Rules for integrands involving trig integral functions

1. $\int u\,\text{SinIntegral}[a+b\,x]\,dx$

   1: $\int \text{SinIntegral}[a+b\,x]\,dx$

Derivation: Integration by parts

Rule:

$$\int \text{SinIntegral}[a+b\,x]\,dx \to \frac{(a+b\,x)\,\text{SinIntegral}[a+b\,x]}{b}+\frac{\text{Cos}[a+b\,x]}{b}$$

Program code:

```
Int[SinIntegral[a_.+b_.*x_],x_Symbol] :=
  (a+b*x)*SinIntegral[a+b*x]/b + Cos[a+b*x]/b/;
FreeQ[{a,b},x]
```

```
Int[CosIntegral[a_.+b_.*x_],x_Symbol] :=
  (a+b*x)*CosIntegral[a+b*x]/b - Sin[a+b*x]/b /;
FreeQ[{a,b},x]
```

2. $\int (c+d\,x)^m\,\text{SinIntegral}[a+b\,x]\,dx$

   1: $\int \frac{\text{SinIntegral}[b\,x]}{x}\,dx$

Basis: $\text{SinIntegral}[z] = \frac{1}{2}\,i\,(\text{ExpIntegralE}[1,-i\,z]-\text{ExpIntegralE}[1,i\,z]+\text{Log}[-i\,z]-\text{Log}[i\,z])$

Basis: $\text{CosIntegral}[z] = \frac{1}{2}\,(-\text{ExpIntegralE}[1,-i\,z]-\text{ExpIntegralE}[1,i\,z]-\text{Log}[-i\,z]-\text{Log}[i\,z]+2\,\text{Log}[z])$

Rule:

$$\int \frac{\text{SinIntegral}[b\,x]}{x}\,dx \to$$

$$\frac{1}{2} b x \,\text{HypergeometricPFQ}[\{1, 1, 1\}, \{2, 2, 2\}, -i b x] + \frac{1}{2} b x \,\text{HypergeometricPFQ}[\{1, 1, 1\}, \{2, 2, 2\}, i b x]$$

Program code:

```
Int[SinIntegral[b_.*x_]/x_,x_Symbol] :=
  1/2*b*x*HypergeometricPFQ[{1,1,1},{2,2,2},-I*b*x] +
  1/2*b*x*HypergeometricPFQ[{1,1,1},{2,2,2},I*b*x] /;
FreeQ[b,x]
```

```
Int[CosIntegral[b_.*x_]/x_,x_Symbol] :=
  -1/2*I*b*x*HypergeometricPFQ[{1,1,1},{2,2,2},-I*b*x] +
  1/2*I*b*x*HypergeometricPFQ[{1,1,1},{2,2,2},I*b*x] +
  EulerGamma*Log[x] +
  1/2*Log[b*x]^2 /;
FreeQ[b,x]
```

2: $\int (c + d x)^m \,\text{SinIntegral}[a + b x] \, dx$ when $m \neq -1$

Derivation: Integration by parts

Rule: If $m \neq -1$, then

$$\int (c + d x)^m \,\text{SinIntegral}[a + b x] \, dx \longrightarrow \frac{(c + d x)^{m+1} \,\text{SinIntegral}[a + b x]}{d (m + 1)} - \frac{b}{d (m + 1)} \int \frac{(c + d x)^{m+1} \,\text{Sin}[a + b x]}{a + b x} \, dx$$

Program code:

```
Int[(c_.+d_.*x_)^m_.*SinIntegral[a_.+b_.*x_],x_Symbol] :=
  (c+d*x)^(m+1)*SinIntegral[a+b*x]/(d*(m+1)) -
  b/(d*(m+1))*Int[(c+d*x)^(m+1)*Sin[a+b*x]/(a+b*x),x] /;
FreeQ[{a,b,c,d,m},x] && NeQ[m,-1]
```

```
Int[(c_.+d_.*x_)^m_.*CosIntegral[a_.+b_.*x_],x_Symbol] :=
  (c+d*x)^(m+1)*CosIntegral[a+b*x]/(d*(m+1)) -
  b/(d*(m+1))*Int[(c+d*x)^(m+1)*Cos[a+b*x]/(a+b*x),x] /;
FreeQ[{a,b,c,d,m},x] && NeQ[m,-1]
```

2. $\int u\,\text{SinIntegral}[a+b\,x]^2\,dx$

1: $\int \text{SinIntegral}[a+b\,x]^2\,dx$

Derivation: Integration by parts

Rule:

$$\int \text{SinIntegral}[a+b\,x]^2\,dx \to \frac{(a+b\,x)\,\text{SinIntegral}[a+b\,x]^2}{b} - 2\int \text{Sin}[a+b\,x]\,\text{SinIntegral}[a+b\,x]\,dx$$

Program code:

```
Int[SinIntegral[a_.+b_.*x_]^2,x_Symbol] :=
  (a+b*x)*SinIntegral[a+b*x]^2/b -
  2*Int[Sin[a+b*x]*SinIntegral[a+b*x],x] /;
FreeQ[{a,b},x]
```

```
Int[CosIntegral[a_.+b_.*x_]^2,x_Symbol] :=
  (a+b*x)*CosIntegral[a+b*x]^2/b -
  2*Int[Cos[a+b*x]*CosIntegral[a+b*x],x] /;
FreeQ[{a,b},x]
```

2. $\int (c+d\,x)^m\,\text{SinIntegral}[a+b\,x]^2\,dx$

1: $\int x^m\,\text{SinIntegral}[b\,x]^2\,dx$ when $m\in\mathbb{Z}^+$

Derivation: Integration by parts

Rule: If $m\in\mathbb{Z}^+$, then

$$\int x^m\,\text{SinIntegral}[b\,x]^2\,dx \longrightarrow \frac{x^{m+1}\,\text{SinIntegral}[b\,x]^2}{m+1}-\frac{2}{m+1}\int x^m\,\text{Sin}[b\,x]\,\text{SinIntegral}[b\,x]\,dx$$

Program code:

```
Int[x_^m_.*SinIntegral[b_.*x_]^2,x_Symbol] :=
  x^(m+1)*SinIntegral[b*x]^2/(m+1) -
  2/(m+1)*Int[x^m*Sin[b*x]*SinIntegral[b*x],x] /;
FreeQ[b,x] && IGtQ[m,0]
```

```
Int[x_^m_.*CosIntegral[b_.*x_]^2,x_Symbol] :=
  x^(m+1)*CosIntegral[b*x]^2/(m+1) -
  2/(m+1)*Int[x^m*Cos[b*x]*CosIntegral[b*x],x] /;
FreeQ[b,x] && IGtQ[m,0]
```

2: $\int (c+d\,x)^m\,\text{SinIntegral}[a+b\,x]^2\,dx$ when $m\in\mathbb{Z}^+$

Derivation: Iterated integration by parts

Rule: If $m\in\mathbb{Z}^+$, then

$$\int (c+d\,x)^m\,\text{SinIntegral}[a+b\,x]^2\,dx \longrightarrow$$

$$\frac{(a+b\,x)\,(c+d\,x)^m\,\text{SinIntegral}[a+b\,x]^2}{b\,(m+1)}-$$
$$\frac{2}{m+1}\int(c+d\,x)^m\,\text{Sin}[a+b\,x]\,\text{SinIntegral}[a+b\,x]\,dx+\frac{(b\,c-a\,d)\,m}{b\,(m+1)}\int(c+d\,x)^{m-1}\,\text{SinIntegral}[a+b\,x]^2\,dx$$

Program code:

```
Int[(c_.+d_.*x_)^m_.*SinIntegral[a_+b_.*x_]^2,x_Symbol] :=
  (a+b*x)*(c+d*x)^m*SinIntegral[a+b*x]^2/(b*(m+1)) -
  2/(m+1)*Int[(c+d*x)^m*Sin[a+b*x]*SinIntegral[a+b*x],x] +
  (b*c-a*d)*m/(b*(m+1))*Int[(c+d*x)^(m-1)*SinIntegral[a+b*x]^2,x] /;
FreeQ[{a,b,c,d},x] && IGtQ[m,0]
```

```
Int[(c_.+d_.*x_)^m_.*CosIntegral[a_+b_.*x_]^2,x_Symbol] :=
  (a+b*x)*(c+d*x)^m*CosIntegral[a+b*x]^2/(b*(m+1)) -
  2/(m+1)*Int[(c+d*x)^m*Cos[a+b*x]*CosIntegral[a+b*x],x] +
  (b*c-a*d)*m/(b*(m+1))*Int[(c+d*x)^(m-1)*CosIntegral[a+b*x]^2,x] /;
FreeQ[{a,b,c,d},x] && IGtQ[m,0]
```

x: $\int x^m \operatorname{SinIntegral}[a+b\,x]^2\,dx$ when $m+2\in\mathbb{Z}^-$

Derivation: Inverted integration by parts

Rule: If $m+2\in\mathbb{Z}^-$, then

$$\int x^m \operatorname{SinIntegral}[a+b\,x]^2\,dx \to \frac{b\,x^{m+2}\operatorname{SinIntegral}[a+b\,x]^2}{a\,(m+1)}+\frac{x^{m+1}\operatorname{SinIntegral}[a+b\,x]^2}{m+1}-\frac{2\,b}{a\,(m+1)}\int x^{m+1}\operatorname{Sin}[a+b\,x]\operatorname{SinIntegral}[a+b\,x]\,dx-\frac{b\,(m+2)}{a\,(m+1)}\int x^{m+1}\operatorname{SinIntegral}[a+b\,x]^2\,dx$$

Program code:

```
(* Int[x_^m_.*SinIntegral[a_+b_.*x_]^2,x_Symbol] :=
  b*x^(m+2)*SinIntegral[a+b*x]^2/(a*(m+1)) +
  x^(m+1)*SinIntegral[a+b*x]^2/(m+1) -
  2*b/(a*(m+1))*Int[x^(m+1)*Sin[a+b*x]*SinIntegral[a+b*x],x] -
  b*(m+2)/(a*(m+1))*Int[x^(m+1)*SinIntegral[a+b*x]^2,x] /;
FreeQ[{a,b},x] && ILtQ[m,-2] *)
```

```
(* Int[x_^m_.*CosIntegral[a_+b_.*x_]^2,x_Symbol] :=
  b*x^(m+2)*CosIntegral[a+b*x]^2/(a*(m+1)) +
  x^(m+1)*CosIntegral[a+b*x]^2/(m+1) -
  2*b/(a*(m+1))*Int[x^(m+1)*Cos[a+b*x]*CosIntegral[a+b*x],x] -
  b*(m+2)/(a*(m+1))*Int[x^(m+1)*CosIntegral[a+b*x]^2,x] /;
FreeQ[{a,b},x] && ILtQ[m,-2] *)
```

3. $\int u\,\text{Sin}[a+b\,x]\,\text{SinIntegral}[c+d\,x]\,dx$

1: $\int \text{Sin}[a+b\,x]\,\text{SinIntegral}[c+d\,x]\,dx$

Reference: G&R 5.32.2

Reference: G&R 5.31.1

Derivation: Integration by parts

Rule:

$$\int \text{Sin}[a+b\,x]\,\text{SinIntegral}[c+d\,x]\,dx \longrightarrow -\frac{\text{Cos}[a+b\,x]\,\text{SinIntegral}[c+d\,x]}{b}+\frac{d}{b}\int \frac{\text{Cos}[a+b\,x]\,\text{Sin}[c+d\,x]}{c+d\,x}\,dx$$

Program code:

```
Int[Sin[a_.+b_.*x_]*SinIntegral[c_.+d_.*x_],x_Symbol] :=
  -Cos[a+b*x]*SinIntegral[c+d*x]/b +
  d/b*Int[Cos[a+b*x]*Sin[c+d*x]/(c+d*x),x] /;
FreeQ[{a,b,c,d},x]
```

```
Int[Cos[a_.+b_.*x_]*CosIntegral[c_.+d_.*x_],x_Symbol] :=
  Sin[a+b*x]*CosIntegral[c+d*x]/b -
  d/b*Int[Sin[a+b*x]*Cos[c+d*x]/(c+d*x),x] /;
FreeQ[{a,b,c,d},x]
```

2. $\int (e+f x)^m \operatorname{Sin}[a+b x] \operatorname{SinIntegral}[c+d x] \, dx$

1: $\int (e+f x)^m \operatorname{Sin}[a+b x] \operatorname{SinIntegral}[c+d x] \, dx$ when $m \in \mathbb{Z}^+$

Derivation: Integration by parts

Rule: If $m \in \mathbb{Z}^+$, then

$$\int (e+f x)^m \operatorname{Sin}[a+b x] \operatorname{SinIntegral}[c+d x] \, dx \to$$
$$-\frac{(e+f x)^m \operatorname{Cos}[a+b x] \operatorname{SinIntegral}[c+d x]}{b}+\frac{d}{b} \int \frac{(e+f x)^m \operatorname{Cos}[a+b x] \operatorname{Sin}[c+d x]}{c+d x} \, dx+\frac{f m}{b} \int (e+f x)^{m-1} \operatorname{Cos}[a+b x] \operatorname{SinIntegral}[c+d x] \, dx$$

Program code:

```
Int[(e_.+f_.*x_)^m_.*Sin[a_.+b_.*x_]*SinIntegral[c_.+d_.*x_],x_Symbol] :=
  -(e+f*x)^m*Cos[a+b*x]*SinIntegral[c+d*x]/b +
  d/b*Int[(e+f*x)^m*Cos[a+b*x]*Sin[c+d*x]/(c+d*x),x] +
  f*m/b*Int[(e+f*x)^(m-1)*Cos[a+b*x]*SinIntegral[c+d*x],x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0]
```

```
Int[(e_.+f_.*x_)^m_.*Cos[a_.+b_.*x_]*CosIntegral[c_.+d_.*x_],x_Symbol] :=
  (e+f*x)^m*Sin[a+b*x]*CosIntegral[c+d*x]/b -
  d/b*Int[(e+f*x)^m*Sin[a+b*x]*Cos[c+d*x]/(c+d*x),x] -
  f*m/b*Int[(e+f*x)^(m-1)*Sin[a+b*x]*CosIntegral[c+d*x],x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0]
```

2: $\int (e+f x)^m \operatorname{Sin}[a+b x] \operatorname{SinIntegral}[c+d x] \, dx$ when $m+1 \in \mathbb{Z}^-$

Derivation: Inverted integration by parts

Rule: If $m+1 \in \mathbb{Z}^-$, then

$$\int (e+f x)^m \operatorname{Sin}[a+b x] \operatorname{SinIntegral}[c+d x] \, dx \longrightarrow$$

$$\frac{(e+f x)^{m+1} \operatorname{Sin}[a+b x] \operatorname{SinIntegral}[c+d x]}{f (m+1)} -$$

$$\frac{d}{f (m+1)} \int \frac{(e+f x)^{m+1} \operatorname{Sin}[a+b x] \operatorname{Sin}[c+d x]}{c+d x} \, dx - \frac{b}{f (m+1)} \int (e+f x)^{m+1} \operatorname{Cos}[a+b x] \operatorname{SinIntegral}[c+d x] \, dx$$

Program code:

```
Int[(e_.+f_.*x_)^m_*Sin[a_.+b_.*x_]*SinIntegral[c_.+d_.*x_],x_Symbol] :=
  (e+f*x)^(m+1)*Sin[a+b*x]*SinIntegral[c+d*x]/(f*(m+1)) -
  d/(f*(m+1))*Int[(e+f*x)^(m+1)*Sin[a+b*x]*Sin[c+d*x]/(c+d*x),x] -
  b/(f*(m+1))*Int[(e+f*x)^(m+1)*Cos[a+b*x]*SinIntegral[c+d*x],x] /;
FreeQ[{a,b,c,d,e,f},x] && ILtQ[m,-1]
```

```
Int[(e_.+f_.*x_)^m_.*Cos[a_.+b_.*x_]*CosIntegral[c_.+d_.*x_],x_Symbol] :=
  (e+f*x)^(m+1)*Cos[a+b*x]*CosIntegral[c+d*x]/(f*(m+1)) -
  d/(f*(m+1))*Int[(e+f*x)^(m+1)*Cos[a+b*x]*Cos[c+d*x]/(c+d*x),x] +
  b/(f*(m+1))*Int[(e+f*x)^(m+1)*Sin[a+b*x]*CosIntegral[c+d*x],x] /;
FreeQ[{a,b,c,d,e,f},x] && ILtQ[m,-1]
```

4. $\int u \operatorname{Cos}[a+b x] \operatorname{SinIntegral}[c+d x] \, dx$

1: $\int \operatorname{Cos}[a+b x] \operatorname{SinIntegral}[c+d x] \, dx$

Reference: G&R 5.32.1

Reference: G&R 5.31.2

Derivation: Integration by parts

Rule:

$$\int \operatorname{Cos}[a+b x] \operatorname{SinIntegral}[c+d x] \, dx \longrightarrow \frac{\operatorname{Sin}[a+b x] \operatorname{SinIntegral}[c+d x]}{b} - \frac{d}{b} \int \frac{\operatorname{Sin}[a+b x] \operatorname{Sin}[c+d x]}{c+d x} \, dx$$

Program code:

```
Int[Cos[a_.+b_.*x_]*SinIntegral[c_.+d_.*x_],x_Symbol] :=
  Sin[a+b*x]*SinIntegral[c+d*x]/b -
  d/b*Int[Sin[a+b*x]*Sin[c+d*x]/(c+d*x),x] /;
FreeQ[{a,b,c,d},x]
```

```
Int[Sin[a_.+b_.*x_]*CosIntegral[c_.+d_.*x_],x_Symbol] :=
  -Cos[a+b*x]*CosIntegral[c+d*x]/b +
  d/b*Int[Cos[a+b*x]*Cos[c+d*x]/(c+d*x),x] /;
FreeQ[{a,b,c,d},x]
```

2. $\int (e + f x)^m \operatorname{Cos}[a + b x] \operatorname{SinIntegral}[c + d x] \, dx$

1: $\int (e + f x)^m \operatorname{Cos}[a + b x] \operatorname{SinIntegral}[c + d x] \, dx$ when $m \in \mathbb{Z}^+$

Derivation: Integration by parts

Rule: If $m \in \mathbb{Z}^+$, then

$$\int (e + f x)^m \operatorname{Cos}[a + b x] \operatorname{SinIntegral}[c + d x] \, dx \to$$
$$\frac{(e + f x)^m \operatorname{Sin}[a + b x] \operatorname{SinIntegral}[c + d x]}{b} - \frac{d}{b} \int \frac{(e + f x)^m \operatorname{Sin}[a + b x] \operatorname{Sin}[c + d x]}{c + d x} \, dx - \frac{f m}{b} \int (e + f x)^{m-1} \operatorname{Sin}[a + b x] \operatorname{SinIntegral}[c + d x] \, dx$$

Program code:

```
Int[(e_.+f_.*x_)^m_.*Cos[a_.+b_.*x_]*SinIntegral[c_.+d_.*x_],x_Symbol] :=
  (e+f*x)^m*Sin[a+b*x]*SinIntegral[c+d*x]/b -
  d/b*Int[(e+f*x)^m*Sin[a+b*x]*Sin[c+d*x]/(c+d*x),x] -
  f*m/b*Int[(e+f*x)^(m-1)*Sin[a+b*x]*SinIntegral[c+d*x],x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0]
```

```
Int[(e_.+f_.*x_)^m_.*Sin[a_.+b_.*x_]*CosIntegral[c_.+d_.*x_],x_Symbol] :=
  -(e+f*x)^m*Cos[a+b*x]*CosIntegral[c+d*x]/b +
  d/b*Int[(e+f*x)^m*Cos[a+b*x]*Cos[c+d*x]/(c+d*x),x] +
  f*m/b*Int[(e+f*x)^(m-1)*Cos[a+b*x]*CosIntegral[c+d*x],x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0]
```

2: $\int (e + f x)^m \operatorname{Cos}[a + b x] \operatorname{SinIntegral}[c + d x] \, dx$ when $m + 1 \in \mathbb{Z}^-$

Derivation: Inverted integration by parts

Rule: If $m + 1 \in \mathbb{Z}^-$, then

$$\int (e+f x)^m \operatorname{Cos}[a+b x] \operatorname{SinIntegral}[c+d x] \, dx \longrightarrow$$
$$\frac{(e+f x)^{m+1} \operatorname{Cos}[a+b x] \operatorname{SinIntegral}[c+d x]}{f (m+1)} - \frac{d}{f (m+1)} \int \frac{(e+f x)^{m+1} \operatorname{Cos}[a+b x] \operatorname{Sin}[c+d x]}{c+d x} \, dx + \frac{b}{f (m+1)} \int (e+f x)^{m+1} \operatorname{Sin}[a+b x] \operatorname{SinIntegral}[c+d x] \, dx$$

Program code:

```
Int[(e_.+f_.*x_)^m_.*Cos[a_.+b_.*x_]*SinIntegral[c_.+d_.*x_],x_Symbol] :=
  (e+f*x)^(m+1)*Cos[a+b*x]*SinIntegral[c+d*x]/(f*(m+1)) -
  d/(f*(m+1))*Int[(e+f*x)^(m+1)*Cos[a+b*x]*Sin[c+d*x]/(c+d*x),x] +
  b/(f*(m+1))*Int[(e+f*x)^(m+1)*Sin[a+b*x]*SinIntegral[c+d*x],x] /;
FreeQ[{a,b,c,d,e,f},x] && ILtQ[m,-1]
```

```
Int[(e_.+f_.*x_)^m_*Sin[a_.+b_.*x_]*CosIntegral[c_.+d_.*x_],x_Symbol] :=
  (e+f*x)^(m+1)*Sin[a+b*x]*CosIntegral[c+d*x]/(f*(m+1)) -
  d/(f*(m+1))*Int[(e+f*x)^(m+1)*Sin[a+b*x]*Cos[c+d*x]/(c+d*x),x] -
  b/(f*(m+1))*Int[(e+f*x)^(m+1)*Cos[a+b*x]*CosIntegral[c+d*x],x] /;
FreeQ[{a,b,c,d,e,f},x] && ILtQ[m,-1]
```

5. $\int u \,\text{SinIntegral}[d\,(a+b\,\text{Log}[c\,x^n])]\,dx$

1: $\int \text{SinIntegral}[d\,(a+b\,\text{Log}[c\,x^n])]\,dx$

Derivation: Integration by parts

Basis: $\partial_x \text{SinIntegral}[d\,(a+b\,\text{Log}[c\,x^n])] == \frac{b\,d\,n\,\text{Sin}[d\,(a+b\,\text{Log}[c\,x^n])]}{x\,(d\,(a+b\,\text{Log}[c\,x^n]))}$

Rule: If $m \neq -1$, then

$$\int \text{SinIntegral}[d\,(a+b\,\text{Log}[c\,x^n])]\,dx \rightarrow x\,\text{SinIntegral}[d\,(a+b\,\text{Log}[c\,x^n])] - b\,d\,n \int \frac{\text{Sin}[d\,(a+b\,\text{Log}[c\,x^n])]}{d\,(a+b\,\text{Log}[c\,x^n])}\,dx$$

Program code:

```
Int[SinIntegral[d_.*(a_.+b_.*Log[c_.*x_^n_.])],x_Symbol] :=
  x*SinIntegral[d*(a+b*Log[c*x^n])] - b*d*n*Int[Sin[d*(a+b*Log[c*x^n])]/(d*(a+b*Log[c*x^n])),x] /;
FreeQ[{a,b,c,d,n},x]
```

```
Int[CosIntegral[d_.*(a_.+b_.*Log[c_.*x_^n_.])],x_Symbol] :=
  x*CosIntegral[d*(a+b*Log[c*x^n])] - b*d*n*Int[Cos[d*(a+b*Log[c*x^n])]/(d*(a+b*Log[c*x^n])),x] /;
FreeQ[{a,b,c,d,n},x]
```

2: $\int \frac{\text{SinIntegral}[d\,(a+b\,\text{Log}[c\,x^n])]}{x}\,dx$

Derivation: Integration by substitution

Basis: $\frac{F[\text{Log}[c\,x^n]]}{x} == \frac{1}{n}\,\text{Subst}[F[x],\,x,\,\text{Log}[c\,x^n]]\,\partial_x \text{Log}[c\,x^n]$

Rule:

$$\int \frac{\text{SinIntegral}[d\,(a + b\,\text{Log}[c\,x^n])]}{x}\,dx \to \frac{1}{n}\,\text{Subst}[\text{SinIntegral}[d\,(a + b\,x)],\,x,\,\text{Log}[c\,x^n]]$$

Program code:

```
Int[F_[d_.*(a_.+b_.*Log[c_.*x_^n_.])]/x_,x_Symbol] :=
  1/n*Subst[F[d*(a+b*x)],x,Log[c*x^n]] /;
FreeQ[{a,b,c,d,n},x] && MemberQ[{SinIntegral,CosIntegral},x]
```

3: $\int (e\,x)^m\,\text{SinIntegral}[d\,(a + b\,\text{Log}[c\,x^n])]\,dx$ when $m \neq -1$

Derivation: Integration by parts

Basis: $\partial_x\,\text{SinIntegral}[d\,(a + b\,\text{Log}[c\,x^n])] == \frac{b\,d\,n\,\text{Sin}[d\,(a+b\,\text{Log}[c\,x^n])]}{x\,(d\,(a+b\,\text{Log}[c\,x^n]))}$

Rule: If $m \neq -1$, then

$$\int (e\,x)^m\,\text{SinIntegral}[d\,(a + b\,\text{Log}[c\,x^n])]\,dx \to \frac{(e\,x)^{m+1}\,\text{SinIntegral}[d\,(a + b\,\text{Log}[c\,x^n])]}{e\,(m+1)} - \frac{b\,d\,n}{m+1}\int \frac{(e\,x)^m\,\text{Sin}[d\,(a + b\,\text{Log}[c\,x^n])]}{d\,(a + b\,\text{Log}[c\,x^n])}\,dx$$

Program code:

```
Int[(e_.*x_)^m_.*SinIntegral[d_.*(a_.+b_.*Log[c_.*x_^n_.])],x_Symbol] :=
  (e*x)^(m+1)*SinIntegral[d*(a+b*Log[c*x^n])]/(e*(m+1)) -
  b*d*n/(m+1)*Int[(e*x)^m*Sin[d*(a+b*Log[c*x^n])]/(d*(a+b*Log[c*x^n])),x] /;
FreeQ[{a,b,c,d,e,m,n},x] && NeQ[m,-1]
```

```
Int[(e_.*x_)^m_.*CosIntegral[d_.*(a_.+b_.*Log[c_.*x_^n_.])],x_Symbol] :=
  (e*x)^(m+1)*CosIntegral[d*(a+b*Log[c*x^n])]/(e*(m+1)) -
  b*d*n/(m+1)*Int[(e*x)^m*Cos[d*(a+b*Log[c*x^n])]/(d*(a+b*Log[c*x^n])),x] /;
FreeQ[{a,b,c,d,e,m,n},x] && NeQ[m,-1]
```