20240919 作业1

1.(3) 记 $z = i^{8} - 4i^{21} + i$

$= 1 - 4i + i = 1 - 3i$

$\therefore \operatorname{Re} z = 1$，$\operatorname{Im} z = -3$，$\arg z = \arctan(-3)$，$\bar{z} = 1 + 3i$

(4)，记 $z = \left(\frac{1+\sqrt{3}i}{2}\right)^{5}$

$= \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{5}$

$= \left(\cos\frac{5}{3}\pi + i\sin\frac{5}{3}\pi\right)$

$= \frac{1}{2} - \frac{\sqrt{3}}{2}i$

$\therefore \operatorname{Re} z = \frac{1}{2}$，$\operatorname{Im} z = -\frac{\sqrt{3}}{2}$，$\arg z = -\frac{\pi}{3}$，$\bar{z} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

2.(1)，记 $z = \frac{(\sqrt{3}+i)(2-2i)}{(\sqrt{3}-i)(2+2i)}$

$= \frac{4\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\left(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})\right)}{4\sqrt{2}\left(\cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})\right)\left(\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4})\right)}$

$= \frac{\cos(-\frac{\pi}{12}) + i\sin(-\frac{\pi}{12})}{\cos(\frac{\pi}{12}) + i\sin(\frac{\pi}{12})}$

$= \cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$

$= \frac{\sqrt{3}}{2} - \frac{1}{2}i$

三角形式 $z = \cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$，指数形式 $z = e^{i(-\frac{\pi}{6})}$

(2)，记原式 $= z$.

$z = \frac{\cos 10\theta + i\sin 10\theta}{\cos 9\theta - i\sin 9\theta}$

$= \frac{\cos 10\theta + i\sin 10\theta}{\cos(-9\theta) + i\sin(-9\theta)}$

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$= \cos 19\theta + i\sin 19\theta.$

指数表示：$z = e^{19i\theta}$

3. (1). 原式 $= \left(2\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)\right)^{12}$.

$= 2^{12}\left(\cos(-2\pi) + i\sin(-2\pi)\right)$

$= 4096$

(2). 原式 $= \sqrt{\sqrt{6}\cdot e^{i\frac{\pi}{4}}}$

$= (6)^{\frac{1}{4}} e^{i\left(\frac{\frac{\pi}{4}+2k\pi}{2}\right)}$, $k = 0$ 或 1

即 原式 $= (6)^{\frac{1}{4}} e^{\frac{\pi}{8}i}$ 或 $(6)^{\frac{1}{4}} e^{\frac{9}{8}\pi i}$

4. $z^2 - 3(1+i)z + 5i = 0.$

$z^2 - 3(1+i)z + \frac{9}{4}(1+i)^2 + \frac{1}{2}i = 0$

$\left(z - \frac{3}{2}(1+i)\right)^2 = -\frac{1}{2}i$

$z - \frac{3}{2}(1+i) = \frac{\sqrt{2}}{2}e^{-\frac{\pi}{4}}$ 或 $\frac{\sqrt{2}}{2}e^{\frac{3}{4}\pi}$

即 $z - \frac{3}{2}(1+i) = \frac{1}{2} - \frac{1}{2}i$ 或 $\frac{1}{2}i - \frac{1}{2}$

即：$z = 2+i$ 或 $1+2i$

5. 19：记 $z = e^{i\theta} + e^{2i\theta} + \cdots + e^{ni\theta}$

$= \dfrac{e^{i\theta} - e^{(n+1)i\theta}}{1 - e^{i\theta}}$

$\cancel{= \dfrac{(e^{i\theta} - e^{(n+1)i\theta})(1+e^{i\theta})}{1 - e^{2i\theta}}}$

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$$\cancel{= \frac{e^{i\theta}+e^{2i\theta}-e^{(n+1)i\theta}-e^{(n+2)i\theta}}{1-e^{2i\theta}}}$$

$$\cancel{= \frac{(\cos\theta+\cos 2\theta-\cos(n+1)\theta-\cos(n+2)\theta)+i(\sin\theta+\sin 2\theta-\sin(n+1)\theta-\sin(n+2)\theta)}{1-\cos 2\theta-i\sin 2\theta}}$$

$$\cancel{=}$$

$$\cancel{= \frac{}{1-}}$$

∴ 则 $\bar{z} = \dfrac{e^{-i\theta}-e^{-(n+1)i\theta}}{1-e^{-i\theta}}$

$$\mathrm{Re}\, z = \frac{z+\bar{z}}{2}$$

$$= \frac{1}{2}\cdot\frac{e^{i\theta}+e^{-i\theta}+e^{ni\theta}+e^{-ni\theta}-(e^{(n+1)i\theta}+e^{-(n+1)i\theta})-2}{2-e^{i\theta}-e^{-i\theta}}$$

$$= \frac{1}{2}\cdot\frac{2\cos\theta+2\cos n\theta-2\cos(n+1)\theta-2}{2-2\cos\theta}$$

$$= \frac{\cos\theta+\cos n\theta-\cos(n+1)\theta-1}{2-2\cos\theta}$$

$$\mathrm{Im}\, z = \frac{z-\bar{z}}{2i}$$

$$= \frac{1}{2i}\cdot\frac{e^{i\theta}-e^{-i\theta}+e^{ni\theta}-e^{-ni\theta}-(e^{(n+1)i\theta}-e^{-(n+1)i\theta})}{2-e^{i\theta}-e^{-i\theta}}$$

$$= \frac{\sin\theta+\sin n\theta-\sin(n+1)\theta}{2-2\cos\theta}$$

又 ∵ $z = \cos\theta+\cos 2\theta+\cdots+\cos n\theta+i(\sin\theta+\cdots+\sin n\theta)$

∴ $\sin\theta+\sin 2\theta+\cdots+\sin n\theta = \dfrac{\sin\theta+\sin n\theta-\sin(n+1)\theta}{2-2\cos\theta}$

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$$\cos\theta + \cos 2\theta + \cdots + \cos n\theta = \frac{\cos\theta + \cos n\theta - \cos(n+1)\theta - 1}{2 - 2\cos\theta}$$