1.4. The power set.

Defn. $\mathcal{P}(A) = \{X \mid X \subseteq A\}$.

e.g. 5. $\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, A\}$.

$|\mathcal{P}(A)| = 8 = 2^3$. $|A| = 3$.

Thrm 2.

$|A| = n \Rightarrow |\mathcal{P}(A)| = 2^n$.

- $\bigcup_{i=1}^{n} A_i$ $\bigcap_{i=1}^{n} A_i$

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

e.g. 6. (a) $A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12\}$.

(b) $C \cap (A \cup B)$

$= (C \cap A) \cup (C \cap B)$

$= \{2\} \cup \{3\} = \{2, 3\}$.

(c) $C \cup (A \cap B) = C \cup \{6, 12\}$.

$= \cancel{(C \cup A) \cap (C \cup B)}$.

$= \cancel{\{2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 17, 19, 23\}}$.

$\cancel{\cap \{2, 3,}$

1.6 Set differences.

* $A \setminus B = \{x \mid x \in A.\ x \notin B\}$.

* $A' = \{x \mid x \in U.\ x \notin A.\ = U \setminus A$

* $A \setminus B = A \cap B'$

* $A \triangle B = \{x \mid x \in (A \cup B).\ x \notin (A \cap B)\}$.

$A \triangle B = (A \cup B) \setminus (A \cap B)$. $\cancel{(A \cup B) \cap (A' \cup B')}$.

$A \triangle B = (A \setminus B) \cup (B \setminus A)$.

* De Morgan' laws.

- $(A\cup B)' = A'\cap B'$.
- $(A\cap B)' = A'\cup B'$.

Ex 7.

a) $= (A\cap A')\cap(B'\cap B)$
$= \phi\cap\phi = \phi$

b) $= \phi$

c) $= (A\cap A')\cup(A\cap B)$
$= \phi\cup(A\cap B)$
$= A\cap B$

d) $= (A\cap A')\cup(A\cap B')\cap(A\cap C')$.
$= \phi\cup(A\cap B')\cap(A\cap C')$.
$= A\cap(B'\cap C')$.
$= A\cap(B\cup C)'$.

EX. 8.

a) $(A\cup B)'\cap C' = A'\cap B'\cap C'$.

b). 这门的人才是

c)

Ex. 9.

a) $A\setminus(B\cup C) = (A\setminus B)\cap(A\setminus C)$.
$= A\cap(B\cup C)' = A\cap(B'\cap C')$. $= (A\cap B'\cap C')$

b). $(A\cap B)\setminus C = A\cap B\cap C'$.
$= (A\cap C')\cap(B\cap C')$. $= (A\setminus C)\cap(B\setminus C)$.

c). $(A\setminus B)\setminus C = (A\cap B')\cap C' = A\cap(B\cup C)'$

Ex 1.

9. $A = \{a \mid a^2(a+1)(a-1) = 0\}$. $B = \{0, 1, 2, 4, 9, \dots\}$.
$= \{0, -1, 1\}$.

a. $A\setminus B =$

- Symmetric difference:

$$A \triangle B = (A \cup B) \setminus (A \cap B)$$
$$= (A \setminus B) \cup (B \setminus A)$$

- <u>Antisymmetric relation</u>.

$(x, y) \in R,\ (y, x) \in R \Rightarrow x = y.$

- $$\begin{tikzcd}
 & aRb \arrow[dl] & \\
 {[a]=[b]} \arrow[rr] & & {[a] \cap [b] \neq \emptyset} \arrow[ul]
\end{tikzcd}$$

- $\underset{\text{domain}}{f : A} \to \underset{\text{codomain}}{B}$.

  $y = f(\underset{\text{input/ preimage}}{x})$: y is the image /output.

- f^{-1} is a fcn. $\Leftrightarrow$ f is bijection.