Checking Termination of Logic Programs with Function Symbols Through Linear Constraints

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The bottom-up evaluation of this program never terminates, producing $nat(0), nat(s(0)), nat(s(s(0))), \ldots$.

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- Establishing if a program has a terminating bottom-up evaluation is an **undecidable problem**;
- Recent work has focused on finding sufficient conditions for the termination of logic programs.

Argument-restricted criterion [Lierler&Lifschitz, ICLP'09]

Computes, for each predicate argument, an upper bound of the depth of terms that may occur in that argument.

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$\mathtt{q}[1] \longrightarrow 0$, $\mathtt{p}[1] \longrightarrow 1$

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- **Pros**:
  - Simple: it just needs to compute a particular level mapping for each argument;
  - Efficient: Computing an argument ranking (if exists) requires polynomial time.
- **Cons**:
  - Only characterizes **the depth of terms** inside arguments **alone**;
  - No distinction between different function symbols;
  - **Few practical (terminating) programs** are recognized.

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  - $\texttt{size}(\texttt{head}(r_2)) = 0 + 1$;
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- The overall size of the head will never increase **w.r.t. the body**.

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If *all* rules satisfy this condition, the program is terminating.

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Definition (Firing Graph)

Directed graph whose nodes are the rules of the program and there is an edge from r_1 to r_2 if r_1 fires r_2.

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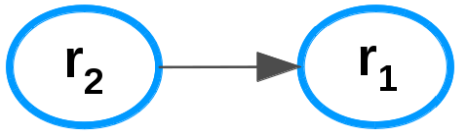

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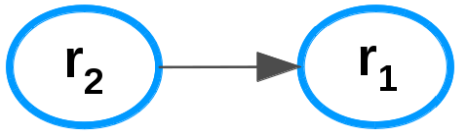

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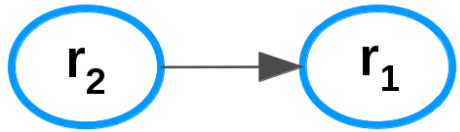

The program is not argument-restricted.

However, rule r_1 will never "trigger" r_2.

Definition (Firing Graph)

Directed graph whose nodes are the rules of the program and there is an edge from r_1 to r_2 if r_1 fires r_2.

There is no cyclic execution of r_1 and r_2.

Goal: Check termination of the SCCs of the Firing Graph.

Term size

Intuition: The size of a term is a template for all possible sizes the term may have during the program evaluation.

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size(t,r) is the size of term t w.r.t. rule r

Atom size

The size of an atom is a **linear combination of its terms sizes**.

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Given a rule r and an atom $A = p(t_1, \ldots, t_n)$ in r, the **size of** A is:

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They will be chosen depending on the program structure.

Rule-bounded programs

Definition

A program $\mathcal{P}$ is *rule-bounded* if for every SCC $\mathcal{C}$, every rule r in $\mathcal{C}$ is such that:

$$size(body(r), r) \geq size(head(r), r)$$

for some fixed set of parameters α_i.

Rule-bounded programs

Example

Consider the length program:

r_1 : $\texttt{len}([\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}], 0).$
r_2 : $\texttt{len}(\texttt{Tail}, \mathbf{s}(\texttt{N})) \leftarrow \texttt{len}(\mathbf{list}(\texttt{Head}, \texttt{Tail}), \texttt{N}).$

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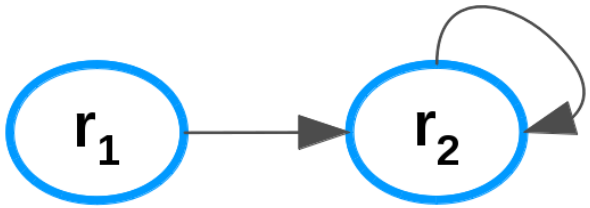

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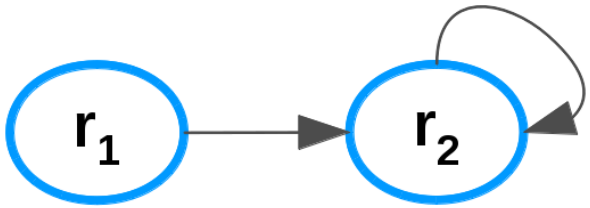

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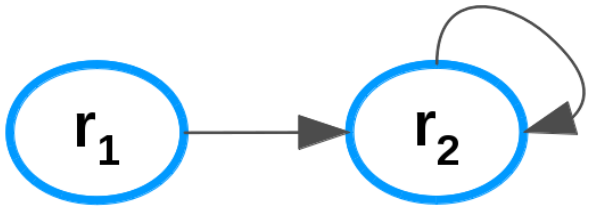

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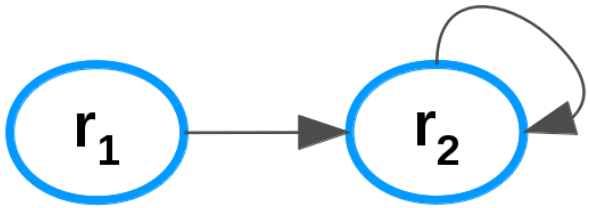

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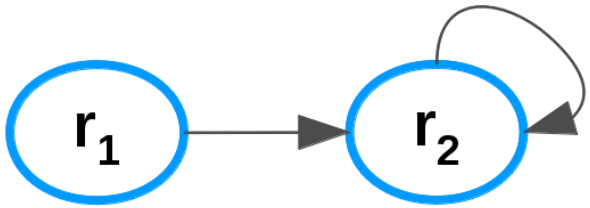

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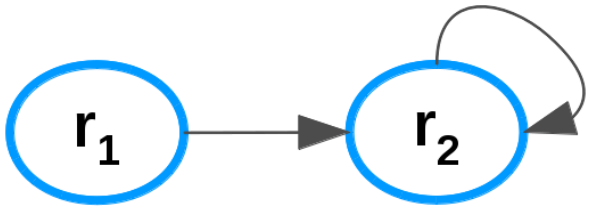

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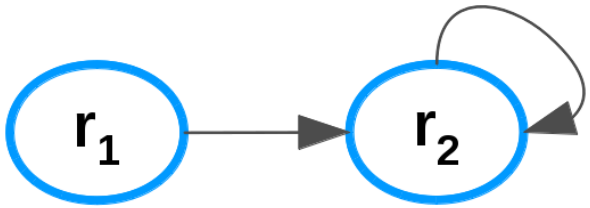

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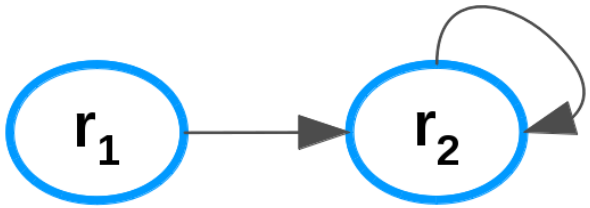

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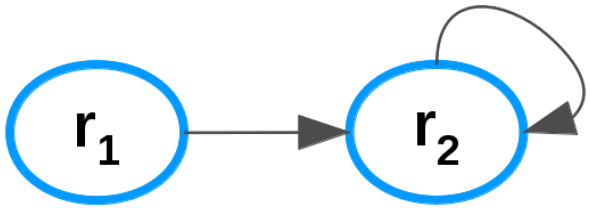

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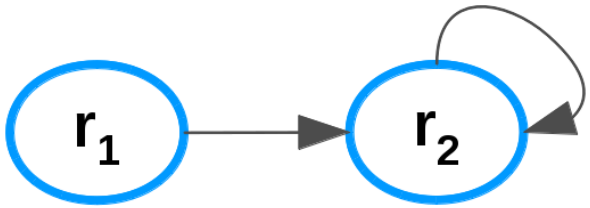

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We can choose $\alpha_1 = \alpha_2 = 1 \Rightarrow$ the program is rule-bounded.

Rule-bounded programs

Theorem

The bottom-up evaluation of a rule-bounded program always terminates.

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Theorem

The complexity of checking whether a program is rule-bounded is NP.

Examples of rule-bounded programs

Example (Bubble sort)

$$
\begin{array}{lll}
r_0: & \texttt{sort(List,[],[])} & \leftarrow \quad \texttt{input(List).} \\
r_1: & \texttt{sort([Y|T],[X|Temp],Sorted)} & \leftarrow \quad \texttt{sort([X|[Y|T]]),Temp,Sorted),X} \leq \texttt{Y.} \\
r_2: & \texttt{sort([X|T],[Y|Temp],Sorted)} & \leftarrow \quad \texttt{sort([X|[Y|T]]),Temp,Sorted),Y} < \texttt{X.} \\
r_3: & \texttt{sort(Temp,[],[X|Sorted])} & \leftarrow \quad \texttt{sort([X],Temp,Sorted).}
\end{array}
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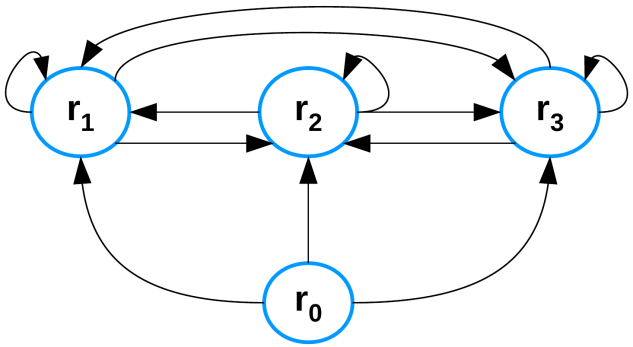

Figure: Bubble sort Firing Graph

$$\text{SCC } \mathcal{C} = \{r_1, r_2, r_3\}$$

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$$
\begin{cases}
\alpha_1 \cdot (4+x+y+t) + \alpha_2 \cdot temp + \alpha_3 \cdot sorted \geq \\
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\end{cases}
$$

A possible solution is $\alpha_1 = 2$, $\alpha_2 = 2$, $\alpha_3 = 1$

Examples of rule-bounded programs

Example (Tree visit)

$r_0 : \texttt{visit}(\texttt{Tree}, [\,], [\,]) \leftarrow \texttt{input}(\texttt{Tree}).$

$r_1 : \texttt{visit}(\texttt{Left}, [\texttt{Root}|\texttt{Visited}], [\texttt{Right}|\texttt{ToVisit}]) \leftarrow \texttt{visit}(\texttt{tree}(\texttt{Root}, \texttt{Left}, \texttt{Right}), \texttt{Visited}, \texttt{ToVisit}).$

$r_2 : \texttt{visit}(\texttt{Next}, \texttt{Visited}, \texttt{ToVisit}) \leftarrow \texttt{visit}(\texttt{nil}, \texttt{Visited}, [\texttt{Next}|\texttt{ToVisit}]).$

$$\begin{cases} \alpha_1 \cdot (3 + \mathit{root} + \mathit{left} + \mathit{right}) + \alpha_2 \cdot \mathit{visited} + \alpha_3 \cdot \mathit{tovisit} \geq \\ \qquad \alpha_1 \cdot \mathit{left} + \alpha_2 \cdot (2 + \mathit{root} + \mathit{visited}) + \alpha_3 \cdot (2 + \mathit{right} + \mathit{tovisit}) \\ \alpha_1 \cdot 0 + \alpha_2 \cdot \mathit{visited} + \alpha_3 \cdot (2 + \mathit{next} + \mathit{tovisit}) \geq \\ \qquad \alpha_1 \cdot \mathit{next} + \alpha_2 \cdot \mathit{visited} + \alpha_3 \cdot \mathit{tovisit} \end{cases}$$

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$r_1 : \texttt{visit}(\texttt{Left}, [\texttt{Root}|\texttt{Visited}], [\texttt{Right}|\texttt{ToVisit}]) \leftarrow \texttt{visit}(\texttt{tree}(\texttt{Root}, \texttt{Left}, \texttt{Right}), \texttt{Visited}, \texttt{ToVisit}).$

$r_2 : \texttt{visit}(\texttt{Next}, \texttt{Visited}, \texttt{ToVisit}) \leftarrow \texttt{visit}(\texttt{nil}, \texttt{Visited}, [\texttt{Next}|\texttt{ToVisit}]).$

$$\begin{cases} \alpha_1 \cdot (3 + \mathit{root} + \mathit{left} + \mathit{right}) + \alpha_2 \cdot \mathit{visited} + \alpha_3 \cdot \mathit{tovisit} \geq \\ \qquad \alpha_1 \cdot \mathit{left} + \alpha_2 \cdot (2 + \mathit{root} + \mathit{visited}) + \alpha_3 \cdot (2 + \mathit{right} + \mathit{tovisit}) \\ \alpha_1 \cdot 0 + \alpha_2 \cdot \mathit{visited} + \alpha_3 \cdot (2 + \mathit{next} + \mathit{tovisit}) \geq \\ \qquad \alpha_1 \cdot \mathit{next} + \alpha_2 \cdot \mathit{visited} + \alpha_3 \cdot \mathit{tovisit} \end{cases}$$

A possible solution is $\alpha_1 = 2$, $\alpha_2 = 1$, $\alpha_3 = 2$

Dealing with non-increasing cycles

Example

$$r_1 : \mathtt{p(X,Y)} \leftarrow \mathtt{q(f(X),Y)}.$$
$$r_2 : \mathtt{q(W,f(Z))} \leftarrow \mathtt{p(W,Z)}.$$

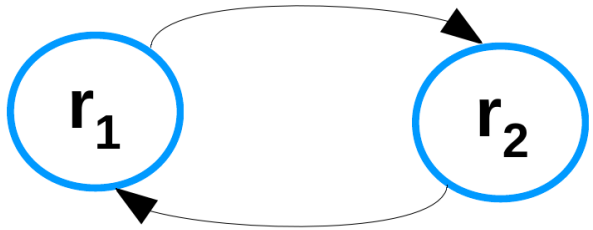

Figure: Firing Graph

The program terminates, but...

Dealing with non-increasing cycles

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$$r_1 : \texttt{p(X,Y)} \leftarrow \texttt{q(f(X),Y)}.$$
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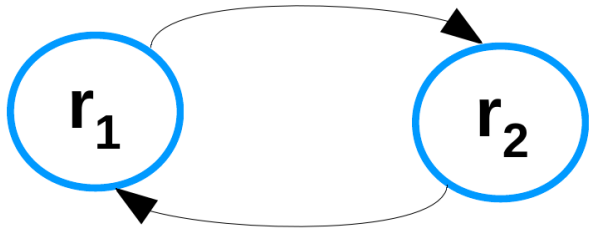

Figure: Firing Graph

The program terminates, but...

Rule r_2 increases its head size $\Rightarrow$ program is not rule-bounded.

Dealing with non-increasing cycles

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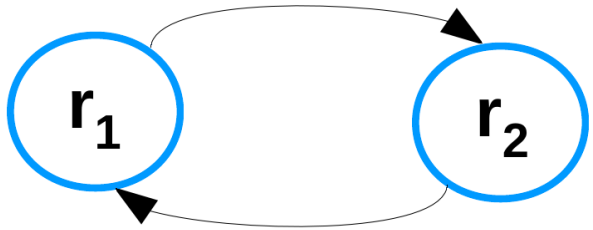

Figure: Firing Graph

The program terminates, but...

Rule r_2 increases its head size $\Rightarrow$ program is not rule-bounded.

But the **cycle** does not increase the size of propagated values.

Dealing with non-increasing cycles

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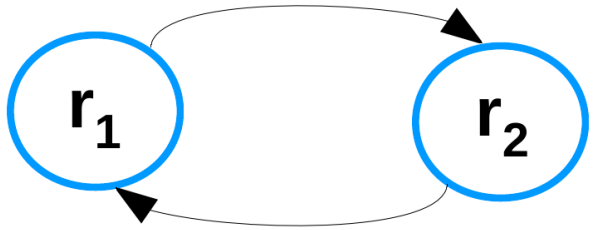

Figure: Firing Graph

We can rewrite the cycle r_1, r_2 as a single rule:

Dealing with non-increasing cycles

Example

$$r_1 : \mathtt{p(X,Y)} \leftarrow \mathtt{q(f(X),Y)}.$$
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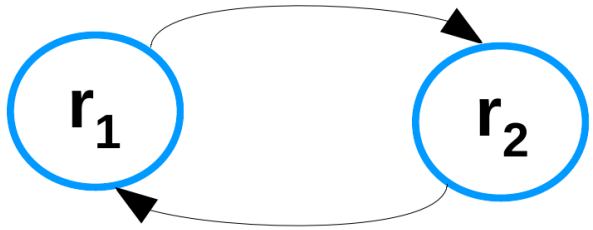

Figure: Firing Graph

We can rewrite the cycle r_1, r_2 as a single rule:

$$\underbrace{\mathtt{q(f(X),Y)} \longrightarrow \mathtt{p(X,Y)}}_{r_1}$$

Dealing with non-increasing cycles

Example

$$r_1 : \mathtt{p(X,Y)} \leftarrow \mathtt{q(f(X),Y)}.$$
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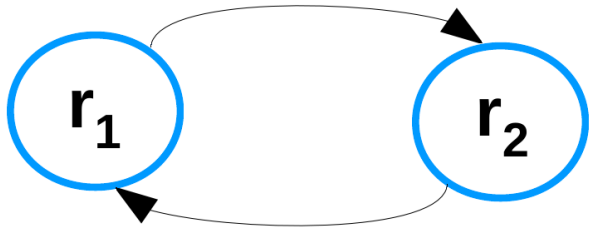

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$$\underbrace{\mathtt{q(f(X),Y)} \rightarrow \mathtt{p(X,Y)}}_{r_1} \Rightarrow \underbrace{\mathtt{p(W,Z)} \rightarrow \mathtt{q(W,f(Z))}}_{r_2}$$

Dealing with non-increasing cycles

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$$r_1 : \texttt{p(X,Y)} \leftarrow \texttt{q(f(X),Y)}.$$
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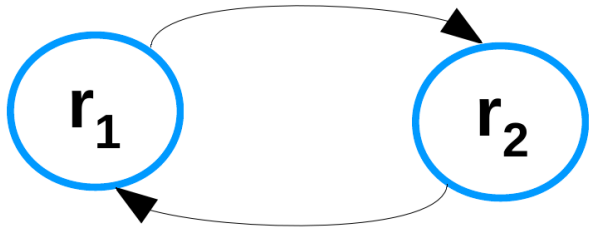

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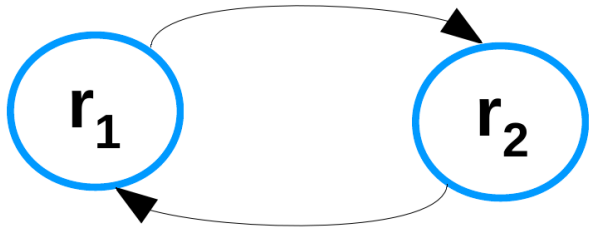

Figure: Firing Graph

We can rewrite the cycle r_1, r_2 as a single rule:

$$\underbrace{\texttt{q(f(X),Y)} \longrightarrow \texttt{p(X,Y)}}_{r_1} \Rightarrow \underbrace{\texttt{p(W,Z)} \longrightarrow \texttt{q(W,f(Z))}}_{r_2}$$

$$\texttt{X/W, Y/Z}$$

Dealing with non-increasing cycles

Example

$$r_1 : \mathtt{p(X,Y)} \leftarrow \mathtt{q(f(X),Y)}.$$
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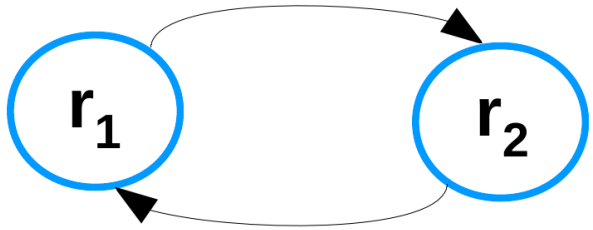

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We can rewrite the cycle r_1, r_2 as a single rule:

$$\underbrace{\mathtt{q(f(X),Y)} \longrightarrow \mathtt{p(X,Y)}}_{r_1} \Rightarrow \underbrace{\mathtt{p(W,Z)} \longrightarrow \mathtt{q(W,f(Z))}}_{r_2}$$

$$\mathtt{X/W,\ Y/Z}$$

$$\underbrace{\mathtt{q(f(W),Z)} \longrightarrow \mathtt{q(W,f(Z))}}_{r_{12}}$$

Dealing with non-increasing cycles

Example

$$r_1 : \mathtt{p(X,Y)} \leftarrow \mathtt{q(f(X),Y)}.$$
$$r_2 : \mathtt{q(W,f(Z))} \leftarrow \mathtt{p(W,Z)}.$$

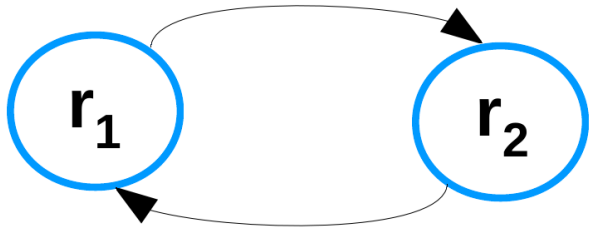


Figure: Firing Graph

We can rewrite the cycle r_1, r_2 as a single rule:

$$\underbrace{\mathtt{q(f(X),Y)} \rightarrow \mathtt{p(X,Y)}}_{r_1} \Rightarrow \underbrace{\mathtt{p(W,Z)} \rightarrow \mathtt{q(W,f(Z))}}_{r_2}$$

$$\mathtt{X/W,\ Y/Z}$$

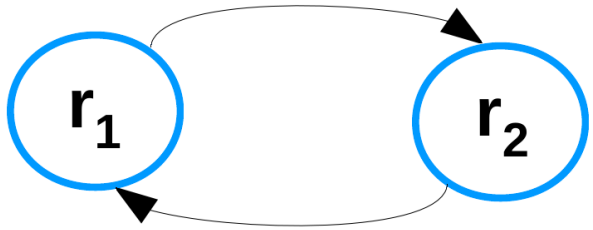
$$\underbrace{\mathtt{q(f(W),Z)} \rightarrow \mathtt{q(W,f(Z))}}_{r_{12}}$$

$$\alpha_{q_1} \cdot (1 + w) + \alpha_{q_2} \cdot z \geq \alpha_{q_1} \cdot w + \alpha_{q_2} \cdot (1 + z)$$

Dealing with non-increasing cycles

Example

$$r_1 : \mathtt{p(X,Y)} \leftarrow \mathtt{q(f(X),Y)}.$$
$$r_2 : \mathtt{q(W,f(Z))} \leftarrow \mathtt{p(W,Z)}.$$



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$$\mathtt{X/W,\ Y/Z}$$

$$\underbrace{\mathtt{q(f(W),Z)} \rightarrow \mathtt{q(W,f(Z))}}_{r_{12}}$$

$$\alpha_{q_1} \cdot (1 + w) + \alpha_{q_2} \cdot z \geq \alpha_{q_1} \cdot w + \alpha_{q_2} \cdot (1 + z)$$

$$\alpha_{q_1} \geq \alpha_{q_2}$$

Dealing with non-increasing cycles

Example

$$r_1 : \mathtt{p(X,Y)} \leftarrow \mathtt{q(f(X),Y)}.$$
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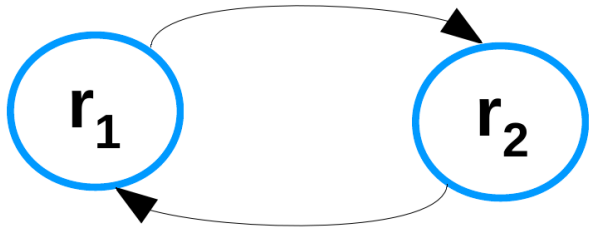

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$$\mathtt{X/W,\ Y/Z}$$

$$\underbrace{\mathtt{q(f(W),Z)} \rightarrow \mathtt{q(W,f(Z))}}_{r_{12}}$$

$$\alpha_{q_1} \cdot (1 + w) + \alpha_{q_2} \cdot z \geq \alpha_{q_1} \cdot w + \alpha_{q_2} \cdot (1 + z)$$

$$\alpha_{q_1} \geq \alpha_{q_2} \qquad \alpha_{q_1} = \alpha_{q_2} = 1$$

Conclusion

Contributions:

- Using linear constraints for checking bottom-up termination;
- The technique is complementary to the other techniques that analyze single arguments;
- The technique recognizes several practical logic programs;

Future work:

- Combine rule-bounded programs with other techniques in the literature;
- Complexity analysis of the proposed technique (there may be many tractable cases);
- Study the termination problem for programs with interpreted function symbols (none of the currently known techniques support them).

Some references

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2. Logic programming with function symbols: checking termination of bottom-up evaluation through program adornments, *Greco et al.*, ICLP '13 (to appear in TPLP journal).
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6. Computable functions in ASP: Theory and Implementation, *Calimeri et al.*, ICLP '08.
7. Termination detection in logic programs using argument sizes, *Kirack Sohn and Allen Van Gelder*, PODS '91.

Thank you for your attention!