






the 2019 Exam Assessing Readiness ()

Wait, who are you?

Name:

email address:

Directions:

- Try to complete as much of the  as you can in one time block, spending no more than 4 hours on it. It's okay if you don't finish.
 - You are not allowed to use outside sources (including other humans and the internet) in your work on the . Cheating is not cool. Seriously. There's an honor pledge at the end of the .
 - Please show your work right on the ; sentences are a bonus. Actually, sentences are a really useful way of communicating your understanding, so *hint*: use them.
 - When you're done, take your , and any additional pages you want to include, and either
 - scan this to one PDF file and send the result to director@mathily.org, or
 - mail it all to...

MathILy c/o dr. sarah-marie belcastro
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 - Questions? Contact dr. sarah-marie belcastro at director@mathily.org.
-



Gnirts are strings of s and s, with but a single rule: No two s can be next to each other.

The 2-gnirts are , , . There are three 2-gnirts.

The 3-gnirts are , , , _____, _____. There are ____ 3-gnirts.

The 4-gnirts are , , ...

..., , . There are ____ 4-gnirts.

How many 1-gnirts are there?







How many 5-gnirts are there? How about 6-gnirts?



Explain how to produce a list of 4-gnirts using lists of shorter gnirts.

How many k -gnirts are there? Explain.

Rearranging Gnirt-lists

We can write the 2-gnirts as a list

where each 2-gnirt differs from the next by only one symbol:      

(It's silly, but we can write the 1-gnirts this way as well:  )

Do the same for the the 3-gnirts: _____    _____ _____   

How does the 2-gnirt list come into play here?

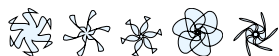
Try it for the 4-gnirts:

_____ _____         _____     _____    

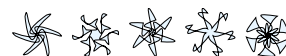
How is this list related to previous gnirt lists?

Explain how to construct this list in general (that is, for k -gnirts).

Why does it always work?



BRRR!



In honor of the recent very cold temperatures across much of the northern United States, we bring you **b**oring **r**ecurrence **r**elation **r**earrangements. Please fill in the blanks; we apologize for the temporary tedium.

$$T_n = T_{n-2} + T_{n-4} + \boxed{} + \cdots + T_2 + T_0.$$

$$T_{n-2} = \boxed{} + T_{n-6} + \boxed{} + \cdots + \boxed{} + \boxed{}.$$

$$\text{Therefore } T_n = \boxed{} T_{\boxed{}}.$$

$$Z_n = Z_{n-1} + 3Z_{n-3} + 3\boxed{} + \cdots + 3Z_1.$$

$$Z_{n-2} = \boxed{} + \boxed{}\boxed{} + \boxed{}\boxed{} + \cdots + \boxed{}\boxed{}.$$

$$\text{Therefore } Z_n = \boxed{} + \boxed{}\boxed{} + \boxed{}.$$

Zero Magic

Fill in the cells of this square with $a, -a, b, -b, c, -c, d, -d$ so that

- All letters represent positive integers,
- No two of those integers are equal,
- Every row sums to 0, and
- Every column sums to 0.

	0	

What are the possible sums for the main diagonals? Explain.

How many ways are there to choose a, b, c, d to fill in the square?

There is an interesting relationship between two of a, b, c, d . Which two, and what is the relationship?

Fill in the cells of this square with integers that obey the rules given above.

	0	

Can you fill in the cells of this square with really different* values than the square on the left? Do so, or explain why it's not possible.

	0	

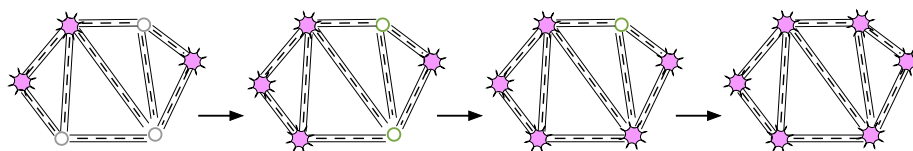
*“really different” could mean lots of things. We mean *all* of them.

***!

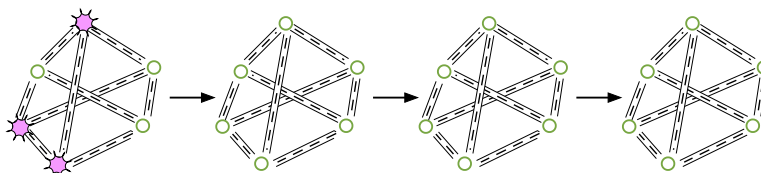
Contagion

***!

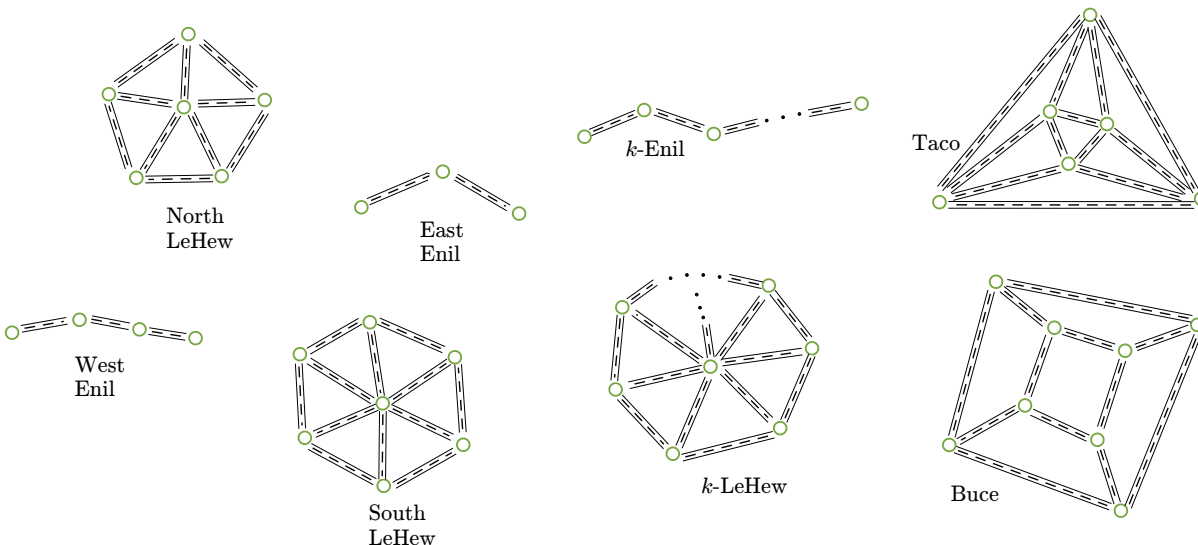
Pricklyrasp plague[†] spreads in an interesting way: When all cities connected by roads to a plague-ridden city—except one—are also plague-ridden, that last city will become infected with Pricklyrasp plague as well. For example, in the region of Chartra, here's how the plague spread:



Show what will happen in the region of Spekh:



Economical bioterrorists want to choose the smallest number of cities to infect while still eventually overwhelming the region with Pricklyrasp plague. Please find that smallest number for each of these regions.



Economical bioterrorists *must* infect at least as many cities in a region R as _____, called $L(R)$.

In any region, the greatest number of cities that economical bioterrorists *might* need to infect is _____, called $G(R)$.

Give an example of a region where economical bioterrorists must infect more than $L(R)$ cities, but less than $G(R)$ cities.

Give an example of a region where $L(R) = G(R)$.

[†] Pricklyrasp plague is characterized by a prickly feeling in the elbows accompanied by a raspy voice.

More Gnirtiness

Please fill in this table:

number	gnirt	sum	number	gnirt	sum
1		1	11		$__ + __$
2		2	$______$		$__ + __ + 1$
$______$		3	13	$______$	$______$
4	$______$	$3 + 1$	$______$	$______$	$13 + 1$
$______$		5	15		
6		$5 + 1$	16		
7		$__ + __$	17		
8	$______$	$______$	18		
$______$	$______$	$__ + __$	19		
$______$	$______$	$__ + __$	20		

Which numbers have a single term in the sum?
 And what property do the corresponding gnirts have?

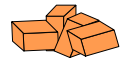
Describe how to produce the sum and gnirt associated with the number 59.

Given any number n , describe how to produce the sum and gnirt associated with the number n .

What sum corresponds to the non-gnirt ? How about ?
 What numbers are associated to and ?
 Why are we glad that gnirts never have two s next to each other?

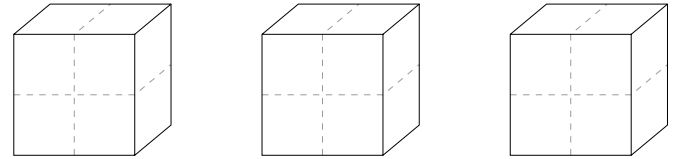


Baby Carrots in Lunchboxes

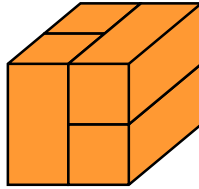


At Precision Vegetable School, every student has a lunchbox for hir laser-cut $2 \times 1 \times 1$ baby carrots.

First graders are assigned $2 \times 2 \times 1$ lunchboxes. Please demonstrate all possible ways a first-grader can pack hir 2 lasercut baby carrots into hir lunchbox.

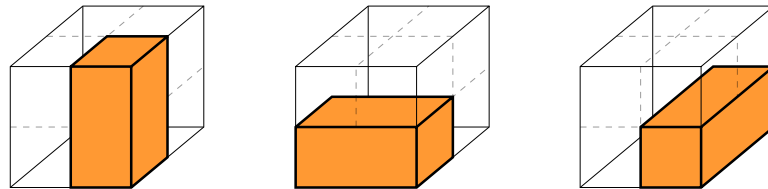


Second graders are assigned $2 \times 2 \times 2$ lunchboxes. Here is one way for a second-grader to pack hir 4 lasercut baby carrots into hir lunchbox.



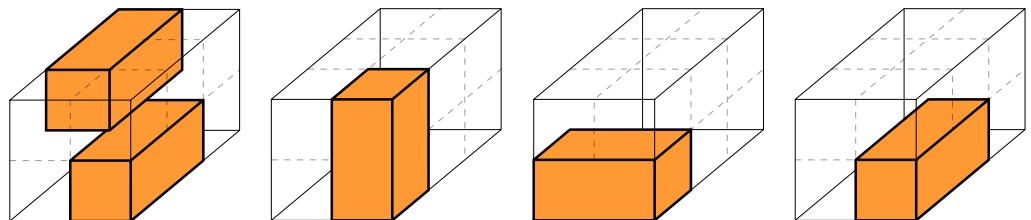
How many lasercut baby carrots may a k th-grader eat at lunch?

Please complete these partial packings.



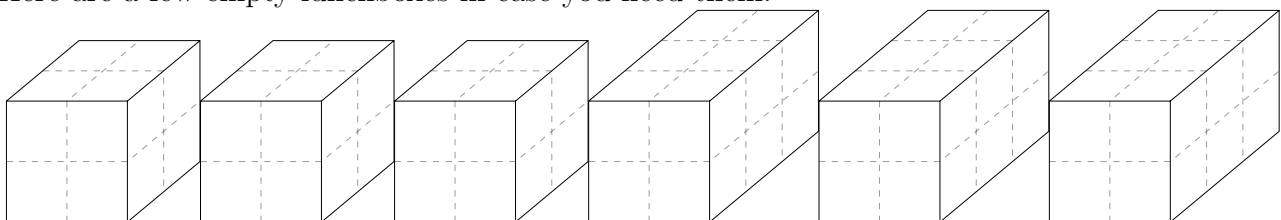
How many days can a second-grader go to school without repeating a lunchbox carrot packing?

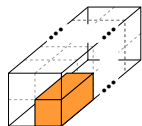
Third graders are assigned $2 \times 2 \times 3$ lunchboxes. Please complete these partial packings.



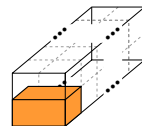
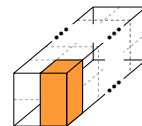
How many days can a third-grader go to school without repeating a lunchbox carrot packing?

Here are a few empty lunchboxes in case you need them:

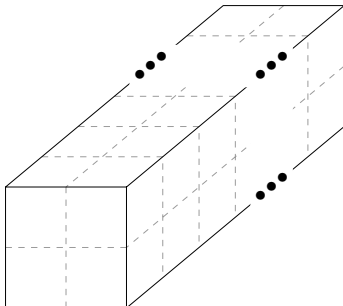




Baby Carrots: The Upper Grades




A k th-grade student at Precision Vegetable School is assigned a $2 \times 2 \times k$ lunchbox for hir lasercut $2 \times 1 \times 1$ baby carrots.




Write C_k for the number of school days a k th-grader can go to school without repeating a lunchbox carrot packing.

Find and explain an expression for C_k in terms of information about younger students.

For what grade does C_k exceed the length of the school year? Does that happen for an earlier or later grade than it becomes unreasonable to eat all the assigned carrots at lunch?

Now that you're done with the , give a little bit of feedback: Had you seen any of this material before? Did you think any of the problems, or parts thereof, were particularly engaging or excellent? Other thoughts to share?

Honesty Pledge: Sign below to indicate that you did not collaborate, give help, or receive help from any sources other than the MathILy director. (And you did not lend anyone your )

Your signature:

Print your name, too: