min(0, f(t) - \psi), \tag{9}$$

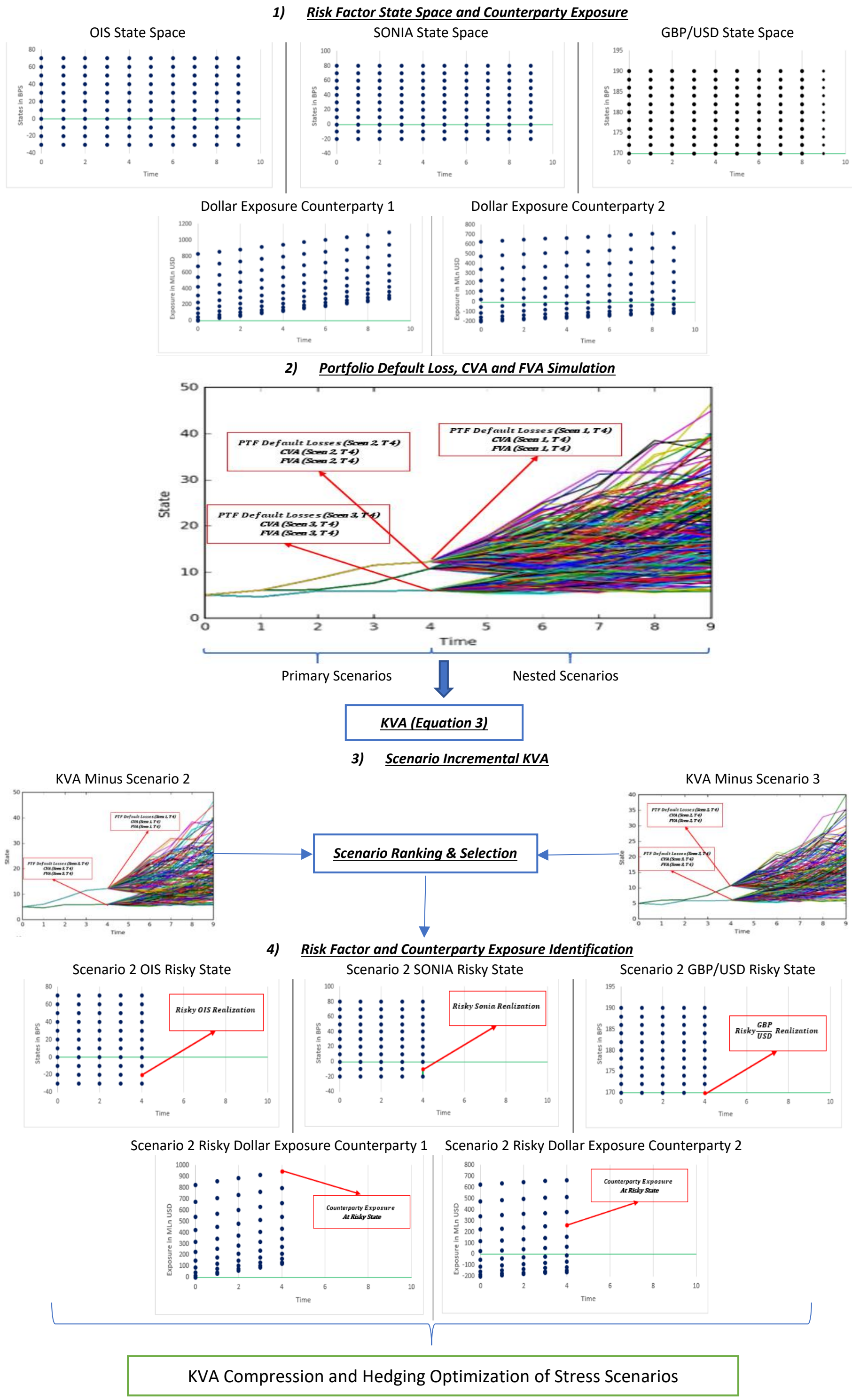

Figure 5: Reverse Stress Testing engine.

where $f(t)$ is the infinitesimal forward rate, and ψ (equal to 20 bp in our case study) is a bound calibrated econometrically. The process ρ_t is constrained to take non-negative values (apart for a handful of basis points expressed in ψ), the parameter β is adjustable, and the calibrated term structure $\sigma(t)$ is constrained to be of the form of a shifted exponential.

3.2 Distorted Risk Appetite

Table 1 compares time-0 XVA metrics computed with the two interest rate models and, as a common credit component, the distance-to-default model with stochastic volatility of Albanese and Vidler (2007) (still assuming market risk perfectly hedged and counterparty credit risk unhedged). As could expected, the KVA appears as the most model sensitive metric. In Figure 6, we compare the economic capital term structure projected with the SD model and the one projected with the HW model. We notice that the economic capital term structure generated with the HW model (in blue) is visibly more pronounced, signaling an overall higher economic capital demand for the portfolio.

Table 1: Comparison of the time-0 XVA metrics (in million USD) between the SD and HW models. FVA (RHO) accounts only for the re-hypothecation option, where FVA (RHO + EC + CVA + FVA) deems reserve and economic capital as a fungible source of funding (cf. Crépey et al. (2019)).

	SD	HW	Model Risk
CVA	242	248	2.5%
FVA (RHO)	126	109	-13%
FVA (RHO + EC + CVA + FVA)	62	45	-27%
KVA	275	388	41%

We recall that, in the reverse stress testing exercise, we isolate the scenarios that correspond to the highest risk capital demand. The different set of stress scenarios affects the location of the 97.5% quantile which underlies the definition of economic capital and KVA.

The comparison between Figures 2 and 7 shows the distorted actionable signals that econometrically unrealistic models can provide to end-users, as extreme scenarios and their hedges are very receptive to model assumptions. The HW model clusters the bulk of the portfolio risk in the short term. In contrast, the SD model disperses the risk more homogeneously over the lifetime of the OTC book, while skewing only slightly the risk on shorter maturities. In the SD model extreme states are low but positive or nearly positive rates (see Figure 2). In contrast, in the HW model, extreme scenarios are deep, unrealistic, negative rates (see Figure 7).

It is straightforward to note how low-quality models can turn in a double-edged sword to the end-user looking for actionable insights. However, macroeconomic hedging

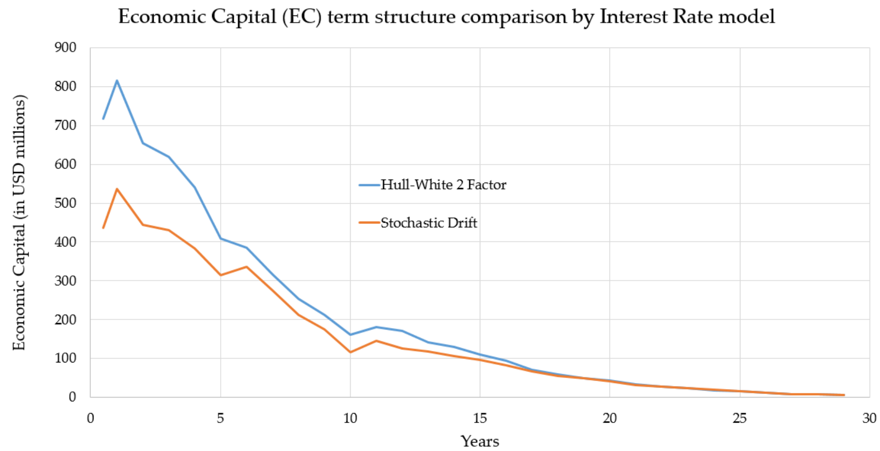

Figure 6: Comparison of the economic capital term structures computed with the HW and the SD models.

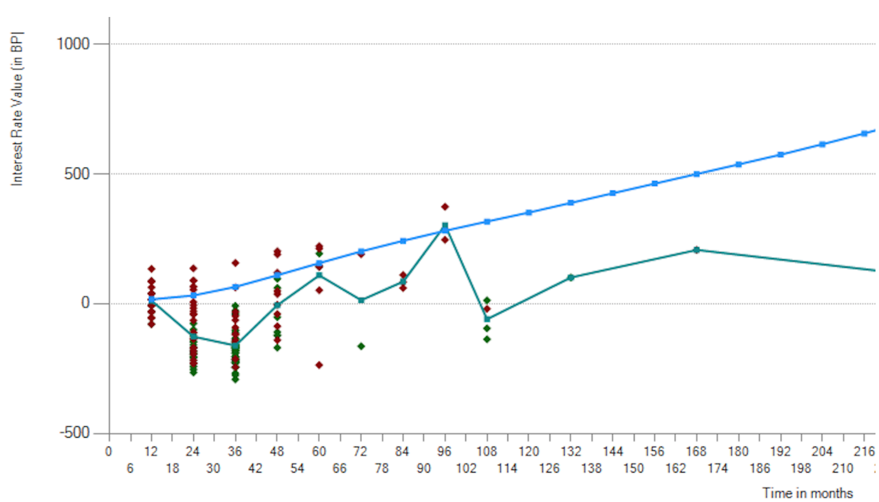

Figure 7: Analog of Figure 2 using the HW model, where deep negative rates attract most of the risk.

ratios implied from the HW model do not just provide a false sense of security to the end-user. External agents can easily exploit the potential distorted appetite that the HW model generates (e.g., in the range between -100 and -350 BPS in Figure 7) if the bank is willing to pay the HW price for the purchase of protection against unreachable economic states in the market.

3.3 Alpha-leakage and the VaR/ES Myopia

In Albanese, Crépey, and Iabichino (2020), we give examples of lifetime projections of model risk for an exotic derivative based on a state-space analysis of possible outcomes, which is useful for investigation purposes. The corresponding analysis is based on the following Doob-Meyer decomposition for the value process of the trade plus hedge package subject to recalibration:

$$\delta V_t = \delta N_t + \delta R_t + \delta A_t,$$

where δN_t is the return of the martingale component of the process related to the intrinsic risk factors that are explicitly modeled, while $\delta R_t + \delta A_t$ is the Doob-Meyer decomposition of the valuation change due to model recalibration.

In perfect markets without model risk, valuations would be martingales, i.e. $\delta V_t = \delta N_t$, which is mitigated by delta and gamma hedging, and $\delta R_t = \delta A_t = 0$, for each date t. However, in the presence of model risk, the latter equalities break.

The no longer vanishing martingale return δR_t therein, due to recalibration, can be managed by vega-hedging, i.e. by offsetting sensitivities to model parameters (see Hagan (2002)). However, there is a fundamental difference between delta hedging and vega hedging. While delta hedging is mathematically rigorous, in the sense that it guarantees replication in a diffusion model that does not necessitate recalibrations, vega hedging addresses precisely recalibration risk and is not rigorous. Even if the recalibrated parameters follow a diffusion, vega hedging does not guarantee replication. Vega hedging is only meant to reduce the volatility of the strategy valuation V in the short term, but it is totally ineffective on the alpha term in the Doob Meyer decomposition.

In particular, there will be a finite variation predictable remainder δA_t that expresses alpha-leakage. It captures model risk and can not be mitigated by dynamic hedging strategies, it can only be prevented by using models that embed the correct market views.

Figure 8 refers to the case study in Albanese, Crépey, and Iabichino (2020), concerning a callable range accrual in the USD 10-year swap rate over the corridor $[0\%, 4\%]$ (whereby the bank receives whenever the index is outside the corridor). In this case study, counterparty credit risk is ignored. The focus is on unhedged market/model risk. Then, for such a callable range accrual, the HW-model implies far longer stopping times to call decisions and higher deltas, in the "hope" (or wrong model belief) that interest rates will fall deeply below 0%, where the contract is precious to the bank. The SD model expresses a very different view, as these states are unachievable. In this

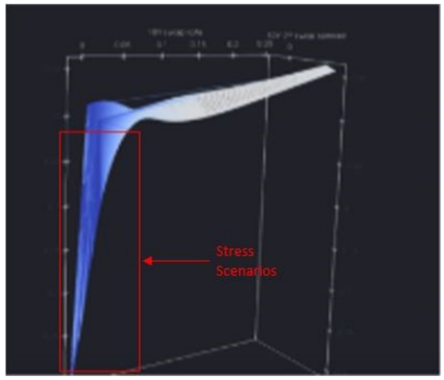

Figure 8: State space plot for the valuation of excessive hedges shortened for a callable range accrual priced with a Hull-White model, as compared to the valuations of an SD model (color-code: blue for negative values, white for intermediate).

case, when the 10-year swap rate approaches 0, the SD model indicates that an early exercise is optimal, as the probability to come out from the range is immaterial. If seen from the SD model, low rates are stress scenarios of extreme market risk at the bottom of the credit cycle, which makes optimal an early exercise and forces to unwind the hedge (even at a large loss for buying back the zero floor hedges that have become very expensive), instead to keep paying (in the corridor) the hope that the rates will follow in unreachable profitable levels, as suggested by the HW model.

The impact of recalibration risk is particularly sizeable for unrealistic models. If stress scenarios are not correctly calibrated, i.e. if the valuation model breaks down and is not reliable in stress scenarios, then on a bridge between the spot state variables and a solvency scenario, the model is bound to be subject to frequent recalibrations, as the hedging portfolio value process deviates from martingality.

As the HW model is gradually recalibrated, the systematic alpha leakage gives rise to potentially significant losses. However, a VaR model predicated on the HW model would not detect the leakage. VaR models are only a function of quadratic variation of returns and, as long as hedge ratios are computed consistently using the HW model, VaR will consistently under-estimate risk.

Revising the VaR metric with an expected shortfall (ES) metric or a Stressed VaR metric does not address the concern either since also in this case first moment of the return distribution (i.e. the drift in the Doob-Meyer decomposition) is caused by daily recalibration and is neglected by construction in both variations. Therefore, VaR, ES or Stressed Var models will be blind to alpha leakages up until the time when accrued losses are no longer sustainable, and all the portfolio necessitates to be unwound. For this reason, a VaR predicated on a model that does not calibrate to stress scenarios should not be considered as reliable for the purpose of market risk monitoring.

3.4 Model Risk and Bayesian KVA

In the above example, delayed exercise decisions and over-valuation for low levels of interest rates approaching zero lead to situations whereby trade unwinding results into a substantial mark-down combined with a loss as hedging trades are unwound. These losses are neither dividends nor credit or funding losses, they are model risk losses. Unlike the former, there is no rigorous framework for reserving against model risk losses. The only meaningful course of action is not to use invalid models with recalibration risk under stress scenarios.

However, market data do not uniquely determine valid models. Once the set of realistic models, among which a sound portfolio optimization for regulated entities can be carried out, is identified, the residual model risk can be accounted for through Bayesian averages over the valuations generated in this set, in the vein of Black and Litterman (1990, 1992), of economic capital projections and the ensuing KVA metric. These methods involve generating scenario sets with several different measures to find aggregate metrics of interest, such as VaR (see e.g. Siu, Tong, and Yang (2004)) or KVA, as in this article. In the KVA context, the Bayesian definition is the same as (3), except that the aggregation underlying (1)–(2) is over the composite scenario set. Taking a Bayesian average among valid models by generating a composite scenario set based on multiple model specifications typically increases capital requirements. The difference derives from model risk and can be used as a model risk capital buffer. Note that the Bayesian KVA mixing bears on the paths simulated in different co-calibrated models, as opposed to mixing the prices in these models in the case of a standard AVA (additional valuation adjustment).

4 Credit and Trading Limits

In addition to place validity constraints on models, reverse stress testing also places constraints on risk appetites. The possibility to identify the counterparties that exacerbate the capital demand (see Figure 4) sets the base to formulate a new credit limit metric based on counterparty incremental KVA.

The potential future exposure (PFE)[2] defines the risk of a given counterparty based on potential future extreme mark-to-market (MtM) losses. To prevent risk concentrations which might jeopardize bank solvency, credit risk officers use PFE-like metrics to assess the maximum total exposure over the lifetime of a trade or a netting set they are willing to face. However, counterparty specific PFE-like metrics are limited metrics since counterparty credit losses related to an individual counterparty are just one facet of the risk. Correlated losses triggered by credit contagion events or convexity concentrations for exotic derivatives have the potential to threaten solvency and are

[2]An extreme quantile (typically ranging between 95% and 99%) of the counterparty specific time-t MtM distribution defines the PFE. The risk metric used for credit limit monitoring is typically the maximum PFE ($\max_{t\in[0,T]}(\mathrm{PFE}(t))$, where T is either the residual life of the netting set or its first year of credit exposure.

amplified by risk concentration, but escape a PFE based analysis. Fully collateralized derivative portfolios result in a zero PFE although there is collateral funding risk.

Taking the difference between the KVA of the entire portfolio and the KVA of the portfolio obtained by removing the counterparty c, we compute the incremental counterparty KVA (Δ_cKVA). The Δ_cKVA, as well as the PFE, assumes going-concern, and that the portfolio is held on a run-off basis. However, we find the Δ_cKVA useful for setting limits on the counterparty specific idiosyncratic risk appetite, as it reflects all the potential sources of future losses and it carries the intuitive meaning of the incremental demand of capital generated by a counterparty. Accounting for the full term structure of economic capital, as opposed to just the current value, is in line with regulatory directives, such as CRR 292.7(b) (see European Banking Association (2013)), which explicitly prescribes to monitor and potentially capitalize the concentration of exposure beyond the first year of credit exposure.

The higher risk sensitivity of Δ_cKVA makes KVA based credit limits a strong addition to PFE based systems. Under full implementation of the BCBS 261 directive on Uncleared Margin Rules, where most client's trades will be collateralized, it may well be that the null PFE will be a too narrow metric to set risk appetites, while capital consumption may still be material. The benefits of using Δ_cKVA metrics in setting idiosyncratic risk appetites are visible in Figure 9, which depicts the comparison between the PFE and the Δ_cKVA for the whole set of counterparties of the analyzed portfolio. Figure 9 shows that the metrics are virtually unrelated, the more risk-sensitive Δ_cKVA measure captures details related to wrong-way risk, portfolio effects and XVA volatility that the PFE misses. Furthermore, we note that the Δ_cKVA can be low in situations where the PFE is material. These netting sets, while being penalized from a PFE perspective, are de-facto beneficial as they reduce the overall risk of the derivative portfolio. On the opposite end of the spectrum, there are netting sets characterized by a smaller PFE, but material Δ_cKVA, signaling pronounced potential losses that go beyond MtM losses.

Having fixed a credit limit method (i.e. the PFE or the Δ_cKVA), we compare results derived from the usage of the HW or the SD model. From Figure 10 we observe that the PFE points scatter roughly along the diagonal, which indicate that this metric is fairly indifferent to the model of choice. However Figure 11, which considers the Δ_cKVA instead, hides a different story. The Δ_cKVA is quite sensitive to the radically different nature of stress scenarios (see Section 3.2). We conclude that XVA analytics are highly informative tools to carry out a reverse stress testing analysis, but only at the condition that realistic, high-quality models are used for the dual purposes of scenario generation and derivative pricing.

A similar, more granular approach can be used to define trade incremental KVA (cf. Albanese et al. (2019, Section 4.2)). In particular, since the KVA is a portfolio metric, the trade incremental KVA is capable of distinguishing between trades, which are costly in terms of capital, from overall risk-reducing trades. The characteristics of the portfolio held by the bank at trade inception drive discrimination processes. Therefore, a trading desk may enter into a trade and then look for another (internal

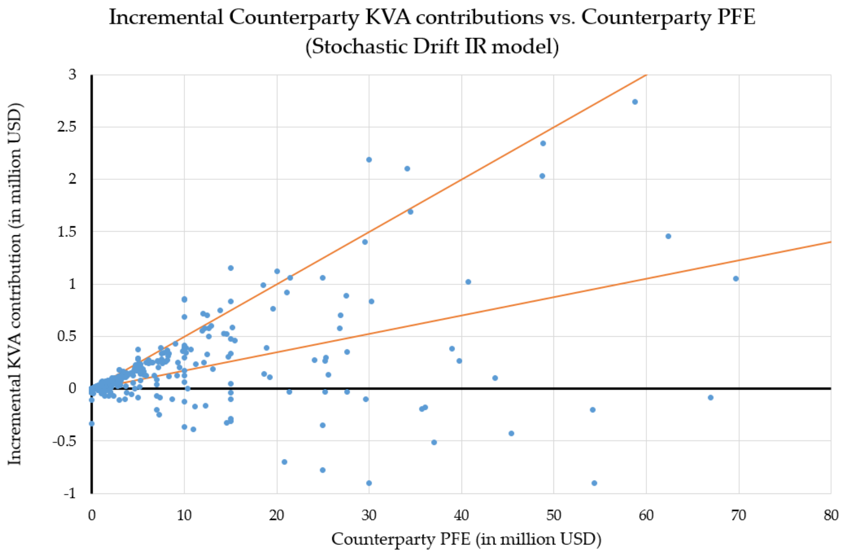

Figure 9: Δ_cKVA metrics and PFE using the SD model (see (9)). The orange lines denote the area within which the intermediary 50% of the data lie.

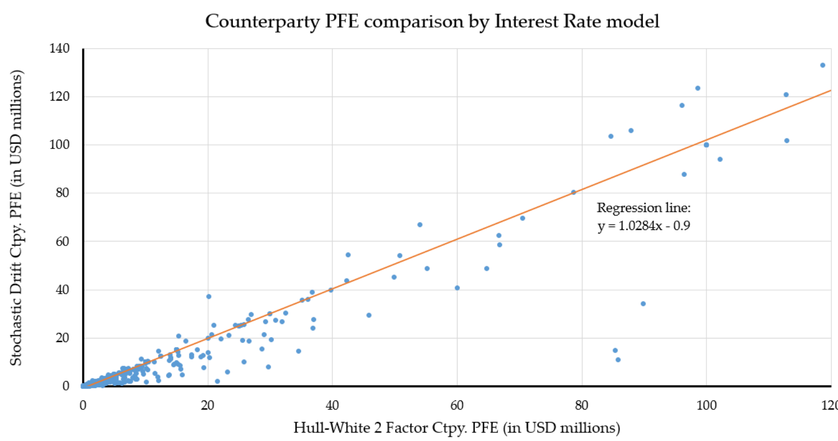

Figure 10: PFE metrics obtained with the HW model (x-axis) versus the same metric obtained with the SD model (y-axis). Note that the regression line (in orange) has an angular coefficient very close to 1, indicating a lack of sensitivity of the PFE metric.

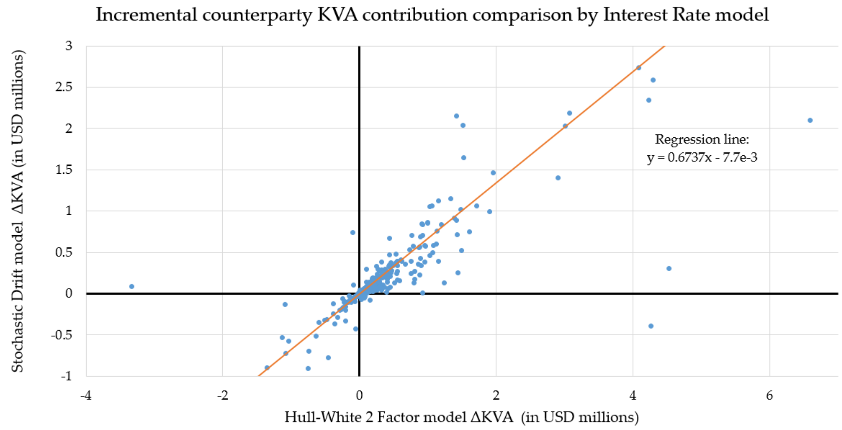

Figure 11: Δ_cKVA metrics obtained with the HW model (x-axis) versus the same metric obtained with the SD model (y-axis). Note that the regression line (in orange) now has an angular coefficient relatively far from 1, indicating a systematic bias due to model risk of the KVA metric.

or external) desk whose portfolio is particularly suited to accommodate this trade. The two desks may then look for an agreement and split the advantages arising from reduced capital and XVA costs at the portfolio level (cf. Albanese, Chataigner, and Crépey (2019, Section 5)).

5 Conclusion

In this article, we introduced a new framework for reverse stress testing. We find that scenario differentials for economic capital based KVA metrics are economically meaningful to rank and select stress scenarios out of an extensive simulation sample. The approach reveals both the impact of macro-economic risk factors connected to the credit cycle and of idiosyncratic weaknesses and risk concentrations for the specific portfolio at hand. The analysis of the latter can reveal the weakness of a model, such as the Hull-White model with unrealistic, deeply negative interest rates. We also present a new metric for credit limit monitoring in terms of counterparty incremental KVA. This metric directly quantifies the solvency and the capital impact of trading decisions, it is more risk-sensitive than PFE, and it naturally accounts for concentration risk at the whole portfolio level.

A The Core Equity Tier 1 Capital Process

As explained in more detail in the references given in Section 2.1 (and assuming here no collateral exchanges between the bank and its clients for notational simplicity), the (unilateral) credit valuation adjustment, computed assuming that the bank does not default, is given by

$$\begin{aligned}
\mathrm{CVA}_t &= \mathbb{E}_t\left[\sum_i \mathbb{1}_{\{t<\tau_i<\infty\}}(1-R_i)(\mathrm{MtM}^i_{\tau_i})^+\right]\\
&= \mathbb{E}_t\int_t^\infty \left[\sum_i (1-R_i)(\mathrm{MtM}^i_s)^+\boldsymbol{\delta}_{\tau_i}(ds)\right] \qquad (10)\\
&= \mathbb{E}_t\int_t^\infty \left[\sum_i \mathbb{1}_{\{t<\tau_i\}} e^{-\int_t^s \lambda^i_\zeta d\zeta}\lambda^i_s(1-R_i)(\mathrm{MtM}^i_s)^+ ds\right],
\end{aligned}$$

where sums run over netting sets i, MtM^i_s and λ^i_s are the time-s default-free value of the i-th netting set and default intensity of the corresponding counterparty, τ_i and R_i are the default time and recovery rate of the latter, and $\boldsymbol{\delta}_{\tau_i}$ denotes a Dirac mass at time τ_i. The formula in the third line of (10) assumes some conditional independence between τ_i and the remaining information in the model. Note that even this intensity-based formula depends on τ_i itself (beyond its intensity λ_i), via the default indicator there. Hence, a rigorous XVA implementation requires an atomic simulation of default times (as opposed to a mere diffusion of their intensities, see e.g. Albanese, Armenti, and Crépey (2020, Section 7) for more details).

The funding valuation adjustment (FVA), defined, as initiated by Albanese, Andersen, and Iabichino (2015), as the cost of funding to shareholders for variation margin cash attracted by future scenarios where the bank is a net poster, is given by

$$\begin{aligned}
\mathrm{FVA}_t = \mathbb{E}_t\int_t^\infty & \\
&\left[\mathbb{1}_{\{s<\tau_B\}}\lambda^B_s\left(\sum_i \mathrm{MtM}^i_s\mathbb{1}_{\{s<\tau_i\}} - \mathrm{EC}_s - \mathrm{CVA}(s) - \mathrm{FVA}(s)\right)^+ ds\right], \qquad (11)
\end{aligned}$$

where λ^B_s is the funding spread of the bank at time s, τ_B is the time of default of the bank, and EC is defined as in (1), with

$$\mathrm{CET1}_t = \mathrm{CET1}_0 - \int_0^t \big(dD(s) + dF(s)\big) - \mathrm{CVA}_t - \mathrm{FVA}_t. \qquad (12)$$

Here $dD(s)$ and $dF(s)$ are shorthand notation for the bracketed integrands in the second line of (10) and in (11), i.e. the future losses realized by the bank due to defaults and funding expenses. In case the counterparty default and risky funding expenses are hedged, the accordingly modified expressions should be substituted for D and/or F in the above. In case of residual, unhedged market risk, and/or hedge of CVA/FVA volatility, the corresponding P&Ls should be added to (12).

The coupling between FVA and EC (as the latter gauges CET1, including FVA, fluctations) can be disentangled by Picard iterations as detailed in Crépey et al. (2019).

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