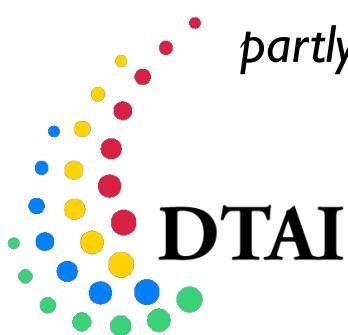


# Probability, Rules and Learning

Luc De Raedt  
*RuleML 2014*

*partly based on a joint tutorial with Angelika Kimmig*



# Purpose of this talk

Explore the use of probability in the context of rules, in  
the context of RuleML

My interpretation of RuleML, it is about

rules

reasoning about objects and relationships

applications in semantic web

emerging interest in uncertainty (eg. Vojtas & Bobek)

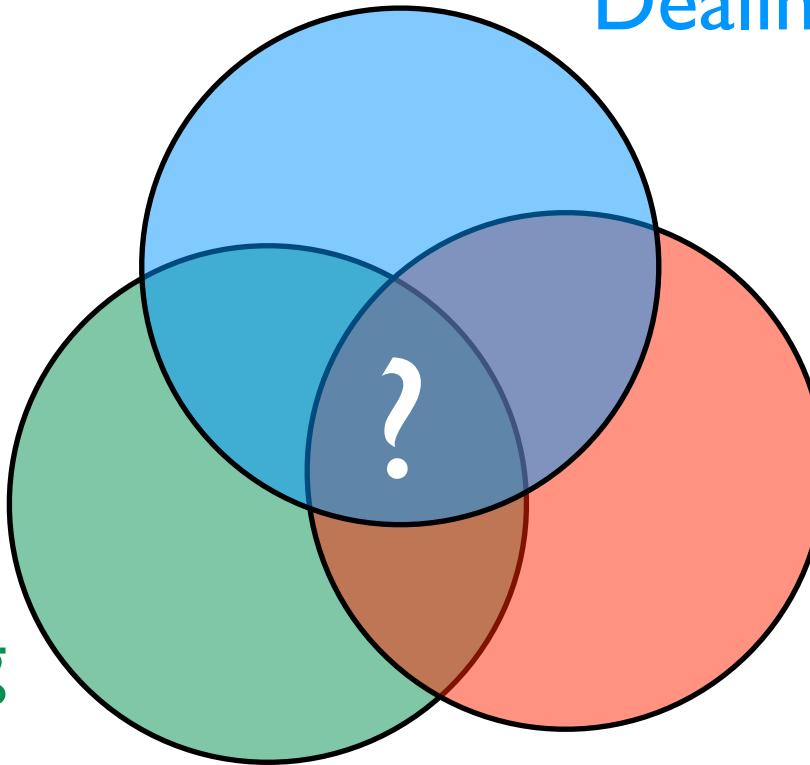
# (Wild) conjecture ?

- A lot of what you (seem to) do may be reformulated in a probabilistic logic / context
- The deterministic setting is obtained as a special case
- Still many challenges ahead ...

# A key question in AI:

Reasoning with relational data

- logic
- databases
- programming
- ...



Dealing with uncertainty

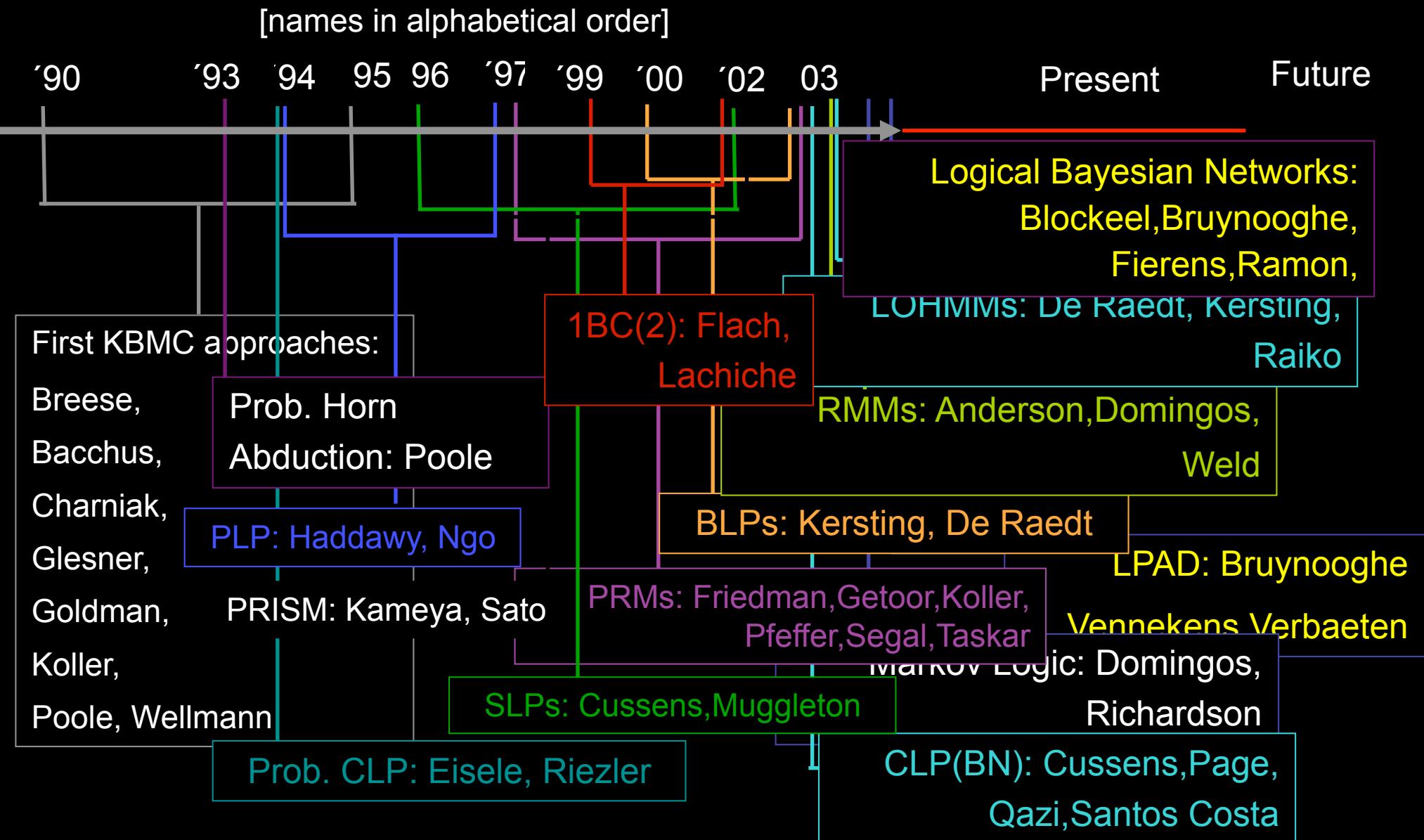
- probability theory
- graphical models
- ...

Learning

- parameters
- structure

Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

# Some SRL formalisms

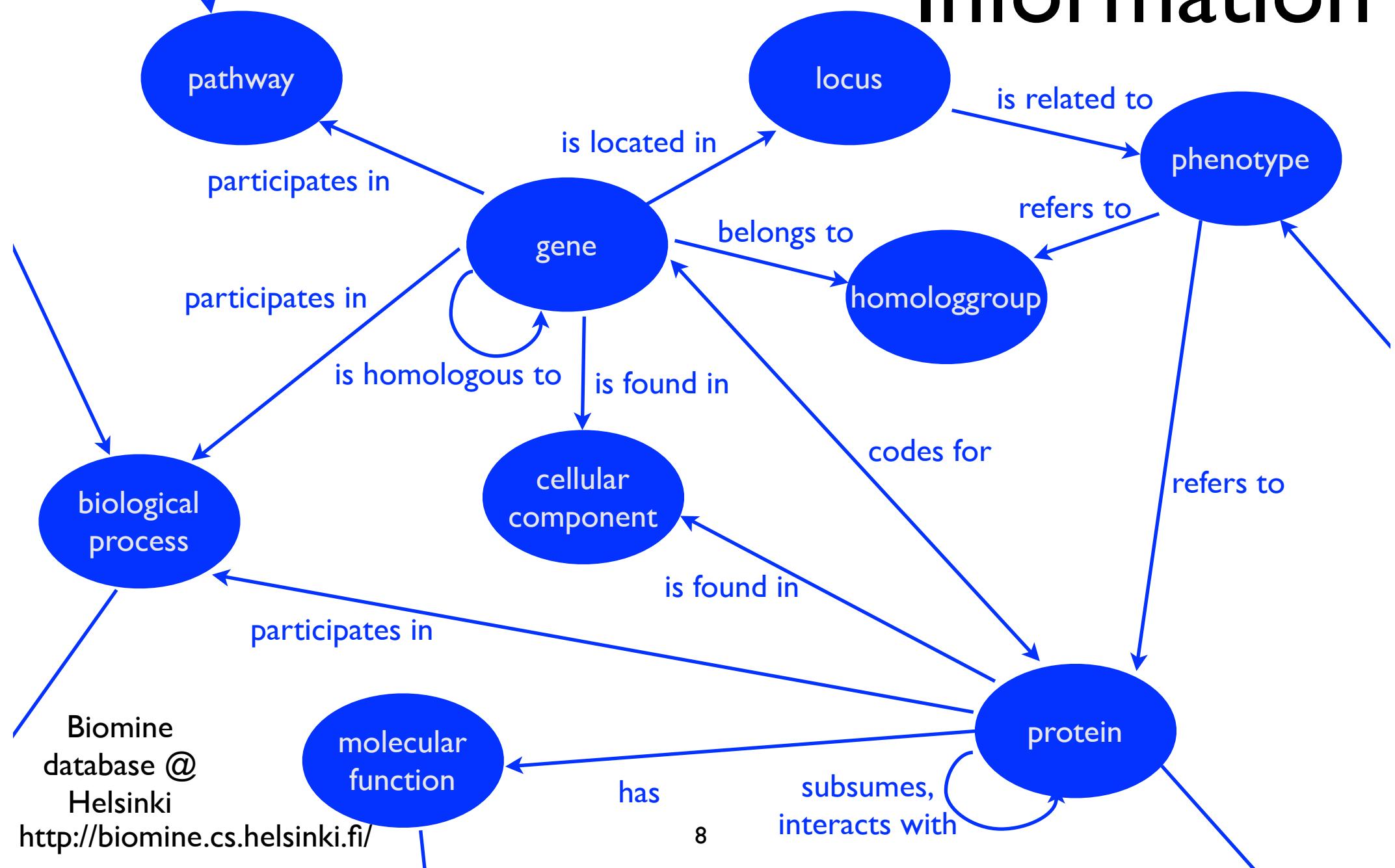


# Overview

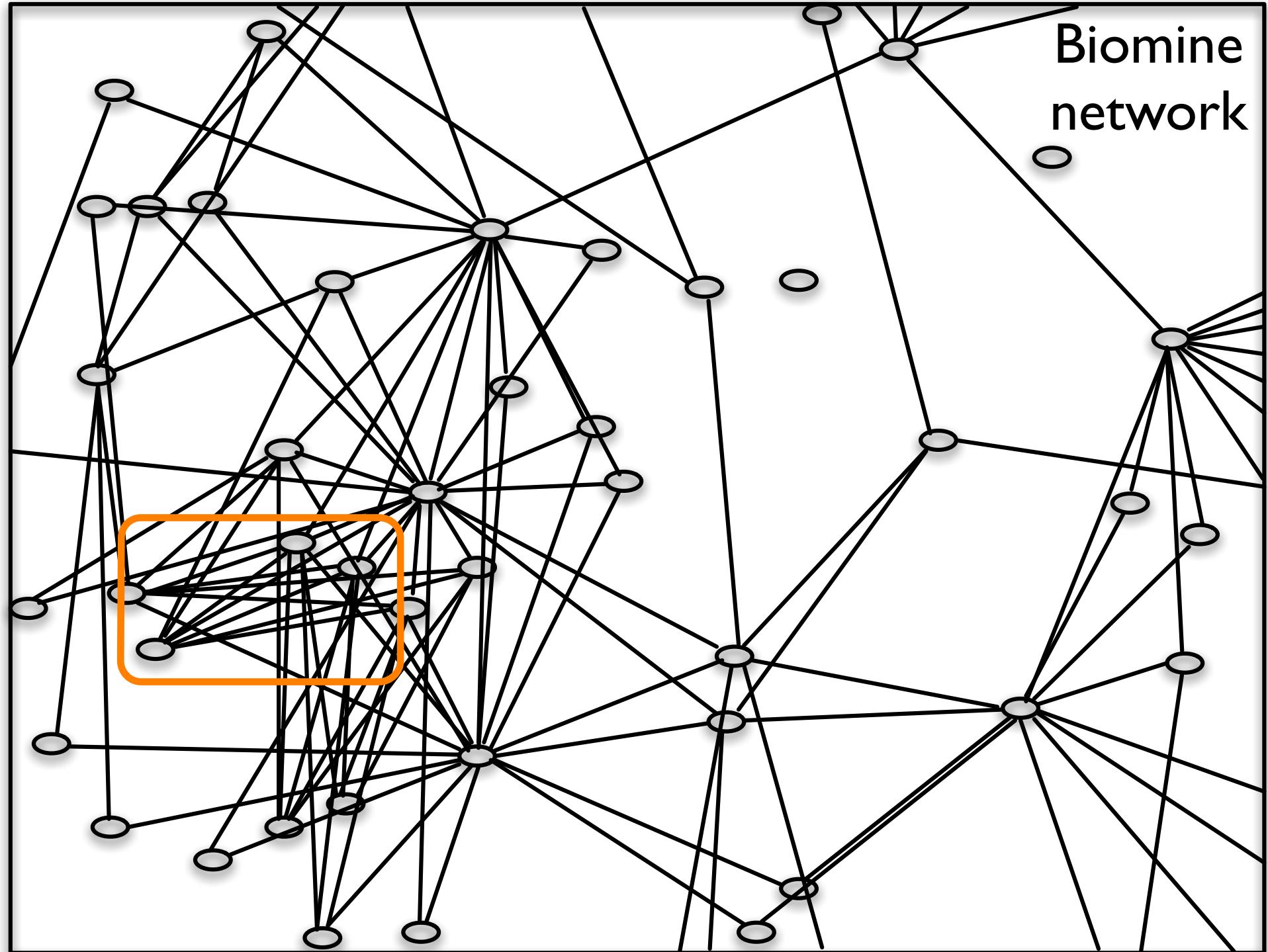
- Part I : Basic probabilistic Prolog framework & relation to alternative frameworks -- the rules and probabilities
- Part II : Inference (short) -- the reasoning
- Part III : Probabilistic rule learning (ProbFOIL) -- the learning
- Part IV: Dynamics & Continuous distributions for Relational Tracking (in Robotics)
- *Focus on ProbLog line of research at KU Leuven*

# PART I: Intro to Probabilistic Prologs

# Networks of Uncertain Information



Biomine  
network



Notch receptor processing

BiologicalProcess

GO:GO:0007220

BiologicalProces

-participates\_in  
0.220

presenilin 2  
Gene  
EntrezGene:81751

Gene

presenilin 2  
Gene  
EntrezGene:81751

Notch receptor processing  
BiologicalProcess  
GO:GO:0007220

integral to nuclear inner  
CellularComponent  
GO:GO:0005639

-participates\_in  
0.219

is\_homologated\_in  
0.530198

-participates\_in  
0.229

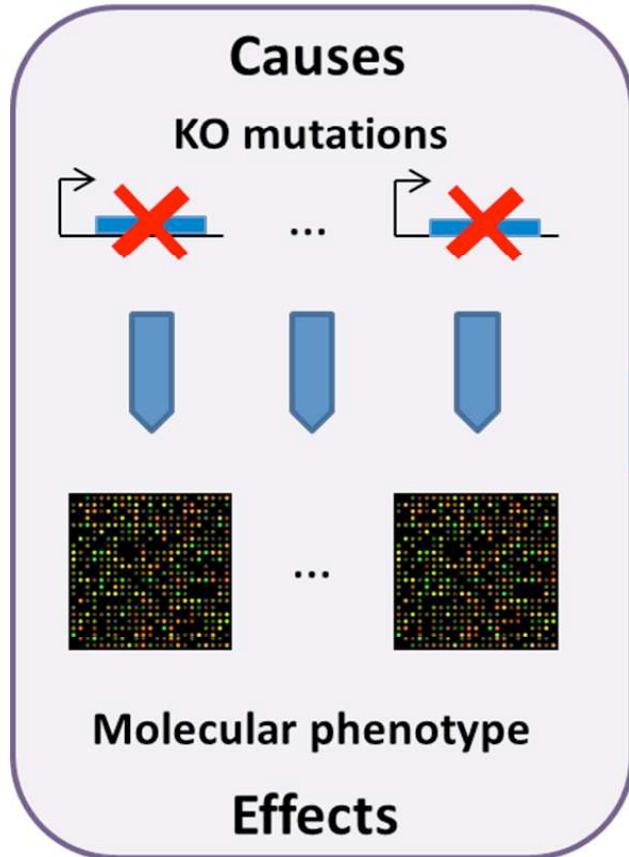
participates\_in  
0.192

-participates\_in  
0.207

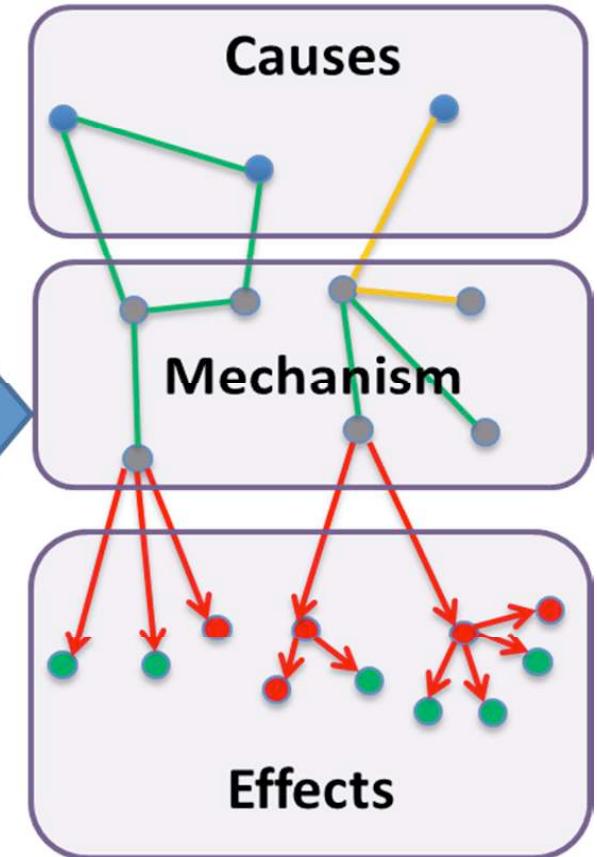
-participates\_in  
0.238

is\_homologated\_in  
0.505010

# Phenetic



DT-ProbLog  
decision theoretic version



- Causes: Mutations
  - All related to similar phenotype
- Effects: Differentially expressed genes
- 27 000 cause effect pairs

- Interaction network:
  - 3063 nodes
    - Genes
    - Proteins
  - 16794 edges
    - Molecular interactions
    - Uncertain

- Goal: connect causes to effects through common subnetwork
  - = Find mechanism
- Techniques:
  - DTProbLog
  - Approximate inference

Can we find the mechanism connecting causes to effects?

# Example: Information Extraction

instance	iteration	date learned	confidence
<a href="#">kelly andrews</a> is a <a href="#">female</a>	826	29-mar-2014	98.7  
<a href="#">investment next year</a> is an <a href="#">economic sector</a>	829	10-apr-2014	95.3  
<a href="#">shibenik</a> is a <a href="#">geopolitical entity</a> that is an organization	829	10-apr-2014	97.2  
<a href="#">quality web design work</a> is a <a href="#">character trait</a>	826	29-mar-2014	91.0  
<a href="#">mercedes benz cls by carlsson</a> is an <a href="#">automobile manufacturer</a>	829	10-apr-2014	95.2  
<a href="#">social work</a> is an academic program <a href="#">at the university rutgers university</a>	827	02-apr-2014	93.8  
<a href="#">dante wrote</a> the book <a href="#">the divine comedy</a>	826	29-mar-2014	93.8  
<a href="#">willie aames</a> was <a href="#">born in the city los angeles</a>	831	16-apr-2014	100.0  
<a href="#">kitt peak</a> is a mountain <a href="#">in the state or province arizona</a>	831	16-apr-2014	96.9  
<a href="#">greenwich</a> is a park <a href="#">in the city london</a>	831	16-apr-2014	100.0  

instances for many  
different relations

degree of certainty

# Graphs & Randomness

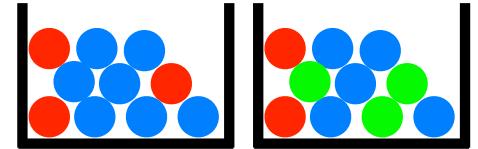
ProbLog, Phenny, Prism, ICL, Probabilistic  
Databases, ...

- all based on a “random graph” model

Stochastic Logic Programs, ProPPR, PCFGs, ...

- based on a “random walk” model
- connected to PageRank
- not the subject of this talk !

ProbLog by example:



# A bit of gambling

h

- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

0.4 :: heads .

**probabilistic fact:** heads is true with probability 0.4 and false with 0.6  
**annotated disjunction:** first ball is red with probability 0.3 and blue with 0.7

0.3 :: col(1,red) ; 0.7 :: col(1,blue) <- true .

0.2 :: col(2,red) ; 0.3 :: col(2,green) ;  
0.5 :: col(2,blue) <- true .

**annotated disjunction:** second ball is red with

probability 0.2, green with 0.3 and blue with 0.5  
**logical rule encoding consequences**  
win :- col(1,C), col(2,C) . background knowledge

# Questions

```
0.4 :: heads.
```

```
0.3 :: col(1,red) ; 0.7 :: col(1,blue) <- true.
```

```
0.2 :: col(2,red) ; 0.3 :: col(2,green) ; 0.5 :: col(2,blue) <- true.
```

```
win :- heads, col(_,red).
```

```
win :- col(1,C), col(2,C).
```

## marginal probability

- Probability of **win**?

## conditional probability

- Probability of **win** given **col(2,green)**?

- Most probable world where **win** is true?

## MPE inference

# Possible Worlds

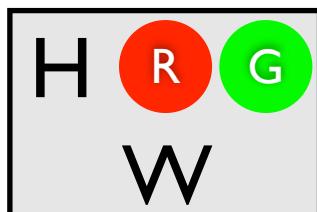
```
0.4 :: heads.
```

```
0.3 :: col(1,red) ; 0.7 :: col(1,blue) <- true
```

```
0.2 :: col(2,red) ; 0.3 :: col(2,green) ; 0.5 :: col(2,blue) <- true.
```

```
win :- heads, col(_,red).  
win :- col(1,C), col(2,C).
```

$$0.4 \times 0.3 \times 0.3$$



# Possible Worlds

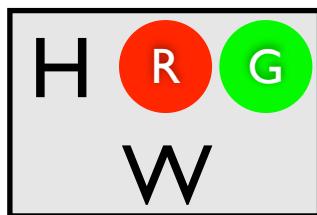
```
0.4 :: heads.
```

```
0.3 :: col(1,red) ; 0.7 :: col(1,blue) <- true.
```

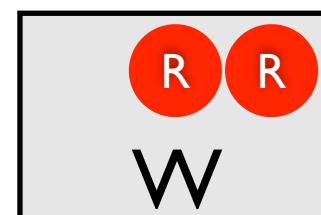
```
0.2 :: col(2,red) ; 0.3 :: col(2,green) ; 0.5 :: col(2,blue) <- true.
```

```
win :- heads, col(_,red).  
win :- col(1,C), col(2,C).
```

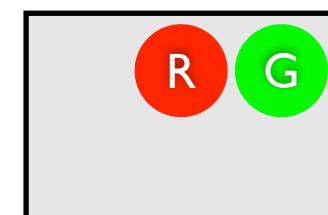
$$0.4 \times 0.3 \times 0.3$$



$$(1-0.4) \times 0.3 \times 0.2$$

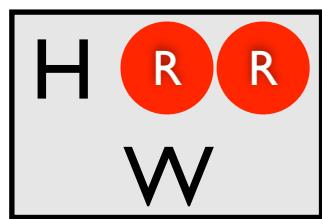


$$(1-0.4) \times 0.3 \times 0.3$$

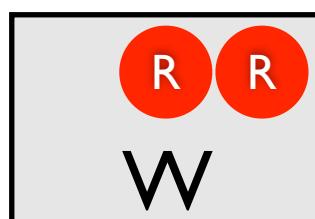


# All Possible Worlds

0.024



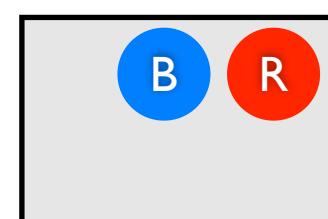
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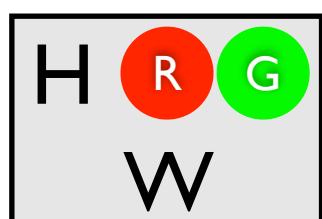
0.056



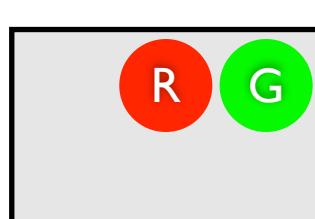
0.084



0.036



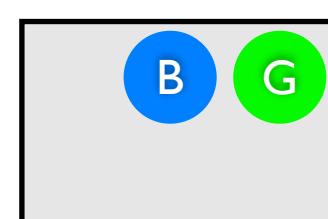
0.054



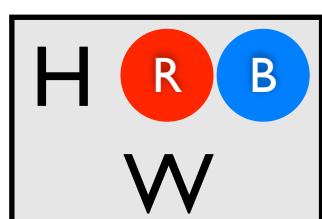
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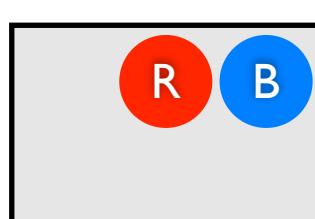
0.126



0.060



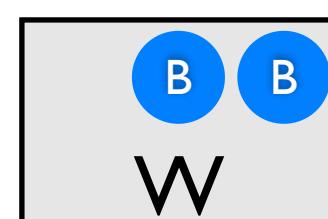
0.090



0.140



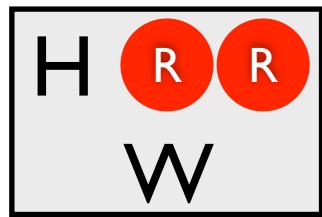
0.210



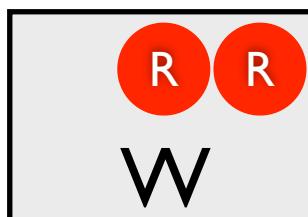
# Most likely world where col(2,blue) is false?

MPE Inference

0.024



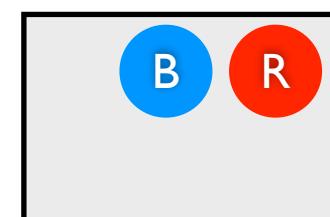
0.036



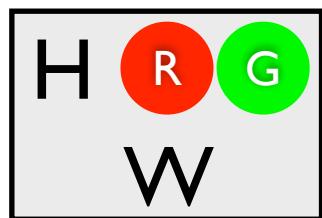
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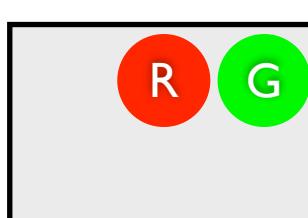
0.084



0.036



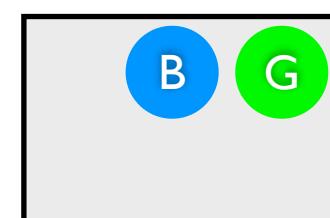
0.054



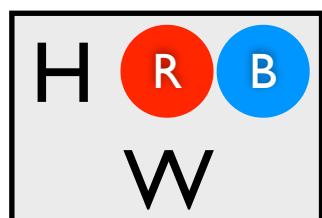
0.084



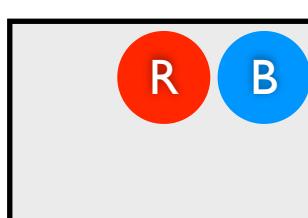
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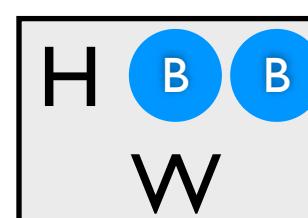
0.060



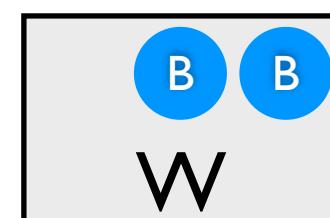
0.090



0.140



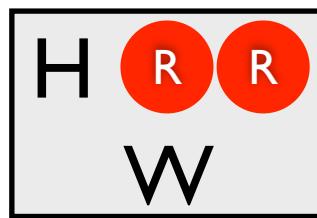
0.210



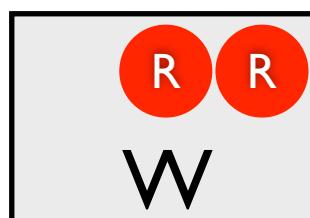
$$P(\underline{\text{win}}) = \Sigma = 0.562$$

Marginal  
Probability

0.024



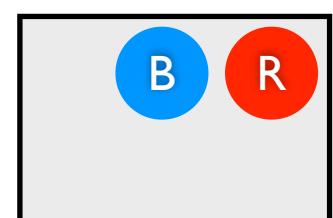
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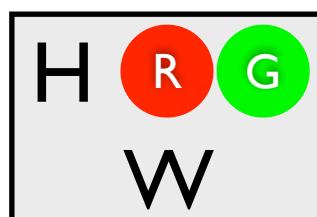
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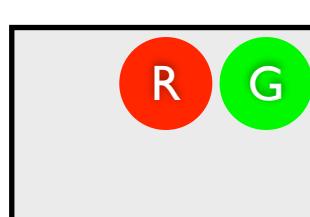
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0.036



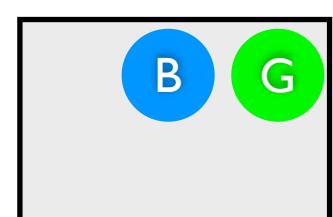
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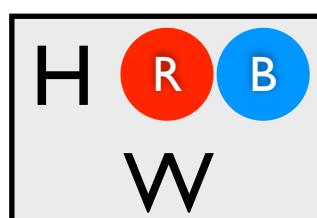
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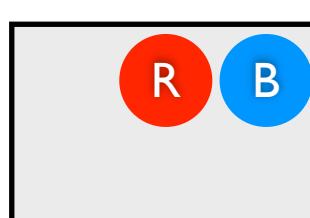
0.126



0.060



0.090



0.140



0.210

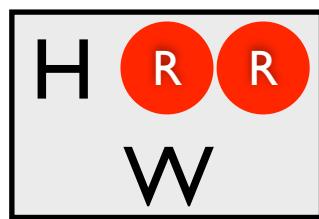


$$P(\text{win} | \underline{\text{col}(2, \text{green})}) = \frac{\Sigma}{\Sigma}$$

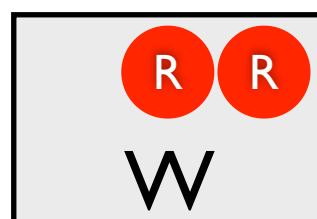
$$= P(\text{win} \wedge \underline{\text{col}(2, \text{green})}) / P(\underline{\text{col}(2, \text{green})})$$

Conditional  
Probability

0.024



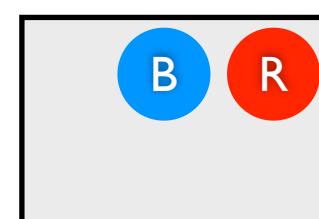
0.036



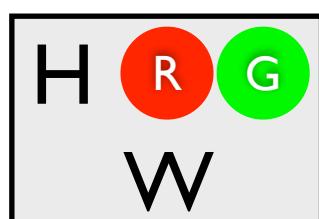
0.056



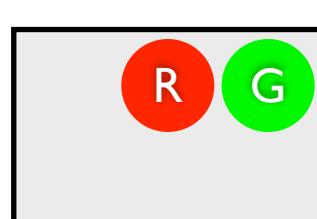
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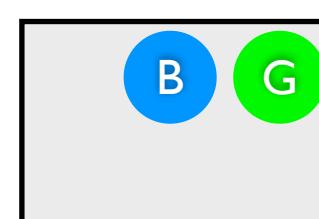
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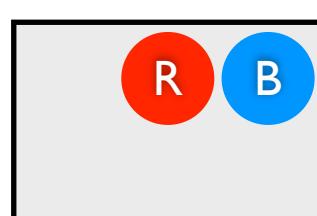
0.126



0.060



0.090



0.140



0.210



# Distribution Semantics

## (with probabilistic facts)

[Sato, ICLP 95]

$$P(Q) = \frac{\sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)}{\text{probability of possible world}}$$

query

subset of probabilistic facts

sum over possible worlds where Q is true

Prolog rules

# cProbLog: constraints on possible worlds

```
weight(skis, 6).  
weight(boots, 4).  
weight(helmet, 3).  
weight(gloves, 2).  
  
P::pack(Item) :-  
    weight(Item, Weight),  
    P is 1.0/Weight.
```

```
excess(Limit) :- ...
```

```
not excess(10).  
pack(helmet) v pack(boots).
```

**constraints**  
as FOL formulas  
treat as evidence

distribution  
normalized distribution  
over all possible  
over restricted set of  
possible worlds

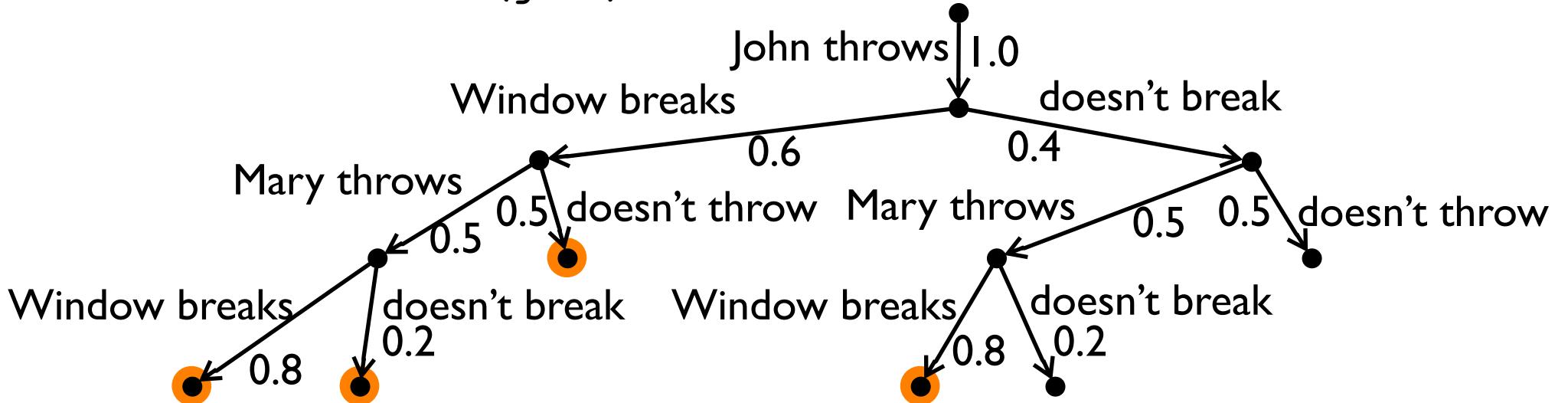
sbh e(10)	sb g e(10)	sbh e(10)	sb
s h g e(10)	s g	s h	s
bhg	b g	bh	b
hg	g	h	

# Alternative view: CP-Logic

```
throws(john) .
0.5 :: throws(mary) .
```

```
0.8 :: break <- throws(mary) .
0.6 :: break <- throws(john) .
```

probabilistic causal laws



$$P(\text{break}) = 0.6 \times 0.5 \times 0.8 + 0.6 \times 0.5 \times 0.2 + 0.6 \times 0.5 + 0.4 \times 0.5 \times 0.8$$

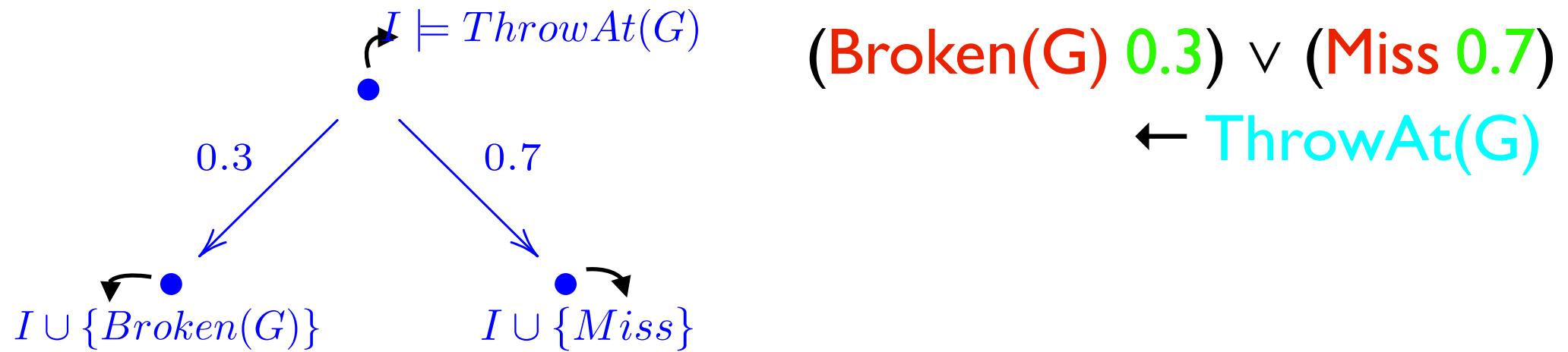
# CP-logic [Vennekens et al.]

E.g., “**throwing** a rock at a glass **breaks** it with probability **0.3** and **misses** it with probability **0.7**”

$(\text{Broken}(G):0.3) \vee (\text{Miss } 0.7) \leftarrow \text{ThrowAt}(G).$

Note that the actual non-deterministic event (“rock flying at glass”) is implicit

# Semantics



Probability tree is an execution model of theory iff:

- Each tree-transition **matches** causal law
- The tree cannot be extended

Each execution model defines the same probability distribution over final states

# Distributional Clauses (DC)

- Discrete- and continuous-valued random variables

## **random variable** with Gaussian distribution

```
length(Obj) ~ gaussian(6.0, 0.45) :- type(Obj, glass).
```

```
stackable(OBot, OTop) :-
```

```
    ≈length(OBot) ≥ ≈length(OTop),
```

```
    ≈width(OBot) ≥ ≈width(OTop).
```

**comparing** values of  
random variables

```
ontype(Obj, plate) ~ finite([0 : glass, 0.0024 : cup,  
                             0 : pitcher, 0.8676 : plate,  
                             0.0284 : bowl, 0 : serving,  
                             0.1016 : none])
```

```
:- obj(Obj), on(Obj, O2), type(O2, plate).
```

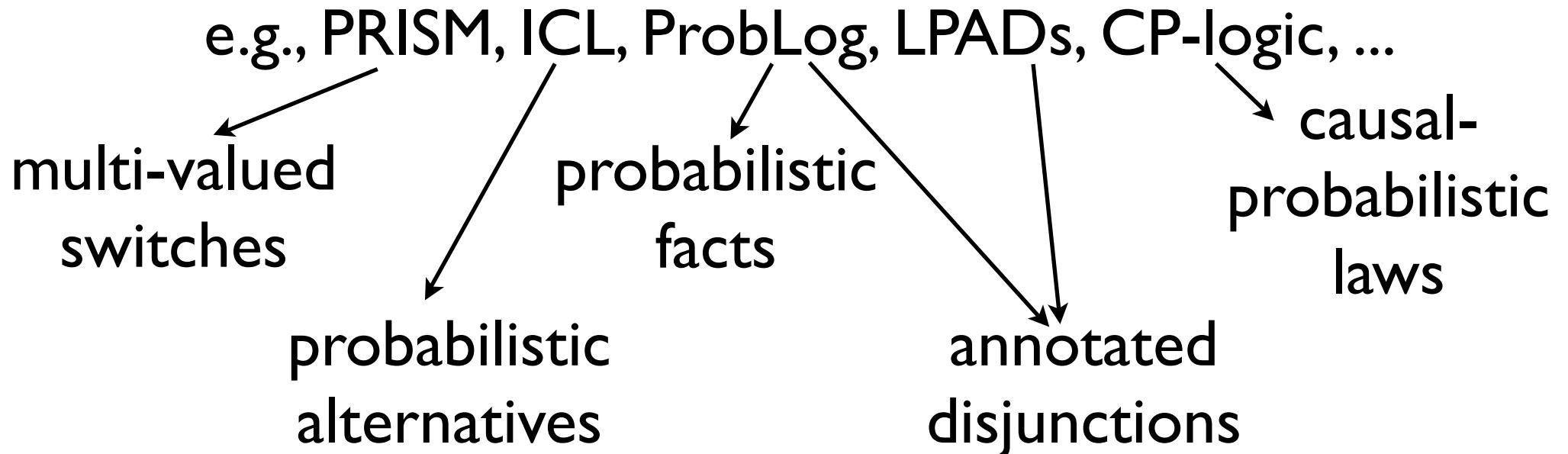


## **random variable** with discrete distribution

# Probabilistic Logic Programming

Distribution Semantics [Sato, ICLP 95]:  
probabilistic choices + logic program  
→ distribution over possible worlds

OVERVIEW paper [Kimmig, De Raedt, Arxiv]



# Probabilistic databases

programming versus database query language  
different types of queries

ProducesProduct		
Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn		
Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

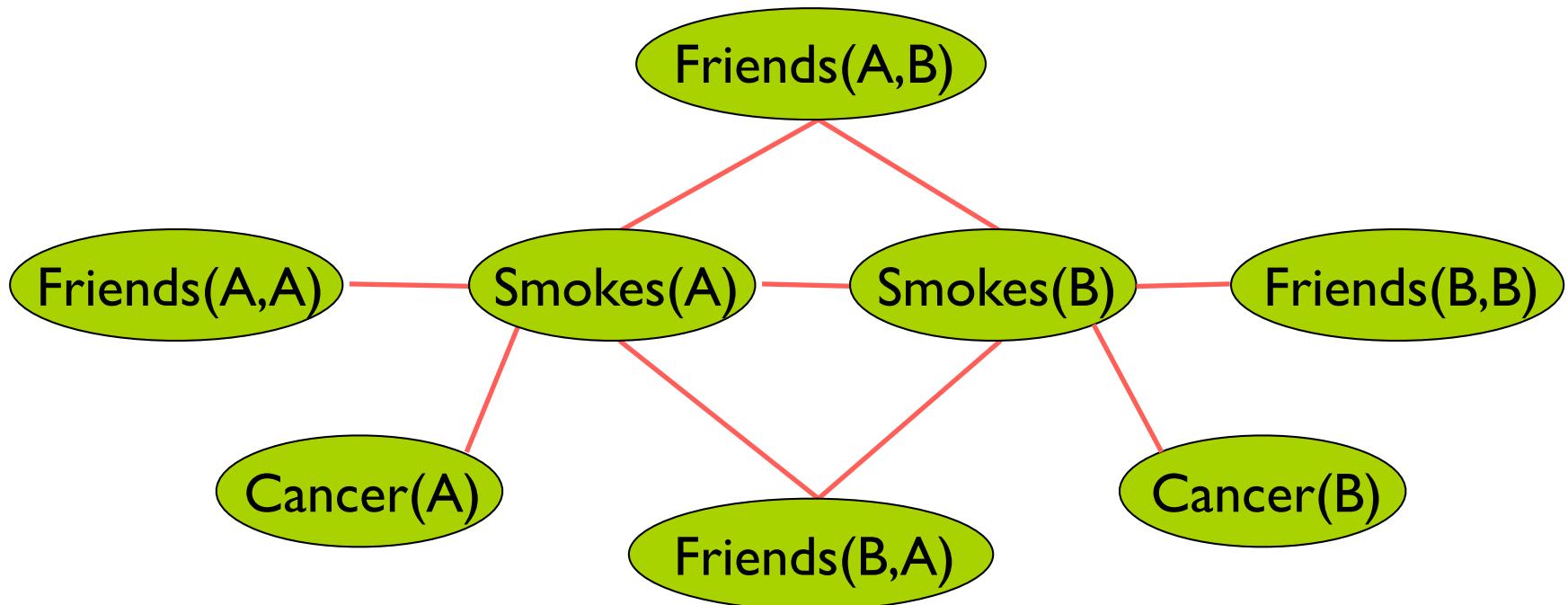
# Probabilistic Programs

- Distributional clauses / PLP similar in spirit
  - to e.g. BLOG, ... but embedded in existing logic and programming language
  - to e.g. Church but use of logic instead of functional programming ...
  - natural possible world semantics and link with prob. databases.
  - somewhat harder to do meta-programming

# Markov Logic

- |     |                                                                                                      |
|-----|------------------------------------------------------------------------------------------------------|
| 1.5 | $\forall x \text{ Smokes}(x) \Rightarrow \text{Cancer}(x)$                                           |
| 1.1 | $\forall x, y \text{ Friends}(x, y) \Rightarrow (\text{Smokes}(x) \Leftrightarrow \text{Smokes}(y))$ |

Suppose we have two constants: **Anna** (A) and **Bob** (B)



# Markov Logic

## Key differences

- programming language
- soft constraints
- Pro(b)log uses least-fix point semantics
  - can express transitive closure of relation
  - this cannot be expressed in FOL (and Markov Logic), requires second order logic
  - $p(X,Y) :- p(X,Z), p(Z,Y).$

# Take away message

## Key insight

- Taisuke Sato, Distribution Semantics and David Poole

## Upgrading logic / rules

- unify basic notions in logic and in probability theory
- ground atoms become random variables
- retain the rules and the logic

# PART II: Inference

# Inference in PLP

- As in Prolog and logic programming
  - proof-based
- As in Answer Set Programming
  - model based
- As in Probabilistic Programming
  - sampling

# Inference

1. using proofs
2. using models

knowledge compilation

**Given:**

program

queries

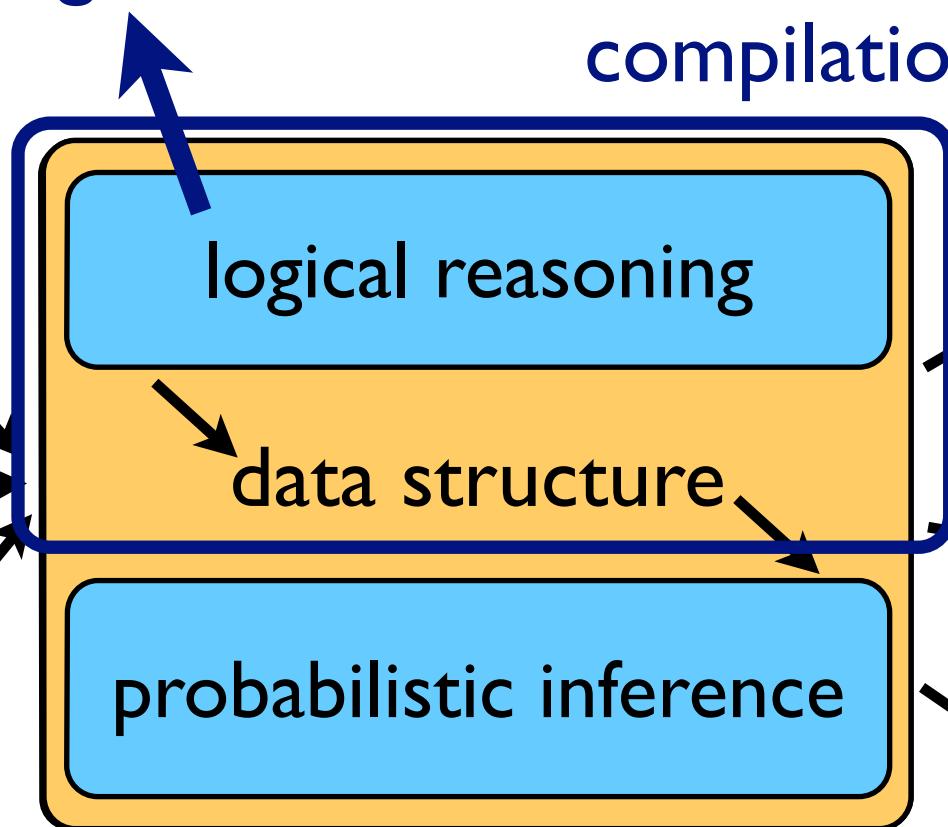
evidence

**Find:**

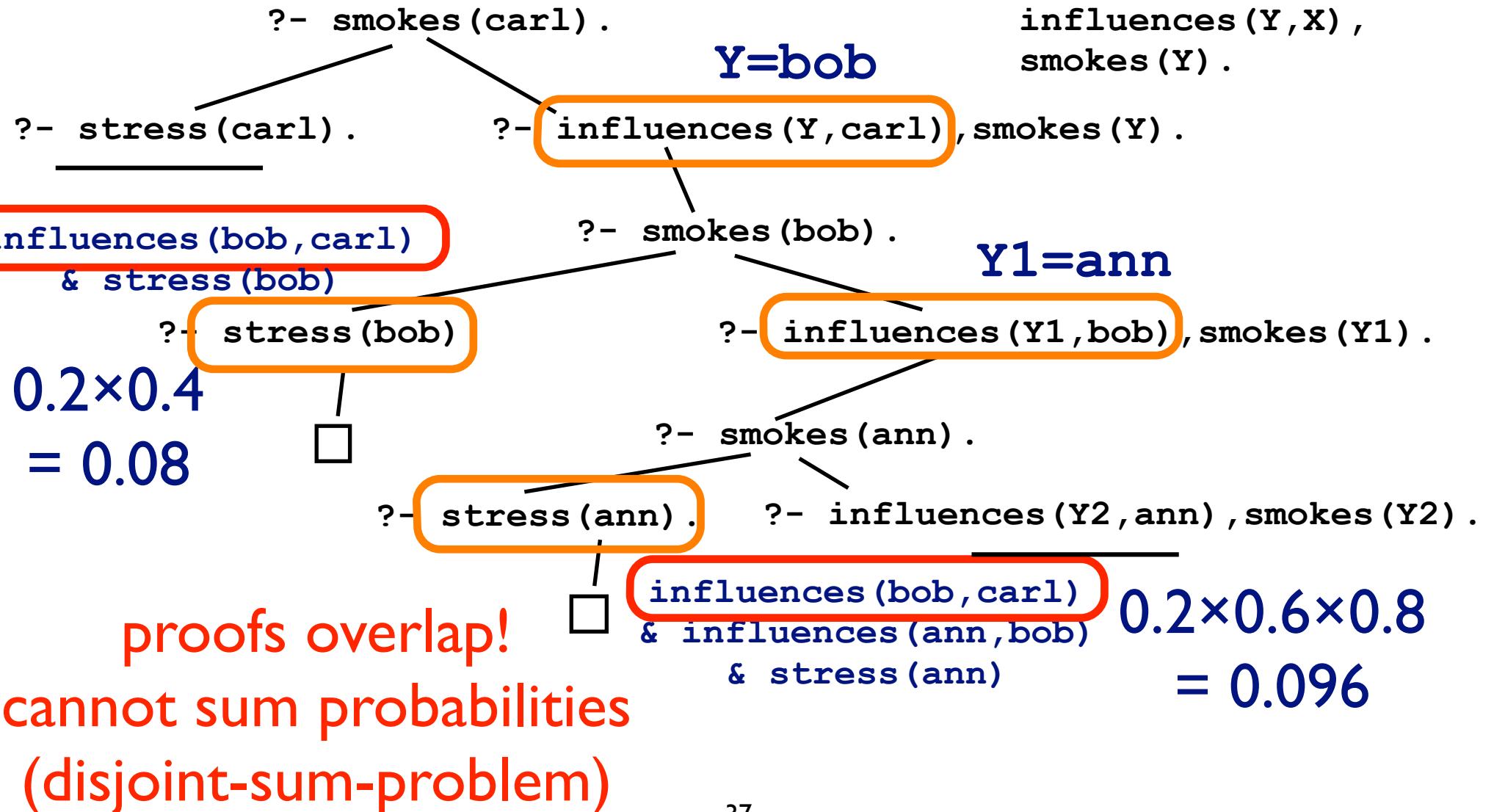
marginal probabilities

conditional probabilities

MPE state



# Proofs in ProbLog



# Disjoint-Sum-Problem

possible worlds

solution: knowledge compilation

infl(bob, carl) & infl(ann, bob) & st(ann) & \+st(bob)	0.05 / 6
infl(bob, carl) & infl(ann, bob) & st(ann) & st(bob)	0.0384
infl(bob, carl) & \+infl(ann, bob) & st(ann) & st(bob)	0.0256
infl(bob, carl) & infl(ann, bob) & \+st(ann) & st(bob)	0.0096
infl(bob, carl) & \+infl(ann, bob) & \+st(ann) & st(bob)	0.0064
<hr/>	
... influences(bob, carl) & stress(bob)	$\sum = 0.1376$

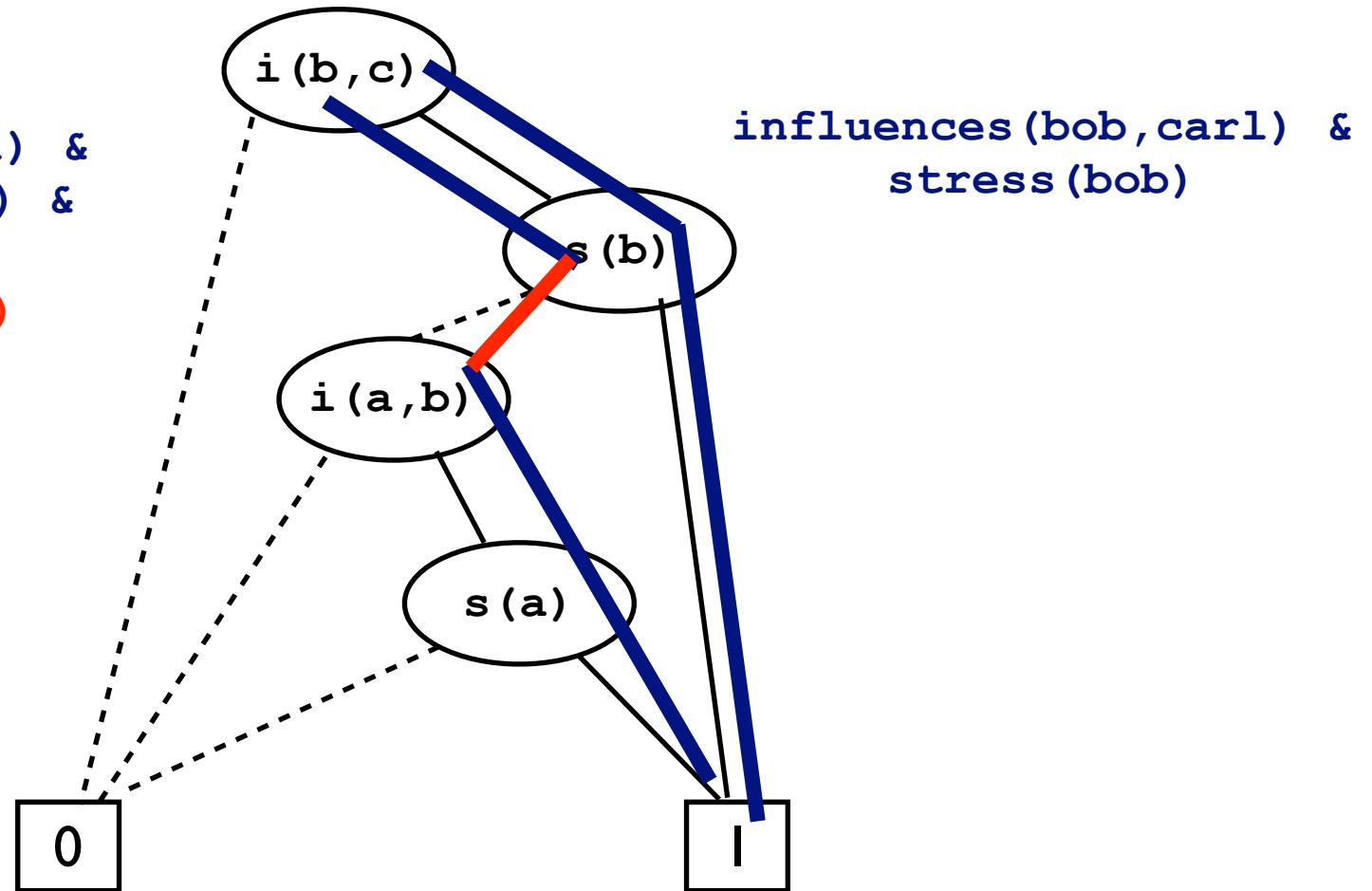
sum of proof probabilities:  $0.096 + 0.08 = 0.1760$

# Binary Decision Diagrams

[Bryant 86]

influences (bob, carl) &  
influences (ann, bob) &  
stress (ann)  
**& not stress (bob)**

influences (bob, carl) &  
stress (bob)



# Binary Decision Diagrams

$$0.8 \times 0.0 + 0.2 \times 0.688 = 0.1376$$

0.8

i (b, c)

yes

$$0.6 \times 0.48 + 0.4 \times 1.0 = 0.688$$

0.6

s (b)

influences (bob, carl) ?

$$0.4 \times 0.0 + 0.6 \times 0.8 = 0.48$$

0.4

i (a, b)

yes

influences (ann, bob) ?

$$0.2 \times 0.0 + 0.8 \times 1.0 = 0.8$$

0.2

s (a)

yes

yes

stress (ann) ?

0

0.0

1

1.0

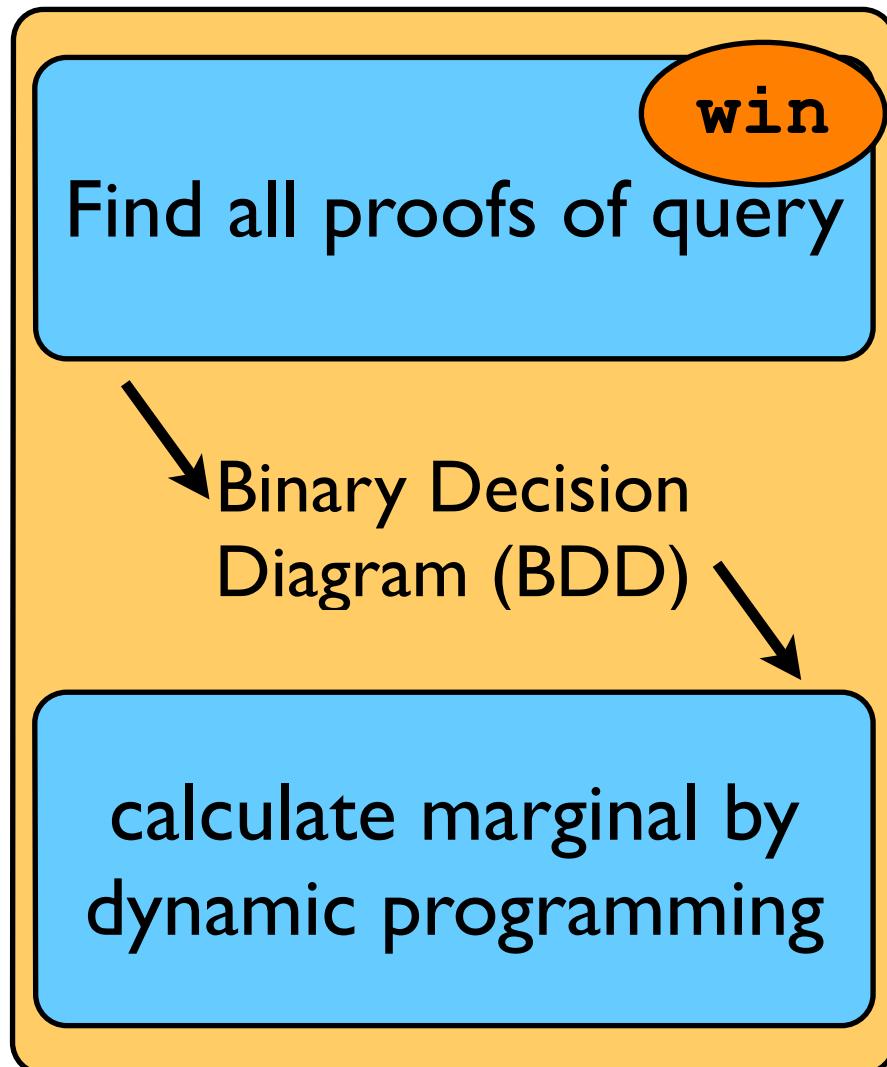
probability of  
smokes (c) ?

smokes (c) = i (b, c) & s (b)  $\vee$   
i (b, c) & i (a, b) & s (a)

# Initial Approach

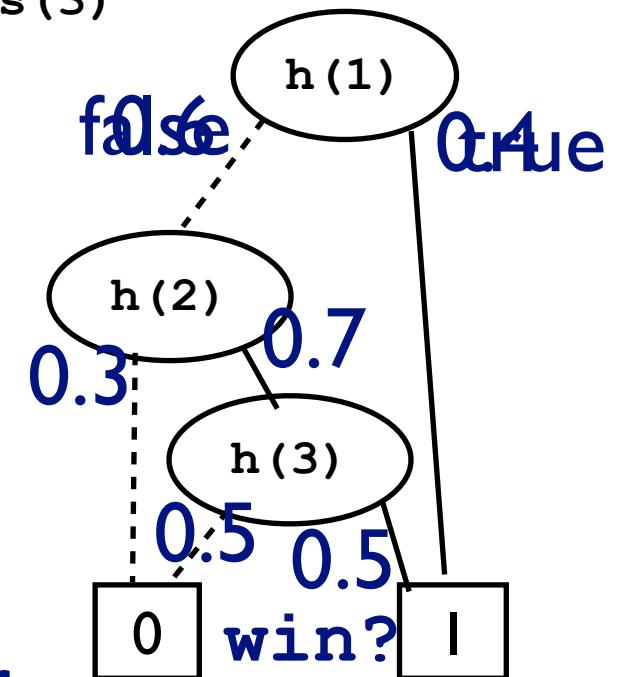
(ProbLogI & others)

```
0.4 : heads(1).  
0.7 : heads(2).  
0.5 : heads(3).  
win :- heads(1).  
win :- heads(2),heads(3).
```



heads(1)  
heads(2) & heads(3)

$P(\text{win}) =$   
probability of  
reaching I-leaf



# Answering Questions

1. using proofs
2. using models

**Given:**

program

queries

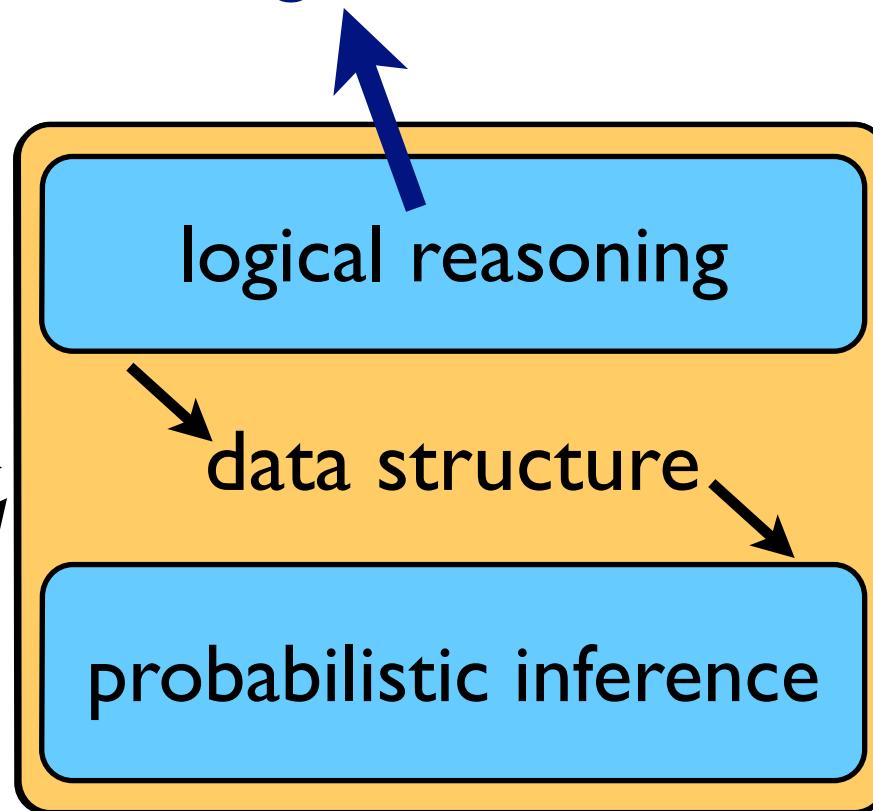
evidence

**Find:**

marginal  
probabilities

conditional  
probabilities

MPE state



# Current Approach (ProbLog2)

```
0.4 :: heads(1).  
0.7 :: heads(2).  
0.5 :: heads(3).  
win :- heads(1).  
win :- heads(2),  
      heads(3).
```

Find relevant ground program for queries & evidence

win

Weighted CNF

use weighted model counting / satisfiability

win :- heads(1).  
win :- heads(2), heads(3).

win  $\leftrightarrow$  h(1)  $\vee$  (h(2)  $\wedge$  h(3))  
may require loop-breaking

$(\neg \text{win} \vee \text{h}(1) \vee \text{h}(2))$   
 $\wedge (\neg \text{win} \vee \text{h}(1) \vee \text{h}(3))$   
 $\wedge (\text{win} \vee \neg \text{h}(1))$   
 $\wedge (\text{win} \vee \neg \text{h}(2) \vee \neg \text{h}(3))$

use  
standard  
tool

$\text{h}(1) \rightarrow 0.4$        $\text{h}(2) \rightarrow 0.7$        $\text{h}(3) \rightarrow 0.5$   
 $\neg \text{h}(1) \rightarrow 0.6$        $\neg \text{h}(2) \rightarrow 0.3$        $\neg \text{h}(3) \rightarrow 0.5$

# ProbLog → CNF

```
?- smokes(carl) .
```

```
0.8::stress(ann) .  
0.4::stress(bob) .  
0.6::influences(ann,bob) .  
0.2::influences(bob,carl) .
```

```
smokes(X) :- stress(X) .  
smokes(X) :-  
    influences(Y,X) ,  
    smokes(Y) .
```

- Find relevant ground rules by backward reasoning

```
smokes(carl) :- influences(bob,carl) , smokes(bob) .  
smokes(bob) :- stress(bob) .  
smokes(bob) :- influences(ann,bob) , smokes(ann) .  
smokes(ann) :- stress(ann) .
```

- Convert to propositional logic formula

may require  
loop-breaking

$$\begin{aligned} \text{sm}(c) &\leftrightarrow (\text{i}(b,c) \wedge \text{sm}(b)) \\ \wedge \text{sm}(b) &\leftrightarrow (\text{st}(b) \vee (\text{i}(a,b) \wedge \text{sm}(a))) \\ \wedge \text{sm}(a) &\leftrightarrow \text{st}(a) \end{aligned}$$

- Rewrite in CNF (as usual)

# Weighted

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

propositional formula in conjunctive normal form (CNF)

given by ProbLog program & query

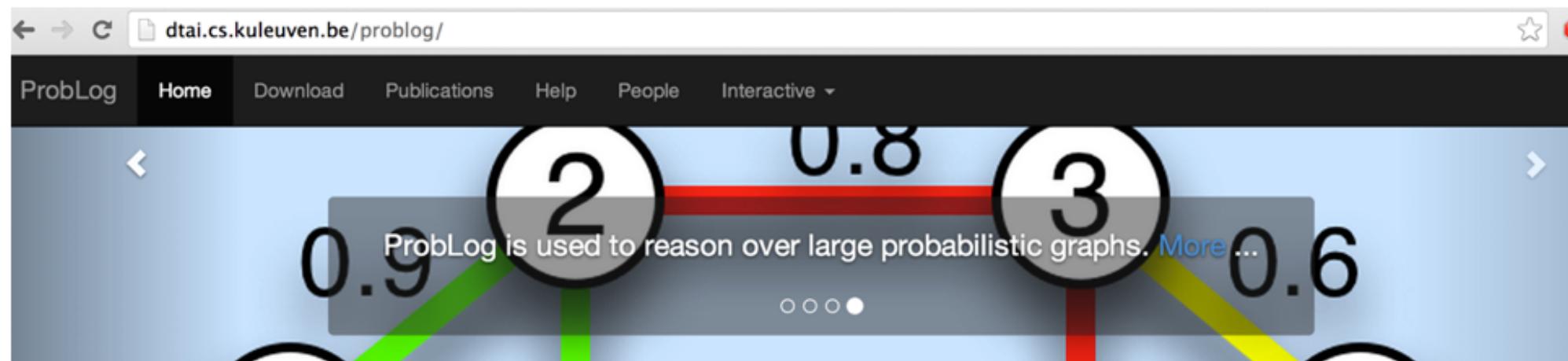
$$WMC(\phi) = \sum_{I_V \models \phi} \prod_{l \in I_V} w(l)$$

interpretations (truth  
value assignments) of  
propositional variables

possible worlds

weight  
of literal

for  $p::f$ ,  
 $w(f) = p$   
 $w(\text{not } f) = 1-p$



## Introduction.

Probabilistic logic programs are logic programs in which some of the facts are annotated with probabilities.

ProbLog is a tool that allows you to intuitively build programs that do not only encode **complex interactions** between a large sets of **heterogenous components** but also the inherent **uncertainties** that are present in real-life situations.

The engine tackles several tasks such as computing the marginals given evidence and learning from (partial) interpretations. ProbLog is a suite of efficient algorithms for various inference tasks. It is based on a conversion of the program and the queries and evidence to a weighted Boolean formula. This allows us to reduce the inference tasks to well-studied tasks such as weighted model counting, which can be solved using state-of-the-art methods known from the graphical model and knowledge compilation literature.

## The Language. Probabilistic Logic Programming.

ProbLog makes it easy to express complex, probabilistic models.

```
0.3::stress(X) :- person(X).  
0.2::influences(X,Y) :- person(X), person(Y).
```

# Take-away message

- Inference is hard ( $\#P$ -complete, WMC)
- Focus of a lot of research
- A lot of progress with usable implementations, but many challenges remain (like lifted inference)

# Part III: Rule learning

# Information Extraction in NELL

instance	iteration	date learned	confidence
<a href="#">kelly andrews</a> is a <a href="#">female</a>	826	29-mar-2014	98.7  
<a href="#">investment next year</a> is an <a href="#">economic sector</a>	829	10-apr-2014	95.3  
<a href="#">shibenik</a> is a <a href="#">geopolitical entity</a> that is an organization	829	10-apr-2014	97.2  
<a href="#">quality web design work</a> is a <a href="#">character trait</a>	826	29-mar-2014	91.0  
<a href="#">mercedes benz cls by carlsson</a> is an <a href="#">automobile manufacturer</a>	829	10-apr-2014	95.2  
<a href="#">social work</a> is an academic program <a href="#">at the university rutgers university</a>	827	02-apr-2014	93.8  
<a href="#">dante wrote</a> the book <a href="#">the divine comedy</a>	826	29-mar-2014	93.8  
<a href="#">willie aames</a> was <a href="#">born in the city los angeles</a>	831	16-apr-2014	100.0  
<a href="#">kitt peak</a> is a mountain <a href="#">in the state or province arizona</a>	831	16-apr-2014	96.9  
<a href="#">greenwich</a> is a park <a href="#">in the city london</a>	831	16-apr-2014	100.0  

instances for many different relations

degree of certainty

# Rule learning in NELL (I)

- Original approach
  - Make probabilistic data deterministic
  - run classic rule-learner (variant of FOIL)
  - re-introduce probabilities on learned rules and predict

# Rule learning in NELL (2)

- Newer Page Rank Based Approach (Cohen et al. CIKM, Arxiv) -- ProPPR
  - Change the underlying model, from random graph / database to random walk one;
  - No longer “degree of belief” assigned to facts;
  - more like stochastic logic programs
  - Learn rules / parameters

# Probabilistic Rule Learning

- Learn the rules directly in a PLP setting
- Generalize relational learning and inductive logic programming directly towards probabilistic setting
- Traditional rule learning/ILP as a special case
- Apply to probabilistic databases like NELL

# Quinlan's Playtennis

ex	outlook ok	temperature ok	humidity ok	wind ok	class
1	t	t	f	f	- +
2	f	t	f	t	- +
3	t	f	f	f	- -
4	f	f	t	f	- -
...					
...					

# Our Windsurfing Example

ex	pop	windok	sunshine	class
1	0,7	0,5	0,7	0,9
2	0,6	0,7	0,6	0,85
3	0,4	0,3	0,4	0,45
4	0,3	0,7	0,2	0,3
...				
...				

pop = Probability of Precipitation

# Differences

- Observations (features) are uncertain
- Class is uncertain as well
- This type of data occurs naturally in applications in
  - image / video analysis
  - text processing and the web
  - life sciences (e.g., Muggleton et al. MLJ 09)
  - probabilistic databases

# Rule learning

In the logical setting

playtennis :- outlook=ok, wind=ok

playtennis :- outlook=ok, humidity=ok

In the probabilistic case

surfing :- not pop, windok

surfing :- not pop, sunshine

both a declarative and a probabilistic interpretation

# Computing Probabilities

Consider the rules

surfing :- not pop, windok

surfing :- not pop, sunshine

The example 0.2::pop, 0.7::windok, 0.6::sunshine. Then

$$P(\text{surfing}) = P( (\text{not pop and windok}) \text{ or } (\text{not pop and sunshine}))$$

$$= P( (\text{not pop and windok}) \text{ or } (\text{not pop and sunshine and not windok}))$$

$$= 0.8 \times 0.7 + 0.8 \times 0.6 \times 0.3 \quad \text{disjoint sum problem}$$

# In ProbLog (I)

## Basic Setting

surfing(X) :- not pop(X), windok(X)

surfing(X) :- not pop(X) and sunshine(X)

0.2::pop(e1). 0.7::windok(e1). 0.6::sunshine(e1).

H

B

?-P(surfing(e1)).

e

gives  $0.8 \times 0.7 + 0.8 \times 0.6 \times 0.3 = P(B \cup H |= e)$

# In ProbLog (2)

## Extended Setting

p1::surfing(X) :- not pop(X) and windok(X).

H

p2::surfing(X) :- not pop(X) and sunshine(X).

0.2::pop(e1). 0.7::windok(e1). 0.6::sunshine(e1).

B

?-P(surfing(e1)).

e

gives  $0.8 \times 0.7 \times p1 + 0.8 \times 0.6 \times 0.3 \times p2 = P(B \cup H |= e)$

# Inductive Probabilistic Logic Programs

**Given**

a set of example facts  $e \in E$  together with the probability  $p$  that they hold

a background theory  $B$  in ProbLog

a hypothesis space  $L$  (a set of clauses)

**Find**

$$\arg \min_H loss(H, B, E) = \arg \min_H \sum_{e_i \in E} |P_s(B \cup H \vdash e) - p_i|$$

# Observations

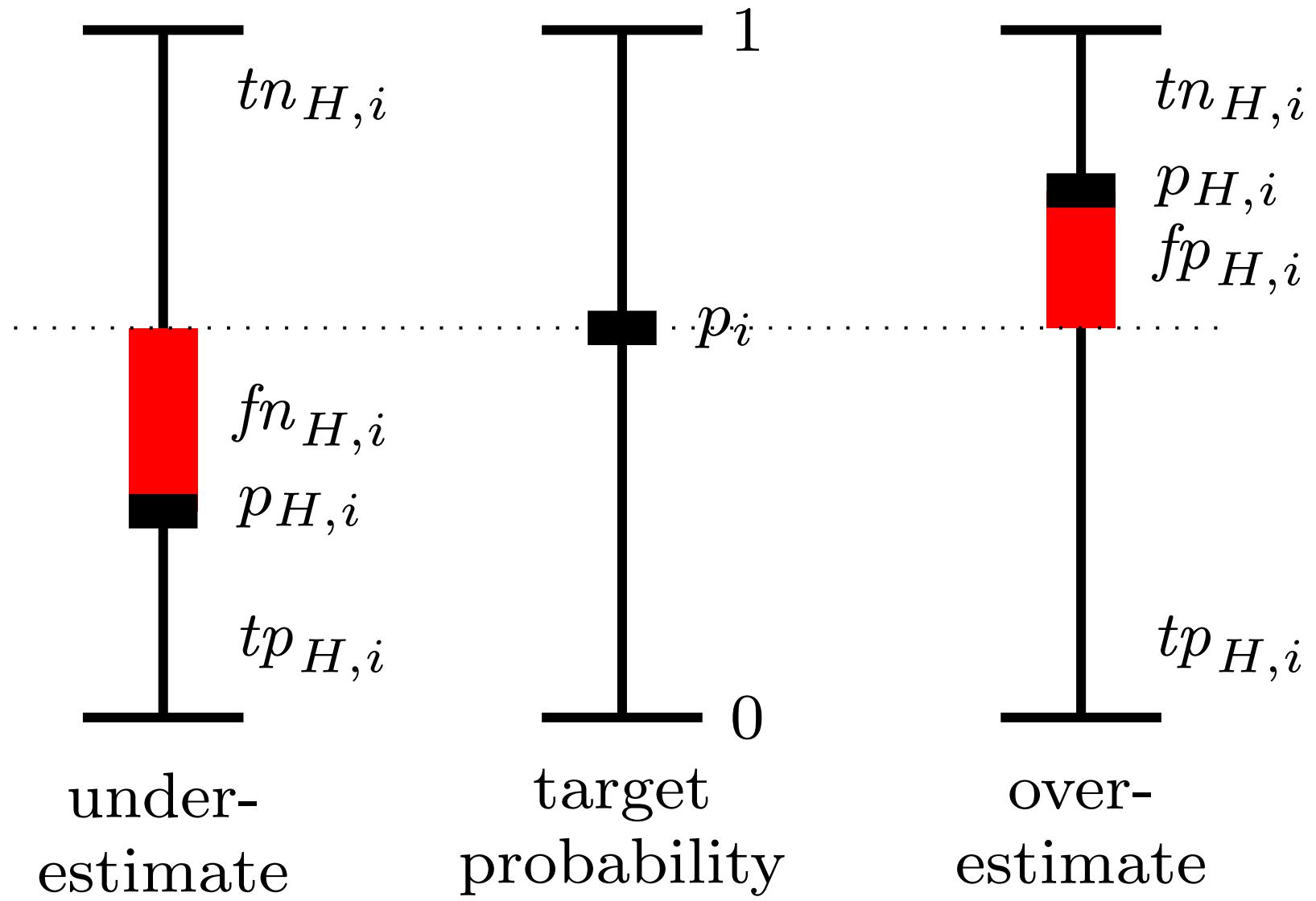
Propositional versus first order

- traditional rule learning = propositional
- inductive logic programming = first order

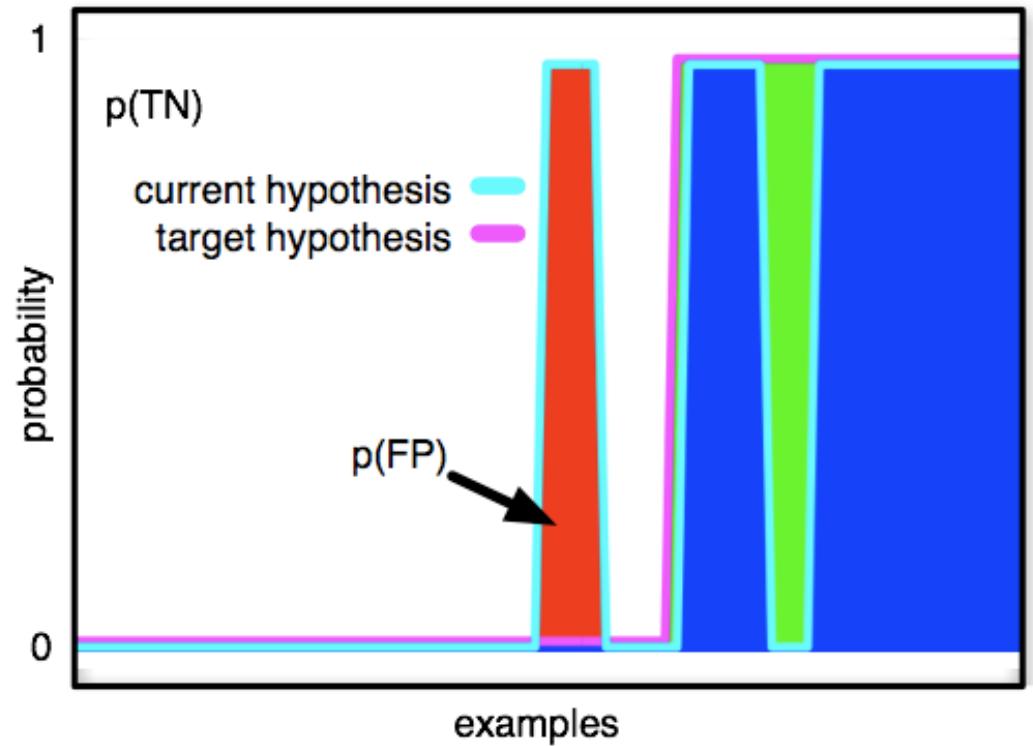
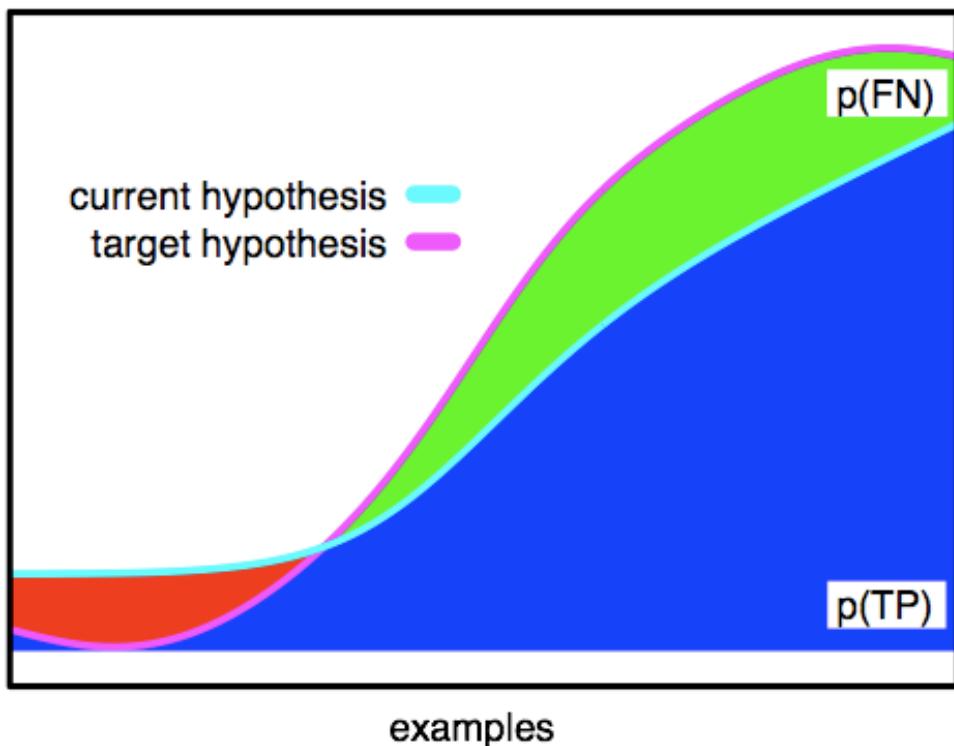
Deterministic case

- all probabilities 0 or 1
- traditional rule learning / ILP as special case

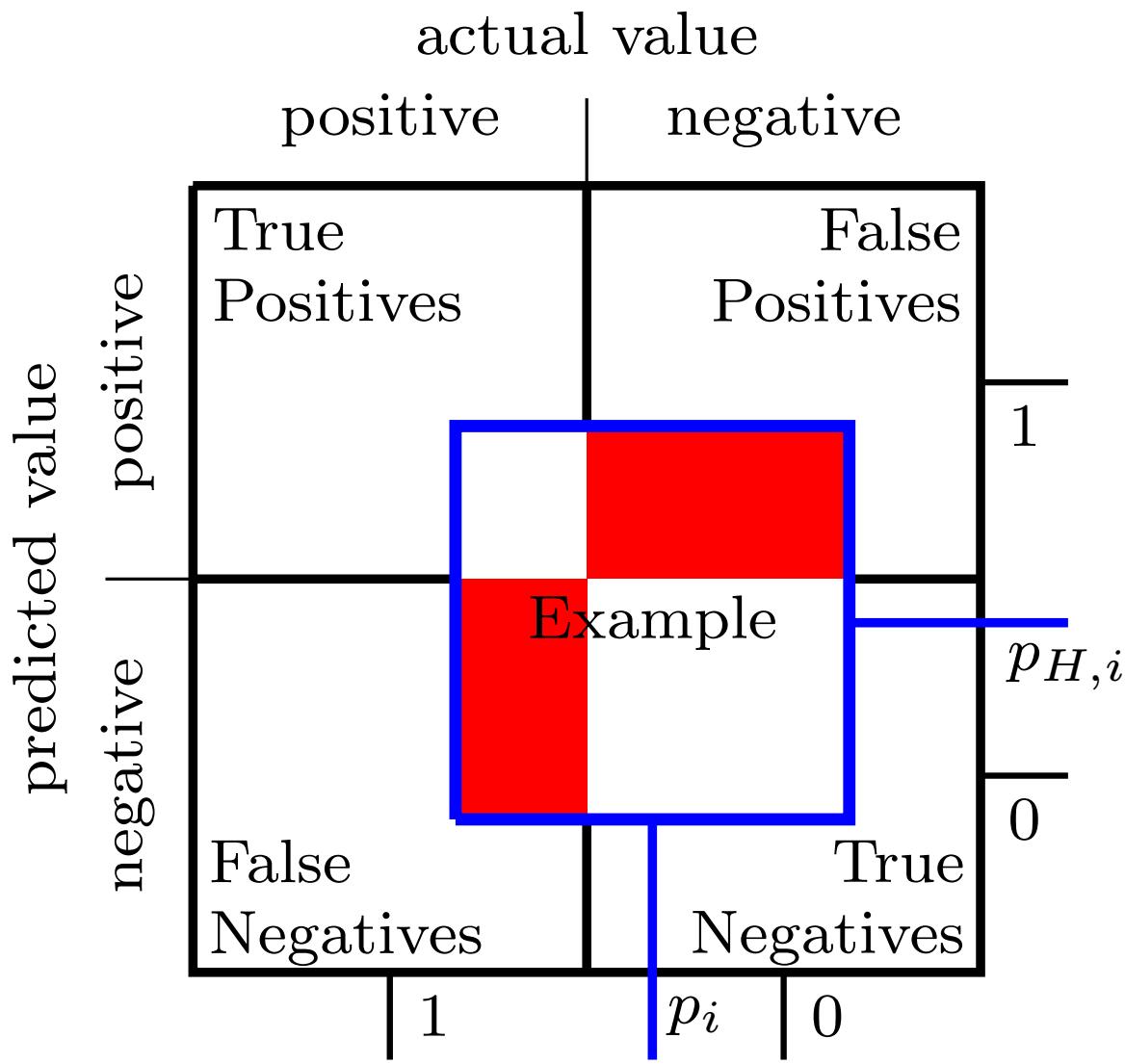
# Analysis



# Analysis



# Contingency Table



# Rule learning

## Interesting properties

- adding a rule is monotonic, this can only increase the probability of an example
- adding a condition to a rule is anti-monotonic, this can only decrease the probability of an example
- several rules may be needed to cover an example
  - use all examples all of the time (do not delete them while learning), do not forget the positives
  - disjoint sum problem

# ProbFOIL

Quinlan's well-known FOIL algorithm combined with ProbLog and probabilistic examples and background knowledge

Essentially a vanilla sequential covering algorithm with m-estimate as local score and accuracy as global score.

(But other variations based on e.g. Fuernkranz tutorial are possible ... )

# Criteria

$$\begin{aligned}precision &= \frac{TP}{TP + FP} \\m\text{-estimate} &= \frac{TP + m \cdot \frac{P}{N}}{TP + FP + m} && \text{local score} \\recall &= \frac{TP}{TP + FN} \\accuracy &= \frac{TP + TN}{TP + TN + FP + FN} && \text{global score}\end{aligned}$$

Avoiding overfitting using significance test

# ProbFOIL

---

**Algorithm 1** The ProbFOIL<sup>+</sup> learning algorithm

---

```
1: function PROBFOIL+(target)                                ▷ target is the target predicate
2:    $H := \emptyset$ 
3:   while true do
4:     clause := LEARNRULE( $H$ , target)
5:     if GLOBALSCORE( $H$ ) < GLOBALSCORE( $H \cup \{clauses\}$ ) then
6:        $H := H \cup \{clauses\}$ 
7:     else
8:       return  $H$ 
9: function LEARNRULE( $H$ , target)
10:   candidates :=  $\{x :: target \leftarrow true\}$                          ▷ Start with an empty (probabilistic) body
11:   bestrule :=  $(x :: target \leftarrow true)$ 
12:   while candidates  $\neq \emptyset$  do                                         ▷ Grow rule
13:     nextcandidates :=  $\emptyset$ 
14:     for all  $x :: target \leftarrow body \in candidates$  do
15:       for all literal  $\in \rho(target \leftarrow body)$  do                      ▷ Generate all refinements
16:         if not REJECTREFINEMENT( $H$ , bestrule,  $x :: target \leftarrow body$ ) then      ▷ Reject unsuited
    refinements
17:           nextcandidates := nextcandidates  $\cup \{x :: target \leftarrow body \wedge l\}$ 
18:           if LOCALSCORE ( $H$ ,  $x :: target \leftarrow body \wedge literal$ ) > LOCALSCORE( $H$ , bestrule) then
19:             bestrule :=  $(x :: target \leftarrow body \wedge literal)$                       ▷ Update best rule
20:           candidates := nextcandidates
21:   return bestrule
```

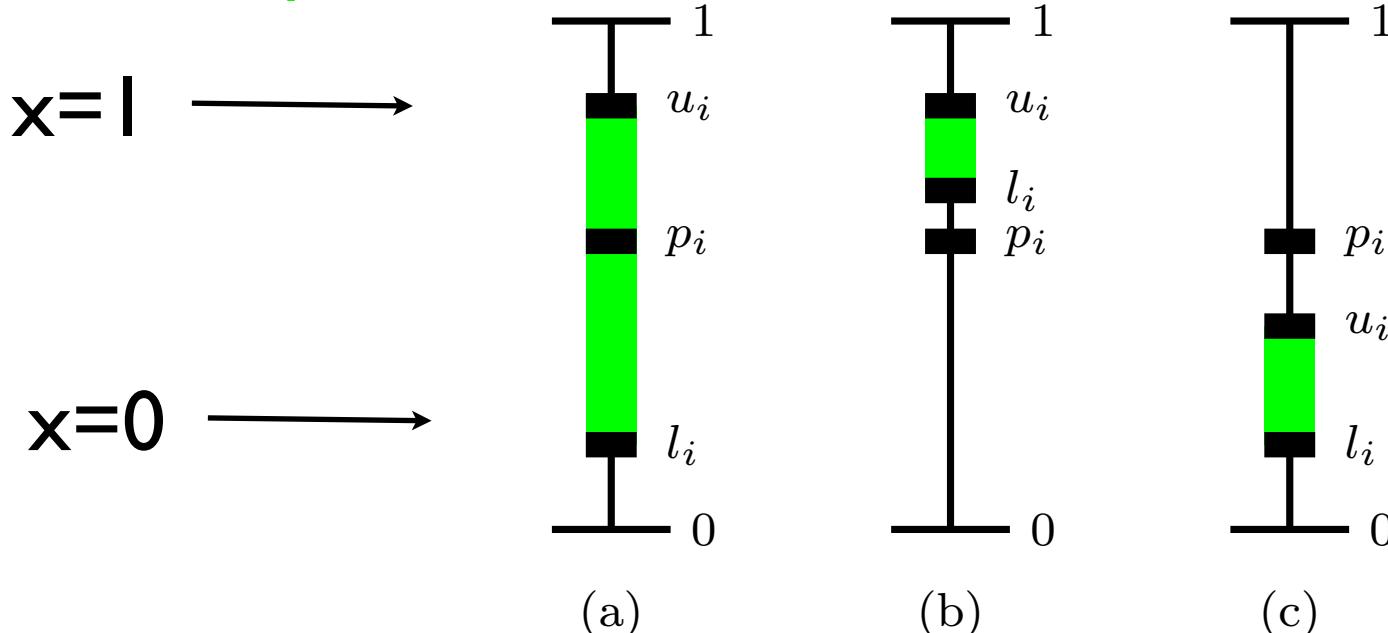
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# Extended rule learning

Learn rules with probability  $x:: \text{head} :- \text{body}$

What changes ?

- value of  $x$  *determines prob. of coverage of example*



# Extended rule learning

Express local score as a function of  $x$

Compute optimal value of  $x$

# Implementation Optimizations

Incremental grounding

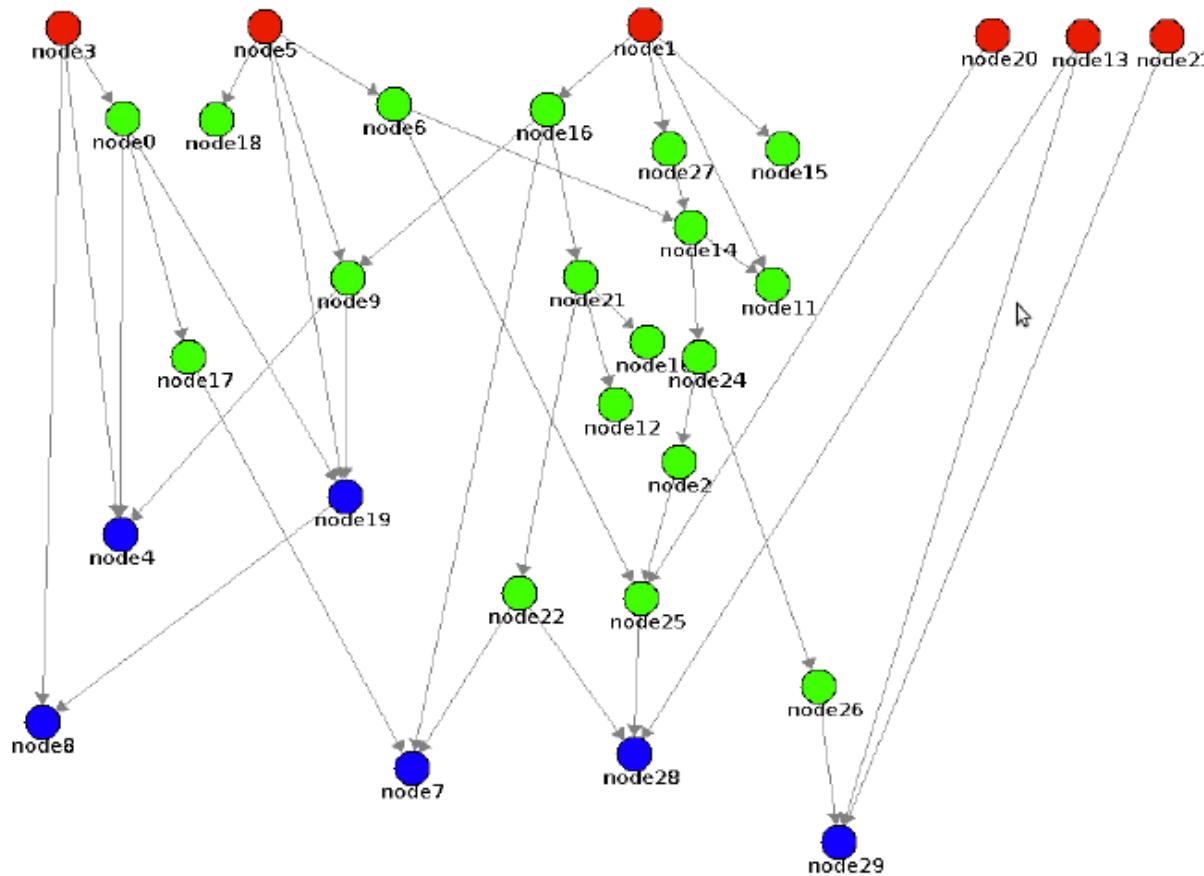
Simplified CNF conversion to ProbLog

Sometimes direct calculation of probabilities

Even simpler when propositional data only

Some language bias (range-restricted)

# Experiments on Bayesian Net



# Results

Table 1: Mean absolute error on network A with CPTs  $\sim \text{Beta}(\alpha, \beta)$ , averaged over all target attributes. Observed nodes are independent.

$\alpha/\beta$	1.0000	0.1000	0.0100	0.0010	0.0001	0.00001
ZeroR	0.054	0.11	0.11	0.13	0.12	0.12
LinearRegression	$7.7 \times 10^{-3}$	0.027	0.024	0.025	0.024	0.024
MultilayerPerceptron	$1.8 \times 10^{-3}$	$8.5 \times 10^{-3}$	$6.3 \times 10^{-3}$	$5.8 \times 10^{-3}$	$5.7 \times 10^{-3}$	$5.7 \times 10^{-3}$
M5P	$1.7 \times 10^{-3}$	$6.7 \times 10^{-3}$	$4.2 \times 10^{-3}$	$4.2 \times 10^{-3}$	$4.0 \times 10^{-3}$	$4.0 \times 10^{-3}$
M5P -R -M 4.0	0.013	0.031	0.026	0.029	0.028	0.028
SMOreg	$7.7 \times 10^{-3}$	0.027	0.024	0.026	0.025	0.025
ProbFOIL(1,15,0.0,rel)	0.069	0.051	$5.9 \times 10^{-4}$	$1.6 \times 10^{-7}$	$1.6 \times 10^{-7}$	$1.6 \times 10^{-7}$
ProbFOIL <sup>+</sup> (1,1,0.0,rel)	$1.8 \times 10^{-3}$	$3.0 \times 10^{-3}$	$10.0 \times 10^{-5}$	$1.6 \times 10^{-7}$	$1.6 \times 10^{-7}$	$1.6 \times 10^{-7}$

Observed Nodes Independent

ProbFOIL better on deterministic data

Regression learners better than BASIC ProbFOIL

Extended ProbFOIL : best of both worlds

# Results

$\alpha/\beta$	1.0000	0.1000	0.0100	0.0010	0.0001	0.00001
LinearRegression	$2.6 \times 10^{-3}$	0.018	0.021	0.020	0.018	0.018
MultilayerPerceptron	$4 \times 10^{-4}$	$3.1 \times 10^{-3}$	$5.7 \times 10^{-3}$	$3.9 \times 10^{-3}$	$3.3 \times 10^{-3}$	$3.3 \times 10^{-3}$
M5P	$7 \times 10^{-4}$	$4.9 \times 10^{-3}$	$6.5 \times 10^{-3}$	$5.4 \times 10^{-3}$	$4.4 \times 10^{-3}$	$4.4 \times 10^{-3}$
M5P -R -M 4.0	$5.2 \times 10^{-3}$	0.021	0.023	0.025	0.024	0.024
SMOreg	$2.6 \times 10^{-3}$	0.017	0.021	0.020	0.017	0.017
ProbFOIL(1,10,0.0,rel)	0.015	0.012	$1.9 \times 10^{-3}$	$9.4 \times 10^{-8}$	$4.2 \times 10^{-8}$	$4.2 \times 10^{-8}$
ProbFOIL <sup>+</sup> (1,5,0.0,rel)	$3.9 \times 10^{-3}$	$3.9 \times 10^{-3}$	$5.3 \times 10^{-4}$	$2.8 \times 10^{-7}$	$4.2 \times 10^{-8}$	$4.2 \times 10^{-8}$

Observed Nodes Dependent / Full Observability

ProbFOIL better on deterministic data  
Regression learners better than BASIC ProbFOIL  
Extended ProbFOIL : best of both worlds

# Results

Table 4: Mean absolute error on network B with CPTs  $\sim \text{Beta}(\alpha, \beta)$ , averaged over all target attributes. Observed nodes are **dependent**. There is **partial** observability.

$\alpha/\beta$	1.0000	0.1000	0.0100	0.0010	0.0001	0.00001
ZeroR	0.023	0.077	0.085	0.093	0.096	0.096
LinearRegression	$4.8 \times 10^{-3}$	0.019	0.026	0.032	0.034	0.034
MultilayerPerceptron	$1.2 \times 10^{-3}$	$2.9 \times 10^{-3}$	$9.7 \times 10^{-3}$	0.020	0.031	0.032
M5P	$1.6 \times 10^{-3}$	$6.1 \times 10^{-3}$	0.022	0.027	0.027	0.027
M5P -R -M 4.0	$6.2 \times 10^{-3}$	0.022	0.033	0.040	0.040	0.040
SMOreg	$4.5 \times 10^{-3}$	0.018	0.022	0.029	0.034	0.034
ProbFOIL(1,15,0.0,rel)	0.020	0.023	0.012	0.015	0.015	0.015
ProbFOIL <sup>+</sup> (1,10,0.0,rel)	$9.5 \times 10^{-3}$	0.011	0.011	0.013	0.013	0.013

Observed Nodes Dependent / Partial Observability

ProbFOIL better on deterministic data

Regression learners sometimes better than ProbFOIL

# Information Extraction in NELL

instance	iteration	date learned	confidence
<a href="#">kelly andrews</a> is a <a href="#">female</a>	826	29-mar-2014	98.7  
<a href="#">investment next year</a> is an <a href="#">economic sector</a>	829	10-apr-2014	95.3  
<a href="#">shibenik</a> is a <a href="#">geopolitical entity</a> that is an organization	829	10-apr-2014	97.2  
<a href="#">quality web design work</a> is a <a href="#">character trait</a>	826	29-mar-2014	91.0  
<a href="#">mercedes benz cls by carlsson</a> is an <a href="#">automobile manufacturer</a>	829	10-apr-2014	95.2  
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<a href="#">willie aames</a> was <a href="#">born in the city los angeles</a>	831	16-apr-2014	100.0  
<a href="#">kitt peak</a> is a mountain <a href="#">in the state or province arizona</a>	831	16-apr-2014	96.9  
<a href="#">greenwich</a> is a park <a href="#">in the city london</a>	831	16-apr-2014	100.0  

instances for many different relations

degree of certainty

# NELL

Table 5: Number of facts per predicate (NELL athlete dataset)

athletecoach(person, person)	18	athleteplaysforteam(person, team)	721
athleteplayssport(person, sport)	1921	teamplaysinleague(team, league)	1085
athleteplaysinleague(person, league)	872	athletesoknownas(person, name)	17
coachesinleague(person, league)	93	coachesteam(person, team)	132
teamhomestadium(team, stadium)	198	teamplayssport(team, sport)	359
athleteplayssportsteamposition(person, position)	255	athletehomestadium(person, stadium)	187
athlete(person)	1909	attraction(stadium)	2
coach(person)	624	female(person)	2
male(person)	7	hobby(sport)	5
organization(league)	1	person(person)	2
personafrica(person)	1	personasia(person)	4
personaustralia(person)	22	personcanada(person)	1
personeurope(person)	1	personmexico(person)	108
personus(person)	6	sport(sport)	36
sportsleague(league)	18	sportsteam(team)	1330
sportsteamposition(position)	22	stadiummorevenue(stadium)	171

# NELL

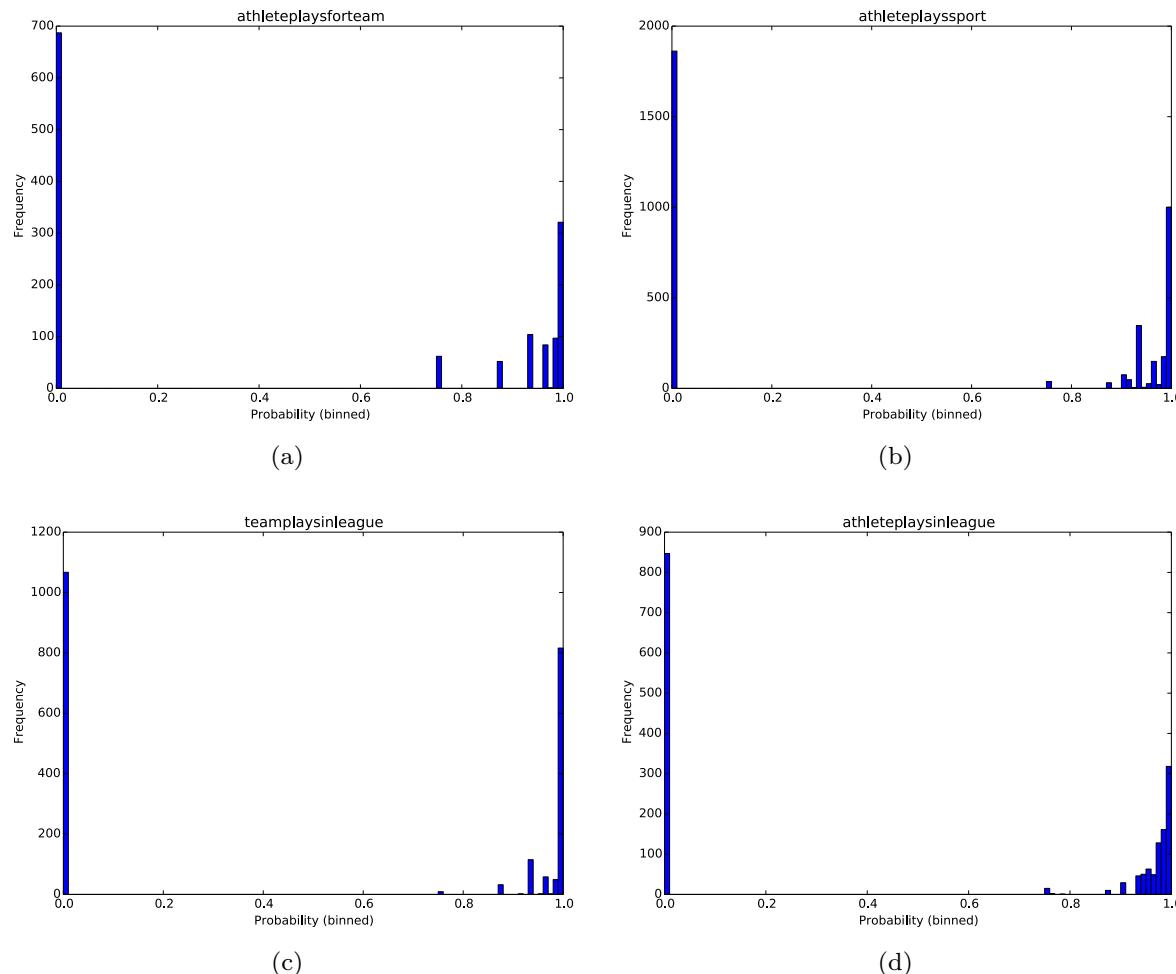


Fig. 5: Histogram of probabilities for each of the binary predicates with more than 500 facts: (a) *athleteplaysforteam*; (b) *athleteplayssport*; (c) *teamplaysinleague*; and, (d) *athletesplaysinleague*.

#### 5.4.2 *athletesplaysport*(*person*,*sport*)

```
0.98604::athletesplaysport(A,B) ← athleteplaysforteam(V_2,V_1),  
    athleteplaysinleague(A,V_3), coachesinleague(V_2,V_3),  
    teamplaysport(V_1,B), athleteplaysforteam(A,V_1).  
0.907::athletesplaysport(A,B) ← athleteplaysforteam(V_2,V_1), teammate(V_2,A),  
    teamplaysport(V_1,B).  
0.91817::athletesplaysport(A,B) ← coachesteam(A,V_1), teamplaysport(V_1,B).
```

Listing 2: Learned rules for the `athletesplaysport` predicate.

#### 5.4.3 *teamplaysinleague*(*team*,*league*)

```
0.95848::teamplaysinleague(A,B) ← athleteplaysforteam(V_1,V_2),  
    coachesinleague(V_1,B), teamplaysagainstteam(V_2,A).  
0.92240::teamplaysinleague(A,B) ← athleteplaysinleague(V_1,B),  
    coachesteam(V_1,V_2), teamplaysagainstteam(V_2,A),  
    athleteplaysforteam(V_1,V_2), athleteledsportsteam(V_1,V_2).  
1.0::teamplaysinleague(A,B) ← athleteledsportsteam(V_1,A),  
    athleteplaysinleague(V_1,B).  
0.99998::teamplaysinleague(A,B) ← coachesinleague(V_1,B), coachwontrophy(V_1,V_2),  
    teamwontrophy(A,V_2).  
1.0::teamplaysinleague(A,B) ← athleteplaysinleague(V_1,B), coachesteam(V_1,V_2),  
    teamplaysagainstteam(V_2,A).
```

Listing 3: Learned rules for the `teamplaysinleague` predicate.

#### 5.4.5 *teamplaysagainstteam*(*team*,*team*)

```
0.9375::teamplaysagainstteam(A,B) ← teamwontrophy(A,V_1), teamwontrophy(B,V_1).  
0.58662::teamplaysagainstteam(A,B) ← athleteplaysforteam(V_1,A),  
coachesteam(V_1,V_2), teamplayssport(B,V_3), teamplayssport(V_2,V_3).
```

Listing 5: Learned rules for the **athleteplaysinleague** predicate.

#### 5.4.1 *athleteplaysforteam*(*person*,*team*)

```
0.9375::athleteplaysforteam(A,B) ← athleteledsportsteam(A,B).  
0.9675::athleteplaysforteam(A,B) ← athleteledsportsteam(A,V_1),  
teamplaysagainstteam(B,V_1).  
0.79391::athleteplaysforteam(A,B) ← athleteplaysinleague(A,V_1),  
teamplaysinleague(B,V_1).
```

Listing 1: Learned rules for the **athleteplaysforteam** predicate.

# Take away message

Rule learning applies / generalizes naturally to probabilistic data and databases.

# Parameter Learning

e.g., webpage classification model

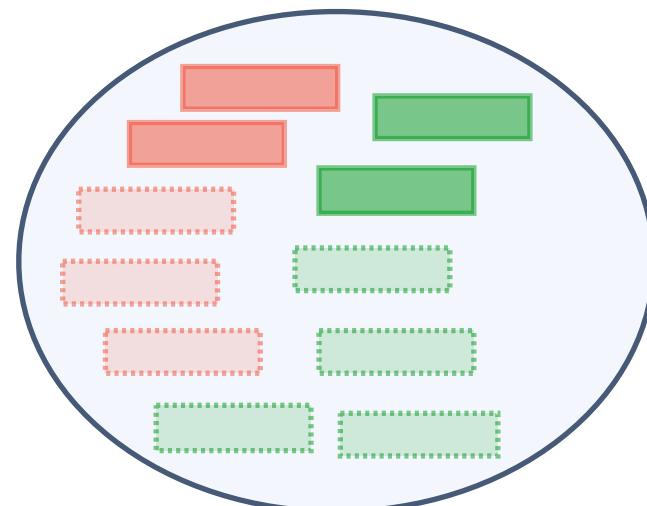
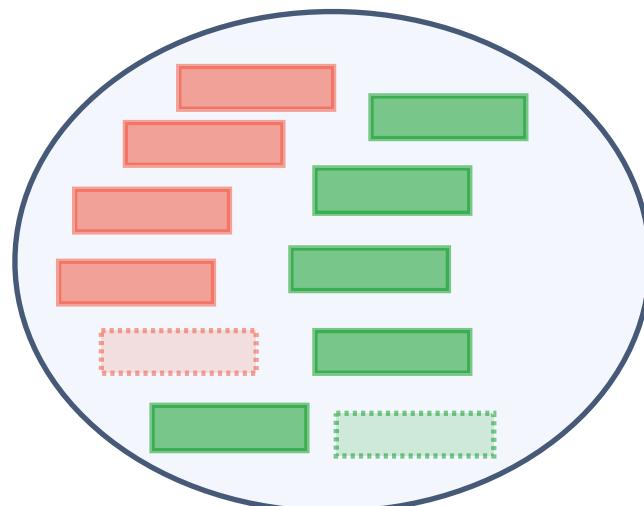
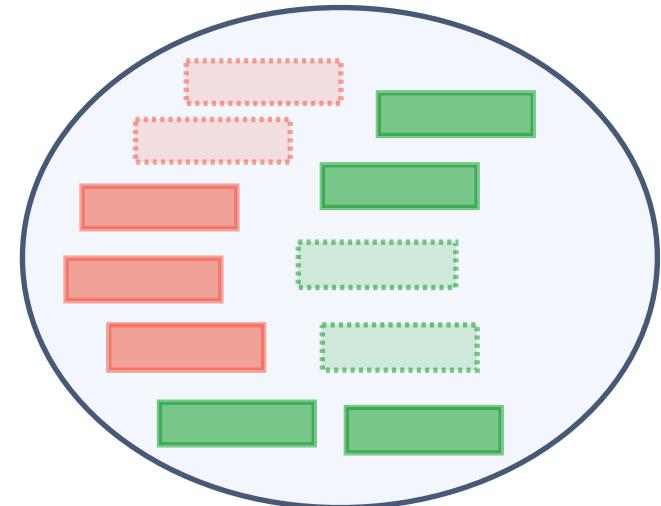
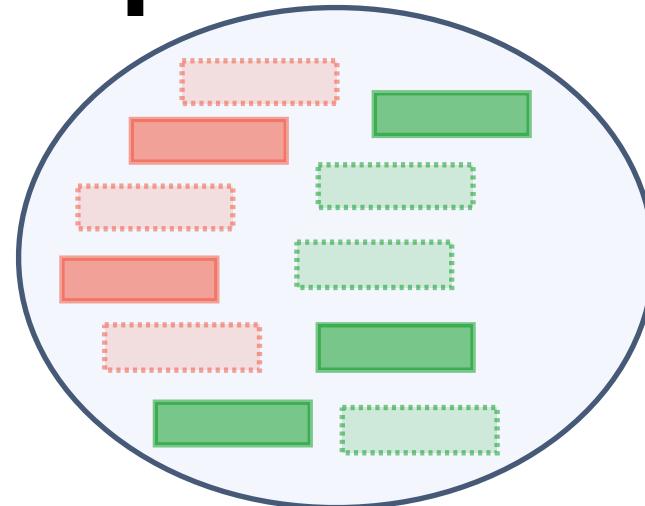
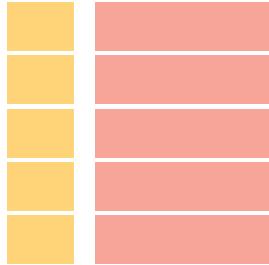
for each *CLASS1*, *CLASS2* and each *WORD*

```
?? :: link_class(Source,Target,CLASS1,CLASS2).  
?? :: word_class(WORD,CLASS).
```

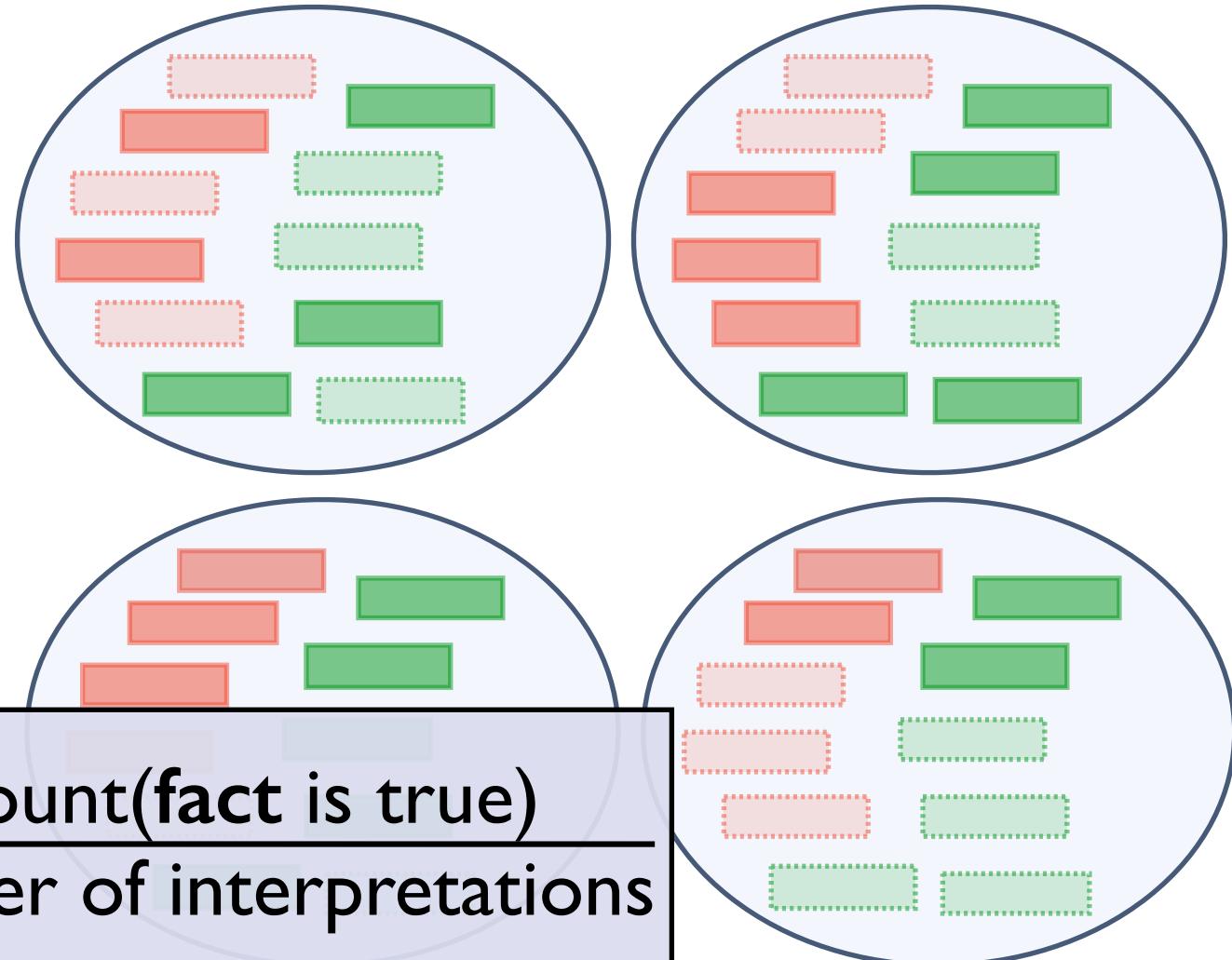
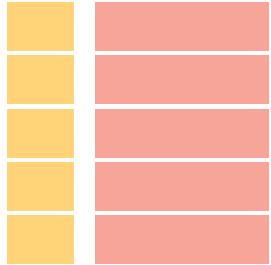
```
class(Page,C)  :-  has_word(Page,W), word_class(W,C).
```

```
class(Page,C)      :-  links_to(OtherPage,Page),  
                      class(OtherPage,OtherClass),  
                      link_class(OtherPage,Page,OtherClass,C).
```

# Sampling Interpretations



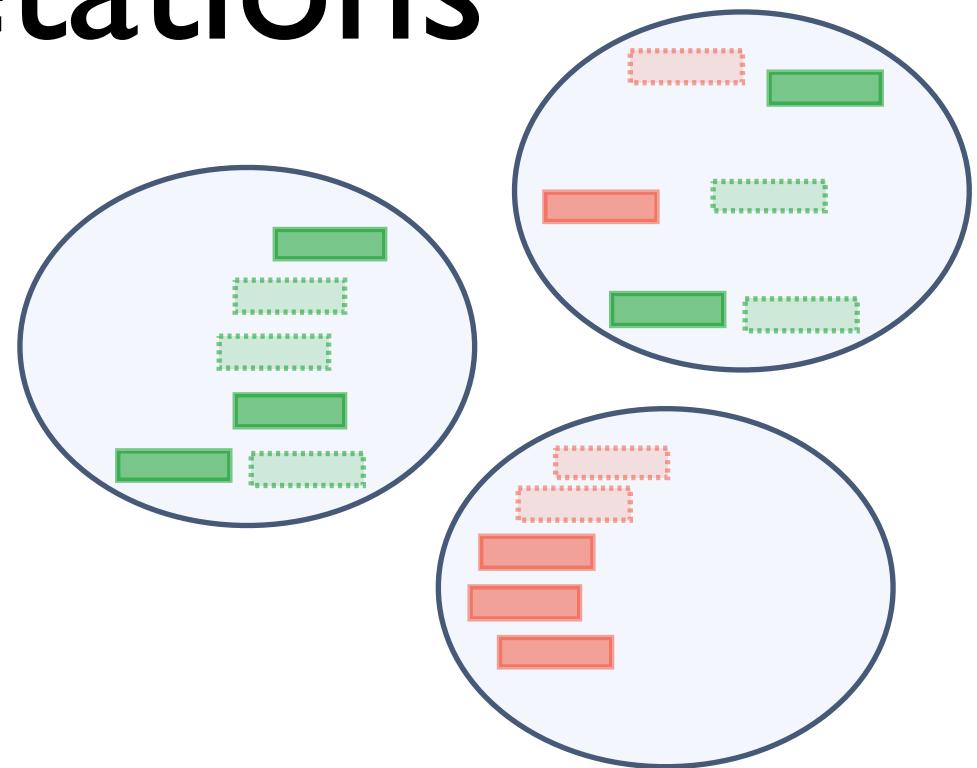
# Parameter Estimation



$$p(\text{fact}) = \frac{\text{count}(\text{fact is true})}{\text{Number of interpretations}}$$

# Learning from partial interpretations

- Not all facts observed
- Soft-EM
- use **expected count instead of count**
- $P(Q | E)$  -- conditional queries !



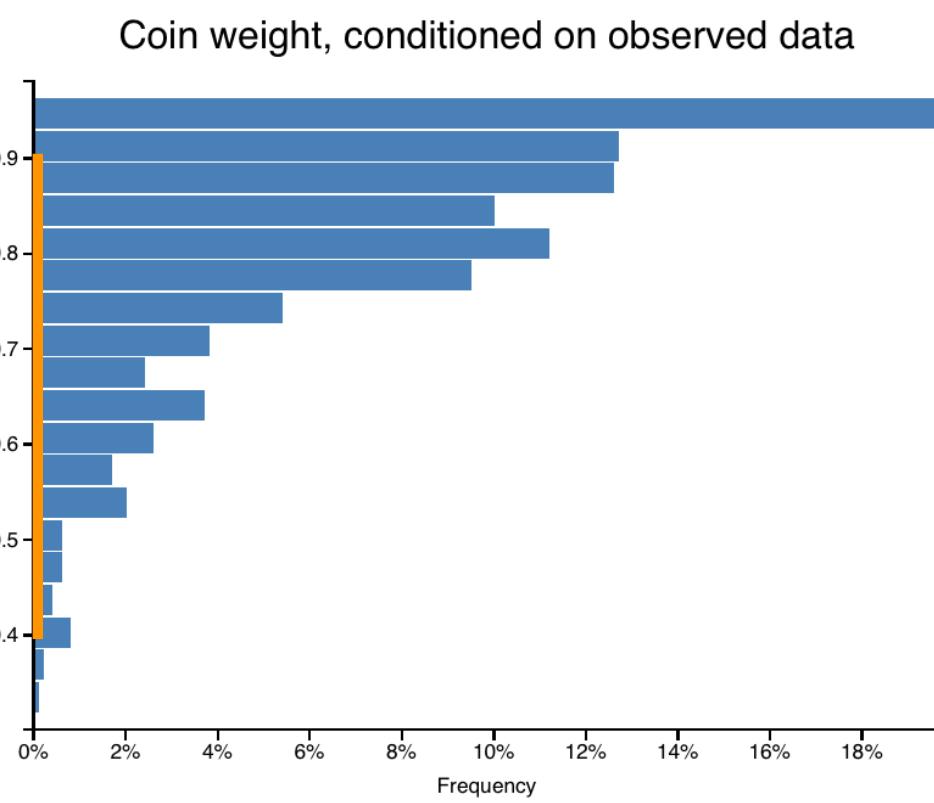
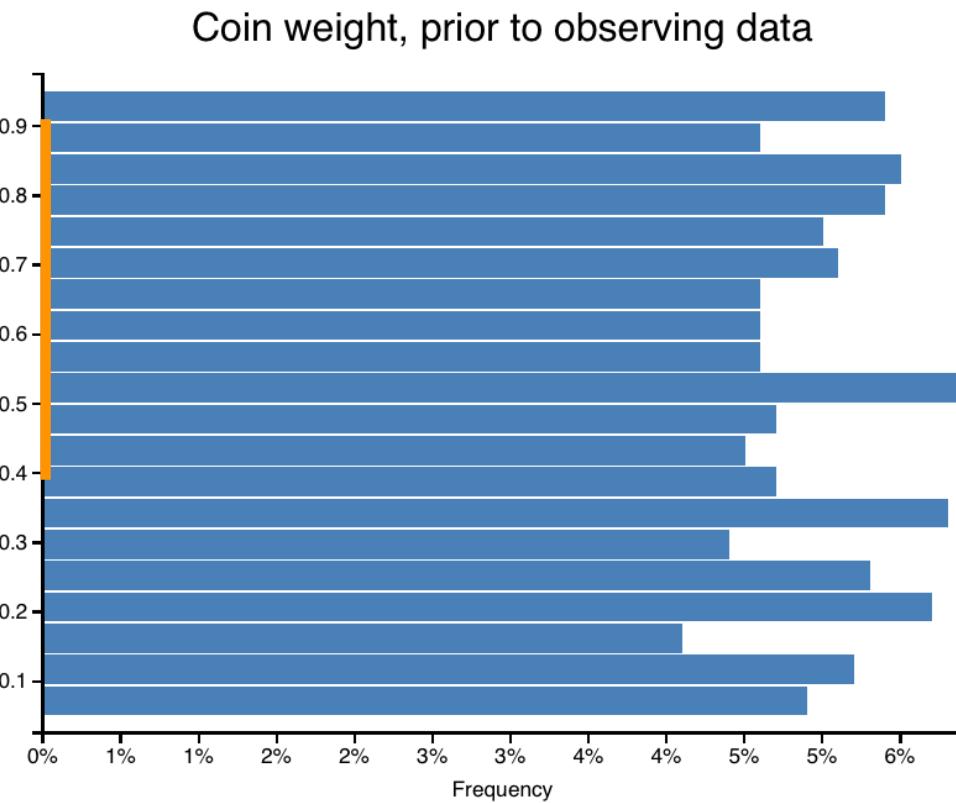
# Bayesian Parameter Learning

- Learning as inference (e.g., Church)
- Prior distributions for parameters
- Given data, find most likely parameter values

# Example

[from probmods.org]

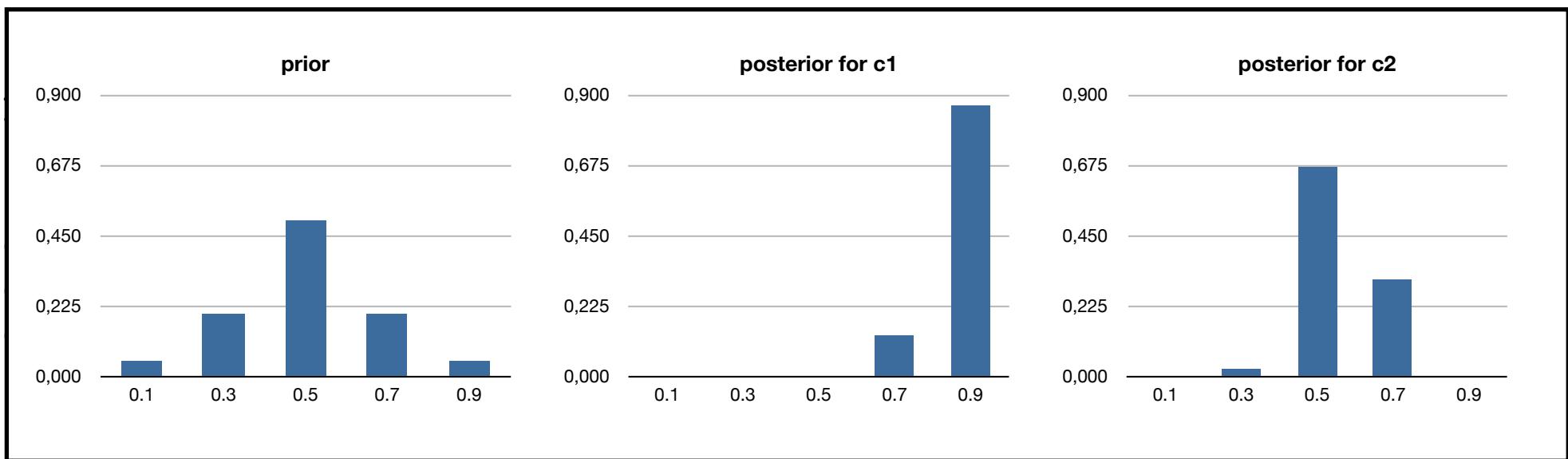
- Flipping a coin with unknown weight
- Prior: uniform distribution on  $[0, 1]$
- Observation: 5x heads in a row
- Sampling from Church model:



# ProbLog Example

**prior**

```
0.05::weight(C,0.1); 0.2::weight(C,0.3); 0.5::weight(C,0.5);  
0.2::weight(C,0.7); 0.05::weight(C,0.9) <- coin(C).
```



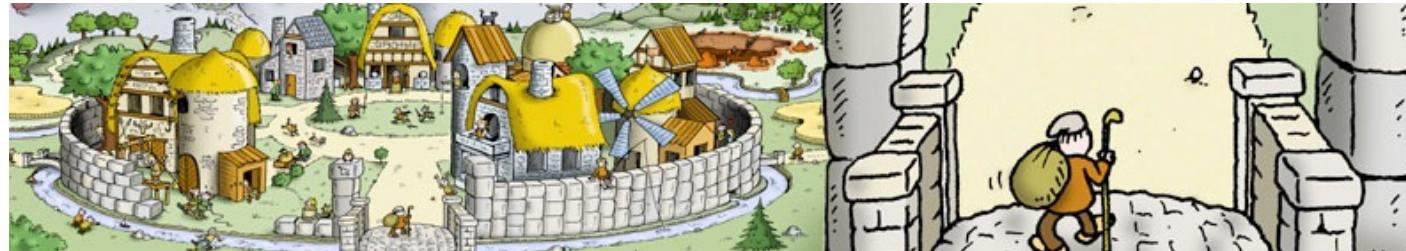
query(weight(C,X)) :- coin(C), param(X). **ask for posterior**

```
evidence(data(c1,[h,h,h,h,h,h,h,h,h,h,h]),true).  
evidence(data(c2,[h,t,h,h,h,h,t,t,h,t,t,h]),true).
```

**data**

# Part IV: Dynamics

# Dynamics: Evolving Networks



- *Travian*: A massively multiplayer real-time strategy game
  - Commercial game run by TravianGames GmbH
  - ~3.000.000 players spread over different “worlds”
  - ~25.000 players in one world

[Thon et al., MLJ II, ECML 08]



# World Dynamics

Fragment of world with

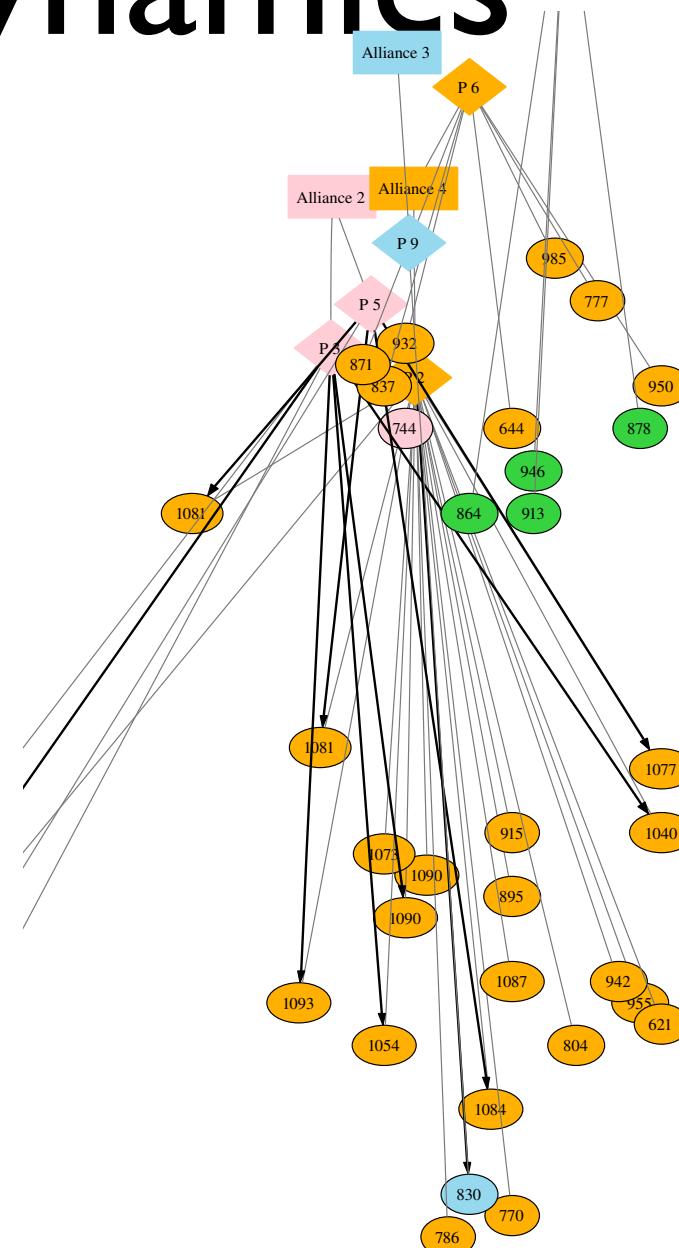
- ~10 alliances
- ~200 players
- ~600 cities

alliances color-coded

Can we build a model  
of this world ?

Can we use it for playing  
better ?

[Thon, Landwehr, De Raedt, ECML08]



# World Dynamics

Fragment of world with

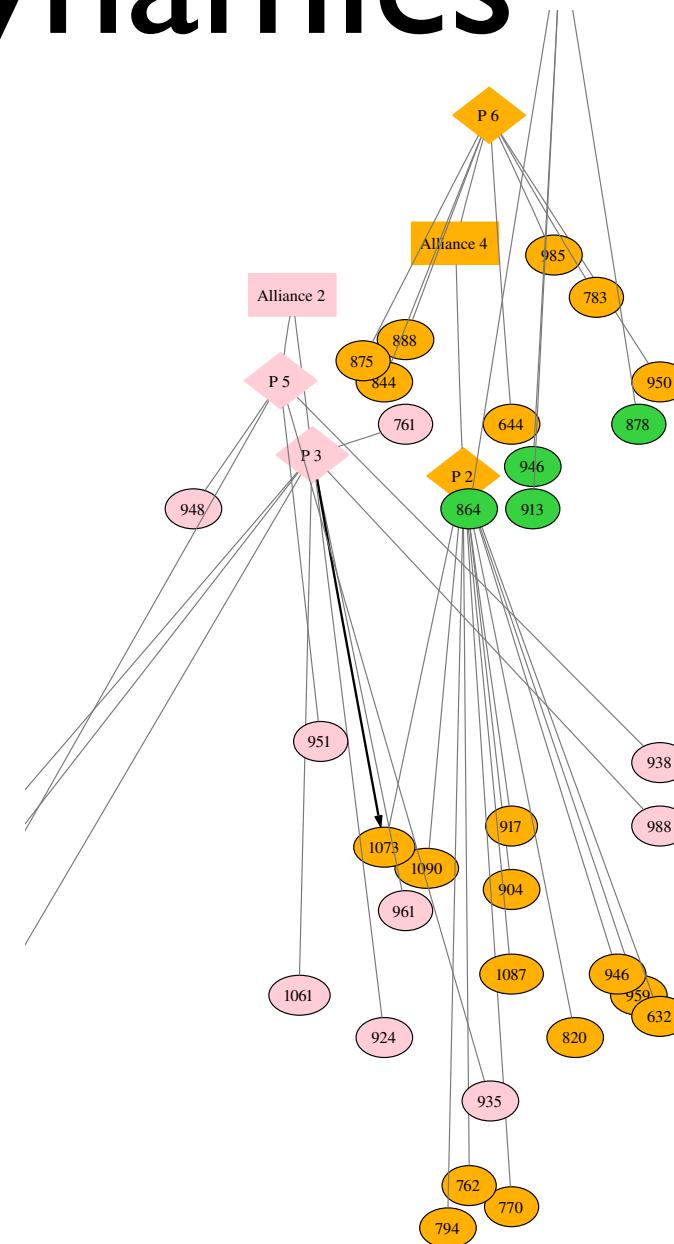
- ~10 alliances
- ~200 players
- ~600 cities

alliances color-coded

Can we build a model  
of this world ?

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[Thon, Landwehr, De Raedt, ECML08]



# World Dynamics

Fragment of world with

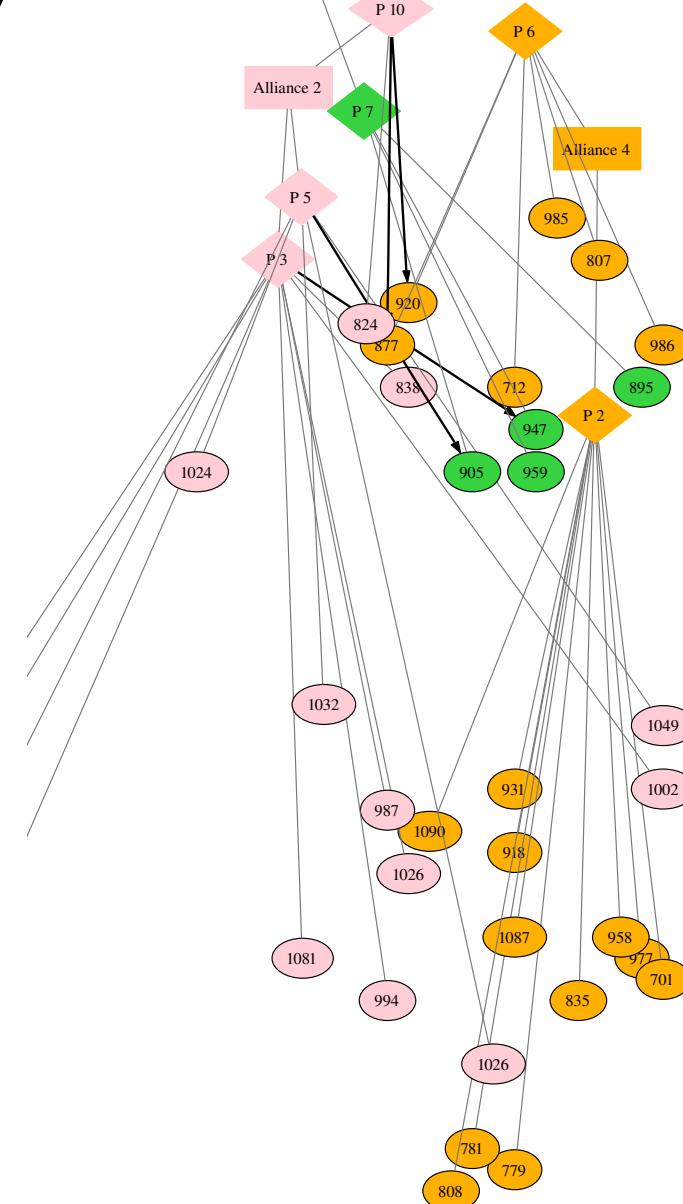
- ~10 alliances
- ~200 players
- ~600 cities

alliances color-coded

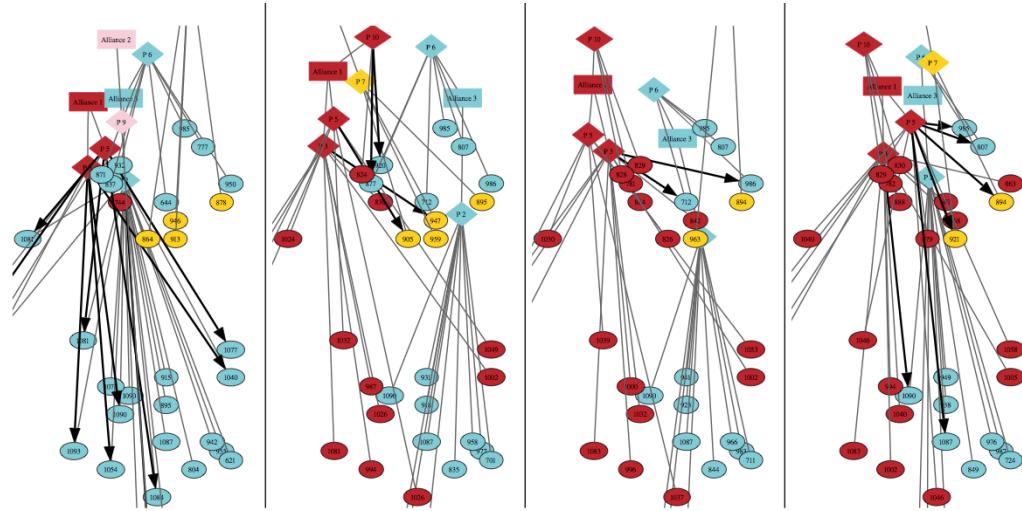
Can we build a model  
of this world ?

Can we use it for playing  
better ?

[Thon, Landwehr, De Raedt, ECML08]



# Causal Probabilistic Time-Logic (CPT-L)



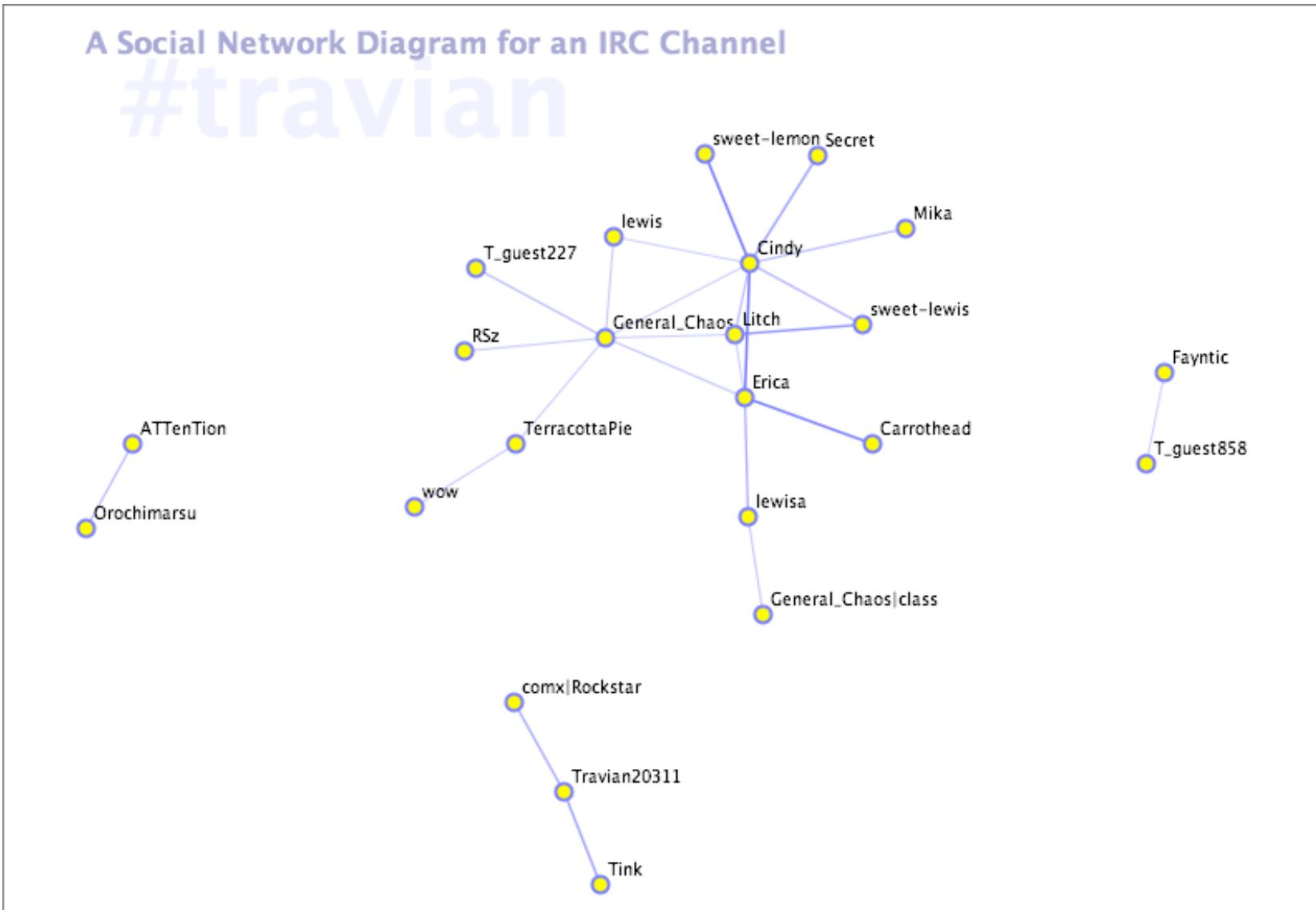
how does the world change over time?

one of the **effects** holds at time T+1

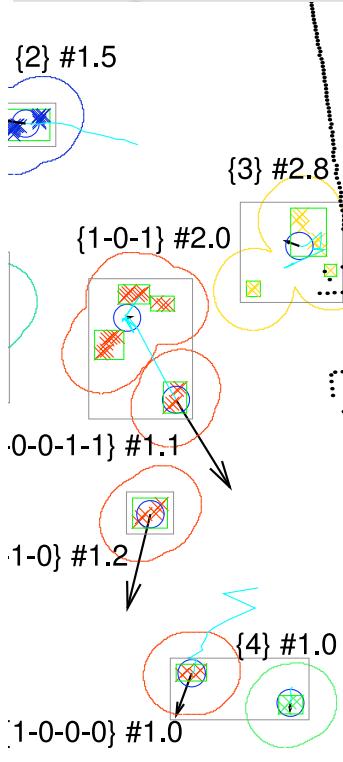
```
0.4 :: conquest(Attacker, C) ; 0.6 :: nil <-
    city(C, Owner) , city(C2, Attacker) , close(C, C2) .
```

if **cause** holds at time T

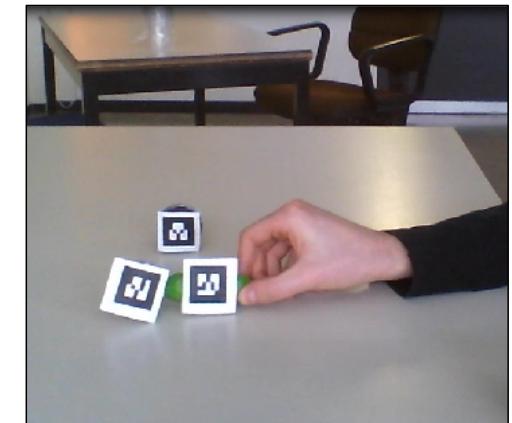
# Social Network of Chats



# Relational Tracking



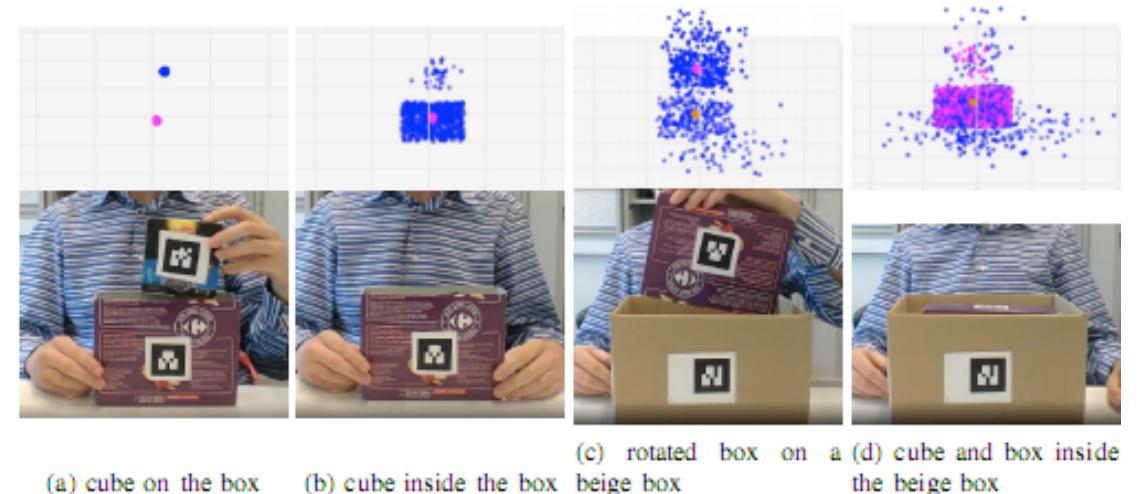
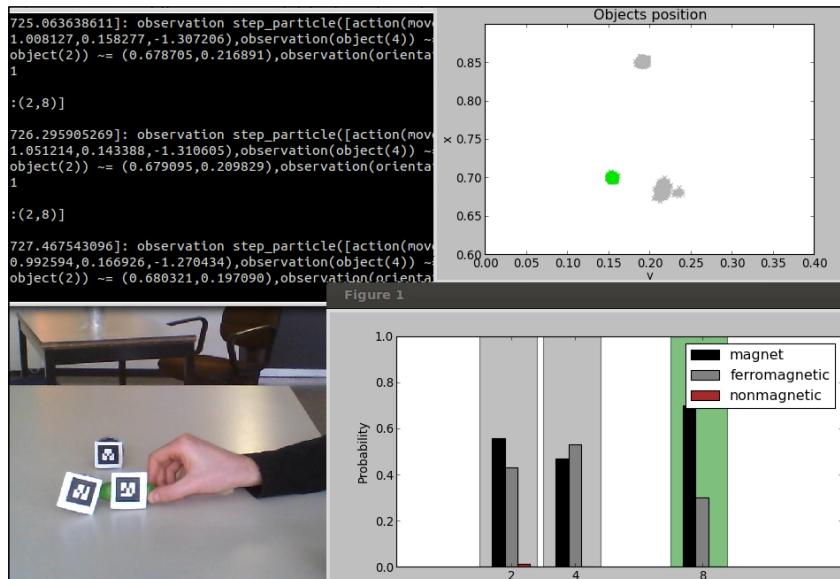
- Track people or objects over time? Even if temporarily hidden?
- Recognize activities?
- Infer object properties?



# Relational State Estimation over Time

## Magnetism scenario

- object tracking
- category estimation from interactions



## Box scenario

- object tracking even when invisible
- estimate spatial relations

# Speed 0x Queries (updated every 5 steps)

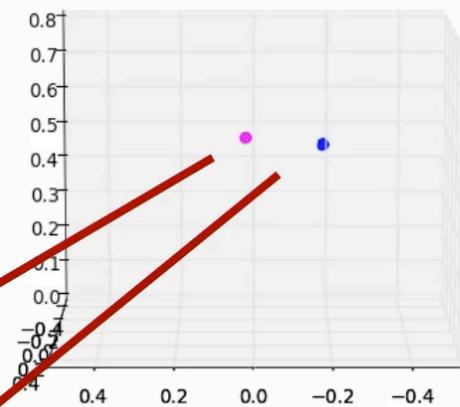
```
[]  
  
on(X,Y):  
[1.0:(3,(table)),1.0:(4,(table))]  
  
inside(X,Y):  
[]  
  
tr_inside(X,Y):  
[]
```



Box ID=4

Cube ID=3

## Particles

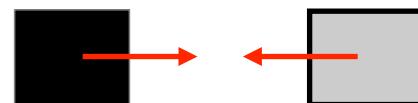


# Magnetic scenario

- 3 object types: magnetic, ferromagnetic, nonmagnetic

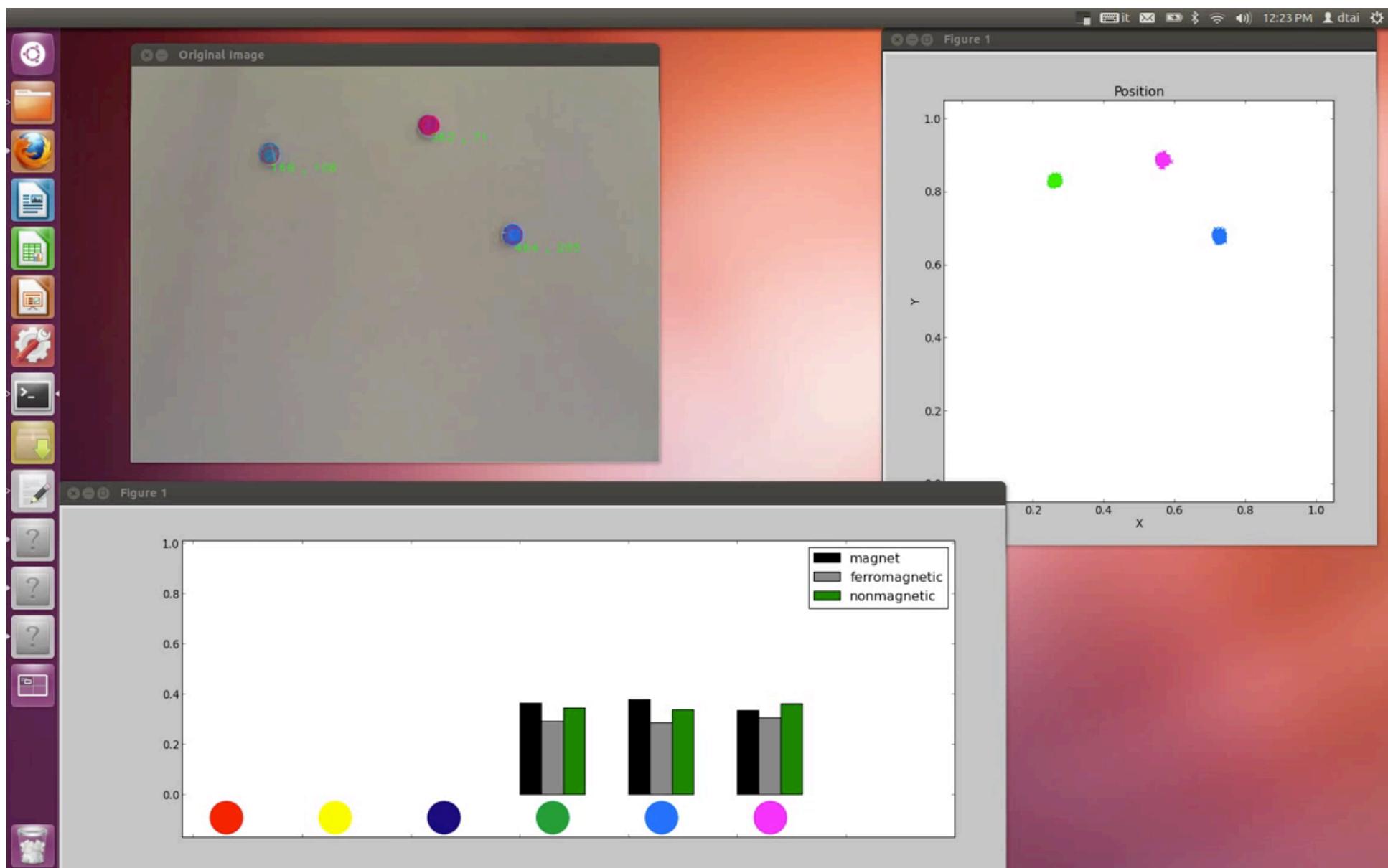


- Nonmagnetic objects do not interact
- A magnet and a ferromagnetic object attract each other



- Magnetic force that depends on the distance
- If an object is held magnetic force is compensated.





# Magnetic scenario

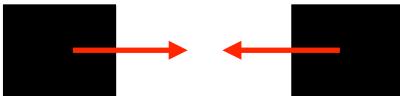
- 3 object types: magnetic, ferromagnetic, nonmagnetic

$\text{type}(X)_t \sim \text{finite}([1/3:\text{magnet}, 1/3:\text{ferromagnetic}, 1/3:\text{nonmagnetic}]) \leftarrow \text{object}(X).$

- 2 magnets attract or repulse

$\text{interaction}(A,B)_t \sim \text{finite}([0.5:\text{attraction}, 0.5:\text{repulsion}]) \leftarrow \text{object}(A), \text{object}(B), A < B, \text{type}(A)_t = \text{magnet}, \text{type}(B)_t = \text{magnet}.$

- Next position after attraction

$\text{pos}(A)_{t+1} \sim \text{gaussian}(\text{middlepoint}(A,B)_t, \text{Cov}) \leftarrow$   
  
 $\text{near}(A,B)_t, \text{not}(\text{held}(A)), \text{not}(\text{held}(B)),$   
 $\text{interaction}(A,B)_t = \text{attr},$   
 $c/\text{dist}(A,B)_t^2 > \text{friction}(A)_t.$



$\text{pos}(A)_{t+1} \sim \text{gaussian}(\text{pos}(A)_t, \text{Cov}) \leftarrow \text{not}(\text{attraction}(A,B)).$

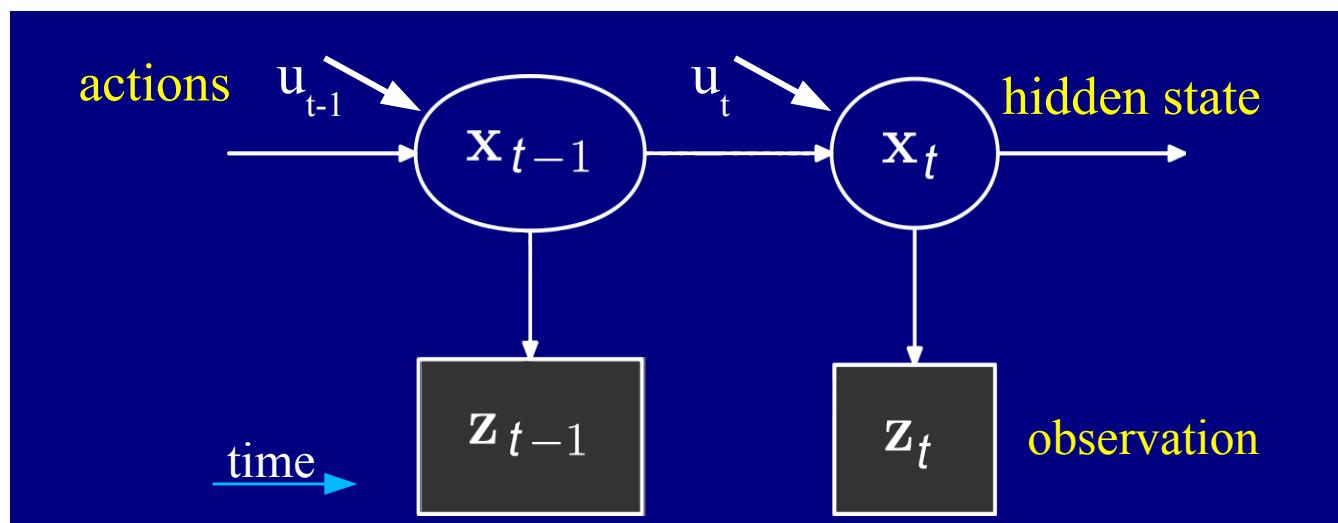
# Dynamic Distributional Clauses

Prior distribution  $p(x_0)$

State transition model  $p(x_t|x_{t-1}, u_t)$

Measurement model  $p(z_t|x_t)$

Other rules:  $p(x'_t|x''_t)$



# Ongoing Work

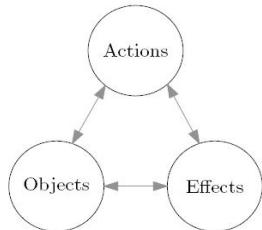
- Online parameter learning [Nitti, ICRA 2014]
- Integration with planning
- Larger Experiments
- Applications in robotics (also to learn affordances)

# Take-away message

- Probabilistic rules also apply to dynamic environments
- Topic of ongoing work
- Scalability is a challenge

# Learning relational affordances

Learn probabilistic model

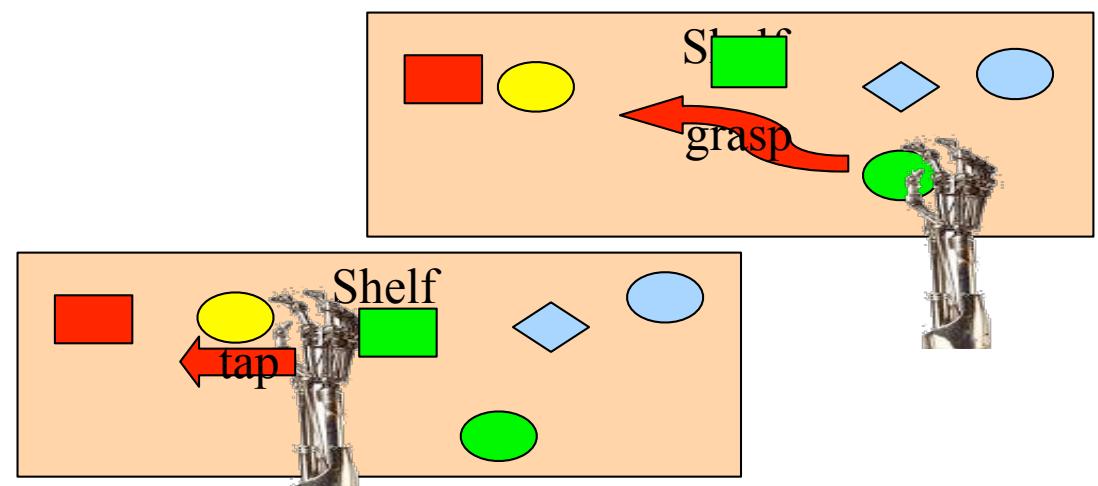
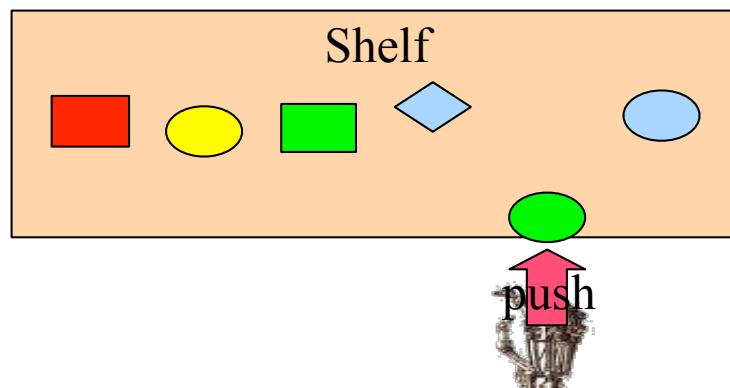


Inputs	Outputs	Function
$(O, A)$	$E$	Effect prediction
$(O, E)$	$A$	Action recognition/planning
$(A, E)$	$O$	Object recognition/selection

Learning relational affordances  
between two objects  
(learnt by experience)

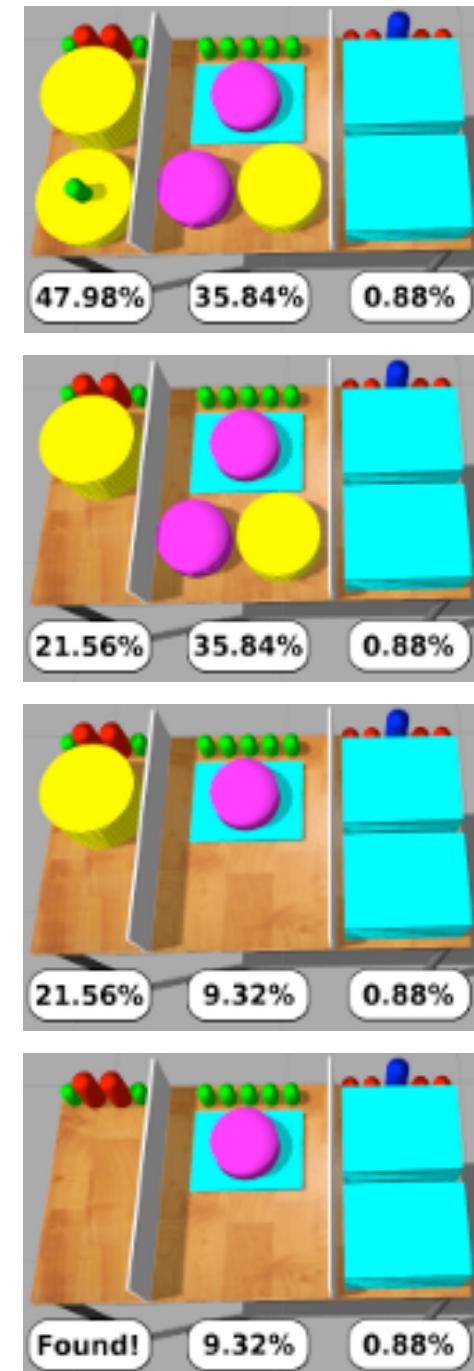
From two object interactions  
Generalize to N

*Moldovan et al. ICRA 12, 13, 14*



# Occluded Object Search

- How to achieve a specific configuration of objects on the shelf?
- Where's the orange mug?
- Where's something to serve soup in?
- Models of objects and their spatial arrangement



[Moldovan et al. 14]

# ProbLog for activity recognition from video



CAVIAR-INRIA human activity dataset

28 videos  
≈ 26.500 frames

- Separation between low-level events (LLE) and high-level events (HLE)
  - LLE: *walking, running, active, inactive, abrupt*
  - HLE: *meeting, moving, fighting, leaving\_object*
- Probabilistic Logic approach: *Event Calculus in ProbLog* (Prob-EC) to infer the high-level events from an **algebra** of low-level events.
- Example:

```
initiatedAt(fighting(P1, P2) = true, T) ←  
    happensAt(abrupt(P1), T),  
    holdsAt(close(P1, P2, 44) = true, T),  
    not happensAt(inactive(P2), T).
```

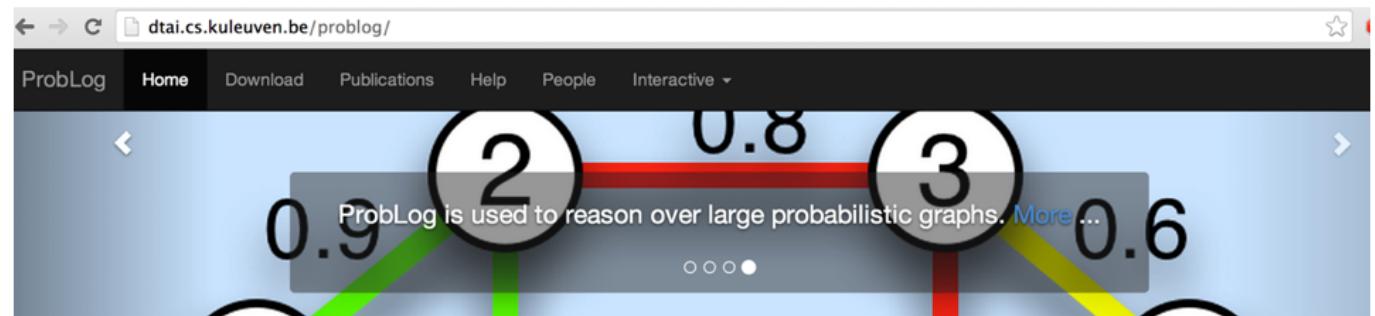
# Conclusions

- Probabilistic rules and logic
- Learning and inference
- Many challenges remain:
  - scalability and inference
  - users getting used to probabilities (datasets)
  - learning
- Argued that it applies to a lot of issues for RuleML ?

Maurice Bruynooghe  
Bart Demoen  
**Anton Dries**  
Daan Fierens  
Jason Filippou  
Bernd Gutmann  
Manfred Jaeger  
Gerda Janssens  
Kristian Kersting  
**Angelika Kimmig**  
Theofrastos Mantadelis  
Wannes Meert  
Bogdan Moldovan  
Siegfried Nijssen  
**Davide Nitti**  
Joris Renkens  
Kate Revoredo  
Ricardo Rocha  
Vitor Santos Costa  
Dimitar Shterionov  
**Ingo Thon**  
Hannu Toivonen  
**Guy Van den Broeck**  
**Mathias Verbeke**  
Jonas Vlasselaer

# Thanks !

<http://dtai.cs.kuleuven.be/problog>



## Introduction.

Probabilistic logic programs are logic programs in which some of the facts are annotated with probabilities.

ProbLog is a tool that allows you to intuitively build programs that do not only encode **complex interactions** between a large sets of **heterogenous components** but also the inherent **uncertainties** that are present in real-life situations.

The engine tackles several tasks such as computing the marginals given evidence and learning from (partial) interpretations. ProbLog is a suite of efficient algorithms for various inference tasks. It is based on a conversion of the program and the queries and evidence to a weighted Boolean formula. This allows us to reduce the inference tasks to well-studied tasks such as weighted model counting, which can be solved using state-of-the-art methods known from the graphical model and knowledge compilation literature.

## The Language. Probabilistic Logic Programming.

ProbLog makes it easy to express complex, probabilistic models.

```
0.3::stress(X) :- person(X).  
0.2::influences(X,Y) :- person(X), person(Y).
```

# PLP Systems

- **PRISM** <http://sato-www.cs.titech.ac.jp/prism/>
- **ProbLog2** <http://dtai.cs.kuleuven.be/problog/>
- **Yap Prolog** <http://www.dcc.fc.up.pt/~vsc/Yap/> includes
  - **ProbLog1**
  - **cplint** <https://sites.google.com/a/unife.it/ml/cplint>
  - **CLP(BN)**
  - **LP2**
- **PITA** in XSB Prolog <http://xsb.sourceforge.net/>
- **AILog2** <http://artint.info/code/ailog/ailog2.html>
- **SLPs** <http://stoics.org.uk/~nicos/sware/pepl>
- **contdist** <http://www.cs.sunysb.edu/~cram/contdist/>
- **DC** <https://code.google.com/p/distributional-clauses>
- **WFOMC** <http://dtai.cs.kuleuven.be/ml/systems/wfomc>

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