

Rules for integrands of the form $(a + b \sinh[c + d x^n])^p$

$$1. \int (a + b \sinh[c + d x^n])^p dx \text{ when } n \in \mathbb{Z} \wedge p \in \mathbb{Z}$$

$$1. \int (a + b \sinh[c + d x^n])^p dx \text{ when } n - 1 \in \mathbb{Z}^+ \wedge p \in \mathbb{Z}^+$$

$$1: \int \sinh[c + d x^n] dx \text{ when } n - 1 \in \mathbb{Z}^+$$

Derivation: Algebraic expansion

$$\text{Basis: } \sinh[z] == \frac{e^z}{2} - \frac{e^{-z}}{2}$$

$$\text{Basis: } \cosh[z] == \frac{e^z}{2} + \frac{e^{-z}}{2}$$

Rule: If $n - 1 \in \mathbb{Z}^+$, then

$$\int \sinh[c + d x^n] dx \rightarrow \frac{1}{2} \int e^{c+d x^n} dx - \frac{1}{2} \int e^{-c-d x^n} dx$$

Program code:

```
Int[Sinh[c_+d_*x_^n_],x_Symbol] :=
  1/2*Int[E^(c+d*x^n),x] - 1/2*Int[E^(-c-d*x^n),x] /;
FreeQ[{c,d},x] && IGtQ[n,1]
```

```
Int[Cosh[c_+d_*x_^n_],x_Symbol] :=
  1/2*Int[E^(c+d*x^n),x] + 1/2*Int[E^(-c-d*x^n),x] /;
FreeQ[{c,d},x] && IGtQ[n,1]
```

2: $\int (a + b \sinh[c + d x^n])^p dx$ when $n - 1 \in \mathbb{Z}^+ \wedge p - 1 \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule: If $n - 1 \in \mathbb{Z}^+ \wedge p - 1 \in \mathbb{Z}^+$, then

$$\int (a + b \sinh[c + d x^n])^p dx \rightarrow \int \text{TrigReduce}[(a + b \sinh[c + d x^n])^p, x] dx$$

Program code:

```
Int[(a_.+b_.*Sinh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(a+b*Sinh[c+d*x^n])^p,x],x] /;
FreeQ[{a,b,c,d},x] && IGtQ[n,1] && IGtQ[p,1]
```

```
Int[(a_.+b_.*Cosh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(a+b*Cosh[c+d*x^n])^p,x],x] /;
FreeQ[{a,b,c,d},x] && IGtQ[n,1] && IGtQ[p,1]
```

2: $\int (a + b \sinh[c + d x^n])^p dx$ when $n \in \mathbb{Z}^- \wedge p \in \mathbb{Z}$

Derivation: Integration by substitution

Basis: If $n \in \mathbb{Z}$, then $F[x^n] = -\text{Subst}\left[\frac{F[x^{-n}]}{x^2}, x, \frac{1}{x}\right] \partial_x \frac{1}{x}$

Rule: If $n \in \mathbb{Z}^- \wedge p \in \mathbb{Z}$, then

$$\int (a + b \sinh[c + d x^n])^p dx \rightarrow -\text{Subst}\left[\int \frac{(a + b \sinh[c + d x^{-n}])^p}{x^2} dx, x, \frac{1}{x}\right]$$

Program code:

```
Int[(a_.+b_.*Sinh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  -Subst[Int[(a+b*Sinh[c+d*x^(-n)])^p/x^2,x],x,1/x] /;
FreeQ[{a,b,c,d},x] && ILtQ[n,0] && IntegerQ[p]
```

```
Int[(a_.+b_.*Cosh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  -Subst[Int[(a+b*Cosh[c+d*x^(-n)])^p/x^2,x],x,1/x] /;
FreeQ[{a,b,c,d},x] && ILtQ[n,0] && IntegerQ[p]
```

2: $\int (a + b \sinh[c + d x^n])^p dx$ when $n \in \mathbb{F} \wedge p \in \mathbb{Z}$

Derivation: Integration by substitution

Basis: If $k \in \mathbb{Z}^+$, then $F[x^n] = k \text{Subst}[x^{k-1} F[x^{k^n}], x, x^{1/k}] \partial_x x^{1/k}$

Rule: If $n \in \mathbb{F} \wedge p \in \mathbb{Z}$, let $k = \text{Denominator}[n]$, then

$$\int (a + b \sinh[c + d x^n])^p dx \rightarrow k \text{Subst}\left[\int x^{k-1} (a + b \sinh[c + d x^{k^n}])^p dx, x, x^{1/k}\right]$$

Program code:

```
Int[(a_.+b_.*Sinh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Module[{k=Denominator[n]},
    k*Subst[Int[x^(k-1)*(a+b*Sinh[c+d*x^(k*n)])^p,x],x,x^(1/k)]] /;
  FreeQ[{a,b,c,d},x] && FractionQ[n] && IntegerQ[p]
```

```
Int[(a_.+b_.*Cosh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Module[{k=Denominator[n]},
    k*Subst[Int[x^(k-1)*(a+b*Cosh[c+d*x^(k*n)])^p,x],x,x^(1/k)]] /;
  FreeQ[{a,b,c,d},x] && FractionQ[n] && IntegerQ[p]
```

3. $\int (a + b \sinh[c + d x^n])^p dx$ when $p \in \mathbb{Z}^+$

1: $\int \sinh[c + d x^n] dx$

Derivation: Algebraic expansion

Basis: $\sinh[z] == \frac{e^z}{2} - \frac{e^{-z}}{2}$

Basis: $\cosh[z] == \frac{e^z}{2} + \frac{e^{-z}}{2}$

Rule:

$$\int \sinh[c + d x^n] dx \rightarrow \frac{1}{2} \int e^{c+d x^n} dx - \frac{1}{2} \int e^{-c-d x^n} dx$$

Program code:

```
Int[Sinh[c_.+d_.**x_^n_],x_Symbol] :=
  1/2*Int[E^(c+d*x^n),x] - 1/2*Int[E^(-c-d*x^n),x] /;
FreeQ[{c,d,n},x]
```

```
Int[Cosh[c_.+d_.**x_^n_],x_Symbol] :=
  1/2*Int[E^(c+d*x^n),x] + 1/2*Int[E^(-c-d*x^n),x] /;
FreeQ[{c,d,n},x]
```

2: $\int (a + b \sinh[c + d x^n])^p dx$ when $p \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule: If $p \in \mathbb{Z}^+$, then

$$\int (a + b \sinh[c + d x^n])^p dx \rightarrow \int \text{TrigReduce}[(a + b \sinh[c + d x^n])^p, x] dx$$

Program code:

```
Int[(a_.+b_.*Sinh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(a+b*Sinh[c+d*x^n])^p,x],x] /;
FreeQ[{a,b,c,d,n},x] && IGtQ[p,0]
```

```
Int[(a_.+b_.*Cosh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(a+b*Cosh[c+d*x^n])^p,x],x] /;
FreeQ[{a,b,c,d,n},x] && IGtQ[p,0]
```

S: $\int (a + b \sinh[c + d u^n])^p dx$ when $p \in \mathbb{Z} \wedge u = e + f x$

Derivation: Integration by substitution

Rule: If $p \in \mathbb{Z} \wedge u = e + f x$, then

$$\int (a + b \sinh[c + d u^n])^p dx \rightarrow \frac{1}{f} \text{Subst}\left[\int (a + b \sinh[c + d x^n])^p dx, x, u\right]$$

Program code:

```
Int[(a_.+b_.*Sinh[c_.+d_.*u_^n_])^p_,x_Symbol] :=
  1/Coefficient[u,x,1]*Subst[Int[(a+b*Sinh[c+d*x^n])^p,x],x,u] /;
FreeQ[{a,b,c,d,n},x] && IntegerQ[p] && LinearQ[u,x] && NeQ[u,x]
```

```
Int[(a_.+b_.*Cosh[c_.+d_.*u_^n_])^p_,x_Symbol] :=
  1/Coefficient[u,x,1]*Subst[Int[(a+b*Cosh[c+d*x^n])^p,x],x,u] /;
FreeQ[{a,b,c,d,n},x] && IntegerQ[p] && LinearQ[u,x] && NeQ[u,x]
```

X: $\int (a + b \sinh[c + d u^n])^p dx$

Rule:

$$\int (a + b \sinh[c + d u^n])^p dx \rightarrow \int (a + b \sinh[c + d u^n])^p dx$$

Program code:

```
Int[(a_.+b_.*Sinh[c_.+d_.*u_^n_])^p_,x_Symbol] :=
  Unintegrable[(a+b*Sinh[c+d*u^n])^p,x] /;
FreeQ[{a,b,c,d,n,p},x] && LinearQ[u,x]
```

```
Int[(a_.+b_.*Cosh[c_.+d_.*u_^n_])^p_,x_Symbol] :=
  Unintegrable[(a+b*Cosh[c+d*u^n])^p,x] /;
FreeQ[{a,b,c,d,n,p},x] && LinearQ[u,x]
```

N: $\int (a + b \sinh[u])^p dx$ when $u = c + d x^n$

Derivation: Algebraic normalization

Rule: If $u = c + d x^n$, then

$$\int (a + b \sinh[u])^p dx \rightarrow \int (a + b \sinh[c + d x^n])^p dx$$

Program code:

```
Int[(a_.+b_.*Sinh[u_])^p_,x_Symbol] :=
  Int[(a+b*Sinh[ExpandToSum[u,x]])^p,x] /;
FreeQ[{a,b,p},x] && BinomialQ[u,x] && Not[BinomialMatchQ[u,x]]
```

```
Int[(a_.+b_.*Cosh[u_])^p_,x_Symbol] :=
  Int[(a+b*Cosh[ExpandToSum[u,x]])^p,x] /;
FreeQ[{a,b,p},x] && BinomialQ[u,x] && Not[BinomialMatchQ[u,x]]
```

Rules for integrands of the form $(e x)^m (a + b \sinh[c + d x^n])^p$

1. $\int (e x)^m (a + b \sinh[c + d x^n])^p dx$ when $\frac{m+1}{n} \in \mathbb{Z}$

1. $\int x^m (a + b \sinh[c + d x^n])^p dx$ when $\frac{m+1}{n} \in \mathbb{Z}$

$$1. \int \frac{\text{Sinh}[c + d x^n]}{x} dx$$

$$\text{1: } \int \frac{\text{Sinh}[d x^n]}{x} dx$$

Derivation: Primitive rule

$$\text{Basis: SinhIntegral}'[z] == \frac{\text{Sinh}[z]}{z}$$

Rule:

$$\int \frac{\text{Sinh}[d x^n]}{x} dx \rightarrow \frac{\text{SinhIntegral}[d x^n]}{n}$$

Program code:

```
Int[Sinh[d_.*x_^n_]/x_,x_Symbol] :=
  SinhIntegral[d*x^n]/n /;
FreeQ[{d,n},x]
```

```
Int[Cosh[d_.*x_^n_]/x_,x_Symbol] :=
  CoshIntegral[d*x^n]/n /;
FreeQ[{d,n},x]
```

2: $\int \frac{\sinh[c + d x^n]}{x} dx$

Derivation: Algebraic expansion

Basis: $\sinh[w + z] = \sinh[w] \cosh[z] + \cosh[w] \sinh[z]$

Rule:

$$\int \frac{\sinh[c + d x^n]}{x} dx \rightarrow \sinh[c] \int \frac{\cosh[d x^n]}{x} dx + \cosh[c] \int \frac{\sinh[d x^n]}{x} dx$$

Program code:

```
Int[Sinh[c_+d_.*x_^n_]/x_,x_Symbol] :=
  Sinh[c]*Int[Cosh[d*x^n]/x,x] + Cosh[c]*Int[Sinh[d*x^n]/x,x] /;
FreeQ[{c,d,n},x]
```

```
Int[Cosh[c_+d_.*x_^n_]/x_,x_Symbol] :=
  Cosh[c]*Int[Cosh[d*x^n]/x,x] + Sinh[c]*Int[Sinh[d*x^n]/x,x] /;
FreeQ[{c,d,n},x]
```

2: $\int x^m (a + b \sinh[c + d x^n])^p dx$ when $\frac{m+1}{n} \in \mathbb{Z} \wedge (p = 1 \vee m = n - 1 \vee p \in \mathbb{Z} \wedge \frac{m+1}{n} > 0)$

Derivation: Integration by substitution

Basis: If $\frac{m+1}{n} \in \mathbb{Z}$, then $x^m F[x^n] = \frac{1}{n} \text{Subst}[x^{\frac{m+1}{n}-1} F[x], x, x^n] \partial_x x^n$

Rule: If $\frac{m+1}{n} \in \mathbb{Z} \wedge (p = 1 \vee m = n - 1 \vee p \in \mathbb{Z} \wedge \frac{m+1}{n} > 0)$, then

$$\int x^m (a + b \sinh[c + d x^n])^p dx \rightarrow \frac{1}{n} \text{Subst}\left[\int x^{\frac{m+1}{n}-1} (a + b \sinh[c + d x])^p dx, x, x^n\right]$$

Program code:

```
Int[x_^m_.*(a_.+b_.*Sinh[c_.+d_.*x_^n_])^p_.,x_Symbol] :=
  1/n*Subst[Int[x^(Simplify[(m+1)/n]-1)*(a+b*Sinh[c+d*x])^p,x],x,x^n] /;
FreeQ[{a,b,c,d,m,n,p},x] && IntegerQ[Simplify[(m+1)/n]] && (EqQ[p,1] || EqQ[m,n-1] || IntegerQ[p] && GtQ[Simplify[(m+1)/n],0])
```

```
Int[x_^m_.*(a_.+b_.*Cosh[c_.+d_.*x_^n_])^p_.,x_Symbol] :=
  1/n*Subst[Int[x^(Simplify[(m+1)/n]-1)*(a+b*Cosh[c+d*x])^p,x],x,x^n] /;
FreeQ[{a,b,c,d,m,n,p},x] && IntegerQ[Simplify[(m+1)/n]] && (EqQ[p,1] || EqQ[m,n-1] || IntegerQ[p] && GtQ[Simplify[(m+1)/n],0])
```

2: $\int (e x)^m (a + b \sinh[c + d x^n])^p dx$ when $\frac{m+1}{n} \in \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(e x)^m}{x^n} = 0$

Rule: If $\frac{m+1}{n} \in \mathbb{Z}$, then

$$\int (e x)^m (a + b \sinh[c + d x^n])^p dx \rightarrow \frac{e^{\text{IntPart}[m]} (e x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (a + b \sinh[c + d x^n])^p dx$$

Program code:

```
Int[(e*x_)^m*(a_+b_.*Sinh[c_+d_.*x_^n_])^p_,x_Symbol] :=
  e^IntPart[m]*(e*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*Sinh[c+d*x^n])^p,x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && IntegerQ[Simplify[(m+1)/n]]
```

```
Int[(e*x_)^m*(a_+b_.*Cosh[c_+d_.*x_^n_])^p_,x_Symbol] :=
  e^IntPart[m]*(e*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*Cosh[c+d*x^n])^p,x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && IntegerQ[Simplify[(m+1)/n]]
```

$$2. \int (e x)^m (a + b \sinh[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge n \in \mathbb{Z}$$

$$1. \int (e x)^m (a + b \sinh[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge n \in \mathbb{Z}^+$$

$$1. \int (e x)^m \sinh[c + d x^n] dx$$

$$\text{1: } \int (e x)^m \sinh[c + d x^n] dx \text{ when } n \in \mathbb{Z}^+ \wedge 0 < n < m + 1$$

Reference: CRC 392, A&S 4.3.119

Reference: CRC 396, A&S 4.3.123

Derivation: Integration by parts

Basis: If $n \in \mathbb{Z}$, then $(e x)^m \sinh[c + d x^n] = -\frac{e^{n-1} (e x)^{m-n+1}}{d n} \partial_x \cosh[c + d x^n]$

Rule: If $n \in \mathbb{Z}^+ \wedge 0 < n < m + 1$, then

$$\int (e x)^m \sinh[c + d x^n] dx \rightarrow \frac{e^{n-1} (e x)^{m-n+1} \cosh[c + d x^n]}{d n} - \frac{e^n (m - n + 1)}{d n} \int (e x)^{m-n} \cosh[c + d x^n] dx$$

Program code:

```
Int[(e_.**x_)^m_.**Sinh[c_.+d_.**x_^n_],x_Symbol] :=
  e^(n-1)*(e*x)^(m-n+1)*Cosh[c+d*x^n]/(d*n) -
  e^n*(m-n+1)/(d*n)*Int[(e*x)^(m-n)*Cosh[c+d*x^n],x] /;
FreeQ[{c,d,e},x] && IGtQ[n,0] && LtQ[0,n,m+1]
```

```
Int[(e_.**x_)^m_.**Cosh[c_.+d_.**x_^n_],x_Symbol] :=
  e^(n-1)*(e*x)^(m-n+1)*Sinh[c+d*x^n]/(d*n) -
  e^n*(m-n+1)/(d*n)*Int[(e*x)^(m-n)*Sinh[c+d*x^n],x] /;
FreeQ[{c,d,e},x] && IGtQ[n,0] && LtQ[0,n,m+1]
```

2: $\int (e x)^m \sinh[c + d x^n] dx$ when $n \in \mathbb{Z}^+ \wedge m < -1$

Reference: CRC 405, A&S 4.3.120

Reference: CRC 406, A&S 4.3.124

Derivation: Integration by parts

Rule: If $n \in \mathbb{Z}^+ \wedge m < -1$, then

$$\int (e x)^m \sinh[c + d x^n] dx \rightarrow \frac{(e x)^{m+1} \sinh[c + d x^n]}{e^{n(m+1)}} - \frac{d n}{e^n (m+1)} \int (e x)^{m+n} \cosh[c + d x^n] dx$$

Program code:

```
Int[(e_.**x_)^m_*Sinh[c_.+d_.**x_^n_],x_Symbol] :=
  (e*x)^(m+1)*Sinh[c+d*x^n]/(e*(m+1)) -
  d*n/(e^n*(m+1))*Int[(e*x)^(m+n)*Cosh[c+d*x^n],x] /;
FreeQ[{c,d,e},x] && IGtQ[n,0] && LtQ[m,-1]
```

```
Int[(e_.**x_)^m_*Cosh[c_.+d_.**x_^n_],x_Symbol] :=
  (e*x)^(m+1)*Cosh[c+d*x^n]/(e*(m+1)) -
  d*n/(e^n*(m+1))*Int[(e*x)^(m+n)*Sinh[c+d*x^n],x] /;
FreeQ[{c,d,e},x] && IGtQ[n,0] && LtQ[m,-1]
```

$$\mathbf{3:} \int (e x)^m \operatorname{Sinh}[c+d x^n] dx \text{ when } n \in \mathbb{Z}^+$$

Derivation: Algebraic expansion

$$\text{Basis: } \operatorname{Sinh}[z] == \frac{e^z}{2} - \frac{e^{-z}}{2}$$

$$\text{Basis: } \operatorname{Cosh}[z] == \frac{e^z}{2} + \frac{e^{-z}}{2}$$

Rule: If $n \in \mathbb{Z}^+$, then

$$\int (e x)^m \operatorname{Sinh}[c+d x^n] dx \rightarrow \frac{1}{2} \int (e x)^m e^{c+d x^n} dx - \frac{1}{2} \int (e x)^m e^{-c-d x^n} dx$$

Program code:

```
Int[(e_.**x_)^m_.*Sinh[c_.+d_.**x_^n_],x_Symbol] :=
  1/2*Int[(e**x)^m**E^(c+d*x^n),x] - 1/2*Int[(e**x)^m**E^(-c-d*x^n),x] /;
FreeQ[{c,d,e,m},x] && IGtQ[n,0]
```

```
Int[(e_.**x_)^m_.*Cosh[c_.+d_.**x_^n_],x_Symbol] :=
  1/2*Int[(e**x)^m**E^(c+d*x^n),x] + 1/2*Int[(e**x)^m**E^(-c-d*x^n),x] /;
FreeQ[{c,d,e,m},x] && IGtQ[n,0]
```

$$2. \int (e x)^m (a+b \operatorname{Sinh}[c+d x^n])^p dx \text{ when } p > 1$$

$$\mathbf{1:} \int \frac{\operatorname{Sinh}[a+b x^n]^p}{x^n} dx \text{ when } (n \mid p) \in \mathbb{Z} \wedge p > 1 \wedge n \neq 1$$

Derivation: Integration by parts

Rule: If $(n \mid p) \in \mathbb{Z} \wedge p > 1 \wedge n \neq 1$, then

$$\int \frac{\text{Sinh}[a + b x^n]^p}{x^n} dx \rightarrow -\frac{\text{Sinh}[a + b x^n]^p}{(n-1) x^{n-1}} + \frac{b n p}{n-1} \int \text{Sinh}[a + b x^n]^{p-1} \text{Cosh}[a + b x^n] dx$$

Program code:

```
Int[x_^m_.*Sinh[a_+b_*x_^n_]^p_,x_Symbol] :=
  -Sinh[a+b*x^n]^p/((n-1)*x^(n-1)) +
  b*n*p/(n-1)*Int[Sinh[a+b*x^n]^(p-1)*Cosh[a+b*x^n],x] /;
FreeQ[{a,b},x] && IntegersQ[n,p] && EqQ[m+n,0] && GtQ[p,1] && NeQ[n,1]
```

```
Int[x_^m_.*Cosh[a_+b_*x_^n_]^p_,x_Symbol] :=
  -Cosh[a+b*x^n]^p/((n-1)*x^(n-1)) +
  b*n*p/(n-1)*Int[Cosh[a+b*x^n]^(p-1)*Sinh[a+b*x^n],x] /;
FreeQ[{a,b},x] && IntegersQ[n,p] && EqQ[m+n,0] && GtQ[p,1] && NeQ[n,1]
```

2: $\int x^m \text{Sinh}[a + b x^n]^p dx$ when $m - 2n + 1 == 0 \wedge p > 1$

Reference: G&R 2.471.1b' special case when $m - 2n + 1 == 0$

Reference: G&R 2.471.1a' special case with $m - 2n + 1 == 0$

Rule: If $m - 2n + 1 == 0 \wedge p > 1$, then

$$\int x^m \text{Sinh}[a + b x^n]^p dx \rightarrow -\frac{n \text{Sinh}[a + b x^n]^p}{b^2 n^2 p^2} + \frac{x^n \text{Cosh}[a + b x^n] \text{Sinh}[a + b x^n]^{p-1}}{b n p} - \frac{p-1}{p} \int x^m \text{Sinh}[a + b x^n]^{p-2} dx$$

Program code:

```
Int[x_^m_.*Sinh[a_+b_*x_^n_]^p_,x_Symbol] :=
  -n*Sinh[a+b*x^n]^p/(b^2*n^2*p^2) +
  x^n*Cosh[a+b*x^n]*Sinh[a+b*x^n]^(p-1)/(b*n*p) -
  (p-1)/p*Int[x^m*Sinh[a+b*x^n]^(p-2),x] /;
FreeQ[{a,b,m,n},x] && EqQ[m-2*n+1] && GtQ[p,1]
```



```

Int[x_^m_.*Cosh[a_+b_.*x_^n_]^p_,x_Symbol] :=
  -n*Cosh[a+b*x^n]^p/(b^2*n^2*p^2) +
  x^n*Sinh[a+b*x^n]*Cosh[a+b*x^n]^(p-1)/(b*n*p) +
  (p-1)/p*Int[x^m*Cosh[a+b*x^n]^(p-2),x] /;
FreeQ[{a,b,m,n},x] && EqQ[m-2*n+1] && GtQ[p,1]

```

3: $\int x^m \sinh[a + b x^n]^p dx$ when $(m | n) \in \mathbb{Z} \wedge p > 1 \wedge 0 < 2n < m+1$

Reference: G&R 2.471.1b'

Reference: G&R 2.631.3'

Rule: If $(m | n) \in \mathbb{Z} \wedge p > 1 \wedge 0 < 2n < m+1$, then

$$\begin{aligned}
 & \int x^m \sinh[a + b x^n]^p dx \rightarrow \\
 & - \frac{(m-n+1) x^{m-2n+1} \sinh[a + b x^n]^p}{b^2 n^2 p^2} + \frac{x^{m-n+1} \cosh[a + b x^n] \sinh[a + b x^n]^{p-1}}{b n p} - \\
 & \frac{p-1}{p} \int x^m \sinh[a + b x^n]^{p-2} dx + \frac{(m-n+1)(m-2n+1)}{b^2 n^2 p^2} \int x^{m-2n} \sinh[a + b x^n]^p dx
 \end{aligned}$$

Program code:

```

Int[x_^m_.*Sinh[a_+b_.*x_^n_]^p_,x_Symbol] :=
  -(m-n+1)*x^(m-2*n+1)*Sinh[a+b*x^n]^p/(b^2*n^2*p^2) +
  x^(m-n+1)*Cosh[a+b*x^n]*Sinh[a+b*x^n]^(p-1)/(b*n*p) -
  (p-1)/p*Int[x^m*Sinh[a+b*x^n]^(p-2),x] +
  (m-n+1)*(m-2*n+1)/(b^2*n^2*p^2)*Int[x^(m-2*n)*Sinh[a+b*x^n]^p,x] /;
FreeQ[{a,b},x] && IntegersQ[m,n] && GtQ[p,1] && LtQ[0,2*n,m+1]

```

```

Int[x_^m_.*Cosh[a_+b_*x_^n_]^p_,x_Symbol] :=
  -(m-n+1)*x^(m-2*n+1)*Cosh[a+b*x^n]^p/(b^2*n^2*p^2) +
  x^(m-n+1)*Sinh[a+b*x^n]*Cosh[a+b*x^n]^(p-1)/(b*n*p) +
  (p-1)/p*Int[x^m*Cosh[a+b*x^n]^(p-2),x] +
  (m-n+1)*(m-2*n+1)/(b^2*n^2*p^2)*Int[x^(m-2*n)*Cosh[a+b*x^n]^p,x] /;
FreeQ[{a,b},x] && IntegersQ[m,n] && GtQ[p,1] && LtQ[0,2*n,m+1]

```

4: $\int x^m \sinh[a + b x^n]^p dx$ when $(m | n) \in \mathbb{Z} \wedge p > 1 \wedge 0 < 2n < 1 - m \wedge m + n + 1 \neq 0$

Reference: G&R 2.475.1'

Reference: G&R 2.475.2'

Rule: If $(m | n) \in \mathbb{Z} \wedge p > 1 \wedge 0 < 2n < 1 - m \wedge m + n + 1 \neq 0$, then

$$\begin{aligned}
 & \int x^m \sinh[a + b x^n]^p dx \rightarrow \\
 & \frac{x^{m+1} \sinh[a + b x^n]^p}{m+1} - \frac{b n p x^{m+n+1} \cosh[a + b x^n] \sinh[a + b x^n]^{p-1}}{(m+1)(m+n+1)} + \\
 & \frac{b^2 n^2 p^2}{(m+1)(m+n+1)} \int x^{m+2n} \sinh[a + b x^n]^p dx + \frac{b^2 n^2 p(p-1)}{(m+1)(m+n+1)} \int x^{m+2n} \sinh[a + b x^n]^{p-2} dx
 \end{aligned}$$

Program code:

```

Int[x_^m_.*Sinh[a_+b_*x_^n_]^p_,x_Symbol] :=
  x^(m+1)*Sinh[a+b*x^n]^p/(m+1) -
  b*n*p*x^(m+n+1)*Cosh[a+b*x^n]*Sinh[a+b*x^n]^(p-1)/((m+1)*(m+n+1)) +
  b^2*n^2*p^2/((m+1)*(m+n+1))*Int[x^(m+2*n)*Sinh[a+b*x^n]^p,x] +
  b^2*n^2*p*(p-1)/((m+1)*(m+n+1))*Int[x^(m+2*n)*Sinh[a+b*x^n]^(p-2),x] /;
FreeQ[{a,b},x] && IntegersQ[m,n] && GtQ[p,1] && LtQ[0,2*n,1-m] && NeQ[m+n+1,0]

```

```

Int[x_^m_.*Cosh[a_+b_.*x_^n_]^p_,x_Symbol] :=
  x^(m+1)*Cosh[a+b*x^n]^p/(m+1) -
  b*n*p*x^(m+n+1)*Sinh[a+b*x^n]*Cosh[a+b*x^n]^(p-1)/((m+1)*(m+n+1)) +
  b^2*n^2*p^2/((m+1)*(m+n+1))*Int[x^(m+2*n)*Cosh[a+b*x^n]^p,x] -
  b^2*n^2*p*(p-1)/((m+1)*(m+n+1))*Int[x^(m+2*n)*Cosh[a+b*x^n]^(p-2),x] /;
FreeQ[{a,b},x] && IntegerQ[m,n] && GtQ[p,1] && LtQ[0,2*n,1-m] && NeQ[m+n+1,0]

```

5: $\int (e x)^m (a + b \sinh[c + d x^n])^p dx$ when $p \in \mathbb{Z} \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{F}$

Derivation: Integration by substitution

Basis: If $k \in \mathbb{Z}^+$, then $(e x)^m F[x] = \frac{k}{e} \text{Subst}[x^{k(m+1)-1} F[\frac{x^k}{e}], x, (e x)^{1/k}] \partial_x (e x)^{1/k}$

Rule: If $p \in \mathbb{Z} \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{F}$, let $k = \text{Denominator}[m]$, then

$$\int (e x)^m (a + b \sinh[c + d x^n])^p dx \rightarrow \frac{k}{e} \text{Subst}\left[\int x^{k(m+1)-1} \left(a + b \sinh\left[c + \frac{d x^{kn}}{e^n}\right]\right)^p dx, x, (e x)^{1/k}\right]$$

Program code:

```

Int[(e_.*x_)^m_*(a_+b_.*Sinh[c_+d_.*x_^n_] )^p_,x_Symbol] :=
  With[{k=Denominator[m]},
    k/e*Subst[Int[x^(k*(m+1)-1)*(a+b*Sinh[c+d*x^(k*n)/e^n])^p,x],x,(e*x)^(1/k)]] /;
FreeQ[{a,b,c,d,e},x] && IntegerQ[p] && IGtQ[n,0] && FractionQ[m]

```

```

Int[(e_.*x_)^m_*(a_+b_.*Cosh[c_+d_.*x_^n_] )^p_,x_Symbol] :=
  With[{k=Denominator[m]},
    k/e*Subst[Int[x^(k*(m+1)-1)*(a+b*Cosh[c+d*x^(k*n)/e^n])^p,x],x,(e*x)^(1/k)]] /;
FreeQ[{a,b,c,d,e},x] && IntegerQ[p] && IGtQ[n,0] && FractionQ[m]

```

$$\mathbf{6:} \int (e x)^m (a+b \sinh[c+d x^n])^p dx \text{ when } p-1 \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}^+$$

Derivation: Algebraic expansion

Rule: If $p-1 \in \mathbb{Z}^+ \wedge n \in \mathbb{Z}^+$, then

$$\int (e x)^m (a+b \sinh[c+d x^n])^p dx \rightarrow \int (e x)^m \text{TrigReduce}[(a+b \sinh[c+d x^n])^p, x] dx$$

Program code:

```
Int[(e.*x_)^m_.*(a_.+b_.*Sinh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(e*x)^m,(a+b*Sinh[c+d*x^n])^p,x],x] /;
FreeQ[{a,b,c,d,e,m},x] && IGtQ[p,1] && IGtQ[n,0]
```

```
Int[(e.*x_)^m_.*(a_.+b_.*Cosh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(e*x)^m,(a+b*Cosh[c+d*x^n])^p,x],x] /;
FreeQ[{a,b,c,d,e,m},x] && IGtQ[p,1] && IGtQ[n,0]
```

$$3. \int (e x)^m (a + b \sinh[c + d x^n])^p dx \text{ when } p < -1$$

$$1: \int x^m \sinh[a + b x^n]^p dx \text{ when } m - 2n + 1 = 0 \wedge p < -1 \wedge p \neq -2$$

Reference: G&R 2.477.1 special case when $m - 2n + 1 = 0$

Reference: G&R 2.477.2' special case with $m - 2n + 1 = 0$

Rule: If $m - 2n + 1 = 0 \wedge p < -1 \wedge p \neq -2$, then

$$\int x^m \sinh[a + b x^n]^p dx \rightarrow \frac{x^n \cosh[a + b x^n] \sinh[a + b x^n]^{p+1}}{b n (p+1)} - \frac{n \sinh[a + b x^n]^{p+2}}{b^2 n^2 (p+1) (p+2)} - \frac{p+2}{p+1} \int x^m \sinh[a + b x^n]^{p+2} dx$$

Program code:

```
Int[x_^m_.*Sinh[a_.+b_.*x_^n_]^p_,x_Symbol] :=
  x^n*Cosh[a+b*x^n]*Sinh[a+b*x^n]^(p+1)/(b*n*(p+1)) -
  n*Sinh[a+b*x^n]^(p+2)/(b^2*n^2*(p+1)*(p+2)) -
  (p+2)/(p+1)*Int[x^m*Sinh[a+b*x^n]^(p+2),x] /;
FreeQ[{a,b,m,n},x] && EqQ[m-2*n+1,0] && LtQ[p,-1] && NeQ[p,-2]
```

```
Int[x_^m_.*Cosh[a_.+b_.*x_^n_]^p_,x_Symbol] :=
  -x^n*Sinh[a+b*x^n]*Cosh[a+b*x^n]^(p+1)/(b*n*(p+1)) +
  n*Cosh[a+b*x^n]^(p+2)/(b^2*n^2*(p+1)*(p+2)) +
  (p+2)/(p+1)*Int[x^m*Cosh[a+b*x^n]^(p+2),x] /;
FreeQ[{a,b,m,n},x] && EqQ[m-2*n+1,0] && LtQ[p,-1] && NeQ[p,-2]
```

$$\mathbf{2:} \int x^m \sinh[a + b x^n]^p dx \text{ when } (m | n) \in \mathbb{Z} \wedge p < -1 \wedge p \neq -2 \wedge 0 < 2n < m+1$$

Reference: G&R 2.477.1

Reference: G&R 2.477.2

Rule: If $(m | n) \in \mathbb{Z} \wedge p < -1 \wedge p \neq -2 \wedge 0 < 2n < m+1$, then

$$\int x^m \sinh[a + b x^n]^p dx \rightarrow \frac{x^{m-n+1} \cosh[a + b x^n] \sinh[a + b x^n]^{p+1}}{b n (p+1)} - \frac{(m-n+1) x^{m-2n+1} \sinh[a + b x^n]^{p+2}}{b^2 n^2 (p+1)(p+2)} - \frac{p+2}{p+1} \int x^m \sinh[a + b x^n]^{p+2} dx + \frac{(m-n+1)(m-2n+1)}{b^2 n^2 (p+1)(p+2)} \int x^{m-2n} \sinh[a + b x^n]^{p+2} dx$$

Program code:

```
Int[x_^m_.*Sinh[a_+b_*x_^n_]^p_,x_Symbol] :=
  x^(m-n+1)*Cosh[a+b*x^n]*Sinh[a+b*x^n]^(p+1)/(b*n*(p+1)) -
  (m-n+1)*x^(m-2*n+1)*Sinh[a+b*x^n]^(p+2)/(b^2*n^2*(p+1)*(p+2)) -
  (p+2)/(p+1)*Int[x^m*Sinh[a+b*x^n]^(p+2),x] +
  (m-n+1)*(m-2*n+1)/(b^2*n^2*(p+1)*(p+2))*Int[x^(m-2*n)*Sinh[a+b*x^n]^(p+2),x] /;
FreeQ[{a,b},x] && IntegersQ[m,n] && LtQ[p,-1] && NeQ[p,-2] && LtQ[0,2*n,m+1]
```

```
Int[x_^m_.*Cosh[a_+b_*x_^n_]^p_,x_Symbol] :=
  -x^(m-n+1)*Sinh[a+b*x^n]*Cosh[a+b*x^n]^(p+1)/(b*n*(p+1)) +
  (m-n+1)*x^(m-2*n+1)*Cosh[a+b*x^n]^(p+2)/(b^2*n^2*(p+1)*(p+2)) +
  (p+2)/(p+1)*Int[x^m*Cosh[a+b*x^n]^(p+2),x] -
  (m-n+1)*(m-2*n+1)/(b^2*n^2*(p+1)*(p+2))*Int[x^(m-2*n)*Cosh[a+b*x^n]^(p+2),x] /;
FreeQ[{a,b},x] && IntegersQ[m,n] && LtQ[p,-1] && NeQ[p,-2] && LtQ[0,2*n,m+1]
```

$$2. \int (e x)^m (a + b \sinh[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge n \in \mathbb{Z}^-$$

$$1. \int (e x)^m (a + b \sinh[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{Q}$$

$$\textcolor{red}{1}: \int x^m (a + b \sinh[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{Z}$$

Derivation: Integration by substitution

Basis: If $n \in \mathbb{Z} \wedge m \in \mathbb{Z}$, then $x^m F[x^n] = -\text{Subst}\left[\frac{F[\frac{x^{-n}}{x^{m+2}}], x, \frac{1}{x}}{\partial_x \frac{1}{x}}\right]$

Rule: If $p \in \mathbb{Z} \wedge n \in \mathbb{Z}^- \wedge m \notin \mathbb{Q}$, then

$$\int x^m (a + b \sinh[c + d x^n])^p dx \rightarrow -\text{Subst}\left[\int \frac{(a + b \sinh[c + d x^{-n}])^p}{x^{m+2}} dx, x, \frac{1}{x}\right]$$

Program code:

```
Int[x_^m_.*(a_.+b_.*Sinh[c_.+d_.*x_^n_])^p_.,x_Symbol] :=
  -Subst[Int[(a+b*Sinh[c+d*x^(-n)])^p/x^(m+2),x],x,1/x] /;
FreeQ[{a,b,c,d},x] && IntegerQ[p] && ILtQ[n,0] && IntegerQ[m]
```

```
Int[x_^m_.*(a_.+b_.*Cosh[c_.+d_.*x_^n_])^p_.,x_Symbol] :=
  -Subst[Int[(a+b*Cosh[c+d*x^(-n)])^p/x^(m+2),x],x,1/x] /;
FreeQ[{a,b,c,d},x] && IntegerQ[p] && ILtQ[n,0] && IntegerQ[m]
```

$$\mathbf{2:} \int (e x)^m (a+b \sinh[c+d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{F}$$

Derivation: Integration by substitution

Basis: If $n \in \mathbb{Z} \wedge k > 1$, then $(e x)^m F[x^n] = -\frac{k}{e} \text{Subst}\left[\frac{F[e^{-n} x^{-k n}]}{x^{k(m+1)+1}}, x, \frac{1}{(e x)^{1/k}}\right] \partial_x \frac{1}{(e x)^{1/k}}$

Rule: If $p \in \mathbb{Z} \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{F}$, let $k = \text{Denominator}[m]$, then

$$\int (e x)^m (a+b \sinh[c+d x^n])^p dx \rightarrow -\frac{k}{e} \text{Subst}\left[\int \frac{(a+b \sinh[c+d e^{-n} x^{-k n}])^p}{x^{k(m+1)+1}} dx, x, \frac{1}{(e x)^{1/k}}\right]$$

Program code:

```
Int[(e_.**x_)^m_*(a_.+b_.**Sinh[c_.+d_.**x_^n_])^p_,x_Symbol] :=
  With[{k=Denominator[m]},
    -k/e*Subst[Int[(a+b*Sinh[c+d/(e^n*x^(k*n))])^p/x^(k*(m+1)+1),x],x,1/(e*x)^(1/k)]] /;
  FreeQ[{a,b,c,d,e},x] && IntegerQ[p] && ILtQ[n,0] && FractionQ[m]
```

```
Int[(e_.**x_)^m_*(a_.+b_.**Cosh[c_.+d_.**x_^n_])^p_,x_Symbol] :=
  With[{k=Denominator[m]},
    -k/e*Subst[Int[(a+b*Cosh[c+d/(e^n*x^(k*n))])^p/x^(k*(m+1)+1),x],x,1/(e*x)^(1/k)]] /;
  FreeQ[{a,b,c,d,e},x] && IntegerQ[p] && ILtQ[n,0] && FractionQ[m]
```


$$\mathbf{2:} \int (e x)^m (a+b \sinh[c+d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge n \in \mathbb{Z}^- \wedge m \notin \mathbb{Q}$$

Derivation: Piecewise constant extraction and integration by substitution

$$\text{Basis: } \partial_x \left((e x)^m (x^{-1})^m \right) = 0$$

$$\text{Basis: } F[x] = -\text{Subst} \left[\frac{F\left[\frac{x^{-1}}{x^2}\right]}{x^2}, x, \frac{1}{x} \right] \partial_x \frac{1}{x}$$

Rule: If $p \in \mathbb{Z} \wedge n \in \mathbb{Z}^- \wedge m \notin \mathbb{Q}$, then

$$\int (e x)^m (a+b \sinh[c+d x^n])^p dx \rightarrow (e x)^m (x^{-1})^m \int \frac{(a+b \sinh[c+d x^{-n}])^p}{(x^{-1})^m} dx \rightarrow - (e x)^m (x^{-1})^m \text{Subst} \left[\int \frac{(a+b \sinh[c+d x^{-n}])^p}{x^{m+2}} dx, x, \frac{1}{x} \right]$$

Program code:

```
Int[(e.*x_)^m_*(a_.+b_.*Sinh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  -(e*x)^m*(x^(-1))^m*Subst[Int[(a+b*Sinh[c+d*x^(-n)])^p/x^(m+2),x],x,1/x] /;
FreeQ[{a,b,c,d,e,m},x] && IntegerQ[p] && ILtQ[n,0] && Not[RationalQ[m]]
```

```
Int[(e.*x_)^m_*(a_.+b_.*Cosh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  -(e*x)^m*(x^(-1))^m*Subst[Int[(a+b*Cosh[c+d*x^(-n)])^p/x^(m+2),x],x,1/x] /;
FreeQ[{a,b,c,d,e,m},x] && IntegerQ[p] && ILtQ[n,0] && Not[RationalQ[m]]
```

$$3. \int (e x)^m (a + b \sinh[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge n \in \mathbb{F}$$

$$1: \int x^m (a + b \sinh[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge n \in \mathbb{F}$$

Derivation: Integration by substitution

Basis: If $k \in \mathbb{Z}^+$, then $x^m F[x^n] = k \text{Subst}[x^{k(m+1)-1} F[x^{kn}], x, x^{1/k}] \partial_x x^{1/k}$

Rule: If $p \in \mathbb{Z} \wedge n \in \mathbb{F}$, let $k = \text{Denominator}[n]$, then

$$\int x^m (a + b \sinh[c + d x^n])^p dx \rightarrow k \text{Subst}\left[\int x^{k(m+1)-1} (a + b \sinh[c + d x^{kn}])^p dx, x, x^{1/k}\right]$$

Program code:

```
Int[x_^m_.*(a_.+b_.*Sinh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Module[{k=Denominator[n]},
    k*Subst[Int[x^(k*(m+1)-1)*(a+b*Sinh[c+d*x^(k*n)])^p,x],x,x^(1/k)]] /;
  FreeQ[{a,b,c,d,m},x] && IntegerQ[p] && FractionQ[n]
```

```
Int[x_^m_.*(a_.+b_.*Cosh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Module[{k=Denominator[n]},
    k*Subst[Int[x^(k*(m+1)-1)*(a+b*Cosh[c+d*x^(k*n)])^p,x],x,x^(1/k)]] /;
  FreeQ[{a,b,c,d,m},x] && IntegerQ[p] && FractionQ[n]
```

2: $\int (e x)^m (a + b \sinh[c + d x^n])^p dx$ when $p \in \mathbb{Z} \wedge n \in \mathbb{F}$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(e x)^m}{x^m} = 0$

Rule: If $p \in \mathbb{Z} \wedge n \in \mathbb{F}$, then

$$\int (e x)^m (a + b \sinh[c + d x^n])^p dx \rightarrow \frac{e^{\text{IntPart}[m]} (e x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (a + b \sinh[c + d x^n])^p dx$$

Program code:

```
Int[(e*x_)^m*(a_+b_.*Sinh[c_+d_.*x_^n_])^p_,x_Symbol] :=
  e^IntPart[m]*(e*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*Sinh[c+d*x^n])^p,x] /;
FreeQ[{a,b,c,d,e,m},x] && IntegerQ[p] && FractionQ[n]
```

```
Int[(e*x_)^m*(a_+b_.*Cosh[c_+d_.*x_^n_])^p_,x_Symbol] :=
  e^IntPart[m]*(e*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*Cosh[c+d*x^n])^p,x] /;
FreeQ[{a,b,c,d,e,m},x] && IntegerQ[p] && FractionQ[n]
```

$$4. \int (e x)^m (a + b \sinh[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge m \neq -1 \wedge \frac{n}{m+1} \in \mathbb{Z}^+$$

$$1: \int x^m (a + b \sinh[c + d x^n])^p dx \text{ when } p \in \mathbb{Z} \wedge m \neq -1 \wedge \frac{n}{m+1} \in \mathbb{Z}^+$$

Derivation: Integration by substitution

Basis: If $\frac{n}{m+1} \in \mathbb{Z}$, then $x^m F[x^n] = \frac{1}{m+1} \text{Subst}[F[x^{\frac{n}{m+1}}], x, x^{m+1}] \partial_x x^{m+1}$

Rule: If $p \in \mathbb{Z} \wedge m \neq -1 \wedge \frac{n}{m+1} \in \mathbb{Z}^+$, then

$$\int x^m (a + b \sinh[c + d x^n])^p dx \rightarrow \frac{1}{m+1} \text{Subst}\left[\int (a + b \sinh[c + d x^{\frac{n}{m+1}}])^p dx, x, x^{m+1}\right]$$

Program code:

```
Int[x_^m.*(a_.+b_.*Sinh[c_.+d_.*x_^n_])^p_.,x_Symbol] :=
  1/(m+1)*Subst[Int[(a+b*Sinh[c+d*x^Simplify[n/(m+1)]])^p,x],x,x^(m+1)] /;
FreeQ[{a,b,c,d,m,n},x] && IntegerQ[p] && NeQ[m,-1] && IGtQ[Simplify[n/(m+1)],0] && Not[IntegerQ[n]]
```

```
Int[x_^m.*(a_.+b_.*Cosh[c_.+d_.*x_^n_])^p_.,x_Symbol] :=
  1/(m+1)*Subst[Int[(a+b*Cosh[c+d*x^Simplify[n/(m+1)]])^p,x],x,x^(m+1)] /;
FreeQ[{a,b,c,d,m,n},x] && IntegerQ[p] && NeQ[m,-1] && IGtQ[Simplify[n/(m+1)],0] && Not[IntegerQ[n]]
```

2: $\int (e x)^m (a + b \sinh[c + d x^n])^p dx$ when $p \in \mathbb{Z} \wedge m \neq -1 \wedge \frac{n}{m+1} \in \mathbb{Z}^+$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(e x)^m}{x^m} = 0$

Rule: If $p \in \mathbb{Z} \wedge m \neq -1 \wedge \frac{n}{m+1} \in \mathbb{Z}^+$, then

$$\int (e x)^m (a + b \sinh[c + d x^n])^p dx \rightarrow \frac{e^{\text{IntPart}[m]} (e x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (a + b \sinh[c + d x^n])^p dx$$

Program code:

```
Int[(e*x_)^m*(a_.+b_.*Sinh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  e^IntPart[m]*(e*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*Sinh[c+d*x^n])^p,x] /;
FreeQ[{a,b,c,d,e,m,n},x] && IntegerQ[p] && NeQ[m,-1] && IGtQ[Simplify[n/(m+1)],0] && Not[IntegerQ[n]]
```

```
Int[(e*x_)^m*(a_.+b_.*Cosh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  e^IntPart[m]*(e*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*Cosh[c+d*x^n])^p,x] /;
FreeQ[{a,b,c,d,e,m,n},x] && IntegerQ[p] && NeQ[m,-1] && IGtQ[Simplify[n/(m+1)],0] && Not[IntegerQ[n]]
```

5. $\int (e x)^m (a + b \sinh[c + d x^n])^p dx$ when $p \in \mathbb{Z}^+$

1: $\int (e x)^m \sinh[c + d x^n] dx$

Derivation: Algebraic expansion

Basis: $\sinh[z] == \frac{e^z}{2} - \frac{e^{-z}}{2}$

Basis: $\cosh[z] == \frac{e^z}{2} + \frac{e^{-z}}{2}$

Rule:

$$\int (e x)^m \sinh[c + d x^n] dx \rightarrow \frac{1}{2} \int (e x)^m e^{c+d x^n} dx - \frac{1}{2} \int (e x)^m e^{-c-d x^n} dx$$

Program code:

```
Int[(e_.**x_)^m_.*Sinh[c_.+d_.**x_^n_],x_Symbol] :=
  1/2*Int[(e*x)^m*E^(c+d*x^n),x] - 1/2*Int[(e*x)^m*E^(-c-d*x^n),x] /;
FreeQ[{c,d,e,m,n},x]
```

```
Int[(e_.**x_)^m_.*Cosh[c_.+d_.**x_^n_],x_Symbol] :=
  1/2*Int[(e*x)^m*E^(c+d*x^n),x] + 1/2*Int[(e*x)^m*E^(-c-d*x^n),x] /;
FreeQ[{c,d,e,m,n},x]
```

2: $\int (e x)^m (a + b \sinh[c + d x^n])^p dx$ when $p \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule: If $p \in \mathbb{Z}^+$, then

$$\int (e x)^m (a + b \sinh[c + d x^n])^p dx \rightarrow \int (e x)^m \text{TrigReduce}[(a + b \sinh[c + d x^n])^p, x] dx$$

Program code:

```
Int[(e.*x_)^m_.*(a_.+b_.*Sinh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(e*x)^m,(a+b*Sinh[c+d*x^n])^p,x],x] /;
FreeQ[{a,b,c,d,e,m,n},x] && IGtQ[p,0]
```

```
Int[(e.*x_)^m_.*(a_.+b_.*Cosh[c_.+d_.*x_^n_])^p_,x_Symbol] :=
  Int[ExpandTrigReduce[(e*x)^m,(a+b*Cosh[c+d*x^n])^p,x],x] /;
FreeQ[{a,b,c,d,e,m,n},x] && IGtQ[p,0]
```

S: $\int x^m (a + b \sinh[c + d u^n])^p dx$ when $u = f + g x \wedge m \in \mathbb{Z}$

Derivation: Integration by substitution

Basis: If $m \in \mathbb{Z}$, then $x^m F[f + g x] = \frac{1}{g^{m+1}} \text{Subst}[(x - f)^m F[x], x, f + g x] \partial_x (f + g x)$

Rule: If $u = f + g x \wedge m \in \mathbb{Z}$, then

$$\int x^m (a + b \sinh[c + d u^n])^p dx \rightarrow \frac{1}{g^{m+1}} \text{Subst}\left[\int (x - f)^m (a + b \sinh[c + d x^n])^p dx, x, u\right]$$

Program code:

```
Int[x_^m_.*(a_.+b_.*Sinh[c_.+d_.*u_^n_])^p_,x_Symbol] :=
  1/Coefficient[u,x,1]^(m+1)*Subst[Int[(x-Coefficient[u,x,0])^m*(a+b*Sinh[c+d*x^n])^p,x],x,u] /;
FreeQ[{a,b,c,d,n,p},x] && LinearQ[u,x] && NeQ[u,x] && IntegerQ[m]
```

```
Int[x_^m_.*(a_.+b_.*Cosh[c_.+d_.*u_^n_])^p_,x_Symbol] :=
  1/Coefficient[u,x,1]^(m+1)*Subst[Int[(x-Coefficient[u,x,0])^m*(a+b*Cosh[c+d*x^n])^p,x],x,u] /;
FreeQ[{a,b,c,d,n,p},x] && LinearQ[u,x] && NeQ[u,x] && IntegerQ[m]
```


X: $\int (e x)^m (a + b \sinh[c + d u^n])^p dx$ when $u = f + g x$

Rule:

$$\int (e x)^m (a + b \sinh[c + d u^n])^p dx \rightarrow \int (e x)^m (a + b \sinh[c + d u^n])^p dx$$

Program code:

```
Int[(e.*x_)^m_.*(a_.+b_.*Sinh[c_.+d_.*u_^n_])^p_.,x_Symbol] :=
  Unintegrable[(e*x)^m*(a+b*Sinh[c+d*u^n])^p,x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && LinearQ[u,x]
```

```
Int[(e.*x_)^m_.*(a_.+b_.*Cosh[c_.+d_.*u_^n_])^p_.,x_Symbol] :=
  Unintegrable[(e*x)^m*(a+b*Cosh[c+d*u^n])^p,x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && LinearQ[u,x]
```

N: $\int (e x)^m (a + b \sinh[u])^p dx$ when $u = c + d x^n$

Derivation: Algebraic normalization

Rule: If $u = c + d x^n$, then

$$\int (e x)^m (a + b \sinh[u])^p dx \rightarrow \int (e x)^m (a + b \sinh[c + d x^n])^p dx$$

Program code:

```
Int[(e.*x_)^m_.*(a_.+b_.*Sinh[u_])^p_.,x_Symbol] :=
  Int[(e*x)^m*(a+b*Sinh[ExpandToSum[u,x]])^p,x] /;
FreeQ[{a,b,e,m,p},x] && BinomialQ[u,x] && Not[BinomialMatchQ[u,x]]
```

```

Int[(e*x_)^m_.*(a_+b_.*Cosh[u_])^p_,x_Symbol] :=
  Int[(e*x)^m*(a+b*Cosh[ExpandToSum[u,x]])^p,x] /;
FreeQ[{a,b,e,m,p},x] && BinomialQ[u,x] && Not[BinomialMatchQ[u,x]]

```

Rules for integrands of the form $x^m \sinh[a + b x^n]^p \cosh[a + b x^n]$

1: $\int x^{n-1} \sinh[a + b x^n]^p \cosh[a + b x^n] dx$ when $p \neq -1$

Derivation: Power rule for integration

Rule: If $p \neq -1$, then

$$\int x^{n-1} \sinh[a + b x^n]^p \cosh[a + b x^n] dx \rightarrow \frac{\sinh[a + b x^n]^{p+1}}{b n (p+1)}$$

Program code:

```

Int[x_^m_.*Sinh[a_+b_.*x_^n_]^p_.*Cosh[a_+b_.*x_^n_],x_Symbol] :=
  Sinh[a+b*x^n]^(p+1)/(b*n*(p+1)) /;
FreeQ[{a,b,m,n,p},x] && EqQ[m,n-1] && NeQ[p,-1]

```

```

Int[x_^m_.*Cosh[a_+b_.*x_^n_]^p_.*Sinh[a_+b_.*x_^n_],x_Symbol] :=
  Cosh[a+b*x^n]^(p+1)/(b*n*(p+1)) /;
FreeQ[{a,b,m,n,p},x] && EqQ[m,n-1] && NeQ[p,-1]

```

2: $\int x^m \sinh[a + b x^n]^p \cosh[a + b x^n] dx$ when $0 < n < m + 1 \wedge p \neq -1$

Reference: G&R 2.479.6

Reference: G&R 2.479.3

Derivation: Integration by parts

Basis: $x^m \cosh[a + b x^n] \sinh[a + b x^n]^p = x^{m-n+1} \partial_x \frac{\sinh[a + b x^n]^{p+1}}{b n (p+1)}$

Rule: If $0 < n < m + 1 \wedge p \neq -1$, then

$$\int x^m \sinh[a + b x^n]^p \cosh[a + b x^n] dx \rightarrow \frac{x^{m-n+1} \sinh[a + b x^n]^{p+1}}{b n (p+1)} - \frac{m-n+1}{b n (p+1)} \int x^{m-n} \sinh[a + b x^n]^{p+1} dx$$

Program code:

```
Int[x_^m_.*Sinh[a_.+b_.*x_^n_]^p_.*Cosh[a_.+b_.*x_^n_],x_Symbol] :=
  x^(m-n+1)*Sinh[a+b*x^n]^(p+1)/(b*n*(p+1)) -
  (m-n+1)/(b*n*(p+1))*Int[x^(m-n)*Sinh[a+b*x^n]^(p+1),x] /;
FreeQ[{a,b,p},x] && LtQ[0,n,m+1] && NeQ[p,-1]
```

```
Int[x_^m_.*Cosh[a_.+b_.*x_^n_]^p_.*Sinh[a_.+b_.*x_^n_],x_Symbol] :=
  x^(m-n+1)*Cosh[a+b*x^n]^(p+1)/(b*n*(p+1)) -
  (m-n+1)/(b*n*(p+1))*Int[x^(m-n)*Cosh[a+b*x^n]^(p+1),x] /;
FreeQ[{a,b,p},x] && LtQ[0,n,m+1] && NeQ[p,-1]
```