Homework #4

MATH 60062/70062: Mathematical Statistics II

Please submit your answers as a **PDF** file to Blackboard by **9:15 a.m. on April 5**. Please show your work and write legibly. Your grade will be based on the correctness of your answers and the clarity with which you express them.

1. (15 points) Suppose that
$$g_T(t \mid \theta) = h(t)c(\theta)e^{w(\theta)t}$$
is a one-parameter Exponential family for the random variable T. Show that this family has a monotone likelihood ratio (MLR) if $w(\theta)$ is a nondecreasing function of θ.

2. (10 points) Suppose that $\boldsymbol{X} = (X_1, \dots, X_n)$ is a random sample from $\mathcal{N}(\mu, \sigma_0^2)$, where $-\infty < \mu < \infty$ and σ_0^2 is known. Show that the family of the distributions for $T(\boldsymbol{X}) = \bar{X}$ has an MLR.

3. (15 points) Suppose that $X_1, \dots, X_n$ are iid with PDF
$$f_X(x \mid \theta) = \theta x^{\theta-1} I(0 < x < 1).$$
Find a most powerful level α test for
$$H_0 : \theta = 1 \quad \text{versus} \quad H_1 : \theta = 2.$$

4. (40 points) Suppose that $X_1, \dots, X_n$ are iid with PDF
$$f_X(x \mid \theta) = \frac{1}{\theta} e^{-x/\theta} I(x > 0),$$
where $\theta > 0$. This is an Exponential distribution with mean θ.

   a. (15 points) Find a most powerful level α test for
   $$H_0 : \theta = 1 \quad \text{versus} \quad H_1 : \theta = 2.$$

   b. (5 points) Show that $T = \sum_{i=1}^n X_i$ is sufficient for θ.

   c. (20 points) Find a uniformly most powerful level α test for
   $$H_0 : \theta \geq \theta_0 \quad \text{versus} \quad H_1 : \theta < \theta_0.$$

5. (20 points) Suppose that $X_1, \dots, X_n$ are iid Pois(θ), where $\theta > 0$. Find a uniformly most powerful level α test for
$$H_0 : \theta \leq \theta_0 \quad \text{versus} \quad H_1 : \theta > \theta_0.$$