FM2 Assignment #28 Name: Maggie.

1. Lebron James's probability of making a free throw is 75%. After practice one day, he decides he must make 50 free throws before he can go home. How many free throw attempts should he expect to take?

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$$\frac{50}{75\%} = 66.\dot{6} < 67.$$

So 67 attempts are expected.

2. Prove that the complement of a complete bipartite graph does not possess a spanning tree.

Suppose $K_{m,n}$ is a complete bipartite graph.
The complement of $K_{m,n}$ is a disconnected graph with the complete graph K_m and the complete graph K_n.
Since ~~the~~ the complement is not connected, a spanning tree that contains all the vertices does not exist. $\square$.

3. Let $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \\ 5 & 5 & 5 & 0 \end{pmatrix}$. Find all row vectors $\vec{y}$ such that $\vec{y}A^T = (2 \quad 2 \quad 3)$.

$$[\vec{y}A^T]^T = (2\ 2\ 3)^T$$

$$A(\vec{y})^T = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 2 \\ 4 & 3 & 2 & 1 & 2 \\ 5 & 5 & 5 & 0 & 3 \end{array}\right).$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0.6 \\ 0 & 0 & 0 & 1 & 0.2 \end{array}\right)$$

{using technology}.

Therefore, let $y_3 = s$,

$y_1 = y_3 = s$,

$y_2 = -2y_3 + 0.6 = -2s + 0.6$

$y_4 = 0.2$.

$$\text{So } \vec{y}^T = \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} s + \begin{pmatrix} 0 \\ 0.6 \\ 0 \\ 0.2 \end{pmatrix}$$

$$\vec{y} = (1\ -2\ 1\ 0)s + (0\ \ 0.6\ \ 0\ \ 0.2),$$

$s \in \mathbb{R}$.

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4. Solve the differential equation $\cos x \frac{dy}{dx} + y\cos^2 x \csc x = \sin 2x$ where $x \in]-\pi/2, \pi/2[$.

$$\frac{dy}{dx} + y \cdot \frac{\cos x}{\sin x} = 2\sin x.$$

So the I.F is $e^{\int \frac{\cos x}{\sin x} dx} = \sin x$.

So $(\sin x \cdot y)' = 2\sin^2 x$.

$$\sin x \cdot y = \int 2\sin^2 x \, dx$$

$$= \int 1 - \cos 2x \, dx$$

$$= x - \frac{\sin 2x}{2} + C_1$$

$$y = \frac{x}{\sin x} - \cos x + C_1 \cdot \frac{1}{\sin x}$$

$$y = x \cdot \csc x - \cos x + C_1 \csc x.$$

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5. Let X be the score on the throw of a fair die.

(a) Show that the pgf for X is $G(t) = \frac{1}{6}t(1-t^6)(1-t)^{-1}$.

$$G(t) = \frac{1}{6}t^1 + \frac{1}{6}t^2 + \cdots + \frac{1}{6}t^6$$

$$= \frac{1}{6}(t^1 + t^2 + \cdots + t^6)$$

$$= \frac{1}{6}\left(t \cdot \frac{1-t^6}{1-t}\right)$$

$$= \frac{1}{6}t(1-t^6)(1-t)^{-1}.$$

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(b) Hence determine the probability of a sum of 14 when four fair dice are thrown.

$$G^4(t) = \left[\frac{1}{6}t(1-t^6)(1-t)^{-1}\right]^4$$

$$= \left(\frac{1}{6}\right)^4 t^4 \left(\frac{1-t^6}{1-t}\right)^4$$

Since we want to know the coefficient of t^{14}, we only need to know the coefficient of $t^{14-4} = t^{10}$.

and since $\left(\frac{1-t^6}{1-t}\right)^4 = \frac{1-4t^6+6t^{12}-4t^{18}+t^{24}}{(1-t)^4}$,

we only need to know the coeffecient of t^{10} in $(1-t)^{-4}$, and t^4 in $-4t^6(1-t)^{-4}$; which is

$$\binom{-4}{10}(-1)^{10} - 4\binom{-4}{4}(-1)^6 = 146.$$

Therefore, the coefficient of t^{14} in $G^4(t)$ is $\frac{146}{1296} = \boxed{\frac{73}{648}}$, which is the probability.

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FM2 Assignment #28

$\frac{12}{10}$ Excellent! Name: Ruiyan Maggie Huang.

1. Females enter Superstore at an average rate of two a minute and males enter it at an average rate of one a minute. Find the probability that three people enter Superstore in a given minute. Could you have got your answer in another way?

Let X count the # of females entering in a minute, and Y count the # of males entering. Then $X \sim Po(2)$. $Y \sim Po(1)$.

Method #1: So the total # of people is $X+Y \sim Po(1+2)$, using technology,

$P(X+Y=3) = \boxed{0.224}$ (3 s.f.)

Method #2:

$P(X=0) \times P(Y=3) + P(X=1) \times P(Y=2) + P(X=2) \times P(Y=1) + P(X=3) \times P(Y=0)$

$= \boxed{0.224}$ (3 s.f.) Using technology.

2. Consider the matrix $\begin{pmatrix} k & 1 & 1 \\ k & 2 & k-1 \\ k & 0 & k-2 \end{pmatrix}$. For which values of k is the matrix invertible?

$\det = k\begin{vmatrix} 2 & k-1 \\ 0 & k-2 \end{vmatrix} - 1\begin{vmatrix} k & k-1 \\ k & k-2 \end{vmatrix} + 1\begin{vmatrix} k & 2 \\ k & 0 \end{vmatrix}$

$= k(2k-4) - (k^2-2k-k^2+k) + (0-2k)$.

$= 2k^2-4k+k-2k$

$= 2k^2-5k$,

if invertible, $\det \neq 0$, $2k^2-5k \neq 0$.

$k \neq 0$ and $k \neq \frac{5}{2}$. $k \in \mathbb{R}$.

3. The joint probability distribution for X and Y is given by $P(X = x, Y = y) = \frac{xy}{18}$, $(x, y) \in \{1, 2\} \times \{1, 2, 3\}$.

(a) Tabulate the joint probability distribution for X and Y.

X \ Y	1	2	3
1	$\frac{1}{18}$	$\frac{1}{9}$	$\frac{1}{6}$
2	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{3}$

(b) Tabulate the probability distribution for $X + Y$.

x+y	2	3	4	5
$P(X+Y=x+y)$	$\frac{1}{18}$	$\frac{2}{9}$	$\frac{7}{18}$	$\frac{1}{3}$

$\frac{1}{6}$

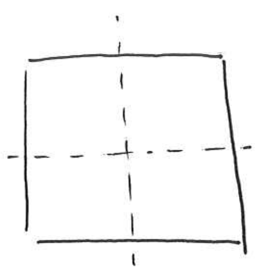
4. As we saw in class, the Poisson distribution can be derived as the limiting case of a binomial distribution with $p = \mu/n$ and $q = 1 - \mu/n$. Write down the pgf for the binomial distribution and show that the pgf for the Poisson distribution can be obtained from this by letting n go to infinity.

let $G(t)$ be the pgf for $X \sim B(n, \frac{\mu}{n})$, and let $H(t)$ be the pgf for $X \sim Po(\mu)$.

$$G(t) = (q+pt)^n = \left(1 - \frac{\mu}{n} + \frac{\mu t}{n}\right)^n = \left(1 - \frac{\mu(1-t)}{n}\right)^n.$$

As $n \to \infty$, $G(t) = e^{\mu(t-1)} = H(t)$. //

5. Use the pigeon hole principle to show that some pair of any five points in a unit square will be at most $\dfrac{1}{\sqrt{2}}$ units apart.



Divide the unit square into 4 equal parts as shown.

let the 4 areas be the holes and the 5 points be the pigeons.

It follows that there must be at least 2 points in the same area.

The furthest distance between these two points occurs when they are at the opposite vertex,

with a distance of $\frac{1}{2}\sqrt{2} = \frac{1}{\sqrt{2}}$ units.

Therefore, any 5 points will be at most $\frac{1}{\sqrt{2}}$ units apart. //.

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