

Rules for integrands of the form $(a + b \sin[c + d (e + f x)^n])^p$

$$1. \int (a + b \sin[c + d (e + f x)^n])^p dx \text{ when } p \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}$$

$$1. \int (a + b \sin[c + d (e + f x)^n])^p dx \text{ when } p \in \mathbb{Z}^+ \wedge n - 1 \in \mathbb{Z}^+$$

$$1. \int \sin[c + d (e + f x)^n] dx \text{ when } n - 1 \in \mathbb{Z}^+$$

$$1. \int \sin[c + d (e + f x)^2] dx$$

$$\textcolor{red}{1}: \int \sin[d (e + f x)^2] dx$$

Derivation: Primitive rule

$$\text{Basis: } \text{FresnelS}'[z] == \sin\left[\frac{\pi z^2}{2}\right]$$

Rule:

$$\int \sin[d (e + f x)^2] dx \rightarrow \frac{\sqrt{\frac{\pi}{2}}}{f \sqrt{d}} \text{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{d} (e + f x)\right]$$

Program code:

```
Int[Sin[d.*(e_.+f_.**x_)^2],x_Symbol] :=
  Sqrt[Pi/2]/(f*Rt[d,2])*FresnelS[Sqrt[2/Pi]*Rt[d,2]*(e+f*x)] /;
FreeQ[{d,e,f},x]
```

```
Int[Cos[d.*(e_.+f_.**x_)^2],x_Symbol] :=
  Sqrt[Pi/2]/(f*Rt[d,2])*FresnelC[Sqrt[2/Pi]*Rt[d,2]*(e+f*x)] /;
FreeQ[{d,e,f},x]
```

$$2: \int \sin[c + d (e + f x)^2] dx$$

Derivation: Algebraic expansion

$$\text{Basis: } \sin[w + z] == \sin[w] \cos[z] + \cos[w] \sin[z]$$

$$\text{Basis: } \cos[w + z] == \cos[w] \cos[z] - \sin[w] \sin[z]$$

Note: Although not essential, this rule produces antiderivatives in terms of Fresnel integrals instead of complex error functions.

Rule:

$$\int \sin[c + d (e + f x)^2] dx \rightarrow \sin[c] \int \cos[d (e + f x)^2] dx + \cos[c] \int \sin[d (e + f x)^2] dx$$

Program code:

```
Int[Sin[c_+d_.*(e_+f_.*x_)^2],x_Symbol] :=
  Sin[c]*Int[Cos[d*(e+f*x)^2],x] + Cos[c]*Int[Sin[d*(e+f*x)^2],x] /;
FreeQ[{c,d,e,f},x]
```

```
Int[Cos[c_+d_.*(e_+f_.*x_)^2],x_Symbol] :=
  Cos[c]*Int[Cos[d*(e+f*x)^2],x] - Sin[c]*Int[Sin[d*(e+f*x)^2],x] /;
FreeQ[{c,d,e,f},x]
```

2: $\int \sin[c + d (e + f x)^n] dx$ when $n - 2 \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Basis: $\sin[z] == \frac{1}{2} i e^{-i z} - \frac{1}{2} i e^{i z}$

Basis: $\cos[z] == \frac{1}{2} e^{-i z} + \frac{1}{2} e^{i z}$

Rule: If $n - 2 \in \mathbb{Z}^+$, then

$$\int \sin[c + d (e + f x)^n] dx \rightarrow \frac{i}{2} \int e^{-c - d i (e + f x)^n} dx - \frac{i}{2} \int e^{c + d i (e + f x)^n} dx$$

Program code:

```
Int[Sin[c_.+d_.*(e_.+f_.*x_)^n_],x_Symbol] :=
  I/2*Int[E^(-c*I-d*I*(e+f*x)^n),x] - I/2*Int[E^(c*I+d*I*(e+f*x)^n),x] /;
FreeQ[{c,d,e,f},x] && IGtQ[n,2]
```

```
Int[Cos[c_.+d_.*(e_.+f_.*x_)^n_],x_Symbol] :=
  1/2*Int[E^(-c*I-d*I*(e+f*x)^n),x] + 1/2*Int[E^(c*I+d*I*(e+f*x)^n),x] /;
FreeQ[{c,d,e,f},x] && IGtQ[n,2]
```

2: $\int (a + b \sin[c + d (e + f x)^n])^p dx$ when $p - 1 \in \mathbb{Z}^+ \wedge n - 1 \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule: If $p - 1 \in \mathbb{Z}^+ \wedge n - 1 \in \mathbb{Z}^+$, then

$$\int (a + b \sin[c + d (e + f x)^n])^p dx \rightarrow \int \text{TrigReduce}[(a + b \sin[c + d (e + f x)^n])^p, x] dx$$

Program code:

```
Int[(a_.+b_.*Sin[c_.+d_.*(e_.+f_.*x_)^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(a+b*SIN[c+d*(e+f*x)^n])^p,x],x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[p,1] && IGtQ[n,1]
```

```
Int[(a_.+b_.*Cos[c_.+d_.*(e_.+f_.*x_)^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(a+b*COS[c+d*(e+f*x)^n])^p,x],x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[p,1] && IGtQ[n,1]
```

2: $\int (a + b \sin[c + d (e + f x)^n])^p dx$ when $p \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}^-$

Derivation: Integration by substitution

Basis: If $n \in \mathbb{Z}$, then $F[(e + f x)^n] = -\frac{1}{f} \text{Subst}\left[\frac{F[x^{-n}]}{x^2}, x, \frac{1}{e + f x}\right] \partial_x \frac{1}{e + f x}$

Rule: If $p \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}^-$, then

$$\int (a + b \sin[c + d (e + f x)^n])^p dx \rightarrow -\frac{1}{f} \text{Subst}\left[\int \frac{(a + b \sin[c + d x^{-n}])^p}{x^2} dx, x, \frac{1}{e + f x}\right]$$

Program code:

```
Int[(a_.+b_.*Sin[c_.+d_.*(e_.+f_.*x_)^n_])^p_,x_Symbol] :=
  -1/f*Subst[Int[(a+b*SIN[c+d*x^(-n)])^p/x^2,x],x,1/(e+f*x)] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[p,0] && ILtQ[n,0] && EqQ[n,-2]
```

```
Int[(a_.+b_.*Cos[c_.+d_.*(e_.+f_.*x_)^n_])^p_,x_Symbol] :=
  -1/f*Subst[Int[(a+b*COS[c+d*x^(-n)])^p/x^2,x],x,1/(e+f*x)] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[p,0] && ILtQ[n,0] && EqQ[n,-2]
```

2: $\int (a + b \sin[c + d (e + f x)^n])^p dx$ when $p \in \mathbb{Z}^+ \wedge \frac{1}{n} \in \mathbb{Z}$

Derivation: Integration by substitution

Basis: If $-1 \leq n \leq 1$, then $F[(e + f x)^n] = \frac{1}{n f} \text{Subst}[x^{1/n-1} F[x], x, (e + f x)^n] \partial_x (e + f x)^n$

Rule: If $p \in \mathbb{Z}^+ \wedge \frac{1}{n} \in \mathbb{Z}$, then

$$\int (a + b \sin[c + d (e + f x)^n])^p dx \rightarrow \frac{1}{n f} \text{Subst}\left[\int x^{1/n-1} (a + b \sin[c + d x])^p dx, x, (e + f x)^n\right]$$

Program code:

```
Int[(a_.+b_.*Sin[c_.+d_.*(e_.+f_.*x_)^n_])^p_,x_Symbol] :=
  1/(n*f)*Subst[Int[x^(1/n-1)*(a+b*Sin[c+d*x])^p,x],x,(e+f*x)^n] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[p,0] && IntegerQ[1/n]
```

```
Int[(a_.+b_.*Cos[c_.+d_.*(e_.+f_.*x_)^n_])^p_,x_Symbol] :=
  1/(n*f)*Subst[Int[x^(1/n-1)*(a+b*Cos[c+d*x])^p,x],x,(e+f*x)^n] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[p,0] && IntegerQ[1/n]
```

3: $\int (a + b \sin[c + d (e + f x)^n])^p dx$ when $p \in \mathbb{Z}^+ \wedge n \in \mathbb{F}$

Derivation: Integration by substitution

Basis: If $k \in \mathbb{Z}^+$, then $F[(e + f x)^n] = \frac{k}{f} \text{Subst}[x^{k-1} F[x^{k n}], x, (e + f x)^{1/k}] \partial_x (e + f x)^{1/k}$

Rule: If $p \in \mathbb{Z}^+ \wedge n \in \mathbb{F}$, let $k = \text{Denominator}[n]$, then

$$\int (a + b \sin[c + d (e + f x)^n])^p dx \rightarrow \frac{k}{f} \text{Subst}\left[\int x^{k-1} (a + b \sin[c + d x^{k n}])^p dx, x, (e + f x)^{1/k}\right]$$

Program code:

```
Int[(a_.+b_.*Sin[c_.+d_.*(e_.+f_.*x_)^n_])^p_,x_Symbol] :=
  Module[{k=Denominator[n]},
    k/f*Subst[Int[x^(k-1)*(a+b*Sin[c+d*x^(k*n)])^p,x],x,(e+f*x)^(1/k)]] /;
  FreeQ[{a,b,c,d,e,f},x] && IGtQ[p,0] && FractionQ[n]
```

```
Int[(a_.+b_.*Cos[c_.+d_.*(e_.+f_.*x_)^n_])^p_,x_Symbol] :=
  Module[{k=Denominator[n]},
    k/f*Subst[Int[x^(k-1)*(a+b*Cos[c+d*x^(k*n)])^p,x],x,(e+f*x)^(1/k)]] /;
  FreeQ[{a,b,c,d,e,f},x] && IGtQ[p,0] && FractionQ[n]
```

4. $\int (a + b \sin[c + d (e + f x)^n])^p dx$ when $p \in \mathbb{Z}^+$

1: $\int \sin[c + d (e + f x)^n] dx$

Derivation: Algebraic expansion

Basis: $\sin[z] == \frac{1}{2} i e^{-i z} - \frac{1}{2} i e^{i z}$

Basis: $\cos[z] == \frac{1}{2} e^{-i z} + \frac{1}{2} e^{i z}$

Rule:

$$\int \sin[c + d (e + f x)^n] dx \rightarrow \frac{i}{2} \int e^{-c i - d i (e + f x)^n} dx - \frac{i}{2} \int e^{c i + d i (e + f x)^n} dx$$

Program code:

```
Int[Sin[c_+d_*(e_+f_*x_)^n_],x_Symbol] :=
  1/2*Int[E^(-c*I-d*I*(e+f*x)^n),x] - 1/2*Int[E^(c*I+d*I*(e+f*x)^n),x] /;
FreeQ[{c,d,e,f,n},x]
```

```
Int[Cos[c_+d_*(e_+f_*x_)^n_],x_Symbol] :=
  1/2*Int[E^(-c*I-d*I*(e+f*x)^n),x] + 1/2*Int[E^(c*I+d*I*(e+f*x)^n),x] /;
FreeQ[{c,d,e,f,n},x]
```


2: $\int (a + b \sin[c + d (e + f x)^n])^p dx$ when $p - 1 \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule: If $p - 1 \in \mathbb{Z}^+$, then

$$\int (a + b \sin[c + d (e + f x)^n])^p dx \rightarrow \int \text{TrigReduce}[(a + b \sin[c + d (e + f x)^n])^p] dx$$

Program code:

```
Int[(a_.+b_.*Sin[c_.+d_.*(e_.+f_.*x_)^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(a+b*SIN[c+d*(e+f*x)^n])^p,x],x] /;
FreeQ[{a,b,c,d,e,f,n},x] && IGtQ[p,1]
```

```
Int[(a_.+b_.*Cos[c_.+d_.*(e_.+f_.*x_)^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(a+b*COS[c+d*(e+f*x)^n])^p,x],x] /;
FreeQ[{a,b,c,d,e,f,n},x] && IGtQ[p,1]
```

X: $\int (a + b \sin[c + d (e + f x)^n])^p dx$

Rule:

$$\int (a + b \sin[c + d (e + f x)^n])^p dx \rightarrow \int (a + b \sin[c + d (e + f x)^n])^p dx$$

Program code:

```
Int[(a_.+b_.*Sin[c_.+d_.*(e_.+f_.*x_)^n_])^p_,x_Symbol] :=
  Unintegrable[(a+b*SIN[c+d*(e+f*x)^n])^p,x] /;
FreeQ[{a,b,c,d,e,f,n,p},x]
```

```
Int[(a_.+b_.*Cos[c_.+d_.*(e_.+f_.*x_)^n_])^p_,x_Symbol] :=
  Unintegrable[(a+b*Cos[c+d*(e+f*x)^n])^p,x] /;
FreeQ[{a,b,c,d,e,f,n,p},x]
```

N. $\int (a + b \sin[u])^p dx$

1: $\int (a + b \sin[c + d u^n])^p dx$ when $u == e + f x$

Derivation: Algebraic normalization

Rule: If $u == e + f x$, then

$$\int (a + b \sin[c + d u^n])^p dx \rightarrow \int (a + b \sin[c + d (e + f x)^n])^p dx$$

Program code:

```
Int[(a_.+b_.*Sin[c_.+d_.*u_^n_])^p_,x_Symbol] :=
  Int[(a+b*SIN[c+d*ExpandToSum[u,x]^n])^p,x] /;
FreeQ[{a,b,c,d,n,p},x] && LinearQ[u,x] && Not[LinearMatchQ[u,x]]
```

```
Int[(a_.+b_.*Cos[c_.+d_.*u_^n_])^p_,x_Symbol] :=
  Int[(a+b*COS[c+d*ExpandToSum[u,x]^n])^p,x] /;
FreeQ[{a,b,c,d,n,p},x] && LinearQ[u,x] && Not[LinearMatchQ[u,x]]
```

2: $\int (a + b \sin[u])^p dx$ when $u = c + d x^n$

Derivation: Algebraic normalization

Rule: If $u = c + d x^n$, then

$$\int (a + b \sin[u])^p dx \rightarrow \int (a + b \sin[c + d x^n])^p dx$$

Program code:

```
Int[(a_.+b_.*Sin[u_])^p_,x_Symbol] :=
  Int[(a+b*SIN[ExpandToSum[u,x]])^p,x] /;
FreeQ[{a,b,p},x] && BinomialQ[u,x] && Not[BinomialMatchQ[u,x]]
```

```
Int[(a_.+b_.*Cos[u_])^p_,x_Symbol] :=
  Int[(a+b*COS[ExpandToSum[u,x]])^p,x] /;
FreeQ[{a,b,p},x] && BinomialQ[u,x] && Not[BinomialMatchQ[u,x]]
```

Rules for integrands of the form $(e x)^m (a + b \sin[c + d x^n])^p$

$$1. \int (e x)^m (a + b \sin[c + d x^n])^p dx \text{ when } \frac{m+1}{n} \in \mathbb{Z}$$

$$1. \int x^m (a + b \sin[c + d x^n])^p dx \text{ when } \frac{m+1}{n} \in \mathbb{Z}$$

$$1. \int \frac{\sin[c + d x^n]}{x} dx$$

$$1: \int \frac{\sin[d x^n]}{x} dx$$

Derivation: Primitive rule

$$\text{Basis: } \text{SinIntegral}'[z] == \frac{\text{Sin}[z]}{z}$$

Rule:

$$\int \frac{\sin[d x^n]}{x} dx \rightarrow \frac{\text{SinIntegral}[d x^n]}{n}$$

Program code:

```
Int[Sin[d_.*x_^n_]/x_,x_Symbol] :=
  SinIntegral[d*x^n]/n /;
FreeQ[{d,n},x]
```

```
Int[Cos[d_.*x_^n_]/x_,x_Symbol] :=
  CosIntegral[d*x^n]/n /;
FreeQ[{d,n},x]
```

2: $\int \frac{\sin[c + d x^n]}{x} dx$

Derivation: Algebraic expansion

Basis: $\sin[w + z] = \sin[w] \cos[z] + \cos[w] \sin[z]$

Rule:

$$\int \frac{\sin[c + d x^n]}{x} dx \rightarrow \sin[c] \int \frac{\cos[d x^n]}{x} dx + \cos[c] \int \frac{\sin[d x^n]}{x} dx$$

Program code:

```
Int[Sin[c_+d_.*x_^n_]/x_,x_Symbol] :=
  Sin[c]*Int[Cos[d*x^n]/x,x] + Cos[c]*Int[Sin[d*x^n]/x,x] /;
FreeQ[{c,d,n},x]
```

```
Int[Cos[c_+d_.*x_^n_]/x_,x_Symbol] :=
  Cos[c]*Int[Cos[d*x^n]/x,x] - Sin[c]*Int[Sin[d*x^n]/x,x] /;
FreeQ[{c,d,n},x]
```

2: $\int x^m (a + b \sin[c + d x^n])^p dx$ when $\frac{m+1}{n} \in \mathbb{Z} \wedge (p = 1 \vee m = n - 1 \vee p \in \mathbb{Z} \wedge \frac{m+1}{n} > 0)$

Derivation: Integration by substitution

Basis: If $\frac{m+1}{n} \in \mathbb{Z}$, then $x^m F[x^n] = \frac{1}{n} \text{Subst}[x^{\frac{m+1}{n}-1} F[x], x, x^n] \partial_x x^n$

Rule: If $\frac{m+1}{n} \in \mathbb{Z} \wedge (p = 1 \vee m = n - 1 \vee p \in \mathbb{Z} \wedge \frac{m+1}{n} > 0)$, then

$$\int x^m (a + b \sin[c + d x^n])^p dx \rightarrow \frac{1}{n} \text{Subst}\left[\int x^{\frac{m+1}{n}-1} (a + b \sin[c + d x])^p dx, x, x^n\right]$$

Program code:

```
Int[x_^m_.*(a_.+b_.*Sin[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  1/n*Subst[Int[x^(Simplify[(m+1)/n]-1)*(a+b*Sin[c+d*x])^p,x],x,x^n] /;
FreeQ[{a,b,c,d,m,n,p},x] && IntegerQ[Simplify[(m+1)/n]] && (EqQ[p,1] || EqQ[m,n-1] || IntegerQ[p] && GtQ[Simplify[(m+1)/n],0])
```

```
Int[x_^m_.*(a_.+b_.*Cos[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  1/n*Subst[Int[x^(Simplify[(m+1)/n]-1)*(a+b*Cos[c+d*x])^p,x],x,x^n] /;
FreeQ[{a,b,c,d,m,n,p},x] && IntegerQ[Simplify[(m+1)/n]] && (EqQ[p,1] || EqQ[m,n-1] || IntegerQ[p] && GtQ[Simplify[(m+1)/n],0])
```

2: $\int (e x)^m (a + b \sin[c + d x^n])^p dx$ when $\frac{m+1}{n} \in \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(e x)^m}{x^m} = 0$

Rule: If $\frac{m+1}{n} \in \mathbb{Z}$, then

$$\int (e x)^m (a + b \sin[c + d x^n])^p dx \rightarrow \frac{e^{\text{IntPart}[m]} (e x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (a + b \sin[c + d x^n])^p dx$$

Program code:

```
Int[(e*x_)^m*(a_+b_.*Sin[c_+d_.*x_^n_])^p_,x_Symbol] :=
  e^IntPart[m]*(e*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*Sin[c+d*x^n])^p,x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && IntegerQ[Simplify[(m+1)/n]]
```

```
Int[(e*x_)^m*(a_+b_.*Cos[c_+d_.*x_^n_])^p_,x_Symbol] :=
  e^IntPart[m]*(e*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*Cos[c+d*x^n])^p,x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && IntegerQ[Simplify[(m+1)/n]]
```

$$2. \int (e x)^m (a + b \sin[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge n \in \mathbb{Z}$$

$$1. \int (e x)^m (a + b \sin[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge n \in \mathbb{Z}^+$$

$$1. \int (e x)^m \sin[c + d x^n] dx$$

$$\text{1: } \int x^{\frac{n}{2}-1} \sin[a + b x^n] dx$$

Derivation: Integration by substitution

$$\text{Basis: } x^{\frac{n}{2}-1} F[x^n] = \frac{2}{n} \text{Subst}\left[F[x^2], x, x^{\frac{n}{2}}\right] \partial_x x^{\frac{n}{2}}$$

Note: Although not essential, this rule produces antiderivatives in terms of Fresnel integrals instead of complex error functions.

Rule:

$$\int x^{\frac{n}{2}-1} \sin[a + b x^n] dx \rightarrow \frac{2}{n} \text{Subst}\left[\int \sin[a + b x^2] dx, x, x^{\frac{n}{2}}\right]$$

Program code:

```
Int[x^m_.*Sin[a_.+b_.*x^n_],x_Symbol] :=
  2/n*Subst[Int[Sin[a+b*x^2],x],x,x^(n/2)] /;
FreeQ[{a,b,m,n},x] && EqQ[m,n/2-1]
```

```
Int[x^m_.*Cos[a_.+b_.*x^n_],x_Symbol] :=
  2/n*Subst[Int[Cos[a+b*x^2],x],x,x^(n/2)] /;
FreeQ[{a,b,m,n},x] && EqQ[m,n/2-1]
```


$$\mathbf{2:} \int (e x)^m \sin[c + d x^n] dx \text{ when } n \in \mathbb{Z}^+ \wedge 0 < n < m + 1$$

Reference: CRC 392, A&S 4.3.119

Reference: CRC 396, A&S 4.3.123

Derivation: Integration by parts

Basis: If $n \in \mathbb{Z}$, then $(e x)^m \sin[c + d x^n] == -\frac{e^{n-1} (e x)^{m-n+1}}{d n} \partial_x \cos[c + d x^n]$

Rule: If $n \in \mathbb{Z}^+ \wedge 0 < n < m + 1$, then

$$\int (e x)^m \sin[c + d x^n] dx \rightarrow -\frac{e^{n-1} (e x)^{m-n+1} \cos[c + d x^n]}{d n} + \frac{e^n (m-n+1)}{d n} \int (e x)^{m-n} \cos[c + d x^n] dx$$

Program code:

```
Int[(e_.**x_)^m_.*Sin[c_.+d_.**x_^n_],x_Symbol] :=
  -e^(n-1)*(e*x)^(m-n+1)*Cos[c+d*x^n]/(d*n) +
  e^n*(m-n+1)/(d*n)*Int[(e*x)^(m-n)*Cos[c+d*x^n],x] /;
FreeQ[{c,d,e},x] && IGtQ[n,0] && LtQ[n,m+1]
```

```
Int[(e_.**x_)^m_.*Cos[c_.+d_.**x_^n_],x_Symbol] :=
  e^(n-1)*(e*x)^(m-n+1)*Sin[c+d*x^n]/(d*n) -
  e^n*(m-n+1)/(d*n)*Int[(e*x)^(m-n)*Sin[c+d*x^n],x] /;
FreeQ[{c,d,e},x] && IGtQ[n,0] && LtQ[n,m+1]
```

3: $\int (e x)^m \sin[c + d x^n] dx$ when $n \in \mathbb{Z}^+ \wedge m < -1$

Reference: CRC 405, A&S 4.3.120

Reference: CRC 406, A&S 4.3.124

Derivation: Integration by parts

Rule: If $n \in \mathbb{Z}^+ \wedge m < -1$, then

$$\int (e x)^m \sin[c + d x^n] dx \rightarrow \frac{(e x)^{m+1} \sin[c + d x^n]}{e (m+1)} - \frac{d n}{e^n (m+1)} \int (e x)^{m+n} \cos[c + d x^n] dx$$

Program code:

```
Int[(e_.**x_)^m_*Sin[c_.+d_.**x_^n_],x_Symbol] :=
  (e*x)^(m+1)*Sin[c+d*x^n]/(e*(m+1)) -
  d*n/(e^n*(m+1))*Int[(e*x)^(m+n)*Cos[c+d*x^n],x] /;
FreeQ[{c,d,e},x] && IGtQ[n,0] && LtQ[m,-1]
```

```
Int[(e_.**x_)^m_*Cos[c_.+d_.**x_^n_],x_Symbol] :=
  (e*x)^(m+1)*Cos[c+d*x^n]/(e*(m+1)) +
  d*n/(e^n*(m+1))*Int[(e*x)^(m+n)*Sin[c+d*x^n],x] /;
FreeQ[{c,d,e},x] && IGtQ[n,0] && LtQ[m,-1]
```

4: $\int (e x)^m \sin[c + d x^n] dx$ when $n \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Basis: $\sin[z] = \frac{1}{2} i e^{-i z} - \frac{1}{2} i e^{i z}$

Basis: $\cos[z] = \frac{1}{2} e^{-i z} + \frac{1}{2} e^{i z}$

Rule: If $n \in \mathbb{Z}^+$, then

$$\int (e x)^m \sin[c + d x^n] dx \rightarrow \frac{i}{2} \int (e x)^m e^{-c i - d i x^n} dx - \frac{i}{2} \int (e x)^m e^{c i + d i x^n} dx$$

Program code:

```
Int[(e.*x_)^m_.*Sin[c_.+d_.*x_^n_],x_Symbol] :=
  I/2*Int[(e*x)^m*E^(-c*I-d*I*x^n),x] - I/2*Int[(e*x)^m*E^(c*I+d*I*x^n),x] /;
FreeQ[{c,d,e,m},x] && IGtQ[n,0]
```

```
Int[(e.*x_)^m_.*Cos[c_.+d_.*x_^n_],x_Symbol] :=
  1/2*Int[(e*x)^m*E^(-c*I-d*I*x^n),x] + 1/2*Int[(e*x)^m*E^(c*I+d*I*x^n),x] /;
FreeQ[{c,d,e,m},x] && IGtQ[n,0]
```

2. $\int (e x)^m (a + b \sin[c + d x^n])^p dx$ when $p > 1$

0: $\int x^m \sin[a + b x^n]^2 dx$

Derivation: Algebraic expansion

Basis: $\sin[z]^2 = \frac{1}{2} - \frac{\cos[2z]}{2}$

Rule:

$$\int x^m \sin[a + b x^n]^2 dx \rightarrow \frac{1}{2} \int x^m dx - \frac{1}{2} \int x^m \cos[2a + 2b x^n] dx$$

Program code:

```
Int[x_^m_.*Sin[a_.+b_.*x_^n_/2]^2,x_Symbol] :=
  1/2*Int[x^m,x] - 1/2*Int[x^m*Cos[2*a+b*x^n],x] /;
FreeQ[{a,b,m,n},x]
```

```
Int[x_^m_.*Cos[a_.+b_.*x_^n_/2]^2,x_Symbol] :=
  1/2*Int[x^m,x] + 1/2*Int[x^m*Cos[2*a+b*x^n],x] /;
FreeQ[{a,b,m,n},x]
```

$$\mathbf{1:} \int x^m \sin[a + b x^n]^p dx \text{ when } p - 1 \in \mathbb{Z}^+ \wedge m + n = 0 \wedge n \neq 1 \wedge n \in \mathbb{Z}$$

Derivation: Integration by parts

Rule: If $p - 1 \in \mathbb{Z}^+ \wedge m + n = 0 \wedge n \neq 1 \wedge n \in \mathbb{Z}$, then

$$\int x^m \sin[a + b x^n]^p dx \rightarrow \frac{x^{m+1} \sin[a + b x^n]^p}{m+1} - \frac{b n p}{m+1} \int \sin[a + b x^n]^{p-1} \cos[a + b x^n] dx$$

Program code:

```
Int[x_^m_.*Sin[a_.+b_.*x_^n_]^p_,x_Symbol] :=
  x^(m+1)*Sin[a+b*x^n]^p/(m+1) -
  b*n*p/(m+1)*Int[Sin[a+b*x^n]^(p-1)*Cos[a+b*x^n],x] /;
FreeQ[{a,b},x] && IGtQ[p,1] && EqQ[m+n,0] && NeQ[n,1] && IntegerQ[n]
```

```
Int[x_^m_.*Cos[a_.+b_.*x_^n_]^p_,x_Symbol] :=
  x^(m+1)*Cos[a+b*x^n]^p/(m+1) +
  b*n*p/(m+1)*Int[Cos[a+b*x^n]^(p-1)*Sin[a+b*x^n],x] /;
FreeQ[{a,b},x] && IGtQ[p,1] && EqQ[m+n,0] && NeQ[n,1] && IntegerQ[n]
```

2: $\int x^m \sin[a + b x^n]^p dx$ when $m - 2n + 1 == 0 \wedge p > 1$

Reference: G&R 2.631.2' special case when $m - 2n + 1 == 0$

Reference: G&R 2.631.3' special case when $m - 2n + 1 == 0$

Rule: If $m - 2n + 1 == 0 \wedge p > 1$, then

$$\int x^m \sin[a + b x^n]^p dx \rightarrow \frac{n \sin[a + b x^n]^p}{b^2 n^2 p^2} - \frac{x^n \cos[a + b x^n] \sin[a + b x^n]^{p-1}}{b n p} + \frac{p-1}{p} \int x^m \sin[a + b x^n]^{p-2} dx$$

Program code:

```
Int[x_^m_.*Sin[a_+b_*x_^n_]^p_,x_Symbol] :=
  n*Sin[a+b*x^n]^p/(b^2*n^2*p^2) -
  x^n*cos[a+b*x^n]*Sin[a+b*x^n]^(p-1)/(b*n*p) +
  (p-1)/p*Int[x^m*Sin[a+b*x^n]^(p-2),x] /;
FreeQ[{a,b,m,n},x] && EqQ[m-2*n+1,0] && GtQ[p,1]
```

```
Int[x_^m_.*Cos[a_+b_*x_^n_]^p_,x_Symbol] :=
  n*Cos[a+b*x^n]^p/(b^2*n^2*p^2) +
  x^n*Sin[a+b*x^n]*Cos[a+b*x^n]^(p-1)/(b*n*p) +
  (p-1)/p*Int[x^m*Cos[a+b*x^n]^(p-2),x] /;
FreeQ[{a,b,m,n},x] && EqQ[m-2*n+1,0] && GtQ[p,1]
```

3: $\int x^m \sin[a + b x^n]^p dx$ when $p > 1 \wedge n \in \mathbb{Z}^+ \wedge m - 2n + 1 \in \mathbb{Z}^+$

Reference: G&R 2.631.2'

Reference: G&R 2.631.3'

Rule: If $p > 1 \wedge n \in \mathbb{Z}^+ \wedge m - 2n + 1 \in \mathbb{Z}^+$, then

$$\int x^m \sin[a + b x^n]^p dx \rightarrow \frac{(m-n+1) x^{m-2n+1} \sin[a + b x^n]^p}{b^2 n^2 p^2} - \frac{x^{m-n+1} \cos[a + b x^n] \sin[a + b x^n]^{p-1}}{b n p} + \frac{p-1}{p} \int x^m \sin[a + b x^n]^{p-2} dx - \frac{(m-n+1)(m-2n+1)}{b^2 n^2 p^2} \int x^{m-2n} \sin[a + b x^n]^p dx$$

Program code:

```
Int[x_^m_.*Sin[a_+b_*x_^n_]^p_,x_Symbol] :=
  (m-n+1)*x^(m-2*n+1)*Sin[a+b*x^n]^p/(b^2*n^2*p^2) -
  x^(m-n+1)*Cos[a+b*x^n]*Sin[a+b*x^n]^(p-1)/(b*n*p) +
  (p-1)/p*Int[x^m*Sin[a+b*x^n]^(p-2),x] -
  (m-n+1)*(m-2*n+1)/(b^2*n^2*p^2)*Int[x^(m-2*n)*Sin[a+b*x^n]^p,x] /;
FreeQ[{a,b},x] && GtQ[p,1] && IGtQ[n,0] && IGtQ[m,2*n-1]
```

```
Int[x_^m_.*Cos[a_+b_*x_^n_]^p_,x_Symbol] :=
  (m-n+1)*x^(m-2*n+1)*Cos[a+b*x^n]^p/(b^2*n^2*p^2) +
  x^(m-n+1)*Sin[a+b*x^n]*Cos[a+b*x^n]^(p-1)/(b*n*p) +
  (p-1)/p*Int[x^m*Cos[a+b*x^n]^(p-2),x] -
  (m-n+1)*(m-2*n+1)/(b^2*n^2*p^2)*Int[x^(m-2*n)*Cos[a+b*x^n]^p,x] /;
FreeQ[{a,b},x] && GtQ[p,1] && IGtQ[n,0] && IGtQ[m,2*n-1]
```

$$4: \int x^m \sin[a + b x^n]^p dx \text{ when } p > 1 \wedge n \in \mathbb{Z}^+ \wedge m + 2n - 1 \in \mathbb{Z}^- \wedge m + n + 1 \neq 0$$

Reference: G&R 2.638.1'

Reference: G&R 2.638.2'

Rule: If $p > 1 \wedge n \in \mathbb{Z}^+ \wedge m + 2n - 1 \in \mathbb{Z}^- \wedge m + n + 1 \neq 0$, then

$$\int x^m \sin[a + b x^n]^p dx \rightarrow \frac{x^{m+1} \sin[a + b x^n]^p}{m+1} - \frac{b n p x^{m+n+1} \cos[a + b x^n] \sin[a + b x^n]^{p-1}}{(m+1)(m+n+1)} - \frac{b^2 n^2 p^2}{(m+1)(m+n+1)} \int x^{m+2n} \sin[a + b x^n]^p dx + \frac{b^2 n^2 p(p-1)}{(m+1)(m+n+1)} \int x^{m+2n} \sin[a + b x^n]^{p-2} dx$$

Program code:

```
Int[x_^m_.*Sin[a_+b_.*x_^n_]^p_,x_Symbol] :=
  x^(m+1)*Sin[a+b*x^n]^p/(m+1) -
  b*n*p*x^(m+n+1)*Cos[a+b*x^n]*Sin[a+b*x^n]^(p-1)/((m+1)*(m+n+1)) -
  b^2*n^2*p^2/((m+1)*(m+n+1))*Int[x^(m+2*n)*Sin[a+b*x^n]^p,x] +
  b^2*n^2*p*(p-1)/((m+1)*(m+n+1))*Int[x^(m+2*n)*Sin[a+b*x^n]^(p-2),x] /;
FreeQ[{a,b},x] && GtQ[p,1] && IGtQ[n,0] && ILtQ[m,-2*n+1] && NeQ[m+n+1,0]
```

```
Int[x_^m_.*Cos[a_+b_.*x_^n_]^p_,x_Symbol] :=
  x^(m+1)*Cos[a+b*x^n]^p/(m+1) +
  b*n*p*x^(m+n+1)*Sin[a+b*x^n]*Cos[a+b*x^n]^(p-1)/((m+1)*(m+n+1)) -
  b^2*n^2*p^2/((m+1)*(m+n+1))*Int[x^(m+2*n)*Cos[a+b*x^n]^p,x] +
  b^2*n^2*p*(p-1)/((m+1)*(m+n+1))*Int[x^(m+2*n)*Cos[a+b*x^n]^(p-2),x] /;
FreeQ[{a,b},x] && GtQ[p,1] && IGtQ[n,0] && ILtQ[m,-2*n+1] && NeQ[m+n+1,0]
```


$$\mathbf{5:} \int (e x)^m (a+b \sin[c+d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{F}$$

Derivation: Integration by substitution

Basis: If $k \in \mathbb{Z}^+$, then $(e x)^m F[x] = \frac{k}{e} \text{Subst}\left[x^{k(m+1)-1} F\left[\frac{x^k}{e}\right], x, (e x)^{1/k}\right] \partial_x (e x)^{1/k}$

Rule: If $p \in \mathbb{Z} \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{F}$, let $k = \text{Denominator}[m]$, then

$$\int (e x)^m (a+b \sin[c+d x^n])^p dx \rightarrow \frac{k}{e} \text{Subst}\left[\int x^{k(m+1)-1} \left(a+b \sin\left[c+\frac{d x^{kn}}{e^n}\right]\right)^p dx, x, (e x)^{1/k}\right]$$

Program code:

```
Int[(e_.**x_)^m_*(a_.+b_.*Sin[c_.+d_.**x_^n_])^p_,x_Symbol] :=
  With[{k=Denominator[m]},
    k/e*Subst[Int[x^(k*(m+1)-1)*(a+b*Sin[c+d*x^(k*n)/e^n])^p,x],x,(e*x)^(1/k)]] /;
  FreeQ[{a,b,c,d,e},x] && IntegerQ[p] && IGtQ[n,0] && FractionQ[m]
```

```
Int[(e_.**x_)^m_*(a_.+b_.*Cos[c_.+d_.**x_^n_])^p_,x_Symbol] :=
  With[{k=Denominator[m]},
    k/e*Subst[Int[x^(k*(m+1)-1)*(a+b*Cos[c+d*x^(k*n)/e^n])^p,x],x,(e*x)^(1/k)]] /;
  FreeQ[{a,b,c,d,e},x] && IntegerQ[p] && IGtQ[n,0] && FractionQ[m]
```

$$\mathbf{6:} \int (e x)^m (a+b \sin[c+d x^n])^p dx \text{ when } p-1 \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}^+$$

Derivation: Algebraic expansion

Rule: If $p-1 \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}^+$, then

$$\int (e x)^m (a+b \sin[c+d x^n])^p dx \rightarrow \int (e x)^m \text{TrigReduce}[(a+b \sin[c+d x^n])^p, x] dx$$

Program code:

```
Int[(e.*x_)^m_.*(a_.+b_.*Sin[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(e*x)^m,(a+b*Sin[c+d*x^n])^p,x],x] /;
FreeQ[{a,b,c,d,e,m},x] && IGtQ[p,1] && IGtQ[n,0]
```

```
Int[(e.*x_)^m_.*(a_.+b_.*Cos[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(e*x)^m,(a+b*Cos[c+d*x^n])^p,x],x] /;
FreeQ[{a,b,c,d,e,m},x] && IGtQ[p,1] && IGtQ[n,0]
```

$$3. \int (e x)^m (a+b \sin[c+d x^n])^p dx \text{ when } p < -1$$

$$1: \int x^m \sin[a+b x^n]^p dx \text{ when } m-2n+1 \equiv 0 \wedge p < -1 \wedge p \neq -2$$

Reference: G&R 2.643.1' special case when $m-2n+1 \equiv 0$

Reference: G&R 2.643.2' special case when $m-2n+1 \equiv 0$

Rule: If $m-2n+1 \equiv 0 \wedge p < -1 \wedge p \neq -2$, then

$$\int x^m \sin[a+b x^n]^p dx \rightarrow \frac{x^n \cos[a+b x^n] \sin[a+b x^n]^{p+1}}{b n (p+1)} - \frac{n \sin[a+b x^n]^{p+2}}{b^2 n^2 (p+1) (p+2)} + \frac{p+2}{p+1} \int x^m \sin[a+b x^n]^{p+2} dx$$

Program code:

```
Int[x_^m_.*Sin[a_+b_.*x_^n_]^p_,x_Symbol] :=
  x^n*Cos[a+b*x^n]*Sin[a+b*x^n]^(p+1)/(b*n*(p+1)) -
  n*Sin[a+b*x^n]^(p+2)/(b^2*n^2*(p+1)*(p+2)) +
  (p+2)/(p+1)*Int[x^m*Sin[a+b*x^n]^(p+2),x] /;
FreeQ[{a,b,m,n},x] && EqQ[m-2*n+1,0] && LtQ[p,-1] && NeQ[p,-2]
```

```
Int[x_^m_.*Cos[a_+b_.*x_^n_]^p_,x_Symbol] :=
  -x^n*Sin[a+b*x^n]*Cos[a+b*x^n]^(p+1)/(b*n*(p+1)) -
  n*Cos[a+b*x^n]^(p+2)/(b^2*n^2*(p+1)*(p+2)) +
  (p+2)/(p+1)*Int[x^m*Cos[a+b*x^n]^(p+2),x] /;
FreeQ[{a,b,m,n},x] && EqQ[m-2*n+1,0] && LtQ[p,-1] && NeQ[p,-2]
```

$$\mathbf{2:} \int x^m \sin[a + b x^n]^p dx \text{ when } (m | n) \in \mathbb{Z} \wedge p < -1 \wedge p \neq -2 \wedge 0 < 2n < m+1$$

Reference: G&R 2.643.1'

Reference: G&R 2.643.2

Rule: If $(m | n) \in \mathbb{Z} \wedge p < -1 \wedge p \neq -2 \wedge 0 < 2n < m+1$, then

$$\int x^m \sin[a + b x^n]^p dx \rightarrow \frac{x^{m-n+1} \cos[a + b x^n] \sin[a + b x^n]^{p+1}}{b n (p+1)} - \frac{(m-n+1) x^{m-2n+1} \sin[a + b x^n]^{p+2}}{b^2 n^2 (p+1)(p+2)} + \frac{p+2}{p+1} \int x^m \sin[a + b x^n]^{p+2} dx + \frac{(m-n+1)(m-2n+1)}{b^2 n^2 (p+1)(p+2)} \int x^{m-2n} \sin[a + b x^n]^{p+2} dx$$

Program code:

```
Int[x_^m_.*Sin[a_.+b_.*x_^n_]^p_,x_Symbol] :=
  x^(m-n+1)*Cos[a+b*x^n]*Sin[a+b*x^n]^(p+1)/(b*n*(p+1)) -
  (m-n+1)*x^(m-2*n+1)*Sin[a+b*x^n]^(p+2)/(b^2*n^2*(p+1)*(p+2)) +
  (p+2)/(p+1)*Int[x^m*Sin[a+b*x^n]^(p+2),x] +
  (m-n+1)*(m-2*n+1)/(b^2*n^2*(p+1)*(p+2))*Int[x^(m-2*n)*Sin[a+b*x^n]^(p+2),x] /;
FreeQ[{a,b},x] && LtQ[p,-1] && NeQ[p,-2] && IGtQ[n,0] && IGtQ[m,2*n-1]
```

```
Int[x_^m_.*Cos[a_.+b_.*x_^n_]^p_,x_Symbol] :=
  -x^(m-n+1)*Sin[a+b*x^n]*Cos[a+b*x^n]^(p+1)/(b*n*(p+1)) -
  (m-n+1)*x^(m-2*n+1)*Cos[a+b*x^n]^(p+2)/(b^2*n^2*(p+1)*(p+2)) +
  (p+2)/(p+1)*Int[x^m*Cos[a+b*x^n]^(p+2),x] +
  (m-n+1)*(m-2*n+1)/(b^2*n^2*(p+1)*(p+2))*Int[x^(m-2*n)*Cos[a+b*x^n]^(p+2),x] /;
FreeQ[{a,b},x] && LtQ[p,-1] && NeQ[p,-2] && IGtQ[n,0] && IGtQ[m,2*n-1]
```

$$2. \int (e x)^m (a + b \sin[c + d x^n])^p dx \text{ when } p \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}^-$$

$$1. \int (e x)^m (a + b \sin[c + d x^n])^p dx \text{ when } p \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{Q}$$

$$\textcolor{red}{1}: \int x^m (a + b \sin[c + d x^n])^p dx \text{ when } p \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{Z}$$

Derivation: Integration by substitution

Basis: If $n \in \mathbb{Z} \wedge m \in \mathbb{Z}$, then $x^m F[x^n] = -\text{Subst}\left[\frac{F[\frac{x^{-n}}{x^{m+2}}], x, \frac{1}{x}}{\partial_x \frac{1}{x}}\right]$

Rule: If $p \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{Z}$, then

$$\int x^m (a + b \sin[c + d x^n])^p dx \rightarrow -\text{Subst}\left[\int \frac{(a + b \sin[c + d x^{-n}])^p}{x^{m+2}} dx, x, \frac{1}{x}\right]$$

Program code:

```
Int[x_^m_.*(a_.+b_.*Sin[c_.+d_.*x_^n_])^p_.,x_Symbol] :=
  -Subst[Int[(a+b*Sin[c+d*x^(-n)])^p/x^(m+2),x],x,1/x] /;
FreeQ[{a,b,c,d},x] && IGtQ[p,0] && ILtQ[n,0] && IntegerQ[m] && EqQ[n,-2]
```

```
Int[x_^m_.*(a_.+b_.*Cos[c_.+d_.*x_^n_])^p_.,x_Symbol] :=
  -Subst[Int[(a+b*Cos[c+d*x^(-n)])^p/x^(m+2),x],x,1/x] /;
FreeQ[{a,b,c,d},x] && IGtQ[p,0] && ILtQ[n,0] && IntegerQ[m] && EqQ[n,-2]
```

$$\mathbf{2:} \int (e x)^m (a+b \sin[c+d x^n])^p dx \text{ when } p \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{F}$$

Derivation: Integration by substitution

Basis: If $n \in \mathbb{Z} \wedge k > 1$, then $(e x)^m F[x^n] = -\frac{k}{e} \text{Subst}\left[\frac{F[e^{-n} x^{-k n}]}{x^{k(m+1)+1}}, x, \frac{1}{(e x)^{1/k}}\right] \partial_x \frac{1}{(e x)^{1/k}}$

Rule: If $p \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{F}$, let $k = \text{Denominator}[m]$, then

$$\int (e x)^m (a+b \sin[c+d x^n])^p dx \rightarrow -\frac{k}{e} \text{Subst}\left[\int \frac{(a+b \sin[c+d e^{-n} x^{-k n}])^p}{x^{k(m+1)+1}} dx, x, \frac{1}{(e x)^{1/k}}\right]$$

Program code:

```
Int[(e_.**x_)^m_*(a_.+b_.*Sin[c_.+d_.**x_^n_])^p_,x_Symbol] :=
  With[{k=Denominator[m]},
    -k/e*Subst[Int[(a+b*Sin[c+d/(e^n*x^(k*n))])^p/x^(k*(m+1)+1),x],x,1/(e*x)^(1/k)]] /;
  FreeQ[{a,b,c,d,e},x] && IGtQ[p,0] && ILtQ[n,0] && FractionQ[m]
```

```
Int[(e_.**x_)^m_*(a_.+b_.*Cos[c_.+d_.**x_^n_])^p_,x_Symbol] :=
  With[{k=Denominator[m]},
    -k/e*Subst[Int[(a+b*Cos[c+d/(e^n*x^(k*n))])^p/x^(k*(m+1)+1),x],x,1/(e*x)^(1/k)]] /;
  FreeQ[{a,b,c,d,e},x] && IGtQ[p,0] && ILtQ[n,0] && FractionQ[m]
```

$$\mathbf{2:} \int (e x)^m (a+b \sin[c+d x^n])^p dx \text{ when } p \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}^- \wedge m \notin \mathbb{Q}$$

Derivation: Piecewise constant extraction and integration by substitution

$$\text{Basis: } \partial_x \left((e x)^m (x^{-1})^m \right) = 0$$

$$\text{Basis: } F[x] = -\text{Subst} \left[\frac{F\left[\frac{x^{-1}}{x^2}\right]}{x^2}, x, \frac{1}{x} \right] \partial_x \frac{1}{x}$$

Rule: If $p \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}^- \wedge m \notin \mathbb{Q}$, then

$$\int (e x)^m (a+b \sin[c+d x^n])^p dx \rightarrow (e x)^m (x^{-1})^m \int \frac{(a+b \sin[c+d x^{-n}])^p}{(x^{-1})^m} dx \rightarrow - (e x)^m (x^{-1})^m \text{Subst} \left[\int \frac{(a+b \sin[c+d x^{-n}])^p}{x^{m+2}} dx, x, \frac{1}{x} \right]$$

Program code:

```
Int[(e.*x_)^m_*(a_.+b_.*Sin[c_.+d_.*x_^n_])^p_.,x_Symbol] :=
  -(e*x)^m*(x^(-1))^m*Subst[Int[(a+b*Sin[c+d*x^(-n)])^p/x^(m+2),x],x,1/x] /;
FreeQ[{a,b,c,d,e,m},x] && IGtQ[p,0] && ILtQ[n,0] && Not[RationalQ[m]]
```

```
Int[(e.*x_)^m_*(a_.+b_.*Cos[c_.+d_.*x_^n_])^p_.,x_Symbol] :=
  -(e*x)^m*(x^(-1))^m*Subst[Int[(a+b*Cos[c+d*x^(-n)])^p/x^(m+2),x],x,1/x] /;
FreeQ[{a,b,c,d,e,m},x] && IGtQ[p,0] && ILtQ[n,0] && Not[RationalQ[m]]
```

$$3. \int (e x)^m (a + b \sin[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge n \in \mathbb{F}$$

$$1: \int x^m (a + b \sin[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge n \in \mathbb{F}$$

Derivation: Integration by substitution

Basis: If $k \in \mathbb{Z}^+$, then $x^m F[x^n] = k \text{Subst}[x^{k(m+1)-1} F[x^{k n}], x, x^{1/k}] \partial_x x^{1/k}$

Rule: If $p \in \mathbb{Z} \wedge n \in \mathbb{F}$, let $k = \text{Denominator}[n]$, then

$$\int x^m (a + b \sin[c + d x^n])^p dx \rightarrow k \text{Subst}\left[\int x^{k(m+1)-1} (a + b \sin[c + d x^{k n}])^p dx, x, x^{1/k}\right]$$

Program code:

```
Int[x_^m_.*(a_.+b_.*Sin[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Module[{k=Denominator[n]},
    k*Subst[Int[x^(k*(m+1)-1)*(a+b*Sin[c+d*x^(k*n)])^p,x],x,x^(1/k)] /;
    FreeQ[{a,b,c,d,m},x] && IntegerQ[p] && FractionQ[n]
```

```
Int[x_^m_.*(a_.+b_.*Cos[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Module[{k=Denominator[n]},
    k*Subst[Int[x^(k*(m+1)-1)*(a+b*Cos[c+d*x^(k*n)])^p,x],x,x^(1/k)] /;
    FreeQ[{a,b,c,d,m},x] && IntegerQ[p] && FractionQ[n]
```


2: $\int (e x)^m (a + b \sin[c + d x^n])^p dx$ when $p \in \mathbb{Z} \wedge n \in \mathbb{F}$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(e x)^m}{x^m} = 0$

Rule: If $p \in \mathbb{Z} \wedge n \in \mathbb{F}$, then

$$\int (e x)^m (a + b \sin[c + d x^n])^p dx \rightarrow \frac{e^{\text{IntPart}[m]} (e x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (a + b \sin[c + d x^n])^p dx$$

Program code:

```
Int[(e*x_)^m*(a_+b_.*Sin[c_+d_.*x_^n_])^p_,x_Symbol] :=
  e^IntPart[m]*(e*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*Sin[c+d*x^n])^p,x] /;
FreeQ[{a,b,c,d,e,m},x] && IntegerQ[p] && FractionQ[n]
```

```
Int[(e*x_)^m*(a_+b_.*Cos[c_+d_.*x_^n_])^p_,x_Symbol] :=
  e^IntPart[m]*(e*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*Cos[c+d*x^n])^p,x] /;
FreeQ[{a,b,c,d,e,m},x] && IntegerQ[p] && FractionQ[n]
```

$$4. \int (e x)^m (a + b \sin[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge m \neq -1 \wedge \frac{n}{m+1} \in \mathbb{Z}^+$$

$$1: \int x^m (a + b \sin[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge m \neq -1 \wedge \frac{n}{m+1} \in \mathbb{Z}^+$$

Derivation: Integration by substitution

Basis: If $\frac{n}{m+1} \in \mathbb{Z}$, then $x^m F[x^n] = \frac{1}{m+1} \text{Subst}[F[x^{\frac{n}{m+1}}], x, x^{m+1}] \partial_x x^{m+1}$

Rule: If $p \in \mathbb{Z} \wedge m \neq -1 \wedge \frac{n}{m+1} \in \mathbb{Z}^+$, then

$$\int x^m (a + b \sin[c + d x^n])^p dx \rightarrow \frac{1}{m+1} \text{Subst}\left[\int (a + b \sin[c + d x^{\frac{n}{m+1}}])^p dx, x, x^{m+1}\right]$$

Program code:

```
Int[x_^m_.*(a_.+b_.*Sin[c_.+d_.*x_^n_])^p_.,x_Symbol] :=
  1/(m+1)*Subst[Int[(a+b*Sin[c+d*x^Simplify[n/(m+1)])]^p,x],x,x^(m+1)] /;
FreeQ[{a,b,c,d,m,n},x] && IntegerQ[p] && NeQ[m,-1] && IGtQ[Simplify[n/(m+1)],0] && Not[IntegerQ[n]]
```

```
Int[x_^m_.*(a_.+b_.*Cos[c_.+d_.*x_^n_])^p_.,x_Symbol] :=
  1/(m+1)*Subst[Int[(a+b*Cos[c+d*x^Simplify[n/(m+1)])]^p,x],x,x^(m+1)] /;
FreeQ[{a,b,c,d,m,n},x] && IntegerQ[p] && NeQ[m,-1] && IGtQ[Simplify[n/(m+1)],0] && Not[IntegerQ[n]]
```

2: $\int (e x)^m (a+b \sin[c+d x^n])^p dx$ when $p \in \mathbb{Z} \wedge m \neq -1 \wedge \frac{n}{m+1} \in \mathbb{Z}^+$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(e x)^m}{x^m} = 0$

Rule: If $p \in \mathbb{Z} \wedge m \neq -1 \wedge \frac{n}{m+1} \in \mathbb{Z}^+$, then

$$\int (e x)^m (a+b \sin[c+d x^n])^p dx \rightarrow \frac{e^{\text{IntPart}[m]} (e x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (a+b \sin[c+d x^n])^p dx$$

Program code:

```
Int[(e*x_)^m_*(a_+b_.*Sin[c_+d_.*x_^n_])^p_,x_Symbol] :=
  e^IntPart[m]*(e*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*Sin[c+d*x^n])^p,x] /;
FreeQ[{a,b,c,d,e,m,n},x] && IntegerQ[p] && NeQ[m,-1] && IGtQ[Simplify[n/(m+1)],0] && Not[IntegerQ[n]]
```

```
Int[(e*x_)^m_*(a_+b_.*Cos[c_+d_.*x_^n_])^p_,x_Symbol] :=
  e^IntPart[m]*(e*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*Cos[c+d*x^n])^p,x] /;
FreeQ[{a,b,c,d,e,m,n},x] && IntegerQ[p] && NeQ[m,-1] && IGtQ[Simplify[n/(m+1)],0] && Not[IntegerQ[n]]
```

5. $\int (e x)^m (a + b \sin[c + d x^n])^p dx$ when $p \in \mathbb{Z}^+$

1: $\int (e x)^m \sin[c + d x^n] dx$

Derivation: Algebraic expansion

Basis: $\sin[z] == \frac{1}{2} i e^{-i z} - \frac{1}{2} i e^{i z}$

Basis: $\cos[z] == \frac{1}{2} e^{-i z} + \frac{1}{2} e^{i z}$

Rule:

$$\int (e x)^m \sin[c + d x^n] dx \rightarrow \frac{i}{2} \int (e x)^m e^{-c i - d i x^n} dx - \frac{i}{2} \int (e x)^m e^{c i + d i x^n} dx$$

Program code:

```
Int[(e.*x_)^m_.*Sin[c_.+d_.*x_^n_],x_Symbol] :=
  I/2*Int[(e*x)^m*E^(-c*I-d*I*x^n),x] - I/2*Int[(e*x)^m*E^(c*I+d*I*x^n),x] /;
FreeQ[{c,d,e,m,n},x]
```

```
Int[(e.*x_)^m_.*Cos[c_.+d_.*x_^n_],x_Symbol] :=
  1/2*Int[(e*x)^m*E^(-c*I-d*I*x^n),x] + 1/2*Int[(e*x)^m*E^(c*I+d*I*x^n),x] /;
FreeQ[{c,d,e,m,n},x]
```

2: $\int (e x)^m (a + b \sin[c + d x^n])^p dx$ when $p \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule: If $p \in \mathbb{Z}^+$, then

$$\int (e x)^m (a + b \sin[c + d x^n])^p dx \rightarrow \int (e x)^m \text{TrigReduce}[(a + b \sin[c + d x^n])^p, x] dx$$

Program code:

```
Int[(e_.**x_)^m_.*(a_.+b_.*Sin[c_.+d_.**x_^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(e*x)^m,(a+b*sin[c+d*x^n])^p,x],x] /;
FreeQ[{a,b,c,d,e,m,n},x] && IGtQ[p,0]
```

```
Int[(e_.**x_)^m_.*(a_.+b_.*Cos[c_.+d_.**x_^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(e*x)^m,(a+b*cos[c+d*x^n])^p,x],x] /;
FreeQ[{a,b,c,d,e,m,n},x] && IGtQ[p,0]
```

X: $\int (e x)^m (a + b \sin[c + d x^n])^p dx$

Rule:

$$\int (e x)^m (a + b \sin[c + d x^n])^p dx \rightarrow \int (e x)^m (a + b \sin[c + d x^n])^p dx$$

Program code:

```
Int[(e_.**x_)^m_.*(a_.+b_.*Sin[c_.+d_.**x_^n_])^p_,x_Symbol] :=
  Unintegrable[(e*x)^m*(a+b*sin[c+d*x^n])^p,x] /;
FreeQ[{a,b,c,d,e,m,n,p},x]
```

```
Int[(e.*x_)^m_.*(a_.+b_.*Cos[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Unintegrable[(e*x)^m*(a+b*Cos[c+d*x^n])^p,x] /;
FreeQ[{a,b,c,d,e,m,n,p},x]
```

N: $\int (e x)^m (a + b \sin[u])^p dx$ when $u == c + d x^n$

Derivation: Algebraic normalization

Rule: If $u == c + d x^n$, then

$$\int (e x)^m (a + b \sin[u])^p dx \rightarrow \int (e x)^m (a + b \sin[c + d x^n])^p dx$$

Program code:

```
Int[(e.*x_)^m_.*(a_.+b_.*Sin[u_])^p_,x_Symbol] :=
  Int[(e*x)^m*(a+b*SIN[ExpandToSum[u,x]])^p,x] /;
FreeQ[{a,b,e,m,p},x] && BinomialQ[u,x] && Not[BinomialMatchQ[u,x]]
```

```
Int[(e.*x_)^m_.*(a_.+b_.*Cos[u_])^p_,x_Symbol] :=
  Int[(e*x)^m*(a+b*COS[ExpandToSum[u,x]])^p,x] /;
FreeQ[{a,b,e,m,p},x] && BinomialQ[u,x] && Not[BinomialMatchQ[u,x]]
```

Rules for integrands of the form $(g + h x)^m (a + b \sin[c + d (e + f x)^n])^p$

1: $\int (g + h x)^m (a + b \sin[c + d (e + f x)^n])^p dx$ when $p \in \mathbb{Z}^+ \wedge \frac{1}{n} \in \mathbb{Z}$

Derivation: Integration by substitution

Basis: If $-1 \leq n \leq 1$, then $(g + h x)^m F[(e + f x)^n] = \frac{1}{n f} \text{Subst}[x^{1/n-1} \left(g - \frac{e h}{f} + \frac{h x^{1/n}}{f}\right)^m F[x], x, (e + f x)^n] \partial_x (e + f x)^n$

Rule: If $p \in \mathbb{Z}^+ \wedge \frac{1}{n} \in \mathbb{Z}$, then

$$\int (g + h x)^m (a + b \sin[c + d (e + f x)^n])^p dx \rightarrow \frac{1}{n f} \text{Subst}\left[\int (a + b \sin[c + d x])^p \text{ExpandIntegrand}\left[x^{1/n-1} \left(g - \frac{e h}{f} + \frac{h x^{1/n}}{f}\right)^m, x\right] dx, x, (e + f x)^n\right]$$

Program code:

```
Int[(g_.+h_.**x_)^m_.*(a_.+b_.*Sin[c_.+d_.*(e_.+f_.**x_)^n_])^p_.,x_Symbol] :=
  1/(n*f)*Subst[Int[ExpandIntegrand[(a+b*SIN[c+d*x])^p,x^(1/n-1)*(g-e*h/f+h*x^(1/n)/f)^m,x],x],x,(e+f*x)^n] /;
FreeQ[{a,b,c,d,e,f,g,h,m},x] && IGtQ[p,0] && IntegerQ[1/n]
```

```
Int[(g_.+h_.**x_)^m_.*(a_.+b_.*Cos[c_.+d_.*(e_.+f_.**x_)^n_])^p_.,x_Symbol] :=
  1/(n*f)*Subst[Int[ExpandIntegrand[(a+b*COS[c+d*x])^p,x^(1/n-1)*(g-e*h/f+h*x^(1/n)/f)^m,x],x],x,(e+f*x)^n] /;
FreeQ[{a,b,c,d,e,f,g,h,m},x] && IGtQ[p,0] && IntegerQ[1/n]
```

x: $\int (g + h x)^m (a + b \sin[c + d (e + f x)^n])^p dx$ when $p \in \mathbb{Z}^+ \wedge m \in \mathbb{Z} \wedge \frac{1}{n} \in \mathbb{Z}$

Derivation: Integration by substitution

Basis: If $m \in \mathbb{Z} \wedge \frac{1}{n} \in \mathbb{Z}$, then $(g + h x)^m F[(e + f x)^n] = \frac{1}{n f^{m+1}} \text{Subst}[x^{1/n-1} (f g - e h + h x^{1/n})^m F[x], x, (e + f x)^n] \partial_x (e + f x)^n$

Rule: If $p \in \mathbb{Z}^+ \wedge m \in \mathbb{Z} \wedge \frac{1}{n} \in \mathbb{Z}$, then

$$\int (g + h x)^m (a + b \sin[c + d (e + f x)^n])^p dx \rightarrow \frac{1}{n f^{m+1}} \text{Subst}\left[\int (a + b \sin[c + d x])^p \text{ExpandIntegrand}[x^{1/n-1} (f g - e h + h x^{1/n})^m, x] dx, x, (e + f x)^n\right]$$

Program code:

```
(* Int[(g_.+h_.*x_)^m_.*(a_.+b_.*Sin[c_.+d_.*(e_.+f_.*x_)^n_])^p_.,x_Symbol] :=
  1/(n*f^(m+1))*Subst[Int[ExpandIntegrand[(a+b*SIN[c+d*x])^p,x^(1/n-1)*(f*g-e*h+h*x^(1/n))^m,x],x,(e+f*x)^n] /;
FreeQ[{a,b,c,d,e,f,g,h},x] && IGtQ[p,0] && IntegerQ[m] && IntegerQ[1/n] *)
```

```
(* Int[(g_.+h_.*x_)^m_.*(a_.+b_.*Cos[c_.+d_.*(e_.+f_.*x_)^n_])^p_.,x_Symbol] :=
  1/(n*f^(m+1))*Subst[Int[ExpandIntegrand[(a+b*cos[c+d*x])^p,x^(1/n-1)*(f*g-e*h+h*x^(1/n))^m,x],x,(e+f*x)^n] /;
FreeQ[{a,b,c,d,e,f,g,h},x] && IGtQ[p,0] && IntegerQ[m] && IntegerQ[1/n] *)
```

2: $\int (g + h x)^m (a + b \sin[c + d (e + f x)^n])^p dx$ when $p \in \mathbb{Z}^+ \wedge m \in \mathbb{Z}^+$

Derivation: Integration by substitution

Basis: If $m \in \mathbb{Z} \wedge k \in \mathbb{Z}^+$, then $(g + h x)^m F[(e + f x)^n] = \frac{k}{f^{m+1}} \text{Subst}[x^{k-1} (f g - e h + h x^k)^m F[x^k], x, (e + f x)^{1/k}] \partial_x (e + f x)^{1/k}$

Rule: If $p \in \mathbb{Z}^+ \wedge m \in \mathbb{Z}^+$, let $k = \text{Denominator}[n]$, then

$$\int (g + h x)^m (a + b \sin[c + d (e + f x)^n])^p dx \rightarrow$$

$$\frac{k}{f^{m+1}} \text{Subst}\left[\int (a+b \sin[c+d x^{kn}])^p \text{ExpandIntegrand}\left[x^{k-1} (f g - e h + h x^k)^m, x\right] dx, x, (e+f x)^{1/k}\right]$$

Program code:

```
Int[(g_.+h_.**x_)^m_.*(a_.+b_.*Sin[c_.+d_.*(e_.+f_.**x_)^n_])^p_.,x_Symbol] :=
Module[{k=If[FractionQ[n],Denominator[n],1]},
k/f^(m+1)*Subst[Int[ExpandIntegrand[(a+b*Sin[c+d*x^(k*n)])^p,x^(k-1)*(f*g-e*h+h*x^k)^m,x],x],x,(e+f*x)^(1/k)]] /;
FreeQ[{a,b,c,d,e,f,g,h},x] && IGtQ[p,0] && IGtQ[m,0]
```

```
Int[(g_.+h_.**x_)^m_.*(a_.+b_.*Cos[c_.+d_.*(e_.+f_.**x_)^n_])^p_.,x_Symbol] :=
Module[{k=If[FractionQ[n],Denominator[n],1]},
k/f^(m+1)*Subst[Int[ExpandIntegrand[(a+b*Cos[c+d*x^(k*n)])^p,x^(k-1)*(f*g-e*h+h*x^k)^m,x],x],x,(e+f*x)^(1/k)]] /;
FreeQ[{a,b,c,d,e,f,g,h},x] && IGtQ[p,0] && IGtQ[m,0]
```

3: $\int (g + h x)^m (a + b \sin[c + d (e + f x)^n])^p dx$ when $p \in \mathbb{Z}^+ \wedge f g - e h = 0$

Derivation: Integration by substitution

Basis: If $f g - e h = 0$, then $(g + h x)^m F[e + f x] = \frac{1}{f} \text{Subst}\left[\left(\frac{h x}{f}\right)^m F[x], x, e + f x\right] \partial_x (e + f x)$

Note: If $p \in \mathbb{Z}^+$, then $\left(\frac{h x}{f}\right)^m (a + b \sin[c + d x^n])^p$ is integrable wrt x .

Rule: If $p \in \mathbb{Z}^+ \wedge f g - e h = 0$, then

$$\int (g + h x)^m (a + b \sin[c + d (e + f x)^n])^p dx \rightarrow \frac{1}{f} \text{Subst}\left[\int \left(\frac{h x}{f}\right)^m (a + b \sin[c + d x^n])^p dx, x, e + f x\right]$$

Program code:

```
Int[(g_.+h_.**x_)^m_.*(a_.+b_.*Sin[c_.+d_.*(e_.+f_.**x_)^n_])^p_.,x_Symbol] :=
1/f*Subst[Int[(h*x/f)^m*(a+b*Sin[c+d*x^n])^p,x],x,e+f*x] /;
FreeQ[{a,b,c,d,e,f,g,h,m},x] && IGtQ[p,0] && EqQ[f*g-e*h,0]
```

```
Int[(g_.+h_.**x_)^m_.*(a_.+b_.*Cos[c_.+d_.*(e_.+f_.**x_)^n_])^p_,x_Symbol] :=
  1/f*Subst[Int[(h*x/f)^m*(a+b*Cos[c+d*x^n])^p,x],x,e+f*x] /;
FreeQ[{a,b,c,d,e,f,g,h,m},x] && IGtQ[p,0] && EqQ[f*g-e*h,0]
```

X: $\int (g + h x)^m (a + b \sin[c + d (e + f x)^n])^p dx$

Rule:

$$\int (g + h x)^m (a + b \sin[c + d (e + f x)^n])^p dx \rightarrow \int (g + h x)^m (a + b \sin[c + d (e + f x)^n])^p dx$$

Program code:

```
Int[(g_.+h_.**x_)^m_.*(a_.+b_.*Sin[c_.+d_.*(e_.+f_.**x_)^n_])^p_,x_Symbol] :=
  Unintegrable[(g+h*x)^m*(a+b*SIN[c+d*(e+f*x)^n])^p,x] /;
FreeQ[{a,b,c,d,e,f,g,h,m,n,p},x]
```

```
Int[(g_.+h_.**x_)^m_.*(a_.+b_.*Cos[c_.+d_.*(e_.+f_.**x_)^n_])^p_,x_Symbol] :=
  Unintegrable[(g+h*x)^m*(a+b*COS[c+d*(e+f*x)^n])^p,x] /;
FreeQ[{a,b,c,d,e,f,g,h,m,n,p},x]
```

N: $\int v^m (a + b \sin[c + d u^n])^p dx$ when $u == e + f x \wedge v == g + h x$

Derivation: Algebraic normalization

Rule: If $u == e + f x \wedge v == g + h x$, then

$$\int v^m (a + b \sin[c + d u^n])^p dx \rightarrow \int (g + h x)^m (a + b \sin[c + d (e + f x)^n])^p dx$$

Program code:

```
Int[v_^m_.*(a_.+b_.*Sin[c_.+d_.*u_^n_])^p_,x_Symbol] :=
  Int[ExpandToSum[v,x]^m*(a+b*Sin[c+d*ExpandToSum[u,x]^n])^p,x] /;
FreeQ[{a,b,c,d,m,n,p},x] && LinearQ[u,x] && LinearQ[v,x] && Not[LinearMatchQ[u,x] && LinearMatchQ[v,x]]
```

```
Int[v_^m_.*(a_.+b_.*Cos[c_.+d_.*u_^n_])^p_,x_Symbol] :=
  Int[ExpandToSum[v,x]^m*(a+b*Cos[c+d*ExpandToSum[u,x]^n])^p,x] /;
FreeQ[{a,b,c,d,m,n,p},x] && LinearQ[u,x] && LinearQ[v,x] && Not[LinearMatchQ[u,x] && LinearMatchQ[v,x]]
```

Rules for integrands of the form $x^m \sin[a + b x^n]^p \cos[a + b x^n]$

1. $\int x^m \sin[a + b x^n]^p \cos[a + b x^n] dx$ when $p \neq -1$

1: $\int x^{n-1} \sin[a + b x^n]^p \cos[a + b x^n] dx$ when $p \neq -1$

Derivation: Power rule for integration

Rule: If $p \neq -1$, then

$$\int x^{n-1} \sin[a + b x^n]^p \cos[a + b x^n] dx \rightarrow \frac{\sin[a + b x^n]^{p+1}}{b n (p+1)}$$

Program code:

```
Int[x_^m_.*Sin[a_.+b_.*x_^n_.]^p_.*Cos[a_.+b_.*x_^n_.],x_Symbol] :=
  Sin[a+b*x^n]^(p+1)/(b*n*(p+1)) /;
FreeQ[{a,b,m,n,p},x] && EqQ[m,n-1] && NeQ[p,-1]
```

```
Int[x_^m_.*Cos[a_.+b_.*x_^n_.]^p_.*Sin[a_.+b_.*x_^n_.],x_Symbol] :=
  -Cos[a+b*x^n]^(p+1)/(b*n*(p+1)) /;
FreeQ[{a,b,m,n,p},x] && EqQ[m,n-1] && NeQ[p,-1]
```

2: $\int x^m \sin[a + b x^n]^p \cos[a + b x^n] dx$ when $0 < n < m + 1 \wedge p \neq -1$

Reference: G&R 2.645.6

Reference: G&R 2.645.3

Derivation: Integration by parts

Basis: $x^m \sin[a + b x^n]^p \cos[a + b x^n] = x^{m-n+1} \partial_x \frac{\sin[a + b x^n]^{p+1}}{b n (p+1)}$

Rule: If $0 < n < m + 1 \wedge p \neq -1$, then

$$\int x^m \sin[a + b x^n]^p \cos[a + b x^n] dx \rightarrow \frac{x^{m-n+1} \sin[a + b x^n]^{p+1}}{b n (p+1)} - \frac{m-n+1}{b n (p+1)} \int x^{m-n} \sin[a + b x^n]^{p+1} dx$$

Program code:

```
Int[x_^m_.*Sin[a_+b_*x_^n_.]^p_.*Cos[a_+b_*x_^n_.],x_Symbol] :=
  x^(m-n+1)*Sin[a+b*x^n]^(p+1)/(b*n*(p+1)) -
  (m-n+1)/(b*n*(p+1))*Int[x^(m-n)*Sin[a+b*x^n]^(p+1),x] /;
FreeQ[{a,b,p},x] && LtQ[0,n,m+1] && NeQ[p,-1]
```

```
Int[x_^m_.*Cos[a_+b_*x_^n_.]^p_.*Sin[a_+b_*x_^n_.],x_Symbol] :=
  -x^(m-n+1)*Cos[a+b*x^n]^(p+1)/(b*n*(p+1)) +
  (m-n+1)/(b*n*(p+1))*Int[x^(m-n)*Cos[a+b*x^n]^(p+1),x] /;
FreeQ[{a,b,p},x] && LtQ[0,n,m+1] && NeQ[p,-1]
```