# GRAPH REVIEW.

- A vetex w/ deg = 1 is pendant
- Handshaking:

$$\sum_{v \in V} \deg(v) = 2e.$$

- An undirected graph can only have an even # of odd vertices.
- Digraph : directed graph.

$e = (a, b)$. a: initial vertex. b: terminal vertex

$(a, b) = (c, d) \Rightarrow a = c$, $b = d$

$\Rightarrow$ parallel

- In-deg. : $\deg^-(v)$. $v$ as terminal $v$.

out-deg : $\deg^+(v)$ $v$ as initial $v$.

- Complete graph $K_n$.
- complement of $G$ : $G'$.

* $K_n'$ : null graph.

- Bipartite graphs. : a partition exists.

complete bipartite graph : $K_{m,n}$.

- walk : may repeat edge & vertex

trail : no edge appears more than once.

Circuit : trail begin & end at the same vertex.

path : no vertex is visited > 1

cycle : path which --- at same vertex.

- Regular graph : ($r$-regular : $\deg v = r$.).

all vertices have the same degree.

- # of walks of length $n$ from $v_i$ to $v_j$ is given by the $(i,j)$th entry of $A_G^n$.
- simple graph : no multi edge, no loop.
- $G = (V, E)$. simple connected.

$a, b \in G$, not adjacent.

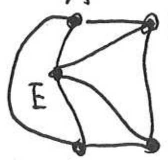
$G + ab = G_1 \Rightarrow G_1$ has a cycle containing $ab$.

- When an edge is removed from a cycle in a connected graph, the result is still connected.

- G : connected
  Eulerian trail : every edge appear only once.
  Eulerian circuit : every edge --- once.
  ↓
  Eulerian graph.

  e.g.


(√)




(×).

- Thrm.
  G connected.
  G has Eulerian circuit $\Leftrightarrow$ every ~~ver~~ vertex has an even deg.
- Thrm
  G connected.
  G ha Eulerian trail but no circuit
  $\Leftrightarrow$ it has exactly 2 vertices of odd deg.
- G : connected.
  Hamiltonian path : path containing all vertices of G.
  Hamiltonian cycle : $\rightarrow$ Hamiltonian graph.



- Dirac's thrm.
  $|V| = n \geq 3$, $\deg(A) \geq \frac{n}{2}$. $\forall A \in V$.
  $\rightarrow$ ~~Hit~~ Hamiltonian cycle.
  $\Rightarrow$ $\deg(A) + \deg(B) \geq n$. A.B non adjacent.
  (generalization).
- if $m \neq n$, G cannot have a Hamiltonian cycle.
  if ~~m~~ m, n differ by 2 or more, no H path
- $v - e + f = 2$.
  proof : $\begin{cases} \text{case 1: no cycle} \\ \text{case 2: has cycle.} \end{cases}$
- $e \leq 3v - 6$. $(2e \geq 3f)$. $\rightarrow$ $K_5$ is nonplanar
- if no circuit of lengths 3.
  $e \leq 2v - 4$. $(2e \geq 4f)$ $\rightarrow$ $K_{3,3}$ is nonplanar.
- Homeomorphic :
  can be obtained ~~by~~ from the same graph by elementary subdivisions.
- (Kuratowski's thrm)
  non-planar $\Leftrightarrow$ a subgraph homeo to $K_5$ / $K_{3,3}$.

$\rightarrow$ Continued.

- G: simple. connected.

  $$v-1 \le e \le \frac{v(v-1)}{2}$$

- In a simple, connected graph.
  there are $\ge 2$ vertices of same deg.
  (pf by PHP).

- Any subgraph of a bipartite graph is bipartite.

- T. $|V| \ge 2$. $\rightarrow$ $\binom{n}{2}$ different paths in T.

# TREES & ALGORITHMS REVIEW

- T: connected, simple, no cycles.
- T is a tree $\Leftrightarrow$ $\exists$ unique, simple, path btw any pair of $v$.
- rooted tree.
  - w/ children : <u>internal vertices</u>
  - w/o chidren : <u>leaf</u>. (deg = 1).
- T $e = v - 1$ (induction)
- <u>Spanning tree</u> (H)
  - G: connected.
  - $H \leq G$. H contains every $v$ in G.
- Every connected graph has a spanning tree. (pf by moving edges in a cycle).
- To Find a spanning tree:
  - edge Removal.
  - edge Addition