UNIVERSIDAD TECNOLÓGICA DE PANAMÀ
FACULTAD DE CIENCIAS Y TECNOLOGÌA
DEPARTAMENTO DE CIENCIAS EXACTAS
Parcial N° 1 – II Semestre
CÁLCULO II

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Enviar en archivo PDF, con letras claras y firmes, no imágenes borrosas. Lo que no se ve claro no se puede evaluar.

Evalúe las siguientes integrales:

1. $\int_{-1}^{7}(|x-2|-3)dx$ (8 puntos)
2. $\int_{0}^{1}(5^x-3^x)\,dx$ (8 puntos)
3. $\int \frac{x^2+4x}{x^3+6x^2+5}\,dx$ (7 puntos)
4. $\int_{1}^{2} t^2\sqrt{t^3+1}\,dt$ (12 puntos)
5. $\int_{-\pi}^{\pi} cos^2 x \,\mathrm{sen}\, x\, dx$ (8 puntos)
6. $\int cot^5 2x\, csc^2 2x\, dx$ (6 puntos)
7. $\cancel{\int_1^5 \frac{x}{\sqrt{x-3}}dx}$ $\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx$ (12 puntos)
8. $\int(1+\sec\pi x)^2\sec\pi x\tan\pi x\,dx$ (7 puntos)

Resuelva los siguientes problemas:

Construya las gráficas. Luego dibuje el rectángulo del elemento de área en la figura y marque $f(c_i)$ y $g(c_i)$ con sus funciones correspondientes. Escriba la definición de área con límite y luego la integral definida para calcularla.

9. Determine el área de la región limitada por las gráficas de $f(y)=y^2-2$ y $g(y)=6-y^2$ (18 puntos)

10. Determine el área de la región comprendida por la gráfica de $f(x)=6-x-x^2$ y la recta $y=0$. (16 puntos)

1) $\int_{-1}^{7}(|x-2|-3)\,dx = \int_{-1}^{2}\left[-(x-2)-3\right]dx + \int_{2}^{7}\left[(x-2)-3\right]dx = \int_{-1}^{2}(-x+2-3)\,dx + \int_{2}^{7}(x-5)\,dx$

$|x-2| = \begin{cases} -(x-2) & x<2 \\ (x-2) & x>2 \end{cases}$

$x-2\geq 0$

$x>2$

$= \int_{-1}^{2}(-x-1)\,dx + \int_{2}^{7}(x-5)\,dx = -\int_{-1}^{2}x\,dx - \int_{-1}^{2}dx + \int_{2}^{7}x\,dx - 5\int_{2}^{7}dx$

$= \left[-\frac{x^2}{2} - x\right]_{-1}^{2} + \left[\frac{x^2}{2} - 5x\right]_{2}^{7} = \left[-\frac{(2)^2}{2} - 2\right] - \left[-\frac{(-1)^2}{2} - (-1)\right] + \left[\frac{(7)^2}{2} - 5(7)\right] - \left[\frac{(2)^2}{2} - 5(2)\right] = \left(-4 + \frac{1}{2} - 1\right) + \left(\frac{49}{2} - 35 - 2 + 10\right)$

$= -\frac{9}{2} + \left(-\frac{5}{2}\right) = -\frac{9}{2} - \frac{5}{2} = -\frac{14}{2} = \boxed{-7}$

2) $\int_{0}^{1}(5^x - 3^x)\,dx = \int_{0}^{1}(5^u - 3^u)\,du = \int_{0}^{1}5^u\,du - \int_{0}^{1}3^u\,du = \left[\frac{5^u}{\ln 5} - \frac{3^u}{\ln 3}\right]_0 = \left[\frac{5^x}{\ln 5} - \frac{3^x}{\ln 3}\right]_0^1$

$u=x$

$du=dx$

$= \left[\frac{5^{(1)}}{\ln 5} - \frac{3^{(1)}}{\ln 3}\right] - \left[\frac{5^{(0)}}{\ln 5} - \frac{3^{(0)}}{\ln 3}\right] = \frac{5}{\ln 5} - \frac{3}{\ln 3} - \frac{1}{\ln 5} + \frac{1}{\ln 3} = \boxed{\frac{4}{\ln 5} - \frac{2}{\ln 3}} \approx 0.66$

3) $\int \frac{x^2+4x}{x^3+6x^2+5}\,dx = \int \frac{\frac{du}{3}}{u} = \int \frac{du}{3u} = \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u| + C = \boxed{\frac{1}{3}\ln|x^3+6x^2+5| + C}$

$u = x^3+6x^2+5$

$du = (3x^2+12x)\,dx$

$\frac{du}{3} = (x^2+4x)\,dx$

4) $\int_{1}^{2} t^2\sqrt{t^3+1}\,dt = \frac{1}{3}\int_{1}^{2} du\sqrt{u} = \frac{1}{3}\int_{1}^{2}u^{1/2}\,du = \left[\frac{2u^{3/2}}{9}\right]_1^2 = \left[\frac{2\sqrt{(t^3+1)^3}}{9}\right]_1^2$

$u = t^3+1$

$du = 3t^2\,dt$

$\frac{du}{3} = t^2\,dt$

$= \left[\frac{2\sqrt{((2)^3+1)^3}}{9}\right] - \left[\frac{2\sqrt{((1)^3+1)^3}}{9}\right] = \left[\frac{2\sqrt{729}}{9} - \frac{2\sqrt{8}}{9}\right] = \left[\frac{2(27)}{9} - \frac{2\sqrt{8}}{9}\right]$

$= 6 - \frac{2(2\sqrt{2})}{9} = \boxed{6 - \frac{4\sqrt{2}}{9}}$

5) $\int_{-\pi}^{\pi} \cos^2 x \operatorname{sen} x\, dx = \int_{-\pi}^{\pi} (\cos x)^2 \operatorname{sen} x\, dx = \int_{-\pi}^{\pi} u\, du = \left[\frac{u^2}{2}\right]_{-\pi}^{\pi}$

$u = (\cos x)^2$

$du = 2(-\operatorname{sen} x)\, dx$

$\frac{du}{-2} = \operatorname{sen} x\, dx$

$= \left[\frac{((\cos x)^2)^2}{2}\right]_{-\pi}^{\pi} = \left[\frac{(\cos x)^4}{2}\right]_{-\pi}^{\pi} = \left[\frac{(\cos(\pi))^4}{2}\right] - \left[\frac{(\cos(-\pi))^4}{2}\right]$

$= \frac{1}{2} - \frac{1}{2} = 0$

6) $\int \cot^5 2x \csc^2 2x\, dx = \int u^5 \frac{du}{-2} = \frac{1}{-2}\int u^5\, du = -\frac{1}{2}\cdot\frac{u^6}{6} + C = \frac{-u^6}{12} + C =$

$u = \cot 2x$

$du = -2\csc^2 2x\, dx$

$\frac{du}{-2} = \csc^2 2x\, dx$

$\frac{-(\cot 2x)^6}{12} + C$

7) $\int_1^5 \frac{x}{\sqrt{2x-1}}\, dx = \int_1^5 \frac{\left(\frac{u+1}{2}\right)}{u^{1/2}} \frac{du}{2} = \int_1^5 \frac{\frac{u+1}{2}}{2u^{1/2}}\, du = \int_1^5 \frac{u+1}{4u^{1/2}}\, du = \frac{1}{4}\int_1^5 \frac{u+1}{u^{1/2}}\, du$

$u = 2x-1 \quad \Big| \quad x = \frac{u+1}{2}$

$du = 2dx$

$\frac{du}{2} = dx$

$= \frac{1}{4}\int_1^5 (u^{1/2} + u^{-1/2})\, du = \frac{1}{4}\left[\int_1^5 u^{1/2}\, du + \int_1^5 u^{-1/2}\, du\right] = \frac{1}{4}\left[\frac{2u^{3/2}}{3} + 2u^{1/2}\right]_1^5$

$= \left[\frac{u^{3/2}}{6} + \frac{u^{1/2}}{2}\right]_1^5 = \left[\frac{\sqrt{(2x-1)^3}}{6} + \frac{\sqrt{2x-1}}{2}\right]_1^5 = \left[\frac{\sqrt{(2(5)-1)^3}}{6} + \frac{\sqrt{2(5)-1}}{2}\right] - \left[\frac{\sqrt{(2(1)-1)^3}}{6} + \frac{\sqrt{2(1)-1}}{2}\right]$

$= \left[\frac{27}{6} + \frac{3}{2}\right] - \left[\frac{1}{6} + \frac{1}{2}\right] = \frac{27}{6} + \frac{3}{2} - \frac{1}{6} - \frac{1}{2} = \frac{26}{6} + 1 = \frac{26}{6} + \frac{6}{6} = \frac{32}{6} = \frac{16}{3}$

8) $\int (1+\sec\pi x)^2 \sec\pi x \tan\pi x\, dx = \int u^2 \frac{du}{\pi} = \frac{1}{\pi}\int u^2\, du = \frac{1}{2}\cdot\frac{u^3}{3} + C = \frac{u^3}{6} + C$

$u = 1 + \sec\pi x$

$du = \pi \sec\pi x \tan\pi x\, dx$

$\frac{du}{\pi} = \sec\pi x \tan\pi x\, dx$

$= \frac{(1+\sec\pi x)^3}{6} + C$

9) $f(y) = y^2 - 2 \qquad g(y) = 6 - y^2$

$f(y) = y^2 - 2 \quad V(-2,0)$

$y = \frac{-b}{2a} = \frac{-0}{2(1)} = 0$

$y^2 - 2 = 0 \quad | \quad x = y^2 - 2$

$y^2 = 2 \quad | \quad x = (0)^2 - 2$

$y = \pm\sqrt{2} \quad | \quad x = -2$

± 1.41

$g(y) = 6 - y^2 \quad V(6,0)$

$y = \frac{-b}{2a} = \frac{-0}{2(-1)} = 0$

$6 - y^2 = 0 \quad | \quad x = 6 - y^2$

$6 = y^2 \quad | \quad x = 6 - (0)^2$

$y = \pm\sqrt{6} \quad | \quad x = 6$

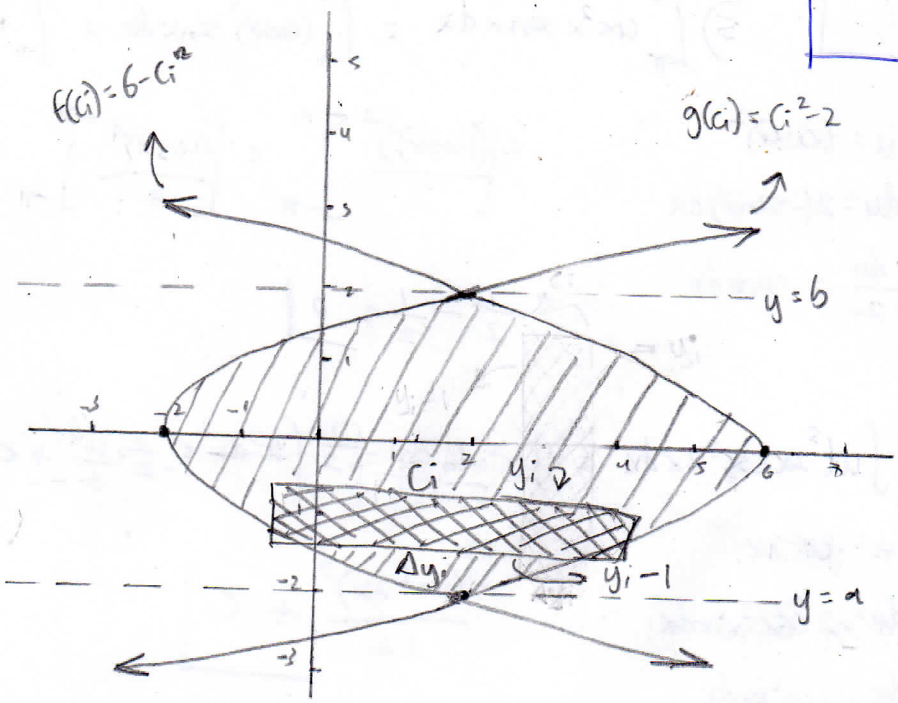
± 2.45

$y^2 - 2 = 6 - y^2$

$2y^2 = 8$

$y^2 = 4$

$y = \pm 2$ intersectan las dos gráficas



$$A = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \left[(6 - c_i^2) - (c_i^2 - 2)\right] \Delta y_i$$

$$A = \int_{-2}^{2} (6 - y^2 - y^2 + 2)\, dy$$

$$= \int_{-2}^{2} (-2y^2 + 8)\, dy$$

$$= -2\int_{-2}^{2} y^2\, dy + 8\int_{-2}^{2} dy$$

$$= \left[\frac{-2y^3}{3} + 8y\right]_{-2}^{2}$$

$$= \left[\frac{-2(2)^3}{3} + 8(2)\right] - \left[\frac{-2(-2)^3}{3} + 8(-2)\right]$$

$$= \frac{-16}{3} + 16 - \frac{16}{3} + 16 = \frac{-32}{3} + 32 = \frac{64}{3}\, u^2$$

10) $f(x) = 6 - x - x^2 \qquad y = 0$

$f(x) = 6 - x - x^2 \qquad V\left(\frac{-1}{2}, \frac{25}{4}\ \text{o}\ 6.25\right)$

$x = \frac{-b}{2a} = \frac{1}{2(-1)} = \frac{-1}{2}$

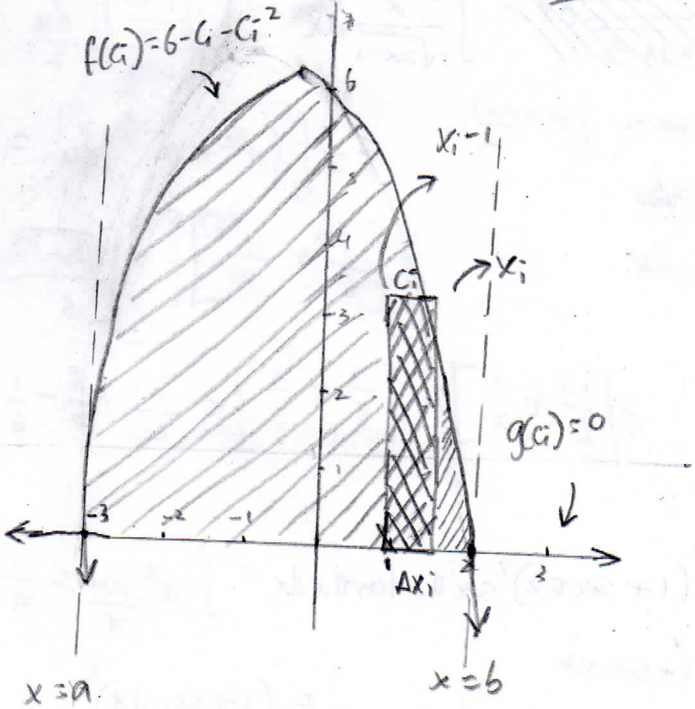
$0 = 6 - x - x^2 \quad | \quad y = 6 - 0 - (0)^2$

$0 = -(x^2 + x - 6) \quad | \quad y = 6$

$0 = (x-2)(x+3)$

$x = 2 \quad x = -3$

con eje y e intersección con la otra recta



$$A = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \left[(6 - c_i - c_i^2) - 0\right] \Delta x_i$$

$$A = \int_{-3}^{2} (6 - x - x^2)\, dx$$

$$= 6\int_{-3}^{2} dx - \int_{-3}^{2} x\, dx - \int_{-3}^{2} x^2\, dx$$

$$= \left[6x - \frac{x^2}{2} - \frac{x^3}{3}\right]_{-3}^{2}$$

$$= \left[6(2) - \frac{(2)^2}{2} - \frac{(2)^3}{3}\right] - \left[6(-3) - \frac{(-3)^2}{2} - \frac{(-3)^3}{3}\right]$$

$$= \left(10 - \frac{8}{3}\right) - \left(-18 - \frac{9}{2} + 9\right)$$

$$= 10 - \frac{8}{3} + 9 + \frac{9}{2}$$

$$= \frac{125}{6}\, u^2$$