

Rules for integrands of the form $(c + dx)^m (F^g(e+fx))^n (a + b (F^g(e+fx))^n)^p$

1. $\int (c + dx)^m (F^g(e+fx))^n (a + b (F^g(e+fx))^n)^p dx$

1: $\int \frac{(c + dx)^m (F^g(e+fx))^n}{a + b (F^g(e+fx))^n} dx$ when $m \in \mathbb{Z}^+$

Derivation: Integration by parts

Basis: $\frac{(F^g(e+fx))^n}{a + b (F^g(e+fx))^n} = \partial_x \frac{\text{Log}\left[1 + \frac{b (F^g(e+fx))^n}{a}\right]}{b f g n \text{Log}[F]}$

Rule: If $m \in \mathbb{Z}^+$, then

$$\int \frac{(c + dx)^m (F^g(e+fx))^n}{a + b (F^g(e+fx))^n} dx \rightarrow \frac{(c + dx)^m}{b f g n \text{Log}[F]} \text{Log}\left[1 + \frac{b (F^g(e+fx))^n}{a}\right] - \frac{d m}{b f g n \text{Log}[F]} \int (c + dx)^{m-1} \text{Log}\left[1 + \frac{b (F^g(e+fx))^n}{a}\right] dx$$

Program code:

```
Int[(c_.+d_.**x_)^m_.*(F_^(g_.*(e_.+f_.**x_)))^n_./(a_+b_.*(F_^(g_.*(e_.+f_.**x_)))^n_.),x_Symbol] :=
(c+d*x)^m/(b*f*g*n*Log[F])*Log[1+b*(F^(g*(e+f*x)))^n/a] -
d*m/(b*f*g*n*Log[F])*Int[(c+d*x)^(m-1)*Log[1+b*(F^(g*(e+f*x)))^n/a],x] /;
FreeQ[{F,a,b,c,d,e,f,g,n},x] && IGtQ[m,0]
```

2: $\int (c + dx)^m (F^g(e+fx))^n (a + b (F^g(e+fx))^n)^p dx$ when $p \neq -1$

Derivation: Integration by parts

Basis: $(F^g(e+fx))^n (a + b (F^g(e+fx))^n)^p = \partial_x \frac{(a + b (F^g(e+fx))^n)^{p+1}}{b f g n (p+1) \text{Log}[F]}$

Rule: If $p \neq -1$, then

$$\int (c + dx)^m (F^g(e+fx))^n (a + b (F^g(e+fx))^n)^p dx \rightarrow$$

$$\frac{(c+dx)^m (a+b(F^g(e+fx)))^{p+1}}{b f g n (p+1) \text{Log}[F]} - \frac{d m}{b f g n (p+1) \text{Log}[F]} \int (c+dx)^{m-1} (a+b(F^g(e+fx)))^{p+1} dx$$

Program code:

```
Int[(c_+d_.**x_)^m_.*(F_^(g_.*(e_+f_.**x_)))^n_.*(a_+b_.*(F_^(g_.*(e_+f_.**x_)))^n_.)^p_,x_Symbol] :=
(c+d*x)^m*(a+b*(F^(g*(e+f*x))))^n^(p+1)/(b*f*g*n*(p+1)*Log[F]) -
d*m/(b*f*g*n*(p+1)*Log[F])*Int[(c+d*x)^(m-1)*(a+b*(F^(g*(e+f*x))))^n^(p+1),x] /;
FreeQ[{F,a,b,c,d,e,f,g,m,n,p},x] && NeQ[p,-1]
```

x: $\int (c+dx)^m (F^g(e+fx))^n (a+b(F^g(e+fx)))^p dx$

Rule:

$$\int (c+dx)^m (F^g(e+fx))^n (a+b(F^g(e+fx)))^p dx \rightarrow \int (c+dx)^m (F^g(e+fx))^n (a+b(F^g(e+fx)))^p dx$$

Program code:

```
Int[(c_+d_.**x_)^m_.*(F_^(g_.*(e_+f_.**x_)))^n_.*(a_+b_.*(F_^(g_.*(e_+f_.**x_)))^n_.)^p_,x_Symbol] :=
Unintegrable[(c+d*x)^m*(F^(g*(e+f*x))))^n*(a+b*(F^(g*(e+f*x))))^p,x] /;
FreeQ[{F,a,b,c,d,e,f,g,m,n,p},x]
```

2: $\int (c+dx)^m (k G^{j(h+ix)})^q (a+b(F^g(e+fx)))^p dx$ when $f g n \text{Log}[F] - i j q \text{Log}[G] = 0$

Derivation: Piecewise constant extraction

Basis: If $f g n \text{Log}[F] - i j q \text{Log}[G] = 0$, then $\partial_x \frac{(k G^{j(h+ix)})^q}{(F^g(e+fx))^n} = 0$

Rule: If $f g n \text{Log}[F] - i j q \text{Log}[G] = 0$, then

$$\int (c+dx)^m (k G^{j(h+ix)})^q (a+b(F^j(g(e+fx)))^n)^p dx \rightarrow \frac{(k G^{j(h+ix)})^q}{(F^j(g(e+fx)))^n} \int (c+dx)^m (F^j(g(e+fx)))^n (a+b(F^j(g(e+fx)))^n)^p dx$$

Program code:

```
Int[(c_+d_.**x_)^m_.*(k_.*G_^(j_.*(h_.+i_.**x_)))^q_.*(a_.+b_.*(F_^(g_.*(e_.+f_.**x_)))^n_.)^p_,x_Symbol] :=
  (k*G^(j*(h+i*x)))^q/(F^(g*(e+f*x)))^n*Int[(c+d*x)^m*(F^(g*(e+f*x)))^n*(a+b*(F^(g*(e+f*x)))^n)^p,x] /;
FreeQ[{F,a,b,c,d,e,f,g,h,i,j,k,m,n,p,q},x] && EqQ[f*g*n*Log[F]-i*j*q*Log[G],0] && NeQ[(k*G^(j*(h+i*x)))^q-(F^(g*(e+f*x)))^n,0]
```