Práctica de Integrales exponenciales y logarítmicas

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Cálculo I
11L702

1) $\int \frac{dx}{x+3} = \int \frac{dv}{v} = \ln|v| + c = \ln|x+3| + c$

$v = x+3$

$dv = dx$

2) $\int \frac{x\,dx}{3x^2-4} = \int \frac{6}{6}\frac{x\,dx}{3x^2-4} = \frac{1}{6}\int \frac{6x\,dx}{3x^2-4} = \frac{1}{6}\int \frac{dv}{v} = \frac{1}{6}\ln|v| + c = \frac{1}{6}\ln|3x^2-4| + c$

$v = 3x^2-4$

$dv = 6x\,dx$

3) $\int \frac{4x\,dx}{2x^2-6} = \int \frac{4x\,dx}{2(x^2-3)} = \int \frac{2x\,dx}{x^2-3} = \int \frac{dv}{v} = \ln|v| + c = \ln|x^2-3| + c$

$v = x^2-3$

$dv = 2x\,dx$

4) $\int \frac{\cos ax}{1-\operatorname{sen} ax}\,dx = \int \frac{\cos ax}{v}\,\frac{dv}{-a\cos ax} = \frac{1}{-a}\int \frac{dv}{v} = \frac{1}{a}\ln|v| + c = -\frac{1}{a}\ln|1-\operatorname{sen} ax| + c$

$v = 1 - \operatorname{sen} ax$

$dv = -a\cos ax\,dx$

$\frac{dv}{a\cos ax} = dx$

5) $\int \frac{4x+2}{x+2}\,dx = \int \frac{4x+2}{v}\,dv = \int \frac{4(v-2)+2}{v}\,dv = \int \frac{4v-8+2}{v}\,dv = \int \frac{4v-6}{v}\,dv = \int 4\,dv - \int \frac{6}{v}\,dv$

$v = x+2 \Rightarrow x = v-2$

$dv = dx$

$= 4x - 6\ln|v| + c = 4x - \ln|x+2|^6 + c$

6) $\int \frac{dy}{y\ln^2 y} = \int \frac{du}{u^2} = \int u^{-2}\,du = \frac{u^{-1}}{-1} = -u^{-1} = -\frac{1}{\ln y} + c$

$u = \ln y$

$du = \frac{1}{y}\,dy$

7) $\int \frac{dx}{2x\ln 3x} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln u + c = \frac{1}{2}\ln|\ln 3x| + C$

$u = \ln 3x$

$du = \frac{3}{3x} dx = \frac{dx}{x}$

8) $\int \frac{\ln x}{x} dx = \int \frac{\ln x}{x} x\,dv = \int v\,dv = \frac{v^2}{2} = \frac{\ln|x|^2}{2} + c$

$v = \ln x$

$dv = \frac{1}{x} dx$

$x\,dv = dx$

9) $\int \frac{1}{3-2x} dx = \int \frac{1}{v} \frac{dv}{-2} = -\frac{1}{2}\int \frac{dv}{v} = -\frac{1}{2}\ln|v| + c = -\frac{1}{2}\ln|3-2x| + c$

$v = 3-2x$

$dv = -2\,dx$

$\frac{dv}{-2} = dx$

10) $\int \frac{3x^2}{5x^3-1} dx = \frac{1}{5}\int \frac{15x^2}{v} dx = \frac{1}{5}\int \frac{dv}{v} = \frac{1}{5}\ln|v| + c = \frac{1}{5}\ln|5x^3-1| + C$

$v = 5x^3-1$

$dv = 15x^2 dv$

11) $\int e^{ax+b} dx = \int e^v \frac{dv}{a} = \frac{1}{a}\int e^v dv = \frac{1}{a}e^v + c = \frac{1}{a}e^{ax+b} + C$

$v = ax+b$

$dv = a\,dx$

$\frac{dv}{a} = dx$

12) $\int 3^{2x} dx = \int 3^v \frac{dv}{2} = \frac{1}{2}\int 3^v dv = \frac{1}{2}\frac{3^v}{\ln 3} + c = \frac{3^{2x}}{2\ln 3} + c$

$v = 2x$

$dv = 2\,dx$

$\frac{dv}{2} = dx$

13) $\int \sqrt[3]{e^x} dx = \int e^{x/3} dx = \int e^v 3\,dv = 3\int e^v dv = 3e^v + c = 3e^{x/3} + C$

$v = \frac{x}{3}$

$dv = \frac{3}{9} dx$

$dv = \frac{1}{3} dx \rightarrow 3\,dv = dx$

14) $\int \frac{dx}{5^{4x}} = \int \frac{dx}{625^x} = \int \frac{1}{625^x} dx = \int \left(\frac{1}{625}\right)^x dx = \frac{\left(\frac{1}{625}\right)^x}{\ln\left(\frac{1}{625}\right)} = -\frac{1}{4\ln(5)\,625^x} + C$

$v = 5^{4x}$

15) $\int x^2(3 - e^{x^3})\,dx = \int (3x^2 - x^2 e^{x^3})\,dx = \int 3x^2\,dx - \int x^2 e^{x^3}\,dx = 3\int x^2\,dx - \int x^2 e^{x^3}\,dx$

$= \frac{3x^3}{3} - \int x^2 e^v \frac{dv}{3x^2} = x^3 - \frac{1}{3}\int e^v\,dv = x^3 - \frac{1}{3}e^v + C = x^3 - \frac{e^{x^3}}{3} + C$

$v = x^3$

$dv = 3x^2\,dx$

$\frac{dv}{3x^2} = dx$

16) $\int \frac{dt}{\sqrt[5]{e^{2t}}} = \int \frac{dt}{e^{2t/5}} = \int \frac{5\,dv}{2e^{2t/5}} = \int \frac{5\,dv}{2v} = \frac{5}{2}\int \frac{dv}{v} = \frac{5}{2}\int v^{-2}\,dv = \frac{5}{2}\frac{v^{-1}}{-1} = -\frac{5}{2e^{2t/5}} + C$

$v = e^{2t/5}$

$dv = \frac{2}{5}e^{2t/5}\,dt$

$\frac{5\,dv}{2e^{2t/5}} = dt$

17) $\int e^{1/\sec 2x} \operatorname{sen} 2x\,dx = \int e^{\cos 2x} \operatorname{sen} 2x\,dx = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{\cos 2x} + C$

$= -\frac{1}{2}e^{1/\sec 2x} + C$

$u = \cos 2x$

$du = -\operatorname{sen} 2x\,(2)\,dx$

$du = -2\operatorname{sen} 2x\,dx$

$-\frac{1}{2}du = \operatorname{sen} 2x\,dx$

18) $\int \frac{t^3}{e^{2t^4}}\,dt = \int \frac{t^3}{e^v}\frac{dv}{8t^3} = \frac{1}{8}\int \frac{dv}{e^v} = \frac{1}{8}\int e^{-v}\,dv = -\frac{1}{8}e^v + C = -\frac{1}{8}e^{2t^4} + C$

$u = 2t^4$

$du = 8t^3\,dx$

$\frac{dv}{8t^3} = dx$

19) $\int x\,5^{x^2}\,dx = \int x\,5^{x^2}\frac{dv}{2x} = \frac{1}{2}\int 5^{x^2}\,dv = \frac{1}{2}\int 5^v\,dv = \frac{1}{2}\frac{5^v}{\ln|5|} + C = \frac{5^{x^2}}{2\ln|5|} + C$

$v = x^2$

$dv = 2x\,dx$

$\frac{dv}{2x} = dx$

20) $\int (e^{2x} - e^{-2x})^2 dx$

$\int (e^{4x} - 2 + e^{-4x}) dx = \int e^{4x} - \int 2 + \int e^{-4x}$

$(e^{2x} - e^{-2x})(e^{2x} - e^{-2x})$

$= \frac{e^{4x}}{4} - 2x - \frac{1}{4e^{4x}} + C$