Práctica Razón de cambio

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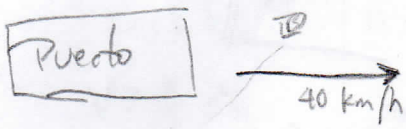
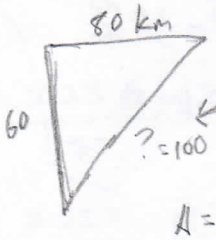
Cálculo I
1IL702
3-750-1980

1)





← después de dos horas

A | 30 km/h ↓

A = 30 km/h * 2h = 60 km

B = 40 km/h * 2h = 80 km

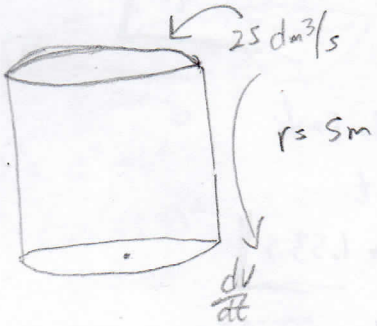
$c^2 = a^2 + b^2$

$c^2 = 60^2 + 80^2$

$c^2 = 3600 + 6400$

$c^2 = 10000$ → $c = 100$ → $\frac{100\,km}{2h} = V = 50\,km/h$

2)



Razón de cambio de h

$5m\left(\frac{10\,km}{1\,m}\right) = 50\,dm$

$V = \pi r^2 h$

$V = \pi (50\,dm)^2 h$

$V = \pi\, 2500\,dm^2 h$

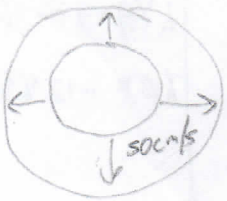
$\frac{dV}{dt} = 2500\,dm^2\,\pi\,\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{2500\,dm^2}\frac{\pi}{}\frac{dV}{dt}$

$\frac{dh}{dt} = \frac{1}{2500\,dm^2}\pi\,(25\,dm^3/s)$

$\frac{dh}{dt} = \frac{1}{100\pi}\,dm/s$

3)



$A = \pi r^2$

$V = 50\,cm/s \rightarrow \left(\frac{1\,m}{100\,cm}\right) = 0{,}5\,m/s$ $\quad r = 3m$

$\frac{dr}{dt} = 0{,}5\,m/s$

$\frac{dA}{dt} = ?$

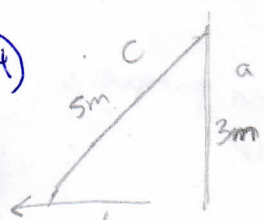
$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(3m)(0{,}5m/s)$

$\frac{dA}{dt} = 3\pi\,m/s$

4)



$\frac{db}{dt} = 1.2m/s$ / $\frac{dc}{dt} = 0$ ← Es la hipotenusa, por lo tanto es constante

× Encontramos b

$c^2 = a^2 + b^2$

$c^2 - a^2 = b^2$

$25 - 9 = b^2$

$16 = b^2$

$b = 4$

$c^2 = a^2 + b^2$

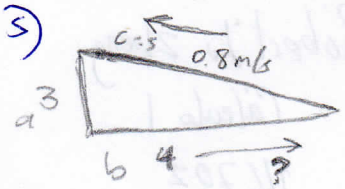
$2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt}$

$2(5)(0) = 2(3)\frac{da}{dt} + 2(4)(1.2m/s)$

$-9.6m/s = 6\frac{da}{dt}$

$\frac{da}{dt} = \frac{-9.6m/s}{6}$ → $-1.6m/s$

5)



$\frac{dc}{dt} = 0.8\,m/s \qquad \frac{da}{dt} = 0 \qquad \frac{db}{dt} = ?$

$c^2 = a^2 + b^2$

$c^2 = a + b$

$c^2 = 25$

$c = 5$

$b^2 = c^2 - a^2$

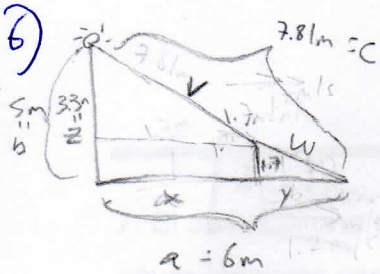
$2b\frac{db}{dt} = 2c\frac{dc}{dt} - 2a\frac{da}{dt}$ ($\frac{da}{dt} \to 0$)

$2(4)\frac{db}{dt} = 2(5)(0.8)$

$8\frac{db}{dt} = 8$

$\frac{db}{dt} = -1\,m/s$

6)



$\frac{dx}{dt} = 1.2\,m/s \qquad \frac{dy}{dt} = ?$

semejanza de triángulos

$\frac{y}{1.7} = \frac{x+a}{5} \to 5y = 1.7a \to 5y = 1.7(x+y)$

$5y = 1.7x + 1.7y$

$3.3y = 1.7x$

$y = \frac{1.7}{3.3}x$

$\frac{dy}{dt} = \frac{1.7}{3.3}\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{1.7}{3.3}(1.2\,m/s)$

$\frac{dy}{dt} = 0.618\,m/s$

7)

$h(t) = 20t - 4.9t^2$

$h'(t) = 20 - 2(4.9)t$

a) Después de 1s

$h'(t) = 20 - 2(4.9)(1)$

$h'(t) = 10.2\,m/s$

b) t cuando 5m/s

$5 = 20 - 9.8t$

$\frac{-15}{-9.8} = t \to 1.53\,s$

c) Máximo → cuando la recta tangente es 0.

$h'(t) = 20 - 9.8t$

$0 = 20 - 9.8t$

$\frac{-20}{-9.8} = t \to t = 2.04$

$h(t) = 20(2.04) - 4.9(2.04)^2$

$h(t) = 20.41\,m$

d) Velocidad cuando 12m

$12 = 20t - 4.9t^2$

$4.9t^2 - 20t + 12 = 0$

$t^2 - 4.08t + 2.5 = 0$

$t_1 = 0.73 \qquad t_2 = 3.35$

$t_1 = 0.73$

$h'(t) = 20 - 2(4.9)(0.73)$

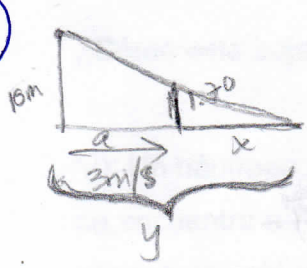
$h'(t) = 12.85\,m/s$

$t_2 = 3.35$

$h'(t) = 20 - 2(4.9)(3.35)$

$h'(t) = -12.83\,m/s$

8)



$\frac{da}{dt} = 3\,m/s \qquad \frac{dx}{dt} = ?$

$\frac{x}{1.7} = \frac{a+x}{10} \to x = \frac{1.7a + 1.7x}{10} \to 10x = 1.7a + 1.7x = 8.3x = 1.7a$

$x = \frac{1.7}{8.3}a \to \frac{dx}{dt} = \frac{1.7}{8.3}\frac{dA}{dt} \to \frac{dx}{dt} = \frac{1.7}{8.3}(3\,m/s) = 0.61\,m/s$

$a + x = (3 + 0.61)\,m/s = 3.61\,m/s$ ← se le suma porque es el extremo

9. a)

semejanza de triángulos $\frac{dV_i}{dt} = 0.16 m^3/min$

$\frac{H}{R} = \frac{h}{r} \rightarrow \frac{5}{1.5} = \frac{h}{r} \rightarrow \frac{1.5}{5} = \frac{r}{h} \rightarrow r = \frac{1.5h}{5}$

$V = \frac{\pi}{3} r^2 h$

$= \frac{\pi}{3}\left(\frac{1.5h}{5}\right)^2 h$

$V = \frac{\pi}{3} \frac{2.25h^3}{25}$

$V = \frac{2.25\pi}{75} h^3$

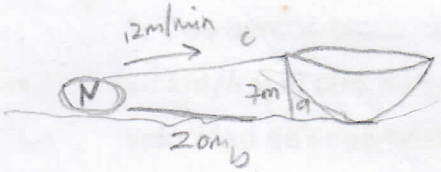
$\frac{dV}{dt} = \frac{2.25\pi}{75} 3h^2 \frac{dh}{dt}$

$0.16 = \frac{2.25\pi}{75} 3(4) \frac{dh}{dt}$

$\frac{0.16 \times 75}{27\pi} = \frac{dh}{dt} \rightarrow \frac{12}{27\pi} = \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{4}{9\pi} \rightarrow$ disminuye $= \frac{dh}{dt} = -\frac{4 m/min}{9\pi}$

10)



$\frac{dc}{dt} = 12 m/min$

$\frac{da}{dt} = 0$

$c^2 = a^2 + b^2$

$2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt}$

$2(\sqrt{449}m)(12 m/min) = 2(7m)(0) + 2(20m)\frac{db}{dt}$

$24\sqrt{449}\ m/min = 40m \frac{db}{dt}$

$\frac{db}{dt} = \frac{24\sqrt{449}\ m^2/min}{40m} \rightarrow \frac{3\sqrt{449}}{5} m/min$