Setup. Consider a two-period setting. A household owns a property with initial value $V > 0$ and has income y. In the second period the property suffers a proportional depreciation shock $\delta \in [0, 1]$ distributed according to F. The household has increasing and concave utility $U(\cdot)$ over terminal wealth.

An insurance contract is (p, δ^*), where $p \geq 0$ is the premium and $\delta^* \in [0, 1]$ is a threshold (franchise/trigger level). Indemnity is

$$c(\delta; \delta^*) = \begin{cases} 0, & \delta < \delta^*, \\ \delta V, & \delta \geq \delta^*, \end{cases}$$

so the contract fully covers the realized loss when δ exceeds the threshold.

Insurance market is competitive and insurers are risk-neutral, which implies actuarially fair pricing (zero expected profit).

Insurer profit and zero-profit premium schedule. Under (p, δ^*), the insurer receives p and pays $c(\delta; \delta^*)$. Expected profit is

$$\begin{aligned} \Omega(p, \delta^*) &= \int_0^{\delta^*} p \, dF(\delta) + \int_{\delta^*}^1 \big(p - \delta V\big) \, dF(\delta) \\ &= p - V \int_{\delta^*}^1 \delta \, dF(\delta). \end{aligned}$$

Competition implies $\Omega(p, \delta^*) = 0$, hence

$$p(\delta^*) = V \int_{\delta^*}^1 \delta \, dF(\delta) = V \, \mathbb{E}[\delta \, \mathbf{1}\{\delta \geq \delta^*\}] \,. \tag{1}$$

Household terminal wealth. Given (p, δ^*), terminal wealth is

$$W(\delta; \delta^*, p) = \begin{cases} y - p + V - \delta V, & \delta < \delta^*, \\ y - p + V, & \delta \geq \delta^*. \end{cases}$$

Impose $p = p(\delta^*)$ from (1), and write $W(\delta; \delta^*) \equiv W(\delta; \delta^*, p(\delta^*))$.

Household problem. The household chooses $\delta^* \in [0, 1]$ (equivalently chooses (p, δ^*) subject to (1)) to maximize expected utility:

$$\max_{\delta^* \in [0,1]} \; \mathbb{E}\big[U\big(W(\delta; \delta^*)\big)\big] \,. \tag{2}$$

Optimality of full insurance. Consider $\delta^* = 0$. Since $\delta \in [0,1]$, The indemnity is paid in every state and terminal wealth is constant:

$$W(\delta; 0) = y - p(0) + V, \qquad p(0) = V \int_0^1 \delta \, dF(\delta) = V \, \mathbb{E}[\delta].$$

Define

$$W_0 \equiv y + V - p(0) = y + V - V \, \mathbb{E}[\delta]. \tag{3}$$

Fix any $\delta^* \in (0,1]$ and define

$$K(\delta^*) \equiv p(0) - p(\delta^*) = V \int_0^{\delta^*} \delta \, dF(\delta).$$

which stands for the extra premium paid when changing the deductible from δ^* to 0.

Using (1) and (3), wealth under δ^* can be rewritten as

$$\begin{aligned} W(\delta; \delta^*) &= \begin{cases} y + V - p(\delta^*) - \delta V, & \delta < \delta^*, \\ y + V - p(\delta^*), & \delta \geq \delta^*, \end{cases} \\ &= \big(y + V - p(0)\big) + \big(p(0) - p(\delta^*)\big) - \delta V \mathbf{1}\{\delta < \delta^*\} \\ &= W_0 + \varepsilon(\delta; \delta^*), \end{aligned}$$

where

$$\varepsilon(\delta; \delta^*) \equiv K(\delta^*) - \delta V \mathbf{1}\{\delta < \delta^*\}. \tag{4}$$

Its expectation is

$$\mathbb{E}[\varepsilon(\delta; \delta^*)] = K(\delta^*) - V \int_0^{\delta^*} \delta \, dF(\delta) = 0.$$

Thus $\mathbb{E}[W(\delta; \delta^*)] = W_0$ for each δ^*.

Since U is concave, Jensen's inequality yields

$$\mathbb{E}\big[U\big(W_0 + \varepsilon\big)\big] \leq U(W_0 + \mathbb{E}[\varepsilon]) = U(W_0) = \mathbb{E}[U(W(\delta; 0))].$$

Therefore, for all $\delta^* \in [0,1]$,

$$\mathbb{E}\big[U\big(W(\delta; \delta^*)\big)\big] \leq \mathbb{E}\big[U\big(W(\delta; 0)\big)\big] \,. \tag{5}$$

Hence $\delta^* = 0$ is optimal.

If U is strictly concave and F assigns positive probability to $(0, \delta^*)$, then

Jensen is strict for any $\delta^* > 0$, implying uniqueness:

$$\delta^*_{opt} = 0, \qquad p_{opt} = p(0) = V\,\mathbb{E}[\delta].$$

Interpretation. Under competitive, risk-neutral, zero-profit pricing, the premium equals expected indemnity (actuarially fair). Moving from any $\delta^* > 0$ to $\delta^* = 0$ replaces a risky terminal-wealth distribution with a constant at the same mean. By concavity of U, full insurance is optimal.