Rules for integrands of the form $(d+e\,x)^m \operatorname{Sinh}\left[a+b\,x+c\,x^2\right]^n$

1. $\int \operatorname{Sinh}\left[a+b\,x+c\,x^2\right]^n\,dx$

   1: $\int \operatorname{Sinh}\left[a+b\,x+c\,x^2\right]\,dx$

Derivation: Algebraic expansion

Basis: $\operatorname{Sinh}[z] == \frac{e^z}{2} - \frac{e^{-z}}{2}$

Rule:

$$\int \operatorname{Sinh}\left[a+b\,x+c\,x^2\right]\,dx \longrightarrow \frac{1}{2}\int e^{a+b\,x+c\,x^2}\,dx - \frac{1}{2}\int e^{-a-b\,x-c\,x^2}\,dx$$

Program code:

```
Int[Sinh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  1/2*Int[E^(a+b*x+c*x^2),x] - 1/2*Int[E^(-a-b*x-c*x^2),x] /;
FreeQ[{a,b,c},x]
```

```
Int[Cosh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  1/2*Int[E^(a+b*x+c*x^2),x] + 1/2*Int[E^(-a-b*x-c*x^2),x] /;
FreeQ[{a,b,c},x]
```

2: $\int \text{Sinh}\left[a+b\,x+c\,x^2\right]^n\,dx$ when $n\in\mathbb{Z}\,\land\,n>1$

Derivation: Algebraic expansion

Rule: If $n\in\mathbb{Z}\,\land\,n>1$, then

$$\int \text{Sinh}\left[a+b\,x+c\,x^2\right]^n\,dx\,\longrightarrow\,\int \text{TrigReduce}\left[\text{Sinh}\left[a+b\,x+c\,x^2\right]^n\right]\,dx$$

Program code:

```
Int[Sinh[a_.+b_.*x_+c_.*x_^2]^n_,x_Symbol] :=
  Int[ExpandTrigReduce[Sinh[a+b*x+c*x^2]^n,x],x] /;
FreeQ[{a,b,c},x] && IGtQ[n,1]
```

```
Int[Cosh[a_.+b_.*x_+c_.*x_^2]^n_,x_Symbol] :=
  Int[ExpandTrigReduce[Cosh[a+b*x+c*x^2]^n,x],x] /;
FreeQ[{a,b,c},x] && IGtQ[n,1]
```

3: $\int \text{Sinh}[v]^n \, dx$ when $n \in \mathbb{Z}^+ \land v == a + b x + c x^2$

Derivation: Algebraic normalization

Rule: If $n \in \mathbb{Z}^+ \land v == a + b x + c x^2$, then

$$\int \text{Sinh}[v]^n \, dx \longrightarrow \int \text{Sinh}\left[a + b x + c x^2\right]^n \, dx$$

Program code:

```
Int[Sinh[v_]^n_.,x_Symbol] :=
  Int[Sinh[ExpandToSum[v,x]]^n,x] /;
IGtQ[n,0] && QuadraticQ[v,x] && Not[QuadraticMatchQ[v,x]]
```

```
Int[Cosh[v_]^n_.,x_Symbol] :=
  Int[Cosh[ExpandToSum[v,x]]^n,x] /;
IGtQ[n,0] && QuadraticQ[v,x] && Not[QuadraticMatchQ[v,x]]
```

2. $\int (d + e\,x)^m \,\mathrm{Sinh}\left[a + b\,x + c\,x^2\right]^n \,dx$

   1. $\int (d + e\,x)^m \,\mathrm{Sinh}\left[a + b\,x + c\,x^2\right] dx$

      1. $\int (d + e\,x)^m \,\mathrm{Sinh}\left[a + b\,x + c\,x^2\right] dx$ when $m > 0$

         1. $\int (d + e\,x) \,\mathrm{Sinh}\left[a + b\,x + c\,x^2\right] dx$

            1: $\int (d + e\,x) \,\mathrm{Sinh}\left[a + b\,x + c\,x^2\right] dx$ when $b\,e - 2\,c\,d == 0$

Rule: If $b\,e - 2\,c\,d == 0$, then

$$\int (d + e\,x) \,\mathrm{Sinh}\left[a + b\,x + c\,x^2\right] dx \longrightarrow \frac{e\,\mathrm{Cosh}\left[a + b\,x + c\,x^2\right]}{2\,c}$$

Program code:

```
Int[(d_.+e_.*x_)*Sinh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  e*Cosh[a+b*x+c*x^2]/(2*c) /;
FreeQ[{a,b,c,d,e},x] && EqQ[b*e-2*c*d,0]
```

```
Int[(d_.+e_.*x_)*Cosh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  e*Sinh[a+b*x+c*x^2]/(2*c) /;
FreeQ[{a,b,c,d,e},x] && EqQ[b*e-2*c*d,0]
```

2: $\int (d+e\,x)\,\mathrm{Sinh}\left[a+b\,x+c\,x^2\right]\,dx$ when $b\,e-2\,c\,d\neq 0$

Rule: If $b\,e-2\,c\,d\neq 0$, then

$$\int (d+e\,x)\,\mathrm{Sinh}\left[a+b\,x+c\,x^2\right]\,dx\;\longrightarrow\;\frac{e\,\mathrm{Cosh}\left[a+b\,x+c\,x^2\right]}{2\,c}-\frac{b\,e-2\,c\,d}{2\,c}\int \mathrm{Sinh}\left[a+b\,x+c\,x^2\right]\,dx$$

Program code:

```
Int[(d_.+e_.*x_)*Sinh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  e*Cosh[a+b*x+c*x^2]/(2*c) -
  (b*e-2*c*d)/(2*c)*Int[Sinh[a+b*x+c*x^2],x] /;
FreeQ[{a,b,c,d,e},x] && NeQ[b*e-2*c*d,0]
```

```
Int[(d_.+e_.*x_)*Cosh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  e*Sinh[a+b*x+c*x^2]/(2*c) -
  (b*e-2*c*d)/(2*c)*Int[Cosh[a+b*x+c*x^2],x] /;
FreeQ[{a,b,c,d,e},x] && NeQ[b*e-2*c*d,0]
```

2. $\int (d+e\,x)^m \,\text{Sinh}\left[a+b\,x+c\,x^2\right] dx$ when $m>1$

1: $\int (d+e\,x)^m \,\text{Sinh}\left[a+b\,x+c\,x^2\right] dx$ when $m>1 \land b\,e-2\,c\,d==0$

Rule: If $m>1 \land b\,e-2\,c\,d==0$, then

$$\int (d+e\,x)^m \,\text{Sinh}\left[a+b\,x+c\,x^2\right] dx \longrightarrow \frac{e\,(d+e\,x)^{m-1}\,\text{Cosh}\left[a+b\,x+c\,x^2\right]}{2\,c}+\frac{e^2\,(m-1)}{2\,c}\int (d+e\,x)^{m-2}\,\text{Cosh}\left[a+b\,x+c\,x^2\right] dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*Sinh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  e*(d+e*x)^(m-1)*Cosh[a+b*x+c*x^2]/(2*c) -
  e^2*(m-1)/(2*c)*Int[(d+e*x)^(m-2)*Cosh[a+b*x+c*x^2],x] /;
FreeQ[{a,b,c,d,e},x] && GtQ[m,1] && EqQ[b*e-2*c*d,0]
```

```
Int[(d_.+e_.*x_)^m_*Cosh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  e*(d+e*x)^(m-1)*Sinh[a+b*x+c*x^2]/(2*c) -
  e^2*(m-1)/(2*c)*Int[(d+e*x)^(m-2)*Sinh[a+b*x+c*x^2],x] /;
FreeQ[{a,b,c,d,e},x] && GtQ[m,1] && EqQ[b*e-2*c*d,0]
```

2: $\int (d+e\,x)^m \operatorname{Sinh}\left[a+b\,x+c\,x^2\right] dx$ when $m>1 \land b\,e-2\,c\,d\neq 0$

Rule: If $m>1 \land b\,e-2\,c\,d\neq 0$, then

$$\int (d+e\,x)^m \operatorname{Sinh}\left[a+b\,x+c\,x^2\right] dx \longrightarrow$$

$$\frac{e\,(d+e\,x)^{m-1} \operatorname{Cosh}\left[a+b\,x+c\,x^2\right]}{2\,c}-\frac{b\,e-2\,c\,d}{2\,c}\int (d+e\,x)^{m-1} \operatorname{Sinh}\left[a+b\,x+c\,x^2\right] dx-\frac{e^2\,(m-1)}{2\,c}\int (d+e\,x)^{m-2} \operatorname{Cosh}\left[a+b\,x+c\,x^2\right] dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*Sinh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  e*(d+e*x)^(m-1)*Cosh[a+b*x+c*x^2]/(2*c) -
  (b*e-2*c*d)/(2*c)*Int[(d+e*x)^(m-1)*Sinh[a+b*x+c*x^2],x] -
  e^2*(m-1)/(2*c)*Int[(d+e*x)^(m-2)*Cosh[a+b*x+c*x^2],x] /;
FreeQ[{a,b,c,d,e},x] && GtQ[m,1] && NeQ[b*e-2*c*d,0]
```

```
Int[(d_.+e_.*x_)^m_*Cosh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  e*(d+e*x)^(m-1)*Sinh[a+b*x+c*x^2]/(2*c) -
  (b*e-2*c*d)/(2*c)*Int[(d+e*x)^(m-1)*Cosh[a+b*x+c*x^2],x] -
  e^2*(m-1)/(2*c)*Int[(d+e*x)^(m-2)*Sinh[a+b*x+c*x^2],x] /;
FreeQ[{a,b,c,d,e},x] && GtQ[m,1] && NeQ[b*e-2*c*d,0]
```

2. $\int (d+e\,x)^m \,\mathrm{Sinh}\left[a+b\,x+c\,x^2\right] dx$ when $m<-1$

1: $\int (d+e\,x)^m \,\mathrm{Sinh}\left[a+b\,x+c\,x^2\right] dx$ when $m<-1 \land b\,e-2\,c\,d==0$

Rule: If $m<-1 \land b\,e-2\,c\,d==0$, then

$$\int (d+e\,x)^m \,\mathrm{Sinh}\left[a+b\,x+c\,x^2\right] dx \longrightarrow \frac{(d+e\,x)^{m+1}\,\mathrm{Sinh}\left[a+b\,x+c\,x^2\right]}{e\,(m+1)}-\frac{2\,c}{e^2\,(m+1)}\int (d+e\,x)^{m+2}\,\mathrm{Cosh}\left[a+b\,x+c\,x^2\right] dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*Sinh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  (d+e*x)^(m+1)*Sinh[a+b*x+c*x^2]/(e*(m+1)) -
  2*c/(e^2*(m+1))*Int[(d+e*x)^(m+2)*Cosh[a+b*x+c*x^2],x] /;
FreeQ[{a,b,c,d,e},x] && LtQ[m,-1] && EqQ[b*e-2*c*d,0]
```

```
Int[(d_.+e_.*x_)^m_*Cosh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  (d+e*x)^(m+1)*Cosh[a+b*x+c*x^2]/(e*(m+1)) -
  2*c/(e^2*(m+1))*Int[(d+e*x)^(m+2)*Sinh[a+b*x+c*x^2],x] /;
FreeQ[{a,b,c,d,e},x] && LtQ[m,-1] && EqQ[b*e-2*c*d,0]
```

2: $\int (d+e\,x)^m \operatorname{Sinh}[a+b\,x+c\,x^2]\,dx$ when $m<-1\land b\,e-2\,c\,d\neq 0$

Rule: If $m<-1\land b\,e-2\,c\,d\neq 0$, then

$$\int (d+e\,x)^m \operatorname{Sinh}[a+b\,x+c\,x^2]\,dx \longrightarrow$$

$$\frac{(d+e\,x)^{m+1}\operatorname{Sinh}[a+b\,x+c\,x^2]}{e\,(m+1)}-\frac{b\,e-2\,c\,d}{e^2\,(m+1)}\int (d+e\,x)^{m+1}\operatorname{Cosh}[a+b\,x+c\,x^2]\,dx-\frac{2\,c}{e^2\,(m+1)}\int (d+e\,x)^{m+2}\operatorname{Cosh}[a+b\,x+c\,x^2]\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*Sinh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  (d+e*x)^(m+1)*Sinh[a+b*x+c*x^2]/(e*(m+1)) -
  (b*e-2*c*d)/(e^2*(m+1))*Int[(d+e*x)^(m+1)*Cosh[a+b*x+c*x^2],x] -
  2*c/(e^2*(m+1))*Int[(d+e*x)^(m+2)*Cosh[a+b*x+c*x^2],x] /;
FreeQ[{a,b,c,d,e},x] && LtQ[m,-1] && NeQ[b*e-2*c*d,0]
```

```
Int[(d_.+e_.*x_)^m_*Cosh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  (d+e*x)^(m+1)*Cosh[a+b*x+c*x^2]/(e*(m+1)) -
  (b*e-2*c*d)/(e^2*(m+1))*Int[(d+e*x)^(m+1)*Sinh[a+b*x+c*x^2],x] -
  2*c/(e^2*(m+1))*Int[(d+e*x)^(m+2)*Sinh[a+b*x+c*x^2],x] /;
FreeQ[{a,b,c,d,e},x] && LtQ[m,-1] && NeQ[b*e-2*c*d,0]
```

3: $\int (d+e\,x)^m \,\text{Sinh}\left[a+b\,x+c\,x^2\right] dx$

Rule:

$$\int (d+e\,x)^m \,\text{Sinh}\left[a+b\,x+c\,x^2\right] dx \;\longrightarrow\; \int (d+e\,x)^m \,\text{Sinh}\left[a+b\,x+c\,x^2\right] dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_.*Sinh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  Unintegrable[(d+e*x)^m*Sinh[a+b*x+c*x^2],x] /;
FreeQ[{a,b,c,d,e,m},x]
```

```
Int[(d_.+e_.*x_)^m_.*Cosh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  Unintegrable[(d+e*x)^m*Cosh[a+b*x+c*x^2],x] /;
FreeQ[{a,b,c,d,e,m},x]
```

2: $\int (d+e\,x)^m \,\text{Sinh}\left[a+b\,x+c\,x^2\right]^n dx$ when $n \in \mathbb{Z} \land n>1$

Derivation: Algebraic expansion

Rule: If $n \in \mathbb{Z} \land n>1$, then

$$\int (d+e\,x)^m \,\text{Sinh}\left[a+b\,x+c\,x^2\right]^n dx \;\longrightarrow\; \int (d+e\,x)^m \,\text{TrigReduce}\left[\text{Sinh}\left[a+b\,x+c\,x^2\right]^n\right] dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_.*Sinh[a_.+b_.*x_+c_.*x_^2]^n_,x_Symbol] :=
  Int[ExpandTrigReduce[(d+e*x)^m,Sinh[a+b*x+c*x^2]^n,x],x] /;
FreeQ[{a,b,c,d,e,m},x] && IGtQ[n,1]
```

```
Int[(d_.+e_.*x_)^m_.*Cosh[a_.+b_.*x_+c_.*x_^2]^n_,x_Symbol] :=
  Int[ExpandTrigReduce[(d+e*x)^m,Cosh[a+b*x+c*x^2]^n,x],x] /;
FreeQ[{a,b,c,d,e,m},x] && IGtQ[n,1]
```

3: $\int u^m \operatorname{Sinh}[v]^n \, dx$ when $n \in \mathbb{Z}^+ \land u == d + e x \land v == a + b x + c x^2$

Derivation: Algebraic normalization

Rule: If $n \in \mathbb{Z}^+ \land u == d + e x \land v == a + b x + c x^2$, then

$$\int u^m \operatorname{Sinh}[v]^n \, dx \longrightarrow \int (d + e x)^m \operatorname{Sinh}\left[a + b x + c x^2\right]^n dx$$

Program code:

```
Int[u_^m_.*Sinh[v_]^n_.,x_Symbol] :=
  Int[ExpandToSum[u,x]^m*Sinh[ExpandToSum[v,x]]^n,x] /;
FreeQ[m,x] && IGtQ[n,0] && LinearQ[u,x] && QuadraticQ[v,x] && Not[LinearMatchQ[u,x] && QuadraticMatchQ[v,x]]
```

```
Int[u_^m_.*Cosh[v_]^n_.,x_Symbol] :=
  Int[ExpandToSum[u,x]^m*Cosh[ExpandToSum[v,x]]^n,x] /;
FreeQ[m,x] && IGtQ[n,0] && LinearQ[u,x] && QuadraticQ[v,x] && Not[LinearMatchQ[u,x] && QuadraticMatchQ[v,x]]
```