M-36 Probability 4 : Venn Diagrams and Abstract Problems

Answer on this sheet. Venn Diagrams will often help!

1. A and B are independent events such that $p(A) = 0.9$ and $p(B) = 0.3$. Find:

   (a) $p(A \cap B')$

   (b) $p((A \cup B)')$

2. For events C and D it is known that:

   $p(C) = 0.7$ $\quad p(C' \cap D') = 0.25$
   $p(D) = 0.2$

   (a) Find $p(C \cap D')$.

   (b) Explain why C and D are not independent events.

3. Given that $p(E') = p(F) = 0.6$; $p(E \cap F) = 0.24$

   (a) Write down p(E).

   (b) Explain why E and F are independent.

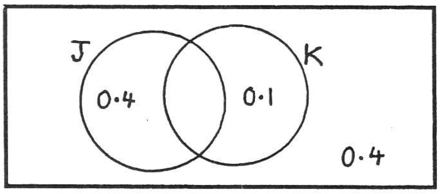
   (c) Explain why E and F are not mutually exclusive.

   (d) Find $p(E \cup F')$.

4. Independent events G and H are such that $p(G \cap H') = 0.12$ and $p(G' \cap H) = 0.42$.

   Make a Venn Diagram, labelling $p(G \cap H) = x$. Hence make an equation and find two possible values of x.

5. In the Venn Diagram below, are events J and K independent? Explain your answer.



6. For events K and L $p(K \cup L) = 2 \times p(K \cap L)$. Given also $(K \cap L') = 0.1$ and $(K' \cap L) = 0.2$ find:

   (a) $p(K \cap L)$

   (b) $p(K')$

7. The probability that in two consecutive trials event M occurs in the first trial and does not occur in the second is $\frac{2}{9}$.
   Given that $p(M) < 0.5$, find the probability that in two consecutive trials event M occurs in the first trial and in the second trial.

M-37 Probability 5 : Conditional Probability

1. **1 2 4 7 11 16 22 29**

   A number is chosen at random from the above list of eight numbers. Find:

   (a) p(it is even | it is not a multiple of 4)

   (b) p(it is less than 15 | it is greater than 5)

   (c) p(it is less than 5 | it is less than 15)

   (d) p(it lies between 10 and 20 | it lies between 5 and 25)

2. By considering the sample space, calculate these conditional probabilities when two unbiased 6-sided dice are thrown:

   (a) p(total is 8 or more | at least one 6 is visible)

   (b) p(at least one die shows a 2 | the total is less than 6)

   (c) p(there are no 5s visible | there are no 6s visible)

   (d) p(both dice show the same number | the total is 12)

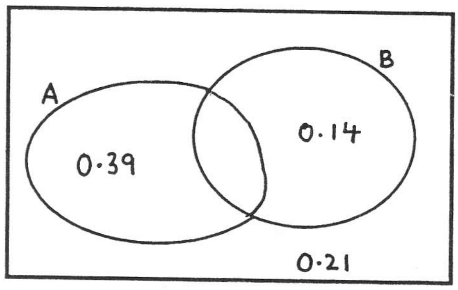
   (e) p(there's exactly one 6 | there are no 1s and 2s)

3. In the Venn Diagram to the right, find

   (a) p(A) (b) p(A | B)

   (c) p(B) (d) p(B | A)

   (e) What single word describes the relationship between events A and B?



4. For events A and B it is known that: $p(A' \cap B') = 0.35$ $p(A) = 0.25$ $p(B) = 0.6$.

   Find: (a) $p(A \cap B)$ (b) $p(A \mid B)$ (c) $p(B' \mid A')$

5. In a bag are 5 red counters and 3 blue counters. A counter is taken and not replaced, then another counter is taken. Using a tree diagram or otherwise find the probability that:

   (a) the second counter is blue

   (b) the first counter was red, given that the second counter is blue.

6. Where I live, the probability of the sun shining on as given day is 0.6. If the sun is shining there is a 0.85 probability that I'll go to the beach. If it's not shining there is a 0.3 probability that I'll go to the beach.

   Yesterday I went to the beach. Find the probability that the sun was shining.

M-38 Probability 6 : Discrete Random Variables and Expectation

1. Find the mean of each discrete random variable X below:

(a)

x	1	2	3
p(X=x)	0.2	0.5	0.3

(b)

x	2	3	4
p(X=x)	0.1	0.8	0.1

(c)

x	-1	1	4	6
p(X=x)	0.4	0.15	0.2	0.25

(d) Here, find k first:

x	1	3	4	6
p(X=x)	2k	0.15	k	0.4

2. (a) In distribution (a) of Q1, find $p(X \neq 3)$.

(b) In distribution (b) of Q1, find $p(X^2 \geq 1)$.

(c) Explain why this table does **not** represent a discrete random variable:

x	1	2	3	4
p(X=x)	0.6	0.1	0.2	0.2

3. A Fibonnacci die is unbiased, six-sided and labelled with these numbers: 1, 2, 3, 5, 8, 13. What is the expected score?

4. Find the value of k in the discrete distribution table below and hence find the mean of the distribution:

x	0	1	3	5
p(X=x)	0.2	k	2k	0.35

5. (a) Copy and complete (in terms of k) this table for a discrete random variable:

x	1	2	3
p(X=x)	0.2	1-k	

(b) What range of values can k take? Give your answer in the form $a \leq k \leq b$ $(a, b \in \mathbb{Q})$.

(c) Find (in terms of k) the mean of the distribution.

6. X is a discrete random variable which can only take the three values X=1, X=2 and X=4.

It is known that p(X=2) = 0.3 and that the mean of the distribution is 2.8.

Find p(X=1).

7. In a simple gambling game you pay €1 to play. You throw two unbiased dice. If your total score is 10 or more you get €4 (and if you don't you lose your €1, of course).

Let X be the discrete random variable "your profit in €".

(a) Copy and complete the table below and find E(X).

x	-1	4
p(X=x)		

(b) If you played the game 1000 times, what would you expect your approximate total profit/loss to be?

8. Two unbiased tetrahedral (four-sided) dice are thrown and the scores are added. Copy and complete this probability distribution table and calculate $p(X > 4)$.

x	2	3	4	5	6	7	8
p(X=x)	$\frac{1}{16}$				$\frac{3}{16}$		

M-39 The Binomial Distribution

1. 10 coins are thrown. Find the probability of getting:

   (a) exactly two heads
   (b) exactly eight heads
   (c) three or more heads
   (d) exactly five heads
   (e) more heads than tails.

2. Five people write down a day of the week at random. Find the probability that:

   (a) nobody writes Sunday
   (b) everbody writes Sunday
   (c) exactly one person writes Sunday
   (d) at least one person writes Sunday.

3. Five six-sided dice are thrown. Find the probability of getting:

   (a) no 6s
   (b) at least one 6
   (c) no 5s or 6s *HINT: the probability of getting a 5 or a 6 is* $\frac{1}{3}$.
   (d) four or more 6s
   (e) four or more of the same number. *HINT: use your answer to (d).*

4. (a) In Q1, write down the mean number of heads.

   (b) In Q2, write down the mean number of people writing Sunday.

   (c) In Q3, write down the mean number of 6s.

5. N coins are thrown. Find the probability that exactly half show tails if:

   (a) $N = 2$ (b) $N = 6$ (c) $N = 20$.

6. An experiment consists of throwing 12 tetrahedral dice (unbiased four-sided dice with faces numbered **1, 2, 3** and **4**). Find the probability of getting the mean number of **3**s.

7. The distribution of digits in the decimal expansion of

   $$\pi = 3.14159265...$$

   appears to be completely random.

   A 10-digit piece of the expansion is selected at random. Find the probability that it contains:

   (a) two or more 7s
   (b) exactly one 9
   (c) five 6s.

8. A bag contains a very large number (millions, say!) of coloured counters. There are twice as many red as there are blue, and twice as many blue as there are yellow.

   If I choose four counters at random, find the probability of getting:

   (a) at least one blue
   (b) all reds
   (c) two or more yellows.

9. A light-bulb manufacturer knows that the probability of any individual bulb being faulty is 0.015. Bulbs are packed in boxes of 12.

   (a) Find the probability that a box chosen at random contains:

   (i) one or more faulty bulb
   (ii) two or more faulty bulbs.

   (b) The manufacturer wants to reduce the probability of any individual bulb being faulty in such a way that the probability of a random box containing one or more faulty bulb falls below 5%. To what value must they reduce this individual probability (from 0.015)?

10. If a certain experiment is repeated 5 times p(at least one success) = 0.8.

    If the experiment is repeated 10 times, find p(at least one success).

M-40 The Normal Distribution 1

1. A fruit grower knows that the weights of his apples are normally distributed with mean 100g and standard deviation 20g.
   Find the probability that the mass of a random apple

   (a) is less than 110g

   (b) is less than 150g

   (c) is more than 70g

   (d) is between 90g and 110g

   (e) is between 84g and 107g

   (f) is between 80g and 85g

2. A machine packs potatoes into bags whose masses are normally distributed with mean 1060g and standard deviation 45g.

   (a) Find the probability that a bag chosen at random has a mass of less than 1kg.

   (b) 200 bags are delivered to a supermarket. How many would you expect to weigh more than 1050g?

   (c) Two bags are chosen at random. What is the probability that both weight between 1020g and 1070g?

3. Percentage scores in a mathematics test are normally distributed with mean 52 and standard deviation 18.

   (a) A score of 83 or more achieves a grade 7. What percentage of scores will receive a grade 7?

   (b) A score between 43 and 53 achieves a grade 4. The International School of Ruritania has 43 candidates for the test. How many candidates would be expected to achieve a grade 4?

   (c) A score of less than 20 receives a grade 1. What percentage of candidates receive a grade 1?

4. In another normally distributed test the mean is 37 points and the standard deviation is 7.5. The pass mark is 27.

   In a certain region 259 students passed the exam. How many would you expect to have failed?

5. X is a continuous random variable and $X \sim N(5, 1)$. Find:

   (a) $p(X > 6.2)$

   (b) $p(4.3 < X < 5.3)$

   (c) $p(|X - 5| \le 1)$

   (d) $p(|X - 4| \le 1)$

6. Some people believe that intelligence is a quantity that can me measured by pencil and paper tests. These tests produce a score called the IQ (Intelligence Quotient). IQs of the population have a mean of 100.

   Assume the above is reasonable (quite a big assumption!) **and** that the standard deviation of the IQs is 24 **and** that IQs are normally distributed.

   (a) What percentage of the population has an IQ between 110 and 135?

   (b) There's an organization of supposedly brainy people called MENSA which only has members whose IQs are 140 or more. What percentage of the population can join MENSA?

   You'll need a GDC for these:

   (c) **CHILD GENIUS HAS IQ OF 200** screams the newspaper headline. Taking the population of the world as 6 billion people, how many will have an IQ of 200 or more.

   (d) Taking the population of the world as 6 billion people, how many will have an IQ of 250 or more.

M-41 The Normal Distribution 2

1. Assume a normal distribution in questions (a) to (c) below.

   (a) The mean height of children of a certain age is 136cm. 12% of children have a height of 145cm or more. Find the standard deviation of the heights.

   (b) The standard deviation of weights of loaves of bread is 20g. Only 1% of loaves weigh less than 500g. Find the mean weight of the loaves.

   (c) The weights of cauliflowers are normally distributed with mean 0.85kg. 74% of cauliflowers have weights less than 1.1kg. Find:

   (i) the standard deviation of cauliflowers' masses,

   (ii) what percentage of cauliflowers have mass greater than 1kg.

   (d) It is suspected that the scores in a test are normally distributed. 30% of students score less than 108 marks on the test, and 20% score more than 154 marks.

   (i) Find the mean and standard deviation of the scores, if they are normally distributed,

   (ii) 60% of students score more than 117 marks. Does this fact appear to be reasonably consistent with the idea that the scores are normally distributed as above?

2. Given $Z \sim N(10, 1)$, find the value of k such that $p(|Z-10| \leq k) = 0.3$.

3. In a certain country the weights of new-born babies are normally distributed. 10% of babies weigh more than 4.2kg. 30% of babies weigh less than 2.8kg. Find:

   (a) the mean and standard deviation of the weights

   (b) the percentage of new-born babies weighing between 3 and 4kg.

4. A company which sells jars of spice knows that the masses of spice in its jars are normally districted with mean 80g and standard deviation 3g.

   (a) The labels on the jars say **Contents weigh 75g**. What percentage of jars will in fact weigh less than 75g?

   (b) The company wishes to ensure than no more than 0.5% of jars are underweight.

   (i) If the mean remains constant at 80g, to what value should it reduce the standard deviation?

   (ii) If the standard deviation remains constant at 3g, to what value should it increase the mean?

5. In a certain normal distribution with mean 60, 35% of all values lie between 50 and 80. Use your GDC to find

   (a) the variance, (b) the percentage of values that are over 100.