

Rules for integrands of the form $(a + b x^n)^p \sin[c + d x]$

1: $\int (a + b x^n)^p \sin[c + d x] dx$ when $p \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule: If $p \in \mathbb{Z}^+$, then

$$\int (a + b x^n)^p \sin[c + d x] dx \rightarrow \int \sin[c + d x] \text{ExpandIntegrand}[(a + b x^n)^p, x] dx$$

```
Int[(a_+b_.*x_^n_)^p_.*Sin[c_+d_.*x_],x_Symbol] :=
  Int[ExpandIntegrand[Sin[c+d*x],(a+b*x^n)^p,x],x] /;
FreeQ[{a,b,c,d,n},x] && IGtQ[p,0]
```

```
Int[(a_+b_.*x_^n_)^p_.*Cos[c_+d_.*x_],x_Symbol] :=
  Int[ExpandIntegrand[Cos[c+d*x],(a+b*x^n)^p,x],x] /;
FreeQ[{a,b,c,d,n},x] && IGtQ[p,0]
```

2. $\int (a + b x^n)^p \sin[c + d x] dx$ when $p \in \mathbb{Z}^- \wedge n \in \mathbb{Z}$

1. $\int (a + b x^n)^p \sin[c + d x] dx$ when $p \in \mathbb{Z}^- \wedge n \in \mathbb{Z}^+$

1: $\int (a + b x^n)^p \sin[c + d x] dx$ when $p + 1 \in \mathbb{Z}^- \wedge n - 2 \in \mathbb{Z}^+$

Derivation: Integration by parts

Basis: $\partial_x \frac{(a + b x^n)^{p+1}}{b n (p+1)} = x^{n-1} (a + b x^n)^p$

Basis: $\partial_x (x^{-n+1} \sin[c + d x]) = -(n-1) x^{-n} \sin[c + d x] + d x^{-n+1} \cos[c + d x]$

Rule: If $p + 1 \in \mathbb{Z}^- \wedge n - 2 \in \mathbb{Z}^+$, then

$$\int (a + b x^n)^p \sin[c + d x] dx \rightarrow$$

$$\frac{x^{-n+1} (a+b x^n)^{p+1} \sin[c+d x]}{b n (p+1)} - \frac{-n+1}{b n (p+1)} \int x^{-n} (a+b x^n)^{p+1} \sin[c+d x] dx - \frac{d}{b n (p+1)} \int x^{-n+1} (a+b x^n)^{p+1} \cos[c+d x] dx$$

Program code:

```
Int[(a+b_.**x_^n_)^p_*Sin[c_.+d_.*x_],x_Symbol] :=
  x^(-n+1)*(a+b*x^n)^(p+1)*Sin[c+d*x]/(b*n*(p+1)) -
  (-n+1)/(b*n*(p+1))*Int[x^(-n)*(a+b*x^n)^(p+1)*Sin[c+d*x],x] -
  d/(b*n*(p+1))*Int[x^(-n+1)*(a+b*x^n)^(p+1)*Cos[c+d*x],x] /;
FreeQ[{a,b,c,d},x] && ILtQ[p,-1] && IGtQ[n,2]
```

```
Int[(a+b_.**x_^n_)^p_*Cos[c_.+d_.*x_],x_Symbol] :=
  x^(-n+1)*(a+b*x^n)^(p+1)*Cos[c+d*x]/(b*n*(p+1)) -
  (-n+1)/(b*n*(p+1))*Int[x^(-n)*(a+b*x^n)^(p+1)*Cos[c+d*x],x] +
  d/(b*n*(p+1))*Int[x^(-n+1)*(a+b*x^n)^(p+1)*Sin[c+d*x],x] /;
FreeQ[{a,b,c,d},x] && ILtQ[p,-1] && IGtQ[n,2]
```

2: $\int (a+b x^n)^p \sin[c+d x] dx$ when $p \in \mathbb{Z}^- \wedge n \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule: If $p \in \mathbb{Z}^- \wedge n \in \mathbb{Z}^+$, then

$$\int (a+b x^n)^p \sin[c+d x] dx \rightarrow \int \sin[c+d x] \text{ExpandIntegrand}[(a+b x^n)^p, x] dx$$

```
Int[(a+b_.**x_^n_)^p_*Sin[c_.+d_.*x_],x_Symbol] :=
  Int[ExpandIntegrand[Sin[c+d*x],(a+b*x^n)^p,x],x] /;
FreeQ[{a,b,c,d},x] && ILtQ[p,0] && IGtQ[n,0] && (EqQ[n,2] || EqQ[p,-1])
```

```
Int[(a+b_.**x_^n_)^p_*Cos[c_.+d_.*x_],x_Symbol] :=
  Int[ExpandIntegrand[Cos[c+d*x],(a+b*x^n)^p,x],x] /;
FreeQ[{a,b,c,d},x] && ILtQ[p,0] && IGtQ[n,0] && (EqQ[n,2] || EqQ[p,-1])
```

2: $\int (a + b x^n)^p \sin[c + d x] dx$ when $p \in \mathbb{Z}^- \wedge n \in \mathbb{Z}^-$

Derivation: Algebraic simplification

Basis: If $p \in \mathbb{Z}$, then $(a + b x^n)^p = x^{np} (b + a x^{-n})^p$

Rule: If $p \in \mathbb{Z}^- \wedge n \in \mathbb{Z}^-$, then

$$\int (a + b x^n)^p \sin[c + d x] dx \rightarrow \int x^{np} (b + a x^{-n})^p \sin[c + d x] dx$$

```
Int[(a_+b_.*x_^n_)^p_*Sin[c_+d_.*x_],x_Symbol] :=
  Int[x^(n*p)*(b+a*x^(-n))^p_*Sin[c+d*x],x] /;
FreeQ[{a,b,c,d},x] && ILtQ[p,0] && ILtQ[n,0]
```

```
Int[(a_+b_.*x_^n_)^p_*Cos[c_+d_.*x_],x_Symbol] :=
  Int[x^(n*p)*(b+a*x^(-n))^p_*Cos[c+d*x],x] /;
FreeQ[{a,b,c,d},x] && ILtQ[p,0] && ILtQ[n,0]
```

X: $\int (a + b x^n)^p \sin[c + d x] dx$

Rule:

$$\int (a + b x^n)^p \sin[c + d x] dx \rightarrow \int (a + b x^n)^p \sin[c + d x] dx$$

Program code:

```
Int[(a_+b_.*x_^n_)^p_*Sin[c_+d_.*x_],x_Symbol] :=
  Unintegrable[(a+b*x^n)^p_*Sin[c+d*x],x] /;
FreeQ[{a,b,c,d,n,p},x]
```

```
Int[(a_+b_.*x_^n_)^p_*Cos[c_+d_.*x_],x_Symbol] :=
  Unintegrable[(a+b*x^n)^p*Cos[c+d*x],x] /;
FreeQ[{a,b,c,d,n,p},x]
```

Rules for integrands of the form $(e x)^m (a + b x^n)^p \sin[c + d x]$

1: $\int (e x)^m (a + b x^n)^p \sin[c + d x] dx$ when $p \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule: If $p \in \mathbb{Z}^+$, then

$$\int (e x)^m (a + b x^n)^p \sin[c + d x] dx \rightarrow \int \sin[c + d x] \text{ExpandIntegrand}[(e x)^m (a + b x^n)^p, x] dx$$

```
Int[(e_.*x_)^m_.*(a_+b_.*x_^n_)^p_.*Sin[c_+d_.*x_],x_Symbol] :=
  Int[ExpandIntegrand[Sin[c+d*x],(e*x)^m*(a+b*x^n)^p,x],x] /;
FreeQ[{a,b,c,d,e,m,n},x] && IGtQ[p,0]
```

```
Int[(e_.*x_)^m_.*(a_+b_.*x_^n_)^p_.*Cos[c_+d_.*x_],x_Symbol] :=
  Int[ExpandIntegrand[Cos[c+d*x],(e*x)^m*(a+b*x^n)^p,x],x] /;
FreeQ[{a,b,c,d,e,m,n},x] && IGtQ[p,0]
```

2: $\int (e x)^m (a+b x^n)^p \sin[c+d x] dx$ when $p+1 \in \mathbb{Z}^- \wedge m = n-1 \wedge (n \in \mathbb{Z} \vee e > 0)$

Derivation: Integration by parts

Basis: If $m = n-1 \wedge (n \in \mathbb{Z} \vee e > 0)$, then $\partial_x \frac{e^m (a+b x^n)^{p+1}}{b n (p+1)} = (e x)^m (a+b x^n)^p$

Rule: If $p+1 \in \mathbb{Z}^- \wedge m = n-1 \wedge (n \in \mathbb{Z} \vee e > 0)$, then

$$\int (e x)^m (a+b x^n)^p \sin[c+d x] dx \rightarrow \frac{e^m (a+b x^n)^{p+1} \sin[c+d x]}{b n (p+1)} - \frac{d e^m}{b n (p+1)} \int (a+b x^n)^{p+1} \cos[c+d x] dx$$

```
Int[(e_.**x_)^m_.*(a_+b_.**x_^n_)^p_*Sin[c_+d_.**x_],x_Symbol] :=
  e^m*(a+b*x^n)^(p+1)*Sin[c+d*x]/(b*n*(p+1)) -
  d*e^m/(b*n*(p+1))*Int[(a+b*x^n)^(p+1)*Cos[c+d*x],x] /;
FreeQ[{a,b,c,d,e,m,n},x] && ILtQ[p,-1] && EqQ[m,n-1] && (IntegerQ[n] || GtQ[e,0])
```

```
Int[(e_.**x_)^m_.*(a_+b_.**x_^n_)^p_*Cos[c_+d_.**x_],x_Symbol] :=
  e^m*(a+b*x^n)^(p+1)*Cos[c+d*x]/(b*n*(p+1)) +
  d*e^m/(b*n*(p+1))*Int[(a+b*x^n)^(p+1)*Sin[c+d*x],x] /;
FreeQ[{a,b,c,d,e,m,n},x] && ILtQ[p,-1] && EqQ[m,n-1] && (IntegerQ[n] || GtQ[e,0])
```

3. $\int x^m (a + b x^n)^p \sin[c + d x] dx$ when $p \in \mathbb{Z}^- \wedge (m | n) \in \mathbb{Z}$

1. $\int x^m (a + b x^n)^p \sin[c + d x] dx$ when $p \in \mathbb{Z}^- \wedge n \in \mathbb{Z}^+$

1: $\int x^m (a + b x^n)^p \sin[c + d x] dx$ when $p + 1 \in \mathbb{Z}^- \wedge n \in \mathbb{Z}^+ \wedge (m - n + 1 > 0 \vee n > 2)$

Derivation: Integration by parts

Basis: $\partial_x \frac{(a+b x^n)^{p+1}}{b n (p+1)} = x^{n-1} (a + b x^n)^p$

Basis: $\partial_x (x^{m-n+1} \sin[c + d x]) = (m - n + 1) x^{m-n} \sin[c + d x] + d x^{m-n+1} \cos[c + d x]$

Rule: If $p + 1 \in \mathbb{Z}^- \wedge n \in \mathbb{Z}^+ \wedge (m - n + 1 > 0 \vee n > 2)$, then

$$\int x^m (a + b x^n)^p \sin[c + d x] dx \rightarrow \frac{x^{m-n+1} (a + b x^n)^{p+1} \sin[c + d x]}{b n (p + 1)} - \frac{m - n + 1}{b n (p + 1)} \int x^{m-n} (a + b x^n)^{p+1} \sin[c + d x] dx - \frac{d}{b n (p + 1)} \int x^{m-n+1} (a + b x^n)^{p+1} \cos[c + d x] dx$$

Program code:

```
Int[x^m_.*(a+b_.*x^n_)^p_*Sin[c_.+d_.*x_],x_Symbol] :=
  x^(m-n+1)*(a+b*x^n)^(p+1)*Sin[c+d*x]/(b*n*(p+1)) -
  (m-n+1)/(b*n*(p+1))*Int[x^(m-n)*(a+b*x^n)^(p+1)*Sin[c+d*x],x] -
  d/(b*n*(p+1))*Int[x^(m-n+1)*(a+b*x^n)^(p+1)*Cos[c+d*x],x] /;
FreeQ[{a,b,c,d,m},x] && ILtQ[p,-1] && IGtQ[n,0] && (GtQ[m-n+1,0] || GtQ[n,2]) && RationalQ[m]
```

```
Int[x^m_.*(a+b_.*x^n_)^p_*Cos[c_.+d_.*x_],x_Symbol] :=
  x^(m-n+1)*(a+b*x^n)^(p+1)*Cos[c+d*x]/(b*n*(p+1)) -
  (m-n+1)/(b*n*(p+1))*Int[x^(m-n)*(a+b*x^n)^(p+1)*Cos[c+d*x],x] +
  d/(b*n*(p+1))*Int[x^(m-n+1)*(a+b*x^n)^(p+1)*Sin[c+d*x],x] /;
FreeQ[{a,b,c,d,m},x] && ILtQ[p,-1] && IGtQ[n,0] && (GtQ[m-n+1,0] || GtQ[n,2]) && RationalQ[m]
```

2: $\int x^m (a+b x^n)^p \sin[c+d x] dx$ when $p \in \mathbb{Z}^- \wedge n \in \mathbb{Z}^+$

Derivation: Algebraic expansion

Rule: If $p \in \mathbb{Z}^- \wedge n \in \mathbb{Z}^+$, then

$$\int x^m (a+b x^n)^p \sin[c+d x] dx \rightarrow \int \sin[c+d x] \text{ExpandIntegrand}[x^m (a+b x^n)^p, x] dx$$

```
Int[x_^m_.*(a_+b_.x_^n_)^p_*Sin[c_+d_.x_],x_Symbol] :=
  Int[ExpandIntegrand[Sin[c+d*x],x^m*(a+b*x^n)^p,x],x] /;
FreeQ[{a,b,c,d,m},x] && ILtQ[p,0] && IGtQ[n,0] && (EqQ[n,2] || EqQ[p,-1]) && IntegerQ[m]
```

```
Int[x_^m_.*(a_+b_.x_^n_)^p_*Cos[c_+d_.x_],x_Symbol] :=
  Int[ExpandIntegrand[Cos[c+d*x],x^m*(a+b*x^n)^p,x],x] /;
FreeQ[{a,b,c,d,m},x] && ILtQ[p,0] && IGtQ[n,0] && (EqQ[n,2] || EqQ[p,-1]) && IntegerQ[m]
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2: $\int x^m (a+b x^n)^p \sin[c+d x] dx$ when $p \in \mathbb{Z}^- \wedge n \in \mathbb{Z}^-$

Derivation: Algebraic simplification

Basis: If $p \in \mathbb{Z}$, then $(a+b x^n)^p = x^{np} (b+a x^{-n})^p$

Rule: If $p \in \mathbb{Z}^- \wedge n \in \mathbb{Z}^-$, then

$$\int x^m (a+b x^n)^p \sin[c+d x] dx \rightarrow \int x^{m+np} (b+a x^{-n})^p \sin[c+d x] dx$$

```
Int[x_^m_.*(a_+b_.x_^n_)^p_*Sin[c_+d_.x_],x_Symbol] :=
  Int[x^(m+n*p)*(b+a*x^(-n))^p_*Sin[c+d*x],x] /;
FreeQ[{a,b,c,d,m},x] && ILtQ[p,0] && ILtQ[n,0]
```

```
Int[x_^m_.*(a_+b_.*x_^n_)^p_*Cos[c_+d_.*x_],x_Symbol] :=
  Int[x^(m+n*p)*(b+a*x^(-n))^p*Cos[c+d*x],x] /;
FreeQ[{a,b,c,d,m},x] && ILtQ[p,0] && ILtQ[n,0]
```

X: $\int (e x)^m (a + b x^n)^p \sin[c + d x] dx$

Rule:

$$\int (e x)^m (a + b x^n)^p \sin[c + d x] dx \rightarrow \int (e x)^m (a + b x^n)^p \sin[c + d x] dx$$

Program code:

```
Int[(e_.*x_)^m_.*(a_+b_.*x_^n_)^p_.*Sin[c_+d_.*x_],x_Symbol] :=
  Unintegrable[(e*x)^m*(a+b*x^n)^p*sin[c+d*x],x] /;
FreeQ[{a,b,c,d,e,m,n,p},x]
```

```
Int[(e_.*x_)^m_.*(a_+b_.*x_^n_)^p_.*Cos[c_+d_.*x_],x_Symbol] :=
  Unintegrable[(e*x)^m*(a+b*x^n)^p*cos[c+d*x],x] /;
FreeQ[{a,b,c,d,e,m,n,p},x]
```