Práctica 2.6

Robert Lu Zheng
3-750-1980
Cálculo 1
1IL702

5)

$y = 2x^2 + 1$; $\frac{dx}{dt} = 2cm/s$

$y' = 2(2)xx'$
$= 4(2)x = 8x$

a) $x = -1$: $y' = 8x = 8(-1) = -8$

b) $x = 0$ $y' = 8(0) = 0$

c) $x = 1$ $y' = 8x = 8(1) = 8$

7) $y = \tan x$; $\frac{dx}{dt} = 3$ pies/s

$y' = \tan x$
$y' = \sec^2 x\, x'$
$y' = \sec^2 x\,(3)$
$y' = 3\sec^2 x$

a) $x = -\frac{\pi}{3}$ $y' = 3\sec^2\left(-\frac{\pi}{3}\right) = 12$ pies/s

b) $x = -\frac{\pi}{4}$ $y' = 3\sec^2\left(-\frac{\pi}{4}\right) = 6$ pies/s

c) $x = 0$ $y' = 3\sec^2(0) = 3$ pies/s

9) $y = ax + b$

Si x es constante, ¿y tambien lo hace a razón constante? ¿Lo hace a misma razón que x?.

Sí, cuando x es constante, y también es constante. Y cambia diferente a x excepto cuando $a = 1$.

11) $\frac{dA}{dt} = ?$

$r' = 4 cm/min$

$A = \pi r^2$
$A' = 2\pi r r'$
$A' = 2\pi r(4)$
$A' = 8\pi r$

a) $r = 8$ $A' = 8\pi(8) = 64\pi\ cm^2/min$

b) $r = 32$ $A' = 8\pi(32) = 256\pi\ cm^2/min$

13) Esfera

$r' = 3$ pulg / min

$V = \frac{4}{3}\pi r^3$
$V' = \frac{4}{3}\pi 3r^2 r'$
$V' = 4\pi r^2 r'$ → $V' = 4\pi r^2(3)$
$V' = 12\pi r^2$

a) $r = 9$ y $r = 36$

$V = 12\pi(9)^2$
$V = 972\ pulg^3/min$

$V = 12\pi(36)^2$
$V = 15552\ pulg^2/min$

b) Porque la variable independiente r es exponencial a la dos.

15) $l' = 6cm/s$ Cubo

$V = l^3$
$V' = 3l^2 l'$ → $V' = 18l^2$

a) $l = 2$
$V' = 18(4)$
$= 72\ cm^2/s$

b) 10cm
$V' = 18(100)$
$V' = 1800\ cm^2/s$

17) $V' = 10 \text{ pies}^3/\text{min}$ cono

$d = 3h$

$h = 15 \quad h' = ?$

$d = 2r$

$3h = 2r$

$r = \frac{3h}{2} = \left(\frac{3(15)}{2}\right) = 22.5 \simeq \frac{45}{2}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{45}{2}\right)^2 h$

$V = \frac{1}{3}\pi \frac{2025}{4} h$

$V = \frac{675\pi}{4} h$

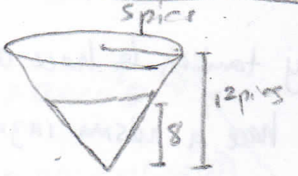
$V' = \frac{675\pi}{4} h'$

$10 = \frac{675\pi}{4} h'$

$\frac{40}{675\pi} = h'$

$\boxed{h' = \frac{40}{675\pi} \text{ m/min}}$

18)



$V'_{conico} = 10 \text{ pies}^3/\text{min}$

$h = 8 \text{ pies} \quad h' = ?$

$\frac{5}{12} = \frac{x}{8} \qquad x = \frac{10}{3}$

$x = \frac{40}{12}$

$V = \frac{\pi r^2 h}{3}$

$V = \frac{\pi\left(\frac{10}{3}\right)^2 h}{3}$

$V = \frac{\pi\left(\frac{100}{9}\right)h}{3}$

$V = \pi\left(\frac{100}{27}\right)h$

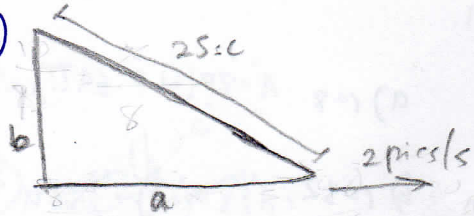
$V' = \pi\left(\frac{100}{27}\right)h'$

$\frac{27\pi V'}{100} = h'$

$\frac{27\pi(10)}{100} = h'$

$\boxed{h' = \frac{27\pi}{10} \text{ pies/min}}$

21)



$a' = 2 \text{ pies/s}$

$c' = 0$

a) Cambio en la parte superior cuando la base está a 7, 15, 24 pies

$a = 7$

$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$

$b^2 = (25)^2 - (7)^2$

$b^2 = 576$

$b = 24$

$2cc' = 2aa' + 2bb'$

$2(25)(0) = 2(7)(2) + 2(24)b'$

$0 = 28 + 48b'$

$-\frac{28}{48} = b'$

$\boxed{b' = \frac{7}{12} \text{ pies/s}}$

$a = 15$

$b^2 = c^2 - a^2$

$b^2 = (25)^2 - (15)^2$

$b^2 = 400$

$b = 20$

$2cc' = 2aa' + 2bb'$

$0 = 2(15)(2) + 2(20)b'$

$\frac{-60}{40} = b'$

$\boxed{b' = \frac{3}{2} \text{ pies/s}}$

$a = 24$

$b^2 = c^2 - a^2$

$b^2 = (25)^2 - (24)^2$

$b^2 = 49$

$b = 7$

$0 = 2aa' + 2bb'$

$= 2(24)(2) + 2(7)b'$

$= 96 + 14b'$

$\frac{-96}{14} = b'$

$\boxed{-\frac{48}{7} \text{ pies/s} = b'}$

b) $\frac{dA}{dt}$ cuando $a = 7$

$A = \frac{b \cdot h}{2}$

$A = \frac{a \cdot b}{2}$

$A' = \frac{a' \cdot b + b' a}{2}$

$A' = \frac{(2)(24) + \frac{7}{12}(7)}{2}$

$A' = \frac{48 + \frac{49}{12}}{2}$

$\frac{\frac{576 + 49}{12}}{2} = \boxed{\frac{625}{24} \text{ pies}^2/\text{s}}$

c) $\frac{d\theta}{dt}$ cuando $a = 7$

$\tan\theta = \frac{a}{b}$

$\sec^2\theta = \frac{a'b - b'a}{b^2}$

$\sec^2\theta = \frac{(2)(24) - \left(\frac{7}{12}\right)(7)}{(24)^2}$

$\sec^2\theta = \frac{48 - \frac{49}{12}}{576}$

$\sec^2 = \frac{\frac{576 - 49}{12}}{576}$

$\sec^2\theta = \frac{527}{6912}$

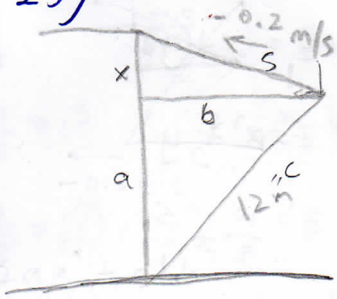
$\sec\theta = \sqrt{\frac{527}{6912}}$

$\theta' = \sec^{-1}\left(\sqrt{\frac{527}{6912}}\right)$

$\theta' = \frac{1}{12} \text{ rad/s}$

23)*

$s' = -0.2\ m/s$



$x = 12 - a$

$x = 12 - 6$

$x = 6$

$a = 6$

$b^2 = c^2 - a^2$

$b^2 = 144 - 36$

$b^2 = 108$

$b = \sqrt{108}$

$s^2 = x^2 + b^2$

$s^2 = 36 + 108$

$s^2 = 144$

$s = 12$

$x' = ?$

$b' = ?$

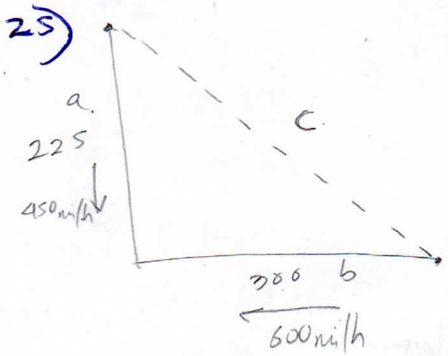
$s^2 = x^2 + b^2$

$2ss' = 2xx' + 2bb'$

$2(12)(-0.2) = 2(12)x' + 2\sqrt{108}\,b'$

$-4.8 = 12x' + 2\sqrt{108}\,b'$

25)



$c' = ?$

$a = 225\ mi$

$a' = 450\ mi/h$

$b = 300\ mi$

$b' = 600\ mi/h$

$c^2 = a^2 + b^2$

$c^2 = (225)^2 + (300)^2$

$c^2 = 50625 + 90000$

$c^2 = 140625$

$c = 375$

$2cc' = 2aa' + 2bb'$

$2(375)c' = 2(225)(450) + 2(300)(600)$

$750c' = 202500 + 360000$

$c' = \frac{562500}{750}$

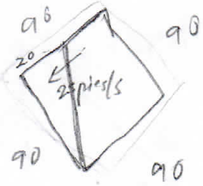
$c' = 750\ mi/h$

$375\,mi\left(\frac{1h}{750\,mi}\right) = 0.5h = 30\,min$

a) La distancia entre ellos se reduce a 750 mi/h.

b) El operador dispone de 30 minutos antes que colisionen.

27)



$c^2 = a^2 + b^2$

$c^2 = (20)^2 + (90)^2$

$c^2 = 400 + 8100$

$c^2 = 8500 \Rightarrow c = 92.2$

$2cc' = 2aa' + 2bb'$

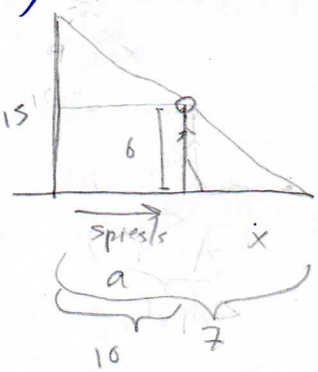
$2(92.2)c' = 2(20)(25) + \cancel{2(90)(0)}^{0}$

$184.4c' = 1000$

$c' = 5.42\ pies/s$

Su distancia cambia a 5.42 pies/s respecto a home.

29)



$a' = 5$ pies/s

$$\frac{x}{a+x} = \frac{6}{15}$$

$$x = \frac{6(a+x)}{15}$$

$$x = \frac{6a}{15} + \frac{6x}{15}$$

$$x - \frac{6x}{15} = \frac{6a}{15}$$

$$\frac{9x}{\cancel{15}} = \frac{6a'}{\cancel{15}}$$

$$9x = 6a$$

$$x = \frac{6}{9}a$$

$$x' = \frac{6}{9}a'$$

$$x' = \frac{6}{9}(5) = \boxed{\frac{10}{3}\text{ pies/s}}$$

rapidez que está cambiando la longitud de su sombra

$$z = x + y$$

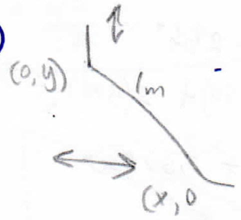
$$z' = a' + y'$$

$$z' = \frac{10}{3} + \frac{5}{1}$$

$$z' = \boxed{\frac{25}{3}\text{ pies/s}}$$

la punta de la sombra se aleja a este cambio.

31)



$$x(t) = \frac{1}{2}\operatorname{sen}\frac{\pi t}{6}$$

A) Duración de un ciclo completo de la varilla

$$0 = \frac{1}{2}\operatorname{sen}\frac{\pi t}{6}$$

$$2 \cdot 0 = \operatorname{sen}\frac{\pi t}{6}$$

$$\frac{\pi t}{6} = \operatorname{sen}^{-1}(0)$$

$$\frac{\pi t}{6} = 0$$ ← cada vez que $t = 6n$ ← cualquier entero.

Un ciclo es ida y vuelta, por lo tanto, es la segunda llegada a 0.

$$\boxed{t = 6(2) = 12\text{ s}}$$

b) Punto más bajo de la varilla superior.

Como los ciclos son iguales, es cuando alcanza la mitad de distancia la varilla inferior

$$x(t) = \frac{1}{2}\operatorname{sen}\frac{\pi(3)}{6}$$

$$x = 0.5$$

$$y^2 = 1 - 0.5$$

$$y^2 = 0.5$$

$$y = \sqrt{0.5} \approx 0.71\text{ m}$$

33) $S = 4\pi r^2$ Demostrar que el radio decrece constante

$V = \frac{4}{3}\pi r^3$

$S = V' = y(4\pi r^2)$

$V' = \frac{4}{\cancel{3}}\pi(\cancel{3}) r^2 r' \longrightarrow$

$V' = (4\pi r^2) r' \longrightarrow \boxed{r' = y}$

35) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ $R' = ?$

$R_1 = 50$ ohm

$R_2 = 75$ ohm

$R_1' = 1$ ohm/s

$R_2' = 0.5$ ohm/s

$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2}$

$R = \frac{R_1 R_2}{R_2 + R_1}$

$R = \frac{(50)(75)}{50+75}$

$R = \frac{3750}{125} = 30$ ohm

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$\frac{-R'}{R^2} = \frac{-R_1'}{R_1^2} - \frac{R_2'}{R_2^2}$

$-R' = \left(\frac{R_1'}{R_1^2} + \frac{R_2'}{R_2^2}\right)900$

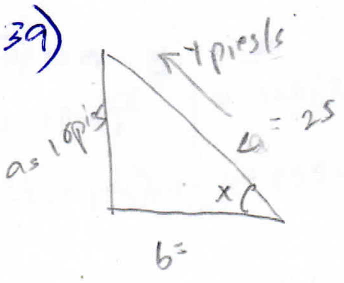
$R' = \frac{900R_1'}{R_1^2} + \frac{900R_2'}{R_2^2}$

$R' = \frac{900(1)}{2500} + \frac{900(0.5)}{5625}$

$R' = 0.36 + 0.08$

$R' = 0.44\ \Omega/s$

39)



$\operatorname{sen} x = \frac{10}{c}$

$x' \cos x = \frac{-10c'}{c^2}$

$x' = \frac{-10c'}{c^2} \sec x$

$\frac{dc}{dt} = -1$

$b^2 = c^2 - a^2$

$b^2 = 25^2 - 10^2$

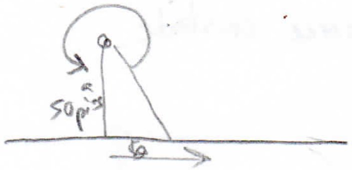
$b^2 = 525$

$b = \sqrt{525}$

$x' = \frac{-10(-1)}{(25)^2}\left(\frac{25}{\sqrt{525}}\right)$

$x' = \frac{10}{25\sqrt{525}} \longrightarrow 0.017$ rad/s

41)



gira a 30 rev/min

$30 \text{ rev/min} \left(\frac{360°}{1 \text{ rev}}\right) = \frac{d\theta}{dt}$ 10800 grados/min

$\tan\theta = \frac{b}{a}$

$b = a\tan\theta$

$b'(\theta) = a\sec^2\theta \cdot \theta'$

$b'(\theta) = \frac{50}{\cos^2\theta} 2\pi(10800)$

$\omega = \frac{\theta}{t}$ $\omega = 2\pi f$

$\theta = \omega t$

$\theta = 2\pi f t$

$\theta'(t) = 2\pi f t$

$\theta'(t) = 2\pi f$

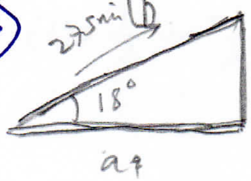
formula

$b'(30°) = \frac{200\pi}{3}$ pies/s

$b'(60°) = 200\pi$ pies/s

$b(70°) = 427.44\pi$ pies/s

43)



$\text{sen}(18) = \frac{b'}{275}$

$b' = 275\,\text{sen}(18)$

$b' = 84.97$ mi/h