M-38 Solutions to Discrete Random Variables

1. a) $E(X) = 1(0.2) + 2(0.5) + 3(0.3)$
$= 0.2 + 1 + 0.9$
$= \underline{2.1}$

b) $E(X) = 2(0.1) + 3(0.8) + 4(0.1)$
$= 0.2 + 2.4 + 0.4$
$= \underline{3.0}$

c) $E(X) = -1(0.4) + 1(0.15) + 4(0.2) + 6(0.25)$
$= -0.4 + 0.15 + 0.8 + 1.5$
$= \underline{2.05}$

d) $1 = 2k + 0.15 + k + 0.4$
$1 = 3k + 0.55$
$0.45 = 3k$
$k = \underline{0.15}$

$E(X) = 1(0.3) + 3(0.15) + 4(0.15) + 6(0.4)$
$= 0.3 + 0.45 + 0.60 + 2.4$
$= \underline{3.75}$

2a) $P(X \neq 3) = 1 - 0.3$
$= \underline{0.7}$

b)

X^2	4	9	16
$P(X=x)$	0.1	0.8	0.1

So $P(X^2 \geq 1) = 1$

c) Because the sum of probability is 1.1 which is longer than 1.

3. $E(X) = 1(\frac{1}{6}) + 2(\frac{1}{6}) + 3(\frac{1}{6}) + 5(\frac{1}{6}) + 8(\frac{1}{6}) + 13(\frac{1}{6})$
$= \frac{1}{6}(1+2+3+5+8+13)$
$= \frac{1}{6}(32)$
$= \underline{\frac{16}{3}}$

4. $0.2 + k + 2k + 0.35 = 1$
$3k + 0.55 = 1$
$3k = 0.45$
$k = 0.15$

$E(X) = 0(0.2) + 1(0.15) + 3(0.3) + 5(0.35)$
$= 0.15 + 0.9 + 1.75$
$= \underline{2.80}$

5a)

x	1	2	3
$P(X=x)$	0.2	$1-k$	$1-0.2-1+k$ $= k-0.2$

(b) $0 \leq k - 0.2 \leq 1 - 0.2$
$0.2 \leq k \leq 1$
$\underline{\frac{1}{5} \leq k \leq 1}$.

(c) $E(X) = 1(0.2) + 2(1-k) + 3(k-0.2)$
$= 0.2 + 2 - 2k + 3k - 0.6$
$= \underline{k + 1.6}$

6.

X	1	2	4
$P(X=x)$	k	0.3	$1-0.3-k$ $= 0.7-k$

$E(X) = 2.8$
$2.8 = 1(k) + 2(0.3) + 4(0.7-k)$
$2.8 = k + 0.6 + 2.8 - 4k$
$0 = 0.6 - 3k$
$3k = 0.6$
$\underline{k = 0.2}$

So $\underline{P(X=1) = 0.2}$

7.

X	-1	4
$P(X=x)$	$\frac{30}{36}$	$\frac{6}{36}$

+	1	2	3	4	5	6
1						7
2					7	
3				7		
4			7			10
5		7			10	11
6	7			10	11	12

$E(X) = -1(\frac{30}{36}) + 4(\frac{6}{36})$
$= \frac{-30 + 24}{36}$
$= \frac{-6}{36}$
$= \underline{-\frac{1}{6}}$

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7b) Expected gain $= \in(\frac{-1}{6})1000$
$= \frac{-1000}{6}$
Expected loss of $\in \frac{1000}{6}$.

8.

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

From the table

X	2	3	4	5	6	7	8
$P(X=x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

You could find a function to fill up the table above but the easier way is to have a table as above.

$P(X > 4) = 1 - P(X=2) - P(X=3) - P(4)$
$= 1 - \frac{1+2+3}{16}$
$= \frac{10}{16}$
$= \underline{\frac{5}{8}}$