Rules for integrands of the form $(a + b \operatorname{Sec}[e + f x])^m (d \operatorname{Sec}[e + f x])^n (A + B \operatorname{Sec}[e + f x])$

1. $\int (a + b \operatorname{Sec}[e + f x]) (d \operatorname{Sec}[e + f x])^n (A + B \operatorname{Sec}[e + f x]) \, dx$ when $A b - a B \neq 0$

   1: $\int (a + b \operatorname{Sec}[e + f x]) (d \operatorname{Sec}[e + f x])^n (A + B \operatorname{Sec}[e + f x]) \, dx$ when $A b - a B \neq 0 \land n \leq -1$

Derivation: Nondegenerate secant recurrence 1a with $A \to a A$, $B \to A b + a B$, $C \to b B$, $m \to 0$, $p \to 0$

Rule: If $A b - a B \neq 0 \land n \leq -1$, then

$$\int (a + b \operatorname{Sec}[e + f x]) (d \operatorname{Sec}[e + f x])^n (A + B \operatorname{Sec}[e + f x]) \, dx \longrightarrow$$
$$-\frac{A a \operatorname{Tan}[e + f x] (d \operatorname{Sec}[e + f x])^n}{f n} + \frac{1}{d n} \int (d \operatorname{Sec}[e + f x])^{n+1} (n (B a + A b) + (B b n + A a (n + 1)) \operatorname{Sec}[e + f x]) \, dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  A*a*Cot[e+f*x]*(d*Csc[e+f*x])^n/(f*n) +
  1/(d*n)*Int[(d*Csc[e+f*x])^(n+1)*Simp[n*(B*a+A*b)+(B*b*n+A*a*(n+1))*Csc[e+f*x],x],x] /;
FreeQ[{a,b,d,e,f,A,B},x] && NeQ[A*b-a*B,0] && LeQ[n,-1]
```

2: $\int (a+b\,\mathrm{Sec}[e+f\,x])\,(d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,n\nleq -1$

Derivation: Nondegenerate secant recurrence 1b with $A\to a\,A,\ B\to A\,b+a\,B,\ C\to b\,B,\ m\to 0,\ p\to 0$

Rule: If $A\,b-a\,B\neq 0\ \land\ n\nleq -1$, then

$$\int (a+b\,\mathrm{Sec}[e+f\,x])\,(d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])\,dx\longrightarrow$$
$$\frac{b\,B\,\mathrm{Tan}[e+f\,x]\,(d\,\mathrm{Sec}[e+f\,x])^n}{f\,(n+1)}+\frac{1}{n+1}\int (d\,\mathrm{Sec}[e+f\,x])^n\,(A\,a\,(n+1)+B\,b\,n+(A\,b+B\,a)\,(n+1)\,\mathrm{Sec}[e+f\,x])\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])*(d_.*csc[e_.+f_.*x_])^n_.*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -b*B*Cot[e+f*x]*(d*Csc[e+f*x])^n/(f*(n+1)) +
  1/(n+1)*Int[(d*Csc[e+f*x])^n*Simp[A*a*(n+1)+B*b*n+(A*b+B*a)*(n+1)*Csc[e+f*x],x],x] /;
FreeQ[{a,b,d,e,f,A,B},x] && NeQ[A*b-a*B,0] && Not[LeQ[n,-1]]
```

2. $\int \mathrm{Sec}[e+f x]\,(a+b\,\mathrm{Sec}[e+f x])^m\,(A+B\,\mathrm{Sec}[e+f x])\,dx$ when $A b-a B\neq 0$

1: $\int \frac{\mathrm{Sec}[e+f x]\,(A+B\,\mathrm{Sec}[e+f x])}{a+b\,\mathrm{Sec}[e+f x]}\,dx$ when $A b-a B\neq 0$

Derivation: Algebraic expansion

Basis: $\frac{A+B z}{a+b z}==\frac{B}{b}+\frac{A b-a B}{b\,(a+b z)}$

Rule: If $A b-a B\neq 0$, then

$$\int \frac{\mathrm{Sec}[e+f x]\,(A+B\,\mathrm{Sec}[e+f x])}{a+b\,\mathrm{Sec}[e+f x]}\,dx\;\longrightarrow\;\frac{B}{b}\int \mathrm{Sec}[e+f x]\,dx+\frac{A b-a B}{b}\int \frac{\mathrm{Sec}[e+f x]}{a+b\,\mathrm{Sec}[e+f x]}\,dx$$

Program code:

```
Int[csc[e_.+f_.*x_]*(A_+B_.*csc[e_.+f_.*x_])/(a_+b_.*csc[e_.+f_.*x_]),x_Symbol] :=
  B/b*Int[Csc[e+f*x],x] + (A*b-a*B)/b*Int[Csc[e+f*x]/(a+b*Csc[e+f*x]),x] /;
FreeQ[{a,b,e,f,A,B},x] && NeQ[A*b-a*B,0]
```

2. $\int \mathrm{Sec}[e+f x]\,(a+b\,\mathrm{Sec}[e+f x])^m\,(A+B\,\mathrm{Sec}[e+f x])\,dx$ when $A b-a B\neq 0\,\wedge\,a^2-b^2==0$

1: $\int \mathrm{Sec}[e+f x]\,(a+b\,\mathrm{Sec}[e+f x])^m\,(A+B\,\mathrm{Sec}[e+f x])\,dx$ when $A b-a B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,a B m+A b\,(m+1)==0$

Derivation: Singly degenerate secant recurrence 2a with $A\to-\frac{a B m}{b\,(m+1)}$, $n\to 0$, $p\to 0$

Derivation: Singly degenerate secant recurrence 2c with $A\to-\frac{a B m}{b\,(m+1)}$, $n\to 0$, $p\to 0$

Note: If $a^2-b^2==0\,\wedge\,a B m+A b\,(m+1)==0$, then $m+1\neq 0$.

Rule: If $A b-a B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,a B m+A b\,(m+1)==0$, then

$$\int \text{Sec}[e+f x]\,(a+b\,\text{Sec}[e+f x])^m\,(A+B\,\text{Sec}[e+f x])\,dx \,\rightarrow\, \frac{B\,\text{Tan}[e+f x]\,(a+b\,\text{Sec}[e+f x])^m}{f\,(m+1)}$$

Program code:

```
Int[csc[e_.+f_.*x_]*(a_+b_.*csc[e_.+f_.*x_])^m_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -B*Cot[e+f*x]*(a+b*Csc[e+f*x])^m/(f*(m+1)) /;
FreeQ[{a,b,A,B,e,f,m},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && EqQ[a*B*m+A*b*(m+1),0]
```

2. $\int \text{Sec}[e+f x]\,(a+b\,\text{Sec}[e+f x])^m\,(A+B\,\text{Sec}[e+f x])\,dx$ when $A b-a B\neq 0\,\land\,a^2-b^2==0\,\land\,a B m+A b\,(m+1)\neq 0$

1: $\int \text{Sec}[e+f x]\,(a+b\,\text{Sec}[e+f x])^m\,(A+B\,\text{Sec}[e+f x])\,dx$ when $A b-a B\neq 0\,\land\,a^2-b^2==0\,\land\,a B m+A b\,(m+1)\neq 0\,\land\,m<-\frac{1}{2}$

Derivation: Singly degenerate secant recurrence 2a with $n\rightarrow 0,\ p\rightarrow 0$

Rule: If $A b-a B\neq 0\,\land\,a^2-b^2==0\,\land\,a B m+A b\,(m+1)\neq 0\,\land\,m<-\frac{1}{2}$, then

$$\int \text{Sec}[e+f x]\,(a+b\,\text{Sec}[e+f x])^m\,(A+B\,\text{Sec}[e+f x])\,dx \,\rightarrow$$
$$-\frac{(A b-a B)\,\text{Tan}[e+f x]\,(a+b\,\text{Sec}[e+f x])^m}{a f\,(2 m+1)}+\frac{a B m+A b\,(m+1)}{a b\,(2 m+1)}\int \text{Sec}[e+f x]\,(a+b\,\text{Sec}[e+f x])^{m+1}\,dx$$

Program code:

```
Int[csc[e_.+f_.*x_]*(a_+b_.*csc[e_.+f_.*x_])^m_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  (A*b-a*B)*Cot[e+f*x]*(a+b*Csc[e+f*x])^m/(a*f*(2*m+1)) +
  (a*B*m+A*b*(m+1))/(a*b*(2*m+1))*Int[Csc[e+f*x]*(a+b*Csc[e+f*x])^(m+1),x] /;
FreeQ[{a,b,A,B,e,f},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && NeQ[a*B*m+A*b*(m+1),0] && LtQ[m,-1/2]
```

2: $\int \text{Sec}[e+f x]\ (a+b\,\text{Sec}[e+f x])^m\ (A+B\,\text{Sec}[e+f x])\,dx$ when $A b-a B\neq 0\land a^2-b^2==0\land a B m+A b\ (m+1)\neq 0\land m\nless -\frac{1}{2}$

Derivation: Singly degenerate secant recurrence 2c with $n\to 0,\ p\to 0$

Rule: If $A b-a B\neq 0\land a^2-b^2==0\land a B m+A b\ (m+1)\neq 0\land m\nless -\frac{1}{2}$, then

$$\int \text{Sec}[e+f x]\ (a+b\,\text{Sec}[e+f x])^m\ (A+B\,\text{Sec}[e+f x])\,dx\ \longrightarrow$$
$$\frac{B\,\text{Tan}[e+f x]\ (a+b\,\text{Sec}[e+f x])^m}{f\ (m+1)}+\frac{a B m+A b\ (m+1)}{b\ (m+1)}\int \text{Sec}[e+f x]\ (a+b\,\text{Sec}[e+f x])^m\,dx$$

Program code:

```
Int[csc[e_.+f_.*x_]*(a_+b_.*csc[e_.+f_.*x_])^m_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -B*Cot[e+f*x]*(a+b*Csc[e+f*x])^m/(f*(m+1)) +
  (a*B*m+A*b*(m+1))/(b*(m+1))*Int[Csc[e+f*x]*(a+b*Csc[e+f*x])^m,x] /;
FreeQ[{a,b,A,B,e,f,m},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && NeQ[a*B*m+A*b*(m+1),0] && Not[LtQ[m,-1/2]]
```

3. $\int \operatorname{Sec}[e+f x]\,(a+b\operatorname{Sec}[e+f x])^m\,(A+B\operatorname{Sec}[e+f x])\,dx$ when $A b-a B\neq 0\,\wedge\,a^2-b^2\neq 0$

1: $\int \operatorname{Sec}[e+f x]\,(a+b\operatorname{Sec}[e+f x])^m\,(A+B\operatorname{Sec}[e+f x])\,dx$ when $A b-a B\neq 0\,\wedge\,a^2-b^2\neq 0\,\wedge\,m>0$

Reference: G&R 2.551.1 inverted

Derivation: Nondegenerate secant recurrence 1b with $A\to a A,\ B\to A b+a B,\ C\to b B,\ m\to 0,\ n\to n-1,\ p\to 0$

Rule: If $A b-a B\neq 0\,\wedge\,a^2-b^2\neq 0\,\wedge\,m>0$, then

$$\int \operatorname{Sec}[e+f x]\,(a+b\operatorname{Sec}[e+f x])^m\,(A+B\operatorname{Sec}[e+f x])\,dx\,\longrightarrow$$

$$\frac{B\operatorname{Tan}[e+f x]\,(a+b\operatorname{Sec}[e+f x])^m}{f\,(m+1)}+\frac{1}{m+1}\int \operatorname{Sec}[e+f x]\,(a+b\operatorname{Sec}[e+f x])^{m-1}\,(b B m+a c\,(m+1)+(a B m+A b\,(m+1))\operatorname{Sec}[e+f x])\,dx$$

Program code:

```
Int[csc[e_.+f_.*x_]*(a_+b_.*csc[e_.+f_.*x_])^m_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -B*Cot[e+f*x]*(a+b*Csc[e+f*x])^m/(f*(m+1)) +
  1/(m+1)*Int[Csc[e+f*x]*(a+b*Csc[e+f*x])^(m-1)*Simp[b*B*m+a*A*(m+1)+(a*B*m+A*b*(m+1))*Csc[e+f*x],x],x] /;
FreeQ[{a,b,A,B,e,f},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0] && GtQ[m,0]
```

2: $\int \operatorname{Sec}[e+f x]\,(a+b\operatorname{Sec}[e+f x])^m\,(A+B\operatorname{Sec}[e+f x])\,dx$ when $A b-a B\neq 0\,\wedge\,a^2-b^2\neq 0\,\wedge\,m<-1$

Reference: G&R 2.551.1

Derivation: Nondegenerate secant recurrence 1a with $C\to 0,\ n\to 0,\ p\to 0$

Rule: If $A b-a B\neq 0\,\wedge\,a^2-b^2\neq 0\,\wedge\,m<-1$, then

$$\int \operatorname{Sec}[e+f x]\,(a+b\operatorname{Sec}[e+f x])^m\,(A+B\operatorname{Sec}[e+f x])\,dx\,\longrightarrow$$

$$\frac{(A b - a B) \operatorname{Tan}[e + f x] (a + b \operatorname{Sec}[e + f x])^{m+1}}{f (m + 1) (a^2 - b^2)} +$$
$$\frac{1}{(m + 1) (a^2 - b^2)} \int \operatorname{Sec}[e + f x] (a + b \operatorname{Sec}[e + f x])^{m+1} ((a A - b B) (m + 1) - (A b - a B) (m + 2) \operatorname{Sec}[e + f x]) \, dx$$

Program code:

```
Int[csc[e_.+f_.*x_]*(a_+b_.*csc[e_.+f_.*x_])^m_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -(A*b-a*B)*Cot[e+f*x]*(a+b*Csc[e+f*x])^(m+1)/(f*(m+1)*(a^2-b^2)) +
  1/((m+1)*(a^2-b^2))*Int[Csc[e+f*x]*(a+b*Csc[e+f*x])^(m+1)*Simp[(a*A-b*B)*(m+1)-(A*b-a*B)*(m+2)*Csc[e+f*x],x],x] /;
FreeQ[{a,b,A,B,e,f},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0] && LtQ[m,-1]
```

3. $\int \frac{\operatorname{Sec}[e + f x] (A + B \operatorname{Sec}[e + f x])}{\sqrt{a + b \operatorname{Sec}[e + f x]}} \, dx$ when $A b - a B \neq 0 \land a^2 - b^2 \neq 0$

1: $\int \frac{\operatorname{Sec}[e + f x] (A + B \operatorname{Sec}[e + f x])}{\sqrt{a + b \operatorname{Sec}[e + f x]}} \, dx$ when $a^2 - b^2 \neq 0 \land A^2 - B^2 == 0$

Derivation: Piecewise constant extraction and integration by substitution

Basis: $\partial_x \left(\frac{1}{\operatorname{Tan}[e+f x]} \sqrt{\frac{b (1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b (1+\operatorname{Sec}[e+f x])}{a-b}} \right) == 0$

Basis: $\operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x] F[\operatorname{Sec}[e + f x]] == \frac{1}{f} \operatorname{Subst}[F[x], x, \operatorname{Sec}[e + f x]] \partial_x \operatorname{Sec}[e + f x]$

Rule: If $a^2 - b^2 \neq 0$, then

$$\int \frac{\operatorname{Sec}[e + f x] (A + B \operatorname{Sec}[e + f x])}{\sqrt{a + b \operatorname{Sec}[e + f x]}} \, dx \longrightarrow \frac{A b - a B}{b \operatorname{Tan}[e + f x]}$$

$$\sqrt{\frac{b\,(1-\mathrm{Sec}[e+f\,x])}{a+b}}\,\sqrt{-\frac{b\,(1+\mathrm{Sec}[e+f\,x])}{a-b}}\int\frac{\mathrm{Sec}[e+f\,x]\,\mathrm{Tan}[e+f\,x]\,\sqrt{-\frac{b\,B}{a\,A-b\,B}-\frac{A\,b\,\mathrm{Sec}[e+f\,x]}{a\,A-b\,B}}}{\sqrt{a+b\,\mathrm{Sec}[e+f\,x]}\,\sqrt{\frac{b\,B}{a\,A+b\,B}-\frac{A\,b\,\mathrm{Sec}[e+f\,x]}{a\,A+b\,B}}}\,dx$$

$$\rightarrow\frac{A\,b-a\,B}{b\,f\,\mathrm{Tan}[e+f\,x]}\sqrt{\frac{b\,(1-\mathrm{Sec}[e+f\,x])}{a+b}}\,\sqrt{-\frac{b\,(1+\mathrm{Sec}[e+f\,x])}{a-b}}\,\mathrm{Subst}\Big[\int\frac{\sqrt{-\frac{b\,B}{a\,A-b\,B}-\frac{A\,b\,x}{a\,A-b\,B}}}{\sqrt{a+b\,x}\,\sqrt{\frac{b\,B}{a\,A+b\,B}-\frac{A\,b\,x}{a\,A+b\,B}}}\,dx,\,x,\,\mathrm{Sec}[e+f\,x]\Big]$$

$$\rightarrow\frac{2\,(A\,b-a\,B)\,\sqrt{a+\frac{b\,B}{A}}\,\sqrt{\frac{b\,(1-\mathrm{Sec}[e+f\,x])}{a+b}}\,\sqrt{-\frac{b\,(1+\mathrm{Sec}[e+f\,x])}{a-b}}}{b^2\,f\,\mathrm{Tan}[e+f\,x]}\,\mathrm{EllipticE}\Big[\mathrm{ArcSin}\Big[\frac{\sqrt{a+b\,\mathrm{Sec}[e+f\,x]}}{\sqrt{a+\frac{b\,B}{A}}}\Big],\,\frac{a\,A+b\,B}{a\,A-b\,B}\Big]$$

Program code:

```
Int[csc[e_.+f_.*x_]*(A_+B_.*csc[e_.+f_.*x_])/Sqrt[a_+b_.*csc[e_.+f_.*x_]],x_Symbol] :=
  -2*(A*b-a*B)*Rt[a+b*B/A,2]*Sqrt[b*(1-Csc[e+f*x])/(a+b)]*Sqrt[-b*(1+Csc[e+f*x])/(a-b)]/(b^2*f*Cot[e+f*x])*
    EllipticE[ArcSin[Sqrt[a+b*Csc[e+f*x]]/Rt[a+b*B/A,2]],(a*A+b*B)/(a*A-b*B)] /;
FreeQ[{a,b,e,f,A,B},x] && NeQ[a^2-b^2,0] && EqQ[A^2-B^2,0]
```

2: $\int\frac{\mathrm{Sec}[e+f\,x]\,(A+B\,\mathrm{Sec}[e+f\,x])}{\sqrt{a+b\,\mathrm{Sec}[e+f\,x]}}\,dx$ when $a^2-b^2\neq 0\,\land\,A^2-B^2\neq 0$

Derivation: Algebraic expansion

Basis: $A+B\,z=A-B+B\,(1+z)$

Rule: If $a^2-b^2\neq 0\,\land\,A^2-B^2\neq 0$, then

$$\int \frac{\text{Sec}[e+f x]\,(A+B\,\text{Sec}[e+f x])}{\sqrt{a+b\,\text{Sec}[e+f x]}}\,dx \to (A-B)\int \frac{\text{Sec}[e+f x]}{\sqrt{a+b\,\text{Sec}[e+f x]}}\,dx + B\int \frac{\text{Sec}[e+f x]\,(1+\text{Sec}[e+f x])}{\sqrt{a+b\,\text{Sec}[e+f x]}}\,dx$$

Program code:

```
Int[csc[e_.+f_.*x_]*(A_+B_.*csc[e_.+f_.*x_])/Sqrt[a_+b_.*csc[e_.+f_.*x_]],x_Symbol] :=
  (A-B)*Int[Csc[e+f*x]/Sqrt[a+b*Csc[e+f*x]],x] +
  B*Int[Csc[e+f*x]*(1+Csc[e+f*x])/Sqrt[a+b*Csc[e+f*x]],x] /;
FreeQ[{a,b,e,f,A,B},x] && NeQ[a^2-b^2,0] && NeQ[A^2-B^2,0]
```

4: $\int \text{Sec}[e+f x]\,(a+b\,\text{Sec}[e+f x])^m\,(A+B\,\text{Sec}[e+f x])\,dx$ when $A b-a B\neq 0 \land a^2-b^2\neq 0 \land A^2-B^2==0 \land 2 m\notin \mathbb{Z}$

Derivation: Integration by substitution

Rule: If $A b-a B\neq 0 \land a^2-b^2\neq 0 \land A^2-B^2==0 \land 2 m\notin \mathbb{Z}$, then

$$\int \text{Sec}[e+f x]\,(a+b\,\text{Sec}[e+f x])^m\,(A+B\,\text{Sec}[e+f x])\,dx \to$$

$$-\frac{2\sqrt{2}\,A\,(a+b\,\text{Sec}[e+f x])^m\,(A-B\,\text{Sec}[e+f x])\,\sqrt{\frac{A+B\,\text{Sec}[e+f x]}{A}}}{B\,f\,\text{Tan}[e+f x]\,\left(\frac{A\,(a+b\,\text{Sec}[e+f x])}{a A+b B}\right)^m}\,\text{AppellF1}\!\left[\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{A-B\,\text{Sec}[e+f x]}{2 A},\frac{b\,(A-B\,\text{Sec}[e+f x])}{A b+a B}\right]$$

Program code:

```
Int[csc[e_.+f_.*x_]*(a_+b_.*csc[e_.+f_.*x_])^m_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  2*Sqrt[2]*A*(a+b*Csc[e+f*x])^m*(A-B*Csc[e+f*x])*Sqrt[(A+B*Csc[e+f*x])/A]/(B*f*Cot[e+f*x]*(A*(a+b*Csc[e+f*x])/(a*A+b*B))^m)*
    AppellF1[1/2,-(1/2),-m,3/2,(A-B*Csc[e+f*x])/(2*A),(b*(A-B*Csc[e+f*x]))/(A*b+a*B)] /;
FreeQ[{a,b,A,B,e,f},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0] && EqQ[A^2-B^2,0] && Not[IntegerQ[2*m]]
```

5: $\int \mathrm{Sec}[e+f\,x]\,(a+b\,\mathrm{Sec}[e+f\,x])^m\,(A+B\,\mathrm{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0$

Derivation: Algebraic expansion

Basis: $A+B\,z == \frac{A\,b-a\,B}{b}+\frac{B}{b}\,(a+b\,z)$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0$, then

$$\int \mathrm{Sec}[e+f\,x]\,(a+b\,\mathrm{Sec}[e+f\,x])^m\,(A+B\,\mathrm{Sec}[e+f\,x])\,dx \longrightarrow \frac{A\,b-a\,B}{b}\int \mathrm{Sec}[e+f\,x]\,(a+b\,\mathrm{Sec}[e+f\,x])^m\,dx+\frac{B}{b}\int \mathrm{Sec}[e+f\,x]\,(a+b\,\mathrm{Sec}[e+f\,x])^{m+1}\,dx$$

Program code:

```
Int[csc[e_.+f_.*x_]*(a_+b_.*csc[e_.+f_.*x_])^m_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  (A*b-a*B)/b*Int[Csc[e+f*x]*(a+b*Csc[e+f*x])^m,x] + B/b*Int[Csc[e+f*x]*(a+b*Csc[e+f*x])^(m+1),x] /;
FreeQ[{a,b,A,B,e,f,m},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0]
```

3. $\int \text{Sec}[e+f x]^2 (a+b \text{Sec}[e+f x])^m (A+B \text{Sec}[e+f x]) \, dx$ when $A b - a B \neq 0$

1: $\int \text{Sec}[e+f x]^2 (a+b \text{Sec}[e+f x])^m (A+B \text{Sec}[e+f x]) \, dx$ when $A b - a B \neq 0 \land a^2 - b^2 == 0 \land m < -\frac{1}{2}$

Derivation: ???

Rule: If $A b - a B \neq 0 \land a^2 - b^2 == 0 \land m < -\frac{1}{2}$, then

$$\int \text{Sec}[e+f x]^2 (a+b \text{Sec}[e+f x])^m (A+B \text{Sec}[e+f x]) \, dx \longrightarrow$$
$$\frac{(A b - a B) \text{Tan}[e+f x] (a+b \text{Sec}[e+f x])^m}{b f (2 m+1)} + \frac{1}{b^2 (2 m+1)} \int \text{Sec}[e+f x] (a+b \text{Sec}[e+f x])^{m+1} (m (A b - a B) + b B (2 m+1) \text{Sec}[e+f x]) \, dx$$

Program code:

```
Int[csc[e_.+f_.*x_]^2*(a_+b_.*csc[e_.+f_.*x_])^m_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -(A*b-a*B)*Cot[e+f*x]*(a+b*Csc[e+f*x])^m/(b*f*(2*m+1)) +
  1/(b^2*(2*m+1))*Int[Csc[e+f*x]*(a+b*Csc[e+f*x])^(m+1)*Simp[A*b*m-a*B*m+b*B*(2*m+1)*Csc[e+f*x],x],x] /;
FreeQ[{a,b,e,f,A,B},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && LtQ[m,-1/2]
```

2: $\int \text{Sec}[e+f x]^2 (a+b \text{Sec}[e+f x])^m (A+B \text{Sec}[e+f x]) \, dx$ when $A b - a B \neq 0 \land a^2 - b^2 \neq 0 \land m < -1$

Derivation: Nondegenerate secant recurrence 1a with $A \to a A$, $B \to A b + a B$, $C \to b B$, $m \to 0$, $p \to 0$

Rule: If $A b - a B \neq 0 \land a^2 - b^2 \neq 0 \land m < -1$, then

$$\int \text{Sec}[e+f x]^2 (a+b \text{Sec}[e+f x])^m (A+B \text{Sec}[e+f x]) \, dx \longrightarrow$$
$$-\frac{a (A b - a B) \text{Tan}[e+f x] (a+b \text{Sec}[e+f x])^{m+1}}{b f (m+1) (a^2 - b^2)} -$$

$$\frac{1}{b\,(m+1)\,(a^2-b^2)}\int \text{Sec}[e+f\,x]\,(a+b\,\text{Sec}[e+f\,x])^{m+1}\,\left(b\,(A\,b-a\,B)\,(m+1)-\left(a\,A\,b\,(m+2)-B\,(a^2+b^2\,(m+1))\right)\text{Sec}[e+f\,x]\right)dx$$

Program code:

```
Int[csc[e_.+f_.*x_]^2*(a_+b_.*csc[e_.+f_.*x_])^m_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  a*(A*b-a*B)*Cot[e+f*x]*(a+b*Csc[e+f*x])^(m+1)/(b*f*(m+1)*(a^2-b^2)) -
  1/(b*(m+1)*(a^2-b^2))*Int[Csc[e+f*x]*(a+b*Csc[e+f*x])^(m+1)*
    Simp[b*(A*b-a*B)*(m+1)-(a*A*b*(m+2)-B*(a^2+b^2*(m+1)))*Csc[e+f*x],x],x] /;
FreeQ[{a,b,e,f,A,B},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0] && LtQ[m,-1]
```

3: $\int \text{Sec}[e+f\,x]^2\,(a+b\,\text{Sec}[e+f\,x])^m\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\wedge\,m\not<-1$

Derivation: Nondegenerate secant recurrence 1b with $A\to a\,A,\ B\to A\,b+a\,B,\ C\to b\,B,\ m\to 0,\ p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\wedge\,m\not<-1$, then

$$\int \text{Sec}[e+f\,x]^2\,(a+b\,\text{Sec}[e+f\,x])^m\,(A+B\,\text{Sec}[e+f\,x])\,dx\;\longrightarrow$$

$$\frac{B\,\text{Tan}[e+f\,x]\,(a+b\,\text{Sec}[e+f\,x])^{m+1}}{b\,f\,(m+2)}+\frac{1}{b\,(m+2)}\int \text{Sec}[e+f\,x]\,(a+b\,\text{Sec}[e+f\,x])^m\,\left(b\,B\,(m+1)+(A\,b\,(m+2)-a\,B)\,\text{Sec}[e+f\,x]\right)dx$$

Program code:

```
Int[csc[e_.+f_.*x_]^2*(a_+b_.*csc[e_.+f_.*x_])^m_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -B*Cot[e+f*x]*(a+b*Csc[e+f*x])^(m+1)/(b*f*(m+2)) +
  1/(b*(m+2))*Int[Csc[e+f*x]*(a+b*Csc[e+f*x])^m*Simp[b*B*(m+1)+(A*b*(m+2)-a*B)*Csc[e+f*x],x],x] /;
FreeQ[{a,b,e,f,A,B,m},x] && NeQ[A*b-a*B,0] && Not[LtQ[m,-1]]
```

4. $\int (a + b \operatorname{Sec}[e + f x])^m (d \operatorname{Sec}[e + f x])^n (A + B \operatorname{Sec}[e + f x]) \, dx$ when $A b - a B \neq 0 \land a^2 - b^2 == 0$

   1. $\int (a + b \operatorname{Sec}[e + f x])^m (d \operatorname{Sec}[e + f x])^n (A + B \operatorname{Sec}[e + f x]) \, dx$ when $A b - a B \neq 0 \land a^2 - b^2 == 0 \land m + n + 1 == 0$

      1: $\int (a + b \operatorname{Sec}[e + f x])^m (d \operatorname{Sec}[e + f x])^n (A + B \operatorname{Sec}[e + f x]) \, dx$ when $A b - a B \neq 0 \land a^2 - b^2 == 0 \land m + n + 1 == 0 \land a A m - b B n == 0$

Rule: If $A b - a B \neq 0 \land a^2 - b^2 == 0 \land m + n + 1 == 0 \land a A m - b B n == 0$, then

$$\int (a + b \operatorname{Sec}[e + f x])^m (d \operatorname{Sec}[e + f x])^n (A + B \operatorname{Sec}[e + f x]) \, dx \to -\frac{A \operatorname{Tan}[e + f x] (a + b \operatorname{Sec}[e + f x])^m (d \operatorname{Sec}[e + f x])^n}{f n}$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  A*Cot[e+f*x]*(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^n/(f*n) /;
FreeQ[{a,b,d,e,f,A,B,m,n},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && EqQ[m+n+1,0] && EqQ[a*A*m-b*B*n,0]
```

2. $\int (a+b\,\mathrm{Sec}[e+f\,x])^m\,(d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2==0\,\land\,m+n+1==0\,\land\,a\,A\,m-b\,B\,n\neq 0$

   1: $\int (a+b\,\mathrm{Sec}[e+f\,x])^m\,(d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2==0\,\land\,m+n+1==0\,\land\,m\leq -1$

Derivation: Singly degenerate secant recurrence 2b with $m\to -n-2,\ p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2==0\,\land\,m+n+1==0\,\land\,m\leq -1$, then

$$\int (a+b\,\mathrm{Sec}[e+f\,x])^m\,(d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])\,dx\,\longrightarrow$$

$$\frac{(A\,b-a\,B)\,\mathrm{Tan}[e+f\,x]\,(a+b\,\mathrm{Sec}[e+f\,x])^m\,(d\,\mathrm{Sec}[e+f\,x])^n}{b\,f\,(2\,m+1)}+\frac{(a\,A\,m+b\,B\,(m+1))}{a^2\,(2\,m+1)}\int (a+b\,\mathrm{Sec}[e+f\,x])^{m+1}\,(d\,\mathrm{Sec}[e+f\,x])^n\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -(A*b-a*B)*Cot[e+f*x]*(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^n/(b*f*(2*m+1)) +
  (a*A*m+b*B*(m+1))/(a^2*(2*m+1))*Int[(a+b*Csc[e+f*x])^(m+1)*(d*Csc[e+f*x])^n,x] /;
FreeQ[{a,b,d,e,f,A,B,n},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && EqQ[m+n+1,0] && LeQ[m,-1]
```

2: $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,m+n+1==0\,\wedge\,m\nleq -1$

Derivation: Singly degenerate secant recurrence 1c with $m\to -n-2,\ p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,m+n+1==0\,\wedge\,m\nleq -1$, then

$$\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx\,\longrightarrow$$
$$-\frac{A\,\text{Tan}[e+f\,x]\,(a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n}{f\,n}-\frac{(a\,A\,m-b\,B\,n)}{b\,d\,n}\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^{n+1}\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  A*Cot[e+f*x]*(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^n/(f*n) -
  (a*A*m-b*B*n)/(b*d*n)*Int[(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^(n+1),x] /;
FreeQ[{a,b,d,e,f,A,B,m,n},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && EqQ[m+n+1,0] && Not[LeQ[m,-1]]
```

2. $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,m\geq\frac{1}{2}$

1. $\int \sqrt{a+b\,\text{Sec}[e+f\,x]}\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0$

1: $\int \sqrt{a+b\,\text{Sec}[e+f\,x]}\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,A\,b\,(2\,n+1)+2\,a\,B\,n==0$

Derivation: Singly degenerate secant recurrence 1a with $B\to -\frac{A\,b\,(3+2\,n)}{2\,a\,(1+n)},\ m\to\frac{1}{2},\ p\to 0$

Derivation: Singly degenerate secant recurrence 1b with $B\to -\frac{A\,b\,(3+2\,n)}{2\,a\,(1+n)},\ m\to\frac{1}{2},\ p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,A\,b\,(2\,n+1)+2\,a\,B\,n==0$, then

$$\int \sqrt{a+b\,\text{Sec}[e+f\,x]}\,\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx \to \frac{2\,b\,B\,\text{Tan}[e+f\,x]\,\,(d\,\text{Sec}[e+f\,x])^n}{f\,(2\,n+1)\,\sqrt{a+b\,\text{Sec}[e+f\,x]}}$$

Program code:

```
Int[Sqrt[a_+b_.*csc[e_.+f_.*x_]]*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -2*b*B*Cot[e+f*x]*(d*Csc[e+f*x])^n/(f*(2*n+1)*Sqrt[a+b*Csc[e+f*x]]) /;
FreeQ[{a,b,d,e,f,A,B,n},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && EqQ[A*b*(2*n+1)+2*a*B*n,0]
```

2. $\int \sqrt{a+b\,\text{Sec}[e+f\,x]}\,\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,A\,b\,(2\,n+1)+2\,a\,B\,n\neq 0$

1: $\int \sqrt{a+b\,\text{Sec}[e+f\,x]}\,\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,A\,b\,(2\,n+1)+2\,a\,B\,n\neq 0\,\wedge\,n<0$

Derivation: Singly degenerate secant recurrence 1a with $m\to\frac{1}{2}$, $p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,A\,b\,(2\,n+1)+2\,a\,B\,n\neq 0\,\wedge\,n<0$, then

$$\int \sqrt{a+b\,\text{Sec}[e+f\,x]}\,\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx \to$$

$$-\frac{A\,b^2\,\text{Tan}[e+f\,x]\,\,(d\,\text{Sec}[e+f\,x])^n}{a\,f\,n\,\sqrt{a+b\,\text{Sec}[e+f\,x]}}+\frac{(A\,b\,(2\,n+1)+2\,a\,B\,n)}{2\,a\,d\,n}\int \sqrt{a+b\,\text{Sec}[e+f\,x]}\,\,(d\,\text{Sec}[e+f\,x])^{n+1}\,dx$$

Program code:

```
Int[Sqrt[a_+b_.*csc[e_.+f_.*x_]]*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  A*b^2*Cot[e+f*x]*(d*Csc[e+f*x])^n/(a*f*n*Sqrt[a+b*Csc[e+f*x]]) +
  (A*b*(2*n+1)+2*a*B*n)/(2*a*d*n)*Int[Sqrt[a+b*Csc[e+f*x]]*(d*Csc[e+f*x])^(n+1),x] /;
FreeQ[{a,b,d,e,f,A,B},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && NeQ[A*b*(2*n+1)+2*a*B*n,0] && LtQ[n,0]
```

2: $\int \sqrt{a+b\,\mathrm{Sec}[e+f\,x]}\,\left(d\,\mathrm{Sec}[e+f\,x]\right)^n\,\left(A+B\,\mathrm{Sec}[e+f\,x]\right)\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2==0\,\land\,A\,b\,(2\,n+1)+2\,a\,B\,n\neq 0\,\land\,n\not<0$

Derivation: Singly degenerate secant recurrence 1b with $m\to\frac{1}{2}$, $p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2==0\,\land\,A\,b\,(2\,n+1)+2\,a\,B\,n\neq 0\,\land\,n\not<0$, then

$$\int \sqrt{a+b\,\mathrm{Sec}[e+f\,x]}\,\left(d\,\mathrm{Sec}[e+f\,x]\right)^n\,\left(A+B\,\mathrm{Sec}[e+f\,x]\right)\,dx\,\longrightarrow$$

$$\frac{2\,b\,B\,\mathrm{Tan}[e+f\,x]\,\left(d\,\mathrm{Sec}[e+f\,x]\right)^n}{f\,(2\,n+1)\,\sqrt{a+b\,\mathrm{Sec}[e+f\,x]}}+\frac{A\,b\,(2\,n+1)+2\,a\,B\,n}{b\,(2\,n+1)}\int\sqrt{a+b\,\mathrm{Sec}[e+f\,x]}\,\left(d\,\mathrm{Sec}[e+f\,x]\right)^n\,dx$$

Program code:

```
Int[Sqrt[a_+b_.*csc[e_.+f_.*x_]]*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -2*b*B*Cot[e+f*x]*(d*Csc[e+f*x])^n/(f*(2*n+1)*Sqrt[a+b*Csc[e+f*x]]) +
  (A*b*(2*n+1)+2*a*B*n)/(b*(2*n+1))*Int[Sqrt[a+b*Csc[e+f*x]]*(d*Csc[e+f*x])^n,x] /;
FreeQ[{a,b,d,e,f,A,B,n},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && NeQ[A*b*(2*n+1)+2*a*B*n,0] && Not[LtQ[n,0]]
```

2. $\int \left(a+b\,\mathrm{Sec}[e+f\,x]\right)^m\,\left(d\,\mathrm{Sec}[e+f\,x]\right)^n\,\left(A+B\,\mathrm{Sec}[e+f\,x]\right)\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2==0\,\land\,m>\frac{1}{2}$

1: $\int \left(a+b\,\mathrm{Sec}[e+f\,x]\right)^m\,\left(d\,\mathrm{Sec}[e+f\,x]\right)^n\,\left(A+B\,\mathrm{Sec}[e+f\,x]\right)\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2==0\,\land\,m>\frac{1}{2}\,\land\,n<-1$

Derivation: Singly degenerate secant recurrence 1a with $p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2==0\,\land\,m>\frac{1}{2}\,\land\,n<-1$, then

$$\int \left(a+b\,\mathrm{Sec}[e+f\,x]\right)^m\,\left(d\,\mathrm{Sec}[e+f\,x]\right)^n\,\left(A+B\,\mathrm{Sec}[e+f\,x]\right)\,dx\,\longrightarrow$$

$$-\frac{a\,A\,\mathrm{Tan}[e+f\,x]\,\left(a+b\,\mathrm{Sec}[e+f\,x]\right)^{m-1}\,\left(d\,\mathrm{Sec}[e+f\,x]\right)^n}{f\,n}-$$

$$\frac{b}{a\,d\,n}\int(a+b\,\text{Sec}[e+f\,x])^{m-1}\,(d\,\text{Sec}[e+f\,x])^{n+1}\,(a\,A\,(m-n-1)-b\,B\,n-(a\,B\,n+A\,b\,(m+n))\,\text{Sec}[e+f\,x])\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  a*A*Cot[e+f*x]*(a+b*Csc[e+f*x])^(m-1)*(d*Csc[e+f*x])^n/(f*n) -
  b/(a*d*n)*Int[(a+b*Csc[e+f*x])^(m-1)*(d*Csc[e+f*x])^(n+1)*Simp[a*A*(m-n-1)-b*B*n-(a*B*n+A*b*(m+n))*Csc[e+f*x],x],x] /;
FreeQ[{a,b,d,e,f,A,B},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && GtQ[m,1/2] && LtQ[n,-1]
```

2: $\int(a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2==0\,\land\,m>\frac{1}{2}\,\land\,n\nless -1$

Derivation: Singly degenerate secant recurrence 1b with p → 0

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2==0\,\land\,m>\frac{1}{2}\,\land\,n\nless -1$, then

$$\int(a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx\,\longrightarrow$$

$$\frac{b\,B\,\text{Tan}[e+f\,x]\,(a+b\,\text{Sec}[e+f\,x])^{m-1}\,(d\,\text{Sec}[e+f\,x])^n}{f\,(m+n)}+$$

$$\frac{1}{d\,(m+n)}\int(a+b\,\text{Sec}[e+f\,x])^{m-1}\,(d\,\text{Sec}[e+f\,x])^n\,(a\,A\,d\,(m+n)+B\,(b\,d\,n)+(A\,b\,d\,(m+n)+a\,B\,d\,(2\,m+n-1))\,\text{Sec}[e+f\,x])\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -b*B*Cot[e+f*x]*(a+b*Csc[e+f*x])^(m-1)*(d*Csc[e+f*x])^n/(f*(m+n)) +
  1/(d*(m+n))*Int[(a+b*Csc[e+f*x])^(m-1)*(d*Csc[e+f*x])^n*Simp[a*A*d*(m+n)+B*(b*d*n)+(A*b*d*(m+n)+a*B*d*(2*m+n-1))*Csc[e+f*x],x],x]
FreeQ[{a,b,d,e,f,A,B,n},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && GtQ[m,1/2] && Not[LtQ[n,-1]]
```

3. $\int (a+b\,\mathrm{Sec}[e+f\,x])^m\,(d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,m<-\frac{1}{2}$

1: $\int (a+b\,\mathrm{Sec}[e+f\,x])^m\,(d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,m<-\frac{1}{2}\,\wedge\,n>0$

Derivation: Singly degenerate secant recurrence 2a with $p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,m<-\frac{1}{2}\,\wedge\,n>0$, then

$$\int (a+b\,\mathrm{Sec}[e+f\,x])^m\,(d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])\,dx\,\longrightarrow$$

$$-\frac{d\,(A\,b-a\,B)\,\mathrm{Tan}[e+f\,x]\,(a+b\,\mathrm{Sec}[e+f\,x])^m\,(d\,\mathrm{Sec}[e+f\,x])^{n-1}}{a\,f\,(2\,m+1)}-$$

$$\frac{1}{a\,b\,(2\,m+1)}\int (a+b\,\mathrm{Sec}[e+f\,x])^{m+1}\,(d\,\mathrm{Sec}[e+f\,x])^{n-1}\,(A\,(a\,d\,(n-1))-B\,(b\,d\,(n-1))-d\,(a\,B\,(m-n+1)+A\,b\,(m+n))\,\mathrm{Sec}[e+f\,x])\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  d*(A*b-a*B)*Cot[e+f*x]*(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^(n-1)/(a*f*(2*m+1)) -
  1/(a*b*(2*m+1))*Int[(a+b*Csc[e+f*x])^(m+1)*(d*Csc[e+f*x])^(n-1)*
    Simp[A*(a*d*(n-1))-B*(b*d*(n-1))-d*(a*B*(m-n+1)+A*b*(m+n))*Csc[e+f*x],x],x] /;
FreeQ[{a,b,d,e,f,A,B},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && LtQ[m,-1/2] && GtQ[n,0]
```

2: $\int (a+b\,\mathrm{Sec}[e+f\,x])^m\,(d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,m<-\frac{1}{2}\,\wedge\,n\not>0$

Derivation: Singly degenerate secant recurrence 2b with $p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,m<-\frac{1}{2}\,\wedge\,n\not>0$, then

$$\int (a+b\,\mathrm{Sec}[e+f\,x])^m\,(d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])\,dx\,\longrightarrow$$

$$\frac{(A\,b-a\,B)\,\mathrm{Tan}[e+f\,x]\,(a+b\,\mathrm{Sec}[e+f\,x])^m\,(d\,\mathrm{Sec}[e+f\,x])^n}{b\,f\,(2\,m+1)}-$$

$$\frac{1}{a^2\,(2\,m+1)}\int(a+b\,\text{Sec}[e+f\,x])^{m+1}\,(d\,\text{Sec}[e+f\,x])^n\,(b\,B\,n-a\,A\,(2\,m+n+1)+(A\,b-a\,B)\,(m+n+1)\,\text{Sec}[e+f\,x])\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -(A*b-a*B)*Cot[e+f*x]*(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^n/(b*f*(2*m+1)) -
  1/(a^2*(2*m+1))*Int[(a+b*Csc[e+f*x])^(m+1)*(d*Csc[e+f*x])^n*
    Simp[b*B*n-a*A*(2*m+n+1)+(A*b-a*B)*(m+n+1)*Csc[e+f*x],x],x] /;
FreeQ[{a,b,d,e,f,A,B,n},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && LtQ[m,-1/2] && Not[GtQ[n,0]]
```

4: $\int(a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,n>1$

Derivation: Singly degenerate secant recurrence 2c with p → 0

Rule: If $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2==0\,\wedge\,n>1$, then

$$\int(a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx\,\longrightarrow$$
$$\frac{B\,d\,\text{Tan}[e+f\,x]\,(a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^{n-1}}{f\,(m+n)}+$$
$$\frac{d}{b\,(m+n)}\int(a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^{n-1}\,(b\,B\,(n-1)+(A\,b\,(m+n)+a\,B\,m)\,\text{Sec}[e+f\,x])\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -B*d*Cot[e+f*x]*(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^(n-1)/(f*(m+n)) +
  d/(b*(m+n))*Int[(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^(n-1)*Simp[b*B*(n-1)+(A*b*(m+n)+a*B*m)*Csc[e+f*x],x],x] /;
FreeQ[{a,b,d,e,f,A,B,m},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && GtQ[n,1]
```

5: $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2==0\,\land\,n<0$

Derivation: Singly degenerate secant recurrence 1c with $p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2==0\,\land\,n<0$, then

$$\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx\,\longrightarrow$$

$$-\frac{A\,\text{Tan}[e+f\,x]\,(a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n}{f\,n}-$$

$$\frac{1}{b\,d\,n}\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^{n+1}\,(a\,A\,m-b\,B\,n-A\,b\,(m+n+1)\,\text{Sec}[e+f\,x])\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  A*Cot[e+f*x]*(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^n/(f*n) -
  1/(b*d*n)*Int[(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^(n+1)*Simp[a*A*m-b*B*n-A*b*(m+n+1)*Csc[e+f*x],x],x] /;
FreeQ[{a,b,d,e,f,A,B,m},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0] && LtQ[n,0]
```

6: $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2==0$

Derivation: Algebraic expansion

Baisi: $A+B\,z==\frac{A\,b-a\,B}{b}+\frac{B\,(a+b\,z)}{b}$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2==0$, then

$$\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx\,\longrightarrow$$
$$\frac{A\,b-a\,B}{b}\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,dx+\frac{B}{b}\int (a+b\,\text{Sec}[e+f\,x])^{m+1}\,(d\,\text{Sec}[e+f\,x])^n\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  (A*b-a*B)/b*Int[(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^n,x] +
  B/b*Int[(a+b*Csc[e+f*x])^(m+1)*(d*Csc[e+f*x])^n,x] /;
FreeQ[{a,b,d,e,f,A,B,m},x] && NeQ[A*b-a*B,0] && EqQ[a^2-b^2,0]
```

5. $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0$

1. $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,m>1$

1: $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,m>1\,\land\,n\leq -1$

Derivation: Nondegenerate secant recurrence 1a with $A\to a\,A,\ B\to A\,b+a\,B,\ C\to b\,B,\ m\to m-1,\ p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,m>1\,\land\,n\leq -1$, then

$$\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx\,\longrightarrow$$

$$-\frac{a\,A\,\mathrm{Tan}[e+f\,x]\,(a+b\,\mathrm{Sec}[e+f\,x])^{m-1}\,(d\,\mathrm{Sec}[e+f\,x])^{n}}{f\,n}+$$
$$\frac{1}{d\,n}\int(a+b\,\mathrm{Sec}[e+f\,x])^{m-2}\,(d\,\mathrm{Sec}[e+f\,x])^{n+1}\cdot$$
$$\left(a\,(a\,B\,n-A\,b\,(m-n-1))+\left(2\,a\,b\,B\,n+A\,\left(b^2\,n+a^2\,(1+n)\right)\right)\mathrm{Sec}[e+f\,x]+b\,(b\,B\,n+a\,A\,(m+n))\,\mathrm{Sec}[e+f\,x]^2\right)dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  a*A*Cot[e+f*x]*(a+b*Csc[e+f*x])^(m-1)*(d*Csc[e+f*x])^n/(f*n) +
  1/(d*n)*Int[(a+b*Csc[e+f*x])^(m-2)*(d*Csc[e+f*x])^(n+1)*
     Simp[a*(a*B*n-A*b*(m-n-1))+(2*a*b*B*n+A*(b^2*n+a^2*(1+n)))*Csc[e+f*x]+b*(b*B*n+a*A*(m+n))*Csc[e+f*x]^2,x],x] /;
FreeQ[{a,b,d,e,f,A,B},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0] && GtQ[m,1] && LeQ[n,-1]
```

2: $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,m>1\,\land\,n\nleq -1$

Derivation: Nondegenerate secant recurrence 1b with $A\to a\,A$, $B\to A\,b+a\,B$, $C\to b\,B$, $m\to m-1$, $p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,m>1\,\land\,n\nleq -1$, then

$$\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx\,\longrightarrow$$
$$\frac{b\,B\,\text{Tan}[e+f\,x]\,(a+b\,\text{Sec}[e+f\,x])^{m-1}\,(d\,\text{Sec}[e+f\,x])^n}{f\,(m+n)}+$$
$$\frac{1}{m+n}\int (a+b\,\text{Sec}[e+f\,x])^{m-2}\,(d\,\text{Sec}[e+f\,x])^n\cdot$$
$$\left(a^2\,A\,(m+n)+a\,b\,B\,n+\left(a\,(2\,A\,b+a\,B)\,(m+n)+b^2\,B\,(m+n-1)\right)\,\text{Sec}[e+f\,x]+b\,(A\,b\,(m+n)+a\,B\,(2\,m+n-1))\,\text{Sec}[e+f\,x]^2\right)\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -b*B*Cot[e+f*x]*(a+b*Csc[e+f*x])^(m-1)*(d*Csc[e+f*x])^n/(f*(m+n)) +
  1/(m+n)*Int[(a+b*Csc[e+f*x])^(m-2)*(d*Csc[e+f*x])^n*
     Simp[a^2*A*(m+n)+a*b*B*n+(a*(2*A*b+a*B)*(m+n)+b^2*B*(m+n-1))*Csc[e+f*x]+b*(A*b*(m+n)+a*B*(2*m+n-1))*Csc[e+f*x]^2,x],x] /;
FreeQ[{a,b,d,e,f,A,B,n},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0] && GtQ[m,1] && Not[IGtQ[n,1] && Not[IntegerQ[m]]]
```

2. $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,m<-1$

1. $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,m<-1\,\land\,n>0$

1: $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,m<-1\,\land\,0<n<1$

Derivation: Nondegenerate secant recurrence 1a with $C\to 0$, $p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,m<-1\,\land\,0<n<1$, then

$$\int (a + b\,\text{Sec}[e + f\,x])^m\,(d\,\text{Sec}[e + f\,x])^n\,(A + B\,\text{Sec}[e + f\,x])\,dx \longrightarrow$$
$$\frac{d\,(A\,b - a\,B)\,\text{Tan}[e + f\,x]\,(a + b\,\text{Sec}[e + f\,x])^{m+1}\,(d\,\text{Sec}[e + f\,x])^{n-1}}{f\,(m + 1)\,(a^2 - b^2)} +$$
$$\frac{1}{(m + 1)\,(a^2 - b^2)} \int (a + b\,\text{Sec}[e + f\,x])^{m+1}\,(d\,\text{Sec}[e + f\,x])^{n-1} \cdot$$
$$(d\,(n - 1)\,(A\,b - a\,B) + d\,(a\,A - b\,B)\,(m + 1)\,\text{Sec}[e + f\,x] - d\,(A\,b - a\,B)\,(m + n + 1)\,\text{Sec}[e + f\,x]^2)\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -d*(A*b-a*B)*Cot[e+f*x]*(a+b*Csc[e+f*x])^(m+1)*(d*Csc[e+f*x])^(n-1)/(f*(m+1)*(a^2-b^2)) +
  1/((m+1)*(a^2-b^2))*Int[(a+b*Csc[e+f*x])^(m+1)*(d*Csc[e+f*x])^(n-1)*
    Simp[d*(n-1)*(A*b-a*B)+d*(a*A-b*B)*(m+1)*Csc[e+f*x]-d*(A*b-a*B)*(m+n+1)*Csc[e+f*x]^2,x],x] /;
FreeQ[{a,b,d,e,f,A,B},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0] && LtQ[m,-1] && LtQ[0,n,1]
```

2: $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,m<-1\,\land\,n>1$

Derivation: Nondegenerate secant recurrence 1a with $A\to a\,A,\ B\to A\,b+a\,B,\ C\to b\,B,\ m\to m-1,\ p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,m<-1\,\land\,n>1$, then

$$\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx\,\longrightarrow$$
$$-\left(\left(a\,d^2\,(A\,b-a\,B)\,\text{Tan}[e+f\,x]\,(a+b\,\text{Sec}[e+f\,x])^{m+1}\,(d\,\text{Sec}[e+f\,x])^{n-2}\right)\Big/\left(b\,f\,(m+1)\,\left(a^2-b^2\right)\right)\right)-$$
$$\frac{d}{b\,(m+1)\,\left(a^2-b^2\right)}\int (a+b\,\text{Sec}[e+f\,x])^{m+1}\,(d\,\text{Sec}[e+f\,x])^{n-2}\cdot$$
$$\left(a\,d\,(A\,b-a\,B)\,(n-2)+b\,d\,(A\,b-a\,B)\,(m+1)\,\text{Sec}[e+f\,x]-\left(a\,A\,b\,d\,(m+n)-d\,B\,\left(a^2\,(n-1)+b^2\,(m+1)\right)\right)\,\text{Sec}[e+f\,x]^2\right)\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  a*d^2*(A*b-a*B)*Cot[e+f*x]*(a+b*Csc[e+f*x])^(m+1)*(d*Csc[e+f*x])^(n-2)/(b*f*(m+1)*(a^2-b^2)) -
  d/(b*(m+1)*(a^2-b^2))*Int[(a+b*Csc[e+f*x])^(m+1)*(d*Csc[e+f*x])^(n-2)*
    Simp[a*d*(A*b-a*B)*(n-2)+b*d*(A*b-a*B)*(m+1)*Csc[e+f*x]-(a*A*b*d*(m+n)-d*B*(a^2*(n-1)+b^2*(m+1)))*Csc[e+f*x]^2,x],x] /;
FreeQ[{a,b,d,e,f,A,B},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0] && LtQ[m,-1] && GtQ[n,1]
```

2: $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,m<-1\,\land\,n\not>0$

Derivation: Nondegenerate secant recurrence 1c with $C\to 0,\ p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,m<-1\,\land\,n\not>0$, then

$$\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx\,\longrightarrow$$
$$-\frac{b\,(A\,b-a\,B)\,\text{Tan}[e+f\,x]\,(a+b\,\text{Sec}[e+f\,x])^{m+1}\,(d\,\text{Sec}[e+f\,x])^n}{a\,f\,(m+1)\,\left(a^2-b^2\right)}+$$

$$\frac{1}{a\,(m+1)\,\left(a^2-b^2\right)}\int(a+b\,\text{Sec}[e+f\,x])^{m+1}\,(d\,\text{Sec}[e+f\,x])^n\cdot$$
$$\left(A\,\left(a^2\,(m+1)-b^2\,(m+n+1)\right)+a\,b\,B\,n-a\,(A\,b-a\,B)\,(m+1)\,\text{Sec}[e+f\,x]+b\,(A\,b-a\,B)\,(m+n+2)\,\text{Sec}[e+f\,x]^2\right)\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  b*(A*b-a*B)*Cot[e+f*x]*(a+b*Csc[e+f*x])^(m+1)*(d*Csc[e+f*x])^n/(a*f*(m+1)*(a^2-b^2)) +
  1/(a*(m+1)*(a^2-b^2))*Int[(a+b*Csc[e+f*x])^(m+1)*(d*Csc[e+f*x])^n*
    Simp[A*(a^2*(m+1)-b^2*(m+n+1))+a*b*B*n-a*(A*b-a*B)*(m+1)*Csc[e+f*x]+b*(A*b-a*B)*(m+n+2)*Csc[e+f*x]^2,x],x] /;
FreeQ[{a,b,d,e,f,A,B,n},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0] && LtQ[m,-1] && Not[ILtQ[m+1/2,0] && ILtQ[n,0]]
```

3. $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,0<m<1$

1: $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,0<m<1\,\land\,n>0$

Derivation: Nondegenerate secant recurrence 1b with $A\to A\,c,\ B\to B\,c+A\,d,\ C\to B\,d,\ n\to n-1,\ p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,0<m<1\,\land\,n>0$, then

$$\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx\,\longrightarrow$$
$$\frac{B\,d\,\text{Tan}[e+f\,x]\,(a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^{n-1}}{f\,(m+n)}+$$
$$\frac{d}{m+n}\int (a+b\,\text{Sec}[e+f\,x])^{m-1}\,(d\,\text{Sec}[e+f\,x])^{n-1}\cdot$$
$$\left(a\,B\,(n-1)+(b\,B\,(m+n-1)+a\,A\,(m+n))\,\text{Sec}[e+f\,x]+(a\,B\,m+A\,b\,(m+n))\,\text{Sec}[e+f\,x]^2\right)dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -B*d*Cot[e+f*x]*(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^(n-1)/(f*(m+n)) +
  d/(m+n)*Int[(a+b*Csc[e+f*x])^(m-1)*(d*Csc[e+f*x])^(n-1)*
    Simp[a*B*(n-1)+(b*B*(m+n-1)+a*A*(m+n))*Csc[e+f*x]+(a*B*m+A*b*(m+n))*Csc[e+f*x]^2,x],x] /;
FreeQ[{a,b,d,e,f,A,B},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0] && LtQ[0,m,1] && GtQ[n,0]
```

2: $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,0<m<1\,\land\,n\leq -1$

Derivation: Nondegenerate secant recurrence 1a with $C\to 0,\ p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,0<m<1\,\land\,n\leq -1$, then

$$\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx\,\longrightarrow$$

$$-\frac{A\,\text{Tan}[e+f\,x]\,(a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n}{f\,n}-\frac{1}{d\,n}\int(a+b\,\text{Sec}[e+f\,x])^{m-1}\,(d\,\text{Sec}[e+f\,x])^{n+1}\cdot\left(A\,b\,m-a\,B\,n-(b\,B\,n+a\,A\,(n+1))\,\text{Sec}[e+f\,x]-A\,b\,(m+n+1)\,\text{Sec}[e+f\,x]^2\right)\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  A*Cot[e+f*x]*(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^n/(f*n) -
  1/(d*n)*Int[(a+b*Csc[e+f*x])^(m-1)*(d*Csc[e+f*x])^(n+1)*
    Simp[A*b*m-a*B*n-(b*B*n+a*A*(n+1))*Csc[e+f*x]-A*b*(m+n+1)*Csc[e+f*x]^2,x],x] /;
FreeQ[{a,b,d,e,f,A,B},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0] && LtQ[0,m,1] && LeQ[n,-1]
```

4: $\int(a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,n>1\,\land\,m+n\neq 0$

Derivation: Nondegenerate secant recurrence 1b with $A\to a\,A,\ B\to A\,b+a\,B,\ C\to b\,B,\ m\to m-1,\ p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,n>1\,\land\,m+n\neq 0$, then

$$\int(a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx\longrightarrow$$

$$\frac{B\,d^2\,\text{Tan}[e+f\,x]\,(a+b\,\text{Sec}[e+f\,x])^{m+1}\,(d\,\text{Sec}[e+f\,x])^{n-2}}{b\,f\,(m+n)}+\frac{d^2}{b\,(m+n)}\int(a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^{n-2}\,\left(a\,B\,(n-2)+B\,b\,(m+n-1)\,\text{Sec}[e+f\,x]+(A\,b\,(m+n)-a\,B\,(n-1))\,\text{Sec}[e+f\,x]^2\right)\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  -B*d^2*Cot[e+f*x]*(a+b*Csc[e+f*x])^(m+1)*(d*Csc[e+f*x])^(n-2)/(b*f*(m+n)) +
  d^2/(b*(m+n))*Int[(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^(n-2)*
    Simp[a*B*(n-2)+B*b*(m+n-1)*Csc[e+f*x]+(A*b*(m+n)-a*B*(n-1))*Csc[e+f*x]^2,x],x] /;
FreeQ[{a,b,d,e,f,A,B,m},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0] && GtQ[n,1] && NeQ[m+n,0] && Not[IGtQ[m,1]]
```

5: $\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,n\leq -1$

Derivation: Nondegenerate secant recurrence 1c with $C\to 0,\ p\to 0$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0\,\land\,n\leq -1$, then

$$\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^n\,(A+B\,\text{Sec}[e+f\,x])\,dx\,\longrightarrow$$
$$-\frac{A\,\text{Tan}[e+f\,x]\,(a+b\,\text{Sec}[e+f\,x])^{m+1}\,(d\,\text{Sec}[e+f\,x])^n}{a\,f\,n}+$$
$$\frac{1}{a\,d\,n}\int (a+b\,\text{Sec}[e+f\,x])^m\,(d\,\text{Sec}[e+f\,x])^{n+1}\,\left(a\,B\,n-A\,b\,(m+n+1)+A\,a\,(n+1)\,\text{Sec}[e+f\,x]+A\,b\,(m+n+2)\,\text{Sec}[e+f\,x]^2\right)dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  A*Cot[e+f*x]*(a+b*Csc[e+f*x])^(m+1)*(d*Csc[e+f*x])^n/(a*f*n) +
  1/(a*d*n)*Int[(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^(n+1)*
    Simp[a*B*n-A*b*(m+n+1)+A*a*(n+1)*Csc[e+f*x]+A*b*(m+n+2)*Csc[e+f*x]^2,x],x] /;
FreeQ[{a,b,d,e,f,A,B,m},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0] && LeQ[n,-1]
```

6: $\int \frac{A+B\,\text{Sec}[e+f\,x]}{\sqrt{d\,\text{Sec}[e+f\,x]}\,\sqrt{a+b\,\text{Sec}[e+f\,x]}}\,dx$ when $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0$

Derivation: Algebraic expansion

Basis: $\frac{A+B\,z}{\sqrt{d\,z}\,\sqrt{a+b\,z}}==\frac{A\,\sqrt{a+b\,z}}{a\,\sqrt{d\,z}}-\frac{(A\,b-a\,B)\,\sqrt{d\,z}}{a\,d\,\sqrt{a+b\,z}}$

Rule: If $A\,b-a\,B\neq 0\,\land\,a^2-b^2\neq 0$, then

$$\int \frac{A + B \operatorname{Sec}[e + f x]}{\sqrt{d \operatorname{Sec}[e + f x]}\ \sqrt{a + b \operatorname{Sec}[e + f x]}}\, dx \to \frac{A}{a} \int \frac{\sqrt{a + b \operatorname{Sec}[e + f x]}}{\sqrt{d \operatorname{Sec}[e + f x]}}\, dx - \frac{A b - a B}{a d} \int \frac{\sqrt{d \operatorname{Sec}[e + f x]}}{\sqrt{a + b \operatorname{Sec}[e + f x]}}\, dx$$

Program code:

```
Int[(A_+B_.*csc[e_.+f_.*x_])/(Sqrt[d_.*csc[e_.+f_.*x_]]*Sqrt[a_+b_.*csc[e_.+f_.*x_]]),x_Symbol] :=
  A/a*Int[Sqrt[a+b*Csc[e+f*x]]/Sqrt[d*Csc[e+f*x]],x] -
  (A*b-a*B)/(a*d)*Int[Sqrt[d*Csc[e+f*x]]/Sqrt[a+b*Csc[e+f*x]],x] /;
FreeQ[{a,b,d,e,f,A,B},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0]
```

7: $\int \frac{\sqrt{d \operatorname{Sec}[e + f x]}\ (A + B \operatorname{Sec}[e + f x])}{\sqrt{a + b \operatorname{Sec}[e + f x]}}\, dx$ when $A b - a B \neq 0 \land a^2 - b^2 \neq 0$

Derivation: Algebraic expansion

Rule: If $A b - a B \neq 0 \land a^2 - b^2 \neq 0$, then

$$\int \frac{\sqrt{d \operatorname{Sec}[e + f x]}\ (A + B \operatorname{Sec}[e + f x])}{\sqrt{a + b \operatorname{Sec}[e + f x]}}\, dx \to A \int \frac{\sqrt{d \operatorname{Sec}[e + f x]}}{\sqrt{a + b \operatorname{Sec}[e + f x]}}\, dx + \frac{B}{d} \int \frac{(d \operatorname{Sec}[e + f x])^{3/2}}{\sqrt{a + b \operatorname{Sec}[e + f x]}}\, dx$$

Program code:

```
Int[Sqrt[d_.*csc[e_.+f_.*x_]]*(A_+B_.*csc[e_.+f_.*x_])/Sqrt[a_+b_.*csc[e_.+f_.*x_]],x_Symbol] :=
  A*Int[Sqrt[d*Csc[e+f*x]]/Sqrt[a+b*Csc[e+f*x]],x] +
  B/d*Int[(d*Csc[e+f*x])^(3/2)/Sqrt[a+b*Csc[e+f*x]],x] /;
FreeQ[{a,b,d,e,f,A,B},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0]
```

8: $\int \frac{\sqrt{a+b\,\text{Sec}[e+f\,x]}\,(A+B\,\text{Sec}[e+f\,x])}{\sqrt{d\,\text{Sec}[e+f\,x]}}\,dx$ when $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2\neq 0$

Derivation: Algebraic expansion

Basis: $\frac{A+B\,z}{\sqrt{d\,z}} == \frac{B\,\sqrt{d\,z}}{d}+\frac{A}{\sqrt{d\,z}}$

Rule: If $A\,b-a\,B\neq 0\,\wedge\,a^2-b^2\neq 0$, then

$$\int \frac{\sqrt{a+b\,\text{Sec}[e+f\,x]}\,(A+B\,\text{Sec}[e+f\,x])}{\sqrt{d\,\text{Sec}[e+f\,x]}}\,dx \to \frac{B}{d}\int \sqrt{a+b\,\text{Sec}[e+f\,x]}\,\sqrt{d\,\text{Sec}[e+f\,x]}\,dx+A\int \frac{\sqrt{a+b\,\text{Sec}[e+f\,x]}}{\sqrt{d\,\text{Sec}[e+f\,x]}}\,dx$$

Program code:

```
Int[Sqrt[a_+b_.*csc[e_.+f_.*x_]]*(A_+B_.*csc[e_.+f_.*x_])/Sqrt[d_.*csc[e_.+f_.*x_]],x_Symbol] :=
  B/d*Int[Sqrt[a+b*Csc[e+f*x]]*Sqrt[d*Csc[e+f*x]],x] +
  A*Int[Sqrt[a+b*Csc[e+f*x]]/Sqrt[d*Csc[e+f*x]],x] /;
FreeQ[{a,b,d,e,f,A,B},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0]
```

9: $\int \frac{(d \operatorname{Sec}[e+f x])^n (A+B \operatorname{Sec}[e+f x])}{a+b \operatorname{Sec}[e+f x]} dx$ when $A b-a B \neq 0 \wedge a^2-b^2 \neq 0$

Derivation: Algebraic expansion

Basis: $\frac{A+B z}{a+b z} == \frac{A}{a} - \frac{(A b-a B)(d z)}{a d (a+b z)}$

Rule: If $A b-a B \neq 0 \wedge a^2-b^2 \neq 0$, then

$$\int \frac{(d \operatorname{Sec}[e+f x])^n (A+B \operatorname{Sec}[e+f x])}{a+b \operatorname{Sec}[e+f x]} dx \longrightarrow \frac{A}{a} \int (d \operatorname{Sec}[e+f x])^n dx - \frac{A b-a B}{a d} \int \frac{(d \operatorname{Sec}[e+f x])^{n+1}}{a+b \operatorname{Sec}[e+f x]} dx$$

Program code:

```
Int[(d_.*csc[e_.+f_.*x_])^n_*(A_+B_.*csc[e_.+f_.*x_])/(a_+b_.*csc[e_.+f_.*x_]),x_Symbol] :=
  A/a*Int[(d*Csc[e+f*x])^n,x] - (A*b-a*B)/(a*d)*Int[(d*Csc[e+f*x])^(n+1)/(a+b*Csc[e+f*x]),x] /;
FreeQ[{a,b,d,e,f,A,B,n},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0]
```

X: $\int (a+b \operatorname{Sec}[e+f x])^m (d \operatorname{Sec}[e+f x])^n (A+B \operatorname{Sec}[e+f x]) dx$ when $A b-a B \neq 0 \wedge a^2-b^2 \neq 0$

Rule: If $A b-a B \neq 0 \wedge a^2-b^2 \neq 0$, then

$$\int (a+b \operatorname{Sec}[e+f x])^m (d \operatorname{Sec}[e+f x])^n (A+B \operatorname{Sec}[e+f x]) dx \longrightarrow$$
$$\int (a+b \operatorname{Sec}[e+f x])^m (d \operatorname{Sec}[e+f x])^n (A+B \operatorname{Sec}[e+f x]) dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_*(d_.*csc[e_.+f_.*x_])^n_.*(A_+B_.*csc[e_.+f_.*x_]),x_Symbol] :=
  Unintegrable[(a+b*Csc[e+f*x])^m*(d*Csc[e+f*x])^n*(A+B*Csc[e+f*x]),x] /;
FreeQ[{a,b,d,e,f,A,B,m,n},x] && NeQ[A*b-a*B,0] && NeQ[a^2-b^2,0]
```

Rules for integrands of the form $(a+b\,\mathrm{Sec}[e+f\,x])^m\,(c+d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])^p$

1. $\int(a+b\,\mathrm{Sec}[e+f\,x])^m\,(c+d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])^p\,dx$ when $b\,c+a\,d==0\land a^2-b^2==0$

x: $\int(a+b\,\mathrm{Sec}[e+f\,x])^m\,(c+d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])^p\,dx$ when $b\,c+a\,d==0\land a^2-b^2==0\land m\in\mathbb{Z}$

Derivation: Algebraic simplification

Basis: If $b\,c+a\,d==0\land a^2-b^2==0$, then $(a+b\,\mathrm{Sec}[z])\,(c+d\,\mathrm{Sec}[z])==-a\,c\,\mathrm{Tan}[z]^2$

Rule: If $b\,c+a\,d==0\land a^2-b^2==0\land m\in\mathbb{Z}$, then

$$\int(a+b\,\mathrm{Sec}[e+f\,x])^m\,(c+d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])^p\,dx\;\rightarrow\;(-a\,c)^m\int\mathrm{Tan}[e+f\,x]^{2m}\,(c+d\,\mathrm{Sec}[e+f\,x])^{n-m}\,(A+B\,\mathrm{Sec}[e+f\,x])^p\,dx$$

Program code:

```
(* Int[(a_+b_.*csc[e_.+f_.*x_])^m_.*(c_+d_.*csc[e_.+f_.*x_])^n_.*(A_.+B_.*csc[e_.+f_.*x_])^p_.,x_Symbol] :=
  (-a*c)^m*Int[Cot[e+f*x]^(2*m)*(c+d*Csc[e+f*x])^(n-m)*(A+B*Csc[e+f*x])^p,x] /;
FreeQ[{a,b,c,d,e,f,A,B,n,p},x] && EqQ[b*c+a*d,0] && EqQ[a^2-b^2,0] && IntegerQ[m] &&
  Not[IntegerQ[n] && (LtQ[m,0] && GtQ[n,0] || LtQ[0,n,m] || LtQ[m,n,0])] *)
```

1: $\int(a+b\,\mathrm{Sec}[e+f\,x])^m\,(c+d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])^p\,dx$ when $b\,c+a\,d==0\land a^2-b^2==0\land(m\mid n\mid p)\in\mathbb{Z}$

Derivation: Algebraic simplification

Basis: If $b\,c+a\,d==0\land a^2-b^2==0$, then $(a+b\,\mathrm{Sec}[z])\,(c+d\,\mathrm{Sec}[z])==-a\,c\,\mathrm{Tan}[z]^2$

Rule: If $b\,c+a\,d==0\land a^2-b^2==0\land(m\mid n\mid p)\in\mathbb{Z}$, then

$$\int(a+b\,\mathrm{Sec}[e+f\,x])^m\,(c+d\,\mathrm{Sec}[e+f\,x])^n\,(A+B\,\mathrm{Sec}[e+f\,x])^p\,dx\;\rightarrow\;(-a\,c)^m\int\mathrm{Tan}[e+f\,x]^{2m}\,(c+d\,\mathrm{Sec}[e+f\,x])^{n-m}\,(A+B\,\mathrm{Sec}[e+f\,x])^p\,dx$$

$$\longrightarrow\ (-a\,c)^m \int \frac{\mathrm{Sin}[e+f\,x]^{2\,m}\,(d+c\,\mathrm{Cos}[e+f\,x])^{n-m}\,(B+A\,\mathrm{Cos}[e+f\,x])^p}{\mathrm{Cos}[e+f\,x]^{m+n+p}}\,dx$$

Program code:

```
Int[(a_+b_.*csc[e_.+f_.*x_])^m_.*(c_+d_.*csc[e_.+f_.*x_])^n_.*(A_.+B_.*csc[e_.+f_.*x_])^p_.,x_Symbol] :=
  (-a*c)^m*Int[Cos[e+f*x]^(2*m)*(d+c*Sin[e+f*x])^(n-m)*(B+A*Sin[e+f*x])^p/Sin[e+f*x]^(m+n+p),x] /;
FreeQ[{a,b,c,d,e,f,A,B,n,p},x] && EqQ[b*c+a*d,0] && EqQ[a^2-b^2,0] && IntegersQ[m,n,p]
```