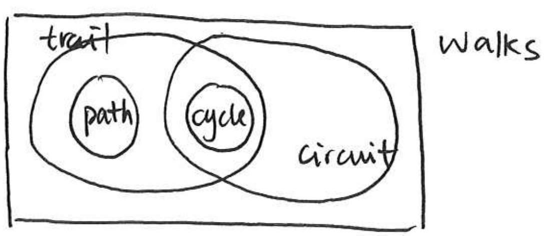
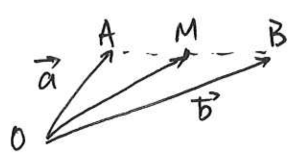
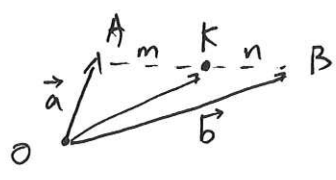
- Equivalence Relation:
  reflexive, symmmetric, transitive.
- $(x^{-1} * y)^{-1} = y^{-1} * x$.
- Absolute convergence → convergence.
  can be used while determining the interval of conv.
- bijection
  - injection : $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
  - surjection : $\exists x \ \forall f(x)$.
- right coset : $\{ha \mid h \in H\}$
- $\lim_{n \to \infty} \frac{\sin x}{x} = 1$
- L'Hopital's rule applies only when $\frac{f(x)}{g(x)}$ is in indeterminate form.
- Integral test:
  f cont, ↓, positive. $\sum_{n=1}^{\infty} a_n$ conv. $\Leftrightarrow \int_1^{\infty} f(x)dx$ conv.
- For a conv. alternating series, $|R_n| = |S - S_n| \leq a_{n+1}$.
- Taylor series: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!} (x-c)^n$. $\xrightarrow{c=0}$ MacLaurin.
  ↑ $n \to \infty$
  Taylor poly. and $f(x) = P(x) + R(x)$. $R_n(x) = \frac{f^{(n+1)}(b)}{(n+1)!} (x-c)^{n+1}$.
  where b is btw x and c, inclusive.
- Eulerian : edge.
  Hamiltonian : vertex.
- Eulerian
  - trail : has exactly 2 vertices w/ odd deg.
  - circut : $\deg(v) \equiv 0 \pmod 2 \quad \forall v \in V$.

- In a simple, connected graph,
  (1) $v-1 \le e \le \frac{v(v-1)}{2}$
  (2) there are ≥ 2 vertices of same deg. (PHP)
- Any subgraph of a bipartite graph is bipartite.
- T. $|V| \geq 2 \longrightarrow \binom{n}{2}$ different paths in T, because $\exists!$ simple path btw any pair of v.
- Ceva: 交于一点, $=1$
  Menelaus: 一条线, $=-1$.
- $\det(AB) = \det(A)\det(B)$
- $\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du.$
- Euler's relation $v-e+f=2$.
- G: connected, simple, planar, $e \le 3v-6$.
  if no circuits of length 3, $e \le 2v-4$.
- Bipartite: no cycle of length 3.
- 

  walk (v+e) — trail (v) → circuit
  walk (v+e) — path (/) → cycle.

- 

  $\vec{OM} = \frac{\vec{a}+\vec{b}}{2}$

  $\vec{OK} = \frac{m}{m+n}\vec{b} + \frac{n}{m+n}\vec{a}$

- $B \setminus C = B \cap C'$
- $\lim_{x \to 0} \left(1+\frac{1}{x}\right)^x = e$

- $\circlearrowleft \theta$: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

  reflection in line $y = (\tan\theta)x$. : $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$.

- $X \sim B(n, p)$:

  $P(X = x) = \binom{n}{x} p^x q^{n-x}$. ; $G(t) = (q + pt)^n$.

  $E(x) = G'(1) = np$. ; $Var(x) = G''(1) + G'(1) - [G'(1)]^2$
  $= npq$.

- $X \sim Geo(p)$:

  $P(X = x) = q^{x-1} p$; $G(t) = \dfrac{pt}{1 - qt}$

  $E(x) = G'(1) = \dfrac{1}{p}$; $Var(x) = \dfrac{q}{p^2}$.

- $X \sim NB(r, p)$:

  $P(X = x) = \binom{x-1}{r-1} p^r q^{x-r}$;

  $G(t) = p^r t^r (1 - qt)^{-r}$;

  $E(x) = G'(1) = \dfrac{r}{p}$. ; $Var(x) = \dfrac{rq}{p^2}$.

- $X \sim Po(\mu)$:

  $P(X = x) = \dfrac{e^{-\mu} \mu^x}{x!}$

  $E(x) = \mu$; $Var(x) = \mu$.

- pgf for $X + Y$ is $G(t) \cdot H(t)$

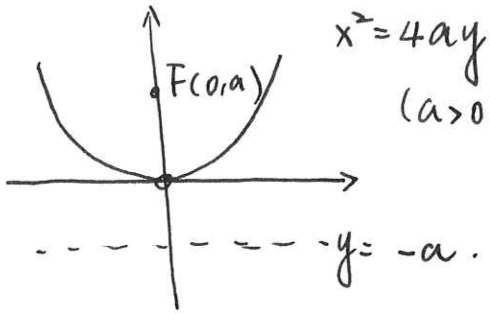
  If $X \sim Po(\mu_x)$, $Y \sim Po(\mu_y)$, $X + Y \sim Po(\mu_x + \mu_y)$.

- $E(X + Y) = E(X) + E(Y)$.

  If independent, $E(XY) = E(X)E(Y)$.

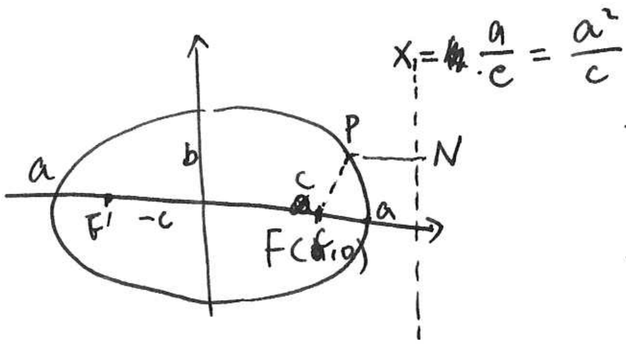
  $Var(X + Y) = Var(X) + Var(Y)$.

CONIC SECTIONS.



$x^2 = 4ay.$

$(a > 0).$

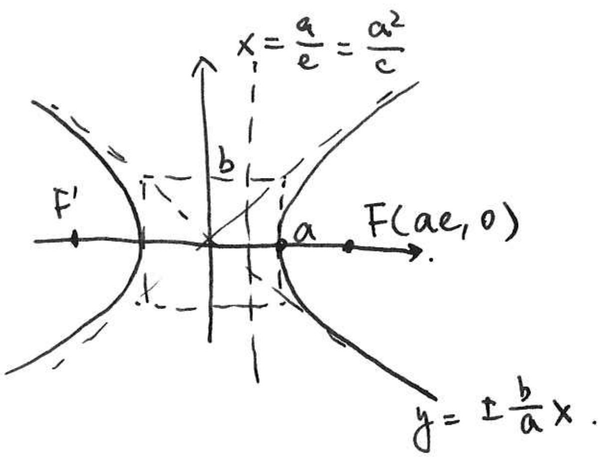


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

$PF + PF' = 2a.$

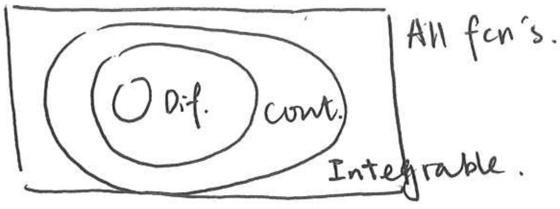
$$\frac{PF}{PN} = e = \frac{c}{a}$$

$b^2 = a^2(1-e^2)$, $c = ea.$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

$|PF - PF'| = 2a.$

- 

- $$C_n = \frac{f^{(n)}(a)}{n!}$$

- $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$ where x_i^* is any # chosen from the interval $[x_{i-1}, x_i]$.

- $$\frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

- Δ ellipse < 0
  Δ parabola $= 0$
  Δ hyperbola > 0.

  $\Delta = b^2 - 4ac$.