

Lecture 12: Populations, Random Samples, and Statistics

Mathematical Statistics I, MATH 60061/70061

Tuesday October 19, 2021

Reference: Casella & Berger, 5.1-5.2

Sample mean and sample variance

Let X_1, \dots, X_n be a random sample from a population with mean μ and variance $\sigma^2 < \infty$. Then

- ① $E(\bar{X}) = \mu$,
- ② $\text{Var}(\bar{X}) = \sigma^2/n$,
- ③ $E(S^2) = \sigma^2$.

The statistic \bar{X} is an **unbiased** estimator of μ , and S^2 is an **unbiased** estimator of σ^2 .

Question: Is S also an unbiased estimator of σ ?

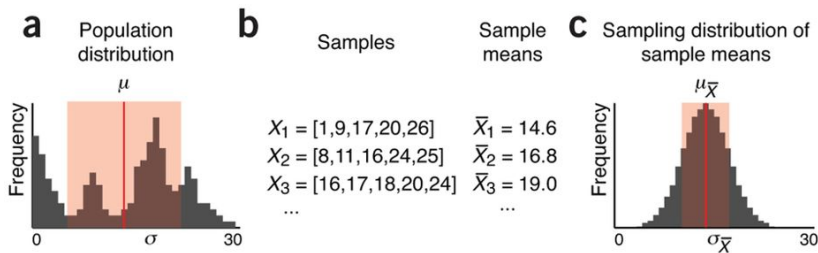
Biasness of sample standard deviation

Note: S is *biased* for σ unless $\sigma = 0$, since

$$E(S) = E(\sqrt{S^2}) \leq \sqrt{E(S^2)} = \sigma$$

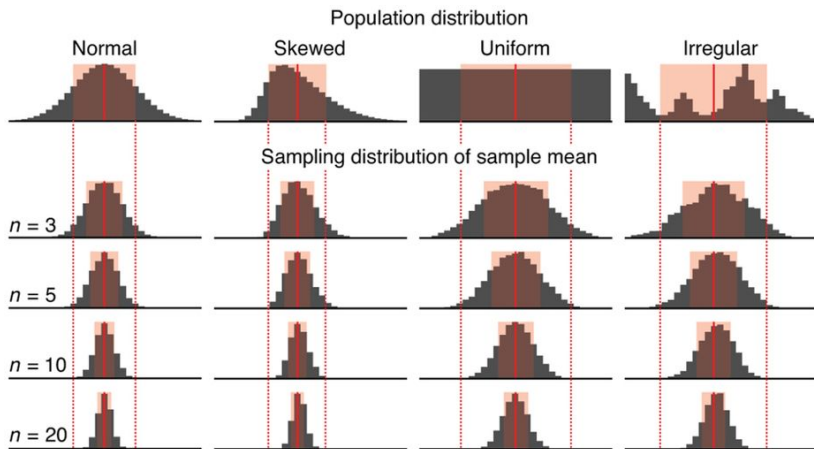
by Jensen's inequality, and the inequality is strict unless S^2 is a constant (which only happens when $\sigma = 0$).

Population and sample



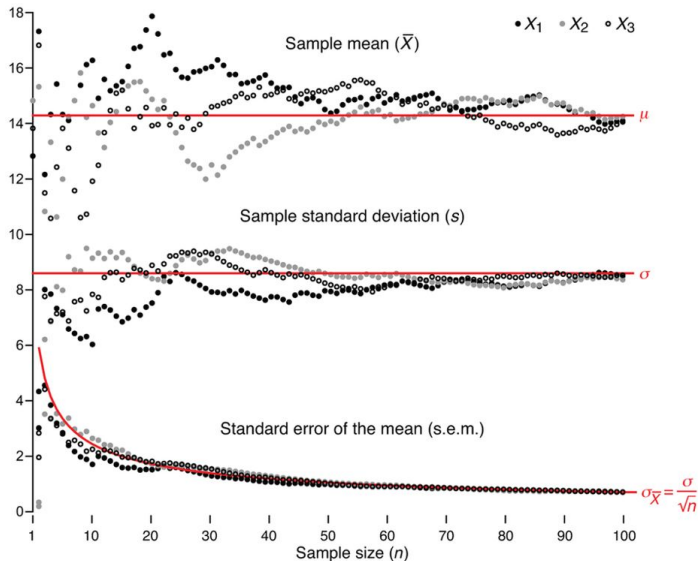
Source: "Points of Significance" series, Nature Methods

Sampling distribution



Source: "Points of Significance" series, Nature Methods

Sample estimates



Source: "Points of Significance" series, Nature Methods