

Rules for integrands of the form $u (a + b \operatorname{ArcSin}[c x])^n$

$$1. \int (d + e x)^m (a + b \operatorname{ArcSin}[c x])^n dx$$

$$1. \int (d + e x)^m (a + b \operatorname{ArcSin}[c x])^n dx \text{ when } n \in \mathbb{Z}^+$$

$$1: \int \frac{(a + b \operatorname{ArcSin}[c x])^n}{d + e x} dx \text{ when } n \in \mathbb{Z}^+$$

Derivation: Integration by substitution

$$\text{Basis: } \frac{1}{d + e x} = \text{Subst}\left[\frac{\cos[x]}{c d + e \sin[x]}, x, \operatorname{ArcSin}[c x]\right] \partial_x \operatorname{ArcSin}[c x]$$

Note: $\frac{(a + b x)^n \cos[x]}{c d + e \sin[x]}$ is not integrable unless $n \in \mathbb{Z}^+$.

Rule: If $n \in \mathbb{Z}^+$, then

$$\int \frac{(a + b \operatorname{ArcSin}[c x])^n}{d + e x} dx \rightarrow \text{Subst}\left[\int \frac{(a + b x)^n \cos[x]}{c d + e \sin[x]} dx, x, \operatorname{ArcSin}[c x]\right]$$

Program code:

```
Int[(a_.+b_.*ArcSin[c_.*x_])^n_./(d_.+e_.*x_),x_Symbol] :=
  Subst[Int[(a+b*x)^n*Cos[x]/(c*d+e*SIn[x]),x],x,ArcSin[c*x]] /;
FreeQ[{a,b,c,d,e},x] && IGtQ[n,0]
```

```
Int[(a_.+b_.*ArcCos[c_.*x_])^n_./(d_.+e_.*x_),x_Symbol] :=
  -Subst[Int[(a+b*x)^n*SIn[x]/(c*d+e*Cos[x]),x],x,ArcCos[c*x]] /;
FreeQ[{a,b,c,d,e},x] && IGtQ[n,0]
```

2: $\int (d+ex)^m (a+b \operatorname{ArcSin}[cx])^n dx$ when $n \in \mathbb{Z}^+ \wedge m \neq -1$

Reference: G&R 2.831, CRC 453, A&S 4.4.65

Reference: G&R 2.832, CRC 454, A&S 4.4.67

Derivation: Integration by parts

Basis: If $m \neq -1$, then $(d+ex)^m = \partial_x \frac{(d+ex)^{m+1}}{e(m+1)}$

Rule: If $n \in \mathbb{Z}^+ \wedge m \neq -1$, then

$$\int (d+ex)^m (a+b \operatorname{ArcSin}[cx])^n dx \rightarrow \frac{(d+ex)^{m+1} (a+b \operatorname{ArcSin}[cx])^n}{e(m+1)} - \frac{b c n}{e(m+1)} \int \frac{(d+ex)^{m+1} (a+b \operatorname{ArcSin}[cx])^{n-1}}{\sqrt{1-c^2 x^2}} dx$$

Program code:

```
Int[(d+_e_.**x_)^m_.*(a+_b_.*ArcSin[c_.**x_])^n_,x_Symbol] :=
  (d+e*x)^(m+1)*(a+b*ArcSin[c*x])^n/(e*(m+1)) -
  b*c*n/(e*(m+1))*Int[(d+e*x)^(m+1)*(a+b*ArcSin[c*x])^(n-1)/Sqrt[1-c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,m},x] && IGtQ[n,0] && NeQ[m,-1]
```

```
Int[(d+_e_.**x_)^m_.*(a+_b_.*ArcCos[c_.**x_])^n_,x_Symbol] :=
  (d+e*x)^(m+1)*(a+b*ArcCos[c*x])^n/(e*(m+1)) +
  b*c*n/(e*(m+1))*Int[(d+e*x)^(m+1)*(a+b*ArcCos[c*x])^(n-1)/Sqrt[1-c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,m},x] && IGtQ[n,0] && NeQ[m,-1]
```

2. $\int (d+ex)^m (a+b \operatorname{ArcSin}[cx])^n dx$ when $m \in \mathbb{Z}^+$

1: $\int (d+ex)^m (a+b \operatorname{ArcSin}[cx])^n dx$ when $m \in \mathbb{Z}^+ \wedge n < -1$

Derivation: Algebraic expansion

Rule: If $m \in \mathbb{Z}^+ \wedge n < -1$, then

$$\int (d+ex)^m (a+b \operatorname{ArcSin}[cx])^n dx \rightarrow \int \operatorname{ExpandIntegrand}[(d+ex)^m (a+b \operatorname{ArcSin}[cx])^n, x] dx$$

Program code:

```
Int[(d_+e_.**x_)^m_.*(a_.+b_.*ArcSin[c_.**x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x)^m*(a+b*ArcSin[c*x])^n,x],x] /;
FreeQ[{a,b,c,d,e},x] && IGtQ[m,0] && LtQ[n,-1]
```

```
Int[(d_+e_.**x_)^m_.*(a_.+b_.*ArcCos[c_.**x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x)^m*(a+b*ArcCos[c*x])^n,x],x] /;
FreeQ[{a,b,c,d,e},x] && IGtQ[m,0] && LtQ[n,-1]
```

$$\mathbf{2:} \int (d + e x)^m (a + b \operatorname{ArcSin}[c x])^n dx \text{ when } m \in \mathbb{Z}^+$$

Derivation: Integration by substitution

$$\text{Basis: } F[x] := \frac{1}{c} F\left[\frac{\sin(\operatorname{ArcSin}[cx])}{c}\right] \cos[\operatorname{ArcSin}[c x]] \partial_x \operatorname{ArcSin}[c x]$$

Note: If $m \in \mathbb{Z}^+$, then $(a + b x)^n \cos[x] (c d + e \sin[x])^m$ is integrable in closed-form.

Rule: If $m \in \mathbb{Z}^+$, then

$$\int (d + e x)^m (a + b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{1}{c^{m+1}} \operatorname{Subst}\left[\int (a + b x)^n \cos[x] (c d + e \sin[x])^m dx, x, \operatorname{ArcSin}[c x]\right]$$

Program code:

```
Int[(d_+e_.x_)^m_.*(a_+b_.*ArcSin[c_.x_])^n_,x_Symbol] :=
  1/c^(m+1)*Subst[Int[(a+b*x)^n*Cos[x]*(c*d+e*Sin[x])^m,x],x,ArcSin[c*x]] /;
FreeQ[{a,b,c,d,e,n},x] && IGtQ[m,0]
```

```
Int[(d_+e_.x_)^m_.*(a_+b_.*ArcCos[c_.x_])^n_,x_Symbol] :=
  -1/c^(m+1)*Subst[Int[(a+b*x)^n*Sin[x]*(c*d+e*Cos[x])^m,x],x,ArcCos[c*x]] /;
FreeQ[{a,b,c,d,e,n},x] && IGtQ[m,0]
```

$$2. \int P_x (a + b \operatorname{ArcSin}[c x])^n dx$$

$$\mathbf{1:} \int P_x (a + b \operatorname{ArcSin}[c x]) dx$$

Derivation: Integration by parts

Rule: Let $u = \int P_x dx$, then

$$\int P_x (a + b \operatorname{ArcSin}[c x]) \, dx \rightarrow u (a + b \operatorname{ArcSin}[c x]) - b c \int \frac{u}{\sqrt{1 - c^2 x^2}} \, dx$$

Program code:

```
Int[Px_*(a_.+b_.*ArcSin[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[ExpandExpression[Px,x],x]},
    Dist[a+b*ArcSin[c*x],u,x] - b*c*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x] /;
  FreeQ[{a,b,c},x] && PolynomialQ[Px,x]
```

```
Int[Px_*(a_.+b_.*ArcCos[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[ExpandExpression[Px,x],x]},
    Dist[a+b*ArcCos[c*x],u,x] + b*c*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x] /;
  FreeQ[{a,b,c},x] && PolynomialQ[Px,x]
```

x: $\int P_x (a + b \operatorname{ArcSin}[c x])^n \, dx$ when $n \in \mathbb{Z}^+$

Derivation: Integration by parts

Rule: If $n \in \mathbb{Z}^+$, let $u = \int P_x \, dx$, then

$$\int P_x (a + b \operatorname{ArcSin}[c x])^n \, dx \rightarrow u (a + b \operatorname{ArcSin}[c x])^n - b c n \int \frac{u (a + b \operatorname{ArcSin}[c x])^{n-1}}{\sqrt{1 - c^2 x^2}} \, dx$$

Program code:

```
(* Int[Px_*(a_.+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  With[{u=IntHide[Px,x]},
    Dist[(a+b*ArcSin[c*x])^n,u,x] - b*c*n*Int[SimplifyIntegrand[u*(a+b*ArcSin[c*x])^(n-1)/Sqrt[1-c^2*x^2],x],x] /;
  FreeQ[{a,b,c},x] && PolynomialQ[Px,x] && IGtQ[n,0] *)
```

```
(* Int[Px*(a_.+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  With[{u=IntHide[Px,x]},
    Dist[(a+b*ArcCos[c*x])^n,u,x] + b*c*n*Int[SimplifyIntegrand[u*(a+b*ArcCos[c*x])^(n-1)/Sqrt[1-c^2*x^2],x],x] /;
    FreeQ[{a,b,c},x] && PolynomialQ[Px,x] && IGtQ[n,0] *)
```

2: $\int P_x (a + b \operatorname{ArcSin}[c x])^n dx$ when $n \neq 1$

Derivation: Algebraic expansion

Rule: If $n \neq 1$, then

$$\int P_x (a + b \operatorname{ArcSin}[c x])^n dx \rightarrow \int \operatorname{ExpandIntegrand}[P_x (a + b \operatorname{ArcSin}[c x])^n, x] dx$$

Program code:

```
Int[Px*(a_.+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[Px*(a+b*ArcSin[c*x])^n,x],x] /;
  FreeQ[{a,b,c,n},x] && PolynomialQ[Px,x]
```

```
Int[Px*(a_.+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[Px*(a+b*ArcCos[c*x])^n,x],x] /;
  FreeQ[{a,b,c,n},x] && PolynomialQ[Px,x]
```

3. $\int P_x (d + e x)^m (a + b \operatorname{ArcSin}[c x])^n dx$ when $n \in \mathbb{Z}^+$

1: $\int P_x (d + e x)^m (a + b \operatorname{ArcSin}[c x]) dx$

Derivation: Integration by parts

Rule: Let $u = \int P_x (d + e x)^m dx$, then

$$\int P_x (d+ex)^m (a+b \operatorname{ArcSin}[cx]) \, dx \rightarrow u (a+b \operatorname{ArcSin}[cx]) - bc \int \frac{u}{\sqrt{1-c^2 x^2}} \, dx$$

Program code:

```
Int[Px_*(d_+e_.*x_)^m_.*(a_+b_.*ArcSin[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[Px*(d+e*x)^m,x]},
    Dist[a+b*ArcSin[c*x],u,x] - b*c*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x]] /;
  FreeQ[{a,b,c,d,e,m},x] && PolynomialQ[Px,x]
```

```
Int[Px_*(d_+e_.*x_)^m_.*(a_+b_.*ArcCos[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[Px*(d+e*x)^m,x]},
    Dist[a+b*ArcCos[c*x],u,x] + b*c*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x]] /;
  FreeQ[{a,b,c,d,e,m},x] && PolynomialQ[Px,x]
```

2: $\int (f+gx)^p (d+ex)^m (a+b \operatorname{ArcSin}[cx])^n \, dx$ when $(n \mid p) \in \mathbb{Z}^+ \wedge m \in \mathbb{Z}^- \wedge m+p+1 < 0$

Derivation: Integration by parts

Note: If $p \in \mathbb{Z}^+ \wedge m \in \mathbb{Z}^- \wedge m+p+1 < 0$, then $\int (f+gx)^p (d+ex)^m \, dx$ is a rational function.

Rule: If $(n \mid p) \in \mathbb{Z}^+ \wedge m \in \mathbb{Z}^- \wedge m+p+1 < 0$, let $u = \int (f+gx)^p (d+ex)^m \, dx$, then

$$\int (f+gx)^p (d+ex)^m (a+b \operatorname{ArcSin}[cx])^n \, dx \rightarrow u (a+b \operatorname{ArcSin}[cx])^n - bc n \int \frac{u (a+b \operatorname{ArcSin}[cx])^{n-1}}{\sqrt{1-c^2 x^2}} \, dx$$

Program code:

```
Int[(f_+g_.*x_)^p_.*(d_+e_.*x_)^m_.*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  With[{u=IntHide[(f+g*x)^p*(d+e*x)^m,x]},
    Dist[(a+b*ArcSin[c*x])^n,u,x] - b*c*n*Int[SimplifyIntegrand[u*(a+b*ArcSin[c*x])^(n-1)/Sqrt[1-c^2*x^2],x],x]] /;
  FreeQ[{a,b,c,d,e,f,g},x] && IGtQ[n,0] && IGtQ[p,0] && ILtQ[m,0] && LtQ[m+p+1,0]
```

```
Int[(f_.+g_.x_)^p.*(d_.+e_.x_)^m.*(a_.+b_.*ArcCos[c_.x_])^n,x_Symbol] :=
  With[{u=IntHide[(f+g*x)^p*(d+e*x)^m,x]},
    Dist[(a+b*ArcCos[c*x])^n,u,x] + b*c*n*Int[SimplifyIntegrand[u*(a+b*ArcCos[c*x])^(n-1)/Sqrt[1-c^2*x^2],x],x] /;
    FreeQ[{a,b,c,d,e,f,g},x] && IGtQ[n,0] && IGtQ[p,0] && ILtQ[m,0] && LtQ[m+p+1,0]
```

3: $\int \frac{(f + g x + h x^2)^p (a + b \operatorname{ArcSin}[c x])^n}{(d + e x)^2} dx$ when $(n \mid p) \in \mathbb{Z}^+ \wedge e g - 2 d h == 0$

Derivation: Integration by parts

Note: If $p \in \mathbb{Z}^+ \wedge e g - 2 d h == 0$, then $\int \frac{(f+g x+h x^2)^p}{(d+e x)^2} dx$ is a rational function.

Rule: If $(n \mid p) \in \mathbb{Z}^+ \wedge e g - 2 d h == 0$, let $u = \int \frac{(f+g x+h x^2)^p}{(d+e x)^2} dx$, then

$$\int \frac{(f + g x + h x^2)^p (a + b \operatorname{ArcSin}[c x])^n}{(d + e x)^2} dx \rightarrow u (a + b \operatorname{ArcSin}[c x])^n - b c n \int \frac{u (a + b \operatorname{ArcSin}[c x])^{n-1}}{\sqrt{1 - c^2 x^2}} dx$$

Program code:

```
Int[(f_.+g_.x_+h_.x_^2)^p.*(a_.+b_.*ArcSin[c_.x_])^n/(d_.+e_.x_)^2,x_Symbol] :=
  With[{u=IntHide[(f+g*x+h*x^2)^p/(d+e*x)^2,x]},
    Dist[(a+b*ArcSin[c*x])^n,u,x] - b*c*n*Int[SimplifyIntegrand[u*(a+b*ArcSin[c*x])^(n-1)/Sqrt[1-c^2*x^2],x],x] /;
    FreeQ[{a,b,c,d,e,f,g,h},x] && IGtQ[n,0] && IGtQ[p,0] && EqQ[e*g-2*d*h,0]
```

```
Int[(f_.+g_.x_+h_.x_^2)^p.*(a_.+b_.*ArcCos[c_.x_])^n/(d_.+e_.x_)^2,x_Symbol] :=
  With[{u=IntHide[(f+g*x+h*x^2)^p/(d+e*x)^2,x]},
    Dist[(a+b*ArcCos[c*x])^n,u,x] + b*c*n*Int[SimplifyIntegrand[u*(a+b*ArcCos[c*x])^(n-1)/Sqrt[1-c^2*x^2],x],x] /;
    FreeQ[{a,b,c,d,e,f,g,h},x] && IGtQ[n,0] && IGtQ[p,0] && EqQ[e*g-2*d*h,0]
```


4: $\int P_x (d+ex)^m (a+b \arcsin[cx])^n dx$ when $n \in \mathbb{Z}^+ \wedge m \in \mathbb{Z}$

Derivation: Algebraic expansion

Rule: If $n \in \mathbb{Z}^+ \wedge m \in \mathbb{Z}$, then

$$\int P_x (d+ex)^m (a+b \arcsin[cx])^n dx \rightarrow \int \text{ExpandIntegrand}[P_x (d+ex)^m (a+b \arcsin[cx])^n, x] dx$$

Program code:

```
Int[Px_*(d_+e_.*x_)^m_.*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[Px*(d+e*x)^m*(a+b*ArcSin[c*x])^n,x],x] /;
FreeQ[{a,b,c,d,e},x] && PolynomialQ[Px,x] && IGtQ[n,0] && IntegerQ[m]
```

```
Int[Px_*(d_+e_.*x_)^m_.*(a_+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[Px*(d+e*x)^m*(a+b*ArcCos[c*x])^n,x],x] /;
FreeQ[{a,b,c,d,e},x] && PolynomialQ[Px,x] && IGtQ[n,0] && IntegerQ[m]
```

4. $\int (f+gx)^m (d+ex^2)^p (a+b \arcsin[cx])^n dx$ when $c^2 d+e=0 \wedge m \in \mathbb{Z} \wedge p-\frac{1}{2} \in \mathbb{Z}$

1. $\int (f+gx)^m (d+ex^2)^p (a+b \arcsin[cx])^n dx$ when $c^2 d+e=0 \wedge m \in \mathbb{Z} \wedge p-\frac{1}{2} \in \mathbb{Z} \wedge d>0$

1: $\int (f+gx)^m (d+ex^2)^p (a+b \arcsin[cx])^n dx$ when $c^2 d+e=0 \wedge m \in \mathbb{Z}^+ \wedge p+\frac{1}{2} \in \mathbb{Z}^- \wedge d>0 \wedge (m<-2p-1 \vee m>3)$

Derivation: Integration by parts

Note: If $m \in \mathbb{Z} \wedge p+\frac{1}{2} \in \mathbb{Z} \wedge 0 < m < -2p-1$, then $\int (f+gx)^m (d+ex^2)^p dx$ is an algebraic function.

Rule: If $c^2 d+e=0 \wedge m \in \mathbb{Z}^+ \wedge p+\frac{1}{2} \in \mathbb{Z}^- \wedge d>0 \wedge (m<-2p-1 \vee m>3)$, let

$u = \int (f+gx)^m (d+ex^2)^p dx$, then

$$\int (f+gx)^m (d+ex^2)^p (a+b \arcsin[cx]) dx \rightarrow u (a+b \arcsin[cx]) - bc \int \frac{u}{\sqrt{1-c^2x^2}} dx$$

Program code:

```
Int[(f+g._.x_)^m_.*(d+e._.x_^2)^p_.*(a_+b_.*ArcSin[c._.x_]),x_Symbol] :=
  With[{u=IntHide[(f+g*x)^m*(d+e*x^2)^p,x]},
    Dist[a+b*ArcSin[c*x],u,x] - b*c*Int[Dist[1/Sqrt[1-c^2*x^2],u,x],x] /;
    FreeQ[{a,b,c,d,e,f,g},x] && EqQ[c^2*d+e,0] && IGtQ[m,0] && ILtQ[p+1/2,0] && GtQ[d,0] && (LtQ[m,-2*p-1] || GtQ[m,3])
```

```
Int[(f+g._.x_)^m_.*(d+e._.x_^2)^p_.*(a_+b_.*ArcCos[c._.x_]),x_Symbol] :=
  With[{u=IntHide[(f+g*x)^m*(d+e*x^2)^p,x]},
    Dist[a+b*ArcCos[c*x],u,x] + b*c*Int[Dist[1/Sqrt[1-c^2*x^2],u,x],x] /;
    FreeQ[{a,b,c,d,e,f,g},x] && EqQ[c^2*d+e,0] && IGtQ[m,0] && ILtQ[p+1/2,0] && GtQ[d,0] && (LtQ[m,-2*p-1] || GtQ[m,3])
```

2:

$$\int (f+gx)^m (d+ex^2)^p (a+b \arcsin[cx])^n dx \text{ when } c^2 d+e = 0 \wedge m \in \mathbb{Z}^+ \wedge p + \frac{1}{2} \in \mathbb{Z} \wedge d > 0 \wedge n \in \mathbb{Z}^+ \wedge (m = 1 \vee p > 0 \vee (n = 1 \wedge p > -1) \vee (m = 2 \wedge p < -2))$$

Derivation: Algebraic expansion

Rule: If $c^2 d + e = 0 \wedge m \in \mathbb{Z} \wedge p + \frac{1}{2} \in \mathbb{Z} \wedge d > 0 \wedge n \in \mathbb{Z}^+ \wedge m > 0$, then

$$\int (f+gx)^m (d+ex^2)^p (a+b \arcsin[cx])^n dx \rightarrow \int (d+ex^2)^p (a+b \arcsin[cx])^n \text{ExpandIntegrand}[(f+gx)^m, x] dx$$

Program code:

```
Int[(f+g._.x_)^m_.*(d+e._.x_^2)^p_.*(a_+b_.*ArcSin[c._.x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x^2)^p*(a+b*ArcSin[c*x])^n,(f+g*x)^m,x],x] /;
  FreeQ[{a,b,c,d,e,f,g},x] && EqQ[c^2*d+e,0] && IGtQ[m,0] && IntegerQ[p+1/2] && GtQ[d,0] && IGtQ[n,0] &&
  (m==1 || p>0 || n==1 && p>-1 || m==2 && p<-2)
```

```

Int[(f+g_.**x_)^m_.*(d+e_.**x_^2)^p_*(a_.+b_.*ArcCos[c_.**x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x^2)^p*(a+b*ArcCos[c*x])^n,(f+g*x)^m,x],x] /;
FreeQ[{a,b,c,d,e,f,g},x] && EqQ[c^2*d+e,0] && IGtQ[m,0] && IntegerQ[p+1/2] && GtQ[d,0] && IGtQ[n,0] &&
(m==1 || p>0 || n==1 && p>-1 || m==2 && p<-2)

```

$$3. \int (f+gx)^m (d+ex^2)^p (a+b \operatorname{ArcSin}[cx])^n dx \text{ when } c^2 d+e=0 \wedge m \in \mathbb{Z} \wedge p+\frac{1}{2} \in \mathbb{Z}^+ \wedge d>0$$

$$1: \int (f+gx)^m \sqrt{d+ex^2} (a+b \operatorname{ArcSin}[cx])^n dx \text{ when } c^2 d+e=0 \wedge m \in \mathbb{Z}^- \wedge d>0 \wedge n \in \mathbb{Z}^+$$

Derivation: Integration by parts

Basis: If $c^2 d+e=0 \wedge d>0$, then $\frac{(a+b \operatorname{ArcSin}[cx])^n}{\sqrt{d+ex^2}} = \partial_x \frac{(a+b \operatorname{ArcSin}[cx])^{n+1}}{bc\sqrt{d}(n+1)}$

Rule: If $c^2 d+e=0 \wedge m \in \mathbb{Z}^- \wedge d>0 \wedge n \in \mathbb{Z}^+$, then

$$\int (f+gx)^m \sqrt{d+ex^2} (a+b \operatorname{ArcSin}[cx])^n dx \rightarrow \frac{(f+gx)^m (d+ex^2) (a+b \operatorname{ArcSin}[cx])^{n+1}}{bc\sqrt{d}(n+1)} - \frac{1}{bc\sqrt{d}(n+1)} \int (dgm+2efx+eg(m+2)x^2) (f+gx)^{m-1} (a+b \operatorname{ArcSin}[cx])^{n+1} dx$$

Program code:

```

Int[(f+g_.**x_)^m_*Sqrt[d+e_.**x_^2]*(a_.+b_.*ArcSin[c_.**x_])^n_,x_Symbol] :=
  (f+g*x)^m*(d+e*x^2)*(a+b*ArcSin[c*x])^(n+1)/(b*c*Sqrt[d]*(n+1)) -
  1/(b*c*Sqrt[d]*(n+1))*Int[(d*g*m+2*e*f*x+e*g*(m+2)*x^2)*(f+g*x)^(m-1)*(a+b*ArcSin[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f,g},x] && EqQ[c^2*d+e,0] && ILtQ[m,0] && GtQ[d,0] && IGtQ[n,0]

```

```

Int[(f+g_.**x_)^m_*Sqrt[d+e_.**x_^2]*(a_.+b_.*ArcCos[c_.**x_])^n_,x_Symbol] :=
  -(f+g*x)^m*(d+e*x^2)*(a+b*ArcCos[c*x])^(n+1)/(b*c*Sqrt[d]*(n+1)) +
  1/(b*c*Sqrt[d]*(n+1))*Int[(d*g*m+2*e*f*x+e*g*(m+2)*x^2)*(f+g*x)^(m-1)*(a+b*ArcCos[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f,g},x] && EqQ[c^2*d+e,0] && ILtQ[m,0] && GtQ[d,0] && IGtQ[n,0]

```

$$\mathbf{2:} \int (f+gx)^m (d+ex^2)^p (a+b \arcsin(cx))^n dx \text{ when } c^2 d+e=0 \wedge m \in \mathbb{Z} \wedge p+\frac{1}{2} \in \mathbb{Z}^+ \wedge d>0 \wedge n \in \mathbb{Z}^+$$

Derivation: Algebraic expansion

Rule: If $c^2 d+e=0 \wedge m \in \mathbb{Z} \wedge p+\frac{1}{2} \in \mathbb{Z}^+ \wedge d>0 \wedge n \in \mathbb{Z}^+$, then

$$\int (f+gx)^m (d+ex^2)^p (a+b \arcsin(cx))^n dx \rightarrow \int \sqrt{d+ex^2} (a+b \arcsin(cx))^n \text{ExpandIntegrand}[(f+gx)^m (d+ex^2)^{p-1/2}, x] dx$$

Program code:

```
Int[(f+_g_.*x_)^m_.*(d+_e_.*x_^2)^p_.*(a+_b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[Sqrt[d+_e*x^2]*(a+_b*ArcSin[c*x])^n,(f+_g*x)^m*(d+_e*x^2)^(p-1/2),x],x] /;
  FreeQ[{a,b,c,d,e,f,g},x] && EqQ[c^2*d+_e,0] && IntegerQ[m] && IGtQ[p+1/2,0] && GtQ[d,0] && IGtQ[n,0]
```

```
Int[(f+_g_.*x_)^m_.*(d+_e_.*x_^2)^p_.*(a+_b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[Sqrt[d+_e*x^2]*(a+_b*ArcCos[c*x])^n,(f+_g*x)^m*(d+_e*x^2)^(p-1/2),x],x] /;
  FreeQ[{a,b,c,d,e,f,g},x] && EqQ[c^2*d+_e,0] && IntegerQ[m] && IGtQ[p+1/2,0] && GtQ[d,0] && IGtQ[n,0]
```

$$\mathbf{3:} \int (f+gx)^m (d+ex^2)^p (a+b \arcsin(cx))^n dx \text{ when } c^2 d+e=0 \wedge m \in \mathbb{Z}^- \wedge p-\frac{1}{2} \in \mathbb{Z}^+ \wedge d>0 \wedge n \in \mathbb{Z}^+$$

Derivation: Integration by parts

Basis: If $c^2 d+e=0 \wedge d>0$, then $\frac{(a+b \arcsin(cx))^n}{\sqrt{d+ex^2}} = \partial_x \frac{(a+b \arcsin(cx))^{n+1}}{bc \sqrt{d} (n+1)}$

Rule: If $c^2 d+e=0 \wedge m \in \mathbb{Z}^- \wedge p-\frac{1}{2} \in \mathbb{Z}^+ \wedge d>0 \wedge n \in \mathbb{Z}^+$, then

$$\int (f+gx)^m (d+ex^2)^p (a+b \arcsin(cx))^n dx \rightarrow \frac{(f+gx)^m (d+ex^2)^{p+\frac{1}{2}} (a+b \arcsin(cx))^{n+1}}{bc \sqrt{d} (n+1)} -$$

$$\frac{1}{b c \sqrt{d} (n+1)} \int (f+g x)^{m-1} (a+b \operatorname{ArcSin}[c x])^{n+1} \operatorname{ExpandIntegrand}\left[(d g m+e f(2 p+1) x+e g(m+2 p+1) x^2)(d+e x^2)^{p-\frac{1}{2}}, x\right] dx$$

Program code:

```
Int[(f_+g_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  (f+g*x)^m*(d+e*x^2)^(p+1/2)*(a+b*ArcSin[c*x])^(n+1)/(b*c*Sqrt[d]*(n+1)) -
  1/(b*c*Sqrt[d]*(n+1))*
  Int[ExpandIntegrand[(f+g*x)^(m-1)*(a+b*ArcSin[c*x])^(n+1), (d*g*m+e*f*(2*p+1)*x+e*g*(m+2*p+1)*x^2)*(d+e*x^2)^(p-1/2), x], x] /;
FreeQ[{a,b,c,d,e,f,g},x] && EqQ[c^2*d+e,0] && ILtQ[m,0] && IGtQ[p-1/2,0] && GtQ[d,0] && IGtQ[n,0]
```

```
Int[(f_+g_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  -(f+g*x)^m*(d+e*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n+1)/(b*c*Sqrt[d]*(n+1)) +
  1/(b*c*Sqrt[d]*(n+1))*
  Int[ExpandIntegrand[(f+g*x)^(m-1)*(a+b*ArcCos[c*x])^(n+1), (d*g*m+e*f*(2*p+1)*x+e*g*(m+2*p+1)*x^2)*(d+e*x^2)^(p-1/2), x], x] /;
FreeQ[{a,b,c,d,e,f,g},x] && EqQ[c^2*d+e,0] && ILtQ[m,0] && IGtQ[p-1/2,0] && GtQ[d,0] && IGtQ[n,0]
```

$$4. \int (f+gx)^m (d+ex^2)^p (a+b \operatorname{ArcSin}[cx])^n dx \text{ when } c^2 d+e = 0 \wedge m \in \mathbb{Z} \wedge p - \frac{1}{2} \in \mathbb{Z}^- \wedge d > 0$$

$$1. \int \frac{(f+gx)^m (a+b \operatorname{ArcSin}[cx])^n}{\sqrt{d+ex^2}} dx \text{ when } c^2 d+e = 0 \wedge m \in \mathbb{Z} \wedge d > 0$$

$$1: \int \frac{(f+gx)^m (a+b \operatorname{ArcSin}[cx])^n}{\sqrt{d+ex^2}} dx \text{ when } c^2 d+e = 0 \wedge m \in \mathbb{Z}^+ \wedge d > 0 \wedge n < -1$$

Derivation: Integration by parts

$$\text{Basis: If } c^2 d+e = 0 \wedge d > 0, \text{ then } \frac{(a+b \operatorname{ArcSin}[cx])^n}{\sqrt{d+ex^2}} = \partial_x \frac{(a+b \operatorname{ArcSin}[cx])^{n+1}}{bc \sqrt{d} (n+1)}$$

Rule: If $c^2 d+e = 0 \wedge m \in \mathbb{Z} \wedge d > 0 \wedge m > 0 \wedge n < -1$, then

$$\int \frac{(f+gx)^m (a+b \operatorname{ArcSin}[cx])^n}{\sqrt{d+ex^2}} dx \rightarrow \frac{(f+gx)^m (a+b \operatorname{ArcSin}[cx])^{n+1}}{bc \sqrt{d} (n+1)} - \frac{gm}{bc \sqrt{d} (n+1)} \int (f+gx)^{m-1} (a+b \operatorname{ArcSin}[cx])^{n+1} dx$$

Program code:

```
Int[(f+_g_.*x_)^m_.*(a_+_b_.*ArcSin[c_.*x_])^n_/Sqrt[d+_e_.*x_^2],x_Symbol] :=
  (f+g*x)^m*(a+b*ArcSin[c*x])^(n+1)/(b*c*Sqrt[d]*(n+1)) -
  g*m/(b*c*Sqrt[d]*(n+1))*Int[(f+g*x)^(m-1)*(a+b*ArcSin[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f,g},x] && EqQ[c^2*d+e,0] && IGtQ[m,0] && GtQ[d,0] && LtQ[n,-1]
```

```
Int[(f+_g_.*x_)^m_.*(a_+_b_.*ArcCos[c_.*x_])^n_/Sqrt[d+_e_.*x_^2],x_Symbol] :=
  -(f+g*x)^m*(a+b*ArcCos[c*x])^(n+1)/(b*c*Sqrt[d]*(n+1)) +
  g*m/(b*c*Sqrt[d]*(n+1))*Int[(f+g*x)^(m-1)*(a+b*ArcCos[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f,g},x] && EqQ[c^2*d+e,0] && IGtQ[m,0] && GtQ[d,0] && LtQ[n,-1]
```

$$\text{2: } \int \frac{(f+gx)^m (a+b \operatorname{ArcSin}[cx])^n}{\sqrt{d+ex^2}} dx \text{ when } c^2 d + e == 0 \wedge m \in \mathbb{Z} \wedge d > 0 \wedge (m > 0 \vee n \in \mathbb{Z}^+)$$

Derivation: Integration by substitution

Basis: If $c^2 d + e == 0 \wedge d > 0$, then $\frac{F[x]}{\sqrt{d+ex^2}} == \frac{1}{c\sqrt{d}} \operatorname{Subst}\left[F\left[\frac{\operatorname{Sin}[x]}{c}\right], x, \operatorname{ArcSin}[cx]\right] \partial_x \operatorname{ArcSin}[cx]$

Rule: If $c^2 d + e == 0 \wedge m \in \mathbb{Z} \wedge d > 0 \wedge (m > 0 \vee n \in \mathbb{Z}^+)$, then

$$\int \frac{(f+gx)^m (a+b \operatorname{ArcSin}[cx])^n}{\sqrt{d+ex^2}} dx \rightarrow \frac{1}{c^{m+1} \sqrt{d}} \operatorname{Subst}\left[\int (a+bx)^n (cf+g \operatorname{Sin}[x])^m dx, x, \operatorname{ArcSin}[cx]\right]$$

Program code:

```
Int[(f_+g_.*x_)^m_.*(a_+b_.*ArcSin[c_.*x_])^n_/Sqrt[d_+e_.*x_^2],x_Symbol] :=
  1/(c^(m+1)*Sqrt[d])*Subst[Int[(a+b*x)^n*(c*f+g*Sin[x])^m,x],x,ArcSin[c*x]] /;
FreeQ[{a,b,c,d,e,f,g,n},x] && EqQ[c^2*d+e,0] && IntegerQ[m] && GtQ[d,0] && (GtQ[m,0] || IGtQ[n,0])
```

```
Int[(f_+g_.*x_)^m_.*(a_+b_.*ArcCos[c_.*x_])^n_/Sqrt[d_+e_.*x_^2],x_Symbol] :=
  -1/(c^(m+1)*Sqrt[d])*Subst[Int[(a+b*x)^n*(c*f+g*Cos[x])^m,x],x,ArcCos[c*x]] /;
FreeQ[{a,b,c,d,e,f,g,n},x] && EqQ[c^2*d+e,0] && IntegerQ[m] && GtQ[d,0] && (GtQ[m,0] || IGtQ[n,0])
```

2: $\int (f+gx)^m (d+ex^2)^p (a+b \operatorname{ArcSin}[cx])^n dx$ when $c^2 d+e=0 \wedge m \in \mathbb{Z} \wedge p+\frac{1}{2} \in \mathbb{Z}^- \wedge d>0 \wedge n \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule: If $c^2 d+e=0 \wedge m \in \mathbb{Z} \wedge p+\frac{1}{2} \in \mathbb{Z}^- \wedge d>0 \wedge n \in \mathbb{Z}^+$, then

$$\int (f+gx)^m (d+ex^2)^p (a+b \operatorname{ArcSin}[cx])^n dx \rightarrow \int \frac{(a+b \operatorname{ArcSin}[cx])^n}{\sqrt{d+ex^2}} \operatorname{ExpandIntegrand}[(f+gx)^m (d+ex^2)^{p+1/2}, x] dx$$

Program code:

```
Int[(f+_g_.*x_)^m_.*(d+_e_.*x_^2)^p_.*(a+_b_.*ArcSin[c_.*x_])^n_.,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*ArcSin[c*x])^n/Sqrt[d+e*x^2], (f+g*x)^m*(d+e*x^2)^(p+1/2), x], x] /;
FreeQ[{a,b,c,d,e,f,g}, x] && EqQ[c^2*d+e, 0] && IntegerQ[m] && ILtQ[p+1/2, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

```
Int[(f+_g_.*x_)^m_.*(d+_e_.*x_^2)^p_.*(a+_b_.*ArcCos[c_.*x_])^n_.,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*ArcCos[c*x])^n/Sqrt[d+e*x^2], (f+g*x)^m*(d+e*x^2)^(p+1/2), x], x] /;
FreeQ[{a,b,c,d,e,f,g}, x] && EqQ[c^2*d+e, 0] && IntegerQ[m] && ILtQ[p+1/2, 0] && GtQ[d, 0] && IGtQ[n, 0]
```


2: $\int (f+gx)^m (d+ex^2)^p (a+b \operatorname{ArcSin}[cx])^n dx$ when $c^2 d + e = 0 \wedge m \in \mathbb{Z} \wedge p - \frac{1}{2} \in \mathbb{Z} \wedge d \neq 0$

Derivation: Piecewise constant extraction

Basis: If $c^2 d + e = 0$, then $\partial_x \frac{(d+ex^2)^p}{(1-c^2 x^2)^p} = 0$

Rule: If $c^2 d + e = 0 \wedge m \in \mathbb{Z} \wedge p - \frac{1}{2} \in \mathbb{Z} \wedge d \neq 0$, then

$$\int (f+gx)^m (d+ex^2)^p (a+b \operatorname{ArcSin}[cx])^n dx \rightarrow \frac{d^{\operatorname{IntPart}[p]} (d+ex^2)^{\operatorname{FracPart}[p]}}{(1-c^2 x^2)^{\operatorname{FracPart}[p]}} \int (f+gx)^m (1-c^2 x^2)^p (a+b \operatorname{ArcSin}[cx])^n dx$$

Program code:

```
Int[(f+_.*x_)^m_.*(d+_.*x_^2)^p_.*(a+_.*b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  d^IntPart[p]*(d+e*x^2)^FracPart[p]/(1-c^2*x^2)^FracPart[p]*Int[(f+g*x)^m*(1-c^2*x^2)^p*(a+b*ArcSin[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,n},x] && EqQ[c^2*d+e,0] && IntegerQ[m] && IntegerQ[p-1/2] && Not[GtQ[d,0]]
```

```
Int[(f+_.*x_)^m_.*(d+_.*x_^2)^p_.*(a+_.*b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  d^IntPart[p]*(d+e*x^2)^FracPart[p]/(1-c^2*x^2)^FracPart[p]*Int[(f+g*x)^m*(1-c^2*x^2)^p*(a+b*ArcCos[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,n},x] && EqQ[c^2*d+e,0] && IntegerQ[m] && IntegerQ[p-1/2] && Not[GtQ[d,0]]
```

$$5. \int \text{Log}[h(f+gx)^m] (d+ex^2)^p (a+b \arcsin[cx])^n dx \text{ when } c^2 d + e = 0 \wedge p - \frac{1}{2} \in \mathbb{Z}$$

$$1. \int \text{Log}[h(f+gx)^m] (d+ex^2)^p (a+b \arcsin[cx])^n dx \text{ when } c^2 d + e = 0 \wedge p - \frac{1}{2} \in \mathbb{Z} \wedge d > 0$$

$$1: \int \frac{\text{Log}[h(f+gx)^m] (a+b \arcsin[cx])^n}{\sqrt{d+ex^2}} dx \text{ when } c^2 d + e = 0 \wedge d > 0 \wedge n \in \mathbb{Z}^+$$

Derivation: Integration by parts

Basis: If $c^2 d + e = 0 \wedge d > 0$, then $\frac{(a+b \arcsin[cx])^n}{\sqrt{d+ex^2}} = \partial_x \frac{(a+b \arcsin[cx])^{n+1}}{bc \sqrt{d} (n+1)}$

Note: If $n \in \mathbb{Z}^+$, then $\frac{(a+b \arcsin[cx])^{n+1}}{f+gx}$ is integrable in closed-form.

Rule: If $c^2 d + e = 0 \wedge d > 0 \wedge n \in \mathbb{Z}^+$, then

$$\int \frac{\text{Log}[h(f+gx)^m] (a+b \arcsin[cx])^n}{\sqrt{d+ex^2}} dx \rightarrow \frac{\text{Log}[h(f+gx)^m] (a+b \arcsin[cx])^{n+1}}{bc \sqrt{d} (n+1)} - \frac{gm}{bc \sqrt{d} (n+1)} \int \frac{(a+b \arcsin[cx])^{n+1}}{f+gx} dx$$

Program code:

```
Int[Log[h_.*(f_+g_.x_)^m_.]*(a_+b_.*ArcSin[c_.x_])^n_./Sqrt[d_+e_.x_^2],x_Symbol] :=
  Log[h*(f+g*x)^m]*(a+b*ArcSin[c*x])^(n+1)/(b*c*Sqrt[d]*(n+1)) -
  g*m/(b*c*Sqrt[d]*(n+1))*Int[(a+b*ArcSin[c*x])^(n+1)/(f+g*x),x] /;
FreeQ[{a,b,c,d,e,f,g,h,m},x] && EqQ[c^2*d+e,0] && GtQ[d,0] && IGtQ[n,0]
```

```
Int[Log[h_.*(f_+g_.x_)^m_.]*(a_+b_.*ArcCos[c_.x_])^n_./Sqrt[d_+e_.x_^2],x_Symbol] :=
  -Log[h*(f+g*x)^m]*(a+b*ArcCos[c*x])^(n+1)/(b*c*Sqrt[d]*(n+1)) +
  g*m/(b*c*Sqrt[d]*(n+1))*Int[(a+b*ArcCos[c*x])^(n+1)/(f+g*x),x] /;
FreeQ[{a,b,c,d,e,f,g,h,m},x] && EqQ[c^2*d+e,0] && GtQ[d,0] && IGtQ[n,0]
```

2: $\int \text{Log}[h(f+gx)^m] (d+ex^2)^p (a+b \text{ArcSin}[cx])^n dx$ when $c^2 d + e = 0 \wedge p - \frac{1}{2} \in \mathbb{Z} \wedge d \neq 0$

Derivation: Piecewise constant extraction

Basis: If $c^2 d + e = 0$, then $\partial_x \frac{(d+ex^2)^p}{(1-c^2 x^2)^p} = 0$

Rule: If $c^2 d + e = 0 \wedge p - \frac{1}{2} \in \mathbb{Z} \wedge d \neq 0$, then

$$\int \text{Log}[h(f+gx)^m] (d+ex^2)^p (a+b \text{ArcSin}[cx])^n dx \rightarrow \frac{d^{\text{IntPart}[p]} (d+ex^2)^{\text{FracPart}[p]}}{(1-c^2 x^2)^{\text{FracPart}[p]}} \int \text{Log}[h(f+gx)^m] (1-c^2 x^2)^p (a+b \text{ArcSin}[cx])^n dx$$

Program code:

```
Int[Log[h_.*(f_+g_.*x_)^m_.*(d_+e_.*x_)^p_.*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  d^IntPart[p]*(d+e*x^2)^FracPart[p]/(1-c^2*x^2)^FracPart[p]*Int[Log[h*(f+g*x)^m]*(1-c^2*x^2)^p*(a+b*ArcSin[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,h,m,n},x] && EqQ[c^2*d+e,0] && IntegerQ[p-1/2] && Not[GtQ[d,0]]
```

```
Int[Log[h_.*(f_+g_.*x_)^m_.*(d_+e_.*x_)^p_.*(a_+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  d^IntPart[p]*(d+e*x^2)^FracPart[p]/(1-c^2*x^2)^FracPart[p]*Int[Log[h*(f+g*x)^m]*(1-c^2*x^2)^p*(a+b*ArcCos[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,h,m,n},x] && EqQ[c^2*d+e,0] && IntegerQ[p-1/2] && Not[GtQ[d,0]]
```

$$6. \int (d + ex)^m (f + gx)^m (a + b \operatorname{ArcSin}[cx])^n dx$$

$$1: \int (d + ex)^m (f + gx)^m (a + b \operatorname{ArcSin}[cx]) dx \text{ when } m + \frac{1}{2} \in \mathbb{Z}^-$$

Derivation: Integration by parts

Rule: If $m + \frac{1}{2} \in \mathbb{Z}^-$, let $u = \int (d + ex)^m (f + gx)^m dx$, then

$$\int (d + ex)^m (f + gx)^m (a + b \operatorname{ArcSin}[cx]) dx \rightarrow u (a + b \operatorname{ArcSin}[cx]) - b c \int \frac{u}{\sqrt{1 - c^2 x^2}} dx$$

Program code:

```
Int[(d_+e_.*x_)^m_*(f_+g_.*x_)^m_*(a_+b_.*ArcSin[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(d+e*x)^m*(f+g*x)^m,x]},
    Dist[a+b*ArcSin[c*x],u,x] - b*c*Int[Dist[1/Sqrt[1-c^2*x^2],u,x],x] /;
  FreeQ[{a,b,c,d,e,f,g},x] && ILtQ[m+1/2,0]
```

```
Int[(d_+e_.*x_)^m_*(f_+g_.*x_)^m_*(a_+b_.*ArcCos[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(d+e*x)^m*(f+g*x)^m,x]},
    Dist[a+b*ArcCos[c*x],u,x] + b*c*Int[Dist[1/Sqrt[1-c^2*x^2],u,x],x] /;
  FreeQ[{a,b,c,d,e,f,g},x] && ILtQ[m+1/2,0]
```

2: $\int (d+ex)^m (f+gx)^m (a+b \operatorname{ArcSin}[cx])^n dx$ when $m \in \mathbb{Z}$

Derivation: Algebraic expansion

Rule: If $m \in \mathbb{Z}$, then

$$\int (d+ex)^m (f+gx)^m (a+b \operatorname{ArcSin}[cx])^n dx \rightarrow \int \operatorname{ExpandIntegrand}[(d+ex)^m (f+gx)^m (a+b \operatorname{ArcSin}[cx])^n, x] dx$$

Program code:

```
Int[(d_+e_.*x_)^m_.*(f_+g_.*x_)^m_.*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x)^m*(f+g*x)^m*(a+b*ArcSin[c*x])^n,x],x] /;
FreeQ[{a,b,c,d,e,f,g,n},x] && IntegerQ[m]
```

```
Int[(d_+e_.*x_)^m_.*(f_+g_.*x_)^m_.*(a_+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x)^m*(f+g*x)^m*(a+b*ArcCos[c*x])^n,x],x] /;
FreeQ[{a,b,c,d,e,f,g,n},x] && IntegerQ[m]
```

7: $\int u (a + b \operatorname{ArcSin}[c x]) \, dx$ when $\int u \, dx$ is free of inverse functions

Derivation: Integration by parts

Rule: Let $v = \int u \, dx$, if v is free of inverse functions, then

$$\int u (a + b \operatorname{ArcSin}[c x]) \, dx \rightarrow v (a + b \operatorname{ArcSin}[c x]) - b c \int \frac{v}{\sqrt{1 - c^2 x^2}} \, dx$$

Program code:

```
Int[u_*(a_.+b_.*ArcSin[c_.*x_]),x_Symbol] :=
  With[{v=IntHide[u,x]},
    Dist[a+b*ArcSin[c*x],v,x] - b*c*Int[SimplifyIntegrand[v/Sqrt[1-c^2*x^2],x],x] /;
    InverseFunctionFreeQ[v,x] /;
    FreeQ[{a,b,c},x]
```

```
Int[u_*(a_.+b_.*ArcCos[c_.*x_]),x_Symbol] :=
  With[{v=IntHide[u,x]},
    Dist[a+b*ArcCos[c*x],v,x] + b*c*Int[SimplifyIntegrand[v/Sqrt[1-c^2*x^2],x],x] /;
    InverseFunctionFreeQ[v,x] /;
    FreeQ[{a,b,c},x]
```

$$8. \int P_x u (a + b \operatorname{ArcSin}[c x])^n dx$$

$$1: \int P_x (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d + e = 0 \wedge p - \frac{1}{2} \in \mathbb{Z}$$

Derivation: Algebraic expansion

Rule: If $c^2 d + e = 0 \wedge p - \frac{1}{2} \in \mathbb{Z}$, then

$$\int P_x (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n dx \rightarrow \int \operatorname{ExpandIntegrand}[P_x (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n, x] dx$$

Program code:

```
Int[Px*(d+e.*x^2)^p*(a_.+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  With[{u=ExpandIntegrand[Px*(d+e*x^2)^p*(a+b*ArcSin[c*x])^n,x]},
    Int[u,x] /;
    SumQ[u] /;
    FreeQ[{a,b,c,d,e,n},x] && PolynomialQ[Px,x] && EqQ[c^2*d+e,0] && IntegerQ[p-1/2]
```

```
Int[Px*(d+e.*x^2)^p*(a_.+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  With[{u=ExpandIntegrand[Px*(d+e*x^2)^p*(a+b*ArcCos[c*x])^n,x]},
    Int[u,x] /;
    SumQ[u] /;
    FreeQ[{a,b,c,d,e,n},x] && PolynomialQ[Px,x] && EqQ[c^2*d+e,0] && IntegerQ[p-1/2]
```

2: $\int P_x (f + g (d + e x^2)^p)^m (a + b \operatorname{ArcSin}[c x])^n dx$ when $c^2 d + e = 0 \wedge p + \frac{1}{2} \in \mathbb{Z}^+ \wedge (m | n) \in \mathbb{Z}$

Derivation: Algebraic expansion

Rule: If $c^2 d + e = 0 \wedge p + \frac{1}{2} \in \mathbb{Z}^+ \wedge (m | n) \in \mathbb{Z}$, then

$$\int P_x (f + g (d + e x^2)^p)^m (a + b \operatorname{ArcSin}[c x])^n dx \rightarrow \int \operatorname{ExpandIntegrand}[P_x (f + g (d + e x^2)^p)^m (a + b \operatorname{ArcSin}[c x])^n, x] dx$$

Program code:

```
Int[Px_.*(f_+g_.*(d_+e_.*x_^2)^p_)^m_.*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  With[{u=ExpandIntegrand[Px*(f+g*(d+e*x^2)^p)^m*(a+b*ArcSin[c*x])^n,x]},
    Int[u,x] /;
    SumQ[u] /;
    FreeQ[{a,b,c,d,e,f,g},x] && PolynomialQ[Px,x] && EqQ[c^2*d+e,0] && IGtQ[p+1/2,0] && IntegersQ[m,n]
```

```
Int[Px_.*(f_+g_.*(d_+e_.*x_^2)^p_)^m_.*(a_+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  With[{u=ExpandIntegrand[Px*(f+g*(d+e*x^2)^p)^m*(a+b*ArcCos[c*x])^n,x]},
    Int[u,x] /;
    SumQ[u] /;
    FreeQ[{a,b,c,d,e,f,g},x] && PolynomialQ[Px,x] && EqQ[c^2*d+e,0] && IGtQ[p+1/2,0] && IntegersQ[m,n]
```


9. $\int \text{RF}_x u (a + b \text{ArcSin}[c x])^n dx$ when $n \in \mathbb{Z}^+$

1. $\int \text{RF}_x (a + b \text{ArcSin}[c x])^n dx$ when $n \in \mathbb{Z}^+$

1: $\int \text{RF}_x \text{ArcSin}[c x]^n dx$ when $n \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule: If $n \in \mathbb{Z}^+$, then

$$\int \text{RF}_x \text{ArcSin}[c x]^n dx \rightarrow \int \text{ArcSin}[c x]^n \text{ExpandIntegrand}[\text{RF}_x, x] dx$$

Program code:

```
Int[RFx_*ArcSin[c_*x_]^n_,x_Symbol] :=
  With[{u=ExpandIntegrand[ArcSin[c*x]^n,RFx,x]},
    Int[u,x] /;
    SumQ[u] /;
    FreeQ[c,x] && RationalFunctionQ[RFx,x] && IGtQ[n,0]
```

```
Int[RFx_*ArcCos[c_*x_]^n_,x_Symbol] :=
  With[{u=ExpandIntegrand[ArcCos[c*x]^n,RFx,x]},
    Int[u,x] /;
    SumQ[u] /;
    FreeQ[c,x] && RationalFunctionQ[RFx,x] && IGtQ[n,0]
```

2: $\int \text{RF}_x (a + b \text{ArcSin}[c x])^n dx$ when $n \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule: If $n \in \mathbb{Z}^+$, then

$$\int \text{RF}_x (a + b \text{ArcSin}[c x])^n dx \rightarrow \int \text{ExpandIntegrand}[\text{RF}_x (a + b \text{ArcSin}[c x])^n, x] dx$$

Program code:

```
Int[RFx*(a+b_*ArcSin[c_*x_])^n_.,x_Symbol] :=
  Int[ExpandIntegrand[RFx*(a+b*ArcSin[c*x])^n,x],x] /;
FreeQ[{a,b,c},x] && RationalFunctionQ[RFx,x] && IGtQ[n,0]
```

```
Int[RFx*(a+b_*ArcCos[c_*x_])^n_.,x_Symbol] :=
  Int[ExpandIntegrand[RFx*(a+b*ArcCos[c*x])^n,x],x] /;
FreeQ[{a,b,c},x] && RationalFunctionQ[RFx,x] && IGtQ[n,0]
```

$$2. \int \text{RF}_x (d + e x^2)^p (a + b \text{ArcSin}[c x])^n dx \text{ when } n \in \mathbb{Z}^+ \wedge c^2 d + e = 0 \wedge p - \frac{1}{2} \in \mathbb{Z}$$

$$1: \int \text{RF}_x (d + e x^2)^p \text{ArcSin}[c x]^n dx \text{ when } n \in \mathbb{Z}^+ \wedge c^2 d + e = 0 \wedge p - \frac{1}{2} \in \mathbb{Z}$$

Derivation: Algebraic expansion

Rule: If $n \in \mathbb{Z}^+ \wedge c^2 d + e = 0 \wedge p - \frac{1}{2} \in \mathbb{Z}$, then

$$\int \text{RF}_x (d + e x^2)^p \text{ArcSin}[c x]^n dx \rightarrow \int (d + e x^2)^p \text{ArcSin}[c x]^n \text{ExpandIntegrand}[\text{RF}_x, x] dx$$

Program code:

```
Int[RFx_*(d_+e_.*x_^2)^p_*ArcSin[c_.*x_]^n_,x_Symbol] :=
  With[{u=ExpandIntegrand[(d+e*x^2)^p*ArcSin[c*x]^n,RFx,x]},
    Int[u,x] /;
    SumQ[u] /;
    FreeQ[{c,d,e},x] && RationalFunctionQ[RFx,x] && IGtQ[n,0] && EqQ[c^2*d+e,0] && IntegerQ[p-1/2]
```

```
Int[RFx_*(d_+e_.*x_^2)^p_*ArcCos[c_.*x_]^n_,x_Symbol] :=
  With[{u=ExpandIntegrand[(d+e*x^2)^p*ArcCos[c*x]^n,RFx,x]},
    Int[u,x] /;
    SumQ[u] /;
    FreeQ[{c,d,e},x] && RationalFunctionQ[RFx,x] && IGtQ[n,0] && EqQ[c^2*d+e,0] && IntegerQ[p-1/2]
```

2: $\int \text{RF}_x (d + e x^2)^p (a + b \text{ArcSin}[c x])^n dx$ when $n \in \mathbb{Z}^+ \wedge c^2 d + e = 0 \wedge p - \frac{1}{2} \in \mathbb{Z}$

Derivation: Algebraic expansion

Rule: If $n \in \mathbb{Z}^+ \wedge c^2 d + e = 0 \wedge p - \frac{1}{2} \in \mathbb{Z}$, then

$$\int \text{RF}_x (d + e x^2)^p (a + b \text{ArcSin}[c x])^n dx \rightarrow \int (d + e x^2)^p \text{ExpandIntegrand}[\text{RF}_x (a + b \text{ArcSin}[c x])^n, x] dx$$

Program code:

```
Int[RFx*(d_+e_.*x_^2)^p_*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x^2)^p,RFx*(a+b*ArcSin[c*x])^n,x],x] /;
FreeQ[{a,b,c,d,e},x] && RationalFunctionQ[RFx,x] && IGtQ[n,0] && EqQ[c^2*d+e,0] && IntegerQ[p-1/2]
```

```
Int[RFx*(d_+e_.*x_^2)^p_*(a_+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x^2)^p,RFx*(a+b*ArcCos[c*x])^n,x],x] /;
FreeQ[{a,b,c,d,e},x] && RationalFunctionQ[RFx,x] && IGtQ[n,0] && EqQ[c^2*d+e,0] && IntegerQ[p-1/2]
```

U: $\int u (a + b \text{ArcSin}[c x])^n dx$

Rule:

$$\int u (a + b \text{ArcSin}[c x])^n dx \rightarrow \int u (a + b \text{ArcSin}[c x])^n dx$$

Program code:

```
Int[u_*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  Unintegrable[u*(a+b*ArcSin[c*x])^n,x] /;
FreeQ[{a,b,c,n},x]
```

```
Int[u_.*(a_+b_.*ArcCos[c_.*x_])^n_.,x_Symbol] :=  
  Unintegrable[u*(a+b*ArcCos[c*x])^n,x] /;  
FreeQ[{a,b,c,n},x]
```