

Rules for integrands involving inverse sines and cosines

1. $\int u (a + b \operatorname{ArcSin}[c + d x])^n dx$

1: $\int (a + b \operatorname{ArcSin}[c + d x])^n dx$

Derivation: Integration by substitution

Rule:

$$\int (a + b \operatorname{ArcSin}[c + d x])^n dx \rightarrow \frac{1}{d} \operatorname{Subst}\left[\int (a + b \operatorname{ArcSin}[x])^n dx, x, c + d x\right]$$

Program code:

```
Int[(a_.+b_.*ArcSin[c_+d_.*x_])^n_,x_Symbol] :=  
  1/d*Subst[Int[(a+b*ArcSin[x])^n,x],x,c+d*x] /;  
FreeQ[{a,b,c,d,n},x]
```

```
Int[(a_.+b_.*ArcCos[c_+d_.*x_])^n_,x_Symbol] :=  
  1/d*Subst[Int[(a+b*ArcCos[x])^n,x],x,c+d*x] /;  
FreeQ[{a,b,c,d,n},x]
```

$$2: \int (e + f x)^m (a + b \operatorname{ArcSin}[c + d x])^n dx$$

Derivation: Integration by substitution

Rule:

$$\int (e + f x)^m (a + b \operatorname{ArcSin}[c + d x])^n dx \rightarrow \frac{1}{d} \operatorname{Subst}\left[\int \left(\frac{d e - c f}{d} + \frac{f x}{d}\right)^m (a + b \operatorname{ArcSin}[x])^n dx, x, c + d x\right]$$

Program code:

```
Int[(e_+f_.*x_)^m_.*(a_+b_.*ArcSin[c_+d_.*x_])^n_,x_Symbol] :=
  1/d*Subst[Int[((d*e-c*f)/d+f*x/d)^m*(a+b*ArcSin[x])^n,x],x,c+d*x] /;
FreeQ[{a,b,c,d,e,f,m,n},x]
```

```
Int[(e_+f_.*x_)^m_.*(a_+b_.*ArcCos[c_+d_.*x_])^n_,x_Symbol] :=
  1/d*Subst[Int[((d*e-c*f)/d+f*x/d)^m*(a+b*ArcCos[x])^n,x],x,c+d*x] /;
FreeQ[{a,b,c,d,e,f,m,n},x]
```

3: $\int (A + Bx + Cx^2)^p (a + b \operatorname{ArcSin}[c + dx])^n dx$ when $B(1 - c^2) + 2Ac d = 0 \wedge 2cC - Bd = 0$

Derivation: Integration by substitution

Basis: If $B(1 - c^2) + 2Ac d = 0 \wedge 2cC - Bd = 0$, then $A + Bx + Cx^2 = -\frac{C}{d^2} + \frac{C}{d^2}(c + dx)^2$

Rule: If $B(1 - c^2) + 2Ac d = 0 \wedge 2cC - Bd = 0$, then

$$\int (A + Bx + Cx^2)^p (a + b \operatorname{ArcSin}[c + dx])^n dx \rightarrow \frac{1}{d} \operatorname{Subst}\left[\int \left(-\frac{C}{d^2} + \frac{Cx^2}{d^2}\right)^p (a + b \operatorname{ArcSin}[x])^n dx, x, c + dx\right]$$

Program code:

```
Int[(A_+B_.*x_+C_.*x_^2)^p_.*(a_.+b_.*ArcSin[c_+d_.*x_])^n_.,x_Symbol] :=
  1/d*Subst[Int[(-C/d^2+C/d^2*x^2)^p*(a+b*ArcSin[x])^n,x],x,c+d*x] /;
FreeQ[{a,b,c,d,A,B,C,n,p},x] && EqQ[B*(1-c^2)+2*A*c*d,0] && EqQ[2*C*C-B*d,0]
```

```
Int[(A_+B_.*x_+C_.*x_^2)^p_.*(a_.+b_.*ArcCos[c_+d_.*x_])^n_.,x_Symbol] :=
  1/d*Subst[Int[(-C/d^2+C/d^2*x^2)^p*(a+b*ArcCos[x])^n,x],x,c+d*x] /;
FreeQ[{a,b,c,d,A,B,C,n,p},x] && EqQ[B*(1-c^2)+2*A*c*d,0] && EqQ[2*C*C-B*d,0]
```

$$\mathbf{4:} \int (e + f x)^m (A + B x + C x^2)^p (a + b \operatorname{ArcSin}[c + d x])^n dx \text{ when } B(1 - c^2) + 2 A c d = 0 \wedge 2 c C - B d = 0$$

Derivation: Integration by substitution

Basis: If $B(1 - c^2) + 2 A c d = 0 \wedge 2 c C - B d = 0$, then $A + B x + C x^2 = -\frac{C}{d^2} + \frac{C}{d^2} (c + d x)^2$

Rule: If $B(1 - c^2) + 2 A c d = 0 \wedge 2 c C - B d = 0$, then

$$\int (e + f x)^m (A + B x + C x^2)^p (a + b \operatorname{ArcSin}[c + d x])^n dx \rightarrow \frac{1}{d} \operatorname{Subst}\left[\int \left(\frac{d e - c f}{d} + \frac{f x}{d}\right)^m \left(-\frac{C}{d^2} + \frac{C x^2}{d^2}\right)^p (a + b \operatorname{ArcSin}[x])^n dx, x, c + d x\right]$$

Program code:

```
Int[(e_.+f_.**x_)^m_.*(A_.+B_.**x_+C_.**x_^2)^p_.*(a_.+b_.**ArcSin[c_+d_.**x_])^n_,x_Symbol] :=
  1/d*Subst[Int[((d*e-c*f)/d+f*x/d)^m*(-C/d^2+C/d^2*x^2)^p*(a+b*ArcSin[x])^n,x],x,c+d*x] /;
FreeQ[{a,b,c,d,e,f,A,B,C,m,n,p},x] && EqQ[B*(1-c^2)+2*A*c*d,0] && EqQ[2*c*C-B*d,0]
```

```
Int[(e_.+f_.**x_)^m_.*(A_.+B_.**x_+C_.**x_^2)^p_.*(a_.+b_.**ArcCos[c_+d_.**x_])^n_,x_Symbol] :=
  1/d*Subst[Int[((d*e-c*f)/d+f*x/d)^m*(-C/d^2+C/d^2*x^2)^p*(a+b*ArcCos[x])^n,x],x,c+d*x] /;
FreeQ[{a,b,c,d,e,f,A,B,C,m,n,p},x] && EqQ[B*(1-c^2)+2*A*c*d,0] && EqQ[2*c*C-B*d,0]
```

$$2. \int (a + b \operatorname{ArcSin}[c + d x^2])^n dx \text{ when } c^2 = 1$$

$$1. \int (a + b \operatorname{ArcSin}[c + d x^2])^n dx \text{ when } c^2 = 1 \wedge n > 0$$

$$1. \int \sqrt{a + b \operatorname{ArcSin}[c + d x^2]} dx \text{ when } c^2 = 1$$

$$\mathbf{1:} \int \sqrt{a + b \operatorname{ArcSin}[c + d x^2]} dx \text{ when } c^2 = 1$$

Derivation: Integration by parts

Rule: If $c^2 = 1$, then

$$\begin{aligned}
\int \sqrt{a + b \operatorname{ArcSin}[c + d x^2]} \, dx &\rightarrow x \sqrt{a + b \operatorname{ArcSin}[c + d x^2]} - b d \int \frac{x^2}{\sqrt{-2 c d x^2 - d^2 x^4} \sqrt{a + b \operatorname{ArcSin}[c + d x^2]}} \, dx \\
&\rightarrow x \sqrt{a + b \operatorname{ArcSin}[c + d x^2]} - \\
&\frac{\sqrt{\pi} x \left(\cos\left[\frac{a}{2b}\right] + c \sin\left[\frac{a}{2b}\right] \right) \operatorname{FresnelC}\left[\sqrt{\frac{c}{\pi b}} \sqrt{a + b \operatorname{ArcSin}[c + d x^2]}\right] + \sqrt{\pi} x \left(\cos\left[\frac{a}{2b}\right] - c \sin\left[\frac{a}{2b}\right] \right) \operatorname{FresnelS}\left[\sqrt{\frac{c}{\pi b}} \sqrt{a + b \operatorname{ArcSin}[c + d x^2]}\right]}{\sqrt{\frac{c}{b}} \left(\cos\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] - c \sin\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] \right)} + \frac{\sqrt{\pi} x \left(\cos\left[\frac{a}{2b}\right] - c \sin\left[\frac{a}{2b}\right] \right) \operatorname{FresnelS}\left[\sqrt{\frac{c}{\pi b}} \sqrt{a + b \operatorname{ArcSin}[c + d x^2]}\right]}{\sqrt{\frac{c}{b}} \left(\cos\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] - c \sin\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] \right)}
\end{aligned}$$

Program code:

```

Int[Sqrt[a_.+b_.*ArcSin[c+d_.*x^2]],x_Symbol] :=
  x*Sqrt[a+b*ArcSin[c+d*x^2]] -
  Sqrt[Pi]**x*(Cos[a/(2*b)]+c*Sin[a/(2*b)])*FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a+b*ArcSin[c+d*x^2]]]/
  (Sqrt[c/b]*(Cos[ArcSin[c+d*x^2]/2]-c*Sin[ArcSin[c+d*x^2]/2])) +
  Sqrt[Pi]**x*(Cos[a/(2*b)]-c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a+b*ArcSin[c+d*x^2]]]/
  (Sqrt[c/b]*(Cos[ArcSin[c+d*x^2]/2]-c*Sin[ArcSin[c+d*x^2]/2])) /;
FreeQ[{a,b,c,d},x] && EqQ[c^2,1]

```

2. $\int \sqrt{a + b \operatorname{ArcCos}[c + d x^2]} \, dx$ when $c^2 = 1$

1: $\int \sqrt{a + b \operatorname{ArcCos}[1 + d x^2]} \, dx$

Rule:

$$\begin{aligned}
&\int \sqrt{a + b \operatorname{ArcCos}[1 + d x^2]} \, dx \rightarrow \\
&- \frac{2 \sqrt{a + b \operatorname{ArcCos}[1 + d x^2]} \sin\left[\frac{1}{2} \operatorname{ArcCos}[1 + d x^2]\right]^2}{d x} + \\
&- \frac{1}{\sqrt{\frac{1}{b}} d x} 2 \sqrt{\pi} \sin\left[\frac{a}{2b}\right] \sin\left[\frac{1}{2} \operatorname{ArcCos}[1 + d x^2]\right] \operatorname{FresnelC}\left[\sqrt{\frac{1}{\pi b}} \sqrt{a + b \operatorname{ArcCos}[1 + d x^2]}\right] +
\end{aligned}$$

$$\frac{1}{\sqrt{\frac{1}{b} dx}} 2 \sqrt{\pi} \cos\left[\frac{a}{2b}\right] \sin\left[\frac{1}{2} \arccos[1 + dx^2]\right] \text{FresnelS}\left[\sqrt{\frac{1}{\pi b}} \sqrt{a + b \arccos[1 + dx^2]}\right]$$

Program code:

```
Int[Sqrt[a_+b_.*ArcCos[1+d_.*x_^2]],x_Symbol] :=
-2*Sqrt[a+b*ArcCos[1+d*x^2]]*Sin[ArcCos[1+d*x^2]/2]^2/(d*x) -
2*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1+d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sqrt[a+b*ArcCos[1+d*x^2]]]/(Sqrt[1/b]*d*x) +
2*Sqrt[Pi]*Cos[a/(2*b)]*Sin[ArcCos[1+d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a+b*ArcCos[1+d*x^2]]]/(Sqrt[1/b]*d*x) /;
FreeQ[{a,b,d},x]
```

$$2: \int \sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]} \, dx$$

Rule:

$$\int \sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]} \, dx \rightarrow$$

$$\frac{2 \sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]} \operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcCos}[-1 + d x^2]\right]^2}{d x} -$$

$$\frac{1}{\sqrt{\frac{1}{b} d x}} 2 \sqrt{\pi} \operatorname{Cos}\left[\frac{a}{2 b}\right] \operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcCos}[-1 + d x^2]\right] \operatorname{FresnelC}\left[\sqrt{\frac{1}{\pi b}} \sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]}\right] -$$

$$\frac{1}{\sqrt{\frac{1}{b} d x}} 2 \sqrt{\pi} \operatorname{Sin}\left[\frac{a}{2 b}\right] \operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcCos}[-1 + d x^2]\right] \operatorname{FresnelS}\left[\sqrt{\frac{1}{\pi b}} \sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]}\right]$$

Program code:

```
Int[Sqrt[a_.+b_.*ArcCos[-1+d_.*x^2]],x_Symbol] :=
  2*Sqrt[a+b*ArcCos[-1+d*x^2]]*Cos[(1/2)*ArcCos[-1+d*x^2]]^2/(d*x) -
  2*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1+d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sqrt[a+b*ArcCos[-1+d*x^2]]]/(Sqrt[1/b]*d*x) -
  2*Sqrt[Pi]*Sin[a/(2*b)]*Cos[ArcCos[-1+d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a+b*ArcCos[-1+d*x^2]]]/(Sqrt[1/b]*d*x) /;
FreeQ[{a,b,d},x]
```

$$2: \int (a + b \operatorname{ArcSin}[c + d x^2])^n \, dx \text{ when } c^2 = 1 \wedge n > 1$$

Derivation: Integration by parts twice

$$\text{Basis: If } c^2 = 1, \text{ then } \partial_x (a + b \operatorname{ArcSin}[c + d x^2])^n = \frac{2 b d n x (a + b \operatorname{ArcSin}[c + d x^2])^{n-1}}{\sqrt{-2 c d x^2 - d^2 x^4}}$$

Basis: $\frac{x^2}{\sqrt{-d x^2 (2 c + d x^2)}} = -\partial_x \frac{\sqrt{-2 c d x^2 - d^2 x^4}}{d^2 x}$

Rule: If $c^2 = 1 \wedge n > 1$, then

$$\int (a + b \operatorname{ArcSin}[c + d x^2])^n dx \rightarrow x (a + b \operatorname{ArcSin}[c + d x^2])^n - 2 b d n \int \frac{x^2 (a + b \operatorname{ArcSin}[c + d x^2])^{n-1}}{\sqrt{-2 c d x^2 - d^2 x^4}} dx$$

$$\rightarrow x (a + b \operatorname{ArcSin}[c + d x^2])^n + \frac{2 b n \sqrt{-2 c d x^2 - d^2 x^4} (a + b \operatorname{ArcSin}[c + d x^2])^{n-1}}{d x} - 4 b^2 n (n-1) \int (a + b \operatorname{ArcSin}[c + d x^2])^{n-2} dx$$

Program code:

```
Int[(a_.+b_.*ArcSin[c_+d_.x_^2])^n_,x_Symbol] :=
  x*(a+b*ArcSin[c+d*x^2])^n +
  2*b*n*Sqrt[-2*c*d*x^2-d^2*x^4]*(a+b*ArcSin[c+d*x^2])^(n-1)/(d*x) -
  4*b^2*n*(n-1)*Int[(a+b*ArcSin[c+d*x^2])^(n-2),x] /;
FreeQ[{a,b,c,d},x] && EqQ[c^2,1] && GtQ[n,1]
```

```
Int[(a_.+b_.*ArcCos[c_+d_.x_^2])^n_,x_Symbol] :=
  x*(a+b*ArcCos[c+d*x^2])^n -
  2*b*n*Sqrt[-2*c*d*x^2-d^2*x^4]*(a+b*ArcCos[c+d*x^2])^(n-1)/(d*x) -
  4*b^2*n*(n-1)*Int[(a+b*ArcCos[c+d*x^2])^(n-2),x] /;
FreeQ[{a,b,c,d},x] && EqQ[c^2,1] && GtQ[n,1]
```

2. $\int (a + b \operatorname{ArcSin}[c + d x^2])^n dx$ when $c^2 = 1 \wedge n < 0$

1. $\int \frac{1}{a + b \operatorname{ArcSin}[c + d x^2]} dx$ when $c^2 = 1$

1: $\int \frac{1}{a + b \operatorname{ArcSin}[c + d x^2]} dx$ when $c^2 = 1$

Rule: If $c^2 = 1$, then

$$\int \frac{1}{a + b \operatorname{ArcSin}[c + d x^2]} dx \rightarrow$$

$$-\frac{x \left(c \cos\left[\frac{a}{2b}\right] - \sin\left[\frac{a}{2b}\right] \right) \operatorname{CosIntegral}\left[\frac{c}{2b} (a + b \operatorname{ArcSin}[c + d x^2])\right]}{2b \left(\cos\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] - c \sin\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] \right)} - \frac{x \left(c \cos\left[\frac{a}{2b}\right] + \sin\left[\frac{a}{2b}\right] \right) \operatorname{SinIntegral}\left[\frac{c}{2b} (a + b \operatorname{ArcSin}[c + d x^2])\right]}{2b \left(\cos\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] - c \sin\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] \right)}$$

Program code:

```
Int[1/(a_.+b_.*ArcSin[c_+d_.*x_^2]),x_Symbol] :=
  -x*(c*Cos[a/(2*b)]-Sin[a/(2*b)])*CosIntegral[(c/(2*b))*(a+b*ArcSin[c+d*x^2])]/
    (2*b*(Cos[ArcSin[c+d*x^2]/2]-c*Sin[ArcSin[c+d*x^2]/2])) -
  x*(c*Cos[a/(2*b)]+Sin[a/(2*b)])*SinIntegral[(c/(2*b))*(a+b*ArcSin[c+d*x^2])]/
    (2*b*(Cos[ArcSin[c+d*x^2]/2]-c*Sin[ArcSin[c+d*x^2]/2])) /;
FreeQ[{a,b,c,d},x] && EqQ[c^2,1]
```

$$2. \int \frac{1}{a + b \operatorname{ArcCos}[c + d x^2]} dx \text{ when } c^2 = 1$$

$$1: \int \frac{1}{a + b \operatorname{ArcCos}[1 + d x^2]} dx$$

Rule:

$$\int \frac{1}{a + b \operatorname{ArcCos}[1 + d x^2]} dx \rightarrow$$

$$\frac{x \cos\left[\frac{a}{2b}\right] \operatorname{CosIntegral}\left[\frac{1}{2b} (a + b \operatorname{ArcCos}[1 + d x^2])\right]}{\sqrt{2} b \sqrt{-d x^2}} + \frac{x \sin\left[\frac{a}{2b}\right] \operatorname{SinIntegral}\left[\frac{1}{2b} (a + b \operatorname{ArcCos}[1 + d x^2])\right]}{\sqrt{2} b \sqrt{-d x^2}}$$

Program code:

```
Int[1/(a_.+b_.*ArcCos[1+d_.*x_^2]),x_Symbol] :=
  x*Cos[a/(2*b)]*CosIntegral[(a+b*ArcCos[1+d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[-d*x^2]) +
  x*Sin[a/(2*b)]*SinIntegral[(a+b*ArcCos[1+d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[-d*x^2]) /;
FreeQ[{a,b,d},x]
```

2: $\int \frac{1}{a + b \operatorname{ArcCos}[-1 + d x^2]} dx$

Rule:

$$\int \frac{1}{a + b \operatorname{ArcCos}[-1 + d x^2]} dx \rightarrow \frac{x \operatorname{Sin}\left[\frac{a}{2b}\right] \operatorname{CosIntegral}\left[\frac{1}{2b} (a + b \operatorname{ArcCos}[-1 + d x^2])\right]}{\sqrt{2} b \sqrt{d x^2}} - \frac{x \operatorname{Cos}\left[\frac{a}{2b}\right] \operatorname{SinIntegral}\left[\frac{1}{2b} (a + b \operatorname{ArcCos}[-1 + d x^2])\right]}{\sqrt{2} b \sqrt{d x^2}}$$

Program code:

```
Int[1/(a_+b_*ArcCos[-1+d_*x_^2]),x_Symbol] :=
  x*Sin[a/(2*b)]*CosIntegral[(a+b*ArcCos[-1+d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[d*x^2]) -
  x*Cos[a/(2*b)]*SinIntegral[(a+b*ArcCos[-1+d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[d*x^2]) /;
FreeQ[{a,b,d},x]
```

$$2. \int \frac{1}{\sqrt{a + b \operatorname{ArcSin}[c + d x^2]}} dx \text{ when } c^2 = 1$$

$$1: \int \frac{1}{\sqrt{a + b \operatorname{ArcSin}[c + d x^2]}} dx \text{ when } c^2 = 1$$

Rule: If $c^2 = 1$, then

$$\int \frac{1}{\sqrt{a + b \operatorname{ArcSin}[c + d x^2]}} dx \rightarrow$$

$$- \left(\left(\sqrt{\pi} x \left(\cos\left[\frac{a}{2b}\right] - c \sin\left[\frac{a}{2b}\right] \right) \operatorname{FresnelC}\left[\frac{1}{\sqrt{bc}} \sqrt{\pi} \sqrt{a + b \operatorname{ArcSin}[c + d x^2]}\right] \right) / \left(\sqrt{bc} \left(\cos\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] - c \sin\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] \right) \right) \right) -$$

$$\left(\sqrt{\pi} x \left(\cos\left[\frac{a}{2b}\right] + c \sin\left[\frac{a}{2b}\right] \right) \operatorname{FresnelS}\left[\frac{1}{\sqrt{bc}} \sqrt{\pi} \sqrt{a + b \operatorname{ArcSin}[c + d x^2]}\right] \right) / \left(\sqrt{bc} \left(\cos\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] - c \sin\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] \right) \right)$$

Program code:

```
Int[1/Sqrt[a_.+b_.*ArcSin[c_.+d_.x^2]],x_Symbol] :=
  -Sqrt[Pi]**(Cos[a/(2*b)]-c*Sin[a/(2*b)])*FresnelC[1/(Sqrt[b*c])*Sqrt[Pi])*Sqrt[a+b*ArcSin[c+d*x^2]]]/
  (Sqrt[b*c]*(Cos[ArcSin[c+d*x^2]/2]-c*Sin[ArcSin[c+d*x^2]/2])) -
  Sqrt[Pi]**(Cos[a/(2*b)]+c*Sin[a/(2*b)])*FresnelS[(1/(Sqrt[b*c])*Sqrt[Pi])*Sqrt[a+b*ArcSin[c+d*x^2]]]/
  (Sqrt[b*c]*(Cos[ArcSin[c+d*x^2]/2]-c*Sin[ArcSin[c+d*x^2]/2])) /;
```

FreeQ[{a,b,c,d},x] && EqQ[c^2,1]

$$2. \int \frac{1}{\sqrt{a + b \operatorname{ArcCos}[c + d x^2]}} dx \text{ when } c^2 = 1$$

$$1: \int \frac{1}{\sqrt{a + b \operatorname{ArcCos}[1 + d x^2]}} dx$$

Rule:

$$\int \frac{1}{\sqrt{a + b \operatorname{ArcCos}[1 + d x^2]}} dx \rightarrow$$

$$-\frac{1}{d x} 2 \sqrt{\frac{\pi}{b}} \cos\left[\frac{a}{2 b}\right] \sin\left[\frac{1}{2} \operatorname{ArcCos}[1 + d x^2]\right] \operatorname{FresnelC}\left[\sqrt{\frac{1}{\pi b}} \sqrt{a + b \operatorname{ArcCos}[1 + d x^2]}\right] -$$

$$\frac{1}{d x} 2 \sqrt{\frac{\pi}{b}} \sin\left[\frac{a}{2 b}\right] \sin\left[\frac{1}{2} \operatorname{ArcCos}[1 + d x^2]\right] \operatorname{FresnelS}\left[\sqrt{\frac{1}{\pi b}} \sqrt{a + b \operatorname{ArcCos}[1 + d x^2]}\right]$$

Program code:

```
Int[1/Sqrt[a_.+b_.*ArcCos[1+d_.*x_^2]],x_Symbol] :=
  -2*Sqrt[Pi/b]*Cos[a/(2*b)]*Sin[ArcCos[1+d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sqrt[a+b*ArcCos[1+d*x^2]]]/(d*x) -
  2*Sqrt[Pi/b]*Sin[a/(2*b)]*Sin[ArcCos[1+d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a+b*ArcCos[1+d*x^2]]]/(d*x) /;
FreeQ[{a,b,d},x]
```

2: $\int \frac{1}{\sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]}} dx$

Rule:

$$\int \frac{1}{\sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]}} dx \rightarrow$$

$$\frac{1}{d x} 2 \sqrt{\frac{\pi}{b}} \sin\left[\frac{a}{2 b}\right] \cos\left[\frac{1}{2} \operatorname{ArcCos}[-1 + d x^2]\right] \operatorname{FresnelC}\left[\sqrt{\frac{1}{\pi b}} \sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]}\right] -$$

$$\frac{1}{d x} 2 \sqrt{\frac{\pi}{b}} \cos\left[\frac{a}{2 b}\right] \cos\left[\frac{1}{2} \operatorname{ArcCos}[-1 + d x^2]\right] \operatorname{FresnelS}\left[\sqrt{\frac{1}{\pi b}} \sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]}\right]$$

Program code:

```
Int[1/Sqrt[a_.+b_.*ArcCos[-1+d_.*x_^2]],x_Symbol] :=
  2*Sqrt[Pi/b]*Sin[a/(2*b)]*Cos[ArcCos[-1+d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sqrt[a+b*ArcCos[-1+d*x^2]]]/(d*x) -
  2*Sqrt[Pi/b]*Cos[a/(2*b)]*Cos[ArcCos[-1+d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a+b*ArcCos[-1+d*x^2]]]/(d*x) /;
FreeQ[{a,b,d},x]
```

$$3. \int (a + b \operatorname{ArcSin}[c + d x^2])^n dx \text{ when } c^2 = 1 \wedge n < -1$$

$$1. \int \frac{1}{(a + b \operatorname{ArcSin}[c + d x^2])^{3/2}} dx \text{ when } c^2 = 1$$

$$1: \int \frac{1}{(a + b \operatorname{ArcSin}[c + d x^2])^{3/2}} dx \text{ when } c^2 = 1$$

Derivation: Integration by parts

$$\text{Basis: If } c^2 = 1, \text{ then } -\frac{b dx}{\sqrt{-2cdx^2 - d^2x^4} (a + b \operatorname{ArcSin}[c + dx^2])^{3/2}} = \partial_x \frac{1}{\sqrt{a + b \operatorname{ArcSin}[c + dx^2]}}$$

Rule: If $c^2 = 1$, then

$$\begin{aligned} \int \frac{1}{(a + b \operatorname{ArcSin}[c + d x^2])^{3/2}} dx &\rightarrow -\frac{\sqrt{-2cdx^2 - d^2x^4}}{b dx \sqrt{a + b \operatorname{ArcSin}[c + d x^2]}} - \frac{d}{b} \int \frac{x^2}{\sqrt{-2cdx^2 - d^2x^4} \sqrt{a + b \operatorname{ArcSin}[c + d x^2]}} dx \\ &\rightarrow -\frac{\sqrt{-2cdx^2 - d^2x^4}}{b dx \sqrt{a + b \operatorname{ArcSin}[c + d x^2]}} - \\ &\left(\left(\frac{c}{b} \right)^{3/2} \sqrt{\pi} x \left(\cos\left[\frac{a}{2b}\right] + c \sin\left[\frac{a}{2b}\right] \right) \operatorname{FresnelC}\left[\sqrt{\frac{c}{\pi b}} \sqrt{a + b \operatorname{ArcSin}[c + d x^2]}\right] \right) / \left(\cos\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] - c \sin\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] \right) + \\ &\left(\left(\frac{c}{b} \right)^{3/2} \sqrt{\pi} x \left(\cos\left[\frac{a}{2b}\right] - c \sin\left[\frac{a}{2b}\right] \right) \operatorname{FresnelS}\left[\sqrt{\frac{c}{\pi b}} \sqrt{a + b \operatorname{ArcSin}[c + d x^2]}\right] \right) / \left(\cos\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] - c \sin\left[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]\right] \right) \end{aligned}$$

Program code:

```
Int[1/(a_+b_.*ArcSin[c_+d_.*x^2])^(3/2),x_Symbol] :=
  -Sqrt[-2*c*d*x^2-d^2*x^4]/(b*d*x*Sqrt[a+b*ArcSin[c+d*x^2]]) -
  (c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)]+c*Sin[a/(2*b)])*FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a+b*ArcSin[c+d*x^2]]]/
  (Cos[(1/2)*ArcSin[c+d*x^2]]-c*Sin[ArcSin[c+d*x^2]/2]) +
  (c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)]-c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a+b*ArcSin[c+d*x^2]]]/
  (Cos[(1/2)*ArcSin[c+d*x^2]]-c*Sin[ArcSin[c+d*x^2]/2]) /;
FreeQ[{a,b,c,d},x] && EqQ[c^2,1]
```

$$2. \int \frac{1}{(a + b \operatorname{ArcCos}[c + d x^2])^{3/2}} dx \text{ when } c^2 = 1$$

$$\text{1: } \int \frac{1}{(a + b \operatorname{ArcCos}[1 + d x^2])^{3/2}} dx$$

Derivation: Integration by parts

$$\text{Basis: } \frac{b dx}{\sqrt{-2 d x^2 - d^2 x^4} (a + b \operatorname{ArcCos}[1 + d x^2])^{3/2}} = \partial_x \frac{1}{\sqrt{a + b \operatorname{ArcCos}[1 + d x^2]}}$$

Rule:

$$\begin{aligned} \int \frac{1}{(a + b \operatorname{ArcCos}[1 + d x^2])^{3/2}} dx &\rightarrow \frac{\sqrt{-2 d x^2 - d^2 x^4}}{b d x \sqrt{a + b \operatorname{ArcCos}[1 + d x^2]}} + \frac{d}{b} \int \frac{x^2}{\sqrt{-2 d x^2 - d^2 x^4} \sqrt{a + b \operatorname{ArcCos}[1 + d x^2]}} dx \\ &\rightarrow \frac{\sqrt{-2 d x^2 - d^2 x^4}}{b d x \sqrt{a + b \operatorname{ArcCos}[1 + d x^2]}} - \\ &\quad \frac{1}{d x} 2 \left(\frac{1}{b} \right)^{3/2} \sqrt{\pi} \sin\left[\frac{a}{2b}\right] \sin\left[\frac{1}{2} \operatorname{ArcCos}[1 + d x^2]\right] \operatorname{FresnelC}\left[\sqrt{\frac{1}{\pi b}} \sqrt{a + b \operatorname{ArcCos}[1 + d x^2]}\right] + \\ &\quad \frac{1}{d x} 2 \left(\frac{1}{b} \right)^{3/2} \sqrt{\pi} \cos\left[\frac{a}{2b}\right] \sin\left[\frac{1}{2} \operatorname{ArcCos}[1 + d x^2]\right] \operatorname{FresnelS}\left[\sqrt{\frac{1}{\pi b}} \sqrt{a + b \operatorname{ArcCos}[1 + d x^2]}\right] \end{aligned}$$

Program code:

```
Int[1/(a_+b_.*ArcCos[1+d_.*x^2])^(3/2),x_Symbol] :=
  Sqrt[-2*d*x^2-d^2*x^4]/(b*d*x*Sqrt[a+b*ArcCos[1+d*x^2]]) -
  2*(1/b)^(3/2)*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1+d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sqrt[a+b*ArcCos[1+d*x^2]]]/(d*x) +
  2*(1/b)^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*Sin[ArcCos[1+d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a+b*ArcCos[1+d*x^2]]]/(d*x) /;
FreeQ[{a,b,d},x]
```

$$\mathbf{2:} \int \frac{1}{(a + b \operatorname{ArcCos}[-1 + d x^2])^{3/2}} dx$$

Derivation: Integration by parts

$$\text{Basis: } \frac{b dx}{\sqrt{2 d x^2 - d^2 x^4} (a + b \operatorname{ArcCos}[-1 + d x^2])^{3/2}} = \partial_x \frac{1}{\sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]}}$$

Rule:

$$\begin{aligned} \int \frac{1}{(a + b \operatorname{ArcCos}[-1 + d x^2])^{3/2}} dx &\rightarrow \frac{\sqrt{2 d x^2 - d^2 x^4}}{b d x \sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]}} + \frac{d}{b} \int \frac{x^2}{\sqrt{2 d x^2 - d^2 x^4} \sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]}} dx \\ &\rightarrow \frac{\sqrt{2 d x^2 - d^2 x^4}}{b d x \sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]}} - \\ &\quad \frac{1}{d x} 2 \left(\frac{1}{b} \right)^{3/2} \sqrt{\pi} \cos\left[\frac{a}{2b}\right] \cos\left[\frac{1}{2} \operatorname{ArcCos}[-1 + d x^2]\right] \operatorname{FresnelC}\left[\sqrt{\frac{1}{\pi b}} \sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]}\right] - \\ &\quad \frac{1}{d x} 2 \left(\frac{1}{b} \right)^{3/2} \sqrt{\pi} \sin\left[\frac{a}{2b}\right] \cos\left[\frac{1}{2} \operatorname{ArcCos}[-1 + d x^2]\right] \operatorname{FresnelS}\left[\sqrt{\frac{1}{\pi b}} \sqrt{a + b \operatorname{ArcCos}[-1 + d x^2]}\right] \end{aligned}$$

Program code:

```
Int[1/(a_.+b_.*ArcCos[-1+d_.*x_^2])^(3/2),x_Symbol] :=
  Sqrt[2*d*x^2-d^2*x^4]/(b*d*x*Sqrt[a+b*ArcCos[-1+d*x^2]]) -
  2*(1/b)^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1+d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sqrt[a+b*ArcCos[-1+d*x^2]]]/(d*x) -
  2*(1/b)^(3/2)*Sqrt[Pi]*Sin[a/(2*b)]*Cos[ArcCos[-1+d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a+b*ArcCos[-1+d*x^2]]]/(d*x) /;
FreeQ[{a,b,d},x]
```

$$2. \int \frac{1}{(a + b \operatorname{ArcSin}[c + d x^2])^2} dx \text{ when } c^2 = 1$$

$$1: \int \frac{1}{(a + b \operatorname{ArcSin}[c + d x^2])^2} dx \text{ when } c^2 = 1$$

Derivation: Integration by parts

$$\text{Basis: If } c^2 = 1, \text{ then } -\frac{2 b d x}{\sqrt{-2 c d x^2 - d^2 x^4} (a + b \operatorname{ArcSin}[c + d x^2])^2} = \partial_x \frac{1}{a + b \operatorname{ArcSin}[c + d x^2]}$$

Rule: If $c^2 = 1$, then

$$\begin{aligned} \int \frac{1}{(a + b \operatorname{ArcSin}[c + d x^2])^2} dx &\rightarrow -\frac{\sqrt{-2 c d x^2 - d^2 x^4}}{2 b d x (a + b \operatorname{ArcSin}[c + d x^2])} - \frac{d}{2 b} \int \frac{x^2}{\sqrt{-2 c d x^2 - d^2 x^4} (a + b \operatorname{ArcSin}[c + d x^2])} dx \\ &\rightarrow -\frac{\sqrt{-2 c d x^2 - d^2 x^4}}{2 b d x (a + b \operatorname{ArcSin}[c + d x^2])} - \frac{x (\cos[\frac{a}{2b}] + c \sin[\frac{a}{2b}]) \operatorname{CosIntegral}[\frac{c}{2b} (a + b \operatorname{ArcSin}[c + d x^2])] }{4 b^2 (\cos[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]] - c \sin[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]])} + \\ &\quad \frac{x (\cos[\frac{a}{2b}] - c \sin[\frac{a}{2b}]) \operatorname{SinIntegral}[\frac{c}{2b} (a + b \operatorname{ArcSin}[c + d x^2])] }{4 b^2 (\cos[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]] - c \sin[\frac{1}{2} \operatorname{ArcSin}[c + d x^2]])} \end{aligned}$$

Program code:

```
Int[1/(a_+b_.*ArcSin[c_+d_.*x^2])^2,x_Symbol] :=
  -Sqrt[-2*c*d*x^2-d^2*x^4]/(2*b*d*x*(a+b*ArcSin[c+d*x^2])) -
  x*(Cos[a/(2*b)]+c*Sin[a/(2*b)])*CosIntegral[(c/(2*b))*(a+b*ArcSin[c+d*x^2])]/
  (4*b^2*(Cos[ArcSin[c+d*x^2]/2]-c*Sin[ArcSin[c+d*x^2]/2])) +
  x*(Cos[a/(2*b)]-c*Sin[a/(2*b)])*SinIntegral[(c/(2*b))*(a+b*ArcSin[c+d*x^2])]/
  (4*b^2*(Cos[ArcSin[c+d*x^2]/2]-c*Sin[ArcSin[c+d*x^2]/2])) /;
FreeQ[{a,b,c,d},x] && EqQ[c^2,1]
```


$$2. \int \frac{1}{(a + b \operatorname{ArcCos}[c + d x^2])^2} dx \text{ when } c^2 = 1$$

$$1: \int \frac{1}{(a + b \operatorname{ArcCos}[1 + d x^2])^2} dx$$

Rule:

$$\int \frac{1}{(a + b \operatorname{ArcCos}[1 + d x^2])^2} dx \rightarrow$$

$$\frac{\sqrt{-2 d x^2 - d^2 x^4}}{2 b d x (a + b \operatorname{ArcCos}[1 + d x^2])} + \frac{x \sin\left[\frac{a}{2b}\right] \operatorname{CosIntegral}\left[\frac{1}{2b} (a + b \operatorname{ArcCos}[1 + d x^2])\right]}{2 \sqrt{2} b^2 \sqrt{-d x^2}} - \frac{x \cos\left[\frac{a}{2b}\right] \operatorname{SinIntegral}\left[\frac{1}{2b} (a + b \operatorname{ArcCos}[1 + d x^2])\right]}{2 \sqrt{2} b^2 \sqrt{-d x^2}}$$

Program code:

```
Int[1/(a_.*b_.*ArcCos[1+d_.*x_^2])^2,x_Symbol] :=
  Sqrt[-2*d*x^2-d^2*x^4]/(2*b*d*x*(a+b*ArcCos[1+d*x^2])) +
  x*Sin[a/(2*b)]*CosIntegral[(a+b*ArcCos[1+d*x^2])/(2*b)]/(2*Sqrt[2]*b^2*Sqrt[(-d)*x^2]) -
  x*Cos[a/(2*b)]*SinIntegral[(a+b*ArcCos[1+d*x^2])/(2*b)]/(2*Sqrt[2]*b^2*Sqrt[(-d)*x^2]) /;
FreeQ[{a,b,d},x]
```

$$2: \int \frac{1}{(a + b \operatorname{ArcCos}[-1 + d x^2])^2} dx$$

Rule:

$$\int \frac{1}{(a + b \operatorname{ArcCos}[-1 + d x^2])^2} dx \rightarrow$$

$$\frac{\sqrt{2 d x^2 - d^2 x^4}}{2 b d x (a + b \operatorname{ArcCos}[-1 + d x^2])} - \frac{x \cos\left[\frac{a}{2b}\right] \operatorname{CosIntegral}\left[\frac{1}{2b} (a + b \operatorname{ArcCos}[-1 + d x^2])\right]}{2 \sqrt{2} b^2 \sqrt{d x^2}} - \frac{x \sin\left[\frac{a}{2b}\right] \operatorname{SinIntegral}\left[\frac{1}{2b} (a + b \operatorname{ArcCos}[-1 + d x^2])\right]}{2 \sqrt{2} b^2 \sqrt{d x^2}}$$

Program code:

```
Int[1/(a_.+b_.*ArcCos[-1+d_.*x_^2])^2,x_Symbol] :=
  Sqrt[2*d*x^2-d^2*x^4]/(2*b*d*x*(a+b*ArcCos[-1+d*x^2])) -
  x*Cos[a/(2*b)]*CosIntegral[(a+b*ArcCos[-1+d*x^2])/(2*b)]/(2*Sqrt[2]*b^2*Sqrt[d*x^2]) -
  x*Sin[a/(2*b)]*SinIntegral[(a+b*ArcCos[-1+d*x^2])/(2*b)]/(2*Sqrt[2]*b^2*Sqrt[d*x^2]) /;
FreeQ[{a,b,d},x]
```

3: $\int (a + b \operatorname{ArcSin}[c + d x^2])^n dx$ when $c^2 = 1 \wedge n < -1 \wedge n \neq -2$

Derivation: Inverted integration by parts twice

Rule: If $c^2 = 1 \wedge n < -1 \wedge n \neq -2$, then

$$\int (a + b \operatorname{ArcSin}[c + d x^2])^n dx \rightarrow \frac{x (a + b \operatorname{ArcSin}[c + d x^2])^{n+2}}{4 b^2 (n+1) (n+2)} + \frac{\sqrt{-2 c d x^2 - d^2 x^4} (a + b \operatorname{ArcSin}[c + d x^2])^{n+1}}{2 b d (n+1) x} - \frac{1}{4 b^2 (n+1) (n+2)} \int (a + b \operatorname{ArcSin}[c + d x^2])^{n+2} dx$$

Program code:

```
Int[(a_.+b_.*ArcSin[c_+d_.*x_^2])^n_,x_Symbol] :=
  x*(a+b*ArcSin[c+d*x^2])^(n+2)/(4*b^2*(n+1)*(n+2)) +
  Sqrt[-2*c*d*x^2-d^2*x^4]*(a+b*ArcSin[c+d*x^2])^(n+1)/(2*b*d*(n+1)*x) -
  1/(4*b^2*(n+1)*(n+2))*Int[(a+b*ArcSin[c+d*x^2])^(n+2),x] /;
FreeQ[{a,b,c,d},x] && EqQ[c^2,1] && LtQ[n,-1] && NeQ[n,-2]
```

```
Int[(a_.+b_.*ArcCos[c_+d_.*x_^2])^n_,x_Symbol] :=
  x*(a+b*ArcCos[c+d*x^2])^(n+2)/(4*b^2*(n+1)*(n+2)) -
  Sqrt[-2*c*d*x^2-d^2*x^4]*(a+b*ArcCos[c+d*x^2])^(n+1)/(2*b*d*(n+1)*x) -
  1/(4*b^2*(n+1)*(n+2))*Int[(a+b*ArcCos[c+d*x^2])^(n+2),x] /;
FreeQ[{a,b,c,d},x] && EqQ[c^2,1] && LtQ[n,-1] && NeQ[n,-2]
```

3: $\int \frac{\text{ArcSin}[a x^p]^n}{x} dx$ when $n \in \mathbb{Z}^+$

Derivation: Integration by substitution

Basis: $\frac{\text{ArcSin}[a x^p]^n}{x} == \frac{1}{p} \text{ArcSin}[a x^p]^n \text{Cot}[\text{ArcSin}[a x^p]] \partial_x \text{ArcSin}[a x^p]$

Rule: If $n \in \mathbb{Z}^+$, then

$$\int \frac{\text{ArcSin}[a x^p]^n}{x} dx \rightarrow \frac{1}{p} \text{Subst}\left[\int x^n \text{Cot}[x] dx, x, \text{ArcSin}[a x^p]\right]$$

Program code:

```
Int[ArcSin[a_.*x_^p_]^n_/x_,x_Symbol] :=
  1/p*Subst[Int[x^n*Cot[x],x],x,ArcSin[a*x^p]] /;
FreeQ[{a,p},x] && IGtQ[n,0]
```

```
Int[ArcCos[a_.*x_^p_]^n_/x_,x_Symbol] :=
  -1/p*Subst[Int[x^n*Tan[x],x],x,ArcCos[a*x^p]] /;
FreeQ[{a,p},x] && IGtQ[n,0]
```

4: $\int u \operatorname{ArcSin}\left[\frac{c}{a + b x^n}\right]^m dx$

Derivation: Algebraic simplification

Basis: $\operatorname{ArcSin}[z] == \operatorname{ArcCsc}\left[\frac{1}{z}\right]$

Rule:

$$\int u \operatorname{ArcSin}\left[\frac{c}{a + b x^n}\right]^m dx \rightarrow \int u \operatorname{ArcCsc}\left[\frac{a}{c} + \frac{b x^n}{c}\right]^m dx$$

Program code:

```
Int[u_*ArcSin[c_/(a_+b_*x_^n_)]^m_,x_Symbol] :=
  Int[u*ArcCsc[a/c+b*x^n/c]^m,x] /;
FreeQ[{a,b,c,n,m},x]
```

```
Int[u_*ArcCos[c_/(a_+b_*x_^n_)]^m_,x_Symbol] :=
  Int[u*ArcSec[a/c+b*x^n/c]^m,x] /;
FreeQ[{a,b,c,n,m},x]
```

$$5: \int \frac{\text{ArcSin}[\sqrt{1+b x^2}]^n}{\sqrt{1+b x^2}} dx$$

Derivation: Piecewise constant extraction and integration by substitution

$$\text{Basis: } \partial_x \frac{\sqrt{-b x^2}}{x} = 0$$

$$\text{Basis: } \frac{x \text{ArcSin}[\sqrt{1+b x^2}]^n}{\sqrt{-b x^2} \sqrt{1+b x^2}} = \frac{1}{b} \text{Subst} \left[\frac{\text{ArcSin}[x]^n}{\sqrt{1-x^2}}, x, \sqrt{1+b x^2} \right] \partial_x \sqrt{1+b x^2}$$

Rule:

$$\begin{aligned} \int \frac{\text{ArcSin}[\sqrt{1+b x^2}]^n}{\sqrt{1+b x^2}} dx &\rightarrow \frac{\sqrt{-b x^2}}{x} \int \frac{x \text{ArcSin}[\sqrt{1+b x^2}]^n}{\sqrt{-b x^2} \sqrt{1+b x^2}} dx \\ &\rightarrow \frac{\sqrt{-b x^2}}{b x} \text{Subst} \left[\int \frac{\text{ArcSin}[x]^n}{\sqrt{1-x^2}} dx, x, \sqrt{1+b x^2} \right] \end{aligned}$$

Program code:

```
Int[ArcSin[Sqrt[1+b_.*x_^2]]^n_/Sqrt[1+b_.*x_^2],x_Symbol] :=
  Sqrt[-b*x^2]/(b*x)*Subst[Int[ArcSin[x]^n/Sqrt[1-x^2],x],x,Sqrt[1+b*x^2]] /;
FreeQ[{b,n},x]
```

```
Int[ArcCos[Sqrt[1+b_.*x_^2]]^n_/Sqrt[1+b_.*x_^2],x_Symbol] :=
  Sqrt[-b*x^2]/(b*x)*Subst[Int[ArcCos[x]^n/Sqrt[1-x^2],x],x,Sqrt[1+b*x^2]] /;
FreeQ[{b,n},x]
```

6: $\int u f^{c \operatorname{ArcSin}[a+bx]^n} dx$ when $n \in \mathbb{Z}^+$

Derivation: Integration by substitution

Basis:

$$F[x, \operatorname{ArcSin}[a+bx]] = \frac{1}{b} \operatorname{Subst}\left[F\left[-\frac{a}{b} + \frac{\sin[x]}{b}, x\right] \cos[x], x, \operatorname{ArcSin}[a+bx]\right] \partial_x \operatorname{ArcSin}[a+bx]$$

Rule: If $n \in \mathbb{Z}^+$, then

$$\int u f^{c \operatorname{ArcSin}[a+bx]^n} dx \rightarrow \frac{1}{b} \operatorname{Subst}\left[\int \operatorname{Subst}\left[u, x, -\frac{a}{b} + \frac{\sin[x]}{b}\right] f^{c x^n} \cos[x] dx, x, \operatorname{ArcSin}[a+bx]\right]$$

Program code:

```
Int[u_.*f^(c_.*ArcSin[a_.*b_.*x_]^n_),x_Symbol] :=
  1/b*Subst[Int[ReplaceAll[u,x->-a/b+Sin[x]/b]*f^(c*x^n)*Cos[x],x],x,ArcSin[a+b*x]] /;
FreeQ[{a,b,c,f},x] && IGtQ[n,0]
```

```
Int[u_.*f^(c_.*ArcCos[a_.*b_.*x_]^n_),x_Symbol] :=
  -1/b*Subst[Int[ReplaceAll[u,x->-a/b+Cos[x]/b]*f^(c*x^n)*Sin[x],x],x,ArcCos[a+b*x]] /;
FreeQ[{a,b,c,f},x] && IGtQ[n,0]
```

7. $\int v(a+b \operatorname{ArcSin}[u]) dx$ when u is free of inverse functions

1. $\int v(a+b \operatorname{ArcSin}[u]) dx$ when u is free of inverse functions

1: $\int \operatorname{ArcSin}[a x^2 + b \sqrt{c+d x^2}] dx$ when $b^2 c = 1$

Derivation: Integration by parts and piecewise constant extraction

Basis: If $b^2 c = 1$, then $1 - (a x^2 + b \sqrt{c+d x^2})^2 = -x^2 (b^2 d + a^2 x^2 + 2 a b \sqrt{c+d x^2})$

$$\text{Basis: } \partial_x \frac{x \sqrt{b^2 d + a^2 x^2 + 2 a b \sqrt{c + d x^2}}}{\sqrt{-x^2 (b^2 d + a^2 x^2 + 2 a b \sqrt{c + d x^2})}} = 0$$

Note: The resulting integrand is of the form $x F[x^2]$ which can be integrated by substitution.

Rule: If $b^2 c = 1$, then

$$\begin{aligned} \int \text{ArcSin}[a x^2 + b \sqrt{c + d x^2}] dx &\rightarrow x \text{ArcSin}[a x^2 + b \sqrt{c + d x^2}] - \int \frac{x^2 (b d + 2 a \sqrt{c + d x^2})}{\sqrt{c + d x^2} \sqrt{-x^2 (b^2 d + a^2 x^2 + 2 a b \sqrt{c + d x^2})}} dx \\ &\rightarrow x \text{ArcSin}[a x^2 + b \sqrt{c + d x^2}] - \frac{x \sqrt{b^2 d + a^2 x^2 + 2 a b \sqrt{c + d x^2}}}{\sqrt{-x^2 (b^2 d + a^2 x^2 + 2 a b \sqrt{c + d x^2})}} \int \frac{x (b d + 2 a \sqrt{c + d x^2})}{\sqrt{c + d x^2} \sqrt{b^2 d + a^2 x^2 + 2 a b \sqrt{c + d x^2}}} dx \end{aligned}$$

Program code:

```
Int[ArcSin[a_.*x^2+b_.*Sqrt[c_+d_.*x^2]],x_Symbol] :=
  x*ArcSin[a*x^2+b*Sqrt[c+d*x^2]] -
  x*Sqrt[b^2*d+a^2*x^2+2*a*b*Sqrt[c+d*x^2]]/Sqrt[(-x^2)*(b^2*d+a^2*x^2+2*a*b*Sqrt[c+d*x^2])] *
  Int[x*(b*d+2*a*Sqrt[c+d*x^2])/(Sqrt[c+d*x^2]*Sqrt[b^2*d+a^2*x^2+2*a*b*Sqrt[c+d*x^2]]),x] /;
FreeQ[{a,b,c,d},x] && EqQ[b^2*c,1]
```

```
Int[ArcCos[a_.*x^2+b_.*Sqrt[c_+d_.*x^2]],x_Symbol] :=
  x*ArcCos[a*x^2+b*Sqrt[c+d*x^2]] +
  x*Sqrt[b^2*d+a^2*x^2+2*a*b*Sqrt[c+d*x^2]]/Sqrt[(-x^2)*(b^2*d+a^2*x^2+2*a*b*Sqrt[c+d*x^2])] *
  Int[x*(b*d+2*a*Sqrt[c+d*x^2])/(Sqrt[c+d*x^2]*Sqrt[b^2*d+a^2*x^2+2*a*b*Sqrt[c+d*x^2]]),x] /;
FreeQ[{a,b,c,d},x] && EqQ[b^2*c,1]
```


2: $\int \text{ArcSin}[u] \, dx$ when **u is free of inverse functions**

Derivation: Integration by parts

Rule: If u is free of inverse functions, then

$$\int \text{ArcSin}[u] \, dx \rightarrow x \text{ArcSin}[u] - \int \frac{x \partial_x u}{\sqrt{1-u^2}} \, dx$$

Program code:

```
Int[ArcSin[u_],x_Symbol] :=
  x*ArcSin[u] -
  Int[SimplifyIntegrand[x*D[u,x]/Sqrt[1-u^2],x],x] /;
InverseFunctionFreeQ[u,x] && Not[FunctionOfExponentialQ[u,x]]
```

```
Int[ArcCos[u_],x_Symbol] :=
  x*ArcCos[u] +
  Int[SimplifyIntegrand[x*D[u,x]/Sqrt[1-u^2],x],x] /;
InverseFunctionFreeQ[u,x] && Not[FunctionOfExponentialQ[u,x]]
```

2: $\int (c + dx)^m (a + b \operatorname{ArcSin}[u]) dx$ when $m \neq -1 \wedge u$ is free of inverse functions

Derivation: Integration by parts

Rule: If $m \neq -1 \wedge u$ is free of inverse functions, then

$$\int (c + dx)^m (a + b \operatorname{ArcSin}[u]) dx \rightarrow \frac{(c + dx)^{m+1} (a + b \operatorname{ArcSin}[u])}{d(m+1)} - \frac{b}{d(m+1)} \int \frac{(c + dx)^{m+1} \partial_x u}{\sqrt{1-u^2}} dx$$

Program code:

```
Int[(c_.+d_.x_)^m_.*(a_.+b_.*ArcSin[u_]),x_Symbol] :=
  (c+d*x)^(m+1)*(a+b*ArcSin[u])/(d*(m+1)) -
  b/(d*(m+1))*Int[SimplifyIntegrand[(c+d*x)^(m+1)*D[u,x]/Sqrt[1-u^2],x],x] /;
FreeQ[{a,b,c,d,m},x] && NeQ[m,-1] && InverseFunctionFreeQ[u,x] && Not[FunctionOfQ[(c+d*x)^(m+1),u,x]] && Not[FunctionOfExponentialQ[u,
```

```
Int[(c_.+d_.x_)^m_.*(a_.+b_.*ArcCos[u_]),x_Symbol] :=
  (c+d*x)^(m+1)*(a+b*ArcCos[u])/(d*(m+1)) +
  b/(d*(m+1))*Int[SimplifyIntegrand[(c+d*x)^(m+1)*D[u,x]/Sqrt[1-u^2],x],x] /;
FreeQ[{a,b,c,d,m},x] && NeQ[m,-1] && InverseFunctionFreeQ[u,x] && Not[FunctionOfQ[(c+d*x)^(m+1),u,x]] && Not[FunctionOfExponentialQ[u,
```

3: $\int v (a + b \operatorname{ArcSin}[u]) \, dx$ when u and $\int v \, dx$ are free of inverse functions

Derivation: Integration by parts

Rule: If u is free of inverse functions, let $w = \int v \, dx$, if w is free of inverse functions, then

$$\int v (a + b \operatorname{ArcSin}[u]) \, dx \rightarrow w (a + b \operatorname{ArcSin}[u]) - b \int \frac{w \partial_x u}{\sqrt{1-u^2}} \, dx$$

Program code:

```
Int[v_*(a_.+b_.*ArcSin[u_]),x_Symbol] :=
  With[{w=IntHide[v,x]},
    Dist[(a+b*ArcSin[u]),w,x] -
    b*Int[SimplifyIntegrand[w*D[u,x]/Sqrt[1-u^2],x],x] /;
    InverseFunctionFreeQ[w,x] /;
    FreeQ[{a,b},x] && InverseFunctionFreeQ[u,x] && Not[MatchQ[v, (c_.+d_.**x)^m_. /; FreeQ[{c,d,m},x]]]
```

```
Int[v_*(a_.+b_.*ArcCos[u_]),x_Symbol] :=
  With[{w=IntHide[v,x]},
    Dist[(a+b*ArcCos[u]),w,x] +
    b*Int[SimplifyIntegrand[w*D[u,x]/Sqrt[1-u^2],x],x] /;
    InverseFunctionFreeQ[w,x] /;
    FreeQ[{a,b},x] && InverseFunctionFreeQ[u,x] && Not[MatchQ[v, (c_.+d_.**x)^m_. /; FreeQ[{c,d,m},x]]]
```