Práctica 4.2

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Cálculo I
7IL702

$\int \sqrt{1-4y}\, dy = \int u^{1/2} \frac{du}{-4} = -\frac{1}{4}\int u^{1/2} du = -\frac{1}{4}\cdot\frac{u^{3/2}}{\frac{3}{2}} = -\frac{1}{6}u^{3/2} + C$

$u = 1-4y$

$du = -4\,dy$

$= -\frac{1}{6}(1-4y)^{3/2} + C$

$-\frac{du}{4} = dy$

3) $\int x^3\sqrt{x^2-9}\, dx = \int x^3 u^{1/2} \frac{du}{2x} = \frac{1}{2}\int x^2 u^{1/2} du = \frac{1}{2}\int (\sqrt{u+9})^2 u^{1/2} du = \frac{1}{2}\int u^{3/2} + 9u^{1/2}\, du$

$u = x^2-9 \rightarrow \sqrt{u+9} = x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$= \frac{1}{2}\left(\int u^{3/2} du + \int 9u^{1/2} du\right) = \frac{1}{2}\frac{u^{5/2}}{\frac{5}{2}} + \frac{1}{2}\cdot 9\frac{u^{1/2}}{\frac{3}{2}} = \frac{1}{5}u^{5/2} + 3u^{1/2}$

$= \frac{1}{5}(x^2-9)^{5/2} + 3(x^2-9)^{1/2} + C$

5) $\int x^2(x^3-1)^{10}\, dx = \int \frac{3}{3}x^2(x^3-1)^{10} dx = \frac{1}{3}\int 3x^2(x^3-1)^{10} dx = \frac{1}{3}\int du\,(u)^{10} = \frac{1}{3}\frac{u^{11}}{11} = \frac{(x^3-1)^{11}}{33} + C$

$u = x^3-1$

$du = 3x^2\,dx$

7) $\int \frac{y^3}{(1-2y^4)^5}\, dy = \int \frac{-8}{8}\frac{y^3}{(1-2y^4)^5} dy = \frac{1}{8}\int -8y^3(1-2y^4)^{-5} dy = \frac{1}{8}\int du\,(u)^{-5} = \frac{1}{8}\frac{u^{-4}}{-4} = -\frac{1}{32}u^{-4}$

$u = 1-2y^4$

$du = -8y^3\,dy$

$= \frac{1}{-32}(1-2y^4)^{-4} + C$

9) $\int (x^2-4x+4)^{4/3}\, dx = \int u^{4/3}\frac{du}{2x-4} = \int u^{4/3}\frac{du}{2\sqrt{u+4}-4} = \frac{1}{2}\int \frac{u^{4/3}}{u^{1/2}} du = \frac{1}{2}\int u^{\frac{11}{6}} du = \frac{1}{2}\frac{u^{19/6}}{\frac{19}{6}} = \frac{3}{19}u^{19/6}$

$u = x^2-4x+4$

$u = (x-2)^2 \rightarrow \sqrt{u}+2 = x$

$\frac{du}{2x-4} = dx$

$= \frac{3}{9}(x^2-4x+4)^{19/6} + C$

11) $\int x\sqrt{x+2}\,dx$

$u = x+2 \rightarrow x = u-2$

$du = dx$

$\int (v-2)\sqrt{v}\,dv = \int u^{3/2} - 2u^{1/2}\,du = \int u^{3/2}\,du - 2\int u^{1/2}\,du = \frac{u^{5/2}}{\frac{5}{2}} - 2\frac{u^{3/2}}{\frac{3}{2}}$

$= \frac{2u^{5/2}}{5} - \frac{4u^{3/2}}{3} = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

13) $\int \frac{2r}{(t-r)^7}\,dr = 2\int r(t-r)^{-7}\,dr = 2\int (t-v)u^{-7}\,dv = 2\int u^{-7}t - u^{-6}\,du = 2t\int u^{-7}\,du - \int u^{-6}\,du$

$u = t-r \rightarrow r = t-v$

$du = dr$

$= 2t\frac{u^{-6}}{-6} - \frac{2u^{-5}}{-5} = -\frac{1}{3}tu^{-6} - \frac{2u^{-5}}{-5} = -\frac{1}{3}t(t-r)^{-6} - \frac{2(t-r)^{-5}}{-5} + C$

15) $\int \sqrt{3-2x}\,x^2\,dx = \int \sqrt{u}\left(\frac{3-u}{2}\right)^2\frac{du}{-2} = -\frac{1}{2}\int \sqrt{u}\left(\frac{9-6u+u^2}{4}\right)du = -\frac{1}{2}\int \frac{9u^{1/2} - 6u^{3/2} + u^{5/2}}{4}\,du$

$u = 3-2x \rightarrow \frac{3-u}{2} = x$

$du = -2dx$

$\frac{du}{-2} = dx$

$= -\frac{1}{8}9\int u^{1/2}\,du + \frac{1}{8}6\int u^{3/2}\,du - \frac{1}{8}\int u^{5/2}\,du$

$= -\frac{9}{8}\frac{u^{3/2}}{\frac{3}{2}} + \frac{3}{4}\frac{u^{5/2}}{\frac{5}{2}} - \frac{1}{8}\frac{u^{7/2}}{\frac{7}{2}} = -\frac{3}{4}u^{3/2} + \frac{3}{10}u^{5/2} - \frac{1}{28}u^{7/2}$

$= -\frac{3}{4}(3-2x)^{3/2} + \frac{3}{10}(3-2x)^{5/2} - \frac{1}{28}(3-2x)^{7/2} + C$

17) $\int \cos 4\theta\,d\theta = \int \cos u\,\frac{du}{4} = \frac{1}{4}\int \cos u\,du = \frac{1}{4}\operatorname{sen} u + c = \frac{1}{4}\operatorname{sen} 4\theta + c$

$u = 4\theta$

$du = 4d\theta$

$\frac{du}{4} = d\theta$

19) $\int 6x^2 \operatorname{sen} x^3\,dx = \int \frac{2}{2}6x^2\operatorname{sen} x^3\,dx = 2\int 3x^2\operatorname{sen} x^3\,dx = 2\int du\operatorname{sen} u = -2\cos u = -2\cos x^3 + c$

$u = x^3$

$du = 3x^2dx$

21) $\int \sec^2 5x\,dx = \int \sec^2 u\,\frac{du}{5} = \frac{1}{5}\int \sec^2 u\,du = \frac{1}{5}\int (\sec u)^2\,du = \frac{1}{5}\frac{(\sec u)^3}{3} = \frac{1}{15}\sec^3 u$

$u = 5x$

$du = 5dx$

$\frac{du}{5} = dx$

$= \frac{1}{15}\sec^3 5x + C$

23) $\int y \csc 3y^2 \cot 3y^2 \, dy = \int \frac{6}{6} y \csc u \cot u \, dy = \frac{1}{6}\int 6y \csc u \cot u \, dy$

$3y^2$

$= \frac{1}{6}\int du \csc u \cot u = \frac{1}{6}(-\csc u) = -\frac{1}{6}\csc 3y^2 + C$

$= 6y\,dy$

) $\int \cos x (2+\operatorname{sen} x)^5 \, dx = \int du\, u^5 = \frac{u^6}{6} = \frac{(2+\operatorname{sen} x)^6}{6} + C$

$u = 2 + \operatorname{sen} x$

$du = \cos x \, dx$

27) $\int \sqrt{1+\frac{1}{3x}} \frac{dx}{x^2} = \int \sqrt{\frac{3x+1}{3x}} \frac{dx}{x^2} = \int u^{1/2} \frac{3x^2 du}{x^2} = 3\int u^{1/2} du = 3\frac{u^{3/2}}{\frac{3}{2}} = 2u^{3/2}$.

$u = \frac{3x+1}{3x}$

$= 2\left(\frac{3x+1}{3x}\right)^{3/2} + C$

$du = \frac{9x - 9x + 3}{9x^2}$

$du = \frac{1}{3x^2} dx$

$3x^2 du = dx$

29) $\int 2\operatorname{sen} x \sqrt[3]{1+\cos x} \, dx = 2\int \operatorname{sen} x \, u^{1/3} \frac{du}{-\operatorname{sen} x} = -2\int u^{1/3} du = -2\frac{u^{4/3}}{\frac{4}{3}} = -\frac{3}{2}u^{4/3} + C$

$u = 1 + \cos x$

$du = -\operatorname{sen} x \, dx$

$= -\frac{3}{2}(1+\cos x)^{4/3} + C$

$\frac{du}{-\operatorname{sen} x} = dx$

31) $\int \cos^3 t \operatorname{sen} t \, dt = \int (\cos t)^3 \operatorname{sen} t \, dt = \int u^3 \operatorname{sen} t \frac{du}{-\operatorname{sen} t} = -\int u^3 du = -\frac{u^4}{4} = -\frac{\cos^4 t}{4} + C$

$u = \cos t$

$du = -\operatorname{sen} t \, dt$

$\frac{du}{-\operatorname{sen} t} = dt$

$(\tan^2 u + 1)(\tan^2 u + 1)$

$\tan^4 u + 2\tan^2 u + 1$

33) $\int (\tan 2x + \cot 2x)^2 dx = \int \left(\frac{\tan 2x}{1} + \frac{1}{\tan 2x}\right)^2 dx = \int \left(\frac{\tan^2 2x + 1}{\tan 2x}\right)^2 dx = \int \frac{\tan^4 2x + 2\tan^2 2x + 1}{\tan 2x} dx$

$u = \tan 2x$

$= \int \tan^2 2x \, dx + \int 2\, dx + \int \frac{1}{\tan 2x} dx$

$u =$

$\frac{du}{2} = dx$

35) $\int \frac{x^2+2x}{\sqrt{x^3+3x^2+1}}\,dx = \int \frac{3}{3}\frac{x^2+2x}{\sqrt{x^3+3x^2+1}}\,dx = \frac{1}{3}\int \frac{3x^2+6x}{\sqrt{x^3+3x^2+1}}\,dx = \frac{1}{3}\int \frac{3x^2+6x}{u}\,\frac{2(x^3+3x^2+1)^{1/2}}{3x^2+6x}\,du$

$u = \sqrt{x^3+3x^2+1}$

$u = (x^3+3x^2+1)^{1/2}$

$du = \frac{1}{2}(x^3+3x^2+1)^{-1/2}(3x^2+6x)dx$

$\frac{2(x^3+3x^2+1)^{1/2}}{3x^2+6x}\,du = dx$

$= \frac{2}{3}\int \frac{u^{1/2}}{u}\,du = \frac{2}{3}\int u^{-1/2}\,du = \frac{2}{3}\frac{u^{1/2}}{\frac{1}{2}} = \frac{4}{3}u^{1/2}$

$= \frac{4}{3}\sqrt{x^3+3x^2+1}+C$

37) $\int \frac{y+3}{(3-y)^{2/3}}\,dy = \int \frac{-(3-y)}{(3-y)^{2/3}}\,du = \int \frac{(3-y)\,dy}{(3-y)^{2/3}} = \int \frac{u}{u^{2/3}}\,dy = \int u^{1/3}\,dy = \frac{u^{4/3}}{\frac{4}{3}}\,dy$

$u = 3-y$

$du = -dy$

$-du = dy$

$= \frac{3u^{4/3}}{4} = \frac{3(3-y)^{4/3}}{4}+C$

39) $\int \frac{(r^{1/3}+2)^4}{\sqrt[3]{r^2}}\,dr = \int \frac{u^4}{\sqrt[3]{r^2}}\,3\sqrt[3]{r^2}\,du = 3\int u^4\,du = \frac{3u^5}{5} = \frac{3(r^{1/3}+2)^5}{5}+C$

$u = r^{1/3}+2$

$du = \frac{1}{3}r^{-2/3}\,dr$

$3r^{2/3}\,du = dr$

$\frac{3}{2} - \frac{2}{2} = \frac{1}{2}$

41) $\int \frac{x^3}{(x^2+4)^{3/2}}\,dx = \int \frac{x^3}{u^{3/2}}\,\frac{du}{2x} = \frac{1}{2}\int \frac{x^2}{u^{3/2}}\,du = \frac{1}{2}\int \frac{x^2+4-4}{u^{3/2}}\,du = \frac{1}{2}\int \frac{u-4}{u^{3/2}}\,du$

$u = x^2+4$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$= \frac{1}{2}\int u^{-1/2}\,du - \frac{1}{2}\int 4u^{-3/2}\,du = \frac{1}{2}\frac{u^{1/2}}{\frac{1}{2}} - 2\frac{u^{-1/2}}{-\frac{1}{2}} = u^{1/2}+u^{-1/2} = \sqrt{x^2+4}+\frac{1}{\sqrt{x^2+4}}+C$

43) $\int \operatorname{sen}x\,\operatorname{sen}(\cos x)\,dx = \int \operatorname{sen}x\,\operatorname{sen}(\cos x)\,\frac{du}{-\operatorname{sen}x} = -\int \operatorname{sen}u\,du = \cos u + C = \cos(\cos x)+C$

$u = \cos x$

$du = -\operatorname{sen}x\,dx$

$\frac{du}{-\operatorname{sen}x} = dx$