

Maggie.

MATHEMATICS
HIGHER LEVEL
PAPER 3 – CALCULUS

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the *Mathematics HL and Further Mathematics HL formula booklet* is required for this paper.
- The maximum mark for this examination paper is [60 marks].

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 14]

The function f is defined on the domain $\left] -\frac{\pi}{2}, \frac{\pi}{2} \right[$ by $f(x) = \ln(1 + \sin x)$.

(a) Show that
$$f''(x) = -\frac{1}{(1 + \sin x)}$$
.

[4 marks]

- (b) (i) Find the Maclaurin series for f(x) up to and including the term in x^4 .
 - (ii) Explain briefly why your result shows that f is neither an even function nor an odd function.

[7 marks]

(c) Determine the value of $\lim_{x\to 0} \frac{\ln(1+\sin x)-x}{x^2}$.

[3 marks]

2. [Maximum mark: 8]

Consider the differential equation

$$x \frac{dy}{dx} = y + \sqrt{x^2 - y^2}, x > 0, x^2 > y^2.$$

(a) Show that this is a homogeneous differential equation.

[1 mark]

(b) Find the general solution, giving your answer in the form y = f(x).

[7 marks]



1+ 1x2-42

dvt d

1. (a)
$$f'(x) = \frac{1}{1+\sin x} \cdot \cos x$$

 $f''(x) = \frac{-\sin x (1+\sin x) - \cos^2 x}{(1+\sin x)^2}$
 $= \frac{-\sin x - 1}{(1+\sin x)^2}$
 $= -\frac{1}{1+\sin x}$. 1.

(b) (i)
$$P_4(x) = f(x) + f'(x) \times + \frac{f'(x)}{2} \times^2 + \dots + \frac{f'(x)}{2} \times^4$$

$$= 0 + \times -\frac{1}{2} \times^2 + \frac{1}{6} \times^3 - \frac{2}{24} \times^4$$

$$= \times -\frac{1}{2} \times^2 + \frac{1}{6} \times^3 - \frac{1}{12} \times^4.$$

$$P_{4}(-x) = -x - \frac{1}{2}x^{2} - \frac{1}{6}x^{3} - \frac{1}{12}x^{4}$$

which is neither even nor odd. I make explanation

(c)
$$\lim_{x\to 0} \frac{\ln(|+\sin x|) - x}{x^2}$$

= $\lim_{x\to 0} \frac{x - \frac{1}{2}x^2 + O(x^3) - x}{x^2}$

= $\lim_{x\to 0} \frac{-\frac{1}{2}x^2 + O(x^3)}{x^2}$

= $\lim_{x\to 0} \frac{-\frac{1}{2}x^2 + O(x^3)}{x^2}$

$$= \left[-\frac{1}{2} \right]$$
2. (a)
$$\frac{dy}{dx} = \frac{y + \sqrt{x^2 - y^2}}{x^2}$$

= x + \(\int \) \(\text{vwich is homogeneous.} \)

dy = vx.

$$\frac{dy}{dx} = v + \sqrt{1-v^2}$$

$$\frac{dv}{dx} \times + x = x + \sqrt{1-v^2}$$

$$\frac{dv}{dx} \times + x = x + \sqrt{1-v^2}$$

$$\int \frac{1}{\sqrt{1-v^2}} dv = \int \frac{1}{x} dx.$$

$$arcsin \frac{1}{x} = lnx + c,$$

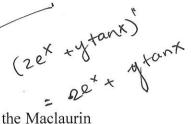
$$\frac{1}{x} = sin(lncx). \quad c = e^{c} ... / (see^{c}x + lsee^{c}x +$$

3. [Maximum mark: 15]

Consider the differential equation

$$\frac{dy}{dx} = 2e^x + y \tan x$$
, given that $y = 1$ when $x = 0$.

The domain of the function y is $\left[0, \frac{\pi}{2}\right[$.

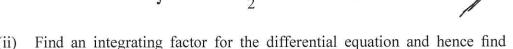


(a) By finding the values of successive derivatives when x = 0, find the Maclaurin series for y as far as the term in x^3 .

[6 marks]

(b) (i) Differentiate the function $e^x(\sin x + \cos x)$ and hence show that

$$\int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x) + c.$$



[9 marks]

4. [Maximum mark: 10]

Let $f(x) = 2x + |x|, x \in \mathbb{R}$.

dy tanx)

dy tanx)

dy tanx)

dy tanx)

dy tanx)

dy tanx)

(a) Prove that f is continuous but not differentiable at the point (0, 0).

[7 marks]

(b) Determine the value of $\int_{-a}^{a} f(x) dx$ where a > 0.

the solution in the form y = f(x).

tanx + secix.

[3 marks]

5. [Maximum mark: 13]

Z/Y

Consider the infinite series $\sum_{n=1}^{\infty} \frac{(n-1)x^n}{n^2 \times 2^n}.$

A dra y

d sell.

(a) Find the radius of convergence.

[4 marks]

(b) Find the interval of convergence.

d tanx

[9 marks]

2 seix. secxtanx

(b) (i)
$$\frac{d}{dx} e^{x} (\sin x + \cos x)$$

$$= e^{x} (\sin x + \cos x) + e^{x} \frac{d}{dx} (\sin x + \cos x)$$

$$= e^{x} (\sin x + \cos x + \cos x - \sin x)$$

$$= 2e^{x} \cos x$$
Thurfore,
$$\int 2e^{x} \cos x dx = e^{x} (\sin x + \cos x) + C,$$

$$\Leftrightarrow \int e^{x} \cos x dx = \frac{1}{2} e^{x} (\sin x + \cos x) + C,$$
where $c = \frac{1}{2} c_{1}$.

(ii)
$$\frac{dy}{dx} = -\tan x \cdot y = 2e^{x}$$
.
Since $(\ln \cos x)' = -\tan x$,
 $\cos x$ is an integratry factor.
Hence, $(\cos x \cdot y)' = 2e^{x} \cos x$.
 $y = \cos x = 2 \int e^{x} \cos x \, dx$
 $y = e^{x} (\sin x + \cos x) + c$, $\cot x$.

4. (a)
$$\lim_{x\to 0} f(x) = \lim_{x\to 0} (2x + |x|) = 0$$
.
 $f(0) = 2(0) + |0| = 0$.
So $\lim_{x\to 0} f(x) = f(0)$, therefore f continuous.

However, f is not different value be cause $\lim_{h \to 0} \frac{f(o+h) \notin -f(o)}{h} = \lim_{h \to 0} \frac{2h+|h|}{h}$ $= 2 + \lim_{h \to 0} \frac{|h|}{h}$ $= 1 + \lim_{h \to 0} \frac{|h|}{h}$ since $\lim_{h \to 0} \frac{|h|}{h} = 1 \neq \lim_{h \to 0} \frac{|h|}{h} = -1$,

timit does not exist, and thus f is not differentiable. (b) $\int_{-a}^{a} 2x + |x| dx$ $= \int_{-a}^{a} 2x dx + \int_{-a}^{a} |x| dx$ $= \left[x^{2}\right]_{-a}^{a} + \left[\frac{1}{2}x^{2}\right]_{0}^{a} + \left[-\frac{1}{2}x^{2}\right]_{-a}^{c}$ $= a^{2} - a^{2} + \frac{1}{2}a^{2} + \frac{1}{2}a^{2}$ $= a^{2} - a^{2} + \frac{1}{2}a^{2} + \frac{1}{2}a^{2}$

J. (a) when x = -1, we get. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n \ln^2}$, apply alternating series test, $\lim_{n \to \infty} \frac{n-1}{n^2} = 0$, and since $\lim_{n \to \infty} \frac{n}{n^2} = 0$, and since $\lim_{n \to \infty} \frac{n}{n^2} = 0$, and since $\lim_{n \to \infty} \frac{n}{n^2} = 0$, and \lim_{n

Then $\sum_{n=1}^{\infty} \frac{n-1}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n} - \beta$, is divergent.

Therefore, R= 1, interval of convergence is [-1,1[.