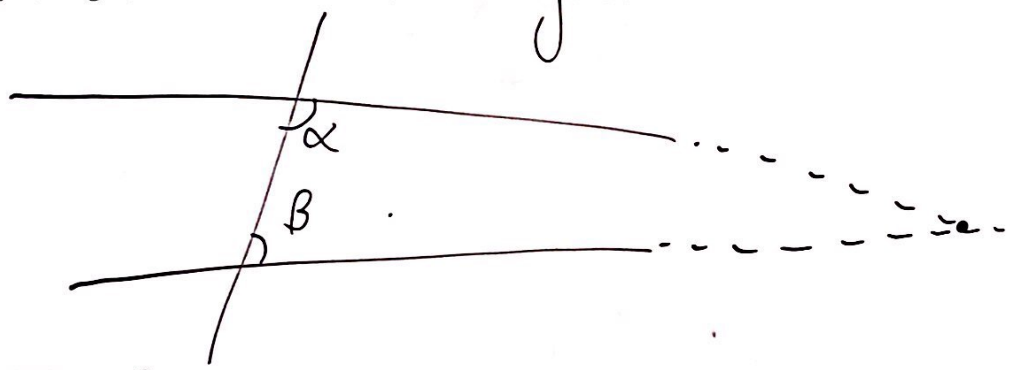
# 1. Euclid's axioms and results

EUCLID'S AXIOMS:

1. It's possible to draw a straight line from any point to any point.
2. It's possible to extend a finite straight line continuously in a straight line
3. It's possible to describe a circle with any center and any radius
4. All right angles are equal to each other.

5. Parallel postulate: If a straight line falling across two other straight lines makes internal angles less than two right angles, then the two other lines, being extended to infinity, will meet on the side of the two angles



$\alpha + \beta < \pi$

Dfn
line
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to it.

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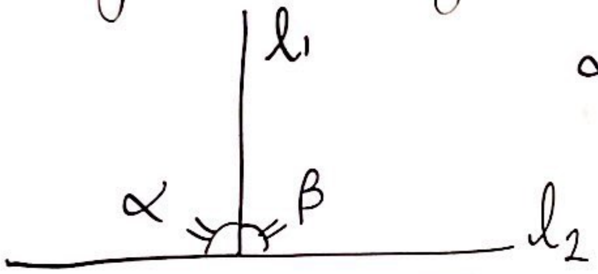
Dfn: When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right. The straight standing on the other is perpendicular to it.




$\alpha = \beta \Rightarrow \alpha$ and $\beta$ are right angles

$l_1 \perp l_2$

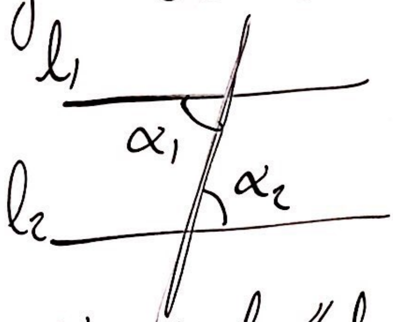
Euclide's common notions: For length of segment, angles sizes and areas

1. $a=b,\ b=c \Rightarrow a=c$
2. $a=b,\ c=d \Rightarrow a+c=b+d$
3. $a=b,\ c=d \Rightarrow a-c=b-d$
4. If a and are the same, then $a=b$
5. $a+b>a \quad (b>0)$

**Playfair's axiome:** In a plane, given a line and a point not on this line, there is at most on parallel line to the given one passing through P.

**Dfn:** Parallel lines are lines, that being on the same plane, do not meet one another.

**Lemma 1:** If a straight line falling across 2 straight lines makes internal angles equal, then the lines are //




$\alpha_1 = \alpha_2 \Rightarrow l_1 \parallel l_2$

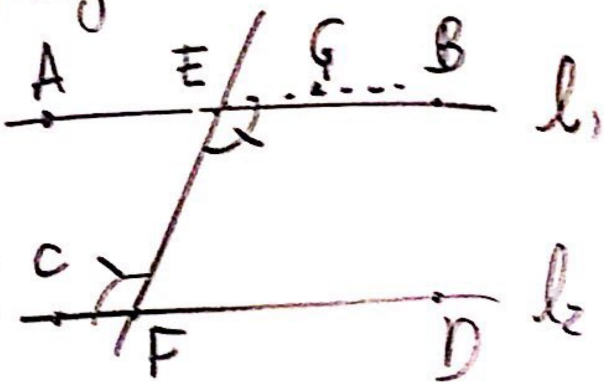
Lemma 2: If we admit PP, lines // to the same line are either equal, either // one to the other.

Statement: PP $\Leftrightarrow$ Playfair's axiom

Proof:

$\Rightarrow$) By lemma, if two lines are // to a third one and pass through P, they are $=$.

$\Leftarrow$): Suppose that
$\angle BEF + \angle EFD < \pi$.




Take G s.t $\angle EFC = \angle FGC$

$\Rightarrow CD \parallel EG$

$\angle BEF < \angle EFC$

$\Rightarrow \angle BEF < \angle GEF$

$\Rightarrow l_1 \neq EG$

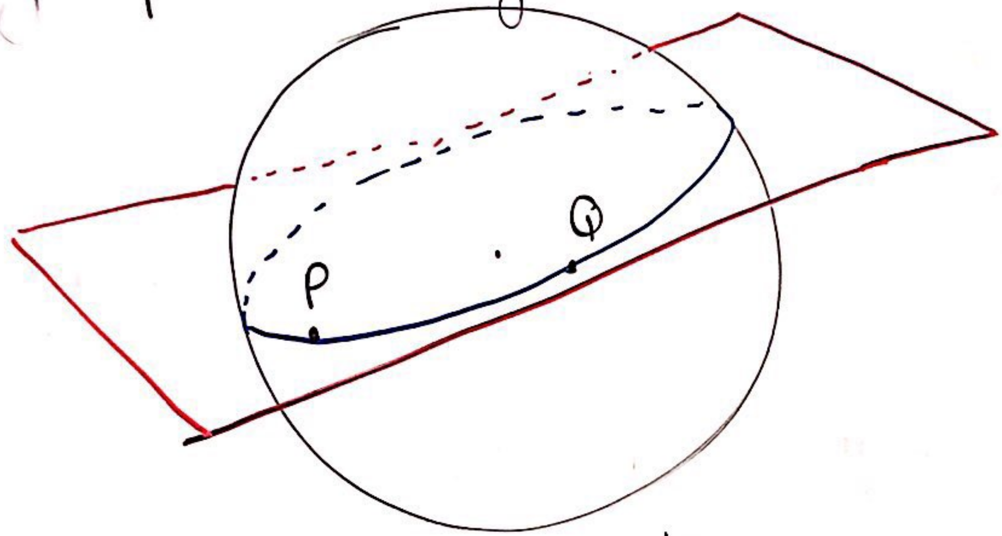
$\Rightarrow l_1 \not\parallel l_2$

$\Rightarrow l_1$ and $l_2$ will meet.

2. Sperical geometry

Consider $S^2 = \{(x, y, z) \mid x^2 + y^2 + z^2 = 1\}$

**Dfn**: A great circle is the circle cut out of $S^2$ by a plane throug $(0, 0, 0)$




Let $P$ and $Q$ be one $S^2$

Either of the arcs of a great circle with endpoint $P$ and $Q$ is said to be a line joining and $Q$

The distance between P and Q is the length of the shortest line PQ

Rmk: The distance between the points is equal to the angle (in rad) they subtend at the center of $S^2$

Which of the Euclid's axiome are true?

1. ✓ (but not unique)
2. ✓

5. <u>Parallel</u> <u>postulate</u>: If a straight line falling across two other straight lines makes internal angles less than two right angles, then the two other lines, being extended to infinity, will meet on the side of the two angles


3. With a given line on $S^2$ as radius ✓
   Else, only possible for radius $\leq \pi$

4. ✓

5. ✓ Playfair's axiom ✓

→ No parallel lines!