# Práctica 3.7

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Cálculo I
1IL702

3) Dos números positivos que satisfagan condiciones

La suma es $S$ y el producto es un máximo

$n_1, n_2$

$n_1 + n_2 = S$

$n_2 = S - n_1$

$P = n_1 * n_2$

$P = n_1(S - n_1)$

$P = n_1 S - 2n_1^2$

$P' = S - 2n_1$

$P'' = -2$

$0 = S - 2n_1$

$-S = -2n_1$

$n_1 = \frac{S}{2}$ ← máximo

$n_2 = \frac{S}{1} - \frac{S}{2}$

$n_2 = \frac{2S - S}{2}$

$n_2 = \frac{S}{2}$

Dos números $= n_1 = \frac{S}{2}$, $n_2 = \frac{S}{2}$

7) Suma del primer número y el doble del segundo es 108 y el producto es el máximo

$x + 2y = 108$

$x = 108 - 2y$

$P = x * y$

$P = (108 - 2y)y$

$P = 108y - 2y^2$

$P' = 108 - 4y$

$P'' = -4$

$0 = 108 - 4y$

$-108 = -4y$

$27 = y$ ← máxima

$x = 108 - 2y$

$x = 108 - 2(27)$

$x = 108 - 54$

$x = 54$

$x = 54, y = 27$

9) Largo y ancho de un rectángulo que tiene perímetro dado y área máxima

$Pe = 2l + 2h$

$2l = Pe - 2h$

$l = \frac{Pe - 2h}{2}$

$Pe = 80m$

$A = l \times h$

$A = \left(\frac{Pe - 2h}{2}\right)h$

$A = \frac{hPe - 2h^2}{2}$

$A = (hPe - 2h^2)2^{-1}$

$A' = (hPe - 2h^2)'2^{-1} + \cancel{2^{-1\prime}(hPe - 2h^2)}^{0}$

$A' = \frac{(Pe - 4h)}{2}$

$A' = (Pe - 4h)2^{-1}$

$A'' = (Pe - 4h)'2^{-1} + \cancel{2^{-1\prime}(Pe - 4h)}^{0}$

$A'' = \frac{-4}{2} = -2$

$0 = \frac{Pe - 4h}{2}$

$0 = Pe - 4h$

$-Pe = -4h$

$\frac{Pe}{4} = h$

$h = \frac{Pe}{4}$

$h = \frac{80}{4}$

$h = 20$

$l = \frac{80 - 2(20)}{2}$

$l = \frac{40}{2}$

$l = 20$

Por lo tanto largo y ancho = 20

11) Largo y ancho de un rectángulo que tiene área dada y un perímetro mínimo

Área = 32 pies$^2$

$A = l \times h$

$32 = l \times h$

$l = \frac{32}{h}$

$P = 2h + 2l$

$P = 2h + 2\left(\frac{32}{h}\right)$

$P = \frac{2h}{1} + \frac{64}{h}$

$P = \frac{2h^2 + 64}{h}$

$P = (2h^2+64)h^{-1}$

$P' = (2h^2+64)' h^{-1} + h^{-1'}(2h^2+64)$

$P' = \frac{4h}{h} + \frac{(-1)(2h^2+64)}{h^2}$

$P' = 4 + \frac{-2h^2-64}{h^2}$

$0 = \frac{4}{1} + \frac{-2h^2-64}{h^2}$

$0 = \frac{4h^2-2h^2-64}{h^2}$

$0 = 2h^2 - 64$

$64 = 2h^2$

$32 = h^2 = \sqrt{32} = 4\sqrt{2}$

$l = \frac{32}{h}$

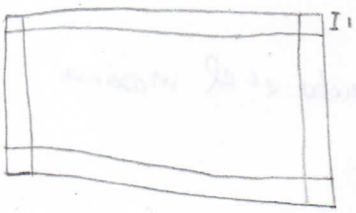
$= \frac{32}{4\sqrt{2}}$

$= \frac{8}{\sqrt{2}}$

$l = 4\sqrt{2}$

Por lo tanto $l$ y $h = 4\sqrt{2}$ pies

17)



$A = 30$ pulg$^2$

$30 = lw$

$l = \frac{30}{w}$

$A = (l+2)(w+2)$

$A = \left(\frac{30}{w} + \frac{2}{1}\right)(w+2)$

$A = \frac{30+2w}{w}(w+2)$

$A = 2w + 34 + \frac{60}{w}$

$A = 2w + 34 + \frac{60}{w}$

$A' = \frac{2}{1} + \frac{60}{w^2}$

$0 = \frac{2w^2+60}{w^2}$

$0 = 2w^2 + 60$

$\frac{-60}{2} = w^2$

$-30 = w^2$ ← debe ser positivo

$\sqrt{30} = w$

$l = \frac{30}{\sqrt{30}}$

$l = \sqrt{30}$

$A = (\sqrt{30}+2)(\sqrt{30}+2)$

dimensiones

18) 36 pulg$^2$

$36 = lw$ $\quad \frac{3}{2}$ ← marco

$l = \frac{36}{w}$

$A = (l+3)(w+3)$

$A = \left(\frac{36}{w} + 3\right)(w+3)$

$A = 3w + 45 + \frac{108}{w}$

$A' = 3 + \frac{108}{w^2}$

$0 = \frac{3w^2+108}{w^2}$

$0 = 3w^2 + 108$

$-108 = 3w^2$

$36 = w^2$ ← debe ser positivo

$w = 6$

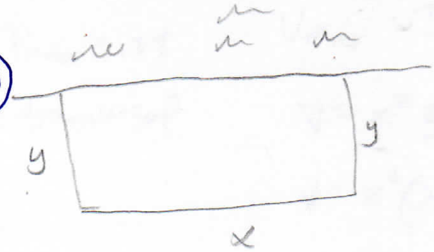
$l = \frac{36}{6}$

$l = 6$

$A = (6+3)(6+3)$

$A = (9)(9)$

dimensiones

19) 

$A = 245{,}000 \, m^2$

$A = xy$

$\frac{245000}{y} = x$

$P = 2y + x$

$P = 2y + \frac{245000}{y}$

$P = \frac{2y^2 + 245000}{y}$

$P = (2y^2 + 245000)\, y^{-1}$

$P' = (2y^2+245000)'\, y^{-1} + y^{-1\,\prime}(2y^2+245000)$

$P' = \frac{4\cancel{y}\,y}{\cancel{y}} + \frac{(-1)(2y^2+245000)}{y^2}$

$P' = 4 + \frac{-2y^2+245000}{y^2}$

$0 = \frac{4}{1} + \frac{-2y^2 - 245000}{y^2}$

$= \frac{4y^2 - 2y^2 - 245000}{y^2}$

$0 = 2y^2 - 245000$

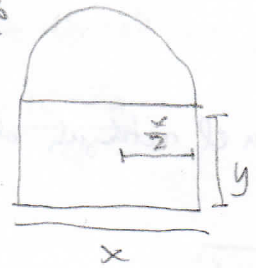
$245000 = 2y^2$

$122500 = y^2$

$y = \pm 350 \qquad y = 350 \, m$

debe ser positivo

$\frac{245000}{350} = x \rightarrow 700 \, m$

21) 

Área máxima

Perímetro = 16 pies

$r = \frac{x}{2}$

$P = 2x + 2y + \pi r$

$P = 2x + 2y + \pi \frac{x}{2}$

$P = \frac{5\pi x}{2} + 2y$

$16 = \frac{5\pi x}{2} + 2y$

$16 - \frac{5\pi x}{2} = 2y$

$y = \frac{8}{1} - \frac{5\pi x}{4}$

$y = \frac{32 - 5\pi x}{4}$

$A = xy + \frac{\pi r^2}{2}$

$A = xy + \frac{\pi\left(\frac{x^2}{4}\right)}{2}$

$A = \frac{xy}{1} + \frac{\pi x^2}{8}$

$A = \frac{8xy + \pi x^2}{8}$

$A = \frac{8x\left(\frac{32 - 5\pi x}{4}\right) + \pi x^2}{8}$

$A = \frac{64x - 10\pi x^2 + \pi x^2}{8}$

$A = \frac{64x - 9\pi x^2}{8}$

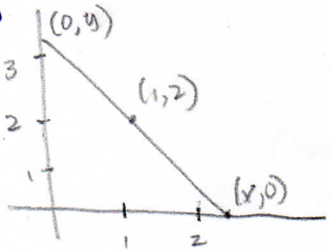
$A = (64 - 9\pi x^2)\, 8^{-1}$

$A' = (64 - 9\pi x^2)'\, 8^{-1} + \cancel{8^{-1\,\prime}(64 - 9\pi x^2)}$ 0

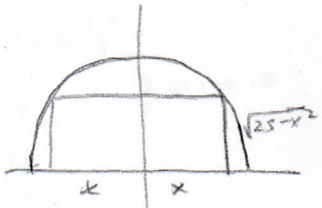
$A' = \frac{-18\pi x}{8}$

$0 = \frac{-18\pi x}{8}$

23)




25) Rectángulo delimitado por el eje x. Y el semicírculo $y=\sqrt{25-x^2}$. Ancho y largo debe tener el rectángulo de manera que su área sea un máximo?




$A = x \cdot y \qquad y=\sqrt{25-x^2}$

$A = 2x \cdot \sqrt{25-x^2}$

$A = 2x(25-x^2)^{1/2}$

$A' = (2x)'(25-x^2)^{1/2} + (25-x^2)^{1/2 '} \cdot 2x$

$= 2\sqrt{25-x^2} + \frac{1}{2}(25-x^2)^{-1/2}(-2x)(2x)$

$= \frac{2\sqrt{25-x^2}}{1} + \frac{-2x^2}{\sqrt{25-x^2}}$

$= \frac{2(25-x^2)-2x^2}{\sqrt{25-x^2}}$

$A' = \frac{50-2x^2-2x^2}{\sqrt{25-x^2}}$

$0 = \frac{50-4x^2}{\sqrt{25-x^2}}$

$0 = 50-4x^2$

$-50 = -4x^2$

$\pm\sqrt{\frac{50}{4}} = x$

$\frac{5\sqrt{2}}{2} = x$

$0 = 2x$

$0 = \left(\frac{5\sqrt{2}}{\cancel{2}}\right)\cancel{2}$

$\boxed{0 = 5\sqrt{2}}$

$y = \sqrt{25-x^2}$

$y = \sqrt{25-\left(\frac{5\sqrt{2}}{2}\right)^2}$

$y = \sqrt{25-\frac{25(2)}{4}}$

$y = \sqrt{\frac{25}{1}-\frac{50}{4}}$

$y = \sqrt{\frac{100-50}{4}}$

$y = \sqrt{\frac{50}{4}}$

$\boxed{y = \frac{5\sqrt{2}}{2}}$

29) $P_{máx} = 108$ $\quad V_{máx} = ?$ ← dimensions

Como es transversal

$P = 8x + 4y$

$108 = 8x + 4y$

$108 - 8x = 4y$

$27 - 2x = y$

$27 - 2(9) = y$

$27 - 18 = y$

$y = 9$

$V = x^2 y$

$V = x^2(27 - 2x)$

$V = 27x^2 - 2x^3$

$V' = 54x - 6x^2$

$0 = 54x - 6x^2$

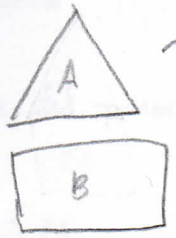
$\frac{-54x}{-6x} = \frac{-6x^2}{-6x}$

$x = 9$ ← máximo

Dimensiones máximas

(Alto = 9, Ancho = 9, Profundo = 9) pulgadas

35) $P = 10$ Dimensiones del triángulo y cuadrado que hacen el área mínima




$3a + 4b = 10$

$b = \frac{10 - 3a}{4}$

$A = \frac{\sqrt{3}}{4}a^2 + b^2$

$A = \frac{\sqrt{3}}{4}a^2 + \left(\frac{10-3a}{4}\right)^2$

$= \frac{\sqrt{3}}{4}a^2 + \frac{100 - 60a + 9a^2}{16}$

$= \frac{(4\sqrt{3} + 9)a^2 - 60a + 100}{16}$

$A' = \frac{2(4\sqrt{3} + 9)a - 60}{16}$

$2(4\sqrt{3} + 9)a = 60$

$a = \frac{60}{8\sqrt{3} + 18} = 1.88$

$b = \frac{10 - 3a}{4} = \frac{10 - 3(1.88)}{4} = 1.09$