

## Rules for integrands of the form $(d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n$

1.  $\int (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n dx$  when  $c^2 d + e = 0$

$$1. \int \frac{(a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \text{ when } c^2 d + e = 0$$

$$1. \int \frac{(a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \text{ when } c^2 d + e = 0 \wedge d > 0$$

$$\text{**x:** } \int \frac{(a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \text{ when } c^2 d + e = 0 \wedge d > 0$$

Derivation: Integration by substitution

Basis: If  $c^2 d + e = 0 \wedge d > 0$ , then  $\frac{F(\operatorname{ArcSin}[c x])}{\sqrt{d + e x^2}} = \frac{1}{c \sqrt{d}} \operatorname{Subst}[F[x], x, \operatorname{ArcSin}[c x]] \partial_x \operatorname{ArcSin}[c x]$

Rule: If  $c^2 d + e = 0 \wedge d > 0$ , then

$$\int \frac{(a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \rightarrow \frac{1}{c \sqrt{d}} \operatorname{Subst}\left[\int (a + b x)^n dx, x, \operatorname{ArcSin}[c x]\right]$$

Program code:

```
(* Int[(a_.+b_.*ArcSin[c_.*x_])^n_/Sqrt[d_+e_.*x_^2],x_Symbol] :=
  1/(c*Sqrt[d])*Subst[Int[(a+b*x)^n,x],x,ArcSin[c*x]] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && GtQ[d,0] *)
```

```
(* Int[(a_.+b_.*ArcCos[c_.*x_])^n_/Sqrt[d_+e_.*x_^2],x_Symbol] :=
  -1/(c*Sqrt[d])*Subst[Int[(a+b*x)^n,x],x,ArcCos[c*x]] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && GtQ[d,0] *)
```

$$1: \int \frac{1}{\sqrt{d+e x^2} (a+b \operatorname{ArcSin}[c x])} dx \text{ when } c^2 d+e == 0 \wedge d > 0$$

Derivation: Integration by substitution

Rule: If  $c^2 d+e == 0 \wedge d > 0$ , then

$$\int \frac{1}{\sqrt{d+e x^2} (a+b \operatorname{ArcSin}[c x])} dx \rightarrow \frac{\operatorname{Log}[a+b \operatorname{ArcSin}[c x]]}{b c \sqrt{d}}$$

Program code:

```
Int[1/(Sqrt[d_+e_.*x_^2]*(a_+b_.*ArcSin[c_.*x_])),x_Symbol] :=
  Log[a+b*ArcSin[c*x]]/(b*c*Sqrt[d]) /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[d,0]
```

```
Int[1/(Sqrt[d_+e_.*x_^2]*(a_+b_.*ArcCos[c_.*x_])),x_Symbol] :=
  -Log[a+b*ArcCos[c*x]]/(b*c*Sqrt[d]) /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[d,0]
```

$$2: \int \frac{(a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \text{ when } c^2 d + e == 0 \wedge d > 0 \wedge n \neq -1$$

Derivation: Integration by substitution

Rule: If  $c^2 d + e == 0 \wedge d > 0 \wedge n \neq -1$ , then

$$\int \frac{(a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \rightarrow \frac{(a + b \operatorname{ArcSin}[c x])^{n+1}}{b c \sqrt{d} (n+1)}$$

Program code:

```
Int[(a_.+b_.*ArcSin[c_.*x_])^n_/Sqrt[d_+e_.x_^2],x_Symbol] :=
  (a+b*ArcSin[c*x])^(n+1)/(b*c*Sqrt[d]*(n+1)) /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && GtQ[d,0] && NeQ[n,-1]
```

```
Int[(a_.+b_.*ArcCos[c_.*x_])^n_/Sqrt[d_+e_.x_^2],x_Symbol] :=
  -(a+b*ArcCos[c*x])^(n+1)/(b*c*Sqrt[d]*(n+1)) /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && GtQ[d,0] && NeQ[n,-1]
```

$$2: \int \frac{(a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \text{ when } c^2 d + e == 0 \wedge d \neq 0$$

Derivation: Piecewise constant extraction

Basis: If  $c^2 d + e == 0$ , then  $\partial_x \frac{\sqrt{1-c^2 x^2}}{\sqrt{d+e x^2}} == 0$

Rule: If  $c^2 d + e == 0 \wedge d \neq 0$ , then

$$\int \frac{(a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \rightarrow \frac{\sqrt{1 - c^2 x^2}}{\sqrt{d + e x^2}} \int \frac{(a + b \operatorname{ArcSin}[c x])^n}{\sqrt{1 - c^2 x^2}} dx$$

Program code:

```
Int[(a_.+b_.*ArcSin[c_.*x_])^n_/Sqrt[d_+e_.*x_^2],x_Symbol] :=
  Sqrt[1-c^2*x^2]/Sqrt[d+e*x^2]*Int[(a+b*ArcSin[c*x])^n/Sqrt[1-c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && Not[GtQ[d,0]]
```

```
Int[(a_.+b_.*ArcCos[c_.*x_])^n_/Sqrt[d_+e_.*x_^2],x_Symbol] :=
  Sqrt[1-c^2*x^2]/Sqrt[d+e*x^2]*Int[(a+b*ArcCos[c*x])^n/Sqrt[1-c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && Not[GtQ[d,0]]
```

2.  $\int (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n dx$  when  $c^2 d + e = 0 \wedge n > 0$

**1:**  $\int (d + e x^2)^p (a + b \operatorname{ArcSin}[c x]) dx$  when  $c^2 d + e = 0 \wedge p \in \mathbb{Z}^+$

Derivation: Integration by parts

Rule: If  $c^2 d + e = 0 \wedge p \in \mathbb{Z}^+$ , let  $u \rightarrow \int (d + e x^2)^p dx$ , then

$$\int (d + e x^2)^p (a + b \operatorname{ArcSin}[c x]) dx \rightarrow u (a + b \operatorname{ArcSin}[c x]) - b c \int \frac{u}{\sqrt{1 - c^2 x^2}} dx$$

Program code:

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSin[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(d+e*x^2)^p,x]},
    Dist[a+b*ArcSin[c*x],u,x] - b*c*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[p,0]
```

```

Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCos[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(d+e*x^2)^p,x]},
    Dist[a+b*ArcCos[c*x],u,x] + b*c*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x] /;
    FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[p,0]

```

$$2. \int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e=0 \wedge n>0 \wedge p>0$$

$$\textcolor{red}{x}: \int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e=0 \wedge n>0 \wedge p>0 \wedge (p \in \mathbb{Z} \vee d>0)$$

Derivation: Inverted integration by parts

Rule: If  $c^2 d+e=0 \wedge n>0 \wedge p>0 \wedge (p \in \mathbb{Z} \vee d>0)$ , then

$$\begin{aligned}
 & \int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \\
 & \frac{x (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n}{2 p+1} + \\
 & \frac{2 d p}{2 p+1} \int (d+e x^2)^{p-1} (a+b \operatorname{ArcSin}[c x])^n dx - \frac{b c n d^p}{2 p+1} \int x (1-c^2 x^2)^{p-\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n-1} dx
 \end{aligned}$$

Program code:

```

(* Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSin[c_.*x_])^n_.,x_Symbol] :=
  x*(d+e*x^2)^p*(a+b*ArcSin[c*x])^n/(2*p+1) +
  2*d*p/(2*p+1)*Int[(d+e*x^2)^(p-1)*(a+b*ArcSin[c*x])^n,x] -
  b*c*n*d^p/(2*p+1)*Int[x*(1-c^2*x^2)^(p-1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
  FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0] && (IntegerQ[p] || GtQ[d,0]) *)

```

```

(* Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCos[c_.*x_])^n_.,x_Symbol] :=
  x*(d+e*x^2)^p*(a+b*ArcCos[c*x])^n/(2*p+1) +
  2*d*p/(2*p+1)*Int[(d+e*x^2)^(p-1)*(a+b*ArcCos[c*x])^n,x] +
  b*c*n*d^p/(2*p+1)*Int[x*(1-c^2*x^2)^(p-1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
  FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0] && (IntegerQ[p] || GtQ[d,0]) *)

```

$$2. \int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e=0 \wedge n>0 \wedge p>0$$

$$1: \int \sqrt{d+e x^2} (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e=0 \wedge n>0$$

Derivation: Inverted integration by parts

Note: The piecewise constant factor in the second integral reduces the degree of  $d$  in the resulting antiderivative.

Rule: If  $c^2 d+e=0 \wedge n>0$ , then

$$\int \sqrt{d+e x^2} (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{x \sqrt{d+e x^2} (a+b \operatorname{ArcSin}[c x])^n}{2} - \frac{b c n \sqrt{d+e x^2}}{2 \sqrt{1-c^2 x^2}} \int x (a+b \operatorname{ArcSin}[c x])^{n-1} dx + \frac{\sqrt{d+e x^2}}{2 \sqrt{1-c^2 x^2}} \int \frac{(a+b \operatorname{ArcSin}[c x])^n}{\sqrt{1-c^2 x^2}} dx$$

Program code:

```
Int[Sqrt[d+_e_*x_^2]*(a+_b_*ArcSin[c_*x_])^n_,x_Symbol] :=
  x*Sqrt[d+e*x^2]*(a+b*ArcSin[c*x])^n/2 -
  b*c*n*Sqrt[d+e*x^2]/(2*Sqrt[1-c^2*x^2])*Int[x*(a+b*ArcSin[c*x])^(n-1),x] +
  Sqrt[d+e*x^2]/(2*Sqrt[1-c^2*x^2])*Int[(a+b*ArcSin[c*x])^n/Sqrt[1-c^2*x^2],x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0]
```

```
Int[Sqrt[d+_e_*x_^2]*(a+_b_*ArcCos[c_*x_])^n_,x_Symbol] :=
  x*Sqrt[d+e*x^2]*(a+b*ArcCos[c*x])^n/2 +
  b*c*n*Sqrt[d+e*x^2]/(2*Sqrt[1-c^2*x^2])*Int[x*(a+b*ArcCos[c*x])^(n-1),x] +
  Sqrt[d+e*x^2]/(2*Sqrt[1-c^2*x^2])*Int[(a+b*ArcCos[c*x])^n/Sqrt[1-c^2*x^2],x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0]
```

**2:**  $\int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx$  when  $c^2 d+e == 0 \wedge n > 0 \wedge p > 0$

Derivation: Inverted integration by parts

Rule: If  $c^2 d+e == 0 \wedge n > 0 \wedge p > 0$ , then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow$$

$$\frac{x (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n}{2p+1} + \frac{2dp}{2p+1} \int (d+e x^2)^{p-1} (a+b \operatorname{ArcSin}[c x])^n dx -$$

$$\frac{b c n d^{\operatorname{IntPart}[p]} (d+e x^2)^{\operatorname{FracPart}[p]}}{(2p+1) (1-c^2 x^2)^{\operatorname{FracPart}[p]}} \int x (1-c^2 x^2)^{p-\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n-1} dx$$

Program code:

```
Int[(d+_e_.*x^2)^p_.*(a+_b_.*ArcSin[c_.*x_])^n_.,x_Symbol] :=
  x*(d+e*x^2)^p*(a+b*ArcSin[c*x])^n/(2*p+1) +
  2*d*p/(2*p+1)*Int[(d+e*x^2)^(p-1)*(a+b*ArcSin[c*x])^n,x] -
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/((2*p+1)*(1-c^2*x^2)^FracPart[p])*
  Int[x*(1-c^2*x^2)^(p-1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0]
```

```
Int[(d+_e_.*x^2)^p_.*(a+_b_.*ArcCos[c_.*x_])^n_.,x_Symbol] :=
  x*(d+e*x^2)^p*(a+b*ArcCos[c*x])^n/(2*p+1) +
  2*d*p/(2*p+1)*Int[(d+e*x^2)^(p-1)*(a+b*ArcCos[c*x])^n,x] +
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/((2*p+1)*(1-c^2*x^2)^FracPart[p])*
  Int[x*(1-c^2*x^2)^(p-1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0]
```

**3.**  $\int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx$  when  $c^2 d+e == 0 \wedge n > 0 \wedge p < -1$

$$1. \int \frac{(a + b \operatorname{ArcSin}[c x])^n}{(d + e x^2)^{3/2}} dx \text{ when } c^2 d + e = 0 \wedge n > 0$$

$$1: \int \frac{(a + b \operatorname{ArcSin}[c x])^n}{(d + e x^2)^{3/2}} dx \text{ when } c^2 d + e = 0 \wedge n > 0 \wedge d > 0$$

Derivation: Integration by parts

$$\text{Basis: } \frac{1}{(d+e x^2)^{3/2}} = \partial_x \frac{x}{d \sqrt{d+e x^2}}$$

Rule: If  $c^2 d + e = 0 \wedge n > 0 \wedge d > 0$ , then

$$\int \frac{(a + b \operatorname{ArcSin}[c x])^n}{(d + e x^2)^{3/2}} dx \rightarrow \frac{x (a + b \operatorname{ArcSin}[c x])^n}{d \sqrt{d + e x^2}} - \frac{b c n}{\sqrt{d}} \int \frac{x (a + b \operatorname{ArcSin}[c x])^{n-1}}{d + e x^2} dx$$

Program code:

```
Int[(a_.+b_.*ArcSin[c_.*x_])^n_./(d_+e_.*x_^2)^(3/2),x_Symbol] :=
  x*(a+b*ArcSin[c*x])^n/(d*Sqrt[d+e*x^2]) -
  b*c*n/Sqrt[d]*Int[x*(a+b*ArcSin[c*x])^(n-1)/(d+e*x^2),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[d,0]
```

```
Int[(a_.+b_.*ArcCos[c_.*x_])^n_./(d_+e_.*x_^2)^(3/2),x_Symbol] :=
  x*(a+b*ArcCos[c*x])^n/(d*Sqrt[d+e*x^2]) +
  b*c*n/Sqrt[d]*Int[x*(a+b*ArcCos[c*x])^(n-1)/(d+e*x^2),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[d,0]
```



$$\text{2: } \int \frac{(a + b \operatorname{ArcSin}[c x])^n}{(d + e x^2)^{3/2}} dx \text{ when } c^2 d + e == 0 \wedge n > 0$$

Derivation: Integration by parts and piecewise constant extraction

$$\text{Basis: } \frac{1}{(d+e x^2)^{3/2}} == \partial_x \frac{x}{d \sqrt{d+e x^2}}$$

$$\text{Basis: If } c^2 d + e == 0, \text{ then } \partial_x \frac{\sqrt{1-c^2 x^2}}{\sqrt{d+e x^2}} == 0$$

Rule: If  $c^2 d + e == 0 \wedge n > 0$ , then

$$\int \frac{(a + b \operatorname{ArcSin}[c x])^n}{(d + e x^2)^{3/2}} dx \rightarrow \frac{x (a + b \operatorname{ArcSin}[c x])^n}{d \sqrt{d + e x^2}} - \frac{b c n \sqrt{1 - c^2 x^2}}{d \sqrt{d + e x^2}} \int \frac{x (a + b \operatorname{ArcSin}[c x])^{n-1}}{1 - c^2 x^2} dx$$

Program code:

```
Int[(a_.+b_.*ArcSin[c_.*x_])^n_./(d_+e_.*x_^2)^(3/2),x_Symbol] :=
  x*(a+b*ArcSin[c*x])^n/(d*Sqrt[d+e*x^2]) -
  b*c*n*Sqrt[1-c^2*x^2]/(d*Sqrt[d+e*x^2])*Int[x*(a+b*ArcSin[c*x])^(n-1)/(1-c^2*x^2),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0]
```

```
Int[(a_.+b_.*ArcCos[c_.*x_])^n_./(d_+e_.*x_^2)^(3/2),x_Symbol] :=
  x*(a+b*ArcCos[c*x])^n/(d*Sqrt[d+e*x^2]) +
  b*c*n*Sqrt[1-c^2*x^2]/(d*Sqrt[d+e*x^2])*Int[x*(a+b*ArcCos[c*x])^(n-1)/(1-c^2*x^2),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0]
```

$$2. \int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e=0 \wedge n>0 \wedge p<-1 \wedge p \neq -\frac{3}{2}$$

$$1: \int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e=0 \wedge n>0 \wedge p<-1 \wedge p \neq -\frac{3}{2} \wedge (p \in \mathbb{Z} \vee d>0)$$

Rule: If  $c^2 d+e=0 \wedge n>0 \wedge p<-1 \wedge p \neq -\frac{3}{2} \wedge (p \in \mathbb{Z} \vee d>0)$ , then

$$\begin{aligned} & \int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \\ & -\frac{x (d+e x^2)^{p+1} (a+b \operatorname{ArcSin}[c x])^n}{2 d (p+1)} + \\ & \frac{2 p+3}{2 d (p+1)} \int (d+e x^2)^{p+1} (a+b \operatorname{ArcSin}[c x])^n dx + \frac{b c n d^p}{2 (p+1)} \int x (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n-1} dx \end{aligned}$$

Program code:

```
(* Int[(d_+e_.*x_^2)^p_*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  -x*(d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n/(2*d*(p+1)) +
  (2*p+3)/(2*d*(p+1))*Int[(d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n,x] +
  b*c*n*d^p/(2*(p+1))*Int[x*(1-c^2*x^2)^(p+1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && NeQ[p,-3/2] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
(* Int[(d_+e_.*x_^2)^p_*(a_+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  -x*(d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n/(2*d*(p+1)) +
  (2*p+3)/(2*d*(p+1))*Int[(d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n,x] -
  b*c*n*d^p/(2*(p+1))*Int[x*(1-c^2*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && NeQ[p,-3/2] && (IntegerQ[p] || GtQ[d,0]) *)
```

$$2: \int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e=0 \wedge n>0 \wedge p<-1 \wedge p \neq -\frac{3}{2}$$

Rule: If  $c^2 d+e=0 \wedge n>0 \wedge p<-1 \wedge p \neq -\frac{3}{2}$ , then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow$$

$$-\frac{x (d+e x^2)^{p+1} (a+b \operatorname{ArcSin}[c x])^n}{2 d (p+1)} + \frac{2 p+3}{2 d (p+1)} \int (d+e x^2)^{p+1} (a+b \operatorname{ArcSin}[c x])^n dx +$$

$$\frac{b c n d^{\operatorname{IntPart}[p]} (d+e x^2)^{\operatorname{FracPart}[p]}}{2 (p+1) (1-c^2 x^2)^{\operatorname{FracPart}[p]}} \int x (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n-1} dx$$

Program code:

```
Int[(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSin[c_.*x_])^n_.,x_Symbol] :=
  -x*(d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n/(2*d*(p+1)) +
  (2*p+3)/(2*d*(p+1))*Int[(d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n,x] +
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(2*(p+1)*(1-c^2*x^2)^FracPart[p])*
  Int[x*(1-c^2*x^2)^(p+1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && NeQ[p,-3/2]
```

```
Int[(d_+e_.*x_^2)^p_*(a_.+b_.*ArcCos[c_.*x_])^n_.,x_Symbol] :=
  -x*(d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n/(2*d*(p+1)) +
  (2*p+3)/(2*d*(p+1))*Int[(d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n,x] -
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(2*(p+1)*(1-c^2*x^2)^FracPart[p])*
  Int[x*(1-c^2*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && NeQ[p,-3/2]
```

$$4: \int \frac{(a + b \operatorname{ArcSin}[c x])^n}{d + e x^2} dx \text{ when } c^2 d + e = 0 \wedge n \in \mathbb{Z}^+$$

Derivation: Integration by substitution

Basis: If  $c^2 d + e = 0$ , then  $\frac{1}{d+e x^2} = \frac{1}{c d} \operatorname{Sec}[\operatorname{ArcSin}[c x]] \partial_x \operatorname{ArcSin}[c x]$

Note: If  $n \in \mathbb{Z}^+$ , then  $(a + b x)^n \operatorname{Sec}[x]$  is integrable in closed-form.

Rule: If  $c^2 d + e = 0 \wedge n \in \mathbb{Z}^+$ , then

$$\int \frac{(a + b \operatorname{ArcSin}[c x])^n}{d + e x^2} dx \rightarrow \frac{1}{c d} \operatorname{Subst}\left[\int (a + b x)^n \operatorname{Sec}[x] dx, x, \operatorname{ArcSin}[c x]\right]$$

Program code:

```
Int[(a_.+b_.*ArcSin[c_.*x_])^n_./(d_.+e_.*x_^2),x_Symbol] :=
  1/(c*d)*Subst[Int[(a+b*x)^n*Sec[x],x],x,ArcSin[c*x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[n,0]
```

```
Int[(a_.+b_.*ArcCos[c_.*x_])^n_./(d_.+e_.*x_^2),x_Symbol] :=
  -1/(c*d)*Subst[Int[(a+b*x)^n*Csc[x],x],x,ArcCos[c*x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[n,0]
```

$$3. \int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e == 0 \wedge n < -1$$

$$1: \int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e == 0 \wedge n < -1 \wedge (p \in \mathbb{Z} \vee d > 0)$$

Derivation: Integration by parts

$$\text{Basis: } \frac{(a+b \operatorname{ArcSin}[c x])^n}{\sqrt{1-c^2 x^2}} == \partial_x \frac{(a+b \operatorname{ArcSin}[c x])^{n+1}}{b c (n+1)}$$

Rule: If  $c^2 d+e == 0 \wedge n < -1 \wedge (p \in \mathbb{Z} \vee d > 0)$ , then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{d^p (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n+1}}{b c (n+1)} + \frac{c d^p (2p+1)}{b (n+1)} \int x (1-c^2 x^2)^{p-\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n+1} dx$$

Program code:

```
(* Int[(d_+e_.*x_^2)^p_.*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  d^p*(1-c^2*x^2)^(p+1/2)*(a+b*ArcSin[c*x])^(n+1)/(b*c*(n+1)) +
  c*d^p*(2*p+1)/(b*(n+1))*Int[x*(1-c^2*x^2)^(p-1/2)*(a+b*ArcSin[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,p},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
(* Int[(d_+e_.*x_^2)^p_.*(a_+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  -d^p*(1-c^2*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n+1)/(b*c*(n+1)) -
  c*d^p*(2*p+1)/(b*(n+1))*Int[x*(1-c^2*x^2)^(p-1/2)*(a+b*ArcCos[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,p},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && (IntegerQ[p] || GtQ[d,0]) *)
```

$$2: \int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e == 0 \wedge n < -1$$

Derivation: Integration by parts and piecewise constant extraction

Basis:  $\frac{(a+b \operatorname{ArcSin}[c x])^n}{\sqrt{1-c^2 x^2}} == \partial_x \frac{(a+b \operatorname{ArcSin}[c x])^{n+1}}{b c (n+1)}$

Basis: If  $c^2 d + e == 0$ , then  $\partial_x \frac{(d+e x^2)^p}{(1-c^2 x^2)^p} == 0$

Rule: If  $c^2 d + e == 0 \wedge n < -1$ , then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow$$

$$\frac{\sqrt{1-c^2 x^2} (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^{n+1}}{b c (n+1)} + \frac{c (2p+1)}{b (n+1)} \int \frac{x (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^{n+1}}{\sqrt{1-c^2 x^2}} dx \rightarrow$$

$$\frac{\sqrt{1-c^2 x^2} (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^{n+1}}{b c (n+1)} + \frac{c (2p+1) d^{\operatorname{IntPart}[p]} (d+e x^2)^{\operatorname{FracPart}[p]}}{b (n+1) (1-c^2 x^2)^{\operatorname{FracPart}[p]}} \int x (1-c^2 x^2)^{p-\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n+1} dx$$

Program code:

```
Int[(d+_e_.*x_^2)^p_.*(a_+_b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  Sqrt[1-c^2*x^2]*(d+e*x^2)^p*(a+b*ArcSin[c*x])^(n+1)/(b*c*(n+1)) +
  c*(2*p+1)*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(b*(n+1)*(1-c^2*x^2)^FracPart[p])*
  Int[x*(1-c^2*x^2)^(p-1/2)*(a+b*ArcSin[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,p},x] && EqQ[c^2*d+e,0] && LtQ[n,-1]
```

```
Int[(d+_e_.*x_^2)^p_.*(a_+_b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  -Sqrt[1-c^2*x^2]*(d+e*x^2)^p*(a+b*ArcCos[c*x])^(n+1)/(b*c*(n+1)) -
  c*(2*p+1)*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(b*(n+1)*(1-c^2*x^2)^FracPart[p])*
  Int[x*(1-c^2*x^2)^(p-1/2)*(a+b*ArcCos[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,p},x] && EqQ[c^2*d+e,0] && LtQ[n,-1]
```

4.  $\int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx$  when  $c^2 d+e == 0 \wedge 2p \in \mathbb{Z}^+$

**1:**  $\int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx$  when  $c^2 d+e == 0 \wedge 2p \in \mathbb{Z}^+ \wedge (p \in \mathbb{Z} \vee d > 0)$

Derivation: Integration by substitution

Basis: If  $c^2 d+e == 0 \wedge (p \in \mathbb{Z} \vee d > 0)$ , then  $(d+e x^2)^p = \frac{d^p}{c} \operatorname{Cos}[\operatorname{ArcSin}[c x]]^{2p+1} \partial_x \operatorname{ArcSin}[c x]$

Note: If  $2p \in \mathbb{Z}^+$ , then  $(a+b x)^n \cos[x]^{2p+1}$  is integrable in closed-form.

Rule: If  $c^2 d+e == 0 \wedge 2p \in \mathbb{Z}^+ \wedge (p \in \mathbb{Z} \vee d > 0)$ , then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{d^p}{c} \operatorname{Subst}\left[\int (a+b x)^n \operatorname{Cos}[x]^{2p+1} dx, x, \operatorname{ArcSin}[c x]\right]$$

Program code:

```
Int[(d_+e_.*x_^2)^p_.*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  d^p/c*Subst[Int[(a+b*x)^n*Cos[x]^(2*p+1),x],x,ArcSin[c*x]] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && IGtQ[2*p,0] && (IntegerQ[p] || GtQ[d,0])
```

```
Int[(d_+e_.*x_^2)^p_.*(a_+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  -d^p/c*Subst[Int[(a+b*x)^n*Sin[x]^(2*p+1),x],x,ArcCos[c*x]] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && IGtQ[2*p,0] && (IntegerQ[p] || GtQ[d,0])
```

$$\mathbf{2:} \int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e \neq 0 \wedge 2p \in \mathbb{Z}^+ \wedge \neg (p \in \mathbb{Z} \vee d > 0)$$

Derivation: Piecewise constant extraction

$$\text{Basis: If } c^2 d+e \neq 0, \text{ then } \partial_x \frac{\sqrt{d+e x^2}}{\sqrt{1-c^2 x^2}} \neq 0$$

Rule: If  $c^2 d+e \neq 0 \wedge 2p \in \mathbb{Z}^+ \wedge \neg (p \in \mathbb{Z} \vee d > 0)$ , then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{d^{p-\frac{1}{2}} \sqrt{d+e x^2}}{\sqrt{1-c^2 x^2}} \int (1-c^2 x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx$$

Program code:

```
Int[(d+_.*x_^2)^p_.*(a+_.*b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  d^(p-1/2)*Sqrt[d+e*x^2]/Sqrt[1-c^2*x^2]*Int[(1-c^2*x^2)^p*(a+b*ArcSin[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && IGtQ[2*p,0] && Not[IntegerQ[p] || GtQ[d,0]]
```

```
Int[(d+_.*x_^2)^p_.*(a+_.*b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  d^(p-1/2)*Sqrt[d+e*x^2]/Sqrt[1-c^2*x^2]*Int[(1-c^2*x^2)^p*(a+b*ArcCos[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && IGtQ[2*p,0] && Not[IntegerQ[p] || GtQ[d,0]]
```

$$2. \int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e \neq 0$$

$$\mathbf{1:} \int (d+e x^2)^p (a+b \operatorname{ArcSin}[c x]) dx \text{ when } c^2 d+e \neq 0 \wedge (p \in \mathbb{Z}^+ \vee p + \frac{1}{2} \in \mathbb{Z}^-)$$

Derivation: Integration by parts

Rule: If  $c^2 d+e \neq 0 \wedge (p \in \mathbb{Z}^+ \vee p + \frac{1}{2} \in \mathbb{Z}^-)$ , let  $u \rightarrow \int (d+e x^2)^p dx$ , then



$$\int (d+e x^2)^p (a+b \arcsin[c x]) dx \rightarrow u (a+b \arcsin[c x]) - b c \int \frac{u}{\sqrt{1-c^2 x^2}} dx$$

### Program code:

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSin[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(d+e*x^2)^p,x]},
    Dist[a+b*ArcSin[c*x],u,x] - b*c*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x] /;
    FreeQ[{a,b,c,d,e},x] && NeQ[c^2*d+e,0] && (IGtQ[p,0] || ILtQ[p+1/2,0])
```

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCos[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(d+e*x^2)^p,x]},
    Dist[a+b*ArcCos[c*x],u,x] + b*c*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x] /;
    FreeQ[{a,b,c,d,e},x] && NeQ[c^2*d+e,0] && (IGtQ[p,0] || ILtQ[p+1/2,0])
```

**2:**  $\int (d+e x^2)^p (a+b \arcsin[c x])^n dx$  when  $c^2 d+e \neq 0 \wedge p \in \mathbb{Z} \wedge (p > 0 \vee n \in \mathbb{Z}^+)$

### Derivation: Algebraic expansion

Rule: If  $c^2 d+e \neq 0 \wedge p \in \mathbb{Z} \wedge (p > 0 \vee n \in \mathbb{Z}^+)$ , then

$$\int (d+e x^2)^p (a+b \arcsin[c x])^n dx \rightarrow \int (a+b \arcsin[c x])^n \text{ExpandIntegrand}[(d+e x^2)^p, x] dx$$

### Program code:

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*ArcSin[c*x])^n,(d+e*x^2)^p,x],x] /;
  FreeQ[{a,b,c,d,e,n},x] && NeQ[c^2*d+e,0] && IntegerQ[p] && (GtQ[p,0] || IGtQ[n,0])
```

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*ArcCos[c*x])^n,(d+e*x^2)^p,x],x] /;
  FreeQ[{a,b,c,d,e,n},x] && NeQ[c^2*d+e,0] && IntegerQ[p] && (GtQ[p,0] || IGtQ[n,0])
```

**U:**  $\int (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n dx$

**Rule:**

$$\int (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n dx \rightarrow \int (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n dx$$

**Program code:**

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSin[c_.*x_])^n_.,x_Symbol] :=
  Unintegrable[(d+e*x^2)^p*(a+b*ArcSin[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,n,p},x]
```

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCos[c_.*x_])^n_.,x_Symbol] :=
  Unintegrable[(d+e*x^2)^p*(a+b*ArcCos[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,n,p},x]
```

**Rules for integrands of the form  $(d + e x)^p (f + g x)^q (a + b \operatorname{ArcSin}[c x])^n$**

**1:**  $\int (d + e x)^p (f + g x)^q (a + b \operatorname{ArcSin}[c x])^n dx$  when  $e f + d g == 0 \wedge c^2 d^2 - e^2 == 0 \wedge (p | q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge d > 0 \wedge \frac{g}{e} < 0$

**Derivation: Algebraic expansion**

**Basis:** If  $e f + d g == 0 \wedge c^2 d^2 - e^2 == 0 \wedge d > 0 \wedge \frac{g}{e} < 0$ , then

$$(d + e x)^p (f + g x)^q = \left(-\frac{d^2 g}{e}\right)^q (d + e x)^{p-q} (1 - c^2 x^2)^q$$

**Rule:** If  $e f + d g == 0 \wedge c^2 d^2 - e^2 == 0 \wedge (p | q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge d > 0 \wedge \frac{g}{e} < 0$ , then

$$\int (d+e x)^p (f+g x)^q (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \left(-\frac{d^2 g}{e}\right)^q \int (d+e x)^{p-q} (1-c^2 x^2)^q (a+b \operatorname{ArcSin}[c x])^n dx$$

Program code:

```
Int[(d_+e_.*x_)^p_*(f_+g_.*x_)^q_*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  (-d^2*g/e)^q*Int[(d+e*x)^(p-q)*(1-c^2*x^2)^q*(a+b*ArcSin[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,n},x] && EqQ[e*f+d*g,0] && EqQ[c^2*d^2-e^2,0] && HalfIntegerQ[p,q] && GeQ[p-q,0] && GtQ[d,0] && LtQ[g/e,0]
```

```
Int[(d_+e_.*x_)^p_*(f_+g_.*x_)^q_*(a_+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  (-d^2*g/e)^q*Int[(d+e*x)^(p-q)*(1-c^2*x^2)^q*(a+b*ArcCos[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,n},x] && EqQ[e*f+d*g,0] && EqQ[c^2*d^2-e^2,0] && HalfIntegerQ[p,q] && GeQ[p-q,0] && GtQ[d,0] && LtQ[g/e,0]
```

2:  $\int (d+e x)^p (f+g x)^q (a+b \operatorname{ArcSin}[c x])^n dx$  when  $e f + d g == 0 \wedge c^2 d^2 - e^2 == 0 \wedge (p | q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge \neg (d > 0 \wedge \frac{g}{e} < 0)$

Derivation: Piecewise constant extraction

Basis: If  $e f + d g == 0 \wedge c^2 d^2 - e^2 == 0$ , then  $\partial_x \frac{(d+e x)^q (f+g x)^q}{(1-c^2 x^2)^q} == 0$

Rule: If  $e f + d g == 0 \wedge c^2 d^2 - e^2 == 0 \wedge (p | q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge \neg (d > 0 \wedge \frac{g}{e} < 0)$ , then

$$\int (d+e x)^p (f+g x)^q (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{(d+e x)^q (f+g x)^q}{(1-c^2 x^2)^q} \int (d+e x)^{p-q} (1-c^2 x^2)^q (a+b \operatorname{ArcSin}[c x])^n dx$$

Program code:

```
Int[(d_+e_.*x_)^p_*(f_+g_.*x_)^q_*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  (d+e*x)^q*(f+g*x)^q/(1-c^2*x^2)^q*Int[(d+e*x)^(p-q)*(1-c^2*x^2)^q*(a+b*ArcSin[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,n},x] && EqQ[e*f+d*g,0] && EqQ[c^2*d^2-e^2,0] && HalfIntegerQ[p,q] && GeQ[p-q,0]
```

```

Int[(d_+e_.*x_)^p_*(f_+g_.*x_)^q_*(a_.+b_.*ArcCos[c_.*x_])^n_.,x_Symbol] :=
  (d+e*x)^q*(f+g*x)^q/(1-c^2*x^2)^q*Int[(d+e*x)^(p-q)*(1-c^2*x^2)^q*(a+b*ArcCos[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,n},x] && EqQ[e*f+d*g,0] && EqQ[c^2*d^2-e^2,0] && HalfIntegerQ[p,q] && GeQ[p-q,0]

```