How to draw a blow-up picture?

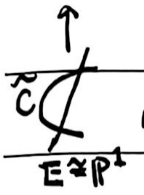
Date

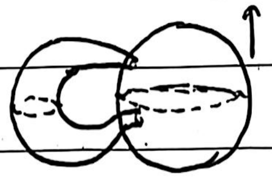
This is an interesting note for fun, just as someone draw Vien diagram in cubes than disks, or ~~Vak~~ Vakil can use Vien diagram to represent spectral sequence. Such pictures is told by Prof. Yang Zhou.

First, we're unable to draw real ($\mathbb{R}$) pictures, we consider the easiest case of blow-up (we're not talking about real blow-up!):

E.g.1.

C  cubic nodal curve $\subset \mathbb{P}^2$ $\leftrightarrow$  singular torus $\subset \mathbb{CP}^2$



$\tilde{C}$ $\subset \mathbb{P}^1 \times \mathbb{P}^2$ $\longleftrightarrow$  can be embedded to $\mathbb{R}^3$

$E \cong \mathbb{P}^1$

$\tilde{C} \cong C - \{pt\}$ $E \cong \mathbb{CP}^1 \cong S^2$ $\subset \mathbb{CP}^1 \times \mathbb{CP}^2$

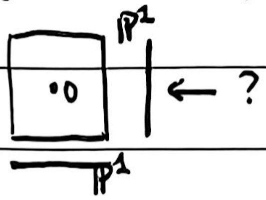
$\cong$ tube 

But the intersection behaviour of $\tilde{C}$ and E is $\tilde{C}$ intersects E at two isolated pts and transversally, but it's impossible in $\mathbb{R}^3$

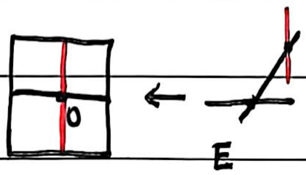
$\Rightarrow$ can't be embedded to $\mathbb{R}^3$!

We can't draw a real picture, but use flase dimension to represent

E.g.2. Another simplest example: $Bl_0(\mathbb{P}^1 \times \mathbb{P}^1)$



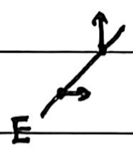
We draw two divisors pass through 0



as E parametizes normal directions of $\{0\} \subset \mathbb{P}^1 \times \mathbb{P}^1$

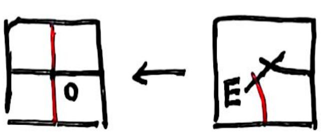
directions form projective ones $E \cong \mathbb{P}^1$

A good picture can be found, like this:



but it's too complicated, and can't be generalized into higher dimensional

We draw picture simply as



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We show these pictures by more examples:

E.g.3. The picture of deformation to normal cone in previous notes;

E.g.4. The del Pezzo surface of degree 5

$\cong Bl_{3pts}(\mathbb{P}^1\times\mathbb{P}^1) \cong Bl_{4pts}(\mathbb{P}^2)$

We compute it and draw by wall-crossing:

- $\mathbb{P}^1\times\mathbb{P}^1 = \overline{M}_{0,\vec{v}}$, $\vec{v}=(1-\varepsilon,1-\varepsilon,1-\varepsilon,\eta,\eta)$, $0<\eta\ll\varepsilon\ll 1$

and $\overline{M}_{0,5}\to\overline{M}_{0,\vec{v}}$ is blow up three points (easy)

- $\mathbb{P}^2=\overline{H}_{2,\vec{v}}$, $\vec{v}=(1-\varepsilon,1-\varepsilon,1-\varepsilon,1-\varepsilon,\eta)$, $0<\eta\ll\varepsilon\ll 1$

and $\overline{H}_{2,5}\to\overline{H}_{2,\vec{v}}$ is blow up four points (we'll draw it)

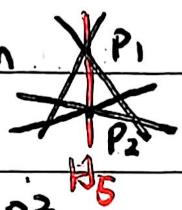
where $\overline{H}_{d,n}$ parametize hyperplane arrangement $H_1+\dots+H_n$ in $\mathbb{P}^d$

$\overline{H}_{2,\vec{v}}$ has first 4 hyperplane fixed, last one H_5 flexible $\in(\mathbb{P}^2)^\vee\cong\mathbb{P}^2$


to ensure stability: no triple intersection

thus blow-up four pts of $\mathbb{P}^2 \Leftrightarrow$ blow-up four choices of H_5

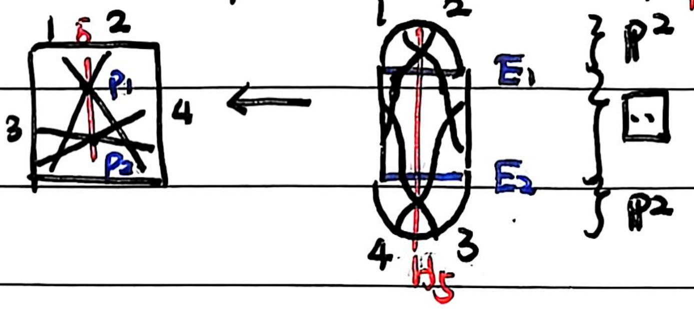
What's special is triple intersection



"special"

and we blow-up p_1, p_2 in $\mathbb{P}^2$ has four choices of H_5

$\Rightarrow$



we only blow-up p_1, p_2 than H_5 as H_5 is divisor, blow-up it is blow-up nothing

except in the intersected points p_1, p_2

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