Rules for integrands of the form $(f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n$

1. $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n \, dx$ when $e == c^2 d$

   1. $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n \, dx$ when $e == c^2 d \wedge n > 0$

      1. $\int x (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n \, dx$ when $e == c^2 d \wedge n > 0$

         1: $\int \frac{x (a + b \operatorname{ArcSinh}[c x])^n}{d + e x^2} \, dx$ when $e == c^2 d \wedge n \in \mathbb{Z}^+$

Derivation: Integration by substitution

Basis: If $e == c^2 d$, then $\frac{x}{d+e x^2} == \frac{1}{e} \operatorname{Subst}[\operatorname{Tanh}[x], x, \operatorname{ArcSinh}[c x]] \partial_x \operatorname{ArcSinh}[c x]$

Basis: If $c^2 d + e == 0$, then $\frac{x}{d+e x^2} == \frac{1}{e} \operatorname{Subst}[\operatorname{Coth}[x], x, \operatorname{ArcCosh}[c x]] \partial_x \operatorname{ArcCosh}[c x]$

Note: If $n \in \mathbb{Z}^+$, then $(a + b x)^n \operatorname{Tanh}[x]$ is integrable in closed-form.

Rule: If $e == c^2 d \wedge n \in \mathbb{Z}^+$, then

$$\int \frac{x (a + b \operatorname{ArcSinh}[c x])^n}{d + e x^2} \, dx \to \frac{1}{e} \operatorname{Subst}\left[\int (a + b x)^n \operatorname{Tanh}[x] \, dx, x, \operatorname{ArcSinh}[c x]\right]$$

Program code:

```
Int[x_*(a_.+b_.*ArcSinh[c_.*x_])^n_./(d_+e_.*x_^2),x_Symbol] :=
  1/e*Subst[Int[(a+b*x)^n*Tanh[x],x],x,ArcSinh[c*x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && IGtQ[n,0]
```

```
Int[x_*(a_.+b_.*ArcCosh[c_.*x_])^n_./(d_+e_.*x_^2),x_Symbol] :=
  1/e*Subst[Int[(a+b*x)^n*Coth[x],x],x,ArcCosh[c*x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[n,0]
```

2. $\int x \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right)^n \mathbb{d}x$ when $e == c^2\,d \wedge n > 0 \wedge p \neq -1$

1: $\int x \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right)^n \mathbb{d}x$ when $e == c^2\,d \wedge n > 0 \wedge p \neq -1 \wedge \left(p \in \mathbb{Z} \vee d > 0\right)$

Derivation: Integration by parts

Rule: If $e == c^2\,d \wedge n > 0 \wedge p \neq -1 \wedge \left(p \in \mathbb{Z} \vee d > 0\right)$, then

$$\int x \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right)^n \mathbb{d}x \longrightarrow \frac{\left(d + e\,x^2\right)^{p+1} \left(a + b\,\text{ArcSinh}[c\,x]\right)^n}{2\,e\,(p+1)} - \frac{b\,n\,d^p}{2\,c\,(p+1)} \int \left(1 + c^2\,x^2\right)^{p+\frac{1}{2}} \left(a + b\,\text{ArcSinh}[c\,x]\right)^{n-1} \mathbb{d}x$$

Program code:

```
(* Int[x_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  (d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n/(2*e*(p+1)) -
  b*n*d^p/(2*c*(p+1))*Int[(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,p},x] && EqQ[e,c^2*d] && GtQ[n,0] && NeQ[p,-1] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
Int[x_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (d+e*x^2)^(p+1)*(a+b*ArcCosh[c*x])^n/(2*e*(p+1)) -
  b*n*(-d)^p/(2*c*(p+1))*Int[(1+c*x)^(p+1/2)*(-1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && NeQ[p,-1] && IntegerQ[p]
```

```
(* Int[x_*(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(2*e1*e2*(p+1)) -
  b*n*(-d1*d2)^(p-1/2)/(2*c*(p+1))*Int[(-1+c^2*x^2)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,p},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && IntegerQ[p+1/2] && (GtQ[d1,0] && LtQ[d2,0]) *)
```

2: $\int x \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right)^n \mathbb{d}x$ when $e == c^2\,d \wedge n > 0 \wedge p \neq -1$

Derivation: Integration by parts and piecewise constant extraction

Basis: If $e == c^2\,d$, then $\partial_x \frac{\left(d+e\,x^2\right)^p}{\left(1+c^2\,x^2\right)^p} == 0$

Rule: If $e == c^2\,d \wedge n > 0 \wedge p \neq -1$, then

$$\int x \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right)^n \mathbb{d}x \longrightarrow \frac{\left(d + e\,x^2\right)^{p+1} \left(a + b\,\text{ArcSinh}[c\,x]\right)^n}{2\,e\,(p+1)} - \frac{b\,n\,d^{\text{IntPart}[p]} \left(d + e\,x^2\right)^{\text{FracPart}[p]}}{2\,c\,(p+1)\left(1 + c^2\,x^2\right)^{\text{FracPart}[p]}} \int \left(1 + c^2\,x^2\right)^{p+\frac{1}{2}} \left(a + b\,\text{ArcSinh}[c\,x]\right)^{n-1} \mathbb{d}x$$

Program code:

```
Int[x_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  (d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n/(2*e*(p+1)) -
  b*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(2*c*(p+1)*(1+c^2*x^2)^FracPart[p])*
    Int[(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,p},x] && EqQ[e,c^2*d] && GtQ[n,0] && NeQ[p,-1]
```

```
Int[x_*(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(2*e1*e2*(p+1)) -
  b*n*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(2*c*(p+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(-1+c^2*x^2)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,p},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && NeQ[p,-1] && IntegerQ[p+1/2]
```

```
Int[x_*(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(2*e1*e2*(p+1)) -
  b*n*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(2*c*(p+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(1+c*x)^(p+1/2)*(-1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,p},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && NeQ[p,-1]
```

2. $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$ when $e == c^2\,d \wedge n > 0 \wedge m+2\,p+3 == 0$

1: $\int \frac{(a+b\,\text{ArcSinh}[c\,x])^n}{x\,(d+e\,x^2)}\,dx$ when $e == c^2\,d \wedge n \in \mathbb{Z}^+$

Derivation: Integration by substitution

Basis: If $e == c^2\,d$, then $\frac{1}{x\,(d+e\,x^2)} == \frac{1}{d}\,\text{Subst}\left[\frac{1}{\text{Cosh}[x]\,\text{Sinh}[x]}, x, \text{ArcSinh}[c\,x]\right]\partial_x\,\text{ArcSinh}[c\,x]$

Rule: If $e == c^2\,d \wedge n \in \mathbb{Z}^+$, then

$$\int \frac{(a+b\,\text{ArcSinh}[c\,x])^n}{x\,(d+e\,x^2)}\,dx \to \frac{1}{d}\,\text{Subst}\left[\int \frac{(a+b\,x)^n}{\text{Cosh}[x]\,\text{Sinh}[x]}\,dx, x, \text{ArcSinh}[c\,x]\right]$$

Program code:

```
Int[(a_.+b_.*ArcSinh[c_.*x_])^n_./(x_*(d_+e_.*x_^2)),x_Symbol] :=
  1/d*Subst[Int[(a+b*x)^n/(Cosh[x]*Sinh[x]),x],x,ArcSinh[c*x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && IGtQ[n,0]
```

```
Int[(a_.+b_.*ArcCosh[c_.*x_])^n_./(x_*(d_+e_.*x_^2)),x_Symbol] :=
  -1/d*Subst[Int[(a+b*x)^n/(Cosh[x]*Sinh[x]),x],x,ArcCosh[c*x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[n,0]
```

2. $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$ when $e == c^2\,d \wedge n > 0 \wedge m+2\,p+3 == 0 \wedge m \neq -1$

1: $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$ when $e == c^2\,d \wedge n > 0 \wedge m+2\,p+3 == 0 \wedge m \neq -1 \wedge (p \in \mathbb{Z} \vee d > 0)$

Derivation: Integration by parts

Basis: If $m+2\,p+3 == 0$, then $(f\,x)^m\,(d+e\,x^2)^p == \partial_x \frac{(f\,x)^{m+1}\,(d+e\,x^2)^{p+1}}{d\,f\,(m+1)}$

Rule: If $e == c^2\,d \wedge n > 0 \wedge m+2\,p+3 == 0 \wedge m \neq -1 \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx \rightarrow \frac{(f\,x)^{m+1}\,(d+e\,x^2)^{p+1}\,(a+b\,\text{ArcSinh}[c\,x])^n}{d\,f\,(m+1)} - \frac{b\,c\,n\,d^p}{f\,(m+1)} \int (f\,x)^{m+1}\,(1+c^2\,x^2)^{p+\frac{1}{2}}\,(a+b\,\text{ArcSinh}[c\,x])^{n-1}\,dx$$

Program code:

```
(* Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n/(d*f*(m+1)) -
  b*c*n*d^p/(f*(m+1))*Int[(f*x)^(m+1)*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m,p},x] && EqQ[e,c^2*d] && GtQ[n,0] && EqQ[m+2*p+3,0] && NeQ[m,-1] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcCosh[c*x])^n/(d*f*(m+1)) +
  b*c*n*(-d)^p/(f*(m+1))*Int[(f*x)^(m+1)*(1+c*x)^(p+1/2)*(-1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && EqQ[m+2*p+3,0] && NeQ[m,-1] && IntegerQ[p]
```

```
(* Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(d1*d2*f*(m+1)) +
  b*c*n*(-d1*d2)^p/(f*(m+1))*Int[(f*x)^(m+1)*(-1+c^2*x^2)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m,p},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && EqQ[m+2*p+3,0] &&
  NeQ[m,-1] && IntegerQ[p+1/2] && (GtQ[d1,0] && LtQ[d2,0]) *)
```

2: $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$ when $e == c^2\,d \wedge n > 0 \wedge m+2\,p+3 == 0 \wedge m \neq -1$

Derivation: Integration by parts and piecewise constant extraction

Basis: If $m+2\,p+3 == 0$, then $(f\,x)^m\,(d+e\,x^2)^p == \partial_x \frac{(f\,x)^{m+1}\,(d+e\,x^2)^{p+1}}{d\,f\,(m+1)}$

Basis: If $e == c^2\,d$, then $\partial_x \frac{(d+e\,x^2)^p}{(1+c^2\,x^2)^p} == 0$

Rule: If $e == c^2\,d \wedge n > 0 \wedge m+2\,p+3 == 0 \wedge m \neq -1$, then

$$\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx \to \frac{(f\,x)^{m+1}\,(d+e\,x^2)^{p+1}\,(a+b\,\text{ArcSinh}[c\,x])^n}{d\,f\,(m+1)} - \frac{b\,c\,n\,d^{\text{IntPart}[p]}\,(d+e\,x^2)^{\text{FracPart}[p]}}{f\,(m+1)\,(1+c^2\,x^2)^{\text{FracPart}[p]}} \int (f\,x)^{m+1}\,(1+c^2\,x^2)^{p+\frac{1}{2}}\,(a+b\,\text{ArcSinh}[c\,x])^{n-1}\,dx$$

Program code:

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n/(d*f*(m+1)) -
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(f*(m+1)*(1+c^2*x^2)^FracPart[p])*
    Int[(f*x)^(m+1)*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m,p},x] && EqQ[e,c^2*d] && GtQ[n,0] && EqQ[m+2*p+3,0] && NeQ[m,-1]
```

```
Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(d1*d2*f*(m+1)) +
  b*c*n*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(f*(m+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^(m+1)*(-1+c^2*x^2)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m,p},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && EqQ[m+2*p+3,0] && NeQ[m,-1] && IntegerQ[p+1/2]
```

```
Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(d1*d2*f*(m+1)) +
  b*c*n*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(f*(m+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^(m+1)*(1+c*x)^(p+1/2)*(-1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m,p},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && EqQ[m+2*p+3,0] && NeQ[m,-1]
```

3. $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])\,dx$ when $e == c^2\,d \wedge p \in \mathbb{Z}^+$

1. $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])\,dx$ when $e == c^2\,d \wedge p \in \mathbb{Z}^+ \wedge \frac{m-1}{2} \in \mathbb{Z}^-$

1: $\int \frac{(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])}{x}\,dx$ when $e == c^2\,d \wedge p \in \mathbb{Z}^+$

Derivation: Inverted integration by parts

Rule: If $e == c^2\,d \wedge p \in \mathbb{Z}^+$, then

$$\int \frac{(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])}{x}\,dx \longrightarrow \frac{(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])}{2\,p} - \frac{b\,c\,d^p}{2\,p}\int (1+c^2\,x^2)^{p-\frac{1}{2}}\,dx + d\int \frac{(d+e\,x^2)^{p-1}\,(a+b\,\text{ArcSinh}[c\,x])}{x}\,dx$$

Program code:

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])/x_,x_Symbol] :=
  (d+e*x^2)^p*(a+b*ArcSinh[c*x])/(2*p) -
  b*c*d^p/(2*p)*Int[(1+c^2*x^2)^(p-1/2),x] +
  d*Int[(d+e*x^2)^(p-1)*(a+b*ArcSinh[c*x])/x,x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && IGtQ[p,0]
```

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_])/x_,x_Symbol] :=
  (d+e*x^2)^p*(a+b*ArcCosh[c*x])/(2*p) -
  b*c*(-d)^p/(2*p)*Int[(1+c*x)^(p-1/2)*(-1+c*x)^(p-1/2),x] +
  d*Int[(d+e*x^2)^(p-1)*(a+b*ArcCosh[c*x])/x,x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[p,0]
```

2: $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])\,dx$ when $e == c^2\,d \wedge p \in \mathbb{Z}^+ \wedge \frac{m+1}{2} \in \mathbb{Z}^-$

Derivation: Inverted integration by parts

Rule: If $e == c^2\,d \wedge p \in \mathbb{Z}^+ \wedge \frac{m+1}{2} \in \mathbb{Z}^-$, then

$$\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])\,dx \to \frac{(f\,x)^{m+1}\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])}{f\,(m+1)} - \frac{b\,c\,d^p}{f\,(m+1)}\int (f\,x)^{m+1}\,(1+c^2\,x^2)^{p-\frac{1}{2}}\,dx - \frac{2\,e\,p}{f^2\,(m+1)}\int (f\,x)^{m+2}\,(d+e\,x^2)^{p-1}\,(a+b\,\text{ArcSinh}[c\,x])\,dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_]),x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcSinh[c*x])/(f*(m+1)) -
  b*c*d^p/(f*(m+1))*Int[(f*x)^(m+1)*(1+c^2*x^2)^(p-1/2),x] -
  2*e*p/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^(p-1)*(a+b*ArcSinh[c*x]),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[e,c^2*d] && IGtQ[p,0] && ILtQ[(m+1)/2,0]
```

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_]),x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcCosh[c*x])/(f*(m+1)) -
  b*c*(-d)^p/(f*(m+1))*Int[(f*x)^(m+1)*(1+c*x)^(p-1/2)*(-1+c*x)^(p-1/2),x] -
  2*e*p/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^(p-1)*(a+b*ArcCosh[c*x]),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && IGtQ[p,0] && ILtQ[(m+1)/2,0]
```

2: $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])\,dx$ when $e == c^2\,d \wedge p \in \mathbb{Z}^+$

Derivation: Integration by parts

Rule: If $e == c^2\,d \wedge p \in \mathbb{Z}^+$, let $u = \int (f\,x)^m\,(d+e\,x^2)^p\,dx$, then

$$\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])\,dx \longrightarrow u\,(a+b\,\text{ArcSinh}[c\,x]) - b\,c\int \frac{u}{\sqrt{1+c^2\,x^2}}\,dx$$

Program code:

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(f*x)^m*(d+e*x^2)^p,x]},
  Dist[a+b*ArcSinh[c*x],u,x] - b*c*Int[SimplifyIntegrand[u/Sqrt[1+c^2*x^2],x],x]] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[e,c^2*d] && IGtQ[p,0]
```

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(f*x)^m*(d+e*x^2)^p,x]},
  Dist[a+b*ArcCosh[c*x],u,x] - b*c*Int[SimplifyIntegrand[u/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x],x]] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && IGtQ[p,0]
```

4. $\int x^m \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right) dx$ when $e == c^2\,d \wedge p - \frac{1}{2} \in \mathbb{Z} \wedge \left(\frac{m+1}{2} \in \mathbb{Z}^+ \vee \frac{m+2\,p+3}{2} \in \mathbb{Z}^-\right)$

1: $\int x^m \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right) dx$ when $e == c^2\,d \wedge p - \frac{1}{2} \in \mathbb{Z} \wedge \left(\frac{m+1}{2} \in \mathbb{Z}^+ \vee \frac{m+2\,p+3}{2} \in \mathbb{Z}^-\right) \wedge p \neq -\frac{1}{2} \wedge d > 0$

Derivation: Integration by parts

Note: If $p - \frac{1}{2} \in \mathbb{Z} \wedge \left(\frac{m+1}{2} \in \mathbb{Z}^+ \vee \frac{m+2\,p+3}{2} \in \mathbb{Z}^-\right)$, then $\int x^m \left(1 + c^2\,x^2\right)^p dx$ is an algebraic function not involving logarithms, inverse trig or inverse hyperbolic functions.

Rule: If $e == c^2\,d \wedge p - \frac{1}{2} \in \mathbb{Z} \wedge \left(\frac{m+1}{2} \in \mathbb{Z}^+ \vee \frac{m+2\,p+3}{2} \in \mathbb{Z}^-\right) \wedge p \neq -\frac{1}{2} \wedge d > 0$, let $u = \int x^m \left(1 + c^2\,x^2\right)^p dx$, then

$$\int x^m \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right) dx \longrightarrow d^p\,u \left(a + b\,\text{ArcSinh}[c\,x]\right) - b\,c\,d^p \int \frac{u}{\sqrt{1 + c^2\,x^2}}\,dx$$

Program code:

```
Int[x_^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSinh[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[x^m*(1+c^2*x^2)^p,x]},
  Dist[d^p*(a+b*ArcSinh[c*x]),u,x] - b*c*d^p*Int[SimplifyIntegrand[u/Sqrt[1+c^2*x^2],x],x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && IntegerQ[p-1/2] && (IGtQ[(m+1)/2,0] || ILtQ[(m+2*p+3)/2,0]) && NeQ[p,-1/2] && GtQ[d,0]
```

```
Int[x_^m_*(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[x^m*(1+c*x)^p*(-1+c*x)^p,x]},
  Dist[(-d1*d2)^p*(a+b*ArcCosh[c*x]),u,x] - b*c*(-d1*d2)^p*Int[SimplifyIntegrand[u/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x],x]] /;
FreeQ[{a,b,c,d1,e1,d2,e2},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && IntegerQ[p-1/2] &&
  (IGtQ[(m+1)/2,0] || ILtQ[(m+2*p+3)/2,0]) && NeQ[p,-1/2] && GtQ[d1,0] && LtQ[d2,0]
```

2: $\int x^m \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right) dx$ when $e == c^2\,d \wedge p + \frac{1}{2} \in \mathbb{Z}^+ \wedge \left(\frac{m+1}{2} \in \mathbb{Z}^+ \vee \frac{m+2\,p+3}{2} \in \mathbb{Z}^-\right)$

Derivation: Integration by parts and piecewise constant extraction

Note: If $p + \frac{1}{2} \in \mathbb{Z} \wedge \left(\frac{m+1}{2} \in \mathbb{Z}^+ \vee \frac{m+2\,p+3}{2} \in \mathbb{Z}^-\right)$, then $\int x^m \left(1 + c^2\,x^2\right)^p dx$ is an algebraic function not involving logarithms, inverse trig or inverse hyperbolic functions.

Rule: If $e == c^2\,d \wedge p + \frac{1}{2} \in \mathbb{Z}^+ \wedge \left(\frac{m+1}{2} \in \mathbb{Z}^+ \vee \frac{m+2\,p+3}{2} \in \mathbb{Z}^-\right)$, let $u = \int x^m \left(1 + c^2\,x^2\right)^p dx$, then

$$\int x^m \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right) dx \to \left(a + b\,\text{ArcSinh}[c\,x]\right) \int x^m \left(d + e\,x^2\right)^p dx - \frac{b\,c\,d^{p-\frac{1}{2}}\sqrt{d + e\,x^2}}{\sqrt{1 + c^2\,x^2}} \int \frac{u}{\sqrt{1 + c^2\,x^2}}\,dx$$

Program code:

```
Int[x_^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSinh[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[x^m*(1+c^2*x^2)^p,x]},
  (a+b*ArcSinh[c*x])*Int[x^m*(d+e*x^2)^p,x] -
  b*c*d^(p-1/2)*Sqrt[d+e*x^2]/Sqrt[1+c^2*x^2]*Int[SimplifyIntegrand[u/Sqrt[1+c^2*x^2],x],x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && IGtQ[p+1/2,0] && (IGtQ[(m+1)/2,0] || ILtQ[(m+2*p+3)/2,0])
```

```
Int[x_^m_*(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[x^m*(1+c*x)^p*(-1+c*x)^p,x]},
  (a+b*ArcCosh[c*x])*Int[x^m*(d1+e1*x)^p*(d2+e2*x)^p,x] -
  b*c*(-d1*d2)^(p-1/2)*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]/(Sqrt[1+c*x]*Sqrt[-1+c*x])*
    Int[SimplifyIntegrand[u/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x],x]] /;
FreeQ[{a,b,c,d1,e1,d2,e2},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && IGtQ[p+1/2,0] && (IGtQ[(m+1)/2,0] || ILtQ[(m+2*p+3)/2,0])
```

5. $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e == c^2 d \wedge n > 0 \wedge p > 0$

1. $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e == c^2 d \wedge n > 0 \wedge p > 0 \wedge m < -1$

1: $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e == c^2 d \wedge n > 0 \wedge p > 0 \wedge m < -1 \wedge (p \in \mathbb{Z} \vee d > 0)$

Derivation: Inverted integration by parts

Rule: If $e == c^2 d \wedge n > 0 \wedge p > 0 \wedge m < -1 \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx \longrightarrow \frac{(f x)^{m+1} (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n}{f (m + 1)} - \frac{2 e p}{f^2 (m + 1)} \int (f x)^{m+2} (d + e x^2)^{p-1} (a + b \operatorname{ArcSinh}[c x])^n dx - \frac{b c n d^p}{f (m + 1)} \int (f x)^{m+1} (1 + c^2 x^2)^{p-\frac{1}{2}} (a + b \operatorname{ArcSinh}[c x])^{n-1} dx$$

Program code:

```
(* Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcSinh[c*x])^n/(f*(m+1)) -
  2*e*p/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^(p-1)*(a+b*ArcSinh[c*x])^n,x] -
  b*c*n*d^p/(f*(m+1))*Int[(f*x)^(m+1)*(1+c^2*x^2)^(p-1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[e,c^2*d] && GtQ[n,0] && GtQ[p,0] && LtQ[m,-1] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcCosh[c*x])^n/(f*(m+1)) -
  2*e*p/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^(p-1)*(a+b*ArcCosh[c*x])^n,x] -
  b*c*n*(-d)^p/(f*(m+1))*Int[(f*x)^(m+1)*(1+c*x)^(p-1/2)*(-1+c*x)^(p-1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0] && LtQ[m,-1] && IntegerQ[p]
```

```
(* Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^n/(f*(m+1)) -
  2*e1*e2*p/(f^2*(m+1))*Int[(f*x)^(m+2)*(d1+e1*x)^(p-1)*(d2+e2*x)^(p-1)*(a+b*ArcCosh[c*x])^n,x] -
  b*c*n*(-d1*d2)^p/(f*(m+1))*Int[(f*x)^(m+1)*(-1+c^2*x^2)^(p-1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && GtQ[p,0] && LtQ[m,-1] &&
  IntegerQ[p-1/2] && (GtQ[d1,0] && LtQ[d2,0]) *)
```

2. $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e == c^2 d \wedge n > 0 \wedge p > 0 \wedge m < -1$

1: $\int (f x)^m \sqrt{d + e x^2} (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e == c^2 d \wedge n > 0 \wedge m < -1$

Derivation: Inverted integration by parts

Note: The piecewise constant factor in the second integral reduces the degree of d in the resulting antiderivative.

Rule: If $e == c^2 d \wedge n > 0 \wedge m < -1$, then

$$\int (f x)^m \sqrt{d + e x^2} (a + b \operatorname{ArcSinh}[c x])^n dx \rightarrow$$

$$\frac{(f x)^{m+1} \sqrt{d + e x^2} (a + b \operatorname{ArcSinh}[c x])^n}{f (m + 1)} -$$

$$\frac{b c n \sqrt{d + e x^2}}{f (m + 1) \sqrt{1 + c^2 x^2}} \int (f x)^{m+1} (a + b \operatorname{ArcSinh}[c x])^{n-1} dx - \frac{c^2 \sqrt{d + e x^2}}{f^2 (m + 1) \sqrt{1 + c^2 x^2}} \int \frac{(f x)^{m+2} (a + b \operatorname{ArcSinh}[c x])^n}{\sqrt{1 + c^2 x^2}} dx$$

Program code:

```
Int[(f_.*x_)^m_*Sqrt[d_+e_.*x_^2]*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*Sqrt[d+e*x^2]*(a+b*ArcSinh[c*x])^n/(f*(m+1)) -
  b*c*n*Sqrt[d+e*x^2]/(f*(m+1)*Sqrt[1+c^2*x^2])*Int[(f*x)^(m+1)*(a+b*ArcSinh[c*x])^(n-1),x] -
  c^2*Sqrt[d+e*x^2]/(f^2*(m+1)*Sqrt[1+c^2*x^2])*Int[(f*x)^(m+2)*(a+b*ArcSinh[c*x])^n/Sqrt[1+c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[e,c^2*d] && GtQ[n,0] && LtQ[m,-1]
```

```
Int[(f_.*x_)^m_*Sqrt[d1_+e1_.*x_]*Sqrt[d2_+e2_.*x_]*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]*(a+b*ArcCosh[c*x])^n/(f*(m+1)) -
  b*c*n*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]/(f*(m+1)*Sqrt[1+c*x]*Sqrt[-1+c*x])*
    Int[(f*x)^(m+1)*(a+b*ArcCosh[c*x])^(n-1),x] -
  c^2*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]/(f^2*(m+1)*Sqrt[1+c*x]*Sqrt[-1+c*x])*
    Int[((f*x)^(m+2)*(a+b*ArcCosh[c*x])^n)/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && LtQ[m,-1]
```

2: $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$ when $e == c^2\,d \wedge n > 0 \wedge p > 0 \wedge m < -1$

Derivation: Inverted integration by parts

Rule: If $e == c^2\,d \wedge n > 0 \wedge p > 0 \wedge m < -1$, then

$$\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx \longrightarrow$$
$$\frac{(f\,x)^{m+1}\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n}{f\,(m+1)} - \frac{2\,e\,p}{f^2\,(m+1)}\int (f\,x)^{m+2}\,(d+e\,x^2)^{p-1}\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx -$$
$$\frac{b\,c\,n\,d^{\text{IntPart}[p]}\,(d+e\,x^2)^{\text{FracPart}[p]}}{f\,(m+1)\,(1+c^2\,x^2)^{\text{FracPart}[p]}}\int (f\,x)^{m+1}\,(1+c^2\,x^2)^{p-\frac{1}{2}}\,(a+b\,\text{ArcSinh}[c\,x])^{n-1}\,dx$$

Program code:

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcSinh[c*x])^n/(f*(m+1)) -
  2*e*p/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^(p-1)*(a+b*ArcSinh[c*x])^n,x] -
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(f*(m+1)*(1+c^2*x^2)^FracPart[p])*
    Int[(f*x)^(m+1)*(1+c^2*x^2)^(p-1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[e,c^2*d] && GtQ[n,0] && GtQ[p,0] && LtQ[m,-1]
```

```
Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^n/(f*(m+1)) -
  2*e1*e2*p/(f^2*(m+1))*Int[(f*x)^(m+2)*(d1+e1*x)^(p-1)*(d2+e2*x)^(p-1)*(a+b*ArcCosh[c*x])^n,x] -
  b*c*n*(-d1*d2)^(p-1/2)*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]/(f*(m+1)*Sqrt[1+c*x]*Sqrt[-1+c*x])*
    Int[(f*x)^(m+1)*(-1+c^2*x^2)^(p-1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && GtQ[p,0] && LtQ[m,-1] && IntegerQ[p-1/2]
```

2. $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e == c^2 d \wedge n > 0 \wedge p > 0 \wedge m \not< -1$

1: $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e == c^2 d \wedge n > 0 \wedge p > 0 \wedge m \not< -1 \wedge (p \in \mathbb{Z} \vee d > 0)$

Derivation: Inverted integration by parts

Rule: If $e == c^2 d \wedge n > 0 \wedge p > 0 \wedge m \not< -1 \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx \longrightarrow \frac{(f x)^{m+1} (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n}{f (m + 2 p + 1)} + \frac{2 d p}{m + 2 p + 1} \int (f x)^m (d + e x^2)^{p-1} (a + b \operatorname{ArcSinh}[c x])^n dx - \frac{b c n d^p}{f (m + 2 p + 1)} \int (f x)^{m+1} (1 + c^2 x^2)^{p-\frac{1}{2}} (a + b \operatorname{ArcSinh}[c x])^{n-1} dx$$

Program code:

```
(* Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcSinh[c*x])^n/(f*(m+2*p+1)) +
  2*d*p/(m+2*p+1)*Int[(f*x)^m*(d+e*x^2)^(p-1)*(a+b*ArcSinh[c*x])^n,x] -
  b*c*n*d^p/(f*(m+2*p+1))*Int[(f*x)^(m+1)*(1+c^2*x^2)^(p-1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[e,c^2*d] && GtQ[n,0] && GtQ[p,0] && Not[LtQ[m,-1]] && (IntegerQ[p] || GtQ[d,0]) && (RationalQ[m] ||
```

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcCosh[c*x])^n/(f*(m+2*p+1)) +
  2*d*p/(m+2*p+1)*Int[(f*x)^m*(d+e*x^2)^(p-1)*(a+b*ArcCosh[c*x])^n,x] -
  b*c*n*(-d)^p/(f*(m+2*p+1))*Int[(f*x)^(m+1)*(1+c*x)^(p-1/2)*(-1+c*x)^(p-1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0] && Not[LtQ[m,-1]] && IntegerQ[p] && (RationalQ[m] || EqQ[n,1])
```

2. $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n \, dx$ when $e == c^2 d \wedge n > 0 \wedge p > 0 \wedge m \not< -1$

1: $\int (f x)^m \sqrt{d + e x^2} (a + b \operatorname{ArcSinh}[c x])^n \, dx$ when $e == c^2 d \wedge n > 0 \wedge m \not< -1$

Derivation: Inverted integration by parts

Note: The piecewise constant factor in the second integral reduces the degree of d in the resulting antiderivative.

Rule: If $e == c^2 d \wedge n > 0 \wedge m \not< -1$, then

$$\int (f x)^m \sqrt{d + e x^2} (a + b \operatorname{ArcSinh}[c x])^n \, dx \rightarrow \frac{(f x)^{m+1} \sqrt{d + e x^2} (a + b \operatorname{ArcSinh}[c x])^n}{f (m + 2)} -$$

$$\frac{b c n \sqrt{d + e x^2}}{f (m + 2) \sqrt{1 + c^2 x^2}} \int (f x)^{m+1} (a + b \operatorname{ArcSinh}[c x])^{n-1} \, dx + \frac{\sqrt{d + e x^2}}{(m + 2) \sqrt{1 + c^2 x^2}} \int \frac{(f x)^m (a + b \operatorname{ArcSinh}[c x])^n}{\sqrt{1 + c^2 x^2}} \, dx$$

Program code:

```
Int[(f_.*x_)^m_*Sqrt[d_+e_.*x_^2]*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*Sqrt[d+e*x^2]*(a+b*ArcSinh[c*x])^n/(f*(m+2)) -
  b*c*n*Sqrt[d+e*x^2]/(f*(m+2)*Sqrt[1+c^2*x^2])*Int[(f*x)^(m+1)*(a+b*ArcSinh[c*x])^(n-1),x] +
  Sqrt[d+e*x^2]/((m+2)*Sqrt[1+c^2*x^2])*Int[(f*x)^m*(a+b*ArcSinh[c*x])^n/Sqrt[1+c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[e,c^2*d] && GtQ[n,0] && Not[LtQ[m,-1]] && (RationalQ[m] || EqQ[n,1])
```

```
Int[(f_.*x_)^m_*Sqrt[d1_+e1_.*x_]*Sqrt[d2_+e2_.*x_]*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]*(a+b*ArcCosh[c*x])^n/(f*(m+2)) -
  b*c*n*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]/(f*(m+2)*Sqrt[1+c*x]*Sqrt[-1+c*x])*Int[(f*x)^(m+1)*(a+b*ArcCosh[c*x])^(n-1),x] -
  Sqrt[d1+e1*x]*Sqrt[d2+e2*x]/((m+2)*Sqrt[1+c*x]*Sqrt[-1+c*x])*Int[(f*x)^m*(a+b*ArcCosh[c*x])^n/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && Not[LtQ[m,-1]] && (RationalQ[m] || EqQ[n,1])
```

2: $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$ when $e == c^2\,d \wedge n > 0 \wedge p > 0 \wedge m \not< -1$

Derivation: Inverted integration by parts

Rule: If $e == c^2\,d \wedge n > 0 \wedge p > 0 \wedge m \not< -1$, then

$$\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx \longrightarrow$$

$$\frac{(f\,x)^{m+1}\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n}{f\,(m+2\,p+1)} + \frac{2\,d\,p}{m+2\,p+1}\int (f\,x)^m\,(d+e\,x^2)^{p-1}\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx -$$

$$\frac{b\,c\,n\,d^{\text{IntPart}[p]}\,(d+e\,x^2)^{\text{FracPart}[p]}}{f\,(m+2\,p+1)\,(1+c^2\,x^2)^{\text{FracPart}[p]}}\int (f\,x)^{m+1}\,(1+c^2\,x^2)^{p-\frac{1}{2}}\,(a+b\,\text{ArcSinh}[c\,x])^{n-1}\,dx$$

Program code:

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^p*(a+b*ArcSinh[c*x])^n/(f*(m+2*p+1)) +
  2*d*p/(m+2*p+1)*Int[(f*x)^m*(d+e*x^2)^(p-1)*(a+b*ArcSinh[c*x])^n,x] -
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(f*(m+2*p+1)*(1+c^2*x^2)^FracPart[p])*
    Int[(f*x)^(m+1)*(1+c^2*x^2)^(p-1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[e,c^2*d] && GtQ[n,0] && GtQ[p,0] && Not[LtQ[m,-1]] && (RationalQ[m] || EqQ[n,1])
```

```
Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^n/(f*(m+2*p+1)) +
  2*d1*d2*p/(m+2*p+1)*Int[(f*x)^m*(d1+e1*x)^(p-1)*(d2+e2*x)^(p-1)*(a+b*ArcCosh[c*x])^n,x] -
  b*c*n*(-d1*d2)^(p-1/2)*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]/(f*(m+2*p+1)*Sqrt[1+c*x]*Sqrt[-1+c*x])*
    Int[(f*x)^(m+1)*(-1+c^2*x^2)^(p-1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && GtQ[p,0] && Not[LtQ[m,-1]] &&
  IntegerQ[p-1/2] && (RationalQ[m] || EqQ[n,1])
```

6. $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e == c^2 d \wedge n > 0 \wedge m < -1 \wedge m \in \mathbb{Z}$

1: $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e == c^2 d \wedge n > 0 \wedge m < -1 \wedge m \in \mathbb{Z} \wedge (p \in \mathbb{Z} \vee d > 0)$

Rule: If $e == c^2 d \wedge n > 0 \wedge m < -1 \wedge m \in \mathbb{Z} \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx \rightarrow \frac{(f x)^{m+1} (d + e x^2)^{p+1} (a + b \operatorname{ArcSinh}[c x])^n}{d f (m+1)} - \frac{c^2 (m + 2 p + 3)}{f^2 (m+1)} \int (f x)^{m+2} (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx - \frac{b c n d^p}{f (m+1)} \int (f x)^{m+1} (1 + c^2 x^2)^{p+\frac{1}{2}} (a + b \operatorname{ArcSinh}[c x])^{n-1} dx$$

Programcode:

```
(* Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n/(d*f*(m+1)) -
  c^2*(m+2*p+3)/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^p*(a+b*ArcSinh[c*x])^n,x] -
  b*c*n*d^p/(f*(m+1))*Int[(f*x)^(m+1)*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[e,c^2*d] && GtQ[n,0] && LtQ[m,-1] && IntegerQ[m] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcCosh[c*x])^n/(d*f*(m+1)) +
  c^2*(m+2*p+3)/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^p*(a+b*ArcCosh[c*x])^n,x] +
  b*c*n*(-d)^p/(f*(m+1))*Int[(f*x)^(m+1)*(1+c*x)^(p+1/2)*(-1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[m,-1] && IntegerQ[m] && IntegerQ[p]
```

2: $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$ when $e == c^2\,d \wedge n > 0 \wedge m < -1 \wedge m \in \mathbb{Z}$

Rule: If $e == c^2\,d \wedge n > 0 \wedge m < -1 \wedge m \in \mathbb{Z}$, then

$$\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx \longrightarrow$$
$$\frac{(f\,x)^{m+1}\,(d+e\,x^2)^{p+1}\,(a+b\,\text{ArcSinh}[c\,x])^n}{d\,f\,(m+1)} - \frac{c^2\,(m+2\,p+3)}{f^2\,(m+1)}\int (f\,x)^{m+2}\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx -$$
$$\frac{b\,c\,n\,d^{\text{IntPart}[p]}\,(d+e\,x^2)^{\text{FracPart}[p]}}{f\,(m+1)\,(1+c^2\,x^2)^{\text{FracPart}[p]}}\int (f\,x)^{m+1}\,(1+c^2\,x^2)^{p+\frac{1}{2}}\,(a+b\,\text{ArcSinh}[c\,x])^{n-1}\,dx$$

Programcode:

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n/(d*f*(m+1)) -
  c^2*(m+2*p+3)/(f^2*(m+1))*Int[(f*x)^(m+2)*(d+e*x^2)^p*(a+b*ArcSinh[c*x])^n,x] -
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(f*(m+1)*(1+c^2*x^2)^FracPart[p])*
    Int[(f*x)^(m+1)*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[e,c^2*d] && GtQ[n,0] && LtQ[m,-1] && IntegerQ[m]
```

```
Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(d1*d2*f*(m+1)) +
  c^2*(m+2*p+3)/(f^2*(m+1))*Int[(f*x)^(m+2)*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^n,x] +
  b*c*n*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(f*(m+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^(m+1)*(-1+c^2*x^2)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,p},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && LtQ[m,-1] && IntegerQ[m] && IntegerQ[p+1/2]
```

```
Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (f*x)^(m+1)*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(d1*d2*f*(m+1)) +
  c^2*(m+2*p+3)/(f^2*(m+1))*Int[(f*x)^(m+2)*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^n,x] +
  b*c*n*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(f*(m+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^(m+1)*(1+c*x)^(p+1/2)*(-1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,p},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && LtQ[m,-1] && IntegerQ[m]
```

7. $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n \, dx$ when $e == c^2 d \wedge n > 0 \wedge p < -1$

1. $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n \, dx$ when $e == c^2 d \wedge n > 0 \wedge p < -1 \wedge m > 1$

1: $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n \, dx$ when $e == c^2 d \wedge n > 0 \wedge p < -1 \wedge m > 1 \wedge (p \in \mathbb{Z} \vee d > 0)$

Derivation: Integration by parts

Basis: $x (d + e x^2)^p == \partial_x \frac{(d+e x^2)^{p+1}}{2 e (p+1)}$

Rule: If $e == c^2 d \wedge n > 0 \wedge p < -1 \wedge m > 1 \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n \, dx \to \frac{f (f x)^{m-1} (d + e x^2)^{p+1} (a + b \operatorname{ArcSinh}[c x])^n}{2 e (p + 1)} - \frac{f^2 (m-1)}{2 e (p+1)} \int (f x)^{m-2} (d + e x^2)^{p+1} (a + b \operatorname{ArcSinh}[c x])^n \, dx - \frac{b f n d^p}{2 c (p+1)} \int (f x)^{m-1} (1 + c^2 x^2)^{p+\frac{1}{2}} (a + b \operatorname{ArcSinh}[c x])^{n-1} \, dx$$

Program code:

```
(* Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  f*(f*x)^(m-1)*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n/(2*e*(p+1)) -
  f^2*(m-1)/(2*e*(p+1))*Int[(f*x)^(m-2)*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n,x] -
  b*f*n*d^p/(2*c*(p+1))*Int[(f*x)^(m-1)*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[e,c^2*d] && GtQ[n,0] && LtQ[p,-1] && GtQ[m,1] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  f*(f*x)^(m-1)*(d+e*x^2)^(p+1)*(a+b*ArcCosh[c*x])^n/(2*e*(p+1)) -
  f^2*(m-1)/(2*e*(p+1))*Int[(f*x)^(m-2)*(d+e*x^2)^(p+1)*(a+b*ArcCosh[c*x])^n,x] -
  b*f*n*(-d)^p/(2*c*(p+1))*Int[(f*x)^(m-1)*(1+c*x)^(p+1/2)*(-1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && GtQ[m,1] && IntegerQ[p]
```

2: $\int (f x)^m \left(d + e x^2\right)^p \left(a + b\,\text{ArcSinh}[c x]\right)^n dx$ when $e == c^2 d \wedge n > 0 \wedge p < -1 \wedge m > 1$

Derivation: Integration by parts

Basis: $x \left(d + e x^2\right)^p == \partial_x \frac{\left(d+e x^2\right)^{p+1}}{2 e (p+1)}$

Rule: If $e == c^2 d \wedge n > 0 \wedge p < -1 \wedge m > 1$, then

$$\int (f x)^m \left(d + e x^2\right)^p \left(a + b\,\text{ArcSinh}[c x]\right)^n dx \rightarrow$$

$$\frac{f (f x)^{m-1} \left(d + e x^2\right)^{p+1} \left(a + b\,\text{ArcSinh}[c x]\right)^n}{2 e (p+1)} - \frac{f^2 (m-1)}{2 e (p+1)} \int (f x)^{m-2} \left(d + e x^2\right)^{p+1} \left(a + b\,\text{ArcSinh}[c x]\right)^n dx -$$

$$\frac{b f n d^{\text{IntPart}[p]} \left(d + e x^2\right)^{\text{FracPart}[p]}}{2 c (p+1) \left(1 + c^2 x^2\right)^{\text{FracPart}[p]}} \int (f x)^{m-1} \left(1 + c^2 x^2\right)^{p+\frac{1}{2}} \left(a + b\,\text{ArcSinh}[c x]\right)^{n-1} dx$$

Program code:

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  f*(f*x)^(m-1)*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n/(2*e*(p+1)) -
  f^2*(m-1)/(2*e*(p+1))*Int[(f*x)^(m-2)*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n,x] -
  b*f*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(2*c*(p+1)*(1+c^2*x^2)^FracPart[p])*
    Int[(f*x)^(m-1)*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[e,c^2*d] && GtQ[n,0] && LtQ[p,-1] && GtQ[m,1]
```

```
Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  f*(f*x)^(m-1)*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(2*e1*e2*(p+1)) -
  f^2*(m-1)/(2*e1*e2*(p+1))*Int[(f*x)^(m-2)*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n,x] -
  b*f*n*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(2*c*(p+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^(m-1)*(-1+c^2*x^2)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && LtQ[p,-1] && GtQ[m,1] && IntegerQ[p+1/2]
```

```
Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  f*(f*x)^(m-1)*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(2*e1*e2*(p+1)) -
  f^2*(m-1)/(2*e1*e2*(p+1))*Int[(f*x)^(m-2)*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n,x] -
  b*f*n*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(2*c*(p+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^(m-1)*(1+c*x)^(p+1/2)*(-1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && LtQ[p,-1] && Not[IntegerQ[p]] && GtQ[m,1]
```

2. $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e == c^2 d \wedge n > 0 \wedge p < -1 \wedge m \neq 1$

1: $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e == c^2 d \wedge n > 0 \wedge p < -1 \wedge m \neq 1 \wedge (p \in \mathbb{Z} \vee d > 0)$

Rule: If $e == c^2 d \wedge n > 0 \wedge p < -1 \wedge m \neq 1 \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx \rightarrow$$
$$-\frac{(f x)^{m+1} (d + e x^2)^{p+1} (a + b \operatorname{ArcSinh}[c x])^n}{2 d f (p + 1)} +$$
$$\frac{m + 2 p + 3}{2 d (p + 1)} \int (f x)^m (d + e x^2)^{p+1} (a + b \operatorname{ArcSinh}[c x])^n dx + \frac{b c n d^p}{2 f (p + 1)} \int (f x)^{m+1} (1 + c^2 x^2)^{p+\frac{1}{2}} (a + b \operatorname{ArcSinh}[c x])^{n-1} dx$$

Program code:

```
(* Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  -(f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n/(2*d*f*(p+1)) +
  (m+2*p+3)/(2*d*(p+1))*Int[(f*x)^m*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n,x] +
  b*c*n*d^p/(2*f*(p+1))*Int[(f*x)^(m+1)*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[e,c^2*d] && GtQ[n,0] && LtQ[p,-1] && Not[GtQ[m,1]] && (IntegerQ[p] || GtQ[d,0]) &&
  (IntegerQ[m] || IntegerQ[p] || EqQ[n,1]) *)
```

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  -(f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcCosh[c*x])^n/(2*d*f*(p+1)) +
  (m+2*p+3)/(2*d*(p+1))*Int[(f*x)^m*(d+e*x^2)^(p+1)*(a+b*ArcCosh[c*x])^n,x] -
  b*c*n*(-d)^p/(2*f*(p+1))*Int[(f*x)^(m+1)*(1+c*x)^(p+1/2)*(-1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && Not[GtQ[m,1]] && IntegerQ[p]
```

2: $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n \, dx$ when $e == c^2 d \wedge n > 0 \wedge p < -1 \wedge m \neq 1$

Rule: If $e == c^2 d \wedge n > 0 \wedge p < -1 \wedge m \neq 1$, then

$$\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n \, dx \rightarrow$$
$$-\frac{(f x)^{m+1} (d + e x^2)^{p+1} (a + b \operatorname{ArcSinh}[c x])^n}{2 d f (p+1)} + \frac{m + 2p + 3}{2 d (p+1)} \int (f x)^m (d + e x^2)^{p+1} (a + b \operatorname{ArcSinh}[c x])^n \, dx +$$
$$\frac{b c n d^{\text{IntPart}[p]} (d + e x^2)^{\text{FracPart}[p]}}{2 f (p+1) (1 + c^2 x^2)^{\text{FracPart}[p]}} \int (f x)^{m+1} (1 + c^2 x^2)^{p+\frac{1}{2}} (a + b \operatorname{ArcSinh}[c x])^{n-1} \, dx$$

Program code:

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  -(f*x)^(m+1)*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n/(2*d*f*(p+1)) +
  (m+2*p+3)/(2*d*(p+1))*Int[(f*x)^m*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n,x] +
  b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(2*f*(p+1)*(1+c^2*x^2)^FracPart[p])*
    Int[(f*x)^(m+1)*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[e,c^2*d] && GtQ[n,0] && LtQ[p,-1] && Not[GtQ[m,1]] && (IntegerQ[m] || IntegerQ[p] || EqQ[n,1])
```

```
Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  -(f*x)^(m+1)*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(2*d1*d2*f*(p+1)) +
  (m+2*p+3)/(2*d1*d2*(p+1))*Int[(f*x)^m*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n,x] -
  b*c*n*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(2*f*(p+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^(m+1)*(-1+c^2*x^2)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && LtQ[p,-1] && Not[GtQ[m,1]] &&
  (IntegerQ[m] || EqQ[n,1]) && IntegerQ[p+1/2]
```

```
Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  -(f*x)^(m+1)*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(2*d1*d2*f*(p+1)) +
  (m+2*p+3)/(2*d1*d2*(p+1))*Int[(f*x)^m*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n,x] -
  b*c*n*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(2*f*(p+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^(m+1)*(1+c*x)^(p+1/2)*(-1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && LtQ[p,-1] && Not[GtQ[m,1]] &&
  (IntegerQ[m] || IntegerQ[p] || EqQ[n,1])
```

8. $\int \frac{(f x)^m (a + b\,\text{ArcSinh}[c x])^n}{\sqrt{d + e x^2}}\,dx$ when $e == c^2 d \wedge n > 0$

1. $\int \frac{(f x)^m (a + b\,\text{ArcSinh}[c x])^n}{\sqrt{d + e x^2}}\,dx$ when $e == c^2 d \wedge n > 0 \wedge m > 1$

1: $\int \frac{(f x)^m (a + b\,\text{ArcSinh}[c x])^n}{\sqrt{d + e x^2}}\,dx$ when $e == c^2 d \wedge n > 0 \wedge m > 1 \wedge d > 0$

Rule: If $e == c^2 d \wedge n > 0 \wedge m > 1 \wedge d > 0$, then

$$\int \frac{(f x)^m (a + b\,\text{ArcSinh}[c x])^n}{\sqrt{d + e x^2}}\,dx \rightarrow \frac{f (f x)^{m-1} \sqrt{d + e x^2} (a + b\,\text{ArcSinh}[c x])^n}{e m} - \frac{b f n}{c m \sqrt{d}} \int (f x)^{m-1} (a + b\,\text{ArcSinh}[c x])^{n-1}\,dx - \frac{f^2 (m-1)}{c^2 m} \int \frac{(f x)^{m-2} (a + b\,\text{ArcSinh}[c x])^n}{\sqrt{d + e x^2}}\,dx$$

Program code:

```
(* Int[(f_.*x_)^m_*(a_.+b_.*ArcSinh[c_.*x_])^n_./Sqrt[d_+e_.*x_^2],x_Symbol] :=
  f*(f*x)^(m-1)*Sqrt[d+e*x^2]*(a+b*ArcSinh[c*x])^n/(e*m) -
  b*f*n*Sqrt[1+c^2*x^2]/(c*m*Sqrt[d+e*x^2])*Int[(f*x)^(m-1)*(a+b*ArcSinh[c*x])^(n-1),x] -
  f^2*(m-1)/(c^2*m)*Int[((f*x)^(m-2)*(a+b*ArcSinh[c*x])^n)/Sqrt[d+e*x^2],x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[e,c^2*d] && GtQ[n,0] && GtQ[m,1] && GtQ[d,0] && IntegerQ[m] *)
```

```
(* Int[(f_.*x_)^m_*(a_.+b_.*ArcCosh[c_.*x_])^n_./(Sqrt[d1_+e1_.*x_]*Sqrt[d2_+e2_.*x_]),x_Symbol] :=
  f*(f*x)^(m-1)*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]*(a+b*ArcCosh[c*x])^n/(e1*e2*m) +
  b*f*n*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]/(c*d1*d2*m*Sqrt[1+c*x]*Sqrt[-1+c*x])*Int[(f*x)^(m-1)*(a+b*ArcCosh[c*x])^(n-1),x] +
  f^2*(m-1)/(c^2*m)*Int[(f*x)^(m-2)*(a+b*ArcCosh[c*x])^n/(Sqrt[d1+e1*x]*Sqrt[d2+e2*x]),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && GtQ[m,1] && IntegerQ[m] *)
```

2: $\int \frac{(f x)^m (a + b \operatorname{ArcSinh}[c x])^n}{\sqrt{d + e x^2}} dx$ when $e == c^2 d \wedge n > 0 \wedge m > 1$

Rule: If $e == c^2 d \wedge n > 0 \wedge m > 1$, then

$$\int \frac{(f x)^m (a + b \operatorname{ArcSinh}[c x])^n}{\sqrt{d + e x^2}} dx \rightarrow \frac{f (f x)^{m-1} \sqrt{d + e x^2} (a + b \operatorname{ArcSinh}[c x])^n}{e m} - \frac{b f n \sqrt{1 + c^2 x^2}}{c m \sqrt{d + e x^2}} \int (f x)^{m-1} (a + b \operatorname{ArcSinh}[c x])^{n-1} dx - \frac{f^2 (m-1)}{c^2 m} \int \frac{(f x)^{m-2} (a + b \operatorname{ArcSinh}[c x])^n}{\sqrt{d + e x^2}} dx$$

Program code:

```
Int[(f_.*x_)^m_*(a_.+b_.*ArcSinh[c_.*x_])^n_./Sqrt[d_+e_.*x_^2],x_Symbol] :=
  f*(f*x)^(m-1)*Sqrt[d+e*x^2]*(a+b*ArcSinh[c*x])^n/(e*m) -
  b*f*n*Sqrt[1+c^2*x^2]/(c*m*Sqrt[d+e*x^2])*Int[(f*x)^(m-1)*(a+b*ArcSinh[c*x])^(n-1),x] -
  f^2*(m-1)/(c^2*m)*Int[((f*x)^(m-2)*(a+b*ArcSinh[c*x])^n)/Sqrt[d+e*x^2],x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[e,c^2*d] && GtQ[n,0] && GtQ[m,1] && IntegerQ[m]
```

```
Int[(f_.*x_)^m_*(a_.+b_.*ArcCosh[c_.*x_])^n_./(Sqrt[d1_+e1_.*x_]*Sqrt[d2_+e2_.*x_]),x_Symbol] :=
  f*(f*x)^(m-1)*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]*(a+b*ArcCosh[c*x])^n/(e1*e2*m) +
  b*f*n*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]/(c*d1*d2*m*Sqrt[1+c*x]*Sqrt[-1+c*x])*Int[(f*x)^(m-1)*(a+b*ArcCosh[c*x])^(n-1),x] +
  f^2*(m-1)/(c^2*m)*Int[(f*x)^(m-2)*(a+b*ArcCosh[c*x])^n/(Sqrt[d1+e1*x]*Sqrt[d2+e2*x]),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && GtQ[m,1] && IntegerQ[m]
```

2: $\int \frac{x^m \left(a + b\,\text{ArcSinh}[c\,x]\right)^n}{\sqrt{d + e\,x^2}}\,dx$ when $e == c^2\,d \wedge d > 0 \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{Z}$

Derivation: Integration by substitution

Basis: If $e == c^2\,d \wedge d > 0 \wedge m \in \mathbb{Z}$, then $\frac{x^m}{\sqrt{d+e\,x^2}} == \frac{1}{c^{m+1}\sqrt{d}}\,\text{Subst}[\text{Sinh}[x]^m, x, \text{ArcSinh}[c\,x]]\,\partial_x \text{ArcSinh}[c\,x]$

Note: If $n \in \mathbb{Z}^+$, then $(a + b\,x)^n\,\text{Sinh}[x]$ is integrable in closed-form.

Rule: If $e == c^2\,d \wedge d > 0 \wedge n \in \mathbb{Z}^+ \wedge m \in \mathbb{Z}$, then

$$\int \frac{x^m \left(a + b\,\text{ArcSinh}[c\,x]\right)^n}{\sqrt{d + e\,x^2}}\,dx \to \frac{1}{c^{m+1}\sqrt{d}}\,\text{Subst}\left[\int (a + b\,x)^n\,\text{Sinh}[x]^m\,dx, x, \text{ArcSinh}[c\,x]\right]$$

Program code:

```
Int[x_^m_*(a_.+b_.*ArcSinh[c_.*x_])^n_./Sqrt[d_+e_.*x_^2],x_Symbol] :=
  1/(c^(m+1)*Sqrt[d])*Subst[Int[(a+b*x)^n*Sinh[x]^m,x],x,ArcSinh[c*x]] /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && GtQ[d,0] && IGtQ[n,0] && IntegerQ[m]
```

```
Int[x_^m_*(a_.+b_.*ArcCosh[c_.*x_])^n_./(Sqrt[d1_+e1_.*x_]*Sqrt[d2_+e2_.*x_]),x_Symbol] :=
  1/(c^(m+1)*Sqrt[-d1*d2])*Subst[Int[(a+b*x)^n*Cosh[x]^m,x],x,ArcCosh[c*x]] /;
FreeQ[{a,b,c,d1,e1,d2,e2},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && IGtQ[n,0] && GtQ[d1,0] && LtQ[d2,0] && IntegerQ[m]
```

3: $\int \frac{(f\,x)^m \left(a + b\,\text{ArcSinh}[c\,x]\right)}{\sqrt{d + e\,x^2}}\,dx$ when $e == c^2\,d \wedge d > 0 \wedge m \notin \mathbb{Z}$

Rule: If $e == c^2\,d \wedge d > 0 \wedge m \notin \mathbb{Z}$, then

$$\int \frac{(f\,x)^m \left(a + b\,\text{ArcSinh}[c\,x]\right)}{\sqrt{d + e\,x^2}}\,dx \to$$

$$\frac{1}{\sqrt{d}\ f\ (m+1)}(f\,x)^{m+1}\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{Hypergeometric2F1}\left[\frac{1}{2},\ \frac{1+m}{2},\ \frac{3+m}{2},\ -c^2\,x^2\right] -$$
$$\left(b\,c\,(f\,x)^{m+2}\,\text{HypergeometricPFQ}\left[\left\{1,\ 1+\frac{m}{2},\ 1+\frac{m}{2}\right\},\ \left\{\frac{3}{2}+\frac{m}{2},\ 2+\frac{m}{2}\right\},\ -c^2\,x^2\right]\right)\Big/\left(\sqrt{d}\ f^2\ (m+1)\ (m+2)\right)$$

Program code:

```
Int[(f_.*x_)^m_*(a_.+b_.*ArcSinh[c_.*x_])/Sqrt[d_+e_.*x_^2],x_Symbol] :=
  (f*x)^(m+1)*(a+b*ArcSinh[c*x])*Hypergeometric2F1[1/2,(1+m)/2,(3+m)/2,-c^2*x^2]/(Sqrt[d]*f*(m+1)) -
  b*c*(f*x)^(m+2)*HypergeometricPFQ[{1,1+m/2,1+m/2},{3/2+m/2,2+m/2},-c^2*x^2]/(Sqrt[d]*f^2*(m+1)*(m+2)) /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[e,c^2*d] && GtQ[d,0] && Not[IntegerQ[m]]
```

```
Int[(f_.*x_)^m_*(a_.+b_.*ArcCosh[c_.*x_])/(Sqrt[d1_+e1_.*x_]*Sqrt[d2_+e2_.*x_]),x_Symbol] :=
  (f*x)^(m+1)*Sqrt[1-c^2*x^2]*(a+b*ArcCosh[c*x])*Hypergeometric2F1[1/2,(1+m)/2,(3+m)/2,c^2*x^2]/
    (f*(m+1)*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]) +
  b*c*(f*x)^(m+2)*HypergeometricPFQ[{1,1+m/2,1+m/2},{3/2+m/2,2+m/2},c^2*x^2]/(Sqrt[-d1*d2]*f^2*(m+1)*(m+2)) /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[d1,0] && LtQ[d2,0] && Not[IntegerQ[m]]
```

4: $\int \frac{(f x)^m (a + b\,\text{ArcSinh}[c x])^n}{\sqrt{d + e x^2}}\,dx$ when $e == c^2 d \wedge n > 0 \wedge d \not> 0$

Derivation: Piecewise constant extraction

Basis: If $e == c^2 d$, then $\partial_x \frac{\sqrt{1+c^2 x^2}}{\sqrt{d+e x^2}} == 0$

Rule: If $e == c^2 d \wedge n > 0 \wedge d \not> 0$, then

$$\int \frac{(f x)^m (a + b\,\text{ArcSinh}[c x])^n}{\sqrt{d + e x^2}}\,dx \to \frac{\sqrt{1 + c^2 x^2}}{\sqrt{d + e x^2}} \int \frac{(f x)^m (a + b\,\text{ArcSinh}[c x])^n}{\sqrt{1 + c^2 x^2}}\,dx$$

Program code:

```
Int[(f_.*x_)^m_*(a_.+b_.*ArcSinh[c_.*x_])^n_./Sqrt[d_+e_.*x_^2],x_Symbol] :=
  Sqrt[1+c^2*x^2]/Sqrt[d+e*x^2]*Int[(f*x)^m*(a+b*ArcSinh[c*x])^n/Sqrt[1+c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[e,c^2*d] && GtQ[n,0] && Not[GtQ[d,0]] && (IntegerQ[m] || EqQ[n,1])
```

```
Int[(f_.*x_)^m_*(a_.+b_.*ArcCosh[c_.*x_])^n_./(Sqrt[d1_+e1_.*x_]*Sqrt[d2_+e2_.*x_]),x_Symbol] :=
  Sqrt[1+c*x]*Sqrt[-1+c*x]/(Sqrt[d1+e1*x]*Sqrt[d2+e2*x])*Int[(f*x)^m*(a+b*ArcCosh[c*x])^n/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && Not[GtQ[d1,0] && LtQ[d2,0]] && (IntegerQ[m] || E
```

9. $\int (f x)^m (d + e x^2)^p (a + b\,\text{ArcSinh}[c x])^n\,dx$ when $e == c^2 d \wedge n > 0 \wedge m > 1 \wedge m + 2p + 1 \neq 0$

1: $\int (f x)^m (d + e x^2)^p (a + b\,\text{ArcSinh}[c x])^n\,dx$ when $e == c^2 d \wedge n > 0 \wedge m > 1 \wedge m + 2p + 1 \neq 0 \wedge (p \in \mathbb{Z} \vee d > 0)$

Rule: If $e == c^2 d \wedge n > 0 \wedge m > 1 \wedge m + 2p + 1 \neq 0 \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f x)^m (d + e x^2)^p (a + b\,\text{ArcSinh}[c x])^n\,dx \to \frac{f (f x)^{m-1} (d + e x^2)^{p+1} (a + b\,\text{ArcSinh}[c x])^n}{e (m + 2p + 1)} -$$

$$\frac{f^2\,(m-1)}{c^2\,(m+2\,p+1)}\int (f\,x)^{m-2}\,\left(d+e\,x^2\right)^p\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,dx-\frac{b\,f\,n\,d^p}{c\,(m+2\,p+1)}\int (f\,x)^{m-1}\,\left(1+c^2\,x^2\right)^{p+\frac{1}{2}}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^{n-1}\,dx$$

Program code:

```
(* Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  f*(f*x)^(m-1)*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n/(e*(m+2*p+1)) -
  f^2*(m-1)/(c^2*(m+2*p+1))*Int[(f*x)^(m-2)*(d+e*x^2)^p*(a+b*ArcSinh[c*x])^n,x] -
  b*f*n*d^p/(c*(m+2*p+1))*Int[(f*x)^(m-1)*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[e,c^2*d] && GtQ[n,0] && GtQ[m,1] && NeQ[m+2*p+1,0] && (IntegerQ[p] || GtQ[d,0]) && IntegerQ[m] *)
```

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  f*(f*x)^(m-1)*(d+e*x^2)^(p+1)*(a+b*ArcCosh[c*x])^n/(e*(m+2*p+1)) +
   f^2*(m-1)/(c^2*(m+2*p+1))*Int[(f*x)^(m-2)*(d+e*x^2)^p*(a+b*ArcCosh[c*x])^n,x] -
  b*f*n*(-d)^p/(c*(m+2*p+1))*Int[(f*x)^(m-1)*(1+c*x)^(p+1/2)*(-1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[m,1] && NeQ[m+2*p+1,0] && IntegerQ[p] && IntegerQ[m]
```

2: $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n \, dx$ when $e == c^2 d \wedge n > 0 \wedge m > 1 \wedge m + 2p + 1 \neq 0$

Rule: If $e == c^2 d \wedge n > 0 \wedge m > 1 \wedge m + 2p + 1 \neq 0$, then

$$\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n \, dx \rightarrow$$
$$\frac{f (f x)^{m-1} (d + e x^2)^{p+1} (a + b \operatorname{ArcSinh}[c x])^n}{e (m + 2p + 1)} - \frac{f^2 (m-1)}{c^2 (m + 2p + 1)} \int (f x)^{m-2} (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n \, dx -$$
$$\frac{b f n d^{\operatorname{IntPart}[p]} (d + e x^2)^{\operatorname{FracPart}[p]}}{c (m + 2p + 1) (1 + c^2 x^2)^{\operatorname{FracPart}[p]}} \int (f x)^{m-1} (1 + c^2 x^2)^{p+\frac{1}{2}} (a + b \operatorname{ArcSinh}[c x])^{n-1} \, dx$$

Program code:

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  f*(f*x)^(m-1)*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n/(e*(m+2*p+1)) -
  f^2*(m-1)/(c^2*(m+2*p+1))*Int[(f*x)^(m-2)*(d+e*x^2)^p*(a+b*ArcSinh[c*x])^n,x] -
  b*f*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(c*(m+2*p+1)*(1+c^2*x^2)^FracPart[p])*
    Int[(f*x)^(m-1)*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e,f,p},x] && EqQ[e,c^2*d] && GtQ[n,0] && GtQ[m,1] && NeQ[m+2*p+1,0] && IntegerQ[m]
```

```
Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  f*(f*x)^(m-1)*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(e1*e2*(m+2*p+1)) +
  f^2*(m-1)/(c^2*(m+2*p+1))*Int[(f*x)^(m-2)*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^n,x] -
  b*f*n*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(c*(m+2*p+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^(m-1)*(-1+c^2*x^2)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,p},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && GtQ[m,1] && NeQ[m+2*p+1,0] && IntegerQ[m] && Int
```

```
Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  f*(f*x)^(m-1)*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(e1*e2*(m+2*p+1)) +
  f^2*(m-1)/(c^2*(m+2*p+1))*Int[(f*x)^(m-2)*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^n,x] -
  b*f*n*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(c*(m+2*p+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^(m-1)*(1+c*x)^(p+1/2)*(-1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,p},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[n,0] && GtQ[m,1] && NeQ[m+2*p+1,0] && IntegerQ[m]
```

2. $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e == c^2 d \wedge n < -1$

1. $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e == c^2 d \wedge n < -1 \wedge m + 2 p + 1 == 0$

1: $\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e == c^2 d \wedge n < -1 \wedge m + 2 p + 1 == 0 \wedge (p \in \mathbb{Z} \vee d > 0)$

Derivation: Integration by parts

Basis: $\frac{(a+b \operatorname{ArcSinh}[c x])^n}{\sqrt{1+c^2 x^2}} == \partial_x \frac{(a+b \operatorname{ArcSinh}[c x])^{n+1}}{b c (n+1)}$

Rule: If $e == c^2 d \wedge n < -1 \wedge m + 2 p + 1 == 0 \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f x)^m (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx \rightarrow$$

$$\frac{d^p (f x)^m (1 + c^2 x^2)^{p+\frac{1}{2}} (a + b \operatorname{ArcSinh}[c x])^{n+1}}{b c (n + 1)} - \frac{f m d^p}{b c (n + 1)} \int (f x)^{m-1} (1 + c^2 x^2)^{p-\frac{1}{2}} (a + b \operatorname{ArcSinh}[c x])^{n+1} dx$$

Program code:

```
(* Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_,x_Symbol] :=
  d^p*(f*x)^m*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n+1)/(b*c*(n+1)) -
  f*m*d^p/(b*c*(n+1))*Int[(f*x)^(m-1)*(1+c^2*x^2)^(p-1/2)*(a+b*ArcSinh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f,m,p},x] && EqQ[e,c^2*d] && LtQ[n,-1] && EqQ[m+2*p+1,0] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_,x_Symbol] :=
  (f*x)^m*Sqrt[1+c*x]*Sqrt[-1+c*x]*(d+e*x^2)^p*(a+b*ArcCosh[c*x])^(n+1)/(b*c*(n+1)) +
  f*m*(-d)^p/(b*c*(n+1))*Int[(f*x)^(m-1)*(1+c*x)^(p-1/2)*(-1+c*x)^(p-1/2)*(a+b*ArcCosh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f,m,p},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && EqQ[m+2*p+1,0] && IntegerQ[p]
```

2: $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$ when $e == c^2\,d \wedge n < -1 \wedge m+2\,p+1 == 0$

Derivation: Integration by parts

Basis: $\frac{(a+b\,\text{ArcSinh}[c\,x])^n}{\sqrt{1+c^2\,x^2}} == \partial_x \frac{(a+b\,\text{ArcSinh}[c\,x])^{n+1}}{b\,c\,(n+1)}$

Rule: If $e == c^2\,d \wedge n < -1 \wedge m+2\,p+1 == 0$, then

$$\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx \to \frac{(f\,x)^m\sqrt{1+c^2\,x^2}\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^{n+1}}{b\,c\,(n+1)} - \frac{f\,m\,d^{\text{IntPart}[p]}\,(d+e\,x^2)^{\text{FracPart}[p]}}{b\,c\,(n+1)\,(1+c^2\,x^2)^{\text{FracPart}[p]}}\int (f\,x)^{m-1}\,(1+c^2\,x^2)^{p-\frac{1}{2}}\,(a+b\,\text{ArcSinh}[c\,x])^{n+1}\,dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_,x_Symbol] :=
  (f*x)^m*Sqrt[1+c^2*x^2]*(d+e*x^2)^p*(a+b*ArcSinh[c*x])^(n+1)/(b*c*(n+1)) -
  f*m*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(b*c*(n+1)*(1+c^2*x^2)^FracPart[p])*
    Int[(f*x)^(m-1)*(1+c^2*x^2)^(p-1/2)*(a+b*ArcSinh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f,m,p},x] && EqQ[e,c^2*d] && LtQ[n,-1] && EqQ[m+2*p+1,0]
```

```
Int[(f_.*x_)^m_.*(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_,x_Symbol] :=
  (f*x)^m*Sqrt[1+c*x]*Sqrt[-1+c*x]*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^(n+1)/(b*c*(n+1)) +
  f*m*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(b*c*(n+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^(m-1)*(-1+c^2*x^2)^(p-1/2)*(a+b*ArcCosh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m,p},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && LtQ[n,-1] && EqQ[m+2*p+1,0] && IntegerQ[p-1/2]
```

```
Int[(f_.*x_)^m_.*(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_,x_Symbol] :=
  (f*x)^m*Sqrt[1+c*x]*Sqrt[-1+c*x]*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^(n+1)/(b*c*(n+1)) +
  f*m*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(b*c*(n+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^(m-1)*(1+c*x)^(p-1/2)*(-1+c*x)^(p-1/2)*(a+b*ArcCosh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m,p},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && LtQ[n,-1] && EqQ[m+2*p+1,0]
```

2: $\int \frac{(f\,x)^m\,(a+b\,\text{ArcSinh}[c\,x])^n}{\sqrt{d+e\,x^2}}\,dx$ when $e == c^2\,d \wedge n < -1 \wedge d > 0$

Derivation: Integration by parts

Basis: If $e == c^2\,d \wedge d > 0$, then $\frac{(a+b\,\text{ArcSinh}[c\,x])^n}{\sqrt{d+e\,x^2}} == \partial_x \frac{(a+b\,\text{ArcSinh}[c\,x])^{n+1}}{b\,c\,\sqrt{d}\,(n+1)}$

Rule: If $e == c^2\,d \wedge n < -1 \wedge d > 0$, then

$$\int \frac{(f\,x)^m\,(a+b\,\text{ArcSinh}[c\,x])^n}{\sqrt{d+e\,x^2}}\,dx \to \frac{(f\,x)^m\,(a+b\,\text{ArcSinh}[c\,x])^{n+1}}{b\,c\,\sqrt{d}\,(n+1)} - \frac{f\,m}{b\,c\,\sqrt{d}\,(n+1)} \int (f\,x)^{m-1}\,(a+b\,\text{ArcSinh}[c\,x])^{n+1}\,dx$$

Program code:

```
Int[(f_.*x_)^m_.*(a_.+b_.*ArcSinh[c_.*x_])^n_/Sqrt[d_+e_.*x_^2],x_Symbol] :=
  (f*x)^m*(a+b*ArcSinh[c*x])^(n+1)/(b*c*Sqrt[d]*(n+1)) -
  f*m/(b*c*Sqrt[d]*(n+1))*Int[(f*x)^(m-1)*(a+b*ArcSinh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[e,c^2*d] && LtQ[n,-1] && GtQ[d,0]
```

```
Int[(f_.*x_)^m_.*(a_.+b_.*ArcCosh[c_.*x_])^n_/(Sqrt[d1_+e1_.*x_]*Sqrt[d2_+e2_.*x_]),x_Symbol] :=
  (f*x)^m*(a+b*ArcCosh[c*x])^(n+1)/(b*c*Sqrt[-d1*d2]*(n+1)) -
  (f*m)/(b*c*Sqrt[-d1*d2]*(n+1))*Int[(f*x)^(m-1)*(a+b*ArcCosh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && LtQ[n,-1] && GtQ[d1,0] && LtQ[d2,0]
```

x: $\int \frac{(f x)^m (a + b\,\text{ArcSinh}[c x])^n}{\sqrt{d + e x^2}}\,dx$ when $e == c^2 d \wedge n < -1 \wedge d \not> 0$

Derivation: Piecewise constant extraction

Basis: If $e == c^2 d$, then $\partial_x \frac{\sqrt{1+c^2 x^2}}{\sqrt{d+e x^2}} == 0$

Rule: If $e == c^2 d \wedge n < -1 \wedge d \not> 0$, then

$$\int \frac{(f x)^m (a + b\,\text{ArcSinh}[c x])^n}{\sqrt{d + e x^2}}\,dx \to \frac{\sqrt{1 + c^2 x^2}}{\sqrt{d + e x^2}} \int \frac{(f x)^m (a + b\,\text{ArcSinh}[c x])^n}{\sqrt{1 + c^2 x^2}}\,dx$$

Program code:

```
(* Int[(f_.*x_)^m_.*(a_.+b_.*ArcSinh[c_.*x_])^n_/Sqrt[d_+e_.*x_^2],x_Symbol] :=
  Sqrt[1+c^2*x^2]/Sqrt[d+e*x^2]*Int[(f*x)^m*(a+b*ArcSinh[c*x])^n/Sqrt[1+c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,f,m},x] && EqQ[e,c^2*d] && LtQ[n,-1] && Not[GtQ[d,0]] *)
```

```
(* Int[(f_.*x_)^m_.*(a_.+b_.*ArcCosh[c_.*x_])^n_/(Sqrt[d1_+e1_.*x_]*Sqrt[d2_+e2_.*x_]),x_Symbol] :=
  Sqrt[1+c*x]*Sqrt[-1+c*x]/(Sqrt[d1+e1*x]*Sqrt[d2+e2*x])*Int[(f*x)^m*(a+b*ArcCosh[c*x])^n/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && LtQ[n,-1] && Not[GtQ[d1,0] && LtQ[d2,0]] *)
```

3. $\int (f x)^m (d + e x^2)^p (a + b\,\text{ArcSinh}[c x])^n\,dx$ when $e == c^2 d \wedge n < -1 \wedge m + 3 \in \mathbb{Z}^+ \wedge 2 p \in \mathbb{Z}^+$

   1: $\int (f x)^m (d + e x^2)^p (a + b\,\text{ArcSinh}[c x])^n\,dx$ when $e == c^2 d \wedge n < -1 \wedge m + 3 \in \mathbb{Z}^+ \wedge 2 p \in \mathbb{Z}^+ \wedge (p \in \mathbb{Z} \vee d > 0)$

Derivation: Integration by parts

Basis: $\frac{(a+b\,\text{ArcSinh}[c x])^n}{\sqrt{1+c^2 x^2}} == \partial_x \frac{(a+b\,\text{ArcSinh}[c x])^{n+1}}{b c (n+1)}$

Rule: If $e == c^2 d \wedge n < -1 \wedge m + 3 \in \mathbb{Z}^+ \wedge 2 p \in \mathbb{Z}^+ \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx \;\to\; \frac{d^p\,(f\,x)^m\,(1+c^2\,x^2)^{p+\frac{1}{2}}\,(a+b\,\text{ArcSinh}[c\,x])^{n+1}}{b\,c\,(n+1)} - \frac{f\,m\,d^p}{b\,c\,(n+1)}\int (f\,x)^{m-1}\,(1+c^2\,x^2)^{p-\frac{1}{2}}\,(a+b\,\text{ArcSinh}[c\,x])^{n+1}\,dx - \frac{c\,d^p\,(m+2\,p+1)}{b\,f\,(n+1)}\int (f\,x)^{m+1}\,(1+c^2\,x^2)^{p-\frac{1}{2}}\,(a+b\,\text{ArcSinh}[c\,x])^{n+1}\,dx$$

Program code:

```
(* Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_,x_Symbol] :=
  d^p*(f*x)^m*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n+1)/(b*c*(n+1)) -
  f*m*d^p/(b*c*(n+1))*Int[(f*x)^(m-1)*(1+c^2*x^2)^(p-1/2)*(a+b*ArcSinh[c*x])^(n+1),x] -
  c*d^p*(m+2*p+1)/(b*f*(n+1))*Int[(f*x)^(m+1)*(1+c^2*x^2)^(p-1/2)*(a+b*ArcSinh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[e,c^2*d] && LtQ[n,-1] && IGtQ[m,-3] && IGtQ[2*p,0] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_,x_Symbol] :=
  (f*x)^m*Sqrt[1+c*x]*Sqrt[-1+c*x]*(d+e*x^2)^p*(a+b*ArcCosh[c*x])^(n+1)/(b*c*(n+1)) +
  f*m*(-d)^p/(b*c*(n+1))*Int[(f*x)^(m-1)*(1+c*x)^(p-1/2)*(-1+c*x)^(p-1/2)*(a+b*ArcCosh[c*x])^(n+1),x] -
  c*(-d)^p*(m+2*p+1)/(b*f*(n+1))*Int[(f*x)^(m+1)*(1+c*x)^(p-1/2)*(-1+c*x)^(p-1/2)*(a+b*ArcCosh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && IGtQ[m,-3] && IGtQ[p,0]
```

2: $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$ when $e == c^2\,d \wedge n < -1 \wedge m+3 \in \mathbb{Z}^+ \wedge 2\,p \in \mathbb{Z}^+$

Derivation: Integration by parts

Basis: $\frac{(a+b\,\text{ArcSinh}[c\,x])^n}{\sqrt{1+c^2\,x^2}} == \partial_x \frac{(a+b\,\text{ArcSinh}[c\,x])^{n+1}}{b\,c\,(n+1)}$

Rule: If $e == c^2\,d \wedge n < -1 \wedge m+3 \in \mathbb{Z}^+ \wedge 2\,p \in \mathbb{Z}^+$, then

$$\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx \;\to\; \frac{(f\,x)^m\,\sqrt{1+c^2\,x^2}\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^{n+1}}{b\,c\,(n+1)} -$$

$$\frac{f\,m\,d^{\text{IntPart}[p]}\left(d+e\,x^2\right)^{\text{FracPart}[p]}}{b\,c\,(n+1)\left(1+c^2\,x^2\right)^{\text{FracPart}[p]}}\int (f\,x)^{m-1}\left(1+c^2\,x^2\right)^{p-\frac{1}{2}}\left(a+b\,\text{ArcSinh}[c\,x]\right)^{n+1}\mathbb{d}x -$$
$$\frac{c\,(m+2\,p+1)\,d^{\text{IntPart}[p]}\left(d+e\,x^2\right)^{\text{FracPart}[p]}}{b\,f\,(n+1)\left(1+c^2\,x^2\right)^{\text{FracPart}[p]}}\int (f\,x)^{m+1}\left(1+c^2\,x^2\right)^{p-\frac{1}{2}}\left(a+b\,\text{ArcSinh}[c\,x]\right)^{n+1}\mathbb{d}x$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_,x_Symbol] :=
  (f*x)^m*Sqrt[1+c^2*x^2]*(d+e*x^2)^p*(a+b*ArcSinh[c*x])^(n+1)/(b*c*(n+1)) -
  f*m*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(b*c*(n+1)*(1+c^2*x^2)^FracPart[p])*
    Int[(f*x)^(m-1)*(1+c^2*x^2)^(p-1/2)*(a+b*ArcSinh[c*x])^(n+1),x] -
  c*(m+2*p+1)*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(b*f*(n+1)*(1+c^2*x^2)^FracPart[p])*
    Int[(f*x)^(m+1)*(1+c^2*x^2)^(p-1/2)*(a+b*ArcSinh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,f},x] && EqQ[e,c^2*d] && LtQ[n,-1] && IGtQ[m,-3] && IGtQ[2*p,0]
```

```
Int[(f_.*x_)^m_.*(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_,x_Symbol] :=
  (f*x)^m*Sqrt[1+c*x]*Sqrt[-1+c*x]*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^(n+1)/(b*c*(n+1)) +
  f*m*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(b*c*(n+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^(m-1)*(-1+c^2*x^2)^(p-1/2)*(a+b*ArcCosh[c*x])^(n+1),x] -
  c*(m+2*p+1)*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(b*f*(n+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^(m+1)*(-1+c^2*x^2)^(p-1/2)*(a+b*ArcCosh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && LtQ[n,-1] && IGtQ[m,-3] && IGtQ[p+1/2,0]
```

3. $\int x^m \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right)^n dx$ when $e == c^2\,d \wedge 2\,p \in \mathbb{Z} \wedge p > -1 \wedge m \in \mathbb{Z}^+$

1: $\int x^m \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right)^n dx$ when $e == c^2\,d \wedge 2\,p \in \mathbb{Z} \wedge p > -1 \wedge m \in \mathbb{Z}^+ \wedge \left(p \in \mathbb{Z} \vee d > 0\right)$

Derivation: Integration by substitution

Basis: $F[x] == \frac{1}{c}\,\text{Subst}\left[F\left[\frac{\text{Sinh}[x]}{c}\right]\text{Cosh}[x], x, \text{ArcSinh}[c\,x]\right]\partial_x \text{ArcSinh}[c\,x]$

Basis: If $e == c^2\,d \wedge m \in \mathbb{Z} \wedge \left(p \in \mathbb{Z} \vee d > 0\right)$, then
$x^m \left(d + e\,x^2\right)^p == \frac{d^p}{c^{m+1}}\,\text{Subst}\left[\text{Sinh}[x]^m\,\text{Cosh}[x]^{2\,p+1}, x, \text{ArcSinh}[c\,x]\right]\partial_x \text{ArcSinh}[c\,x]$

Rule: If $e == c^2\,d \wedge 2\,p \in \mathbb{Z} \wedge p > -1 \wedge m \in \mathbb{Z}^+ \wedge \left(p \in \mathbb{Z} \vee d > 0\right)$, then

$$\int x^m \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right)^n dx \to \frac{d^p}{c^{m+1}}\,\text{Subst}\left[\int \left(a + b\,x\right)^n \text{Sinh}[x]^m\,\text{Cosh}[x]^{2\,p+1}\,dx, x, \text{ArcSinh}[c\,x]\right]$$

Program code:

```
Int[x_^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  d^p/c^(m+1)*Subst[Int[(a+b*x)^n*Sinh[x]^m*Cosh[x]^(2*p+1),x],x,ArcSinh[c*x]] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[e,c^2*d] && IntegerQ[2*p] && GtQ[p,-1] && IGtQ[m,0] && (IntegerQ[p] || GtQ[d,0])
```

```
Int[x_^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (-d)^p/c^(m+1)*Subst[Int[(a+b*x)^n*Cosh[x]^m*Sinh[x]^(2*p+1),x],x,ArcCosh[c*x]] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && IGtQ[p,0] && IGtQ[m,0]
```

```
Int[x_^m_.*(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (-d1*d2)^p/c^(m+1)*Subst[Int[(a+b*x)^n*Cosh[x]^m*Sinh[x]^(2*p+1),x],x,ArcCosh[c*x]] /;
FreeQ[{a,b,c,d1,e1,d2,e2,n},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && IntegerQ[p+1/2] && GtQ[p,-1] && IGtQ[m,0] && (GtQ[d1,0] && LtQ
```

2: $\int x^m \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right)^n \mathbb{d}x$ when $e == c^2\,d \wedge 2\,p \in \mathbb{Z} \wedge p > -1 \wedge m \in \mathbb{Z}^+ \wedge \neg \left(p \in \mathbb{Z} \vee d > 0\right)$

Derivation: Piecewise constant extraction

Basis: If $e == c^2\,d$, then $\partial_x \frac{(d+e\,x^2)^p}{(1+c^2\,x^2)^p} == 0$

Rule: If $e == c^2\,d \wedge 2\,p \in \mathbb{Z} \wedge p > -1 \wedge m \in \mathbb{Z}^+ \wedge \neg \left(p \in \mathbb{Z} \vee d > 0\right)$, then

$$\int x^m \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right)^n \mathbb{d}x \longrightarrow \frac{d^{\text{IntPart}[p]} \left(d + e\,x^2\right)^{\text{FracPart}[p]}}{\left(1 + c^2\,x^2\right)^{\text{FracPart}[p]}} \int x^m \left(1 + c^2\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right)^n \mathbb{d}x$$

Program code:

```
Int[x_^m_.*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSinh[c_.*x_])^n_,x_Symbol] :=
  d^IntPart[p]*(d+e*x^2)^FracPart[p]/(1+c^2*x^2)^FracPart[p]*Int[x^m*(1+c^2*x^2)^p*(a+b*ArcSinh[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[e,c^2*d] && IntegerQ[2*p] && GtQ[p,-1] && IGtQ[m,0] && Not[(IntegerQ[p] || GtQ[d,0])]
```

```
Int[x_^m_.*(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/((1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[x^m*(1+c*x)^p*(-1+c*x)^p*(a+b*ArcCosh[c*x])^n,x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,n},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && IntegerQ[2*p] && GtQ[p,-1] && IGtQ[m,0] &&
  Not[IntegerQ[p] || GtQ[d1,0] && LtQ[d2,0]]
```

4: $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$ when $e == c^2\,d \wedge d > 0 \wedge p+\frac{1}{2} \in \mathbb{Z}^+ \wedge \frac{m+1}{2} \notin \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule: If $e == c^2\,d \wedge d > 0 \wedge p+\frac{1}{2} \in \mathbb{Z}^+ \wedge \frac{m+1}{2} \notin \mathbb{Z}^+$, then

$$\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx \to \int \frac{(a+b\,\text{ArcSinh}[c\,x])^n}{\sqrt{d+e\,x^2}}\,\text{ExpandIntegrand}\left[(f\,x)^m\,(d+e\,x^2)^{p+\frac{1}{2}},\,x\right]dx$$

Program code:

```
Int[(f_.*x_)^m_*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*ArcSinh[c*x])^n/Sqrt[d+e*x^2],(f*x)^m*(d+e*x^2)^(p+1/2),x],x] /;
FreeQ[{a,b,c,d,e,f,m,n},x] && EqQ[e,c^2*d] && GtQ[d,0] && IGtQ[p+1/2,0] && Not[IGtQ[(m+1)/2,0]] && (EqQ[m,-1] || EqQ[m,-2])
```

```
Int[(f_.*x_)^m_*(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*ArcCosh[c*x])^n/(Sqrt[d1+e1*x]*Sqrt[d2+e2*x]),(f*x)^m*(d1+e1*x)^(p+1/2)*(d2+e2*x)^(p+1/2),x],x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m,n},x] && EqQ[e1-c*d1,0] && EqQ[e2+c*d2,0] && GtQ[d1,0] && LtQ[d2,0] && IGtQ[p+1/2,0] && Not[IGtQ[(m+1)/
  (EqQ[m,-1] || EqQ[m,-2])
```

2. $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$ when $e \neq c^2\,d$

0: $\int (f\,x)^m\,(d+e\,x^2)\,(a+b\,\text{ArcCosh}[c\,x])\,dx$ when $c^2\,d+e \neq 0 \wedge m \neq -1 \wedge m \neq -3$

Derivation: Integration by parts

Note: This rule can be removed when integrands of the form $(d+e\,x)^m\,(f+g\,x)^m\,\left(a+c\,x^2\right)^p$ when $e\,f+d\,g == 0$ are integrated without first resorting to piecewise constant extraction.

Rule: If $c^2\,d+e \neq 0 \wedge m \neq -1 \wedge m \neq -3$, then

$$\int (f\,x)^m\,(d+e\,x^2)\,(a+b\,\text{ArcCosh}[c\,x])\,dx \longrightarrow \frac{d\,(f\,x)^{m+1}\,(a+b\,\text{ArcCosh}[c\,x])}{f\,(m+1)} + \frac{e\,(f\,x)^{m+3}\,(a+b\,\text{ArcCosh}[c\,x])}{f^3\,(m+3)} - \frac{b\,c}{f\,(m+1)\,(m+3)}\int \frac{(f\,x)^{m+1}\,(d\,(m+3)+e\,(m+1)\,x^2)}{\sqrt{1+c\,x}\,\sqrt{-1+c\,x}}\,dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)*(a_.+b_.*ArcCosh[c_.*x_]),x_Symbol] :=
  d*(f*x)^(m+1)*(a+b*ArcCosh[c*x])/(f*(m+1)) +
  e*(f*x)^(m+3)*(a+b*ArcCosh[c*x])/(f^3*(m+3)) -
  b*c/(f*(m+1)*(m+3))*Int[(f*x)^(m+1)*(d*(m+3)+e*(m+1)*x^2)/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x] /;
FreeQ[{a,b,c,d,e,f,m},x] && NeQ[c^2*d+e,0] && NeQ[m,-1] && NeQ[m,-3]
```

1: $\int x \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right) \mathbb{d}x$ when $e \neq c^2\,d \wedge p \neq -1$

Derivation: Integration by parts

Basis:: If $p \neq -1$, then $x \left(d + e\,x^2\right)^p == \partial_x \frac{\left(d+e\,x^2\right)^{p+1}}{2\,e\,(p+1)}$

Rule: If $e \neq c^2\,d \wedge p \neq -1$, then

$$\int x \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right) \mathbb{d}x \longrightarrow \frac{\left(d + e\,x^2\right)^{p+1} \left(a + b\,\text{ArcSinh}[c\,x]\right)}{2\,e\,(p+1)} - \frac{b\,c}{2\,e\,(p+1)} \int \frac{\left(d + e\,x^2\right)^{p+1}}{\sqrt{1 + c^2\,x^2}} \mathbb{d}x$$

Program code:

```
Int[x_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_]),x_Symbol] :=
  (d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])/(2*e*(p+1)) - b*c/(2*e*(p+1))*Int[(d+e*x^2)^(p+1)/Sqrt[1+c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,p},x] && NeQ[e,c^2*d] && NeQ[p,-1]
```

```
Int[x_*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_]),x_Symbol] :=
  (d+e*x^2)^(p+1)*(a+b*ArcCosh[c*x])/(2*e*(p+1)) - b*c/(2*e*(p+1))*Int[(d+e*x^2)^(p+1)/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x] /;
FreeQ[{a,b,c,d,e,p},x] && NeQ[c^2*d+e,0] && NeQ[p,-1]
```

2: $\int (f\,x)^m \left(d + e\,x^2\right)^p \left(a + b\,\text{ArcSinh}[c\,x]\right) \mathbb{d}x$ when $e \neq c^2\,d \wedge p \in \mathbb{Z} \wedge \left(p > 0 \vee \frac{m-1}{2} \in \mathbb{Z}^+ \wedge m + p \leq 0\right)$

Derivation: Integration by parts

Note: If $\frac{m-1}{2} \in \mathbb{Z}^+ \wedge p \in \mathbb{Z}^- \wedge m + p \geq 0$, then $\int (f\,x)^m \left(d + e\,x^2\right)^p$ is a rational function.

Rule: If $e \neq c^2\,d \wedge p \in \mathbb{Z} \wedge \left(p > 0 \vee \frac{m-1}{2} \in \mathbb{Z}^+ \wedge m + p \leq 0\right)$, let $u = \int (f\,x)^m \left(d + e\,x^2\right)^p \mathbb{d}x$, then

$$\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])\,dx \;\to\; u\,(a+b\,\text{ArcSinh}[c\,x]) - b\,c\int \frac{u}{\sqrt{1+c^2\,x^2}}\,dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(f*x)^m*(d+e*x^2)^p,x]},
  Dist[a+b*ArcSinh[c*x],u,x] - b*c*Int[SimplifyIntegrand[u/Sqrt[1+c^2*x^2],x],x]] /;
FreeQ[{a,b,c,d,e,f,m},x] && NeQ[e,c^2*d] && IntegerQ[p] && (GtQ[p,0] || IGtQ[(m-1)/2,0] && LeQ[m+p,0])
```

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(f*x)^m*(d+e*x^2)^p,x]},
  Dist[a+b*ArcCosh[c*x],u,x] - b*c*Int[SimplifyIntegrand[u/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x],x]] /;
FreeQ[{a,b,c,d,e,f,m},x] && NeQ[c^2*d+e,0] && IntegerQ[p] && (GtQ[p,0] || IGtQ[(m-1)/2,0] && LeQ[m+p,0])
```

3: $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$ when $e \neq c^2\,d \wedge n \in \mathbb{Z}^+ \wedge p \in \mathbb{Z} \wedge m \in \mathbb{Z}$

Derivation: Algebraic expansion

Rule: If $e \neq c^2\,d \wedge n \in \mathbb{Z}^+ \wedge p \in \mathbb{Z} \wedge m \in \mathbb{Z}$, then

$$\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx \;\to\; \int (a+b\,\text{ArcSinh}[c\,x])^n\,\text{ExpandIntegrand}[(f\,x)^m\,(d+e\,x^2)^p,\,x]\,dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*ArcSinh[c*x])^n,(f*x)^m*(d+e*x^2)^p,x],x] /;
FreeQ[{a,b,c,d,e,f},x] && NeQ[e,c^2*d] && IGtQ[n,0] && IntegerQ[p] && IntegerQ[m]
```

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*ArcCosh[c*x])^n,(f*x)^m*(d+e*x^2)^p,x],x] /;
FreeQ[{a,b,c,d,e,f},x] && NeQ[c^2*d+e,0] && IGtQ[n,0] && IntegerQ[p] && IntegerQ[m]
```

X: $\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$

Rule:

$$\int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx \;\rightarrow\; \int (f\,x)^m\,(d+e\,x^2)^p\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$$

Program code:

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  Unintegrable[(f*x)^m*(d+e*x^2)^p*(a+b*ArcSinh[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,m,n,p},x]
```

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  Unintegrable[(f*x)^m*(d+e*x^2)^p*(a+b*ArcCosh[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,m,n,p},x] && IntegerQ[p]
```

```
Int[(f_.*x_)^m_.*(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  Unintegrable[(f*x)^m*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^n,x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,f,m,n,p},x]
```

Rules for integrands of the form $(h\,x)^m\,(d+e\,x)^p\,(f+g\,x)^q\,(a+b\,\text{ArcSinh}[c\,x])^n$

1: $\int (h\,x)^m\,(d+e\,x)^p\,(f+g\,x)^q\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$ when $e\,f+d\,g==0 \wedge c^2\,d^2+e^2==0 \wedge (p\mid q)\in\mathbb{Z}+\frac{1}{2} \wedge p-q\geq 0 \wedge d>0 \wedge \frac{g}{e}<0$

Derivation: Algebraic expansion

Basis: If $e\,f+d\,g==0 \wedge c^2\,d^2+e^2==0 \wedge d>0 \wedge \frac{g}{e}<0$, then

$$(d+e\,x)^p\,(f+g\,x)^q==\left(-\frac{d^2\,g}{e}\right)^q\,(d+e\,x)^{p-q}\,\left(1+c^2\,x^2\right)^q$$

Rule: If $e\,f + d\,g == 0 \wedge c^2\,d^2 + e^2 == 0 \wedge (p \mid q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge d > 0 \wedge \frac{g}{e} < 0$, then

$$\int (h\,x)^m\,(d + e\,x)^p\,(f + g\,x)^q\,(a + b\,\text{ArcSinh}[c\,x])^n\,dx \rightarrow \left(-\frac{d^2\,g}{e}\right)^q \int (h\,x)^m\,(d + e\,x)^{p-q}\,(1 + c^2\,x^2)^q\,(a + b\,\text{ArcSinh}[c\,x])^n\,dx$$

Program code:

```
Int[(h_.*x_)^m_.*(d_+e_.*x_)^p_*(f_+g_.*x_)^q_*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  (-d^2*g/e)^q*Int[(h*x)^m*(d+e*x)^(p-q)*(1+c^2*x^2)^q*(a+b*ArcSinh[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,h,m,n},x] && EqQ[e*f+d*g,0] && EqQ[c^2*d^2+e^2,0] && HalfIntegerQ[p,q] && GeQ[p-q,0] && GtQ[d,0] && LtQ[g/e,0]
```

2: $\int (h\,x)^m\,(d + e\,x)^p\,(f + g\,x)^q\,(a + b\,\text{ArcSinh}[c\,x])^n\,dx$ when $e\,f + d\,g == 0 \wedge c^2\,d^2 + e^2 == 0 \wedge (p \mid q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge \neg \left(d > 0 \wedge \frac{g}{e} < 0\right)$

Derivation: Piecewise constant extraction

Basis: If $e\,f + d\,g == 0 \wedge c^2\,d^2 + e^2 == 0$, then $\partial_x \frac{(d+e\,x)^q\,(f+g\,x)^q}{(1+c^2\,x^2)^q} == 0$

Rule: If $e\,f + d\,g == 0 \wedge c^2\,d^2 + e^2 == 0 \wedge (p \mid q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge \neg \left(d > 0 \wedge \frac{g}{e} < 0\right)$, then

$$\int (h\,x)^m\,(d + e\,x)^p\,(f + g\,x)^q\,(a + b\,\text{ArcSinh}[c\,x])^n\,dx \rightarrow$$

$$\frac{\left(-\frac{d^2\,g}{e}\right)^{\text{IntPart}[q]}\,(d + e\,x)^{\text{FracPart}[q]}\,(f + g\,x)^{\text{FracPart}[q]}}{(1 + c^2\,x^2)^{\text{FracPart}[q]}} \int (h\,x)^m\,(d + e\,x)^{p-q}\,(1 + c^2\,x^2)^q\,(a + b\,\text{ArcSinh}[c\,x])^n\,dx$$

Program code:

```
Int[(h_.*x_)^m_.*(d_+e_.*x_)^p_*(f_+g_.*x_)^q_*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  (-d^2*g/e)^IntPart[q]*(d+e*x)^FracPart[q]*(f+g*x)^FracPart[q]/(1+c^2*x^2)^FracPart[q]*
    Int[(h*x)^m*(d+e*x)^(p-q)*(1+c^2*x^2)^q*(a+b*ArcSinh[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,h,m,n},x] && EqQ[e*f+d*g,0] && EqQ[c^2*d^2+e^2,0] && HalfIntegerQ[p,q] && GeQ[p-q,0]
```

```
Int[(f_.*x_)^m_.*(d_+e_.*x_^2)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (-d)^IntPart[p]*(d+e*x^2)^FracPart[p]/((1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
    Int[(f*x)^m*(1+c*x)^p*(-1+c*x)^p*(a+b*ArcCosh[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,m,n,p},x] && EqQ[c^2*d+e,0] && Not[IntegerQ[p]]
```