Homework #4

MATH 60061/70061: Mathematical Statistics I

Please submit your answers as a **PDF** file to Blackboard by **9:15 a.m. on December 14**. Please show your work and write legibly. Your grade will be based on the correctness of your answers and the clarity with which you express them.

1. (15 points) Let X_n have a χ^2_n distribution. Find a Normal approximation to the distribution of X_n, as $n \to \infty$.
2. (40 points) Suppose $X_1, \ldots, X_n$ are an iid sample from an Exponential distribution with mean θ.
   a. (10 points) Find a Normal approximation to the distribution of the sample mean $\bar{X}$.
   b. (15 points) For a given function g, suppose that $g'(\theta)$ exists and $g'(\theta) \neq 0$. Find a Normal approximation to the distribution of $g(\bar{X})$.
   c. (15 points) Find a function g such that the variance of the Normal approximation of $g(\bar{X})$ does not depend on θ.
3. (15 points) Suppose $X_1, \ldots, X_n$ are an iid sample from an Exponential distribution with mean θ. Show that $T(\boldsymbol{X}) = \bar{X}$ is a sufficient statistic.
4. (15 points) Suppose $X_1, \ldots, X_n$ are an iid sample from $\mathcal{N}(\mu, \sigma^2)$. Show that $\boldsymbol{T} = \boldsymbol{T}(\boldsymbol{X}) = (\sum_{i=1}^n X_i, \sum_{i=1}^n X_i^2)$ is sufficient for (μ, σ^2).
5. (15 points) Suppose $X_1, \ldots, X_n$ are iid from

$$f_X(x \mid \sigma) = \frac{1}{2\sigma} e^{-|x|/\sigma} I(x \in \mathbb{R}).$$

Show that

$$S(\boldsymbol{X}) = \frac{\sum_{i=1}^k |X_i|}{\sum_{i=1}^n |X_i|}$$

is ancillary.