Rules for integrands of the form $(a + b\,x)^m$

1. $\int x^m \,dx$

   1: $\int \frac{1}{x} \,dx$

   Reference: G&R 2.01.2, CRC 9, A&S 3.3.15

   Derivation: Reciprocal rule for integration

   Rule 1.1.1.1.1.1:

   $$\int \frac{1}{x} \,dx \longrightarrow \text{Log}[x]$$

   Program code:

```
Int[1/x_,x_Symbol] :=
  Log[x]
```

2: $\int x^m \, dx$ when $m \neq -1$

Reference: G&R 2.01.1, CRC 7, A&S 3.3.14

Derivation: Power rule for integration

Rule 1.1.1.1.1.2: If $m \neq -1$, then

$$\int x^m \, dx \rightarrow \frac{x^{m+1}}{m+1}$$

Program code:

```
Int[x_^m_.,x_Symbol] :=
  x^(m+1)/(m+1) /;
FreeQ[m,x] && NeQ[m,-1]
```

2. $\int (a + b\,x)^m \,dx$

1: $\int \frac{1}{a + b\,x} \,dx$

Reference: G&R 2.111.1.2, CRC 27, A&S 3.3.15

Derivation: Reciprocal rule for integration

Rule 1.1.1.1.2.1:

$$\int \frac{1}{a + b\,x} \,dx \to \frac{\text{Log}[a + b\,x]}{b}$$

Program code:

```
Int[1/(a_+b_.*x_),x_Symbol] :=
  Log[RemoveContent[a+b*x,x]]/b /;
FreeQ[{a,b},x]
```

2: $\int (a+b\,x)^m\,dx$ when $m \neq -1$

Reference: G&R 2.111.1.1, CRC 23, A&S 3.3.14

Derivation: Power rule for integration

Rule 1.1.1.1.2.2: If $m \neq -1$, then

$$\int (a+b\,x)^m\,dx \;\longrightarrow\; \frac{(a+b\,x)^{m+1}}{b\,(m+1)}$$

Program code:

```
Int[(a_.+b_.*x_)^m_,x_Symbol] :=
  (a+b*x)^(m+1)/(b*(m+1)) /;
FreeQ[{a,b,m},x] && NeQ[m,-1]
```

S: $\int (a+b\,u)^m\,dx$ when $u == c+d\,x$

Derivation: Integration by substitution

Rule 1.1.1.1.S: If $u == c+d\,x$, then

$$\int (a+b\,u)^m\,dx \;\longrightarrow\; \frac{1}{d}\,\text{Subst}\left[\int (a+b\,x)^m\,dx,\, x,\, u\right]$$

Program code:

```
Int[(a_.+b_.*u_)^m_,x_Symbol] :=
  1/Coefficient[u,x,1]*Subst[Int[(a+b*x)^m,x],x,u] /;
FreeQ[{a,b,m},x] && LinearQ[u,x] && NeQ[u,x]
```