

Rules for integrands involving hyperbolic integral functions

1. $\int u \operatorname{SinhIntegral}[a + b x] \, dx$

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Derivation: Integration by parts

Rule:

$$\int \operatorname{SinhIntegral}[a + b x] \, dx \rightarrow \frac{(a + b x) \operatorname{SinhIntegral}[a + b x]}{b} - \frac{\operatorname{Cosh}[a + b x]}{b}$$

Program code:

```
Int[SinhIntegral[a_.+b_.x_],x_Symbol] :=
  (a+b*x)*SinhIntegral[a+b*x]/b - Cosh[a+b*x]/b;
FreeQ[{a,b},x]
```

```
Int[CoshIntegral[a_.+b_.x_],x_Symbol] :=
  (a+b*x)*CoshIntegral[a+b*x]/b - Sinh[a+b*x]/b /;
FreeQ[{a,b},x]
```

2. $\int (c + d x)^m \operatorname{SinhIntegral}[a + b x] \, dx$

1: $\int \frac{\operatorname{SinhIntegral}[b x]}{x} \, dx$

Basis: $\operatorname{SinhIntegral}[z] = -\frac{1}{2} (\operatorname{ExpIntegralE}[1, -z] - \operatorname{ExpIntegralE}[1, z] + \operatorname{Log}[-z] - \operatorname{Log}[z])$

Basis: $\operatorname{CoshIntegral}[z] = -\frac{1}{2} (\operatorname{ExpIntegralE}[1, -z] + \operatorname{ExpIntegralE}[1, z] + \operatorname{Log}[-z] - \operatorname{Log}[z])$

Rule:

$$\int \frac{\operatorname{SinhIntegral}[b x]}{x} \, dx \rightarrow$$

$$\frac{1}{2} b x \operatorname{HypergeometricPFQ}\left[\{1, 1, 1\}, \{2, 2, 2\}, -b x\right] + \frac{1}{2} b x \operatorname{HypergeometricPFQ}\left[\{1, 1, 1\}, \{2, 2, 2\}, b x\right]$$

Program code:

```
Int[SinhIntegral[b_.x_]/x_,x_Symbol] :=
  1/2*b*x*HypergeometricPFQ[{1,1,1},{2,2,2},-b*x] +
  1/2*b*x*HypergeometricPFQ[{1,1,1},{2,2,2},b*x] /;
FreeQ[b,x]
```

```
Int[CoshIntegral[b_.x_]/x_,x_Symbol] :=
  -1/2*b*x*HypergeometricPFQ[{1,1,1},{2,2,2},-b*x] +
  1/2*b*x*HypergeometricPFQ[{1,1,1},{2,2,2},b*x] +
  EulerGamma*Log[x] +
  1/2*Log[b*x]^2 /;
FreeQ[b,x]
```

2: $\int (c + d x)^m \operatorname{SinhIntegral}[a + b x] dx$ when $m \neq -1$

Derivation: Integration by parts

Rule: If $m \neq -1$, then

$$\int (c + d x)^m \operatorname{SinhIntegral}[a + b x] dx \rightarrow \frac{(c + d x)^{m+1} \operatorname{SinhIntegral}[a + b x]}{d (m + 1)} - \frac{b}{d (m + 1)} \int \frac{(c + d x)^{m+1} \operatorname{Sinh}[a + b x]}{a + b x} dx$$

Program code:

```
Int[(c_.+d_.x_)^m_.SinhIntegral[a_.+b_.x_],x_Symbol] :=
  (c+d*x)^(m+1)*SinhIntegral[a+b*x]/(d*(m+1)) -
  b/(d*(m+1))*Int[(c+d*x)^(m+1)*Sinh[a+b*x]/(a+b*x),x] /;
FreeQ[{a,b,c,d,m},x] && NeQ[m,-1]
```

```

Int[(c_.+d_.**x_)^m_.*CoshIntegral[a_.+b_.**x_],x_Symbol] :=
  (c+d*x)^(m+1)*CoshIntegral[a+b*x]/(d*(m+1)) -
  b/(d*(m+1))*Int[(c+d*x)^(m+1)*Cosh[a+b*x]/(a+b*x),x] /;
FreeQ[{a,b,c,d,m},x] && NeQ[m,-1]

```

2. $\int u \operatorname{SinhIntegral}[a + b x]^2 dx$

1: $\int \operatorname{SinhIntegral}[a + b x]^2 dx$

Derivation: Integration by parts

Rule:

$$\int \operatorname{SinhIntegral}[a + b x]^2 dx \rightarrow \frac{(a + b x) \operatorname{SinhIntegral}[a + b x]^2}{b} - 2 \int \operatorname{Sinh}[a + b x] \operatorname{SinhIntegral}[a + b x] dx$$

Program code:

```

Int[SinhIntegral[a_.+b_.**x_]^2,x_Symbol] :=
  (a+b*x)*SinhIntegral[a+b*x]^2/b -
  2*Int[Sinh[a+b*x]*SinhIntegral[a+b*x],x] /;
FreeQ[{a,b},x]

```

```

Int[CoshIntegral[a_.+b_.**x_]^2,x_Symbol] :=
  (a+b*x)*CoshIntegral[a+b*x]^2/b -
  2*Int[Cosh[a+b*x]*CoshIntegral[a+b*x],x] /;
FreeQ[{a,b},x]

```

$$2. \int (c + d x)^m \operatorname{SinhIntegral}[a + b x]^2 dx$$

$$1: \int x^m \operatorname{SinhIntegral}[b x]^2 dx \text{ when } m \in \mathbb{Z}^+$$

Derivation: Integration by parts

Rule: If $m \in \mathbb{Z}^+$, then

$$\int x^m \operatorname{SinhIntegral}[b x]^2 dx \rightarrow \frac{x^{m+1} \operatorname{SinhIntegral}[b x]^2}{m+1} - \frac{2}{m+1} \int x^m \operatorname{Sinh}[b x] \operatorname{SinhIntegral}[b x] dx$$

Program code:

```
Int[x_^m_.*SinhIntegral[b_.*x_]^2,x_Symbol] :=
  x^(m+1)*SinhIntegral[b*x]^2/(m+1) -
  2/(m+1)*Int[x^m*Sinh[b*x]*SinhIntegral[b*x],x] /;
FreeQ[b,x] && IGtQ[m,0]
```

```
Int[x_^m_.*CoshIntegral[b_.*x_]^2,x_Symbol] :=
  x^(m+1)*CoshIntegral[b*x]^2/(m+1) -
  2/(m+1)*Int[x^m*Cosh[b*x]*CoshIntegral[b*x],x] /;
FreeQ[b,x] && IGtQ[m,0]
```

$$2: \int (c + d x)^m \operatorname{SinhIntegral}[a + b x]^2 dx \text{ when } m \in \mathbb{Z}^+$$

Derivation: Iterated integration by parts

Rule: If $m \in \mathbb{Z}^+$, then

$$\int (c + d x)^m \operatorname{SinhIntegral}[a + b x]^2 dx \rightarrow$$

$$\frac{(a + b x) (c + d x)^m \operatorname{SinhIntegral}[a + b x]^2}{b (m + 1)} - \frac{2}{m + 1} \int (c + d x)^m \operatorname{Sinh}[a + b x] \operatorname{SinhIntegral}[a + b x] dx + \frac{(b c - a d) m}{b (m + 1)} \int (c + d x)^{m-1} \operatorname{SinhIntegral}[a + b x]^2 dx$$

Program code:

```
Int[(c_+d_.**x_)^m_.*SinhIntegral[a_+b_.**x_]^2,x_Symbol] :=
  (a+b*x)*(c+d*x)^m*SinhIntegral[a+b*x]^2/(b*(m+1)) -
  2/(m+1)*Int[(c+d*x)^m*Sinh[a+b*x]*SinhIntegral[a+b*x],x] +
  (b*c-a*d)*m/(b*(m+1))*Int[(c+d*x)^(m-1)*SinhIntegral[a+b*x]^2,x] /;
FreeQ[{a,b,c,d},x] && IGtQ[m,0]
```

```
Int[(c_+d_.**x_)^m_.*CoshIntegral[a_+b_.**x_]^2,x_Symbol] :=
  (a+b*x)*(c+d*x)^m*CoshIntegral[a+b*x]^2/(b*(m+1)) -
  2/(m+1)*Int[(c+d*x)^m*Cosh[a+b*x]*CoshIntegral[a+b*x],x] +
  (b*c-a*d)*m/(b*(m+1))*Int[(c+d*x)^(m-1)*CoshIntegral[a+b*x]^2,x] /;
FreeQ[{a,b,c,d},x] && IGtQ[m,0]
```

x: $\int x^m \text{SinhIntegral}[a + b x]^2 dx$ when $m + 2 \in \mathbb{Z}^-$

Derivation: Inverted integration by parts

Rule: If $m + 2 \in \mathbb{Z}^-$, then

$$\int x^m \text{SinhIntegral}[a + b x]^2 dx \rightarrow \frac{b x^{m+2} \text{SinhIntegral}[a + b x]^2}{a(m+1)} + \frac{x^{m+1} \text{SinhIntegral}[a + b x]^2}{m+1} - \frac{2b}{a(m+1)} \int x^{m+1} \text{Sinh}[a + b x] \text{SinhIntegral}[a + b x] dx - \frac{b(m+2)}{a(m+1)} \int x^{m+1} \text{SinhIntegral}[a + b x]^2 dx$$

Program code:

```
(* Int[x_^m_.*SinhIntegral[a_+b_.*x_]^2,x_Symbol] :=
  b*x^(m+2)*SinhIntegral[a+b*x]^2/(a*(m+1)) +
  x^(m+1)*SinhIntegral[a+b*x]^2/(m+1) -
  2*b/(a*(m+1))*Int[x^(m+1)*Sinh[a+b*x]*SinhIntegral[a+b*x],x] -
  b*(m+2)/(a*(m+1))*Int[x^(m+1)*SinhIntegral[a+b*x]^2,x] /;
FreeQ[{a,b},x] && ILtQ[m,-2] *)
```

```
(* Int[x_^m_.*CoshIntegral[a_+b_.*x_]^2,x_Symbol] :=
  b*x^(m+2)*CoshIntegral[a+b*x]^2/(a*(m+1)) +
  x^(m+1)*CoshIntegral[a+b*x]^2/(m+1) -
  2*b/(a*(m+1))*Int[x^(m+1)*Cosh[a+b*x]*CoshIntegral[a+b*x],x] -
  b*(m+2)/(a*(m+1))*Int[x^(m+1)*CoshIntegral[a+b*x]^2,x] /;
FreeQ[{a,b},x] && ILtQ[m,-2] *)
```

$$3. \int u \sinh[a + b x] \operatorname{SinhIntegral}[c + d x] \, dx$$

$$1: \int \sinh[a + b x] \operatorname{SinhIntegral}[c + d x] \, dx$$

Derivation: Integration by parts

Rule:

$$\int \sinh[a + b x] \operatorname{SinhIntegral}[c + d x] \, dx \rightarrow \frac{\cosh[a + b x] \operatorname{SinhIntegral}[c + d x]}{b} - \frac{d}{b} \int \frac{\cosh[a + b x] \sinh[c + d x]}{c + d x} \, dx$$

Program code:

```
Int[Sinh[a_.+b_.*x_]*SinhIntegral[c_.+d_.*x_],x_Symbol] :=
  Cosh[a+b*x]*SinhIntegral[c+d*x]/b -
  d/b*Int[Cosh[a+b*x]*Sinh[c+d*x]/(c+d*x),x] /;
FreeQ[{a,b,c,d},x]
```

```
Int[Cosh[a_.+b_.*x_]*CoshIntegral[c_.+d_.*x_],x_Symbol] :=
  Sinh[a+b*x]*CoshIntegral[c+d*x]/b -
  d/b*Int[Sinh[a+b*x]*Cosh[c+d*x]/(c+d*x),x] /;
FreeQ[{a,b,c,d},x]
```

$$2. \int (e + f x)^m \sinh[a + b x] \operatorname{SinhIntegral}[c + d x] \, dx$$

$$1: \int (e + f x)^m \sinh[a + b x] \operatorname{SinhIntegral}[c + d x] \, dx \text{ when } m \in \mathbb{Z}^+$$

Derivation: Integration by parts

Rule: If $m \in \mathbb{Z}^+$, then

$$\int (e + f x)^m \sinh[a + b x] \operatorname{SinhIntegral}[c + d x] \, dx \rightarrow$$

$$\frac{(e + f x)^m \cosh[a + b x] \operatorname{SinhIntegral}[c + d x]}{b} - \frac{d}{b} \int \frac{(e + f x)^m \cosh[a + b x] \operatorname{Sinh}[c + d x]}{c + d x} dx - \frac{f m}{b} \int (e + f x)^{m-1} \cosh[a + b x] \operatorname{SinhIntegral}[c + d x] dx$$

Program code:

```
Int[(e_.+f_.**x_)^m_.*Sinh[a_.+b_.**x_]*SinhIntegral[c_.+d_.**x_],x_Symbol] :=
  (e+f*x)^m*Cosh[a+b*x]*SinhIntegral[c+d*x]/b -
  d/b*Int[(e+f*x)^m*Cosh[a+b*x]*Sinh[c+d*x]/(c+d*x),x] -
  f*m/b*Int[(e+f*x)^(m-1)*Cosh[a+b*x]*SinhIntegral[c+d*x],x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0]
```

```
Int[(e_.+f_.**x_)^m_.*Cosh[a_.+b_.**x_]*CoshIntegral[c_.+d_.**x_],x_Symbol] :=
  (e+f*x)^m*Sinh[a+b*x]*CoshIntegral[c+d*x]/b -
  d/b*Int[(e+f*x)^m*Sinh[a+b*x]*Cosh[c+d*x]/(c+d*x),x] -
  f*m/b*Int[(e+f*x)^(m-1)*Sinh[a+b*x]*CoshIntegral[c+d*x],x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0]
```


2: $\int (e + f x)^m \sinh[a + b x] \operatorname{SinhIntegral}[c + d x] dx$ when $m + 1 \in \mathbb{Z}^-$

Derivation: Inverted integration by parts

Rule: If $m + 1 \in \mathbb{Z}^-$, then

$$\int (e + f x)^m \sinh[a + b x] \operatorname{SinhIntegral}[c + d x] dx \rightarrow$$

$$\frac{(e + f x)^{m+1} \sinh[a + b x] \operatorname{SinhIntegral}[c + d x]}{f (m + 1)} -$$

$$\frac{d}{f (m + 1)} \int \frac{(e + f x)^{m+1} \sinh[a + b x] \sinh[c + d x]}{c + d x} dx - \frac{b}{f (m + 1)} \int (e + f x)^{m+1} \cosh[a + b x] \operatorname{SinhIntegral}[c + d x] dx$$

Program code:

```
Int[(e_.+f_.**x_)^m_*Sinh[a_.+b_.**x_]*SinhIntegral[c_.+d_.**x_],x_Symbol] :=
  (e+f*x)^(m+1)*Sinh[a+b*x]*SinhIntegral[c+d*x]/(f*(m+1)) -
  d/(f*(m+1))*Int[(e+f*x)^(m+1)*Sinh[a+b*x]*Sinh[c+d*x]/(c+d*x),x] -
  b/(f*(m+1))*Int[(e+f*x)^(m+1)*Cosh[a+b*x]*SinhIntegral[c+d*x],x] /;
FreeQ[{a,b,c,d,e,f},x] && ILtQ[m,-1]
```

```
Int[(e_.+f_.**x_)^m_*Cosh[a_.+b_.**x_]*CoshIntegral[c_.+d_.**x_],x_Symbol] :=
  (e+f*x)^(m+1)*Cosh[a+b*x]*CoshIntegral[c+d*x]/(f*(m+1)) -
  d/(f*(m+1))*Int[(e+f*x)^(m+1)*Cosh[a+b*x]*Cosh[c+d*x]/(c+d*x),x] -
  b/(f*(m+1))*Int[(e+f*x)^(m+1)*Sinh[a+b*x]*CoshIntegral[c+d*x],x] /;
FreeQ[{a,b,c,d,e,f},x] && ILtQ[m,-1]
```

$$4. \int u \cosh[a + b x] \operatorname{SinhIntegral}[c + d x] dx$$

$$1: \int \cosh[a + b x] \operatorname{SinhIntegral}[c + d x] dx$$

Derivation: Integration by parts

Rule:

$$\int \cosh[a + b x] \operatorname{SinhIntegral}[c + d x] dx \rightarrow \frac{\sinh[a + b x] \operatorname{SinhIntegral}[c + d x]}{b} - \frac{d}{b} \int \frac{\sinh[a + b x] \sinh[c + d x]}{c + d x} dx$$

Program code:

```
Int[Cosh[a_.+b_.*x_]*SinhIntegral[c_.+d_.*x_],x_Symbol] :=
  Sinh[a+b*x]*SinhIntegral[c+d*x]/b -
  d/b*Int[Sinh[a+b*x]*Sinh[c+d*x]/(c+d*x),x] /;
FreeQ[{a,b,c,d},x]
```

```
Int[Sinh[a_.+b_.*x_]*CoshIntegral[c_.+d_.*x_],x_Symbol] :=
  Cosh[a+b*x]*CoshIntegral[c+d*x]/b -
  d/b*Int[Cosh[a+b*x]*Cosh[c+d*x]/(c+d*x),x] /;
FreeQ[{a,b,c,d},x]
```

$$2. \int (e + f x)^m \cosh[a + b x] \operatorname{SinhIntegral}[c + d x] dx$$

$$1: \int (e + f x)^m \cosh[a + b x] \operatorname{SinhIntegral}[c + d x] dx \text{ when } m \in \mathbb{Z}^+$$

Derivation: Integration by parts

Rule: If $m \in \mathbb{Z}^+$, then

$$\int (e + f x)^m \cosh[a + b x] \operatorname{SinhIntegral}[c + d x] dx \rightarrow$$

$$\frac{(e+fx)^m \sinh[ax] \operatorname{SinhIntegral}[c+dx]}{b} - \frac{d}{b} \int \frac{(e+fx)^m \sinh[ax] \sinh[c+dx]}{c+dx} dx - \frac{fm}{b} \int (e+fx)^{m-1} \sinh[ax] \operatorname{SinhIntegral}[c+dx] dx$$

Program code:

```
Int[(e_.+f_.**x_)^m_.*Cosh[a_.+b_.**x_]*SinhIntegral[c_.+d_.**x_],x_Symbol] :=
  (e+f*x)^m*Sinh[a+b*x]*SinhIntegral[c+d*x]/b -
  d/b*Int[(e+f*x)^m*Sinh[a+b*x]*Sinh[c+d*x]/(c+d*x),x] -
  f*m/b*Int[(e+f*x)^(m-1)*Sinh[a+b*x]*SinhIntegral[c+d*x],x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0]
```

```
Int[(e_.+f_.**x_)^m_.*Sinh[a_.+b_.**x_]*CoshIntegral[c_.+d_.**x_],x_Symbol] :=
  (e+f*x)^m*Cosh[a+b*x]*CoshIntegral[c+d*x]/b -
  d/b*Int[(e+f*x)^m*Cosh[a+b*x]*Cosh[c+d*x]/(c+d*x),x] -
  f*m/b*Int[(e+f*x)^(m-1)*Cosh[a+b*x]*CoshIntegral[c+d*x],x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0]
```

2: $\int (e + f x)^m \cosh[a + b x] \operatorname{SinhIntegral}[c + d x] dx$ when $m + 1 \in \mathbb{Z}^-$

Derivation: Inverted integration by parts

Rule: If $m + 1 \in \mathbb{Z}^-$, then

$$\int (e + f x)^m \cosh[a + b x] \operatorname{SinhIntegral}[c + d x] dx \rightarrow$$

$$\frac{(e + f x)^{m+1} \cosh[a + b x] \operatorname{SinhIntegral}[c + d x]}{f (m + 1)} -$$

$$\frac{d}{f (m + 1)} \int \frac{(e + f x)^{m+1} \cosh[a + b x] \sinh[c + d x]}{c + d x} dx - \frac{b}{f (m + 1)} \int (e + f x)^{m+1} \sinh[a + b x] \operatorname{SinhIntegral}[c + d x] dx$$

Program code:

```
Int[(e_.+f_.*x_)^m_.*Cosh[a_.+b_.*x_] * SinhIntegral[c_.+d_.*x_], x_Symbol] :=
  (e+f*x)^(m+1)*Cosh[a+b*x]*SinhIntegral[c+d*x]/(f*(m+1)) -
  d/(f*(m+1))*Int[(e+f*x)^(m+1)*Cosh[a+b*x]*Sinh[c+d*x]/(c+d*x), x] -
  b/(f*(m+1))*Int[(e+f*x)^(m+1)*Sinh[a+b*x]*SinhIntegral[c+d*x], x] /;
FreeQ[{a,b,c,d,e,f}, x] && ILtQ[m, -1]
```

```
Int[(e_.+f_.*x_)^m_.*Sinh[a_.+b_.*x_] * CoshIntegral[c_.+d_.*x_], x_Symbol] :=
  (e+f*x)^(m+1)*Sinh[a+b*x]*CoshIntegral[c+d*x]/(f*(m+1)) -
  d/(f*(m+1))*Int[(e+f*x)^(m+1)*Sinh[a+b*x]*Cosh[c+d*x]/(c+d*x), x] -
  b/(f*(m+1))*Int[(e+f*x)^(m+1)*Cosh[a+b*x]*CoshIntegral[c+d*x], x] /;
FreeQ[{a,b,c,d,e,f}, x] && ILtQ[m, -1]
```

$$5. \int u \operatorname{SinhIntegral}[d(a + b \operatorname{Log}[c x^n])] dx$$

$$1: \int \operatorname{SinhIntegral}[d(a + b \operatorname{Log}[c x^n])] dx$$

Derivation: Integration by parts

$$\text{Basis: } \partial_x \operatorname{SinhIntegral}[d(a + b \operatorname{Log}[c x^n])] = \frac{b d n \operatorname{Sinh}[d(a + b \operatorname{Log}[c x^n])]}{x (d(a + b \operatorname{Log}[c x^n]))}$$

Rule: If $m \neq -1$, then

$$\int \operatorname{SinhIntegral}[d(a + b \operatorname{Log}[c x^n])] dx \rightarrow x \operatorname{SinhIntegral}[d(a + b \operatorname{Log}[c x^n])] - b d n \int \frac{\operatorname{Sinh}[d(a + b \operatorname{Log}[c x^n])]}{d(a + b \operatorname{Log}[c x^n])} dx$$

Program code:

```
Int[SinhIntegral[d.*(a_.+b_.Log[c_.*x_^n_.]),x_Symbol] :=
  x*SinhIntegral[d*(a+b*Log[c*x^n])] - b*d*n*Int[Sinh[d*(a+b*Log[c*x^n])]/(d*(a+b*Log[c*x^n])),x] /;
FreeQ[{a,b,c,d,n},x]
```

```
Int[CoshIntegral[d.*(a_.+b_.Log[c_.*x_^n_.]),x_Symbol] :=
  x*CoshIntegral[d*(a+b*Log[c*x^n])] - b*d*n*Int[Cosh[d*(a+b*Log[c*x^n])]/(d*(a+b*Log[c*x^n])),x] /;
FreeQ[{a,b,c,d,n},x]
```

$$2: \int \frac{\operatorname{SinhIntegral}[d(a + b \operatorname{Log}[c x^n])]}{x} dx$$

Derivation: Integration by substitution

$$\text{Basis: } \frac{F[\operatorname{Log}[c x^n]]}{x} = \frac{1}{n} \operatorname{Subst}[F[x], x, \operatorname{Log}[c x^n]] \partial_x \operatorname{Log}[c x^n]$$

Rule:

$$\int \frac{\text{SinhIntegral}[d(a + b \log[cx^n])]}{x} dx \rightarrow \frac{1}{n} \text{Subst}[\text{SinhIntegral}[d(a + bx)], x, \log[cx^n]]$$

Program code:

```
Int[F_*(d_.*(a_.*b_.*Log[c_.*x_^n_.]))/x_,x_Symbol] :=
  1/n*Subst[F[d*(a+b*x)],x,Log[c*x^n]] /;
FreeQ[{a,b,c,d,n},x] && MemberQ[{SinhIntegral,CoshIntegral},x]
```

3: $\int (ex)^m \text{SinhIntegral}[d(a + b \log[cx^n])] dx$ when $m \neq -1$

Derivation: Integration by parts

Basis: $\partial_x \text{SinhIntegral}[d(a + b \log[cx^n])] = \frac{bdn \sinh[d(a + b \log[cx^n])]}{x(d(a + b \log[cx^n])})$

Rule: If $m \neq -1$, then

$$\int (ex)^m \text{SinhIntegral}[d(a + b \log[cx^n])] dx \rightarrow \frac{(ex)^{m+1} \text{SinhIntegral}[d(a + b \log[cx^n])]}{e(m+1)} - \frac{bdn}{m+1} \int \frac{(ex)^m \sinh[d(a + b \log[cx^n])]}{d(a + b \log[cx^n])} dx$$

Program code:

```
Int[(e_.*x_)^m_.*SinhIntegral[d_.*(a_.*b_.*Log[c_.*x_^n_.))],x_Symbol] :=
  (e*x)^(m+1)*SinhIntegral[d*(a+b*Log[c*x^n])]/(e*(m+1)) -
  b*d*n/(m+1)*Int[(e*x)^m*Sinh[d*(a+b*Log[c*x^n])]/(d*(a+b*Log[c*x^n])),x] /;
FreeQ[{a,b,c,d,e,m,n},x] && NeQ[m,-1]
```

```
Int[(e_.*x_)^m_.*CoshIntegral[d_.*(a_.*b_.*Log[c_.*x_^n_.))],x_Symbol] :=
  (e*x)^(m+1)*CoshIntegral[d*(a+b*Log[c*x^n])]/(e*(m+1)) -
  b*d*n/(m+1)*Int[(e*x)^m*Cosh[d*(a+b*Log[c*x^n])]/(d*(a+b*Log[c*x^n])),x] /;
FreeQ[{a,b,c,d,e,m,n},x] && NeQ[m,-1]
```