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1L702
Cálculo I
3-750-1980

Derivadas #1 2.1

1) Pendiente

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{2-(-3)} = \boxed{\frac{-3}{5}}$$

5) $f(x) = 3 - 5x \quad (-1, 8)$

$$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{3 - 5(x+\Delta x) - 3 + 5x}{\Delta x} = \frac{\cancel{3} - \cancel{5x} - 5\Delta x - \cancel{3} + \cancel{5x}}{\Delta x} = \frac{-5\cancel{\Delta x}}{\cancel{\Delta x}} = -5$$

9) $f(t) = 3t - t^2 \quad (0,0)$

$$m_{tan} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta t \to 0} \frac{3(t+\Delta t) - (t+\Delta t)^2 - 3t + t^2}{\Delta t} = \frac{3t + 3\Delta t - (t^2 + 2t\Delta t + \Delta t^2) - 3t + t^2}{\Delta t}$$

$$= \frac{3t + 3\Delta t - \cancel{t^2} - 2t\Delta t - \Delta t^2 - \cancel{3t} + \cancel{t^2}}{\Delta t} = \frac{3\Delta t - 2t\Delta t - \Delta t^2}{\Delta t} = \frac{\cancel{\Delta t}(3 - 2t - \Delta t)}{\cancel{\Delta t}} = 3 - 2t - \Delta t = 3 - 2t - 0 =$$

$$m = 3 - 2t \longrightarrow 3 - 2(0) = \boxed{3}$$

11) $f(x) = 7$

$$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \boxed{7}$$

13) $f(x) = -10x$

$$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{-10(x+\Delta x) + 10x}{\Delta x} = \frac{-\cancel{10x} - 10\Delta x + \cancel{10x}}{\Delta x}$$

$$= \frac{-10\cancel{\Delta x}}{\cancel{\Delta x}} = \boxed{-10}$$

15) $h(s) = 3 + \frac{2}{3}s$

$$m = \lim_{\Delta s \to 0} \frac{f(s+\Delta s) - f(s)}{\Delta s} = \frac{3 + \frac{2}{3}(s+\Delta s) - 3 - \frac{2}{3}s}{\Delta s} = \frac{3 + \frac{\cancel{2s} + 2\Delta s}{3} - \frac{9 - \cancel{2s}}{3}}{\Delta s} = \frac{\frac{3}{1} + \frac{2\Delta s - 9}{3}}{\Delta x}$$

$$= \frac{\frac{\cancel{9} + 2\Delta s - \cancel{9}}{3}}{\Delta s} = \frac{\frac{2\Delta s}{3}}{\Delta s} \quad \frac{2\cancel{\Delta s}}{3\cancel{\Delta s}} = \boxed{\frac{2}{3}}$$

17) $f(x) = x^2+x-3$

$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{(x+\Delta x)^2 + x+\Delta x - 3 - x^2 - x + 3}{\Delta x} = \frac{\cancel{x^2} + 2x\Delta x + \Delta x^2 + \cancel{x} + \Delta x - \cancel{3} - \cancel{x^2} - \cancel{x} + \cancel{3}}{\Delta x}$

$= \frac{2x\Delta x + \Delta x^2 + \Delta x}{\Delta x} = \frac{\cancel{\Delta x}(2x + \Delta x + 1)}{\cancel{\Delta x}} = 2x + \Delta x + 1 = 2x + 0 + 1 = \boxed{2x+1}$

19) $f(x) = x^3 - 12x$

$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{(x+\Delta x)^3 - 12(x+\Delta x) - x^3 + 12x}{\Delta x} = \frac{\cancel{x^3} + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - \cancel{12x} - 12\Delta x - \cancel{x^3} + \cancel{12x}}{\Delta x}$

$= \frac{3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - 12\Delta x}{\Delta x} = \frac{\cancel{\Delta x}(3x^2 + 3x\Delta x + \Delta x^2 - 12)}{\cancel{\Delta x}} = 3x^2 + 3x\Delta x + \Delta x^2 - 12 = 3x^2 + \cancel{3x(0)} + \cancel{(0)^2} - 12$

$= \boxed{3x^2 - 12}$

21) $f(x) = \frac{1}{x-1} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{\frac{1}{x+\Delta x - 1} - \frac{1}{x-1}}{\Delta x} = \frac{\frac{\cancel{x} - \cancel{1} - \cancel{x} - \Delta x + \cancel{1}}{x^2 - 2x + x\Delta x + 1}}{\Delta x} = \frac{\frac{\Delta x}{x^2 - 2x + x\Delta x - \Delta x + 1}}{\Delta x}$

$= \frac{-\Delta x}{(x^2 - 2x + x\Delta x - \Delta x + 1)\Delta x} = \frac{1}{x^2 - 2x + x\Delta x - \Delta x + 1} = \frac{1}{x^2 - 2x + \cancel{x(0)} - \cancel{(0)} + 1} = \frac{1}{x^2 - 2x + 1} = \boxed{\frac{1}{(x-1)^2}}$

33) $f(x) = x^2$ | $2x - y + 1 = 0$

$2x - y + 1 = 0$
$2x + 1 = y$
$m = 2$

$f'(x) = 2x$
$\frac{2x}{2} = \frac{2}{2} = x = 1$

$= (1)^2$
$y = 1$

$P(1,1)$
$y - 1 = 2(x-1)$

$y - 1 = 2x - 2$
$\boxed{y = 2x - 1}$

35) $f(x) = x^3$ | $3x - y + 1 = 0$

$y = 3x + 1$
$m = 3$

$f'(x) = 3x^2$
$3x^2 = 3$

$x^2 = 1$
$x = \pm 1$

$y = x^3$
$y = (\pm 1)^3$
$y = 1$

$P(\pm 1, 1)$
$(y-1) = 3(x \pm 1)$
$y - 1 = 3x \pm 3$

$y = 3x \pm 3 + 1$

$y = 3x + 3 + 1$
$\boxed{y = 3x + 4}$

$y = 3x - 3 + 1$
$\boxed{y = 3x - 2}$

37) $f(x) = \frac{1}{\sqrt{x}}$ | $x + 2y - 6 = 0$

$2y = -x + 6$
$y = \frac{-1}{2}x + 3$
$m = \frac{-1}{2}$

$f'(x) = -\frac{1}{2x\sqrt{x}}$

$\frac{-1}{2x\sqrt{x}} = \frac{-1}{2}$

$-1 = \frac{-\cancel{2}x\sqrt{x}}{\cancel{2}}$
$-1 = -x\sqrt{x}$
$(-1)^2 = (-x\sqrt{x})^2$

$1 = x^2$
$x = \pm 1$

$y = \frac{1}{\sqrt{x}}$
$= \frac{1}{\sqrt{1}}$
$= \frac{1}{1}$
$= 1$

$P(1,1)$
$(y-1) = \frac{-1}{2}(x-1)$
$y - 1 = \frac{-1}{2}x + \frac{1}{2}$
$\boxed{y = \frac{-1}{2}x + \frac{3}{2}}$

39) La función es constante. Por lo que la pendiente es 1.

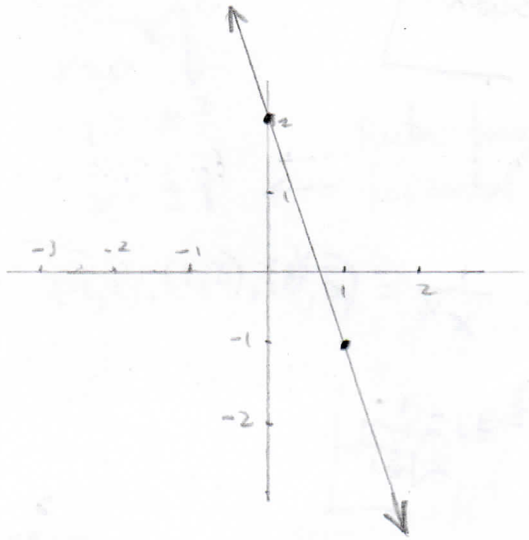
41) La pendiente da 0 cuando $x=4$. Positiva cuando $x>4$. Negativa cuando $x<4$.

53) $f(0)=2$; $f'(x)=-3$ para $-\infty < x < \infty$

$f(x) = -3x+2$
$f(0) = -3(0)+2$
$= 2$

$f'(x) = -3$

x	y
0	2
1	-1



75) $f(x) = \dfrac{2}{x-3}$

$x-3 = 0$
$x = 3$

Números derivables: $\mathbb{R} - \{3\}$

77) $f(x) = (x+4)^{\frac{2}{3}}$

Números en x derivables $= \mathbb{R}$

79) $f(x) = \sqrt{x-1}$

$x-1 \geq 0$ / Números de x derivables $= \boxed{x \geq 1}$
$x \geq 1$

2.2

7) $y = \dfrac{1}{x^5}$

$y' = \dfrac{\cancel{x^5 \cdot (1)'} - 1(x^5)'}{(x^5)^2}$

$y' = \dfrac{-5x^4}{x^{10}} = \boxed{\dfrac{-5}{x^6}}$

9) $f(x) = \sqrt[5]{x}$
$= x^{1/5}$
$= \frac{1}{5} x^{-4/5}$
$= \boxed{\dfrac{1}{5\sqrt[5]{x^4}}}$

17) $s(t) = t^3 + 5t^2 - 3t + 8$
$\boxed{s'(t) = 3t^2 + 10t - 3t}$

19) $y = \frac{\pi}{2} \operatorname{sen}\theta - \cos\theta$
$y' = \cancel{\frac{\pi}{2}' \operatorname{sen}\theta} + \operatorname{sen}\theta' \cdot \frac{\pi}{2} - \cos\theta'$
$\boxed{y' = \frac{\pi}{2} \cos\theta + \operatorname{sen}\theta}$

23) $y = \frac{1}{x} - 3\operatorname{sen} x$

$y = x^{-1} - 3\operatorname{sen} x$

$y' = -1x^{-2} - (3' \operatorname{sen} x + \operatorname{sen} x' \cdot 3)$

$y' = \frac{-1}{x^2} - (3\cos x)$

$y' = \frac{-1}{x^2} - 3\cos x$

25) $y = \frac{5}{2x^2}$

Reescriba $= \frac{5x^{-2}}{2}$

$f'(x) = \frac{5(-2x^{-3})}{2}$

$= \frac{-10x^{-3}}{2} \rightarrow \frac{-5}{x^3}$

27) $y = \frac{6}{(5x)^3}$

Reescriba $= \frac{6x^{-3}}{125}$

$f'(x) = \frac{6}{125} \cdot -3x^{-4}$

$= \frac{-18x^{-4}}{125}$

$= \frac{-18}{125x^4}$

29) $y = \frac{\sqrt{x}}{x}$

R. $= \frac{x^{1/2}}{x} = \frac{1}{x^{1/2}} = x^{-1/2}$

$f'(x) = -\frac{1}{2}x^{-\frac{3}{2}} = \frac{-1}{2\sqrt{x^3}}$

35) $y = (4x+1)^2 \quad | \ (0,1)$

31) $f(x) = \frac{8}{x^2} \quad | \ (2,2)$

$f(x) = 8x^{-2}$

$f'(x) = 8(-2x^{-3}) \rightarrow = -16x^{-3}$

$= \frac{-16}{x^3}$

43) $f(x) = \frac{4x^3+3x^2}{x}$

$f'(x) = \frac{(12x^2+6x)\cdot x - (4x^3+3x^2)}{x^2}$

$= \frac{12x^3+6x^2-4x^3-3x^2}{x^2}$

$= \frac{8x^3+3x^2}{x^2}$

$= \frac{\cancel{x^2}(8x+3)}{\cancel{x^2}}$

$= 8x+3$

47) $y = x(x^2+1)$

$y = x^3 + x$

$y' = 3x^2 + 1$

49) $f(x) = \sqrt{x} - 6\sqrt[3]{x}$

$f(x) = x^{\frac{1}{2}} - 6x^{1/3}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 6\left(\frac{1}{3}\right)x^{-2/3}$

$= \frac{1}{2\sqrt{x}} - \frac{2}{\sqrt[3]{x^2}}$

53) $y = x^4 - 3x^2 + 2 \quad | \ (1,0)$

$y' = 4x^3 - 6x$

$m = 4(1)^3 - 6(1)$

$m = 4 - 6$

$= -2$

$y - y_1 = m(x - x_1)$

$y - 0 = -2(x-1)$

$y = -2x + 2$

$2x + y + 2 = 0$

55) $f(x) = \frac{2}{4\sqrt{x^3}}$

$f(x) = \frac{2x^{-\frac{3}{2}}}{4}$

$f'(x) = \frac{2}{4}\left(\frac{-3}{2}\right)x^{-5/2}$

$= \frac{-3}{4}x^{-5/2}$

$m = \frac{-3}{4\sqrt{x^5}}$

$m = \frac{-3}{4\sqrt{1^5}} = \frac{-3}{4(1)} = \frac{-3}{4}$

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{-3}{4}(x-1)$

$y = -\frac{3}{4}x + \frac{3}{4} + 2$

$y = \frac{-3}{4}x + \frac{11}{4}$

$\frac{3}{4}x + y - \frac{11}{4}$

57) $y = x^4 - 2x^2 + 3$

$y' = 4x^3 - 4x = m$

$m = 0$

$0 = 4x^3 - 4x$

$= x(4x^2 - 4)$

$x = 0$

$1 = x^2$

$x = \pm 1$ ← Recta tangente horizontal en.

$(-1,2), (1,2), (0,3)$

$(6+k)(6+k)$

$36 + 12k - k^2$

59) $y = \frac{1}{x^2}$

$y = x^{-2}$

$y' = -2x^{-3}$

$m = \frac{-2}{x^3}$

$0 = \frac{-2}{x^3}$

no hay tangentes

61) $y = x + \operatorname{sen} x$

$y' = \cos x$

$m = \cos x$

$0 = \cos x$

$x = \pi$

(π, π)

63) $f(x) = k - x^2$ | $y = -6x + 1$

$m = -6$

$f'(x) = k - 2x$

$m = k - 2x$

$-6 = k - 2x$

$-2x = -6 - k$

$x = -\frac{+6-k}{2}$

$x = \frac{6+k}{2}$

$k - x^2 = -6x + 1$

$k - \left(\frac{6+k}{2}\right)^2 = -6\left(\frac{6+k}{2}\right) + 1$

$k - \frac{36 + 12k - k^2}{4} = -18 - 3k + 1$

$\frac{4k - 36 - 12k + k^2}{4} = -17 - 3k$

$-8k - 36 + k^2 = -68 - 12k$

$4k - 36 + k^2 = -68$

$4k + 32 + k^2 = 0$

65) $f(x) = \frac{k}{x}$ | $y = -\frac{3}{4}x + 3$

$f'(x) = \frac{k}{x^2}$

$m = \frac{-k}{x^2} : k = \frac{3x^2}{4}$

$f(x) = y$

$\frac{k}{x} = -\frac{3}{4}x + 3$

$\frac{3x^2}{4} = \frac{-3}{4}x + 3$

$x = 2$

$k = \frac{3(2)^2}{4}$

$k = \frac{3(\cancel{4})}{\cancel{4}}$

$k = 3$

67) $f(x) = kx^3$ | $y = x + 1$

$m = 1$

$f'(x) = k\,3x^2$

$m = 3x^2 k$

$1 = 3x^2 k$

$k = \frac{1}{3x^2}$

$k = \frac{1}{3\left(\frac{-3}{2}\right)^2}$

$= \frac{1}{3\left(\frac{9}{4}\right)}$

$k = \frac{1}{\frac{27}{4}}$

$\boxed{k = \frac{4}{27}}$

$f(x) = y$

$kx^3 = x + 1$

$\frac{1}{3x^2}x^3 = x + 1$

$\frac{x}{3} = x + 1$

$x = 3x + 3$

$2x = -3$

$\boxed{x = \frac{-3}{2}}$