2024.12.26作业.

3.(2). 记 $F(p)=\frac{p^2+p+2}{p^3}=\frac{1}{p}+\frac{1}{p^2}+\frac{2}{p^3}$

$\mathcal{L}^{-1}(e^{-p}F(p))=f(t-1)u(t-1)$

$=(1+(t-1)+(t-1)^2)u(t-1)$

$=(t^2-t+1)u(t-1)$

(4). $\mathcal{L}^{-1}(\frac{2}{p}e^{-p}-\frac{1}{p}e^{-2p})$

$=2u(t-1)-u(t-2)$

4.(2) $\mathcal{L}(e^{-\alpha t}\sin\beta t)$

$=\frac{\beta}{(p+\alpha)^2+\beta^2}$.

$\mathcal{L}(te^{-\alpha t}\sin\beta t)=-(\frac{\beta}{(p+\alpha)^2+\beta^2})'$

$=\frac{2\beta(p+\alpha)}{((p+\alpha)^2+\beta^2)^2}$

5. $\mathcal{L}(e^{-3t}\sin 2t)$

$=\frac{2}{(p+3)^2+4}$

$$\mathcal{L}\left(\frac{1}{t}e^{-3t}\sin 2t\right)=\int_{p}^{+\infty}\frac{2}{(s+3)^2+4}ds.$$

$$=\arctan\left(\frac{s+3}{2}\right)\Big|_{p}^{+\infty}=\frac{\pi}{2}-\arctan\left(\frac{p+3}{2}\right).$$

(6). $\mathcal{L}\left(\int_0^t \frac{e^{-3s}\sin 2s}{s}ds\right)$

$=\frac{1}{p}\mathcal{L}\left(\frac{\sin 2t}{t}e^{-3t}\right)$

$\because \mathcal{L}(\sin 2t\, e^{-3t})=\frac{2}{(p+3)^2+4}$

$\mathcal{L}\left(\frac{1}{t}\sin 2t\, e^{-3t}\right)=\int_{p}^{+\infty}\frac{2}{(s+3)^2+4}ds$

$=\frac{\pi}{2}-\arctan\left(\frac{p+3}{2}\right)$

$\therefore$ 原式 $=\frac{1}{p}\left(\frac{\pi}{2}-\arctan\left(\frac{p+3}{2}\right)\right)$

5. (2). $\because \mathcal{L}(t\sin\omega t)=\frac{2\omega p}{(p^2+\omega^2)^2}$

令 $\omega=2$ 得 $\mathcal{L}(t\sin 2t)=\frac{4p}{(p^2+4)^2}$.

也即: $\mathcal{L}^{-1}\left(\frac{4p}{(p^2+4)^2}\right)=t\sin 2t$

(4). $\frac{p+3}{(p+1)(p-3)}=\frac{-\frac{1}{2}}{p+1}+\frac{\frac{3}{2}}{p-3}=\frac{3}{2}\frac{1}{p-3}-\frac{1}{2}\frac{1}{p+1}$

$\because \mathcal{L}(e^{3t})=\frac{1}{p-3},\ \mathcal{L}(e^{-t})=\frac{1}{p+1}$

$\therefore \mathcal{L}^{-1}\left(\frac{p+3}{(p+1)(p-3)}\right)=\frac{3}{2}e^{3t}-\frac{1}{2}e^{-t}$

(5) $\frac{2p+5}{p^2+4p+13}$

$$=\frac{2(p+2)+1}{(p+2)^2+9}=\frac{2(p+2)}{(p+2)^2+9}+\frac{1}{3}\frac{3}{(p+2)^2+9}$$

$$\therefore \mathcal{L}^{-1}\left(\frac{2p+5}{p^2+4p+13}\right)=2e^{-2t}\cos 3t+\frac{1}{3}e^{-2t}\sin 3t.$$

(6) $\mathcal{L}^{-1}\left(\ln\frac{p^2+1}{p^2}\right)=\mathcal{L}^{-1}\left(\ln(p^2+1)-2\ln p\right)$

$$=\mathcal{L}^{-1}\left(\int\left(\frac{2p}{p^2+1}-\frac{2}{p}\right)dp\right)$$

$$=\frac{1}{t}\mathcal{L}^{-1}\left(\frac{2p}{p^2+1}-\frac{2}{p}\right)$$

$$=\frac{1}{t}\left(2\cos t-2u(t)\right)$$

$$=\frac{2}{t}\left(\cos t-u(t)\right)$$

6.(2) 原式 $=\frac{1}{(p+a)(p-a)(p+b)(p-b)}$，记为 $F(p)$

$$\mathcal{L}^{-1}(F(p))=\frac{e^{ta}}{2a(a+b)(a-b)}+\frac{e^{-ta}}{2a(b-a)(a+b)}$$

$$+\frac{e^{tb}}{2b(b+a)(b-a)}+\frac{e^{-tb}}{2b(a-b)(a+b)}$$

(4). $f(p)=\frac{1}{p(p+ai)^2(p-ai)^2}$

$\therefore Res[f(p)e^{pt},0]=\frac{1}{a^4}$

$Res[f(p)e^{pt},ai]=\frac{it}{4a^3}e^{iat}-\frac{e^{iat}}{2a^4}$

$Res[f(p)e^{pt},-ai]=\frac{-it}{4a^3}e^{-iat}-\frac{e^{-iat}}{2a^4}$.

$\therefore \mathcal{L}^{-1}(f(p))=\frac{1}{a^4}-\frac{t}{2a^3}\cdot\frac{e^{iat}-e^{-iat}}{2i}-\frac{1}{a^4}\cdot\frac{e^{iat}+e^{-iat}}{2}$

$=\frac{1}{a^4}-\frac{t}{2a^3}\sin at-\frac{1}{a^4}\cos at$

7. (2). $f(p)=\frac{1}{p(p+\sqrt{5}i)(p-\sqrt{5}i)}$

$Res[f(p)e^{pt},0]=\frac{1}{5}$

$Res[f(p)e^{pt},\sqrt{5}i]=-\frac{e^{\sqrt{5}it}}{10}$

$Res[f(p)e^{pt},-\sqrt{5}i]=-\frac{e^{-\sqrt{5}it}}{10}$.

$\therefore \mathcal{L}^{-1}(f(p))=\frac{1}{5}-\frac{1}{5}\cdot\frac{e^{\sqrt{5}it}+e^{-\sqrt{5}it}}{2}$

$=\frac{1}{5}(1-\cos\sqrt{5}t)$

(3). 记 $F(p)=\dfrac{p^2+2}{p(p+1)(p+2)}$

$Res[F(p)e^{pt},0]=1$.

$Res[F(p)e^{pt},-1]=-3e^{-t}$.

$Res[F(p)e^{pt},-2]=3e^{-2t}$

$\therefore \mathcal{L}^{-1}(F(p))=1-3e^{-t}+3e^{-2t}$

8. (2). $\sin t * \cos t$

$=\int_0^t \sin s\cos(t-s)ds$

$=\int_0^t \sin s(\cos t\cos s+\sin t\sin s)ds$

$=\int_0^t \frac{1}{2}(\cos t\sin 2s+\sin t(1-\cos 2s))ds$

$=\frac{1}{2}\int_0^t [\sin(2s-t)+\sin t]ds$.

$=\frac{1}{2}(-\frac{1}{2}\cos(2s-t)|_0^t+t\sin t)=\frac{1}{2}t\sin t$

9.(2). $\mathcal{L}^{-1}(F(p))$

$= \mathcal{L}^{-1}(\frac{1}{p}) * (\mathcal{L}^{-1}(\frac{1}{p-1}) * \mathcal{L}^{-1}(\frac{1}{p-2}))$

$= \mathcal{L}^{-1}(\frac{1}{p}) * (e^t * e^{2t})$

$= u(t) * (\int_0^t e^s e^{2(t-s)} ds$

$= u(t) * (e^{2t} - e^t)$

$= \int_0^t e^{2(t-s)} ds - \int_0^t e^{t-s} ds$

$= -\frac{1}{2} e^{2(t-s)} \Big|_0^t + e^{t-s} \Big|_0^t$

$= \frac{1}{2} e^{2t} - e^t + \frac{1}{2}$