Linear
Diophantine Equation. Exercise 1D.3.

1(a) $6x + 51y = 22$

$\gcd(6, 51) = \gcd(2 \times 3, 3 \times 17)$
$= 3.$

Since $3 \nmid 22$ then there are
<u>no</u> $x, y \in \mathbb{Z}$ ~~such that~~
$6x + 51y = 22.$

b) $33x + 14y = 115.$

$\gcd(14, 33) = \gcd(2 \times 7, 3 \times 11)$
$= 1.$

and $1 \mid 115.$

Thus, there exist $x, y \in \mathbb{Z}$
Such that $33x + 14y = 115.$

$33 = 2(14) + 5.$
$14 = 2(5) + 4.$
$5 = 1(4) + 1.$
$4 = 4(1) + 0$

So. $1 = 5 - 4.$
$= 5 - [14 - 2(5)]$
$= 3(5) - 14.$
$= 3[33 - 2(14)] - 14$
$= 3(33) - 6(14) - 14$
$= 3(33) - 7(14).$

So $115[33(3) - 14(7)] = 115.$
$33(345) - 14(805) = 115.$

$x_0 = 345, \quad y_0 = -805$ is a solution.

$x = 345 + 14t$
$y = -805 - 33t \quad ; t \in \mathbb{Z}.$

2a) Want $x, y \in \mathbb{Z}^+$.

a) $18x + 5y = 48$.

$\gcd(5, 18) = 1$.

and $1 \mid 48$

Thus, there exist $x, y \in \mathbb{Z}$ such that $18x + 5y = 48$ but the problem is to obtain $x, y \in \mathbb{Z}^+$.

$$18 = 3(5) + 3$$
$$5 = 1(3) + 2$$
$$3 = 1(2) + 1$$
$$2 = 2(1) + 0$$

So
$$\begin{aligned} 1 &= 3 - 2 \\ &= 3 - [5 - 3] \\ &= 2(3) - 5 \\ &= 2[18 - 3(5)] - 5 \\ &= 2(18) - 6(5) - 5 \\ &= 2(18) - 7(5) \end{aligned}$$

$$48[2(18) - 7(5)] = 48$$
$$96(18) - 336(5) = 48$$

$x_0 = 96$, $y_0 = -336$

$x = 96 + 5t$

$y = -336 - 18t$, $t \in \mathbb{Z}$

we want

$y > 0$ and $x > 0$

$-336 - 18t > 0$ and $96 + 5t > 0$

$-18t > 336$ $\qquad$ $5t > -96$

$t < \frac{336}{(-18)}$ $\qquad$ $t > -\frac{96}{5}$

$t < -18\frac{2}{3}$ $\qquad$ $t > -19\frac{1}{5}$

So $t = -19$.

The only solution is

$x = 96 + 5(-19)$
$= 1$

$y = -336 - 18(-19)$
$= 6$.

Linear Diophantine Equation Ex. 10.3

2d. $158x - 57y = 11$.

$\gcd(57, 158) = 1$

and $1 | 11$

Thus, there exist $x, y \in \mathbb{Z}$ such that $158x - 57y = 11$.

but the problem is to obtain $x, y \in \mathbb{Z}^+$

$$158 = 2(57) + 44$$
$$57 = 1(44) + 13$$
$$44 = 3(13) + 5$$
$$13 = 2(5) + 3$$
$$5 = 1(3) + 2$$
$$3 = 1(2) + 1$$
$$2 = 2(1) + 0.$$

so
$$1 = 3 - 2$$
$$= 3 - (5 - 3)$$
$$= 2(3) - 5$$
$$= 2[13 - 2(5)] - 5$$
$$= 2(13) - 5(5)$$
$$= 2(13) - 5[44 - 3(13)]$$
$$= 17(13) - 5(44)$$
$$= 17[57 - 44] - 44(5)$$
$$= 17(57) - 17(44) - 5(44)$$
$$= 17(57) - 22(44)$$
$$= 17(57) - 22[158 - 2(57)]$$
$$= 17(57) + 44(57) - 22(158)$$
$$= 61(57) - 22(158)$$

$$11[61(57) - 22(158)] = 11$$
$$671(57) - 242(158) = 11$$

so $x_0 = -242$; $y_0 = 671$

For $ax + by = c$

then $a(x) - b(-y) = c$

$x = -242 + 57t$

$-y = -[671 - 158t]$; $t \in \mathbb{Z}$

$= -671 + 158t$

we want $y > 0$ and $x > 0$.

$-671 + 158t > 0$

$158t > 671$

$t > \frac{671}{158}$

$t > 4.2468\ldots$; $t \geq 5$.

and $-242 + 57t > 0$

$57t > +242$

$t > \frac{242}{57}$

$t > 4.2456\ldots$

$t \geq 5$.

So $x = -242 + 57t$;

$-y = -671 + 158t$;

$t \geq 5$, $t \in \mathbb{Z}$.

3. let $7|a$ and $11|b$;
$a, b \in \mathbb{Z}^+$.
Also $a+b = 100$.

From $7|a \Rightarrow a = 7h, h \in \mathbb{Z}^+$.
and $11|b \Rightarrow b = 11k, k \in \mathbb{Z}^+$.
Thus, $7h + 11k = 100$.
$\gcd(7, 11) = 1$.
and $1|100$. so there exist
$h, k \in \mathbb{Z}$ such that
$7h + 11k = 100$.
By inspection $7(8) + 11(4) = 100$.
So one answer is $a = 56$, $b = 44$.

$h = 8 + 11t$;
$k = 4 - 7t$; $t \in \mathbb{Z}$.
want $h, k \in \mathbb{Z}^+$.
$4 - 7t > 0$
$4 > 7t$.
$\frac{4}{7} > t$.
so $t = 0$.

Thus, there is only 1 solution.
$\underline{a = 56, \ b = 44}$.

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Linear Diophantine Eq. Ex. 1D.3.

4. $m + w + c = 20$.

$5m + 4w + 2c = 62$.

Solve for $m, w, c \in \mathbb{Z}^+$

Since $m + w + c = 20$ then $c = 20 - m - w$.

So $5m + 4w + 2c = 62$ becomes

$5m + 4w + 2[20 - m - w] = 62$

$5m + 4w + 40 - 2m - 2w = 62$.

$3m + 2w = 22$.

$\gcd(3,2) = 1$ and $1 \mid 22$

So there exist $m, w \in \mathbb{Z}$ such that $3m + 2w = 22$

By inspection, $3(0) + 2(11) = 22$.

Let $m_0 = 0$, $w_0 = 11$.

$m = 0 + 2t$;

$w = 11 - 3t$; $t \in \mathbb{Z}$.

We want $m > 0$ and $w > 0$.

$11 - 3t > 0$.

$11 > 3t$

$\frac{11}{3} > t$

$t = 1, 2, 3$.

If $t = 1$; $m = 2$; $w = 8$; $c = 10$.

If $t = 2$; $m = 4$; $w = 5$; $c = 11$

If $t = 3$; $m = 6$; $w = 2$; $c = 12$.