

0: $\int u \, dx$

```
If[TrueQ[$LoadShowSteps],

Int[u_,x_Symbol] :=
  Int[DeactivateTrig[u,x],x] /;
SimplifyFlag && FunctionOfTrigOfLinearQ[u,x],

Int[u_,x_Symbol] :=
  Int[DeactivateTrig[u,x],x] /;
FunctionOfTrigOfLinearQ[u,x]]
```

Rules for integrands of the form $(a \sin[e + f x])^m (b \operatorname{Trg}[e + f x])^n$

1. $\int (a \sin[e + f x])^m (b \cos[e + f x])^n \, dx$

1: $\int (a \sin[e + f x])^m (b \cos[e + f x])^n \, dx$ when $m + n + 2 == 0 \wedge m \neq -1$

Reference: G&R 2.510.3, CRC 334a, A&S 4.3.128b with $m + n + 2 == 0$

Reference: G&R 2.510.6, CRC 334b, A&S 4.3.128a with $m + n + 2 == 0$

Rule: If $m + n + 2 == 0 \wedge m \neq -1$, then

$$\int (a \sin[e + f x])^m (b \cos[e + f x])^n \, dx \rightarrow \frac{(a \sin[e + f x])^{m+1} (b \cos[e + f x])^{n+1}}{a b f (m+1)}$$

Program code:

```
Int[(a_.*sin[e_.+f_.*x_])^m_.*(b_.*cos[e_.+f_.*x_])^n_,x_Symbol] :=
  (a*Sin[e+f*x])^(m+1)*(b*Cos[e+f*x])^(n+1)/(a*b*f*(m+1)) /;
FreeQ[{a,b,e,f,m,n},x] && EqQ[m+n+2,0] && NeQ[m,-1]
```

2: $\int (a \sin[e+fx])^m \cos[e+fx]^n dx$ when $\frac{n-1}{2} \in \mathbb{Z}$

Derivation: Integration by substitution

Basis: If $\frac{n-1}{2} \in \mathbb{Z}$, then

$$(a \sin[e+fx])^m \cos[e+fx]^n = \frac{1}{af} \operatorname{Subst} \left[x^m \left(1 - \frac{x^2}{a^2} \right)^{\frac{n-1}{2}}, x, a \sin[e+fx] \right] \partial_x (a \sin[e+fx])$$

Rule: If $\frac{n-1}{2} \in \mathbb{Z}$, then

$$\int (a \sin[e+fx])^m \cos[e+fx]^n dx \rightarrow \frac{1}{af} \operatorname{Subst} \left[\int x^m \left(1 - \frac{x^2}{a^2} \right)^{\frac{n-1}{2}} dx, x, a \sin[e+fx] \right]$$

Program code:

```
Int[(a_.sin[e_.+f_.*x_])^m_.cos[e_.+f_.*x_]^n_,x_Symbol] :=
  1/(a*f)*Subst[Int[x^m*(1-x^2/a^2)^( (n-1)/2 ),x],x,a*Sin[e+f*x]] /;
FreeQ[{a,e,f,m},x] && IntegerQ[(n-1)/2] && Not[IntegerQ[(m-1)/2]] && LtQ[0,m,n]
```

```
Int[(a_.cos[e_.+f_.*x_])^m_.sin[e_.+f_.*x_]^n_,x_Symbol] :=
  -1/(a*f)*Subst[Int[x^m*(1-x^2/a^2)^( (n-1)/2 ),x],x,a*Cos[e+f*x]] /;
FreeQ[{a,e,f,m},x] && IntegerQ[(n-1)/2] && Not[IntegerQ[(m-1)/2]] && GtQ[m,0] && LeQ[m,n]
```

$$3. \int (a \sin[e+fx])^m (b \cos[e+fx])^n dx \text{ when } m > 1$$

$$1: \int (a \sin[e+fx])^m (b \cos[e+fx])^n dx \text{ when } m > 1 \wedge n < -1$$

Reference: G&R 2.510.1

Reference: G&R 2.510.4

Rule: If $m > 1 \wedge n < -1$, then

$$\int (a \sin[e+fx])^m (b \cos[e+fx])^n dx \rightarrow -\frac{a (a \sin[e+fx])^{m-1} (b \cos[e+fx])^{n+1}}{b f (n+1)} + \frac{a^2 (m-1)}{b^2 (n+1)} \int (a \sin[e+fx])^{m-2} (b \cos[e+fx])^{n+2} dx$$

Program code:

```
Int[(a_.sin[e_.+f_.x_])^m_*(b_.cos[e_.+f_.x_])^n_,x_Symbol] :=
  -a*(a*sin[e+f*x])^(m-1)*(b*cos[e+f*x])^(n+1)/(b*f*(n+1)) +
  a^2*(m-1)/(b^2*(n+1))*Int[(a*sin[e+f*x])^(m-2)*(b*cos[e+f*x])^(n+2),x] /;
FreeQ[{a,b,e,f},x] && GtQ[m,1] && LtQ[n,-1] && (IntegersQ[2*m,2*n] || EqQ[m+n,0])
```

```
Int[(a_.cos[e_.+f_.x_])^m_*(b_.sin[e_.+f_.x_])^n_,x_Symbol] :=
  a*(a*cos[e+f*x])^(m-1)*(b*sin[e+f*x])^(n+1)/(b*f*(n+1)) +
  a^2*(m-1)/(b^2*(n+1))*Int[(a*cos[e+f*x])^(m-2)*(b*sin[e+f*x])^(n+2),x] /;
FreeQ[{a,b,e,f},x] && GtQ[m,1] && LtQ[n,-1] && (IntegersQ[2*m,2*n] || EqQ[m+n,0])
```

2: $\int (a \sin[e+fx])^m (b \cos[e+fx])^n dx$ when $m > 1 \wedge m+n \neq 0$

Reference: G&R 2.510.2, CRC 323b, A&S 4.3.127b

Reference: G&R 2.510.5, CRC 323a, A&S 4.3.127a

Rule: If $m > 1 \wedge m+n \neq 0$, then

$$\int (a \sin[e+fx])^m (b \cos[e+fx])^n dx \rightarrow -\frac{a (a \sin[e+fx])^{m-1} (b \cos[e+fx])^{n+1}}{b f (m+n)} + \frac{a^2 (m-1)}{m+n} \int (a \sin[e+fx])^{m-2} (b \cos[e+fx])^n dx$$

Program code:

```
Int[(a_.sin[e_.+f_.x_])^m_*(b_.cos[e_.+f_.x_])^n_,x_Symbol] :=
  -a*(b*cos[e+f*x])^(n+1)*(a*sin[e+f*x])^(m-1)/(b*f*(m+n)) +
  a^2*(m-1)/(m+n)*Int[(b*cos[e+f*x])^n*(a*sin[e+f*x])^(m-2),x] /;
FreeQ[{a,b,e,f,n},x] && GtQ[m,1] && NeQ[m+n,0] && IntegersQ[2*m,2*n]
```

```
Int[(a_.cos[e_.+f_.x_])^m_*(b_.sin[e_.+f_.x_])^n_,x_Symbol] :=
  a*(b*sin[e+f*x])^(n+1)*(a*cos[e+f*x])^(m-1)/(b*f*(m+n)) +
  a^2*(m-1)/(m+n)*Int[(b*sin[e+f*x])^n*(a*cos[e+f*x])^(m-2),x] /;
FreeQ[{a,b,e,f,n},x] && GtQ[m,1] && NeQ[m+n,0] && IntegersQ[2*m,2*n]
```

4: $\int (a \sin[e+fx])^m (b \cos[e+fx])^n dx$ when $m < -1$

Reference: G&R 2.510.3, CRC 334a, A&S 4.3.128b

Reference: G&R 2.510.6, CRC 334b, A&S 4.3.128a

Rule: If $m < -1$, then

$$\int (a \sin[e+fx])^m (b \cos[e+fx])^n dx \rightarrow \frac{(a \sin[e+fx])^{m+1} (b \cos[e+fx])^{n+1}}{a b f (m+1)} + \frac{m+n+2}{a^2 (m+1)} \int (a \sin[e+fx])^{m+2} (b \cos[e+fx])^n dx$$

Program code:

```
Int[(a_.sin[e_.+f_.x_])^m_*(b_.cos[e_.+f_.x_])^n_,x_Symbol] :=
  (b*cos[e+f*x])^(n+1)*(a*sin[e+f*x])^(m+1)/(a*b*f*(m+1)) +
  (m+n+2)/(a^2*(m+1))*Int[(b*cos[e+f*x])^n*(a*sin[e+f*x])^(m+2),x] /;
FreeQ[{a,b,e,f,n},x] && LtQ[m,-1] && IntegersQ[2*m,2*n]
```

```
Int[(a_.cos[e_.+f_.x_])^m_*(b_.sin[e_.+f_.x_])^n_,x_Symbol] :=
  -(b*sin[e+f*x])^(n+1)*(a*cos[e+f*x])^(m+1)/(a*b*f*(m+1)) +
  (m+n+2)/(a^2*(m+1))*Int[(b*sin[e+f*x])^n*(a*cos[e+f*x])^(m+2),x] /;
FreeQ[{a,b,e,f,n},x] && LtQ[m,-1] && IntegersQ[2*m,2*n]
```

5: $\int \sqrt{a \sin[e+fx]} \sqrt{b \cos[e+fx]} dx$

Derivation: Piecewise constant extraction

■ Basis: $\partial_x \frac{\sqrt{a \sin[e+fx]} \sqrt{b \cos[e+fx]}}{\sqrt{\sin[2e+2fx]}} == 0$

Rule:

$$\int \sqrt{a \sin[e+fx]} \sqrt{b \cos[e+fx]} dx \rightarrow \frac{\sqrt{a \sin[e+fx]} \sqrt{b \cos[e+fx]}}{\sqrt{\sin[2e+2fx]}} \int \sqrt{\sin[2e+2fx]} dx$$

Program code:

```
Int[Sqrt[a_.sin[e_.+f_.x_]]*Sqrt[b_.cos[e_.+f_.x_]],x_Symbol] :=
  Sqrt[a*sin[e+f*x]]*Sqrt[b*cos[e+f*x]]/Sqrt[Sin[2*e+2*f*x]]*Int[Sqrt[Sin[2*e+2*f*x]],x] /;
FreeQ[{a,b,e,f},x]
```

6: $\int \frac{1}{\sqrt{a \sin[e+fx]} \sqrt{b \cos[e+fx]}} dx$

Derivation: Piecewise constant extraction

■ Basis: $\partial_x \frac{\sqrt{\sin[2e+2fx]}}{\sqrt{a \sin[e+fx]} \sqrt{b \cos[e+fx]}} == 0$

Rule:

$$\int \frac{1}{\sqrt{a \sin[e+fx]} \sqrt{b \cos[e+fx]}} dx \rightarrow \frac{\sqrt{\sin[2e+2fx]}}{\sqrt{a \sin[e+fx]} \sqrt{b \cos[e+fx]}} \int \frac{1}{\sqrt{\sin[2e+2fx]}} dx$$

Program code:

```
Int[1/(Sqrt[a_.sin[e_.+f_.x_]]*Sqrt[b_.cos[e_.+f_.x_]]),x_Symbol] :=
  Sqrt[Sin[2*e+2*f*x]]/(Sqrt[a*sin[e+fx]]*Sqrt[b*cos[e+fx]])*Int[1/Sqrt[Sin[2*e+2*f*x]],x] /;
FreeQ[{a,b,e,f},x]
```

x: $\int (a \sin[e+fx])^m (b \cos[e+fx])^n dx$ when $m+n=0$

Derivation: Piecewise constant extraction

Basis: If $m+n=0$, then $\partial_x \frac{(a \sin[e+fx])^m (b \cos[e+fx])^n}{(a \tan[e+fx])^m} = 0$

Rule: If $m+n=0$, then

$$\int (a \sin[e+fx])^m (b \cos[e+fx])^n dx \rightarrow \frac{(a \sin[e+fx])^m (b \cos[e+fx])^n}{(a \tan[e+fx])^m} \int (a \tan[e+fx])^m dx$$

Program code:

```
(* Int[(a_.sin[e_.+f_.x_])^m_*(b_.cos[e_.+f_.x_])^n_,x_Symbol] :=
  (a*sin[e+fx])^m*(b*cos[e+fx])^n/(a*tan[e+fx])^m*Int[(a*tan[e+fx])^m,x] /;
FreeQ[{a,b,e,f,m,n},x] && EqQ[m+n,0] *)
```

7: $\int (a \sin[e+fx])^m (b \cos[e+fx])^n dx$ when $m+n=0 \wedge 0 < m < 1$

Derivation: Integration by substitution

Basis: If $-1 < m < 1$, let $k \rightarrow \text{Denominator}[m]$, then

$$\frac{(a \sin[e+fx])^m}{(b \cos[e+fx])^m} == \frac{k a b}{f} \text{Subst} \left[\frac{x^{k(m+1)-1}}{a^2 + b^2 x^{2k}}, x, \frac{(a \sin[e+fx])^{1/k}}{(b \cos[e+fx])^{1/k}} \right] \partial_x \frac{(a \sin[e+fx])^{1/k}}{(b \cos[e+fx])^{1/k}}$$

Note: This rule is analogous to the rule for integrands of the form $(a \tan[e+fx])^m$ when $-1 < m < 1$.

Rule: If $m + n == 0 \wedge 0 < m < 1$, let $k \rightarrow \text{Denominator}[m]$, then

$$\int \frac{(a \sin[e+fx])^m}{(b \cos[e+fx])^m} dx \rightarrow \frac{k a b}{f} \text{Subst} \left[\int \frac{x^{k(m+1)-1}}{a^2 + b^2 x^{2k}} dx, x, \frac{(a \sin[e+fx])^{1/k}}{(b \cos[e+fx])^{1/k}} \right]$$

Program code:

```
Int[(a_.sin[e_.+f_.x_])^m_*(b_.cos[e_.+f_.x_])^n_,x_Symbol] :=
  With[{k=Denominator[m]},
    k*a*b/f*Subst[Int[x^(k*(m+1)-1)/(a^2+b^2*x^(2*k)),x],x,(a*Sin[e+f*x])^(1/k)/(b*Cos[e+f*x])^(1/k)]] /;
  FreeQ[{a,b,e,f},x] && EqQ[m+n,0] && GtQ[m,0] && LtQ[m,1]
```

```
Int[(a_.cos[e_.+f_.x_])^m_*(b_.sin[e_.+f_.x_])^n_,x_Symbol] :=
  With[{k=Denominator[m]},
    -k*a*b/f*Subst[Int[x^(k*(m+1)-1)/(a^2+b^2*x^(2*k)),x],x,(a*Cos[e+f*x])^(1/k)/(b*Sin[e+f*x])^(1/k)]] /;
  FreeQ[{a,b,e,f},x] && EqQ[m+n,0] && GtQ[m,0] && LtQ[m,1]
```

8: $\int (a \sin[e+fx])^m (b \cos[e+fx])^n dx$

Derivation: Piecewise constant extraction and integration by substitution

Basis: $\partial_x \frac{(b \cos[e+fx])^{n-1}}{(\cos[e+fx]^2)^{\frac{n-1}{2}}} == 0$

Basis: $\cos[e+fx] F[a \sin[e+fx]] == \frac{1}{a f} \text{Subst}[F[x], x, a \sin[e+fx]] \partial_x (a \sin[e+fx])$

Note: If $\frac{n}{2} \in \mathbb{Z} \wedge 3m \in \mathbb{Z} \wedge -1 < m < 1$, integration of $x^m (1 - \frac{x^2}{a^2})^{\frac{n-1}{2}}$ results in a complicated antiderivative involving

elliptic integrals and the imaginary unit.

Rule:

$$\begin{aligned} \int (a \sin[e+fx])^m (b \cos[e+fx])^n dx &\rightarrow \frac{b^{2 \operatorname{IntPart}[\frac{n-1}{2}]+1} (b \cos[e+fx])^{2 \operatorname{FracPart}[\frac{n-1}{2}]} \int \cos[e+fx] (a \sin[e+fx])^m (1 - \sin[e+fx]^2)^{\frac{n-1}{2}} dx}{(\cos[e+fx]^2)^{\operatorname{FracPart}[\frac{n-1}{2}]}} \\ &\rightarrow \frac{b^{2 \operatorname{IntPart}[\frac{n-1}{2}]+1} (b \cos[e+fx])^{2 \operatorname{FracPart}[\frac{n-1}{2}]}}{a f (\cos[e+fx]^2)^{\operatorname{FracPart}[\frac{n-1}{2}]}} \operatorname{Subst}\left[\int x^m \left(1 - \frac{x^2}{a^2}\right)^{\frac{n-1}{2}} dx, x, a \sin[e+fx]\right] \\ &\rightarrow \frac{b^{2 \operatorname{IntPart}[\frac{n-1}{2}]+1} (b \cos[e+fx])^{2 \operatorname{FracPart}[\frac{n-1}{2}]} (a \sin[e+fx])^{m+1}}{a f (m+1) (\cos[e+fx]^2)^{\operatorname{FracPart}[\frac{n-1}{2}]}} \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1-n}{2}, \frac{3+m}{2}, \sin[e+fx]^2\right] \end{aligned}$$

Program code:

```
(* Int[(a_.sin[e_.+f_.*x_])^m_*(b_.cos[e_.+f_.*x_])^n_,x_Symbol] :=
  b^(2*IntPart[(n-1)/2]+1)*(b*cos[e+f*x])^(2*FracPart[(n-1)/2])/(a*f*(Cos[e+f*x]^2)^FracPart[(n-1)/2])*
  Subst[Int[x^m*(1-x^2/a^2)^( (n-1)/2),x],x,a*Sin[e+f*x]] /;
FreeQ[{a,b,e,f,m,n},x] && (RationalQ[n] || Not[RationalQ[m]] && (EqQ[b,1] || NeQ[a,1])) *)
```

```
(* Int[(a_.cos[e_.+f_.*x_])^m_*(b_.sin[e_.+f_.*x_])^n_,x_Symbol] :=
  -b^(2*IntPart[(n-1)/2]+1)*(b*sin[e+f*x])^(2*FracPart[(n-1)/2])/(a*f*(Sin[e+f*x]^2)^FracPart[(n-1)/2])*
  Subst[Int[x^m*(1-x^2/a^2)^( (n-1)/2),x],x,a*cos[e+f*x]] /;
FreeQ[{a,b,e,f,m,n},x] *)
```

```
Int[(a_.cos[e_.+f_.*x_])^m_*(b_.sin[e_.+f_.*x_])^n_,x_Symbol] :=
  -b^(2*IntPart[(n-1)/2]+1)*(b*sin[e+f*x])^(2*FracPart[(n-1)/2])*(a*cos[e+f*x])^(m+1)/(a*f*(m+1)*(Sin[e+f*x]^2)^FracPart[(n-1)/2])
  Hypergeometric2F1[(1+m)/2,(1-n)/2,(3+m)/2,Cos[e+f*x]^2] /;
FreeQ[{a,b,e,f,m,n},x] && SimplerQ[n,m]
```

```

Int[ (a_.*sin[e_.+f_.**x_])^m_.*(b_.*cos[e_.+f_.**x_])^n_,x_Symbol] :=
  b^(2*IntPart[(n-1)/2]+1)*(b*cos[e+f*x])^(2*FracPart[(n-1)/2])*(a*sin[e+f*x])^(m+1)/(a*f*(m+1)*(Cos[e+f*x]^2)^FracPart[(n-1)/2])*
  Hypergeometric2F1[(1+m)/2,(1-n)/2,(3+m)/2,Sin[e+f*x]^2] /;
FreeQ[{a,b,e,f,m,n},x]

```

2. $\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx$ when $m \notin \mathbb{Z} \wedge n \notin \mathbb{Z}$

1: $\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx$ when $m-n+2 = 0 \wedge m \neq -1$

Rule: If $m-n+2 = 0 \wedge m \neq -1$, then

$$\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx \rightarrow \frac{b (a \sin[e+fx])^{m+1} (b \sec[e+fx])^{n-1}}{a f (m+1)}$$

Program code:

```

Int[ (a_.*sin[e_.+f_.**x_])^m_.*(b_.*sec[e_.+f_.**x_])^n_,x_Symbol] :=
  b*(a*sin[e+f*x])^(m+1)*(b*sec[e+f*x])^(n-1)/(a*f*(m+1)) /;
FreeQ[{a,b,e,f,m,n},x] && EqQ[m-n+2,0] && NeQ[m,-1]

```

2. $\int (a \sin[e+fx])^m (b \operatorname{Sec}[e+fx])^n dx$ when $n > 1$

1: $\int (a \sin[e+fx])^m (b \operatorname{Sec}[e+fx])^n dx$ when $n > 1 \wedge m > 1$

Rule: If $n > 1 \wedge m > 1$, then

$$\int (a \sin[e+fx])^m (b \operatorname{Sec}[e+fx])^n dx \rightarrow \frac{a b (a \sin[e+fx])^{m-1} (b \operatorname{Sec}[e+fx])^{n-1}}{f (n-1)} - \frac{a^2 b^2 (m-1)}{n-1} \int (a \sin[e+fx])^{m-2} (b \operatorname{Sec}[e+fx])^{n-2} dx$$

Program code:

```
Int[(a_.sin[e_.+f_.x_])^m_*(b_.sec[e_.+f_.x_])^n_,x_Symbol] :=
  a*b*(a*sin[e+f*x])^(m-1)*(b*Sec[e+f*x])^(n-1)/(f*(n-1)) -
  a^2*b^2*(m-1)/(n-1)*Int[(a*sin[e+f*x])^(m-2)*(b*Sec[e+f*x])^(n-2),x] /;
FreeQ[{a,b,e,f},x] && GtQ[n,1] && GtQ[m,1] && IntegersQ[2*m,2*n]
```

2: $\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx$ when $n > 1$

Rule: If $n > 1$, then

$$\frac{n-1}{b^2 (m-n+2)} \int (a \sin[e+fx])^m (b \sec[e+fx])^n dx \rightarrow$$

$$\frac{b (a \sin[e+fx])^{m+1} (b \sec[e+fx])^{n-1}}{a f (n-1)} - \frac{b^2 (m-n+2)}{n-1} \int (a \sin[e+fx])^m (b \sec[e+fx])^{n-2} dx$$

Program code:

```
Int[(a_.sin[e_.+f_.*x_])^m_*(b_.sec[e_.+f_.*x_])^n_,x_Symbol] :=
  (a*sin[e+f*x])^(m+1)*(b*sec[e+f*x])^(n+1)/(a*b*f*(m-n)) -
  (n+1)/(b^2*(m-n))*Int[(a*sin[e+f*x])^m*(b*sec[e+f*x])^(n+2),x] /;
FreeQ[{a,b,e,f,m},x] && GtQ[n,1] && IntegersQ[2*m,2*n]
```

3. $\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx$ when $n < -1$

1: $\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx$ when $n < -1 \wedge m < -1$

Rule: If $n < -1 \wedge m < -1$, then

$$\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx \rightarrow \frac{(a \sin[e+fx])^{m+1} (b \sec[e+fx])^{n+1}}{a b f (m+1)} - \frac{n+1}{a^2 b^2 (m+1)} \int (a \sin[e+fx])^{m+2} (b \sec[e+fx])^{n+2} dx$$

Program code:

```
Int[(a_.*sin[e_.+f_.*x_])^m_*(b_.*sec[e_.+f_.*x_])^n_,x_Symbol] :=
  (a*sin[e+f*x])^(m+1)*(b*sec[e+f*x])^(n+1)/(a*b*f*(m+1)) -
  (n+1)/(a^2*b^2*(m+1))*Int[(a*sin[e+f*x])^(m+2)*(b*sec[e+f*x])^(n+2),x] /;
FreeQ[{a,b,e,f},x] && LtQ[n,-1] && LtQ[m,-1] && IntegersQ[2*m,2*n]
```

2: $\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx$ when $n < -1 \wedge m-n \neq 0$

Rule: If $n < -1 \wedge m-n \neq 0$, then

$$\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx \rightarrow \frac{(a \sin[e+fx])^{m+1} (b \sec[e+fx])^{n+1}}{a b f (m-n)} - \frac{n+1}{b^2 (m-n)} \int (a \sin[e+fx])^m (b \sec[e+fx])^{n+2} dx$$

Program code:

```
Int[(a_.sin[e_.+f_.x_])^m_*(b_.sec[e_.+f_.x_])^n_,x_Symbol] :=
  (a*sin[e+f*x])^(m+1)*(b*sec[e+f*x])^(n+1)/(a*b*f*(m-n)) -
  (n+1)/(b^2*(m-n))*Int[(a*sin[e+f*x])^m*(b*sec[e+f*x])^(n+2),x] /;
FreeQ[{a,b,e,f,m},x] && LtQ[n,-1] && NeQ[m-n,0] && IntegersQ[2*m,2*n]
```

4: $\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx$ when $m > 1 \wedge m-n \neq 0$

Rule: If $m > 1 \wedge m-n \neq 0$, then

$$\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx \rightarrow -\frac{a b (a \sin[e+fx])^{m-1} (b \sec[e+fx])^{n-1}}{f (m-n)} + \frac{a^2 (m-1)}{m-n} \int (a \sin[e+fx])^{m-2} (b \sec[e+fx])^n dx$$

Program code:

```
Int[(a_.sin[e_.+f_.x_])^m_*(b_.sec[e_.+f_.x_])^n_,x_Symbol] :=
  -a*b*(a*sin[e+f*x])^(m-1)*(b*sec[e+f*x])^(n-1)/(f*(m-n)) +
  a^2*(m-1)/(m-n)*Int[(a*sin[e+f*x])^(m-2)*(b*sec[e+f*x])^n,x] /;
FreeQ[{a,b,e,f,n},x] && GtQ[m,1] && NeQ[m-n,0] && IntegersQ[2*m,2*n]
```

5: $\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx$ when $m < -1$

Rule: If $m < -1$, then

$$\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx \rightarrow \frac{b (a \sin[e+fx])^{m+1} (b \sec[e+fx])^{n-1}}{a f (m+1)} + \frac{m-n+2}{a^2 (m+1)} \int (a \sin[e+fx])^{m+2} (b \sec[e+fx])^n dx$$

Program code:

```
Int[(a_.sin[e_.+f_.x_])^m_*(b_.sec[e_.+f_.x_])^n_,x_Symbol] :=
  b*(a*sin[e+f*x])^(m+1)*(b*sec[e+f*x])^(n-1)/(a*f*(m+1)) +
  (m-n+2)/(a^2*(m+1))*Int[(a*sin[e+f*x])^(m+2)*(b*sec[e+f*x])^n,x] /;
FreeQ[{a,b,e,f,n},x] && LtQ[m,-1] && IntegersQ[2*m,2*n]
```

6. $\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx$ when $m \notin \mathbb{Z} \wedge n \notin \mathbb{Z}$

1: $\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx$ when $m - \frac{1}{2} \in \mathbb{Z} \wedge n - \frac{1}{2} \in \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: $\partial_x ((b \cos[e+fx])^n (b \sec[e+fx])^n) = 0$

Rule: If $m - \frac{1}{2} \in \mathbb{Z} \wedge n - \frac{1}{2} \in \mathbb{Z}$, then

$$\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx \rightarrow (b \cos[e+fx])^n (b \sec[e+fx])^n \int \frac{(a \sin[e+fx])^m}{(b \cos[e+fx])^n} dx$$

Program code:

```
Int[(a_.*sin[e_.+f_.*x_])^m_*(b_.*sec[e_.+f_.*x_])^n_,x_Symbol] :=
  (b*cos[e+f*x])^n*(b*sec[e+f*x])^n*Int[(a*sin[e+f*x])^m/(b*cos[e+f*x])^n,x] /;
FreeQ[{a,b,e,f,m,n},x] && IntegerQ[m-1/2] && IntegerQ[n-1/2]
```


2: $\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx$ when $m \notin \mathbb{Z} \wedge n \notin \mathbb{Z} \wedge n < 1$

Derivation: Piecewise constant extraction

Basis: $\partial_x \left((b \cos[e+fx])^{n+1} (b \sec[e+fx])^{n+1} \right) = 0$

Rule: If $m \notin \mathbb{Z} \wedge n \notin \mathbb{Z} \wedge n < 1$, then

$$\int (a \sin[e+fx])^m (b \sec[e+fx])^n dx \rightarrow \frac{1}{b^2} (b \cos[e+fx])^{n+1} (b \sec[e+fx])^{n+1} \int \frac{(a \sin[e+fx])^m}{(b \cos[e+fx])^n} dx$$

Program code:

```
Int[(a_.*sin[e_.+f_.*x_])^m_*(b_.*sec[e_.+f_.*x_])^n_,x_Symbol] :=
  1/b^2*(b*cos[e+f*x])^(n+1)*(b*sec[e+f*x])^(n+1)*Int[(a*sin[e+f*x])^m/(b*cos[e+f*x])^n,x] /;
FreeQ[{a,b,e,f,m,n},x] && Not[IntegerQ[m]] && Not[IntegerQ[n]] && LtQ[n,1]
```

$$\mathbf{3:} \int (a \operatorname{Sin}[e+fx])^m (b \operatorname{Sec}[e+fx])^n dx \text{ when } m \notin \mathbb{Z} \wedge n \notin \mathbb{Z}$$

Derivation: Piecewise constant extraction

$$\text{Basis: } \partial_x \left((b \operatorname{Cos}[e+fx])^{n-1} (b \operatorname{Sec}[e+fx])^{n-1} \right) = 0$$

Rule: If $m \notin \mathbb{Z} \wedge n \notin \mathbb{Z}$, then

$$\int (a \operatorname{Sin}[e+fx])^m (b \operatorname{Sec}[e+fx])^n dx \rightarrow b^2 (b \operatorname{Cos}[e+fx])^{n-1} (b \operatorname{Sec}[e+fx])^{n-1} \int \frac{(a \operatorname{Sin}[e+fx])^m}{(b \operatorname{Cos}[e+fx])^n} dx$$

Program code:

```
Int[(a_.*sin[e_.+f_.**x_])^m_*(b_.*sec[e_.+f_.**x_])^n_,x_Symbol] :=
  b^2*(b*cos[e+f*x])^(n-1)*(b*sec[e+f*x])^(n-1)*Int[(a*sin[e+f*x])^m/(b*cos[e+f*x])^n,x] /;
FreeQ[{a,b,e,f,m,n},x] && Not[IntegerQ[m]] && Not[IntegerQ[n]]
```

3: $\int (a \sin[e+fx])^m (b \operatorname{Csc}[e+fx])^n dx$ when $m \notin \mathbb{Z} \wedge n \notin \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: $\partial_x ((a \sin[e+fx])^n (b \operatorname{Csc}[e+fx])^n) = 0$

Rule: If $m \notin \mathbb{Z} \wedge n \notin \mathbb{Z}$, then

$$\int (a \sin[e+fx])^m (b \operatorname{Csc}[e+fx])^n dx \rightarrow (ab)^{\operatorname{IntPart}[n]} (a \sin[e+fx])^{\operatorname{FracPart}[n]} (b \operatorname{Csc}[e+fx])^{\operatorname{FracPart}[n]} \int (a \sin[e+fx])^{m-n} dx$$

Program code:

```
Int[(a_.sin[e_.+f_.**x_])^m_.*(b_.csc[e_.+f_.**x_])^n_,x_Symbol] :=
  (a*b)^(IntPart[n])*(a*sin[e+f*x])^FracPart[n]*(b*csc[e+f*x])^FracPart[n]*Int[(a*sin[e+f*x])^(m-n),x] /;
  FreeQ[{a,b,e,f,m,n},x] && Not[IntegerQ[m]] && Not[IntegerQ[n]]
```