Solutions to FM2 Test #5

1. Unbiased estimates of the population mean and variance are $\bar{x} = 3$ and $s_{n-1}^2 = 10$ respectively.

2. Let X measure the length in metres of a randomly chosen king fish. Then $X \sim \mathrm{N}(\mu, 0.12^2)$. Since $P(X \leq 0.7) = 0.8$, we conclude $\mu = 0.599$ (3 s.f.).

3. Let X count the number of emails arriving in a 45 minute period. Then $X \sim \mathrm{Po}(3)$. Hence $P(X > 2) = 1 - P(X \leq 2) = 0.577$ (3 s.f.).

4. The confidence interval is $\bar{x} \pm z^* \dfrac{3.2}{\sqrt{400}} = [10.0, 10.6]$ (3 s.f.).

5. Let M and W measure the weights in kilograms of a randomly chosen man and a randomly chosen woman respectively. Then $M \sim \mathrm{N}(70, 10^2)$ and $W \sim \mathrm{N}(60, 5^2)$. Hence the total weight $T \sim \mathrm{N}(530, 575)$. So $P(T > 550) = 0.202$ (3 s.f.).

6. (a) $E(\bar{X}) = 4/p$ (b) So an unbiased estimator of $1/p$ is $\bar{X}/4$.

7. The null hypothesis is $H_0\colon \mu = 64$ and the alternative hypothesis is $H_1\colon \mu \neq 64$. The test statistic is

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$$

   The p-value is $0.000177 \ll 0.05$. Hence there is very strong evidence that the species is not species A.

8. The null hypothesis is $H_0\colon \mu = 48$ and the alternative hypothesis is $H_1\colon \mu > 48$. The test statistic is

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(5)$$

   The p-value is 0.0320. Hence there is evidence at the 5% level of significance that Alice's mean phosphorous level exceeds 48 mg/l.

9. The null hypothesis is $H_0\colon \mu = 0.5$ and the alternative hypothesis is $H_1\colon \mu \neq 0.5$. The test statistic is

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$$

   To calculate the p-value we need to know σ, which is calculated by integration to be $\sqrt{1/12}$, whence the p-value is 0.00661. Hence there is sufficient evidence at the 1% level of significance to reject the null hypothesis in favour of the alternative hypothesis. The student better go back to the drawing board and rethink his random number generator.

10. (a) $\alpha = 0.05$

    (b) Given the null hypothesis, we have $\bar{X} \sim \mathrm{N}(60, 4/100)$. So the acceptance region at the 5% level of significance for the mean of the sample is $]-\infty, 60 + z^* \times 2/10] =]-\infty, 60.329]$. Next $\beta = 0.25 = \text{normalcdf}(-\infty, 60.329, \mu, 0.2)$, whence $\mu = 60.5$ (3 s.f.).