AwesomeMath Test B
March 8-March 31, 2019

PLEASE READ CAREFULLY and THOROUGHLY

DEADLINE

Submit your solutions to admissions@awesomemath.org no later than **Sunday, March 31, 2019 11:59 PM CST**.

INSTRUCTIONS

- This test is for students taking our Level 1 classes up through Level 3, so the test accommodates different ages and experience and is evaluated accordingly.
- We want you to have a positive and successful experience at our camps, therefore when submitting solutions **your work should be yours and yours alone**. The purpose of the test is to ensure that you are placed in classes that fit your level and skills. If you consult outside resources (other people or online), then you are compromising your ability to succeed in our challenging program.
- **Do not be discouraged if you cannot solve all of the questions**. We want to see the solutions you come up with no matter how many problems you solve. Some of the problems involve complex mathematical ideas, but all can be solved using only elementary techniques, admittedly combined in clever ways.
- **Include all significant steps in your reasoning and computation**. We are interested in your ability to present your work, so unsupported answers will receive much less credit than well-reasoned progress towards a solution without a correct answer.
- In this document, you will find a cover sheet and an answer sheet. Print out each one and make several copies of the blank answer sheet. Fill out the top of each answer sheet as you go, and then fill out the cover sheet when you are finished. Start each problem on a new answer sheet.
- **You may handwrite or type your solutions**. If you type your solutions, your cover and answer sheets should still include the same information as shown on the test packet answer sheets (9-digit UIN, First and Last Name, Problem #, etc.)

SUBMISSION REQUIREMENTS

- **Make sure the TEST COVER SHEET is the first page of your submission**, and that it is *completely* filled out.
- Email your solutions to admissions@awesomemath.org.
- The Subject Line of your email should be written in the following format: Test (A/B/C) Solutions, First Name, Last Name, UIN#. Example: Test B Solutions, Jane Doe, UIN 123456789
- **Please submit your solutions as a single pdf document**. Attaching multiple pages or separate jpg, png pages will delay the processing of your submissions

AwesomeMath
making x,y,z as easy as a,b,c

AwesomeMath Admission Test

Cover Sheet

PLEASE PRINT LEGIBLY and DO NOT LEAVE ANY FIELDS BLANK

9-Digit UIN:

Last Name: **First Name**:

Admission Test (check one): [ ] A [ ] B [ ] C

Phone Number:

Email:

Test B

March 8 – 31, 2019

1. Find all triples (p, q, r) of primes such that
$$pqr - 18(p + q + r) = 2019.$$

2. Let $(a_n)_n \geq 0$ be a sequence of positive real numbers such that $a_3 = 2019$ and
$$a_{n+1} = 2^{n-3} a_n (a_n + 1) + 3, \ \ n \geq 0.$$
Find a_0.

3. Find all pairs (x, y) of positive integers such that
$$3(xy + 3)^2 - (3x + 3y)^2 = 2019.$$

4. The 13-digit number $N = abcdefghijklm$ is divisible by 101. Find the digit a knowing that
$$bcdefg + 2019 = hijklm.$$

5. Solve in real numbers the equation
$$\sqrt{1019 - x} - \sqrt[3]{2019 + x} = 16.$$

6. In quadrilateral $ABCD$, $\angle A = \angle B = 60^\circ$, $|AD - BC| = 13$, and $CD = \sqrt{2019}$. Find AB, knowing that it has integer length and that the perimeter of $ABCD$ is as small as possible.

7. If $a \leq b \leq c \leq d$ are positive integers such that
$$a^3 + b^3 + c^3 + d^3 = (a^2 - 1)bcd + 2019,$$
find the least possible value of a.

8. Solve in complex numbers the system of equations
$$\begin{cases} \dfrac{2019}{z} - 2w = 12 - 23i \\ \dfrac{2019}{w} - 2z = 12 + 23i. \end{cases}$$

9. Find the least positive real number r such that the equation
$$\sqrt{x} + \sqrt{y} + r\sqrt{1 - (x + y)} = \frac{\sqrt{2019}}{13}$$
is solvable in real numbers $x, y \in \left(0, \frac{1}{2}\right)$.

10. Consider 2019 points in the plane and label them with numbers from 1 to 2019, each used exactly once. Connect two points if and only if their labels are relatively prime. How many segments are drawn?

AwesomeMath Admission Test Answer Sheet

9-Digit UIN:

First Name: **Last Name**:

Problem Number: **Page** **of**

Write neatly! All work should be inside the box. Do NOT write on the back of the page!