Problemas 2.3

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Calculo 1
3-750-1980

3) $h(t) = \sqrt{t}(1-t^2)$

$h(t) = t^{1/2}(1-t^2)$

$h'(t) = t^{1/2\,\prime}(1-t^2) + (1-t^2)' t^{1/2}$

$= \frac{1}{2}t^{-1/2}(1-t^2) + (-2t\, t^{1/2})$

$= \frac{1-t^2}{2\sqrt{t}} - \frac{2\sqrt{t^3}}{1}$

$= \frac{(1-t^2)-4t^2}{2\sqrt{t}}$

$= \frac{1-t^2-4t^2}{2\sqrt{t}}$

$= \frac{1-5t^2}{2\sqrt{t}}$

$= \boxed{\frac{2\sqrt{t}(1-5t^2)}{4t}}$

7) $f(x) = \frac{x}{x^2+1}$

$f'(x) = \frac{x'(x^2+1) - (x^2+1)' \cdot x}{(x^2+1)^2}$

$= \frac{x^2+1-2x^2}{(x^2+1)^2}$

$= \boxed{\frac{-x^2+1}{(x^2+1)^2}}$

9) $h(x) = \frac{\sqrt{x}}{x^3+1}$

$h(x) = \frac{x^{1/2}}{x^3+1}$

$h'(x) = \frac{x^{1/2\,\prime}(x^3+1) - (x^3+1)' \cdot x^{1/2}}{(x^3+1)^2}$

$= \frac{1}{2}x^{-1/2}(x^3+1) - (3x^2)(x^{1/2})$

$= \frac{x^3+1}{2\sqrt{x}} - \frac{3x^{5/2}}{1}$

$= \frac{x^3+1-6x^3}{2\sqrt{x}}$

$= \frac{-5x^3+1}{2\sqrt{x}}$

$= \frac{-10x^{7/2}+2\sqrt{x}}{4x}$

$= \boxed{\frac{-5x^{7/2}+\sqrt{x}}{2x}}$

11) $g(x) = \frac{\operatorname{sen} x}{x^2}$

$g(x) = (\operatorname{sen} x)(x^{-2})$

$g'(x) = (\operatorname{sen} x)' x^{-2} + (x^{-2})'(\operatorname{sen} x)$

$= (\cos x)(x^{-2}) + (-2x^{-3})(\operatorname{sen} x)$

$= \frac{\cos x}{x^2} - \frac{\operatorname{sen} x}{2x^3}$

$= \frac{2x^3\cos x - x^2 \operatorname{sen} x}{2x^5}$

$= \boxed{\frac{2x\cos x - \operatorname{sen} x}{2x^3}}$

13) $f(x) = (x^3+4x)(3x^2+2x-5)$

$c=0$

$f'(x) = (x^3+4x)'(3x^2+2x-5) + (3x^2+2x-5)'(x^3+4x)$

$= (3x^2+4)(3x^2+2x-5) + (6x+2)(x^3+4x)$

$= (9x^4+6x^3-15x^2+12x^2+8x-20) + (6x^4+24x^2+2x^3+8x)$

$= 9x^4+6x^3-3x^2+8x-20+6x^4+24x^2+2x^3+8x$

$= \boxed{15x^4+8x^3+21x^2+16x-20}$

$f(0) = 15(0)^4 + \cancel{8(0)^3} + 21\cancel{(0)^2} + \cancel{16(0)} - 20$

$= \boxed{-20}$

15) $f(x) = \frac{x^2-4}{x-3}$

$c=1$

$f'(x) = \frac{(x^2-4)'(x-3) - (x-3)'(x^2-4)}{(x-3)^2}$

$= \frac{2x(x-3) - (1)(x^2-4)}{(x-3)^2}$

$= \frac{2x^2-6x-x^2-4}{(x-3)^2}$

$= \frac{x^2-6x-4}{(x-3)^2}$

$f'(1) = \frac{(1)^2-6(1)-4}{(1-3)^2}$

$= \frac{1-6-4}{(-2)^2}$

$= \boxed{\frac{-9}{4}}$

17) $f(x) = x\cos x$

$c = \frac{\pi}{4}$

$f'(x) = x'\cos x + \cos x' x$

$= \cos x + (-\operatorname{sen} x)x$

$= \cos x - x\operatorname{sen} x$

$f'(\frac{\pi}{4}) = \cos\frac{\pi}{4} - \frac{\pi}{4}\operatorname{sen}\frac{\pi}{4}$

$= \frac{1}{\sqrt2} - \frac{\pi}{4}\cdot\frac{1}{\sqrt2}$

$= \frac{1}{\sqrt2} - \frac{\pi}{4\sqrt2}$

$= \frac{4\sqrt2 - \pi\sqrt2}{8}$

$= \boxed{\frac{\sqrt2}{2} - \frac{\pi\sqrt2}{8}}$

21) $y = \frac{6}{7x^2}$

$y = (6)(7x^2)^{-1}$

$y' = 6'(7x^2)^{-1} + (7x^2)^{-1'}(6)$

$= (7x^2)^{-1} + (-7x^2)^{-2}(6)$

$= \frac{1}{7x^2} - \frac{6}{49x^4}$

$= \frac{49x^4 - 42x^2}{343x^6}$

$= \frac{x^2(49x^2-42)}{343x^6}$

$= \boxed{\frac{49x^2-42}{343x^4}}$

23) $y = \frac{4x^{3/2}}{x}$

$y = (4x^{3/2})x^{-1}$

$y' = (4x^{\frac32})'x^{-1} + (x^{-1})'(4x^{\frac32})$

$= (6x^{1/2})(x^{-1}) + (-x^{-2})(4x^{\frac32})$

$= \frac{6x^{1/2}}{x} - \frac{4x^{3/2}}{x^2}$

$= \frac{6x^{5/2} - 4x^{5/2}}{x^3}$

$= \frac{2x^{5/2}}{x^3} = \frac{2}{\sqrt x} = \boxed{\frac{2\sqrt x}{x}}$

25) $(2x^3+5x)(x-3)(x+2)$

$f(x) = 2x^5 - 2x^4 - 7x^3 - 5x^2 - 30x$

$\boxed{f'(x) = 10x^4 - 8x^3 - 21x^2 - 10x - 30}$

19) $y = \frac{x^2+3x}{7}$

$= (x^2+3x)(7^{-1})$

$y' = (x^2+3x)'(7^{-1}) + (7^{-1})'(x^2+3x)$

$= \frac{2x+3}{7} + (-7^{-2})(x^2+3x)$

$= \frac{2x+3}{7} - \frac{x^2+3x}{49}$

$= \frac{98x + 147 - 7x^2 - 21x}{343}$

$= \boxed{\frac{-7x^2+77x+147}{343}}$

29) $f(x) = \frac{3x-1}{\sqrt x} = (3x-1)(x^{-1/2})$

$f'(x) = (3x-1)'(x^{-1/2}) + (x^{-1/2})'(3x-1)$

$= 3x^{-1/2} + (-\frac12 x^{-3/2})(3x-1)$

$= \frac{3}{\sqrt x} - \frac{3x-1}{2\sqrt{x^3}}$

$= \boxed{\frac{6\sqrt{x^3} - 3x\sqrt x + \sqrt x}{2x^2}}$

31) $h(x) = (x^3-2)^2$

$h'(x) = 2(x^3-2)(2x^2)$

$= 4x^2(x^3-2)$

$= \boxed{4x^5 - 8x^2}$

33) $f(x) = \frac{2-\frac1x}{x-3}$

$f(x) = \frac{\frac{2x-1}{x}}{x-3}$

$f(x) = \frac{2x-1}{x^2-3x}$

$f'(x) = \frac{(2x-1)'(x^2-3x) - (x^2-3x)'(2x-1)}{(x^2-3x)^2}$

$= \frac{2(x^2-3x) - (2x-3)(2x-1)}{(x^2-3x)^2}$

$= \frac{2x^2-6x-(4x^2-2x-6x+3)}{(x^2-3x)^2}$

$= \frac{2x^2-6x-4x^2+2x+6x-3}{(x^2-3x)^2}$

$= \boxed{\frac{-2x^2+2x-3}{(x^2-3x)^2}}$

41) $f(t) = \frac{\cos t}{t}$

$f(t) = (\cos t)(t^{-1})$

$= \cos(t)' t^{-1} + (t^{-1})'(\cos(t))$

$= \frac{-\operatorname{sen} t}{t} + (-t^{-2})(\cos t)$

$= \frac{-\operatorname{sen} t}{t} - \frac{\cos(t)}{t^2}$

$= \frac{-t^2 \operatorname{sen} t - t\cos(t)}{t^3}$

$= \boxed{\frac{-t\operatorname{sen} t - \cos(t)}{t^2}}$

51) $f(x) = x^2 \tan x$

$f'(x) = (x^2)' \tan x + \tan x' \, x^2$

$= 2x \tan x + \sec^2 x \, x^2$

$= \boxed{2x \tan x + x^2 \sec^2 x}$

67) $f(x) = \tan x \quad (\frac{\pi}{4}, 1)$ *

Ec. Recta tangente

$f'(x) = \tan x$

$= \sec^2 x$

$m = \sec^2(\frac{\pi}{4}) = \left(\frac{1}{\cos\frac{\pi}{4}}\right)^2$

$= \left(\frac{1}{\frac{1}{\sqrt{2}}}\right)^2 = (\sqrt{2})^2 = \boxed{2}$

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x - \frac{\pi}{4})$

$y - 1 = \frac{2x}{1} - \frac{\pi}{2}$

$\boxed{y = \frac{4x - \pi}{2} + 1}$

47) $y = \frac{3(1 - \operatorname{sen} x)}{2\cos x}$

$y = \frac{3 - 3\operatorname{sen} x}{2\cos x}$

$y' = \frac{(3 - 3\operatorname{sen} x)'(2\cos x) - (2\cos x)'(3 - 3\operatorname{sen} x)}{(2\cos x)^2}$

$y' = \frac{-3\cos x(2\cos x) - (-2\operatorname{sen} x)(3 - 3\operatorname{sen} x)}{(2\cos x)^2}$

$= \frac{-6\cos^2 x - (-6\operatorname{sen} x + 6\operatorname{sen}^2 x)}{(2\cos x)^2}$

$= \frac{-6\cos^2 x + 6\operatorname{sen} x - 6\operatorname{sen}^2 x}{4\cos^2 x}$

$= \boxed{\frac{-3\cos^2 x + 3\operatorname{sen} x - 3\operatorname{sen}^2 x}{2\cos^2 x}}$

53) $y = 2x\operatorname{sen} x + x^2 \cos x$

$y' = (2x' \operatorname{sen} x + \operatorname{sen} x' \, 2x) + (x^{2\prime} \cos x + \cos x' \, x^2)$

$= 2\operatorname{sen} x + 2x\cos x + 2x\cos x - x^2 \operatorname{sen} x$

$= \boxed{2\operatorname{sen} x + 4x\cos x - x^2 \operatorname{sen} x}$

73) $f(x) = \frac{2x - 1}{x^2}$

Recta tangente Horizontal

$f'(x) = \frac{(2x-1)' x^2 - x^{2\prime}(2x-1)}{x^4}$

$= \frac{2x^2 - 4x^2 + 2x}{x^4}$

$= \frac{-2x^2 + 2x}{x^4}$

$0 = \frac{-2x + 2}{x^3}$

$0 = -2x + 2$

$-2x = -2$

$x = 1$

$f(1) = \frac{2(1) - 1}{1^2}$

$f(1) = 1$

$\boxed{P(1,1)}$

63) $f(x) = \frac{x}{x+4} \quad (-5, 5)$ *

Ecuación de la recta tangente

$f'(x) = \frac{x'(x+4) - (x+4)' x}{(x+4)^2}$

$= \frac{x + 4 - x}{(x+4)^2} = \frac{4}{(x+4)^2} = \frac{4}{(-5+4)^2}$

$m = \boxed{4}$

$y - y_1 = m(x - x_1)$

$y - 5 = 4(x + 5)$

$y - 5 = 4x + 20$

$\boxed{y = 4x + 25}$

91) $f(x) = x^4 + 2x^3 - 3x^2 - x$

$f''(x) =$

$f'(x) = 4x^3 + 6x^2 - 6x - 1$

$\boxed{f''(x) = 12x^2 + 12x - 6}$

95) $f(x)=\frac{x}{x-1}$ $\quad f^{99}(x)$

$f'(x)=\frac{x'(x-1)-(x-1)'x}{(x-1)^2}$

$=\frac{\cancel{x}-1-\cancel{x}}{(x-1)^2}$

$f'(x)=\frac{-1}{(x-1)^2}$

$f''(x)=\frac{-1'(x-1)^2-(x-1)^{2'}(-1)}{(x-1)^4}$

$=\boxed{\frac{2x-1}{(x-1)^4}}$

97) $f(x)=x\,\text{sen}x$ $\quad f^{99}(x)=$

$f'(x)=x'\,\text{sen}x+\text{sen}x'\,x$

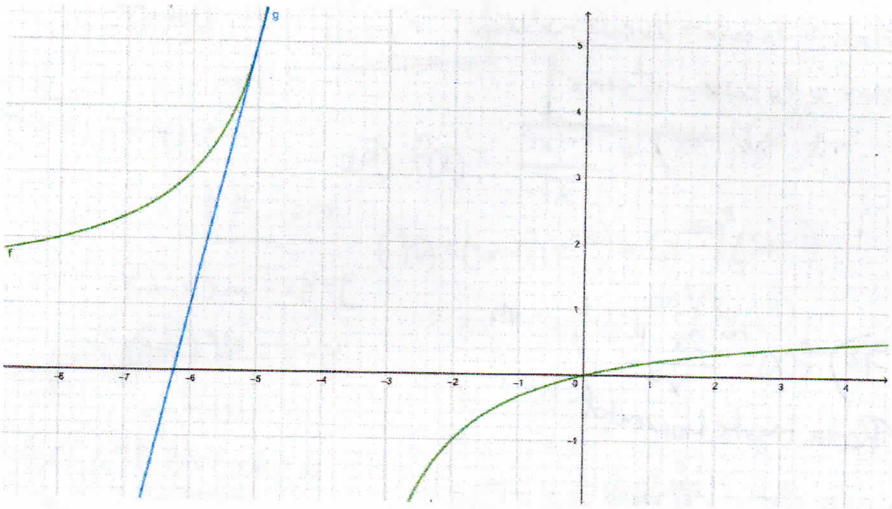
$f'(x)=\text{sen}x+x\cos x$

$f''(x)=\text{sen}x'+x'\cos x+\cos x'\,x$

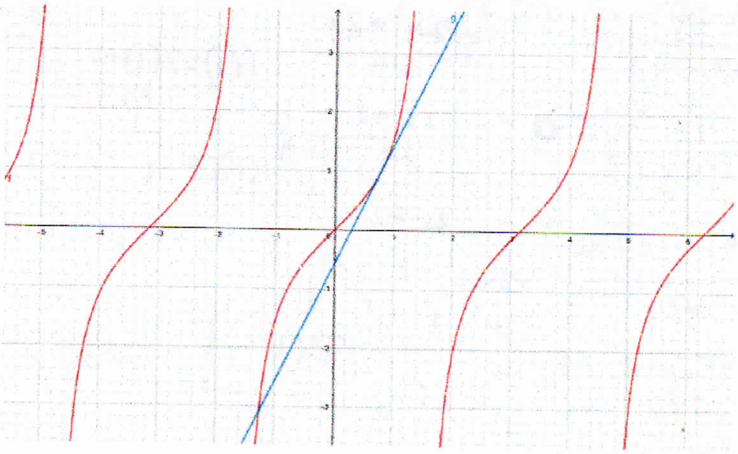
$=\cos x+\cos x-x\,\text{sen}x$

$=\boxed{2\cos x-x\,\text{sen}x}$

Gráfica del problema 65.



Gráfica del problema 67



115) $V(t) = 36 - t^2$

$t = [0, 6]$

Calcular V y Aceleración cuando $t = 3$.

$V(3) = 36 - (3)^2$

$= 36 - 9$

$= 27\ m/s$

$A = -2t$

$A(t) = -2(3)$

$= -6\ m/s^2$

Qué puede decir acerca de la rapidez del objeto cuando la velocidad y aceleración tienen signos opuestos?

La aceleración define la variación de la velocidad. En este caso, al estar negativo, significa que hace decrecer la magnitud de la velocidad.

117) $s(t) = -8.25t^2 + 66t$

Al aplicar los frenos un vehículo viaja a 66 pies/s (45 mi/h).

t	0	1	2	3	4
s(t)	0	57.75	99	123.75	132
v(t)	66	49.5	33	16.5	0
a(t)	-16.5	-16.5	-16.5	-16.5	-16.5

$V(t) = -16.5t + 66$

$A(t) = -16.5$

83) Longitud de rectángulo : $6t + 5$ t = segundos dimensiones = cm

Altura = $\sqrt{t}$

$\frac{dA}{dt} = ?$

A = longitud x altura

$A = (6t+5)\sqrt{t}$

$A = 6\sqrt{t^3} + 5\sqrt{t}$

$A = 6t^{3/2} + 5t^{1/2}$

$A' = (\frac{3}{2})(6)t^{1/2} + 5(\frac{1}{2})t^{-1/2}$

$A' = \frac{9t^{1/2}}{1} + \frac{5}{2\sqrt{t}}$

$A' = \frac{18t + 5}{2\sqrt{t}}\ cm^2/s$

84) Radio $=\sqrt{t+2}$ $\quad t=$ segundos $\quad$ dimensiones: pulgadas

Altura $=\frac{1}{2}\sqrt{t}$

$\frac{dV}{dt}=?$

$V=\pi r^2 h$

$V=\pi\left(\sqrt{t+2}\text{ pulg}\right)^2\left(\frac{1}{2}\sqrt{t}\text{ pulg}\right)$

$V=\pi\left[(t+2)\text{pulg}^2\right]\left(\frac{1}{2}\sqrt{t}\text{ pulg}\right)$

$=\pi\left(\frac{\sqrt{t^3}}{2}+\frac{\sqrt{t}}{1}\text{ pulg}^3\right)$

$V=\pi\left(\frac{\sqrt{t^3}+2\sqrt{t}}{2}\text{ pulg}^3\right)$

$V'=\pi\left(\frac{\sqrt{t^3}+2\sqrt{t}}{2}\right)'\text{ pulg}^3$

$=\pi\left(\frac{t^{3/2}+2t^{1/2}}{2}\right)'\text{ pulg}^3$

$=\pi\left(\frac{2(t^{3/2'}+2t^{1/2'})}{4}\right)\text{ pulg}^3/s$

$=\pi\left(\frac{\frac{3}{2}t^{1/2}+t^{-1/2}}{2}\right)\text{ pulg}^3/s$

$=\pi\left(\frac{\frac{3\sqrt{t}}{2}+\frac{1}{\sqrt{t}}}{2}\right)$

$=\pi\left(\frac{3\sqrt{t}}{4}+\frac{1}{2\sqrt{t}}\right)$

$\frac{dV}{dt}=\frac{6t+4}{8\sqrt{t}}$

$=\boxed{\frac{3t+2}{4\sqrt{t}}\text{ pulg}^3/s}$

85) $C=100\left(\frac{200}{x^2}+\frac{x}{x+30}\right), x\geq 1$

$C'=?$

a) $x=10$

b) $x=15$

c) $x=20$

$C=100\left(\frac{200}{x^2}+\frac{x}{x+30}\right)$

$=100\left(\frac{200x+6000+x^3}{x^3+30x^2}\right)$

$C=100\left(\frac{200x+6000+x^3}{x^3+30x^2}\right)$

$C'=100\left(\frac{(200x+6000+x^3)'(x^3+30x^2)-(x^3+30x^2)'(200x+6000+x^3)}{(x^3+30x^2)^2}\right)$

$C'=100\left(\frac{(200+3x^2)(x^3+30x^2)-(3x^2+60x)(200x+6000+x^3)}{(x^3+30x^2)^2}\right)$

$C'=\frac{-40000x^3-3000x^4+2400000x^2+36000000x}{x^4(x+30)^2}$

$C'(10)=-38.13\$$

$C'(15)=-10.37\$$

$C'(20)=-3.8\$$

El costo por unidad disminuye cuando se piden más componentes.

86) $P(t)=500\left(1+\frac{4t}{50+t^2}\right)$

$t=2h$

$P(t)=500\left(\frac{50+t^2+4t}{50+t^2}\right)$

$P'(t)=-\frac{2000t^2+100000}{(t^2+50)^2}$

$\boxed{P'(2)=\frac{-2000(2)^2+100000}{((2)^2+50)^2}=31.55\text{ bacterias}/h}$