

- ① After considering 10 images of chessboard.  
Now let us consider a image from the 10 images and mark the pixel coordinates and as well as the world co-ordinates of any 6 points on the chess board.

Where, the world coordinates are represented in the format of  $(x_w, y_w, z_w)$  where, the depict the following co-ordinates from the images selected.

$(4, 2.2) (5, 3.2) (6, 3.1) (4, 3.2) (5, 4.0) (6, 4.3)$

Now, considering the pixel coordinates from the images selected they can be represented as following format of  $(x_i, y_i)$

$\hookrightarrow (255, 180) (320, 130) (420, 160) (310, 115) (290, 155) (310, 200)$

Where, the multiplication can be performed among the represented co-ordinates is depicted as below:-

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

After performing the above operation this can result as  $\Rightarrow$

$$x_i = \frac{m_{11}x_w + m_{12}y_w + m_{13}z_w + m_{14}}{m_{31}x_w + m_{32}y_w + m_{33}z_w + m_{34}} \rightarrow \textcircled{1}$$

$$y_i = \frac{m_{21}x_w + m_{22}y_w + m_{23}z_w + m_{24}}{m_{31}x_w + m_{32}y_w + m_{33}z_w + m_{34}} \rightarrow (2)$$

i.e

$$\rightarrow x_w m_{11} + y_w m_{12} + z_w m_{13} + m_{14} - x_i [x_w m_{31} + y_w m_{32} + z_w m_{33} + m_{34}] = 0$$

$$\rightarrow x_w m_{21} + y_w m_{22} + z_w m_{23} + m_{24} - y_i [x_w m_{31} + y_w m_{32} + z_w m_{33} + m_{34}] = 0$$

Now, representation of the following in a matrix format.

$$\begin{bmatrix} x_w & y_w & z_w & 1 & 0 & 0 & 0 & 0 & -x_c x'_w \\ 0 & 0 & 0 & 0 & x_w & y_w & z_w & 1 & -y_c x_w \end{bmatrix}$$

Writing the equations for the '6' points

$$\begin{bmatrix} 4 & 2.2 & 0 & 1 & 0 & 0 & 0 & 0 & -1024 & -588.8 & 0 & -255 \\ 0 & 0 & 0 & 0 & 4 & 2.2 & 0 & 1 & -736 & -423.2 & 0 & -180 \\ 5 & 3.2 & 0 & 1 & 0 & 0 & 0 & 0 & -163 & -1001.6 & 0 & -320 \\ 0 & 0 & 0 & 0 & 5 & 3.2 & 0 & 1 & -655 & -406.1 & 0 & -130 \\ 6 & 3.1 & 0 & 1 & 0 & 0 & 0 & 0 & -2532 & -266 & 0 & -420 \\ 0 & 0 & 0 & 0 & 6 & 3.1 & 0 & 1 & -978 & -489 & 0 & -160 \\ 4 & 3.2 & 0 & 1 & 0 & 0 & 0 & 0 & -1248 & -1060.8 & 0 & -310 \\ 0 & 0 & 0 & 0 & 4 & 3.2 & 0 & 1 & -464 & -394.4 & 0 & -115 \\ 5 & 4.0 & 0 & 1 & 0 & 0 & 0 & 0 & -1445 & -1184.9 & 0 & -290 \\ 0 & 0 & 0 & 0 & 5 & 4.0 & 0 & 1 & -775 & -633.5 & 0 & -155 \\ 6 & 4.3 & 0 & 1 & 0 & 0 & 0 & 0 & -1860 & -133.3 & 0 & -310 \\ 0 & 0 & 0 & 0 & 6 & 4.3 & 0 & 1 & -1212 & -868.3 & 0 & -200 \end{bmatrix}$$



let,

A = Matrix of all known values

Eigen vector corresponding to the smallest eigen value of  $A \cdot A^T$  is Matrix 'M'

i.e Smallest & Least Eigen Value of

$$A \cdot A^T \Rightarrow \lambda = 0.001$$

i.e

$$\begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{bmatrix}$$

i.e

then,

$$= 0.0$$

$$m = \begin{bmatrix} -0.00 & -0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.0015 & 0.0245 & 3.111 \end{bmatrix}$$

QR representation of the Matrix 'M' gives the intrinsic & extrinsic parameters.

QR decomposition of 'm'

$$\text{Intrinsic Matrix} = \begin{bmatrix} 0.413 & -0.02 & 0.10 \\ 0 & 0.532 & -0.07 \\ 0 & 0 & 0.64 \end{bmatrix}$$

$$\text{Extrinsic Matrix} = \begin{bmatrix} -0.34 & 0.25 & 0.4 \\ 0.40 & -0.37 & 0.03 \\ 0.22 & 0.5 & 0.52 \end{bmatrix}$$

Considering, the Pixel density of camera ( $m_x$ ) = 0.175  
 ↳ Comparing the derived Intrinsic matrix with

$$\begin{bmatrix} m.f & 0 & 0.23 \\ 0 & m.f & -0.18 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow m.f = 0.52$$

i.e  $f = \frac{0.52}{0.175} \text{ cm}$

∴ focal length ( $f$ ) = 29.7 cm

i.e Principal point  $T(P_x, P_y) = (0.23, -0.18)$

2. While we select a pair of images as a source and destination.

Where we select a four points from the source image & corresponding from the destination image.

i.e

	P	Q	R	S
Source	(2, 2)	(7, 6)	(9, 3)	(10, 5)

Destination	(1, 2)	(3, 6)	(8, 4)	(5, 3)
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As we know,

$$\underbrace{\begin{bmatrix} x_s y_s & 1 & 0 & 0 & 0 & -x_d x_s & -x_d y_s & -x_d \\ 0 & 0 & 0 & x_s y_s & 1 & -y_d x_s & -y_d y_s & -y_d \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_s y_s & 1 & 0 & 0 & 0 & -x_d x_s & -x_d y_s & -x_d \\ 0 & 0 & 0 & x_s y_s & 1 & -y_d x_s & -y_d y_s & -y_d \end{bmatrix}}_A = \underbrace{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}_H$$



$$AH = 0$$

Where, 'H' is a Homography matrix.

Now, if we simplify the above 'A' with the help of the corresponding X, Y coordinates Eigen vector of smallest eigen value of  $A^T A$  gives the homography matrix 'H'.

$$A = \begin{bmatrix} 4 & 2 & 1 & 0 & 0 & 0 & -4 & -2 & -1 \\ 0 & 0 & 0 & 4 & 2 & 1 & -8 & -4 & -2 \\ 7 & 6 & 1 & 0 & 0 & 0 & -21 & -18 & -3 \\ 0 & 0 & 0 & 7 & 6 & 1 & -42 & -36 & -6 \\ 9 & 3 & 1 & 0 & 0 & 0 & -72 & -24 & -8 \\ 0 & 0 & 0 & 9 & 3 & 1 & -24 & -12 & -4 \\ 10 & 5 & 1 & 0 & 0 & 0 & -50 & -25 & -5 \\ 0 & 0 & 0 & 10 & 5 & 1 & -15 & -15 & -3 \end{bmatrix}$$

When  $A \cdot A^T$  is calculated

$$A^T = \begin{bmatrix} 4 & 0 & 7 & 0 & 9 & 0 & 10 & 0 \\ 2 & 0 & 6 & 0 & 3 & 0 & 5 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 4 & 0 & 7 & 0 & 9 & 0 & 10 \\ 0 & 2 & 0 & 6 & 0 & 3 & 0 & 5 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ -4 & -8 & -21 & -42 & -72 & -24 & -50 & -15 \\ -2 & -4 & -18 & -36 & -24 & -12 & -25 & -15 \\ -1 & -2 & -3 & -6 & -8 & -4 & -5 & -3 \end{bmatrix}$$

$$A \cdot A^T = \begin{bmatrix} 42 & 42 & 164 & 246 & 387 & 124 & 306 & 93 \\ 42 & 105 & 246 & 533 & 688 & 291 & 520 & 237 \\ 164 & 246 & 860 & 1548 & 2050 & 732 & 1616 & 591 \\ 246 & 533 & 1548 & 3182 & 3936 & 1546 & 3030 & 1289 \\ 387 & 688 & 2050 & 3936 & 5915 & 2048 & 4346 & 1464 \\ 124 & 291 & 732 & 1546 & 2048 & 827 & 1520 & 650 \\ 306 & 570 & 1616 & 3030 & 4346 & 1520 & 3276 & 1140 \\ 93 & 237 & 591 & 1289 & 1464 & 658 & 1140 & 555 \end{bmatrix}$$

The Calculated least eigen value of  $A \cdot A^T$  is  
 $\lambda = 0.002$  # The eigen vector correspondingly  
 to the calculated eigen value  $\lambda = 0.002$  is

$$[0.431, 0.427, -0.641, -0.473, -0.710, -0.57, 0.99, 2, 0]$$

$$\text{Homography Matrix (H)} = \begin{bmatrix} 0.431 & 0.427 & -0.641 \\ -0.473 & -0.710 & -0.57 \\ 0.99 & 1 & 0 \end{bmatrix}$$