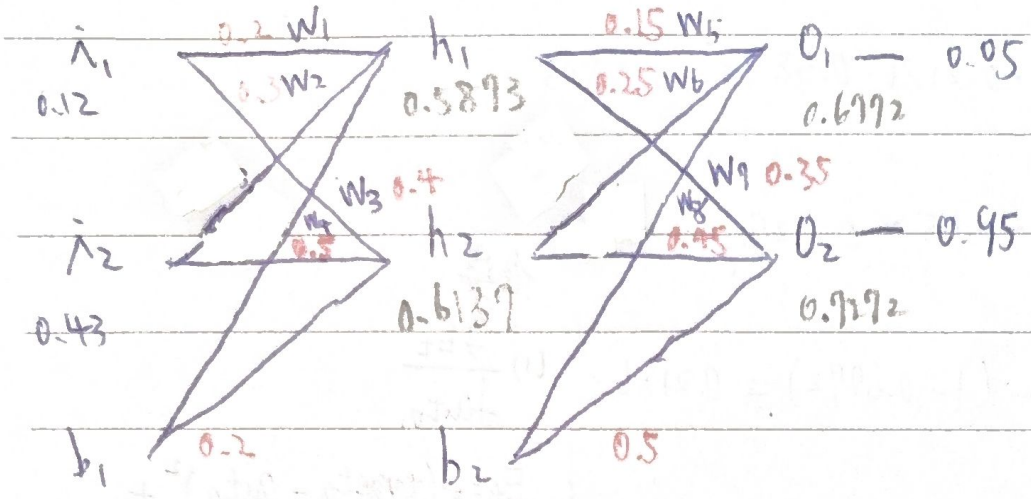

Calculate Errors and adjust weights.

$E_{total} = E_{O_1} + E_{O_2} = 0.2216$

$W_1^+ = 0.1999$

$W_2^+ = 0.2997$

$W_3^+ = 0.3998$

$W_4^+ = 0.4993$

$W_5^+ = 0.1098$

$W_6^+ = 0.2079$

$W_7^+ = 0.363$

$W_8^+ = 0.4636$

$net_{h_1} = i_1 \times W_1 + i_2 \cdot W_2 + b_1 \cdot 0.2$

$= 0.12 \cdot 0.2 + 0.43 \cdot 0.3 + 1 \times 0.2$

$= 1.024 + 0.129 + 0.2$

$= 0.353$

介於0~1

$Out_{h_1} = \frac{1}{1+\exp^{-net_{h_1}}} \doteq 0.5873$

$net_{h_2} = i_1 \cdot W_3 + i_2 \cdot W_4 + b_1 \cdot 0.2$

$= 0.12 \cdot 0.4 + 0.43 \cdot 0.5 + 1 \cdot 0.2$

$= 0.048 + 0.215 + 0.2$

$= 0.463$

$Out_{h_2} = \frac{1}{1+\exp^{-net_{h_2}}} \doteq 0.6139$

$net_{O_1} = h_1 \cdot W_5 + h_2 \cdot W_6 + b_2 \cdot 0.5$

$= 0.5893 \cdot 0.15 + 0.6109 \cdot 0.25 + 1 \cdot 0.5$

$= 0.088095 + 0.152725 + 0.5$

$= 0.74082$

$Out_{O_1} = \frac{1}{1+\exp^{-net_{O_1}}} \doteq 0.6772$

$net_{O_2} = h_1 \cdot W_7 + h_2 \cdot W_8 + b_2 \cdot 0.5$

$= 0.5893 \cdot 0.35 + 0.6109 \cdot 0.45 + 1 \cdot 0.5$

$= 0.205555 + 0.274905 + 0.5$

$= 0.98046$

$Out_{O_2} = \frac{1}{1+\exp^{-net_{O_2}}} \doteq 0.7272$

$E_{O_1} = \frac{1}{2}(target_{O_1} - Out_{O_1})^2$

$= \frac{1}{2}(0.05 - 0.6772)^2$

$\doteq 0.1967$

$E_{O_2} = \frac{1}{2}(target_{O_2} - Out_{O_2})^2$

$\doteq 0.0248$

$E_{total} = E_{O_1} + E_{O_2} = 0.2215$

已知問題: Out_{h_2} 填錯，可能造成後續運算時產生誤差

W_5^+:

$$\frac{\partial E_t}{\partial W_5} = \frac{\partial E_t}{\partial out_{o_1}} \cdot \frac{\partial out_{o_1}}{\partial net_{o_1}} \cdot \frac{\partial net_{o_1}}{\partial W_5} = 0.6272 \cdot 0.2186 \cdot 0.5873 \fallingdotseq 0.0805$$

$$\frac{\partial E_t}{\partial out_{o_1}} = Out_{o_1} - Target_{o_1} = 0.6772 - 0.05 = 0.6272$$

$$\frac{\partial out_{o_1}}{\partial net_{o_1}} = Out_{o_1}(1 - Out_{o_1}) = 0.6772 \cdot (1 - 0.6772) \fallingdotseq 0.2186$$

$$\frac{\partial net_{o_1}}{\partial W_5} = Out_{h_1} = 0.5873$$

$$W_5^+ = W_5 - \eta \cdot \frac{\partial E_t}{\partial W_5} = 0.15 - 0.5 \cdot 0.0805 \fallingdotseq 0.1098 *$$

Eta: learning rate, 設 0.5

推導:

(1) $\frac{\partial E_t}{\partial out_{o_1}}$

$$E_t = \frac{1}{2}(target_{o_1} - Out_{o_1})^2 + \frac{1}{2}(target_{o_2} - Out_{o_2})^2$$

利用 Out_{o_1} 微分

$$\rightarrow 2 \cdot \frac{1}{2}(target_{o_1} - Out_{o_1})^{2-1} \times \left(target_{o_1} - Out_{o_1}\right)' + 0$$

$$\rightarrow (target_{o_1} - Out_{o_1}) \cdot (-1) + 0$$

$$\rightarrow Out_{o_1} - target_{o_1}$$

(2) $\frac{\partial Out_{o_1}}{\partial net_{o_1}}$

$$Out_{o_1} = \frac{1}{1 + \exp^{-net_{o_1}}}$$

① $-net_{o_1} = x$

② 上下同 $\times e^x$ $\rightarrow \frac{e^x}{e^x + 1}$

除法法則 $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x) \cdot g'(x)}{g^2(x)}$

$$f(x) = \frac{1}{1 + e^{-x}} = \frac{e^x}{e^x + 1} \quad \leftarrow f(x) \quad \leftarrow g(x)$$

$$\frac{d}{dx} f(x) = \frac{e^x(e^x + 1) - e^x \cdot e^x}{(1 + e^x)^2} = \frac{e^x}{(1 + e^x)^2} = f(x)(1 - f(x))$$

$$\frac{\partial Out_{o_1}}{\partial net_{o_1}} = Out_{o_1}(1 - Out_{o_1})$$

(3) $\frac{\partial net_{o_1}}{\partial W_5}$

$$net_{o_1} = \underbrace{h_1 W_5}_{h_1} + \underbrace{h_2 W_6}_{0} + \underbrace{b_2 W_{b2}}_{0}$$

$$\frac{\partial net_{o_1}}{\partial W_5} = Out_{h_1}$$

W_6^+: 前面相同

$$\frac{\partial E_t}{\partial W_6} = \frac{\partial E_t}{\partial net_{o_1}} \cdot \frac{\partial net_{o_1}}{\partial W_6} = 0.6272 \cdot 0.2186 \cdot 0.6139 \fallingdotseq 0.0842$$

$$\frac{\partial net_{o_1}}{\partial W_6} = Out_{h_2} = 0.6139$$

$$W_6^+ = W_6 - \eta \cdot \frac{\partial E_t}{\partial W_6} = 0.25 - 0.5 \cdot 0.0842 = 0.2079 *$$

W_7^+:

$$\frac{\partial E_t}{\partial W_7} = \frac{\partial E_t}{\partial out_{o_2}} \cdot \frac{\partial out_{o_2}}{\partial net_{o_2}} \cdot \frac{\partial net_{o_2}}{\partial W_7} = -0.2228 \cdot 0.1984 \cdot 0.5873 \fallingdotseq -0.026$$

$$\frac{\partial E_t}{\partial out_{o_2}} = Out_{o_2} - target_{o_2} = 0.7272 - 0.95 = -0.2228$$

$$\frac{\partial out_{o_2}}{\partial net_{o_2}} = Out_{o_2} \cdot (1 - Out_{o_2}) = 0.7272 \times (1 - 0.7272) \fallingdotseq 0.1984$$

$$\frac{\partial net_{o_2}}{\partial W_7} = Out_{h_1} = 0.5873$$

$$W_7^+ = W_7 - \eta \cdot \frac{\partial E_t}{\partial W_7} = 0.35 - 0.5 \cdot (-0.026) = 0.363 *$$

W_8^+:

$$\frac{\partial E_t}{\partial W_8} = \frac{\partial E_t}{\partial out_{o_2}} \cdot \frac{\partial out_{o_2}}{\partial net_{o_2}} \cdot \frac{\partial net_{o_2}}{\partial W_8} = -0.2228 \cdot 0.1984 \cdot 0.6137 \fallingdotseq -0.0271$$

$$\frac{\partial net_{o_2}}{\partial W_8} = Out_{h_2} = 0.6137$$

$$W_8^+ = W_8 - \eta \cdot \frac{\partial E_t}{\partial W_8} = 0.45 - 0.5 \cdot (-0.0271) \fallingdotseq 0.4636 *$$

前面相同

No.
Date : :

W_1^+, W_2^+:

$$\frac{\partial E_{total}}{\partial W_1} = \frac{\partial E_{total}}{\partial Out_{h_1}} \cdot \frac{\partial Out_{h_1}}{\partial net_{h_1}} \cdot \frac{\partial net_{h_1}}{\partial W_1} = 5.1 \cdot 10^{-3} \cdot 0.2424 \cdot 0.12 \doteq 1.5 \cdot 10^{-4}$$

$$\frac{\partial E_{total}}{\partial Out_{h_1}} = \frac{\partial E_{O_1}}{\partial Out_{h_1}} + \frac{\partial E_{O_2}}{\partial Out_{h_1}} = 0.0206 + (-0.0155) = 5.1 \cdot 10^{-3}$$

$$\frac{\partial E_{O_1}}{\partial Out_{h_1}} = \frac{\partial E_{O_1}}{\partial net_{O_1}} \cdot \frac{\partial net_{O_1}}{\partial Out_{h_1}} = 0.1371 \cdot 0.15 \doteq 0.0206$$

$$\frac{\partial E_{O_1}}{\partial net_{O_1}} = \frac{\partial E_{O_1}}{\partial Out_{O_1}} \cdot \frac{\partial Out_{O_1}}{\partial net_{O_1}} = 0.6272 \cdot 0.2186 \doteq 0.1371$$

$$\frac{\partial E_{O_1}}{\partial Out_{O_1}} = Out_{O_1} - target_{O_1} = 0.6772 - 0.05 = 0.6272$$

$$\frac{\partial Out_{O_1}}{\partial net_{O_1}} = Out_{O_1}(1 - Out_{O_1}) = 0.6772 \cdot (1 - 0.6772) \doteq 0.2186$$

$$\frac{\partial net_{O_1}}{\partial Out_{h_1}} = W_5 = 0.15$$

$$\frac{\partial E_{O_2}}{\partial Out_{h_1}} = \frac{\partial E_{O_2}}{\partial net_{O_2}} \cdot \frac{\partial net_{O_2}}{\partial Out_{h_1}} = -0.0442 \cdot 0.35 \doteq -0.0155$$

$$\frac{\partial E_{O_2}}{\partial net_{O_2}} = \frac{\partial E_{O_2}}{\partial Out_{O_2}} \cdot \frac{\partial Out_{O_2}}{\partial net_{O_2}} = -0.2228 \cdot 0.1984 \doteq -0.0442$$

$$\frac{\partial E_{O_2}}{\partial Out_{O_2}} = Out_{O_2} - target_{O_2} = 0.7272 - 0.95 = -0.2228$$

$$\frac{\partial Out_{O_2}}{\partial net_{O_2}} = Out_{O_2}(1 - Out_{O_2}) = 0.7272 \cdot (1 - 0.7272) \doteq 0.1984$$

$$\frac{\partial net_{O_2}}{\partial Out_{h_1}} = W_7 = 0.35$$

$$\frac{\partial Out_{h_1}}{\partial net_{h_1}} = Out_{h_1}(1 - Out_{h_1}) = 0.5873 \cdot (1 - 0.5873) = 0.2424$$

$$\frac{\partial net_{h_1}}{\partial W_1} = \lambda_1 = 0.12$$

W_1^+

$$W_1 - \eta \cdot \frac{\partial E_t}{\partial W_1} = 0.2 - 0.5 \cdot 1.5 \cdot 10^{-4} \doteq 0.1999 \ *$$

W_2^+

$$W_2 - \eta \cdot \frac{\partial E_t}{\partial W_2} = 0.3 - 0.5 \cdot 5.3 \cdot 10^{-4} \doteq 0.2997 \ *$$

$$\frac{\partial E_t}{\partial W_2} = \frac{\partial E_t}{\partial net_{h_1}} \cdot \frac{\partial net_{h_1}}{\partial W_2} = \frac{\partial E_t}{\partial net_{h_1}} \cdot \lambda_2 = 5.1 \cdot 10^{-3} \cdot 0.2424 \cdot 0.43 = 5.3 \cdot 10^{-4}$$

前面相同

No.
Date : :

W_3^+, W_4^+:

$$\frac{\partial E_{total}}{\partial W_3} = \frac{\partial E_t}{\partial Out_{h_2}} \times \frac{\partial Out_{h_2}}{\partial net_{h_2}} \times \frac{\partial net_{h_2}}{\partial W_3} = 0.0144 \cdot 0.2371 \cdot 0.12 = 4.1 \times 10^{-4}$$

$$\frac{\partial E_t}{\partial Out_{h_2}} = \frac{\partial E_{O_1}}{\partial Out_{h_2}} + \frac{\partial E_{O_2}}{\partial Out_{h_2}} = 0.0343 + (-0.0199) = 0.0144$$

$$\frac{\partial E_{O_1}}{\partial Out_{h_2}} = \frac{\partial E_{O_1}}{\partial net_{O_1}} \cdot \frac{\partial net_{O_1}}{\partial Out_{h_2}} = 0.1371 \cdot 0.25 \doteq 0.0343$$

$$\frac{\partial E_{O_1}}{\partial net_{O_1}} = \frac{\partial E_{O_1}}{\partial Out_{O_1}} \cdot \frac{\partial Out_{O_1}}{\partial net_{O_1}} \doteq 0.6272 \cdot 0.2186 \doteq 0.1371$$

$$\frac{\partial E_{O_1}}{\partial Out_{O_1}} = Out_{O_1} - target_{O_1} = 0.6772 - 0.05 = 0.6272$$

$$\frac{\partial Out_{O_1}}{\partial net_{O_1}} = Out_{O_1}(1 - Out_{O_1}) = 0.6772 - (1 - 0.6772) \doteq 0.2186$$

$$\frac{\partial net_{O_1}}{\partial Out_{h_2}} = W_6 = 0.25$$

$$\frac{\partial E_{O_2}}{\partial Out_{h_2}} = \frac{\partial E_{O_2}}{\partial net_{O_2}} \cdot \frac{\partial net_{O_2}}{\partial Out_{h_2}} = -0.0442 \cdot 0.45 = -0.0199$$

$$\frac{\partial E_{O_2}}{\partial net_{O_2}} = \frac{\partial E_{O_2}}{\partial Out_{O_2}} \cdot \frac{\partial Out_{O_2}}{\partial net_{O_2}} = -0.2228 \cdot 0.1984 \doteq -0.0442$$

$$\frac{\partial E_{O_2}}{\partial Out_{O_2}} = Out_{O_2} - Target_{O_2} = 0.7272 - 0.95 = -0.2228$$

$$\frac{\partial Out_{O_2}}{\partial net_{O_2}} = Out_{O_2}(1 - Out_{O_2}) = 0.7272(1 - 0.7272) = 0.1984$$

$$\frac{\partial net_{O_2}}{\partial Out_{h_2}} = W_8 = 0.45$$

$$\frac{\partial Out_{h_2}}{\partial net_{h_2}} = Out_{h_2}(1 - Out_{h_2}) = 0.6137(1 - 0.6137) \doteq 0.2371$$

$$\frac{\partial net_{h_2}}{\partial W_3} = \lambda_1 = 0.12$$

W_3^+:

$$W_3 - \eta \cdot \frac{\partial E_t}{\partial W_3} = 0.4 - 0.5 \cdot 4.1 \cdot 10^{-4} \doteq 0.3998 \ *$$

W_4^+

$$W_4 - \eta \cdot \frac{\partial E_t}{\partial W_4} = 0.5 - 0.5 \cdot 1.5 \times 10^{-3} \doteq 0.4993 \ *$$

$$\frac{\partial E_t}{\partial W_4} = \frac{\partial E_t}{\partial net_{h_2}} \cdot \frac{\partial net_{h_2}}{\partial W_4} = 0.0144 \cdot 0.2371 \cdot \lambda_2 = 0.0144 \cdot 0.2371 \cdot 0.43 = 1.5 \times 10^{-3}$$

前面相同