HL2 Assignment #16 Name: Maggie. $9\frac{1}{2}$

1. Find the radius of convergence and interval of convergence for the series $\sum_{n=0}^{\infty} n!x^n$.

$\frac{}{10}$ Very good.

By ratio test,

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!\,x^{n+1}}{n!\,x^n}\right| = \lim_{n\to\infty}|(n+1)x.| = \infty. > 1.$$

therefore, the series $\sum_{n=0}^{\infty} n!x^n$ diverges.

So the radius of convergence is 0. and there is no interval of convergence

, we could say $[0,0]$

2. Is it possible to find a power series whose interval of convergence is $]0,\infty[$? Explain.

No. it is impossible.

Suppose there is a power series w/ interval of convergence $]0,\infty[$, and center a.

~~Since~~ According ~~tto~~ to theorem 1,

$a = \frac{0+\infty}{2} = \infty$.

then the radius of convergence

$R = |0-\infty| = \infty$

but on the other hand.

$R = |\infty - \infty| = 0$.

and $0 \neq \infty$.

which is a contradiction.

Therefore,

It is impossible to have a power series with interval of convergence is $]0,\infty[$.

3. Find the domain of the function $f(x) = 1 + \frac{x^3}{2\cdot 3} + \frac{x^6}{2\cdot 3\cdot 5\cdot 6} + \frac{x^9}{2\cdot 3\cdot 5\cdot 6\cdot 8\cdot 9} + \ldots$.

Let $f(x) = \sum_{n=0}^{\infty} a_n$. where $a_0 = 1$.

and $a_{n+1} = a_n \cdot \frac{x^3}{(3n)(3n-1)}$

Applying the ratio test.

when series $\{a_n\}$ converges.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1.$$

Since $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^3}{9n^2-3n}\right|$

$$= \lim_{n\to\infty}\frac{|x|^3}{9n^2-3n}$$

$$= \frac{|x|^3}{\infty} < 1.$$

which is true $\forall x \in \mathbb{R}$.

therefore,

the domain of $f(x)$ is all real numbers.

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4. Show that the function $f(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$ is a solution to the differential equation $f''(x) = f(x)$.

First, we find the interval of convergence of $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$ using ratio test:

$$\lim_{n\to\infty} \left| \frac{x^{2n+2}}{(2n+2)!} \cdot \frac{(2n)!}{x^{2n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x^2}{(2n+1)(2n+2)} \right|$$

$$= \frac{x^2}{\infty} = 0 < 1. \ \forall x \in \mathbb{R}.$$

therefore, $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$ converges $\forall x \in \mathbb{R}$.

Then, according to thrm 2,

$f(x)$ is differentiable on $\mathbb{R}$ and

$f'(x) = \sum_{n=1}^{\infty} 2n \frac{1}{(2n)!} x^{2n-1}$, which also converges $\forall x \in \mathbb{R}$.

Similarly,

$$f''(x) = \sum_{n=1}^{\infty} \frac{2n(2n-1)}{(2n)!} x^{2n-2}.$$

$$= \sum_{n=1}^{\infty} \frac{x^{2n-2}}{(2n-2)!}$$

$$= \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = f(x).$$

$|x| = 1$, e. $\to$ odd

$x = x^{-1}$? (Yeah!!!).

5. Prove that every group of even order contains an odd number of elements of order 2.

$|x| = 2$.

$|x| \neq 2, \neq 1$ x, x^{-1} appear in pairs $\Rightarrow$ even

proof. Let G be a group of even order,

let $a \in G$, $a^2 = e$. i.e. $|a| = 2$. $\leftarrow$ How do we know such an a exists?

therefore, $H = \{e, a\} \leq G$. as it is finite and closed (finite subgroup test)

~~Th~~ According to Lagrange's thrm, all cosets of H has the same number of elmts as H, namely 2.

$\forall b \in G$, $b \notin H$. and $|b| = 2$.

~~then~~ $bH = \{b, ba\}$.

since $b^2 = a^2 = e$. $(ba)^2 = b^2a^2 = e$. requires commutativity.

therefore $|b| = |ba| = 2$ $\forall b \in G, b \notin H, |b| = 2$.

Thus, there are always an even number of elments outside of H have order 2.

since a is also of order 2, there is always in total an odd number of elmts in G of order 2. $\square$

$\frac{1}{3}$

FM2 Assignment #17 Name: Maggie.

$9\frac{1}{2}/10$ Very good

1. Let $f(x) = \cos(x^2)$. Use a series approach to find $f^{(8)}(0)$.

$$f(x) = \cos(x^2) = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} - + \cdots$$

According to thrm 3,

$$f^{(8)}(0) = 8!\, C_8 = 8! \cdot \frac{1}{4!} = 1680.$$

✓

2. Find $\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}}\right)$.

Let $f(x) = \sqrt{x}$. on $[0,1]$.

f has the lower Riemann Sum of

$$L_n = \frac{1}{n}\left(\sqrt{0} + \sqrt{\tfrac{1}{n}} + \sqrt{\tfrac{2}{n}} + \cdots + \sqrt{\tfrac{n-1}{n}}\right).$$

and upper Riemann Sum of

$$U_n = \frac{1}{n}\left(\sqrt{\tfrac{1}{n}} + \sqrt{\tfrac{2}{n}} + \cdots + \sqrt{\tfrac{n}{n}}\right).$$

and ~~since~~ therefore $\lim_{n\to\infty} L_n = \lim_{n\to\infty} U_n$.

So f is integrable on $[0,1]$

and $\int_0^1 f(x)\,dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^1 = \frac{2}{3}$.

Therefore,

$$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\tfrac{1}{n}} + \sqrt{\tfrac{2}{n}} + \cdots + \sqrt{\tfrac{n}{n}}\right) = \lim_{n\to\infty} U_n = \frac{2}{3}.$$

✓

3. Find $\int_0^a x\,dx$ from first principles by taking the limit of a lower Riemann sum and the limit of an upper Riemann sum.

$$L_n = \frac{a}{n}\cdot 0 + \frac{a}{n}\cdot\left(\frac{a}{n}\right) + \cdots + \frac{a}{n}\left(\frac{(n-1)a}{n}\right)$$

$$= \left(\frac{a}{n}\right)^2 (1 + 2 + \cdots + (n-1))$$

$$= \frac{a^2}{n^2}\cdot\frac{n(n-1)}{2} = \frac{a^2(n-1)}{2n}$$

✓

$$U_n = \frac{a}{n}\left(\frac{a}{n}\right) + \frac{a}{n}\left(\frac{2a}{n}\right) + \cdots + \frac{a}{n}\left(\frac{na}{n}\right)$$

$$= \left(\frac{a}{n}\right)^2 (1 + 2 + \cdots + n)$$

$$= \left(\frac{a}{n}\right)^2\left(\frac{(1+n)n}{2}\right) = \frac{a^2(n+1)}{2n}.$$

✓

$$\lim_{n\to\infty} L_n = \frac{a^2}{2} = \lim_{n\to\infty} U_n.$$

so integral exists.

and

$$\int_0^a x\,dx = \lim_{n\to\infty} L_n = \frac{a^2}{2}.$$

✓

/6

4. Find $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \dfrac{3n^2+2k^2}{n^3}$.

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{3n^2+2k^2}{n^3}$$

$$\cancel{= \sum_{k=1}^{\infty} \lim_{n\to\infty} \frac{3n^2+2k^2}{n^3}}$$

$$\cancel{= \sum_{k=1}^{\infty} 0}$$

$$= \lim_{n\to\infty}\left(\sum_{k=1}^{n} \frac{3}{n} + 2\sum_{k=1}^{n} \frac{k^2}{n^3}\right)$$

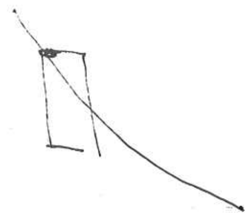
$$= \lim_{n\to\infty}\left(3 + 2\,\frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty}\left(3 + \frac{n(n+1)(2n+1)}{3n^3}\right)$$

$$= 3 + \frac{2}{3}$$

$$= \frac{11}{3}.$$

✓



5. Find $\displaystyle\int_0^1 \frac{1}{1+x}\,dx$ and deduce that

$$\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n-1}\right)=\ln 2.$$

Use $n=100$ with the **sum** and **seq** functions of your calculator to estimate $\ln 2$. Why is your estimate too large?

$$\int_0^1 \frac{1}{1+x} = [\ln(x+1)]_0^1$$

$$= \ln 2 - \ln 1 = \ln 2.$$

and since.

as $\frac{1}{1+x}$ is decreasing

$$u_n\ \cancel{L_n} = \frac{1}{n}\left(1+\frac{1}{1+\frac{1}{n}}+\frac{1}{1+\frac{2}{n}}+\cdots+\frac{1}{1+\frac{n-1}{n}}\right)$$

$$= \frac{1}{n}\left(1+\frac{n}{n+1}+\frac{n}{n+2}+\cdots+\frac{n}{2n-1}\right)$$

$$= \frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n-1}.$$

$$L_n\ U_n = \frac{1}{n}\left(\frac{n}{n+1}+\frac{n}{n+2}+\cdots+\frac{n}{2n}\right)$$

$$= \frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}.$$

$$\lim_{n\to\infty} L_n = \lim_{n\to\infty} U_n = \int_0^1 \frac{1}{1+x}.$$

which means that

$$\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{n+1}+\cdots+\frac{1}{2n-1}\right) = \ln 2.$$

The estimation using technology is 0.696 while ~~the~~ ln 2 is about 0.693.

The estimation is too large because. $n=100<\infty$

So $\frac{1}{n}=\frac{1}{100}>\frac{1}{\infty}=0.$

$$u_{100} > \int_0^1 \frac{1}{1+x}\,dx$$

$$U_{100} > \int_0^1 \frac{1}{1+x}\,dx \quad \frac{1}{2}$$

$3\frac{1}{2}$

FM2 Assignment #18 Name: Maggie.

$\frac{8\frac{1}{2}}{10}$ Good

1. Find $\dfrac{d}{dx}\displaystyle\int_0^{x^2} \cos t\,dt$.

$$\frac{d}{dx}\int_0^{x^2} \cos t\ dt = \cos x^2 \cdot (2x)$$
$$= 2x\cos x^2$$

✓

2. Find $\dfrac{d}{dx}\displaystyle\int_x^{x^2} \cos t\,dt$.

$$\frac{d}{dx}\int_x^{x^2} \cos t\ dt$$
$$= \frac{d}{dx}\left(\int_0^{x^2} \cos t\, dt - \int_0^{x} \cos t\, dt\right).$$
$$= 2x\cos x^2 - \cos x$$

✓

3. For each of the following either explain why the graph cannot exist or draw a graph with the given property.

(a) A bipartite graph which contains K_4.

It cannot exist. ✓

K_4 is a complete graph w/ all degree ~~equat~~ equal to 3, so it is impossible to partition the vertices as all are ~~connected~~. adjacent to each other, or neighbours.

Therefore, there can't be any bipartite graph containing K_4.

why?

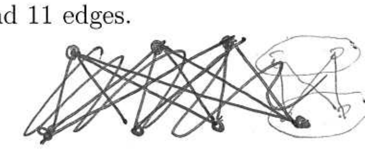
(b) A simple planar bipartite graph with 7 vertices and 11 edges.

$7 = 1+6 = 2+5 = 3+4$

$K_{2,5}$, $= C_4$

6 edges 10 edges 12 edges

For a planar graph,



$v - e + f = 2$,

in this case, there is only 1 face as no cycle can be ~~7~~ created in a bipartite graph, so ✗

$7 - 11 + 1 = -3 \neq 2$, which means it's not planar

So there does not exist a simple, planar graph w/ 7 vertices and 11 edges.

4. Show that for small x, $x \csc x \approx [1 - (x^2/6 - x^4/120)]^{-1}$. Expand the RHS by the binomial expansion and conclude that

$$\csc x \approx \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360}.$$

$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$x \csc x = \frac{x}{\sin x} \approx \cancel{\frac{3!}{x^4} + x^6}$

$\approx \dfrac{1}{1 - \frac{x^2}{6} + \frac{x^4}{120}}$.

$= [1 - (\frac{x^2}{6} - \frac{x^4}{120})]^{-1}$.

Using the bionomial expansion,

$x \operatorname{css} x = 1 - 1 \cdot [\cancel{0} - \frac{x^2}{6} + \frac{x^4}{120}] + [- \frac{x^2}{6} + \frac{x^4}{120}]^2 + \ldots$

$= 1 + \frac{x^2}{6} - \frac{x^4}{120} + \frac{x^4}{36} + O(x^6)$.

So $\csc x \approx \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360}$. ✓

5. Let $f : G \to H$ be a group homomorphism with $K = \ker(f)$.

(a) Show that $gkg^{-1} \in K$ for all $g \in G$ and $k \in K$.

$f(gkg^{-1})$
$= f(g) \cdot f(k) \cdot f(g^{-1})$
$= f(g) \cdot f(g)^{-1}$
$= e'$.
$\Rightarrow gkg^{-1} \in K$. ✓

(b) Deduce that each left coset of K in G is also a right coset.

~~Let $a \in G$. $a \notin K$.~~

Since $gkg^{-1} \in K$.

$gk_1g^{-1} = k_2$ for some $k_2 \in K$.

$gk_1g^{-1} \cdot g = k_2 g$.

$gk_1 = k_2 g$. $\forall k_1 \in K$.

$\{gk_1 \mid k_1 \in K\}$

$\{k_2 g \mid k_2 \in K\}$.

element, not set

and gk_1 is a left coset ✗

rather $gK = \{gk \mid k \in K\}$ is a left coset.

$k_2 g$ is a right coset.

i.e. each left coset is also a right coset.

$1\frac{1}{2} / 3\frac{1}{2}$

FM2 Assignment #19 Name: Maggie $\frac{10}{10}$ Excellent!

1. The linear transformation that maps (x, y) to $(x + ky, y)$ is called a horizontal shear with shear factor k. If M is the matrix for a horizontal shear with shear factor 1, find M^{2019}.

~~$M(x,y) = (x+y, y)$~~

$M\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ y \end{pmatrix}$.

$M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

M^{2019} means that (x, y) is horizontally sheared for 2019 times.

so (x, y) is mapped to $(x+2019y, y)$

so $M^{2019}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+2019y \\ y \end{pmatrix}$

so $M^{2019} = \begin{pmatrix} 1 & 2019 \\ 0 & 1 \end{pmatrix}$ ✓

2. Find T^{-1} for the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x + 3y, 2x + 5y)$.

$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+3y \\ 2x+5y \end{pmatrix}$. Let the matrix be M.

$M = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$

$M^{-1} = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}^{-1}$

$= -1\begin{pmatrix} 5 & -3 \\ -2 & 1 \end{pmatrix}$

$= \begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix}$

So $T^{-1}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5x+3y \\ 2x-y \end{pmatrix}$ ✓

$T^{-1}(x, y) = (-5x+3y, 2x-y)$

3. Find the matrix for projection onto the line $y = (\tan\theta)x$. Describe the kernel of this transformation.

Let the transformation be T, ~~and~~ with Matrix M.

$M = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ 0 & 0 \end{pmatrix}$

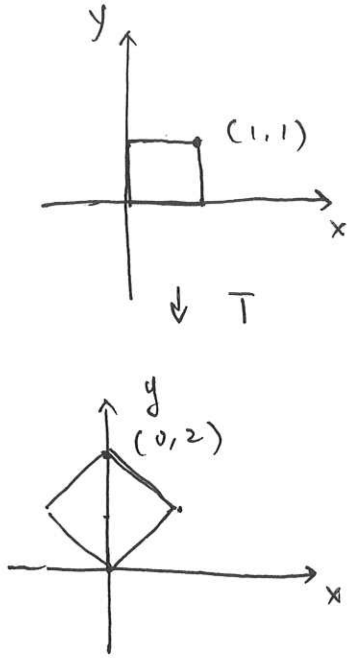
$= \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$. ✓

kernel $(T) = \{(x, y) \mid T(x, y) = (0, 0)\}$

$= \{(x, y) \mid y = (-\cot\theta)x\}$.

$= \{(x, -(\cot\theta)x)\}$. ✓

4. Draw the image of the unit square under the transformation with matrix M= $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. Hence write M as the product of a dilation (enlargement) matrix and a rotation matrix.



$M\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}0\\2\end{pmatrix}$

the linear transformation enlarges the squre by 2 and rotates it anticlockwise for 45°.

so $M = \begin{pmatrix}\cos(45^\circ) & -\sin(45^\circ)\\ \sin(45^\circ) & \cos(45^\circ)\end{pmatrix}\begin{pmatrix}\sqrt{2} & 0\\ 0 & \sqrt{2}\end{pmatrix}$

5. Prove that a group cannot be the union of two of its proper subgroups.

Proof (by contradiction)

Suppose $H_1, H_2 < G$. and $H_1 \cup H_2 = G$.

We claim that $\exists\, a \in G$, s.t $a \in H_1$, $a \notin H_2$.

because otherwise, $H_1 \leq H_2$, $H_1 \cup H_2 = H_2 = G$, which is impossible as $H_2 < G$.

Similarly $\exists\, b \in G$. s.t. $b \in H_2$, $b \notin H_1$.

Because of closure, $ab \in G$.

~~If ab∈~~ WLOG, let $ab \in H_1$.

Then $\exists\, a^{-1} \in H_1$, and because of closure,

$a^{-1}(ab) = b \in H_1$. which is a contradiction.

Therefore, $H_1 \cup H_2 \neq G$. where $H_1, H_2 < G$. $\square$.

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FM2 Assignment #20 Name: Maggie 9/10

1. Must a linear transformation of the plane that preserves areas also preserve lengths?

No.

Because the scale factor for change in area for a linear transformation of a plane is the determinant of its matrix.
and since $f: M \to \det(M)$ is not an injection, the determinants for two different matrices can be the same.
An example would be the linear transformation w/ matrix $\begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ w/ det = 1.

2. Denote the area and circumcircle radius of $\triangle ABC$ by $[ABC]$ and R respectively. Prove that $[ABC] = abc/4R$.

proof.

Recall that $[ABC] = \frac{1}{2}ab\sin C$.

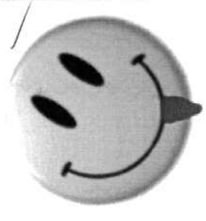
~~Since~~ according to the law of sine,

$\cancel{\frac{a}{\sin A} = 2R}$. $\frac{c}{\sin C} = 2R$

So $\sin C = \frac{c}{2R}$.

therefore, $[ABC] = \frac{1}{2}ab \cdot \frac{c}{2R} = \frac{abc}{4R}$. $\square$.

3. The smiley face on the right is transformed. Match the matrices with the transformed smiley faces.

A-F	image	A-F	image	A-F	image
D		C		E	
A		B		F	

6

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}, \quad E = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad F = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

4. Give a non-identity matrix with the property that $A^T = A^{-1}$. Show that if $A^T = A^{-1}$ then $\det A = \pm 1$. Does the converse hold?

$\det(A) = \det(A^T)$.

$\det(A^{-1}) = \frac{1}{\det(A)}$

missing

If $A^T = A^{-1}$

$\det(A^{-1}) = \det(A^T)$

$= \det(A)$

$= \frac{1}{\det(A)}$

$\det^2(A) = 1$.

$\det(A) = \pm 1$.

ex: $A = \begin{pmatrix} -1 & \cancel{8} \\ 0 & -1 \end{pmatrix}$

$\cancel{\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}}$

$A^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

$A^{-1} = \begin{pmatrix} \frac{d}{\det} & \frac{-c}{\det} \\ \frac{-b}{\det} & \frac{a}{\det} \end{pmatrix}$

det = −1

$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

If $\det(A) = \pm 1$

the $\det(A^T) = \det(A) = \pm 1$.

$\det(A^{-1}) = \frac{1}{\det(A)} = \pm 1$.

So $\det(A^T) = \det(A^{-1}) = \pm 1$.

However, we cannot determine whether $A^T = A^{-1}$.

A counterexample would be $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$.

then $A^T = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ $A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$.

and $A^T \neq A^{-1}$.

So converse not true.

$\frac{1}{2}$

5. Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(2x)^n}{\ln(n+1)}$.

when $x = -\frac{1}{2}$.

$$\sum_{n=1}^{\infty} \frac{(2x)^n}{\ln(n+1)} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}.$$

since $0 < \frac{1}{\ln(n)} < \frac{1}{\ln(n+1)}$,

and $\lim_{n\to\infty} \frac{1}{\ln(n)} = 0$, by alternating series test, it is convergent.

$\frac{1}{2}$

when $x = \frac{1}{2}$,

$$\sum_{n=1}^{\infty} \frac{(2x)^n}{\ln(n+1)} = \sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}.$$

$\sum \frac{1}{\ln(n)}$

$\int \frac{1}{\ln x}\,dx \neq \ln(\ln(x)) + C$.

~~Comparison test with $\sum \frac{1}{n}$~~

Since $\frac{1}{\ln(n+1)}$ is positive, continuous, decreasing $\forall\, n > 0$

Apply integral test,

$$\int_1^{\infty} \frac{1}{\ln(n+1)} = \left[\ln(\ln(n+1))\right]_1^{\infty}$$

$= \infty$.

which means the series diverges,

therefore, the interval of convergence is $[-\frac{1}{2}, \frac{1}{2}[$.

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