

Rules for integrands of the form $(f x)^m (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n$

$$1. \int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d + e = 0$$

$$1. \int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d + e = 0 \wedge n > 0$$

$$1. \int x (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d + e = 0 \wedge n > 0$$

$$1: \int \frac{x (a + b \operatorname{ArcSin}[c x])^n}{d + e x^2} dx \text{ when } c^2 d + e = 0 \wedge n \in \mathbb{Z}^+$$

Derivation: Integration by substitution

Basis: If $c^2 d + e = 0$, then $\frac{x}{d + e x^2} = -\frac{1}{e} \operatorname{Subst}[\operatorname{Tan}[x], x, \operatorname{ArcSin}[c x]] \partial_x \operatorname{ArcSin}[c x]$

Note: If $n \in \mathbb{Z}^+$, then $(a + b x)^n \operatorname{Tan}[x]$ is integrable in closed-form.

Rule: If $c^2 d + e = 0 \wedge n \in \mathbb{Z}^+$, then

$$\int \frac{x (a + b \operatorname{ArcSin}[c x])^n}{d + e x^2} dx \rightarrow -\frac{1}{e} \operatorname{Subst}\left[\int (a + b x)^n \operatorname{Tan}[x] dx, x, \operatorname{ArcSin}[c x]\right]$$

Program code:

```
Int[x*(a_.+b_.*ArcSin[c_.*x_])^n_./(d_.+e_.*x_^2),x_Symbol] :=
  -1/e*Subst[Int[(a+b*x)^n*Tan[x],x],x,ArcSin[c*x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[n,0]
```

```
Int[x*(a_.+b_.*ArcCos[c_.*x_])^n_./(d_.+e_.*x_^2),x_Symbol] :=
  1/e*Subst[Int[(a+b*x)^n*Cot[x],x],x,ArcCos[c*x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[n,0]
```

$$2. \int x (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e = 0 \wedge n > 0 \wedge p \neq -1$$

$$1: \int x (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e = 0 \wedge n > 0 \wedge p \neq -1 \wedge (p \in \mathbb{Z} \vee d > 0)$$

Derivation: Integration by parts

Rule: If $c^2 d+e = 0 \wedge n > 0 \wedge p \neq -1 \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int x (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{(d+e x^2)^{p+1} (a+b \operatorname{ArcSin}[c x])^n}{2 e (p+1)} + \frac{b n d^p}{2 c (p+1)} \int (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n-1} dx$$

Program code:

```
(* Int[x*(d+_.*x_^2)^p_.*(a+_.*b_.*ArcSin[c_.*x_])^n_.,x_Symbol] :=
  (d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n/(2*e*(p+1)) +
  b*n*d^p/(2*c*(p+1))*Int[(1-c^2*x^2)^(p+1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && NeQ[p,-1] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
(* Int[x*(d+_.*x_^2)^p_.*(a+_.*b_.*ArcCos[c_.*x_])^n_.,x_Symbol] :=
  (d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n/(2*e*(p+1)) -
  b*n*d^p/(2*c*(p+1))*Int[(1-c^2*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && NeQ[p,-1] && (IntegerQ[p] || GtQ[d,0]) *)
```

$$\mathbf{2:} \int x (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e=0 \wedge n>0 \wedge p \neq -1$$

Derivation: Integration by parts and piecewise constant extraction

Basis: If $c^2 d+e=0$, then $\partial_x \frac{(d+e x^2)^p}{(1-c^2 x^2)^p} = 0$

Rule: If $c^2 d+e=0 \wedge n>0 \wedge p \neq -1$, then

$$\int x (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{(d+e x^2)^{p+1} (a+b \operatorname{ArcSin}[c x])^n}{2 e (p+1)} + \frac{b n d^{\operatorname{IntPart}[p]} (d+e x^2)^{\operatorname{FracPart}[p]}}{2 c (p+1) (1-c^2 x^2)^{\operatorname{FracPart}[p]}} \int (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n-1} dx$$

Program code:

```
Int[x_*(d_+e_*x_^2)^p_.*(a_+b_*ArcSin[c_*x_])^n_,x_Symbol] :=
  (d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n/(2*e*(p+1)) +
  b*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(2*c*(p+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(1-c^2*x^2)^(p+1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && NeQ[p,-1]
```

```
Int[x_*(d_+e_*x_^2)^p_.*(a_+b_*ArcCos[c_*x_])^n_,x_Symbol] :=
  (d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n/(2*e*(p+1)) -
  b*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(2*c*(p+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(1-c^2*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && NeQ[p,-1]
```

$$2. \int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e == 0 \wedge n > 0 \wedge m+2 p+3 == 0$$

$$1: \int \frac{(a+b \operatorname{ArcSin}[c x])^n}{x (d+e x^2)} dx \text{ when } c^2 d+e == 0 \wedge n \in \mathbb{Z}^+$$

Derivation: Integration by substitution

Basis: If $c^2 d+e == 0$, then $\frac{1}{x (d+e x^2)} == \frac{1}{d} \operatorname{Subst}\left[\frac{1}{\cos[x] \sin[x]}, x, \operatorname{ArcSin}[c x]\right] \partial_x \operatorname{ArcSin}[c x]$

Rule: If $c^2 d+e == 0 \wedge n \in \mathbb{Z}^+$, then

$$\int \frac{(a+b \operatorname{ArcSin}[c x])^n}{x (d+e x^2)} dx \rightarrow \frac{1}{d} \operatorname{Subst}\left[\int \frac{(a+b x)^n}{\cos[x] \sin[x]} dx, x, \operatorname{ArcSin}[c x]\right]$$

Program code:

```
Int[(a_.+b_.*ArcSin[c_.*x_])^n_./(x_*(d_+e_.*x_^2)),x_Symbol] :=
  1/d*Subst[Int[(a+b*x)^n/(Cos[x]*Sin[x]),x],x,ArcSin[c*x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[n,0]
```

```
Int[(a_.+b_.*ArcCos[c_.*x_])^n_./(x_*(d_+e_.*x_^2)),x_Symbol] :=
  -1/d*Subst[Int[(a+b*x)^n/(Cos[x]*Sin[x]),x],x,ArcCos[c*x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[n,0]
```

$$2. \int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \text{ when } c^2 d+e=0 \wedge n>0 \wedge m+2p+3=0 \wedge m \neq -1$$

$$1: \int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \text{ when } c^2 d+e=0 \wedge n>0 \wedge m+2p+3=0 \wedge m \neq -1 \wedge (p \in \mathbb{Z} \vee d>0)$$

Derivation: Integration by parts

Basis: If $m+2p+3=0$, then $(f x)^m (d+e x^2)^p = \partial_x \frac{(f x)^{m+1} (d+e x^2)^{p+1}}{d f (m+1)}$

Rule: If $c^2 d+e=0 \wedge n>0 \wedge m+2p+3=0 \wedge m \neq -1 \wedge (p \in \mathbb{Z} \vee d>0)$, then

$$\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \rightarrow \frac{(f x)^{m+1} (d+e x^2)^{p+1} (a+b \arcsin(c x))^n}{d f (m+1)} - \frac{b c n d^p}{f (m+1)} \int (f x)^{m+1} (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \arcsin(c x))^{n-1} dx$$

Program code:

```
(* Int[(f_.**x_)^m_*(d+_e_.**x_^2)^p_.*(a_+_b_.**ArcSin[c_.**x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n/(d*f*(m+1)) -
  b*c*n*d^p/(f*(m+1))*Int[(f*x)^(m+1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && EqQ[m+2*p+3,0] && NeQ[m,-1] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
(* Int[(f_.**x_)^m_*(d+_e_.**x_^2)^p_.*(a_+_b_.**ArcCos[c_.**x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n/(d*f*(m+1)) +
  b*c*n*d^p/(f*(m+1))*Int[(f*x)^(m+1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && EqQ[m+2*p+3,0] && NeQ[m,-1] && (IntegerQ[p] || GtQ[d,0]) *)
```

$$2: \int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \text{ when } c^2 d+e == 0 \wedge n > 0 \wedge m+2p+3 == 0 \wedge m \neq -1$$

Derivation: Integration by parts and piecewise constant extraction

Basis: If $m+2p+3 == 0$, then $(f x)^m (d+e x^2)^p == \partial_x \frac{(f x)^{m+1} (d+e x^2)^{p+1}}{d f (m+1)}$

Basis: If $c^2 d+e == 0$, then $\partial_x \frac{(d+e x^2)^p}{(1-c^2 x^2)^p} == 0$

Rule: If $c^2 d+e == 0 \wedge n > 0 \wedge m+2p+3 == 0 \wedge m \neq -1$, then

$$\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \rightarrow \frac{(f x)^{m+1} (d+e x^2)^{p+1} (a+b \arcsin(c x))^n}{d f (m+1)} - \frac{b c n d^{\text{IntPart}[p]} (d+e x^2)^{\text{FracPart}[p]}}{f (m+1) (1-c^2 x^2)^{\text{FracPart}[p]}} \int (f x)^{m+1} (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \arcsin(c x))^{n-1} dx$$

Program code:

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n/(d*f*(m+1)) -
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(f*(m+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m+1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && EqQ[m+2*p+3,0] && NeQ[m,-1]
```

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n/(d*f*(m+1)) +
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(f*(m+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m+1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && EqQ[m+2*p+3,0] && NeQ[m,-1]
```

$$3. \int (f x)^m (d+e x^2)^p (a+b \arcsin(c x)) dx \text{ when } c^2 d+e == 0 \wedge p \in \mathbb{Z}^+$$

$$1. \int (f x)^m (d+e x^2)^p (a+b \arcsin(c x)) dx \text{ when } c^2 d+e == 0 \wedge p \in \mathbb{Z}^+ \wedge \frac{m-1}{2} \in \mathbb{Z}^-$$

$$\text{1: } \int \frac{(d+e x^2)^p (a+b \operatorname{ArcSin}[c x])}{x} dx \text{ when } c^2 d+e == 0 \wedge p \in \mathbb{Z}^+$$

Derivation: Inverted integration by parts

Rule: If $c^2 d+e == 0 \wedge p \in \mathbb{Z}^+$, then

$$\int \frac{(d+e x^2)^p (a+b \operatorname{ArcSin}[c x])}{x} dx \rightarrow \frac{(d+e x^2)^p (a+b \operatorname{ArcSin}[c x])}{2p} - \frac{b c d^p}{2p} \int (1-c^2 x^2)^{p-\frac{1}{2}} dx + d \int \frac{(d+e x^2)^{p-1} (a+b \operatorname{ArcSin}[c x])}{x} dx$$

Program code:

```
Int[(d+_.*x_^2)^p_.*(a+_.*b_.*ArcSin[c_.*x_])/x_,x_Symbol] :=
  (d+e*x^2)^p*(a+b*ArcSin[c*x])/(2*p) -
  b*c*d^p/(2*p)*Int[(1-c^2*x^2)^(p-1/2),x] +
  d*Int[(d+e*x^2)^(p-1)*(a+b*ArcSin[c*x])/x,x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[p,0]
```

```
Int[(d+_.*x_^2)^p_.*(a+_.*b_.*ArcCos[c_.*x_])/x_,x_Symbol] :=
  (d+e*x^2)^p*(a+b*ArcCos[c*x])/(2*p) +
  b*c*d^p/(2*p)*Int[(1-c^2*x^2)^(p-1/2),x] +
  d*Int[(d+e*x^2)^(p-1)*(a+b*ArcCos[c*x])/x,x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[p,0]
```

$$\text{2: } \int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x]) dx \text{ when } c^2 d+e == 0 \wedge p \in \mathbb{Z}^+ \wedge \frac{m+1}{2} \in \mathbb{Z}^-$$

Derivation: Inverted integration by parts

Rule: If $c^2 d+e == 0 \wedge p \in \mathbb{Z}^+ \wedge \frac{m+1}{2} \in \mathbb{Z}^-$, then

$$\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x]) dx \rightarrow$$

$$\frac{(f x)^{m+1} (d+e x^2)^p (a+b \arcsin(c x))}{f (m+1)} - \frac{b c d^p}{f (m+1)} \int (f x)^{m+1} (1-c^2 x^2)^{p-\frac{1}{2}} dx - \frac{2 e p}{f^2 (m+1)} \int (f x)^{m+2} (d+e x^2)^{p-1} (a+b \arcsin(c x)) dx$$

Program code:

```
Int[(f_.**x_)^m_*(d_+e_.**x_^2)^p_*(a_.+b_.**ArcSin[c_.**x_]),x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcSin[c*x])/(f*(m+1)) -
  b*c*d^p/(f*(m+1))*Int[(f*x)^(m+1)*(1-c^2*x^2)^(p-1/2),x] -
  2*e*p/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^(p-1)*(a+b*ArcSin[c*x]),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && IGtQ[p,0] && ILtQ[(m+1)/2,0]
```

```
Int[(f_.**x_)^m_*(d_+e_.**x_^2)^p_*(a_.+b_.**ArcCos[c_.**x_]),x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcCos[c*x])/(f*(m+1)) +
  b*c*d^p/(f*(m+1))*Int[(f*x)^(m+1)*(1-c^2*x^2)^(p-1/2),x] -
  2*e*p/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^(p-1)*(a+b*ArcCos[c*x]),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && IGtQ[p,0] && ILtQ[(m+1)/2,0]
```

2: $\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x)) dx$ when $c^2 d+e=0 \wedge p \in \mathbb{Z}^+$

Derivation: Integration by parts

Rule: If $c^2 d+e=0 \wedge p \in \mathbb{Z}^+$, let $u = \int (f x)^m (d+e x^2)^p dx$, then

$$\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x)) dx \rightarrow u (a+b \arcsin(c x)) - b c \int \frac{u}{\sqrt{1-c^2 x^2}} dx$$

Program code:

```
Int[(f_.**x_)^m_*(d_+e_.**x_^2)^p_*(a_.+b_.**ArcSin[c_.**x_]),x_Symbol] :=
  With[{u=IntHide[(f*x)^m*(d+e*x^2)^p,x]},
  Dist[a+b*ArcSin[c*x],u,x] - b*c*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && IGtQ[p,0]
```



```

Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_+b_.*ArcCos[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(f*x)^m*(d+e*x^2)^p,x]},
    Dist[a+b*ArcCos[c*x],u,x] + b*c*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x] /;
    FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && IGtQ[p,0]

```

4. $\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x]) dx$ when $c^2 d+e=0 \wedge p+\frac{1}{2} \in \mathbb{Z}^+ \wedge \left(\frac{m+1}{2} \in \mathbb{Z}^+ \vee \frac{m+2p+3}{2} \in \mathbb{Z}^-\right)$

1: $\int x^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x]) dx$ when $c^2 d+e=0 \wedge p-\frac{1}{2} \in \mathbb{Z} \wedge \left(\frac{m+1}{2} \in \mathbb{Z}^+ \vee \frac{m+2p+3}{2} \in \mathbb{Z}^-\right) \wedge p \neq -\frac{1}{2} \wedge d>0$

Derivation: Integration by parts

Note: If $p-\frac{1}{2} \in \mathbb{Z} \wedge \left(\frac{m+1}{2} \in \mathbb{Z}^+ \vee \frac{m+2p+3}{2} \in \mathbb{Z}^-\right)$, then $\int x^m (1-c^2 x^2)^p dx$ is an algebraic function not involving logarithms, inverse trig or inverse hyperbolic functions.

Rule: If $c^2 d+e=0 \wedge p-\frac{1}{2} \in \mathbb{Z} \wedge \left(\frac{m+1}{2} \in \mathbb{Z}^+ \vee \frac{m+2p+3}{2} \in \mathbb{Z}^-\right) \wedge p \neq -\frac{1}{2} \wedge d>0$, let $u = \int x^m (1-c^2 x^2)^p dx$, then

$$\int x^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x]) dx \rightarrow d^p u (a+b \operatorname{ArcSin}[c x]) - b c d^p \int \frac{u}{\sqrt{1-c^2 x^2}} dx$$

Program code:

```

Int[x_^m_*(d_+e_.*x_^2)^p_*(a_+b_.*ArcSin[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[x^m*(1-c^2*x^2)^p,x]},
    Dist[d^p*(a+b*ArcSin[c*x]),u,x] - b*c*d^p*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x] /;
    FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IntegerQ[p-1/2] && (IGtQ[(m+1)/2,0] || ILtQ[(m+2*p+3)/2,0]) &&
    NeQ[p,-1/2] && GtQ[d,0]

```

```

Int[x_^m_*(d_+e_.*x_^2)^p_*(a_+b_.*ArcCos[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[x^m*(1-c^2*x^2)^p,x]},
    Dist[d^p*(a+b*ArcCos[c*x]),u,x] + b*c*d^p*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x] /;
    FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IntegerQ[p-1/2] && (IGtQ[(m+1)/2,0] || ILtQ[(m+2*p+3)/2,0]) &&
    NeQ[p,-1/2] && GtQ[d,0]

```

$$\mathbf{2:} \int x^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x]) dx \text{ when } c^2 d+e = 0 \wedge p + \frac{1}{2} \in \mathbb{Z}^+ \wedge \left(\frac{m+1}{2} \in \mathbb{Z}^+ \vee \frac{m+2p+3}{2} \in \mathbb{Z}^- \right)$$

Derivation: Integration by parts and piecewise constant extraction

Note: If $p + \frac{1}{2} \in \mathbb{Z} \wedge \left(\frac{m+1}{2} \in \mathbb{Z}^+ \vee \frac{m+2p+3}{2} \in \mathbb{Z}^- \right)$, then $\int x^m (1-c^2 x^2)^p dx$ is an algebraic function not involving logarithms, inverse trig or inverse hyperbolic functions.

Rule: If $c^2 d+e = 0 \wedge p + \frac{1}{2} \in \mathbb{Z}^+ \wedge \left(\frac{m+1}{2} \in \mathbb{Z}^+ \vee \frac{m+2p+3}{2} \in \mathbb{Z}^- \right)$, let $u = \int x^m (1-c^2 x^2)^p dx$, then

$$\int x^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x]) dx \rightarrow (a+b \operatorname{ArcSin}[c x]) \int x^m (d+e x^2)^p dx - \frac{b c d^{p-\frac{1}{2}} \sqrt{d+e x^2}}{\sqrt{1-c^2 x^2}} \int \frac{u}{\sqrt{1-c^2 x^2}} dx$$

Program code:

```
Int[x_^m_*(d_+e_.*x_^2)^p_*(a_+b_.*ArcSin[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[x^m*(1-c^2*x^2)^p,x]},
    (a+b*ArcSin[c*x])*Int[x^m*(d+e*x^2)^p,x] -
    b*c*d^(p-1/2)*Sqrt[d+e*x^2]/Sqrt[1-c^2*x^2]*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x]] /;
  FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[p+1/2,0] && (IGtQ[(m+1)/2,0] || ILtQ[(m+2*p+3)/2,0])
```

```
Int[x_^m_*(d_+e_.*x_^2)^p_*(a_+b_.*ArcCos[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[x^m*(1-c^2*x^2)^p,x]},
    (a+b*ArcCos[c*x])*Int[x^m*(d+e*x^2)^p,x] +
    b*c*d^(p-1/2)*Sqrt[d+e*x^2]/Sqrt[1-c^2*x^2]*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x]] /;
  FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[p+1/2,0] && (IGtQ[(m+1)/2,0] || ILtQ[(m+2*p+3)/2,0])
```

$$5. \int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e == 0 \wedge n > 0 \wedge p > 0$$

$$1. \int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e == 0 \wedge n > 0 \wedge p > 0 \wedge m < -1$$

$$\text{1: } \int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e == 0 \wedge n > 0 \wedge p > 0 \wedge m < -1 \wedge (p \in \mathbb{Z} \vee d > 0)$$

Derivation: Inverted integration by parts

Rule: If $c^2 d+e == 0 \wedge n > 0 \wedge p > 0 \wedge m < -1 \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{(f x)^{m+1} (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n}{f (m+1)} - \frac{2 e p}{f^2 (m+1)} \int (f x)^{m+2} (d+e x^2)^{p-1} (a+b \operatorname{ArcSin}[c x])^n dx - \frac{b c n d^p}{f (m+1)} \int (f x)^{m+1} (1-c^2 x^2)^{p-\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n-1} dx$$

Program code:

```
(* Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_.*(a_+b_.*ArcSin[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcSin[c*x])^n/(f*(m+1)) -
  2*e*p/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^(p-1)*(a+b*ArcSin[c*x])^n,x] -
  b*c*n*d^p/(f*(m+1))*Int[(f*x)^(m+1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0] && LtQ[m,-1] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
(* Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_.*(a_+b_.*ArcCos[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcCos[c*x])^n/(f*(m+1)) -
  2*e*p/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^(p-1)*(a+b*ArcCos[c*x])^n,x] +
  b*c*n*d^p/(f*(m+1))*Int[(f*x)^(m+1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0] && LtQ[m,-1] && (IntegerQ[p] || GtQ[d,0]) *)
```

$$2. \int (f(x))^m (d+e x^2)^p (a+b \arcsin(cx))^n dx \text{ when } c^2 d + e = 0 \wedge n > 0 \wedge p > 0 \wedge m < -1$$

$$1: \int (f(x))^m \sqrt{d+e x^2} (a+b \arcsin(cx))^n dx \text{ when } c^2 d + e = 0 \wedge n > 0 \wedge m < -1$$

Derivation: Inverted integration by parts

Note: The piecewise constant factor in the second integral reduces the degree of d in the resulting antiderivative.

Rule: If $c^2 d + e = 0 \wedge n > 0 \wedge m < -1$, then

$$\int (f(x))^m \sqrt{d+e x^2} (a+b \arcsin(cx))^n dx \rightarrow \frac{(f(x))^{m+1} \sqrt{d+e x^2} (a+b \arcsin(cx))^n}{f(m+1)} - \frac{b c n \sqrt{d+e x^2}}{f(m+1) \sqrt{1-c^2 x^2}} \int (f(x))^{m+1} (a+b \arcsin(cx))^{n-1} dx + \frac{c^2 \sqrt{d+e x^2}}{f^2(m+1) \sqrt{1-c^2 x^2}} \int \frac{(f(x))^{m+2} (a+b \arcsin(cx))^n}{\sqrt{1-c^2 x^2}} dx$$

Program code:

```
Int[(f_.*x_)^m_*Sqrt[d+e_.*x_^2]*(a_.+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*Sqrt[d+e*x^2]*(a+b*ArcSin[c*x])^n/(f*(m+1)) -
  b*c*n*Sqrt[d+e*x^2]/(f*(m+1)*Sqrt[1-c^2*x^2])*Int[(f*x)^(m+1)*(a+b*ArcSin[c*x])^(n-1),x] +
  c^2*Sqrt[d+e*x^2]/(f^2*(m+1)*Sqrt[1-c^2*x^2])*Int[(f*x)^(m+2)*(a+b*ArcSin[c*x])^n/Sqrt[1-c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[m,-1]
```

```
Int[(f_.*x_)^m_*Sqrt[d+e_.*x_^2]*(a_.+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*Sqrt[d+e*x^2]*(a+b*ArcCos[c*x])^n/(f*(m+1)) +
  b*c*n*Sqrt[d+e*x^2]/(f*(m+1)*Sqrt[1-c^2*x^2])*Int[(f*x)^(m+1)*(a+b*ArcCos[c*x])^(n-1),x] +
  c^2*Sqrt[d+e*x^2]/(f^2*(m+1)*Sqrt[1-c^2*x^2])*Int[(f*x)^(m+2)*(a+b*ArcCos[c*x])^n/Sqrt[1-c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[m,-1]
```

2: $\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx$ when $c^2 d+e == 0 \wedge n > 0 \wedge p > 0 \wedge m < -1$

Derivation: Inverted integration by parts

Rule: If $c^2 d+e == 0 \wedge n > 0 \wedge p > 0 \wedge m < -1$, then

$$\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \rightarrow$$

$$\frac{(f x)^{m+1} (d+e x^2)^p (a+b \arcsin(c x))^n}{f (m+1)} - \frac{2 e p}{f^2 (m+1)} \int (f x)^{m+2} (d+e x^2)^{p-1} (a+b \arcsin(c x))^n dx -$$

$$\frac{b c n d^{\text{IntPart}[p]} (d+e x^2)^{\text{FracPart}[p]}}{f (m+1) (1-c^2 x^2)^{\text{FracPart}[p]}} \int (f x)^{m+1} (1-c^2 x^2)^{p-\frac{1}{2}} (a+b \arcsin(c x))^{n-1} dx$$

Program code:

```
Int[(f_.x_)^m_*(d_+e_.x_^2)^p_*(a_+b_.*ArcSin[c_.x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcSin[c*x])^n/(f*(m+1)) -
  2*e*p/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^(p-1)*(a+b*ArcSin[c*x])^n,x] -
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(f*(m+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m+1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0] && LtQ[m,-1]
```

```
Int[(f_.x_)^m_*(d_+e_.x_^2)^p_*(a_+b_.*ArcCos[c_.x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcCos[c*x])^n/(f*(m+1)) -
  2*e*p/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^(p-1)*(a+b*ArcCos[c*x])^n,x] +
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(f*(m+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m+1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0] && LtQ[m,-1]
```

$$2. \int (f(x))^m (d+e x^2)^p (a+b \arcsin(cx))^n dx \text{ when } c^2 d+e = 0 \wedge n > 0 \wedge p > 0 \wedge m \neq -1$$

$$1: \int (f(x))^m (d+e x^2)^p (a+b \arcsin(cx))^n dx \text{ when } c^2 d+e = 0 \wedge n > 0 \wedge p > 0 \wedge m \neq -1 \wedge (p \in \mathbb{Z} \vee d > 0)$$

Derivation: Inverted integration by parts

Rule: If $c^2 d+e = 0 \wedge n > 0 \wedge p > 0 \wedge m \neq -1 \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f(x))^m (d+e x^2)^p (a+b \arcsin(cx))^n dx \rightarrow \frac{(f(x))^{m+1} (d+e x^2)^p (a+b \arcsin(cx))^n}{f'(m+2p+1)} + \frac{2dp}{m+2p+1} \int (f(x))^m (d+e x^2)^{p-1} (a+b \arcsin(cx))^n dx - \frac{bcnd^p}{f'(m+2p+1)} \int (f(x))^{m+1} (1-c^2 x^2)^{p-\frac{1}{2}} (a+b \arcsin(cx))^{n-1} dx$$

Program code:

```
(* Int[(f_.**x_)^m_*(d+_e_.**x_^2)^p_.*(a+_b_.**ArcSin[c_.**x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcSin[c*x])^n/(f*(m+2*p+1)) +
  2*d*p/(m+2*p+1)*Int[(f*x)^m*(d+e*x^2)^(p-1)*(a+b*ArcSin[c*x])^n,x] -
  b*c*n*d^p/(f*(m+2*p+1))*Int[(f*x)^(m+1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0] && Not[LtQ[m,-1]] &&
(IntegerQ[p] || GtQ[d,0]) && (RationalQ[m] || EqQ[n,1]) *)
```

```
(* Int[(f_.**x_)^m_*(d+_e_.**x_^2)^p_.*(a+_b_.**ArcCos[c_.**x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcCos[c*x])^n/(f*(m+2*p+1)) +
  2*d*p/(m+2*p+1)*Int[(f*x)^m*(d+e*x^2)^(p-1)*(a+b*ArcCos[c*x])^n,x] +
  b*c*n*d^p/(f*(m+2*p+1))*Int[(f*x)^(m+1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0] && Not[LtQ[m,-1]] &&
(IntegerQ[p] || GtQ[d,0]) && (RationalQ[m] || EqQ[n,1]) *)
```

$$2. \int (f x)^m (d+e x^2)^p (a+b \arcsin[c x])^n dx \text{ when } c^2 d+e == 0 \wedge n > 0 \wedge p > 0 \wedge m \neq -1$$

$$1: \int (f x)^m \sqrt{d+e x^2} (a+b \arcsin[c x])^n dx \text{ when } c^2 d+e == 0 \wedge n > 0 \wedge m \neq -1$$

Derivation: Inverted integration by parts

Note: The piecewise constant factor in the second integral reduces the degree of d in the resulting antiderivative.

Rule: If $c^2 d+e == 0 \wedge n > 0 \wedge m \neq -1$, then

$$\int (f x)^m \sqrt{d+e x^2} (a+b \arcsin[c x])^n dx \rightarrow \frac{(f x)^{m+1} \sqrt{d+e x^2} (a+b \arcsin[c x])^n}{f (m+2)} - \frac{b c n \sqrt{d+e x^2}}{f (m+2) \sqrt{1-c^2 x^2}} \int (f x)^{m+1} (a+b \arcsin[c x])^{n-1} dx + \frac{\sqrt{d+e x^2}}{(m+2) \sqrt{1-c^2 x^2}} \int \frac{(f x)^m (a+b \arcsin[c x])^n}{\sqrt{1-c^2 x^2}} dx$$

Program code:

```
Int[(f_.*x_)^m_*Sqrt[d+e_.*x_^2]*(a_.+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*Sqrt[d+e*x^2]*(a+b*ArcSin[c*x])^n/(f*(m+2)) -
  b*c*n*Sqrt[d+e*x^2]/(f*(m+2)*Sqrt[1-c^2*x^2])*Int[(f*x)^(m+1)*(a+b*ArcSin[c*x])^(n-1),x] +
  Sqrt[d+e*x^2]/((m+2)*Sqrt[1-c^2*x^2])*Int[(f*x)^m*(a+b*ArcSin[c*x])^n/Sqrt[1-c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && Not[LtQ[m,-1]] && (RationalQ[m] || EqQ[n,1])
```

```
Int[(f_.*x_)^m_*Sqrt[d+e_.*x_^2]*(a_.+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*Sqrt[d+e*x^2]*(a+b*ArcCos[c*x])^n/(f*(m+2)) +
  b*c*n*Sqrt[d+e*x^2]/(f*(m+2)*Sqrt[1-c^2*x^2])*Int[(f*x)^(m+1)*(a+b*ArcCos[c*x])^(n-1),x] +
  Sqrt[d+e*x^2]/((m+2)*Sqrt[1-c^2*x^2])*Int[(f*x)^m*(a+b*ArcCos[c*x])^n/Sqrt[1-c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && Not[LtQ[m,-1]] && (RationalQ[m] || EqQ[n,1])
```

$$2: \int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e == 0 \wedge n > 0 \wedge p > 0 \wedge m \neq -1$$

Derivation: Inverted integration by parts

Rule: If $c^2 d+e == 0 \wedge n > 0 \wedge p > 0 \wedge m \neq -1$, then

$$\begin{aligned} & \int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \\ & \frac{(f x)^{m+1} (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n}{f (m+2p+1)} + \frac{2 d p}{m+2p+1} \int (f x)^m (d+e x^2)^{p-1} (a+b \operatorname{ArcSin}[c x])^n dx - \\ & \frac{b c n d^{\operatorname{IntPart}[p]} (d+e x^2)^{\operatorname{FracPart}[p]}}{f (m+2p+1) (1-c^2 x^2)^{\operatorname{FracPart}[p]}} \int (f x)^{m+1} (1-c^2 x^2)^{p-\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n-1} dx \end{aligned}$$

Program code:

```
Int[(f_.x_)^m_*(d_+e_.x_^2)^p_*(a_+b_.*ArcSin[c_.x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcSin[c*x])^n/(f*(m+2*p+1)) +
  2*d*p/(m+2*p+1)*Int[(f*x)^m*(d+e*x^2)^(p-1)*(a+b*ArcSin[c*x])^n,x] -
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(f*(m+2*p+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m+1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0] && Not[LtQ[m,-1]] && (RationalQ[m] || EqQ[n,1])
```

```
Int[(f_.x_)^m_*(d_+e_.x_^2)^p_*(a_+b_.*ArcCos[c_.x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcCos[c*x])^n/(f*(m+2*p+1)) +
  2*d*p/(m+2*p+1)*Int[(f*x)^m*(d+e*x^2)^(p-1)*(a+b*ArcCos[c*x])^n,x] +
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(f*(m+2*p+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m+1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0] && Not[LtQ[m,-1]] && (RationalQ[m] || EqQ[n,1])
```


6. $\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx$ when $c^2 d+e = 0 \wedge n > 0 \wedge m < -1 \wedge m \in \mathbb{Z}$

1: $\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx$ when $c^2 d+e = 0 \wedge n > 0 \wedge m < -1 \wedge m \in \mathbb{Z} \wedge (p \in \mathbb{Z} \vee d > 0)$

Rule: If $c^2 d+e = 0 \wedge n > 0 \wedge m < -1 \wedge m \in \mathbb{Z} \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{(f x)^{m+1} (d+e x^2)^{p+1} (a+b \operatorname{ArcSin}[c x])^n}{d f (m+1)} + \frac{c^2 (m+2p+3)}{f^2 (m+1)} \int (f x)^{m+2} (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx - \frac{b c n d^p}{f (m+1)} \int (f x)^{m+1} (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n-1} dx$$

Programcode:

```
(* Int[(f_.**x_)^m_*(d_+e_.**x_^2)^p_*(a_.+b_.**ArcSin[c_.**x_])^n_,x_Symbol] :=
  (f**x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n/(d*f*(m+1)) +
  c^2*(m+2*p+3)/(f^2*(m+1))*Int[(f**x)^(m+2)*(d+e*x^2)^p*(a+b*ArcSin[c*x])^n,x] -
  b*c*n*d^p/(f*(m+1))*Int[(f**x)^(m+1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[m,-1] && IntegerQ[m] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
(* Int[(f_.**x_)^m_*(d_+e_.**x_^2)^p_*(a_.+b_.**ArcCos[c_.**x_])^n_,x_Symbol] :=
  (f**x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n/(d*f*(m+1)) +
  c^2*(m+2*p+3)/(f^2*(m+1))*Int[(f**x)^(m+2)*(d+e*x^2)^p*(a+b*ArcCos[c*x])^n,x] +
  b*c*n*d^p/(f*(m+1))*Int[(f**x)^(m+1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[m,-1] && IntegerQ[m] && (IntegerQ[p] || GtQ[d,0]) *)
```

2: $\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx$ when $c^2 d+e = 0 \wedge n > 0 \wedge m < -1 \wedge m \in \mathbb{Z}$

Rule: If $c^2 d+e = 0 \wedge n > 0 \wedge m < -1 \wedge m \in \mathbb{Z}$, then

$$\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow$$

$$\frac{(f x)^{m+1} (d+e x^2)^{p+1} (a+b \arcsin(c x))^n}{d f (m+1)} + \frac{c^2 (m+2 p+3)}{f^2 (m+1)} \int (f x)^{m+2} (d+e x^2)^p (a+b \arcsin(c x))^n dx -$$

$$\frac{b c n d^{\text{IntPart}[p]} (d+e x^2)^{\text{FracPart}[p]}}{f (m+1) (1-c^2 x^2)^{\text{FracPart}[p]}} \int (f x)^{m+1} (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \arcsin(c x))^{n-1} dx$$

Programcode:

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n/(d*f*(m+1)) +
  c^2*(m+2*p+3)/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^p*(a+b*ArcSin[c*x])^n,x] -
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(f*(m+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m+1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[m,-1] && IntegerQ[m]
```

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n/(d*f*(m+1)) +
  c^2*(m+2*p+3)/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^p*(a+b*ArcCos[c*x])^n,x] +
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(f*(m+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m+1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[m,-1] && IntegerQ[m]
```

7. $\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx$ when $c^2 d+e=0 \wedge n>0 \wedge p<-1$

1. $\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx$ when $c^2 d+e=0 \wedge n>0 \wedge p<-1 \wedge m>1$

1: $\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx$ when $c^2 d+e=0 \wedge n>0 \wedge p<-1 \wedge m>1 \wedge (p \in \mathbb{Z} \vee d>0)$

Derivation: Integration by parts

Basis: $x (d+e x^2)^p = \partial_x \frac{(d+e x^2)^{p+1}}{2 e (p+1)}$

Rule: If $c^2 d+e=0 \wedge n>0 \wedge p<-1 \wedge m>1 \wedge (p \in \mathbb{Z} \vee d>0)$, then

$$\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \rightarrow \frac{f (f x)^{m-1} (d+e x^2)^{p+1} (a+b \arcsin(c x))^n}{2 e (p+1)} - \frac{f^2 (m-1)}{2 e (p+1)} \int (f x)^{m-2} (d+e x^2)^{p+1} (a+b \arcsin(c x))^n dx + \frac{b f n d^p}{2 c (p+1)} \int (f x)^{m-1} (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \arcsin(c x))^{n-1} dx$$

Program code:

```
(* Int[(f_.**x_)^m_*(d_+e_.**x_^2)^p_*(a_+b_.**ArcSin[c_.**x_])^n_,x_Symbol] :=
  f*(f**x)^(m-1)*(d+e**x^2)^(p+1)*(a+b**ArcSin[c**x])^n/(2*e*(p+1)) -
  f^2*(m-1)/(2*e*(p+1))*Int[(f**x)^(m-2)*(d+e**x^2)^(p+1)*(a+b**ArcSin[c**x])^n,x] +
  b*f*n*d^p/(2*c*(p+1))*Int[(f**x)^(m-1)*(1-c^2*x^2)^(p+1/2)*(a+b**ArcSin[c**x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && GtQ[m,1] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
(* Int[(f_.**x_)^m_*(d_+e_.**x_^2)^p_*(a_+b_.**ArcCos[c_.**x_])^n_,x_Symbol] :=
  f*(f**x)^(m-1)*(d+e**x^2)^(p+1)*(a+b**ArcCos[c**x])^n/(2*e*(p+1)) -
  f^2*(m-1)/(2*e*(p+1))*Int[(f**x)^(m-2)*(d+e**x^2)^(p+1)*(a+b**ArcCos[c**x])^n,x] -
  b*f*n*d^p/(2*c*(p+1))*Int[(f**x)^(m-1)*(1-c^2*x^2)^(p+1/2)*(a+b**ArcCos[c**x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && GtQ[m,1] && (IntegerQ[p] || GtQ[d,0]) *)
```

2: $\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx$ when $c^2 d+e == 0 \wedge n > 0 \wedge p < -1 \wedge m > 1$

Derivation: Integration by parts

Basis: $x (d+e x^2)^p == \partial_x \frac{(d+e x^2)^{p+1}}{2 e (p+1)}$

Rule: If $c^2 d+e == 0 \wedge n > 0 \wedge p < -1 \wedge m > 1$, then

$$\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \rightarrow \frac{f (f x)^{m-1} (d+e x^2)^{p+1} (a+b \arcsin(c x))^n}{2 e (p+1)} - \frac{f^2 (m-1)}{2 e (p+1)} \int (f x)^{m-2} (d+e x^2)^{p+1} (a+b \arcsin(c x))^n dx +$$

$$\frac{b f n d^{\text{IntPart}[p]} (d+e x^2)^{\text{FracPart}[p]}}{2 c (p+1) (1-c^2 x^2)^{\text{FracPart}[p]}} \int (f x)^{m-1} (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \text{ArcSin}[c x])^{n-1} dx$$

Program code:

```
Int[(f_.**x_)^m_*(d_+e_.**x_^2)^p_*(a_+b_.**ArcSin[c_.**x_])^n_,x_Symbol] :=
  f*(f*x)^(m-1)*(d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n/(2*e*(p+1)) -
  f^2*(m-1)/(2*e*(p+1))*Int[(f*x)^(m-2)*(d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n,x] +
  b*f*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(2*c*(p+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m-1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && GtQ[m,1]
```

```
Int[(f_.**x_)^m_*(d_+e_.**x_^2)^p_*(a_+b_.**ArcCos[c_.**x_])^n_,x_Symbol] :=
  f*(f*x)^(m-1)*(d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n/(2*e*(p+1)) -
  f^2*(m-1)/(2*e*(p+1))*Int[(f*x)^(m-2)*(d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n,x] -
  b*f*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(2*c*(p+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m-1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && GtQ[m,1]
```

$$2. \int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e=0 \wedge n>0 \wedge p<-1 \wedge m \neq 1$$

$$1: \int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e=0 \wedge n>0 \wedge p<-1 \wedge m \neq 1 \wedge (p \in \mathbb{Z} \vee d>0)$$

Rule: If $c^2 d+e=0 \wedge n>0 \wedge p<-1 \wedge m \neq 1 \wedge (p \in \mathbb{Z} \vee d>0)$, then

$$\begin{aligned} & \int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \\ & - \frac{(f x)^{m+1} (d+e x^2)^{p+1} (a+b \operatorname{ArcSin}[c x])^n}{2 d f (p+1)} + \\ & \frac{m+2 p+3}{2 d (p+1)} \int (f x)^m (d+e x^2)^{p+1} (a+b \operatorname{ArcSin}[c x])^n dx + \frac{b c n d^p}{2 f (p+1)} \int (f x)^{m+1} (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n-1} dx \end{aligned}$$

Program code:

```
(* Int[(f_.**x_)^m_*(d_+e_.**x_^2)^p_*(a_.+b_.**ArcSin[c_.**x_])^n_,x_Symbol] :=
- (f**x)^(m+1)*(d+e**x^2)^(p+1)*(a+b**ArcSin[c**x])^n/(2*d*f*(p+1)) +
(m+2*p+3)/(2*d*(p+1))*Int[(f**x)^m*(d+e**x^2)^(p+1)*(a+b**ArcSin[c**x])^n,x] +
b*c*n*d^p/(2*f*(p+1))*Int[(f**x)^(m+1)*(1-c^2*x^2)^(p+1/2)*(a+b**ArcSin[c**x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && Not[GtQ[m,1]] && (IntegerQ[p] || GtQ[d,0]) &&
(IntegerQ[m] || IntegerQ[p] || EqQ[n,1]) *)
```

```
(* Int[(f_.**x_)^m_*(d_+e_.**x_^2)^p_*(a_.+b_.**ArcCos[c_.**x_])^n_,x_Symbol] :=
- (f**x)^(m+1)*(d+e**x^2)^(p+1)*(a+b**ArcCos[c**x])^n/(2*d*f*(p+1)) +
(m+2*p+3)/(2*d*(p+1))*Int[(f**x)^m*(d+e**x^2)^(p+1)*(a+b**ArcCos[c**x])^n,x] -
b*c*n*d^p/(2*f*(p+1))*Int[(f**x)^(m+1)*(1-c^2*x^2)^(p+1/2)*(a+b**ArcCos[c**x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && Not[GtQ[m,1]] && (IntegerQ[p] || GtQ[d,0]) &&
(IntegerQ[m] || IntegerQ[p] || EqQ[n,1]) *)
```

2: $\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx$ when $c^2 d+e == 0 \wedge n > 0 \wedge p < -1 \wedge m \neq 1$

Rule: If $c^2 d+e == 0 \wedge n > 0 \wedge p < -1 \wedge m \neq 1$, then

$$\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \rightarrow$$

$$-\frac{(f x)^{m+1} (d+e x^2)^{p+1} (a+b \arcsin(c x))^n}{2 d f (p+1)} + \frac{m+2 p+3}{2 d (p+1)} \int (f x)^m (d+e x^2)^{p+1} (a+b \arcsin(c x))^n dx +$$

$$\frac{b c n d^{\text{IntPart}[p]} (d+e x^2)^{\text{FracPart}[p]}}{2 f (p+1) (1-c^2 x^2)^{\text{FracPart}[p]}} \int (f x)^{m+1} (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \arcsin(c x))^{n-1} dx$$

Program code:

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
- (f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n/(2*d*f*(p+1)) +
(m+2*p+3)/(2*d*(p+1))*Int[(f*x)^m*(d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n,x] +
b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(2*f*(p+1)*(1-c^2*x^2)^FracPart[p])*
Int[(f*x)^(m+1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && Not[GtQ[m,1]] && (IntegerQ[m] || IntegerQ[p] || EqQ[n,1])
```

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
- (f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n/(2*d*f*(p+1)) +
(m+2*p+3)/(2*d*(p+1))*Int[(f*x)^m*(d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n,x] -
b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(2*f*(p+1)*(1-c^2*x^2)^FracPart[p])*
Int[(f*x)^(m+1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && Not[GtQ[m,1]] && (IntegerQ[m] || IntegerQ[p] || EqQ[n,1])
```

8. $\int \frac{(f x)^m (a+b \arcsin(c x))^n}{\sqrt{d+e x^2}} dx$ when $c^2 d+e == 0 \wedge n > 0$

1. $\int \frac{(f x)^m (a+b \arcsin(c x))^n}{\sqrt{d+e x^2}} dx$ when $c^2 d+e == 0 \wedge n > 0 \wedge m > 1$

$$1: \int \frac{(f x)^m (a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \text{ when } c^2 d + e = 0 \wedge n > 0 \wedge m > 1 \wedge d > 0$$

Rule: If $c^2 d + e = 0 \wedge n > 0 \wedge m > 1 \wedge d > 0$, then

$$\int \frac{(f x)^m (a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \rightarrow \frac{f (f x)^{m-1} \sqrt{d + e x^2} (a + b \operatorname{ArcSin}[c x])^n}{e m} + \frac{b f n}{c m \sqrt{d}} \int (f x)^{m-1} (a + b \operatorname{ArcSin}[c x])^{n-1} dx + \frac{f^2 (m-1)}{c^2 m} \int \frac{(f x)^{m-2} (a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx$$

Program code:

```
(* Int[(f_.**x_)^m_*(a_.+b_.**ArcSin[c_.**x_])^n_/Sqrt[d_+e_.**x_^2],x_Symbol] :=
  f*(f*x)^(m-1)*Sqrt[d+e*x^2]*(a+b*ArcSin[c*x])^n/(e*m) +
  b*f*n/(c*m*Sqrt[d])*Int[(f*x)^(m-1)*(a+b*ArcSin[c*x])^(n-1),x] +
  f^2*(m-1)/(c^2*m)*Int[(f*x)^(m-2)*(a+b*ArcSin[c*x])^n/Sqrt[d+e*x^2],x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[m,1] && GtQ[d,0] && IntegerQ[m] *)
```

```
(* Int[(f_.**x_)^m_*(a_.+b_.**ArcCos[c_.**x_])^n_/Sqrt[d_+e_.**x_^2],x_Symbol] :=
  f*(f*x)^(m-1)*Sqrt[d+e*x^2]*(a+b*ArcCos[c*x])^n/(e*m) -
  b*f*n*Sqrt[1-c^2*x^2]/(c*m*Sqrt[d+e*x^2])*Int[(f*x)^(m-1)*(a+b*ArcCos[c*x])^(n-1),x] +
  f^2*(m-1)/(c^2*m)*Int[(f*x)^(m-2)*(a+b*ArcCos[c*x])^n/Sqrt[d+e*x^2],x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[m,1] && GtQ[d,0] && IntegerQ[m] *)
```

$$2: \int \frac{(f x)^m (a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \text{ when } c^2 d + e = 0 \wedge n > 0 \wedge m > 1$$

Rule: If $c^2 d + e = 0 \wedge n > 0 \wedge m > 1$, then

$$\int \frac{(f x)^m (a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \rightarrow$$

$$\frac{f (f x)^{m-1} \sqrt{d + e x^2} (a + b \operatorname{ArcSin}[c x])^n}{e m} + \frac{b f n \sqrt{1 - c^2 x^2}}{c m \sqrt{d + e x^2}} \int (f x)^{m-1} (a + b \operatorname{ArcSin}[c x])^{n-1} dx + \frac{f^2 (m-1)}{c^2 m} \int \frac{(f x)^{m-2} (a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx$$

Program code:

```
Int[(f_.*x_)^m_*(a_.+b_.*ArcSin[c_.*x_]^n_./Sqrt[d_+e_.*x_^2],x_Symbol] :=
  f*(f*x)^(m-1)*Sqrt[d+e*x^2]*(a+b*ArcSin[c*x])^n/(e*m) +
  b*f*n*Sqrt[1-c^2*x^2]/(c*m*Sqrt[d+e*x^2])*Int[(f*x)^(m-1)*(a+b*ArcSin[c*x])^(n-1),x] +
  f^2*(m-1)/(c^2*m)*Int[((f*x)^(m-2)*(a+b*ArcSin[c*x])^n)/Sqrt[d+e*x^2],x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[m,1] && IntegerQ[m]
```

```
Int[(f_.*x_)^m_*(a_.+b_.*ArcCos[c_.*x_]^n_./Sqrt[d_+e_.*x_^2],x_Symbol] :=
  f*(f*x)^(m-1)*Sqrt[d+e*x^2]*(a+b*ArcCos[c*x])^n/(e*m) -
  b*f*n*Sqrt[1-c^2*x^2]/(c*m*Sqrt[d+e*x^2])*Int[(f*x)^(m-1)*(a+b*ArcCos[c*x])^(n-1),x] +
  f^2*(m-1)/(c^2*m)*Int[((f*x)^(m-2)*(a+b*ArcCos[c*x])^n)/Sqrt[d+e*x^2],x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[m,1] && IntegerQ[m]
```


$$2: \int \frac{x^m (a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \text{ when } c^2 d + e == 0 \wedge d > 0 \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{Z}$$

Derivation: Integration by substitution

Basis: If $c^2 d + e == 0 \wedge d > 0 \wedge m \in \mathbb{Z}$, then $\frac{x^m}{\sqrt{d+e x^2}} = \frac{1}{c^{m+1} \sqrt{d}} \operatorname{Subst}[\operatorname{Sin}[x]^m, x, \operatorname{ArcSin}[c x]] \partial_x \operatorname{ArcSin}[c x]$

Rule: If $c^2 d + e == 0 \wedge d > 0 \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{Z}$, then

$$\int \frac{x^m (a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \rightarrow \frac{1}{c^{m+1} \sqrt{d}} \operatorname{Subst}\left[\int (a + b x)^n \operatorname{Sin}[x]^m dx, x, \operatorname{ArcSin}[c x]\right]$$

Program code:

```
Int[x_^m_*(a_.+b_.*ArcSin[c_.*x_])^n_/Sqrt[d+_e_.*x_^2],x_Symbol] :=
  1/(c^(m+1)*Sqrt[d])*Subst[Int[(a+b*x)^n*Sin[x]^m,x],x,ArcSin[c*x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[d,0] && IGtQ[n,0] && IntegerQ[m]
```

```
Int[x_^m_*(a_.+b_.*ArcCos[c_.*x_])^n_/Sqrt[d+_e_.*x_^2],x_Symbol] :=
  -1/(c^(m+1)*Sqrt[d])*Subst[Int[(a+b*x)^n*Cos[x]^m,x],x,ArcCos[c*x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[d,0] && IGtQ[n,0] && IntegerQ[m]
```

$$3: \int \frac{(f x)^m (a + b \operatorname{ArcSin}[c x])}{\sqrt{d + e x^2}} dx \text{ when } c^2 d + e == 0 \wedge d > 0 \wedge m \notin \mathbb{Z}$$

Rule: If $c^2 d + e == 0 \wedge d > 0 \wedge m \notin \mathbb{Z}$, then

$$\int \frac{(f x)^m (a + b \operatorname{ArcSin}[c x])}{\sqrt{d + e x^2}} dx \rightarrow \frac{(f x)^{m+1} (a + b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{\sqrt{d} f (m+1)}$$

$$\left(b c (f x)^{m+2} \text{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right] \right) / \left(\sqrt{d} f^2 (m+1) (m+2) \right)$$

Program code:

```
Int[(f_.x_)^m_*(a_.+b_.*ArcSin[c_.x_])/Sqrt[d_+e_.x_^2],x_Symbol] :=
  (f*x)^(m+1)*(a+b*ArcSin[c*x])*Hypergeometric2F1[1/2,(1+m)/2,(3+m)/2,c^2*x^2]/(Sqrt[d]*f*(m+1)) -
  b*c*(f*x)^(m+2)*HypergeometricPFQ[{1,1+m/2,1+m/2},{3/2+m/2,2+m/2},c^2*x^2]/(Sqrt[d]*f^2*(m+1)*(m+2)) /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[d,0] && Not[IntegerQ[m]]
```

```
Int[(f_.x_)^m_*(a_.+b_.*ArcCos[c_.x_])/Sqrt[d_+e_.x_^2],x_Symbol] :=
  (f*x)^(m+1)*(a+b*ArcCos[c*x])*Hypergeometric2F1[1/2,(1+m)/2,(3+m)/2,c^2*x^2]/(Sqrt[d]*f*(m+1)) +
  b*c*(f*x)^(m+2)*HypergeometricPFQ[{1,1+m/2,1+m/2},{3/2+m/2,2+m/2},c^2*x^2]/(Sqrt[d]*f^2*(m+1)*(m+2)) /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[d,0] && Not[IntegerQ[m]]
```

4:
$$\int \frac{(f x)^m (a + b \arcsin[c x])^n}{\sqrt{d + e x^2}} dx \text{ when } c^2 d + e == 0 \wedge n > 0 \wedge d \neq 0$$

Derivation: Piecewise constant extraction

Basis: If $c^2 d + e == 0$, then $a_x \frac{\sqrt{1-c^2 x^2}}{\sqrt{d+e x^2}} == 0$

Rule: If $c^2 d + e == 0 \wedge n > 0 \wedge d \neq 0$, then

$$\int \frac{(f x)^m (a + b \arcsin[c x])^n}{\sqrt{d + e x^2}} dx \rightarrow \frac{\sqrt{1 - c^2 x^2}}{\sqrt{d + e x^2}} \int \frac{(f x)^m (a + b \arcsin[c x])^n}{\sqrt{1 - c^2 x^2}} dx$$

Program code:

```
Int[(f_.x_)^m_*(a_.+b_.*ArcSin[c_.x_])^n_/Sqrt[d_+e_.x_^2],x_Symbol] :=
  Sqrt[1-c^2*x^2]/Sqrt[d+e*x^2]*Int[(f*x)^m*(a+b*ArcSin[c*x])^n/Sqrt[1-c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && Not[GtQ[d,0]] && (IntegerQ[m] || EqQ[n,1])
```

```
Int[(f_.*x_)^m_*(a_.+b_.*ArcCos[c_.*x_])^n_./Sqrt[d_+e_.*x_^2],x_Symbol] :=
  Sqrt[1-c^2*x^2]/Sqrt[d+e*x^2]*Int[(f*x)^m*(a+b*ArcCos[c*x])^n/Sqrt[1-c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && Not[GtQ[d,0]] && (IntegerQ[m] || EqQ[n,1])
```

9. $\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx$ when $c^2 d+e = 0 \wedge n > 0 \wedge m > 1 \wedge m+2p+1 \neq 0$

1: $\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx$ when $c^2 d+e = 0 \wedge n > 0 \wedge m > 1 \wedge m+2p+1 \neq 0 \wedge (p \in \mathbb{Z} \vee d > 0)$

Rule: If $c^2 d+e = 0 \wedge n > 0 \wedge m > 1 \wedge m+2p+1 \neq 0 \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{f (f x)^{m-1} (d+e x^2)^{p+1} (a+b \operatorname{ArcSin}[c x])^n}{e (m+2p+1)} + \frac{f^2 (m-1)}{c^2 (m+2p+1)} \int (f x)^{m-2} (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx + \frac{b f n d^p}{c (m+2p+1)} \int (f x)^{m-1} (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n-1} dx$$

Program code:

```
(* Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  f*(f*x)^(m-1)*(d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n/(e*(m+2*p+1)) +
  f^2*(m-1)/(c^2*(m+2*p+1))*Int[(f*x)^(m-2)*(d+e*x^2)^p*(a+b*ArcSin[c*x])^n,x] +
  b*f*n*d^p/(c*(m+2*p+1))*Int[(f*x)^(m-1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[m,1] && NeQ[m+2*p+1,0] && (IntegerQ[p] || GtQ[d,0]) && IntegerQ[m] *)
```

```
(* Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  f*(f*x)^(m-1)*(d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n/(e*(m+2*p+1)) +
  f^2*(m-1)/(c^2*(m+2*p+1))*Int[(f*x)^(m-2)*(d+e*x^2)^p*(a+b*ArcCos[c*x])^n,x] -
  b*f*n*d^p/(c*(m+2*p+1))*Int[(f*x)^(m-1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[m,1] && NeQ[m+2*p+1,0] && (IntegerQ[p] || GtQ[d,0]) && IntegerQ[m] *)
```

$$2: \int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e=0 \wedge n>0 \wedge m>1 \wedge m+2p+1 \neq 0$$

Rule: If $c^2 d+e=0 \wedge n>0 \wedge m>1 \wedge m+2p+1 \neq 0$, then

$$\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow$$

$$\frac{f (f x)^{m-1} (d+e x^2)^{p+1} (a+b \operatorname{ArcSin}[c x])^n}{e (m+2p+1)} + \frac{f^2 (m-1)}{c^2 (m+2p+1)} \int (f x)^{m-2} (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx +$$

$$\frac{b f n d^{\operatorname{IntPart}[p]} (d+e x^2)^{\operatorname{FracPart}[p]}}{c (m+2p+1) (1-c^2 x^2)^{\operatorname{FracPart}[p]}} \int (f x)^{m-1} (1-c^2 x^2)^{p+\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n-1} dx$$

Program code:

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  f*(f*x)^(m-1)*(d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])^n/(e*(m+2*p+1)) +
  f^2*(m-1)/(c^2*(m+2*p+1))*Int[(f*x)^(m-2)*(d+e*x^2)^p*(a+b*ArcSin[c*x])^n,x] +
  b*f*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(c*(m+2*p+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m-1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcSin[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[m,1] && NeQ[m+2*p+1,0] && IntegerQ[m]
```

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  f*(f*x)^(m-1)*(d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])^n/(e*(m+2*p+1)) +
  f^2*(m-1)/(c^2*(m+2*p+1))*Int[(f*x)^(m-2)*(d+e*x^2)^p*(a+b*ArcCos[c*x])^n,x] -
  b*f*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(c*(m+2*p+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m-1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[m,1] && NeQ[m+2*p+1,0] && IntegerQ[m]
```

$$2. \int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e == 0 \wedge n < -1$$

$$1. \int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e == 0 \wedge n < -1 \wedge m+2p+1 == 0$$

$$1: \int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e == 0 \wedge n < -1 \wedge m+2p+1 == 0 \wedge (p \in \mathbb{Z} \vee d > 0)$$

Derivation: Integration by parts

$$\text{Basis: } \frac{(a+b \operatorname{ArcSin}[c x])^n}{\sqrt{1-c^2 x^2}} == \partial_x \frac{(a+b \operatorname{ArcSin}[c x])^{n+1}}{b c (n+1)}$$

Rule: If $c^2 d+e == 0 \wedge n < -1 \wedge m+2p+1 == 0 \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{(f x)^m \sqrt{1-c^2 x^2} (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^{n+1}}{b c (n+1)} - \frac{f m d^p}{b c (n+1)} \int (f x)^{m-1} (1-c^2 x^2)^{p-\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n+1} dx$$

Program code:

```
(* Int[(f_.**x_)^m_.*(d+_e_.**x_^2)^p_.*(a_+_b_.**ArcSin[c_.**x_])^n_,x_Symbol] :=
  (f*x)^m*Sqrt[1-c^2*x^2]*(d+e*x^2)^p*(a+b*ArcSin[c*x])^(n+1)/(b*c*(n+1)) -
  f*m*d^p/(b*c*(n+1))*Int[(f*x)^(m-1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcSin[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f,m,p},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && EqQ[m+2*p+1,0] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
(* Int[(f_.**x_)^m_.*(d+_e_.**x_^2)^p_.*(a_+_b_.**ArcCos[c_.**x_])^n_,x_Symbol] :=
  -(f*x)^m*Sqrt[1-c^2*x^2]*(d+e*x^2)^p*(a+b*ArcCos[c*x])^(n+1)/(b*c*(n+1)) +
  f*m*d^p/(b*c*(n+1))*Int[(f*x)^(m-1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcCos[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f,m,p},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && EqQ[m+2*p+1,0] && (IntegerQ[p] || GtQ[d,0]) *)
```

$$\mathbf{2:} \int (f(x))^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d+e == 0 \wedge n < -1 \wedge m+2p+1 == 0$$

Derivation: Integration by parts

$$\text{Basis: } \frac{(a+b \operatorname{ArcSin}[c x])^n}{\sqrt{1-c^2 x^2}} == \partial_x \frac{(a+b \operatorname{ArcSin}[c x])^{n+1}}{b c (n+1)}$$

Rule: If $c^2 d+e == 0 \wedge n < -1 \wedge m+2p+1 == 0$, then

$$\int (f(x))^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{(f(x))^m \sqrt{1-c^2 x^2} (d+e x^2)^p (a+b \operatorname{ArcSin}[c x])^{n+1}}{b c (n+1)} - \frac{f m d^{\operatorname{IntPart}[p]} (d+e x^2)^{\operatorname{FracPart}[p]}}{b c (n+1) (1-c^2 x^2)^{\operatorname{FracPart}[p]}} \int (f(x))^{m-1} (1-c^2 x^2)^{p-\frac{1}{2}} (a+b \operatorname{ArcSin}[c x])^{n+1} dx$$

Program code:

```
Int[(f_.x_)^m_.*(d+_e_.x_^2)^p_.*(a+_b_.*ArcSin[c_.x_])^n_,x_Symbol] :=
  (f*x)^m*Sqrt[1-c^2*x^2]*(d+e*x^2)^p*(a+b*ArcSin[c*x])^(n+1)/(b*c*(n+1)) -
  f*m*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(b*c*(n+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m-1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcSin[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f,m,p},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && EqQ[m+2*p+1,0]
```

```
Int[(f_.x_)^m_.*(d+_e_.x_^2)^p_.*(a+_b_.*ArcCos[c_.x_])^n_,x_Symbol] :=
  -(f*x)^m*Sqrt[1-c^2*x^2]*(d+e*x^2)^p*(a+b*ArcCos[c*x])^(n+1)/(b*c*(n+1)) +
  f*m*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(b*c*(n+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m-1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcCos[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f,m,p},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && EqQ[m+2*p+1,0]
```

$$2: \int \frac{(f x)^m (a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \text{ when } c^2 d + e == 0 \wedge n < -1 \wedge d > 0$$

Derivation: Integration by parts

$$\text{Basis: If } c^2 d + e == 0 \wedge d > 0, \text{ then } \frac{(a+b \operatorname{ArcSin}[c x])^n}{\sqrt{d+e x^2}} == \partial_x \frac{(a+b \operatorname{ArcSin}[c x])^{n+1}}{b c \sqrt{d} (n+1)}$$

Rule: If $c^2 d + e == 0 \wedge n < -1 \wedge d > 0$, then

$$\int \frac{(f x)^m (a + b \operatorname{ArcSin}[c x])^n}{\sqrt{d + e x^2}} dx \rightarrow \frac{(f x)^m (a + b \operatorname{ArcSin}[c x])^{n+1}}{b c \sqrt{d} (n+1)} - \frac{f m}{b c \sqrt{d} (n+1)} \int (f x)^{m-1} (a + b \operatorname{ArcSin}[c x])^{n+1} dx$$

Program code:

```
Int[(f_.**x_)^m_.*(a_.+b_.*ArcSin[c_.**x_])^n_/Sqrt[d_+e_.**x_^2],x_Symbol] :=
  (f*x)^m*(a+b*ArcSin[c*x])^(n+1)/(b*c*Sqrt[d]*(n+1)) -
  f*m/(b*c*Sqrt[d]*(n+1))*Int[(f*x)^(m-1)*(a+b*ArcSin[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && GtQ[d,0]
```

```
Int[(f_.**x_)^m_.*(a_.+b_.*ArcCos[c_.**x_])^n_/Sqrt[d_+e_.**x_^2],x_Symbol] :=
  -(f*x)^m*(a+b*ArcCos[c*x])^(n+1)/(b*c*Sqrt[d]*(n+1)) +
  f*m/(b*c*Sqrt[d]*(n+1))*Int[(f*x)^(m-1)*(a+b*ArcCos[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && GtQ[d,0]
```

$$3. \int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d + e == 0 \wedge n < -1 \wedge m+3 \in \mathbb{Z}^+ \wedge 2p \in \mathbb{Z}^+$$

$$x: \int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n dx \text{ when } c^2 d + e == 0 \wedge n < -1 \wedge m+3 \in \mathbb{Z}^+ \wedge 2p \in \mathbb{Z}^+ \wedge (p \in \mathbb{Z} \vee d > 0)$$

Derivation: Integration by parts

$$\text{Basis: } \frac{(a+b \operatorname{ArcSin}[c x])^n}{\sqrt{1-c^2 x^2}} == \partial_x \frac{(a+b \operatorname{ArcSin}[c x])^{n+1}}{b c (n+1)}$$

Rule: If $c^2 d + e == 0 \wedge n < -1 \wedge m + 3 \in \mathbb{Z}^+ \wedge 2 p \in \mathbb{Z}^+ \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{(f x)^m \sqrt{1 - c^2 x^2} (d + e x^2)^p (a + b \operatorname{ArcSin}[c x])^{n+1}}{b c (n+1)} - \frac{f m d^p}{b c (n+1)} \int (f x)^{m-1} (1 - c^2 x^2)^{p-\frac{1}{2}} (a + b \operatorname{ArcSin}[c x])^{n+1} dx + \frac{c (m+2 p+1) d^p}{b f (n+1)} \int (f x)^{m+1} (1 - c^2 x^2)^{p-\frac{1}{2}} (a + b \operatorname{ArcSin}[c x])^{n+1} dx$$

Program code:

```
(* Int[(f_.**x_)^m_.*(d_+e_.**x_^2)^p_.*(a_.+b_.**ArcSin[c_.**x_])^n_,x_Symbol] :=
  (f*x)^m*Sqrt[1-c^2*x^2]*(d+e*x^2)^p*(a+b*ArcSin[c*x])^(n+1)/(b*c*(n+1)) -
  f*m*d^p/(b*c*(n+1))*Int[(f*x)^(m-1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcSin[c*x])^(n+1),x] +
  c*(m+2*p+1)*d^p/(b*f*(n+1))*Int[(f*x)^(m+1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcSin[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && IGtQ[m,-3] && IGtQ[2*p,0] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
(* Int[(f_.**x_)^m_.*(d_+e_.**x_^2)^p_.*(a_.+b_.**ArcCos[c_.**x_])^n_,x_Symbol] :=
  -(f*x)^m*Sqrt[1-c^2*x^2]*(d+e*x^2)^p*(a+b*ArcCos[c*x])^(n+1)/(b*c*(n+1)) +
  f*m*d^p/(b*c*(n+1))*Int[(f*x)^(m-1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcCos[c*x])^(n+1),x] -
  c*(m+2*p+1)*d^p/(b*f*(n+1))*Int[(f*x)^(m+1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcCos[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && IGtQ[m,-3] && IGtQ[2*p,0] && (IntegerQ[p] || GtQ[d,0]) *)
```


$$\mathbf{2:} \int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \text{ when } c^2 d+e == 0 \wedge n < -1 \wedge m+3 \in \mathbb{Z}^+ \wedge 2p \in \mathbb{Z}^+$$

Derivation: Integration by parts

$$\text{Basis: } \frac{(a+b \arcsin(c x))^n}{\sqrt{1-c^2 x^2}} == \partial_x \frac{(a+b \arcsin(c x))^{n+1}}{b c (n+1)}$$

Rule: If $c^2 d+e == 0 \wedge n < -1 \wedge m+3 \in \mathbb{Z}^+ \wedge 2p \in \mathbb{Z}^+$, then

$$\begin{aligned} & \int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \rightarrow \\ & \frac{(f x)^m \sqrt{1-c^2 x^2} (d+e x^2)^p (a+b \arcsin(c x))^{n+1}}{b c (n+1)} - \\ & \frac{f m d^{\text{IntPart}[p]} (d+e x^2)^{\text{FracPart}[p]}}{b c (n+1) (1-c^2 x^2)^{\text{FracPart}[p]}} \int (f x)^{m-1} (1-c^2 x^2)^{p-\frac{1}{2}} (a+b \arcsin(c x))^{n+1} dx + \\ & \frac{c (m+2p+1) d^{\text{IntPart}[p]} (d+e x^2)^{\text{FracPart}[p]}}{b f (n+1) (1-c^2 x^2)^{\text{FracPart}[p]}} \int (f x)^{m+1} (1-c^2 x^2)^{p-\frac{1}{2}} (a+b \arcsin(c x))^{n+1} dx \end{aligned}$$

Program code:

```
Int[(f_.x_)^m_.*(d+e_.x_)^p_.*(a_.+b_.*ArcSin[c_.x_])^n_,x_Symbol] :=
  (f*x)^m*Sqrt[1-c^2*x^2]*(d+e*x^2)^p*(a+b*ArcSin[c*x])^(n+1)/(b*c*(n+1)) -
  f*m*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(b*c*(n+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m-1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcSin[c*x])^(n+1),x] +
  c*(m+2*p+1)*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(b*f*(n+1)*(1-c^2*x^2)^FracPart[p])*
  Int[(f*x)^(m+1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcSin[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && IGtQ[m,-3] && IGtQ[2*p,0]
```

```

Int[ (f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  - (f*x)^m*Sqrt[1-c^2*x^2]*(d+e*x^2)^p*(a+b*ArcCos[c*x])^(n+1)/(b*c*(n+1)) +
  f*m*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(b*c*(n+1)*(1-c^2*x^2)^FracPart[p])*
  Int[ (f*x)^(m-1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcCos[c*x])^(n+1),x] -
  c*(m+2*p+1)*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(b*f*(n+1)*(1-c^2*x^2)^FracPart[p])*
  Int[ (f*x)^(m+1)*(1-c^2*x^2)^(p-1/2)*(a+b*ArcCos[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && IGtQ[m,-3] && IGtQ[2*p,0]

```

3. $\int x^m (d+e x^2)^p (a+b \arcsin(cx))^n dx$ when $c^2 d+e = 0 \wedge 2p \in \mathbb{Z} \wedge p > -1 \wedge m \in \mathbb{Z}^+$

1: $\int x^m (d+e x^2)^p (a+b \arcsin(cx))^n dx$ when $c^2 d+e = 0 \wedge 2p \in \mathbb{Z} \wedge p > -1 \wedge m \in \mathbb{Z}^+ \wedge (p \in \mathbb{Z} \vee d > 0)$

Derivation: Integration by substitution

Basis: $F[x] = \frac{1}{c} \text{Subst}[F[\frac{\sin[x]}{c}] \cos[x], x, \arcsin[cx]] \partial_x \arcsin[cx]$

Basis: If $c^2 d+e = 0 \wedge (p \in \mathbb{Z} \vee d > 0) \wedge m \in \mathbb{Z}$, then
 $x^m (d+e x^2)^p = \frac{d^p}{c^{m+1}} \text{Subst}[\sin[x]^m \cos[x]^{2p+1}, x, \arcsin[cx]] \partial_x \arcsin[cx]$

Rule: If $c^2 d+e = 0 \wedge 2p \in \mathbb{Z} \wedge p > -1 \wedge m \in \mathbb{Z}^+ \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int x^m (d+e x^2)^p (a+b \arcsin(cx))^n dx \rightarrow \frac{d^p}{c^{m+1}} \text{Subst}\left[\int (a+bx)^n \sin[x]^m \cos[x]^{2p+1} dx, x, \arcsin[cx]\right]$$

Program code:

```

Int[x_^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  d^p/c^(m+1)*Subst[Int[(a+b*x)^n*Sin[x]^m*Cos[x]^(2*p+1),x],x,ArcSin[c*x]] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && IntegerQ[2*p] && GtQ[p,-1] && IGtQ[m,0] && (IntegerQ[p] || GtQ[d,0])

```

```

Int[x_^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  -d^p/c^(m+1)*Subst[Int[(a+b*x)^n*Cos[x]^m*Sin[x]^(2*p+1),x],x,ArcCos[c*x]] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && IntegerQ[2*p] && GtQ[p,-1] && IGtQ[m,0] && (IntegerQ[p] || GtQ[d,0])

```

$$\mathbf{2:} \int x^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \text{ when } c^2 d+e == 0 \wedge 2p \in \mathbb{Z} \wedge p > -1 \wedge m \in \mathbb{Z}^+ \wedge \neg (p \in \mathbb{Z} \vee d > 0)$$

Derivation: Piecewise constant extraction

Basis: If $c^2 d+e == 0$, then $\partial_x \frac{(d+e x^2)^p}{(1-c^2 x^2)^p} == 0$

$$\text{Basis: } \frac{(d+e x^2)^p}{(1-c^2 x^2)^p} == \frac{d^{\text{IntPart}[p]} (d+e x^2)^{\text{FracPart}[p]}}{(1-c^2 x^2)^{\text{FracPart}[p]}}$$

Rule: If $c^2 d+e == 0 \wedge 2p \in \mathbb{Z} \wedge p > -1 \wedge m \in \mathbb{Z}^+ \wedge \neg (p \in \mathbb{Z} \vee d > 0)$, then

$$\int x^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \rightarrow \frac{d^{\text{IntPart}[p]} (d+e x^2)^{\text{FracPart}[p]}}{(1-c^2 x^2)^{\text{FracPart}[p]}} \int x^m (1-c^2 x^2)^p (a+b \arcsin(c x))^n dx$$

Program code:

```
Int[x_^m_.*(d_+e_.*x_^2)^p_.*(a_+b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  d^IntPart[p]*(d+e*x^2)^FracPart[p]/(1-c^2*x^2)^FracPart[p]*Int[x^m*(1-c^2*x^2)^p*(a+b*ArcSin[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && IntegerQ[2*p] && GtQ[p,-1] && IGtQ[m,0] && Not[(IntegerQ[p] || GtQ[d,0])]
```

```
Int[x_^m_.*(d_+e_.*x_^2)^p_.*(a_+b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  d^IntPart[p]*(d+e*x^2)^FracPart[p]/(1-c^2*x^2)^FracPart[p]*Int[x^m*(1-c^2*x^2)^p*(a+b*ArcCos[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && IntegerQ[2*p] && GtQ[p,-1] && IGtQ[m,0] && Not[(IntegerQ[p] || GtQ[d,0])]
```

$$\mathbf{4:} \int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \text{ when } c^2 d+e == 0 \wedge d > 0 \wedge p + \frac{1}{2} \in \mathbb{Z}^+ \wedge \frac{m+1}{2} \notin \mathbb{Z}^+$$

Derivation: Algebraic expansion

Rule: If $c^2 d+e == 0 \wedge d > 0 \wedge p + \frac{1}{2} \in \mathbb{Z}^+ \wedge \frac{m+1}{2} \notin \mathbb{Z}^+$, then

$$\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \rightarrow \int \frac{(a+b \arcsin(c x))^n}{\sqrt{d+e x^2}} \text{ExpandIntegrand}[(f x)^m (d+e x^2)^{p+\frac{1}{2}}, x] dx$$

Program code:

```
Int[(f_.**x_)^m_*(d_+e_.**x_^2)^p_*(a_.+b_.*ArcSin[c_.**x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*ArcSin[c*x])^n/Sqrt[d+e*x^2], (f*x)^m*(d+e*x^2)^(p+1/2), x], x] /;
  FreeQ[{a,b,c,d,e,f,m,n}, x] && EqQ[c^2*d+e, 0] && GtQ[d, 0] && IGtQ[p+1/2, 0] && Not[IGtQ[(m+1)/2, 0]] && (EqQ[m, -1] || EqQ[m, -2])
```

```
Int[(f_.**x_)^m_*(d_+e_.**x_^2)^p_*(a_.+b_.*ArcCos[c_.**x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*ArcCos[c*x])^n/Sqrt[d+e*x^2], (f*x)^m*(d+e*x^2)^(p+1/2), x], x] /;
  FreeQ[{a,b,c,d,e,f,m,n}, x] && EqQ[c^2*d+e, 0] && GtQ[d, 0] && IGtQ[p+1/2, 0] && Not[IGtQ[(m+1)/2, 0]] && (EqQ[m, -1] || EqQ[m, -2])
```

2. $\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx$ when $c^2 d+e \neq 0$

1: $\int x (d+e x^2)^p (a+b \arcsin(c x)) dx$ when $c^2 d+e \neq 0 \wedge p \neq -1$

Derivation: Integration by parts

Basis:: If $p \neq -1$, then $x (d+e x^2)^p = \partial_x \frac{(d+e x^2)^{p+1}}{2e(p+1)}$

Rule: If $c^2 d+e \neq 0 \wedge p \neq -1$, then

$$\int x (d+e x^2)^p (a+b \arcsin(c x)) dx \rightarrow \frac{(d+e x^2)^{p+1} (a+b \arcsin(c x))}{2e(p+1)} - \frac{bc}{2e(p+1)} \int \frac{(d+e x^2)^{p+1}}{\sqrt{1-c^2 x^2}} dx$$

Program code:

```
Int[x_*(d_+e_.**x_^2)^p_*(a_.+b_.*ArcSin[c_.**x_]),x_Symbol] :=
  (d+e*x^2)^(p+1)*(a+b*ArcSin[c*x])/(2*e*(p+1)) - b*c/(2*e*(p+1))*Int[(d+e*x^2)^(p+1)/Sqrt[1-c^2*x^2], x] /;
  FreeQ[{a,b,c,d,e,p}, x] && NeQ[c^2*d+e, 0] && NeQ[p, -1]
```

```
Int[x_*(d_+e_.*x_^2)^p_.*(a_+b_.*ArcCos[c_.*x_]),x_Symbol] :=
  (d+e*x^2)^(p+1)*(a+b*ArcCos[c*x])/(2*e*(p+1)) + b*c/(2*e*(p+1))*Int[(d+e*x^2)^(p+1)/Sqrt[1-c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,p},x] && NeQ[c^2*d+e,0] && NeQ[p,-1]
```

2: $\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x]) dx$ when $c^2 d+e \neq 0 \wedge p \in \mathbb{Z} \wedge (p > 0 \vee \frac{m-1}{2} \in \mathbb{Z}^+ \wedge m+p \leq 0)$

Derivation: Integration by parts

Note: If $\frac{m-1}{2} \in \mathbb{Z}^+ \wedge p \in \mathbb{Z}^- \wedge m+p \geq 0$, then $\int x^m (d+e x^2)^p$ is a rational function.

Rule: If $c^2 d+e \neq 0 \wedge p \in \mathbb{Z} \wedge (p > 0 \vee \frac{m-1}{2} \in \mathbb{Z}^+ \wedge m+p \leq 0)$, let $u = \int (f x)^m (d+e x^2)^p dx$, then

$$\int (f x)^m (d+e x^2)^p (a+b \operatorname{ArcSin}[c x]) dx \rightarrow u (a+b \operatorname{ArcSin}[c x]) - b c \int \frac{u}{\sqrt{1-c^2 x^2}} dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_+b_.*ArcSin[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(f*x)^m*(d+e*x^2)^p,x]},
    Dist[a+b*ArcSin[c*x],u,x] - b*c*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x] /;
FreeQ[{a,b,c,d,e,f,m},x] && NeQ[c^2*d+e,0] && IntegerQ[p] && (GtQ[p,0] || IGtQ[(m-1)/2,0] && LeQ[m+p,0])
```

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_+b_.*ArcCos[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(f*x)^m*(d+e*x^2)^p,x]},
    Dist[a+b*ArcCos[c*x],u,x] + b*c*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x] /;
FreeQ[{a,b,c,d,e,f,m},x] && NeQ[c^2*d+e,0] && IntegerQ[p] && (GtQ[p,0] || IGtQ[(m-1)/2,0] && LeQ[m+p,0])
```

3: $\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx$ when $c^2 d+e \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p \in \mathbb{Z} \wedge m \in \mathbb{Z}$

Derivation: Algebraic expansion

Rule: If $c^2 d+e \neq 0 \wedge n \in \mathbb{Z}^+ \wedge p \in \mathbb{Z} \wedge m \in \mathbb{Z}$, then

$$\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \rightarrow \int (a+b \arcsin(c x))^n \text{ExpandIntegrand}[(f x)^m (d+e x^2)^p, x] dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d+_e_.*x_^2)^p_.*(a+_b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*ArcSin[c*x])^n,(f*x)^m*(d+e*x^2)^p,x],x] /;
FreeQ[{a,b,c,d,e,f},x] && NeQ[c^2*d+e,0] && IGtQ[n,0] && IntegerQ[p] && IntegerQ[m]
```

```
Int[(f_.*x_)^m_.*(d+_e_.*x_^2)^p_.*(a+_b_.*ArcCos[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*ArcCos[c*x])^n,(f*x)^m*(d+e*x^2)^p,x],x] /;
FreeQ[{a,b,c,d,e,f},x] && NeQ[c^2*d+e,0] && IGtQ[n,0] && IntegerQ[p] && IntegerQ[m]
```

U: $\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx$

Rule:

$$\int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx \rightarrow \int (f x)^m (d+e x^2)^p (a+b \arcsin(c x))^n dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d+_e_.*x_^2)^p_.*(a+_b_.*ArcSin[c_.*x_])^n_,x_Symbol] :=
  Unintegrable[(f*x)^m*(d+e*x^2)^p*(a+b*ArcSin[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,m,n,p},x]
```

```

Int[(f_.**x_)^m_.*(d_+e_.*x_^2)^p_.*(a_+b_.*ArcCos[c_.*x_])^n_.,x_Symbol] :=
  Unintegrable[(f*x)^m*(d+e*x^2)^p*(a+b*ArcCos[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,m,n,p},x]

```

Rules for integrands of the form $(h x)^m (d + e x)^p (f + g x)^q (a + b \operatorname{ArcSin}[c x])^n$

1: $\int (h x)^m (d + e x)^p (f + g x)^q (a + b \operatorname{ArcSin}[c x])^n dx$ when $e f + d g = 0 \wedge c^2 d^2 - e^2 = 0 \wedge (p | q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge d > 0 \wedge \frac{g}{e} < 0$

Derivation: Algebraic normalization

Basis: If $e f + d g = 0 \wedge c^2 d^2 - e^2 = 0 \wedge d > 0 \wedge \frac{g}{e} < 0$, then

$$(d + e x)^p (f + g x)^q = \left(-\frac{d^2 g}{e}\right)^q (d + e x)^{p-q} (1 - c^2 x^2)^q$$

Rule: If $e f + d g = 0 \wedge c^2 d^2 - e^2 = 0 \wedge (p | q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge d > 0 \wedge \frac{g}{e} < 0$, then

$$\int (h x)^m (d + e x)^p (f + g x)^q (a + b \operatorname{ArcSin}[c x])^n dx \rightarrow \left(-\frac{d^2 g}{e}\right)^q \int (h x)^m (d + e x)^{p-q} (1 - c^2 x^2)^q (a + b \operatorname{ArcSin}[c x])^n dx$$

Program code:

```

Int[(h_.**x_)^m_.*(d_+e_.*x_)^p_.*(f_+g_.*x_)^q_.*(a_+b_.*ArcSin[c_.*x_])^n_.,x_Symbol] :=
  (-d^2*g/e)^q*Int[(h*x)^m*(d+e*x)^(p-q)*(1-c^2*x^2)^q*(a+b*ArcSin[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,h,m,n},x] && EqQ[e*f+d*g,0] && EqQ[c^2*d^2-e^2,0] && HalfIntegerQ[p,q] && GeQ[p-q,0] && GtQ[d,0] && LtQ[g/e,0]

```

```

Int[(h_.**x_)^m_.*(d_+e_.*x_)^p_.*(f_+g_.*x_)^q_.*(a_+b_.*ArcCos[c_.*x_])^n_.,x_Symbol] :=
  (-d^2*g/e)^q*Int[(h*x)^m*(d+e*x)^(p-q)*(1-c^2*x^2)^q*(a+b*ArcCos[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,h,m,n},x] && EqQ[e*f+d*g,0] && EqQ[c^2*d^2-e^2,0] && HalfIntegerQ[p,q] && GeQ[p-q,0] && GtQ[d,0] && LtQ[g/e,0]

```

2: $\int (h x)^m (d+e x)^p (f+g x)^q (a+b \operatorname{ArcSin}[c x])^n dx$ when $e f + d g = 0 \wedge c^2 d^2 - e^2 = 0 \wedge (p | q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge \neg (d > 0 \wedge \frac{g}{e} < 0)$

Derivation: Piecewise constant extraction

Basis: If $e f + d g = 0 \wedge c^2 d^2 - e^2 = 0$, then $\partial_x \frac{(d+e x)^q (f+g x)^q}{(1-c^2 x^2)^q} = 0$

Rule: If $e f + d g = 0 \wedge c^2 d^2 - e^2 = 0 \wedge (p | q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge \neg (d > 0 \wedge \frac{g}{e} < 0)$, then

$$\int (d+e x)^p (f+g x)^q (a+b \operatorname{ArcSin}[c x])^n dx \rightarrow \frac{\left(-\frac{d^2 g}{e}\right)^{\operatorname{IntPart}[q]} (d+e x)^{\operatorname{FracPart}[q]} (f+g x)^{\operatorname{FracPart}[q]}}{(1-c^2 x^2)^{\operatorname{FracPart}[q]}} \int (d+e x)^{p-q} (1-c^2 x^2)^q (a+b \operatorname{ArcSin}[c x])^n dx$$

Program code:

```
Int[(h_.**x_)^m_.*(d_+e_.**x_)^p_*(f_+g_.**x_)^q_*(a_+b_.**ArcSin[c_.**x_])^n_,x_Symbol] :=
  (-d^2*g/e)^IntPart[q]*(d+e*x)^FracPart[q]*(f+g*x)^FracPart[q]/(1-c^2*x^2)^FracPart[q]*
  Int[(h*x)^m*(d+e*x)^(p-q)*(1-c^2*x^2)^q*(a+b*ArcSin[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,h,m,n},x] && EqQ[e*f+d*g,0] && EqQ[c^2*d^2-e^2,0] && HalfIntegerQ[p,q] && GeQ[p-q,0]
```

```
Int[(h_.**x_)^m_.*(d_+e_.**x_)^p_*(f_+g_.**x_)^q_*(a_+b_.**ArcCos[c_.**x_])^n_,x_Symbol] :=
  (-d^2*g/e)^IntPart[q]*(d+e*x)^FracPart[q]*(f+g*x)^FracPart[q]/(1-c^2*x^2)^FracPart[q]*
  Int[(h*x)^m*(d+e*x)^(p-q)*(1-c^2*x^2)^q*(a+b*ArcCos[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,h,m,n},x] && EqQ[e*f+d*g,0] && EqQ[c^2*d^2-e^2,0] && HalfIntegerQ[p,q] && GeQ[p-q,0]
```