Understanding ACF and PACF in ARIMA Model

ACF (Autocorrelation Function) and PACF (Partial Autocorrelation Function) are key tools in determining the terms of an ARIMA model. These functions help identify the autoregressive (AR) and moving average (MA) components, allowing for proper model selection.

1. Pure AR Process

In a pure autoregressive process (AR(p)), each value depends on its own past values. The number of lags (p) dictates how many terms are included.

Example: AR(2)

$X_t = 0.6X_{t-1} - 0.3X_{t-2} + \varepsilon_t$
Where ε_t is white noise.

ACF Behavior:
- The ACF decays gradually and smoothly. For AR processes, the correlation propagates indefinitely over time but diminishes as lag increases.

PACF Behavior:
- The PACF cuts off sharply after lag p = 2, indicating the number of lags included in the AR process.

2. Pure MA Process

In a pure moving average process (MA(q)), each value depends only on past errors. The number of lags (q) defines the number of error terms considered.

Example: MA(2)

$X_t = \varepsilon_t + 0.5\varepsilon_{t-1} - 0.3\varepsilon_{t-2}$

ACF Behavior:
- The ACF cuts off sharply after lag q = 2, reflecting the maximum lagged error term in the model.

PACF Behavior:
- The PACF decays gradually and smoothly, with correlations diminishing as the lag increases.

3. Mixed ARMA Process

In a mixed process (ARMA(p, q)), values depend on both past values (AR terms) and past errors (MA terms).

Example: ARMA(1, 1)

$X_t = 0.7X_{t-1} + \varepsilon_t + 0.4\varepsilon_{t-1}$

ACF Behavior:
- The ACF shows a mix of behaviors. It may decay gradually or exhibit a sinusoidal pattern, depending on the combined AR and MA effects.

PACF Behavior:
- The PACF also decays gradually, reflecting the influence of both AR and MA components.

Summary of ACF and PACF Patterns

Process	ACF Pattern	PACF Pattern
Pure AR Process	Gradual decay	Sharp cutoff after p
Pure MA Process	Sharp cutoff after q	Gradual decay
Mixed ARMA Process	Gradual decay	Gradual decay