Personal Statement

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Before my arrival at Fudan University, I had already delved into substantial topics in elementary mathematics and participated in the National High School Mathematics League. However, the scope of high school courses left me with only a limited understanding of proof-based mathematics. Although I wasn't enrolled in the mathematics department during my first year, the engineering faculty required students to take Mathematical Analysis instead of just Calculus, providing a formal introduction to proofs and logical reasoning, which filled the gap left by my high school education and sparked my interest in modern mathematics. I began spending hours in the library exploring various mathematical fields, and when I encountered commutative diagrams, I was struck by their simplicity compared to circuit diagrams. This naive comparison made me realize that mathematics is a powerful tool for understanding the world, where everything, from numbers to shapes, can be interpreted through its lens.

Motivated by this newfound interest, I read *Algebra: Chapter 0* by P. Aluffi during the first winter vacation of my undergraduate studies. The interplay between elementary algebra and category theory, and the sublimation from "concrete nonsense" to "abstract nonsense", deepened my appreciation of the beauty of mathematics. As I worked through proofs and explored structures, I began to realize that I had only scratched the surface of the vast mathematical landscape. Although I was uncertain at the start of my undergraduate journey, my experience with algebra left me with no doubt that I want to dedicate my life to studying and researching mathematics. Consequently, I decided to switch my major to mathematics.

To prepare for this transition, I tackled the Mathematical Analysis notes by Ping Yu from Tsinghua University, which included challenging French-style exercises covering topics such as the Banach-Mazur game, stationary phase methods, and the Takagi curve. I dedicated four to six hours a day to working through these stimulating exercises. Throughout these exercises, the most exciting experience was unfolding an idea and exploring all its consequences, both foreseeable and unforeseeable. Nobody would guess the full intricacies of any field of mathematics from the outset, and counterintuitive results are commonplace as well as instructive. This experience significantly improved my mathematical maturity. Additionally, I took two advanced courses in Complex Analysis and Topology during my second semester, alongside sophomores in the mathematics department. Had I not been proficient in calculus and linear algebra, passed the rigorous and competitive test to switch majors mid-semester, or managed to juggle the eight courses I was taking that semester, my mathematical journey could have ended there.

Fortunately, fate was on my side. One of the professors on the admissions panel was also my Complex Analysis instructor, and she knew me well from my frequent questions in class. Another professor, whose research interests align with harmonic analysis, asked about my favorite theorem, and when I mentioned the stationary phase method, it caught his attention. My strong performance in the interview outweighed my GPA decline and the written test results, and I was admitted to the mathematics department as one of the top 20 students out of over applicants. I continued to engage in academic activities, such as honors courses, reading programs, and postgraduate seminars, which broadened my perspective beyond what I had imagined. Although my GPA was lower than some of my peers due to my precocious focus on advanced topics, I prioritized these courses over the courses I had taught myself.

Initially, my motivation stemmed from formal analysis techniques and beautiful algebraic structures, but over time, my interest in geometry and topology grew. My first exposure to geometry came through Ping Yu's analysis lecture notes, which introduced multivariable calculus via the language of differential manifolds and reformulated theorems such as the implicit function theorem and the Lagrange multiplier method. I became fascinated by geometric and topological objects such as smooth manifolds and CW complexes. As my studies progressed, I encountered a broader landscape of mathematics, where analysis focuses on concrete computations, algebra on abstract structures, while geometry and topology strike a delicate balance between the two. This landscape incorporates techniques from various fields and reveals the potential for unification.

In my sophomore year, I studied algebraic geometry as an extension of my prior algebraic background. By the end of the year, I had delved into differential topology and gauge theory, where I explored the Atiyah-Singer index theorem. I continued exploring these topics, particularly in lower-dimensional aspects, through the topological summer school at Tsinghua University. During this time, I also developed an interest in homological mirror symmetry (HMS) and studied symplectic topology. By my junior year, I found the physical intuitions behind these topics intriguing and studied HMS in greater depth through A-side enumerative geometry. In my senior year, I broadened my focus to number theory and representation theory, particularly their interactions with geometry.

For a long time, I struggled to choose a specific geometric or non-geometric branch of mathematics. However, when I discovered that geometric Langlands integrates much of the knowledge I had accumulated over the past three years, my perspective shifted. However, this discovery did not limit my interest in representation theory; rather, I intend to continue exploring geometric and topological theories, as they are intrinsically interconnected.

My desire to pursue the dream of unification has deepened my appreciation for the works of Langlands, Atiyah, and Kontsevich, whose insights illuminate relationships between diverse mathematical fields. I am also inspired by the profound understanding of structure in the works of Grothendieck, Drinfeld, and Lurie. Both of their works reflect the "l'honneur de l'esprit humain" and continue to drive my exploration of the mathematical world. Inspired by their ideas and insights, I have developed the habit of thinking independently while reading, often attempting to develop ideas on my own, even in well-established areas. This echoes Grothendieck's experience from *Récoltes et Semailles*, where he emphasized the importance of "staying alone" to independently develop and understand theories.

For instance, this month, I had several interesting independent ideas. When studying the correspondence between local systems and representations of the fundamental group, I proposed a proof by viewing it as a continuous limit of the correspondence between Deck transformations and fundamental groups, which coincided with the standard one. I also suggested Beilinson's gluing construction for higher-dimensional cases when first encountering the perversity of sheaves on a curve. Additionally, in a lecture on Witten's approach to the Atiyah-Singer index theorem using supersymmetry, I realized that adding a Pfaffian to the Wiener measure might suggest a stochastic interpretation immediately. These independent reflections, whether rigorous or not, inspired me to pursue more profound and serious mathematics.

My profound interest was further sparked by attending a series of seminars hosted by professors and colleagues in my department. Topics such as p-adic Hodge theory, combinatorial Hodge theory, and microlocal sheaves introduced me to new theories that greatly enhanced my understanding, although they did not directly align with my main interests. To date, I have attended or organized over 10 seminars led by professors, and I have traveled to 3 summer schools and conferences with competitive application processes for limited housing waivers. These experiences allowed me to engage with the latest developments in geometry and topology and collaborate with colleagues.

One of the most impactful academic experiences for me was the reading program with Professor Yang Zhou. After studying the fundamentals of algebraic geometry and a graduate course on moduli spaces and stacks, I worked with him on intersection theory and modern enumerative geometry. We explored Fulton's *intersection theory* and *3264 and All That* by Griffith and Harris, and key papers in enumerative geometry, including Vistoli's work on intersection theory on stacks and Behrand and Fantechi's *intrinsic normal cone* on the construction of virtual fundamental classes. They were presented in weekly two-hour sessions throughout the semester, where I constructed the deformation to the normal cone and computed interesting examples, such as the Chow ring of the Grassmannian using Young diagrams, highlighting their analogy with topological theories. The professor's guidance was invaluable, for instance, when I was struggling with the computation of Chern classes, he advised me to view the splitting principle as analogous to a "Galois extension" of the Chow ring, which helped me understand how to split the Chern polynomial.

Beyond lectures, I regularly engage in reading mathematical textbooks, lecture notes, surveys, and conference papers, and enjoy

informal discussions with my peers. I am equally committed to fostering productive relationships with mentors and colleagues, as well as engaging more actively in academic activities to facilitate collaboration. These thoughts and studies around mathematics have become a habit, a part of my life, which I can no longer separate. I also appreciate other scientific fields, starting with physics, which is closely tied to mathematics, and extending to various other disciplines. I view mathematics as the "infinitesimal" scale, with all other fields serving as approximations on larger scales. Their relationship is fractal-like, with mathematics appearing at all levels in self-similar forms. My other interests, unrelated to science, include music, history, languages, culture, and regular exercise. I find these activities helpful for managing stress and maintaining my physical well-being.

I am eager to further develop my skills and maturity within a rigorous, research-driven community, surrounded by motivated peers and distinguished faculty. I am confident that pursuing a master program will not only challenge me intellectually but also prepare me for the rigorous work required to reach a PhD level. I sincerely hope the admissions committee will find my application suitable for acceptance into the graduate program.