

Rules for integrands of the form $(dx)^m (a + bx^n + cx^{2n})^p$

x. $\int (dx)^m (bx^n + cx^{2n})^p dx$

1. $\int (dx)^m (bx^n + cx^{2n})^p dx$ when $p \in \mathbb{Z}$

1: $\int (dx)^m (bx^n + cx^{2n})^p dx$ when $p \in \mathbb{Z} \wedge (m \in \mathbb{Z} \vee d > 0)$

Derivation: Algebraic simplification

Basis: If $p \in \mathbb{Z}$, then $(bx^n + cx^{2n})^p = x^{np} (b + cx^n)^p$

Rule 1.2.3.2.0.1.1: If $p \in \mathbb{Z} \wedge (m \in \mathbb{Z} \vee d > 0)$, then

$$\int (dx)^m (bx^n + cx^{2n})^p dx \rightarrow d^m \int x^{m+np} (b + cx^n)^p dx$$

Program code:

```
(* Int[(d_.*x_)^m_.*(b_.*x_^n_+c_.*x_^n2_)^p_.,x_Symbol] :=
  d^m*Int[x^(m+n*p)*(b+c*x^n)^p,x] /;
FreeQ[{b,c,d,m,n},x] && EqQ[n2,2*n] && IntegerQ[p] && (IntegerQ[m] || GtQ[d,0]) *)
```

2: $\int (d x)^m (b x^n + c x^{2n})^p dx$ when $p \in \mathbb{Z} \wedge n \in \mathbb{Z}$

Derivation: Algebraic simplification

Basis: If $p \in \mathbb{Z} \wedge n \in \mathbb{Z}$, then $(b x^n + c x^{2n})^p = \frac{1}{d^{n p}} (d x)^{n p} (b + c x^n)^p$

Rule 1.2.3.2.0.1.2: If $p \in \mathbb{Z} \wedge n \in \mathbb{Z}$, then

$$\int (d x)^m (b x^n + c x^{2n})^p dx \rightarrow \frac{1}{d^{n p}} \int (d x)^{m+n p} (b + c x^n)^p dx$$

Program code:

```
(* Int[(d.*x_)^m.*(b.*x_^n+c.*x_^n2_.)^p_,x_Symbol] :=
  1/d^(n*p)*Int[(d*x)^(m+n*p)*(b+c*x^n)^p,x] /;
FreeQ[{b,c,d,m},x] && EqQ[n2,2*n] && IntegerQ[p] && IntegerQ[n] *)
```

$$\mathbf{3:} \int (d x)^m (b x^n + c x^{2n})^p dx \text{ when } p \in \mathbb{Z} \wedge \neg (m \in \mathbb{Z} \vee d > 0)$$

Derivation: Piecewise constant extraction

$$\text{Basis: } \partial_x \frac{(d x)^m}{x^m} == 0$$

Rule 1.2.3.2.0.1.3: If $p \in \mathbb{Z} \wedge \neg (m \in \mathbb{Z} \vee d > 0)$, then

$$\int (d x)^m (b x^n + c x^{2n})^p dx \rightarrow \frac{(d x)^m}{x^m} \int x^{m+n p} (b + c x^n)^p dx$$

Program code:

```
(* Int[(d_.**x_)^m_.*(b_.**x_^n_+c_.**x_^n2_.)^p_,x_Symbol] :=
  (d*x)^m/x^m*Int[x^(m+n*p)*(b+c*x^n)^p,x] /;
FreeQ[{b,c,d,m,n},x] && EqQ[n2,2*n] && IntegerQ[p] && Not[IntegerQ[m] || GtQ[d,0]] *)
```

2: $\int (dx)^m (bx^n + cx^{2n})^p dx$ when $p \notin \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(bx^n + cx^{2n})^p}{(dx)^{np} (b+cx^n)^p} = 0$

Rule 1.2.3.2.0.2: If $p \notin \mathbb{Z}$, then

$$\int (dx)^m (bx^n + cx^{2n})^p dx \rightarrow \frac{(bx^n + cx^{2n})^p}{(dx)^{np} (b+cx^n)^p} \int (dx)^{m+np} (b+cx^n)^p dx$$

Program code:

```
(* Int[(d_.**x_)^m_.*(b_.**x_^n_+c_.**x_^n2_.)^p_,x_Symbol] :=
  (b*x^n+c*x^(2*n))^p/((d*x)^(n*p)*(b+c*x^n)^p)*Int[(d*x)^(m+n*p)*(b+c*x^2)^p,x] /;
FreeQ[{b,c,d,m,n,p},x] && EqQ[n2,2*n] && Not[IntegerQ[p]] *)
```

1: $\int x^m (a + b x^n + c x^{2n})^p dx$ when $m - n + 1 = 0$

Derivation: Integration by substitution

Basis: $x^{n-1} F[x^n] = \frac{1}{n} \text{Subst}[F[x], x, x^n] \partial_x x^n$

Rule 1.2.3.2.1: If $m - n + 1 = 0$, then

$$\int x^m (a + b x^n + c x^{2n})^p dx \rightarrow \frac{1}{n} \text{Subst}\left[\int (a + b x + c x^2)^p dx, x, x^n\right]$$

Program code:

```
Int[x_^m_.*(a_+b_.*x_^n_+c_.*x_^2n_.)^p_,x_Symbol] :=
  1/n*Subst[Int[(a+b*x+c*x^2)^p,x],x,x^n] /;
FreeQ[{a,b,c,m,n,p},x] && EqQ[n2,2*n] && EqQ[Simplify[m-n+1],0]
```

2: $\int (dx)^m (a + b x^n + c x^{2n})^p dx$ when $p \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule 1.2.3.2.2: If $p \in \mathbb{Z}^+$, then

$$\int (dx)^m (a + b x^n + c x^{2n})^p dx \rightarrow \int \text{ExpandIntegrand}[(dx)^m (a + b x^n + c x^{2n})^p, x] dx$$

Program code:

```
Int[(d_.*x_)^m_.*(a_+b_.*x_^n_+c_.*x_^2n_.)^p_,x_Symbol] :=
  Int[ExpandIntegrand[(d*x)^m*(a+b*x^n+c*x^(2*n))^p,x],x] /;
FreeQ[{a,b,c,d,m,n},x] && EqQ[n2,2*n] && IGtQ[p,0] && Not[IntegerQ[Simplify[(m+1)/n]]]
```

3: $\int x^m (a + b x^n + c x^{2n})^p dx$ when $p \in \mathbb{Z}^- \wedge n < 0$

Derivation: Algebraic simplification

Basis: If $p \in \mathbb{Z}$, then $(a + b x^n + c x^{2n})^p = x^{2np} (c + b x^{-n} + a x^{-2n})^p$

Rule 1.2.3.2.3: If $p \in \mathbb{Z}^- \wedge n < 0$, then

$$\int x^m (a + b x^n + c x^{2n})^p dx \rightarrow \int x^{m+2np} (c + b x^{-n} + a x^{-2n})^p dx$$

Program code:

```
Int[x_^m_.*(a_+b_.*x_^n_+c_.*x_^2n_.)^p_,x_Symbol] :=
  Int[x^(m+2*n*p)*(c+b*x^(-n)+a*x^(-2*n))^p,x] /;
FreeQ[{a,b,c,m,n},x] && EqQ[n2,2*n] && ILtQ[p,0] && NegQ[n]
```

4. $\int (d x)^m (a + b x^n + c x^{2n})^p dx$ when $b^2 - 4 a c == 0$

x: $\int (d x)^m (a + b x^n + c x^{2n})^p dx$ when $b^2 - 4 a c == 0 \wedge p \in \mathbb{Z}$

Derivation: Algebraic simplification

Basis: If $b^2 - 4 a c == 0$, then $a + b z + c z^2 == \frac{1}{c} \left(\frac{b}{2} + c z \right)^2$

Rule 1.2.3.2.4.1: If $b^2 - 4 a c == 0 \wedge p \in \mathbb{Z}$, then

$$\int (d x)^m (a + b x^n + c x^{2n})^p dx \rightarrow \frac{1}{c^p} \int (d x)^m \left(\frac{b}{2} + c x^n \right)^{2p} dx$$

Program code:

```
(* Int[(d_.**x_)^m_.*(a_+b_.**x_^n_.+c_.**x_^n2_.)^p_,x_Symbol] :=
  1/c^p*Int[(d*x)^m*(b/2+c*x^n)^(2*p),x] /;
FreeQ[{a,b,c,d,m,n,p},x] && EqQ[n2,2*n] && EqQ[b^2-4*a*c,0] && IntegerQ[p] *)
```

$$2. \int (dx)^m (a + b x^n + c x^{2n})^p dx \text{ when } b^2 - 4ac = 0 \wedge p \notin \mathbb{Z}$$

$$\text{x: } \int (dx)^m (a + b x^n + c x^{2n})^p dx \text{ when } b^2 - 4ac = 0 \wedge p \notin \mathbb{Z} \wedge m + 2n(p+1) + 1 = 0 \wedge p \neq -\frac{1}{2}$$

Derivation: Square trinomial recurrence 2c with $m + 2n(p+1) + 1 = 0$

Rule 1.2.3.2.4.2.1: If $b^2 - 4ac = 0 \wedge p \notin \mathbb{Z} \wedge m + 2n(p+1) + 1 = 0 \wedge p \neq -\frac{1}{2}$, then

$$\int (dx)^m (a + b x^n + c x^{2n})^p dx \rightarrow \frac{(dx)^{m+1} (a + b x^n + c x^{2n})^{p+1}}{2adn(p+1)(2p+1)} - \frac{(dx)^{m+1} (2a + bx^n) (a + b x^n + c x^{2n})^p}{2adn(2p+1)}$$

Program code:

```
(* Int[(d_.**x_)^m_.*(a_+b_.**x_^n_+c_.**x_^n2_.)^p_,x_Symbol] :=
  (d*x)^(m+1)*(a+b*x^n+c*x^(2*n))^(p+1)/(2*a*d*n*(p+1)*(2*p+1)) -
  (d*x)^(m+1)*(2*a+b*x^n)*(a+b*x^n+c*x^(2*n))^p/(2*a*d*n*(2*p+1)) /;
FreeQ[{a,b,c,d,m,n,p},x] && EqQ[n2,2*n] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[p]] && EqQ[m+2*n*(p+1)+1,0] && NeQ[2*p+1,0] *)
```


2: $\int (dx)^m (a+bx^n+cx^{2n})^p dx$ when $b^2 - 4ac = 0 \wedge p \notin \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: If $b^2 - 4ac = 0$, then $\partial_x \frac{(a+bx^n+cx^{2n})^p}{\left(1+\frac{2cx^n}{b}\right)^{2p}} = 0$

Rule 1.2.3.2.4.2.2: If $b^2 - 4ac = 0 \wedge p \notin \mathbb{Z}$, then

$$\int (dx)^m (a+bx^n+cx^{2n})^p dx \rightarrow \frac{a^{\text{IntPart}[p]} (a+bx^n+cx^{2n})^{\text{FracPart}[p]}}{\left(1+\frac{2cx^n}{b}\right)^{2\text{FracPart}[p]}} \int (dx)^m \left(1+\frac{2cx^n}{b}\right)^{2p} dx$$

Program code:

```
Int[(d_.*x_)^m_.*(a_+b_.*x_^n_.+c_.*x_^n2_.)^p_,x_Symbol] :=
  (a+b*x^n+c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2+c*x^n)^(2*FracPart[p]))*Int[(d*x)^m*(b/2+c*x^n)^(2*p),x] /;
FreeQ[{a,b,c,d,m,n,p},x] && EqQ[n2,2*n] && EqQ[b^2-4*a*c,0] && IntegerQ[p-1/2]
```

```
Int[(d_.*x_)^m_.*(a_+b_.*x_^n_.+c_.*x_^n2_.)^p_,x_Symbol] :=
  a^IntPart[p]*(a+b*x^n+c*x^(2*n))^FracPart[p]/(1+2*c*x^n/b)^(2*FracPart[p])*Int[(d*x)^m*(1+2*c*x^n/b)^(2*p),x] /;
FreeQ[{a,b,c,d,m,n,p},x] && EqQ[n2,2*n] && EqQ[b^2-4*a*c,0] && Not[IntegerQ[2*p]]
```

$$5. \int (d x)^m (a + b x^n + c x^{2n})^p dx \text{ when } b^2 - 4 a c \neq 0 \wedge \frac{m+1}{n} \in \mathbb{Z}$$

$$1: \int x^m (a + b x^n + c x^{2n})^p dx \text{ when } b^2 - 4 a c \neq 0 \wedge \frac{m+1}{n} \in \mathbb{Z}$$

Derivation: Integration by substitution

Basis: If $\frac{m+1}{n} \in \mathbb{Z}$, then $x^m F[x^n] = \frac{1}{n} \text{Subst}[x^{\frac{m+1}{n}-1} F[x], x, x^n] \partial_x x^n$

Note: If $n \in \mathbb{Z} \wedge \frac{m+1}{n} \in \mathbb{Z}$, then $m \in \mathbb{Z}$, and $(d x)^m$ automatically evaluates to $d^m x^m$.

Rule 1.2.3.2.5.1: If $b^2 - 4 a c \neq 0 \wedge \frac{m+1}{n} \in \mathbb{Z}$, then

$$\int x^m (a + b x^n + c x^{2n})^p dx \rightarrow \frac{1}{n} \text{Subst}\left[\int x^{\frac{m+1}{n}-1} (a + b x + c x^2)^p dx, x, x^n\right]$$

Program code:

```
Int[x_^m.*(a_+b_.x_^n_+c_.x_^n2_.)^p_,x_Symbol] :=
  1/n*Subst[Int[x^(Simplify[(m+1)/n]-1)*(a+b*x+c*x^2)^p,x],x,x^n] /;
FreeQ[{a,b,c,m,n,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IntegerQ[Simplify[(m+1)/n]]
```

2: $\int (d x)^m (a + b x^n + c x^{2n})^p dx$ when $b^2 - 4 a c \neq 0 \wedge \frac{m+1}{n} \in \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(d x)^m}{x^m} = 0$

Basis: $\frac{(d x)^m}{x^m} = \frac{d^{\text{IntPart}[m]} (d x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}}$

Rule 1.2.3.2.5.2: If $b^2 - 4 a c \neq 0 \wedge \frac{m+1}{n} \in \mathbb{Z}$, then

$$\int (d x)^m (a + b x^n + c x^{2n})^p dx \rightarrow \frac{d^{\text{IntPart}[m]} (d x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (a + b x^n + c x^{2n})^p dx$$

Program code:

```
Int[(d*x_)^m_.*(a_+b_.*x_^n_+c_.*x_^n2_.)^p_,x_Symbol] :=
  d^IntPart[m]*(d*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d,m,n,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IntegerQ[Simplify[(m+1)/n]]
```

$$6. \int (dx)^m (a+bx^n+cx^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}$$

$$1. \int (dx)^m (a+bx^n+cx^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+$$

$$\mathbf{1:} \int x^m (a+bx^n+cx^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{Z} \wedge \text{GCD}[m+1, n] \neq 1$$

Derivation: Integration by substitution

Basis: If $n \in \mathbb{Z} \wedge m \in \mathbb{Z}$, let $k = \text{GCD}[m+1, n]$, then $x^m F[x^n] = \frac{1}{k} \text{Subst}[x^{\frac{m+1}{k}-1} F[x^{n/k}], x, x^k] \partial_x x^k$

Rule 1.2.3.2.6.1.1: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{Z}$, let $k = \text{GCD}[m+1, n]$, if $k \neq 1$, then

$$\int x^m (a+bx^n+cx^{2n})^p dx \rightarrow \frac{1}{k} \text{Subst}\left[\int x^{\frac{m+1}{k}-1} (a+bx^{n/k}+cx^{2n/k})^p dx, x, x^k\right]$$

Program code:

```
Int[x_^m_.*(a_+b_.*x_^n_+c_.*x_^n2_.)^p_,x_Symbol] :=
  With[{k=GCD[m+1,n]},
    1/k*Subst[Int[x^((m+1)/k-1)*(a+b*x^(n/k)+c*x^(2*n/k))^p,x],x,x^k] /;
    k!=1] /;
FreeQ[{a,b,c,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && IntegerQ[m]
```

2: $\int (d x)^m (a + b x^n + c x^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{F}$

Derivation: Integration by substitution

Basis: If $k \in \mathbb{Z}^+$, then $(d x)^m F[x] = \frac{k}{d} \text{Subst}\left[x^{k(m+1)-1} F\left[\frac{x^k}{d}\right], x, (d x)^{1/k}\right] \partial_x (d x)^{1/k}$

Rule 1.2.3.2.6.1.2: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{F}$, let $k = \text{Denominator}[m]$, then

$$\int (d x)^m (a + b x^n + c x^{2n})^p dx \rightarrow \frac{k}{d} \text{Subst}\left[\int x^{k(m+1)-1} \left(a + \frac{b x^{kn}}{d^n} + \frac{c x^{2kn}}{d^{2n}}\right)^p dx, x, (d x)^{1/k}\right]$$

Program code:

```
Int[(d_.**x_)^m_*(a_+b_.**x_^n_+c_.**x_^n2_.)^p_,x_Symbol] :=
  With[{k=Denominator[m]},
    k/d*Subst[Int[x^(k*(m+1)-1)*(a+b*x^(k*n)/d^n+c*x^(2*k*n)/d^(2*n))^p,x],x,(d*x)^(1/k)]] /;
  FreeQ[{a,b,c,d,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && FractionQ[m] && IntegerQ[p]
```

$$3. \int (dx)^m (a+bx^n+cx^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p \in \mathbb{Z}^+$$

$$1: \int (dx)^m (a+bx^n+cx^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p \in \mathbb{Z}^+ \wedge m > n-1 \wedge m+2np+1 \neq 0 \wedge m+n(2p-1)+1 \neq 0$$

Derivation: Trinomial recurrence 1b with $A = 0$, $B = 1$ and $m = m - n$

Rule 1.2.3.2.6.1.3.1: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p \in \mathbb{Z}^+ \wedge m > n-1 \wedge m+2np+1 \neq 0 \wedge m+n(2p-1)+1 \neq 0$, then

$$\frac{\int (dx)^m (a+bx^n+cx^{2n})^p dx \rightarrow \frac{d^{n-1} (dx)^{m-n+1} (a+bx^n+cx^{2n})^p (bnp+c(m+n(2p-1)+1)x^n)}{c(m+2np+1)(m+n(2p-1)+1)} - \frac{npd^n}{c(m+2np+1)(m+n(2p-1)+1)} \int (dx)^{m-n} (a+bx^n+cx^{2n})^{p-1} (ab(m-n+1) - (2ac(m+n(2p-1)+1) - b^2(m+n(p-1)+1))x^n) dx$$

Program code:

```
Int[(d.*x_)^m_.*(a_+b_*x_^n_+c_*x_^n2_)^p_,x_Symbol] :=
d^(n-1)*(d*x)^(m-n+1)*(a+b*x^n+c*x^(2*n))^p*(b*n*p+c*(m+n*(2*p-1)+1)*x^n)/(c*(m+2*n*p+1)*(m+n*(2*p-1)+1)) -
n*p*d^n/(c*(m+2*n*p+1)*(m+n*(2*p-1)+1))*
Int[(d*x)^(m-n)*(a+b*x^n+c*x^(2*n))^(p-1)*Simp[a*b*(m-n+1)-(2*a*c*(m+n*(2*p-1)+1)-b^2*(m+n*(p-1)+1))*x^n,x],x] /;
FreeQ[{a,b,c,d},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && IGtQ[p,0] && GtQ[m,n-1] && NeQ[m+2*n*p+1,0] && NeQ[m+n*(2*p-1)+1,0]
```

$$2: \int (dx)^m (a+bx^n+cx^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p \in \mathbb{Z}^+ \wedge m < -1$$

Reference: G&R 2.160.2

Derivation: Trinomial recurrence 1a with $A = 1$ and $B = 0$

Rule 1.2.3.2.6.1.3.2: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p \in \mathbb{Z}^+ \wedge m < -1$, then

$$\int (d x)^m (a + b x^n + c x^{2n})^p dx \rightarrow \frac{(d x)^{m+1} (a + b x^n + c x^{2n})^p}{d (m+1)} - \frac{n p}{d^n (m+1)} \int (d x)^{m+n} (b + 2 c x^n) (a + b x^n + c x^{2n})^{p-1} dx$$

Program code:

```
Int[(d_.**x_)^m_.*(a_+b_.*x_^n+c_.*x_^n2_.)^p_,x_Symbol] :=
  (d*x)^(m+1)*(a+b*x^n+c*x^(2*n))^p/(d*(m+1)) -
  n*p/(d^n*(m+1))*Int[(d*x)^(m+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^(p-1),x] /;
FreeQ[{a,b,c,d},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && IGtQ[p,0] && LtQ[m,-1]
```

$$\mathbf{3:} \int (d x)^m (a + b x^n + c x^{2n})^p dx \text{ when } b^2 - 4 a c \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p \in \mathbb{Z}^+ \wedge m + 2 n p + 1 \neq 0$$

Derivation: Trinomial recurrence 1a with $A = 0$, $B = 1$ and $m = m - n$

Derivation: Trinomial recurrence 1b with $A = 1$ and $B = 0$

Rule 1.2.3.2.6.1.3.4: If $b^2 - 4 a c \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p \in \mathbb{Z}^+ \wedge m + 2 n p + 1 \neq 0$, then

$$\int (d x)^m (a + b x^n + c x^{2n})^p dx \rightarrow \frac{(d x)^{m+1} (a + b x^n + c x^{2n})^p}{d (m + 2 n p + 1)} + \frac{n p}{m + 2 n p + 1} \int (d x)^m (2 a + b x^n) (a + b x^n + c x^{2n})^{p-1} dx$$

Program code:

```
Int[(d_.**x_)^m_.*(a_+b_.*x_^n+c_.*x_^n2_.)^p_,x_Symbol] :=
  (d*x)^(m+1)*(a+b*x^n+c*x^(2*n))^p/(d*(m+2*n*p+1)) +
  n*p/(m+2*n*p+1)*Int[(d*x)^m*(2*a+b*x^n)*(a+b*x^n+c*x^(2*n))^(p-1),x] /;
FreeQ[{a,b,c,d,m},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && IGtQ[p,0] && NeQ[m+2*n*p+1,0]
```

$$4. \int (dx)^m (a+bx^n+cx^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p+1 \in \mathbb{Z}^-$$

$$1. \int (dx)^m (a+bx^n+cx^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p+1 \in \mathbb{Z}^- \wedge m > n-1$$

$$1: \int (dx)^m (a+bx^n+cx^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p+1 \in \mathbb{Z}^- \wedge n-1 < m \leq 2n-1$$

Derivation: Trinomial recurrence 2a with $A = 1$ and $B = 0$

Derivation: Trinomial recurrence 2b with $A = 0$, $B = 1$ and $m = m - n$

Rule 1.2.3.2.6.1.4.1.1: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p+1 \in \mathbb{Z}^- \wedge n-1 < m \leq 2n-1$, then

$$\frac{\int (dx)^m (a+bx^n+cx^{2n})^p dx \rightarrow \frac{d^{n-1} (dx)^{m-n+1} (b+2cx^n) (a+bx^n+cx^{2n})^{p+1}}{n(p+1)(b^2-4ac)} - \frac{d^n}{n(p+1)(b^2-4ac)} \int (dx)^{m-n} (b(m-n+1)+2c(m+2n(p+1)+1)x^n) (a+bx^n+cx^{2n})^{p+1} dx$$

Program code:

```
Int[(d_.**x_)^m_.*(a_+b_.*x_^n_+c_.*x_^2n_.)^p_,x_Symbol] :=
  d^(n-1)*(d*x)^(m-n+1)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^(p+1)/(n*(p+1)*(b^2-4*a*c)) -
  d^n/(n*(p+1)*(b^2-4*a*c))*
  Int[(d*x)^(m-n)*(b*(m-n+1)+2*c*(m+2*n*(p+1)+1)*x^n)*(a+b*x^n+c*x^(2*n))^(p+1),x] /;
FreeQ[{a,b,c,d},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && ILtQ[p,-1] && GtQ[m,n-1] && LeQ[m,2*n-1]
```

$$2: \int (dx)^m (a+bx^n+cx^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p+1 \in \mathbb{Z}^- \wedge m > 2n-1$$

Derivation: Trinomial recurrence 2a with $A = 0$, $B = 1$ and $m = m - n$

Rule 1.2.3.2.6.1.4.1.2: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p+1 \in \mathbb{Z}^- \wedge m > 2n-1$, then

$$\int (dx)^m (a+bx^n+cx^{2n})^p dx \rightarrow$$

$$-\frac{d^{2n-1} (dx)^{m-2n+1} (2a+bx^n) (a+bx^n+cx^{2n})^{p+1}}{n(p+1)(b^2-4ac)} +$$

$$\frac{d^{2n}}{n(p+1)(b^2-4ac)} \int (dx)^{m-2n} (2a(m-2n+1)+b(m+n(2p+1)+1)x^n) (a+bx^n+cx^{2n})^{p+1} dx$$

Program code:

```
Int[(d_.**x_)^m_.*(a_+b_.**x_^n_+c_.**x_^n2_.)^p_,x_Symbol] :=
  -d^(2*n-1)*(d*x)^(m-2*n+1)*(2*a+b*x^n)*(a+b*x^n+c*x^(2*n))^(p+1)/(n*(p+1)*(b^2-4*a*c)) +
  d^(2*n)/(n*(p+1)*(b^2-4*a*c))*
  Int[(d*x)^(m-2*n)*(2*a*(m-2*n+1)+b*(m+n*(2*p+1)+1)*x^n)*(a+b*x^n+c*x^(2*n))^(p+1),x] /;
FreeQ[{a,b,c,d},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && ILtQ[p,-1] && GtQ[m,2*n-1]
```

$$\mathbf{2:} \int (d x)^m (a + b x^n + c x^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p + 1 \in \mathbb{Z}^-$$

Derivation: Trinomial recurrence 2b with $A = 1$ and $B = 0$

Rule 1.2.3.2.6.1.4.2: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p + 1 \in \mathbb{Z}^-$, then

$$\begin{aligned} & \int (d x)^m (a + b x^n + c x^{2n})^p dx \rightarrow \\ & - \frac{(d x)^{m+1} (b^2 - 2ac + b c x^n) (a + b x^n + c x^{2n})^{p+1}}{a d n (p+1) (b^2 - 4ac)} + \\ & \frac{1}{a n (p+1) (b^2 - 4ac)} \int (d x)^m (a + b x^n + c x^{2n})^{p+1} (b^2 (m+n(p+1)+1) - 2ac(m+2n(p+1)+1) + b c (m+n(2p+3)+1) x^n) dx \end{aligned}$$

Program code:

```
Int[(d_.**x_)^m_.*(a_+b_.**x_^n_+c_.**x_^n2_.)^p_,x_Symbol] :=
- (d*x)^(m+1)*(b^2-2*a*c+b*c*x^n)*(a+b*x^n+c*x^(2*n))^(p+1)/(a*d*n*(p+1)*(b^2-4*a*c)) +
1/(a*n*(p+1)*(b^2-4*a*c))*
Int[(d*x)^m*(a+b*x^n+c*x^(2*n))^(p+1)*Simp[b^2*(m+n*(p+1)+1)-2*a*c*(m+2*n*(p+1)+1)+b*c*(m+n*(2*p+3)+1)*x^n,x],x] /;
FreeQ[{a,b,c,d,m},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && ILtQ[p,-1]
```

$$\mathbf{5:} \int (d x)^m (a + b x^n + c x^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m > 2n - 1 \wedge m + 2np + 1 \neq 0$$

Reference: G&R 2.160.3

Derivation: Trinomial recurrence 3a with $A = 0$, $B = 1$ and $m = m - n$

Note: G&R 2.174.1 is a special case of G&R 2.160.3.

Rule 1.2.3.2.6.1.5: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m > 2n - 1 \wedge m + 2np + 1 \neq 0$, then

$$\int (d x)^m (a + b x^n + c x^{2n})^p dx \rightarrow$$

$$\frac{d^{2n-1} (dx)^{m-2n+1} (a+bx^n+cx^{2n})^{p+1}}{c(m+2n+1)} - \frac{d^{2n}}{c(m+2n+1)} \int (dx)^{m-2n} (a(m-2n+1) + b(m+n(p-1)+1)x^n) (a+bx^n+cx^{2n})^p dx$$

Program code:

```
Int[(d_.**x_)^m_.*(a_+b_.*x_^n_+c_.*x_^n2_.)^p_,x_Symbol] :=
  d^(2*n-1)*(d*x)^(m-2*n+1)*(a+b*x^n+c*x^(2*n))^(p+1)/(c*(m+2*n*p+1)) -
  d^(2*n)/(c*(m+2*n*p+1))*
  Int[(d*x)^(m-2*n)*Simp[a*(m-2*n+1)+b*(m+n*(p-1)+1)*x^n,x]*(a+b*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && GtQ[m,2*n-1] && NeQ[m+2*n*p+1,0] && IntegerQ[p]
```

6: $\int (dx)^m (a+bx^n+cx^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m < -1$

Reference: G&R 2.160.1

Derivation: Trinomial recurrence 3b with $A = 1$ and $B = 0$

Note: G&R 2.161.6 is a special case of G&R 2.160.1.

Rule 1.2.3.2.6.1.6: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m < -1$, then

$$\int (dx)^m (a+bx^n+cx^{2n})^p dx \rightarrow \frac{(dx)^{m+1} (a+bx^n+cx^{2n})^{p+1}}{ad(m+1)} - \frac{1}{ad^n(m+1)} \int (dx)^{m+n} (b(m+n(p+1)+1) + c(m+2n(p+1)+1)x^n) (a+bx^n+cx^{2n})^p dx$$

Program code:

```
Int[(d_.**x_)^m_.*(a_+b_.*x_^n_+c_.*x_^n2_.)^p_,x_Symbol] :=
  (d*x)^(m+1)*(a+b*x^n+c*x^(2*n))^(p+1)/(a*d*(m+1)) -
  1/(a*d^n*(m+1))*Int[(d*x)^(m+n)*(b*(m+n*(p+1)+1)+c*(m+2*n*(p+1)+1)*x^n)*(a+b*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && LtQ[m,-1] && IntegerQ[p]
```

$$7. \int \frac{(dx)^m}{a+bx^n+cx^{2n}} dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+$$

$$1: \int \frac{(dx)^m}{a+bx^n+cx^{2n}} dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m < -1$$

Reference: G&R 2.176, CRC 123

Derivation: Algebraic expansion

$$\text{Basis: } \frac{(dz)^m}{a+bz+cz^2} = \frac{(dz)^m}{a} - \frac{1}{ad} \frac{(dz)^{m+1}(b+cz)}{a+bz+cz^2}$$

Rule 1.2.3.2.6.1.7.1: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m < -1$, then

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}} dx \rightarrow \frac{(dx)^{m+1}}{ad(m+1)} - \frac{1}{ad^n} \int \frac{(dx)^{m+n}(b+cx^n)}{a+bx^n+cx^{2n}} dx$$

Program code:

```
Int[(d.*x_)^m/(a+b.*x_^n+c.*x_^n2_),x_Symbol] :=
  (d*x)^(m+1)/(a*d*(m+1)) -
  1/(a*d^n)*Int[(d*x)^(m+n)*(b+c*x^n)/(a+b*x^n+c*x^(2*n)),x] /;
FreeQ[{a,b,c,d},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && LtQ[m,-1]
```

$$2. \int \frac{(dx)^m}{a + b x^n + c x^{2n}} dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m > 2n - 1$$

$$1: \int \frac{x^m}{a + b x^n + c x^{2n}} dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m > 3n - 1 \wedge m \in \mathbb{Z}$$

Derivation: Algebraic expansion

Rule 1.2.3.2.6.1.7.2.1: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m > 3n - 1 \wedge m \in \mathbb{Z}$, then

$$\int \frac{x^m}{a + b x^n + c x^{2n}} dx \rightarrow \int \text{PolynomialDivide}[x^m, a + b x^n + c x^{2n}, x] dx$$

Program code:

```
Int[x^m/(a+b.*x^n+c.*x^2n.),x_Symbol] :=
  Int[PolynomialDivide[x^m,(a+b*x^n+c*x^(2*n)),x],x] /;
FreeQ[{a,b,c},x] && EqQ[n,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && IGtQ[m,3*n-1]
```

2: $\int \frac{(dx)^m}{a+bx^n+cx^{2n}} dx$ when $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m > 2n - 1$ **Not necessary?**

Reference: G&R 2.174.1, CRC 119

Derivation: Algebraic expansion

Basis: $\frac{(dz)^m}{a+bz+cz^2} = \frac{d^2 (dz)^{m-2}}{c} - \frac{d^2 (dz)^{m-2} (a+bz)}{a+bz+cz^2}$

Rule 1.2.3.2.6.1.7.2.2: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m > 2n - 1$, then

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}} dx \rightarrow \frac{d^{2n-1} (dx)^{m-2n+1}}{c(m-2n+1)} - \frac{d^{2n}}{c} \int \frac{(dx)^{m-2n} (a+bx^n)}{a+bx^n+cx^{2n}} dx$$

Program code:

```
Int[(d_.**x_)^m/(a_+b_.**x_^n+c_.**x_^n2_),x_Symbol] :=
  d^(2*n-1)*(d*x)^(m-2*n+1)/(c*(m-2*n+1)) -
  d^(2*n)/c*Int[(d*x)^(m-2*n)*(a+b*x^n)/(a+b*x^n+c*x^(2*n)),x] /;
FreeQ[{a,b,c,d},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && GtQ[m,2*n-1]
```

$$3. \int \frac{x^m}{a+bx^n+cx^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge \left(\frac{n}{2} \mid m\right) \in \mathbb{Z}^+ \wedge \frac{n}{2} \leq m < 2n \wedge b^2-4ac \neq 0$$

$$1: \int \frac{x^m}{a+bx^n+cx^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge \left(\frac{n}{2} \mid m\right) \in \mathbb{Z}^+ \wedge \frac{3n}{2} \leq m < 2n \wedge b^2-4ac \neq 0$$

Derivation: Algebraic expansion

Basis: If $q \rightarrow \sqrt{\frac{a}{c}}$ and $r \rightarrow \sqrt{2q - \frac{b}{c}}$, then $\frac{z^3}{a+bz^2+cz^4} = \frac{q+rz}{2cr(q+rz+z^2)} - \frac{q-rz}{2cr(q-rz+z^2)}$

Note: If $(a \mid b \mid c) \in \mathbb{R} \wedge b^2-4ac < 0$, then $\frac{a}{c} > 0$ and $2\sqrt{\frac{a}{c}} - \frac{b}{c} > 0$.

Rule 1.2.3.2.6.1.7.3.1: If $b^2-4ac \neq 0 \wedge \left(\frac{n}{2} \mid m\right) \in \mathbb{Z}^+ \wedge \frac{3n}{2} \leq m < 2n \wedge b^2-4ac \neq 0$, let $q \rightarrow \sqrt{\frac{a}{c}}$ and $r \rightarrow \sqrt{2q - \frac{b}{c}}$, then

$$\int \frac{x^m}{a+bx^n+cx^{2n}} dx \rightarrow \frac{1}{2cr} \int \frac{x^{m-3n/2} (q+rx^{n/2})}{q+rx^{n/2}+x^n} dx - \frac{1}{2cr} \int \frac{x^{m-3n/2} (q-rx^{n/2})}{q-rx^{n/2}+x^n} dx$$

Program code:

```
Int[x_^m_/.(a+b_.*x_^n_+c_.*x_^2n_.),x_Symbol] :=
  With[{q=Rt[a/c,2]},
    With[{r=Rt[2*q-b/c,2]},
      1/(2*c*r)*Int[x^(m-3*(n/2))*(q+r*x^(n/2))/(q+r*x^(n/2)+x^n),x] -
      1/(2*c*r)*Int[x^(m-3*(n/2))*(q-r*x^(n/2))/(q-r*x^(n/2)+x^n),x]] /;
  FreeQ[{a,b,c},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n/2,0] && IGtQ[m,0] && GeQ[m,3*n/2] && LtQ[m,2*n] && NegQ[b^2-4*a*c]
```

$$2: \int \frac{x^m}{a+bx^n+cx^{2n}} dx \text{ when } b^2-4ac \neq 0 \wedge \left(\frac{n}{2} \mid m\right) \in \mathbb{Z}^+ \wedge \frac{n}{2} \leq m < \frac{3n}{2} \wedge b^2-4ac \neq 0$$

Derivation: Algebraic expansion

Basis: If $q \rightarrow \sqrt{\frac{a}{c}}$ and $r \rightarrow \sqrt{2q - \frac{b}{c}}$, then $\frac{z}{a+bz^2+cz^4} = \frac{1}{2cr(q-rz+z^2)} - \frac{1}{2cr(q+rz+z^2)}$

Note: If $(a \mid b \mid c) \in \mathbb{R} \wedge b^2 - 4ac < 0$, then $\frac{a}{c} > 0$ and $2\sqrt{\frac{a}{c}} - \frac{b}{c} > 0$.

Rule 1.2.3.2.6.1.7.3.2: If $b^2 - 4ac \neq 0 \wedge \left(\frac{n}{2} \mid m\right) \in \mathbb{Z}^+ \wedge \frac{n}{2} \leq m < \frac{3n}{2} \wedge b^2 - 4ac \neq 0$, let $q \rightarrow \sqrt{\frac{a}{c}}$ and $r \rightarrow \sqrt{2q - \frac{b}{c}}$, then

$$\int \frac{x^m}{a+bx^n+cx^{2n}} dx \rightarrow \frac{1}{2cr} \int \frac{x^{m-n/2}}{q-rx^{n/2}+x^n} dx - \frac{1}{2cr} \int \frac{x^{m-n/2}}{q+rx^{n/2}+x^n} dx$$

Program code:

```
Int[x_^m_/(a_+b_.x_^n_+c_.x_^2n_),x_Symbol] :=
  With[{q=Rt[a/c,2]},
    With[{r=Rt[2*q-b/c,2]},
      1/(2*c*r)*Int[x^(m-n/2)/(q-r*x^(n/2)+x^n),x] -
      1/(2*c*r)*Int[x^(m-n/2)/(q+r*x^(n/2)+x^n),x]] /;
  FreeQ[{a,b,c},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n/2,0] && IGtQ[m,0] && GeQ[m,n/2] && LtQ[m,3*n/2] && NegQ[b^2-4*a*c]
```


4: $\int \frac{(dx)^m}{a+bx^n+cx^{2n}} dx$ when $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m \geq n$

Reference: G&R 2.161.1a & G&R 2.161.3

Derivation: Algebraic expansion

Basis: Let $q \rightarrow \sqrt{b^2 - 4ac}$, then $\frac{(dz)^m}{a+bz+cz^2} = \frac{d}{2} \left(\frac{b}{q} + 1 \right) \frac{(dz)^{m-1}}{\frac{b}{2} + \frac{q}{2} + cz} - \frac{d}{2} \left(\frac{b}{q} - 1 \right) \frac{(dz)^{m-1}}{\frac{b}{2} - \frac{q}{2} + cz}$

■ Rule 1.2.3.2.6.1.7.4: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+ \wedge m \geq n$, let $q \rightarrow \sqrt{b^2 - 4ac}$, then

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}} dx \rightarrow \frac{d^n}{2} \left(\frac{b}{q} + 1 \right) \int \frac{(dx)^{m-n}}{\frac{b}{2} + \frac{q}{2} + cx^n} dx - \frac{d^n}{2} \left(\frac{b}{q} - 1 \right) \int \frac{(dx)^{m-n}}{\frac{b}{2} - \frac{q}{2} + cx^n} dx$$

Program code:

```
Int[(d_.**x_)^m_/(a_+b_.**x_^n_+c_.**x_^n2_.),x_Symbol] :=
  With[{q=Rt[b^2-4*a*c,2]},
    d^n/2*(b/q+1)*Int[(d*x)^(m-n)/(b/2+q/2+c*x^n),x] -
    d^n/2*(b/q-1)*Int[(d*x)^(m-n)/(b/2-q/2+c*x^n),x] /;
  FreeQ[{a,b,c,d},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0] && GeQ[m,n]
```

5: $\int \frac{(dx)^m}{a+bx^n+cx^{2n}} dx$ when $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+$

Reference: G&R 2.161.1a

Derivation: Algebraic expansion

Basis: Let $q \rightarrow \sqrt{b^2 - 4ac}$, then $\frac{1}{a+bz+cz^2} = \frac{c}{q} \frac{1}{\frac{b}{2} - \frac{q}{2} + cz} - \frac{c}{q} \frac{1}{\frac{b}{2} + \frac{q}{2} + cz}$

■ Rule 1.2.3.2.6.1.7.5: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^+$, let $q \rightarrow \sqrt{b^2 - 4ac}$, then

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}} dx \rightarrow \frac{c}{q} \int \frac{(dx)^m}{\frac{b}{2} - \frac{q}{2} + cx^n} dx - \frac{c}{q} \int \frac{(dx)^m}{\frac{b}{2} + \frac{q}{2} + cx^n} dx$$

Program code:

```
Int[(d_.**x_)^m_./(a_+b_.**x_^n_+c_.**x_^n2_),x_Symbol] :=
  With[{q=Rt[b^2-4*a*c,2]},
    c/q*Int[(d*x)^m/(b/2-q/2+c*x^n),x] - c/q*Int[(d*x)^m/(b/2+q/2+c*x^n),x] /;
  FreeQ[{a,b,c,d,m},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IGtQ[n,0]
```

$$2. \int (d x)^m (a + b x^n + c x^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^-$$

$$1. \int (d x)^m (a + b x^n + c x^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{Q}$$

$$\mathbf{1:} \int x^m (a + b x^n + c x^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{Z}$$

Derivation: Integration by substitution

$$\text{Basis: } F[x] = -\text{Subst}\left[\frac{F\left[\frac{x^{-1}}{x^2}\right]}{x^2}, x, \frac{1}{x}\right] \partial_x \frac{1}{x}$$

Rule 1.2.3.2.6.2.1.1: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{Z}$, then

$$\int x^m (a + b x^n + c x^{2n})^p dx \rightarrow -\text{Subst}\left[\int \frac{(a + b x^{-n} + c x^{-2n})^p}{x^{m+2}} dx, x, \frac{1}{x}\right]$$

Program code:

```
Int[x^m.*(a+b.*x^n+c.*x^2n.)^p_,x_Symbol] :=
  -Subst[Int[(a+b*x^(-n)+c*x^(-2*n))^p/x^(m+2),x],x,1/x] /;
FreeQ[{a,b,c,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && ILtQ[n,0] && IntegerQ[m]
```

$$\mathbf{2:} \int (dx)^m (a+bx^n+cx^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{F}$$

Derivation: Integration by substitution

$$\text{Basis: If } n \in \mathbb{Z} \wedge k > 1, \text{ then } (dx)^m F[x^n] = -\frac{k}{d} \text{Subst}\left[\frac{F[d^{-n}x^{-kn}]}{x^{k(m+1)+1}}, x, \frac{1}{(dx)^{1/k}}\right] \partial_x \frac{1}{(dx)^{1/k}}$$

Rule 1.2.3.2.6.2.1.2: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^- \wedge m \in \mathbb{F}$, let $k = \text{Denominator}[m]$, then

$$\int (dx)^m (a+bx^n+cx^{2n})^p dx \rightarrow -\frac{k}{d} \text{Subst}\left[\int \frac{(a+bd^{-n}x^{-kn}+cd^{-2n}x^{-2kn})^p}{x^{k(m+1)+1}} dx, x, \frac{1}{(dx)^{1/k}}\right]$$

Program code:

```
Int[(d_.*x_)^m_.*(a_+b_.*x_^n_+c_.*x_^n2_.)^p_,x_Symbol] :=
  With[{k=Denominator[m]},
    -k/d*Subst[Int[(a+b*d^(-n)*x^(-k*n)+c*d^(-2*n)*x^(-2*k*n))^p/x^(k*(m+1)+1),x],x,1/(d*x)^(1/k)]] /;
  FreeQ[{a,b,c,d,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && ILtQ[n,0] && FractionQ[m]
```

$$\mathbf{2:} \int (dx)^m (a+bx^n+cx^{2n})^p dx \text{ when } b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^- \wedge m \notin \mathbb{Q}$$

Derivation: Piecewise constant extraction and integration by substitution

$$\text{Basis: } \partial_x \left((dx)^m (x^{-1})^m \right) = 0$$

$$\text{Basis: } (dx)^m (x^{-1})^m = d^{\text{IntPart}[m]} (dx)^{\text{FracPart}[m]} (x^{-1})^{\text{FracPart}[m]}$$

$$\text{Basis: } F[x] = -\text{Subst}\left[\frac{F\left[\frac{x^{-1}}{x^2}\right]}{x^2}, x, \frac{1}{x}\right] \partial_x \frac{1}{x}$$

Rule 1.2.3.2.6.2.2: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{Z}^- \wedge m \notin \mathbb{Q}$, then

$$\int (dx)^m (a+bx^n+cx^{2n})^p dx \rightarrow d^{\text{IntPart}[m]} (dx)^{\text{FracPart}[m]} (x^{-1})^{\text{FracPart}[m]} \int \frac{(a+bx^n+cx^{2n})^p}{(x^{-1})^m} dx$$

$$\rightarrow -d^{\text{IntPart}[m]} (dx)^{\text{FracPart}[m]} (x^{-1})^{\text{FracPart}[m]} \text{Subst}\left[\int \frac{(a+bx^{-n}+cx^{-2n})^p}{x^{m+2}} dx, x, \frac{1}{x}\right]$$

Program code:

```
Int[(d_.**x_)^m_*(a_+b_.**x_^n_+c_.**x_^n2_.)^p_,x_Symbol] :=
  -d^IntPart[m]*(d*x)^FracPart[m]*(x^(-1))^FracPart[m]*Subst[Int[(a+b*x^(-n)+c*x^(-2*n))^p/x^(m+2),x],x,1/x] /;
  FreeQ[{a,b,c,d,m,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && ILtQ[n,0] && Not[RationalQ[m]]
```

7. $\int (dx)^m (a+bx^n+cx^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge n \in \mathbb{F}$

1: $\int x^m (a+bx^n+cx^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge n \in \mathbb{F}$

Derivation: Integration by substitution

Basis: If $k \in \mathbb{Z}^+$, then $x^m F[x^n] = k \text{Subst}[x^{k(m+1)-1} F[x^{kn}], x, x^{1/k}] \partial_x x^{1/k}$

Rule 1.2.3.2.7.1: If $b^2 - 4ac \neq 0 \wedge n \in \mathbb{F}$, let $k = \text{Denominator}[n]$, then

$$\int x^m (a+bx^n+cx^{2n})^p dx \rightarrow k \text{Subst}\left[\int x^{k(m+1)-1} (a+bx^{kn}+cx^{2kn})^p dx, x, x^{1/k}\right]$$

Program code:

```
Int[x_^m_*(a_+b_.**x_^n_+c_.**x_^n2_.)^p_,x_Symbol] :=
  With[{k=Denominator[n]},
    k*Subst[Int[x^(k*(m+1)-1)*(a+b*x^(k*n)+c*x^(2*k*n))^p,x],x,x^(1/k)]] /;
  FreeQ[{a,b,c,m,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && FractionQ[n]
```

2: $\int (d x)^m (a + b x^n + c x^{2n})^p dx$ when $b^2 - 4 a c \neq 0 \wedge n \in \mathbb{F}$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(d x)^m}{x^m} == 0$

Basis: $\frac{(d x)^m}{x^m} == \frac{d^{\text{IntPart}[m]} (d x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}}$

Rule 1.2.3.2.7.2: If $b^2 - 4 a c \neq 0 \wedge n \in \mathbb{F}$, then

$$\int (d x)^m (a + b x^n + c x^{2n})^p dx \rightarrow \frac{d^{\text{IntPart}[m]} (d x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (a + b x^n + c x^{2n})^p dx$$

Program code:

```
Int[(d*x_)^m*(a+b_.*x_^n+c_.*x_^n2_.)^p_,x_Symbol] :=
  d^IntPart[m]*(d*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,d,m,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && FractionQ[n]
```

$$8. \int (d x)^m (a + b x^n + c x^{2n})^p dx \text{ when } b^2 - 4 a c \neq 0 \wedge \frac{n}{m+1} \in \mathbb{Z}$$

$$1: \int x^m (a + b x^n + c x^{2n})^p dx \text{ when } b^2 - 4 a c \neq 0 \wedge \frac{n}{m+1} \in \mathbb{Z}$$

Derivation: Integration by substitution

Basis: If $\frac{n}{m+1} \in \mathbb{Z}$, then $x^m F[x^n] = \frac{1}{m+1} \text{Subst}[F[x^{\frac{n}{m+1}}], x, x^{m+1}] \partial_x x^{m+1}$

Rule 1.2.3.2.8.1: If $b^2 - 4 a c \neq 0 \wedge \frac{n}{m+1} \in \mathbb{Z}$

$$\int x^m (a + b x^n + c x^{2n})^p dx \rightarrow \frac{1}{m+1} \text{Subst}\left[\int (a + b x^{\frac{n}{m+1}} + c x^{\frac{2n}{m+1}})^p dx, x, x^{m+1}\right]$$

Program code:

```
Int[x_^m_.*(a_+b_.*x_^n_+c_.*x_^2n_.)^p_,x_Symbol] :=
  1/(m+1)*Subst[Int[(a+b*x^Simplify[n/(m+1)]+c*x^Simplify[2*n/(m+1)])^p,x],x,x^(m+1)] /;
FreeQ[{a,b,c,m,n,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IntegerQ[Simplify[n/(m+1)]] && Not[IntegerQ[n]]
```

2: $\int (dx)^m (a+bx^n+cx^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge \frac{n}{m+1} \in \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(dx)^m}{x^m} = 0$

Basis: $\frac{(dx)^m}{x^m} = \frac{d^{\text{IntPart}[m]} (dx)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}}$

Rule 1.2.3.2.8.2: If $b^2 - 4ac \neq 0 \wedge \frac{n}{m+1} \in \mathbb{Z}$, then

$$\int (dx)^m (a+bx^n+cx^{2n})^p dx \rightarrow \frac{d^{\text{IntPart}[m]} (dx)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (a+bx^n+cx^{2n})^p dx$$

Program code:

```
Int[(d*x_)^m*(a+b_.*x_^n+c_.*x_^2n_.)^p_,x_Symbol] :=
  d^IntPart[m]*(d*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*x^n+c*x^(2*n))^p,x] /;
  FreeQ[{a,b,c,d,m,n,p},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && IntegerQ[Simplify[n/(m+1)]] && Not[IntegerQ[n]]
```

9. $\int (dx)^m (a+bx^n+cx^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge p \in \mathbb{Z}^-$

1: $\int \frac{(dx)^m}{a+bx^n+cx^{2n}} dx$ when $b^2 - 4ac \neq 0$

Reference: G&R 2.161.1a

Derivation: Algebraic expansion

Basis: Let $q = \sqrt{b^2 - 4ac}$, then $\frac{1}{a+bz+cz^2} = \frac{2c}{q} \frac{1}{b-q+2cz} - \frac{2c}{q} \frac{1}{b+q+2cz}$

■

Rule 1.2.3.2.9.1: If $b^2 - 4ac \neq 0$, let $q = \sqrt{b^2 - 4ac}$, then

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}} dx \rightarrow \frac{2c}{q} \int \frac{(dx)^m}{b-q+2cx^n} dx - \frac{2c}{q} \int \frac{(dx)^m}{b+q+2cx^n} dx$$

Program code:

```
Int[(d_.**x_)^m_./(a_+b_.**x_^n_+c_.**x_^n2_.),x_Symbol] :=
  With[{q=Rt[b^2-4*a*c,2]},
    2*c/q*Int[(d*x)^m/(b-q+2*c*x^n),x] -
    2*c/q*Int[(d*x)^m/(b+q+2*c*x^n),x] /;
  FreeQ[{a,b,c,d,m,n},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0]
```

2: $\int (dx)^m (a+bx^n+cx^{2n})^p dx$ when $b^2 - 4ac \neq 0 \wedge p+1 \in \mathbb{Z}^-$

Derivation: Trinomial recurrence 2b with $A = 1$ and $B = 0$

Rule 1.2.3.2.9.2: If $b^2 - 4ac \neq 0 \wedge p+1 \in \mathbb{Z}^-$, then

$$\int (dx)^m (a+bx^n+cx^{2n})^p dx \rightarrow$$

$$- \frac{(dx)^{m+1} (b^2 - 2ac + bcx^n) (a+bx^n+cx^{2n})^{p+1}}{adn(p+1)(b^2 - 4ac)} +$$

$$\frac{1}{an(p+1)(b^2 - 4ac)} \int (dx)^m (a+bx^n+cx^{2n})^{p+1} (b^2(m+n(p+1)+1) - 2ac(m+2n(p+1)+1) + bc(m+n(2p+3)+1)x^n) dx$$

Program code:

```
Int[(d_.**x_)^m_.*(a_+b_.**x_^n_+c_.**x_^n2_.)^p_,x_Symbol] :=
  -(d*x)^(m+1)*(b^2-2*a*c+b*c*x^n)*(a+b*x^n+c*x^(2*n))^(p+1)/(a*d*n*(p+1)*(b^2-4*a*c)) +
  1/(a*n*(p+1)*(b^2-4*a*c))*
  Int[(d*x)^m*(a+b*x^n+c*x^(2*n))^(p+1)*Simp[b^2*(n*(p+1)+m+1)-2*a*c*(m+2*n*(p+1)+1)+b*c*(2*n*p+3*n+m+1)*x^n,x] /;
  FreeQ[{a,b,c,d,m,n},x] && EqQ[n2,2*n] && NeQ[b^2-4*a*c,0] && ILtQ[p+1,0]
```

10: $\int (d x)^m (a + b x^n + c x^{2n})^p dx$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(a + b x^n + c x^{2n})^p}{\left(1 + \frac{2 c x^n}{b + \sqrt{b^2 - 4 a c}}\right)^p \left(1 + \frac{2 c x^n}{b - \sqrt{b^2 - 4 a c}}\right)^p} = 0$

Rule 1.2.3.2.10:

$$\int (d x)^m (a + b x^n + c x^{2n})^p dx \rightarrow \frac{a^{\text{IntPart}[p]} (a + b x^n + c x^{2n})^{\text{FracPart}[p]}}{\left(1 + \frac{2 c x^n}{b + \sqrt{b^2 - 4 a c}}\right)^{\text{FracPart}[p]} \left(1 + \frac{2 c x^n}{b - \sqrt{b^2 - 4 a c}}\right)^{\text{FracPart}[p]}} \int (d x)^m \left(1 + \frac{2 c x^n}{b + \sqrt{b^2 - 4 a c}}\right)^p \left(1 + \frac{2 c x^n}{b - \sqrt{b^2 - 4 a c}}\right)^p dx$$

Program code:

```
Int[(d_.**x_)^m_.*(a_+b_.*x_^n_+c_.*x_^n2_.)^p_,x_Symbol] :=
  a^IntPart[p]*(a+b*x^n+c*x^(2*n))^FracPart[p]/
  ((1+2*c*x^n/(b+Rt[b^2-4*a*c,2]))^FracPart[p]*(1+2*c*x^n/(b-Rt[b^2-4*a*c,2]))^FracPart[p])*
  Int[(d*x)^m*(1+2*c*x^n/(b+Sqrt[b^2-4*a*c]))^p*(1+2*c*x^n/(b-Sqrt[b^2-4*a*c]))^p,x] /;
FreeQ[{a,b,c,d,m,n,p},x] && EqQ[n2,2*n]
```

$$11. \int (dx)^m (a + b x^{-n} + c x^n)^p dx$$

$$1. \int x^m (a + b x^{-n} + c x^n)^p dx$$

$$1: \int x^m (a + b x^{-n} + c x^n)^p dx \text{ when } p \in \mathbb{Z}$$

Derivation: Algebraic normalization

$$\text{Basis: } a + b x^{-n} + c x^n = x^{-n} (b + a x^n + c x^{2n})$$

Rule 1.2.3.2.11.1.1: If $p \in \mathbb{Z}$, then

$$\int x^m (a + b x^{-n} + c x^n)^p dx \rightarrow \int x^{m-np} (b + a x^n + c x^{2n})^p dx$$

Program code:

```
Int[x_^m_.*(a_+b_.*x_^mn_+c_.*x_^n_.)^p_.,x_Symbol] :=
  Int[x^(m-n*p)*(b+a*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,m,n},x] && EqQ[mn,-n] && IntegerQ[p] && PosQ[n]
```

$$2: \int x^m (a + b x^{-n} + c x^n)^p dx \text{ when } p \notin \mathbb{Z}$$

Derivation: Piecewise constant extraction

$$\text{Basis: } \partial_x \frac{x^{np} (a + b x^{-n} + c x^n)^p}{(b + a x^n + c x^{2n})^p} = 0$$

$$\text{Basis: } \frac{x^{np} (a + b x^{-n} + c x^n)^p}{(b + a x^n + c x^{2n})^p} = \frac{x^{n \text{FracPart}[p]} (a + b x^{-n} + c x^n)^{\text{FracPart}[p]}}{(b + a x^n + c x^{2n})^{\text{FracPart}[p]}}$$

Rule 1.2.3.2.11.1.2: If $p \notin \mathbb{Z}$, then

$$\int x^m (a + b x^{-n} + c x^n)^p dx \rightarrow \frac{x^{n \text{FracPart}[p]} (a + b x^{-n} + c x^n)^{\text{FracPart}[p]}}{(b + a x^n + c x^{2n})^{\text{FracPart}[p]}} \int x^{m-np} (b + a x^n + c x^{2n})^p dx$$

Program code:

```
Int[x_^m_.*(a_+b_.**x^mn_+c_.**x^n_.)^p_,x_Symbol] :=
  x^(n*FracPart[p])*(a+b/x^n+c*x^n)^FracPart[p]/(b+a*x^n+c*x^(2*n))^FracPart[p]*Int[x^(m-n*p)*(b+a*x^n+c*x^(2*n))^p,x] /;
FreeQ[{a,b,c,m,n,p},x] && EqQ[mn,-n] && Not[IntegerQ[p]] && PosQ[n]
```

2: $\int (d x)^m (a + b x^{-n} + c x^n)^p dx$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(d x)^m}{x^m} == 0$

Basis: $\frac{(d x)^m}{x^m} == \frac{d^{\text{IntPart}[m]} (d x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}}$

Rule 1.2.3.2.11.2:

$$\int (d x)^m (a + b x^{-n} + c x^n)^p dx \rightarrow \frac{d^{\text{IntPart}[m]} (d x)^{\text{FracPart}[m]}}{x^{\text{FracPart}[m]}} \int x^m (a + b x^{-n} + c x^n)^p dx$$

Program code:

```
Int[(d*x_)^m_.*(a_+b_.**x^mn_+c_.**x^n_.)^p_,x_Symbol] :=
  d^IntPart[m]*(d*x)^FracPart[m]/x^FracPart[m]*Int[x^m*(a+b*x^(-n)+c*x^n)^p,x] /;
FreeQ[{a,b,c,d,m,n,p},x] && EqQ[mn,-n]
```

$$\text{S. } \int u^m (a + b v^n + c v^{2n})^p dx \text{ when } v = d + e x \wedge u = f v$$

$$\text{1: } \int x^m (a + b v^n + c v^{2n})^p dx \text{ when } v = d + e x \wedge m \in \mathbb{Z}$$

Derivation: Integration by substitution

Basis: If $m \in \mathbb{Z}$, then $x^m F[d + e x] = \frac{1}{e^{m+1}} \text{Subst}[(x - d)^m F[x], x, d + e x] \partial_x (d + e x)$

Rule 1.2.3.2.S.1: If $v = d + e x \wedge m \in \mathbb{Z}$, then

$$\int x^m (a + b v^n + c v^{2n})^p dx \rightarrow \frac{1}{e^{m+1}} \text{Subst}\left[\int (x - d)^m (a + b x^n + c x^{2n})^p dx, x, v\right]$$

Program code:

```
Int[x^m.*(a_.+b_.*v_^n_.+c_.*v_^2n_.)^p_,x_Symbol] :=
  1/Coefficient[v,x,1]^(m+1)*Subst[Int[SimplifyIntegrand[(x-Coefficient[v,x,0])^m*(a+b*x^n+c*x^(2*n))^p,x],x],x,v] /;
FreeQ[{a,b,c,n,p},x] && EqQ[n2,2*n] && LinearQ[v,x] && IntegerQ[m] && NeQ[v,x]
```

2: $\int u^m (a + b v^n + c v^{2n})^p dx$ when $v = d + e x$ \wedge $u = f v$

Derivation: Integration by substitution and piecewise constant extraction

Basis: If $u = f v$, then $\partial_x \frac{u^m}{v^m} = 0$

Rule 1.2.3.2.S.2: If $v = d + e x$ \wedge $u = f v$, then

$$\int u^m (a + b v^n + c v^{2n})^p dx \rightarrow \frac{u^m}{e v^m} \text{Subst} \left[\int x^m (a + b x^n + c x^{2n})^p dx, x, v \right]$$

Program code:

```
Int[u_^m.*(a_.+b_.*v_^n+c_.*v_^2n.)^p_,x_Symbol] :=
  u^m/(Coefficient[v,x,1]*v^m)*Subst[Int[x^m*(a+b*x^n+c*x^(2*n))^p,x],x,v] /;
FreeQ[{a,b,c,m,n,p},x] && EqQ[n2,2*n] && LinearPairQ[u,v,x]
```