Simulation Parameters

Ruixuan Zhao, Guitao Yang, Thomas Parisini, and Boli Chen

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Parameters in Section V. Simulation Results

A. Numerical Example

1) Continuous-time system:

$$L_1^\top = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ -3 & -0.4737 & -2.2500 & -4.5789 & 4.4211 & 2.2500 \end{bmatrix},$$
$$L_2^\top = \begin{bmatrix} 0 & -2 & 0 & 0 & 0 & -4 \end{bmatrix},\ L_4^\top = \begin{bmatrix} -3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$L_3^\top = \begin{bmatrix} 0 & 0 & -5.8 & 1.32 & 0 & 0 \\ -0.4706 & 1.1176 & 0 & 0 & -1.0294 & -1.1176 \end{bmatrix}.$$

$$W_{g,1}^* = \begin{bmatrix} -0.0000 & 0.0000 & 0.0000 & 0.0000 \\ -0.0553 & -0.0585 & 0.1280 & -0.1286 \\ -0.3014 & 0.0478 & 0.4924 & 0.3727 \\ 0.1659 & 0.1755 & -0.3839 & 0.3857 \\ 0.3164 & 0.3770 & 0.2168 & -0.1921 \\ -0.8240 & 0.1665 & -0.1827 & -0.1238 \end{bmatrix},$$

$$W_{g,2}^* = \begin{bmatrix} -0.2661 & 0.5686 & -0.2944 & 0.0021 & -0.1848 \\ -0.1328 & 0.0058 & 0.2693 & -0.0385 & 0.4700 \\ 0.1618 & 0.4080 & 0.0537 & -0.0842 & -0.1680 \\ -0.1518 & -0.6434 & -0.3644 & -0.0497 & -0.0318 \\ 0.3903 & 0.0340 & -0.3269 & 0.0048 & 0.1889 \\ 0.0812 & -0.3065 & 0.2591 & -0.0040 & -0.4699 \end{bmatrix}, W_{g,3}^* = \begin{bmatrix} 0.6568 & -0.1979 \\ 0.0398 & 0.7265 \\ 0 & 0 \\ 0 & 0 \\ -0.6568 & 0.1979 \\ 0.3682 & 0.6276 \end{bmatrix},$$

$$W_{g,4}^* = \begin{bmatrix} 0 & -0.0000 & -0.0000 & 0 & 0.0000 \\ 0.1005 & 0.1266 & -0.1338 & 0.2756 & -0.9381 \\ 0.2743 & 0.2154 & 0.0758 & -0.9081 & -0.2192 \\ -0.5484 & -0.6613 & 0.3726 & -0.2267 & -0.2678 \\ 0.2234 & 0.2628 & 0.9151 & 0.2086 & -0.0099 \\ 0.7510 & -0.6567 & -0.0099 & 0.0671 & 0.0130 \end{bmatrix}.$$

2) Discrete-time system:

$$L_1^\top = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ L_3^\top = \begin{bmatrix} 0 & 0 & -1.9754 & 0.0998 & 0 & 0 \end{bmatrix},$$
$$L_2^\top = \begin{bmatrix} -0.2244 & -0.9990 & 0.0252 & -0.0090 & 0.4797 & -2.0659 \end{bmatrix},$$
$$L_4^\top = \begin{bmatrix} -1.0799 & 0.0809 & 0 & 0 & 0.2710 & -1.9582 \end{bmatrix}.$$

$$P_{W_{g,1}^*} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$P_{W_{g,2}^*} = \begin{bmatrix} 0.0984 & 0.9939 & -0.0491 & 0.0025 & -0.0000 & -0.0000 \end{bmatrix},$$
$$P_{W_{g,3}^*} = \begin{bmatrix} -0.0000 & 0.0000 & 0.9987 & -0.0505 & -0.0000 & 0.0000 \end{bmatrix},$$

$$P_{W^*_{g,4}} = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0035 & -0.9957 & 0.0000 & 0.0093 & -0.0923 \\ -0.0000 & -0.0375 & -0.0928 & 0.0000 & -0.0998 & 0.9900 \end{bmatrix}.$$

$$E_1^\top = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$F_1^\top = \begin{bmatrix} 0.9952 & -0.0490 & 0.0008 & -0.0001 & 0.0189 & 0.0001 \end{bmatrix},$$
$$E_2^\top = \begin{bmatrix} 0.0493 & 0.4974 & -0.0082 & 0.0008 & -0.1949 & -0.0008 \end{bmatrix},$$
$$F_2^\top = \begin{bmatrix} -0.0490 & 0.5057 & 0.0081 & -0.0008 & -0.1816 & 0.0008 \end{bmatrix},$$
$$E_3^\top = \begin{bmatrix} 0.0008 & 0.0082 & 0.3325 & -0.0336 & 0.3301 & 0.0336 \end{bmatrix},$$
$$F_3^\top = \begin{bmatrix} 0.0008 & 0.0081 & 0.3338 & 0.0168 & -0.1696 & -0.0168 \end{bmatrix},$$
$$E_4^\top = \begin{bmatrix} -0.0001 & -0.0008 & 0.0168 & 0.9983 & -9.8997 & -0.9983 \\ -0.0008 & -0.0081 & -0.3323 & -0.0167 & 1.0945 & 0.0167 \\ -0.0001 & -0.0008 & -0.0310 & -0.0016 & -9.9155 & 0.0016 \end{bmatrix},$$
$$F_4^\top = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & -0.0000 & 9.9169 & 1.0000 \end{bmatrix}.$$

B. DC microgrid power system

$$L_1 = \begin{bmatrix} -3.1758 & -4.7535 \\ 4.4395 & 4.1561 \\ 0.0028 & 0.0024 \\ -3.1758 & -4.7535 \\ 4.4395 & 4.1561 \\ 0.0028 & 0.0024 \\ -2.8877 & -4.5135 \\ 5.1042 & 5.0237 \\ 0.0032 & 0.0030 \\ -1.7421 & -3.3926 \\ -8.3515 & -11.6198 \\ 27.9406 & 39.3150 \\ -1.3713 & -3.1203 \\ -7.3528 & -10.3185 \\ 27.9413 & 39.3159 \end{bmatrix}, L_2 = \begin{bmatrix} -3.1758 & -4.7535 \\ 4.4395 & 4.1561 \\ 0.0028 & 0.0024 \\ -3.1758 & -4.7535 \\ 4.4395 & 4.1561 \\ 0.0028 & 0.0024 \\ -2.8877 & -4.5135 \\ 5.1042 & 5.0237 \\ 0.0032 & 0.0030 \\ -1.7421 & -3.3926 \\ -8.3515 & -11.6198 \\ 27.9406 & 39.3150 \\ -1.3713 & -3.1203 \\ -7.3528 & -10.3185 \\ 27.9413 & 39.3159 \end{bmatrix}, L_3 = \begin{bmatrix} -2.2685 & -3.1200 \\ 2.9170 & 2.0807 \\ 0.0018 & 0.0012 \\ -2.2685 & -3.1200 \\ 2.9170 & 2.0807 \\ 0.0018 & 0.0012 \\ -2.0822 & -3.0122 \\ 3.3788 & 2.6171 \\ 0.0021 & 0.0015 \\ -1.3246 & -2.4309 \\ -5.8757 & -7.3978 \\ 19.7026 & 25.1533 \\ -1.0886 & -2.3257 \\ -5.1821 & -6.5941 \\ 19.7030 & 25.1538 \end{bmatrix},$$

$$L_4 = \begin{bmatrix} -1.3313 & -2.2917 \\ -3.6974 & -4.8184 \\ 13.6958 & 17.9412 \\ -1.3313 & -2.2917 \\ -3.6974 & -4.8184 \\ 13.6958 & 17.9412 \\ -1.4765 & -2.4021 \\ -4.0011 & -5.1660 \\ 13.6956 & 17.9410 \\ -2.0511 & -2.9183 \\ 3.3682 & 2.5590 \\ 0.0021 & 0.0015 \\ -2.2388 & -3.0432 \\ 2.9113 & 2.0367 \\ 0.0018 & 0.0011 \end{bmatrix}, L_5 = \begin{bmatrix} -1.7941 & -3.2899 \\ -5.2682 & -7.5716 \\ 19.4971 & 28.1406 \\ -1.7941 & -3.2899 \\ -5.2682 & -7.5716 \\ 19.4971 & 28.1406 \\ -2.0157 & -3.5075 \\ -5.7098 & -8.1448 \\ 19.4968 & 28.1403 \\ -2.8773 & -4.4487 \\ 5.1368 & 5.0443 \\ 0.0032 & 0.0030 \\ -3.1667 & -4.7118 \\ 4.4724 & 4.1826 \\ 0.0028 & 0.0025 \end{bmatrix}.$$

$$P_{W_{g,1}^*}^\top = \begin{bmatrix} 0.1823 & -0.0113 & -0.4281 & -0.3697 & -0.1295 & -0.4635 & -0.2414 & -0.1270 \\ 0.3637 & -0.0315 & 0.3668 & -0.4021 & -0.0855 & 0.5120 & -0.0657 & -0.0772 \\ -0.6725 & -0.0242 & -0.0113 & -0.5200 & -0.0729 & 0.0296 & 0.5088 & -0.0374 \\ 0.1941 & -0.0225 & -0.0061 & 0.4140 & 0.1757 & 0.0598 & 0.6216 & 0.1817 \\ 0.0153 & 0.0078 & 0.0202 & -0.0284 & -0.0280 & 0.0299 & -0.0049 & -0.0216 \\ -0.1154 & -0.0061 & -0.0106 & -0.2270 & 0.6551 & 0.0121 & -0.2604 & 0.6556 \\ -0.5254 & -0.0989 & 0.1360 & 0.3232 & -0.0959 & 0.3190 & -0.4692 & -0.1212 \\ 0.0208 & -0.0225 & 0.0105 & -0.0276 & -0.0031 & 0.0219 & 0.0052 & 0.0036 \\ -0.0617 & 0.9926 & 0.0165 & 0.0249 & -0.0071 & 0.0304 & -0.0289 & -0.0087 \\ 0.1000 & 0.0228 & 0.7404 & -0.1903 & -0.0057 & -0.3374 & 0.0122 & 0.0010 \\ 0.0055 & 0.0011 & -0.0077 & -0.0065 & -0.0164 & 0.0145 & 0.0007 & 0.0163 \\ -0.0262 & -0.0025 & -0.0309 & 0.0232 & 0.5047 & -0.0422 & 0.0198 & -0.4956 \\ 0.1985 & 0.0406 & -0.3354 & -0.2482 & -0.0041 & 0.5446 & 0.0506 & 0.0056 \\ -0.0042 & -0.0009 & 0.0094 & 0.0044 & 0.0162 & -0.0122 & -0.0008 & -0.0164 \\ -0.0236 & -0.0027 & -0.0304 & 0.0563 & -0.4953 & -0.0429 & -0.0133 & 0.5023 \end{bmatrix},$$

$$P_{W_{g,2}^*}^\top = \begin{bmatrix} 0.3032 & -0.0089 & -0.4067 & -0.3840 & -0.0695 & -0.4401 & -0.2355 & -0.0678 \\ 0.0163 & 0.0075 & 0.0287 & -0.0246 & -0.0018 & 0.0408 & 0.0153 & 0.0050 \\ 0.2648 & 0.0043 & -0.0275 & 0.7657 & 0.0230 & -0.0551 & -0.5751 & -0.0200 \\ -0.0352 & -0.0271 & -0.0469 & 0.4331 & 0.1716 & 0.0151 & 0.6369 & 0.1793 \\ 0.4839 & -0.0308 & 0.3732 & -0.2260 & 0.0916 & 0.5107 & -0.0790 & 0.0905 \\ 0.0990 & -0.0072 & 0.0723 & 0.1289 & -0.6729 & 0.0892 & 0.2162 & -0.6714 \\ -0.6644 & -0.1012 & 0.1395 & -0.0732 & -0.0609 & 0.3411 & -0.3916 & -0.0685 \\ 0.0344 & -0.0223 & 0.0115 & -0.0048 & -0.0015 & 0.0220 & 0.0010 & 0.0042 \\ -0.0695 & 0.9925 & 0.0173 & -0.0054 & -0.0050 & 0.0327 & -0.0233 & -0.0053 \\ 0.1851 & 0.0240 & 0.7469 & -0.0533 & 0.0098 & -0.3363 & -0.0120 & 0.0099 \\ 0.0092 & 0.0011 & -0.0075 & -0.0007 & -0.0159 & 0.0146 & -0.0004 & 0.0165 \\ -0.0457 & -0.0028 & -0.0330 & -0.0013 & 0.4971 & -0.0434 & 0.0234 & -0.5019 \\ 0.3271 & 0.0425 & -0.3261 & -0.0365 & 0.0110 & 0.5454 & 0.0116 & 0.0106 \\ -0.0069 & -0.0010 & 0.0092 & -0.0000 & 0.0160 & -0.0122 & 0.0000 & -0.0164 \\ -0.0400 & -0.0029 & -0.0317 & 0.0282 & -0.5024 & -0.0431 & -0.0092 & 0.4965 \end{bmatrix},$$

$$P_{W_{g,3}^*}^\top = \begin{bmatrix} 0.2300 & -0.0125 & -0.4648 & -0.3727 & -0.1181 & -0.5058 & -0.0085 & -0.1074 \\ 0.0378 & 0.0095 & 0.0256 & -0.0042 & 0.0007 & 0.0381 & 0.0248 & 0.0074 \\ -0.1100 & -0.0250 & 0.0624 & 0.0710 & -0.1435 & 0.0735 & -0.9465 & -0.1745 \\ -0.0791 & -0.0486 & -0.0509 & 0.7964 & 0.1404 & -0.0207 & 0.0998 & 0.1177 \\ 0.0383 & 0.0098 & 0.0200 & -0.0132 & -0.0278 & 0.0304 & 0.0005 & -0.0217 \\ 0.0404 & -0.0144 & 0.0189 & -0.2716 & 0.6502 & 0.0593 & -0.2114 & 0.6534 \\ -0.5920 & -0.1814 & 0.1819 & -0.3705 & -0.1021 & 0.4017 & 0.1762 & -0.0856 \\ 0.5820 & -0.0942 & 0.2633 & -0.0717 & -0.1054 & 0.3928 & 0.0994 & -0.1008 \\ -0.0985 & 0.9735 & 0.0456 & -0.0410 & -0.0171 & 0.0796 & 0.0266 & -0.0151 \\ 0.2583 & 0.0409 & 0.7471 & -0.0550 & -0.0063 & -0.3252 & -0.0285 & -0.0052 \\ 0.0109 & 0.0019 & -0.0077 & 0.0004 & -0.0164 & 0.0146 & -0.0025 & 0.0160 \\ -0.0566 & -0.0046 & -0.0301 & -0.0051 & 0.5050 & -0.0422 & 0.0183 & -0.4944 \\ 0.3897 & 0.0728 & -0.3337 & 0.0050 & -0.0038 & 0.5483 & -0.0766 & -0.0061 \\ -0.0079 & -0.0017 & 0.0093 & -0.0010 & 0.0162 & -0.0123 & 0.0023 & -0.0162 \\ -0.0577 & -0.0049 & -0.0301 & 0.0268 & -0.4946 & -0.0435 & -0.0117 & 0.5039 \end{bmatrix},$$

$$P_{W^*_{g,4}}^\top = \begin{bmatrix} 0.3712 & 0.0082 & -0.1006 & 0.6107 & 0.1117 & 0.0389 & 0.4476 & 0.1097 \\ 0.0100 & 0.0070 & -0.0002 & -0.0066 & -0.0016 & -0.0032 & -0.0245 & -0.0023 \\ -0.5294 & -0.2349 & 0.0612 & -0.1160 & -0.0100 & 0.0825 & 0.5571 & 0.0149 \\ -0.5746 & -0.0832 & -0.0353 & 0.3175 & 0.0624 & 0.0039 & -0.5507 & 0.0315 \\ -0.0048 & 0.0181 & 0.0001 & -0.0043 & -0.0013 & -0.0009 & 0.0210 & -0.0003 \\ 0.4703 & -0.5461 & 0.0157 & -0.0310 & 0.0073 & 0.0258 & -0.3857 & -0.0063 \\ 0.0616 & 0.0051 & -0.4587 & -0.4403 & -0.1684 & -0.5723 & 0.0807 & -0.1510 \\ -0.0060 & -0.0255 & -0.0019 & 0.0144 & 0.0027 & 0.0010 & 0.0033 & 0.0024 \\ 0.0909 & 0.7964 & -0.0034 & 0.0175 & 0.0072 & 0.0045 & -0.1649 & 0.0005 \\ 0.0679 & 0.0256 & 0.7967 & -0.1014 & -0.0554 & -0.2363 & 0.0303 & -0.0508 \\ -0.1077 & -0.0527 & -0.2377 & 0.4821 & -0.1472 & -0.3552 & -0.0202 & -0.1314 \\ -0.0138 & -0.0028 & -0.0835 & -0.1260 & 0.9598 & -0.1138 & 0.0043 & -0.0360 \\ 0.0424 & 0.0286 & -0.2645 & -0.2109 & -0.0886 & 0.6743 & -0.0111 & -0.0821 \\ -0.0120 & -0.0089 & -0.0147 & 0.0525 & 0.0090 & -0.0538 & -0.0003 & -0.0308 \\ -0.0110 & -0.0014 & -0.0764 & -0.1076 & -0.0352 & -0.1044 & -0.0333 & 0.9670 \end{bmatrix},$$

$$P_{W^*_{g,5}}^\top = \begin{bmatrix} -0.3173 & -0.0562 & -0.0271 & 0.5787 & 0.0589 & 0.1271 & 0.5081 & 0.0598 \\ 0.0251 & 0.0096 & 0.0007 & 0.0029 & -0.0007 & -0.0024 & -0.0083 & -0.0010 \\ -0.6487 & -0.2830 & -0.0071 & -0.4102 & -0.0094 & 0.0077 & -0.0069 & 0.0015 \\ 0.4069 & 0.0305 & -0.0282 & -0.6255 & 0.0156 & 0.0870 & 0.3904 & 0.0449 \\ -0.0234 & 0.0152 & -0.0013 & 0.0013 & -0.0005 & -0.0034 & -0.0033 & -0.0007 \\ 0.5447 & -0.5147 & 0.0580 & 0.2989 & 0.0084 & 0.0631 & -0.1058 & -0.0030 \\ -0.0115 & -0.0087 & -0.4769 & 0.0554 & -0.0691 & -0.6221 & -0.3918 & -0.0827 \\ -0.0012 & -0.0250 & -0.0011 & -0.0042 & 0.0011 & 0.0033 & 0.0152 & 0.0017 \\ 0.0865 & 0.8053 & 0.0131 & 0.1115 & 0.0039 & 0.0194 & -0.0235 & 0.0002 \\ -0.0362 & 0.0116 & 0.7751 & -0.0003 & -0.0271 & -0.2727 & -0.1126 & -0.0305 \\ 0.0047 & -0.0051 & -0.0255 & -0.0000 & -0.0250 & -0.0140 & 0.0395 & 0.0030 \\ 0.0028 & -0.0001 & -0.0401 & -0.0277 & 0.9924 & -0.0555 & -0.0437 & -0.0082 \\ -0.0232 & 0.0152 & -0.3146 & -0.0064 & -0.0455 & 0.5934 & -0.2417 & -0.0528 \\ 0.0743 & -0.0318 & -0.2515 & 0.0009 & -0.0551 & -0.3880 & 0.5882 & -0.0681 \\ 0.0051 & -0.0009 & -0.0473 & -0.0010 & -0.0087 & -0.0674 & -0.0579 & 0.9895 \end{bmatrix}.$$

$$E_1 = \begin{bmatrix} 0.0138 & 0.0178 & -0.0695 & -0.0775 & -0.0097 & -0.0761 & -0.0355 & -0.0088 \\ 0.1826 & -0.0114 & 0.1961 & -0.2329 & -0.0433 & 0.2752 & -0.0282 & -0.0394 \\ -0.1774 & 0.0451 & -0.0011 & -0.1431 & 0.0149 & 0.0114 & 0.1131 & 0.0239 \\ 0.0133 & 0.0162 & -0.0018 & 0.0561 & 0.0418 & 0.0075 & 0.1087 & 0.0433 \\ -0.0227 & 0.0130 & -0.0141 & 0.0078 & 0.0064 & -0.0179 & 0.0031 & 0.0086 \\ -0.0714 & 0.0493 & -0.0065 & -0.0780 & 0.1620 & 0.0001 & -0.0400 & 0.1639 \\ -0.0988 & -0.0021 & 0.0251 & 0.0411 & -0.0074 & 0.0550 & -0.0748 & -0.0113 \\ -0.0329 & 0.0221 & -0.0171 & 0.0024 & 0.0157 & -0.0210 & 0.0082 & 0.0183 \\ -0.0612 & 0.2496 & -0.0014 & -0.0275 & 0.0298 & 0.0034 & 0.0064 & 0.0313 \\ 0.0322 & -0.0013 & 0.1366 & -0.0436 & -0.0061 & -0.0401 & -0.0005 & -0.0051 \\ 0.0127 & 0.0035 & 0.0144 & -0.0236 & -0.0029 & 0.0266 & -0.0015 & 0.0004 \\ 0.0026 & -0.0012 & 0.0018 & -0.0048 & 0.0989 & 0.0029 & 0.0027 & -0.1011 \\ 0.0598 & -0.0065 & -0.0381 & -0.0528 & -0.0117 & 0.1129 & 0.0035 & -0.0102 \\ 0.0103 & 0.0039 & 0.0173 & -0.0219 & -0.0004 & 0.0213 & -0.0016 & -0.0012 \\ 0.0028 & -0.0012 & 0.0016 & 0.0023 & -0.1011 & 0.0023 & -0.0039 & 0.0987 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0.0545 & 0.0191 & -0.0672 & -0.0445 & -0.0254 & -0.0760 & -0.0466 & -0.0261 \\ -0.0169 & 0.0132 & -0.0128 & 0.0169 & -0.0108 & -0.0168 & 0.0001 & -0.0089 \\ 0.0625 & 0.0528 & -0.0066 & 0.2020 & -0.0304 & -0.0138 & -0.1342 & -0.0410 \\ 0.0015 & 0.0157 & -0.0046 & 0.0902 & 0.0154 & 0.0035 & 0.0984 & 0.0155 \\ 0.2657 & -0.0107 & 0.2037 & -0.1115 & 0.0371 & 0.2783 & -0.0384 & 0.0348 \\ 0.0369 & 0.0499 & 0.0192 & 0.0712 & -0.1684 & 0.0230 & 0.0232 & -0.1702 \\ -0.1041 & -0.0021 & 0.0268 & 0.0005 & -0.0196 & 0.0586 & -0.0710 & -0.0218 \\ -0.0137 & 0.0227 & -0.0163 & 0.0293 & -0.0186 & -0.0215 & -0.0048 & -0.0173 \\ -0.0046 & 0.2510 & 0.0021 & 0.0482 & -0.0363 & 0.0034 & -0.0239 & -0.0384 \\ 0.0466 & -0.0011 & 0.1380 & -0.0198 & 0.0070 & -0.0396 & -0.0024 & 0.0068 \\ 0.0248 & 0.0037 & 0.0154 & -0.0060 & -0.0009 & 0.0269 & -0.0047 & 0.0016 \\ 0.0017 & -0.0013 & 0.0017 & -0.0052 & 0.1012 & 0.0028 & 0.0033 & -0.0988 \\ 0.0741 & -0.0064 & -0.0367 & -0.0268 & 0.0133 & 0.1134 & 0.0038 & 0.0135 \\ 0.0220 & 0.0040 & 0.0183 & -0.0049 & 0.0005 & 0.0217 & -0.0049 & -0.0011 \\ 0.0024 & -0.0013 & 0.0017 & 0.0009 & -0.0989 & 0.0024 & -0.0031 & 0.1010 \end{bmatrix},$$

$$E_3 = \begin{bmatrix} 0.0275 & -0.0020 & -0.0828 & -0.0603 & -0.0181 & -0.0924 & -0.0021 & -0.0166 \\ -0.0293 & 0.0069 & -0.0127 & 0.0040 & 0.0059 & -0.0168 & -0.0053 & 0.0078 \\ -0.0352 & 0.0377 & 0.0154 & 0.0041 & -0.0057 & 0.0206 & -0.2412 & -0.0131 \\ -0.0240 & -0.0080 & -0.0138 & 0.1345 & 0.0250 & -0.0115 & 0.0160 & 0.0210 \\ -0.0288 & 0.0069 & -0.0138 & 0.0015 & 0.0033 & -0.0181 & -0.0073 & 0.0053 \\ -0.0051 & 0.0399 & 0.0067 & -0.0645 & 0.1531 & 0.0178 & -0.0942 & 0.1526 \\ -0.0962 & -0.0316 & 0.0316 & -0.0617 & -0.0178 & 0.0679 & 0.0309 & -0.0153 \\ 0.3175 & -0.0334 & 0.1496 & -0.0441 & -0.0553 & 0.2222 & 0.0488 & -0.0542 \\ -0.0238 & 0.2367 & 0.0164 & -0.0196 & 0.0181 & 0.0282 & -0.0452 & 0.0173 \\ 0.0763 & 0.0018 & 0.1409 & -0.0132 & -0.0075 & -0.0328 & 0.0023 & -0.0074 \\ 0.0361 & 0.0159 & 0.0194 & -0.0102 & 0.0018 & 0.0332 & -0.0172 & 0.0042 \\ 0.0054 & 0.0015 & 0.0030 & -0.0045 & 0.1001 & 0.0045 & 0.0015 & -0.0999 \\ 0.1108 & 0.0057 & -0.0330 & -0.0049 & -0.0094 & 0.1212 & -0.0036 & -0.0097 \\ 0.0320 & 0.0152 & 0.0220 & -0.0101 & 0.0040 & 0.0275 & -0.0165 & 0.0023 \\ 0.0046 & 0.0015 & 0.0028 & 0.0019 & -0.0998 & 0.0038 & -0.0047 & 0.1000 \end{bmatrix},$$

$$E_4 = \begin{bmatrix} 0.0514 & -0.0035 & -0.0419 & 0.1460 & 0.0114 & -0.0293 & 0.0726 & 0.0101 \\ -0.0096 & -0.0045 & -0.0240 & 0.0313 & 0.0188 & -0.0378 & -0.0049 & 0.0165 \\ -0.1109 & -0.0494 & 0.0003 & -0.0038 & 0.0002 & -0.0017 & 0.1104 & 0.0045 \\ -0.1062 & -0.0187 & -0.0310 & 0.0972 & 0.0032 & -0.0351 & -0.0939 & -0.0029 \\ -0.0093 & -0.0036 & -0.0241 & 0.0319 & 0.0189 & -0.0376 & 0.0001 & 0.0168 \\ 0.0891 & -0.1116 & -0.0088 & 0.0132 & 0.0037 & -0.0130 & -0.0783 & 0.0003 \\ 0.0030 & -0.0024 & -0.0944 & -0.0430 & -0.0324 & -0.1202 & 0.0120 & -0.0300 \\ -0.0117 & -0.0075 & -0.0268 & 0.0393 & 0.0207 & -0.0416 & -0.0026 & 0.0181 \\ 0.0131 & 0.1570 & -0.0128 & 0.0232 & 0.0036 & -0.0175 & -0.0341 & 0.0016 \\ 0.0148 & 0.0061 & 0.1387 & -0.0333 & -0.0033 & -0.0275 & 0.0056 & -0.0017 \\ -0.0616 & -0.0296 & -0.1353 & 0.2662 & -0.0472 & -0.2128 & -0.0119 & -0.0538 \\ -0.0071 & -0.0015 & -0.0423 & -0.0591 & 0.3353 & -0.0585 & -0.0037 & 0.1367 \\ 0.0164 & 0.0094 & -0.0251 & -0.0769 & -0.0040 & 0.1444 & -0.0002 & -0.0011 \\ 0.0085 & 0.0026 & 0.0212 & -0.0400 & 0.0178 & 0.0263 & 0.0020 & 0.0139 \\ -0.0045 & -0.0002 & -0.0362 & -0.0645 & 0.1377 & -0.0494 & -0.0108 & 0.3392 \end{bmatrix},$$

$$E_5 = \begin{bmatrix} -0.0484 & -0.0111 & -0.0215 & 0.0967 & 0.0001 & -0.0004 & 0.1202 & 0.0018 \\ 0.0069 & -0.0017 & -0.0156 & -0.0010 & 0.0076 & -0.0245 & 0.0326 & 0.0075 \\ -0.1273 & -0.0576 & -0.0101 & -0.0821 & -0.0032 & -0.0110 & 0.0179 & -0.0005 \\ 0.0723 & 0.0033 & -0.0217 & -0.1039 & -0.0072 & -0.0071 & 0.1006 & -0.0007 \\ 0.0020 & -0.0014 & -0.0157 & 0.0007 & 0.0077 & -0.0245 & 0.0332 & 0.0076 \\ 0.1115 & -0.1040 & 0.0029 & 0.0598 & 0.0003 & 0.0001 & -0.0019 & -0.0014 \\ 0.0012 & -0.0026 & -0.0925 & 0.0093 & -0.0119 & -0.1191 & -0.0417 & -0.0130 \\ 0.0050 & -0.0045 & -0.0171 & -0.0008 & 0.0075 & -0.0262 & 0.0405 & 0.0076 \\ 0.0198 & 0.1601 & -0.0062 & 0.0223 & -0.0005 & -0.0088 & 0.0148 & -0.0007 \\ -0.0073 & 0.0028 & 0.1296 & -0.0009 & 0.0265 & -0.0404 & -0.0348 & 0.0259 \\ -0.0050 & 0.0005 & 0.0128 & -0.0018 & 0.0548 & 0.0220 & -0.0463 & 0.0520 \\ 0.0001 & 0.0005 & -0.0166 & -0.0104 & 0.3667 & -0.0225 & -0.0403 & 0.1652 \\ -0.0062 & 0.0038 & -0.0494 & -0.0022 & 0.0350 & 0.1084 & -0.0663 & 0.0337 \\ 0.0416 & -0.0172 & -0.1464 & -0.0000 & -0.0138 & -0.2144 & 0.3238 & -0.0044 \\ 0.0019 & -0.0002 & -0.0229 & -0.0050 & 0.1641 & -0.0318 & -0.0321 & 0.3631 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0.3865 & 0.0375 \\ 0.0375 & 0.6222 \\ 0.0446 & 0.0341 \\ 0.2197 & 0.0301 \\ 0.0301 & 0.0065 \\ 0.0443 & 0.0177 \\ 0.1941 & 0.0068 \\ 0.0317 & 0.0118 \\ 0.0442 & 0.0170 \\ 0.1079 & -0.0693 \\ -0.0856 & -0.0369 \\ -0.0135 & -0.0034 \\ 0.0723 & -0.1023 \\ -0.0821 & -0.0332 \\ -0.0136 & -0.0046 \end{bmatrix}, F_2 = \begin{bmatrix} 0.2197 & 0.0301 \\ 0.0301 & 0.0065 \\ 0.0443 & 0.0177 \\ 0.3865 & 0.0375 \\ 0.0375 & 0.6222 \\ 0.0446 & 0.0341 \\ 0.1941 & 0.0068 \\ 0.0317 & 0.0118 \\ 0.0442 & 0.0170 \\ 0.1079 & -0.0693 \\ -0.0856 & -0.0369 \\ -0.0135 & -0.0034 \\ 0.0723 & -0.1023 \\ -0.0821 & -0.0332 \\ -0.0136 & -0.0046 \end{bmatrix}, F_3 = \begin{bmatrix} 0.1941 & 0.0317 \\ 0.0068 & 0.0118 \\ 0.0327 & 0.0303 \\ 0.1941 & 0.0317 \\ 0.0068 & 0.0118 \\ 0.0327 & 0.0303 \\ 0.3467 & 0.0128 \\ 0.0128 & 0.6682 \\ 0.0329 & 0.0473 \\ 0.1316 & -0.0743 \\ -0.0543 & -0.0435 \\ 0.0019 & -0.0053 \\ 0.1121 & -0.1128 \\ -0.0538 & -0.0391 \\ 0.0018 & -0.0066 \end{bmatrix},$$

$$F_4 = \begin{bmatrix} 0.1079 & -0.0856 \\ -0.0693 & -0.0369 \\ -0.0064 & -0.0122 \\ 0.1079 & -0.0856 \\ -0.0693 & -0.0369 \\ -0.0064 & -0.0122 \\ 0.1316 & -0.0543 \\ -0.0743 & -0.0435 \\ -0.0063 & -0.0118 \\ 0.3750 & 0.0484 \\ 0.0484 & 0.7432 \\ 0.0531 & 0.0831 \\ 0.2421 & 0.0791 \\ 0.0398 & 0.0177 \\ 0.0532 & 0.0624 \end{bmatrix}, F_5 = \begin{bmatrix} 0.0723 & -0.0821 \\ -0.1023 & -0.0332 \\ -0.0229 & -0.0105 \\ 0.0723 & -0.0821 \\ -0.1023 & -0.0332 \\ -0.0229 & -0.0105 \\ 0.1121 & -0.0538 \\ -0.1128 & -0.0391 \\ -0.0228 & -0.0101 \\ 0.2421 & 0.0398 \\ 0.0791 & 0.0177 \\ 0.0735 & 0.0331 \\ 0.4660 & 0.0923 \\ 0.0923 & 0.6899 \\ 0.0748 & 0.0536 \end{bmatrix}.$$

$$\mathcal{L} = \begin{bmatrix} 3.0000 & -1.0000 & 0.0000 & -1.0000 & -1.0000 \\ -1.0000 & 3.0000 & -1.0000 & 0.0000 & -1.0000 \\ 0.0000 & -1.0000 & 2.0000 & -1.0000 & 0.0000 \\ -1.0000 & 0.0000 & -1.0000 & 3.0000 & -1.0000 \\ -1.0000 & -1.0000 & 0.0000 & -1.0000 & 3.0000 \end{bmatrix},$$

$$\mathbf{W} = \begin{bmatrix} 0.1429 & 0.2857 & 0.0000 & 0.2857 & 0.2857 \\ 0.2857 & 0.1429 & 0.2857 & 0.0000 & 0.2857 \\ 0.0000 & 0.2857 & 0.4286 & 0.2857 & 0.0000 \\ 0.2857 & 0.0000 & 0.2857 & 0.1429 & 0.2857 \\ 0.2857 & 0.2857 & 0.0000 & 0.2857 & 0.1429 \end{bmatrix}.$$