Confidence Intervals

1a. *[1 mark]*

The times t, in minutes, taken by a random sample of 75 workers of a company to travel to work can be summarized as follows

$\sum t = 2165$, $\sum t^2 = 76475$.

Let T be the random variable that represents the time taken to travel to work by a worker of this company.

Find unbiased estimates of the mean of T.

1b. *[2 marks]*

Find unbiased estimates of the variance of T.

1c. *[3 marks]*

Assuming that T is normally distributed, find

(i) the 90% confidence interval for the mean time taken to travel to work by the workers of this company,

(ii) the 95% confidence interval for the mean time taken to travel to work by the workers of this company.

1d. *[3 marks]*

Before seeing these results the managing director believed that the mean time was 26 minutes.

Explain whether your answers to part (b) support her belief.

2a. *[3 marks]*

It is known that the standard deviation of the heights of men in a certain country is 15.0 cm.

One hundred men from that country, selected at random, had their heights measured.

The mean of this sample was 185 cm. Calculate a 95% confidence interval for the mean height of the population.

2b. *[4 marks]*

A second random sample of size n is taken from the same population. Find the minimum value of n needed for the width of a 95% confidence interval to be less than 3 cm.

3a. *[4 marks]*

A manufacturer of stopwatches employs a large number of people to time the winner of a 100 metre sprint. It is believed that if the true time of the winner is μ seconds, the times recorded are normally distributed with mean μ seconds and standard deviation 0.03 seconds.

The times, in seconds, recorded by six randomly chosen people are

$$9.765,\ 9.811,\ 9.783,\ 9.797,\ 9.804,\ 9.798.$$

Calculate a 99% confidence interval for μ. Give your answer correct to three decimal places.

3b. *[2 marks]*

Interpret the result found in (a).

3c. *[3 marks]*

Find the confidence level of the interval that corresponds to halving the width of the 99% confidence interval. Give your answer as a percentage to the nearest whole number.

4a. *[4 marks]*

A traffic radar records the speed, v kilometres per hour (km h^{-1}), of cars on a section of a road.

The following table shows a summary of the results for a random sample of 1000 cars whose speeds were recorded on a given day.

Speed	Number of cars
$50 \le v < 60$	5
$60 \le v < 70$	13
$70 \le v < 80$	52
$80 \le v < 90$	68
$90 \le v < 100$	98
$100 \le v < 110$	105
$110 \le v < 120$	289
$120 \le v < 130$	142
$130 \le v < 140$	197
$140 \le v < 150$	31

Using the data in the table,

(i) show that an estimate of the mean speed of the sample is 113.21 km h^{-1};

(ii) find an estimate of the variance of the speed of the cars on this section of the road.

4b. *[2 marks]*

Find the 95% confidence interval, I, for the mean speed.

4c. *[2 marks]*

Let J be the 90% confidence interval for the mean speed.

Without calculating J, explain why $J \subset I$.

5. *[15 marks]*

The length of time, T, in months, that a football manager stays in his job before he is removed can be approximately modelled by a normal distribution with population mean μ and population variance σ^2. An independent sample of five values of T is given below.

6.5, 12.4, 18.2, 3.7, 5.4

(a) Given that $\sigma^2 = 9$,

(i) use the above sample to find the 95 % confidence interval for μ, giving the bounds of the interval to two decimal places;

(ii) find the smallest number of values of T that would be required in a sample for the total width of the 90 % confidence interval for μ to be less than 2 months.

(b) If the value of σ^2 is unknown, use the above sample to find the 95 % confidence interval for μ, giving the bounds of the interval to two decimal places.

Confidence Intervals.

1a. $\mu = \bar{t} = E(t) = \frac{\sum t}{n} = \frac{2165}{75} = 28.9.$

1b. $\sigma^2 = S_{n-1}^2 = \frac{n}{n-1} S_n^2 = \frac{75}{74}\left(\frac{76475}{75} - (28.9)^2\right).$

$= \cancel{188.9}\ 189$

1c. $\bar{x} \pm t^* \frac{S}{\sqrt{n}} \Rightarrow [-7.433, 65.233].\ 90\%.$

$\Rightarrow [-14.56, 72.362]\ 95\%.$

1d. Yes. $S >> \mu$.

2a. $\sigma = 15 \quad n = 100. \quad \bar{x} = 185.$

$\bar{x} \pm \frac{\sigma}{\sqrt{n}} \cdot z^*, \quad 95\% \Rightarrow [182, 188].$

2b.

$z^* \frac{\sigma}{\sqrt{n}} \leq 3 \qquad \sqrt{n} \geq \frac{z^* \cdot \sigma}{3} = 9.80.$

$\Rightarrow n \geq 96.03 \Rightarrow n = 97.$

3a. $\mu, \quad \sigma = 0.03.$

$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} \Rightarrow [9.762, 9.825].$

3b 99% of the time. $\bar{x}$ lies in ↓.

3c. 49.5% $\Rightarrow [9.785, 9.801].$

$\Rightarrow$ [97.9‰, 98.0%].

4a. $\bar{x} = 113.21$

$S = 19.0436.$

4b. $[112.03, 114.39].$