Hypothesis Testing

1. Cherry Cola comes in bottles marked 300 cl. The machine that dispenses the cola fills the bottles to a mean of 303 cl and a standard deviation of 2 cl with the dispensed volume following a normal distribution. A quality control worker takes a random sample of six Cherry Cola bottles and measures the volume of cola to the nearest centilitre.

301 304 305 304 299 306

Is this good evidence that the dispensing machine needs adjustment?

1. Hypothesis

$H_0: \mu = 303.$

$H_1: \mu \neq 303.$

2. Test statistic

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

3. p-value.

p-value = 0.838 (3 s.f.)

4. Conclusion:

There is no ~~difference~~ sufficient evidence at the 5% level of significance. to reject H_0 in favor of H_1.

1. Hypothesis

$H_0: \mu = 303$

$H_1: \mu \neq 303.$

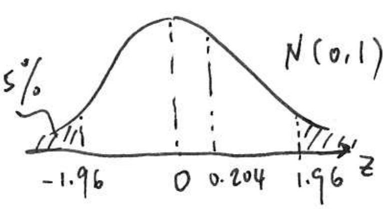
2. Test statistic

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

3. critical value.

for our sample,

$$z = \frac{\bar{x} - 303}{0.2/\sqrt{6}} = 0.204 \text{ (3 s.f.)}$$



4. Conclusion

Since $z_{calc} < z^*$ at the 5% level of significance, there is insufficient evidence to reject H_0 in favor of H_1.

2. The ACME company state that their AA batteries have a mean life-time of 8 hours with a standard deviation of 30 minutes. The Consumer Protection Agency has received several complaints about under-performing ACME batteries from irate consumers. To test the concern that the average life-time of an ACME AA battery is in fact less than 8 hours, the CPA takes a random sample of 100 such batteries and finds an average life-time of 7.8 hours. Is this good evidence to support the complaints against the company?

1. Hypothesis

$H_0: \mu = \cancel{\#80}\ 8$

$H_1: \mu < \cancel{\#80}\ 8$

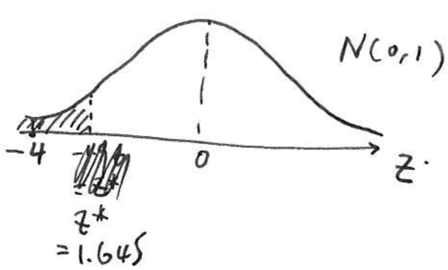
2. Test statistic

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$

by CLT.

3. critical value.

$$\frac{7.8 - 8}{0.5/\sqrt{100}} = -4.$$



Since it's a one-tailed test $z^* = 1.645$ at the 5% level of significance.

4. Conclusion

Since $z_{calc} = -4 < -4^*$, there is sufficient evidence to reject H_0 in favor of H_1.

3. p-value

$p = 3.17 \times 10^{-5} < 0.05$

4. Since $p = 3.17 \times 10^{-5} < 0.05$, there is suff . . . . .

3. Six observations of a continuous random variable X gave the following values.

120.3	122.4	119.8	121.0	122.5	119.6

(a) State any conditions that are necessary for the valid use of the T-test to test a hypothesis about the mean of X. Normal. random sample.

(b) Assuming that the use of the T-test is valid, test the null hypothesis that the mean of X is 120 against the alternative hypothesis that the mean is not 120, using a 5% significance level.

$H_0: \mu = 120$

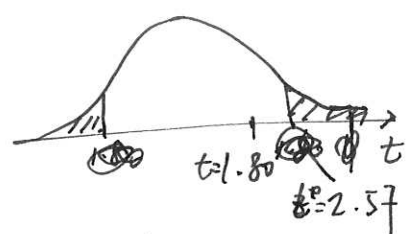
$H_1: \mu \neq 120$

$$\frac{\bar{X}-\mu}{S/\sqrt{n}} = T \sim t(5)$$

p-value $= 0.132 > 0.05$

Hence. insufficient evidence to reject H_0 at 5% ~~sig~~ level of significance.

$t = 1.80$



$\Rightarrow$ insuf.

4. Five candidates attended a special training course to improve their job skills. They were tested before the course started and were tested again at the end of the course. The results are given in the table.

Candidate	1	2	3	4	5
Test score before course	107	93	105	96	92
Test score at end of course	115	94	107	94	98

Assuming that the test does measure the necessary skills taught in the course, use a matched pairs T-test to determine whether the course improved the candidates test performance. State any assumptions you make and show all the necessary steps in your solution.

5. The mean diameter of a type of bearing is supposed to be 2.000 cm. The bearing diameters vary normally with with standard deviation $\sigma = 0.010$ cm. When a lot of the bearings arrives, the consumer takes a random sample of 5 bearings and measures their diameters. The consumer rejects the bearings if the sample diameters is significantly different from 2.000 cm at the 5% significance level.

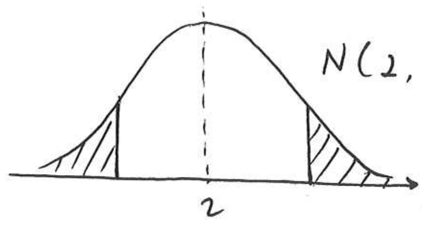
means of the

(a) What is the probability of a Type I error?

(b) What is the probability of a Type II error if the mean diameter is in fact 2.015 cm?

(a) $X \sim N(\mu, \sigma^2)$

$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$



$N(2, \frac{(0.01)^2}{5})$

α = P(Rejecting H_0 | H_0 is true)

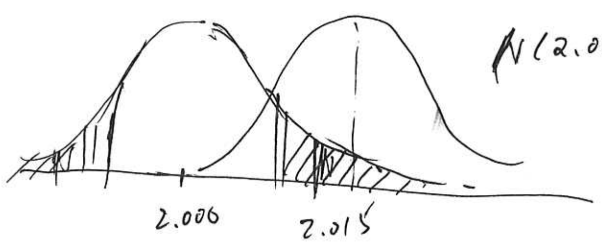
= 5%, which is the level of significance.

(b) ~~β = P(accepting H_0 | H_0 is false)~~

1. Acceptance region:

$2 \pm 1.96 \frac{0.01}{\sqrt{5}} = [1.99123, 2.00877]$

2. β = P(accept H_0 | $\mu = 2.015$)



$N(2.015, \frac{(0.01)^2}{5})$

 $X \sim N(\mu, \sigma^2)$

$n = 100$.

1a. *[1 mark]*

In a large population of hens, the weight of a hen is normally distributed with mean μ kg and standard deviation σ kg. A random sample of 100 hens is taken from the population.

The mean weight for the sample is denoted by $\overline{X}$. $\bar{X} \sim N(\mu, \frac{\sigma^2}{100})$

State the distribution of $\overline{X}$ giving its mean and variance.

1b. *[1 mark]*

The sample values are summarized by $\sum x = 199.8$ and $\sum x^2 = 407.8$ where x kg is the weight of a hen.

Find an unbiased estimate for μ. $= 2.00$

1c. *[2 marks]*

Find an unbiased estimate for σ^2. $= \frac{100}{99}(4.08 - 4) = 0.08 \times \frac{100}{99} = 0.0869.$

1d. *[3 marks]*

Find a 90 % confidence interval for μ. $\bar{x} \pm z_* \frac{\sigma}{\sqrt{n}} \Rightarrow [1.95, 2.05]$

1e. *[2 marks]*

It is found that $\sigma = 0.27$. It is decided to test, at the 1 % level of significance, the null hypothesis $\mu = 1.95$ against the alternative hypothesis $\mu > 1.95$.

Find the p-value for the test. $0.0377 > 0.01$

1f. *[1 mark]*

Write down the conclusion reached. Not reject H_0.

2a. *[1 mark]*

A smartphone's battery life is defined as the number of hours a fully charged battery can be used before the smartphone stops working. A company claims that the battery life of a model of smartphone is, on average, 9.5 hours. To test this claim, an experiment is conducted on a random sample of 20 smartphones of this model. For each smartphone, the battery life, b hours, is measured and the sample mean, $\overline{b}$, calculated. It can be assumed the battery lives are normally distributed with standard deviation 0.4 hours.

State suitable hypotheses for a two-tailed test.

2b. *[4 marks]*

Find the critical region for testing $\overline{b}$ at the 5 % significance level.

2c. *[3 marks]*

It is then found that this model of smartphone has an average battery life of 9.8 hours.

Find the probability of making a Type II error.

2d. *[4 marks]*

Another model of smartphone whose battery life may be assumed to be normally distributed with mean μ hours and standard deviation 1.2 hours is tested. A researcher measures the battery life of six of these smartphones and calculates a confidence interval of [10.2, 11.4] for μ.

Calculate the confidence level of this interval.

3a. *[3 marks]*

Anne is a farmer who grows and sells pumpkins. Interested in the weights of pumpkins produced, she records the weights of eight pumpkins and obtains the following results in kilograms.

7.7 7.5 8.4 8.8 7.3 9.0 7.8 7.6

Assume that these weights form a random sample from a $N(\mu, \sigma^2)$ distribution.

Determine unbiased estimates for μ and σ^2.

3b. *[3 marks]*

Anne claims that the mean pumpkin weight is 7.5 kilograms. In order to test this claim, she sets up the null hypothesis $H_0: \mu = 7.5$.

Use a two-tailed test to determine the p-value for the above results.

3c. *[2 marks]*

Interpret your p-value at the 5% level of significance, justifying your conclusion.

4a. *[1 mark]*

A farmer sells bags of potatoes which he states have a mean weight of 7 kg . An inspector, however, claims that the mean weight is less than 7 kg . In order to test this claim, the inspector takes a random sample of 12 of these bags and determines the weight, x kg , of each bag. He finds that $\sum x = 83.64$; $\sum x^2 = 583.05$. You may assume that the weights of the bags of potatoes can be modelled by the normal distribution $N(\mu, \sigma^2)$.

State suitable hypotheses to test the inspector's claim.

4b. *[3 marks]*

Find unbiased estimates of μ and σ^2.

4c. *[4 marks]*

Carry out an appropriate test and state the p-value obtained.

4d. *[2 marks]*

Using a 10% significance level and justifying your answer, state your conclusion in context.

5a. *[3 marks]*

John rings a church bell 120 times. The time interval, T_i, between two successive rings is a random variable with mean of 2 seconds and variance of $\frac{1}{9}$ seconds^2.

Each time interval, T_i, is independent of the other time intervals. Let $X = \sum_{i=1}^{119} T_i$ be the total time between the first ring and the last ring.

Find

(i) $\mathrm{E}(X)$;

(ii) $\mathrm{Var}(X)$.

5b. *[2 marks]*

Explain why a normal distribution can be used to give an approximate model for X.

5c. *[7 marks]*

Use this model to find the values of A and B such that $\mathrm{P}(A < X < B) = 0.9$, where A and B are symmetrical about the mean of X.

5d. *[5 marks]*

The church vicar subsequently becomes suspicious that John has stopped coming to ring the bell and that he is letting his friend Ray do it. When Ray rings the bell the time interval, T_i has a mean of 2 seconds and variance of $\frac{1}{25}$ seconds^2.

The church vicar makes the following hypotheses:

H_0: Ray is ringing the bell; H_1: John is ringing the bell.

He records four values of X. He decides on the following decision rule:

If $236 \leq X \leq 240$ for all four values of X he accepts H_0, otherwise he accepts H_1.

Calculate the probability that he makes a Type II error.

6a. *[2 marks]*

The owner of a factory is asked to produce bricks of weight 2.2 kg. The quality control manager wishes to test whether or not, on a particular day, the mean weight of bricks being produced is 2.2 kg.

State hypotheses to enable the quality control manager to test the mean weight using a two-tailed test.

6b. *[7 marks]*

He therefore collects a random sample of 20 of these bricks and determines the weight, x kg, of each brick. He produces the following summary statistics.

$$\sum x = 42.0,\ \sum x^2 = 89.2$$

(i) Calculate unbiased estimates of the mean and the variance of the weights of the bricks being produced.

(ii) Assuming that the weights of the bricks are normally distributed, determine the p-value of the above results and state the conclusion in context using a 5% significance level.

6c. *[2 marks]*

The owner is more familiar with using confidence intervals. Determine a 95% confidence interval for the mean weight of bricks produced on that particular day.

7a. *[4 marks]*

Eleven students who had under-performed in a philosophy practice examination were given extra tuition before their final examination. The differences between their final examination marks and their practice examination marks were

$$10, \ -1, \ 6, \ 7, \ -5, \ -5, \ 2, \ -3, \ 8, \ 9, \ -2.$$

Assume that these differences form a random sample from a normal distribution with mean μ and variance σ^2.

Determine unbiased estimates of μ and σ^2.

7b. *[8 marks]*

(i) State suitable hypotheses to test the claim that extra tuition improves examination marks.

(ii) Calculate the p-value of the sample.

(iii) Determine whether or not the above claim is supported at the 5% significance level.

8a. *[3 marks]*

Two species of plant, A and B, are identical in appearance though it is known that the mean length of leaves from a plant of species A is 5.2 cm, whereas the mean length of leaves from a plant of species B is 4.6 cm. Both lengths can be modelled by normal distributions with standard deviation 1.2 cm.

In order to test whether a particular plant is from species A or species B, 16 leaves are collected at random from the plant. The length, x, of each leaf is measured and the mean length evaluated. A one-tailed test of the sample mean, $\overline{X}$, is then performed at the 5% level, with the hypotheses: $H_0: \mu = 5.2$ and $H_1: \mu < 5.2$.

Find the critical region for this test.

8b. *[2 marks]*

It is now known that in the area in which the plant was found 90% of all the plants are of species A and 10% are of species B.

Find the probability that $\overline{X}$ will fall within the critical region of the test.

8c. *[3 marks]*

If, having done the test, the sample mean is found to lie within the critical region, find the probability that the leaves came from a plant of species A.