

$$7) \lim_{x \rightarrow 8^+} \frac{1}{x+8} = \frac{1}{8+8} = \frac{1}{16}$$

$$9) \lim_{x \rightarrow 5^+} \frac{x-5}{x^2-25} = \lim_{x \rightarrow 5^+} \frac{\cancel{x-5}}{(\cancel{x-5})(x+5)} = \lim_{x \rightarrow 5^+} \frac{1}{x+5} = \frac{1}{5+5} = \frac{1}{10}$$

$$11) \lim_{x \rightarrow -3^-} \frac{x}{\sqrt{x^2-9}} = \sqrt{x^2-9} = 0 \quad \sqrt{x^2} - \sqrt{9} = \sqrt{x^2} - \sqrt{9} = x - 3 \quad x = -3 \\ \rightarrow \frac{-3}{\sqrt{(-3)^2-9}} = \frac{-3}{\sqrt{9-9}} = \frac{-3}{\sqrt{0}} = \frac{-3}{0} = -\infty$$

$$13) \lim_{x \rightarrow 0^-} \frac{|x|}{x} = \frac{|-0.001|}{-0.001} = \frac{0.001}{-0.001} = -\infty$$

$$15) \lim_{\Delta x \rightarrow 0^-} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x} = \lim_{\Delta x \rightarrow 0^-} \frac{\frac{x - x - \Delta x}{x^2 + x\Delta x}}{\Delta x} = \lim_{\Delta x \rightarrow 0^-} \frac{-\Delta x}{\cancel{\Delta x}(x^2 + x\Delta x)} = \lim_{\Delta x \rightarrow 0^-} \frac{1}{x^2 + x\Delta x} \\ = \frac{1}{x^2 + x(0)} = \frac{1}{x^2}$$

$$17) \lim_{x \rightarrow 3^-} f(x) = \begin{cases} \frac{x+2}{2}, & x \leq 3 \\ \frac{12-2x}{3}, & x > 3 \end{cases} = \lim_{x \rightarrow 3^-} \frac{x+2}{2} = \frac{3+2}{2} = \frac{5}{2}$$

$$19) \lim_{x \rightarrow 1} f(x) = \begin{cases} x^3 + 1, & x < 1 \\ x + 1, & x \geq 1 \end{cases} \quad \left| \begin{array}{l} \lim_{x \rightarrow 1^-} x^3 + 1 = (1)^3 + 1 = 1 + 1 = 2 \\ \lim_{x \rightarrow 1^+} x + 1 = 1 + 1 = 2 \end{array} \right|$$

$$\lim_{x \rightarrow 1} f(x) = 2$$

← si existe

$$\lim_{x \rightarrow 1^+} x + 1 = 1 + 1 = 2$$

$$21) \lim_{x \rightarrow \pi} \cot x = \lim_{x \rightarrow \pi} \frac{\cos x}{\sin x} = \frac{\cos \pi}{\sin \pi} = \frac{0}{1} = 0$$

$$23) \lim_{x \rightarrow 4^-} (5[x] - 7) = 5(3) - 7 = 15 - 7 = 8$$

$$25) \lim_{x \rightarrow 3} (2 - [-x]) = \lim_{x \rightarrow 3^-} (2 - [x]) = 2 - (-3) = 2 + 3 = 5$$

$$\lim_{x \rightarrow 3^+} (2 - [-x]) = 2 - (-4) = 2 + 4 = 6$$

$$\lim_{x \rightarrow 3} (2 - [-x]) = \neq$$

Analizar la discontinuidad y la continuidad de cada función

$$27) f(x) = \frac{1}{x^2 - 4}$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

Discontinua en $x = \{-2, 2\}$

$$28) f(x) = \frac{x^2 - 1}{x + 1}$$

$$x + 1 = 0$$

$$x = -1$$

Discontinua en $x = -1$

$$29) f(x) = \frac{1}{2}[x] + x$$

$$x = -3, -2, -1, \dots$$

Discontinua en x

$$31) g(x) = \sqrt{49-x^2} \quad [-7, 7]$$

$$33) f(x) = \begin{cases} 3-x, & x \leq 0 \\ 3+\frac{1}{2}x, & x > 0 \end{cases} \quad [-1, 4]$$

$$g(a) = \sqrt{49-(-7)^2} \\ = \sqrt{49-49} \\ = 0$$

$$g(b) = \sqrt{49-(7)^2} \\ = \sqrt{49-49} \\ = 0$$

La función
es continua

$$f(-1) = 3-(-1) \\ = 3+1 \\ = 4$$

$$f(4) = 3 + \frac{1}{2}(4) \\ = 3+2 \\ = 5$$

$$\lim_{x \rightarrow -7^+} g(x) = \sqrt{49-x^2} \\ = \sqrt{49-(-7)^2} \\ = \sqrt{49-49} \\ = 0$$

$$\lim_{x \rightarrow 7^-} g(x) = \sqrt{49-x^2} \\ = \sqrt{49-(7)^2} \\ = \sqrt{49-49} \\ = 0$$

$$\lim_{x \rightarrow -1^+} 3-x \\ = 3-(-1) \\ = 3+1 \\ = 4$$

$$\lim_{x \rightarrow 4^-} 3 + \frac{1}{2}x \\ = 3 + \frac{1}{2}(4) \\ = 3+2 \\ = 5$$

Función Continua

$$35) f(x) = \frac{6}{x}$$

$$37) x^2 - 9$$

$$39) f(x) = \frac{1}{4-x^2}$$

$$x=0$$

Continua para
todo \mathbb{R} .

$$4-x^2=0 \\ 4=x^2 \\ x=\pm 2$$

Discontinua no removible
en $x = \{-2, 2\}$

Discontinua no
removible en $x=0$

$$41) f(x) = 3x - \cos x$$

Continua para
todo \mathbb{R}

$$43) f(x) = \frac{x}{x^2-x}$$

$$x^2-x=0 \quad \text{Discontinua no removible en } x=1$$

$$\frac{x}{x(x-1)} \\ = \frac{1}{x-1}$$

Discontinua removible
en $x=0$

$$45) f(x) = \frac{x}{x^2+1}$$

Continua para
todo \mathbb{R} .

$$47) f(x) = \frac{x+2}{x^2-3x-10}$$

$$x^2-3x-10=0$$

$$(x-5)(x+2)=0$$

$$x=5 \quad x=-2$$

Discontinua no removible
en $x=5$

Discontinua removible
en $x=-2$

$$\frac{x+2}{x^2-3x-10} = 0$$

$$\frac{x+2}{(x-5)(x+2)} = 0$$

$$\frac{1}{x-5} = 0 \\ x=5$$

$$49) f(x) = \frac{|x+7|}{x+7}$$

$$x+7=0$$

$$x=-7$$

Discontinua no removible
en $x=-7$

$$51) f(x) = \begin{cases} x, & x \leq 1 \\ x^2, & x > 1 \end{cases}$$

Continua para todo

\mathbb{R} .

$$53) f(x) = \begin{cases} \frac{1}{2}x + 1, & x \leq 2 \\ 3 - x, & x > 2 \end{cases}$$

Continua para todo

\mathbb{R} .

$$55) f(x) = \begin{cases} \tan \frac{\pi x}{4}, & |x| < 1 \\ x, & |x| \geq 1 \end{cases}$$

Continua para toda

\mathbb{R} .

$$57) f(x) = \csc 2x$$

$\csc(2x)$

$$x = \frac{\pi n}{2}$$

Discontinuidad no removable
para todos $x = \frac{\pi n}{2}$