2024.12.19 作业.

7.1. (1) $\mathcal{L}(f(t))=\int_0^{\infty} f(t)e^{-pt}dt$

$=\int_0^1 te^{-pt}dt+\int_1^3 -4e^{-pt}dt$

$=\left[-\frac{1}{p}\left(te^{-pt}\Big|_0^1-\int_0^1 e^{-pt}dt\right)\right]+\int_1^3 -4e^{-pt}dt$

$=\frac{1}{p^2}-\left(\frac{5}{p}+\frac{1}{p^2}\right)e^{-p}+\frac{4}{p}e^{-3p}$.

(2) $\mathcal{L}[f(t)]=\int_0^{\infty} f(t)e^{-pt}dt$

$=\int_0^{\pi} \sin t\, e^{-pt}dt$

$=\int_0^{\pi}\frac{1}{2i}(e^{it}-e^{-it})e^{-pt}\,dt$

$=\frac{1}{2i}\int_0^{\pi}(e^{(i-p)t}-e^{-(i+p)t})dt$.

$=\frac{1}{2i}\cdot\left[\frac{1}{i-p}\left[e^{(i-p)\pi}-1\right]+\frac{1}{i+p}\left[e^{-(i+p)\pi}-1\right]\right]$

$=\frac{1}{2i}\left(\frac{1}{i-p}(-e^{-p\pi}-1)+\frac{1}{i+p}(-e^{-p\pi}-1)\right)$

$=\frac{1}{2i}\cdot\left(\frac{2i}{-1-p^2}(-e^{-p\pi}-1)\right)=\frac{e^{-p\pi}+1}{1+p^2}$

2.(1). $\sin^2\beta t = \frac{1}{2}(1-\cos 2\beta t)$

$\therefore \mathcal{L}[f(t)] = \frac{1}{2p} - \frac{1}{2}\cdot\frac{p}{p^2+4\beta^2}$.

$= \frac{1}{2p} - \frac{p}{2(p^2+4\beta^2)}$

(2). $f(t) = 3t^{\frac{1}{3}} + 4e^{2t}$.

$\mathcal{L}[f(t)] = \frac{3\Gamma(\frac{4}{3})}{p^{\frac{4}{3}}} + \frac{4}{p-2}$

(3). $f(t) = \sin t\, u(t-2)$

$\mathcal{L}[f(t)] = \int_2^{\infty} \sin t\, e^{-pt} dt$

令 $x = t-2$ $\therefore t = x+2$.

$\int_2^{\infty} \sin t\, e^{-pt} dt = \int_0^{\infty} \sin(x+2) e^{-p(x+2)} dx$

$= e^{-2p} \int_0^{\infty} \sin(x+2) e^{-px} dx$

$= e^{-2p}\left[\int_0^{\infty} \sin x \cos 2\, e^{-px} dx + \int_0^{\infty} \cos x \sin 2\, e^{-px} dx\right]$

$= e^{-2p}\cdot\left[\cos 2\cdot\frac{1}{p^2+1} + \sin 2\cdot\frac{p}{p^2+1}\right]$

$= e^{-2p}\cdot\frac{\cos 2 + p\sin 2}{p^2+1}$

(4). $f(t)=e^{2t}u(t-2)$

$\mathcal{L}[f(t)]=\int_2^{\infty}e^{2t}e^{-pt}dt$

令 $x=t-2$ ∴ $t=x+2$

∴ $\int_2^{+\infty}e^{2t}e^{-pt}dt$

$=\int_0^{+\infty}e^{2(x+2)}e^{-p(x+2)}dx$

$=e^{4-2p}\int_0^{+\infty}e^{2x}e^{-px}dx$

$=e^{4-2p}\cdot\frac{1}{p-2}$

所以 $\mathcal{L}[f(t)]=\frac{e^{4-2p}}{p-2}$