Homework #1

MATH 60062/70062: Mathematical Statistics II

Please submit your answers as a **PDF** file to Blackboard by **9:15 a.m. on February 17**. Please show your work and write legibly. Your grade will be based on the correctness of your answers and the clarity with which you express them.

1. (20 points) Let $X_1, \ldots, X_n$ be a random sample from a population with PDF

$$f(x \mid \theta) = \theta x^{\theta-1}, \quad 0 < x < 1, \quad \theta > 0.$$

   a. Is $\sum_{i=1}^{n} X_i$ sufficient for θ?

   b. Find a complete statistic for θ.

2. (20 points) Suppose that $X_1, \ldots, X_n$ are iid $\text{Unif}[0, \theta]$, where $\theta > 0$.

   a. Find the method of moments estimator of θ.

   b. Find the maximum likelihood estimator of θ.

3. (30 points) Let $X_1, \ldots, X_n$ be a random sample from a population with PDF

$$f(x \mid \theta) = \theta x^{-2}, \quad 0 < \theta \le x < \infty.$$

   a. Using the Factorization Theorem to find a sufficient statistic for θ.

   b. Find the method of moments estimator of θ.

   c. Find the maximum likelihood estimator of θ.

4. (30 points) Suppose that $X \sim \mathcal{N}(\theta, \sigma^2)$, where the prior distribution on θ is $\mathcal{N}(\mu, \tau^2)$, and σ^2, μ, and τ^2 are all known.

   a. (20 points) Show that the posterior distribution of θ is a Normal, with mean and variance given by

$$E(\theta \mid x) = \frac{\tau^2}{\tau^2 + \sigma^2} x + \frac{\sigma^2}{\tau^2 + \sigma^2} \mu,$$

$$\text{Var}(\theta \mid x) = \frac{\sigma^2 \tau^2}{\sigma^2 + \tau^2}.$$

   b. (10 points) Let $X_1, \ldots, X_n$ be a random sample from $\mathcal{N}(\theta, \sigma^2)$. Find the posterior distribution of θ.