Práctica 2.4

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Cálculo
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Descomponer → $y = f(g(x))$

1) $y = (5x-8)^4$

$u = g(x) = 5x - 8$

$y = f(u) = u^4$

3) $y = \sqrt{x^3-7}$

$u = g(x) = x^3 - 7$

$y = f(u) = \sqrt{u}$

5) $y = \csc^3 x = (\csc x)^3$

$u = g(x) = \csc x$

$y = f(u) = u^3$

derivada

9) $g(x) = 3(4-9x)^4$

$g'(x) = 3(4-9x)^{4'}$

$= 3(4)(4-9x)^3(-9)$

$= -108(4-9x)^3$

13) $y = \sqrt[3]{6x^2+1}$

$y = (6x^2+1)^{1/3}$

$y' = \frac{1}{3}(6x^2+1)^{-2/3}(12x)$

$y' = \frac{12x}{3\sqrt[3]{(6x^2+1)^2}}$

$y' = \frac{4x}{\sqrt[3]{(6x^2+1)^2}}$

17) $f(t) = \left(\frac{1}{t-3}\right)^2$

$f'(t) = 2\left(\frac{1}{t-3}\right)\cdot\left(\frac{-1}{(t-3)^2}\right)$

$= \frac{-2}{(t-3)^3}$

21) $y = \frac{1}{\sqrt{3x+5}}$

$y' = \frac{-(\sqrt{3x+5})'}{3x+5}$

$y' = \frac{-(3x+5)^{1/2'}}{3x+5}$

$= \frac{\frac{1}{2}(3x+5)^{-1/2}\cdot(3)}{3x+5}$

$= \frac{\frac{3}{2}(3x+5)^{-1/2}}{3x+5}$

$= \frac{-3}{2\sqrt{3x+5}}$

$= \frac{-3}{2\sqrt[3]{(3x+5)^2}}$

25) $y = x\sqrt{1-x^2}$

$y' = x'\left(\sqrt{1-x^2}\right) + \left(\sqrt{1-x^2}\right)'\cdot x$

$y' = \sqrt{1-x^2} + \left((1-x^2)^{1/2}\right)' x$

$y' = \sqrt{1-x^2} + \frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot(-2x)(x)$

$y' = \sqrt{1-x^2} + \frac{-2x^2}{2\sqrt{1-x^2}}$

$y' = \sqrt{1-x^2} + \frac{-x^2}{\sqrt{1-x^2}}$

$y' = \frac{1-x^2-x^2}{\sqrt{1-x^2}}$

$y' = \frac{1-2x^2}{\sqrt{1-x^2}}$

27) $y = \frac{x}{\sqrt{x^2+1}}$

$y' = \frac{\sqrt{x^2+1} - (x^2+1)^{1/2'}\cdot x}{x^2+1}$

$= \frac{\sqrt{x^2+1} - \frac{1}{2}(x^2+1)^{-1/2}\cdot 2x(x)}{x^2+1}$

$= \frac{\sqrt{x^2+1} - \frac{2x^2}{2\sqrt{x^2+1}}}{x^2+1}$

$= \frac{\frac{2(x^2+1) - 2x^2}{2\sqrt{x^2+1}}}{x^2+1}$

$= \frac{\frac{2x^2+2-2x^2}{2\sqrt{x^2+1}}}{x^2+1}$

$= \frac{\frac{1}{\sqrt{x^2+1}}}{x^2+1} = \frac{1}{\sqrt[3]{(x^2+1)^2}}$

31) $f(v)=\left(\frac{1-2v}{1+v}\right)^3$

$f'(v)=3\left(\frac{1-2v}{1+v}\right)^2\cdot\left(\frac{-2\cdot(1+v)-(1)(1-2v)}{(1+v)^2}\right)$

$=3\left(\frac{1-2v}{1+v}\right)^2\cdot\left(\frac{-2-2v-1+2v}{(1+v)^2}\right)$

$=3\left(\frac{1-2v}{1+v}\right)^2\left(\frac{-3}{(1+v)^2}\right)$

$=\left(\frac{-9}{(1+v)^2}\right)\left(\frac{1-2v}{1+v}\right)^2$

$=\left(\frac{-9}{(1+v)^2}\right)\frac{(1-2v)^2}{(1+v)^2}$

$=\frac{-9(1-2v)^2}{(1+v)^4}$

33) $f(x)=\left((x^2+3)^5+x\right)^2$

$f'(x)=2\left((x^2+3)^5+x\right)\left(5(x^2+3)^4(2x)+1\right)$

$f'(x)=\left(2(x^2+3)^5+2x\right)\left(10x(x^2+3)^4+1\right)$

$=20x(x^2+3)^9+2(x^2+3)^5+20x^2(x^2+3)^4+2x$

45) $g(x)=5\tan 3x$

$g'(x)=5(\tan 3x)'$

$=5(\sec^2 3x\cdot 3)$

$=15\sec^2 3x$

47) $y=\operatorname{sen}(\pi x)^2$

$y'=\cos(\pi x)^2\cdot 2(\pi x)\cdot 1$

$y'=\cos(\pi x)^2\cdot 2\pi x$

$y'=2\pi x\cos(\pi x)^2$

49) $h(x)=\operatorname{sen}2x\cos 2x$

$h(x)=\operatorname{sen}2x\cos 2x$

$h'(x)=(\operatorname{sen}2x)'\cos 2x+(\cos 2x)'\operatorname{sen}2x$

$=\cos 2x(2)\cos 2x+(-\operatorname{sen}2x)(2)\operatorname{sen}2x$

$=2\cos^2 2x-2\operatorname{sen}^2 x$

55) $f(\theta)=\tan^2 5\theta$

$f(\theta)=(\tan 5\theta)^2$

$f'(\theta)=2\tan 5\theta(\sec^2 5\theta)(5)$

$=10\tan 5\theta\sec^2 5\theta$

59) $f(t)=3\sec^2(\pi t-1)$

$f(t)=3(\sec(\pi t-1))^2$

$f'(t)=3(2)(\sec(\pi t-1))(\sec(\pi t-1))(\tan(\pi t-1))(\pi)$

$=6\pi\sec^2(\pi t-1)\tan(\pi t-1)$

63) $y=\operatorname{sen}(\tan 2x)$

$y'=\cos(\tan 2x)\cdot\sec^2 2x\cdot 2x$

$=2x\sec 2x$

E.C. Recta tangente y Gráfica

77) $f(x)=\operatorname{sen}2x\quad(\pi,0)$

$f'(x)=2\cos 2x$

$m=2\cos 2x$

$y-y_1=m(x-x_1)$

$y-0=2\cos 2(\pi)(x-\pi)$

$y=2x-2\pi$

79) $f(x) = \tan^2 x \quad \left(\frac{\pi}{4}, 1\right)$

$f(x) = (\tan x)^2$

$f'(x) = 2\tan x \sec^2 x$

$m = 2\tan x \sec^2 x$

$y - y_1 = m(x - x_1)$

$y - 1 = 2\tan x \sec^2 x \left(x - \frac{\pi}{4}\right)$

$y - 1 = 2\tan\left(\frac{\pi}{4}\right)\sec^2\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)$

$y - 1 = 4\left(x - \frac{\pi}{4}\right)$

$y - 1 = 4x - \pi$

$y = 4x - \pi + 1$

87) Segunda derivada

$f(x) = \frac{1}{x-6}$

$f'(x) = \frac{-1}{(x-6)^2}$

$f''(x) = \frac{-2(x-6)(1)(-1)}{(x-6)^4}$

$= \frac{2}{(x-6)^3}$

81) $f(x) = \sqrt{25 - x^2} \quad (3, 4)$

$f(x) = (25 - x^2)^{1/2}$

$f'(x) = \frac{1}{2}(25 - x^2)^{-1/2}(-2x)$

$= -x(25 - x^2)^{-1/2}$

$m = \frac{-x}{\sqrt{25 - x^2}}$

$y - y_1 = m(x - x_1)$

$y - y_1 = \frac{-x}{\sqrt{25 - x^2}}(x - x_1)$

$y - y_1 = \frac{-3}{\sqrt{25 - (3)^2}}(x - x_1)$

$y - 4 = \frac{-3}{4}(x - 3)$

$y - 4 = \frac{-3x}{4} + \frac{9}{4}$

$y = \frac{-3x}{4} + \frac{9}{4} + \frac{4}{1}$

$y = \frac{-3x}{4} + \frac{25}{4}$

89) $f(x) = \operatorname{sen} x^2$

$f'(x) = \cos(x^2)\,2x$

$f'(x) = 2x\cos x^2$

$f''(x) = (2x)(\cos(x^2))'$

$= (2x)'(\cos(x^2)) + (\cos(x^2))'(2x)$

$= 2\cos x^2 + (-\operatorname{sen} x^2)(2x)(2x)$

$= 2\cos x^2 - \operatorname{sen} x^2\, 4x^2$

$f''(x) = 2\cos x^2 - 4x^2 \operatorname{sen} x^2$