UNIVERSIDAD TECNOLÓGICA DE PANAMÁ
FACULTAD DE CIENCIAS Y TECNOLOGÍA
DEPARTAMENTO DE CIENCIAS EXACTAS
EXAMEN PARCIAL #2
CALCULO II

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Profa. Alba Castillo de Quiel. Grupo: 1IL 112 Fecha: 12 / 10 / 2020

ENVIAR EN PDF, IMÁGENES CLARAS Y TRAZOS OSCUROS

-I. Derive

1. $f(x) = x\,tan^{-1}x + ln\sqrt{1+x^2}$ 8 pts.

2. $h(x) = cos^{-1}(\tanh 4x^2)$ 8 pts.

3. $g(x) = x\,cosh^{-1}x - \sqrt{x^2-1}$ 8 pts.

II. Integre

1. $\int \frac{\text{senh}\,3x}{\sqrt{16-cosh^2 3x}}dx$ 10 pts.

2. $\int_1^4 sech^2 2x\ tanh^4 2x\, dx$ 10 pts.

3. $\int_1^2 \frac{dx}{\sqrt{x^2+4x}}$ 10 pts.

4. $\int \frac{2x}{x^2+2x+10}dx$ 14 pts.

III. Resuelva los siguientes problemas

1. Determine el volumen del sólido de revolución generado al girar la región limitada por la curva $y = 4x - x^2$ y la recta $y = 0$, alrededor del eje y. Tome elementos de área paralelo al eje de revolución. 14 pts

2. Calcule el volumen del sólido generado al girar alrededor de la recta $y = -3$ la región limitada por las dos parábolas $y = x^2$ y $y = 1 + x - x^2$. 18 pts.

$u = x$
$u' = 1$

① $f(x) = x\tan^{-1}x + \ln\sqrt{1+x^2}$

$f'(x) = x'\tan^{-1}x + \tan^{-1}x'\,x + \left(\ln\sqrt{1+x^2}\right)'$

$= \tan^{-1}x + \frac{1}{1+x^2}\,x + \frac{\left((1+x^2)^{1/2}\right)'}{\sqrt{1+x^2}}$

$= \tan^{-1}x + \boxed{\frac{x}{1+x^2}} + \frac{\frac{1}{\cancel{2}}(1+x^2)^{-1/2}(\cancel{2}x)}{\sqrt{1+x^2}}$

$= \tan^{-1}x + \frac{x}{1+x^2} + \frac{x}{\left(\sqrt{1+x^2}\right)\left(\sqrt{1+x^2}\right)}$

$= \tan^{-1}x + \frac{x}{1+x^2} + \frac{x}{1+x^2}$

$= \tan^{-1}x + \frac{2x}{1+x^2}$

② $h(x) = \cos^{-1}(\tanh 4x^2)$

$u = \tanh 4x^2$ | $v = 4x^2$
$u' = (\operatorname{sech}^2 4x^2)\,8x$ | $v' = 8x$

$u' = 8x\operatorname{sech}^2 4x^2$

$h'(x) = \frac{-u'}{\sqrt{1-u^2}}$

$= \frac{-8x\operatorname{sech}^2 4x^2}{\sqrt{1-(\tanh h4x^2)^2}}$

$= \frac{-8x\operatorname{sech}^2 4x^2}{\sqrt{1-\tanh^2 4x^2}}$

③ $g(x) = x\cosh^{-1}x - \sqrt{x^2-1}$ | $u = x$, $u' = 1$

$g'(x) = x'\cosh^{-1}x + \cosh^{-1}x'\,x - \left((x^2-1)^{1/2}\right)'$

$= \cosh^{-1}x + \frac{u'}{\sqrt{u^2-1}} - \frac{1}{\cancel{2}}(x^2-1)^{-1/2}(\cancel{2}x)$

$= \cosh^{-1}x + \cancel{\frac{1}{\sqrt{x^2-1}}} - \cancel{\frac{1}{\sqrt{x^2-1}}} \to 0$

$= \cosh^{-1}x$

Integre

1) $\int \frac{\operatorname{senh} 3x}{\sqrt{16-\cosh^2 3x}}\,dx = \int \frac{du}{3\sqrt{16-u^2}} = \frac{1}{3}\int \frac{du}{\sqrt{a^2-u^2}} = \frac{1}{3}\operatorname{sen}^{-1}\frac{u}{a} + c$

$u^2 = \cosh 3x^2$ | $a=4$

$du = 3\operatorname{senh} 3x\,dx$

$\frac{du}{3} = \operatorname{senh} 3x\,dx$

$= \frac{1}{3}\operatorname{sen}^{-1}\left(\frac{\cosh 3x}{4}\right) + c$

2) $\int_1^4 \operatorname{sech}^2 2x \tanh^4 2x\,dx = \frac{1}{2}\int_1^4 u^4\,du = \frac{1}{2}\left[\frac{u^5}{5}\right]_1^4 = \frac{1}{2}\left[\frac{\tanh 2x}{5}\right]_1^4$

$u = \tanh 2x\,dx$

$du = 2\operatorname{sech}^2 2x\,dx$

$\frac{du}{2} = \operatorname{sech}^2 2x\,dx$

$= \frac{1}{2}\left[\frac{\tanh(2(4))}{5} - \frac{\tanh(2)}{5}\right] = \frac{1}{2}\left[\frac{\tanh 8}{5} - \frac{\tanh 2}{5}\right]$

$= \frac{1}{2}\left[0.199 - 0.192\right] = 3.5\times10^{-3}$

$= \frac{1}{2}\left[7\times10^{-3}\right]$

3) $\int_1^2 \frac{dx}{\sqrt{x^2+4x}} = \int_1^2 \frac{dx}{\sqrt{(x+2)^2-4}} = \int_1^2 \frac{du}{\sqrt{u^2-a^2}} = \left[\cosh^{-1}\left(\frac{u}{a}\right)\right]_1^2$

$x^2+4x+4-4$ $\quad \left(\frac{b}{2}\right)^2 = 4$ $\quad \left(\frac{4}{2}\right)^2 = 4$

$(x+2)^2-4$

$u = x+2$ | $a=2$

$du = dx$

$= \left[\cosh^{-1}\left(\frac{x+2}{2}\right)\right]_1^2 = \cosh^{-1}\left(\frac{2+2}{2}\right) - \cosh^{-1}\left(\frac{1+2}{2}\right)$

$= \cosh^{-1}(2) - \cosh^{-1}\left(\frac{3}{2}\right) = 1.32 - 0.96 = 0.36$

4) $\int \frac{2x}{x^2+2x+10}\,dx = 2\int \frac{x\,dx}{x^2+2x+10} = 2\int \frac{x\,dx}{(x+1)^2+9} = 2\int \frac{u-1\,du}{u^2+9}$

$x^2+2x+10$ $\quad \left(\frac{b}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1$

$x^2+2x+1-1+10$

$(x+1)^2+9$

$a=3$

$u=x+1 \Rightarrow x=u-1$

$du=dx$

$r=u^2+9$

$dr=2u\,du$

$\frac{dr}{2}=u\,du$

$= 2\left[\int \frac{u}{u^2+9}du - \int \frac{du}{u^2+9}\right]$

$= 2\left[\frac{1}{2}\int \frac{dr}{r} - \int \frac{du}{u^2+a^2}\right]$

$= \ln|r| - \frac{2}{a}\tan^{-1}\left(\frac{u}{a}\right) + c$

$= \ln|u^2+9| - \frac{2}{3}\tan^{-1}\left(\frac{x+1}{3}\right) + c$

$= \ln|(x+1)^2+9| - \frac{2}{3}\tan^{-1}\left(\frac{x+1}{3}\right) + c$

$= \ln|x^2+2x+10| - \frac{2}{3}\tan^{-1}\left(\frac{x+1}{3}\right) + c$

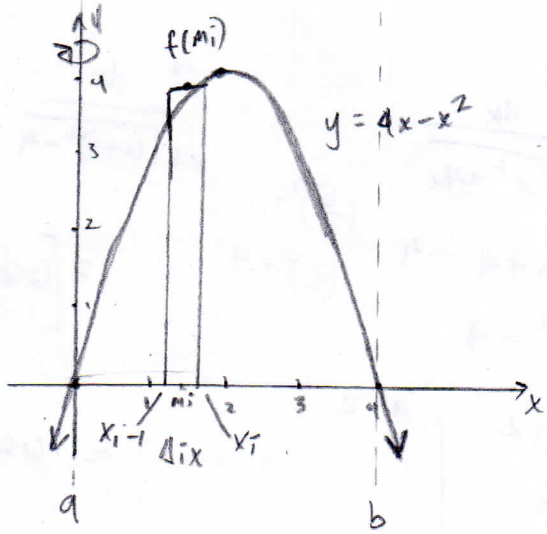
Sólido de revolución

1) $y=4x-x^2 \quad y=0 \quad$ alrededor de y

$x = \frac{-b}{2a} = \frac{-4}{2(-1)} = 2$ $\qquad 0=4x-x^2$

$V(2,4)$ $\qquad 0=x(4-x)$

$x=0 \quad x=4$



$V = \lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} 2\pi m_i (f(m_i))\,\Delta m_i$

$= \lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} 2\pi x(4x-x^2)\,\Delta x$

$= 2\pi \int_0^4 x(4x-x^2)\,dx$

$= 2\pi \int_0^4 (4x^2-x^3)\,dx$

$= 2\pi \left[4\int_0^4 x^2dx - \int_0^4 x^3dx\right]$

$= 2\pi \left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_0^4$

$= 2\pi \left[\frac{4(4)^3}{3} - \frac{(4)^4}{4} - \cancel{\left(\frac{4(0)^3}{3} - \frac{(0)^4}{4}\right)}^{0}\right]$

$= 2\pi\left[\frac{256}{3} - \frac{64}{\cancel{3}}\right] = 2\pi\left[\frac{256-192}{3}\right] = 2\pi\left[\frac{64}{3}\right] = \frac{128}{3}\pi u^3$

volumen

2) Eje de giro $y=-3$

$y=x^2 \qquad y=1+x-x^2$

$y=1+x-x^2$

$-1=1+0-(0)^2$

$y=1$

x^2

$x=\frac{-b}{2a}=\frac{-0}{2(1)}=0$

$V(0,0)$

$1+x-x^2$

$x=\frac{-b}{2a}=\frac{-1}{2(-1)}=\frac{1}{2}$

$0=1+x-x^2$

$V\left(\frac{1}{2},\frac{5}{4}\right)$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-1\pm\sqrt{(1)^2-4(-1)(1)}}{2(-1)}$

$=\frac{-1\pm\sqrt{1+4}}{-2}=\frac{\cancel{-}1\pm\sqrt{5}}{\cancel{-}2}=\frac{1\pm\sqrt{5}}{2}$

Por dualidad de signos, siempre se mantendrá. Por lo tanto, se puede remover el -1 a 1.

$x^2=1+x-x^2$

$2x^2-x-1=0$

$x^2-\frac{x}{2}-\frac{1}{2}=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-\frac{1}{2}\pm\sqrt{\frac{1}{4}-4(1)(-\frac{1}{2})}}{2(1)}$

$=\frac{\frac{1}{2}\pm\sqrt{\frac{1}{4}+2}}{2}$

$=\frac{\frac{1}{2}\pm\sqrt{\frac{9}{4}}}{2}$

$=\frac{\frac{1}{2}\pm\frac{3}{2}}{2}$

$\frac{\frac{1}{2}+\frac{3}{2}}{2}=1 \qquad \frac{\frac{1}{2}-\frac{3}{2}}{2}=-\frac{1}{2}$

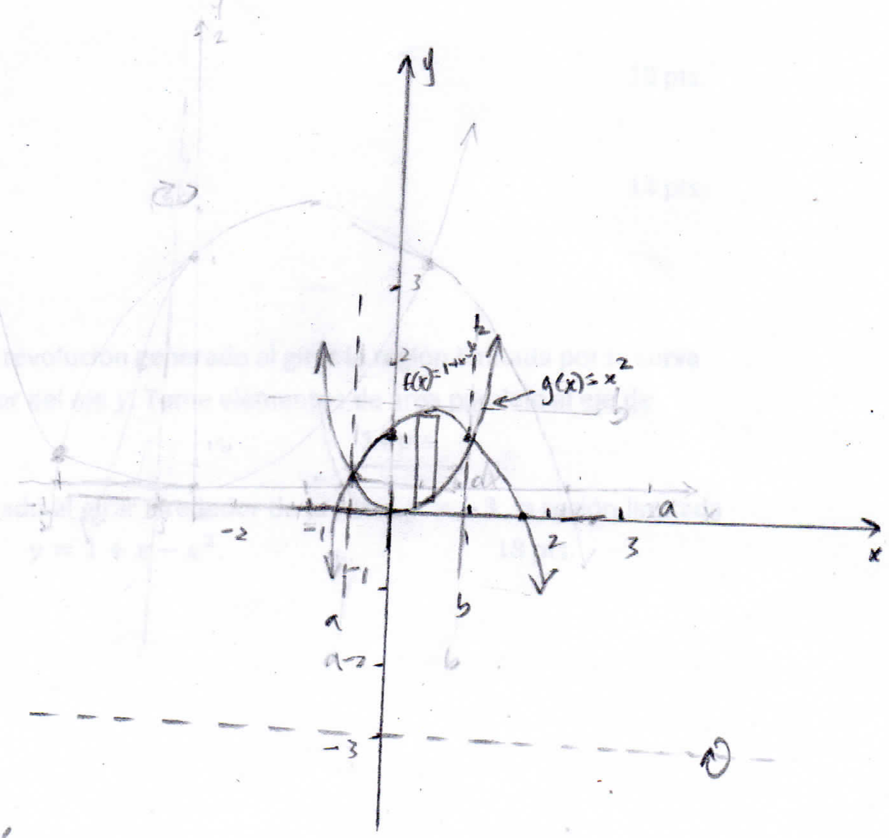
Puntos de intersección entre ambas f(x)s = $(1,1)$ y $\left(-\frac{1}{2},\frac{1}{4}\right)$

$R=-3-(1+x-x^2)$

$=-3-1-x+x^2$

$=-4-x+x^2$

$r=-3-x^2$



$V=\lim_{\|\Delta\|\to 0}\pi\sum_{i=1}^{n}\left[R(x_i)^2-r(x_i)^2\right]\Delta x$

$V=\pi\int_{-1/2}^{1}\left[(-4-x+x^2)^2-(-3-x^2)^2\right]dx$

$=\pi\int_{-1/2}^{1}\left[x^4-2x^3-7x^2+8x+16-(9+6x^2+x^4)\right]dx$

$=\pi\int_{-1/2}^{1}\left[\cancel{x^4}-2x^3-7x^2+8x+16-9-6x^2-\cancel{x^4}\right]dx$

$=\pi\int_{-1/2}^{1}(-2x^3-13x^2+8x+7)\,dx$

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$$= \pi \int_{-\frac{1}{2}}^{1} (-2x^3 - 13x^2 + 8x + 7)\, dx$$

$$= \pi \left[-2\int_{-1/2}^{1} x^3\, dx - 13\int_{-1/2}^{1} x^2\, dx + 8\int_{-1/2}^{1} x\, dx + 7\int_{-1/2}^{1} dx \right]$$

$$= \pi \left[\frac{-x^4}{2} - \frac{13x^3}{3} + 4x^2 + 7x \right]_{-1/2}^{1}$$

$$= \pi \left[\frac{-(1)^4}{2} - \frac{13(1)^3}{3} + 4(1)^2 + 7(1) + \frac{\left(\frac{-1}{2}\right)^4}{2} + \frac{13\left(-\frac{1}{2}\right)^3}{3} - 4\left(\frac{-1}{2}\right)^2 - 7\left(\frac{-1}{2}\right) \right]$$

$$= \pi \left[\frac{-1}{2} - \frac{13}{3} + 4 + 7 + \frac{1}{32} - \frac{13}{24} - 1 + \frac{7}{2} \right]$$

$$= \pi \left[3 - \frac{13}{3} + 10 + \frac{1}{32} - \frac{13}{24} \right]$$

$$= \pi \left[13 - \frac{13}{3} + \frac{1}{32} - \frac{13}{24} \right]$$

$$= \pi \left[\frac{26}{3} + \frac{1}{32} - \frac{13}{24} \right] = \pi \left[\frac{829}{96} - \frac{13}{24} \right] = \frac{259}{32} \pi u^3 \leftarrow \text{volumen}$$