

Rules for integrands of the form $(f x)^m (d + e x^n)^q (a + b x^n + c x^{2n})^p$

0. $\int (f x)^m (e x^n)^q (a + b x^n + c x^{2n})^p dx$

1. $\int (f x)^m (e x^n)^q (a + b x^n + c x^{2n})^p dx$ when $m \in \mathbb{Z} \vee f > 0$

1: $\int (f x)^m (e x^n)^q (a + b x^n + c x^{2n})^p dx$ when $(m \in \mathbb{Z} \vee f > 0) \wedge \frac{m+1}{n} \in \mathbb{Z}$

Derivation: Integration by substitution

Basis: If $\frac{m+1}{n} \in \mathbb{Z}$, then $x^m (e x^n)^q = \frac{1}{e^{\frac{m+1}{n}-1}} x^{n-1} (e x^n)^{q+\frac{m+1}{n}-1}$

Basis: $x^{n-1} F[x^n] = \frac{1}{n} \text{Subst}[F[x], x, x^n] \partial_x x^n$

Rule 1.2.3.4.0.1.1: If $(m \in \mathbb{Z} \vee f > 0) \wedge \frac{m+1}{n} \in \mathbb{Z}$, then

$$\int (f x)^m (e x^n)^q (a + b x^n + c x^{2n})^p dx \rightarrow \frac{f^m}{n e^{\frac{m+1}{n}-1}} \text{Subst}\left[\int (e x)^{q+\frac{m+1}{n}-1} (a + b x + c x^2)^p dx, x, x^n\right]$$

Program code:

```
Int[(f_.**x_)^m_.*(e_.**x_^n_)^q_*(a_+b_.**x_^n_+c_.**x_^n2_)^p_.,x_Symbol] :=
  f^m/(n*e^( (m+1)/n-1))*Subst[Int[(e*x)^(q+(m+1)/n-1)*(a+b*x+c*x^2)^p,x],x,x^n] /;
FreeQ[{a,b,c,e,f,m,n,p,q},x] && EqQ[n2,2*n] && (IntegerQ[m] || GtQ[f,0]) && IntegerQ[Simplify[(m+1)/n]]
```

```
Int[(f_.**x_)^m_.*(e_.**x_^n_)^q_*(a_+c_.**x_^n2_)^p_.,x_Symbol] :=
  f^m/(n*e^( (m+1)/n-1))*Subst[Int[(e*x)^(q+(m+1)/n-1)*(a+c*x^2)^p,x],x,x^n] /;
FreeQ[{a,c,e,f,m,n,p,q},x] && EqQ[n2,2*n] && (IntegerQ[m] || GtQ[f,0]) && IntegerQ[Simplify[(m+1)/n]]
```

$$\mathbf{2:} \int (f x)^m (e x^n)^q (a+b x^n+c x^{2 n})^p dx \text{ when } (m \in \mathbb{Z} \vee f > 0) \wedge \frac{m+1}{n} \notin \mathbb{Z}$$

Derivation: Piecewise constant extraction

$$\text{Basis: } \partial_x \frac{(e x^n)^q}{x^{n q}} = 0$$

Rule 1.2.3.4.0.1.2: If $(m \in \mathbb{Z} \vee f > 0) \wedge \frac{m+1}{n} \notin \mathbb{Z}$, then

$$\int (f x)^m (e x^n)^q (a+b x^n+c x^{2 n})^p dx \rightarrow \frac{f^m e^{\text{IntPart}[q]} (e x^n)^{\text{FracPart}[q]}}{x^{n \text{FracPart}[q]}} \int x^{m+n q} (a+b x^n+c x^{2 n})^p dx$$

Program code:

```
Int[(f_.**x_)^m_.*(e_.**x_^n_)^q_*(a_+b_.**x_^n_+c_.**x_^2n_)^p_,x_Symbol] :=
  f^m*e^IntPart[q]*(e*x^n)^FracPart[q]/x^(n*FracPart[q])*Int[x^(m+n*q)*(a+b*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,e,f,m,n,p,q},x] && EqQ[n2,2*n] && (IntegerQ[m] || GtQ[f,0]) && Not[IntegerQ[Simplify[(m+1)/n]]]
```

```
Int[(f_.**x_)^m_.*(e_.**x_^n_)^q_*(a_+c_.**x_^2n_)^p_,x_Symbol] :=
  f^m*e^IntPart[q]*(e*x^n)^FracPart[q]/x^(n*FracPart[q])*Int[x^(m+n*q)*(a+c*x^(2*n))^p,x] /;
FreeQ[{a,c,e,f,m,n,p,q},x] && EqQ[n2,2*n] && (IntegerQ[m] || GtQ[f,0]) && Not[IntegerQ[Simplify[(m+1)/n]]]
```

2: $\int (f x)^m (e x^n)^q (a + b x^n + c x^{2n})^p dx$ when $m \notin \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(f x)^m}{x^m} = 0$

Rule 1.2.3.4.0.2: If $m \notin \mathbb{Z}$, then

$$\int (f x)^m (e x^n)^q (a + b x^n + c x^{2n})^p dx \rightarrow \frac{f^{\text{IntPart}[m]} (f x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (e x^n)^q (a + b x^n + c x^{2n})^p dx$$

Program code:

```
Int[(f*x_)^m_.*(e_.*x_^n_)^q_*(a_+b_.*x_^n_+c_.*x_^2n_.)^p_,x_Symbol] :=
  f^IntPart[m]*(f*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,e,f,m,n,p,q},x] && EqQ[n2,2*n] && Not[IntegerQ[m]]
```

```
Int[(f*x_)^m_.*(e_.*x_^n_)^q_*(a_+c_.*x_^2n_.)^p_,x_Symbol] :=
  f^IntPart[m]*(f*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(e*x^n)^q*(a+c*x^(2*n))^p,x] /;
FreeQ[{a,c,e,f,m,n,p,q},x] && EqQ[n2,2*n] && Not[IntegerQ[m]]
```

1: $\int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$ when $m-n+1=0$

Derivation: Integration by substitution

Basis: $x^{n-1} F[x^n] = \frac{1}{n} \text{Subst}[F[x], x, x^n] \partial_x x^n$

Rule 1.2.3.4.1: If $m-n+1=0$, then

$$\int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \frac{1}{n} \text{Subst}\left[\int (d+e x)^q (a+b x+c x^2)^p dx, x, x^n\right]$$

Program code:

```
Int[x_^m_.*(d_+e_.*x_^n_)^q_.*(a_+b_.*x_^n_+c_.*x_^n2_)^p_,x_Symbol] :=
  1/n*Subst[Int[(d+e*x)^q*(a+b*x+c*x^2)^p,x],x,x^n] /;
FreeQ[{a,b,c,d,e,m,n,p,q},x] && EqQ[n2,2*n] && EqQ[Simplify[m-n+1],0]
```

```
Int[x_^m_.*(d_+e_.*x_^n_)^q_.*(a_+c_.*x_^n2_)^p_,x_Symbol] :=
  1/n*Subst[Int[(d+e*x)^q*(a+c*x^2)^p,x],x,x^n] /;
FreeQ[{a,c,d,e,m,n,p,q},x] && EqQ[n2,2*n] && EqQ[Simplify[m-n+1],0]
```

2: $\int x^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx$ when $(p \mid q) \in \mathbb{Z} \wedge n < 0$

Derivation: Algebraic expansion

Basis: If $(p \mid q) \in \mathbb{Z}$, then $(d+e x^n)^q (a+b x^n+c x^{2 n})^p = x^{n(2 p+q)} (e+d x^{-n})^q (c+b x^{-n}+a x^{-2 n})^p$

Rule 1.2.3.4.2: If $(p \mid q) \in \mathbb{Z} \wedge n < 0$, then

$$\int x^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx \rightarrow \int x^{m+n(2 p+q)} (e+d x^{-n})^q (c+b x^{-n}+a x^{-2 n})^p dx$$

Program code:

```
Int[x_^m_.*(d_+e_.*x_^n_)^q_.*(a_+b_.*x_^n_+c_.*x_^n2_)^p_,x_Symbol] :=
  Int[x^(m+n*(2*p+q))*(e+d*x^(-n))^q*(c+b*x^(-n)+a*x^(-2*n))^p,x] /;
FreeQ[{a,b,c,d,e,m,n},x] && EqQ[n2,2*n] && IntegersQ[p,q] && NegQ[n]
```

```
Int[x_^m_.*(d_+e_.*x_^n_)^q_.*(a_+c_.*x_^n2_)^p_,x_Symbol] :=
  Int[x^(m+n*(2*p+q))*(e+d*x^(-n))^q*(c+a*x^(-2*n))^p,x] /;
FreeQ[{a,c,d,e,m,n},x] && EqQ[n2,2*n] && IntegersQ[p,q] && NegQ[n]
```

$$3. \int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \text{ when } b^2 - 4ac = 0 \wedge p \notin \mathbb{Z}$$

$$1: \int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \text{ when } b^2 - 4ac = 0 \wedge p \notin \mathbb{Z} \wedge (m \mid n \mid \frac{m+1}{n}) \in \mathbb{Z}^+$$

Derivation: Integration by substitution

Basis: If $\frac{m+1}{n} \in \mathbb{Z}$, then $x^m F[x^n] = \frac{1}{n} \text{Subst}[x^{\frac{m+1}{n}-1} F[x], x, x^n] \partial_x x^n$

Note: If this substitution rule is applied when $m \in \mathbb{Z}^-$, expressions of the form $\text{Log}[x^n]$ rather than $\text{Log}[x]$ may appear in the antiderivative.

Rule 1.2.3.4.3.1: If $b^2 - 4ac = 0 \wedge p \notin \mathbb{Z} \wedge (m \mid n \mid \frac{m+1}{n}) \in \mathbb{Z}^+$, then

$$\int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \frac{1}{n} \text{Subst}\left[\int x^{\frac{m+1}{n}-1} (d+e x)^q (a+b x+c x^2)^p dx, x, x^n\right]$$

Program code:

```
Int[x_^m.*(d+e.*x_^n)^q.*(a+b.*x_^n+c.*x_^n2.)^p,x_Symbol] :=
  1/n*Subst[Int[x^((m+1)/n-1)*(d+e*x)^q*(a+b*x+c*x^2)^p,x],x,x^n] /;
FreeQ[{a,b,c,d,e,p,q},x] && EqQ[n2,2*n] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[p]] && IGtQ[m,0] && IGtQ[n,0] && IGtQ[(m+1)/n,0]
```

2: $\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx$ when $b^2 - 4 a c == 0 \wedge p \notin \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: If $b^2 - 4 a c == 0$, then $\partial_x \frac{(a+b x^n+c x^{2 n})^p}{\left(\frac{b}{2}+c x^n\right)^{2 p}} == 0$

Basis: If $b^2 - 4 a c == 0$, then $\frac{(a+b x^n+c x^{2 n})^p}{\left(\frac{b}{2}+c x^n\right)^{2 p}} == \frac{(a+b x^n+c x^{2 n})^{\text{FracPart}[p]}}{c^{\text{IntPart}[p]} \left(\frac{b}{2}+c x^n\right)^{2 \text{FracPart}[p]}}$

Rule 1.2.3.4.3.2: If $b^2 - 4 a c == 0 \wedge p \notin \mathbb{Z}$, then

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx \rightarrow \frac{(a+b x^n+c x^{2 n})^{\text{FracPart}[p]}}{c^{\text{IntPart}[p]} \left(\frac{b}{2}+c x^n\right)^{2 \text{FracPart}[p]}} \int (f x)^m (d+e x^n)^q \left(\frac{b}{2}+c x^n\right)^{2 p} dx$$

Program code:

```
Int[(f_.**x_)^m_.*(d+_e_.**x_^n_)^q_.*(a+_b_.**x_^n+_c_.**x_^n2_.)^p_,x_Symbol] :=
  (a+b*x^n+c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2+c*x^n)^(2*FracPart[p]))*
  Int[(f*x)^m*(d+e*x^n)^q*(b/2+c*x^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,f,m,n,p,q},x] && EqQ[n2,2*n] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[p]]
```

$$4. \int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx \text{ when } \frac{m+1}{n} \in \mathbb{Z}$$

$$1: \int x^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx \text{ when } \frac{m+1}{n} \in \mathbb{Z}$$

Derivation: Integration by substitution

Basis: If $\frac{m+1}{n} \in \mathbb{Z}$, then $x^m F[x^n] = \frac{1}{n} \text{Subst}\left[x^{\frac{m+1}{n}-1} F[x], x, x^n\right] \partial_x x^n$

Note: If $n \in \mathbb{Z} \wedge \frac{m+1}{n} \in \mathbb{Z}$, then $m \in \mathbb{Z}$, and $(f x)^m$ automatically evaluates to $f^m x^m$.

Rule 1.2.3.4.4.1: If $\frac{m+1}{n} \in \mathbb{Z}$, then

$$\int x^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx \rightarrow \frac{1}{n} \text{Subst}\left[\int x^{\frac{m+1}{n}-1} (d+e x)^q (a+b x+c x^2)^p dx, x, x^n\right]$$

Program code:

```
Int[x_^m_.*(d_+e_.*x_^n_)^q_.*(a_+b_.*x_^n_+c_.*x_^n2_)^p_,x_Symbol] :=
  1/n*Subst[Int[x^(Simplify[(m+1)/n]-1)*(d+e*x)^q*(a+b*x+c*x^2)^p,x],x,x^n] /;
FreeQ[{a,b,c,d,e,m,n,p,q},x] && EqQ[n2,2*n] && IntegerQ[Simplify[(m+1)/n]]
```

```
Int[x_^m_.*(d_+e_.*x_^n_)^q_.*(a_+c_.*x_^n2_)^p_,x_Symbol] :=
  1/n*Subst[Int[x^(Simplify[(m+1)/n]-1)*(d+e*x)^q*(a+c*x^2)^p,x],x,x^n] /;
FreeQ[{a,c,d,e,m,n,p,q},x] && EqQ[n2,2*n] && IntegerQ[Simplify[(m+1)/n]]
```


2: $\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$ when $\frac{m+1}{n} \in \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(f x)^m}{x^m} = 0$

Basis: $\frac{(f x)^m}{x^m} = \frac{f^{\text{IntPart}[m]} (f x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}}$

Rule 1.2.3.4.4.2: If $\frac{m+1}{n} \in \mathbb{Z}$, then

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \frac{f^{\text{IntPart}[m]} (f x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$$

Program code:

```
Int[(f*x_)^m_.*(d_+e_*x_^n_)^q_.*(a_+b_*x_^n_+c_*x_^2n_)^p_,x_Symbol] :=
  f^IntPart[m]*(f*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d,e,f,m,n,p,q},x] && EqQ[n2,2*n] && IntegerQ[Simplify[(m+1)/n]]
```

```
Int[(f*x_)^m_.*(d_+e_*x_^n_)^q_.*(a_+c_*x_^2n_)^p_,x_Symbol] :=
  f^IntPart[m]*(f*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x] /;
FreeQ[{a,c,d,e,f,m,n,p,q},x] && EqQ[n2,2*n] && IntegerQ[Simplify[(m+1)/n]]
```

5. $\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge c d^2 - b d e + a e^2 = 0$

1: $\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge c d^2 - b d e + a e^2 = 0 \wedge p \in \mathbb{Z}$

Derivation: Algebraic simplification

Basis: If $c d^2 - b d e + a e^2 = 0$, then $a + b z + c z^2 = (d + e z) \left(\frac{a}{d} + \frac{c z}{e} \right)$

Rule 1.2.3.4.5.1: If $b^2 - 4ac \neq 0 \wedge c d^2 - b d e + a e^2 = 0 \wedge p \in \mathbb{Z}$, then

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \int (f x)^m (d+e x^n)^{q+p} \left(\frac{a}{d} + \frac{c x^n}{e} \right)^p dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^n_)^q_.*(a_+b_.*x_^n_+c_.*x_^n2_)^p_,x_Symbol] :=
  Int[(f*x)^m*(d+e*x^n)^(q+p)*(a/d+c/e*x^n)^p,x] /;
FreeQ[{a,b,c,d,e,f,m,n,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && EqQ[c*d^2-b*d*e+a*e^2,0] && IntegerQ[p]
```

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^n_)^q_.*(a_+c_.*x_^n2_)^p_,x_Symbol] :=
  Int[(f*x)^m*(d+e*x^n)^(q+p)*(a/d+c/e*x^n)^p,x] /;
FreeQ[{a,c,d,e,f,q,m,n,q},x] && EqQ[n2,2*n] && EqQ[c*d^2+a*e^2,0] && IntegerQ[p]
```

2: $\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$ when $b^2 - 4 a c \neq 0 \wedge c d^2 - b d e + a e^2 = 0 \wedge p \notin \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: If $c d^2 - b d e + a e^2 = 0$, then $\partial_x \frac{(a+b x^n+c x^{2n})^p}{(d+e x^n)^p \left(\frac{a}{d} + \frac{c x^n}{e} \right)^p} = 0$

Basis: If $c d^2 - b d e + a e^2 = 0$, then $\frac{(a+b x^n+c x^{2n})^p}{(d+e x^n)^p \left(\frac{a}{d} + \frac{c x^n}{e} \right)^p} = \frac{(a+b x^n+c x^{2n})^{\text{FracPart}[p]}}{(d+e x^n)^{\text{FracPart}[p]} \left(\frac{a}{d} + \frac{c x^n}{e} \right)^{\text{FracPart}[p]}}$

Rule 1.2.3.4.5.2: If $b^2 - 4 a c \neq 0 \wedge c d^2 - b d e + a e^2 = 0 \wedge p \notin \mathbb{Z}$, then

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \frac{(a+b x^n+c x^{2n})^{\text{FracPart}[p]}}{(d+e x^n)^{\text{FracPart}[p]} \left(\frac{a}{d} + \frac{c x^n}{e} \right)^{\text{FracPart}[p]}} \int (f x)^m (d+e x^n)^{q+p} \left(\frac{a}{d} + \frac{c x^n}{e} \right)^p dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^n_)^q_.*(a_+b_.*x_^n_+c_.*x_^n2_)^p_,x_Symbol] :=
  (a+b*x^n+c*x^(2*n))^FracPart[p]/((d+e*x^n)^FracPart[p]*(a/d+(c*x^n)/e)^FracPart[p])*
  Int[(f*x)^m*(d+e*x^n)^(q+p)*(a/d+c/e*x^n)^p,x] /;
FreeQ[{a,b,c,d,e,f,m,n,p,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && EqQ[c*d^2-b*d*e+a*e^2,0] && Not[IntegerQ[p]]
```

```

Int[(f_.*x_)^m_.*(d_+e_.*x_^n_)^q_.*(a_+c_.*x_^n2_)^p_,x_Symbol] :=
(a+c*x^(2*n))^FracPart[p]/((d+e*x^n)^FracPart[p]*(a/d+(c*x^n)/e)^FracPart[p])*Int[(f*x)^m*(d+e*x^n)^(q+p)*(a/d+c/e*x^n)^p,x] /;
FreeQ[{a,c,d,e,f,m,n,p,q},x] && EqQ[n2,2*n] && EqQ[c*d^2+a*e^2,0] && Not[IntegerQ[p]]

```

6. $\int (f x)^m (d + e x^n)^q (a + b x^n + c x^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}$

1. $\int (f x)^m (d + e x^n)^q (a + b x^n + c x^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+$

1. $\int (f x)^m (d + e x^n)^q (a + b x^n + c x^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge (n | p) \in \mathbb{Z}^+$

1. $\int x^m (d + e x^n)^q (a + b x^n + c x^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge (n | p) \in \mathbb{Z}^+ \wedge (m | q) \in \mathbb{Z} \wedge q < -1$

1: $\int x^m (d + e x^n)^q (a + b x^n + c x^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge (n | p) \in \mathbb{Z}^+ \wedge (m | q) \in \mathbb{Z} \wedge q < -1 \wedge m > 0$

Derivation: Algebraic expansion and binomial recurrence 2b

Note: If $(n | p) \in \mathbb{Z}^+ \wedge (m | q) \in \mathbb{Z} \wedge q < 0$, then $\frac{(-d)^{(m-\text{Mod}[m,n])/n}}{e^{2p+(m-\text{Mod}[m,n])/n}} \sum_{k=0}^{2p} (-d)^k e^{2p-k} P_{2p}[x^n, k]$ is the coefficient of the $x^{\text{Mod}[m,n]} (d + e x^n)^q$ term of the partial fraction expansion of $x^m P_{2p}[x^n] (d + e x^n)^q$.

Note: If $(n | p) \in \mathbb{Z}^+ \wedge (m | q) \in \mathbb{Z} \wedge q < -1 \wedge m > 0$, then

$n e^{2p+(m-\text{Mod}[m,n])/n} (q+1) x^{m-\text{Mod}[m,n]} (a + b x^n + c x^{2n})^p - (-d)^{(m-\text{Mod}[m,n])/n-1} (c d^2 - b d e + a e^2)^p (d (\text{Mod}[m, n] + 1) + e (\text{Mod}[m, n] + n (q+1) + 1) x^n)$ will be divisible by $a + b x^n$.

Note: In the resulting integrand the degree of the polynomial in x^n is at most $q - 1$.

Rule 1.2.3.4.6.1.1.1.1: If $b^2 - 4ac \neq 0 \wedge (n | p) \in \mathbb{Z}^+ \wedge (m | q) \in \mathbb{Z} \wedge q < -1 \wedge m > 0$, then

$$\int x^m (d + e x^n)^q (a + b x^n + c x^{2n})^p dx \rightarrow$$

$$\frac{(-d)^{(m-\text{Mod}[m,n])/n}}{e^{2p+(m-\text{Mod}[m,n])/n}} (c d^2 - b d e + a e^2)^p \int x^{\text{Mod}[m,n]} (d + e x^n)^q dx + \frac{1}{e^{2p+(m-\text{Mod}[m,n])/n}} \int x^{\text{Mod}[m,n]} (d + e x^n)^q \left(e^{2p+(m-\text{Mod}[m,n])/n} x^{m-\text{Mod}[m,n]} (a + b x^n + c x^{2n})^p - (-d)^{(m-\text{Mod}[m,n])/n-1} (c d^2 - b d e + a e^2)^p \right) dx \rightarrow$$

$$\left(\left((-d)^{(m-\text{Mod}[m,n])/n-1} (c d^2 - b d e + a e^2)^p x^{\text{Mod}[m,n]+1} (d + e x^n)^{q+1} \right) / \left(n e^{2p+(m-\text{Mod}[m,n])/n} (q+1) \right) \right) +$$

$$\frac{1}{n e^{2p+(m-\text{Mod}[m,n])/n} (q+1)} \int x^{\text{Mod}[m,n]} (d + e x^n)^{q+1} .$$

$$\left(\frac{1}{d + e x^n} \left(n e^{2p+(m-\text{Mod}[m,n])/n} (q+1) x^{m-\text{Mod}[m,n]} (a + b x^n + c x^{2n})^p - \right. \right.$$

$$\left. \left. (-d)^{(m-\text{Mod}[m,n])/n-1} (c d^2 - b d e + a e^2)^p (d (\text{Mod}[m,n] + 1) + e (\text{Mod}[m,n] + n (q+1) + 1) x^n) \right) \right) dx$$

Program code:

```
Int[x_^m_.*(d_+e_.*x_^n_)^q_*(a_+b_.*x_^n_+c_.*x_^2n_)^p_,x_Symbol] :=
  (-d)^( (m-Mod[m,n])/n-1)*(c*d^2-b*d*e+a*e^2)^p*x^(Mod[m,n]+1)*(d+e*x^n)^(q+1)/(n*e^(2*p+(m-Mod[m,n])/n)*(q+1)) +
  1/(n*e^(2*p+(m-Mod[m,n])/n)*(q+1))*Int[x^Mod[m,n]*(d+e*x^n)^(q+1)*
    ExpandToSum[Together[1/(d+e*x^n)*(n*e^(2*p+(m-Mod[m,n])/n)*(q+1)*x^(m-Mod[m,n])*(a+b*x^n+c*x^(2*n))^p-
      (-d)^( (m-Mod[m,n])/n-1)*(c*d^2-b*d*e+a*e^2)^p*(d*(Mod[m,n]+1)+e*(Mod[m,n]+n*(q+1)+1)*x^n)],x],x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && IGtQ[p,0] && ILtQ[q,-1] && IGtQ[m,0]
```

```
Int[x_^m_.*(d_+e_.*x_^n_)^q_*(a_+c_.*x_^2n_)^p_,x_Symbol] :=
  (-d)^( (m-Mod[m,n])/n-1)*(c*d^2+a*e^2)^p*x^(Mod[m,n]+1)*(d+e*x^n)^(q+1)/(n*e^(2*p+(m-Mod[m,n])/n)*(q+1)) +
  1/(n*e^(2*p+(m-Mod[m,n])/n)*(q+1))*Int[x^Mod[m,n]*(d+e*x^n)^(q+1)*
    ExpandToSum[Together[1/(d+e*x^n)*(n*e^(2*p+(m-Mod[m,n])/n)*(q+1)*x^(m-Mod[m,n])*(a+c*x^(2*n))^p-
      (-d)^( (m-Mod[m,n])/n-1)*(c*d^2+a*e^2)^p*(d*(Mod[m,n]+1)+e*(Mod[m,n]+n*(q+1)+1)*x^n)],x],x] /;
FreeQ[{a,c,d,e},x] && EqQ[n2,2*n] && IGtQ[n,0] && IGtQ[p,0] && ILtQ[q,-1] && IGtQ[m,0]
```

$$\mathbf{2:} \int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \text{ when } b^2-4ac \neq 0 \wedge (n|p) \in \mathbb{Z}^+ \wedge (m|q) \in \mathbb{Z} \wedge q < -1 \wedge m < 0$$

Derivation: Algebraic expansion and binomial recurrence 2b

Note: If $(n|p) \in \mathbb{Z}^+ \wedge (m|q) \in \mathbb{Z} \wedge q < 0$, then $\frac{(-d)^{(m-\text{Mod}[m,n])/n}}{e^{2p+(m-\text{Mod}[m,n])/n}} \sum_{k=0}^{2p} (-d)^k e^{2p-k} p_{2p}[x^n, k]$ is the coefficient of the $x^{\text{Mod}[m,n]} (d+e x^n)^q$ term of the partial fraction expansion of $x^m p_{2p}[x^n] (d+e x^n)^q$.

Note: If $(n|p) \in \mathbb{Z}^+ \wedge (m|q) \in \mathbb{Z} \wedge q < -1 \wedge m < 0$, then

$n(-d)^{-(m-\text{Mod}[m,n])/n+1} e^{2p} (q+1) (a+b x^n+c x^{2n})^p - e^{-(m-\text{Mod}[m,n])/n} (c d^2 - b d e + a e^2)^p x^{-(m-\text{Mod}[m,n])} (d(\text{Mod}[m,n]+1) + e(\text{Mod}[m,n]+n(q+1)+1)x^n)$ will be divisible by $a+b x^n$.

Note: In the resulting integrand the degree of the polynomial in x^n is at most $q-1$.

Rule 1.2.3.4.6.1.1.1.2: If $b^2-4ac \neq 0 \wedge (n|p) \in \mathbb{Z}^+ \wedge (m|q) \in \mathbb{Z} \wedge q < -1 \wedge m < 0$, then

$$\begin{aligned} & \int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \\ & \frac{(-d)^{(m-\text{Mod}[m,n])/n}}{e^{2p+(m-\text{Mod}[m,n])/n}} (c d^2 - b d e + a e^2)^p \int x^{\text{Mod}[m,n]} (d+e x^n)^q dx + \\ & \frac{(-d)^{(m-\text{Mod}[m,n])/n}}{e^{2p}} \int x^m (d+e x^n)^q \left((-d)^{-(m-\text{Mod}[m,n])/n} e^{2p} (a+b x^n+c x^{2n})^p - e^{-(m-\text{Mod}[m,n])/n} (c d^2 - b d e + a e^2)^p x^{-m} \right) dx \rightarrow \\ & \left(\left((-d)^{(m-\text{Mod}[m,n])/n-1} (c d^2 - b d e + a e^2)^p x^{\text{Mod}[m,n]+1} (d+e x^n)^{q+1} \right) / \left(n e^{2p+(m-\text{Mod}[m,n])/n} (q+1) \right) \right) + \\ & \frac{(-d)^{(m-\text{Mod}[m,n])/n-1}}{n e^{2p} (q+1)} \int x^m (d+e x^n)^{q+1} dx . \\ & \left(\frac{1}{d+e x^n} \left(n (-d)^{-(m-\text{Mod}[m,n])/n+1} e^{2p} (q+1) (a+b x^n+c x^{2n})^p - \right. \right. \\ & \left. \left. e^{-(m-\text{Mod}[m,n])/n} (c d^2 - b d e + a e^2)^p x^{-(m-\text{Mod}[m,n])} (d(\text{Mod}[m,n]+1) + e(\text{Mod}[m,n]+n(q+1)+1)x^n) \right) \right) dx \end{aligned}$$

Program code:

```

Int[x_^m_*(d_+e_.*x_^n_)^q_*(a_+b_.*x_^n_+c_.*x_^n2_)^p_,x_Symbol] :=
  (-d)^( (m-Mod[m,n])/n-1)*(c*d^2-b*d*e+a*e^2)^p*x^(Mod[m,n]+1)*(d+e*x^n)^(q+1)/(n*e^(2*p+(m-Mod[m,n])/n)*(q+1)) +
  (-d)^( (m-Mod[m,n])/n-1)/(n*e^(2*p)*(q+1))*Int[x^m*(d+e*x^n)^(q+1)*
    ExpandToSum[Together[1/(d+e*x^n)*(n*(-d)^(-(m-Mod[m,n])/n+1)*e^(2*p)*(q+1)*(a+b*x^n+c*x^(2*n))^p -
      (e^(-(m-Mod[m,n])/n)*(c*d^2-b*d*e+a*e^2)^p*x^(-(m-Mod[m,n])))*(d*(Mod[m,n]+1)+e*(Mod[m,n]+n*(q+1)+1)*x^n))],x],x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && IGtQ[p,0] && ILtQ[q,-1] && ILtQ[m,0]

```

```

Int[x_^m_*(d_+e_.*x_^n_)^q_*(a_+c_.*x_^n2_)^p_,x_Symbol] :=
  (-d)^( (m-Mod[m,n])/n-1)*(c*d^2+a*e^2)^p*x^(Mod[m,n]+1)*(d+e*x^n)^(q+1)/(n*e^(2*p+(m-Mod[m,n])/n)*(q+1)) +
  (-d)^( (m-Mod[m,n])/n-1)/(n*e^(2*p)*(q+1))*Int[x^m*(d+e*x^n)^(q+1)*
    ExpandToSum[Together[1/(d+e*x^n)*(n*(-d)^(-(m-Mod[m,n])/n+1)*e^(2*p)*(q+1)*(a+c*x^(2*n))^p -
      (e^(-(m-Mod[m,n])/n)*(c*d^2+a*e^2)^p*x^(-(m-Mod[m,n])))*(d*(Mod[m,n]+1)+e*(Mod[m,n]+n*(q+1)+1)*x^n))],x],x] /;
FreeQ[{a,c,d,e},x] && EqQ[n2,2*n] && IGtQ[n,0] && IGtQ[p,0] && IntegersQ[m,q] && ILtQ[q,-1] && ILtQ[m,0]

```

$$\mathbf{2:} \int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge (n \mid p) \in \mathbb{Z}^+ \wedge 2np > n-1 \wedge q \notin \mathbb{Z} \wedge m+2np+nq+1 \neq 0$$

Reference: G&R 2.104

Note: This rule is a special case of the Ostrogradskiy-Hermite integration method.

Note: The degree of the polynomial in the resulting integrand is less than $2n$.

Rule 1.2.3.4.6.1.1.2: If $b^2 - 4ac \neq 0 \wedge (n \mid p) \in \mathbb{Z}^+ \wedge 2np > n-1 \wedge q \notin \mathbb{Z} \wedge m+2np+nq+1 \neq 0$, then

$$\begin{aligned} & \int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \\ & \int (f x)^m (d+e x^n)^q ((a+b x^n+c x^{2n})^p - x^{2np}) dx + \frac{c^p}{f^{2np}} \int (f x)^{m+2np} (d+e x^n)^q dx \rightarrow \\ & \frac{c^p (f x)^{m+2np-n+1} (d+e x^n)^{q+1}}{e f^{2np-n+1} (m+2np+nq+1)} + \\ & \frac{1}{e (m+2np+nq+1)} \int (f x)^m (d+e x^n)^q (e (m+2np+nq+1) ((a+b x^n+c x^{2n})^p - c^p x^{2np}) - d c^p (m+2np-n+1) x^{2np-n}) dx \end{aligned}$$

Program code:

```
Int[(f_.x_)^m_.*(d+_e_.x_^n_)^q_.*(a+_b_.x_^n+_c_.x_^2n_.)^p_.,x_Symbol] :=
  c^p*(f*x)^(m+2*n*p-n+1)*(d+e*x^n)^(q+1)/(e*f^(2*n*p-n+1)*(m+2*n*p+n*q+1)) +
  1/(e*(m+2*n*p+n*q+1))*Int[(f*x)^m*(d+e*x^n)^q*
    ExpandToSum[e*(m+2*n*p+n*q+1)*((a+b*x^n+c*x^(2*n))^p-c^p*x^(2*n*p))-d*c^p*(m+2*n*p-n+1)*x^(2*n*p-n),x],x] /;
FreeQ[{a,b,c,d,e,f,m,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && IGtQ[p,0] && GtQ[2*n*p,n-1] &&
Not[IntegerQ[q]] && NeQ[m+2*n*p+n*q+1,0]
```

```

Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_.*(a_+c_.**x_^n2_.)^p_.,x_Symbol] :=
  c^p*(f*x)^(m+2*n*p-n+1)*(d+e*x^n)^(q+1)/(e*f^(2*n*p-n+1)*(m+2*n*p+n*q+1)) +
  1/(e*(m+2*n*p+n*q+1))*Int[(f*x)^m*(d+e*x^n)^q*
    ExpandToSum[e*(m+2*n*p+n*q+1)*((a+c*x^(2*n))^p-c^p*x^(2*n*p))-d*c^p*(m+2*n*p-n+1)*x^(2*n*p-n),x],x] /;
FreeQ[{a,c,d,e,f,m,q},x] && EqQ[n2,2*n] && IGtQ[n,0] && IGtQ[p,0] && GtQ[2*n*p,n-1] &&
  Not[IntegerQ[q]] && NeQ[m+2*n*p+n*q+1,0]

```

3: $\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge (n|p) \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule 1.2.3.4.6.1.1.3: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p \in \mathbb{Z}^+$, then

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \int \text{ExpandIntegrand}[(f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p, x] dx$$

Program code:

```

Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_.*(a_+b_.**x_^n_+c_.**x_^n2_.)^p_.,x_Symbol] :=
  Int[ExpandIntegrand[(f*x)^m*(d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x],x] /;
FreeQ[{a,b,c,d,e,f,m,q},x] && EqQ[n2,2*n] && IGtQ[n,0] && IGtQ[p,0]

```

```

Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_.*(a_+c_.**x_^n2_.)^p_.,x_Symbol] :=
  Int[ExpandIntegrand[(f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x],x] /;
FreeQ[{a,c,d,e,f,m,q},x] && EqQ[n2,2*n] && IGtQ[n,0] && IGtQ[p,0]

```


$$\mathbf{2:} \int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{Z} \wedge \text{GCD}[m+1, n] \neq 1$$

Derivation: Integration by substitution

Basis: If $n \in \mathbb{Z} \wedge m \in \mathbb{Z}$, let $k = \text{GCD}[m+1, n]$, then $x^m F[x^n] = \frac{1}{k} \text{Subst}\left[x^{\frac{m+1}{k}-1} F[x^{n/k}], x, x^k\right] \partial_x x^k$

Rule 1.2.3.4.6.1.2: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{Z}$, let $k = \text{GCD}[m+1, n]$, if $k \neq 1$, then

$$\int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \frac{1}{k} \text{Subst}\left[\int x^{\frac{m+1}{k}-1} (d+e x^{n/k})^q (a+b x^{n/k}+c x^{2n/k})^p dx, x, x^k\right]$$

Program code:

```
Int[x_^m_.*(d_+e_.*x_^n_)^q_.*(a_+b_.*x_^n_+c_.*x_^n2_)^p_,x_Symbol] :=
  With[{k=GCD[m+1,n]},
    1/k*Subst[Int[x^((m+1)/k-1)*(d+e*x^(n/k))^q*(a+b*x^(n/k)+c*x^(2*n/k))^p,x],x,x^k] /;
    k!=1] /;
FreeQ[{a,b,c,d,e,p,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && IntegerQ[m]
```

```
Int[x_^m_.*(d_+e_.*x_^n_)^q_.*(a_+c_.*x_^n2_)^p_,x_Symbol] :=
  With[{k=GCD[m+1,n]},
    1/k*Subst[Int[x^((m+1)/k-1)*(d+e*x^(n/k))^q*(a+c*x^(2*n/k))^p,x],x,x^k] /;
    k!=1] /;
FreeQ[{a,c,d,e,p,q},x] && EqQ[n2,2*n] && IGtQ[n,0] && IntegerQ[m]
```

$$\mathbf{3:} \int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{F}$$

Derivation: Integration by substitution

Basis: If $k \in \mathbb{Z}^+$, then $(f x)^m F[x] = \frac{k}{f} \text{Subst}\left[x^{k(m+1)-1} F\left[\frac{x^k}{f}\right], x, (f x)^{1/k}\right] \partial_x (f x)^{1/k}$

Rule 1.2.3.4.6.1.3: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{F}$, let $k = \text{Denominator}[m]$, then

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx \rightarrow \frac{k}{f} \text{Subst}\left[\int x^{k(m+1)-1} \left(d + \frac{e x^{kn}}{f^n}\right)^q \left(a + \frac{b x^{kn}}{f^n} + \frac{c x^{2kn}}{f^{2n}}\right)^p dx, x, (f x)^{1/k}\right]$$

Program code:

```
Int[(f_.*x_)^m_*(d_+e_.*x_^n_)^q_.*(a_+b_.*x_^n_+c_.*x_^n2_)^p_,x_Symbol] :=
  With[{k=Denominator[m]},
    k/f*Subst[Int[x^(k*(m+1)-1)*(d+e*x^(k*n)/f^n)^q*(a+b*x^(k*n)/f^n+c*x^(2*k*n)/f^(2*n))^p,x],x,(f*x)^(1/k)]] /;
  FreeQ[{a,b,c,d,e,f,p,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && FractionQ[m] && IntegerQ[p]
```

```
Int[(f_.*x_)^m_*(d_+e_.*x_^n_)^q_.*(a_+c_.*x_^n2_)^p_,x_Symbol] :=
  With[{k=Denominator[m]},
    k/f*Subst[Int[x^(k*(m+1)-1)*(d+e*x^(k*n)/f)^q*(a+c*x^(2*k*n)/f)^p,x],x,(f*x)^(1/k)]] /;
  FreeQ[{a,c,d,e,f,p,q},x] && EqQ[n2,2*n] && IGtQ[n,0] && FractionQ[m] && IntegerQ[p]
```

$$4. \int (f x)^m (d+e x^n) (a+b x^n+c x^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+$$

$$1. \int (f x)^m (d+e x^n) (a+b x^n+c x^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p > 0$$

$$1: \int (f x)^m (d+e x^n) (a+b x^n+c x^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p > 0 \wedge m < -1 \wedge m+n(2p+1)+1 \neq 0$$

Derivation: Trinomial recurrence 1a

Rule 1.2.3.4.6.1.4.1.1: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p > 0 \wedge m < -1 \wedge m+n(2p+1)+1 \neq 0$, then

$$\begin{aligned} & \int (f x)^m (d+e x^n) (a+b x^n+c x^{2n})^p dx \rightarrow \\ & \frac{(f x)^{m+1} (a+b x^n+c x^{2n})^p (d(2np+n+m+1)+e(m+1)x^n)}{f(m+1)(m+n(2p+1)+1)} + \\ & \frac{np}{f^n(m+1)(m+n(2p+1)+1)} \int (f x)^{m+n} (a+b x^n+c x^{2n})^{p-1} \cdot \\ & (2ae(m+1)-bd(m+n(2p+1)+1)+(be(m+1)-2cd(m+n(2p+1)+1))x^n) dx \end{aligned}$$

Program code:

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)*(a_+b_.**x_^n_+c_.**x_^n2_)^p_,x_Symbol] :=
  (f*x)^(m+1)*(a+b*x^n+c*x^(2*n))^p*(d*(m+n*(2*p+1)+1)+e*(m+1)*x^n)/(f*(m+1)*(m+n*(2*p+1)+1)) +
  n*p/(f^n*(m+1)*(m+n*(2*p+1)+1))*Int[(f*x)^(m+n)*(a+b*x^n+c*x^(2*n))^(p-1)*
  Simp[2*a*e*(m+1)-b*d*(m+n*(2*p+1)+1)+(b*e*(m+1)-2*c*d*(m+n*(2*p+1)+1))*x^n,x],x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && GtQ[p,0] && LtQ[m,-1] && NeQ[m+n*(2*p+1)+1,0] && IntegerQ
```

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)*(a_+c_.**x_^n2_)^p_,x_Symbol] :=
  (f*x)^(m+1)*(a+c*x^(2*n))^p*(d*(m+n*(2*p+1)+1)+e*(m+1)*x^n)/(f*(m+1)*(m+n*(2*p+1)+1)) +
  2*n*p/(f^n*(m+1)*(m+n*(2*p+1)+1))*Int[(f*x)^(m+n)*(a+c*x^(2*n))^(p-1)*(a*e*(m+1)-c*d*(m+n*(2*p+1)+1))*x^n,x] /;
FreeQ[{a,c,d,e,f},x] && EqQ[n2,2*n] && IGtQ[n,0] && GtQ[p,0] && LtQ[m,-1] && NeQ[m+n*(2*p+1)+1,0] && IntegerQ[p]
```

$$\mathbf{2:} \int (f x)^m (d+e x^n) (a+b x^n+c x^{2n})^p dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p > 0 \wedge m+2np+1 \neq 0 \wedge m+n(2p+1)+1 \neq 0$$

Derivation: Trinomial recurrence 1b

Rule 1.2.3.4.6.1.4.1.2: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p > 0 \wedge m+2np+1 \neq 0 \wedge m+n(2p+1)+1 \neq 0$, then

$$\begin{aligned} & \int (f x)^m (d+e x^n) (a+b x^n+c x^{2n})^p dx \rightarrow \\ & \left(\left((f x)^{m+1} (a+b x^n+c x^{2n})^p (b e n p+c d (m+2 n p+n+1)+c e (2 n p+m+1) x^n) \right) / (c f (m+2 n p+1) (m+n (2 p+1)+1)) \right) + \\ & \quad \frac{n p}{c (m+2 n p+1) (m+n (2 p+1)+1)} \int (f x)^m (a+b x^n+c x^{2n})^{p-1} . \\ & \quad (2 a c d (m+n (2 p+1)+1)-a b e (m+1)+(2 a c e (m+2 n p+1)+b c d (m+n (2 p+1)+1)-b^2 e (m+n p+1)) x^n) dx \end{aligned}$$

Program code:

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)*(a_+b_.**x_^n_+c_.**x_^n2_)^p_,x_Symbol] :=
  (f*x)^(m+1)*(a+b*x^n+c*x^(2*n))^p*(b*e*n*p+c*d*(m+n*(2*p+1)+1)+c*e*(2*n*p+m+1)*x^n)/
  (c*f*(2*n*p+m+1)*(m+n*(2*p+1)+1)) +
  n*p/(c*(2*n*p+m+1)*(m+n*(2*p+1)+1))*Int[(f*x)^m*(a+b*x^n+c*x^(2*n))^(p-1)*
  Simp[2*a*c*d*(m+n*(2*p+1)+1)-a*b*e*(m+1)+(2*a*c*e*(2*n*p+m+1)+b*c*d*(m+n*(2*p+1)+1)-b^2*e*(m+n*p+1))*x^n,x],x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && GtQ[p,0] && NeQ[2*n*p+m+1,0] && NeQ[m+n*(2*p+1)+1,0] &&
```

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)*(a_+c_.**x_^n2_)^p_,x_Symbol] :=
  (f*x)^(m+1)*(a+c*x^n)^p*(c*d*(m+n*(2*p+1)+1)+c*e*(2*n*p+m+1)*x^n)/(c*f*(2*n*p+m+1)*(m+n*(2*p+1)+1)) +
  2*a*n*p/((2*n*p+m+1)*(m+n*(2*p+1)+1))*Int[(f*x)^m*(a+c*x^n)^(p-1)*Simp[d*(m+n*(2*p+1)+1)+e*(2*n*p+m+1)*x^n,x],x] /;
FreeQ[{a,c,d,e,f,m},x] && EqQ[n2,2*n] && IGtQ[n,0] && GtQ[p,0] && NeQ[2*n*p+m+1,0] && NeQ[m+n*(2*p+1)+1,0] && IntegerQ[p]
```

$$2. \int (f x)^m (d+e x^n) (a+b x^n+c x^{2 n})^p dx \text{ when } b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p < -1$$

$$1: \int (f x)^m (d+e x^n) (a+b x^n+c x^{2 n})^p dx \text{ when } b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p < -1 \wedge m > n-1$$

Derivation: Trinomial recurrence 2a

Rule 1.2.3.4.6.1.4.2.1: If $b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p < -1 \wedge m > n-1$, then

$$\frac{\int (f x)^m (d+e x^n) (a+b x^n+c x^{2 n})^p dx \rightarrow \frac{f^{n-1} (f x)^{m-n+1} (a+b x^n+c x^{2 n})^{p+1} (b d-2 a e-(b e-2 c d) x^n)}{n (p+1) (b^2-4 a c)} + \frac{f^n}{n (p+1) (b^2-4 a c)} \int (f x)^{m-n} (a+b x^n+c x^{2 n})^{p+1} ((n-m-1) (b d-2 a e) + (2 n p+2 n+m+1) (b e-2 c d) x^n) dx}$$

Program code:

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)*(a_+b_.**x_^n_+c_.**x_^n2_)^p_.,x_Symbol] :=
  f^(n-1)*(f*x)^(m-n+1)*(a+b*x^n+c*x^(2*n))^(p+1)*(b*d-2*a*e-(b*e-2*c*d)*x^n)/(n*(p+1)*(b^2-4*a*c)) +
  f^n/(n*(p+1)*(b^2-4*a*c))*Int[(f*x)^(m-n)*(a+b*x^n+c*x^(2*n))^(p+1)*
    Simp[(n-m-1)*(b*d-2*a*e)+(2*n*p+2*n+m+1)*(b*e-2*c*d)*x^n,x],x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && LtQ[p,-1] && GtQ[m,n-1] && IntegerQ[p]
```

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)*(a_+c_.**x_^n2_)^p_.,x_Symbol] :=
  f^(n-1)*(f*x)^(m-n+1)*(a+c*x^(2*n))^(p+1)*(a*e-c*d*x^n)/(2*a*c*n*(p+1)) +
  f^n/(2*a*c*n*(p+1))*Int[(f*x)^(m-n)*(a+c*x^(2*n))^(p+1)*(a*e*(n-m-1)+c*d*(2*n*p+2*n+m+1)*x^n),x] /;
FreeQ[{a,c,d,e,f},x] && EqQ[n2,2*n] && IGtQ[n,0] && LtQ[p,-1] && GtQ[m,n-1] && IntegerQ[p]
```

$$\mathbf{2:} \int (f x)^m (d+e x^n) (a+b x^n+c x^{2n})^p dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p < -1$$

Derivation: Trinomial recurrence 2b

Rule 1.2.3.4.6.1.4.2.2: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p < -1$, then

$$\begin{aligned} & \int (f x)^m (d+e x^n) (a+b x^n+c x^{2n})^p dx \rightarrow \\ & - \left((f x)^{m+1} (a+b x^n+c x^{2n})^{p+1} (d(b^2-2ac) - a b e + (b d - 2 a e) c x^n) \right) / (a f n (p+1) (b^2-4ac)) + \\ & \frac{1}{a n (p+1) (b^2-4ac)} \int (f x)^m (a+b x^n+c x^{2n})^{p+1} \cdot \\ & (d(b^2(m+n(p+1)+1) - 2ac(m+2n(p+1)+1)) - a b e(m+1) + c(m+n(2p+3)+1)(b d - 2 a e) x^n) dx \end{aligned}$$

Program code:

```
Int[(f_.**x_)^m_.*(d+_e_.**x_^n_)*(a+_b_.**x_^n+_c_.**x_^n2_)^p_,x_Symbol] :=
- (f**x)^(m+1)*(a+b**x^n+c**x^(2*n))^ (p+1)*(d*(b^2-2*a*c)-a*b*e+(b*d-2*a*e)*c**x^n)/(a*f*n*(p+1)*(b^2-4*a*c)) +
1/(a*n*(p+1)*(b^2-4*a*c))*Int[(f**x)^m*(a+b**x^n+c**x^(2*n))^ (p+1)*
Simp[d*(b^2*(m+n*(p+1)+1)-2*a*c*(m+2*n*(p+1)+1))-a*b*e*(m+1)+c*(m+n*(2*p+3)+1)*(b*d-2*a*e)*x^n,x],x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && LtQ[p,-1] && IntegerQ[p]
```

```
Int[(f_.**x_)^m_.*(d+_e_.**x_^n_)*(a+_c_.**x_^n2_)^p_,x_Symbol] :=
- (f**x)^(m+1)*(a+c**x^(2*n))^ (p+1)*(d+e**x^n)/(2*a*f*n*(p+1)) +
1/(2*a*n*(p+1))*Int[(f**x)^m*(a+c**x^(2*n))^ (p+1)*Simp[d*(m+2*n*(p+1)+1)+e*(m+n*(2*p+3)+1)*x^n,x],x] /;
FreeQ[{a,c,d,e,f,m},x] && EqQ[n2,2*n] && IGtQ[n,0] && LtQ[p,-1] && IntegerQ[p]
```

$$\mathbf{3:} \int (f x)^m (d+e x^n) (a+b x^n+c x^{2n})^p dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m > n-1 \wedge m+n(2p+1)+1 \neq 0$$

Derivation: Trinomial recurrence 3a

Rule 1.2.3.4.6.1.4.3: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m > n-1 \wedge m+n(2p+1)+1 \neq 0$, then

$$\int (f x)^m (d+e x^n) (a+b x^n+c x^{2n})^p dx \rightarrow$$

$$\frac{e f^{n-1} (f x)^{m-n+1} (a+b x^n+c x^{2n})^{p+1}}{c (m+n (2 p+1)+1)} -$$

$$\frac{f^n}{c (m+n (2 p+1)+1)} \int (f x)^{m-n} (a+b x^n+c x^{2n})^p (a e (m-n+1) + (b e (m+n p+1) - c d (m+n (2 p+1)+1)) x^n) dx$$

Program code:

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)*(a_+b_.**x_^n_+c_.**x_^n2_)^p_,x_Symbol] :=
  e*f^(n-1)*(f*x)^(m-n+1)*(a+b*x^n+c*x^(2*n))^(p+1)/(c*(m+n(2*p+1)+1)) -
  f^n/(c*(m+n(2*p+1)+1))*
  Int[(f*x)^(m-n)*(a+b*x^n+c*x^(2*n))^p*Simp[a*e*(m-n+1)+(b*e*(m+n*p+1)-c*d*(m+n(2*p+1)+1))*x^n,x],x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && GtQ[m,n-1] && NeQ[m+n(2*p+1)+1,0] && IntegerQ[p]
```

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)*(a_+c_.**x_^n2_)^p_,x_Symbol] :=
  e*f^(n-1)*(f*x)^(m-n+1)*(a+c*x^(2*n))^(p+1)/(c*(m+n(2*p+1)+1)) -
  f^n/(c*(m+n(2*p+1)+1))*Int[(f*x)^(m-n)*(a+c*x^(2*n))^p*(a*e*(m-n+1)-c*d*(m+n(2*p+1)+1))*x^n,x] /;
FreeQ[{a,c,d,e,f,p},x] && EqQ[n2,2*n] && IGtQ[n,0] && GtQ[m,n-1] && NeQ[m+n(2*p+1)+1,0] && IntegerQ[p]
```

$$\text{4: } \int (f x)^m (d+e x^n) (a+b x^n+c x^{2 n})^p dx \text{ when } b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m < -1$$

Derivation: Trinomial recurrence 3b

Rule 1.2.3.4.6.1.4.4: If $b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m < -1$, then

$$\begin{aligned} & \int (f x)^m (d+e x^n) (a+b x^n+c x^{2 n})^p dx \rightarrow \\ & \frac{d (f x)^{m+1} (a+b x^n+c x^{2 n})^{p+1}}{a f (m+1)} + \\ & \frac{1}{a f^n (m+1)} \int (f x)^{m+n} (a+b x^n+c x^{2 n})^p (a e (m+1) - b d (m+n (p+1) + 1) - c d (m+2 n (p+1) + 1) x^n) dx \end{aligned}$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^n_)*(a_+b_.*x_^n_+c_.*x_^n2_)^p_,x_Symbol] :=
  d*(f*x)^(m+1)*(a+b*x^n+c*x^(2*n))^(p+1)/(a*f*(m+1)) +
  1/(a*f^n*(m+1))*Int[(f*x)^(m+n)*(a+b*x^n+c*x^(2*n))^p*Simp[a*e*(m+1)-b*d*(m+n*(p+1)+1)-c*d*(m+2*n*(p+1)+1)*x^n,x],x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && LtQ[m,-1] && IntegerQ[p]
```

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^n_)*(a_+c_.*x_^n2_)^p_,x_Symbol] :=
  d*(f*x)^(m+1)*(a+c*x^(2*n))^(p+1)/(a*f*(m+1)) +
  1/(a*f^n*(m+1))*Int[(f*x)^(m+n)*(a+c*x^(2*n))^p*(a*e*(m+1)-c*d*(m+2*n*(p+1)+1)*x^n,x],x] /;
FreeQ[{a,c,d,e,f,p},x] && EqQ[n2,2*n] && IGtQ[n,0] && LtQ[m,-1] && IntegerQ[p]
```


$$5. \int \frac{(f x)^m (d+e x^n)}{a+b x^n+c x^{2n}} dx \text{ when } b^2 - 4 a c \neq 0 \wedge n \in \mathbb{Z}^+$$

$$1: \int \frac{(f x)^m (d+e x^n)}{a+b x^n+c x^{2n}} dx \text{ when } b^2 - 4 a c < 0 \wedge \frac{n}{2} \in \mathbb{Z}^+ \wedge 0 < m < n \wedge a c > 0$$

Derivation: Algebraic expansion

Basis: Let $q = \sqrt{a c}$ and $r = \sqrt{2 c q - b c}$, then $\frac{d+e z^2}{a+b z^2+c z^4} = \frac{c}{2 q r} \frac{d r - (c d - e q) z}{q - r z + c z^2} + \frac{c}{2 q r} \frac{d r + (c d - e q) z}{q + r z + c z^2}$

Rule 1.2.3.4.6.1.4.5.1: If $b^2 - 4 a c < 0 \wedge \frac{n}{2} \in \mathbb{Z}^+ \wedge 0 < m < n \wedge a c > 0$, let $q = \sqrt{a c}$, if $2 c q - b c > 0$, let $r = \sqrt{2 c q - b c}$, then

$$\int \frac{(f x)^m (d+e x^n)}{a+b x^n+c x^{2n}} dx \rightarrow \frac{c}{2 q r} \int \frac{(f x)^m (d r - (c d - e q) x^{n/2})}{q - r x^{n/2} + c x^n} dx + \frac{c}{2 q r} \int \frac{(f x)^m (d r + (c d - e q) x^{n/2})}{q + r x^{n/2} + c x^n} dx$$

Program code:

```
Int[(f_.*x_)^m_*(d+e_.*x_^n_)/(a+b_.*x_^n+c_.*x_^2n_),x_Symbol] :=
  With[{q=Rt[a*c,2]},
    With[{r=Rt[2*c*q-b*c,2]},
      c/(2*q*r)*Int[(f*x)^m*Simp[d*r-(c*d-e*q)*x^(n/2),x]/(q-r*x^(n/2)+c*x^n),x] +
      c/(2*q*r)*Int[(f*x)^m*Simp[d*r+(c*d-e*q)*x^(n/2),x]/(q+r*x^(n/2)+c*x^n),x] /;
      Not[LtQ[2*c*q-b*c,0]] /;
      FreeQ[{a,b,c,d,e,f},x] && EqQ[n2,2*n] && LtQ[b^2-4*a*c,0] && IntegersQ[m,n/2] && LtQ[0,m,n] && PosQ[a*c]
```

```
Int[(f_.*x_)^m_*(d+e_.*x_^n_)/(a+c_.*x_^2n_),x_Symbol] :=
  With[{q=Rt[a*c,2]},
    With[{r=Rt[2*c*q,2]},
      c/(2*q*r)*Int[(f*x)^m*Simp[d*r-(c*d-e*q)*x^(n/2),x]/(q-r*x^(n/2)+c*x^n),x] +
      c/(2*q*r)*Int[(f*x)^m*Simp[d*r+(c*d-e*q)*x^(n/2),x]/(q+r*x^(n/2)+c*x^n),x] /;
      Not[LtQ[2*c*q,0]] /;
      FreeQ[{a,c,d,e,f},x] && EqQ[n2,2*n] && GtQ[a*c,0] && IntegersQ[m,n/2] && LtQ[0,m,n]
```

$$2: \int \frac{(f x)^m (d+e x^n)}{a+b x^n+c x^{2 n}} dx \text{ when } b^2-4 a c < 0 \wedge \frac{n}{2}-1 \in \mathbb{Z}^+ \wedge a c > 0$$

Derivation: Algebraic expansion

Basis: Let $q = \sqrt{a c}$ and $r = \sqrt{2 c q - b c}$, then $\frac{d+e z^2}{a+b z^2+c z^4} = \frac{c}{2 q r} \frac{d r - (c d - e q) z}{q - r z + c z^2} + \frac{c}{2 q r} \frac{d r + (c d - e q) z}{q + r z + c z^2}$

Rule 1.2.3.4.6.1.4.5.2: If $b^2-4 a c < 0 \wedge \frac{n}{2}-1 \in \mathbb{Z}^+ \wedge a c > 0$, let $q = \sqrt{a c}$, if $2 c q - b c > 0$, let $r = \sqrt{2 c q - b c}$, then

$$\int \frac{(f x)^m (d+e x^n)}{a+b x^n+c x^{2 n}} dx \rightarrow \frac{c}{2 q r} \int \frac{(f x)^m (d r - (c d - e q) x^{n/2})}{q - r x^{n/2} + c x^n} dx + \frac{c}{2 q r} \int \frac{(f x)^m (d r + (c d - e q) x^{n/2})}{q + r x^{n/2} + c x^n} dx$$

Program code:

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)/(a_+b_.**x_^n_+c_.**x_^n2_),x_Symbol] :=
  With[{q=Rt[a*c,2]},
    With[{r=Rt[2*c*q-b*c,2]},
      c/(2*q*r)*Int[(f*x)^m*(d*r-(c*d-e*q)*x^(n/2))/(q-r*x^(n/2)+c*x^n),x] +
      c/(2*q*r)*Int[(f*x)^m*(d*r+(c*d-e*q)*x^(n/2))/(q+r*x^(n/2)+c*x^n),x] /;
      Not[LtQ[2*c*q-b*c,0]] /;
      FreeQ[{a,b,c,d,e,f,m},x] && EqQ[n2,2*n] && LtQ[b^2-4*a*c,0] && IGtQ[n/2,1] && PosQ[a*c]
```

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)/(a_+c_.**x_^n2_),x_Symbol] :=
  With[{q=Rt[a*c,2]},
    With[{r=Rt[2*c*q,2]},
      c/(2*q*r)*Int[(f*x)^m*(d*r-(c*d-e*q)*x^(n/2))/(q-r*x^(n/2)+c*x^n),x] +
      c/(2*q*r)*Int[(f*x)^m*(d*r+(c*d-e*q)*x^(n/2))/(q+r*x^(n/2)+c*x^n),x] /;
      Not[LtQ[2*c*q,0]] /;
      FreeQ[{a,c,d,e,f,m},x] && EqQ[n2,2*n] && IGtQ[n/2,1] && GtQ[a*c,0]
```

$$\text{3: } \int \frac{(f x)^m (d+e x^n)}{a+b x^n+c x^{2 n}} dx \text{ when } b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}^+$$

Derivation: Algebraic expansion

Basis: Let $q \rightarrow \sqrt{b^2-4 a c}$, then $\frac{d+e z}{a+b z+c z^2} = \left(\frac{e}{2} + \frac{2 c d-b e}{2 q}\right) \frac{1}{\frac{b}{2}-\frac{a}{2}+c z} + \left(\frac{e}{2} - \frac{2 c d-b e}{2 q}\right) \frac{1}{\frac{b}{2}+\frac{a}{2}+c z}$

Rule 1.2.3.4.6.1.4.5.3: If $b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}^+$, let $q \rightarrow \sqrt{b^2-4 a c}$, then

$$\int \frac{(f x)^m (d+e x^n)}{a+b x^n+c x^{2 n}} dx \rightarrow \left(\frac{e}{2} + \frac{2 c d-b e}{2 q}\right) \int \frac{(f x)^m}{\frac{b}{2}-\frac{a}{2}+c x^n} dx + \left(\frac{e}{2} - \frac{2 c d-b e}{2 q}\right) \int \frac{(f x)^m}{\frac{b}{2}+\frac{a}{2}+c x^n} dx$$

Program code:

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)/(a_+b_.**x_^n_+c_.**x_^n2_),x_Symbol] :=
  With[{q=Rt[b^2-4*a*c,2]},
    (e/2+(2*c*d-b*e)/(2*q))*Int[(f*x)^m/(b/2-q/2+c*x^n),x] + (e/2-(2*c*d-b*e)/(2*q))*Int[(f*x)^m/(b/2+q/2+c*x^n),x] /;
  FreeQ[{a,b,c,d,e,f,m},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0]
```

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)/(a_+c_.**x_^n2_),x_Symbol] :=
  With[{q=Rt[-a*c,2]},
    -(e/2+c*d/(2*q))*Int[(f*x)^m/(q-c*x^n),x] + (e/2-c*d/(2*q))*Int[(f*x)^m/(q+c*x^n),x] /;
  FreeQ[{a,c,d,e,f,m},x] && EqQ[n2,2*n] && IGtQ[n,0]
```

$$\begin{aligned}
& 5. \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \\
& 1. \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \in \mathbb{Z} \\
& \text{1: } \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \in \mathbb{Z} \wedge m \in \mathbb{Z}
\end{aligned}$$

Derivation: Algebraic expansion

Rule 1.2.3.4.6.1.5.1.1: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \in \mathbb{Z} \wedge m \in \mathbb{Z}$, then

$$\int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \rightarrow \int \text{ExpandIntegrand}\left[\frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}}, x\right] dx$$

Program code:

```

Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_/.(a_+b_.**x_^n_+c_.**x_^n2_.),x_Symbol] :=
  Int[ExpandIntegrand[(f**x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x],x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && IntegerQ[q] && IntegerQ[m]

```

```

Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_/.(a_+c_.**x_^n2_.),x_Symbol] :=
  Int[ExpandIntegrand[(f**x)^m*(d+e*x^n)^q/(a+c*x^(2*n)),x],x] /;
FreeQ[{a,c,d,e,f,m},x] && EqQ[n2,2*n] && IGtQ[n,0] && IntegerQ[q] && IntegerQ[m]

```

$$\mathbf{2:} \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2 n}} dx \text{ when } b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \in \mathbb{Z} \wedge m \notin \mathbb{Z}$$

Derivation: Algebraic expansion

Rule 1.2.3.4.6.1.5.1.2: If $b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \in \mathbb{Z} \wedge m \notin \mathbb{Z}$, then

$$\int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2 n}} dx \rightarrow \int (f x)^m \text{ExpandIntegrand}\left[\frac{(d+e x^n)^q}{a+b x^n+c x^{2 n}}, x\right] dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^n_)^q_/.(a_+b_.*x_^n_+c_.*x_^2n_),x_Symbol] :=
  Int[ExpandIntegrand[(f*x)^m,(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x],x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && IntegerQ[q] && Not[IntegerQ[m]]
```

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^n_)^q_/.(a_+c_.*x_^2n_),x_Symbol] :=
  Int[ExpandIntegrand[(f*x)^m,(d+e*x^n)^q/(a+c*x^(2*n)),x],x] /;
FreeQ[{a,c,d,e,f,m},x] && EqQ[n2,2*n] && IGtQ[n,0] && IntegerQ[q] && Not[IntegerQ[m]]
```

$$\begin{aligned}
& 2. \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \\
& 1. \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q > 0 \\
& 1. \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q > 0 \wedge m > n-1 \\
& \mathbf{1:} \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q > 0 \wedge m > 2n-1
\end{aligned}$$

Reference: Algebraic expansion

$$\text{Basis: } \frac{d+e x}{a+b x+c x^2} = \frac{c d-b e+c e x}{c^2 x^2} - \frac{a(c d-b e)+(b c d-b^2 e+a c e) x}{c^2 x^2 (a+b x+c x^2)}$$

Rule 1.2.3.4.6.1.5.2.1.1.1: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q > 0 \wedge m > 2n-1$, then

$$\begin{aligned}
& \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \rightarrow \\
& \frac{f^{2n}}{c^2} \int (f x)^{m-2n} (c d-b e+c e x^n) (d+e x^n)^{q-1} dx - \frac{f^{2n}}{c^2} \int \frac{(f x)^{m-2n} (d+e x^n)^{q-1} (a(c d-b e)+(b c d-b^2 e+a c e) x^n)}{a+b x^n+c x^{2n}} dx
\end{aligned}$$

Program code:

```

Int[(f_.**x_)^m_.*(d_.+e_.**x_^n_)^q_/(a_+b_.**x_^n_+c_.**x_^n2_.),x_Symbol] :=
  f^(2*n)/c^2*Int[(f*x)^(m-2*n)*(c*d-b*e+c*e*x^n)*(d+e*x^n)^(q-1),x] -
  f^(2*n)/c^2*Int[(f*x)^(m-2*n)*(d+e*x^n)^(q-1)*Simp[a*(c*d-b*e)+(b*c*d-b^2*e+a*c*e)*x^n,x]/(a+b*x^n+c*x^(2*n)),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && Not[IntegerQ[q]] && GtQ[q,0] && GtQ[m,2*n-1]

```

```

Int[(f_.**x_)^m_.*(d_.+e_.**x_^n_)^q_/(a_+c_.**x_^n2_.),x_Symbol] :=
  f^(2*n)/c*Int[(f*x)^(m-2*n)*(d+e*x^n)^q,x] -
  a*f^(2*n)/c*Int[(f*x)^(m-2*n)*(d+e*x^n)^q/(a+c*x^(2*n)),x] /;
FreeQ[{a,c,d,e,f,q},x] && EqQ[n2,2*n] && IGtQ[n,0] && Not[IntegerQ[q]] && GtQ[m,2*n-1]

```

$$2: \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q > 0 \wedge n-1 < m \leq 2n-1$$

Reference: Algebraic expansion

$$\text{Basis: } \frac{d+ez}{a+bz+cz^2} = \frac{e}{cz} - \frac{ae-(cd-be)z}{cz(a+bz+cz^2)}$$

Rule 1.2.3.4.6.1.5.2.1.1.2: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q > 0 \wedge n-1 < m \leq 2n-1$, then

$$\int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \rightarrow \frac{e f^n}{c} \int (f x)^{m-n} (d+e x^n)^{q-1} dx - \frac{f^n}{c} \int \frac{(f x)^{m-n} (d+e x^n)^{q-1} (a e - (c d - b e) x^n)}{a+b x^n+c x^{2n}} dx$$

Program code:

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_/(a_+b_.**x_^n_+c_.**x_^n2_),x_Symbol] :=
  e*f^n/c*Int[(f*x)^(m-n)*(d+e*x^n)^(q-1),x] -
  f^n/c*Int[(f*x)^(m-n)*(d+e*x^n)^(q-1)*Simp[a*e-(c*d-b*e)*x^n,x]/(a+b*x^n+c*x^(2*n)),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && Not[IntegerQ[q]] && GtQ[q,0] && GtQ[m,n-1] && LeQ[m,2n-1]
```

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_/(a_+c_.**x_^n2_),x_Symbol] :=
  e*f^n/c*Int[(f*x)^(m-n)*(d+e*x^n)^(q-1),x] -
  f^n/c*Int[(f*x)^(m-n)*(d+e*x^n)^(q-1)*Simp[a*e-c*d*x^n,x]/(a+c*x^(2*n)),x] /;
FreeQ[{a,c,d,e,f},x] && EqQ[n2,2*n] && IGtQ[n,0] && Not[IntegerQ[q]] && GtQ[q,0] && GtQ[m,n-1] && LeQ[m,2n-1]
```

$$\mathbf{2:} \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q > 0 \wedge m < 0$$

Reference: Algebraic expansion

$$\text{Basis: } \frac{d+e z}{a+b z+c z^2} = \frac{d}{a} - \frac{z(bd-ae+cdz)}{a(a+bz+cz^2)}$$

Rule 1.2.3.4.6.1.5.2.1.2: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q > 0 \wedge m < 0$, then

$$\int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \rightarrow \frac{d}{a} \int (f x)^m (d+e x^n)^{q-1} dx - \frac{1}{a f^n} \int \frac{(f x)^{m+n} (d+e x^n)^{q-1} (b d-a e+c d x^n)}{a+b x^n+c x^{2n}} dx$$

Program code:

```
Int[(f_.**x_)^m_*(d_.+e_.**x_^n_)^q_/(a_+b_.**x_^n_+c_.**x_^2n_),x_Symbol] :=
  d/a*Int[(f*x)^m*(d+e*x^n)^(q-1),x] -
  1/(a*f^n)*Int[(f*x)^(m+n)*(d+e*x^n)^(q-1)*Simp[b*d-a*e+c*d*x^n,x]/(a+b*x^n+c*x^(2*n)),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && Not[IntegerQ[q]] && GtQ[q,0] && LtQ[m,0]
```

```
Int[(f_.**x_)^m_*(d_.+e_.**x_^n_)^q_/(a_+c_.**x_^2n_),x_Symbol] :=
  d/a*Int[(f*x)^m*(d+e*x^n)^(q-1),x] +
  1/(a*f^n)*Int[(f*x)^(m+n)*(d+e*x^n)^(q-1)*Simp[a*e-c*d*x^n,x]/(a+c*x^(2*n)),x] /;
FreeQ[{a,c,d,e,f},x] && EqQ[n2,2*n] && IGtQ[n,0] && Not[IntegerQ[q]] && GtQ[q,0] && LtQ[m,0]
```


$$\begin{aligned}
& 2. \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q < -1 \\
& 1. \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q < -1 \wedge m > n-1 \\
& \mathbf{1:} \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q < -1 \wedge m > 2n-1
\end{aligned}$$

Reference: Algebraic expansion

$$\text{Basis: } \frac{1}{a+bz+cz^2} = \frac{d^2}{(cd^2-bde+ae^2)z^2} - \frac{(d+ez)(ad+(bd-ae)z)}{(cd^2-bde+ae^2)z^2(ad+bz+cz^2)}$$

Rule 1.2.3.4.6.1.5.2.2.1.1: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q < -1 \wedge m > 2n-1$, then

$$\int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \rightarrow \frac{d^2 f^{2n}}{c d^2-b d e+a e^2} \int (f x)^{m-2n} (d+e x^n)^q dx - \frac{f^{2n}}{c d^2-b d e+a e^2} \int \frac{(f x)^{m-2n} (d+e x^n)^{q+1} (a d+(b d-a e) x^n)}{a+b x^n+c x^{2n}} dx$$

Program code:

```

Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_/(a_+b_.**x_^n_+c_.**x_^n2_),x_Symbol] :=
  d^2*f^(2*n)/(c*d^2-b*d*e+a*e^2)*Int[(f*x)^(m-2*n)*(d+e*x^n)^q,x] -
  f^(2*n)/(c*d^2-b*d*e+a*e^2)*Int[(f*x)^(m-2*n)*(d+e*x^n)^(q+1)*Simp[a*d+(b*d-a*e)*x^n,x]/(a+b*x^n+c*x^(2*n)),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && Not[IntegerQ[q]] && LtQ[q,-1] && GtQ[m,2*n-1]

```

```

Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_/(a_+c_.**x_^n2_),x_Symbol] :=
  d^2*f^(2*n)/(c*d^2+a*e^2)*Int[(f*x)^(m-2*n)*(d+e*x^n)^q,x] -
  a*f^(2*n)/(c*d^2+a*e^2)*Int[(f*x)^(m-2*n)*(d+e*x^n)^(q+1)*(d-e*x^n)/(a+c*x^(2*n)),x] /;
FreeQ[{a,c,d,e,f},x] && EqQ[n2,2*n] && IGtQ[n,0] && Not[IntegerQ[q]] && LtQ[q,-1] && GtQ[m,2*n-1]

```

$$2: \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q < -1 \wedge n-1 < m \leq 2n-1$$

Reference: Algebraic expansion

$$\text{Basis: } \frac{1}{a+bz+cz^2} == -\frac{de}{(cd^2-bde+ae^2)z} + \frac{(d+ez)(ae+cdz)}{(cd^2-bde+ae^2)z(a+bz+cz^2)}$$

Rule 1.2.3.4.6.1.5.2.2.1.2: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q < -1 \wedge n-1 < m \leq 2n-1$, then

$$\int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \rightarrow -\frac{d e f^n}{c d^2-b d e+a e^2} \int (f x)^{m-n} (d+e x^n)^q dx + \frac{f^n}{c d^2-b d e+a e^2} \int \frac{(f x)^{m-n} (d+e x^n)^{q+1} (a e+c d x^n)}{a+b x^n+c x^{2n}} dx$$

Program code:

```
Int[(f_.**x_)^m_.*(d_.+e_.**x_^n_)^q_/(a_+b_.**x_^n_+c_.**x_^2n_),x_Symbol] :=
  -d*e*f^n/(c*d^2-b*d*e+a*e^2)*Int[(f*x)^(m-n)*(d+e*x^n)^q,x] +
  f^n/(c*d^2-b*d*e+a*e^2)*Int[(f*x)^(m-n)*(d+e*x^n)^(q+1)*Simp[a*e+c*d*x^n,x]/(a+b*x^n+c*x^(2*n)),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && Not[IntegerQ[q]] && LtQ[q,-1] && GtQ[m,n-1] && LeQ[m,2*n-1]
```

```
Int[(f_.**x_)^m_.*(d_.+e_.**x_^n_)^q_/(a_+c_.**x_^2n_),x_Symbol] :=
  -d*e*f^n/(c*d^2+a*e^2)*Int[(f*x)^(m-n)*(d+e*x^n)^q,x] +
  f^n/(c*d^2+a*e^2)*Int[(f*x)^(m-n)*(d+e*x^n)^(q+1)*Simp[a*e+c*d*x^n,x]/(a+c*x^(2*n)),x] /;
FreeQ[{a,c,d,e,f},x] && EqQ[n2,2*n] && IGtQ[n,0] && Not[IntegerQ[q]] && LtQ[q,-1] && GtQ[m,n-1] && LeQ[m,2*n-1]
```

$$2: \int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q < -1$$

Derivation: Algebraic expansion

$$\text{Basis: } \frac{1}{a+bz+cz^2} == \frac{e^2}{cd^2-bde+ae^2} + \frac{(d+ez)(cd-be-cez)}{(cd^2-bde+ae^2)(a+bz+cz^2)}$$

Rule 1.2.3.4.6.1.5.2.2.2: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge q < -1$, then

$$\int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2 n}} dx \rightarrow \frac{e^2}{c d^2-b d e+a e^2} \int (f x)^m (d+e x^n)^q dx + \frac{1}{c d^2-b d e+a e^2} \int \frac{(f x)^m (d+e x^n)^{q+1} (c d-b e-c e x^n)}{a+b x^n+c x^{2 n}} dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^n_)^q_/(a_+b_.*x_^n_+c_.*x_^2n_),x_Symbol] :=
  e^2/(c*d^2-b*d*e+a*e^2)*Int[(f*x)^m*(d+e*x^n)^q,x] +
  1/(c*d^2-b*d*e+a*e^2)*Int[(f*x)^m*(d+e*x^n)^(q+1)*Simp[c*d-b*e-c*e*x^n,x]/(a+b*x^n+c*x^(2*n)),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && Not[IntegerQ[q]] && LtQ[q,-1]
```

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^n_)^q_/(a_+c_.*x_^2n_),x_Symbol] :=
  e^2/(c*d^2+a*e^2)*Int[(f*x)^m*(d+e*x^n)^q,x] +
  c/(c*d^2+a*e^2)*Int[(f*x)^m*(d+e*x^n)^(q+1)*(d-e*x^n)/(a+c*x^(2*n)),x] /;
FreeQ[{a,c,d,e,f,m},x] && EqQ[n2,2*n] && IGtQ[n,0] && Not[IntegerQ[q]] && LtQ[q,-1]
```

3: $\int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2 n}} dx$ when $b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge m \in \mathbb{Z}$

Derivation: Algebraic expansion

Basis: If $q = \sqrt{b^2-4 a c}$, then $\frac{1}{a+b z+c z^2} = \frac{2 c}{q (b-q+2 c z)} - \frac{2 c}{q (b+q+2 c z)}$

Rule 1.2.3.4.6.1.5.2.3: If $b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge m \in \mathbb{Z}$, then

$$\int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2 n}} dx \rightarrow \int (d+e x^n)^q \text{ExpandIntegrand}\left[\frac{(f x)^m}{a+b x^n+c x^{2 n}}, x\right] dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^n_)^q_/(a_+b_.*x_^n_+c_.*x_^2n_),x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x^n)^q,(f*x)^m/(a+b*x^n+c*x^(2*n)),x],x] /;
FreeQ[{a,b,c,d,e,f,q,n},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && Not[IntegerQ[q]] && IntegerQ[m]
```

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_/(a_+c_.**x_^n2_.),x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x^n)^q,(f*x)^m/(a+c*x^(2*n)),x],x] /;
FreeQ[{a,c,d,e,f,q,n},x] && EqQ[n2,2*n] && IGtQ[n,0] && Not[IntegerQ[q]] && IntegerQ[m]
```

4: $\int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx$ when $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge m \notin \mathbb{Z}$

Derivation: Algebraic expansion

Basis: If $q = \sqrt{b^2-4ac}$, then $\frac{1}{a+bz+cz^2} = \frac{2c}{q(b-q+2cz)} - \frac{2c}{q(b+q+2cz)}$

Rule 1.2.3.4.6.1.5.2.4: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge q \notin \mathbb{Z} \wedge m \notin \mathbb{Z}$, then

$$\int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \rightarrow \int (f x)^m (d+e x^n)^q \text{ExpandIntegrand}\left[\frac{1}{a+b x^n+c x^{2n}}, x\right] dx$$

Program code:

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_/(a_+b_.**x_^n_+c_.**x_^n2_.),x_Symbol] :=
  Int[ExpandIntegrand[(f*x)^m*(d+e*x^n)^q,1/(a+b*x^n+c*x^(2*n)),x],x] /;
FreeQ[{a,b,c,d,e,f,m,q,n},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && Not[IntegerQ[q]] && Not[IntegerQ[m]]
```

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_/(a_+c_.**x_^n2_.),x_Symbol] :=
  Int[ExpandIntegrand[(f*x)^m*(d+e*x^n)^q,1/(a+c*x^(2*n)),x],x] /;
FreeQ[{a,c,d,e,f,m,q,n},x] && EqQ[n2,2*n] && IGtQ[n,0] && Not[IntegerQ[q]] && Not[IntegerQ[m]]
```

$$\begin{aligned}
6. \int \frac{(f x)^m (a+b x^n+c x^{2n})^p}{d+e x^n} dx & \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \\
1. \int \frac{(f x)^m (a+b x^n+c x^{2n})^p}{d+e x^n} dx & \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p > 0 \wedge m < 0 \\
1: \int \frac{(f x)^m (a+b x^n+c x^{2n})^p}{d+e x^n} dx & \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p > 0 \wedge m < -n
\end{aligned}$$

Reference: Algebraic expansion

$$\text{Basis: } \frac{a+bz+cz^2}{d+ez} = \frac{ad+(bd-ae)z}{d^2} + \frac{(cd^2-bde+ae^2)z^2}{d^2(d+ez)}$$

Rule 1.2.3.4.6.1.6.1.1: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p > 0 \wedge m < -n$, then

$$\begin{aligned}
& \int \frac{(f x)^m (a+b x^n+c x^{2n})^p}{d+e x^n} dx \rightarrow \\
& \frac{1}{d^2} \int (f x)^m (ad+(bd-ae)x^n) (a+b x^n+c x^{2n})^{p-1} dx + \frac{cd^2-bde+ae^2}{d^2 f^{2n}} \int \frac{(f x)^{m+2n} (a+b x^n+c x^{2n})^{p-1}}{d+e x^n} dx
\end{aligned}$$

Program code:

```

Int[(f_.**x_)^m_*(a_.+b_.**x_^n_+c_.**x_^n2_.)^p_./(d_.+e_.**x_^n_),x_Symbol] :=
  1/d^2*Int[(f*x)^m*(a*d+(b*d-a*e)**x^n)*(a+b*x^n+c*x^(2*n))^(p-1),x] +
  (c*d^2-b*d*e+a*e^2)/(d^2*f^(2*n))*Int[(f*x)^(m+2*n)*(a+b*x^n+c*x^(2*n))^(p-1)/(d+e*x^n),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && GtQ[p,0] && LtQ[m,-n]

```

```

Int[(f_.**x_)^m_*(a_.+c_.**x_^n2_.)^p_./(d_.+e_.**x_^n_),x_Symbol] :=
  a/d^2*Int[(f*x)^m*(d-e*x^n)*(a+c*x^(2*n))^(p-1),x] +
  (c*d^2+a*e^2)/(d^2*f^(2*n))*Int[(f*x)^(m+2*n)*(a+c*x^(2*n))^(p-1)/(d+e*x^n),x] /;
FreeQ[{a,c,d,e,f},x] && EqQ[n2,2*n] && IGtQ[n,0] && GtQ[p,0] && LtQ[m,-n]

```

$$\text{2: } \int \frac{(f x)^m (a+b x^n+c x^{2n})^p}{d+e x^n} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p > 0 \wedge m < 0$$

Reference: Algebraic expansion

$$\text{Basis: } \frac{a+bz+cz^2}{d+ez} = \frac{ae+cdz}{de} - \frac{(cd^2-bde+ae^2)z}{de(d+ez)}$$

Rule 1.2.3.4.6.1.6.1.2: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p > 0 \wedge m < 0$, then

$$\int \frac{(f x)^m (a+b x^n+c x^{2n})^p}{d+e x^n} dx \rightarrow \frac{1}{de} \int (f x)^m (ae+cdx^n) (a+b x^n+c x^{2n})^{p-1} dx - \frac{cd^2-bde+ae^2}{def^n} \int \frac{(f x)^{m+n} (a+b x^n+c x^{2n})^{p-1}}{d+e x^n} dx$$

Program code:

```
Int[(f_.**x_)^m_*(a_.+b_.**x_^n_+c_.**x_^n2_.)^p_./(d_.+e_.**x_^n_),x_Symbol] :=
  1/(d*e)*Int[(f*x)^m*(a+e+c*d*x^n)*(a+b*x^n+c*x^(2*n))^(p-1),x] -
  (c*d^2-b*d*e+a*e^2)/(d*e*f^n)*Int[(f*x)^(m+n)*(a+b*x^n+c*x^(2*n))^(p-1)/(d+e*x^n),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && GtQ[p,0] && LtQ[m,0]
```

```
Int[(f_.**x_)^m_*(a_.+c_.**x_^n2_.)^p_./(d_.+e_.**x_^n_),x_Symbol] :=
  1/(d*e)*Int[(f*x)^m*(a+e+c*d*x^n)*(a+c*x^(2*n))^(p-1),x] -
  (c*d^2+a*e^2)/(d*e*f^n)*Int[(f*x)^(m+n)*(a+c*x^(2*n))^(p-1)/(d+e*x^n),x] /;
FreeQ[{a,c,d,e,f},x] && EqQ[n2,2*n] && IGtQ[n,0] && GtQ[p,0] && LtQ[m,0]
```

$$2. \int \frac{(f x)^m (a+b x^n+c x^{2 n})^p}{d+e x^n} dx \text{ when } b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p < -1 \wedge m > 0$$

$$1: \int \frac{(f x)^m (a+b x^n+c x^{2 n})^p}{d+e x^n} dx \text{ when } b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p < -1 \wedge m > n$$

Reference: Algebraic expansion

$$\text{Basis: } \frac{z^2}{d+e z} = -\frac{a d+(b d-a e) z}{c d^2-b d e+a e^2} + \frac{d^2 (a+b z+c z^2)}{(c d^2-b d e+a e^2) (d+e z)}$$

Rule 1.2.3.4.6.1.6.2.1: If $b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p < -1 \wedge m > n$, then

$$\int \frac{(f x)^m (a+b x^n+c x^{2 n})^p}{d+e x^n} dx \rightarrow -\frac{f^2 n}{c d^2-b d e+a e^2} \int (f x)^{m-2 n} (a d+(b d-a e) x^n) (a+b x^n+c x^{2 n})^p dx + \frac{d^2 f^2 n}{c d^2-b d e+a e^2} \int \frac{(f x)^{m-2 n} (a+b x^n+c x^{2 n})^{p+1}}{d+e x^n} dx$$

Program code:

```
Int[(f_.**x_)^m_.*(a_.+b_.**x_^n_+c_.**x_^n2_.)^p_/(d_.+e_.**x_^n_),x_Symbol] :=
  -f^(2*n)/(c*d^2-b*d*e+a*e^2)*Int[(f*x)^(m-2*n)*(a*d+(b*d-a*e)*x^n)*(a+b*x^n+c*x^(2*n))^p,x] +
  d^2*f^(2*n)/(c*d^2-b*d*e+a*e^2)*Int[(f*x)^(m-2*n)*(a+b*x^n+c*x^(2*n))^(p+1)/(d+e*x^n),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && LtQ[p,-1] && GtQ[m,n]
```

```
Int[(f_.**x_)^m_.*(a+c_.**x_^n2_.)^p_/(d_.+e_.**x_^n_),x_Symbol] :=
  -a*f^(2*n)/(c*d^2+a*e^2)*Int[(f*x)^(m-2*n)*(d-e*x^n)*(a+c*x^(2*n))^p,x] +
  d^2*f^(2*n)/(c*d^2+a*e^2)*Int[(f*x)^(m-2*n)*(a+c*x^(2*n))^(p+1)/(d+e*x^n),x] /;
FreeQ[{a,c,d,e,f},x] && EqQ[n2,2*n] && IGtQ[n,0] && LtQ[p,-1] && GtQ[m,n]
```

$$\text{2: } \int \frac{(f x)^m (a+b x^n+c x^{2n})^p}{d+e x^n} dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p < -1 \wedge m > 0$$

Reference: Algebraic expansion

$$\text{Basis: } \frac{z}{d+e z} = \frac{a e+c d z}{c d^2-b d e+a e^2} - \frac{d e (a+b z+c z^2)}{(c d^2-b d e+a e^2) (d+e z)}$$

Rule 1.2.3.4.6.1.6.2.2: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p < -1 \wedge m > 0$, then

$$\int \frac{(f x)^m (a+b x^n+c x^{2n})^p}{d+e x^n} dx \rightarrow \frac{f^n}{c d^2-b d e+a e^2} \int (f x)^{m-n} (a e+c d x^n) (a+b x^n+c x^{2n})^p dx - \frac{d e f^n}{c d^2-b d e+a e^2} \int \frac{(f x)^{m-n} (a+b x^n+c x^{2n})^{p+1}}{d+e x^n} dx$$

Program code:

```
Int[(f_.**x_)^m_.*(a_.+b_.**x_^n_+c_.**x_^n2_.)^p_/(d_.+e_.**x_^n_),x_Symbol] :=
  f^n/(c*d^2-b*d*e+a*e^2)*Int[(f*x)^(m-n)*(a*e+c*d*x^n)*(a+b*x^n+c*x^(2*n))^p,x] -
  d*e*f^n/(c*d^2-b*d*e+a*e^2)*Int[(f*x)^(m-n)*(a+b*x^n+c*x^(2*n))^(p+1)/(d+e*x^n),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && LtQ[p,-1] && GtQ[m,0]
```

```
Int[(f_.**x_)^m_.*(a_+c_.**x_^n2_.)^p_/(d_.+e_.**x_^n_),x_Symbol] :=
  f^n/(c*d^2+a*e^2)*Int[(f*x)^(m-n)*(a*e+c*d*x^n)*(a+c*x^(2*n))^p,x] -
  d*e*f^n/(c*d^2+a*e^2)*Int[(f*x)^(m-n)*(a+c*x^(2*n))^(p+1)/(d+e*x^n),x] /;
FreeQ[{a,c,d,e,f},x] && EqQ[n2,2*n] && IGtQ[n,0] && LtQ[p,-1] && GtQ[m,0]
```


7: $\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge (q \in \mathbb{Z}^+ \vee (m|q) \in \mathbb{Z})$

Derivation: Algebraic expansion

Rule 1.2.3.4.6.1.7: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge (q \in \mathbb{Z}^+ \vee (m|q) \in \mathbb{Z})$, then

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \int (a+b x^n+c x^{2n})^p \text{ExpandIntegrand}[(f x)^m (d+e x^n)^q, x] dx$$

Program code:

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_.*(a_+b_.**x_^n_+c_.**x_^n2_.)^p_,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*x^n+c*x^(2*n))^p,(f*x)^m(d+e*x^n)^q,x],x] /;
FreeQ[{a,b,c,d,e,f,m,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && (IGtQ[q,0] || IntegersQ[m,q])
```

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_.*(a_+c_.**x_^n2_.)^p_,x_Symbol] :=
  Int[ExpandIntegrand[(a+c*x^n)^p,(f*x)^m(d+e*x^n)^q,x],x] /;
FreeQ[{a,c,d,e,f,m,q},x] && EqQ[n2,2*n] && IGtQ[n,0] && IGtQ[q,0]
```

$$2. \int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx \text{ when } b^2 - 4 a c \neq 0 \wedge n \in \mathbb{Z}^-$$

$$1. \int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx \text{ when } b^2 - 4 a c \neq 0 \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{Q}$$

$$\mathbf{1:} \int x^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx \text{ when } b^2 - 4 a c \neq 0 \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{Z}$$

Derivation: Integration by substitution

$$\text{Basis: } F[x] := -\text{Subst}\left[\frac{F\left[\frac{x^{-1}}{x^2}\right]}{x^2}, x, \frac{1}{x}\right] \partial_x \frac{1}{x}$$

Rule 1.2.3.4.6.2.1.1: If $b^2 - 4 a c \neq 0 \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{Z}$, then

$$\int x^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx \rightarrow -\text{Subst}\left[\int \frac{(d+e x^{-n})^q (a+b x^{-n}+c x^{-2 n})^p}{x^{m+2}} dx, x, \frac{1}{x}\right]$$

Program code:

```
Int[x_^m_.*(d_+e_.*x_^n_)^q_.*(a_+b_.*x_^n_+c_.*x_^n2_)^p_,x_Symbol] :=
  -Subst[Int[(d+e*x^(-n))^q*(a+b*x^(-n)+c*x^(-2*n))^p/x^(m+2),x],x,1/x] /;
FreeQ[{a,b,c,d,e,p,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && ILtQ[n,0] && IntegerQ[m]
```

```
Int[x_^m_.*(d_+e_.*x_^n_)^q_.*(a_+c_.*x_^n2_)^p_,x_Symbol] :=
  -Subst[Int[(d+e*x^(-n))^q*(a+c*x^(-2*n))^p/x^(m+2),x],x,1/x] /;
FreeQ[{a,c,d,e,p,q},x] && EqQ[n2,2*n] && ILtQ[n,0] && IntegerQ[m]
```

$$\mathbf{2:} \int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{F}$$

Derivation: Integration by substitution

Basis: If $n \in \mathbb{Z} \wedge g > 1$, then $(f x)^m F[x^n] = -\frac{g}{f} \text{Subst}\left[\frac{F[f^{-n} x^{-g n}]}{x^{g(m+1)+1}}, x, \frac{1}{(f x)^{1/g}}\right] \partial_x \frac{1}{(f x)^{1/g}}$

Rule 1.2.3.4.6.2.1.2: If $b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{F}$, let $g = \text{Denominator}[m]$, then

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow -\frac{g}{f} \text{Subst}\left[\int \frac{(d+e f^{-n} x^{-g n})^q (a+b f^{-n} x^{-g n}+c f^{-2n} x^{-2g n})^p}{x^{g(m+1)+1}} dx, x, \frac{1}{(f x)^{1/g}}\right]$$

Program code:

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_.*(a_+b_.**x_^n_+c_.**x_^n2_.)^p_,x_Symbol] :=
  With[{g=Denominator[m]},
    -g/f*Subst[Int[(d+e*f^(-n))*x^(-g*n)]^q*(a+b*f^(-n))*x^(-g*n)+c*f^(-2*n))*x^(-2*g*n)]^p/x^(g*(m+1)+1),x],x,1/(f*x)^(1/g)] /;
  FreeQ[{a,b,c,d,e,f,p,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IltQ[n,0] && FractionQ[m]
```

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_.*(a_+c_.**x_^n2_.)^p_,x_Symbol] :=
  With[{g=Denominator[m]},
    -g/f*Subst[Int[(d+e*f^(-n))*x^(-g*n)]^q*(a+c*f^(-2*n))*x^(-2*g*n)]^p/x^(g*(m+1)+1),x],x,1/(f*x)^(1/g)] /;
  FreeQ[{a,c,d,e,f,p,q},x] && EqQ[n2,2*n] && IltQ[n,0] && FractionQ[m]
```

$$\mathbf{2:} \int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \text{ when } b^2-4ac \neq 0 \wedge n \in \mathbb{Z}^- \wedge m \notin \mathbb{Q}$$

Derivation: Piecewise constant extraction and integration by substitution

Basis: $\partial_x \left((f x)^m (x^{-1})^m \right) = 0$

Basis: $(f x)^m (x^{-1})^m = f^{\text{IntPart}[m]} (f x)^{\text{FracPart}[m]} (x^{-1})^{\text{FracPart}[m]}$

Basis: $F[x] = -\text{Subst}\left[\frac{F\left[\frac{x^{-1}}{x^2}\right]}{x^2}, x, \frac{1}{x}\right] \partial_x \frac{1}{x}$

Rule 1.2.3.4.6.2.2: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^- \wedge m \notin \mathbb{Q}$, then

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow f^{\text{IntPart}[m]} (f x)^{\text{FracPart}[m]} (x^{-1})^{\text{FracPart}[m]} \int \frac{(d+e x^n)^q (a+b x^n+c x^{2n})^p}{(x^{-1})^m} dx$$

$$\rightarrow -f^{\text{IntPart}[m]} (f x)^{\text{FracPart}[m]} (x^{-1})^{\text{FracPart}[m]} \text{Subst}\left[\int \frac{(d+e x^{-n})^q (a+b x^{-n}+c x^{-2n})^p}{x^{m+2}} dx, x, \frac{1}{x}\right]$$

Program code:

```
Int[(f_.**x_)^m_*(d_+e_.**x_^n_)^q_.*(a_+b_.**x_^n_+c_.**x_^n2_)^p_,x_Symbol] :=
  -f^IntPart[m]*(f*x)^FracPart[m]*(x^(-1))^FracPart[m]*Subst[Int[(d+e*x^(-n))^q*(a+b*x^(-n)+c*x^(-2*n))^p/x^(m+2),x],x,1/x] /;
FreeQ[{a,b,c,d,e,f,m,p,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && ILtQ[n,0] && Not[RationalQ[m]]
```

```
Int[(f_.**x_)^m_*(d_+e_.**x_^n_)^q_.*(a_+c_.**x_^n2_)^p_,x_Symbol] :=
  -f^IntPart[m]*(f*x)^FracPart[m]*(x^(-1))^FracPart[m]*Subst[Int[(d+e*x^(-n))^q*(a+c*x^(-2*n))^p/x^(m+2),x],x,1/x] /;
FreeQ[{a,c,d,e,f,m,p,q},x] && EqQ[n2,2*n] && ILtQ[n,0] && Not[RationalQ[m]]
```

$$7. \int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{F}$$

$$1: \int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{F}$$

Derivation: Integration by substitution

Basis: If $g \in \mathbb{Z}^+$, then $x^m F[x^n] = g \text{ Subst}[x^{g(m+1)-1} F[x^{g n}], x, x^{1/g}] \partial_x x^{1/g}$

Rule 1.2.3.4.7.1: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{F}$, let $g = \text{Denominator}[n]$, then

$$\int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow g \text{ Subst}\left[\int x^{g(m+1)-1} (d+e x^{g n})^q (a+b x^{g n}+c x^{2g n})^p dx, x, x^{1/g}\right]$$

Program code:

```
Int[x_^m_.*(d_+e_.*x_^n_)^q_.*(a_+b_.*x_^n_+c_.*x_^n2_)^p_,x_Symbol] :=
  With[{g=Denominator[n]},
    g*Subst[Int[x^(g*(m+1)-1)*(d+e*x^(g*n))^q*(a+b*x^(g*n)+c*x^(2*g*n))^p,x],x,x^(1/g)]] /;
  FreeQ[{a,b,c,d,e,m,p,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && FractionQ[n]
```

```
Int[x_^m_.*(d_+e_.*x_^n_)^q_.*(a_+c_.*x_^n2_)^p_,x_Symbol] :=
  With[{g=Denominator[n]},
    g*Subst[Int[x^(g*(m+1)-1)*(d+e*x^(g*n))^q*(a+c*x^(2*g*n))^p,x],x,x^(1/g)]] /;
  FreeQ[{a,c,d,e,m,p,q},x] && EqQ[n2,2*n] && FractionQ[n]
```

2: $\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx$ when $b^2 - 4 a c \neq 0 \wedge n \in \mathbb{F}$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(f x)^m}{x^m} == 0$

Basis: $\frac{(f x)^m}{x^m} == \frac{f^{\text{IntPart}[m]} (f x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}}$

Rule 1.2.3.4.7.2: If $b^2 - 4 a c \neq 0 \wedge n \in \mathbb{F}$, then

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx \rightarrow \frac{f^{\text{IntPart}[m]} (f x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p dx$$

Program code:

```
Int[(f*x_)^m*(d+_e_*x_^n_)^q_.*(a+_b_*x_^n+_c_*x_^2n_)^p_,x_Symbol] :=
  f^IntPart[m]*(f*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d,e,f,m,p,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && FractionQ[n]
```

```
Int[(f*x_)^m*(d+_e_*x_^n_)^q_.*(a+_c_*x_^2n_)^p_,x_Symbol] :=
  f^IntPart[m]*(f*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x] /;
FreeQ[{a,c,d,e,f,m,p,q},x] && EqQ[n2,2*n] && FractionQ[n]
```

$$8. \int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge \frac{n}{m+1} \in \mathbb{Z}$$

$$1: \int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge \frac{n}{m+1} \in \mathbb{Z}$$

Derivation: Integration by substitution

Basis: If $\frac{n}{m+1} \in \mathbb{Z}$, then $x^m F[x^n] = \frac{1}{m+1} \text{Subst}[F[x^{\frac{n}{m+1}}], x, x^{m+1}] \partial_x x^{m+1}$

Rule 1.2.3.4.8.1: If $b^2 - 4ac \neq 0 \wedge \frac{n}{m+1} \in \mathbb{Z}$

$$\int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \frac{1}{m+1} \text{Subst}\left[\int (d+e x^{\frac{n}{m+1}})^q (a+b x^{\frac{n}{m+1}}+c x^{\frac{2n}{m+1}})^p dx, x, x^{m+1}\right]$$

Program code:

```
Int[x_^m_.*(d+_e_.*x_^n_)^q_.*(a+_b_.*x_^n+_c_.*x_^2n_)^p_,x_Symbol] :=
  1/(m+1)*Subst[Int[(d+e*x^Simplify[n/(m+1)])^q*(a+b*x^Simplify[n/(m+1)]+c*x^Simplify[2*n/(m+1)])^p,x],x,x^(m+1)] /;
FreeQ[{a,b,c,d,e,m,n,p,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IntegerQ[Simplify[n/(m+1)]] && Not[IntegerQ[n]]
```

```
Int[x_^m_.*(d+_e_.*x_^n_)^q_.*(a+_c_.*x_^2n_)^p_,x_Symbol] :=
  1/(m+1)*Subst[Int[(d+e*x^Simplify[n/(m+1)])^q*(a+c*x^Simplify[2*n/(m+1)])^p,x],x,x^(m+1)] /;
FreeQ[{a,c,d,e,m,n,p,q},x] && EqQ[n2,2*n] && IntegerQ[Simplify[n/(m+1)]] && Not[IntegerQ[n]]
```

$$2: \int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge \frac{n}{m+1} \in \mathbb{Z}$$

Derivation: Piecewise constant extraction

$$\text{Basis: } \partial_x \frac{(f x)^m}{x^m} = 0$$

$$\text{Basis: } \frac{(f x)^m}{x^m} = \frac{f \text{IntPart}[m] (f x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}}$$

Rule 1.2.3.4.8.2: If $b^2 - 4ac \neq 0 \wedge \frac{n}{m+1} \in \mathbb{Z}$, then

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \frac{f^{\text{IntPart}[m]} (f x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$$

Program code:

```
Int[(f*x_)^m_*(d+_e_*x_^n_)^q_*(a+_b_*x_^n+_c_*x_^2n_)^p_,x_Symbol] :=
  f^IntPart[m]*(f*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d,e,f,m,p,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IntegerQ[Simplify[n/(m+1)]] && Not[IntegerQ[n]]
```

```
Int[(f*x_)^m_*(d+_e_*x_^n_)^q_*(a+_c_*x_^2n_)^p_,x_Symbol] :=
  f^IntPart[m]*(f*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x] /;
FreeQ[{a,c,d,e,f,m,p,q},x] && EqQ[n2,2*n] && IntegerQ[Simplify[n/(m+1)]] && Not[IntegerQ[n]]
```

9: $\int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx$ when $b^2 - 4 a c \neq 0$

Derivation: Algebraic expansion

— Basis: If $r = \sqrt{b^2 - 4 a c}$, then $\frac{1}{a+b z+c z^2} = \frac{2 c}{r (b-r+2 c z)} - \frac{2 c}{r (b+r+2 c z)}$

Rule 1.2.3.4.9: If $b^2 - 4 a c \neq 0$, then

$$\int \frac{(f x)^m (d+e x^n)^q}{a+b x^n+c x^{2n}} dx \rightarrow \frac{2 c}{r} \int \frac{(f x)^m (d+e x^n)^q}{b-r+2 c x^n} dx - \frac{2 c}{r} \int \frac{(f x)^m (d+e x^n)^q}{b+r+2 c x^n} dx$$

Program code:

```
Int[(f_*x_)^m_*(d+_e_*x_^n_)^q_/(a+_b_*x_^n+_c_*x_^2n_),x_Symbol] :=
  With[{r=Rt[b^2-4*a*c,2]},
    2*c/r*Int[(f*x)^m*(d+e*x^n)^q/(b-r+2*c*x^n),x] - 2*c/r*Int[(f*x)^m*(d+e*x^n)^q/(b+r+2*c*x^n),x] /;
FreeQ[{a,b,c,d,e,f,m,n,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0]
```



```

Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_/(a_+c_.**x_^n2_),x_Symbol] :=
  With[{r=Rt[-a*c,2]},
    -c/(2*r)*Int[(f*x)^m*(d+e*x^n)^q/(r-c*x^n),x] - c/(2*r)*Int[(f*x)^m*(d+e*x^n)^q/(r+c*x^n),x] /;
  FreeQ[{a,c,d,e,f,m,n,q},x] && EqQ[n2,2*n]

```

10: $\int (f x)^m (d+e x^n) (a+b x^n+c x^{2n})^p dx$ when $b^2-4ac \neq 0 \wedge p+1 \in \mathbb{Z}^-$

Derivation: Trinomial recurrence 2b

Rule 1.2.3.4.10: If $b^2-4ac \neq 0 \wedge p+1 \in \mathbb{Z}^-$, then

$$\begin{aligned}
 & \int (f x)^m (d+e x^n) (a+b x^n+c x^{2n})^p dx \rightarrow \\
 & - \left((f x)^{m+1} (a+b x^n+c x^{2n})^{p+1} (d(b^2-2ac) - a b e + (b d - 2 a e) c x^n) / (a f n (p+1) (b^2-4ac)) \right) + \\
 & \quad \frac{1}{a n (p+1) (b^2-4ac)} \int (f x)^m (a+b x^n+c x^{2n})^{p+1} \cdot \\
 & \quad (d(b^2(m+n(p+1)+1) - 2ac(m+2n(p+1)+1)) - a b e(m+1) + (m+n(2p+3)+1)(b d - 2 a e) c x^n) dx
 \end{aligned}$$

Program code:

```

Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)*(a_+b_.**x_^n_+c_.**x_^n2_)^p_,x_Symbol] :=
  -(f*x)^(m+1)*(a+b*x^n+c*x^(2*n))^(p+1)*(d*(b^2-2*a*c)-a*b*e+(b*d-2*a*e)*c*x^n)/(a*f*n*(p+1)*(b^2-4*a*c)) +
  1/(a*n*(p+1)*(b^2-4*a*c))*Int[(f*x)^m*(a+b*x^n+c*x^(2*n))^(p+1)*
    Simp[d*(b^2*(m+n*(p+1)+1)-2*a*c*(m+2*n*(p+1)+1))-a*b*e*(m+1)+(m+n*(2*p+3)+1)*(b*d-2*a*e)*c*x^n,x],x] /;
  FreeQ[{a,b,c,d,e,f,m,n},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && ILtQ[p+1,0]

```

```

Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)*(a_+c_.**x_^n2_)^p_,x_Symbol] :=
  -(f*x)^(m+1)*(a+c*x^n)^(p+1)*(d+e*x^n)/(2*a*f*n*(p+1)) +
  1/(2*a*n*(p+1))*Int[(f*x)^m*(a+c*x^n)^(p+1)*Simp[d*(m+2*n*(p+1)+1)+e*(m+n*(2*p+3)+1)*x^n,x],x] /;
  FreeQ[{a,c,d,e,f,m,n},x] && EqQ[n2,2*n] && ILtQ[p+1,0]

```

11: $\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$ when $b^2 - 4 a c \neq 0 \wedge (p \in \mathbb{Z}^+ \vee q \in \mathbb{Z}^+)$

Derivation: Algebraic expansion

Rule 1.2.3.4.11: If $b^2 - 4 a c \neq 0 \wedge (p \in \mathbb{Z}^+ \vee q \in \mathbb{Z}^+)$, then

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \int \text{ExpandIntegrand}[(f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p, x] dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^n_)^q_.*(a_+b_.*x_^n_+c_.*x_^n2_)^p_,x_Symbol] :=
  Int[ExpandIntegrand[(f*x)^m*(d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x],x] /;
FreeQ[{a,b,c,d,e,f,m,n,p,q},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && (IGtQ[p,0] || IGtQ[q,0])
```

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^n_)^q_.*(a_+c_.*x_^n2_)^p_,x_Symbol] :=
  Int[ExpandIntegrand[(f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x],x] /;
FreeQ[{a,c,d,e,f,m,n,p,q},x] && EqQ[n2,2*n] && (IGtQ[p,0] || IGtQ[q,0])
```

12: $\int (f x)^m (d+e x^n)^q (a+c x^{2n})^p dx$ when $p \notin \mathbb{Z} \wedge q \in \mathbb{Z}^-$

Derivation: Algebraic expansion

Basis: If $q \in \mathbb{Z}$, then $(d+e x^n)^q = \left(\frac{d}{d^2-e^2 x^{2n}} - \frac{e x^n}{d^2-e^2 x^{2n}} \right)^{-q}$

Note: Resulting integrands are of the form $x^m (a+b x^{2n})^p (c+d x^{2n})^q$ which are integrable in terms of the Appell hypergeometric function .

Rule 1.2.3.4.12: If $p \notin \mathbb{Z} \wedge q \in \mathbb{Z}^-$, then

$$\int (f x)^m (d+e x^n)^q (a+c x^{2n})^p dx \rightarrow \frac{(f x)^m}{x^m} \int x^m (a+c x^{2n})^p \text{ExpandIntegrand} \left[\left(\frac{d}{d^2-e^2 x^{2n}} - \frac{e x^n}{d^2-e^2 x^{2n}} \right)^{-q}, x \right] dx$$

Program code:

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_.*(a_+c_.**x_^n2_)^p_,x_Symbol] :=
  (f**x)^m/x^m*Int[ExpandIntegrand[x^m*(a+c*x^(2*n))^p,(d/(d^2-e^2*x^(2*n))-e*x^n/(d^2-e^2*x^(2*n)))^(-q),x],x] /;
  FreeQ[{a,c,d,e,f,m,n,p},x] && EqQ[n2,2*n] && Not[IntegerQ[p]] && ILtQ[q,0]
```

U: $\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$

Rule 1.2.3.4.X:

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$$

Program code:

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_.*(a_+b_.**x_^n_+c_.**x_^n2_)^p_,x_Symbol] :=
  Unintegrable[(f**x)^m*(d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x] /;
  FreeQ[{a,b,c,d,e,f,m,n,p,q},x] && EqQ[n2,2*n]
```

```
Int[(f_.**x_)^m_.*(d_+e_.**x_^n_)^q_.*(a_+c_.**x_^n2_)^p_,x_Symbol] :=
  Unintegrable[(f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x] /;
FreeQ[{a,c,d,e,f,m,n,p,q},x] && EqQ[n2,2*n]
```

S: $\int u^m (d + e v^n)^q (a + b v^n + c v^{2n})^p dx$ when $v = f + g x \wedge u = h v$

Derivation: Integration by substitution and piecewise constant extraction

Basis: If $u = h v$, then $\partial_x \frac{u^m}{v^m} = 0$

Rule 1.2.3.4.S: If $v = f + g x \wedge u = h v$, then

$$\int u^m (d + e v^n)^q (a + b v^n + c v^{2n})^p dx \rightarrow \frac{u^m}{g v^m} \text{Subst}\left[\int x^m (d + e x^n)^q (a + b x^n + c x^{2n})^p dx, x, v\right]$$

Program code:

```
Int[u_^m_.*(d_+e_.**v_^n_)^q_.*(a_+b_.**v_^n_+c_.**v_^n2_)^p_,x_Symbol] :=
  u^m/(Coefficient[v,x,1]**v^m)*Subst[Int[x^m*(d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x],x,v] /;
FreeQ[{a,b,c,d,e,m,n,p,q},x] && EqQ[n2,2*n] && LinearPairQ[u,v,x] && NeQ[v,x]
```

```
Int[u_^m_.*(d_+e_.**v_^n_)^q_.*(a_+c_.**v_^n2_)^p_,x_Symbol] :=
  u^m/(Coefficient[v,x,1]**v^m)*Subst[Int[x^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x],x,v] /;
FreeQ[{a,c,d,e,m,n,p},x] && EqQ[n2,2*n] && LinearPairQ[u,v,x] && NeQ[v,x]
```

Rules for integrands of the form $(f x)^m (d+e x^n)^q (a+b x^n+c x^{2 n})^p$

1. $\int x^m (d+e x^{-n})^q (a+b x^n+c x^{2 n})^p dx$ when $p \in \mathbb{Z} \vee q \in \mathbb{Z}$

1: $\int x^m (d+e x^{-n})^q (a+b x^n+c x^{2 n})^p dx$ when $q \in \mathbb{Z} \wedge (n > 0 \vee p \notin \mathbb{Z})$

Derivation: Algebraic simplification

Basis: If $q \in \mathbb{Z}$, then $(d+e x^{-n})^q = x^{-n q} (e+d x^n)^q$

Rule: If $q \in \mathbb{Z} \wedge (n > 0 \vee p \notin \mathbb{Z})$, then

$$\int x^m (d+e x^{-n})^q (a+b x^n+c x^{2 n})^p dx \rightarrow \int x^{m-n q} (e+d x^n)^q (a+b x^n+c x^{2 n})^p dx$$

Program code:

```
Int[x_^m_.*(d_+e_.*x_^mn_.)^q_.*(a_+b_.*x_^n_+c_.*x_^n2_.)^p_,x_Symbol] :=
  Int[x^(m-n*q)*(e+d*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && EqQ[n2,2*n] && EqQ[mn,-n] && IntegerQ[q] && (PosQ[n] || Not[IntegerQ[p]])
```

```
Int[x_^m_.*(d_+e_.*x_^mn_.)^q_.*(a_+c_.*x_^n2_.)^p_,x_Symbol] :=
  Int[x^(m+mn*q)*(e+d*x^(-mn))^q*(a+c*x^n2)^p,x] /;
FreeQ[{a,c,d,e,m,mn,p},x] && EqQ[n2,-2*mn] && IntegerQ[q] && (PosQ[n2] || Not[IntegerQ[p]])
```

2: $\int x^m (d+e x^n)^q (a+b x^{-n}+c x^{-2 n})^p dx$ when $p \in \mathbb{Z}$

Derivation: Algebraic simplification

Basis: If $p \in \mathbb{Z}$, then $(a+b x^{-n}+c x^{-2 n})^p = x^{-2 n p} (c+b x^n+a x^{2 n})^p$

Rule: If $p \in \mathbb{Z}$, then

$$\int x^m (d+e x^n)^q (a+b x^{-n}+c x^{-2 n})^p dx \rightarrow \int x^{m-2 n p} (d+e x^n)^q (c+b x^n+a x^{2 n})^p dx$$

Program code:

```
Int[x^m_.*(d+_e_.*x^n_.)^q_.*(a+_b_.*x^mn_.+c_.*x^mn2_.)^p_,x_Symbol] :=
  Int[x^(m-2*n*p)*(d+e*x^n)^q*(c+b*x^n+a*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d,e,m,n,q},x] && EqQ[mn,-n] && EqQ[mn2,2*mn] && IntegerQ[p]
```

```
Int[x^m_.*(d+_e_.*x^n_.)^q_.*(a+_c_.*x^mn2_.)^p_,x_Symbol] :=
  Int[x^(m-2*n*p)*(d+e*x^n)^q*(c+a*x^(2*n))^p,x] /;
FreeQ[{a,c,d,e,m,n,q},x] && EqQ[mn2,-2*n] && IntegerQ[p]
```

2. $\int x^m (d+e x^{-n})^q (a+b x^n+c x^{2 n})^p dx$ when $p \notin \mathbb{Z} \wedge q \notin \mathbb{Z}$

1: $\int x^m (d+e x^{-n})^q (a+b x^n+c x^{2 n})^p dx$ when $p \notin \mathbb{Z} \wedge q \notin \mathbb{Z} \wedge n > 0$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{x^{n q} (d+e x^{-n})^q}{\left(1+\frac{d x^n}{e}\right)^q} = 0$

Rule: If $p \notin \mathbb{Z} \wedge q \notin \mathbb{Z} \wedge n > 0$, then

$$\int x^m (d+e x^{-n})^q (a+b x^n+c x^{2n})^p dx \rightarrow \frac{e^{\text{IntPart}[q]} x^{n \text{FracPart}[q]} (d+e x^{-n})^{\text{FracPart}[q]}}{\left(1+\frac{d x^n}{e}\right)^{\text{FracPart}[q]}} \int x^{m-n q} \left(1+\frac{d x^n}{e}\right)^q (a+b x^n+c x^{2n})^p dx$$

Program code:

```
Int[x_^m_.*(d+_e_.*x_^mn_.)^q_*(a+_b_.*x_^n_+_c_.*x_^n2_.)^p_,x_Symbol] :=
  e^IntPart[q]*x^(n*FracPart[q])*(d+e*x^(-n))^FracPart[q]/(1+d*x^n/e)^FracPart[q]*Int[x^(m-n*q)*(1+d*x^n/e)^q*(a+b*x^n+c*x^(2*n))^p,
  FreeQ[{a,b,c,d,e,m,n,p,q},x] && EqQ[n2,2*n] && EqQ[mn,-n] && Not[IntegerQ[p]] && Not[IntegerQ[q]] && PosQ[n]
```

```
Int[x_^m_.*(d+_e_.*x_^mn_.)^q_*(a+_c_.*x_^n2_.)^p_,x_Symbol] :=
  e^IntPart[q]*x^(-mn*FracPart[q])*(d+e*x^mn)^FracPart[q]/(1+d*x^(-mn)/e)^FracPart[q]*Int[x^(m+mn*q)*(1+d*x^(-mn)/e)^q*(a+c*x^n2)^p,
  FreeQ[{a,c,d,e,m,mn,p,q},x] && EqQ[n2,-2*mn] && Not[IntegerQ[p]] && Not[IntegerQ[q]] && PosQ[n2]
```

x: $\int x^m (d+e x^{-n})^q (a+b x^n+c x^{2n})^p dx$ when $p \notin \mathbb{Z} \wedge q \notin \mathbb{Z} \wedge n > 0$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{x^{n q} (d+e x^{-n})^q}{(e+d x^n)^q} == 0$

Rule: If $p \notin \mathbb{Z} \wedge q \notin \mathbb{Z} \wedge n > 0$, then

$$\int x^m (d+e x^{-n})^q (a+b x^n+c x^{2n})^p dx \rightarrow \frac{x^{n \text{FracPart}[q]} (d+e x^{-n})^{\text{FracPart}[q]}}{(e+d x^n)^{\text{FracPart}[q]}} \int x^{m-n q} (e+d x^n)^q (a+b x^n+c x^{2n})^p dx$$

Program code:

```
(* Int[x_^m_.*(d+_e_.*x_^mn_.)^q_*(a+_b_.*x_^n_+_c_.*x_^n2_.)^p_,x_Symbol] :=
  x^(n*FracPart[q])*(d+e*x^(-n))^FracPart[q]/(e+d*x^n)^FracPart[q]*Int[x^(m-n*q)*(e+d*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x] /;
  FreeQ[{a,b,c,d,e,m,n,p,q},x] && EqQ[n2,2*n] && EqQ[mn,-n] && Not[IntegerQ[p]] && Not[IntegerQ[q]] && PosQ[n] *)
```

```
(* Int[x_^m_.*(d+_e_.*x_^mn_.)^q_*(a+_c_.*x_^n2_.)^p_,x_Symbol] :=
  x^(-mn*FracPart[q])*(d+e*x^mn)^FracPart[q]/(e+d*x^(-mn))^FracPart[q]*Int[x^(m+mn*q)*(e+d*x^(-mn))^q*(a+c*x^n2)^p,x] /;
  FreeQ[{a,c,d,e,m,mn,p,q},x] && EqQ[n2,-2*mn] && Not[IntegerQ[p]] && Not[IntegerQ[q]] && PosQ[n2] *)
```

2: $\int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$ when $p \notin \mathbb{Z} \wedge q \notin \mathbb{Z} \wedge n > 0$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{x^{2n p} (a+b x^n+c x^{2n})^p}{(c+b x^n+a x^{2n})^p} = 0$

Rule: If $p \notin \mathbb{Z} \wedge q \notin \mathbb{Z} \wedge n > 0$, then

$$\int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \frac{x^{2n \text{FracPart}[p]} (a+b x^n+c x^{2n})^{\text{FracPart}[p]}}{(c+b x^n+a x^{2n})^{\text{FracPart}[p]}} \int x^{m-2n p} (d+e x^n)^q (c+b x^n+a x^{2n})^p dx$$

Program code:

```
Int[x_^m_.*(d_+e_.*x_^n_.)^q_.*(a_.+b_.*x_^mn_.+c_.*x_^mn2_.)^p_,x_Symbol] :=
  x^(2*n*FracPart[p])*(a+b*x^(-n)+c*x^(-2*n))^FracPart[p]/(c+b*x^n+a*x^(2*n))^FracPart[p]*
  Int[x^(m-2*n*p)*(d+e*x^n)^q*(c+b*x^n+a*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d,e,m,n,p,q},x] && EqQ[mn,-n] && EqQ[mn2,2*mn] && Not[IntegerQ[p]] && Not[IntegerQ[q]] && PosQ[n]
```

```
Int[x_^m_.*(d_+e_.*x_^n_.)^q_.*(a_.+c_.*x_^mn2_.)^p_,x_Symbol] :=
  x^(2*n*FracPart[p])*(a+c*x^(-2*n))^FracPart[p]/(c+a*x^(2*n))^FracPart[p]*
  Int[x^(m-2*n*p)*(d+e*x^n)^q*(c+a*x^(2*n))^p,x] /;
FreeQ[{a,c,d,e,m,n,p,q},x] && EqQ[mn2,-2*n] && Not[IntegerQ[p]] && Not[IntegerQ[q]] && PosQ[n]
```

3: $\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(f x)^m}{x^m} = 0$

Rule:

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \frac{f^{\text{IntPart}[m]} (f x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$$

Program code:

```
Int[(f*x_)^m_*(d+_e_.*x_^mn_)^q_.*(a+_b_.*x_^n_+_c_.*x_^n2_)^p_,x_Symbol] :=
  f^IntPart[m]*(f*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(d+e*x^mn)^q*(a+b*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d,e,f,m,n,p,q},x] && EqQ[n2,2*n] && EqQ[mn,-n]
```

```
Int[(f*x_)^m_*(d+_e_.*x_^mn_)^q_.*(a+_c_.*x_^n2_)^p_,x_Symbol] :=
  f^IntPart[m]*(f*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(d+e*x^mn)^q*(a+c*x^n2)^p,x] /;
FreeQ[{a,c,d,e,f,m,mn,p,q},x] && EqQ[n2,-2*mn]
```

Rules for integrands of the form $(f x)^m (d+e x^n)^q (a+b x^{-n}+c x^n)^p$

$$1. \int x^m (d+e x^n)^q (a+b x^{-n}+c x^n)^p dx$$

$$\mathbf{1:} \int x^m (d+e x^n)^q (a+b x^{-n}+c x^n)^p dx \text{ when } p \in \mathbb{Z}$$

Derivation: Algebraic normalization

$$\text{Basis: } a+b x^{-n}+c x^n = x^{-n} (b+a x^n+c x^{2n})$$

Rule 1.2.3.4.13.1.1: If $p \in \mathbb{Z}$, then

$$\int x^m (d+e x^n)^q (a+b x^{-n}+c x^n)^p dx \rightarrow \int x^{m-np} (d+e x^n)^q (b+a x^n+c x^{2n})^p dx$$

Program code:

```
Int[x_^m_*(d+_e_.*x_^n_)^q_.*(a+_b_.*x_^mn+_c_.*x_^n2_)^p_,x_Symbol] :=
  Int[x^(m-n*p)*(d+e*x^n)^q*(b+a*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d,e,m,n,q},x] && EqQ[mn,-n] && IntegerQ[p]
```

2: $\int x^m (d+e x^n)^q (a+b x^{-n}+c x^n)^p dx$ when $p \notin \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{x^n p (a+b x^{-n}+c x^n)^p}{(b+a x^n+c x^{2n})^p} == 0$

Basis: $\frac{x^n p (a+b x^{-n}+c x^n)^p}{(b+a x^n+c x^{2n})^p} == \frac{x^n \text{FracPart}[p] (a+b x^{-n}+c x^n)^{\text{FracPart}[p]}}{(b+a x^n+c x^{2n})^{\text{FracPart}[p]}}$

Rule 1.2.3.4.13.1.2: If $p \notin \mathbb{Z}$, then

$$\int x^m (d+e x^n)^q (a+b x^{-n}+c x^n)^p dx \rightarrow \frac{x^n \text{FracPart}[p] (a+b x^{-n}+c x^n)^{\text{FracPart}[p]}}{(b+a x^n+c x^{2n})^{\text{FracPart}[p]}} \int x^{m-n p} (d+e x^n)^q (b+a x^n+c x^{2n})^p dx$$

Program code:

```
Int[x_^m_.*(d_+e_.x_^n_)^q_.*(a_+b_.x_^mn_+c_.x_^n_)^p_,x_Symbol] :=
  x^(n*FracPart[p])*(a+b/x^n+c*x^n)^FracPart[p]/(b+a*x^n+c*x^(2*n))^FracPart[p]*
  Int[x^(m-n*p)*(d+e*x^n)^q*(b+a*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d,e,m,n,p,q},x] && EqQ[mn,-n] && Not[IntegerQ[p]]
```

2: $\int (f x)^m (d+e x^n)^q (a+b x^{-n}+c x^n)^p dx$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(f x)^m}{x^m} == 0$

Basis: $\frac{(f x)^m}{x^m} == \frac{f^{\text{IntPart}[m]} (f x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}}$

Rule 1.2.3.4.13.2:

$$\int (f x)^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx \rightarrow \frac{f^{\text{IntPart}[m]} (f x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (d+e x^n)^q (a+b x^n+c x^{2n})^p dx$$

Program code:

```
Int[(f*x_)^m_.*(d_+e_.*x_^n_)^q_.*(a_+b_.*x_^mn+c_.*x_^n_)^p_,x_Symbol] :=
  f^IntPart[m]*(f*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(d+e*x^n)^q*(a+b*x^(-n)+c*x^n)^p,x] /;
FreeQ[{a,b,c,d,e,f,m,n,p,q},x] && EqQ[mn,-n]
```

Rules for integrands of the form $(f x)^m (d_1 + e_1 x^{n/2})^q (d_2 + e_2 x^{n/2})^q (a + b x^n + c x^{2n})^p$

1. $\int (f x)^m (d_1 + e_1 x^{n/2})^q (d_2 + e_2 x^{n/2})^q (a + b x^n + c x^{2n})^p dx$ when $d_2 e_1 + d_1 e_2 == 0$

1: $\int (f x)^m (d_1 + e_1 x^{n/2})^q (d_2 + e_2 x^{n/2})^q (a + b x^n + c x^{2n})^p dx$ when $d_2 e_1 + d_1 e_2 == 0 \wedge (q \in \mathbb{Z} \vee d_1 > 0 \wedge d_2 > 0)$

Derivation: Algebraic simplification

Basis: If $d_2 e_1 + d_1 e_2 == 0 \wedge (q \in \mathbb{Z} \vee d_1 > 0 \wedge d_2 > 0)$, then $(d_1 + e_1 x^{n/2})^q (d_2 + e_2 x^{n/2})^q == (d_1 d_2 + e_1 e_2 x^n)^q$

Rule: If $d_2 e_1 + d_1 e_2 == 0 \wedge (q \in \mathbb{Z} \vee d_1 > 0 \wedge d_2 > 0)$, then

$$\int (f x)^m (d_1 + e_1 x^{n/2})^q (d_2 + e_2 x^{n/2})^q (a + b x^n + c x^{2n})^p dx \rightarrow \int (f x)^m (d_1 d_2 + e_1 e_2 x^n)^q (a + b x^n + c x^{2n})^p dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d1_+e1_.*x_^non2_)^q_.*(d2_+e2_.*x_^non2_)^q_.*(a_+b_.*x_^n+c_.*x_^n2_)^p_,x_Symbol] :=
  Int[(f*x)^m*(d1*d2+e1*e2*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,n,p,q},x] && EqQ[n2,2*n] && EqQ[non2,n/2] && EqQ[d2*e1+d1*e2,0] && (IntegerQ[q] || GtQ[d1,0] && GtQ[d2,0])
```

2: $\int (f x)^m (d_1 + e_1 x^{n/2})^q (d_2 + e_2 x^{n/2})^q (a + b x^n + c x^{2n})^p dx$ when $d_2 e_1 + d_1 e_2 == 0$

Derivation: Piecewise constant extraction

Basis: If $d_2 e_1 + d_1 e_2 == 0$, then $\partial_x \frac{(d_1 + e_1 x^{n/2})^q (d_2 + e_2 x^{n/2})^q}{(d_1 d_2 + e_1 e_2 x^n)^q} == 0$

Rule: If $d_2 e_1 + d_1 e_2 == 0$, then

$$\frac{\int (f x)^m (d_1 + e_1 x^{n/2})^q (d_2 + e_2 x^{n/2})^q (a + b x^n + c x^{2n})^p dx}{(d_1 d_2 + e_1 e_2 x^n)^{\text{FracPart}[q]}} \rightarrow \int (f x)^m (d_1 d_2 + e_1 e_2 x^n)^q (a + b x^n + c x^{2n})^p dx$$

Program code:

```
Int[(f_.**x_)^m_.*(d1_+e1_.*x_^non2_)^q_.*(d2_+e2_.*x_^non2_)^q_.*(a_.+b_.*x_^n_+c_.*x_^n2_)^p_,x_Symbol] :=
  (d1+e1*x^(n/2))^FracPart[q]*(d2+e2*x^(n/2))^FracPart[q]/(d1*d2+e1*e2*x^n)^FracPart[q]*
  Int[(f*x)^m*(d1*d2+e1*e2*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,n,p,q},x] && EqQ[n2,2*n] && EqQ[non2,n/2] && EqQ[d2*e1+d1*e2,0]
```