Universidad Tecnológica de Panamá
Facultad de Ciencia y Tecnología
Departamento de Ciencias Exactas
Taller N 1 - Cálculo II

Fecha: 4/9/2020

Nombre: Kevin Busch **Cédula: 8-957-1720**
Nombre: Robert Lu Zheng **Cédula: 3-750-1980**
Nombre: Roberto Pineda **Cédula: 8-952-1794**
Nombre: Ricardo Córdoba **Cédula: 8-970-1320**

Prof. Alba Castillo de Quiel

Integre: (60 puntos)

1. $\int (\frac{5}{x^3} + csc^2 4x)dx$
2. $\int \frac{x^2}{(x^3+6)^2}\, dx$
3. $\int x(2x^2+1)7^{(2x^2+1)^2}\, dx$
4. $\int \frac{x^3-6x-20}{x+5} dx$
5. $\int e^{2x} cot\,(e^{2x})dx$
6. $\int_{-1}^{1} \frac{1}{2x+3} dx$
7. $\int_{0}^{2} \frac{x}{\sqrt{1+2x^2}} dx$

Resuelva los problemas **(40 puntos)**

8. **Determine el área de la región limitada por las gráficas $f(x) = -x^2 + 4x + 1$ y $g(x) = x + 1$.**
9. **Determine el área de la región limitada por las gráficas de $f(y) = y^2 + 1$ y $g(y) = y + 3$**

problema 1

Taller #1 Cálculo II

1) $\int \left(\frac{5}{x^3} + \csc^2 4x\right) dx$

$= \int 5x^{-3} dx + \int (\csc 4x)^2 dx = 5\int x^{-3} dx + \int (\csc u)^2 \frac{du}{4} = 5\int x^{-3} dx + \frac{1}{4}\int (\csc u)^2 du$

$u = 4x$

$du = 4dx$

$\frac{du}{4} = dx$

$= \frac{5x^{-2}}{-2} + \frac{1}{4}(-\cot u) + C = \boxed{-\frac{5}{2x^2} - \frac{1}{4}\cot 4x + C}$

Problemas 2 y 3

04-SEP-2020

Calculo II

Taller

$u = x^3+6$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

2. $\int \frac{x^2}{(x^3+6)^2}\,dx$

$= \int \frac{du/3}{u^2} = \frac{1}{3}\int u^{-2}\,du = \frac{1}{3}\left| \frac{u^{-1}}{-1} \right| + C$

$= \frac{1}{-3u} + C = -\frac{1}{3(x^3+6)} + C$

$u = (2x^2+1)^2 = (2x^2+1)(2x^2+1)$

$du = (2x^2+1)(2x^2+1)$

$du = (4x+0)(2x^2+1) + (2x^2+1)(4x+0)$

$du = 8x^3+4x+8x^3+4x$

$du = 8x(2x^2+1)$

$\frac{du}{8} = x(2x^2+1)\,dx$

3. $\int x(2x^2+1)7^{(2x^2+1)^2}\,dx$

$\frac{1}{8}\int 7^u\,du = \frac{7^u}{8\ln 7} + C$

$= \frac{7^{(2x^2+1)^2}}{8\ln 7} + C$

Problema 4

④ $\int \frac{x^2-6x-20}{x+5}\,dx = \int x - 11 + \frac{35}{x+5} = \int x\,dx - 11\int dx + 35\int \frac{du}{u}$

$x^2-6x-20 \div x+5 = x - 11 + \frac{35}{x+5}$

$-x^2 - 5x$

$-11x - 20$

$11x + 55$

35

$u = x+5$

$du = 1\,dx$

$= \frac{x^2}{2} - 11x + 35\ln u = \frac{x^2-22x}{2} + 35\ln(|x+5|) + C$

Problema 5

⑤ $\int e^{2x} \cot(e^{2x})\,dx = \int \cancel{e^{2x}} \cot(u) \cdot \frac{\cancel{e^{-2x}}\,du}{2}$

$= \frac{1}{2}\int \cot u\,du = \frac{1}{2}\int \frac{\cancel{\cos u}}{\operatorname{sen} u} \cdot \frac{1}{\cancel{\cos u}}$

$= \frac{1}{2}\int \frac{1}{v}\,dv = \frac{1}{2}\left[\ln v\right]$

$= \frac{\ln(\operatorname{sen} u)}{2}$

$u = e^{2x}$

$du = 2e^{2x}$

$\frac{du}{2e^{2x}} = \frac{du\,e^{-2x}}{2}$

$v = \operatorname{sen} u$

$dv = \cos u$

$\frac{dv}{\cos u} = du$

Problema 6

(6) $\int_{-1}^{1} \frac{1}{2x+3}\, dx = \int_{-1}^{1} \frac{du/2}{u} =$

$u = 2x+3$

$du = 2\,dx$

$\frac{du}{2} = dx$

$= \int_{-1}^{1} \frac{du}{u}$

$= \frac{1}{2}\left[\ln|u|\right]_{-1}^{1}$

$= \frac{1}{2}\left[\ln|2x+3| + C\right]_{-1}^{1}$

$= \frac{1}{2}\left[\ln|2(1)+3|\right] - \left[\frac{1}{2}\left(\ln|2(-1)+3|\right)\right]$

$= \frac{1}{2}\left[\ln(5)\right] - \left[\frac{1}{2}\left(\ln 1\right)\right]$

$= \frac{1}{2}\left[\ln(5) - \right] - \left[\cancel{\frac{1}{2}(0)}\right]$

$\frac{1}{2}\ln(5)$

Problema 7

(7) $\int_0^2 \frac{x}{\sqrt{1+2x^2}}\,dx = \int_0^2 \frac{x}{\sqrt{u}} \cdot \frac{du}{4x} = \frac{1}{4}\int_0^2 \frac{1}{\sqrt{u}}\,du$

$u = 1+2x^2$

$du = 4x\,dx$

$\frac{du}{4x} = dx$

$= \frac{1}{4}\int_0^2 u^{-1/2}\,du = \frac{1}{4}\left[\frac{2\sqrt{u}}{1}\right] + C = \frac{1}{4}\left[2\sqrt{1+2x^2}\right]$

$= \frac{1}{4}\left[2\sqrt{1+2(2)^2}\right] - \frac{1}{4}\left[2\sqrt{1+2(0)^2}\right]$

$= \frac{1}{4}\left[2\sqrt{9}\right] - \frac{1}{4}\left[2(1)\right] = \frac{3}{2} - \frac{1}{2}$

$= \frac{\cancel{2}}{\cancel{2}} = 1$

Problema 8

8) $F(x) = -x^2 + 4x + 1$, $g(x) = x + 1$

$x = \frac{4}{2(-1)} = \frac{-4}{-2} = +2$

$-(2)^2 + 4(2) + 1$

$-4 + 8 + 1$

$V(2,5)$

$y = 5$

$x + 1 = -x^2 + 4x + 1$

$x^2 - 3x = 0$

$x(x-3) = 0$

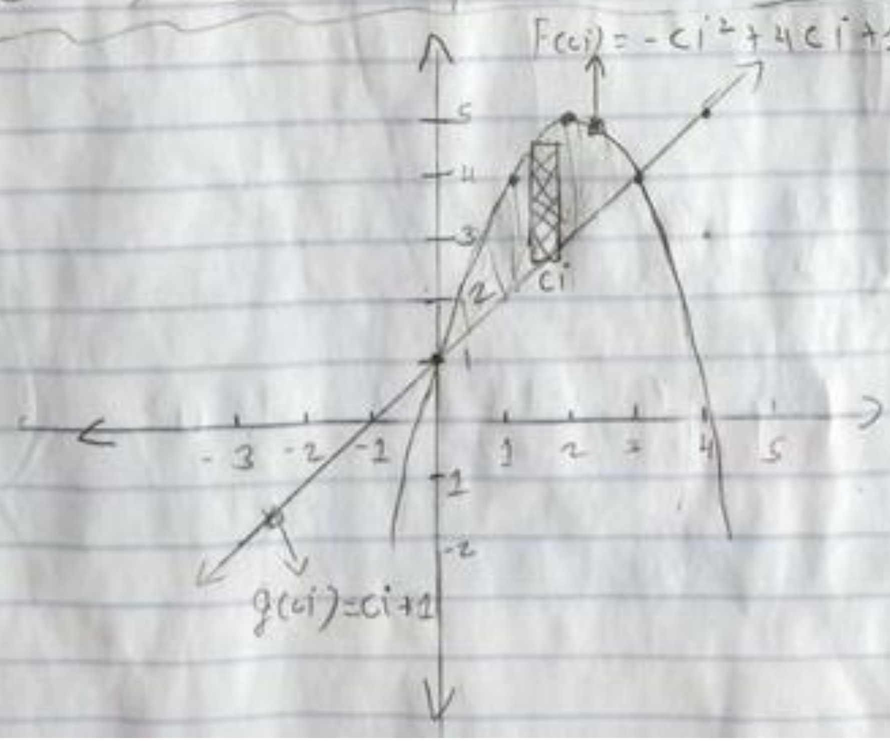
$x - 3 = 0$

$x = 0 \quad x = 3$

$g(0) = 0 + 1 = 1$

$g(3) = 3 + 1 = 4$

$P_1(0,1) \quad P_2(3,4)$



$$A = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} [-ci^2 + 4ci + 1] - [ci + 1] \Delta x_i$$

$$A = \int_0^3 ([-x^2 + 4x + 1] - [x + 1] \, dx$$

$$A = \int_0^3 [-x^2 + 3x] \, dx$$

$$A = \left[-\frac{x^3}{3} + 3\left(\frac{x^2}{2}\right) + c \right]_0^3$$

$$A = \left[\frac{-x^3}{3} + \frac{3x^2}{2} + C \right]_0^3$$

$$A = \left[\frac{-(3)^3}{3} + \frac{3(3)^2}{2} \right] - \left[\cancel{\frac{(0)^3}{3}} + \cancel{\frac{3(0)^2}{2}} \right]$$

$$A = -\frac{27}{3} + \frac{27}{2}$$

$$A = -9 + \frac{27}{2} = \frac{-18 + 27}{2} = \frac{9}{2} \, u^2$$

Problema 9

9) $f(y) = y^2+1$, $g(x) = y+3$

$y = \frac{-b}{2a} = \frac{-0}{2(1)} = 0$ $\quad V(1,0)$

$x = y^2+1$

$0 = y^2+1$

$y^2 = -1$

$y = \sqrt{-1} \rightarrow$ no existe

$0 = y+3$ $\quad (0,-3)$

$y = -3$

$x = y+3$

$x = 0+3$ $\quad (3,0)$

$x = 3$

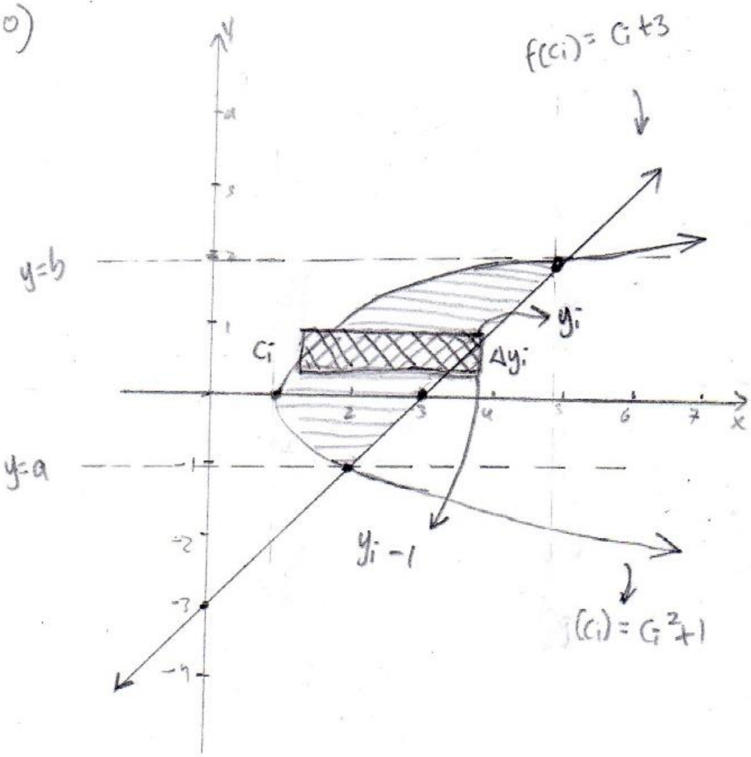
$y^2+1 = y+3$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2 \quad y = -1$ interseccioms

$(5,2);(2,-1)$



$A = \lim_{\|\Delta\| \to 0} \sum_{i=0}^{n} \left[(c_i+3) - (c_i^2+1)\right] \Delta y_i$

$= \int_{-1}^{2} (y+3-y^2-1)\,dy$

$= \int_{-1}^{2} (-y^2+y+2)\,dy$

$= -\int_{-1}^{2} y^2\,dy + \int_{-1}^{2} y\,dy + 2\int_{-1}^{2} dy$

$= \left[\frac{-y^3}{3} + \frac{y^2}{2} + 2y\right]_{-1}^{2}$

$= \left[\frac{-(2)^3}{3} + \frac{(2)^2}{2} + 2(2)\right] - \left[\frac{-(-1)^3}{3} + \frac{(-1)^2}{2} + 2(-1)\right]$

$= \frac{-8}{3} + 6 - \frac{1}{3} - \frac{1}{2} + 2 = -3 - \frac{1}{2} + 8 = 5 - \frac{1}{2} = \boxed{\frac{9}{2} u^2}$