

Rules for integrands of the form $\text{Trig}[d + e x]^m (a + b \sin[d + e x]^n + c \sin[d + e x]^{2n})^p$

$$1. \int (a + b \sin[d + e x]^n + c \sin[d + e x]^{2n})^p dx$$

$$1. \int (a + b \sin[d + e x]^n + c \sin[d + e x]^{2n})^p dx \text{ when } b^2 - 4ac = 0$$

$$1: \int (a + b \sin[d + e x]^n + c \sin[d + e x]^{2n})^p dx \text{ when } b^2 - 4ac = 0 \wedge p \in \mathbb{Z}$$

Derivation: Algebraic simplification

$$\text{Basis: If } b^2 - 4ac = 0, \text{ then } a + b z + c z^2 = \frac{(b+2cz)^2}{4c}$$

Rule: If $b^2 - 4ac = 0 \wedge p \in \mathbb{Z}$, then

$$\int (a + b \sin[d + e x]^n + c \sin[d + e x]^{2n})^p dx \rightarrow \frac{1}{4^p c^p} \int (b + 2c \sin[d + e x]^n)^{2p} dx$$

Program code:

```
Int[(a_.+b_.sin[d_.+e_.x_]^n_.+c_.sin[d_.+e_.x_]^n2_.)^p_. ,x_Symbol] :=
  1/(4^p*c^p)*Int[(b+2*c*Sin[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[n2,2*n] && EqQ[b^2-4*a*c,0] && IntegerQ[p]
```

```
Int[(a_.+b_.cos[d_.+e_.x_]^n_.+c_.cos[d_.+e_.x_]^n2_.)^p_. ,x_Symbol] :=
  1/(4^p*c^p)*Int[(b+2*c*Cos[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[n2,2*n] && EqQ[b^2-4*a*c,0] && IntegerQ[p]
```

$$2: \int (a + b \sin[d + e x]^n + c \sin[d + e x]^{2n})^p dx \text{ when } b^2 - 4ac = 0 \wedge p \notin \mathbb{Z}$$

Derivation: Piecewise constant extraction

$$\text{Basis: If } b^2 - 4ac = 0, \text{ then } \partial_x \frac{(a+bF[x]+cF[x]^2)^p}{(b+2cF[x])^{2p}} = 0$$

Rule: If $b^2 - 4ac = 0 \wedge p \notin \mathbb{Z}$, then

$$\int (a + b \sin[d + e x]^n + c \sin[d + e x]^{2n})^p dx \rightarrow \frac{(a + b \sin[d + e x]^n + c \sin[d + e x]^{2n})^p}{(b + 2 c \sin[d + e x]^n)^{2p}} \int (b + 2 c \sin[d + e x]^n)^{2p} dx$$

Program code:

```
Int[(a_.+b_.*sin[d_.+e_.*x_]^n_.+c_.*sin[d_.+e_.*x_]^n2_.)^p_,x_Symbol] :=
  (a+b*Sin[d+e*x]^n+c*Sin[d+e*x]^(2*n))^p/(b+2*c*Sin[d+e*x]^n)^(2*p)*Int[u*(b+2*c*Sin[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,n,p},x] && EqQ[n2,2*n] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[p]]
```

```
Int[(a_.+b_.*cos[d_.+e_.*x_]^n_.+c_.*cos[d_.+e_.*x_]^n2_.)^p_,x_Symbol] :=
  (a+b*Cos[d+e*x]^n+c*Cos[d+e*x]^(2*n))^p/(b+2*c*Cos[d+e*x]^n)^(2*p)*Int[u*(b+2*c*Cos[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,n,p},x] && EqQ[n2,2*n] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[p]]
```

2. $\int (a + b \sin[d + e x]^n + c \sin[d + e x]^{2n})^p dx$ when $b^2 - 4ac \neq 0$

1: $\int \frac{1}{a + b \sin[d + e x]^n + c \sin[d + e x]^{2n}} dx$ when $b^2 - 4ac \neq 0$

Derivation: Algebraic expansion

Basis: If $q = \sqrt{b^2 - 4ac}$, then $\frac{1}{a+b \sin[d+ex]^n + c \sin[d+ex]^{2n}} = \frac{2c}{q(b-q+2c \sin[d+ex]^n)} - \frac{2c}{q(b+q+2c \sin[d+ex]^n)}$

■ Rule: If $b^2 - 4ac \neq 0$, let $q = \sqrt{b^2 - 4ac}$, then

$$\int \frac{1}{a + b \sin[d + e x]^n + c \sin[d + e x]^{2n}} dx \rightarrow \frac{2c}{q} \int \frac{1}{b - q + 2c \sin[d + e x]^n} dx - \frac{2c}{q} \int \frac{1}{b + q + 2c \sin[d + e x]^n} dx$$

Program code:

```
Int[1/(a_.+b_.*sin[d_.+e_.*x_]^n_.+c_.*sin[d_.+e_.*x_]^(2n_.),x_Symbol] :=
Module[{q=Rt[b^2-4*a*c,2]},
2*c/q*Int[1/(b-q+2*c*Sin[d+e*x]^n),x] -
2*c/q*Int[1/(b+q+2*c*Sin[d+e*x]^n),x] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0]
```

```
Int[1/(a_.+b_.*cos[d_.+e_.*x_]^n_.+c_.*cos[d_.+e_.*x_]^(2n_.),x_Symbol] :=
Module[{q=Rt[b^2-4*a*c,2]},
2*c/q*Int[1/(b-q+2*c*Cos[d+e*x]^n),x] -
2*c/q*Int[1/(b+q+2*c*Cos[d+e*x]^n),x] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0]
```

$$2. \int \sin[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx$$

$$1. \int \sin[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \text{ when } b^2-4ac=0$$

$$1: \int \sin[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \text{ when } b^2-4ac=0 \wedge p \in \mathbb{Z}$$

Derivation: Algebraic simplification

Basis: If $b^2-4ac=0$, then $a+bz+cz^2 = \frac{(b+2cz)^2}{4c}$

Rule: If $b^2-4ac=0 \wedge p \in \mathbb{Z}$, then

$$\int \sin[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \rightarrow \frac{1}{4^p c^p} \int \sin[d+e x]^m (b+2c \sin[d+e x]^n)^{2p} dx$$

Program code:

```
Int[sin[d_+e_.x_]^m_.*(a_+b_.sin[d_+e_.x_]^n_+c_.sin[d_+e_.x_]^(2n_.))^p_,x_Symbol] :=
  1/(4^p*c^p)*Int[Sin[d+e*x]^m*(b+2*c*Sin[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n},x] && EqQ[n2,2*n] && EqQ[b^2-4*a*c,0] && IntegerQ[p]
```

```
Int[cos[d_+e_.x_]^m_.*(a_+b_.cos[d_+e_.x_]^n_+c_.cos[d_+e_.x_]^(2n_.))^p_,x_Symbol] :=
  1/(4^p*c^p)*Int[Cos[d+e*x]^m*(b+2*c*Cos[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n},x] && EqQ[n2,2*n] && EqQ[b^2-4*a*c,0] && IntegerQ[p]
```

$$\mathbf{2:} \int \sin[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2 n})^p dx \text{ when } b^2-4 a c == 0 \wedge p \notin \mathbb{Z}$$

Derivation: Piecewise constant extraction

$$\text{Basis: If } b^2-4 a c == 0, \text{ then } \partial_x \frac{(a+b F[x]+c F[x]^2)^p}{(b+2 c F[x])^{2 p}} == 0$$

Rule: If $b^2-4 a c == 0 \wedge p \notin \mathbb{Z}$, then

$$\int \sin[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2 n})^p dx \rightarrow \frac{(a+b \sin[d+e x]^n+c \sin[d+e x]^{2 n})^p}{(b+2 c \sin[d+e x]^n)^{2 p}} \int \sin[d+e x]^m (b+2 c \sin[d+e x]^n)^{2 p} dx$$

Program code:

```
Int[sin[d_+e_.x_]^m_.*(a_+b_.sin[d_+e_.x_]^n_+c_.sin[d_+e_.x_]^(2n_))^(p_,x_Symbol)] :=
  (a+b*sin[d+e*x]^n+c*sin[d+e*x]^(2*n))^p/(b+2*c*sin[d+e*x]^n)^(2*p)*Int[Sin[d+e*x]^m*(b+2*c*sin[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && EqQ[n2,2*n] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[p]]
```

```
Int[cos[d_+e_.x_]^m_.*(a_+b_.cos[d_+e_.x_]^n_+c_.cos[d_+e_.x_]^(2n_))^(p_,x_Symbol)] :=
  (a+b*cos[d+e*x]^n+c*cos[d+e*x]^(2*n))^p/(b+2*c*cos[d+e*x]^n)^(2*p)*Int[Cos[d+e*x]^m*(b+2*c*cos[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && EqQ[n2,2*n] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[p]]
```

$$2. \int \sin[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2 n})^p dx \text{ when } b^2-4 a c \neq 0$$

$$\mathbf{1:} \int \sin[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2 n})^p dx \text{ when } \frac{m}{2} \in \mathbb{Z} \wedge b^2-4 a c \neq 0 \wedge \frac{n}{2} \in \mathbb{Z} \wedge p \in \mathbb{Z}$$

Derivation: Integration by substitution

$$\text{Basis: } \sin[z]^2 == \frac{1}{1+\cot[z]^2}$$

Basis: If $\frac{m}{2} \in \mathbb{Z}$, then

$$\sin[d+e x]^m F[\sin[d+e x]^2] = -\frac{1}{e} \text{Subst}\left[\frac{F\left[\frac{1}{1+x^2}\right]}{(1+x^2)^{m/2+1}}, x, \cot[d+e x]\right] \partial_x \cot[d+e x]$$

Rule: If $\frac{m}{2} \in \mathbb{Z} \wedge b^2 - 4 a c \neq 0 \wedge \frac{n}{2} \in \mathbb{Z} \wedge p \in \mathbb{Z}$, then

$$\int \sin[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \rightarrow -\frac{1}{e} \text{Subst}\left[\int \frac{(c+b(1+x^2)^{n/2}+a(1+x^2)^n)^p}{(1+x^2)^{m/2+n p+1}} dx, x, \cot[d+e x]\right]$$

Program code:

```
Int[sin[d_+e_.x_]^m_*(a_+b_.sin[d_+e_.x_]^n_+c_.sin[d_+e_.x_]^(2n_)^p_,x_Symbol] :=
Module[{f=FreeFactors[Cot[d+e*x],x]},
-f/e*Subst[Int[ExpandToSum[c+b*(1+x^2)^(n/2)+a*(1+x^2)^n,x]^p/(1+f^2*x^2)^(m/2+n*p+1),x],x,Cot[d+e*x]/f]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[n2,2*n] && IntegerQ[m/2] && NeQ[b^2-4*a*c,0] && IntegerQ[n/2] && IntegerQ[p]
```

```
Int[cos[d_+e_.x_]^m_*(a_+b_.cos[d_+e_.x_]^n_+c_.cos[d_+e_.x_]^(2n_)^p_,x_Symbol] :=
Module[{f=FreeFactors[Tan[d+e*x],x]},
f/e*Subst[Int[ExpandToSum[c+b*(1+x^2)^(n/2)+a*(1+x^2)^n,x]^p/(1+f^2*x^2)^(m/2+n*p+1),x],x,Tan[d+e*x]/f]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[n2,2*n] && IntegerQ[m/2] && NeQ[b^2-4*a*c,0] && IntegerQ[n/2] && IntegerQ[p]
```

2: $\int \sin[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx$ when $b^2 - 4 a c \neq 0 \wedge (m | n | p) \in \mathbb{Z}$

Derivation: Algebraic expansion

Rule: If $b^2 - 4 a c \neq 0 \wedge (m | n | p) \in \mathbb{Z}$, then

$$\int \sin[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \rightarrow \int \text{ExpandTrig}[\sin[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p, x] dx$$

Proeram code:

```
Int[sin[d_+e_.x_]^m_*(a_+b_.sin[d_+e_.x_]^n_+c_.sin[d_+e_.x_]^(2n_)^p_,x_Symbol] :=
Int[ExpandTrig[sin[d+e*x]^m*(a+b*sin[d+e*x]^n+c*sin[d+e*x]^(2*n))^p,x],x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IntegersQ[m,n,p]
```

```

Int[cos[d_+e_.*x_]^m_.*(a_+b_.*cos[d_+e_.*x_]^n_+c_.*cos[d_+e_.*x_]^(2n_))^p_,x_Symbol] :=
  Int[ExpandTrig[cos[d+e*x]^m*(a+b*cos[d+e*x]^n+c*cos[d+e*x]^(2*n))^p,x],x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IntegerQ[m,n,p]

```

$$3. \int \cos[d+ex]^m (a+b \sin[d+ex]^n+c \sin[d+ex]^{2n})^p dx$$

$$1: \int \cos[d+ex]^m (a+b \sin[d+ex]^n+c \sin[d+ex]^{2n})^p dx \text{ when } \frac{m-1}{2} \in \mathbb{Z}$$

Derivation: Integration by substitution

Basis: If $\frac{m-1}{2} \in \mathbb{Z}$, then

$$\cos[d+ex]^m F[\sin[d+ex]] = \frac{1}{e} \text{Subst}\left[\left(1-x^2\right)^{\frac{m-1}{2}} F[x], x, \sin[d+ex]\right] \partial_x \sin[d+ex]$$

Rule: If $\frac{m-1}{2} \in \mathbb{Z}$, then

$$\int \cos[d+ex]^m (a+b \sin[d+ex]^n+c \sin[d+ex]^{2n})^p dx \rightarrow \frac{1}{e} \text{Subst}\left[\int (1-x^2)^{\frac{m-1}{2}} (a+b x^n+c x^{2n})^p dx, x, \sin[d+ex]\right]$$

Program code:

```

Int[cos[d_+e_.*x_]^m_.*(a_+b_.*(f_.*sin[d_+e_.*x_]^(n_+c_.*(f_.*sin[d_+e_.*x_]^(2n_))^(p_))^(p_),x_Symbol] :=
  Module[{g=FreeFactors[Sin[d+e*x],x]},
    g/e*Subst[Int[(1-g^2*x^2)^(m-1)/2*(a+b*(f*g*x)^n+c*(f*g*x)^(2*n))^p,x],x,Sin[d+e*x]/g] /;
FreeQ[{a,b,c,d,e,f,n,p},x] && EqQ[n2,2*n] && IntegerQ[(m-1)/2]

```

```

Int[sin[d_+e_.*x_]^m_.*(a_+b_.*(f_.*cos[d_+e_.*x_]^(n_+c_.*(f_.*cos[d_+e_.*x_]^(2n_))^(p_))^(p_),x_Symbol] :=
  Module[{g=FreeFactors[Cos[d+e*x],x]},
    -g/e*Subst[Int[(1-g^2*x^2)^(m-1)/2*(a+b*(f*g*x)^n+c*(f*g*x)^(2*n))^p,x],x,Cos[d+e*x]/g] /;
FreeQ[{a,b,c,d,e,f,n,p},x] && EqQ[n2,2*n] && IntegerQ[(m-1)/2]

```

$$2. \int \cos[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z}$$

$$1. \int \cos[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4ac=0$$

$$1: \int \cos[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4ac=0 \wedge p \in \mathbb{Z}$$

Derivation: Algebraic simplification

Basis: If $b^2-4ac=0$, then $a+bz+cz^2 = \frac{(b+2cz)^2}{4c}$

Rule: If $\frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4ac=0 \wedge p \in \mathbb{Z}$, then

$$\int \cos[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \rightarrow \frac{1}{4^p c^p} \int \cos[d+e x]^m (b+2c \sin[d+e x]^n)^{2p} dx$$

Program code:

```
Int[cos[d_+e_.x_]^m_*(a_+b_.sin[d_+e_.x_]^n_+c_.sin[d_+e_.x_]^n2_.)^p_,x_Symbol] :=
  1/(4^p*c^p)*Int[Cos[d+e*x]^m*(b+2*c*sin[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n},x] && EqQ[n2,2*n] && Not[IntegerQ[(m-1)/2]] && EqQ[b^2-4*a*c,0] && IntegerQ[p]
```

```
Int[sin[d_+e_.x_]^m_*(a_+b_.cos[d_+e_.x_]^n_+c_.cos[d_+e_.x_]^n2_.)^p_,x_Symbol] :=
  1/(4^p*c^p)*Int[Sin[d+e*x]^m*(b+2*c*cos[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n},x] && EqQ[n2,2*n] && Not[IntegerQ[(m-1)/2]] && EqQ[b^2-4*a*c,0] && IntegerQ[p]
```


$$\mathbf{2:} \int \cos [d+e x]^m (a+b \sin [d+e x]^n+c \sin [d+e x]^{2 n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4 a c=0 \wedge p \notin \mathbb{Z}$$

Derivation: Piecewise constant extraction

$$\text{Basis: If } b^2-4 a c=0, \text{ then } \partial_x \frac{(a+b F[x]+c F[x]^2)^p}{(b+2 c F[x])^{2 p}}=0$$

Rule: If $\frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4 a c=0 \wedge p \notin \mathbb{Z}$, then

$$\int \cos [d+e x]^m (a+b \sin [d+e x]^n+c \sin [d+e x]^{2 n})^p dx \rightarrow \frac{(a+b \sin [d+e x]^n+c \sin [d+e x]^{2 n})^p}{(b+2 c \sin [d+e x]^n)^{2 p}} \int \cos [d+e x]^m (b+2 c \sin [d+e x]^n)^{2 p} dx$$

Program code:

```
Int[cos[d_+e_.x_]^m_*(a_+b_.sin[d_+e_.x_]^n_+c_.sin[d_+e_.x_]^(2n_))^(p_,x_Symbol] :=
  (a+b*sin[d+e*x]^n+c*sin[d+e*x]^(2*n))^p/(b+2*c*sin[d+e*x]^n)^(2*p)*Int[Cos[d+e*x]^m*(b+2*c*sin[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && EqQ[n2,2*n] && Not[IntegerQ[(m-1)/2]] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[p]]
```

```
Int[sin[d_+e_.x_]^m_*(a_+b_.cos[d_+e_.x_]^n_+c_.cos[d_+e_.x_]^(2n_))^(p_,x_Symbol] :=
  (a+b*cos[d+e*x]^n+c*cos[d+e*x]^(2*n))^p/(b+2*c*cos[d+e*x]^n)^(2*p)*Int[Sin[d+e*x]^m*(b+2*c*cos[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && EqQ[n2,2*n] && Not[IntegerQ[(m-1)/2]] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[p]]
```

$$2. \int \cos [d+e x]^m (a+b \sin [d+e x]^n+c \sin [d+e x]^{2 n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4 a c \neq 0$$

$$\mathbf{1:} \int \cos [d+e x]^m (a+b \sin [d+e x]^n+c \sin [d+e x]^{2 n})^p dx \text{ when } \frac{m}{2} \in \mathbb{Z} \wedge b^2-4 a c \neq 0 \wedge \frac{n}{2} \in \mathbb{Z} \wedge p \in \mathbb{Z}$$

Derivation: Integration by substitution

$$\text{Basis: } \sin [z]^2 == \frac{1}{1+\cot [z]^2}$$

$$\text{Basis: } \cos [z]^2 == \frac{\cot [z]^2}{1+\cot [z]^2}$$

Basis: If $\frac{m}{2} \in \mathbb{Z}$, then

$$\cos[d+e x]^m F[\sin[d+e x]^2] = -\frac{1}{e} \text{Subst}\left[\frac{x^m F\left[\frac{1}{1+x^2}\right]}{(1+x^2)^{m/2+1}}, x, \cot[d+e x]\right] \partial_x \cot[d+e x]$$

Rule: If $\frac{m}{2} \in \mathbb{Z} \wedge b^2 - 4ac \neq 0 \wedge \frac{n}{2} \in \mathbb{Z} \wedge p \in \mathbb{Z}$, then

$$\int \cos[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \rightarrow -\frac{1}{e} \text{Subst}\left[\int \frac{x^m (c+b(1+x^2)^{n/2}+a(1+x^2)^n)^p}{(1+x^2)^{m/2+n p+1}} dx, x, \cot[d+e x]\right]$$

Program code:

```
Int[cos[d_+e_.*x_]^m_*(a_+b_.*sin[d_+e_.*x_]^n_+c_.*sin[d_+e_.*x_]^(2n_))^p_,x_Symbol] :=
Module[{f=FreeFactors[Cot[d+e*x],x]},
-f^(m+1)/e*Subst[Int[x^m*ExpandToSum[c+b*(1+x^2)^(n/2)+a*(1+x^2)^n,x]^p/(1+f^2*x^2)^(m/2+n*p+1),x],x,Cot[d+e*x]/f]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[n2,2*n] && IntegerQ[m/2] && NeQ[b^2-4*a*c,0] && IntegerQ[n/2] && IntegerQ[p]
```

```
Int[sin[d_+e_.*x_]^m_*(a_+b_.*cos[d_+e_.*x_]^n_+c_.*cos[d_+e_.*x_]^(2n_))^p_,x_Symbol] :=
Module[{f=FreeFactors[Tan[d+e*x],x]},
f^(m+1)/e*Subst[Int[x^m*ExpandToSum[c+b*(1+x^2)^(n/2)+a*(1+x^2)^n,x]^p/(1+f^2*x^2)^(m/2+n*p+1),x],x,Tan[d+e*x]/f]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[n2,2*n] && IntegerQ[m/2] && NeQ[b^2-4*a*c,0] && IntegerQ[n/2] && IntegerQ[p]
```

$$\mathbf{2:} \int \cos[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \text{ when } \frac{m}{2} \in \mathbb{Z} \wedge b^2-4ac \neq 0 \wedge (n|p) \in \mathbb{Z}$$

Derivation: Algebraic expansion

$$\text{Basis: } \cos[z]^2 == 1 - \sin[z]^2$$

Rule: If $\frac{m}{2} \in \mathbb{Z} \wedge b^2-4ac \neq 0 \wedge (n|p) \in \mathbb{Z}$, then

$$\int \cos[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \rightarrow \int \text{ExpandTrig}[(1-\sin[d+e x]^2)^{m/2} (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p, x] dx$$

Proeram code:

```
Int[cos[d_+e_.x_]^m_.*(a_+b_.sin[d_+e_.x_]^n_+c_.sin[d_+e_.x_]^(2n_))^p_,x_Symbol] :=
  Int[ExpandTrig[(1-sin[d+e*x]^2)^(m/2)*(a+b*sin[d+e*x]^n+c*sin[d+e*x]^(2*n))^p,x],x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[n2,2*n] && IntegerQ[m/2] && NeQ[b^2-4*a*c,0] && IntegersQ[n,p]
```

```
Int[sin[d_+e_.x_]^m_.*(a_+b_.cos[d_+e_.x_]^n_+c_.cos[d_+e_.x_]^(2n_))^p_,x_Symbol] :=
  Int[ExpandTrig[(1-cos[d+e*x]^2)^(m/2)*(a+b*cos[d+e*x]^n+c*cos[d+e*x]^(2*n))^p,x],x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[n2,2*n] && IntegerQ[m/2] && NeQ[b^2-4*a*c,0] && IntegersQ[n,p]
```

$$4. \int \tan[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx$$

$$\mathbf{1:} \int \tan[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \text{ when } \frac{m-1}{2} \in \mathbb{Z}$$

Derivation: Integration by substitution

$$\text{Basis: } \tan[z]^2 == \frac{\sin[z]^2}{1-\sin[z]^2}$$

Basis: If $\frac{m-1}{2} \in \mathbb{Z}$, then

$$\tan[d+e x]^m F[\sin[d+e x]] == \frac{1}{e} \text{Subst}\left[\frac{x^m F[x]}{(1-x^2)^{\frac{m+1}{2}}}, x, \sin[d+e x]\right] \partial_x \sin[d+e x]$$

Rule: If $\frac{m-1}{2} \in \mathbb{Z} \wedge 2 p \in \mathbb{Z}$, then

$$\int \text{Tan}[d+e x]^m (a+b \text{Sin}[d+e x]^n+c \text{Sin}[d+e x]^{2 n})^p dx \rightarrow \frac{1}{e} \text{Subst}\left[\int \frac{x^m (a+b x^n+c x^{2 n})^p}{(1-x^2)^{\frac{m+1}{2}}} dx, x, \text{Sin}[d+e x]\right]$$

Program code:

```
Int[tan[d_+e_.*x_]^m_.*(a_+b_.*(f_.*sin[d_+e_.*x_] )^n_+c_.*(f_.*sin[d_+e_.*x_] )^n2_.)^p_.,x_Symbol] :=
Module[{g=FreeFactors[Sin[d+e*x],x]},
g^(m+1)/e*Subst[Int[x^m*(a+b*(f*g*x)^n+c*(f*g*x)^(2*n))^p/(1-g^2*x^2)^((m+1)/2),x],x,Sin[d+e*x]/g] /;
FreeQ[{a,b,c,d,e,f,n},x] && IntegerQ[(m-1)/2] && IntegerQ[2*p]
```

```
Int[cot[d_+e_.*x_]^m_.*(a_+b_.*(f_.*cos[d_+e_.*x_] )^n_+c_.*(f_.*cos[d_+e_.*x_] )^n2_.)^p_.,x_Symbol] :=
Module[{g=FreeFactors[Cos[d+e*x],x]},
-g^(m+1)/e*Subst[Int[x^m*(a+b*(f*g*x)^n+c*(f*g*x)^(2*n))^p/(1-g^2*x^2)^((m+1)/2),x],x,Cos[d+e*x]/g] /;
FreeQ[{a,b,c,d,e,f,n},x] && IntegerQ[(m-1)/2] && IntegerQ[2*p]
```

$$2. \int \text{Tan}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z}$$

$$1. \int \text{Tan}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4ac=0$$

$$1: \int \text{Tan}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4ac=0 \wedge p \in \mathbb{Z}$$

Derivation: Algebraic simplification

Basis: If $b^2-4ac=0$, then $a+bz+cz^2 = \frac{(b+2cz)^2}{4c}$

Rule: If $\frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4ac=0 \wedge p \in \mathbb{Z}$, then

$$\int \text{Tan}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \rightarrow \frac{1}{4^p c^p} \int \text{Tan}[d+e x]^m (b+2c \sin[d+e x]^n)^{2p} dx$$

Program code:

```
Int[tan[d_+e_.x_]^m_*(a_+b_.sin[d_+e_.x_]^n_+c_.sin[d_+e_.x_]^n2_.)^p_,x_Symbol] :=
  1/(4^p*c^p)*Int[Tan[d+e*x]^m*(b+2*c*sin[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n},x] && EqQ[n2,2*n] && Not[IntegerQ[(m-1)/2]] && EqQ[b^2-4*a*c,0] && IntegerQ[p]
```

```
Int[cot[d_+e_.x_]^m_*(a_+b_.cos[d_+e_.x_]^n_+c_.cos[d_+e_.x_]^n2_.)^p_,x_Symbol] :=
  1/(4^p*c^p)*Int[Cot[d+e*x]^m*(b+2*c*cos[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n},x] && EqQ[n2,2*n] && Not[IntegerQ[(m-1)/2]] && EqQ[b^2-4*a*c,0] && IntegerQ[p]
```

$$\mathbf{2:} \int \tan [d+e x]^m (a+b \sin [d+e x]^n+c \sin [d+e x]^{2 n})^p dx \text { when } \frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4 a c==0 \wedge p \notin \mathbb{Z}$$

Derivation: Piecewise constant extraction

$$\text{Basis: If } b^2-4 a c==0, \text { then } \partial_x \frac{(a+b F[x]+c F[x]^2)^p}{(b+2 c F[x])^{2 p}}==0$$

Rule: If $\frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4 a c==0 \wedge p \notin \mathbb{Z}$, then

$$\int \tan [d+e x]^m (a+b \sin [d+e x]^n+c \sin [d+e x]^{2 n})^p dx \rightarrow \frac{(a+b \sin [d+e x]^n+c \sin [d+e x]^{2 n})^p}{(b+2 c \sin [d+e x]^n)^{2 p}} \int \tan [d+e x]^m (b+2 c \sin [d+e x]^n)^{2 p} dx$$

Program code:

```
Int[tan[d_+e_*x_]^m_*(a_+b_*sin[d_+e_*x_]^n_+c_*sin[d_+e_*x_]^(2n_))^p_,x_Symbol] :=
  (a+b*sin[d+e*x]^n+c*sin[d+e*x]^(2*n))^p/(b+2*c*sin[d+e*x]^n)^(2*p)*Int[Tan[d+e*x]^m_*(b+2*c*sin[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && EqQ[n2,2*n] && Not[IntegerQ[(m-1)/2]] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[p]]
```

```
Int[cot[d_+e_*x_]^m_*(a_+b_*cos[d_+e_*x_]^n_+c_*cos[d_+e_*x_]^(2n_))^p_,x_Symbol] :=
  (a+b*cos[d+e*x]^n+c*cos[d+e*x]^(2*n))^p/(b+2*c*cos[d+e*x]^n)^(2*p)*Int[Cot[d+e*x]^m_*(b+2*c*cos[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && EqQ[n2,2*n] && Not[IntegerQ[(m-1)/2]] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[p]]
```

$$2. \int \tan[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2 n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4 a c \neq 0$$

$$1: \int \tan[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2 n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4 a c \neq 0 \wedge \frac{n}{2} \in \mathbb{Z} \wedge p \in \mathbb{Z}$$

Derivation: Integration by substitution

$$\text{Basis: } \sin[z]^2 == \frac{\tan[z]^2}{1+\tan[z]^2}$$

$$\text{Basis: } \tan[d+e x]^m F[\sin[d+e x]^2] == \frac{1}{e} \text{Subst}\left[\frac{x^m F\left[\frac{x^2}{1+x^2}\right]}{1+x^2}, x, \tan[d+e x]\right] \partial_x \tan[d+e x]$$

Rule: If $\frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4 a c \neq 0 \wedge \frac{n}{2} \in \mathbb{Z} \wedge p \in \mathbb{Z}$, then

$$\int \tan[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2 n})^p dx \rightarrow \frac{1}{e} \text{Subst}\left[\int \frac{x^m (c x^{2 n}+b x^n (1+x^2)^{n/2}+a (1+x^2)^n)^p}{(1+x^2)^{n p+1}} dx, x, \tan[d+e x]\right]$$

Program code:

```
Int[tan[d_+e_*x_]^m_*(a_+b_*sin[d_+e_*x_]^n_+c_*sin[d_+e_*x_]^(2 n_))^p_,x_Symbol] :=
Module[{f=FreeFactors[Tan[d+e*x],x]},
f^(m+1)/e*Subst[Int[x^m*ExpandToSum[c*x^(2*n)+b*x^n*(1+x^2)^(n/2)+a*(1+x^2)^n,x]^p/(1+f^2*x^2)^(n*p+1),x],x,Tan[d+e*x]/f]] /;
FreeQ[{a,b,c,d,e,m},x] && EqQ[n2,2*n] && Not[IntegerQ[(m-1)/2]] && NeQ[b^2-4*a*c,0] && IntegerQ[n/2] && IntegerQ[p]
```

```
Int[cot[d_+e_*x_]^m_*(a_+b_*cos[d_+e_*x_]^n_+c_*cos[d_+e_*x_]^(2 n_))^p_,x_Symbol] :=
Module[{f=FreeFactors[Cot[d+e*x],x]},
-f^(m+1)/e*Subst[Int[x^m*ExpandToSum[c*x^(2*n)+b*x^n*(1+x^2)^(n/2)+a*(1+x^2)^n,x]^p/(1+f^2*x^2)^(n*p+1),x],x,Cot[d+e*x]/f]] /;
FreeQ[{a,b,c,d,e,m},x] && EqQ[n2,2*n] && Not[IntegerQ[(m-1)/2]] && NeQ[b^2-4*a*c,0] && IntegerQ[n/2] && IntegerQ[p]
```

$$\mathbf{2:} \int \tan [d+e x]^m (a+b \sin [d+e x]^n+c \sin [d+e x]^{2 n})^p dx \text { when } \frac{m}{2} \in \mathbb{Z} \wedge b^2-4 a c \neq 0 \wedge (n|p) \in \mathbb{Z}$$

Derivation: Algebraic expansion

$$\text{Basis: } \tan [z]^2 == \frac{\sin [z]^2}{1-\sin [z]^2}$$

Rule: If $\frac{m}{2} \in \mathbb{Z} \wedge b^2-4 a c \neq 0 \wedge (n|p) \in \mathbb{Z}$, then

$$\int \tan [d+e x]^m (a+b \sin [d+e x]^n+c \sin [d+e x]^{2 n})^p dx \rightarrow \int \text{ExpandTrig}\left[\frac{\sin [d+e x]^m (a+b \sin [d+e x]^n+c \sin [d+e x]^{2 n})^p}{(1-\sin [d+e x]^2)^{m/2}}, x\right] dx$$

Proeram code:

```
Int[tan[d_+e_*x_]^m_.*(a_+b_*sin[d_+e_*x_]^n_+c_*sin[d_+e_*x_]^(2*n_)^p_,x_Symbol] :=
  Int[ExpandTrig[sin[d+e*x]^m*(a+b*sin[d+e*x]^n+c*sin[d+e*x]^(2*n))^p/(1-sin[d+e*x]^2)^(m/2),x],x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[n2,2*n] && IntegerQ[m/2] && NeQ[b^2-4*a*c,0] && IntegersQ[n,p]
```

```
Int[cot[d_+e_*x_]^m_.*(a_+b_*cos[d_+e_*x_]^n_+c_*cos[d_+e_*x_]^(2*n_)^p_,x_Symbol] :=
  Int[ExpandTrig[cos[d+e*x]^m*(a+b*cos[d+e*x]^n+c*cos[d+e*x]^(2*n))^p/(1-cos[d+e*x]^2)^(m/2),x],x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[n2,2*n] && IntegerQ[m/2] && NeQ[b^2-4*a*c,0] && IntegersQ[n,p]
```

$$5. \int \cot [d+e x]^m (a+b \sin [d+e x]^n+c \sin [d+e x]^{2 n})^p dx$$

$$\mathbf{1:} \int \cot [d+e x]^m (a+b \sin [d+e x]^n+c \sin [d+e x]^{2 n})^p dx \text { when } \frac{m-1}{2} \in \mathbb{Z}$$

Derivation: Integration by substitution

$$\text{Basis: } \cot [z]^2 == \frac{1-\sin [z]^2}{\sin [z]^2}$$

Basis: If $\frac{m-1}{2} \in \mathbb{Z}$, then

$$\text{Cot}[d+e x]^m \text{F}[\text{Sin}[d+e x]] = \frac{1}{e} \text{Subst}\left[\frac{(1-x^2)^{\frac{m-1}{2}} \text{F}[x]}{x^m}, x, \text{Sin}[d+e x]\right] \partial_x \text{Sin}[d+e x]$$

Rule: If $\frac{m-1}{2} \in \mathbb{Z} \wedge 2p \in \mathbb{Z}$, then

$$\int \text{Cot}[d+e x]^m (a+b \text{Sin}[d+e x]^n+c \text{Sin}[d+e x]^{2n})^p dx \rightarrow \frac{1}{e} \text{Subst}\left[\int \frac{(1-x^2)^{\frac{m-1}{2}} (a+b x^n+c x^{2n})^p}{x^m} dx, x, \text{Sin}[d+e x]\right]$$

Program code:

```
Int[cot[d_+e_.*x_]^m_.*(a_+b_.*(f_.*sin[d_+e_.*x_])^n_+c_.*(f_.*sin[d_+e_.*x_])^n2_.)^p_.,x_Symbol] :=
Module[{g=FreeFactors[Sin[d+e*x],x]},
g^(m+1)/e*Subst[Int[(1-g^2*x^2)^( (m-1)/2)*(a+b*(f*g*x)^n+c*(f*g*x)^(2*n))^p/x^m,x],x,Sin[d+e*x]/g]] /;
FreeQ[{a,b,c,d,e,f,n},x] && IntegerQ[(m-1)/2] && IntegerQ[2*p]
```

```
Int[tan[d_+e_.*x_]^m_.*(a_+b_.*(f_.*cos[d_+e_.*x_])^n_+c_.*(f_.*cos[d_+e_.*x_])^n2_.)^p_.,x_Symbol] :=
Module[{g=FreeFactors[Cos[d+e*x],x]},
-g^(m+1)/e*Subst[Int[(1-g^2*x^2)^( (m-1)/2)*(a+b*(f*g*x)^n+c*(f*g*x)^(2*n))^p/x^m,x],x,Cos[d+e*x]/g]] /;
FreeQ[{a,b,c,d,e,f,n},x] && IntegerQ[(m-1)/2] && IntegerQ[2*p]
```

$$2. \int \text{Cot}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z}$$

$$1. \int \text{Cot}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4ac=0$$

$$1: \int \text{Cot}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4ac=0 \wedge p \in \mathbb{Z}$$

Derivation: Algebraic simplification

Basis: If $b^2-4ac=0$, then $a+bz+cz^2 = \frac{(b+2cz)^2}{4c}$

Rule: If $\frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4ac=0 \wedge p \in \mathbb{Z}$, then

$$\int \text{Cot}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \rightarrow \frac{1}{4^p c^p} \int \text{Cot}[d+e x]^m (b+2c \sin[d+e x]^n)^{2p} dx$$

Program code:

```
Int[cot[d_+e_.*x_]^m_*(a_+b_.*sin[d_+e_.*x_]^n_+c_.*sin[d_+e_.*x_]^(2n_))^p_,x_Symbol] :=
  1/(4^p*c^p)*Int[Cot[d+e*x]^m*(b+2*c*sin[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n},x] && EqQ[n2,2*n] && Not[IntegerQ[(m-1)/2]] && EqQ[b^2-4*a*c,0] && IntegerQ[p]
```

```
Int[tan[d_+e_.*x_]^m_*(a_+b_.*cos[d_+e_.*x_]^n_+c_.*cos[d_+e_.*x_]^(2n_))^p_,x_Symbol] :=
  1/(4^p*c^p)*Int[Tan[d+e*x]^m*(b+2*c*cos[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n},x] && EqQ[n2,2*n] && Not[IntegerQ[(m-1)/2]] && EqQ[b^2-4*a*c,0] && IntegerQ[p]
```

$$\mathbf{2:} \int \text{Cot}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2 n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4 a c==0 \wedge p \notin \mathbb{Z}$$

Derivation: Piecewise constant extraction

$$\text{Basis: If } b^2-4 a c==0, \text{ then } \partial_x \frac{(a+b F[x]+c F[x]^2)^p}{(b+2 c F[x])^{2 p}}==0$$

Rule: If $\frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4 a c==0 \wedge p \notin \mathbb{Z}$, then

$$\int \text{Cot}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2 n})^p dx \rightarrow \frac{(a+b \sin[d+e x]^n+c \sin[d+e x]^{2 n})^p}{(b+2 c \sin[d+e x]^n)^{2 p}} \int \text{Cot}[d+e x]^m (b+2 c \sin[d+e x]^n)^{2 p} dx$$

Program code:

```
Int[cot[d_+e_.x_]^m_*(a_+b_.sin[d_+e_.x_]^n_+c_.sin[d_+e_.x_]^(2n_))^p_,x_Symbol] :=
  (a+b*sin[d+e*x]^n+c*sin[d+e*x]^(2*n))^p/(b+2*c*sin[d+e*x]^n)^(2*p)*Int[Cot[d+e*x]^m*(b+2*c*sin[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && EqQ[n2,2*n] && Not[IntegerQ[(m-1)/2]] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[p]]
```

```
Int[tan[d_+e_.x_]^m_*(a_+b_.cos[d_+e_.x_]^n_+c_.cos[d_+e_.x_]^(2n_))^p_,x_Symbol] :=
  (a+b*cos[d+e*x]^n+c*cos[d+e*x]^(2*n))^p/(b+2*c*cos[d+e*x]^n)^(2*p)*Int[Tan[d+e*x]^m*(b+2*c*cos[d+e*x]^n)^(2*p),x] /;
FreeQ[{a,b,c,d,e,m,n,p},x] && EqQ[n2,2*n] && Not[IntegerQ[(m-1)/2]] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[p]]
```

$$2. \int \text{Cot}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2 n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4 a c \neq 0$$

$$1: \int \text{Cot}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2 n})^p dx \text{ when } \frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4 a c \neq 0 \wedge \frac{n}{2} \in \mathbb{Z} \wedge p \in \mathbb{Z}$$

Derivation: Integration by substitution

$$\text{Basis: } \sin[z]^2 == \frac{1}{1+\text{Cot}[z]^2}$$

$$\text{Basis: } \text{Cot}[d+e x]^m F[\sin[d+e x]^2] == -\frac{1}{e} \text{Subst}\left[\frac{x^m F\left[\frac{1}{1+x^2}\right]}{1+x^2}, x, \text{Cot}[d+e x]\right] \partial_x \text{Cot}[d+e x]$$

Rule: If $\frac{m-1}{2} \notin \mathbb{Z} \wedge b^2-4 a c \neq 0 \wedge \frac{n}{2} \in \mathbb{Z} \wedge p \in \mathbb{Z}$, then

$$\int \text{Cot}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2 n})^p dx \rightarrow -\frac{1}{e} \text{Subst}\left[\int \frac{x^m (c+b (1+x^2)^{n/2}+a (1+x^2)^n)^p}{(1+x^2)^{n p+1}} dx, x, \text{Cot}[d+e x]\right]$$

Program code:

```
Int[cot[d_+e_.*x_]^m_.*(a_+b_.*sin[d_+e_.*x_]^n_+c_.*sin[d_+e_.*x_]^(2n_))^p_,x_Symbol] :=
Module[{f=FreeFactors[Cot[d+e*x],x]},
-f^(m+1)/e*Subst[Int[x^m*ExpandToSum[c+b*(1+f^2*x^2)^(n/2)+a*(1+f^2*x^2)^n,x]^p/(1+f^2*x^2)^(n*p+1),x],x,Cot[d+e*x]/f]] /;
FreeQ[{a,b,c,d,e,m},x] && EqQ[n2,2*n] && IntegerQ[n/2] && IntegerQ[p]
```

```
Int[tan[d_+e_.*x_]^m_.*(a_+b_.*cos[d_+e_.*x_]^n_+c_.*cos[d_+e_.*x_]^(2n_))^p_,x_Symbol] :=
Module[{f=FreeFactors[Tan[d+e*x],x]},
f^(m+1)/e*Subst[Int[x^m*ExpandToSum[c+b*(1+f^2*x^2)^(n/2)+a*(1+f^2*x^2)^n,x]^p/(1+f^2*x^2)^(n*p+1),x],x,Tan[d+e*x]/f]] /;
FreeQ[{a,b,c,d,e,m},x] && EqQ[n2,2*n] && IntegerQ[n/2] && IntegerQ[p]
```

$$\mathbf{2:} \int \text{Cot}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \text{ when } \frac{m}{2} \in \mathbb{Z} \wedge b^2-4ac \neq 0 \wedge (n|p) \in \mathbb{Z}$$

Derivation: Algebraic expansion

$$\text{Basis: } \text{Cot}[z]^2 == \frac{1-\text{Sin}[z]^2}{\text{Sin}[z]^2}$$

Rule: If $\frac{m}{2} \in \mathbb{Z} \wedge b^2-4ac \neq 0 \wedge (n|p) \in \mathbb{Z}$, then

$$\int \text{Cot}[d+e x]^m (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p dx \rightarrow \int \text{ExpandTrig}\left[\frac{(1-\text{Sin}[d+e x]^2)^{m/2} (a+b \sin[d+e x]^n+c \sin[d+e x]^{2n})^p}{\text{Sin}[d+e x]^m}, x\right] dx$$

Proeram code:

```
Int[cot[d_+e_.*x_]^m_.*(a_+b_.*sin[d_+e_.*x_]^n_+c_.*sin[d_+e_.*x_]^(2*n_))^p_.,x_Symbol] :=
  Int[ExpandTrig[(1-sin[d+e*x]^2)^(m/2)*(a+b*sin[d+e*x]^n+c*sin[d+e*x]^(2*n))^p/sin[d+e*x]^m,x],x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[n2,2*n] && IntegerQ[m/2] && NeQ[b^2-4*a*c,0] && IntegersQ[n,p]
```

```
Int[tan[d_+e_.*x_]^m_.*(a_+b_.*cos[d_+e_.*x_]^n_+c_.*cos[d_+e_.*x_]^(2*n_))^p_.,x_Symbol] :=
  Int[ExpandTrig[(1-cos[d+e*x]^2)^(m/2)*(a+b*cos[d+e*x]^n+c*cos[d+e*x]^(2*n))^p/cos[d+e*x]^m,x],x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[n2,2*n] && IntegerQ[m/2] && NeQ[b^2-4*a*c,0] && IntegersQ[n,p]
```

$$6. \int (A + B \sin[d + e x]) (a + b \sin[d + e x] + c \sin[d + e x]^2)^n dx$$

$$1. \int (A + B \sin[d + e x]) (a + b \sin[d + e x] + c \sin[d + e x]^2)^n dx \text{ when } b^2 - 4 a c == 0$$

$$1: \int (A + B \sin[d + e x]) (a + b \sin[d + e x] + c \sin[d + e x]^2)^n dx \text{ when } b^2 - 4 a c == 0 \wedge n \in \mathbb{Z}$$

Derivation: Algebraic simplification

$$\text{Basis: If } b^2 - 4 a c == 0, \text{ then } a + b z + c z^2 == \frac{(b+2 c z)^2}{4 c}$$

Rule: If $b^2 - 4 a c == 0 \wedge n \in \mathbb{Z}$, then

$$\int (A + B \sin[d + e x]) (a + b \sin[d + e x] + c \sin[d + e x]^2)^n dx \rightarrow \frac{1}{4^n c^n} \int (A + B \sin[d + e x]) (b + 2 c \sin[d + e x])^{2 n} dx$$

Program code:

```
Int[(A+B_.*sin[d_.+e_.*x_])*(a+b_.*sin[d_.+e_.*x_]+c_.*sin[d_.+e_.*x_]^2)^n_,x_Symbol] :=
  1/(4^n*c^n)*Int[(A+B*Sin[d+e*x])*(b+2*c*Sin[d+e*x])^(2*n),x] /;
FreeQ[{a,b,c,d,e,A,B},x] && EqQ[b^2-4*a*c,0] && IntegerQ[n]
```

```
Int[(A+B_.*cos[d_.+e_.*x_])*(a+b_.*cos[d_.+e_.*x_]+c_.*cos[d_.+e_.*x_]^2)^n_,x_Symbol] :=
  1/(4^n*c^n)*Int[(A+B*Cos[d+e*x])*(b+2*c*Cos[d+e*x])^(2*n),x] /;
FreeQ[{a,b,c,d,e,A,B},x] && EqQ[b^2-4*a*c,0] && IntegerQ[n]
```

$$\mathbf{2:} \int (A+B \sin [d+e x]) (a+b \sin [d+e x]+c \sin [d+e x]^2)^n dx \text{ when } b^2-4 a c==0 \wedge n \notin \mathbb{Z}$$

Derivation: Piecewise constant extraction

$$\text{Basis: If } b^2-4 a c==0, \text{ then } \partial_x \frac{(a+b F[x]+c F[x]^2)^n}{(b+2 c F[x])^{2 n}}==0$$

Rule: If $b^2-4 a c==0 \wedge n \notin \mathbb{Z}$, then

$$\int (A+B \sin [d+e x]) (a+b \sin [d+e x]+c \sin [d+e x]^2)^n dx \rightarrow \frac{(a+b \sin [d+e x]+c \sin [d+e x]^2)^n}{(b+2 c \sin [d+e x])^{2 n}} \int (A+B \sin [d+e x]) (b+2 c \sin [d+e x])^{2 n} dx$$

Program code:

```
Int[(A+_B_.sin[d+_e_.x_])*(a+_b_.sin[d+_e_.x_]+c_.sin[d+_e_.x_]^2)^n_,x_Symbol] :=
  (a+b*Sin[d+e*x]+c*Sin[d+e*x]^2)^n/(b+2*c*Sin[d+e*x])^(2*n)*Int[(A+B*Sin[d+e*x])*(b+2*c*Sin[d+e*x])^(2*n),x] /;
FreeQ[{a,b,c,d,e,A,B},x] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[n]]
```

```
Int[(A+_B_.cos[d+_e_.x_])*(a+_b_.cos[d+_e_.x_]+c_.cos[d+_e_.x_]^2)^n_,x_Symbol] :=
  (a+b*Cos[d+e*x]+c*Cos[d+e*x]^2)^n/(b+2*c*Cos[d+e*x])^(2*n)*Int[(A+B*Cos[d+e*x])*(b+2*c*Cos[d+e*x])^(2*n),x] /;
FreeQ[{a,b,c,d,e,A,B},x] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[n]]
```

2. $\int (A + B \sin[d + e x]) (a + b \sin[d + e x] + c \sin[d + e x]^2)^n dx$ when $b^2 - 4 a c \neq 0$

1: $\int \frac{A + B \sin[d + e x]}{a + b \sin[d + e x] + c \sin[d + e x]^2} dx$ when $b^2 - 4 a c \neq 0$

Derivation: Algebraic expansion

Basis: If $q = \sqrt{b^2 - 4 a c}$, then $\frac{A+Bz}{a+bz+cz^2} = \left(B + \frac{bB-2Ac}{q}\right) \frac{1}{b+q+2cz} + \left(B - \frac{bB-2Ac}{q}\right) \frac{1}{b-q+2cz}$

■

Rule: If $b^2 - 4 a c \neq 0$, let $q = \sqrt{b^2 - 4 a c}$, then

$$\int \frac{A + B \sin[d + e x]}{a + b \sin[d + e x] + c \sin[d + e x]^2} dx \rightarrow \left(B + \frac{bB - 2Ac}{q}\right) \int \frac{1}{b + q + 2c \sin[d + e x]} dx + \left(B - \frac{bB - 2Ac}{q}\right) \int \frac{1}{b - q + 2c \sin[d + e x]} dx$$

Program code:

```
Int[(A+B_.sin[d_.+e_.x_])/(a_.+b_.sin[d_.+e_.x_]+c_.sin[d_.+e_.x_]^2),x_Symbol] :=
Module[{q=Rt[b^2-4*a*c,2]},
(B+(b*B-2*A*c)/q)*Int[1/(b+q+2*c*sin[d+e*x]),x] +
(B-(b*B-2*A*c)/q)*Int[1/(b-q+2*c*sin[d+e*x]),x] /;
FreeQ[{a,b,c,d,e,A,B},x] && NeQ[b^2-4*a*c,0]
```

```
Int[(A+B_.cos[d_.+e_.x_])/(a_.+b_.cos[d_.+e_.x_]+c_.cos[d_.+e_.x_]^2),x_Symbol] :=
Module[{q=Rt[b^2-4*a*c,2]},
(B+(b*B-2*A*c)/q)*Int[1/(b+q+2*c*cos[d+e*x]),x] +
(B-(b*B-2*A*c)/q)*Int[1/(b-q+2*c*cos[d+e*x]),x] /;
FreeQ[{a,b,c,d,e,A,B},x] && NeQ[b^2-4*a*c,0]
```


$$\mathbf{2:} \int (A+B \sin [d+e x]) (a+b \sin [d+e x]+c \sin [d+e x]^2)^n dx \text{ when } b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}$$

Derivation: Algebraic expansion

Rule: If $b^2-4 a c \neq 0 \wedge n \in \mathbb{Z}$

$$\int (A+B \sin [d+e x]) (a+b \sin [d+e x]+c \sin [d+e x]^2)^n dx \rightarrow \int \text{ExpandTrig}[(A+B \sin [d+e x]) (a+b \sin [d+e x]+c \sin [d+e x]^2)^n, x] dx$$

Program code:

```
Int[(A+B_.*sin[d_.+e_.*x_])*(a_.+b_.*sin[d_.+e_.*x_]+c_.*sin[d_.+e_.*x_]^2)^n_,x_Symbol] :=
  Int[ExpandTrig[(A+B*sin[d+e*x])*(a+b*sin[d+e*x]+c*sin[d+e*x]^2)^n,x],x] /;
FreeQ[{a,b,c,d,e,A,B},x] && NeQ[b^2-4*a*c,0] && IntegerQ[n]
```

```
Int[(A+B_.*cos[d_.+e_.*x_])*(a_.+b_.*cos[d_.+e_.*x_]+c_.*cos[d_.+e_.*x_]^2)^n_,x_Symbol] :=
  Int[ExpandTrig[(A+B*cos[d+e*x])*(a+b*cos[d+e*x]+c*cos[d+e*x]^2)^n,x],x] /;
FreeQ[{a,b,c,d,e,A,B},x] && NeQ[b^2-4*a*c,0] && IntegerQ[n]
```