

Para-1.

a. 3D space $z = f(x, y) = ax + by + c$

$$\frac{dz}{dx} = a \quad \frac{dz}{dy} = b \quad \begin{bmatrix} a \\ b \end{bmatrix}$$

b. $z = f(x) = f(x_1, x_2, \dots, x_n)$

$$= \sum_{i=1}^n a_i(x_i - b_i) + c$$

$$= a_1x_1 + a_2x_2 + \dots + a_nx_n + c$$

$$= \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

c. $f_x(x, y) = \left(\frac{\partial f(x, y)}{\partial x} \right) = 2A(x - x_0)$

$$f_y(x, y) = \left(\frac{\partial f(x, y)}{\partial y} \right) = 2B(y - y_0)$$

$$d. \quad x = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \quad y = (2, 5, 1)$$

$$[3 \times 1] \quad [1 \times 3]$$

$$A = \begin{bmatrix} 4 & 5 & 2 \\ 3 & 1 & 5 \\ 6 & 4 & 3 \end{bmatrix}$$

$$[3 \times 3]$$

$$B = \begin{bmatrix} 3 & 5 \\ 5 & 2 \\ 1 & 4 \end{bmatrix}$$

$$[3 \times 2]$$

$$x^T = [3 \ 1 \ 4] \quad y^T = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 4 & 3 & 6 \\ 5 & 1 & 4 \\ 2 & 5 & 3 \end{bmatrix} \quad B^T = \begin{bmatrix} 3 & 5 & 1 \\ 5 & 2 & 4 \end{bmatrix}$$

$$x \cdot x^T = \begin{bmatrix} 9 \\ 1 \\ 16 \end{bmatrix} \quad x \cdot y^T = \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$$

$$y \cdot x = 15 \quad x \cdot y = 0$$

$$A \cdot x = \begin{bmatrix} 25 \\ 30 \\ 24 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 39 & 38 \\ 19 & 37 \\ 44 & 50 \end{bmatrix}$$

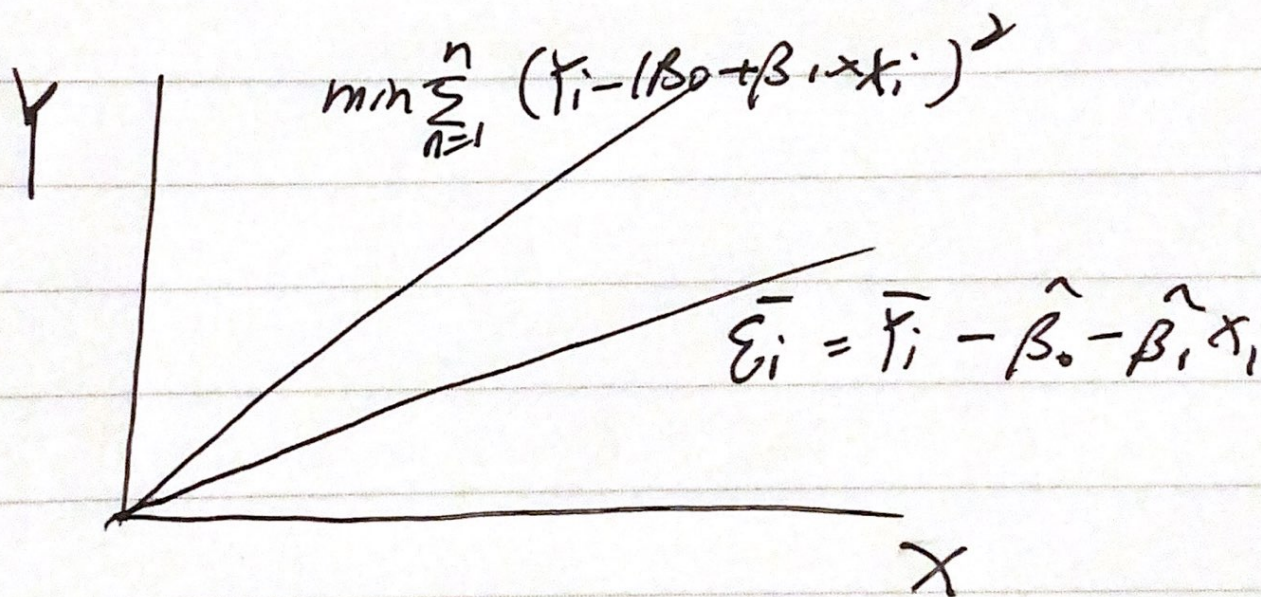
$$B \cdot \text{reshape}(1, 6) = [3 \ 5 \ 5 \ 2 \ 1 \ 4]$$

Part-1: LLS: Single-variable

{ linear function: $y = a_1x_1 + a_2x_2 + \dots + a_nx_n$
linear regression:

$$y = \beta_1x_1 + \beta_2x_2 + \dots + \beta_nx_n + \varepsilon$$

difference: ε



$$SST = \sum_{i=1}^n (y_i - \bar{y})^2$$

$$RSS = \sum_{i=1}^n (y_i - \hat{\beta}_0 - \hat{\beta}_1x_{i1} - \hat{\beta}_2x_{i2} - \dots - \hat{\beta}_px_{ip})^2$$

$$R^2 = \left(1 - \frac{RSS_1}{RSS_0}\right) \times 100\%$$