M41. Normal Distb. 2

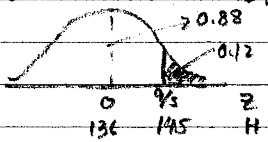
(1)(a) $H \sim N(136, s^2)$.

$P(H > 145) = 0.12$.

s?

$P(H > 145) = P\left(Z > \frac{145-136}{s}\right)$

$0.12 = P\left(Z > \frac{9}{s}\right)$



So $P\left(Z < \frac{9}{s}\right) = 0.88$

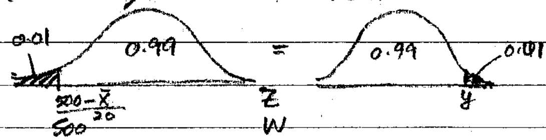
$\frac{9}{s} = 1.1750$ (Table)

$s = \frac{9}{1.1750}$ cm

$s = 7.66$ (3 s.f.) cm

(b) $s = 20g$. $P(W < 500) = 0.01$

$P(W < 500) = P\left(Z < \frac{500-\bar{x}}{20}\right)$



$P(Z < y) = 0.99$

$y = 2.3264$ (Table)

So $\frac{500-\bar{x}}{20} = -2.3264$

$\bar{x} = 20(-2.3264) + 500$.

$\bar{x} \approx 547\,g$. (3s.f.)

(c) $W \sim (0.85, s^2)$.

$P(W < 1.1) = 0.74$.

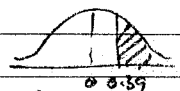
$P\left(Z < \frac{1.1-0.85}{s}\right) = 0.74$

$\frac{1.1-0.85}{s} = 0.6434$ (Table)

(i) $s = \frac{0.25}{0.6434} \approx 0.389\,g$

(ii) $P(W > 1) = P\left(Z > \frac{1-0.85}{0.389}\right)$

$\approx P(Z > 0.39)$



$P(W > 1) \approx 1 - P(Z < 0.39)$

$\approx 1 - 0.6517$

≈ 0.3483

So 34.8 %

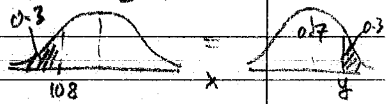
GDC: normalcdf(1, 9E99, 0.85, 0.389)

≈ 0.350 (3s.f.)

(d) $P(X < 108) = 0.3$

$P(X > 154) = 0.2$

$P\left(Z < \frac{108-\bar{x}}{s}\right) = 0.3$



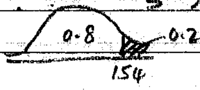
$P(Z < y) = 0.7$

$y = 0.5244$ (Table)

So $\frac{108-\bar{x}}{s} = -0.5244$

$108 - \bar{x} = -0.5244s$. —①

$P\left(Z > \frac{154-\bar{x}}{s}\right) = 0.2$



So $\frac{154-\bar{x}}{s} = 0.8416$ (Table)

$154 - \bar{x} = 0.8416s$ —②

②−① $46 = 1.366\,s$

$s = \frac{46}{1.366}$

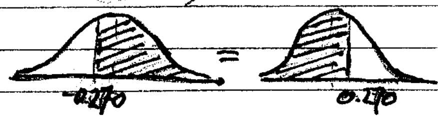
$s \approx 33.7$ (3s.f.)

(d)(i) $\bar{x} = 108 + 0.5244\left(\frac{46}{1.366}\right)$

≈ 126 marks.

(ii) $P(X > 117) = 0.60$.

$P\left(Z > \frac{117-126}{(46/1.366)}\right) \approx P(Z > -0.27)$



$P(X > 117) \approx P(Z < 0.27)$

≈ 0.6064

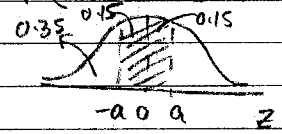
Thus, the fact is consistent with $X \sim N(126, (33.7)^2)$.

(2) $Z \sim N(10, 1)$

$P(|Z - 10| \le k) = 0.3$

$P(-k \le Z - 10 \le k) = 0.3$

$P(10 - k \le Z \le 10 + k) = 0.3$



So $P(Z < a) = 0.65$

$P\left(Z < \frac{a-10}{1}\right) = 0.65$

$a - 10 = 0.3853$

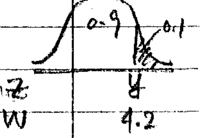
$a = 10 + 0.3853$

So $k = 0.3853$

(3) $W \sim N(\bar{x}, s^2)$

(a) $P(W > 4.2) = 0.1$

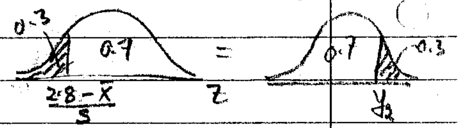
$P(W < 2.8) = 0.3$



$P(Z < y) = 0.9$

$y = 1.2816$ (Table)

So $4.2 - \bar{x} = 1.2816s$ ①



$P(Z < y_2) = 0.5244$

$2.8 - \bar{x} = -0.5244s$ ②

①−②

$1.4 = 1.8068$

$s = \frac{1.4}{1.806}$

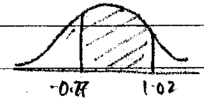
$s \approx 0.775\,kg$ (3s.f.)

$\bar{x} = 4.2 - 1.2816\left(\frac{1.4}{1.806}\right)$

$\approx 3.21\,kg$ (3s.f.)

(b) $P(3 < W < 4) \approx P\left(\frac{3-3.21}{0.775} < Z < \frac{4-3.2}{0.775}\right)$

$\approx P(-0.27 < Z < 1.02)$



$P(3 < W < 4) \approx P(Z < 1.02) - [1 - P(Z < 0.27)]$

$\approx P(Z < 1.02) + P(Z < 0.27) - 1$

$\approx 0.8461 + 0.6064 - 1$

≈ 0.4525

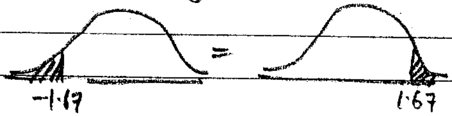
or 45.2 % (3s.f.)

M41

(4) $W \sim N(80, 3^2)$

a) $P(W < 75) = P\left(Z < \frac{75-80}{3}\right)$

$\approx P(Z < -1.67)$



$P(W < 75) = 1 - P(Z < 1.67)$

$= 1 - 0.9525$ (Table)

$= 0.0475$

4.75% (3s.f.)

GDC : normalcdf(-9E99, 75, 80, 3)

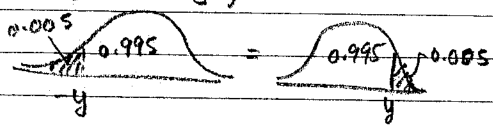
≈ 0.0478

4.78%

(b) $P(W < 75) = 0.005$

(i) $P\left(Z < \frac{75-80}{s}\right) = 0.005$

$P\left(Z < \frac{-5}{s}\right) = 0.005$



$-2.5758 = \frac{-5}{s}$ (Table)

$s = \frac{5}{2.5758}$

$s \approx 1.94$ g (3s.f.)

(ii) $P\left(Z < \frac{75-\bar{x}}{3}\right) = 0.005$

$-2.5758 = \frac{75-\bar{x}}{3}$

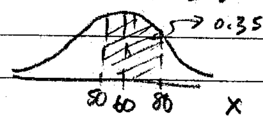
$\bar{x} = 75 + 3(2.5758)$

≈ 82.7 g (3s.f.)

(5) $X \sim N(60, s^2)$

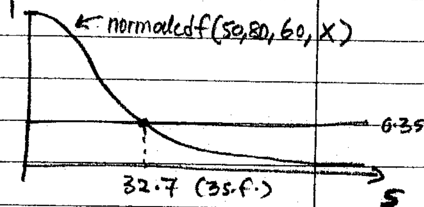
$P(50 < X < 80) = 0.35$

(a)



$\frac{50-60}{s}$ 0 $\frac{80-60}{s}$ Z

$= \frac{-10}{s}$ 0 $\frac{20}{s}$ Z



$Var(X) \approx (32.672444)^2$

≈ 1070 (3s.f.)

(b) $P(X > 100) \approx P\left(Z > \frac{100-60}{32.672}\right)$

$\approx P(Z > 1.22)$

$\approx 1 - P(Z < 1.22)$

$\approx 1 - 0.8888$

≈ 0.1112

So 11.1%

GDC: normalcdf(100, 9E99, 60, 32.672)

≈ 0.110 (3s.f.)

So 11.0% (3s.f.)