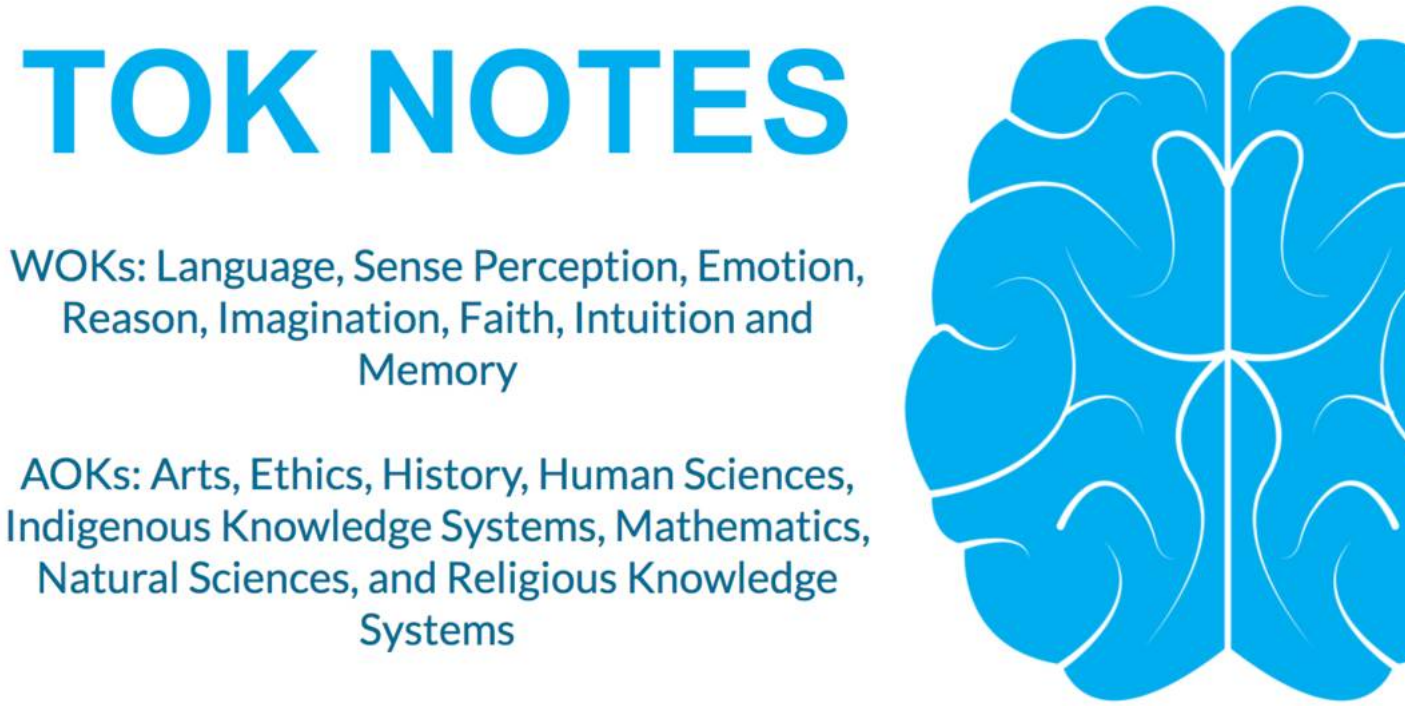
Mathematics Notes

Post 15/17



ToK Notes

Mark as Complete

Mathematics Quotes

- "Mathematics is the abstract key which turns the lock of the universe" (John Polkinghorne).
- "Everything that can be counted does not count. Everything that counts cannot be counted" (Albert Einstein).
- "A mathematician is a machine for turning coffee into theorems" (Paul Erdos).
- "When you have satisfied yourself that the theorem is true, you start proving it" (Arthur Koestler).
- "If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics" Kurt Godel).
- "On each decision, the mathematical analysis only got me to the point where my intuition had to take over." (Robert Jensen)
- "Mathematics is the most beautiful and most powerful creation of the human spirit." (Stefan Banach)
- "The essence of mathematics lies in its freedom" (Georg Cantor).

- "The mathematics is not there till we put it there" (Arthur Eddington).
- "Logic and mathematics are nothing but specialised linguistic structures" (Jean Piaget).
- "Film is one of the three universal languages, the other two: mathematics and music" (Frank Capra).

Mathematics Definitions

- "The abstract science of number, quantity, and space, either as abstract concepts (pure mathematics), or as applied to other disciplines such as physics and engineering (applied mathematics)." (*Oxford Dictionary)*
- "The science of numbers, quantities, and shapes and the relations between them." (*Merriam Webster Dictionary)*
- "The systematic treatment of magnitude,relationships between figures and forms, and relations between quantities expressed symbolically." (*Dictionary.com)*
- "The study of the measurement, properties, and relationships of quantities and sets, using numbers and symbols." (*The Free Dictionary)*

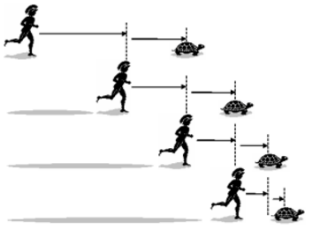
Is math created or discovered?

Eugene Wigner said “The miracle of the appropriateness of the language of mathematics to the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve.” This quote pulls out several aspects of this created verus invented discussion. All phenomena observed in classical electricity and magnetism can be explained by means of just four mathematical equations (Livio). In some cases, Math seems to be so perfectly in-line with natural reality that it must be discovered. Whereas there are other examples of Math where there is no clear relation to reality, suggesting that math is invented. But Mario Livio explains that, "mathematicians [have] developed abstract branches of mathematics with absolutely no applications in mind; yet decades, or sometimes centuries later, physicists discovered that those theories provided necessary mathematical underpinnings for physical phenomena." This again suggests that mathematicians are discovering math.

Also, some theories (explanations of reality) can be mathematically correct, but not correct in the real world. Zeno's Paradox is an example of this. Here's the story:

Zeno’s Paradox: Achilles and the tortoise

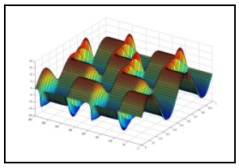
- The question was posed: *How could a tortoise beat greek legend Achilles in a race?* Philosopher Zeno attempted to answer such a question.First, the tortoise is given a head start.
- So, when the race starts, Achilles will have to make up the distance.
- However, in that time, the tortoise would have moved forward by a little bit.
- Then Achilles will have to make up that distance, but in that time the tortoise would have had to move even more.
- Therefore Achilles would never be able to pass the tortoise and the tortoise would win the race.
- Will this theory may seem mathematically correct, we know that in reality, it is impossible. This is an example of how while something things can be explained mathematically, they cannot always be true.



https://www.youtube.com/watch?v=skM37PcZmWE#t=20 ← Good video explaining Zeno’s paradox.

Pure and Applied Mathematics

- **Pure Mathematics** refers to the study of mathematics in itself. In other words, it is math that is done for it’s own sake.
- **Applied Mathematics** is mathematics applied to another discipline, or math that is done for the sake of other subjects. E.g engineering or chemistry.

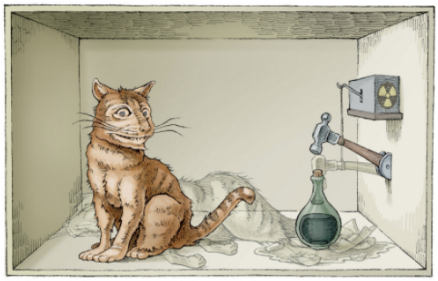


Thought experiments

A thought experiment is an experiment that is carried out purely by imagination. Thought experiments heavily rely on **logic** and **reason** to determine their outcomes.

- Zeno’s paradox (above) is an example of a thought experiment.

Example of a famous thought experiment: Schrödinger's Cat



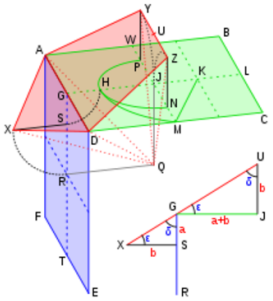
- This is a famous thought experiment that relates to quantum mechanics, which involves applied mathematics and mechanics.
- Basically, a cat is placed in a steel cage with a radioactive substance, a geiger counter attached to a hammer, and a vile of poisonous gas.
- If the radioactive substance decays, it will trigger the geiger counter, drop the hammer and release the poisonous gas, thus killing the cat.
- Quantum mechanics suggests that particles are in a superposition (defined later) of all possible states at once.
- So, the radioactive substance is in a superposition of decayed and not decayed, and therefore the cat is alive and dead at once.
- This is obviously impossible in the real world, however in theoretical quantum mechanics, this thought experiment provides proof for quantum theory. (That particles can exist as two things at once.)
- This experiment thus again **shows how some theories may be viable in terms of mathematical and physical laws, however in reality they are impossible.**

- **Thought experiments allow individuals to experiment in the *theoretical world*.**

https://www.youtube.com/watch?v=d1tn56vWU_g ← Video explaining this thought experiment.

Deductive Reasoning and Mathematics

Mathematics is an area of knowledge in which deductive reasoning (see reasoning notes) is heavily relied on to gain knowledge.



Deductive reasoning is a logical process in which a conclusion is based on the concordance of multiple premises that are generally assumed to be true.

In the case of mathematics, the **premises that are generally assumed to be true** are laws and theories that have been proposed/proven.

Euclid Geometry: Euclid's Five Axioms

Euclid was a greek Mathematician who was best known for his contribution to geometry. His five axioms or laws pertain to geometry and are shown below. These are examples of premises in mathematics that are assumed to be true. Given these axioms, individuals can use deductive reasoning to solve problems and hence gain knowledge.

Euclid's five axioms:-

1. **It shall be possible to draw a straight line joining any two points**
2. **A finite straight line may be extended without limit in either direction**
3. **It shall be possible to draw a circle with a given centre and through a given point**
4. **All right angles are equal to one another**
5. **There is just one straight line through a given point which is parallel to a given line**

Language and Mathematics

There are many arguments of why mathematics should or should not be considered a language.

Why it should:

- Mathematics uses symbols/characters to portray information.
- Mathematical notation is understood and used by majority of the people in the world.
  - It can even be argued that mathematics is a **universal language**.

Why it shouldn't

- Mathematics does not evoke an emotional response.
- Mathematical language is very small as to what it can describe, as compared to other languages. (However the size of a language is only relative)
- Mathematical language is almost a middleman between mathematics and other languages. We don't think in terms of mathematical language, we simply translate our language into it.

Sources

- Livio, Mario. "Math: Discovered, Invented, or Both?" Thought Experiments. PBS.org https://ibmathsresources.com/2014/02/21/is-maths-invented-or-discovered/ 13 Apr 2015

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