Rules for integrands of the form $(a + b \sin[e + f x])^m (c + d \sin[e + f x])^n (A + B \sin[e + f x] + C \sin[e + f x]^2)$

1. $\int (a + b \sin[e + f x])^m (c + d \sin[e + f x]) (A + B \sin[e + f x] + C \sin[e + f x]^2) \, dx$ when $b c - a d \neq 0 \wedge a^2 - b^2 \neq 0$

   1: $\int (a + b \sin[e + f x])^m (c + d \sin[e + f x]) (A + B \sin[e + f x] + C \sin[e + f x]^2) \, dx$ when $b c - a d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge m < -1$

   Derivation: Algebraic expansion, nondegenerate sine recurrence 1c with
   $c \to 1, \, d \to 0, \, A \to c, \, B \to d, \, C \to 0, \, n \to 0, \, p \to 0$ and algebraic simplification

   Basis: $A + B z + C z^2 == \dfrac{A b^2 - a b B + a^2 C}{b^2} + \dfrac{(a + b z)(b B - a C + b C z)}{b^2}$

   Rule: If $b c - a d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge m < -1$, then

   $$\int (a + b \sin[e + f x])^m (c + d \sin[e + f x]) (A + B \sin[e + f x] + C \sin[e + f x]^2) \, dx \longrightarrow$$

   $$\frac{A b^2 - a b B + a^2 C}{b^2} \int (a + b \sin[e + f x])^m (c + d \sin[e + f x]) \, dx + \frac{1}{b^2} \int (a + b \sin[e + f x])^{m+1} (c + d \sin[e + f x]) (b B - a C + b C \sin[e + f x]) \, dx \longrightarrow$$

   $$-\left(\left((b c - a d)(A b^2 - a b B + a^2 C) \cos[e + f x](a + b \sin[e + f x])^{m+1}\right) \Big/ \left(b^2 f (m + 1)(a^2 - b^2)\right)\right) - \frac{1}{b^2 (m + 1)(a^2 - b^2)} \int (a + b \sin[e + f x])^{m+1} \cdot$$

   $$\big(b (m + 1)((b B - a C)(b c - a d) - A b (a c - b d)) +$$
   $$\big(b B (a^2 d + b^2 d (m + 1) - a b c (m + 2)) + (b c - a d)(A b^2 (m + 2) + C (a^2 + b^2 (m + 1)))\big) \sin[e + f x] -$$
   $$b C d (m + 1)(a^2 - b^2) \sin[e + f x]^2 \big) \, dx$$

Program code:

```
Int[(a_.+b_.*sin[e_.+f_.*x_])^m_*(c_.+d_.*sin[e_.+f_.*x_])*(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -(b*c-a*d)*(A*b^2-a*b*B+a^2*C)*Cos[e+f*x]*(a+b*Sin[e+f*x])^(m+1)/(b^2*f*(m+1)*(a^2-b^2)) -
  1/(b^2*(m+1)*(a^2-b^2))*Int[(a+b*Sin[e+f*x])^(m+1)*
    Simp[b*(m+1)*((b*B-a*C)*(b*c-a*d)-A*b*(a*c-b*d))+
    (b*B*(a^2*d+b^2*d*(m+1)-a*b*c*(m+2))+(b*c-a*d)*(A*b^2*(m+2)+C*(a^2+b^2*(m+1))))*Sin[e+f*x]-
    b*C*d*(m+1)*(a^2-b^2)*Sin[e+f*x]^2,x],x] /;
FreeQ[{a,b,c,d,e,f,A,B,C},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && LtQ[m,-1]
```

```
Int[(a_.+b_.*sin[e_.+f_.*x_])^m_*(c_.+d_.*sin[e_.+f_.*x_])*(A_.+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -(b*c-a*d)*(A*b^2+a^2*C)*Cos[e+f*x]*(a+b*Sin[e+f*x])^(m+1)/(b^2*f*(m+1)*(a^2-b^2)) +
  1/(b^2*(m+1)*(a^2-b^2))*Int[(a+b*Sin[e+f*x])^(m+1)*
    Simp[b*(m+1)*(a*C*(b*c-a*d)+A*b*(a*c-b*d))-
      ((b*c-a*d)*(A*b^2*(m+2)+C*(a^2+b^2*(m+1))))*Sin[e+f*x]+
      b*C*d*(m+1)*(a^2-b^2)*Sin[e+f*x]^2,x],x] /;
FreeQ[{a,b,c,d,e,f,A,C},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && LtQ[m,-1]
```

2: $\int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x]) \, (A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2) \, dx$ when $b \, c - a \, d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge m \not< -1$

Derivation: Algebraic expansion, nondegenerate sine recurrence 1b with

$c \to 0$, $d \to 1$, $A \to a \, c$, $B \to b \, c + a \, d$, $C \to b \, d$, $m \to m + 1$, $n \to 0$, $p \to 0$ and algebraic simplification

Basis: $A + B \, z + C \, z^2 == \dfrac{C \, (a+b \, z)^2}{b^2} + \dfrac{A \, b^2 - a^2 \, C + b \, (b \, B - 2 \, a \, C) \, z}{b^2}$

Rule: If $b \, c - a \, d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge m \not< -1$, then

$$\int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x]) \, (A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2) \, dx \longrightarrow$$

$$\frac{C}{b^2} \int (a + b \, Sin[e + f \, x])^{m+2} \, (c + d \, Sin[e + f \, x]) \, dx + \frac{1}{b^2} \int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x]) \, (A \, b^2 - a^2 \, C + b \, (b \, B - 2 \, a \, C) \, Sin[e + f \, x]) \, dx \longrightarrow$$

$$-\frac{C \, d \, Cos[e + f \, x] \, Sin[e + f \, x] \, (a + b \, Sin[e + f \, x])^{m+1}}{b \, f \, (m + 3)} + \frac{1}{b \, (m + 3)} \int (a + b \, Sin[e + f \, x])^m \cdot$$
$$(a \, C \, d + A \, b \, c \, (m + 3) + b \, (B \, c \, (m + 3) + d \, (C \, (m + 2) + A \, (m + 3))) \, Sin[e + f \, x] - (2 \, a \, C \, d - b \, (c \, C + B \, d) \, (m + 3)) \, Sin[e + f \, x]^2) \, dx$$

Program code:

```
Int[(a_.+b_.*sin[e_.+f_.*x_])^m_.*(c_+d_.*sin[e_.+f_.*x_])*(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -C*d*Cos[e+f*x]*Sin[e+f*x]*(a+b*Sin[e+f*x])^(m+1)/(b*f*(m+3)) +
  1/(b*(m+3))*Int[(a+b*Sin[e+f*x])^m*
    Simp[a*C*d+A*b*c*(m+3)+b*(B*c*(m+3)+d*(C*(m+2)+A*(m+3)))*Sin[e+f*x]-(2*a*C*d-b*(c*C+B*d)*(m+3))*Sin[e+f*x]^2,x],x] /;
FreeQ[{a,b,c,d,e,f,A,B,C,m},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && Not[LtQ[m,-1]]
```

```
Int[(a_.+b_.*sin[e_.+f_.*x_])^m_.*(c_+d_.*sin[e_.+f_.*x_])*(A_.+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -C*d*Cos[e+f*x]*Sin[e+f*x]*(a+b*Sin[e+f*x])^(m+1)/(b*f*(m+3)) +
  1/(b*(m+3))*Int[(a+b*Sin[e+f*x])^m*
    Simp[a*C*d+A*b*c*(m+3)+b*d*(C*(m+2)+A*(m+3))*Sin[e+f*x]-(2*a*C*d-b*c*C*(m+3))*Sin[e+f*x]^2,x],x] /;
FreeQ[{a,b,c,d,e,f,A,C,m},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && Not[LtQ[m,-1]]
```

2. $\int (a + b \, \text{Sin}[e + f \, x])^m \, (c + d \, \text{Sin}[e + f \, x])^n \, (A + B \, \text{Sin}[e + f \, x] + C \, \text{Sin}[e + f \, x]^2) \, dx$ when $b \, c + a \, d == 0 \wedge a^2 - b^2 == 0$

1: $\int (a + b \, \text{Sin}[e + f \, x])^m \, (c + d \, \text{Sin}[e + f \, x])^n \, (A + B \, \text{Sin}[e + f \, x] + C \, \text{Sin}[e + f \, x]^2) \, dx$ when $b \, c + a \, d == 0 \wedge a^2 - b^2 == 0 \wedge m < -\frac{1}{2}$

Derivation: Algebraic expansion, singly degenerate sine recurrence 2b with $A \to 1$, $B \to 0$, $p \to 0$ and algebraic simplification

Basis: If $a^2 - b^2 == 0$, then $A + B \, z + C \, z^2 == \frac{a \, A - b \, B + a \, C}{a} + \frac{(a + b \, z) \, (b \, B - a \, C + b \, C \, z)}{b^2}$

Rule: If $b \, c + a \, d == 0 \wedge a^2 - b^2 == 0 \wedge m < -\frac{1}{2}$, then

$$\int (a + b \, \text{Sin}[e + f \, x])^m \, (c + d \, \text{Sin}[e + f \, x])^n \, (A + B \, \text{Sin}[e + f \, x] + C \, \text{Sin}[e + f \, x]^2) \, dx \longrightarrow$$

$$\frac{a \, A - b \, B + a \, C}{a} \int (a + b \, \text{Sin}[e + f \, x])^m \, (c + d \, \text{Sin}[e + f \, x])^n \, dx + \frac{1}{b^2} \int (a + b \, \text{Sin}[e + f \, x])^{m+1} \, (c + d \, \text{Sin}[e + f \, x])^n \, (b \, B - a \, C + b \, C \, \text{Sin}[e + f \, x]) \, dx \longrightarrow$$

$$\frac{1}{2 \, b \, c \, f \, (2 \, m + 1)} (a \, A - b \, B + a \, C) \, \text{Cos}[e + f \, x] \, (a + b \, \text{Sin}[e + f \, x])^m \, (c + d \, \text{Sin}[e + f \, x])^{n+1} -$$

$$\frac{1}{2 \, b \, c \, d \, (2 \, m + 1)} \int (a + b \, \text{Sin}[e + f \, x])^{m+1} \, (c + d \, \text{Sin}[e + f \, x])^n \cdot$$

$$\left(A \left(c^2 \, (m + 1) + d^2 \, (2 \, m + n + 2)\right) - B \, c \, d \, (m - n - 1) - C \left(c^2 \, m - d^2 \, (n + 1)\right) + d \left((A \, c + B \, d) \, (m + n + 2) - c \, C \, (3 \, m - n)\right) \text{Sin}[e + f \, x]\right) \, dx$$

Program code:

```
Int[(a_+b_.*sin[e_.+f_.*x_])^m_*(c_.+d_.*sin[e_.+f_.*x_])^n_.*(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  (a*A-b*B+a*C)*Cos[e+f*x]*(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)/(2*b*c*f*(2*m+1)) -
  1/(2*b*c*d*(2*m+1))*Int[(a+b*Sin[e+f*x])^(m+1)*(c+d*Sin[e+f*x])^n*
    Simp[A*(c^2*(m+1)+d^2*(2*m+n+2))-B*c*d*(m-n-1)-C*(c^2*m-d^2*(n+1))+d*((A*c+B*d)*(m+n+2)-c*C*(3*m-n))*Sin[e+f*x],x],x] /;
FreeQ[{a,b,c,d,e,f,A,B,C,m,n},x] && EqQ[b*c+a*d,0] && EqQ[a^2-b^2,0] && (LtQ[m,-1/2] || EqQ[m+n+2,0] && NeQ[2*m+1,0])
```

```
Int[(a_+b_.*sin[e_.+f_.*x_])^m_*(c_.+d_.*sin[e_.+f_.*x_])^n_.*(A_.+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  (a*A+a*C)*Cos[e+f*x]*(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)/(2*b*c*f*(2*m+1)) -
  1/(2*b*c*d*(2*m+1))*Int[(a+b*Sin[e+f*x])^(m+1)*(c+d*Sin[e+f*x])^n*
    Simp[A*(c^2*(m+1)+d^2*(2*m+n+2))-C*(c^2*m-d^2*(n+1))+d*(A*c*(m+n+2)-c*C*(3*m-n))*Sin[e+f*x],x],x] /;
FreeQ[{a,b,c,d,e,f,A,C,m,n},x] && EqQ[b*c+a*d,0] && EqQ[a^2-b^2,0] && (LtQ[m,-1/2] || EqQ[m+n+2,0] && NeQ[2*m+1,0])
```

2. $\int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^n \, (A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2) \, dx$ when $b \, c + a \, d == 0 \wedge a^2 - b^2 == 0 \wedge m \not< -\frac{1}{2}$

1: $\int \dfrac{(a + b \, Sin[e + f \, x])^m \, (A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2)}{\sqrt{c + d \, Sin[e + f \, x]}} \, dx$ when $b \, c + a \, d == 0 \wedge a^2 - b^2 == 0 \wedge m \not< -\frac{1}{2}$

Derivation: Algebraic expansion and doubly degenerate sine recurrence 2b with $n \to -\frac{1}{2}$, $p \to 0$

Basis: $A + B \, z + C \, z^2 == \dfrac{C \, (e + f \, z + g \, z^2)}{g} - \dfrac{C \, e - A \, g + (C \, f - B \, g) \, z}{g}$

Rule: If $b \, c + a \, d == 0 \wedge a^2 - b^2 == 0 \wedge m \not< -\frac{1}{2}$, then

$$\int \dfrac{(a + b \, Sin[e + f \, x])^m \, (A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2)}{\sqrt{c + d \, Sin[e + f \, x]}} \, dx \;\longrightarrow$$

$$- \dfrac{2 \, C \, Cos[e + f \, x] \, (a + b \, Sin[e + f \, x])^{m+1}}{b \, f \, (2 \, m + 3) \, \sqrt{c + d \, Sin[e + f \, x]}} + \int \dfrac{(a + b \, Sin[e + f \, x])^m \, (A + C + B \, Sin[e + f \, x])}{\sqrt{c + d \, Sin[e + f \, x]}} \, dx$$

Program code:

```
Int[(a_.+b_.*sin[e_.+f_.*x_])^m_.*(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2)/Sqrt[c_.+d_.*sin[e_.+f_.*x_]],x_Symbol] :=
  -2*C*Cos[e+f*x]*(a+b*Sin[e+f*x])^(m+1)/(b*f*(2*m+3)*Sqrt[c+d*Sin[e+f*x]]) +
  Int[(a+b*Sin[e+f*x])^m*Simp[A+C+B*Sin[e+f*x],x]/Sqrt[c+d*Sin[e+f*x]],x] /;
FreeQ[{a,b,c,d,e,f,A,B,C,m},x] && EqQ[b*c+a*d,0] && EqQ[a^2-b^2,0] && Not[LtQ[m,-1/2]]
```

```
Int[(a_.+b_.*sin[e_.+f_.*x_])^m_.*(A_.+C_.*sin[e_.+f_.*x_]^2)/Sqrt[c_.+d_.*sin[e_.+f_.*x_]],x_Symbol] :=
  -2*C*Cos[e+f*x]*(a+b*Sin[e+f*x])^(m+1)/(b*f*(2*m+3)*Sqrt[c+d*Sin[e+f*x]]) +
  (A+C)*Int[(a+b*Sin[e+f*x])^m/Sqrt[c+d*Sin[e+f*x]],x] /;
FreeQ[{a,b,c,d,e,f,A,C,m},x] && EqQ[b*c+a*d,0] && EqQ[a^2-b^2,0] && Not[LtQ[m,-1/2]]
```

2: $\int (a + b \, \text{Sin}[e + f\,x])^m \, (c + d \, \text{Sin}[e + f\,x])^n \, (A + B \, \text{Sin}[e + f\,x] + C \, \text{Sin}[e + f\,x]^2) \, dx$ when $b\,c + a\,d == 0 \wedge a^2 - b^2 == 0 \wedge m \not< -\frac{1}{2} \wedge m + n + 2 \neq 0$

Derivation: Nondegenerate sine recurrence 1b with $p \to 0$ and $a^2 - b^2 == 0$

Derivation: Algebraic expansion and singly degenerate sine recurrence 2c with $A \to c$, $B \to d$, $n \to n + 1$, $p \to 0$

Basis: $A + B\,z + C\,z^2 == \dfrac{C\,(c+d\,z)^2}{d^2} + \dfrac{A\,d^2 - c^2\,C - d\,(2\,c\,C - B\,d)\,z}{d^2}$

Rule: If $b\,c + a\,d == 0 \wedge a^2 - b^2 == 0 \wedge m \not< -\frac{1}{2} \wedge m + n + 2 \neq 0$, then

$$\int (a + b \, \text{Sin}[e + f\,x])^m \, (c + d \, \text{Sin}[e + f\,x])^n \, (A + B \, \text{Sin}[e + f\,x] + C \, \text{Sin}[e + f\,x]^2) \, dx \longrightarrow$$

$$\frac{C}{d^2} \int (a + b \, \text{Sin}[e + f\,x])^m \, (c + d \, \text{Sin}[e + f\,x])^{n+2} \, dx + \frac{1}{d^2} \int (a + b \, \text{Sin}[e + f\,x])^m \, (c + d \, \text{Sin}[e + f\,x])^n \, (A\,d^2 - c^2\,C - d\,(2\,c\,C - B\,d)\,\text{Sin}[e + f\,x]) \, dx \longrightarrow$$

$$- \frac{C \, \text{Cos}[e + f\,x] \, (a + b \, \text{Sin}[e + f\,x])^m \, (c + d \, \text{Sin}[e + f\,x])^{n+1}}{d\,f\,(m + n + 2)} +$$

$$\frac{1}{b\,d\,(m + n + 2)} \int (a + b \, \text{Sin}[e + f\,x])^m \, (c + d \, \text{Sin}[e + f\,x])^n \, (A\,b\,d\,(m + n + 2) + C\,(a\,c\,m + b\,d\,(n + 1)) + (b\,B\,d\,(m + n + 2) - b\,c\,C\,(2\,m + 1))\,\text{Sin}[e + f\,x]) \, dx$$

Program code:

```
Int[(a_+b_.*sin[e_.+f_.*x_])^m_.*(c_.+d_.*sin[e_.+f_.*x_])^n_.*(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -C*Cos[e+f*x]*(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)/(d*f*(m+n+2)) +
  1/(b*d*(m+n+2))*Int[(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^n*
    Simp[A*b*d*(m+n+2)+C*(a*c*m+b*d*(n+1))+(b*B*d*(m+n+2)-b*c*C*(2*m+1))*Sin[e+f*x],x],x] /;
FreeQ[{a,b,c,d,e,f,A,B,C,m,n},x] && EqQ[b*c+a*d,0] && EqQ[a^2-b^2,0] && Not[LtQ[m,-1/2]] && NeQ[m+n+2,0]
```

```
Int[(a_+b_.*sin[e_.+f_.*x_])^m_.*(c_.+d_.*sin[e_.+f_.*x_])^n_.*(A_.+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -C*Cos[e+f*x]*(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)/(d*f*(m+n+2)) +
  1/(b*d*(m+n+2))*Int[(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^n*
    Simp[A*b*d*(m+n+2)+C*(a*c*m+b*d*(n+1))-b*c*C*(2*m+1)*Sin[e+f*x],x],x] /;
FreeQ[{a,b,c,d,e,f,A,C,m,n},x] && EqQ[b*c+a*d,0] && EqQ[a^2-b^2,0] && Not[LtQ[m,-1/2]] && NeQ[m+n+2,0]
```

3. $\int (a + b\,Sin[e + f\,x])^m\,(c + d\,Sin[e + f\,x])^n\,(A + B\,Sin[e + f\,x] + C\,Sin[e + f\,x]^2)\,dx$ when $b\,c - a\,d \neq 0 \wedge a^2 - b^2 == 0 \wedge c^2 - d^2 \neq 0$

1: $\int (a + b\,Sin[e + f\,x])^m\,(c + d\,Sin[e + f\,x])^n\,(A + B\,Sin[e + f\,x] + C\,Sin[e + f\,x]^2)\,dx$ when $b\,c - a\,d \neq 0 \wedge a^2 - b^2 == 0 \wedge c^2 - d^2 \neq 0 \wedge m < -\frac{1}{2}$

Derivation: Algebraic expansion, singly degenerate sine recurrence 2b with A → 1, B → 0, p → 0 and algebraic simplification

Basis: If $a^2 - b^2 == 0$, then $A + B\,z + C\,z^2 == \frac{a\,A - b\,B + a\,C}{a} + \frac{(a + b\,z)\,(b\,B - a\,C + b\,C\,z)}{b^2}$

Rule: If $b\,c - a\,d \neq 0 \wedge a^2 - b^2 == 0 \wedge m < -\frac{1}{2}$, then

$$\int (a + b\,Sin[e + f\,x])^m\,(c + d\,Sin[e + f\,x])^n\,(A + B\,Sin[e + f\,x] + C\,Sin[e + f\,x]^2)\,dx \rightarrow$$

$$\frac{a\,A - b\,B + a\,C}{a}\int (a + b\,Sin[e + f\,x])^m\,(c + d\,Sin[e + f\,x])^n\,dx + \frac{1}{b^2}\int (a + b\,Sin[e + f\,x])^{m+1}\,(c + d\,Sin[e + f\,x])^n\,(b\,B - a\,C + b\,C\,Sin[e + f\,x])\,dx \rightarrow$$

$$(((a\,A - b\,B + a\,C)\,Cos[e + f\,x]\,(a + b\,Sin[e + f\,x])^m\,(c + d\,Sin[e + f\,x])^{n+1})/(f\,(b\,c - a\,d)\,(2\,m + 1))) +$$

$$\frac{1}{b\,(b\,c - a\,d)\,(2\,m + 1)}\int (a + b\,Sin[e + f\,x])^{m+1}\,(c + d\,Sin[e + f\,x])^n \cdot$$

$$(A\,(a\,c\,(m + 1) - b\,d\,(2\,m + n + 2)) + B\,(b\,c\,m + a\,d\,(n + 1)) - C\,(a\,c\,m + b\,d\,(n + 1)) + (d\,(a\,A - b\,B)\,(m + n + 2) + C\,(b\,c\,(2\,m + 1) - a\,d\,(m - n - 1)))\,Sin[e + f\,x])\,dx$$

Program code:

```
Int[(a_+b_.*sin[e_.+f_.*x_])^m_*(c_.+d_.*sin[e_.+f_.*x_])^n_.*(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  (a*A-b*B+a*C)*Cos[e+f*x]*(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)/(f*(b*c-a*d)*(2*m+1)) +
  1/(b*(b*c-a*d)*(2*m+1))*Int[(a+b*Sin[e+f*x])^(m+1)*(c+d*Sin[e+f*x])^n*
    Simp[A*(a*c*(m+1)-b*d*(2*m+n+2))+B*(b*c*m+a*d*(n+1))-C*(a*c*m+b*d*(n+1))+
      (d*(a*A-b*B)*(m+n+2)+C*(b*c*(2*m+1)-a*d*(m-n-1)))*Sin[e+f*x],x],x] /;
FreeQ[{a,b,c,d,e,f,A,B,C,n},x] && NeQ[b*c-a*d,0] && EqQ[a^2-b^2,0] && NeQ[c^2-d^2,0] && LtQ[m,-1/2]
```

```
Int[(a_+b_.*sin[e_.+f_.*x_])^m_*(c_.+d_.*sin[e_.+f_.*x_])^n_.*(A_.+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  a*(A+C)*Cos[e+f*x]*(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)/(f*(b*c-a*d)*(2*m+1)) +
  1/(b*(b*c-a*d)*(2*m+1))*Int[(a+b*Sin[e+f*x])^(m+1)*(c+d*Sin[e+f*x])^n*
    Simp[A*(a*c*(m+1)-b*d*(2*m+n+2))-C*(a*c*m+b*d*(n+1))+
      (a*A*d*(m+n+2)+C*(b*c*(2*m+1)-a*d*(m-n-1)))*Sin[e+f*x],x],x] /;
FreeQ[{a,b,c,d,e,f,A,C,n},x] && NeQ[b*c-a*d,0] && EqQ[a^2-b^2,0] && NeQ[c^2-d^2,0] && LtQ[m,-1/2]
```

2. $\int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^n \, (A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2) \, dx$ when $b \, c - a \, d \neq 0 \wedge a^2 - b^2 == 0 \wedge c^2 - d^2 \neq 0 \wedge m \not< -\frac{1}{2}$

1: $\int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^n$

$(A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2) \, dx$ when $b \, c - a \, d \neq 0 \wedge a^2 - b^2 == 0 \wedge c^2 - d^2 \neq 0 \wedge m \not< -\frac{1}{2} \wedge (n < -1 \vee m + n + 2 == 0)$

Derivation: Algebraic expansion and singly degenerate sine recurrence 1c with A → 1, B → 0, p → 0

Basis: $A + B \, z + C \, z^2 == \frac{c^2 \, C - B \, c \, d + A \, d^2}{d^2} - \frac{(c + d \, z) \, (c \, C - B \, d - C \, d \, z)}{d^2}$

Rule: If $b \, c - a \, d \neq 0 \wedge a^2 - b^2 == 0 \wedge c^2 - d^2 \neq 0 \wedge m \not< -\frac{1}{2} \wedge (n < -1 \vee m + n + 2 == 0)$, then

$$\int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^n \, (A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2) \, dx \longrightarrow$$

$$\frac{c^2 \, C - B \, c \, d + A \, d^2}{d^2} \int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^n \, dx - \frac{1}{d^2} \int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^{n+1} \, (c \, C - B \, d - C \, d \, Sin[e + f \, x]) \, dx \longrightarrow$$

$$- (((c^2 \, C - B \, c \, d + A \, d^2) \, Cos[e + f \, x] \, (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^{n+1}) / (d \, f \, (n + 1) \, (c^2 - d^2))) +$$

$$\frac{1}{b \, d \, (n + 1) \, (c^2 - d^2)} \int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^{n+1} \cdot$$

$$(A \, d \, (a \, d \, m + b \, c \, (n + 1)) + (c \, C - B \, d) \, (a \, c \, m + b \, d \, (n + 1)) + b \, (d \, (B \, c - A \, d) \, (m + n + 2) - C \, (c^2 \, (m + 1) + d^2 \, (n + 1))) \, Sin[e + f \, x]) \, dx$$

Program code:

```
Int[(a_+b_.*sin[e_.+f_.*x_])^m_.*(c_.+d_.*sin[e_.+f_.*x_])^n_*(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -(c^2*C-B*c*d+A*d^2)*Cos[e+f*x]*(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)/(d*f*(n+1)*(c^2-d^2)) +
   1/(b*d*(n+1)*(c^2-d^2))*Int[(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)*
    Simp[A*d*(a*d*m+b*c*(n+1))+(c*C-B*d)*(a*c*m+b*d*(n+1))+b*(d*(B*c-A*d)*(m+n+2)-C*(c^2*(m+1)+d^2*(n+1)))*Sin[e+f*x],x],x] /;
 FreeQ[{a,b,c,d,e,f,A,B,C,m},x] && NeQ[b*c-a*d,0] && EqQ[a^2-b^2,0] && NeQ[c^2-d^2,0] && Not[LtQ[m,-1/2]] && (LtQ[n,-1] || EqQ[m+n+2,
```

```
Int[(a_+b_.*sin[e_.+f_.*x_])^m_.*(c_.+d_.*sin[e_.+f_.*x_])^n_*(A_.+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -(c^2*C+A*d^2)*Cos[e+f*x]*(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)/(d*f*(n+1)*(c^2-d^2)) +
  1/(b*d*(n+1)*(c^2-d^2))*Int[(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)*
    Simp[A*d*(a*d*m+b*c*(n+1))+c*C*(a*c*m+b*d*(n+1))-b*(A*d^2*(m+n+2)+C*(c^2*(m+1)+d^2*(n+1)))*Sin[e+f*x],x],x] /;
FreeQ[{a,b,c,d,e,f,A,C,m},x] && NeQ[b*c-a*d,0] && EqQ[a^2-b^2,0] && NeQ[c^2-d^2,0] && Not[LtQ[m,-1/2]] && (LtQ[n,-1] || EqQ[m+n+2,0])
```

2:

$$\int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^n \, (A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2) \, dx \text{ when } b \, c - a \, d \neq 0 \, \wedge \, a^2 - b^2 == 0 \, \wedge \, c^2 - d^2 \neq 0 \, \wedge \, m \not< -\frac{1}{2} \, \wedge \, m + n + 2 \neq 0$$

Derivation: Nondegenerate sine recurrence 1b with $p \rightarrow 0$ and $a^2 - b^2 == 0$

Derivation: Algebraic expansion and singly degenerate sine recurrence 2c with $A \rightarrow c$, $B \rightarrow d$, $n \rightarrow n + 1$, $p \rightarrow 0$

Basis: $A + B \, z + C \, z^2 == \frac{C \, (c + d \, z)^2}{d^2} + \frac{A \, d^2 - c^2 \, C - d \, (2 \, c \, C - B \, d) \, z}{d^2}$

Rule: If $b \, c - a \, d \neq 0 \, \wedge \, a^2 - b^2 == 0 \, \wedge \, c^2 - d^2 \neq 0 \, \wedge \, m \not< -\frac{1}{2} \, \wedge \, m + n + 2 \neq 0$, then

$$\int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^n \, (A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2) \, dx \; \rightarrow$$

$$\frac{C}{d^2} \int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^{n+2} \, dx + \frac{1}{d^2} \int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^n \, (A \, d^2 - c^2 \, C - d \, (2 \, c \, C - B \, d) \, Sin[e + f \, x]) \, dx \; \rightarrow$$

$$- \frac{C \, Cos[e + f \, x] \, (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^{n+1}}{d \, f \, (m + n + 2)} +$$

$$\frac{1}{b \, d \, (m + n + 2)} \int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^n \, (A \, b \, d \, (m + n + 2) + C \, (a \, c \, m + b \, d \, (n + 1)) + (C \, (a \, d \, m - b \, c \, (m + 1)) + b \, B \, d \, (m + n + 2)) \, Sin[e + f \, x]) \, dx$$

Program code:

```
Int[(a_+b_.*sin[e_.+f_.*x_])^m_.*(c_.+d_.*sin[e_.+f_.*x_])^n_.*(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -C*Cos[e+f*x]*(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)/(d*f*(m+n+2)) +
  1/(b*d*(m+n+2))*Int[(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^n*
    Simp[A*b*d*(m+n+2)+C*(a*c*m+b*d*(n+1))+(C*(a*d*m-b*c*(m+1))+b*B*d*(m+n+2))*Sin[e+f*x],x],x] /;
FreeQ[{a,b,c,d,e,f,A,B,C,m,n},x] && NeQ[b*c-a*d,0] && EqQ[a^2-b^2,0] && NeQ[c^2-d^2,0] && Not[LtQ[m,-1/2]] && NeQ[m+n+2,0]
```

```
Int[(a_+b_.*sin[e_.+f_.*x_])^m_.*(c_.+d_.*sin[e_.+f_.*x_])^n_.*(A_.+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -C*Cos[e+f*x]*(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)/(d*f*(m+n+2)) +
  1/(b*d*(m+n+2))*Int[(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^n*
    Simp[A*b*d*(m+n+2)+C*(a*c*m+b*d*(n+1))+C*(a*d*m-b*c*(m+1))*Sin[e+f*x],x],x] /;
  FreeQ[{a,b,c,d,e,f,A,C,m,n},x] && NeQ[b*c-a*d,0] && EqQ[a^2-b^2,0] && NeQ[c^2-d^2,0] && Not[LtQ[m,-1/2]] && NeQ[m+n+2,0]
```

4. $\int (a + b\, Sin[e + f\, x])^m\, (c + d\, Sin[e + f\, x])^n\, (A + B\, Sin[e + f\, x] + C\, Sin[e + f\, x]^2)\; dx$ when $b\, c - a\, d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge c^2 - d^2 \neq 0$

  1. $\int (a + b\, Sin[e + f\, x])^m\, (c + d\, Sin[e + f\, x])^n\, (A + B\, Sin[e + f\, x] + C\, Sin[e + f\, x]^2)\; dx$ when $b\, c - a\, d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge c^2 - d^2 \neq 0 \wedge m > 0$

    1: $\int (a + b\, Sin[e + f\, x])^m\, (c + d\, Sin[e + f\, x])^n\, (A + B\, Sin[e + f\, x] + C\, Sin[e + f\, x]^2)\; dx$ when $b\, c - a\, d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge c^2 - d^2 \neq 0 \wedge m > 0 \wedge n < -1$

Derivation: Nondegenerate sine recurrence 1a with p → 0

Rule: If $b\, c - a\, d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge c^2 - d^2 \neq 0 \wedge m > 0 \wedge n < -1$, then

$$\int (a + b\, Sin[e + f\, x])^m\, (c + d\, Sin[e + f\, x])^n\, (A + B\, Sin[e + f\, x] + C\, Sin[e + f\, x]^2)\; dx \longrightarrow$$

$$- \big(\big(\big(c^2\, C - B\, c\, d + A\, d^2\big)\, Cos[e + f\, x]\, (a + b\, Sin[e + f\, x])^m\, (c + d\, Sin[e + f\, x])^{n+1}\big) \big/ \big(d\, f\, (n + 1)\, (c^2 - d^2)\big)\big) +$$

$$\frac{1}{d\, (n + 1)\, (c^2 - d^2)} \int (a + b\, Sin[e + f\, x])^{m-1}\, (c + d\, Sin[e + f\, x])^{n+1} \;\cdot$$

$$\big(A\, d\, (b\, d\, m + a\, c\, (n + 1)) + (c\, C - B\, d)\, (b\, c\, m + a\, d\, (n + 1)) -$$
$$(d\, (A\, (a\, d\, (n + 2) - b\, c\, (n + 1)) + B\, (b\, d\, (n + 1) - a\, c\, (n + 2))) - C\, (b\, c\, d\, (n + 1) - a\, (c^2 + d^2\, (n + 1))))\, Sin[e + f\, x] +$$
$$b\, (d\, (B\, c - A\, d)\, (m + n + 2) - C\, (c^2\, (m + 1) + d^2\, (n + 1)))\, Sin[e + f\, x]^2\big)\; dx$$

Program code:

```
Int[(a_.+b_.*sin[e_.+f_.*x_])^m_*(c_.+d_.*sin[e_.+f_.*x_])^n_*(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -(c^2*C-B*c*d+A*d^2)*Cos[e+f*x]*(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)/(d*f*(n+1)*(c^2-d^2)) +
  1/(d*(n+1)*(c^2-d^2))*Int[(a+b*Sin[e+f*x])^(m-1)*(c+d*Sin[e+f*x])^(n+1)*
    Simp[A*d*(b*d*m+a*c*(n+1))+(c*C-B*d)*(b*c*m+a*d*(n+1)) -
      (d*(A*(a*d*(n+2)-b*c*(n+1))+B*(b*d*(n+1)-a*c*(n+2)))-C*(b*c*d*(n+1)-a*(c^2+d^2*(n+1))))*Sin[e+f*x] +
      b*(d*(B*c-A*d)*(m+n+2)-C*(c^2*(m+1)+d^2*(n+1)))*Sin[e+f*x]^2,x],x] /;
  FreeQ[{a,b,c,d,e,f,A,B,C},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0] && GtQ[m,0] && LtQ[n,-1]
```

```
Int[(a_.+b_.*sin[e_.+f_.*x_])^m_*(c_.+d_.*sin[e_.+f_.*x_])^n_*(A_.+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -(c^2*C+A*d^2)*Cos[e+f*x]*(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)/(d*f*(n+1)*(c^2-d^2)) +
  1/(d*(n+1)*(c^2-d^2))*Int[(a+b*Sin[e+f*x])^(m-1)*(c+d*Sin[e+f*x])^(n+1)*
    Simp[A*d*(b*d*m+a*c*(n+1))+c*C*(b*c*m+a*d*(n+1)) -
      (A*d*(a*d*(n+2)-b*c*(n+1))-C*(b*c*d*(n+1)-a*(c^2+d^2*(n+1))))*Sin[e+f*x] -
      b*(A*d^2*(m+n+2)+C*(c^2*(m+1)+d^2*(n+1)))*Sin[e+f*x]^2,x],x] /;
FreeQ[{a,b,c,d,e,f,A,C},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0] && GtQ[m,0] && LtQ[n,-1]
```

2: $\int (a + b \, \text{Sin}[e + f \, x])^m \, (c + d \, \text{Sin}[e + f \, x])^n \, (A + B \, \text{Sin}[e + f \, x] + C \, \text{Sin}[e + f \, x]^2) \, dx$ when $b \, c - a \, d \neq 0 \,\wedge\, a^2 - b^2 \neq 0 \,\wedge\, c^2 - d^2 \neq 0 \,\wedge\, m > 0 \,\wedge\, n \not< -1$

Derivation: Nondegenerate sine recurrence 1b with p → 0

Rule: If $b \, c - a \, d \neq 0 \,\wedge\, a^2 - b^2 \neq 0 \,\wedge\, c^2 - d^2 \neq 0 \,\wedge\, m > 0 \,\wedge\, n \not< -1$, then

$$\int (a + b \, \text{Sin}[e + f \, x])^m \, (c + d \, \text{Sin}[e + f \, x])^n \, (A + B \, \text{Sin}[e + f \, x] + C \, \text{Sin}[e + f \, x]^2) \, dx \;\longrightarrow$$

$$-\frac{C \, \text{Cos}[e + f \, x] \, (a + b \, \text{Sin}[e + f \, x])^m \, (c + d \, \text{Sin}[e + f \, x])^{n+1}}{d \, f \, (m + n + 2)} +$$

$$\frac{1}{d \, (m + n + 2)} \int (a + b \, \text{Sin}[e + f \, x])^{m-1} \, (c + d \, \text{Sin}[e + f \, x])^n \cdot$$

$$\Big(a \, A \, d \, (m + n + 2) + C \, \big(b \, c \, m + a \, d \, (n + 1)\big) +$$

$$\big(d \, (A \, b + a \, B) \, (m + n + 2) - C \, (a \, c - b \, d \, (m + n + 1))\big) \, \text{Sin}[e + f \, x] + \big(C \, (a \, d \, m - b \, c \, (m + 1)) + b \, B \, d \, (m + n + 2)\big) \, \text{Sin}[e + f \, x]^2\Big) \, dx$$

Program code:

```
Int[(a_.+b_.*sin[e_.+f_.*x_])^m_.*(c_.+d_.*sin[e_.+f_.*x_])^n_.*(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -C*Cos[e+f*x]*(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)/(d*f*(m+n+2)) +
  1/(d*(m+n+2))*Int[(a+b*Sin[e+f*x])^(m-1)*(c+d*Sin[e+f*x])^n*
    Simp[a*A*d*(m+n+2)+C*(b*c*m+a*d*(n+1))+
      (d*(A*b+a*B)*(m+n+2)-C*(a*c-b*d*(m+n+1)))*Sin[e+f*x]+
      (C*(a*d*m-b*c*(m+1))+b*B*d*(m+n+2))*Sin[e+f*x]^2,x],x] /;
FreeQ[{a,b,c,d,e,f,A,B,C,n},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0] && GtQ[m,0] &&
  Not[IGtQ[n,0]] && (Not[IntegerQ[m]] || EqQ[a,0] && NeQ[c,0])]
```

```
Int[(a_.+b_.*sin[e_.+f_.*x_])^m_.*(c_.+d_.*sin[e_.+f_.*x_])^n_.*(A_.+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -C*Cos[e+f*x]*(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^(n+1)/(d*f*(m+n+2)) +
  1/(d*(m+n+2))*Int[(a+b*Sin[e+f*x])^(m-1)*(c+d*Sin[e+f*x])^n*
    Simp[a*A*d*(m+n+2)+C*(b*c*m+a*d*(n+1))+(A*b*d*(m+n+2)-C*(a*c-b*d*(m+n+1)))*Sin[e+f*x]+C*(a*d*m-b*c*(m+1))*Sin[e+f*x]^2,x],x] /;
FreeQ[{a,b,c,d,e,f,A,C,n},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0] && GtQ[m,0] &&
  Not[IGtQ[n,0] && (Not[IntegerQ[m]] || EqQ[a,0] && NeQ[c,0])]
```

2. $\int (a + b \, \mathrm{Sin}[e + f \, x])^m \, (c + d \, \mathrm{Sin}[e + f \, x])^n \, (A + B \, \mathrm{Sin}[e + f \, x] + C \, \mathrm{Sin}[e + f \, x]^2) \, dx$ when $b \, c - a \, d \neq 0 \land a^2 - b^2 \neq 0 \land c^2 - d^2 \neq 0 \land m < -1$

   1. $\int \dfrac{A + B \, \mathrm{Sin}[e + f \, x] + C \, \mathrm{Sin}[e + f \, x]^2}{(a + b \, \mathrm{Sin}[e + f \, x])^{3/2} \, \sqrt{c + d \, \mathrm{Sin}[e + f \, x]}} \, dx$ when $b \, c - a \, d \neq 0 \land a^2 - b^2 \neq 0 \land c^2 - d^2 \neq 0$

      1: $\int \dfrac{A + B \, \mathrm{Sin}[e + f \, x] + C \, \mathrm{Sin}[e + f \, x]^2}{(a + b \, \mathrm{Sin}[e + f \, x])^{3/2} \, \sqrt{d \, \mathrm{Sin}[e + f \, x]}} \, dx$ when $a^2 - b^2 \neq 0$

Derivation: Algebraic expansion

Basis: $\dfrac{A + B \, z + C \, z^2}{(a + b \, z)^{3/2} \, \sqrt{d \, z}} \;==\; \dfrac{C \, \sqrt{d \, z}}{b \, d \, \sqrt{a + b \, z}} + \dfrac{A \, b + (b \, B - a \, C) \, z}{b \, (a + b \, z)^{3/2} \, \sqrt{d \, z}}$

Rule: If $a^2 - b^2 \neq 0$, then

$$\int \frac{A + B \, \mathrm{Sin}[e + f \, x] + C \, \mathrm{Sin}[e + f \, x]^2}{(a + b \, \mathrm{Sin}[e + f \, x])^{3/2} \, \sqrt{d \, \mathrm{Sin}[e + f \, x]}} \, dx \;\longrightarrow\; \frac{C}{b \, d} \int \frac{\sqrt{d \, \mathrm{Sin}[e + f \, x]}}{\sqrt{a + b \, \mathrm{Sin}[e + f \, x]}} \, dx + \frac{1}{b} \int \frac{A \, b + (b \, B - a \, C) \, \mathrm{Sin}[e + f \, x]}{(a + b \, \mathrm{Sin}[e + f \, x])^{3/2} \, \sqrt{d \, \mathrm{Sin}[e + f \, x]}} \, dx$$

Program code:

```
Int[(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2)/((a_+b_.*sin[e_.+f_.*x_])^(3/2)*Sqrt[d_.*sin[e_.+f_.*x_]]),x_Symbol] :=
  C/(b*d)*Int[Sqrt[d*Sin[e+f*x]]/Sqrt[a+b*Sin[e+f*x]],x] +
  1/b*Int[(A*b+(b*B-a*C)*Sin[e+f*x])/((a+b*Sin[e+f*x])^(3/2)*Sqrt[d*Sin[e+f*x]]),x] /;
FreeQ[{a,b,d,e,f,A,B,C},x] && NeQ[a^2-b^2,0]
```

```
Int[(A_.+C_.*sin[e_.+f_.*x_]^2)/((a_+b_.*sin[e_.+f_.*x_])^(3/2)*Sqrt[d_.*sin[e_.+f_.*x_]]),x_Symbol] :=
  C/(b*d)*Int[Sqrt[d*Sin[e+f*x]]/Sqrt[a+b*Sin[e+f*x]],x] +
  1/b*Int[(A*b-a*C*Sin[e+f*x])/((a+b*Sin[e+f*x])^(3/2)*Sqrt[d*Sin[e+f*x]]),x] /;
FreeQ[{a,b,d,e,f,A,C},x] && NeQ[a^2-b^2,0]
```

2: $\displaystyle\int \frac{A + B \operatorname{Sin}[e + f\,x] + C \operatorname{Sin}[e + f\,x]^2}{(a + b \operatorname{Sin}[e + f\,x])^{3/2} \sqrt{c + d \operatorname{Sin}[e + f\,x]}}\, dx$ when $b\,c - a\,d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge c^2 - d^2 \neq 0$

Derivation: Algebraic expansion

Basis: $\displaystyle\frac{A + B\,z + C\,z^2}{(a + b\,z)^{3/2}} == \frac{C\,\sqrt{a + b\,z}}{b^2} + \frac{A\,b^2 - a^2\,C + b\,(b\,B - 2\,a\,C)\,z}{b^2\,(a + b\,z)^{3/2}}$

Rule: If $b\,c - a\,d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge c^2 - d^2 \neq 0$, then

$$\int \frac{A + B \operatorname{Sin}[e + f\,x] + C \operatorname{Sin}[e + f\,x]^2}{(a + b \operatorname{Sin}[e + f\,x])^{3/2} \sqrt{c + d \operatorname{Sin}[e + f\,x]}}\, dx \;\longrightarrow\; \frac{C}{b^2} \int \frac{\sqrt{a + b \operatorname{Sin}[e + f\,x]}}{\sqrt{c + d \operatorname{Sin}[e + f\,x]}}\, dx + \frac{1}{b^2} \int \frac{A\,b^2 - a^2\,C + b\,(b\,B - 2\,a\,C) \operatorname{Sin}[e + f\,x]}{(a + b \operatorname{Sin}[e + f\,x])^{3/2} \sqrt{c + d \operatorname{Sin}[e + f\,x]}}\, dx$$

Program code:

```
Int[(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2)/((a_.+b_.*sin[e_.+f_.*x_])^(3/2)*Sqrt[c_.+d_.*sin[e_.+f_.*x_]]),x_Symbol] :=
  C/b^2*Int[Sqrt[a+b*Sin[e+f*x]]/Sqrt[c+d*Sin[e+f*x]],x] +
  1/b^2*Int[(A*b^2-a^2*C+b*(b*B-2*a*C)*Sin[e+f*x])/((a+b*Sin[e+f*x])^(3/2)*Sqrt[c+d*Sin[e+f*x]]),x] /;
FreeQ[{a,b,c,d,e,f,A,B,C},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0]
```

```
Int[(A_.+C_.*sin[e_.+f_.*x_]^2)/((a_.+b_.*sin[e_.+f_.*x_])^(3/2)*Sqrt[c_.+d_.*sin[e_.+f_.*x_]]),x_Symbol] :=
  C/b^2*Int[Sqrt[a+b*Sin[e+f*x]]/Sqrt[c+d*Sin[e+f*x]],x] +
  1/b^2*Int[(A*b^2-a^2*C-2*a*b*C*Sin[e+f*x])/((a+b*Sin[e+f*x])^(3/2)*Sqrt[c+d*Sin[e+f*x]]),x] /;
FreeQ[{a,b,c,d,e,f,A,C},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0]
```

2: $\int \left(a + b\,\mathrm{Sin}[e + f\,x]\right)^m \left(c + d\,\mathrm{Sin}[e + f\,x]\right)^n \left(A + B\,\mathrm{Sin}[e + f\,x] + C\,\mathrm{Sin}[e + f\,x]^2\right) dx$ when $b\,c - a\,d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge c^2 - d^2 \neq 0 \wedge m < -1$

Derivation: Nondegenerate sine recurrence 1c with $p \to 0$

Rule: If $b\,c - a\,d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge c^2 - d^2 \neq 0 \wedge m < -1$, then

$$\int \left(a + b\,\mathrm{Sin}[e + f\,x]\right)^m \left(c + d\,\mathrm{Sin}[e + f\,x]\right)^n \left(A + B\,\mathrm{Sin}[e + f\,x] + C\,\mathrm{Sin}[e + f\,x]^2\right) dx \longrightarrow$$

$$- \left(\left(\left(A\,b^2 - a\,b\,B + a^2\,C\right) \mathrm{Cos}[e + f\,x] \left(a + b\,\mathrm{Sin}[e + f\,x]\right)^{m+1} \left(c + d\,\mathrm{Sin}[e + f\,x]\right)^{n+1} \right) / \left(f\,(m+1)\,(b\,c - a\,d)\,(a^2 - b^2)\right) \right) +$$

$$\frac{1}{(m+1)\,(b\,c - a\,d)\,(a^2 - b^2)} \int \left(a + b\,\mathrm{Sin}[e + f\,x]\right)^{m+1} \left(c + d\,\mathrm{Sin}[e + f\,x]\right)^n \cdot$$

$$\Big((m+1)\,(b\,c - a\,d)\,(a\,A - b\,B + a\,C) + d\,\left(A\,b^2 - a\,b\,B + a^2\,C\right)(m+n+2) -$$
$$\left(c\,\left(A\,b^2 - a\,b\,B + a^2\,C\right) + (m+1)\,(b\,c - a\,d)\,(A\,b - a\,B + b\,C)\right) \mathrm{Sin}[e + f\,x] -$$
$$d\,\left(A\,b^2 - a\,b\,B + a^2\,C\right)(m+n+3)\,\mathrm{Sin}[e + f\,x]^2 \Big) dx$$

Program code:

```
Int[(a_.+b_.*sin[e_.+f_.*x_])^m_*(c_.+d_.*sin[e_.+f_.*x_])^n_*(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -(A*b^2-a*b*B+a^2*C)*Cos[e+f*x]*(a+b*Sin[e+f*x])^(m+1)*(c+d*Sin[e+f*x])^(n+1)/(f*(m+1)*(b*c-a*d)*(a^2-b^2)) +
   1/((m+1)*(b*c-a*d)*(a^2-b^2))*Int[(a+b*Sin[e+f*x])^(m+1)*(c+d*Sin[e+f*x])^n*
     Simp[(m+1)*(b*c-a*d)*(a*A-b*B+a*C)+d*(A*b^2-a*b*B+a^2*C)*(m+n+2) -
       (c*(A*b^2-a*b*B+a^2*C)+(m+1)*(b*c-a*d)*(A*b-a*B+b*C))*Sin[e+f*x] -
       d*(A*b^2-a*b*B+a^2*C)*(m+n+3)*Sin[e+f*x]^2,x],x] /;
FreeQ[{a,b,c,d,e,f,A,B,C,n},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0] && LtQ[m,-1] &&
  (EqQ[a,0] && IntegerQ[m] && Not[IntegerQ[n]] || Not[IntegerQ[2*n] && LtQ[n,-1] && (IntegerQ[n] && Not[IntegerQ[m]] || EqQ[a,0])])
```

```
Int[(a_.+b_.*sin[e_.+f_.*x_])^m_*(c_.+d_.*sin[e_.+f_.*x_])^n_*(A_.+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  -(A*b^2+a^2*C)*Cos[e+f*x]*(a+b*Sin[e+f*x])^(m+1)*(c+d*Sin[e+f*x])^(n+1)/(f*(m+1)*(b*c-a*d)*(a^2-b^2)) +
   1/((m+1)*(b*c-a*d)*(a^2-b^2))*Int[(a+b*Sin[e+f*x])^(m+1)*(c+d*Sin[e+f*x])^n*
     Simp[a*(m+1)*(b*c-a*d)*(A+C)+d*(A*b^2+a^2*C)*(m+n+2) -
       (c*(A*b^2+a^2*C)+b*(m+1)*(b*c-a*d)*(A+C))*Sin[e+f*x] -
       d*(A*b^2+a^2*C)*(m+n+3)*Sin[e+f*x]^2,x],x] /;
FreeQ[{a,b,c,d,e,f,A,C,n},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0] && LtQ[m,-1] &&
  (EqQ[a,0] && IntegerQ[m] && Not[IntegerQ[n]] || Not[IntegerQ[2*n] && LtQ[n,-1] && (IntegerQ[n] && Not[IntegerQ[m]] || EqQ[a,0])])
```

3: $\displaystyle\int \frac{A + B \sin[e + f\,x] + C \sin[e + f\,x]^2}{(a + b \sin[e + f\,x])\,(c + d \sin[e + f\,x])}\,dx$ when $b\,c - a\,d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge c^2 - d^2 \neq 0$

Derivation: Algebraic expansion

Basis: $\displaystyle\frac{A + B z + C z^2}{(a + b z)\,(c + d z)} \;==\; \frac{C}{b\,d} + \frac{A\,b^2 - a\,b\,B + a^2\,C}{b\,(b\,c - a\,d)\,(a + b\,z)} - \frac{c^2\,C - B\,c\,d + A\,d^2}{d\,(b\,c - a\,d)\,(c + d\,z)}$

Rule: If $b\,c - a\,d \neq 0 \,\wedge\, a^2 - b^2 \neq 0 \,\wedge\, c^2 - d^2 \neq 0$, then

$$\int \frac{A + B \sin[e + f\,x] + C \sin[e + f\,x]^2}{(a + b \sin[e + f\,x])\,(c + d \sin[e + f\,x])}\,dx \;\longrightarrow$$

$$\frac{C\,x}{b\,d} + \frac{A\,b^2 - a\,b\,B + a^2\,C}{b\,(b\,c - a\,d)}\int \frac{1}{a + b \sin[e + f\,x]}\,dx - \frac{c^2\,C - B\,c\,d + A\,d^2}{d\,(b\,c - a\,d)}\int \frac{1}{c + d \sin[e + f\,x]}\,dx$$

Program code:

```
Int[(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2)/((a_+b_.*sin[e_.+f_.*x_])*(c_.+d_.*sin[e_.+f_.*x_])),x_Symbol] :=
  C*x/(b*d) +
  (A*b^2-a*b*B+a^2*C)/(b*(b*c-a*d))*Int[1/(a+b*Sin[e+f*x]),x] -
  (c^2*C-B*c*d+A*d^2)/(d*(b*c-a*d))*Int[1/(c+d*Sin[e+f*x]),x] /;
FreeQ[{a,b,c,d,e,f,A,B,C},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0]
```

```
Int[(A_.+C_.*sin[e_.+f_.*x_]^2)/((a_+b_.*sin[e_.+f_.*x_])*(c_.+d_.*sin[e_.+f_.*x_])),x_Symbol] :=
  C*x/(b*d) +
  (A*b^2+a^2*C)/(b*(b*c-a*d))*Int[1/(a+b*Sin[e+f*x]),x] -
  (c^2*C+A*d^2)/(d*(b*c-a*d))*Int[1/(c+d*Sin[e+f*x]),x] /;
FreeQ[{a,b,c,d,e,f,A,C},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0]
```

4: $\displaystyle\int \frac{A + B\,\text{Sin}[e + f\,x] + C\,\text{Sin}[e + f\,x]^2}{\sqrt{a + b\,\text{Sin}[e + f\,x]}\ (c + d\,\text{Sin}[e + f\,x])}\,\text{d}x$ when $b\,c - a\,d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge c^2 - d^2 \neq 0$

Derivation: Algebraic expansion

Basis: $\displaystyle\frac{A + B\,z + C\,z^2}{\sqrt{a + b\,z}\ (c + d\,z)} == \frac{C\,\sqrt{a + b\,z}}{b\,d} - \frac{a\,c\,C - A\,b\,d + (b\,c\,C - b\,B\,d + a\,C\,d)\,z}{b\,d\,\sqrt{a + b\,z}\ (c + d\,z)}$

Rule: If $b\,c - a\,d \neq 0 \wedge a^2 - b^2 \neq 0 \wedge c^2 - d^2 \neq 0$, then

$$\int \frac{A + B\,\text{Sin}[e + f\,x] + C\,\text{Sin}[e + f\,x]^2}{\sqrt{a + b\,\text{Sin}[e + f\,x]}\ (c + d\,\text{Sin}[e + f\,x])}\,\text{d}x \rightarrow$$

$$\frac{C}{b\,d}\int \sqrt{a + b\,\text{Sin}[e + f\,x]}\ \text{d}x - \frac{1}{b\,d}\int \frac{a\,c\,C - A\,b\,d + (b\,c\,C - b\,B\,d + a\,C\,d)\,\text{Sin}[e + f\,x]}{\sqrt{a + b\,\text{Sin}[e + f\,x]}\ (c + d\,\text{Sin}[e + f\,x])}\,\text{d}x$$

Program code:

```
Int[(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2)/(Sqrt[a_.+b_.*sin[e_.+f_.*x_]]*(c_.+d_.*sin[e_.+f_.*x_])),x_Symbol] :=
  C/(b*d)*Int[Sqrt[a+b*Sin[e+f*x]],x] -
  1/(b*d)*Int[Simp[a*c*C-A*b*d+(b*c*C-b*B*d+a*C*d)*Sin[e+f*x],x]/(Sqrt[a+b*Sin[e+f*x]]*(c+d*Sin[e+f*x])),x] /;
FreeQ[{a,b,c,d,e,f,A,B,C},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0]
```

```
Int[(A_.+C_.*sin[e_.+f_.*x_]^2)/(Sqrt[a_.+b_.*sin[e_.+f_.*x_]]*(c_.+d_.*sin[e_.+f_.*x_])),x_Symbol] :=
  C/(b*d)*Int[Sqrt[a+b*Sin[e+f*x]],x] -
  1/(b*d)*Int[Simp[a*c*C-A*b*d+(b*c*C+a*C*d)*Sin[e+f*x],x]/(Sqrt[a+b*Sin[e+f*x]]*(c+d*Sin[e+f*x])),x] /;
FreeQ[{a,b,c,d,e,f,A,C},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0]
```

5: $\displaystyle\int \frac{A + B\,\text{Sin}\left[e + f\,x\right] + C\,\text{Sin}\left[e + f\,x\right]^2}{\sqrt{a + b\,\text{Sin}\left[e + f\,x\right]}\;\sqrt{c + d\,\text{Sin}\left[e + f\,x\right]}}\;dx$ when $b\,c - a\,d \neq 0 \,\wedge\, a^2 - b^2 \neq 0 \,\wedge\, c^2 - d^2 \neq 0$

Derivation: Nondegenerate sine recurrence 1b with $m \to -\frac{1}{2}$, $n \to -\frac{1}{2}$, $p \to 0$

Note: If one of the square root factors does not have a constant term, it is better to raise that factor to the 3/2 power.

Rule: If $b\,c - a\,d \neq 0 \,\wedge\, a^2 - b^2 \neq 0 \,\wedge\, c^2 - d^2 \neq 0$, then

$$\int \frac{A + B\,\text{Sin}\left[e + f\,x\right] + C\,\text{Sin}\left[e + f\,x\right]^2}{\sqrt{a + b\,\text{Sin}\left[e + f\,x\right]}\;\sqrt{c + d\,\text{Sin}\left[e + f\,x\right]}}\;dx \;\longrightarrow$$

$$-\frac{C\,\text{Cos}\left[e + f\,x\right]\,\sqrt{c + d\,\text{Sin}\left[e + f\,x\right]}}{d\,f\,\sqrt{a + b\,\text{Sin}\left[e + f\,x\right]}} \;+$$

$$\frac{1}{2\,d}\int \left(\left(2\,a\,A\,d - C\,(b\,c - a\,d) - 2\,(a\,c\,C - d\,(A\,b + a\,B))\,\text{Sin}\left[e + f\,x\right] + (2\,b\,B\,d - C\,(b\,c + a\,d))\,\text{Sin}\left[e + f\,x\right]^2\right) \middle/ \left((a + b\,\text{Sin}\left[e + f\,x\right])^{3/2}\,\sqrt{c + d\,\text{Sin}\left[e + f\,x\right]}\right)\right)\,dx$$

Program code:

```
Int[(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2)/(Sqrt[a_.+b_.*sin[e_.+f_.*x_]]*Sqrt[c_+d_.*sin[e_.+f_.*x_]]),x_Symbol] :=
  -C*Cos[e+f*x]*Sqrt[c+d*Sin[e+f*x]]/(d*f*Sqrt[a+b*Sin[e+f*x]]) +
  1/(2*d)*Int[1/((a+b*Sin[e+f*x])^(3/2)*Sqrt[c+d*Sin[e+f*x]])*
    Simp[2*a*A*d-C*(b*c-a*d)-2*(a*c*C-d*(A*b+a*B))*Sin[e+f*x]+(2*b*B*d-C*(b*c+a*d))*Sin[e+f*x]^2,x],x] /;
FreeQ[{a,b,c,d,e,f,A,B,C},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0]
```

```
Int[(A_.+C_.*sin[e_.+f_.*x_]^2)/(Sqrt[a_.+b_.*sin[e_.+f_.*x_]]*Sqrt[c_+d_.*sin[e_.+f_.*x_]]),x_Symbol] :=
  -C*Cos[e+f*x]*Sqrt[c+d*Sin[e+f*x]]/(d*f*Sqrt[a+b*Sin[e+f*x]]) +
  1/(2*d)*Int[1/((a+b*Sin[e+f*x])^(3/2)*Sqrt[c+d*Sin[e+f*x]])*
    Simp[2*a*A*d-C*(b*c-a*d)-2*(a*c*C-A*b*d)*Sin[e+f*x]-C*(b*c+a*d)*Sin[e+f*x]^2,x],x] /;
FreeQ[{a,b,c,d,e,f,A,C},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0]
```

6: $\int \dfrac{(d \, Sin[e + f \, x])^n \, (A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2)}{a + b \, Sin[e + f \, x]} \, dx$ when $a^2 - b^2 \neq 0$

Derivation: Algebraic expansion

Basis: $\dfrac{A + B \, z + C \, z^2}{a + b \, z} == \dfrac{b \, B - a \, C}{b^2} + \dfrac{C \, z}{b} + \dfrac{A \, b^2 - a \, b \, B + a^2 \, C}{b^2 \, (a + b \, z)}$

Rule: If $a^2 - b^2 \neq 0$, then

$$\int \dfrac{(d \, Sin[e + f \, x])^n \, (A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2)}{a + b \, Sin[e + f \, x]} \, dx \longrightarrow$$

$$\dfrac{b \, B - a \, C}{b^2} \int (d \, Sin[e + f \, x])^n \, dx + \dfrac{C}{b \, d} \int (d \, Sin[e + f \, x])^{n+1} \, dx + \dfrac{A \, b^2 - a \, b \, B + a^2 \, C}{b^2} \int \dfrac{(d \, Sin[e + f \, x])^n}{a + b \, Sin[e + f \, x]} \, dx$$

Program code:

```
Int[(d_.*sin[e_.+f_.*x_])^n_.*(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2)/(a_+b_.*sin[e_.+f_.*x_]),x_Symbol] :=
  (b*B-a*C)/b^2*Int[(d*Sin[e+f*x])^n,x] +
  C/(b*d)*Int[(d*Sin[e+f*x])^(n+1),x] +
  (A*b^2-a*b*B+a^2*C)/b^2*Int[(d*Sin[e+f*x])^n/(a+b*Sin[e+f*x]),x] /;
FreeQ[{a,b,d,e,f,A,B,C,n},x] && NeQ[a^2-b^2,0]
```

```
Int[(d_.*sin[e_.+f_.*x_])^n_.*(A_.+C_.*sin[e_.+f_.*x_]^2)/(a_+b_.*sin[e_.+f_.*x_]),x_Symbol] :=
  -a*C/b^2*Int[(d*Sin[e+f*x])^n,x] +
  C/(b*d)*Int[(d*Sin[e+f*x])^(n+1),x] +
  (A*b^2+a^2*C)/b^2*Int[(d*Sin[e+f*x])^n/(a+b*Sin[e+f*x]),x] /;
FreeQ[{a,b,d,e,f,A,C,n},x] && NeQ[a^2-b^2,0]
```

U: $\int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^n \, (A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2) \, dx$ when $b \, c - a \, d \neq 0 \, \wedge \, a^2 - b^2 \neq 0 \, \wedge \, c^2 - d^2 \neq 0$

Rule: If $b \, c - a \, d \neq 0 \, \wedge \, a^2 - b^2 \neq 0 \, \wedge \, c^2 - d^2 \neq 0$, then

$$\int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^n \, (A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2) \, dx \longrightarrow$$

$$\int (a + b \, Sin[e + f \, x])^m \, (c + d \, Sin[e + f \, x])^n \, (A + B \, Sin[e + f \, x] + C \, Sin[e + f \, x]^2) \, dx$$

Program code:

```
Int[(a_.+b_.*sin[e_.+f_.*x_])^m_*(c_.+d_.*sin[e_.+f_.*x_])^n_*(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  Unintegrable[(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^n*(A+B*Sin[e+f*x]+C*Sin[e+f*x]^2),x] /;
FreeQ[{a,b,c,d,e,f,A,B,C,m,n},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0]
```

```
Int[(a_.+b_.*sin[e_.+f_.*x_])^m_*(c_.+d_.*sin[e_.+f_.*x_])^n_*(A_.+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  Unintegrable[(a+b*Sin[e+f*x])^m*(c+d*Sin[e+f*x])^n*(A+C*Sin[e+f*x]^2),x] /;
FreeQ[{a,b,c,d,e,f,A,C,m,n},x] && NeQ[b*c-a*d,0] && NeQ[a^2-b^2,0] && NeQ[c^2-d^2,0]
```

Rules for integrands of the form $\left(b\,\mathrm{Sin}[e+f\,x]^p\right)^m\,(c+d\,\mathrm{Sin}[e+f\,x])^n\,\left(A+B\,\mathrm{Sin}[e+f\,x]+C\,\mathrm{Sin}[e+f\,x]^2\right)$

1: $\int\left(b\,\mathrm{Sin}[e+f\,x]^p\right)^m\,(c+d\,\mathrm{Sin}[e+f\,x])^n\,\left(A+B\,\mathrm{Sin}[e+f\,x]+C\,\mathrm{Sin}[e+f\,x]^2\right)\,dx$ when $m \notin \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: $\partial_x \dfrac{\left(b\,\mathrm{Sin}[e+f\,x]^p\right)^m}{\left(b\,\mathrm{Sin}[e+f\,x]\right)^{m\,p}} == 0$

Rule: If $m \notin \mathbb{Z}$, then

$$\int\left(b\,\mathrm{Sin}[e+f\,x]^p\right)^m\,(c+d\,\mathrm{Sin}[e+f\,x])^n\,\left(A+B\,\mathrm{Sin}[e+f\,x]+C\,\mathrm{Sin}[e+f\,x]^2\right)\,dx \longrightarrow$$

$$\frac{\left(b\,\mathrm{Sin}[e+f\,x]^p\right)^m}{\left(b\,\mathrm{Sin}[e+f\,x]\right)^{m\,p}}\int\left(b\,\mathrm{Sin}[e+f\,x]\right)^{m\,p}\,(c+d\,\mathrm{Sin}[e+f\,x])^n\,\left(A+B\,\mathrm{Sin}[e+f\,x]+C\,\mathrm{Sin}[e+f\,x]^2\right)\,dx$$

Program code:

```
Int[(b_.*sin[e_.+f_.*x_]^p_)^m_*(c_.+d_.*sin[e_.+f_.*x_])^n_.*(A_.+B_.*sin[e_.+f_.*x_]+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  (b*Sin[e+f*x]^p)^m/(b*Sin[e+f*x])^(m*p)*Int[(b*Sin[e+f*x])^(m*p)*(c+d*Sin[e+f*x])^n*(A+B*Sin[e+f*x]+C*Sin[e+f*x]^2),x] /;
FreeQ[{b,c,d,e,f,A,B,C,m,n,p},x] && Not[IntegerQ[m]]
```

```
Int[(b_.*cos[e_.+f_.*x_]^p_)^m_*(c_.+d_.*cos[e_.+f_.*x_])^n_.*(A_.+B_.*cos[e_.+f_.*x_]+C_.*cos[e_.+f_.*x_]^2),x_Symbol] :=
  (b*Cos[e+f*x]^p)^m/(b*Cos[e+f*x])^(m*p)*Int[(b*Cos[e+f*x])^(m*p)*(c+d*Cos[e+f*x])^n*(A+B*Cos[e+f*x]+C*Cos[e+f*x]^2),x] /;
FreeQ[{b,c,d,e,f,A,B,C,m,n,p},x] && Not[IntegerQ[m]]
```

```
Int[(b_.*sin[e_.+f_.*x_]^p_)^m_*(c_.+d_.*sin[e_.+f_.*x_])^n_.*(A_.+C_.*sin[e_.+f_.*x_]^2),x_Symbol] :=
  (b*Sin[e+f*x]^p)^m/(b*Sin[e+f*x])^(m*p)*Int[(b*Sin[e+f*x])^(m*p)*(c+d*Sin[e+f*x])^n*(A+C*Sin[e+f*x]^2),x] /;
FreeQ[{b,c,d,e,f,A,C,m,n,p},x] && Not[IntegerQ[m]]
```

```
Int[(b_.*cos[e_.+f_.*x_]^p_)^m_*(c_.+d_.*cos[e_.+f_.*x_])^n_.*(A_.+C_.*cos[e_.+f_.*x_]^2),x_Symbol] :=
    (b*Cos[e+f*x]^p)^m/(b*Cos[e+f*x])^(m*p)*Int[(b*Cos[e+f*x])^(m*p)*(c+d*Cos[e+f*x])^n*(A+C*Cos[e+f*x]^2),x] /;
FreeQ[{b,c,d,e,f,A,C,m,n,p},x] && Not[IntegerQ[m]]
```