Página 67 - 68

Robert Lu Zhang
Cálculo I
7/4/2020
3-750-1980

1) $h(x) = -x^2 + 4x$

a) $\lim_{x \to 4} h(x) = \lim_{x \to 4} -x^2+4x = -(4)^2+4(4) = -16+16 = 0$

b) $\lim_{x \to -1} h(x) = \lim_{x \to -1} -x^2+4x = -(-1)^2+4(-1) = -1-4 = -5$

3) $f(x) = x\cos x$

a) $\lim_{x \to 0} f(x) = \lim_{x \to 0} x\cos x = 0\cos(0) = 0$

b) $\lim_{x \to \pi/3} f(x) = \lim_{x \to \frac{\pi}{3}} x\cos x = \frac{\pi}{3}\cos\frac{\pi}{3} = \frac{\pi}{3}\left(\frac{1}{2}\right) = \frac{\pi}{5}$

5) $\lim_{x \to 2} x^3 = (2)^3 = 8$

7) $\lim_{x \to 0} (2x-1) = 2(0)-1 = -1$

9) $\lim_{x \to -3} (x^2+3x) = (-3)^2+3(-3) = 9-9 = 0$

11) $\lim_{x \to -3} (2x^2+4x+1) = 2(-3)^2+4(-3)+1 = 2(9)-12+1$
$= 18-12+1 = 7$

13) $\lim_{x \to 3} \sqrt{x+1} = \sqrt{3+1} = \sqrt{4} = 2$

15) $\lim_{x \to -4} (x+3)^2 = (-4+3)^2 = (-1)^2 = 1$

17) $\lim_{x \to 2} \frac{1}{x} = \frac{1}{2}$

19) $\lim_{x \to 1} \frac{x}{x^2+4} = \frac{1}{1+4} = \frac{1}{5}$

21) $\lim_{x \to 7} \frac{3x}{\sqrt{x+2}} = \frac{3(7)}{\sqrt{7+2}} = \frac{3(7)}{\sqrt{9}} = \frac{\cancel{3}(7)}{\cancel{3}}$
$= 7$

23) $f(x) = 5-x$ $\quad g(x) = x^3$

a) $\lim_{x \to 1} f(x) = \lim_{x \to 1} 5-x = 5-1 = 4$

b) $\lim_{x \to 4} g(x) = \lim_{x \to 4} x^3 = (4)^3 = 64$

e) $\lim_{x\to 1} g(f(x)) = \lim_{x\to 1} (5-x)^3 = (5-1)^3 = (4)^3 = 64$

25) $f(x) = 4 - x^2 \qquad g(x) = \sqrt{x+1}$

a) $\lim_{x\to 1} f(x) = \lim_{x\to 1} 4-x^2 = 4-1 = 3$

b) $\lim_{x\to 3} g(x) = \lim_{x\to 3} \sqrt{x+1} = \sqrt{3+1} = \sqrt{4} = 2$

c) $\lim_{x\to 1} g(f(x)) = \sqrt{4-x^2+1} = \sqrt{4-(1)^2+1} = \sqrt{4-1+1} = \sqrt{4} = 2$

27) $\lim_{x\to \frac{\pi}{2}} \operatorname{sen} x = \operatorname{sen}\frac{\pi}{2} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

29) $\lim_{x\to 1} \cos\frac{\pi x}{3} = \frac{\cos \pi(1)}{3} = \frac{\cos \pi}{3} = \frac{1}{2}$

31) $\lim_{x\to 0} \sec 2x = \sec 2(0) = \sec 0 = \frac{1}{\cos 0} = \frac{1}{1} = 1$

33) $\lim_{x\to \frac{5\pi}{6}} \operatorname{sen} x = \operatorname{sen}\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

35) $\lim_{x\to 3} \tan\left(\frac{\pi x}{4}\right) = \tan\left(\frac{3\pi}{4}\right) = 1$

39) $\lim_{x\to c} f(x) = 4$

a) $\lim_{x\to c} [f(x)]^3 = (4)^3 = 64$

b) $\lim_{x\to c} \sqrt{f(x)} = \sqrt{4} = 2$

c) $\lim_{x\to c} [3f(x)] = 3(4) = 12$

d) $\lim_{x\to c} [f(x)]^{3/2} = \sqrt{4^3} = \sqrt{64} = 8$

37) $\lim_{x\to c} f(x) = 3 \qquad \lim_{x\to c} g(x) = 2$

a) $\lim_{x\to c} [5g(x)] = 5(2) = 10$

b) $\lim_{x\to c} [f(x) + g(x)] = 3+2 = 5$

c) $\lim_{x\to c} [f(x)g(x)] = (3)(2) = 6$

d) $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{3}{2}$

41) $g(x) = \frac{x^2 - x}{x} = \frac{\cancel{x}(x-1)}{\cancel{x}} = x-1$

a) $\lim_{x\to 0} g(x) =$ No existe el límite

b) $\lim_{x\to -1} g(x) =$ Si existe el límite

43) $g(x) = \frac{x^3 - x}{x-1} = \frac{x(x^2-1)}{x-1} = \frac{x\cancel{(x-1)}(x+1)}{\cancel{x-1}} = x^2+x$

a) $\lim_{x\to 1} g(x) =$ No existe el límite

b) $\lim_{x\to -1} g(x) =$ Sí existe el límite

45) $\lim\limits_{x \to -1} \dfrac{x^2-1}{x+1} \to -1+1=0$

El límite no existe

$\dfrac{(x-1)\cancel{(x+1)}}{\cancel{x+1}} = x-1$

47) $\lim\limits_{x \to 2} \dfrac{x^3-8}{x-2} \to 2-2=0$

El límite no existe

$\dfrac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}} = x^2+2x+4$

49) $\lim\limits_{x \to 0} \dfrac{x}{x^2-x} = \lim\limits_{x\to 0} \dfrac{\cancel{x}}{\cancel{x}(x-1)} = \lim\limits_{x\to 0} \dfrac{1}{x-1}$

$= \dfrac{1}{0-1} = \dfrac{1}{-1} = -1$

51) $\lim\limits_{x \to 4} \dfrac{x-4}{x^2-16} = \lim\limits_{x\to 4} \dfrac{\cancel{x-4}}{\cancel{(x-4)}(x+4)} = \lim\limits_{x\to 4} \dfrac{1}{x+4}$

$= \dfrac{1}{4+4} = \dfrac{1}{8}$

53) $\lim\limits_{x \to -3} \dfrac{x^2+x-6}{x^2-9} = \lim\limits_{x\to -3} \dfrac{\cancel{(x+3)}(x-2)}{\cancel{(x+3)}(x-3)}$

$= \lim\limits_{x\to -3} \dfrac{x-2}{x-3} = \dfrac{-3-2}{-3-3} = \dfrac{-5}{-6} = \dfrac{5}{6}$

55) $\lim\limits_{x \to 4} \dfrac{\sqrt{x+5}-3}{x-4} = \lim\limits_{x\to 4} \dfrac{\sqrt{x+5}-3}{x-4} \cdot \dfrac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$

$= \lim\limits_{x\to 4} \dfrac{x+5-9}{(x-4)(\sqrt{x+5}+3)} = \lim\limits_{x\to 4} \dfrac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x+5}+3)} = \lim\limits_{x\to 4} \dfrac{1}{\sqrt{x+5}+3}$

$= \dfrac{1}{\sqrt{4+5}+3} = \dfrac{1}{\sqrt{9}+3} = \dfrac{1}{3+3} = \dfrac{1}{6}$

57) $\lim\limits_{x \to 0} \dfrac{\sqrt{x+5}-\sqrt{5}}{x} = \lim\limits_{x\to 0} \dfrac{\sqrt{x+5}-\sqrt{5}}{x} \cdot \dfrac{\sqrt{x+5}+\sqrt{5}}{\sqrt{x+5}+\sqrt{5}}$

$= \lim\limits_{x\to 0} \dfrac{x+5-5}{x(\sqrt{x+5}+\sqrt{5})} = \lim\limits_{x\to 0} \dfrac{\cancel{x}}{\cancel{x}(\sqrt{x+5}+\sqrt{5})} = \lim\limits_{x\to 0} \dfrac{1}{\sqrt{x+5}+\sqrt{5}}$

$= \dfrac{1}{\sqrt{5}+\sqrt{5}} = \dfrac{1}{2\sqrt{5}} = \dfrac{2\sqrt{5}}{20} = \dfrac{\sqrt{5}}{10}$

59) $\lim\limits_{x \to 0} \dfrac{\dfrac{1}{3+x} - \dfrac{1}{3}}{x} = \lim\limits_{x\to 0} \dfrac{\dfrac{3-3-x}{9+3x}}{x}$

$= \lim\limits_{x\to 0} \dfrac{-\cancel{x}}{(9+3x)\cancel{x}} = \lim\limits_{x\to 0} -\dfrac{1}{9+3x} = -\dfrac{1}{9+3(0)} = \dfrac{-1}{9}$

61) $\lim\limits_{\Delta x \to 0} \dfrac{2(x+\Delta x)-2x}{\Delta x}$

$= \lim\limits_{\Delta x\to 0} \dfrac{2x+2\Delta x-2x}{\Delta x} = \lim\limits_{\Delta x\to 0} \dfrac{2\cancel{\Delta x}}{\cancel{\Delta x}}$

$= \lim\limits_{\Delta x\to 0} 2$

//

63) $\lim\limits_{\Delta x \to 0} \dfrac{(x+\Delta x)^2-2(x+\Delta x)+1-(x^2-2x+1)}{\Delta x}$

$= \lim\limits_{\Delta x\to 0} \dfrac{\cancel{x^2}+2x\Delta x+\Delta x^2-\cancel{2x}-2\Delta x+\cancel{1}-\cancel{x^2}+\cancel{2x}-\cancel{1}}{\Delta x}$

$= \lim\limits_{\Delta x\to 0} \dfrac{2x\Delta x+\Delta x^2-2\Delta x}{\Delta x} = \lim\limits_{\Delta x\to 0} \dfrac{\cancel{\Delta x}(2x+\Delta x-2)}{\cancel{\Delta x}}$

$= \lim\limits_{\Delta x\to 0} 2x+\Delta x-2 = 2x+0-2 = 2x-2$

65) $\lim\limits_{x\to 0} \frac{\operatorname{sen} x}{5x} = \frac{1}{5} \lim\limits_{x\to 0} \frac{\operatorname{sen} x}{x} = \frac{1}{5}(1) = \frac{1}{5}$

67) $\lim\limits_{x\to 0} \frac{\operatorname{sen} x(1-\cos x)}{x^2} =$

$\lim\limits_{x\to 0} \frac{\operatorname{sen} x}{x} \cdot \lim\limits_{x\to 0} \frac{1-\cos x}{x} = 1 \cdot 0 = 0$

69) $\lim\limits_{x\to 0} \frac{\operatorname{sen}^2 x}{x} = \lim\limits_{x\to 0} \frac{\operatorname{sen} x}{x} \cdot \lim\limits_{x\to 0} \operatorname{sen} x$

$= 1 \cdot 0 = 0$

71) $\lim\limits_{h\to 0} \frac{(1-\cos h)^2}{h} = \lim\limits_{h\to 0} \frac{1-\cos h}{h} \cdot \lim\limits_{h\to 0} 1-\cos h$

$= 1(1-1) = 1 \cdot 0 = 0$

73) $\lim\limits_{x\to \frac{\pi}{2}} \frac{\cos x}{\cot x} = \lim\limits_{x\to \frac{\pi}{2}} \frac{\cos x}{\frac{\cos x}{\operatorname{sen} x}} = \lim\limits_{x\to \frac{\pi}{2}} \frac{\cancel{\cos x} \cdot \operatorname{sen} x}{\cancel{\cos x}}$

$= \lim\limits_{x\to \frac{\pi}{2}} \operatorname{sen} x = \operatorname{sen} \frac{\pi}{2} = 1$

75) $\lim\limits_{t\to 0} \frac{\operatorname{sen} 3t}{2t} = \frac{3}{2} \lim\limits_{t\to 0} \frac{\operatorname{sen} t}{t} = \frac{3}{2}$

77) $\lim\limits_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x} = \lim\limits_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x} \cdot \frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}$

$= \frac{x+2-2}{x(\sqrt{x+2}+\sqrt{2})} = \lim\limits_{x\to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+2}+\sqrt{2})} = \frac{1}{\sqrt{0+2}+\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} = \frac{2\sqrt{2}}{8} = \frac{\sqrt{2}}{4}$

79) $\lim\limits_{x\to 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x}$

$= \lim\limits_{x\to 0} \frac{\frac{2-2-x}{4+2x}}{x} = \lim\limits_{x\to 0} \frac{-x}{(4+2x)x} = \lim\limits_{x\to 0} \frac{1}{4+2x}$

$= \frac{1}{4+2(0)} = \frac{1}{4}$

81) $\lim\limits_{t\to 0} \frac{\operatorname{sen} 3t}{t} = 3 \lim\limits_{t\to 0} \frac{\operatorname{sen} t}{6} = 3 \cdot 1 = 3$

83) $\lim\limits_{x\to 0} \frac{\operatorname{sen} x^2}{x} = \lim\limits_{x\to 0} \frac{(1-\cos x)^2}{\cancel{x}} = 0$

85) $f(x) = 3x-2$

$\lim\limits_{\Delta x\to 0} \frac{3(x+\Delta x)-2 \; -3x+2}{\Delta x} = \lim\limits_{\Delta x\to 0} \frac{\cancel{3x}+3\Delta x-\cancel{2}-\cancel{3x}+\cancel{2}}{\Delta x}$

$= \lim\limits_{\Delta x\to 0} \frac{3\cancel{\Delta x}}{\cancel{\Delta x}} = \lim\limits_{\Delta x\to 0} 3$

87) $f(x) = \frac{1}{x+3}$

$\lim\limits_{\Delta x\to 0} \frac{\frac{1}{x+\Delta x+3} - \frac{1}{x+3}}{\Delta x}$

$\lim\limits_{\Delta x\to 0} \frac{-\cancel{\Delta x}}{(x^2-6x-4\Delta x+9)\cancel{\Delta x}}$

$= \lim\limits_{\Delta x\to 0} \frac{-1}{x^2-6x-4\Delta x+9}$

$= \frac{-1}{9} = \frac{-1}{9}$