UNIVERSIDAD TECNOLÓGICA DE PANAMÀ
FACULTAD DE CIENCIAS Y TECNOLOGÌA
DEPARTAMENTO DE CIENCIAS EXACTAS
Examen Semestral – II Semestre 2020
CÁLCULO II

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ENVIAR EN ARCHIVO PDF, con letras claras y firmes, no imágenes borrosas. Lo que no se ve claro no se puede evaluar. No omitir pasos. Enviar los problemas en el orden que aparecen en la hoja.

I. Resuelva las siguientes integrales

1. $\int e^{-3x} sen5x\, dx$ (Tabular) 16 puntos

2. $\int x^3\sqrt{16-x^2}dx$ 20 puntos

3. $\int \frac{2x^2+x}{(x+1)^2(x^2+x+1)}dx$ 22 puntos

II. Resuelva los siguientes problemas

4. Determine si la integral es convergente o divergente
$\int_{-\infty}^{\infty} \frac{dx}{4x^2+4x+5}$ 16 puntos

5. Determine el volumen del sólido generado al girar la región limitada por la curva $y = \ln x$, el eje x y la recta $x = e^2$.
15 puntos

6. Determine si la serie $\sum_{i=1}^{\infty} \frac{(n+2)!}{n!\, 10^n}$ es convergente o divergente.
12 puntos

1) $\int e^{-3x} \operatorname{sen}5x \, dx$

$u = \operatorname{sen}5x \qquad + \qquad v = e^{-3x} dx$

$5\cos 5x \qquad - \qquad = -\frac{1}{3} e^{-3x} dx$

$-25 \operatorname{sen}5x \qquad = \frac{1}{9} e^{-3x} dx$

$$\int e^{-3x} \operatorname{sen}5x \, dx = -\frac{1}{3} e^{-3x} \operatorname{sen}5x - \frac{5}{9} e^{-3x} \cos 5x - \frac{25}{9} \int e^{-3x} \operatorname{sen}5x \, dx + C$$

$$\int e^{-3x} \operatorname{sen}5x \, dx + \frac{25}{9} \int e^{-3x} \operatorname{sen}5x \, dx = -\frac{1}{3} e^{-3x} \operatorname{sen}5x - \frac{5}{9} e^{-3x} \cos 5x + C$$

$$\frac{34}{9} \int e^{-3x} \operatorname{sen}5x \, dx = -\frac{1}{3} e^{-3x} \operatorname{sen}5x - \frac{5}{9} e^{-3x} \cos 5x + C$$

$$\boxed{\int e^{-3x} \operatorname{sen}5x \, dx = -\frac{9}{34} \left(\frac{1}{3} e^{-3x} \operatorname{sen}5x + \frac{5}{9} e^{-3x} \cos 5x \right) + C}$$

2) $\int x^3 \sqrt{16 - x^2} \, dx$

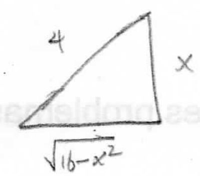
$a = 4$

$u = x$

$x = 4 \operatorname{sen}\theta$

$dx = 4 \cos\theta \, d\theta$

$\operatorname{sen}\theta = \frac{x}{4} \qquad \theta = \operatorname{sen}^{-1}\left(\frac{x}{4}\right)$



$\sqrt{16 - x^2}$

$\sqrt{16 - 16\operatorname{sen}^2\theta}$

$\sqrt{16(1 - \operatorname{sen}^2\theta)}$

$4\sqrt{\cos^2\theta}$

$4\cos\theta$

$= \int (64 \operatorname{sen}^3\theta)(4\cos\theta)(4\cos\theta) \, d\theta$

$= \int (64 \operatorname{sen}^3\theta)(16\cos^2\theta) \, d\theta$

$= 1024 \int (\operatorname{sen}\theta)(\operatorname{sen}^2\theta)(\cos^2\theta) \, d\theta$

$= 1024 \int (\operatorname{sen}\theta)(1 - \cos^2\theta) \cos^2\theta \, d\theta$

$U = \cos\theta$

$du = -\operatorname{sen}\theta \, d\theta$

$-du = \operatorname{sen}\theta \, d\theta$

$= 1024 \int (\operatorname{sen}\theta \cos^2\theta - \operatorname{sen}\theta \cos^4\theta) \, d\theta$

$= 1024 \left[\int \operatorname{sen}\theta \cos^2\theta \, d\theta - \int \operatorname{sen}\theta \cos^4\theta \, d\theta \right]$

$= 1024 \left[-\int u^2 du + \int u^4 du \right]$

$= 1024 \left[-\frac{u^3}{3} + \frac{u^5}{5} \right] + C$

$$= 1024 \left[\frac{\cos^5\theta}{5} - \frac{\cos^3\theta}{3} \right] + C$$

3) $\int \frac{2x^2+x}{(x+1)^2(x^2+x+1)}dx$

$$\left[\frac{2x^2+x}{(x+1)^2(x^2+x+1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+x+1}\right](x+1)^2(x^2+x+1)$$

$$2x^2+x = A(x+1)(x^2+x+1) + B(x^2+x+1) + (Cx+D)(x+1)^2$$

$$2x^2+x = \cancel{Ax^3}+\cancel{2Ax^2}+\cancel{2Ax}+A + \cancel{Bx^2}+\cancel{Bx}+B + \cancel{Cx^3}+\cancel{2Cx^2}+Dx^2+\cancel{Cx}+2Dx+D$$

$$2x^2+x = x^3(A+C) + x^2(2A+B+2C+D) + x(2A+B+C+2D) + A+B+D$$

$$\begin{cases} A+C=0 \\ 2A+B+2C+D=2 \\ 2A+B+C+2D=1 \\ A+B+D=0 \end{cases}$$

$2A+B+2C+D=2$

$2A+B+C+2D=1 \quad (-1)$

$\cancel{2A}+\cancel{B}+2C+D=2$

$-\cancel{2A}-\cancel{B}-C-2D=-1$

$C-D=1$

$C=1+D$

$C=1+1$

$\boxed{C=2}$

$A+C=0$

$A=-C$

$\boxed{A=-2}$

$A+B+D=0$

$-C+B+D=0$

$-(1+D)+B+D=0$

$-1-\cancel{D}+B+\cancel{D}=0$

$-1+B=0$

$\boxed{B=1}$

$2A+B+C+2D=1$

$-2C+B+1+D+2D=1$

$-2C+B+3D=0$

$-2(1+D)+B+3D=0$

$-2-2D+B+3D=0$

$B+D=2$

$1+D=2$

$D=2-1$

$\boxed{D=1}$

$$\int\frac{2x^2+x}{(x+1)^2(x^2+x+1)}dx = -2\int\frac{dx}{x+1} + \int\frac{dx}{(x+1)^2} + \int\frac{(2x+1)dx}{x^2+x+1}$$

$u = x^2+x+1$

$du = (2x+1)dx$

$$= -2\ln|x+1| - \frac{1}{x+1} + \int\frac{du}{u} + C$$

$$= \boxed{-2\ln|x+1| - \frac{1}{x+1} + \ln|x^2+x+1| + C}$$

4) $\int_{-\infty}^{\infty} \frac{dx}{4x^2+4x+5} = \lim_{a\to-\infty} \int_a^0 \frac{dx}{4x^2+4x+5} + \lim_{b\to\infty} \int_0^b \frac{dx}{4x^2+4x+5}$

$\int \frac{dx}{4x^2+4x+5} = \int \frac{dx}{4(x^2+x+\frac{5}{4})} = \int \frac{dx}{4(x^2+x+\frac{1}{4}-\frac{1}{4}+\frac{5}{4})} = \frac{1}{4}\int \frac{dx}{x^2+x+\frac{1}{4}+1} = \frac{1}{4}\int \frac{dx}{(x+\frac{1}{2})^2+1}$

$= \frac{1}{4}\int \frac{du}{u^2+a^2} = \frac{1}{4}\tan^{-1}(x+\frac{1}{2})+C$

$u = x+\frac{1}{2}$

$a = 1$

$= \lim_{a\to-\infty} \left[\frac{1}{4}\tan^{-1}(x+\frac{1}{2})\right]_a^0 + \lim_{b\to\infty} \left[\frac{1}{4}\tan^{-1}(x+\frac{1}{2})\right]_0^b$

$= -\lim_{a\to-\infty} \frac{1}{4}\tan^{-1}(-\infty+\frac{1}{2}) + \frac{1}{4}\tan^{-1}(0+\frac{1}{2}) - \frac{1}{4}\tan^{-1}(0+\frac{1}{2}) + \lim_{b\to\infty} \frac{1}{4}\tan^{-1}(\infty+\frac{1}{2})$

$= -(\frac{1}{4})(-\frac{\pi}{2}) + \cancel{(\frac{1}{4})\tan^{-1}(\frac{1}{2}) - \frac{1}{4}\tan^{-1}(\frac{1}{2})} + (\frac{1}{4})(\frac{\pi}{2})$

$= \frac{\pi}{8} + \frac{\pi}{8} = \boxed{\frac{\pi}{4}}$ ← convergente

5) $y=\ln x$, eje y, y $x=e^2$

$$V = 2\pi \lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} \left[R(x_i)\right]^2 \Delta x$$

$$= 2\pi \int_a^b \left[R(x)\right]^2 dx$$

$$= 2\pi \int_1^{e^2} (\ln x)^2 \, dx$$

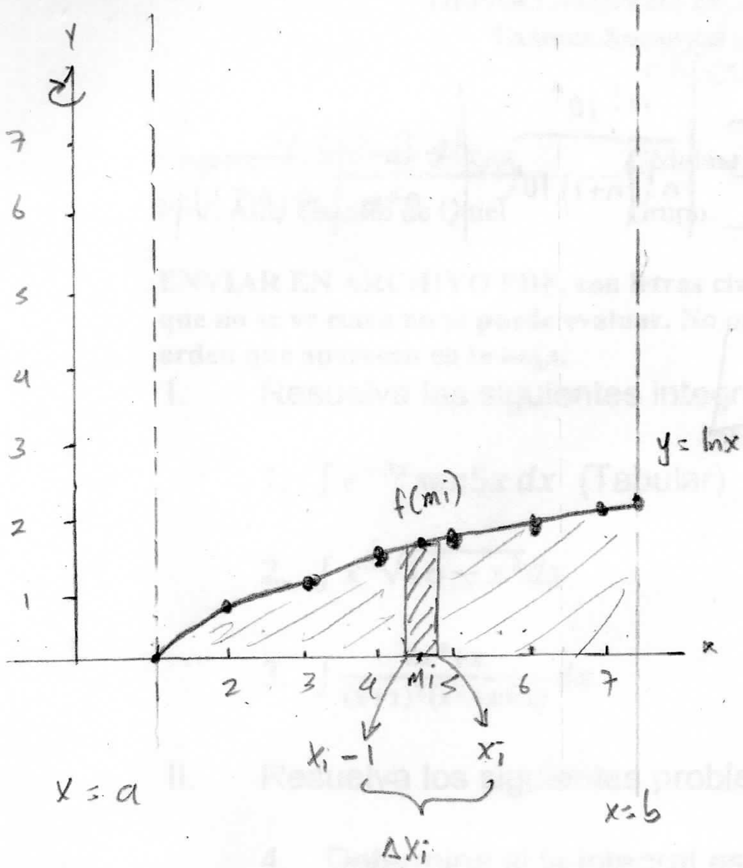
$$= 2\pi \left[x(\ln x)^2 - 2x\ln x + 2x \right]_1^{e^2}$$

$$= 2\pi \left[e^2(\ln e^2)^2 - 2e^2 \ln e^2 + 2e^2 - \left((\ln 1)^2 - 2\ln 1 + 2\right) \right]$$

$$= 2\pi \left[\cancel{4e^2} - \cancel{2e^2(2)} + 2e^2 - (0 - 0 + 2) \right]$$

$$= 2\pi \left[2e^2 - 2 \right] = \boxed{(4\pi e^2 - 4\pi)\, u^3}$$

Volumen de la figura



$$= \int (\ln x)^2 dx \rightarrow = \int (\ln x)^2 (1)\, dx = x(\ln x)^2 - \int x\left(\frac{2}{x}\right) \ln x \, dx$$

$u = (\ln x)^2$ $\quad \int v = dx$

$du = \frac{2}{x} \ln x \, dx$ $\quad v = x$

$$= x(\ln x)^2 - 2\int \ln x \, dx$$

$$= x(\ln x)^2 - 2\left(x\ln x - \int x\left(\frac{1}{x}\right) dx\right)$$

$u = \ln x$ $\quad \int v = dx$

$du = \frac{1}{x} dx$ $\quad v = x$

$$= x(\ln x)^2 - 2\left(x \ln x - \int dx\right)$$

$$= x(\ln x)^2 - 2(x\ln x - x)$$

$$= x(\ln x)^2 - 2x\ln x + 2x + C$$

6) $\sum_{n=1}^{\infty} \frac{(n+2)!}{n!\,10^n} = \sum_{n=1}^{\infty} \frac{(n+1+1)!}{n!\,10^n} = \lim_{n\to\infty} \left| \frac{\frac{(n+2)!}{(n+1)!\,10^{n+1}}}{\frac{(n+2)!}{n!\,10^n}} \right|$

$= \lim_{n\to\infty} \left| \frac{\cancel{(n+2)!}}{(n+1)!\,10^{n+1}} \cdot \frac{n!\,10^n}{\cancel{(n+2)!}} \right| = \lim_{n\to\infty} \left| \frac{n!\,10^n}{n!(n+1)\,10^{n+1}} \right| = \lim_{n\to\infty} \left| \frac{1}{10(n+1)} \right|$

$= \frac{1}{10(\infty+1)} = 0 \leftarrow$ converge