Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2019

Further mathematics

Higher level

Paper 1

18 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 1, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

Calculator notation
The Further Mathematics HL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. (a) because all the vertices are of even degree, ***R1***
G contains an Eulerian circuit ***A1***

Note: Do not award ***R0A1.***

[2 marks]

(b) use of $e \le 3v - 6$ ***M1***
$e = 10$ and $v = 5$ ***A1***
$(10 \not\le 9)$ the inequality is not satisfied ***R1***
therefore G cannot be drawn as a planar graph ***AG***

[3 marks]

(c) the nearest-neighbour path is
$\mathrm{A} \to \mathrm{B} \to \mathrm{D} \to \mathrm{E} \to \mathrm{C} \to \mathrm{A}$ ***(M1)(A1)***
upper bound $= 8 + 9 + 15 + 10 + 11 = 53$ ***A1***

[3 marks]

(d) removing vertex A, the minimum spanning tree of the remaining graph contains the following edges
BD
EC
BE ***(M1)(A1)***
reconnect vertex A with AB and AC ***(M1)***
lower bound $= 9 + 10 + 13 + 8 + 11 = 51$ ***A1***

[4 marks]

Total [12 marks]

2. (a) $f'(x) = \dfrac{2e^x}{2e^x - 1}$ ***(A1)***

$f''(x) = \dfrac{2e^x(2e^x - 1) - 2e^x \cdot 2e^x}{(2e^x - 1)^2} = -\dfrac{2e^x}{(2e^x - 1)^2}$ ***(A1)***

$f'''(x) = -\dfrac{2e^x \cdot (2e^x - 1)^2 - 2e^x \cdot 2(2e^x - 1) \cdot 2e^x}{(2e^x - 1)^4}$ ***(A1)***

$f(0) = 0,\ f'(0) = 2,\ f''(0) = -2,\ f'''(0) = 6$ ***A2***

Note: Award ***A2*** for all correct, ***A1*** for at least two correct.

$f(x) = f(0) + xf'(0) + \dfrac{x^2}{2} f''(0) + \dfrac{x^3}{6} f'''(0) + \ldots$ ***(M1)***

$= 2x - x^2 + x^3 + \ldots$ ***A1***

Note: Allow follow through for the final ***A1*** from their values of $f(0), f'(0), f''(0), f'''(0)$.

[7 marks]

(b) $\lim\limits_{x \to 0} \dfrac{f(x) - 2x}{x^2} = \lim\limits_{x \to 0}(-1 + x + \ldots)$ ***M1***

$= -1$ ***A1***

[2 marks]

Total [9 marks]

3. (a) (i) the eigenvalues satisfy

$$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$$ ***M1***

$(a-\lambda)(d-\lambda)-bc=0$ ***A1***

$\lambda^2-(a+d)\lambda+ad-bc=0$ ***A1***

the condition for real roots is

$(a+d)^2-4(ad-bc)\geq 0$ ***M1***

$(a-d)^2+4bc\geq 0$ ***AG***

(ii) if the matrix is symmetric, $b=c$. In this case, ***M1***

$(a-d)^2+4bc=(a-d)^2+4b^2\geq 0$

because each square term is non-negative ***R1AG***

[6 marks]

(b) (i) the characteristic equation is

$\lambda^2-6\lambda+5=0$ ***M1***

$\lambda=1,5$ ***A1***

(ii) taking $\lambda=1$,

$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ ***M1***

giving eigenvector $=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ ***A1***

taking $\lambda=5$

$$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ ***M1***

giving eigenvector $=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ***A1***

[6 marks]

Total [12 marks]

4. (a) $b^3 + 3b^2 + 2b + 1 = 5(b+2)^2 + 2(b+2) + 1$ ***M1A1***

$b^3 - 2b^2 - 20b - 24 = 0$ ***(A1)***

$b = 6$ ***A1***

[4 marks]

(b) (i) **EITHER**

$N = 6^3 + 3\times 6^2 + 2\times 6 + 1$ ***(M1)***

OR

$N = 5\times 8^2 + 2\times 8 + 1$ ***(M1)***

THEN

$= 337$ ***A1***

(ii) in base 16, $N = 151$ ***(M1)A1***

Note: Accept any valid method for ***M1***.

[4 marks]

Total [8 marks]

5. integrating factor $= \mathrm{e}^{\int 2\tan x \mathrm{d}x}$ ***M1***

$= \mathrm{e}^{2\ln\sec x}$ ***A1***

$= \sec^2 x$ ***A1***

$\sec^2 x \frac{\mathrm{d}y}{\mathrm{d}x} + 2y\tan x \sec^2 x = \sin x \sec^2 x$ ***(M1)***

$\frac{\mathrm{d}}{\mathrm{d}x}\left(y\sec^2 x\right) = \sec x \tan x$ ***A1***

$y\sec^2 x = \sec x + C$ ***A1***

substituting initial conditions, ***M1***

$2 = 1 + C$

$C = 1$ ***A1***

$y = \cos x + \cos^2 x$ ***A1***

Total [9 marks]

6. (a) the identity is 2 ***(M1)***

Element	Order
0	3
1	2
2	1
3	6
4	3
5	6

A3

Note: Award ***A3*** for all correct, ***A2*** for one error, ***A1*** for two errors.

[4 marks]

(b) $S_2 = \{1,\ 2\}$ ***A1***

$S_3 = \{0, 2, 4\}$ ***A1***

[2 marks]

(c) attempt to find at least one coset ***M1***

$0 \to \{0,5\}$; $3 \to \{3,4\}$; $4 \to \{3,4\}$; $5 \to \{0,5\}$ ***A1***

[2 marks]

Total [8 marks]

7. (a) consider $\frac{u_{n+1}}{u_n} = \frac{(-2)^{n+1}x^{n+1}}{\sqrt{n+1}} \times \frac{\sqrt{n}}{(-2)^n x^n}$ ***M1***

$= -2x \times \frac{\sqrt{n}}{\sqrt{n+1}}$ ***A1***

$\to -2x$ as $n \to \infty$ ***A1***

for the series to be convergent,

$|-2x| < 1$ or $|x| < \frac{1}{2}$ ***(M1)***

the radius of convergence is $\frac{1}{2}$ ***A1***

[5 marks]

continued…

Question 7 continued

(b) (i) $\lim\limits_{x\to 0}\dfrac{\ln(1+x)}{x}=\lim\limits_{x\to 0}\dfrac{\frac{1}{(1+x)}}{1}$ ***M1***

$=1$ ***A1***

(ii) use the limit comparison test with $\sum\limits_{n=1}^{\infty}\dfrac{1}{n}$ ***M1***

consider

$\lim\limits_{n\to\infty}\dfrac{\ln(1+\frac{1}{n})}{\frac{1}{n}}=\lim\limits_{x\to 0}\dfrac{\ln(1+x)}{x}$ ***M1***

$=1$ ***A1***

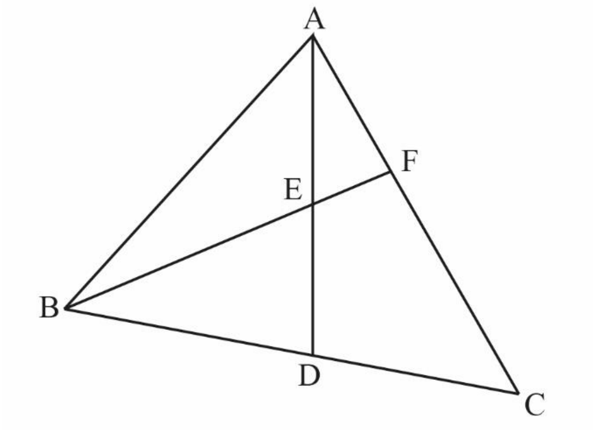
since this limit is finite and non zero and $\sum\limits_{n=1}^{\infty}\dfrac{1}{n}$ diverges ***R1***

the series $\sum\limits_{n=1}^{\infty}\ln\left(1+\dfrac{1}{n}\right)$ diverges ***A1***

[7 marks]

Total [12 marks]

8. (a)



A1

Note: Allow the addition of CE meeting AB.

[1 mark]

continued…

Question 8 continued

(b) using Menelaus' Theorem in triangle ACD with transversal BF and ignoring signs, ***(M1)***

$$\frac{CF}{AF} \times \frac{AE}{ED} \times \frac{DB}{BC} = 1$$ ***A1***

$$\frac{CF}{AF} \times 1 \times \frac{1}{2} = 1$$ ***A1***

$$\frac{CF}{AF} = 2$$ ***A1***

Note: Accept a solution starting with position vectors ***a***, ***b***, ***c***.

[4 marks]

(c) $$\frac{BG}{AG} = 2$$ ***A1***

by symmetry or Ceva's Theorem ***R1***

[2 marks]

Total [7 marks]

9. (a) attempt at row reduction, ***M1***

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 2 \\ 0 & 7 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 4-\mu \end{bmatrix}$$ ***A1A1***

Note: Award ***A1*** for correct row 2 and ***A1*** for correct row 3.

it follows that

$-7 = 4 - \mu$

$\mu = 11$ ***A1***

[4 marks]

(b) (i) **EITHER**

putting $z = \alpha, y = -2\alpha - 1, x = 3 - \alpha$ ***M1A1***

OR

putting $y = \alpha, x = \dfrac{\alpha + 7}{2}, z = -\dfrac{(\alpha + 1)}{2}$ ***M1A1***

continued…

Question 9 continued

(ii) **EITHER**

$x^2+y^2+z^2=(3-\alpha)^2+(-2\alpha-1)^2+\alpha^2$ ***M1***

$=6\alpha^2-2\alpha+10$ ***A1***

valid attempt to minimize this expression using calculus or vertical axis or completing the square, ***M1***

$\alpha=\frac{1}{6}$ ***A1***

OR

$x^2+y^2+z^2=\left(\frac{\alpha+7}{2}\right)^2+\alpha^2+\left(\frac{\alpha+1}{2}\right)^2$ ***M1***

$=\frac{3\alpha^2}{2}+4\alpha+12.5$ ***A1***

valid attempt to minimize this expression using calculus or vertical axis or completing the square, ***M1***

$\alpha=-\frac{4}{3}$ ***A1***

THEN

substituting,

$(x,y,z)=\left(\frac{17}{6},-\frac{4}{3},\frac{1}{6}\right)$ ***A1***

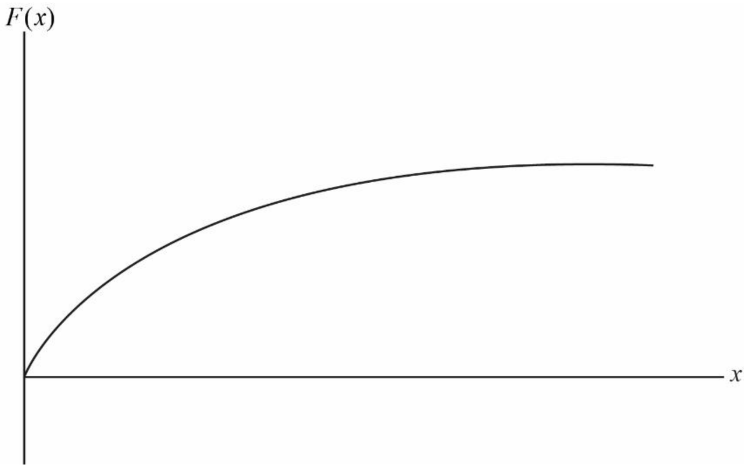
these are the coordinates of the point on the solution line closest to the origin ***R1***

Note: Allow decimal answers that round correctly to 2 significant figures.

[8 marks]

Total [12 marks]

10. (a) (i)



A1

(ii) the mode is zero because the gradient of the graph is greatest where $x = 0$ **R1**

(iii) the median m satisfies $F(m) = 0.5$, ***(M1)***

$$\frac{2}{\pi}\arctan m = \frac{1}{2}$$ ***(A1)***

$$\arctan m = \frac{\pi}{4}$$

$$m = 1$$ ***A1***

[5 marks]

(b) $$f(x) = \frac{2}{\pi}\left(\frac{1}{1+x^2}\right)$$ ***(M1)***

$$\mathrm{E}(X) = \frac{2}{\pi}\int_0^\infty \frac{x}{1+x^2}\mathrm{d}x$$ ***M1***

$$= \left[\frac{1}{\pi}\ln(1+x^2)\right]_0^\infty$$ ***A1***

$\to \infty$ ***A1***

the mean is therefore infinite so the approximation is not valid for this distribution ***R1***

[5 marks]

Total [10 marks]

11. (a) (i) $\begin{vmatrix} 1 & 3 & 2 \\ 5 & 5 & 8 \\ 2 & 1 & 4 \end{vmatrix} = 20 - 8 + 3(16 - 20) + 2(5 - 10) = -10$ ***(M1)A1***

since the determinant is non zero (or the matrix is non singular), ***R1***
the three vectors form a basis for 3-D vectors ***AG***

Note: Allow the use of ref on GDC.

(ii)

$$\begin{bmatrix} 9 \\ 17 \\ 3 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} + \beta \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} + \gamma \begin{bmatrix} 2 \\ 8 \\ 4 \end{bmatrix}$$ ***M1***

solving, $\alpha = 2, \beta = 3, \gamma = -1$ ***A1***

[5 marks]

(b) (i) $\begin{vmatrix} 1 & 3 & 2 \\ 5 & 5 & 8 \\ 2 & 1 & 3 \end{vmatrix} = 0$

since the determinant is zero, ***R1***
S_2 does not form a basis for 3-D vectors ***AG***

Note: Allow the use of ref on GDC.

(ii) the dimension of the subspace is 2 ***A1***

(iii) let

$$\begin{bmatrix} 2 \\ 7 \\ 2 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} + \beta \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} + \gamma \begin{bmatrix} 2 \\ 8 \\ 3 \end{bmatrix}$$ ***M1***

these equations have no solution ***A1***

so the vector $\begin{bmatrix} 2 \\ 7 \\ 2 \end{bmatrix}$ does not belong to the subspace generated by S_2 ***A1***

Note: Allow the omission of one of the α, β, γ terms.

[5 marks]

Total [10 marks]

12. (a) reflexive: $x^2 - x^2 \equiv 0(\text{mod } N)$, it follows that xRx ***R1***
therefore R is reflexive

symmetric: Let xRy so that $x^2 - y^2 \equiv 0(\text{mod } N)$ ***M1***
then it follows that $y^2 - x^2 \equiv 0(\text{mod } N)$ so that yRx ***R1***
therefore R is symmetric

transitive: Let xRy and yRz so that ***M1***
$x^2 - y^2 \equiv 0(\text{mod } N)$ and $y^2 - z^2 \equiv 0(\text{mod } N)$ ***A1***
it follows by adding that $x^2 - z^2 \equiv 0(\text{mod } N)$ therefore xRz ***R1***
therefore R is transitive

since R is reflexive, symmetric and transitive, it follows
that R is an equivalence relation ***AG***

[6 marks]

(b) consider
$(N \pm 1)^2 - 1^2 = N^2 \pm 2N + 1 - 1$ ***M1A1***
$= N^2 \pm 2N \equiv 0(\text{mod } N)$ ***A1***
therefore $N - 1$ and $N + 1$ belong to the same equivalence
class as 1 ***AG***

[3 marks]

Total [9 marks]

13. (a) suppose $f(\boldsymbol{X}) = f(\boldsymbol{Y})$, ie $\boldsymbol{AX} = \boldsymbol{AY}$ ***(M1)***
then $\boldsymbol{A}^{-1}\boldsymbol{AX} = \boldsymbol{A}^{-1}\boldsymbol{AY}$ ***A1***
$\boldsymbol{X} = \boldsymbol{Y}$ ***A1***
since $f(\boldsymbol{X}) = f(\boldsymbol{Y}) \Rightarrow \boldsymbol{X} = \boldsymbol{Y}$, f is an injection ***R1***
now suppose $\boldsymbol{C} \in \boldsymbol{M}$ and consider $f(\boldsymbol{D}) = \boldsymbol{C}$, ie $\boldsymbol{AD} = \boldsymbol{C}$ ***M1***
then $\boldsymbol{D} = \boldsymbol{A}^{-1}\boldsymbol{C}$ ($\boldsymbol{A}^{-1}$ exists since $\boldsymbol{A}$ is non- singular) ***A1***
since given $\boldsymbol{C} \in \boldsymbol{M}$, there exists $\boldsymbol{D} \in \boldsymbol{M}$ such that $f(\boldsymbol{D}) = \boldsymbol{C}$, f is
a surjection ***R1***
therefore f is a bijection ***AG***

[7 marks]

(b) suppose $f(\boldsymbol{X}) = \boldsymbol{Y}$, ie $\boldsymbol{AX} = \boldsymbol{Y}$ ***(M1)***
then $\det(\boldsymbol{A})\det(\boldsymbol{X}) = \det(\boldsymbol{Y})$ ***A1***
since $\det(\boldsymbol{A}) = 0$, it follows that $\det(\boldsymbol{Y}) = 0$ ***A1***
it follows that f is not surjective since the function cannot reach
non-singular matrices ***R1***
therefore f is not a bijection ***AG***

[4 marks]

Total [11 marks]

14. (a) $G_x(t) = \sum P(X = x) \times t^x$ ***M1***

$= \sum_{x=0}^{\infty} \mathrm{e}^{-m} \times \frac{(mt)^x}{x!}$ ***A1***

$= \mathrm{e}^{-m} \times \mathrm{e}^{mt}$ ***A1***

$= \mathrm{e}^{m(t-1)}$ ***AG***

[3 marks]

(b) (i) **Method 1**

$G_y(t) = \mathrm{E}\left(t^Y\right) = \mathrm{E}\left(t^{X_1+2X_2+3X_3}\right)$ ***M1***

$= \mathrm{E}\left(t^{X_1}\right)\mathrm{E}\left(t^{2X_2}\right)\mathrm{E}\left(t^{3X_3}\right)$ ***A1***

$= \mathrm{E}\left(t^{X_1}\right)\mathrm{E}\left(\left(t^2\right)^{X_2}\right)\mathrm{E}\left(\left(t^3\right)^{X_3}\right)$ ***A1***

$= G_x(t)G_x(t^2)G_x(t^3)$ ***A1***

$= \mathrm{e}^{m(t-1)}\mathrm{e}^{m(t^2-1)}\mathrm{e}^{m(t^3-1)}$

$= \mathrm{e}^{-3m}\mathrm{e}^{m(t+t^2+t^3)}$ ***AG***

Method 2

$G_y(t) = G_x(t)G_{2x}(t)G_{3x}(t)$ ***M1***

$G_{2x}(t) = P(X=0) + P(X=1) \times t^2 + P(X=2) \times t^4 + \ldots$ ***M1***

$= G_x(t^2)$ ***A1***

similarly, $G_{3x}(t) = G_x(t^3)$ ***A1***

therefore $G_y(t) = \mathrm{e}^{m(t-1)}\mathrm{e}^{m(t^2-1)}\mathrm{e}^{m(t^3-1)}$

= $= \mathrm{e}^{-3m}\mathrm{e}^{m(t+t^2+t^3)}$ ***AG***

(ii) attempt to expand in a Maclaurin series ***M1***

$\mathrm{e}^{m(t+t^2+t^3)} = \left(1 + m(t+t^2+t^3) + \frac{m^2}{2}(t+t^2+t^3)^2 + \frac{m^3}{6}(t+t^2+t^3)^3 + \frac{m^4}{24}(t+t^2+t^3)^4 + \ldots\right)$ ***A1***

Note: This might be seen as part of $G_y(t)$.

$\mathrm{P}(Y=4) =$ coefficient of t^4 in expansion of $G_y(t)$ ***M1***

$= \mathrm{e}^{-3m}\left(\frac{m^2}{2} + m^2 + \frac{m^3}{6} + \frac{2m^3}{6} + \frac{m^4}{24}\right)$ ***(A1)***

$= \mathrm{e}^{-3m}\left(\frac{3m^2}{2} + \frac{m^3}{2} + \frac{m^4}{24}\right)$ ***A1***

[9 marks]

Total [12 marks]

15. (a) $y - y_1 = m(x - x_1)$ ***A1***

[1 mark]

(b) points of intersection are given by

$x^2 + 2(y_1 + mx - mx_1)^2 = 2$ ***M1***

$x^2 + 2y_1^{\ 2} + 2m^2x^2 + 2m^2x_1^{\ 2} + 4mxy_1 - 4mx_1y_1 - 4m^2xx_1 = 2$ ***M1A1***

$x^2(1+2m^2) + 4mx(y_1 - mx_1) + 2y_1^2 + 2m^2x_1^2 - 4mx_1y_1 - 2 = 0$ ***AG***

[3 marks]

(c) the condition for tangency is $b^2 = 4ac$

$16m^2(y_1 - mx_1)^2 = 4(1+2m^2)(2y_1^2 + 2m^2x_1^2 - 4mx_1y_1 - 2)$ ***M1***

$16m^2y_1^2 - 32m^3x_1y_1 + 16m^4x_1^2 = 8y_1^2 + 8m^2x_1^2 - 16mx_1y_1 - 8$

$+16m^2y_1^2 + 16m^4x_1^2 - 32m^3x_1y_1 - 16m^2$ ***M1A1***

$m^2(x_1^2 - 2) - 2mx_1y_1 + y_1^2 - 1 = 0$ ***AG***

[3 marks]

(d) the condition for perpendicularity is that the product of the roots is -1, ***M1***

$\frac{y_1^2 - 1}{x_1^2 - 2} = -1$ ***A1***

$x_1^2 + y_1^2 = 3$ ***AG***

[2 marks]

Total [9 marks]