Ejercicios Álgebra de Boole Pt 1

2.1 $(I, \max\{, \min, 1-a, 0, 1\}$

$a \vee 0 = \max\{a, 0\} = a \rightarrow$ Se cumple
$a \wedge 1 = \min\{a, 1\} = a \rightarrow$ Se cumple
Identidad

$a \vee (b \vee c) = (a \vee b) \vee c \rightarrow \max\{a, \max(b,c)\} = \max\{\max(a,b), c\} \Rightarrow$

$\Rightarrow \max\{a,b,c\} = \max\{a,b,c\} \rightarrow$ Se cumple. Asociatividad

$a \vee b = b \vee a \rightarrow \max\{a,b) = \max\{b,a\} \rightarrow$ Se cumple Conmutatividad

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \rightarrow \max\{\min(b,c)\} = \min\{\max(a,b), \max(a,c)\}$

Supongamos con un ejemplo $\rightarrow a = 3, b = 2, c = 1$

$\Rightarrow \max\{3, \min\{2,1\}\} = \min\{\max(3,2), \max\{3,1\}\} \Rightarrow \max\{3,1\} = \min\{3,3\}$

$\Rightarrow 3 = 3 \rightarrow$ Se cumple la distributividad.

$a \vee \bar{a} = \max(a, \bar{a}) = \max(a, 1-a)$

Supongamos que $a = 1 \rightarrow \max(1, 1-1) = \max(1,0) = 1$

$a = 0 \rightarrow \max(0, 1-0) = \max(0,1) = 1$

Luego $a \vee \bar{a} = 1$ Se cumple Identidad.

Es un álgebra de Boole

2.3

1. Debido a la propiedad de identidad que dice $a \vee a^* = 1$ y su dualidad $a \wedge a^* = 0$, al ver que $a \vee x = 1$ y $a \wedge x = 0$, podemos afirmar que $x = a^*$

2. Por complementación

$0 \vee 0^* = 1 \rightarrow 0^* = 1$

$1 \vee 1^* = 1 \rightarrow 1^* = 0$

3. $\left.\begin{array}{l} a^* \vee a = a \vee a^* = 1 \\ a^* \wedge a = a \wedge a^* = 0 \end{array}\right\} \xRightarrow{2.3.1} a = (a^*)^*$

4. $a^* = b^* \Rightarrow (a^*)^* = (b^*)^* \Rightarrow a = b$

5. $(a \vee b) \vee (a^* \vee b^*) = (a \vee b \vee a^*) \wedge (a \vee b \vee b^*) = (1 \vee b) \wedge (a \vee 1) = 1 \wedge 1 = 1 =$

$= (a \vee b) \vee (a \vee b)^* \Rightarrow (a^* \vee b^*) = (a \vee b)^*$

$(a \wedge b) \vee (a^* \wedge b^*) = (a \vee a^* \vee b^*) \wedge (b \vee a^* \vee b^*) = (1 \vee b^*) \wedge (a^* \vee 1) =$

$= 1 \wedge 1 = 1 = (a \wedge b) \vee (a \wedge b)^* \Rightarrow (a^* \wedge b^*) = (a \wedge b)^*$

2.4

1. $0 \leq a \leq 1$

$0 \vee a = a \vee 0 = a \Rightarrow 0 \leq a$

$a \vee 1 = 1 \Rightarrow a \leq 1$

2. $a \leq b \Leftrightarrow a \vee b = b \Leftrightarrow a \wedge b = a$

$(a \vee c) \vee (b \vee c) = a \vee c \vee b \vee c = a \vee b \vee c = b \vee c \Rightarrow a \vee c \leq b \vee c$

$(a \wedge c) \wedge (b \wedge c) = a \wedge b \wedge c = a \wedge c \Rightarrow a \wedge c \leq b \wedge c$

3. $a\le b \Rightarrow a\wedge b = a \Rightarrow (a\wedge b)^* = a^* \Rightarrow a^*\vee b^* = a^* \Rightarrow b^*\vee a^* = a^*$

$b^*\le a^* \Rightarrow b^*\vee a^* = a^* \Rightarrow (b\wedge a)^* = a^* \Rightarrow b\wedge a = a \Rightarrow a\wedge b = a \Rightarrow a\le b$

4. $a\wedge b\le c \Leftrightarrow a\le b^*\vee c$

$a\wedge b\wedge c = a\wedge b \Rightarrow$

2.5

1. $a\oplus b = (a\wedge b^*)\vee(a^*\wedge b) = (b^*\wedge a)\vee(b\wedge a^*) = (b\wedge a^*)\vee(b^*\wedge a) = b\oplus a$

2. $a\oplus(b\oplus c) = (a\wedge(b\oplus c)^*)\vee(a^*\wedge(b\oplus c)) = (a\wedge((b\wedge c^*)\vee(b^*\wedge c))^*)\vee$

$(a^*\wedge((b\wedge c^*)\vee(b^*\wedge c))) = (a\wedge(b^*\vee c)\wedge(b\vee c^*))\vee(a^*\wedge b\wedge c^*)\vee(a^*\wedge b^*\wedge c)$

$(a\oplus b)\oplus c = (((a\wedge b^*)\vee(a^*\wedge b))^*\wedge c^*)\vee(((a\wedge b^*)\vee(a^*\wedge b))^*\wedge c) =$

$= ((a^*\vee b)\wedge(a\vee b^*)\wedge c)\vee(a^*\wedge b\wedge c^*)\vee(a^*\wedge b^*\wedge c$

Bu

5. $x\oplus a = b$ si $x = a\oplus b$

$\Rightarrow)\ a\oplus b = a\oplus(x\oplus a) = a\oplus(a\oplus x) = (a\oplus a)\oplus x = 0\oplus x = x\oplus 0 = x$

$\Leftarrow)\ x\oplus a = (a\oplus b)\oplus a = (b\oplus a)\oplus a = b\oplus(a\oplus a) = b\oplus 0 = b$

2.6

1. $70 = 2\cdot 5\cdot 7$, divisores primos con exponente 1 $\Rightarrow$ Es Álgebra de Boole

$At(D(70)) = \{2,5,7\}$

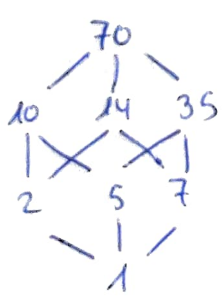
$coAt(D(70)) = \{35,14,10\}$

$2^* = \frac{70}{2} = 35$

$5^* = \frac{70}{5} = 14$

$7^* = \frac{70}{7} = 10$

2.



3. $35\wedge(2\vee 7) = 35\wedge 14 = 7$

$(2\vee 7)\wedge(14\wedge 10) = 14\wedge 2 = 2$

2.7 $D(210) = 2 \cdot 3 \cdot 5 \cdot 7 \rightarrow$ Sí es álgebra de Boole

$30 \vee (15 \wedge 10) = 30 \vee 5 = \underline{\underline{30}}$

$14^* \wedge 21 = \frac{210}{14} \wedge 21 = 15 \wedge 21 = \underline{\underline{3}}$

$(6^* \vee 35)^* \vee 10 = (6 \wedge 35^*) \vee 10 = (6 \wedge \frac{210}{35}) \vee 10 = 6 \vee 10 = \cancel{30}\ \underline{\underline{30}}$

$((3 \vee 10)^* \vee 2)^* = ((3 \vee 10) \wedge 2^*) = 30 \wedge 2^* = 30 \wedge 105 = \cancel{210}\ \underline{\underline{15}}$

$21 = 3 \vee 7 = 105 \wedge 42$ | $35 = 5 \vee 7 = 70 \wedge 105$

2.8

1. $D(2310) \rightarrow At(D(2310)) = \{2, 3, 5, 7, 11\}$

   $CoAt(D(2310)) = \{1155, 770, 462, 330, 210\}$

2. $21 \vee (165 \wedge 77^*) = 21 \vee 15 = \underline{\underline{105}}$

   $770 \wedge (3 \vee 14)^* = 770 \wedge 55 = \underline{\underline{55}}$

   $(15 \vee 110)^* = 15^* \wedge 110^* = 154 \wedge 21 = \underline{\underline{7}}$

   $385 \vee (1155 \wedge 42) = (385 \vee 1155) \wedge (385 \vee 42) = 1155 \wedge 2310 = \underline{\underline{1155}}$

$5 = 5 \vee 5 = (1155 \wedge 770 \wedge 330 \wedge 210)$ | $35 = 5 \vee 7 = 1155 \wedge 770 \wedge 210$ | $154 = 2 \vee 7 \vee 11 = 770 \wedge 462$

$231 = 3 \vee 7 \vee 11 = 1155 \wedge 462$ | $1155 = 3 \vee 5 \vee 7 \vee 11 = 1155 \wedge 1155$

2.9

1. 32 elementos $\rightarrow 32 = 2^{5} \rightarrow$ Tiene 5 átomos

2. Sus coátomos son $\bar{a}_1, \bar{a}_2, \bar{a}_3$ y $\bar{a}_4$

2.10

$n = 105 \vee 42 = 210$

$105^* \vee x = 42 \Rightarrow 2 \vee x = 42 \rightarrow 42 = 2 \vee 3 \vee 7$

$\rightarrow x = 21 = 3 \vee 7$

$\rightarrow x = 42$

2.11

$xy + z^* =$

$= ((x \cdot y)^*)^* + z^* = (x^* + y^*)^* + z^*$

$x \cdot y + z^* = ((x \cdot y + z^*)^*)^* = ((x \cdot y)^* \cdot z)^*$

2.12

Se justifica con las leyes de De Morgan, ya que con el doble complemento de cualquier expresión, sigue siendo lo mismo, pero podemos usarlo para aplicar un solo complemento y transformar las operaciones.

$(a + b)^* = a^* \cdot b^*) \Leftrightarrow (a \cdot b)^* = a^* + b^*$

2.13

- $a^* = (a \wedge a)^* = a \text{ NAND } a = a \uparrow a$

  $a \vee b = (a \vee b)^{**} = ((a \vee b)^*)^* = (a^* \wedge b^*)^* = a^* \uparrow b^* = (a \uparrow a) \uparrow (b \uparrow b)$

  $a \wedge b = (a \wedge b)^{**} = ((a \wedge b)^*)^* = (a \uparrow b)^* = (a \uparrow b) \uparrow (a \uparrow b)$

- $a^* = (a \vee a)^* = a \text{ NOR } a = a \downarrow a$

  $a \vee b = (a \vee b)^{**} = ((a \vee b)^*)^* = (a \downarrow b)^* = (a \downarrow b) \downarrow (a \downarrow b)$

  $a \wedge b = (a \wedge b)^{**} = ((a \wedge b)^*)^* = (a^* \vee b^*)^* = a^* \downarrow b^* = (a \downarrow a) \downarrow (b \downarrow b)$

2.14

$$x\cdot y + z^* = (x^* + y^*)^* + z^* = \left((x^* \downarrow y^*) \downarrow z^*\right)^*$$

$$x\cdot y + z^* = \left((x\cdot y)^* \cdot z^{**}\right)^* = (x \uparrow y) \uparrow z$$

2.2 $a \vee (b \vee c) = (a \vee b) \vee c$

$$a \wedge (a \vee (b \vee c)) = a$$

$$a \wedge ((a \vee b) \vee c) = (a \wedge (a \vee b)) \vee (a \wedge c) = a \vee (a \wedge c) = a$$

$$a^* \wedge (a \vee (b \vee c)) = (a^* \wedge a) \vee (a^* \wedge (b \vee c)) = a^* \wedge (b \vee c)$$

$$a^* \wedge ((a \vee b) \vee c) = (a^* \wedge (a \vee b)) \vee (a^* \wedge c) = ((a^* \wedge a) \vee (a^* \wedge b)) \vee (a^* \wedge c) =$$

$$= (a^* \wedge b) \vee (a^* \wedge c) = a^* \wedge (b \vee c)$$

$$a \vee (b \vee c) = 1 \wedge (a \vee (b \vee c)) = (a \vee a^*) \wedge (a \vee (b \vee c)) =$$

$$= (a \wedge (a \vee (b \vee c))) \vee (a^* \wedge (a \vee (b \vee c))) = (a \wedge ((a \vee b) \vee c)) \vee (a^* \wedge ((a \vee b) \vee c)) =$$

$$= (a \vee a^*) \wedge ((a \vee b) \vee c) = 1 \wedge ((a \vee b) \vee c) = (a \vee b) \vee c$$