Práctica 2.5

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Cálculo
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Derivación implícita

3) $x^{1/2} + y^{1/2} = 16$

$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}y' = 0$

$\frac{1}{2\sqrt{y}}y' = -\frac{1}{2\sqrt{x}}$

$y' = -\frac{\cancel{2}\sqrt{y}}{\cancel{2}\sqrt{x}}$

$\boxed{y' = -\sqrt{\frac{y}{x}}}$

7) $x^3y^3 - y = x$

$(x^3)'y^3 + (y^3)'x^3 - y' = x'$

$3x^2y^3 + 3y^2y'x^3 - y' = 1$

$3y^2y'x^3 - y' = 1 - 3x^2y^3$

$y'(3y^2x^3 - 1) = 1 - 3x^2y^3$

$\boxed{y' = \frac{1 - 3x^2y^3}{3y^2x^3 - 1}}$

9) $x^3 - 3x^2y + 2xy^2 = 12$

$3x^2 - 3(x^{2'}y + y'x^2) + 2(x'y^2 + y^{2'}x) = 0$

$3x^2 - 3(2xy + x^2y') + 2(y^2 + 2yy'x) = 0$

$3x^2 - 6xy - 3x^2y' + 2y^2 + 4yy'x = 0$

$3x^2y' + 4yy'x = -3x^2 + 6xy - 2y^2$

$y'(3x^2 + 4yx) = -3x^2 + 6xy - 2y^2$

$\boxed{y' = \frac{-3x^2 + 6xy - 2y^2}{-3x^2 + 4yx}}$

13) $\operatorname{sen} x = x(1 + \tan y)$

$(\operatorname{sen} x)' = x'(1 + \tan y) + (1 + \tan y)'x$

$\cos x = 1 + \tan y + (\sec^2 y\, y')x$

$\cos x = 1 + \tan y + x\sec^2 y\, y'$

$\cos x - 1 - \tan y = x\sec^2 y\, y'$

$\boxed{y' = \frac{\cos x - 1 - \tan y}{x\sec^2 y}}$

25) $(x+y)^3 = x^3 + y^3$ $\quad (-1, 1)$

$(x+y)^{3'} = x^{3'} + y^{3'}$

$3(x+y)^2(y') = 3x^2 + 3y^2y'$

$3y'(x+y)^2 - 3y^2y' = 3x^2$

$y'(3(x+y)^2 - 3y^2) = 3x^2$

$\boxed{y' = \frac{3x^2}{3(x+y)^2 - 3y^2}}$

$y' = \frac{3(-1)^2}{3(-1+1)^2 - 3(1)^2}$

$= \frac{3}{-3}$

$\boxed{= -1}$

29) $\tan(x+y) = x$ $\quad (0,0)$

$\tan(x+y)' = x'$

$\sec^2(x+y)(1+y') = 1$

$1 + y' = \frac{1}{\sec^2(x+y)}$

$\boxed{y' = \frac{1}{\sec^2(x+y)} - 1}$

$y' = \frac{1}{\sec^2(0)} \cancel{\;}^{1} - 1$

$\boxed{y' = 0}$

31) Pendiente de la Recta tangente

$$(x^2+y^2)^2 = 4x^2y \qquad P(1,1)$$

$$2(x^2+y^2)(2x+2yy') = 4(2xy + y'x^2)$$

$$(2x^2+2y^2)(2x+2yy') = 8xy + 4y'x^2$$

$$4x^3 + 4x^2yy' + 4xy^2 + 4y^3y' = 8xy + 4y'x^2$$

$$4x^2yy' + 4y^3y' - 4y'x^2 = 8xy - 4x^3 - 4xy^2$$

$$y'(4x^2y + 4y^3 - 4x^2) = 8xy - 4x^3 - 4xy^2$$

$$y' = \frac{8xy - 4x^3 - 4xy^2}{4x^2y + 4y^3 - 4x^2}$$

$$y' = \frac{\cancel{4}(2xy - x^3 - xy^2)}{\cancel{4}(x^2y + y^3 - x^2)} = \boxed{\frac{2xy - x^3 - xy^2}{x^2y + y^3 - x^2}}$$

$$= \frac{2(1)(1) - (1)^3 - (1)(1)^2}{(1)^2(1) + (1)^3 - (1)^2} = \frac{2-1-1}{1+1-1}$$

$$= \frac{0}{2} = 0$$

37) E.C Recta tangente

$$x^2y^2 - 9x^2 - 4y^2 = 0 \qquad P(-4, 2\sqrt{3})$$

$$x^{2'}y^2 + y^{2'}x^2 - 9x^{2'} - 4y^{2'} = 0$$

$$2xy^2 + 2yy'x^2 - 18x - 8yy' = 0$$

$$2yy'x^2 - 8yy' = 18x - 2xy^2$$

$$y'(2yx^2 - 8y) = 18x - 2xy^2$$

$$y' = \frac{18x - 2xy^2}{2yx^2 - 8y}$$

$$= \frac{\cancel{2}(9x - xy^2)}{\cancel{2}(yx^2 - 4y)}$$

$$y' = \frac{9x - xy^2}{yx^2 - 4y} \rightarrow m = \frac{\sqrt{3}}{6}$$

$$y - y_1 = \frac{\sqrt{3}}{6}(x - x_1)$$

$$y - 2\sqrt{3} = \frac{\sqrt{3}}{6}(x+4)$$

$$6y - 12\sqrt{3} = x\sqrt{3} + 4\sqrt{3}$$

$$\boxed{6y - x\sqrt{3} - 16\sqrt{3}}$$

39)

$$3(x^2+y^2)^2 = 100(x^2-y^2) \qquad P(4,2)$$

$$3(x^2+y^2)^{2'} = 100(x^2-y^2)'$$

$$3(2)(x^2+y^2)(2x+2yy') = 100(2x - 2yy')$$

$$(6x^2+6y^2)(2x+2yy') = 200x - 200yy'$$

$$12x^3 + 12x^2yy' + 12y^2x + 12x^3y' = 200x - 200yy'$$

$$12x^2yy' + 12x^3y' + 200yy' = 200x - 12x^3 - 12y^2x$$

$$y'(12x^2y + 12x^3 + 200y) = 200x - 12x^3 - 12y^2x$$

$$y' = \frac{200x - 12x^3 - 12y^2x}{12x^2y + 12x^3 + 200y}$$

$$= \frac{\cancel{4}(50x - 3x^3 - 3y^2x)}{\cancel{4}(3x^2y + 3x^3 + 50y)}$$

$$= \frac{50x - 3x^3 - 3y^2x}{3x^2y + 3x^3 + 50y}$$

$$m = \frac{-2}{11}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{2}{11}(x - 4)$$

$$11y - 22 = -2x + 8$$

$$\boxed{11y + 2x - 30 = 0}$$

) Elipse Recta tangente

a) $P(1,2)$

$\frac{x^2}{2}+\frac{y^2}{8}=1$

$\frac{4x}{4}+\frac{16yy'}{64}=0$

$x+\frac{2yy'}{8}=0$

$x+\frac{yy'}{4}=0$

$\frac{yy'}{4}=-x$

$yy'=-4x$

$y'=\frac{-4x}{y}$

$m=\frac{-4(1)}{2}$

$=\frac{-4}{2}$

$m=-2$

$y-y_1=m(x-x_1)$

$y-2=-2(x-1)$

$y-2=-2x+2$

$\boxed{y=-2x+4}$

b) E.C Recta tangente a la elipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ en (X_0,Y_0) es $\frac{X_0X}{a^2}+\frac{Y_0Y}{b^2}=1$

Puntos donde $m=0$ ó $m=\nexists$

5) $25x^2+16y^2+200x-160y+400=0$

$25x^{2'}+16y^{2'}+200x'-160y'+400'=0$

$50x+32yy'+200-160y'=0$

$32yy'-160y'=-200-50x$

$y'(32y-160)=-200-50x$

$y'=\frac{-200-50x}{32y-160}=m$

$0=\frac{-200-50x}{32y-160}=-200-50x=0$

$-50x=200$

$x=-4$

$25(-4)^2+200(4)+400=160y-16y^2$

$-400+800+\cancel{400}=16(10y-y^2)$

$\frac{800}{16}=10y-y^2$

$-y^2+10y-50=0$

$-(y^2-10y+50)=0$