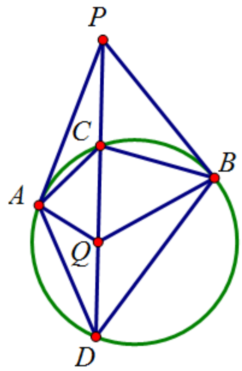
第十一讲　调和点列初步



练1. 连 AB，$\triangle PBC \backsim \triangle PDB \Rightarrow \dfrac{BD}{BC}=\dfrac{PD}{PB}$.

同理可得 $\dfrac{AD}{AC}=\dfrac{PD}{PA}$.

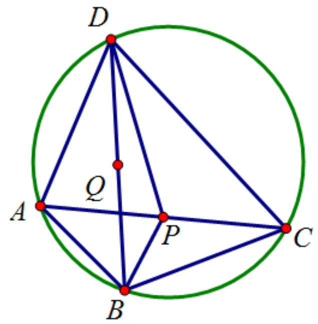
$\because PA=PB \therefore \dfrac{BD}{AD}=\dfrac{BC}{AC}$.

$\because \angle BAC=\angle PBC=\angle DAQ, \angle ABC=\angle ADQ$.

$\therefore \Delta ABC \sim \Delta ADQ \therefore \dfrac{BC}{AC}=\dfrac{DQ}{AQ} \therefore \dfrac{BD}{AD}=\dfrac{DQ}{AQ}$.

$\because \angle DAQ=\angle PBC=\angle BDQ \therefore \triangle ADQ \backsim \triangle DBQ$.

$\therefore \angle DBQ=\angle ADQ=\angle PAC$.



练2. 延长 DP 交圆于另一点 E，则 $\angle CPE=\angle DPA=\angle BPA$.

又 P 是线段 AC 的中点，故 $AB=CE \Rightarrow \angle CDP=\angle BDA$.

又 $\angle ABD=\angle PCD \therefore \triangle ABD \backsim \triangle PCD$.

$\therefore \dfrac{AB}{BD}=\dfrac{PC}{CD}$, 即 $AB \cdot CD=PC \cdot BD$.

$\therefore AB \cdot CD=\dfrac{1}{2}AC \cdot BD=AC \cdot \left(\dfrac{1}{2}BD\right)=AC \cdot BQ$.

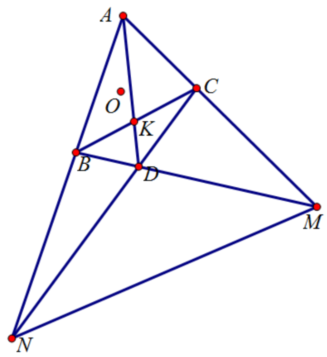
即 $\dfrac{AB}{AC}=\dfrac{BQ}{CD}$.

又 $\angle ABQ=\angle ACD \therefore \triangle ABQ \backsim \triangle ACD \quad \therefore \angle QAB=\angle DAC$.

延长线段 AQ 与圆交于另一点 F，则 $\angle CAB=\angle DAF, \therefore BC=DF$.

又$\because Q$ 为 BD 中点，所以 $\angle CQB=\angle DQF$.

又 $\angle AQB=\angle DQF \therefore \angle AQB=\angle CQB$.



练3. 令 AD 交圆 O 于 E，CE 交 AB 于 P，BE 交 AC 于 Q.

易得 $PQ/\!/MN$.

于是 MN、AD 调和分割 BC，同理 PQ 亦然，

则 $PQ/\!/MN/\!/BC$，从而 K 为 BC 中点，矛盾.

故 $ABCD$ 共圆.