Ejercicios Recurrencia LMD

1.6 $x_0 = 1,\ x_1 = 2,\ x_n = 4 + x_{n-2}$ para todo $n \geq 2$, llegar a $X_n = \frac{1}{2}(4n+1+(-1)^n)$

$\cancel{X_n - X_{n-2} = 4 \to 4 = b^n p(n) \Rightarrow b = 1,\ p(n) = 4,\ gr(p) = 0 \to (x-1)}$

$\cancel{x^2 - 1 = 0 \to (x-1)(x+1)(x-1) \to S_n = A + B\cdot(-1)^n + C\cdot 4^n}$

$\cancel{x_2 = 4 + 1 = 8}$

$x_0 = \frac{1}{2}(4\cdot 0 + 1 + (-1)^0) = \frac{1}{2}\cdot(2) = 1$

$x_1 = \frac{1}{2}(4 + 1 - 1) = \frac{1}{2}\cdot 4 = 2$

$X_n = 4 + X_{n-2} = 4 + \frac{1}{2}(4(n-2)+1+(-1)^{n-2}) = \frac{1}{2}8 + \frac{1}{2}(4(n-2)+1+(-1)^{n-2}) =$

$= \frac{1}{2}8 + \frac{1}{2}\cdot(4n - 8 + 1 (-1)^{n-2}) = \frac{1}{2}(8 + 4n - 8 + 1 + (-1)^{n-2}) = \frac{1}{2}(4n+1+(-1)^{n-2})$

$(-1)^{n-2} = (-1)^n$ debido a que no pierde la paridad luego $\to = \frac{1}{2}(4n+1+(-1)^n)$

1.7

1. $x_n = 4n + 1$

$x_{n-1} = 4(n-1)+1 = 4n-3$

$\to \left.\begin{array}{l} x_n - x_{n-1} = 4 \\ x_{n-1} - x_{n-2} = 4 \end{array}\right\} \to \boxed{x_n - 2x_{n-1} + x_{n-2} = 0}$

2. $y_n = 2^n + n$

$y_{n-1} = 2^{n-1} + n - 1$

$\cancel{y_n - y_{n-1} = 2^n + n - 2^{n-1} - n + 1 \ldots}$

$\to$

$\cancel{\to y_n - y_{n-1} = 2^n - 2^{n-1} + 1}$

$\to 2y_{n-1} = 2^n + 2n - 2 \Big\} \to \left.\begin{array}{l} Y_n - 2Y_{n-1} = -n + 2 \\ Y_{n-1} - 2Y_{n-2} = -n - 1 + 2 \end{array}\right\} \left.\begin{array}{l} Y_n - 3Y_{n-1} + 2Y_{n-2} = 1 \\ Y_{n-1} - 3Y_{n-2} + 2Y_{n-3} = 1 \end{array}\right\}$

$\to \boxed{Y_n - 4Y_{n-1} + 5Y_{n-2} - 2Y_{n-3} = 0}$

3. $Z_n = 2^n + 3^n(n+1)$

$Z_{n-1} = 2^{n-1} + 3^{n-1}(n)$ $\rightarrow$ $\cancel{Z_n - 2Z_{n-1} = 3^n +}$

$\cdot 3$ $\cancel{3Z_{n-1} = 3 \cdot 2^{n-1} + 3^{n-1}}$

$3Z_{n-1} = 3 \cdot 2^{n-1} + 3^n n$ $\rightarrow Z_n - 3Z_{n-1} = 2^{n-1} + 3^n$

$Z_{n-1} - 3Z_{n-2} = 2^{n-2} + 3^{n-1}$ $\quad Z_n - 6Z_{n-1} + 9Z_{n-2} = 3 \cdot 2^{n-2}$

$\cdot 3$ $\quad 3Z_{n-1} - 9Z_{n-2} = 3 \cdot 2^{n-2} + 3^n$ $\quad Z_{n-1} - 6Z_{n-2} + 9Z_{n-3} = 3 \cdot 2^{n-3}$

$\cdot 2$

$\cancel{8Z_{n-1} - 48Z_{n-2} + 72Z_{n-3} = 3 \cdot 2}$

$2Z_{n-1} - 12Z_{n-2} + 18Z_{n-3} = 3 \cdot 2^{n-2}$ $\rightarrow$ $\boxed{Z_n - 8Z_{n-1} + 21Z_{n-2} - 18Z_{n-3} = 0}$

1.8

1. $F_{n+2} > 2 \cdot F_n$ para todo $n \geq 2$

Para $n=2 \rightarrow F_2 = 1$, $F_4 = 3 \rightarrow 3 > 2 \cdot 1 \rightarrow$ Se cumple

$F_{n+3} \Rightarrow F_{n+2} + F_{n+1}$ $\rightarrow$ Demostrar que $F_n > n$, para $n \geq 2$ ¿$F_n + 2F_{n-1} = 2F_{n+1}$?

Suponemos para $n \rightarrow F_{n+1} = F_n + F_{n-1} \rightarrow$ La suma es > 0

Aplicando H.I $\rightarrow F_{n+1} > 0$

Se cumple para $\forall n \in \mathbb{N}$ tq $n \geq 2 \rightarrow F_n \geq F_{n+1}$

2. $\sum_{i=0}^{n} (F_i)^2 = F_n \cdot F_{n+1}$ para todo $n \geq 0$

Suponemos para $0 = n$; $0^2 = 0 \cdot 1 \rightarrow 0 = 0 \rightarrow$ Se cumple

Para $n+1 \rightarrow \sum_{i=0}^{n+1} (F_i)^2 = \underbrace{\sum_{i=0}^{i=n} F_i^2}_{\text{Por H.I } F_n \cdot F_{n+1}} + F_{n+1}^2 \Rightarrow F_n F_{n+1} + F_{n+1}^2$

$F_{n+1} \cdot (F_{n+2}) \rightarrow F_{n+3}$

Por tanto para $\forall n \in \mathbb{N}$ tq $n \geq 0$

3. 5 divide a F_{5n} para todo $n \geq 0$

Para $n=0 \to 0|5$ Se cumple

Para $n+1 \to F_{5n+5} \to \underbrace{F_{5n+4}}_{F_{5n+3}+F_{5n+1}} + F_{5n+3} \to \begin{cases} 2F_{5n+3}+F_{5n+1} \\ 2(F_{5n+2}+F_{5n+1}) \end{cases}$

$\Rightarrow 3F_{5n+2}+2F_{5n+1}$; $3(F_{5n+1}+F_{5n})+2F_{5n+1}$; $5F_{5n+1}+3F_{5n}$

Mult de cinco

La suma de 2 mult de 5, es mult de 5.

5. $mcd(F_n, F_{n+1}) = 1$ para todo $n \geq 0$

~~Supongo 2mcd~~

Consideramos $mcd(F_{n+1}, F_{n+2}) = 1 = mcd(F_{n+1}, F_{n+1}+F_n)$

$mcd(F_{n+1}, F_{n+1}+F_n) = 1$

$mcd(F_{n+1}, F_n) = 1$, por la propiedad del mcd.

~~Osea~~ Para Se cumple que $\forall n \in \mathbb{N}$, $mcd(F_n, F_{n+1}) = 1$

1.9

1. $x_0 = 1,\ x_1 = 1,\ x_n = 2x_{n-1} - x_{n-2}$ para $n \geq 2$

$x_n - 2x_{n-1} - x_{n-2} = 0$

$x^2 - 2x - 1 = (x-1)^2 \to$ Sol general $= \{A \cdot n + B\}$

$x_0 \to 1 = A \cdot 0 + B$
$x_1 \to 1 = A \cdot 1 + B$ $\to \boxed{B=1}$ $A + B = 1 \to \boxed{A=0}$ $\to$ donde $\boxed{x_n = 1}$

2. $x_0 = 1,\ x_1 = 2,\ x_n = 5x_{n-1} - 6x_{n-2}$ para $n \geq 2$

$x_n - 5x_{n-1} - 6x_{n-2} = 0$

$x^2 - 5x - 6 = (x-3)(x-2) \to S_n = A \cdot 3^n + B \cdot 2^n$

$x_0 \to 1 = A + B$
$x_1 \to 2 = 3A + 2B$ $\boxed{B = 1 \quad A = 0} \to \boxed{x_n = 2^n}$

3. $x_0 = 1,\ x_1 = 1,\ x_n = 3x_{n-1} + 4x_{n-2}$ para $n \geq 2$

$x_n - 3x_{n-1} - 4x_{n-2} = 0$

$x^2 - 3x - 4 = (x-4)(x+1) \to S_n = A \cdot 4^n + B \cdot (-1)^n$

$1 = A + B$
$1 = 4A - B$ $\to \boxed{A = 2/5 \quad B = 3/5} \to \boxed{x_n = \frac{2}{5} \cdot 4^n + \frac{3}{5} \cdot (-1)^n}$

4. $x_0 = 1,\ x_1 = 2,\ x_n = -x_{n-1} + 6x_{n-2}$ para $n \geq 2$.

$x_n + x_{n-1} - 6x_{n-2}$

$x^2 + x - 6 = (x-2)(x+3) \to S_n = A \cdot 2^n + B \cdot 3^n$

$1 = A + B$
$2 = 2A + 3B$ $\boxed{A = 1 \quad B = 0} \to \boxed{x_n = 2^n}$

5. $x_0 = 0,\ x_1 = 1,\ x_n = 2x_{n-1} - 2x_{n-2}$ para $n \geq 2$

$$x_n - 2x_{n-1} + 2x_{n-2} = 0$$

$$x^2 - 2x + 2 = (x-(1+i))(x-(1-i)) \qquad S_n = A(1+i)^n + B(1-i)^n$$

$$\left.\begin{array}{l} 0 = A + B \\ 1 = A(1+i) + B(1-i) \end{array}\right\} \boxed{\begin{array}{l} A = \frac{-i}{2} \\ B = \frac{i}{2} \end{array}} \quad \text{donde} \quad \boxed{X_n = \frac{-i}{2}\left((1+i)^n - (1-i)^n\right)}$$

6. $x_0 = 5,\ x_1 = 12,\ x_n = 6x_{n-1} - 9x_{n-2}$ para ≥ 2

$$x_n - 6x_{n-1} + 9x_{n-2} = 0$$

$$x^2 - 6x + 9 = (x-3)^2 \rightarrow S_n = (A \cdot n + B) \cdot 3^n$$

$$\left.\begin{array}{l} 5 = \cancel{A}B \\ 12 = (A + B) \cdot 3 \end{array}\right\} \rightarrow \boxed{\begin{array}{l} A = -1 \\ B = 5 \end{array}} \quad \text{donde} \quad \boxed{X_n = (-n+5) \cdot 3^n}$$

7. $x_0 = 1,\ x_1 = 1,\ x_2 = 2,\ x_n = 5x_{n-1} - 8x_{n-2} + 4x_{n-3}$ para $n \geq 3$

$$x_n - 5x_{n-1} + 8x_{n-2} - 4x_{n-3} = 0$$

$$x^3 - 5x^2 + 8x - 4 = (x-1)(x-2)^2 \rightarrow S_n = (A \cdot n + B)2^n + C$$

$$\left.\begin{array}{l} 1 = B + C \\ 1 = (A+B)2 + C \\ 2 = (2A+B)4 + C \end{array}\right\} \boxed{\begin{array}{l} A = 1/2 \\ B = -1 \\ C = 2 \end{array}} \rightarrow \boxed{X_n = \left(\frac{n}{2} - 1\right) \cdot 2^n + 2}$$

8. $x_0 = 1,\ x_1 = 1,\ x_2 = 2,\ x_n = x_{n-1} + x_{n-2} - x_{n-3}$ para $n \geq 3$

$$x_n - x_{n-1} - x_{n-2} + x_{n-3}$$

$$x^3 - x^2 - x + x = (x-1)^2(x+1) \rightarrow S_n = An + B + C(-1)^n$$

$$\left.\begin{array}{l} 1 = B + C \\ 1 = A + B - C \\ 2 = 2A + B + C \end{array}\right\} \boxed{\begin{array}{l} A = 1/2 \\ B = 3/4 \\ C = 1/4 \end{array}} \rightarrow \boxed{X_n = \frac{n}{2} + \frac{3}{4} + \frac{1}{4} \cdot (-1)^n}$$

9. $x_0 = 0,\ x_1 = 1,\ x_2 = 3,\ x_n = -2x_{n-1} + x_{n-2} + 2x_{n-3}$ para $x \geq 3$

$x_n + 2x_{n-1} - x_{n-2} - 2x_{n-3} = 0$

$x^3 + 2x^2 - x - 2 = 0 \rightarrow (x-1)(x+1)(x+2) \rightarrow S_n = A + B\cdot(-1)^n + C\cdot(-2)^n$

$$\left.\begin{array}{l} 0 = A + B + C \\ 1 = A - B - 2C \\ 3 = A + B + 4C \end{array}\right\} \quad \boxed{\begin{array}{l} A = 1 \\ B = -2 \\ C = 1 \end{array}} \quad \text{donde} \quad \boxed{x_n = 1 + -2\cdot(-1)^n + (-2)^n}$$

10. $x_0 = 1,\ x_1 = 1,\ x_2 = 3,\ x_n = 4x_{n-1} - 5x_{n-2} + 2x_{n-3}$ para ≥ 3

$x_n - 4x_{n-1} + 5x_{n-2} - 2x_{n-3} = 0$

$x^3 - 4x^2 + 5x - 2 = 0 \rightarrow (x-2)(x-1)^2 \rightarrow S_n = A\cdot 2^n + \{Bn + C\}$

$$\left.\begin{array}{l} 1 = A + C \\ 1 = 2A + B + C \\ 3 = 4A + 2B + C \end{array}\right\} \quad \boxed{\begin{array}{l} A = 2 \\ B = -2 \\ C = -1 \end{array}} \quad \text{donde} \quad \boxed{x_n = 2^{n+1} + -2n - 1}$$

11. $x_0 = 1,\ x_1 = 3,\ x_2 = 7,\ x_n = 3x_{n-1} - 3x_{n-2} + x_{n-3}$ para $n \geq 3$

$x_n - 3x_{n-1} + 3x_{n-2} - x_{n-3} = 0$

$x^3 - 3y^2 + 3x - 1 = 0 \rightarrow (x-1)^3 \rightarrow S_n = An^2 + Bn + C$

$$\left.\begin{array}{l} 1 = C \\ 3 = A + B + C \\ 7 = 4A + 2B + C \end{array}\right\} \quad \boxed{\begin{array}{l} A = 1 \\ B = 1 \\ C = 1 \end{array}} \quad \text{donde} \quad \boxed{x_n = n^2 + n + 1}$$

12. $x_0 = 0,\ x_n = 2x_{n-1} + 1$ para ≥ 1

$x_n - 2x_{n-1} = 1 \quad \rightarrow 1 = b^n p(n) \rightarrow b = 1,\ p = 1,\ gr(n) = 0$

$x - 2 = 0 \rightarrow (x-2)(x-1) \rightarrow S_n = A\cdot 2^n + B$

$x_1 = 2\cdot x_0 + 1 = 1$

$$\left.\begin{array}{l} 0 = A + B \\ 1 = 2A + B \end{array}\right\} \quad \boxed{\begin{array}{l} A = 1 \\ B = -1 \end{array}} \quad \text{donde} \quad \boxed{x_n = 2^n - 1}$$

13. $x_0 = 1$, $x_n = x_{n-1} + n$ para $n \geq 1$

$X_n - X_{n-1} = n \rightarrow b^n p(n) \Rightarrow b = 1$, $p(n) = n$, $gr(p) = 1$

$x - 1 = 0 \rightarrow (x-1)(x-1)^2 = (x-1)^3 \rightarrow S_n = An^2 + Bn + C$

$X_1 = X_0 + 1 = 1+1 = 2$; $X_2 = 2 + 2 = 4$

$$\left.\begin{array}{l} 1 = C \\ 2 = A + B + C \\ 4 = 4A + 2B + C \end{array}\right\} \boxed{\begin{array}{l} A = 1/2 \\ B = 1/2 \\ C = 1 \end{array}}$$

donde $\boxed{X_n = \frac{n^2}{2} + \frac{n}{2} + 1} = \boxed{\frac{1}{2}(n^2 + n + 2)}$

14. $X_0 = 1$, $X_n = 2X_{n-1} + n$ para $n \geq 1$

$X_n - 2X_{n-1} = n \rightarrow b^n p(n) \Rightarrow b = 1$, $p(n) = n$, $gr(p) = 1$

$x - 2 = 0 \rightarrow (x-2)(x-1)^2 \rightarrow S_n = A \cdot 2^n + Bn + C$

$X_1 = 2 \cdot 1 + 1 = 3$; $X_2 = 2 \cdot 3 + 1 = 7$

$$\left.\begin{array}{l} 1 = A + C \\ 3 = 2A + B + C \\ 7 = 4A + 2B + C \end{array}\right\} \boxed{\begin{array}{l} A = 2 \\ B = 0 \\ C = -1 \end{array}}$$

donde $\boxed{X_n = 2^{n+1} - 1}$

15. $X_0 = 0$, $X_n - 2X_{n-1} = 3^n$ para $n \geq 1$

$x - 2 = 3^n \rightarrow b^n \cdot p(n) \Rightarrow b = 3$, $p(n) = 1$, $gr(p) = 0$

$\hookrightarrow (x-2)(x-3) \rightarrow S_n = A \cdot 2^n + B \cdot 3^n$

$X_1 = 3$

$$\left.\begin{array}{l} 0 = A + B \\ 3 = 2A + 3B \end{array}\right\} \boxed{\begin{array}{l} A = -3 \\ B = 3 \end{array}}$$

donde $\boxed{X_n = -3 \cdot 2^n + 3^{n+1}}$

16. $X_0 = 0$, $X_n - 2X_{n-1} = (n+1) \cdot 3^n$ para $n \geq 2$

$x - 2 = (n+1) \cdot 3^n \rightarrow b^n \cdot p(n) \rightarrow b = 3$, $p(n) = n+1$, $gr(p) = 1$

$\hookrightarrow (x-2)(x-3)^2 \rightarrow S_n = A \cdot 2^n + (Bn + C) \cdot 3^n$

$X_1 = 6$

$X_2 = 39$

$$\hookrightarrow \left.\begin{array}{l} 0 = A + C \\ 6 = 2A + 3B + 3C \\ 39 = 4A + 18B + 9C \end{array}\right\} \boxed{\begin{array}{l} A = 3 \\ B = 3 \\ C = -3 \end{array}}$$

donde $\boxed{X_n = 3 \cdot 2^n + (3n - 3) \cdot 3^n = (2^n + (n-1) \cdot 3^n) \cdot 3}$