Personal Statement

Yifu Zhou

August 30, 2025

1 Introduction

This is my personal statement concluding my interest in geometry and topology up to junior and anticipating an increasing deeper understanding in later studies.

Modern geometry and topology have grown to such a large scale with various faces now, these "faces" concern various settings differing with each other greatly, calling for quite different techniques. Hence this PS is for explaining my taste to which faces I prefer, but let's discuss a general philosophy for studying geometry next in a not formal style.

Hence, this introduction is based only on my memory and impression on my mind, it's more close to a story than an article.
Another reason is that I should admit that my foundations of some particular fields are not solid due to some lack in my undergraduate studies, which I'd like to fill these big frameworks from various programs during my postgraduate studies next.
Hence, I apologize for some vague or false places in this personal statement.

Different settings come from different additional structures and require methods at different levels of fineness to extract enough data to study them. More additional structures will admit more invariants, which produce more interesting geometry: these structures provide more rigidity in order to detour the difficulty of classification under a too coarse identification, and hence give richer invariants by algebraic or analysis frameworks. These additional structures are motivated by two reasons: perusing for classification in a relatively easier sense, and handling a naturally limited class of geometric objects from different areas. But these so-called "two" reasons are nothing more than your opinion on what is artificial and what is natural.

If we travel through the history of mathematics, then invariants keep track of its development for detecting different isomorphic classes, but later the so-called "invariants" are not numbers or polynomials anymore but lifted into categorical level or factor through some moduli spaces arose from original geometric object. At the same time, the demand of how fine we need the invariant to be, for

example, detecting the unknot, is lifted up to the demand of how abundant the geometry of the moduli and the algebraic structure of category are. But now it's still mysterious for mathematicians to determine whether such a given exquisite structure, which may come from physics or by "magic", suffice to reflect back to original geometry and topology or not.
One more thing worth mentioning is that these two intermediate structures are related with each other closely: one has the moduli space of objects of a given category with additional geometric structure and the groupoid of a geometric object.

Following the trace of the development of geometry, I'll list the "faces" I'm interested, learned or planned to learn later, which will be discussed in depth next, each costs one section. Here we just give some discussion far from rigorous, but state more precise mathematical settings and concrete problems I interested in the next sections.

Low-dimensional topology:
Algebraic topology studies the most general geometry objects–topological spaces, but there is no doubt that we need to add restrictions to it to detour many pathological examples, hence we equip the space with a "good model" to do homological/homotopical algebra, always simplicial. (The Model category may be a parallel trial, but to be honest, I know few about homotopical theory.) These primary invariants motivate the Poincare conjecture, which is our main line in this story, asking how these invariants can determine the geometric object itself if we focus on some kinds of manifold, i.e. how fine these invariants can be. In the view of homotopical theory, manifolds have good cover by geodesic convex neighborhoods, then we have its nerves including all simplicial data. Furthermore, if we want an analogue for CW decomposition, we have handle body decomposition for manifolds.

The Poincare conjecture states that: if a closed n manifold is homotopical to the n sphere, then it's isomorphic to the n sphere. Here we refer to the word "manifold" as topological or smooth, and then the "isomorphic" is referred to as homeomorphic or diffeomorphic, respectively.
It took more than a century to solve the topological case, but the most surprising thing was the first step around the 1960s: the h-corbordism (or Whitney trick) in dimension greater than 5 works directly prove the Poincare conjecture in these cases, showing an unexpected flexibility in higher dimensional geometric topology, one way to cancel such triviality is adding more structures.
The four-dimensional topological h-corbordism was proven later in 1982 by Freedman, but, as I said above, adding the smooth structure will break this, as smooth structures on a given manifold can be too "wild". For example, the four-dimensional Euclidean space admits infinite smooth structures. Hence, the smooth four-dimensional Poincare conjecture is still a mystery.
As for the three-dimensional case, due to a classical theorem saying that any three-dimensional manifold admits unique smooth structure, the story of topo-

logical and smooth category are equivalent and both stopped at 2003, when Perelman proved Thurston geometrization theorem via geometric analysis, more precisely, the Ricci flow, which is not my interest yet.

What attracts me most are the concrete surgeries in this story, which occur everywhere and serve as a main technique. They're full of geometric insight but may tightening by analysis languages, including moves of knots, handle decompositions or Kirby calculus, and so on. These fundamental blocks make up the topological toolbox, which would realize a change from "protagonist" to "cameo" in the next story.
An interesting thing is that, the rigidity given more to the geometric category, admissible surgeries are less due to more structures need to be preserved, for example, in symplectic category we can't do cutting and gluing so rashly but we can use Luttinger surgery; in algebraic or complex analytic category surgery is even more impossible but only blowing up can be used. Hence, less so-called "geometric insight" via flexible operations can be found under more rigidity.

The so-called "rigidity" and "softness" are nothing more than the small and large scales of automorphism groups. For example, the isometry group is usually finite-dimensional, or even zero-dimensional, i.e. discrete; but the diffeomorphism group or symplectomorphism is usually infinite-dimensional. In the moduli space's viewpoint, automorphism groups determine to what extent we can deform such a manifold with additional structure. For example, the Riemannian surface with genus g and marking point n has the automorphism group whose dimension can be negative, when the genus is larger, the dimension decreases until a finite group, thus the rigidity increases; same thing happens when n larger, thus adding markings is called rigidification.
Thus, when the automorphism is smaller, the local shape of the moduli space can determine more global properties, which is just what we say "rigidity" for **single** geometric object. In conclusion, "rigidity" and "softness" behave the same in the single and moduli levels harmonically.
When structures become increasingly more abundant, the concepts adapted in different settings behave like "upgrading" in games. For example, the monodromy in topological settings is upgraded into the holonomy on curved Riemannian manifolds, consisting of two counterparts of obstructions, which corresponds to the Berry phase in physics.

Another reason is that lower-dimensional cases provide basic examples, to predict or to embed into higher-dimensional cases. Even in the two-dimensional case, i.e. surfaces case, can reflect abundant and interesting geometry: compact surfaces are decomposed into $\mathbb{S}^2$, $\mathbb{T}^2$ and $\mathbb{RP}^2$ via connected sum; as for the oriented case we needed, only $\mathbb{S}^2$ and $\mathbb{T}^2$. Then we can classify them into "no holes", "one hole" and "more than one hole", equivalent to classification under many different settings: Fano, Calabi-Yau and general type; or positive, zero and negative; or spherical, Euclidean and hyperbolic; or abelian, non-abelian and anabelian… Although we can't expect these classifications to hold for higher

dimension anymore, these classifications are separately useful.
The moduli problem that arose from these extreme easy but nontrivial examples can be pretty interesting, for example, the Teichmuller spaces, as a synthetic place of hyperbolic geometry, algebraic geometry, dynamic, representation theory, even the modular forms and number theory. Hyperbolic geometry, as a fundamental and pretty important part of geometric topology, is designed to be the "general" part of geometric topology, and I'm interested in its concrete theory too.

Gauge theory:
When we focus on "wild" smooth four-dimensional world, the direct method, served as "protagonist" above, is replaced by indirect way of constructing moduli spaces via global analysis, which was first magically discovered by Donaldson in his Ph.D thesis, motivated by physics, more precisely, instanton theory or Yang-Mills theory. Such a path does show our philosophy of collecting data in a more coherent way: it generalizes global analysis on manifolds to vector bundles, enriching the target of usual functions to vector bundle valued in order to grab more data. Later, the Seiberg-Witten theory, or monopole theory, came into our stage with an occasionally and naturally closed property discovered in the moduli. Due to this automatic closeness, who came from behind is now more popular.

The relationship of global analysis and geometry and topology is a story much earlier, starting from the Gauss-Bonnet formula in differential geometry, and developed to index theorems for computing dimension, or other invariants, of moduli spaces now. In philosophy, it's an "infinitesimal" viewpoint to study the global geometry. Generally, it can be viewed as the pair between K-homology and K-cohomology (K-theory) compatible with the pair between periodic cyclic homology and periodic cyclic cohomology, where the former is given by analytic index and the latter is the topological index, forming a communicative diagram connected by the Chern character from the K side to the cyclic side. Such a formalism of using commutative diagrams to represent formulas is also used in the Grothendieck-Riemann-Roch formula, but settings are replaced by K-theory and Chow.
The story above can be lifted to the categorical level by considering the K-theory of a triangulated category, and then the result formula will compute the dimension of the moduli of objects of this category.

But the story of classical geometric analysis is only the study of its tangent bundle and the exponential map, replace the tangent bundle by some specific vector bundles, then we expect that we get a finer theory.
Hence further, for vector bundles with more structures depending on the smooth structure of the base manifold as connections here, parameterizing these structures is expected to reflect smooth structures of the given base manifold, just analogous to that parameterizing vector bundles themselves can reflect topological invariants of the base manifold.

Although such a leading principle, or even a slogan, may not be rigorous and convincing, at least it's convincing that such method is effective in constructing refined invariants from these moduli spaces, for example, Donaldson and Seiberg-Witten invariants for four-dimensional manifolds.

From the viewpoint of path integral formalism, it's not surprising that we can use physics to detect geometric structure: the "holes" of base manifold make two paths with same start and end points have different propagators; hence, this topological datum can be seen in the same manner as some kinds of homology theory, for example, the BRST (Becchi-Rouet-Stora-Tyutin) homology, or anomaly in physical language. And with additional structure put, it's detected by changing the Lagrangian in path integral.
Various quantum field theories induced by varying the additional structure in base manifolds are believed and verified to be powerful in different branches of mathematics. As mathematicians, we can simply view these theories as studies of well-chosen moduli spaces with physical meaning; or equivalently, the geometry provides the most precise and beautiful framework for physics. It shows that the symmetry of particles in our real world is also natural in the mathematical world, for example, the condition of spin is a kind of "2-oriented" condition, i.e. making the moduli become orientable. These physics occurs everywhere, even out of geometry, which may be unbelievable at first discovery, making a deep impression on mathematics. Although we do be able to explain "motivation" we perused in these cases respectively, we can't find a general reason for such a phenomenon: behaving like the character in a drama, physics under unified framework comes into the pretty different stages of mathematics. Although the eagerness of finding such "general reason" may be meaningless, these entanglements do make a deep impression on me: if two closely related physics phenomena induce different mathematical results, then there should be a bridge to be discovered between them.
However, at least, I suppose that we can't stop at the level of taking physics as black-box, but understanding how it works. Hence, I plan to penetrate into physics deeper in the later study, meanderingly through the path of my study of mathematics.

Now the moduli of connections modulo gauge equivalence need to be compact, the compactification is given by stability conditions: the boundary need to be added is collection of the degenerate, or equivalently, the limited ones of its interior. Hence, one can understand the moduli of monopoles are automatically compact is a pretty rare and surprising thing.
In this story, one shall note that the stability of category corresponds to the compactness of moduli of stable objects.

However, just as the same meaning as Atiyah said to Bott after he discovered index formula, we only know how to prove and compute it, but don't know why it's true, there should be a deeper discovery to explain this. The reason why the four-dimensional differential topology is so strange and challenging is still mysterious, at least for me.

Another main line of gauge theory is its relation with GIT (Geometric invariant theory), by the Hitchin-Kobayashi correspondence, saying that the gauge-theoretic moduli space is isomorphic to the moduli space of vector bundles with poly-stable conditions, which is a bridge of symplectic reduction and GIT quotient in infinite dimensional cases, called Kempf-Ness theorem in finite dimensional cases.
Strengthening the stability to stable we can get submoduli spaces called Mumford-Narashimhan-Seshadri, which are more familiar with algebraic geometers, via another bridge of Riemann-Hilbert related to some kinds of representation of fundamental group. Such a style of results concerning the representation of fundamental groups is this main line of gauge theory, leading to more studies on Higgs bundles and non-abelian Hodge correspondences.

Symplectic geometry:
The starting point is the equivalence between handle decomposition and Morse theory, and different Morse functions related topologically by handle slide, thus viewing them built by handle bodies or level sets is essentially expected as the same thing. This also reflects in homology level, Morse homology is usually same as topological ones. Now we take the Morse viewpoint into our use in some moduli space coming from a symplectic manifold, with some functional as "Morse function", which varies in different problems, and consider its Morse homology. Recall that Morse-type homology has three ingredients: critical points of Morse function, an index for grading and an equation for flow lines, they're chosen differently and cleverly in abundant scenes.
Then these Morse-type homologies used in the lower-dimensional topology are renamed differently, as "XX" Floer homology to memorize Floer's first discovery of infinite-dimensional Morse theory. Just as the Morse homology can be viewed as a coherent algebraic form of Morse theory, these Floer homology can be viewed as a coherent algebraic form of gauge theory.
Counting the J-holomorphic curves can be viewed as a simplified case of counting flow lines in Floer theory to defining the differential, but with homogeneous Cauchy-Riemann equations.

It's Weinstein's philosophy that "All are Lagrangians". By choosing a certain type of Lagrangian in symplectic moduli spaces, all topological Floer theories in gauge theory are expected to coincide with Lagrangian Floer theory. However, such equivalence can be hopeless to be proven now, for example, the Atiyah-Floer conjecture, but it sometimes works for some simpler moduli spaces, too.
Unfortunately, with too many technical restrictions, such as the compactness and transversality of the moduli needed when defining differential, of Lagrangian Floer theory put in both the symplectic manifold and its Lagrangians, defining a category with objects consisting of Lagrangians of given type and their morphisms consisting of their correspondence or intersection, more precisely, their Lagrangian Floer theory, is still hopeless. For example, for the noncompact La-

grangians, the wrapped ones are needed, motivated by imitating the cotangent bundle. These categorifications of the Floer theory are called Fukaya categories.

Had we abstracted our mind and lifted our eyes several times by grabbing geometric data increasingly coherent, there is no doubt that our target should be algebraic.

Starting at here, I'd like to lift all homological algebra, both chain-level and homology-level, into categorical settings coherently. The A_∞-structure arises naturally in Morse theory as chain-level, with these higher associativity being given by counting "broken flow lines", which is compactification of the counting of flow lines, i.e. defining the differential. With respect to the complexity of the A_∞-structure, there is no doubt that the chain-level contains more data than the homology-level, thus one developed the classical Donaldson-Fukaya category whose morphisms are Floer homology into the Fukaya category whose morphisms are chains. As we expected, it has better properties, for example, finding generators.

In such categorical level, we have many representation-theoretic methods applied to these category, as they usually occur as some module category. For example, how can we pass chain-level categories to homology-level categories by localizations, with additional structure of chain-level ones preserved? And how can we find the split generators of the category if exists? These categorical operations are motivated by module category studied in representation theory, which is also useful in the study of algebraic derived categories, and these generators are expected to behave well under mirror symmetry with precise geometric meaning. Such algebraic work in the Fukaya category corresponding to geometric operations is viewed as a noncommunicative geometry method.

It's worth to mention the noncommunicative method applied in such a categorical level, the original idea of noncommunicative geometry is studying the algebra of functions instead of studying the geometric object itself, then we can enhance this idea to the sheaf $\mathcal{O}$, then the category of all $\mathcal{O}$-modules. Such a module category naturally inherits the original spirit and techniques of noncommunicative geometry, for example, taking the K-cohomology and the Hochschild cohomology for the triangulated category coming from these. The interaction between the algebra and geometry, the derived category and geometric object do provide a deeper revisit of mirror symmetry, and so on.

A side story is the prequel to the categorical level of the noncommunicative method, which also contains brilliant mathematics. They defined various algebraic structures on the algebra of some kinds of functions for different geometric structures: metric, smooth, and so on. Then we can move our eyes from geometry to algebra safely if our algebraic structure is chosen well, equivalently, we use the algebra replace our concept of "space", which may motivated by quantum mechanics. Such motivation may have different philosophical explanations, but for mathematics, what makes a deep impression on me is the thesis of Connes, who told us that the non-communication creates an additional "god-given" time axis, via the classification of factors of von Neumann algebras.

The study of the sheaf category is another approach toward Fukaya category, started at Kashiwara and Schapira's microlocal study to sheaves, more precisely, the microsupport of some kinds of sheaves on cotangent bundle is coisotropic. It has ample algebraic theory, related to D-module, cluster algebra, and so on, thus which is expected to provide an alternative to tighten the construction of Fukaya category. It had be generalize to a kind of Weinstein manifolds which are close to cotangent bundles, to be equivalent to the wrapped ones.
From this viewpoint, homological mirror symmetry can be explained as the correspondence of the constructible sheaves on a real manifold and the coherent sheaves on the mirror of its cotangent bundle.
It turns out that sheaf-theoretic method is powerful when we handle problems about Lagrangian filling of Legendrian knots, as the Legendrians are served as the intersection of Lagrangians with the sphere cotangent bundle in a cotangent bundle. Its correspondence to cluster algebra is attractive to me.

The generality of symplectic geometry lies in two roads: the first is that the moduli space usually carries a symplectic form given by the cup product of cohomology, which is just the tangent space of moduli by deformation theory; the second is the Lagrangian correspondence, or specially, the symplectic reduction, which works in the construction of moduli space, for example, the Kempf-Ness theorem. One can also pursue deeper reasons such as the mirror symmetry, as the other side is algebraic, which doesn't need to attach any importance to it, but this is another story.
Due to this motivation, my interest on symplectic geometry doesn't lie in explicit geometry and topology of certain types of symplectic manifolds but general abstract theories, especially the algebraic methods, which are expected to be applied into some place where symplectic form is discovered naturally, and the problem we concern can be rewritten as Lagrangian correspondence. For example, the well-known Heegaard-Floer theory focuses on the symplectic manifold coming from the symmetric product of the knot surface, and its two special Lagrangians are products of generators of the first homology of two handlebodies divided by the knot surface; this Lagrangian Floer theory is relatively simple and close to combination.

As for my interest in mirror symmetry, I postpone it to the next story. One has claimed that the mirror symmetry can only indicate the "rigid" part of symplectic geometry, I can't agree more, as there is concrete example showing that the Fukaya category can recover the symplectic manifold itself. However, now my interest in this area is still in the "rigid" part and mirror symmetry.
Another thing from physics is the viewpoint of open-closed strings, which I take as one of the main differences between symplectic geometry and algebraic geometry. We can simply under it as the open case is bounded while the closed case doesn't have boundary, thus in symplectic geometry, the open string is more usual as we need the boundary condition lies in the Lagrangians, serving as D-branes ("D" means Dirichlet boundary condition.); but the GW (Gromov-

Witten) theory, both algebraic and symplectic, is closed. The closed-to-open map maps from the quantum cohomology to the Floer cohomology, after categorification, it's to the Hochschild cohomology of Fukaya category, which is roughly realized by "cutting" the closed string (loop) to open one (segment) by D-branes.

Algebraic geometry:
The fundamental problem of algebraic geometry is classification under algebraic isomorphism, but due to its complexity, we study classification under birational equivalence, which is coarser. Geographically, fixing several birational invariants as the coordinates of a map, we have two questions to ask. Is a point on the map empty? If not, what does the moduli modulo isomorphism look like? Hence, the central problem of "geometric" algebraic geometry is studying the moduli space, besides the imitation of topological theories into the algebraic framework with higher rigidity, such as the intersection theory on algebraic setting and so on.

Mirror symmetry predicted such a correspondence via the A-model and B-model in the S-duality occurring in superstring theory, on pairs of Calabi-Yau manifolds: enumerative geometry and Hodge theory, symplectic geometry and complex algebraic geometry, derived Fukaya category and derived category of coherent sheaves, coming into their stage in different levels. Outside the Calabi-Yau cases, there are also non-geometric objects with potential serving as the mirror of non-Calabi-Yau manifolds. Just as what we said about the classification into Fano, Calabi-Yau and general type, generalizing the subtle Calabi-Yau case to more general cases and coming back to this special case is FJRW (Fan-Javis-Ruan-Witten) theory.

The harmony of algebraic and analytic is presented in not only conclusions but also techniques. For example, when we construct and prove good properties of moduli spaces, the fundamental question is taking limits, in algebraic language, the valuation criteria, then it's realized by degeneration of an algebraic family; the boundary of moduli space in analytic cases is formalized by real codimension 1 boundary which is not admitted in algebraic setting, but replaced by boundary divisors. Another important example is the analogue of algebraic groups and Lie groups, and their actions and quotients, respectively.
However, it's admitted that the algebraic side, thanks to the abstract framework, has much more advantages than the analytic story: the infinitesimal in algebraic moduli problems has precise description modeled by the ring of dual numbers or other Artinian algebras, although we still have to face technical problems of compactness and transversality. Equivalently, the derived structures on algebraic objects are simpler than what we expected to exist in different geometry, hence the try of so-called "derived differential geometry" instead of derived algebraic geometry is still an unending story.

Pursuing a well-behaved moduli space acquires three conditions: automorphisms not too bad, compact or proper, and transversal or good deformation properties. The first is realized by GIT methods, for example, the rigidification of the curves by marking points. A pleasing viewpoint of this rigidification is **viewing markings as punctures**, which diffeomorphic to the real dimension 1 boundary can be placed in Lagrangians in the symplectic setting, which is said to have made a deep impression on Grothendieck's anabelian geometry. The second is discussed briefly above, and the third is too hard and too technical in most cases.

Counting problem is also occurring in both areas, which acquires enough rigidity to make the number finite, in symplectic setting it's Cauchy-Riemann equations, in algebraic setting it's algebraic condition. From the algebraic viewpoint, it's important to have a process of switching our mind of counting embedded curves to counting parametrized curves, more precisely, counting maps. The former is more considered in algebraic geometry as we view the embedded sub-object as the ideal sheaf structure, the latter is more considered in differential geometry due to the lack of sheaf language, here our counting of stable maps in GW theory is a harmony of algebraic and symplectic worlds too, which both fallen into the A-model side, while counting sheaves, for example, the DT (Donaldson-Thomas) theory, are more believed to fall into the B-model side. I suppose these two viewpoints are linked closely by DT-GW correspondence, or MNOP (Maulik-Nekrasov-Okounkov-Pandharipande) conjecture, which can also be thought of as mirror symmetry.
Such a switching of viewpoint corresponds to the equivalence of QFT and string theory, i.e. the fields on spacetime and worldsheets. For example, the worldsheet is real 2 dimensional spanned by one dimension of time and one dimension of real dimension 1 string, thus may be fitted into algebraic curves or J-holomorphic curves.
I prefer strings or worldsheets as they are much more concrete and drawable than bundles or sheaves.

Out of the world of classical geometry of "spaces", it's Grothendieck's philosophy that we can consider geometric objects over more general objects, if we put the family of objects side away but consider over non-algebraic fields. Then with Galois actions equipped, the study of a given object is divided into two parts: the geometric part and the Galois action of its algebraic closure part. This can be more precisely shown in the etale fundamental group written as an "entanglement" of the topological fundamental group and the absolute Galois group. From here we first glimpse into the arithmetic geometry, which I know few things about.
However, p-adic ones, or more general non-archimedean local fields and other classical "arithmetic" objects, turn out to be useful in various geometry settings, as far as I know, the family Floer theory by Abouzaid and the quantum D-modules by Iritani school, which is helpful in a try of Kontsevich toward the irrationality of cubic fourfolds. It's just due to the simple technique reason: the convergence behaves much better in local and non-archimedean case than in the analytic settings.

My interest in algebraic geometric side majorly lies in the analogous, or essentially intersected, or dual part of geometric or topological theories, which is more down-to-earth, as otherwise the rigidity of algebraic world will make it harder to imagine through concrete examples, which are either trivial or too complicated. But what makes algebraic geometry more attractive than classical geometry is the use of abstract and coherent languages. It provides a completely new and higher viewpoint to revisit geometry, might via functors, might via sheaves and so on which helps a lot in the dream of unification thanks to Grothendieck. With such a challenging task of switching our mind in different levels of perspective, I prefer the theory of moduli spaces and enumerative problems; I also like concrete models such as toric models, tropical method applied to intersection theory and mirror symmetry and so on, but it's something hard for me to keep the geometric insight in my mind when I study related topics on the minimal model program, thus I can only take the birational geometry as something technical in construction of moduli spaces.

Compared with the symplectic side, the B-model derived category can recover and determine more geometry of the geometric object itself due to its rigidity. We can begin this story from the Abel-Jacobi theory studying the complex, or algebraic structures on topological objects: the Hodge structure on cohomologies corresponds to the algebraic structure on geometric objects; then the Torelli-type theorems are lifted to categorical level by expecting that the derived category can recover almost all data of geometry itself.
Such a type of results are also a stage of representation-theoretic methods and can be viewed as noncommunicative methods. For example, the Beilison theorem says that the exceptional sequence of $\mathbb{P}^n$ is $\mathcal{O}$, $\mathcal{O}(1)$, $\ldots$, $\mathcal{O}(n)$, and the tilting object is given by the direct sum of these, and we can also give a quiver to serve as the path algebra of the endomorphism of the tilting module. Hence, it gives a simple description of the derived category by tilting theory.

Derived geometry:
Due to under both the algebraic or analytic settings, the singular and stacky issues occur naturally, the derived formalism is expected to deal with them by detouring along higher structures which allow us to detect higher equivalence relations. This is what Kontsevich said in his famous hidden smoothness principle. Such a principle can also be used to explain why the theory is powerful in intersection theory, for example the construction of virtual fundamental classes: smoothness of moduli is the same thing as the perturbation toward transversal, thus adding derived structure is expected to give hidden perturbation data to it, i.e. hidden intersection class and so on.
Motivated by the asymptotic expansion, or the homological perturbation theory developed in settings such as BV (Batalin-Vilkovisky) formalism, in analysis or physics, I suppose that the derived formalism should be as powerful as the Taylor expansion in analysis in a pure algebraic setting, which serves as the "approximation" by well-behaved ones–chosen as smooth ones here and via simplicial

approximation language. Such motivation also makes it expected to be useful in physics.
This language is well developed in algebraic settings, but there are also some trials in analytic or symplectic settings. However, the algebraic nature of simplicial language, which is the fundamental block of derived language, makes it seem hard to be built.

However, restricted by my poor knowledge to the language of simplicial methods, I can't give any detail of this theory–in fact, even to describe the object we study acquires huge details of homotopy theory. I plan to study its application to moduli stack of principal bundles in the geometric Langlands program and intersection theory, which do provide a new language to revisit classical geometric problems. The former one is motivated by the fact that the representation theory had taken increasingly important participation in the modern study of geometry with group action and quotient everywhere. The latter one is more familiar with geometers in constructing virtual fundamental class mentioned above, which means recovering the intersection data in general case even when the real case is exceptional, as the deformation acquired is controlled by complexes, or cohomology, which is derived in nature.

In conclusion, there are so many analogues between mathematics and physics; and between different specific fields in mathematics. These analogues, which are called the "metaphor" by Manin, can be seen as a combination of the similarity of philosophy and the essential difference of natural settings or languages. I suppose what attracts mathematicians the most is the spirit, existing and staying invariant when our journey passed different worlds in mathematics, like the pillar in the middle of a spiral stairs. Hence, who pursuing mathematics itself shouldn't be manacled masochistically by any particular field, but try our best to dig and appreciate the hole mathematics we prefer: although they're not essentially same, we can revisit them together in their spirit is the same.
Starting from here, I'd like to discuss more rigorous "mathematics" instead "philosophy" or "metaphor", and present examples I know.

2 Low-dimensional topology

To be continued ...

3 Gauge theory

4 Symplectic geometry

5 Algebraic geometry