M40 The Normal Distribution I [using standard normal curve table]

(1) let W = weight of apples

$W \sim N(100, 20^2)$

a) $P(W<110) = P(Z < \frac{110-100}{20})$

$= P(Z < \frac{1}{2})$

$= 0.6915$

GDC $P(W<110) = 0.6915$

using normalcdf $(-9E99, 110, 100, 20)$

b) $P(W<150) = P(Z < \frac{150-100}{20})$

$= P(Z < \frac{50}{20})$

$= P(Z < 2.5)$

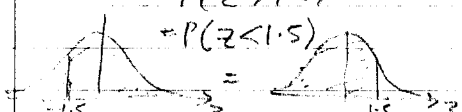
$= \underline{0.9938}$

c) $P(W>70) = P(Z > \frac{70-100}{20})$

$= P(Z > \frac{-30}{20})$

$= P(Z > -1.5)$

$= P(Z < 1.5)$



$P(W>70) = \underline{0.9332}$

GDC: normalcdf $(70, 9E99, 100, 20)$

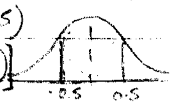
d) $P(90<W<110) = P(\frac{90-100}{20} < Z < \frac{110-100}{20})$

$= P(-\frac{1}{2} < Z < \frac{1}{2})$

$= P(-0.5 < Z < 0.5)$

$= P(Z<0.5) - P(Z<-0.5)$

$= P(Z<0.5) - [1-P(Z<0.5)]$



$= 2P(Z<0.5) - 1$

$= 2(0.6915) - 1$

$= \underline{0.383}$

normalcdf $(90, 110, 100, 20)$.

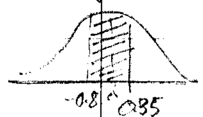
(1e) $P(84<W<107)$

$= P(\frac{84-100}{20} < Z < \frac{107-100}{20})$

$= P(\frac{-16}{20} < Z < \frac{7}{20})$

$= P(-0.8 < Z < 0.35)$

$= P(Z<0.35) - P(Z<-0.8)$



$P(84<W<107) = P(Z<0.35) - [1-P(Z<0.8)]$

$= 0.6368 + 0.7881 - 1$

$= \underline{0.4249}$

normalcdf $(84, 107, 100, 20)$.

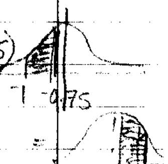
(1f) $P(80<W<85) = P(\frac{80-100}{20} < Z < \frac{85-100}{20})$

$= P(-1 < Z < \frac{-15}{20})$

$= P(-1 < Z < -0.75)$

$= P(0.75 < Z < 1)$

$= P(Z<1) - P(Z<0.75)$



$= 0.8413 - 0.7734$

$= \underline{0.0679}$

normalcdf $(80, 85, 100, 20)$

(2) $W \sim N(1060, 45^2)$

a) $P(W<1000) = P(Z < \frac{1000-1060}{45})$

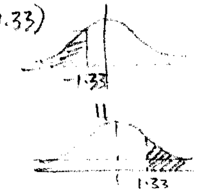
$= P(Z < \frac{-60}{45})$

$\approx P(Z < -1.33)$

$\approx 1 - P(Z<1.33)$

$\approx 1 - 0.9082$

$\approx \underline{0.0918}$



a better estimate is obtained from

normalcdf $(-9E99, 1000, 1060, 45) \approx \underline{0.0912}$

b) $P(W>1050) = P(Z > \frac{1050-1060}{45})$

$= P(Z > \frac{-10}{45})$

$\approx P(Z > -0.22)$

$P(W>1050) \approx P(Z<0.22)$

≈ 0.5871

So $200(0.5871) \approx 117$

A better estimate is obtained from $200 \times$ normalcdf $(1050, 9E99, 1060, 45)$

≈ 118 (3 s.f.) [117.5859031...]

2c) $P(1020<W<1070) = P(\frac{1020-1060}{45} < Z < \frac{1070-1060}{45})$

$= P(\frac{-40}{45} < Z < \frac{10}{45})$

$\approx P(-0.89 < Z < 0.22)$

$\approx P(Z<0.22) - P(Z<-0.89)$

$\approx P(Z<0.22) - [1-P(Z<0.89)]$

$\approx P(Z<0.22) + P(Z<0.89) - 1$

$\approx 0.5871 + 0.8133 - 1$

≈ 0.4004

P(both bags b/w 1020g & 1070g) $\approx 0.4004(0.4004)$

$\approx \underline{0.160}$

(2c) GDC

normalcdf $(1020, 1070, 1060, 45)$

≈ 0.4009

So P(both bags b/w 1020 & 1070g)

$\approx (0.4009)^2$

$\approx \underline{0.161}$ (3 s.f.)

(3) $m \sim N(52, 18^2)$

(a) $P(m>83) = P(Z > \frac{83-52}{18})$

$\approx P(Z > 1.72)$

$P(m>83) \approx 1 - P(Z<1.72)$

$\approx 1 - 0.9573$

≈ 0.0427

4.27% will receive grade 7.

GDC

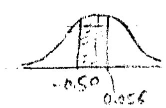
normalcdf $(83, 9E99, 52, 18)$

≈ 0.0425.

or 4.25% obtains 7

(b) $P(43<m<53) = P(\frac{43-52}{18} < Z < \frac{53-52}{18})$

$\approx P(-0.5 < Z < 0.056)$



$P(43<m<53) \approx P(Z<0.056) - [1-P(Z<0.5)]$

$\approx P(Z<0.056) + P(Z<0.5) - 1$

$\approx 0.5219 + 0.6915 - 1$

mean of $P(Z<0.05)$ and $P(Z<0.06)$

≈ 0.2134

So $0.2134(43) \approx 9$ people.

N40.

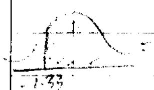
(3c) $P(m<20) = P(Z < \frac{20-52}{18})$
$\approx P(Z<-1.78)$


=


$P(m<20) \approx 1 - P(Z<1.78)$
$\approx 1 - 0.9625$
≈ 0.0375.

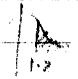
3.75% of candidates receive 1
GDC: normalcdf(-9E99, 20, 52, 18) ≈ 0.0377

(4) $m \sim N(37, 7.5^2)$
$P(m>27) = P(Z > \frac{27-37}{7.5})$
$\approx P(Z>-1.33)$


=

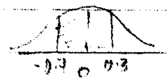

$P(m>27) \approx P(Z<1.33)$
≈ 0.9082
$0.9082N = 259$
$N \approx 285$
So $0.0918(285) \approx 26$
people to fail.

(5) $X \sim N(5, 1^2)$
a) $P(X>6.2) = P(Z > \frac{6.2-5}{1})$
$= P(Z>1.2)$



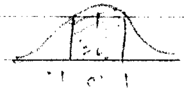
$= 1 - P(Z<1.2)$
$= 1 - 0.8849$
$= 0.1151$

(5b) $P(4.3<X<5.3) = P(4.3-5 < Z < 5.3-5)$
$= P(-0.7<Z<0.3)$



$P(4.3<X<5.3) = P(Z<0.3) - [1-P(Z<0.7)]$
$= P(Z<0.3) + P(Z<0.7) - 1$
$= 0.6179 + 0.7580 - 1$
$= 0.3759$

(5c) $P(|X-5| \le 1)$
$= P(-1 \le X-5 \le 1)$
$= P(4 \le X \le 6)$
$= P(4-5 \le Z \le 6-5)$
$= P(-1 \le Z \le 1)$

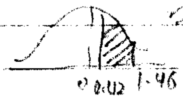


$P(|X-5| \le 1) = P(Z \le 1) - [1 - P(Z<1)]$
$= 2P(Z \le 1) - 1$
$= 2(0.8413) - 1$
≈ 0.6826

(5d) $P(|X-4| \le 1)$
$= P(-1 \le X-4 \le 1)$
$= P(-1+4 \le X \le 1+4)$
$= P(3 \le X \le 5)$
$= P(3-5 \le Z \le 5-5)$
$= P(-2 \le Z \le 0)$

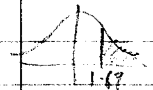
$P(|X-4| \le 1) = P(Z \le 2) - P(Z \le 0)$
$= 0.9773 - 0.5$
$= 0.4773$

(6) $q \sim (100, 24^2)$
(a) $P(110<q<135) = P(\frac{110-100}{24} < Z < \frac{135-100}{24})$
$\approx P(0.42 < Z < 1.46)$
$\approx P(Z<1.46) - P(Z<0.42)$
$\approx 0.9279 - 0.6628$
≈ 0.2651



26.5% of the population.
GDC: normalcdf(110, 135, 100, 24)
≈ 0.2661

(b) $P(q>140) = P(Z > \frac{140-100}{24})$
$\approx P(Z>1.67)$
$\approx 1 - P(Z<1.67)$
$\approx 1 - 0.9525$
≈ 0.0475



4.75% of the population
GDC: normalcdf(140, 9E99, 100, 24)
≈ 0.0478

(6c) $P(q>200) = P(Z > \frac{200-100}{24})$
$= P(Z > \frac{100}{24})$
$\overset{GDC}{\approx} 1.546 \times 10^{-5}$
$6 \times 10^9 \times 1.546 \times 10^{-5} \approx 92800$ (3s.f.)

(6d) $P(q>250) \overset{GDC}{\approx} 2.06 \times 10^{-10}$
$6 \times 10^9 \times 2.06 \times 10^{-10} \approx 1$.