

Rules for normalizing to known sine integrands

$$1. \int u (c \operatorname{Trig}[a + b x])^m (d \operatorname{Trig}[a + b x])^n dx \text{ when } \operatorname{KnownSineIntegrandQ}[u, x]$$

$$1. \int u (c \operatorname{Tan}[a + b x])^m (d \operatorname{Trig}[a + b x])^n dx \text{ when } \operatorname{KnownSineIntegrandQ}[u, x]$$

$$1: \int u (c \operatorname{Tan}[a + b x])^m (d \operatorname{Sin}[a + b x])^n dx \text{ when } \operatorname{KnownSineIntegrandQ}[u, x] \wedge m \notin \mathbb{Z}$$

Derivation: Piecewise constant extraction

$$\text{Basis: } \partial_x \frac{(c \operatorname{Tan}[a + b x])^m (d \operatorname{Cos}[a + b x])^m}{(d \operatorname{Sin}[a + b x])^m} == 0$$

Rule: If $\operatorname{KnownSineIntegrandQ}[u, x] \wedge m \notin \mathbb{Z}$, then

$$\int u (c \operatorname{Tan}[a + b x])^m (d \operatorname{Sin}[a + b x])^n dx \rightarrow \frac{(c \operatorname{Tan}[a + b x])^m (d \operatorname{Cos}[a + b x])^m}{(d \operatorname{Sin}[a + b x])^m} \int \frac{u (d \operatorname{Sin}[a + b x])^{m+n}}{(d \operatorname{Cos}[a + b x])^m} dx$$

Program code:

```
Int[u*(c_.*tan[a_.+b_.*x_])^m_.*(d_.*sin[a_.+b_.*x_])^n_.,x_Symbol] :=
  (c*Tan[a+b*x])^m*(d*Cos[a+b*x])^m/(d*Sin[a+b*x])^m*Int[ActivateTrig[u]*(d*Sin[a+b*x])^(m+n)/(d*Cos[a+b*x])^m,x] /;
FreeQ[{a,b,c,d,m,n},x] && KnownSineIntegrandQ[u,x] && Not[IntegerQ[m]]
```

$$\mathbf{2:} \int u \left(c \tan[a + b x] \right)^m \left(d \cos[a + b x] \right)^n dx \text{ when } \text{KnownSineIntegrandQ}[u, x] \wedge m \notin \mathbb{Z}$$

Derivation: Piecewise constant extraction

$$\text{Basis: } \partial_x \frac{(c \tan[a + b x])^m (d \cos[a + b x])^m}{(d \sin[a + b x])^m} == 0$$

Rule: If $\text{KnownSineIntegrandQ}[u, x] \wedge m \notin \mathbb{Z}$, then

$$\int u \left(c \tan[a + b x] \right)^m \left(d \cos[a + b x] \right)^n dx \rightarrow \frac{(c \tan[a + b x])^m (d \cos[a + b x])^m}{(d \sin[a + b x])^m} \int \frac{u (d \sin[a + b x])^m}{(d \cos[a + b x])^{m-n}} dx$$

Program code:

```
Int[u_*(c_.*tan[a_.*b_.*x_])^m_.*(d_.*cos[a_.*b_.*x_])^n_.,x_Symbol] :=
  (c*Tan[a+b*x])^m*(d*Cos[a+b*x])^m/(d*Sin[a+b*x])^m*Int[ActivateTrig[u]*(d*Sin[a+b*x])^m/(d*Cos[a+b*x])^(m-n),x] /;
FreeQ[{a,b,c,d,m,n},x] && KnownSineIntegrandQ[u,x] && Not[IntegerQ[m]]
```

2. $\int u (c \cot[a + b x])^m (d \operatorname{Trig}[a + b x])^n dx$ when $\text{KnownSineIntegrandQ}[u, x]$

1: $\int u (c \cot[a + b x])^m (d \sin[a + b x])^n dx$ when $\text{KnownSineIntegrandQ}[u, x] \wedge m \notin \mathbb{Z}$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(c \cot[a + b x])^m (d \sin[a + b x])^m}{(d \cos[a + b x])^m} = 0$

Rule: If $\text{KnownSineIntegrandQ}[u, x] \wedge m \notin \mathbb{Z}$, then

$$\int u (c \cot[a + b x])^m (d \sin[a + b x])^n dx \rightarrow \frac{(c \cot[a + b x])^m (d \sin[a + b x])^m}{(d \cos[a + b x])^m} \int \frac{u (d \cos[a + b x])^m}{(d \sin[a + b x])^{m-n}} dx$$

Program code:

```
Int[u_*(c_.*cot[a_+b_*x_])^m_.*(d_.*sin[a_+b_*x_])^n_.,x_Symbol] :=
  (c*Cot[a+b*x])^m*(d*Sin[a+b*x])^m/(d*Cos[a+b*x])^m*Int[ActivateTrig[u]*(d*Cos[a+b*x])^m/(d*Sin[a+b*x])^(m-n),x] /;
FreeQ[{a,b,c,d,m,n},x] && KnownSineIntegrandQ[u,x] && Not[IntegerQ[m]]
```

$$2: \int u (c \cot[a + b x])^m (d \cos[a + b x])^n dx \text{ when } \text{KnownSineIntegrandQ}[u, x] \wedge m \notin \mathbb{Z}$$

Derivation: Piecewise constant extraction

$$\text{Basis: } \partial_x \frac{(c \cot[a + b x])^m (d \sin[a + b x])^m}{(d \cos[a + b x])^m} == 0$$

Rule: If $\text{KnownSineIntegrandQ}[u, x] \wedge m \notin \mathbb{Z}$, then

$$\int u (c \cot[a + b x])^m (d \cos[a + b x])^n dx \rightarrow \frac{(c \cot[a + b x])^m (d \sin[a + b x])^m}{(d \cos[a + b x])^m} \int \frac{u (d \cos[a + b x])^{m+n}}{(d \sin[a + b x])^m} dx$$

Program code:

```
Int[u_*(c_.*cot[a_.*b_.*x_])^m_.*(d_.*cos[a_.*b_.*x_])^n_.,x_Symbol] :=
  (c*Cot[a+b*x])^m*(d*Sin[a+b*x])^m/(d*Cos[a+b*x])^m*Int[ActivateTrig[u]*(d*Cos[a+b*x])^(m+n)/(d*Sin[a+b*x])^m,x] /;
FreeQ[{a,b,c,d,m,n},x] && KnownSineIntegrandQ[u,x] && Not[IntegerQ[m]]
```

3: $\int u (c \sec[a + b x])^m (d \cos[a + b x])^n dx$ when **KnownSineIntegrandQ**[u, x]

Derivation: Piecewise constant extraction

Basis: $\partial_x ((c \sec[a + b x])^m (d \cos[a + b x])^m) = 0$

Rule: If **KnownSineIntegrandQ**[u, x], then

$$\int u (c \sec[a + b x])^m (d \cos[a + b x])^n dx \rightarrow (c \sec[a + b x])^m (d \cos[a + b x])^m \int u (d \cos[a + b x])^{n-m} dx$$

Program code:

```
Int[u*(c_.*sec[a_.+b_.*x_])^m_.*(d_.*cos[a_.+b_.*x_])^n_,x_Symbol] :=
  (c*Sec[a+b*x])^m*(d*cos[a+b*x])^m*Int[ActivateTrig[u]*(d*cos[a+b*x])^(n-m),x] /;
FreeQ[{a,b,c,d,m,n},x] && KnownSineIntegrandQ[u,x]
```

4: $\int u (c \operatorname{Csc}[a + b x])^m (d \operatorname{Sin}[a + b x])^n dx$ when **KnownSineIntegrandQ**[u, x]

Derivation: Piecewise constant extraction

Basis: $\partial_x ((c \operatorname{Csc}[a + b x])^m (d \operatorname{Sin}[a + b x])^m) = 0$

Rule: If **KnownSineIntegrandQ**[u, x], then

$$\int u (c \operatorname{Csc}[a + b x])^m (d \operatorname{Sin}[a + b x])^n dx \rightarrow (c \operatorname{Csc}[a + b x])^m (d \operatorname{Sin}[a + b x])^m \int u (d \operatorname{Sin}[a + b x])^{n-m} dx$$

Program code:

```
Int[u*(c_.*sec[a_.+b_.*x_])^m_.*(d_.*cos[a_.+b_.*x_])^n_,x_Symbol] :=
  (c*Csc[a+b*x])^m*(d*Sin[a+b*x])^m*Int[ActivateTrig[u]*(d*Sin[a+b*x])^(n-m),x] /;
FreeQ[{a,b,c,d,m,n},x] && KnownSineIntegrandQ[u,x]
```

2. $\int u (c \operatorname{Trig}[a + b x])^m dx$ when $m \notin \mathbb{Z} \wedge \operatorname{KnownSineIntegrandQ}[u, x]$

1: $\int u (c \operatorname{Tan}[a + b x])^m dx$ when $m \notin \mathbb{Z} \wedge \operatorname{KnownSineIntegrandQ}[u, x]$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(c \operatorname{Tan}[a + b x])^m (c \operatorname{Cos}[a + b x])^m}{(c \operatorname{Sin}[a + b x])^m} == 0$

Rule: If $m \notin \mathbb{Z} \wedge \operatorname{KnownSineIntegrandQ}[u, x]$, then

$$\int u (c \operatorname{Tan}[a + b x])^m dx \rightarrow \frac{(c \operatorname{Tan}[a + b x])^m (c \operatorname{Cos}[a + b x])^m}{(c \operatorname{Sin}[a + b x])^m} \int \frac{u (c \operatorname{Sin}[a + b x])^m}{(c \operatorname{Cos}[a + b x])^m} dx$$

Program code:

```
Int[u_*(c_.*tan[a_+b_*x_])^m_,x_Symbol] :=
  (c*Tan[a+b*x])^m*(c*Cos[a+b*x])^m/(c*Sin[a+b*x])^m*Int[ActivateTrig[u]*(c*Sin[a+b*x])^m/(c*Cos[a+b*x])^m,x] /;
FreeQ[{a,b,c,m},x] && Not[IntegerQ[m]] && KnownSineIntegrandQ[u,x]
```

2: $\int u (c \cot[a + b x])^m dx$ when $m \notin \mathbb{Z} \wedge \text{KnownSineIntegrandQ}[u, x]$

Derivation: Piecewise constant extraction

Basis: $\partial_x \frac{(c \cot[a + b x])^m (c \sin[a + b x])^m}{(c \cos[a + b x])^m} == 0$

Rule: If $m \notin \mathbb{Z} \wedge \text{KnownSineIntegrandQ}[u, x]$, then

$$\int u (c \cot[a + b x])^m dx \rightarrow \frac{(c \cot[a + b x])^m (c \sin[a + b x])^m}{(c \cos[a + b x])^m} \int \frac{u (c \cos[a + b x])^m}{(c \sin[a + b x])^m} dx$$

Program code:

```
Int[u_*(c_.*cot[a_.*b_.*x_])^m_.,x_Symbol] :=
  (c*Cot[a+b*x])^m*(c*Sin[a+b*x])^m/(c*Cos[a+b*x])^m*Int[ActivateTrig[u]*(c*Cos[a+b*x])^m/(c*Sin[a+b*x])^m,x] /;
FreeQ[{a,b,c,m},x] && Not[IntegerQ[m]] && KnownSineIntegrandQ[u,x]
```


3: $\int u (c \sec[a + b x])^m dx$ when $m \notin \mathbb{Z} \wedge \text{KnownSineIntegrandQ}[u, x]$

Derivation: Piecewise constant extraction

Basis: $\partial_x ((c \sec[a + b x])^m (c \cos[a + b x])^m) = 0$

Rule: If $m \notin \mathbb{Z} \wedge \text{KnownSineIntegrandQ}[u, x]$, then

$$\int u (c \sec[a + b x])^m dx \rightarrow (c \sec[a + b x])^m (c \cos[a + b x])^m \int \frac{u}{(c \cos[a + b x])^m} dx$$

Program code:

```
Int[u_*(c_.*sec[a_.*b_.*x_])^m_.,x_Symbol] :=
  (c*Sec[a+b*x])^m*(c*cos[a+b*x])^m*Int[ActivateTrig[u]/(c*cos[a+b*x])^m,x] /;
FreeQ[{a,b,c,m},x] && Not[IntegerQ[m]] && KnownSineIntegrandQ[u,x]
```

4: $\int u (c \operatorname{Csc}[a + b x])^m dx$ when $m \notin \mathbb{Z} \wedge \text{KnownSineIntegrandQ}[u, x]$

Derivation: Piecewise constant extraction

Basis: $\partial_x ((c \operatorname{Csc}[a + b x])^m (c \operatorname{Sin}[a + b x])^m) = 0$

Rule: If $m \notin \mathbb{Z} \wedge \text{KnownSineIntegrandQ}[u, x]$, then

$$\int u (c \operatorname{Csc}[a + b x])^m dx \rightarrow (c \operatorname{Csc}[a + b x])^m (c \operatorname{Sin}[a + b x])^m \int \frac{u}{(c \operatorname{Sin}[a + b x])^m} dx$$

Program code:

```
Int[u_*(c_.*csc[a_+b_*x_])^m_,x_Symbol] :=
  (c*Csc[a+b*x])^m*(c*Sin[a+b*x])^m*Int[ActivateTrig[u]/(c*Sin[a+b*x])^m,x] /;
FreeQ[{a,b,c,m},x] && Not[IntegerQ[m]] && KnownSineIntegrandQ[u,x]
```

3. $\int u (A + B \operatorname{Csc}[a + b x]) \, dx$ when **KnownSineIntegrandQ**[u, x]

1: $\int u (c \operatorname{Sin}[a + b x])^n (A + B \operatorname{Csc}[a + b x]) \, dx$ when **KnownSineIntegrandQ**[u, x]

Derivation: Algebraic normalization

Rule: If **KnownSineIntegrandQ**[u, x], then

$$\int u (c \operatorname{Sin}[a + b x])^n (A + B \operatorname{Csc}[a + b x]) \, dx \rightarrow c \int u (c \operatorname{Sin}[a + b x])^{n-1} (B + A \operatorname{Sin}[a + b x]) \, dx$$

Program code:

```
Int[u*(c_.*sin[a_.+b_.*x_])^n_.*(A_.+B_.*csc[a_.+b_.*x_]),x_Symbol] :=
  c*Int[ActivateTrig[u]*(c*Sin[a+b*x])^(n-1)*(B+A*Sin[a+b*x]),x] /;
FreeQ[{a,b,c,A,B,n},x] && KnownSineIntegrandQ[u,x]
```

```
Int[u*(c_.*cos[a_.+b_.*x_])^n_.*(A_.+B_.*sec[a_.+b_.*x_]),x_Symbol] :=
  c*Int[ActivateTrig[u]*(c*Cos[a+b*x])^(n-1)*(B+A*Cos[a+b*x]),x] /;
FreeQ[{a,b,c,A,B,n},x] && KnownSineIntegrandQ[u,x]
```

2: $\int u (A + B \operatorname{Csc}[a + b x]) \, dx$ when `KnownSineIntegrandQ[u, x]`

Derivation: Algebraic normalization

Rule: If `KnownSineIntegrandQ[u, x]`, then

$$\int u (A + B \operatorname{Csc}[a + b x]) \, dx \rightarrow \int \frac{u (B + A \operatorname{Sin}[a + b x])}{\operatorname{Sin}[a + b x]} \, dx$$

Program code:

```
Int[u_*(A_+B_.*csc[a_+b_.*x_]),x_Symbol] :=
  Int[ActivateTrig[u]*(B+A*Sin[a+b*x])/Sin[a+b*x],x] /;
FreeQ[{a,b,A,B},x] && KnownSineIntegrandQ[u,x]
```

```
Int[u_*(A_+B_.*sec[a_+b_.*x_]),x_Symbol] :=
  Int[ActivateTrig[u]*(B+A*Cos[a+b*x])/Cos[a+b*x],x] /;
FreeQ[{a,b,A,B},x] && KnownSineIntegrandQ[u,x]
```

4. $\int u (A + B \csc[a + b x] + C \csc[a + b x]^2) dx$ when **KnownSineIntegrandQ**[u, x]

1: $\int u (c \sin[a + b x])^n (A + B \csc[a + b x] + C \csc[a + b x]^2) dx$ when **KnownSineIntegrandQ**[u, x]

Derivation: Algebraic normalization

Rule: If **KnownSineIntegrandQ**[u, x], then

$$\int u (c \sin[a + b x])^n (A + B \csc[a + b x] + C \csc[a + b x]^2) dx \rightarrow c^2 \int u (c \sin[a + b x])^{n-2} (C + B \sin[a + b x] + A \sin[a + b x]^2) dx$$

Program code:

```
Int[u.*(c.*sin[a_.+b_.*x_])^n.*(A_.+B_.*csc[a_.+b_.*x_]^2),x_Symbol] :=
  c^2*Int[ActivateTrig[u]*(c*Sin[a+b*x])^(n-2)*(C+B*Sin[a+b*x]+A*Sin[a+b*x]^2),x] /;
FreeQ[{a,b,c,A,B,C,n},x] && KnownSineIntegrandQ[u,x]
```

```
Int[u.*(c.*cos[a_.+b_.*x_])^n.*(A_.+B_.*sec[a_.+b_.*x_]^2),x_Symbol] :=
  c^2*Int[ActivateTrig[u]*(c*Cos[a+b*x])^(n-2)*(C+B*Cos[a+b*x]+A*Cos[a+b*x]^2),x] /;
FreeQ[{a,b,c,A,B,C,n},x] && KnownSineIntegrandQ[u,x]
```

```
Int[u.*(c.*sin[a_.+b_.*x_])^n.*(A_.+C_.*csc[a_.+b_.*x_]^2),x_Symbol] :=
  c^2*Int[ActivateTrig[u]*(c*Sin[a+b*x])^(n-2)*(C+A*Sin[a+b*x]^2),x] /;
FreeQ[{a,b,c,A,C,n},x] && KnownSineIntegrandQ[u,x]
```

```
Int[u.*(c.*cos[a_.+b_.*x_])^n.*(A_.+C_.*sec[a_.+b_.*x_]^2),x_Symbol] :=
  c^2*Int[ActivateTrig[u]*(c*Cos[a+b*x])^(n-2)*(C+A*Cos[a+b*x]^2),x] /;
FreeQ[{a,b,c,A,C,n},x] && KnownSineIntegrandQ[u,x]
```

2: $\int u (A + B \operatorname{Csc}[a + b x] + C \operatorname{Csc}[a + b x]^2) dx$ when **KnownSineIntegrandQ**[u, x]

Derivation: Algebraic normalization

Rule: If **KnownSineIntegrandQ**[u, x], then

$$\int u (A + B \operatorname{Csc}[a + b x] + C \operatorname{Csc}[a + b x]^2) dx \rightarrow \int \frac{u (C + B \sin[a + b x] + A \sin[a + b x]^2)}{\sin[a + b x]^2} dx$$

Program code:

```
Int[u*(A_.+B_.*csc[a_.+b_.*x_]+C_.*csc[a_.+b_.*x_]^2),x_Symbol] :=
  Int[ActivateTrig[u]*(C+B*Sin[a+b*x]+A*Sin[a+b*x]^2)/Sin[a+b*x]^2,x] /;
FreeQ[{a,b,A,B,C},x] && KnownSineIntegrandQ[u,x]
```

```
Int[u*(A_.+B_.*sec[a_.+b_.*x_]+C_.*sec[a_.+b_.*x_]^2),x_Symbol] :=
  Int[ActivateTrig[u]*(C+B*Cos[a+b*x]+A*Cos[a+b*x]^2)/Cos[a+b*x]^2,x] /;
FreeQ[{a,b,A,B,C},x] && KnownSineIntegrandQ[u,x]
```

```
Int[u*(A_.+C_.*csc[a_.+b_.*x_]^2),x_Symbol] :=
  Int[ActivateTrig[u]*(C+A*Sin[a+b*x]^2)/Sin[a+b*x]^2,x] /;
FreeQ[{a,b,A,C},x] && KnownSineIntegrandQ[u,x]
```

```
Int[u*(A_.+C_.*sec[a_.+b_.*x_]^2),x_Symbol] :=
  Int[ActivateTrig[u]*(C+A*Cos[a+b*x]^2)/Cos[a+b*x]^2,x] /;
FreeQ[{a,b,A,C},x] && KnownSineIntegrandQ[u,x]
```

5: $\int u (A + B \sin[a + b x] + C \csc[a + b x]) \, dx$

Derivation: Algebraic normalization

Rule:

$$\int u (A + B \sin[a + b x] + C \csc[a + b x]) \, dx \rightarrow \int \frac{u (C + A \sin[a + b x] + B \sin[a + b x]^2)}{\sin[a + b x]} \, dx$$

Program code:

```
Int[u*(A_.+B_.*sin[a_.+b_.*x_]+C_.*csc[a_.+b_.*x_]),x_Symbol] :=
  Int[ActivateTrig[u]*(C+A*Sin[a+b*x]+B*Sin[a+b*x]^2)/Sin[a+b*x],x] /;
FreeQ[{a,b,A,B,C},x]
```

```
Int[u*(A_.+B_.*cos[a_.+b_.*x_]+C_.*sec[a_.+b_.*x_]),x_Symbol] :=
  Int[ActivateTrig[u]*(C+A*Cos[a+b*x]+B*Cos[a+b*x]^2)/Cos[a+b*x],x] /;
FreeQ[{a,b,A,B,C},x]
```

6: $\int u \left(A \sin[a + b x]^n + B \sin[a + b x]^{n+1} + C \sin[a + b x]^{n+2} \right) dx$

Derivation: Algebraic normalization

Rule:

$$\int u \left(A \sin[a + b x]^n + B \sin[a + b x]^{n+1} + C \sin[a + b x]^{n+2} \right) dx \rightarrow \int u \sin[a + b x]^n \left(A + B \sin[a + b x] + C \sin[a + b x]^2 \right) dx$$

Program code:

```
Int[u_*(A_.*sin[a_.*b_.*x_]^n_+B_.*sin[a_.*b_.*x_]^n1_+C_.*sin[a_.*b_.*x_]^n2_),x_Symbol] :=
  Int[ActivateTrig[u]*Sin[a+b*x]^n*(A+B*Ssin[a+b*x]+C*Ssin[a+b*x]^2),x] /;
FreeQ[{a,b,A,B,C,n},x] && EqQ[n1,n+1] && EqQ[n2,n+2]
```

```
Int[u_*(A_.*cos[a_.*b_.*x_]^n_+B_.*cos[a_.*b_.*x_]^n1_+C_.*cos[a_.*b_.*x_]^n2_),x_Symbol] :=
  Int[ActivateTrig[u]*Cos[a+b*x]^n*(A+B*Ccos[a+b*x]+C*Ccos[a+b*x]^2),x] /;
FreeQ[{a,b,A,B,C,n},x] && EqQ[n1,n+1] && EqQ[n2,n+2]
```