FM2 Assignment #26

Name: Maggie $\frac{10}{12}$ Excellent!

1. The solution to the differential equation $\frac{dy}{dx} + 5y = 0$ for which $y = 2$ when $x = 0$ can be written in the form $y = Ae^{kx}$. Find the values of A and k.

$\int \frac{1}{-5y}\,dy = \int 1\,dx$

$-\frac{1}{5}\ln|y| = x + C_1$

$|y| = e^{-5x} \cdot e^{C_2}, \quad C_2 = -5C_1$

$|y| = C_3 \cdot e^{-5x}, \quad C_3 = e^{C_2} > 0$

So $y = Ae^{-5x}$,

when $x = 0$,

$y = A = 2$

so $\boxed{A = 2, \; k = -5}$

2. A fair coin is tossed until five heads occur. Find the probability that the fifth head occurs on the tenth toss.

Let X count the number of throws for the fifth head,

Then $X \sim NB(5, \frac{1}{2})$

$P(X=10) = \binom{9}{4}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5 = \frac{9 \times \cancel{8} \times 7 \times \cancel{6}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 2^{10}} = \frac{63}{2^9} = \boxed{\frac{63}{512}}$

3. An isomorphism from a group to itself is called an *automorphism*. Show that the function $f : \mathbb{C}^* \to \mathbb{C}^*$ with rule $f(z) = z^*$ is an automorphism of the group $(C^*, \times)$.

proof. First, we want to show that f is a bijection.
As for every $z^* = a - bi$, we can find $z = f^{-1}(z^*) = a + bi$, it is surjective;
If $f(z) = f(w) = a - bi$, then $z = a + bi = w$, thus injectivity.
Therefore, f is a bijection.

Now, let $z = a + bi$, $w = c + di$,

$f(z) \cdot f(w) = (a - bi)(c - di) = (ac - bd) - (ad + bc)i$

$f(z \cdot w) = f((a+bi)(c+di)) = f((ac - bd) + (ad + bc)i)$

$= (ac - bd) - (ad + bc)i$,

so $f(z) \cdot f(w) = f(z \cdot w)$. Hence, f is an automorphism //.

4

4. The random variable X has probability generating function $G(t) = \dfrac{t}{3-2t}$, mean μ and variance σ^2. Find $P(|X-\mu| < \sigma)$.

$G(t) = \dfrac{\frac{t}{3}}{1-\frac{2}{3}t}$, which satisfies the Geometric distribution. $X \sim G(\frac{1}{3})$.

Geo $(\frac{1}{3})$

So $\mu = E(t) = G'(1) = \dfrac{1}{(\frac{1}{3})} = 3$.

and $\sigma^2 = Var(x) = G''(1) + G'(1) - [G'(1)]^2$

$= \dfrac{q}{p^2} = \dfrac{(\frac{2}{3})}{(\frac{1}{3})^2} = 6$.

therefore, $P(|X-3| < \sqrt{6}) = P(3-\sqrt{6} < 1 \le X \le 5 < 3+\sqrt{6})$

$G(t) = \dfrac{\frac{t}{3}}{1-\frac{2}{3}t} = \frac{1}{3}t + (\frac{1}{3})(\frac{2}{3})t^2 + (\frac{1}{3})(\frac{2}{3})^2 t^3 + \ldots$

So $P(|x-3| < \sqrt{6}) = \frac{1}{3}(1 + \frac{2}{3} + (\frac{2}{3})^2 + \ldots + (\frac{2}{3})^4) = \frac{1}{3}\left(\dfrac{1-\frac{32}{243}}{\frac{1}{3}}\right) = \boxed{\frac{211}{243}}$

5. Newton's law of cooling states that a body cools at a rate proportional to the difference between the temperature of the body and its surroundings. Sherlock Holmes finds that the core temperature of a corpse is 17 °C at 6.30 am and three hours later that this temperature has fallen to 11 °C. If the temperature of the surroundings had been approximately 5 °C throughout the night and the normal living body temperature is 37 °C, what did Holmes estimate as the time of death?

$\frac{\delta T}{\delta t}$ $\quad v \propto \delta t$

let T be the ~~the~~ temperature of the ~~ab~~ body, t be the time,

then $\dfrac{dT}{dt} = k(T-5)$, where k is a constant.

$\frac{1}{k}\int \frac{1}{T-5}\,dT = \int 1\,dt$

$\frac{1}{k}\ln|T-5| = t + C_1$, $T-5>0$,

$T = C_2 e^{kt} + 5$, $C_2 = e^{kC_1}$

Since $37 = C_2 + 5$, $C_2 = 32$.

So $\begin{cases} 6 = C_2 e^{k(t+3)} \\ 12 = C_2 e^{kt} \end{cases}$

$\frac{1}{2} = e^{3k}$

$k = \frac{1}{3}\ln(\frac{1}{2})$

So $12 = 32e^{\frac{1}{3}\ln(\frac{1}{2})\cdot t}$.

$t = \dfrac{\ln(\frac{12}{32})}{\frac{1}{3}\ln(\frac{1}{2})} = 4.25h$.

So the death is around $6.5 - 4.25 = 2.25$, which is $\boxed{2{:}15\text{ am}}$.

4

FM2 Assignment #27 Name: Maggie. $\frac{10}{10}$ Excellent!

1. A radioactive source emits 482 alpha particles in two hours. What is the probability that the source will emit more than three alpha particles in the next minute?

Let X count the number of alpha particles emitted in every minute.

$\frac{482}{2\times 60} = \frac{241}{60}$, so $X \sim Po\left(\frac{241}{60}\right)$

$P(X>3) = 1 - \text{poissoncdf}\left(\frac{241}{60}, 3\right)$

$= 1 - 0.430$

$= 0.570$ (3 s.f.)

2. Prove that a non-Abelian group must contain a proper subgroup.

Consider the contrapositive: every group without a proper subgroup must be Abelian.

Let G be a group with no proper subgroup.

① if $G = \{e\}$, it is abelian.

② suppose $G \neq \{e\}$, then $\exists\, a \in G$, $a \neq e$. Consider $a^0, a^1, a^2, a^3, \ldots$, since G does not have proper subgroup, $\langle a \rangle = G$, which means G is cyclic, and every cyclic group is abelian.

Therefore, G must be abelian, and thus the statement is proved. □.

3. The angle between the asymptotes of the hyperbola $x^2 - k^2y^2 = k^2$ is $\pi/3$. Find the equations of the directrices.

$\frac{x^2}{k^2} - y^2 = 1$.

The asymptotes are

$y = \pm \frac{1}{k} x$.

So $\tan\left(\frac{\pi}{6}\right) = \frac{1}{k}$

$k_1 = \sqrt{3}$, OR,

$\tan\left(\frac{\pi}{3}\right) = \frac{1}{k}$

$k_2 = \frac{\sqrt{3}}{3}$

so $\frac{x^2}{3} - y^2 = 1$ OR $\frac{x^2}{\frac{1}{3}} - y^2 = 1$

$c_1 = \sqrt{3+1} = 2$. OR $c_2 = \sqrt{\frac{1}{3}+1} = \frac{2}{3}\sqrt{3}$

Therefore,

the directrices are

$x = \pm \frac{3}{2}$ OR

$x = \pm \frac{\frac{1}{3}}{\frac{2}{3}\sqrt{3}} = \pm \frac{\sqrt{3}}{6}$

6

4. Suppose $X \sim \mathrm{Po}(\ln 2)$. Find the probability that X is even.

$$P(X \text{ is even}) = \frac{e^{-\lambda}\cdot\lambda^0}{0!} + \frac{e^{-\lambda}\cdot\lambda^2}{2!} + \frac{e^{-\lambda}\cdot\lambda^4}{4!} + \cdots$$

$$= e^{-\lambda}\cdot(e^{\lambda}+e^{-\lambda})\times\frac{1}{2}$$

$$= \frac{e^0+e^{-2\lambda}}{2}$$

$$= \frac{1+e^{-2\lambda}}{2}.$$

$$= \frac{1+e^{-2\ln 2}}{2} = \boxed{\frac{5}{8}}$$

5. A cylindrical water tank has its axis vertical. The area of its base is $2\,\mathrm{m}^2$. Initially the tank is empty. Starting at time $t=0$, water is poured into the tank at a constant rate of $0.2\,\mathrm{m}^3\mathrm{s}^{-1}$, and leaks out a small hole in the base at a rate of $0.1x\,\mathrm{m}^3\mathrm{s}^{-1}$, where x m is depth of the water in the tank at time t. Show that

$$20\frac{dx}{dt} = 2 - x,$$

and solve the differential equation to obtain x as a function of t. Deduce that the depth of the water in the tank never exceeds 2 m, and find in the form $a \ln b$ for integers a and b, how long the water takes to reach a depth of 1 m.

since $\Delta x\cdot 2 = 0.2\Delta t - 0.1x\Delta t$,

So $\frac{\Delta x}{\Delta t} = 0.1 - 0.05x$,

$20\frac{dx}{dt} = 2 - x$.

$20\int\frac{1}{2-x}\,dx = \int 1\,dt.$

$-20\ln|2-x| = t + C_1$

$2-x = e^{-\frac{1}{20}t}\cdot e^{C_2}$, $C_2 = \frac{-C_1}{20}$

$x = 2 - Ae^{-\frac{1}{20}t}$, $A = e^{C_2}$.

$t=0$,

$x = 2 - A = 0$, $A = 2$,

So $x = 2-2e^{-\frac{1}{20}t}$.

As $e^{-\frac{1}{20}t} > 0$,

$2-2e^{-\frac{1}{20}t} < 2$.

So $x<2$, depth of water never exceeds 2m.

when $x=1$,

$e^{-\frac{1}{20}t} = \frac{1}{2}$

$e^{\frac{t}{20}} = 2$

$t = 20\ln 2$,

so $\begin{cases} a = 20 \\ b = 2 \end{cases}$.

4

FM2 Assignment #28 Name: Maggie.

1. Lebron James's probability of making a free throw is 75%. After practice one day, he decides he must make 50 free throws before he can go home. How many free throw attempts should he expect to take?

$\frac{12}{12}$ Excellent!

$$\frac{50}{75\%} = 66.\dot{6} < 67.$$

So 67 attempts are expected.

2. Prove that the complement of a complete bipartite graph does not possess a spanning tree.

Suppose $K_{m,n}$ is a complete bipartite graph.
The complement of $K_{m,n}$ is a disconnected graph with the complete graph K_m and the complete graph K_n.
Since the complement is not connected, a spanning tree that contains all the vertices does not exist. $\square$.

3. Let $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \\ 5 & 5 & 5 & 0 \end{pmatrix}$. Find all row vectors $\vec{y}$ such that $\vec{y}A^T = (2 \quad 2 \quad 3)$.

$$[\vec{y}A^T]^T = (2\ 2\ 3)^T$$

$$A(\vec{y})^T = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 2 \\ 4 & 3 & 2 & 1 & 2 \\ 5 & 5 & 5 & 0 & 3 \end{array}\right).$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0.6 \\ 0 & 0 & 0 & 1 & 0.2 \end{array}\right)$$

{using technology}.

Therefore, let $y_3 = s$,

$y_1 = y_3 = s$,

$y_2 = -2y_3 + 0.6 = -2s + 0.6$

$y_4 = 0.2$.

So $\vec{y}^T = \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} s + \begin{pmatrix} 0 \\ 0.6 \\ 0 \\ 0.2 \end{pmatrix}$

$\vec{y} = (1 \ -2 \ 1 \ 0)s + (0 \ \ 0.6 \ \ 0 \ \ 0.2)$,

$s \in \mathbb{R}$.

6

4. Solve the differential equation $\cos x \frac{dy}{dx} + y\cos^2 x \csc x = \sin 2x$ where $x \in]-\pi/2, \pi/2[$.

$$\frac{dy}{dx} + y \cdot \frac{\cos x}{\sin x} = 2\sin x.$$

So the I.F is $e^{\int \frac{\cos x}{\sin x} dx} = \sin x.$

So $(\sin x \cdot y)' = 2\sin^2 x.$

$$\sin x \cdot y = \int 2\sin^2 x \, dx$$

$$= \int 1 - \cos 2x \, dx$$

$$= x - \frac{\sin 2x}{2} + C_1$$

$$y = \frac{x}{\sin x} - \cos x + C_1 \cdot \frac{1}{\sin x}$$

$$y = x \cdot \csc x - \cos x + C_1 \csc x.$$

2

5. Let X be the score on the throw of a fair die.

(a) Show that the pgf for X is $G(t) = \frac{1}{6}t(1-t^6)(1-t)^{-1}$.

$$G(t) = \frac{1}{6}t^1 + \frac{1}{6}t^2 + \cdots + \frac{1}{6}t^6.$$

$$= \frac{1}{6}(t^1 + t^2 + \cdots + t^6)$$

$$= \frac{1}{6}\left(t \cdot \frac{1-t^6}{1-t}\right)$$

$$= \frac{1}{6}t(1-t^6)(1-t)^{-1}.$$

1

(b) Hence determine the probability of a sum of 14 when four fair dice are thrown.

$$G^4(t) = \left[\frac{1}{6}t(1-t^6)(1-t)^{-1}\right]^4$$

$$= \left(\frac{1}{6}\right)^4 t^4 \left(\frac{1-t^6}{1-t}\right)^4$$

Since we want to know the coefficient of t^{14}, we only need to know the coefficient of $t^{14-4} = t^{10}$.

and since $\left(\frac{1-t^6}{1-t}\right)^4 = \frac{1 - 4t^6 + 6t^{12} - 4t^{18} + t^{24}}{(1-t)^4}$,

we only need to know the coefficient of t^{10} in $(1-t)^{-4}$, and t^4 in $-4t^6(1-t)^{-4}$; which is

$$\binom{-4}{10}(-1)^{10} - 4\binom{-4}{4}(-1)^6 = 146.$$

Therefore, the coefficient of t^{14} in $G^4(t)$ is $\frac{146}{1296} = \boxed{\frac{73}{648}}$, which is the probability.

4

FM2 Assignment #28

$\frac{10}{10}$ Excellent!

Name: Ruiyan Maggie Huang.

1. Females enter Superstore at an average rate of two a minute and males enter it at an average rate of one a minute. Find the probability that three people enter Superstore in a given minute. Could you have got your answer in another way?

Let X count the # of females entering in a minute, and Y count the # of males entering. Then $X \sim Po(2)$. $Y \sim Po(1)$.

Method #1: So the total # of people is $X+Y \sim Po(1+2)$, using technology,

$P(X+Y=3) = \boxed{0.224}$ (3 s.f.)

Method #2:

$P(X=0) \times P(Y=3) + P(X=1) \times P(Y=2) + P(X=2) \times P(Y=1) + P(X=3) \times P(Y=0)$

$= \boxed{0.224}$ (3 s.f.) Using technology.

2. Consider the matrix $\begin{pmatrix} k & 1 & 1 \\ k & 2 & k-1 \\ k & 0 & k-2 \end{pmatrix}$. For which values of k is the matrix invertible?

$\det = k \begin{vmatrix} 2 & k-1 \\ 0 & k-2 \end{vmatrix} - 1 \begin{vmatrix} k & k-1 \\ k & k-2 \end{vmatrix} + 1 \begin{vmatrix} k & 2 \\ k & 0 \end{vmatrix}$

$= k(2k-4) - (k^2-2k-k^2+k) + (0-2k)$.

$= 2k^2-4k+k-2k$

$= 2k^2-5k$,

if invertible, $\det \neq 0$, $2k^2-5k \neq 0$.

$k \neq 0$ and $k \neq \frac{5}{2}$. $k \in \mathbb{R}$.

3. The joint probability distribution for X and Y is given by $P(X = x, Y = y) = \frac{xy}{18}$, $(x, y) \in \{1, 2\} \times \{1, 2, 3\}$.

(a) Tabulate the joint probability distribution for X and Y.

X \ Y	1	2	3
1	$\frac{1}{18}$	$\frac{1}{9}$	$\frac{1}{6}$
2	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{3}$

(b) Tabulate the probability distribution for $X + Y$.

$x+y$	2	3	4	5
$P(X+Y=x+y)$	$\frac{1}{18}$	$\frac{2}{9}$	$\frac{7}{18}$	$\frac{1}{3}$

$\frac{}{6}$

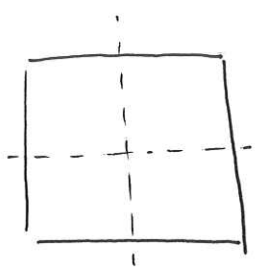
4. As we saw in class, the Poisson distribution can be derived as the limiting case of a binomial distribution with $p = \mu/n$ and $q = 1 - \mu/n$. Write down the pgf for the binomial distribution and show that the pgf for the Poisson distribution can be obtained from this by letting n go to infinity.

let $G(t)$ be the pgf for $X \sim B(n, \frac{\mu}{n})$, and let $H(t)$ be the pgf for $X \sim Po(\mu)$.

$$G(t) = (q+pt)^n = \left(1 - \frac{\mu}{n} + \frac{\mu t}{n}\right)^n = \left(1 - \frac{\mu(1-t)}{n}\right)^n.$$

As $n \to \infty$, $G(t) = e^{\mu(t-1)} = H(t)$. //

5. Use the pigeon hole principle to show that some pair of any five points in a unit square will be at most $\frac{1}{\sqrt{2}}$ units apart.



Divide the unit square into 4 equal parts as shown.

let the 4 areas be the holes and the 5 points be the pigeons.

It follows that there must be at least 2 points in the same area.

The furthest distance between these two points occurs when they are at the opposite vertex,

with a distance of $\frac{1}{2}\sqrt{2} = \frac{1}{\sqrt{2}}$ units.

Therefore, any 5 points will be at most $\frac{1}{\sqrt{2}}$ units apart. //.

4

FM2 Assignment #30 Name: Ruiyan Maggie 10/10 Excellent!

1. Use Rolle's theorem to show that the equation $x^3 + x + c = 0$ where c is a constant cannot have more than one real zero.

Suppose the opposite is true. i.e. both x_1, x_2 are solutions to the equation $f(x)=0$, where $f(x) = x^3+x+c$.

So $f(x_1) = f(x_2) = 0$. By Rolle's theorem, $\exists$ a constant k between x_1 and x_2, such that $f'(k) = 0$.

$f'(x) = 3x^2+1 \geq 1$. therefore, such k can not exist, which is a contradiction. Thus proves the result. $\square$.

2. Let $G = (V, E)$ be the graph with $V = \{1, 2, 3, 4, 5, 6\}$ and $uv \in E$ if $|u - v|$ is odd. To which well known graph is G isomorphic?

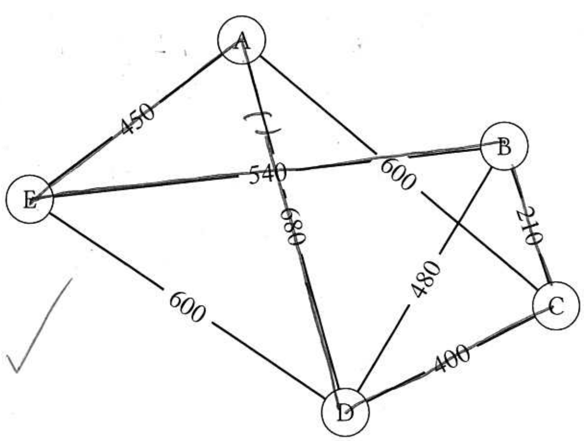
Define the relation R on V is $|u-v|$ is even, $u, v \in V$.

This partitions V into two sets $\{1,3,5\}$ and $\{2,4,6\}$.

Therefore, since $uv \in E$ if $|u-v|$ is odd, there is no edge connected within each partition. However, each vertex is connected to all the vertices in the other partion, which is isomorphic to the graph $K_{3,3}$.

3. Use Kruskal's algorithm in table form to find a minimum spanning tree and its weight in the weighted graph below.

n	edge	weight
1	BC	210
2	CD	400
3	AE	450
4	BE	540



Therefore, the MST is AEBCD,

with weight $210+400+450+540 = 1600$

6

4. Let X and Y be independent random variables with $X \sim \mathrm{Po}(3)$ and $Y \sim \mathrm{Po}(2)$.

(a) Find $\mathrm{E}(2X + 3Y)$ and $\mathrm{Var}(2X + 3Y)$.

$$E(2X+3Y) = 2E(X) + 3E(Y)$$
$$= 2\times3+3\times2 = \underline{12}.$$

$$Var(2X+3Y) = 4\,Var(X)+9\,Var(Y)$$
$$= 4\times3+9\times2$$
$$= 12+18 = \underline{30}$$

(b) Hence state with a reason whether or not $2X + 3Y$ has a Poisson distribution.

2X+3Y is not a Poisson Distribution,

because $E(2X+3Y) \neq Var(2X+3Y)$.

5. Evaluate $\lim_{x\to 0}\left[\frac{\ln(1+x)}{x}\right]^{1/x}$.

$$\lim_{x\to 0}\left[\frac{\ln(1+x)}{x}\right]^{\frac{1}{x}}$$
$$= \lim_{x\to 0}\left[\ln(1+x)^{\frac{1}{x}}\right]^{\frac{1}{x}}$$
$$= \lim_{x\to 0}(\ln e)^{\frac{1}{x}} = 1^{\frac{1}{x}} = 1^{\infty}$$

$$\lim_{x\to 0}\left[\frac{\ln(1+x)}{x}\right]^{\frac{1}{x}}$$
$$= \left[\lim_{x\to 0}\frac{\ln(1+x)}{x}\right]^{\infty}, \quad \text{use L'H.}$$
$$= \left[\lim_{x\to 0}\frac{1}{x+1}\right]^{\infty}$$
$$= \lim_{x\to 0}\left(\frac{1}{x+1}\right)^{\frac{1}{x}}.$$

Let $y = \lim_{x\to 0}\left(\frac{1}{x+1}\right)^{\frac{1}{x}}$,

then $\frac{1}{y} = \lim_{x\to 0}(x+1)^{\frac{1}{x}}$, $\ln\left(\frac{1}{y}\right) = \lim_{x\to 0}\frac{1}{x}\ln(x+1)$, use L'H.

~~determinate form,~~ $= \lim_{x\to 0}\frac{1}{x+1} = 1$.

So $\ln y = \lim_{x\to 0}\frac{x}{\ln(1+x)}\left(\ln(1+x)\ln x + \frac{1}{x+1}\cdot\frac{1}{x}\right)$

$\lim_{x\to 0}$ So $\ln\left(\frac{1}{y}\right) = 1$.

$y = \frac{1}{e}$

See attached paper.

Let $y = \left[\frac{\ln(1+x)}{x} \right]^{\frac{1}{x}}$

$$\lim_{x\to 0} y = \lim_{x\to 0} \left[\frac{x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots}{x} \right]^{\frac{1}{x}}$$

$$= \lim_{x\to 0} \left[1 - \frac{x}{2} + \frac{x^2}{3} + \cdots \right]^{\frac{1}{x}}$$

$$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{1}{x} \ln \left[1 - \frac{x}{2} + \frac{x^2}{3} + O(x^4) \right] \quad , \text{L'H.}$$

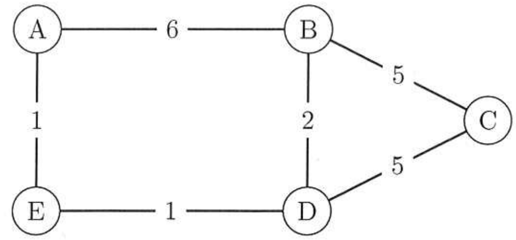
$$= \lim_{x\to 0} \frac{1}{1 - \frac{x}{2} + O(x^2)} \cdot \left[-\frac{1}{2} + O(x) \right]$$

$$= -\frac{1}{2}.$$

So $\lim_{x\to 0} y = e^{-\frac{1}{2}} = \lim_{x\to 0} \left[\frac{\ln(1+x)}{x} \right]^{\frac{1}{x}}$.

Dijkstra's Algorithm Name: Maggie. 6/6

1. Use Dijkstra's algorithm in table form to find the shortest path from A to C in the weighted graph below.

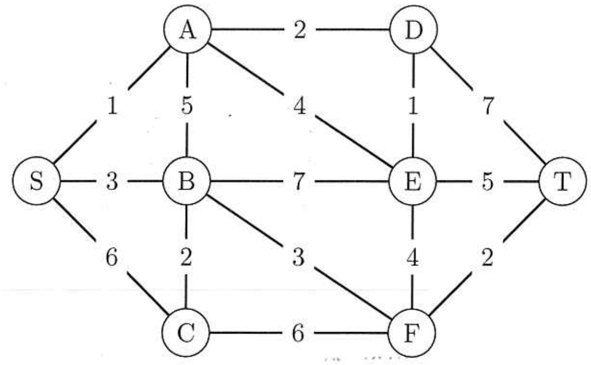


	A	B	C	D	E
A	$\boxed{0}$	~~6~~			1
E		6		2	$\boxed{1}$
D		4	7	$\boxed{2}$	
B		$\boxed{4}$	7		
C			$\boxed{7}$.		

~~$\Rightarrow w(AEDC) = 7$~~

so the shortest path is AEDC, ✓
and its weight is 7.

2. Use Dijkstra's algorithm in table form to find the shortest path from S to T in the weighted graph below.



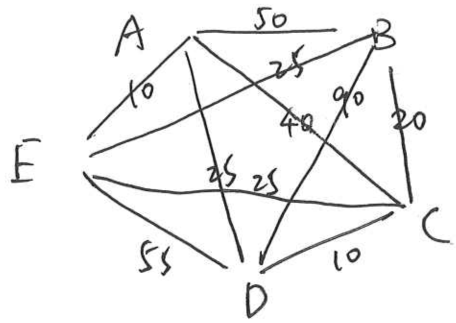
	S	A	B	C	D	E	F	T.
S	$\boxed{0}$	~~1~~	~~3~~	6				
A		$\boxed{1}$	3	6	3	5		
B			$\boxed{3}$	~~5~~	~~3~~	~~5~~	~~6~~	
C				$\boxed{5}$	3	5	6	
F					3	5	$\boxed{6}$	~~8~~
E					3	$\boxed{5}$		8
D					$\boxed{3}$			8
T								$\boxed{8}$

So the shortest path is SBFT, ✓
$w(SBFT) = 8$

/4

3. Draw the weighted graph corresponding to the table of weights below. Now grow a shortest path spanning tree rooted at A to find the shortest path from A to each of the other vertices.

	A	B	C	D	E
A	–	50	40	25	10
B	50	–	20	90	25
C	40	20	–	10	25
D	25	90	10	–	55
E	10	25	25	55	–



	A	B	C	D	E
A	$\boxed{0}$	50	40	25	10
E		35	35	25	$\boxed{10}$
D		35	35	$\boxed{25}$	
B		$\boxed{35}$	35		
C			$\boxed{35}$.		

To B: ~~AE~~. $w(AEB) = 35$

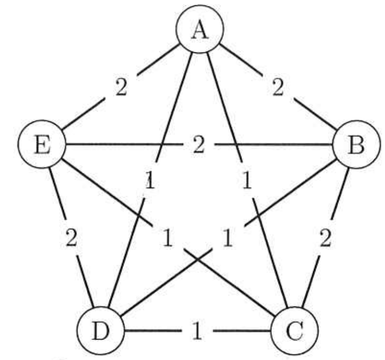
To C: $w(AEC) = 35$

To D: $w(AD) = 25$

To E: $w(AE) = 10$.

Chinese Postman Problem Name: Maggie $5\frac{1}{2}$ / 4

1. Solve the Chinese postman problem and find the weight of the solution for the weighted graph below.



Since all the vertices are of even degree, there exists a Eularian ~~path~~. circuit.

One possibility is ~~AEDBAD~~ AEDCBADBECA.

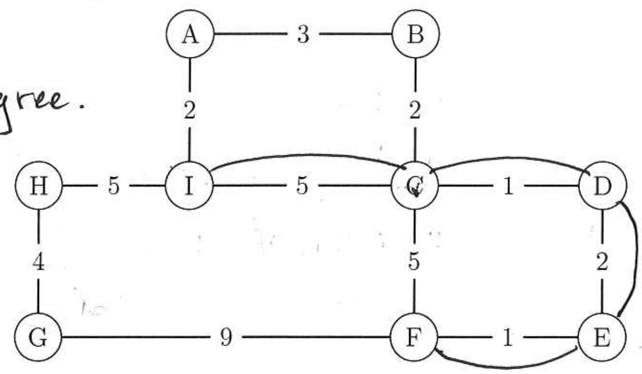
The weight is the total weight of the graph,

$2+2+1+2+2+1+1+2+1+1$.

$= 15$.

✓

2. Solve the Chinese postman problem and find the weight of the solution for the weighted graph below.



Since I and F are of odd degree. We first find the shortest path between them. which we spot as ICDEF, with weight 9.

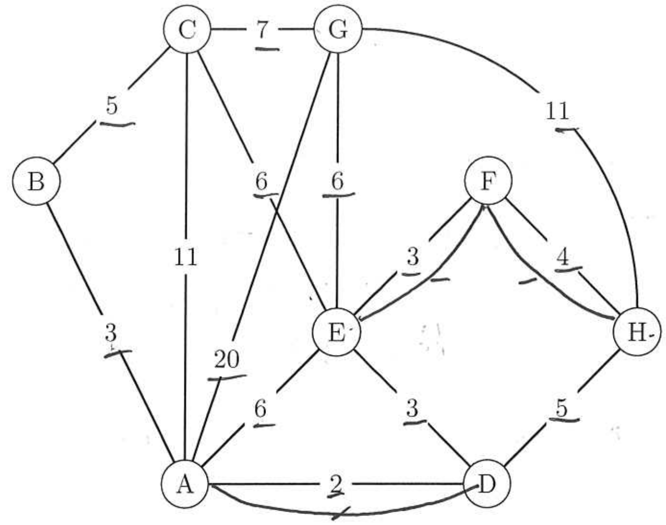
So by adding parallel edges to the original graph, we get a new graph with all even-degree vertices. We find ~~an~~ Eulerian circuit to be

~~GIABGIH~~ CIABCIHGFEDCDEFC.

The total weight is ~~39~~ 39 + 9 = 48.

4

3. Solve the Chinese postman problem and find the weight of the solution for the weighted graph below.



$$\left.\begin{array}{c}10\\+\\20\\+\\20\end{array}\right\}50 \quad \left.\begin{array}{c}+\\10\\+\\20.\\+\\10\\+\\11\end{array}\right\}51$$

Vertices with odd degrees are: A, E, D, H.

Pairing	Shortest path	Weight	
AE/DH	ADE / DH	5+5=10	
AD/EH	AD / EFH	2+7=9.	✓
AH/ED	AH / ED	7+3=10	

So the graph becomes what's drawn above, with a Eulerian circuit.

ABCGHDACEDAGEFHFEA.

which has a total weight of ~~101~~ 92 + 9 = 110.