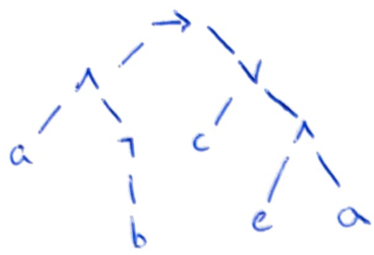
Ejercicios Lógica Proposicional

4.1 y 4.2

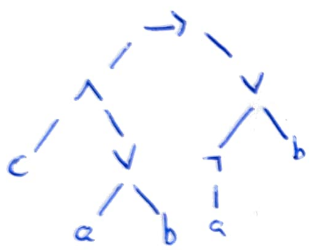
1.) $a \wedge \neg b \to c \vee (e \wedge a)$

Subfórmulas: $\{a \wedge \neg b \to c \vee (e \wedge a), a \wedge \neg b, a, \neg b, b, c \vee (e \wedge a), c, e \wedge a, e, a\}$



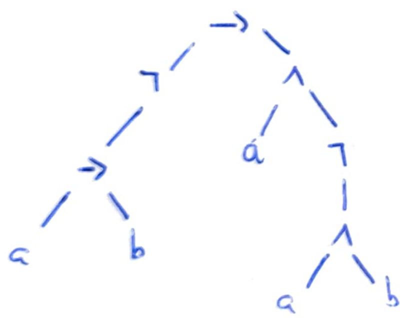
2.) $c \wedge (a \vee b) \to \neg a \vee b$

Subfórmulas: $\{a, b, c, a \vee b, \neg a, c \wedge (a \vee b), \neg a \vee b, c \wedge (a \vee b) \to \neg a \vee b\}$



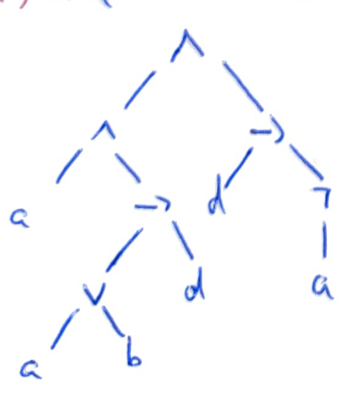
3.) $\neg(a \to b) \to a \wedge \neg(a \wedge b)$

Subfórmulas: $\{a, b, a \wedge b, a \to b, \neg(a \wedge b), \neg(a \to b), a \wedge \neg(a \wedge b), \neg(a \to b) \to a \wedge \neg(a \wedge b)$

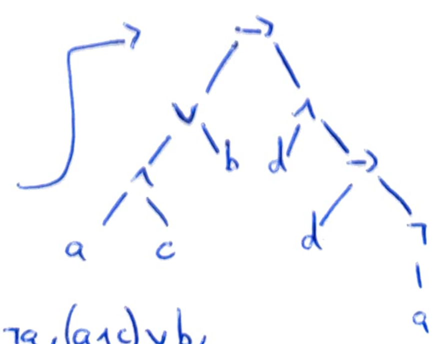


4.) $a \wedge (a \vee b \to d) \wedge (d \to \neg a)$

Subfórmulas: $\{a, b, a \vee b, d, a \vee b \to d, \neg a, a \wedge (a \vee b \to d), d \to \neg a, a \wedge (a \vee b \to d) \wedge (d \to \neg a)\}$



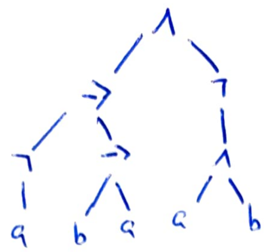
5.) $(a \wedge c) \vee b \to d \wedge (d \to \neg a)$



Subfórmulas: $\{a, c, d, \neg a, a \wedge c, d \to \neg a, (a \wedge c) \vee b, d \wedge (d \to \neg a), (a \wedge c) \vee b \to d \wedge (d \to \neg a)\}$

6. $\neg a \to (b \to a) \land \neg(a \land b)$

Subfórmulas $= \{a, b, \neg a, b \to a, a \land b, \neg a \to (b \to a), \neg(a \land b), \neg a \to (b \to a) \land \neg(a \land b)\}$

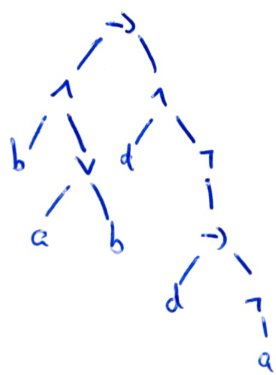


7. $(a \land \neg(b \to c \lor e)) \lor a$

Subfórmulas $= \{c, e, c \lor e, b, b \to c \lor e, a, \neg(b \to c \lor e), a \land \neg(b \to c \lor e), (a \land \neg(b \to c \lor e)) \lor a\}$

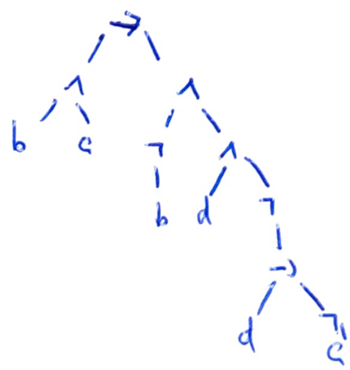
8. $b \land (a \lor b) \to d \land \neg(d \to \neg a)$

Subfórmulas $= \{a, d, \neg a, d \to \neg a, b, a \lor b, \neg(d \to \neg a), b \land (a \lor b), d \land \neg(d \to \neg a), b \land (a \lor b) \to d \land \neg(d \to \neg a)\}$



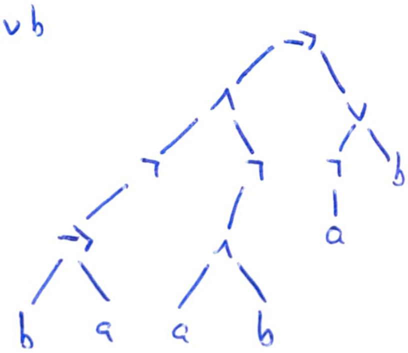
9. $b \land a \to (\neg b \to d \land \neg(d \to \neg a))$

Subfórmulas $= \{a, d, \neg a, d \to \neg a, b, \neg(d \to \neg a), \neg b, d \land \neg(d \to \neg a), b \land a, \neg b \land d \land \neg(d \to \neg a), b \land a \to (\neg b \to d \land \neg(d \to \neg a))\}$



10. $\neg(b \to a) \land \neg(a \land b) \to \neg a \lor b$

Subfórmulas: $\{b, a, b \to a, a \land b, \neg(b \to a), \neg(a \land b), \neg a, \neg(b \to a) \land \neg(a \land b), \neg a \lor b, \neg(b \to a) \land \neg(a \land b) \to \neg a \lor b\}$



4.3

1. $\alpha \vee \gamma = 1 \vee 0 = 1$

2. $\alpha \wedge \gamma = 1 \wedge 0 = 0$

3. $\neg\alpha \wedge \neg\gamma = \neg 1 \wedge \neg 0 = 0 \wedge 1 = 0$

4. $\alpha \leftrightarrow \neg\beta \vee \gamma = 1 \leftrightarrow \neg 1 \vee 0 = 1 \leftrightarrow 0 \vee 0 = 1 \leftrightarrow 0 = 0$

5. $\beta \vee \neg\gamma \rightarrow \alpha = 1 \vee 1 \rightarrow 1 = 1 \rightarrow 1 = 1$

6. $\beta \vee \alpha \rightarrow (\beta \rightarrow \neg\gamma) = 1 \vee 1 \rightarrow (1 \rightarrow 1) = 1$

7. $(\beta \leftrightarrow \neg\alpha) \leftrightarrow (\alpha \leftrightarrow \gamma) = 0 \leftrightarrow 0 = 1$

8. $(\beta \rightarrow \alpha) \rightarrow ((\alpha \rightarrow \gamma) \rightarrow (\neg\gamma \rightarrow \beta)) = 1 \rightarrow (1 \rightarrow 1) = 1$

4.4

$\alpha \leftrightarrow \beta = 1$

$1 + \alpha + \alpha\beta = 1$

$\alpha + \alpha\beta = 0$

$\alpha = \alpha\beta$

1) $\alpha \vee \gamma \rightarrow \beta \vee \gamma = (\alpha + \gamma + \alpha\gamma) \rightarrow (\beta + \gamma + \beta\gamma) =$

$= 1 + \alpha + \gamma + \alpha\gamma + (\alpha + \gamma + \alpha\gamma) \cdot (\beta + \gamma + \beta\gamma) =$

$= 1 + \alpha + \cancel{\gamma} + \cancel{\alpha\gamma} + \alpha\beta + \cancel{\alpha\gamma} + \alpha\beta\gamma + \cancel{\beta\gamma} + \cancel{\gamma} + \cancel{\beta\gamma} + \cancel{\alpha\beta\gamma} + \alpha\gamma + \cancel{\alpha\beta\gamma} =$

$= 1 + \underset{\cancel{\alpha\beta}}{\underset{\parallel}{\alpha}} + \cancel{\alpha\beta} + \underset{\cancel{\alpha\beta\gamma}}{\underset{\parallel}{\alpha\gamma}} + \cancel{\alpha\beta\gamma} = 1$

2) $\alpha \wedge \gamma \rightarrow \beta \wedge \gamma = (\alpha \cdot \gamma) \rightarrow (\beta\gamma) = 1 + \underset{\cancel{\alpha\beta\gamma}}{\underset{\parallel}{\alpha\gamma}} + \cancel{\alpha\beta\gamma} = 1$

3) $\neg\alpha \wedge \beta \leftrightarrow \alpha \vee \beta = (1+\alpha) \wedge \beta \leftrightarrow (\alpha + \beta + \alpha\beta) = \cancel{(1+\alpha + \beta + \beta + \alpha\beta) \leftrightarrow}$

$= (\beta + \alpha\beta) \leftrightarrow (\alpha + \beta + \alpha\beta) = 1 + \cancel{\beta} + \cancel{\alpha\beta} + \alpha + \cancel{\beta} + \cancel{\alpha\beta} = 1 + \alpha$

Si $\alpha = 1 = 1 + 1 = 0$

$\alpha = 0 = 1 + 0 = 1$

4.5

$\alpha \leftrightarrow \beta = 0$

$1+\alpha+\beta = 0$

$\alpha+\beta = 1$

$\alpha = 1+\beta$

1.) $\alpha \wedge \beta = \alpha\cdot\beta = (1+\beta)\beta = \cancel{\beta}+\cancel{\beta} = \underline{\underline{0}}$

2.) $\alpha \vee \beta = 1+\alpha+\alpha\cdot\beta = \cancel{1}+\cancel{1}+\cancel{\beta}+\cancel{\beta}+\beta = \underline{\underline{\beta}}$

3.) $\alpha \to \beta = \alpha+\beta+\alpha\beta = 1+\cancel{\beta}+\cancel{\beta}+\cancel{\beta}+\cancel{\beta} = \underline{\underline{1}}$

4.) $\alpha\wedge\gamma \leftrightarrow \beta\wedge\gamma = \alpha\gamma \leftrightarrow \beta\gamma = 1+\alpha\gamma+\beta\gamma = 1+(1+\beta)\gamma+\beta\gamma =$

$= 1+\gamma+\cancel{\beta\gamma}+\cancel{\beta\gamma} = 1+\gamma \to$ Si $\gamma=0 = 1+0 = \underline{\underline{1}}$
Si $\gamma = 1 = 1+1 = \underline{\underline{0}}$

4.6

~~$\alpha \leftrightarrow \beta = 0$~~ $\quad \alpha\to\beta = 1$

~~$\alpha = 1+\beta$~~ $\quad \alpha = \alpha\beta$

1.) $\alpha\wedge\beta = \alpha\cdot\beta = \underline{\underline{\alpha}}$

2.) $\alpha\vee\beta = \underset{\overset{\parallel}{\cancel{\alpha\beta}}}{\alpha}+\beta+\cancel{\alpha\beta} = \underline{\underline{\beta}}$

3.) $\alpha\to\beta = 1+\underset{\overset{\parallel}{\cancel{\alpha\beta}}}{\alpha}+\cancel{\alpha\beta} = \underline{\underline{1}}$

4.) $\alpha\wedge\gamma \leftrightarrow \beta\wedge\gamma = \alpha\gamma \leftrightarrow \beta\gamma = 1+\alpha\gamma+\beta\gamma = 1+\alpha\beta\gamma+\beta\gamma = \beta\wedge\gamma\to\alpha$

4.7

1.) $\alpha \leftrightarrow \alpha\vee\alpha = \alpha \leftrightarrow \cancel{1}\alpha+\cancel{\alpha}+\cancel{\alpha} = 1+\cancel{\alpha}+\cancel{\alpha} = 1 \to$ Tautología

2.) $(\alpha\to\beta)\wedge(\beta\to\gamma)\to(\alpha\to\gamma) = (1+\alpha+\alpha\beta)\wedge(1+\beta+\beta\gamma)\to(1+\alpha+\alpha\gamma) =$

$= (1+\beta+\beta\gamma+\alpha+\cancel{\alpha\beta}+\cancel{\alpha\beta\gamma}+\cancel{\alpha\beta}+\alpha\beta+\cancel{\alpha\beta\gamma}) \to (1+\alpha+\alpha\gamma) =$

$= \cancel{1}+\cancel{1}+\cancel{\beta}+\cancel{\beta\gamma}+\cancel{\alpha}+\cancel{\alpha\beta}+1+\cancel{\beta}+\cancel{\beta\gamma}+\cancel{\alpha}+\cancel{\alpha\beta}+\cancel{\alpha}+\cancel{\alpha\beta}+\cancel{\alpha\beta\gamma}+\cancel{\alpha}+\cancel{\alpha\beta}+\cancel{\alpha\gamma}+\cancel{\alpha\beta\gamma}+\cancel{\alpha\beta\gamma}+\cancel{\alpha\gamma}+$
$\cancel{\alpha\beta\gamma} = 1 \to$ Tautología

3.) $(\alpha\to\beta)\wedge\beta\to\alpha = 1+(\alpha\to\beta)\beta+(\alpha\to\beta)\beta\alpha = 1+(1+\alpha+\alpha\beta)\beta+(1+\alpha+\alpha\beta)\alpha\beta =$

$= 1+\beta+\alpha\beta+\cancel{\alpha\beta}+\cancel{\alpha\beta}+\cancel{\alpha\beta}+\cancel{\alpha\beta} = 1+\beta+\alpha\beta \to$ Es contingente.

4.) $\neg\alpha \to \alpha \wedge \beta = (1+\alpha) \to \alpha\beta = \cancel{1} + \cancel{1} + \alpha + \cancel{\alpha\beta} + \cancel{\alpha\beta} = \alpha \to$ Contingente

5.) $\alpha \wedge \neg(\alpha \vee \beta) = \alpha \wedge \neg(\alpha+\beta+\alpha\beta) = \alpha \wedge (1+\alpha+\beta+\alpha\beta) = \cancel{\alpha}+\cancel{\alpha}+\cancel{\alpha\beta}+\cancel{\alpha\beta} = 0$

Es una contradicción

6.) $\neg\alpha \leftrightarrow (\alpha \to \neg\alpha) = 1+\alpha \leftrightarrow (1+\cancel{\alpha}+\cancel{\alpha}+\alpha) = 1 + \cancel{1}+\cancel{\alpha}+\cancel{1}+\cancel{\alpha} \to$ Tautología

7.) $(\alpha \to \beta) \leftrightarrow \neg\alpha \vee \beta = (1+\alpha+\alpha\beta) \leftrightarrow (1+\alpha)\vee\beta = (1+\alpha+\alpha\beta) \leftrightarrow 1+\alpha+\cancel{\beta}+\cancel{\beta}+\alpha\beta$

$= \underline{\underline{1}} \to$ Tautología

8.) $(\alpha \to \beta) \leftrightarrow \neg(\alpha \wedge \neg\beta) = (1+\alpha+\alpha\beta) \leftrightarrow (1+\alpha+\alpha\beta) = 1 \to$ Tautología

4.10

a) $a \to b \equiv \neg a \to \neg b \Rightarrow 1+a+ab \equiv 1+\cancel{a}+\cancel{1}+\cancel{1}+b+\cancel{a}+ab \to$ No es cierto

b) $a \leftrightarrow b \equiv \neg a \leftrightarrow \neg b \Rightarrow 1+a+b \equiv 1+a+\cancel{1}+b+\cancel{1} \to$ Sí es cierto

c) $(a \vee b) \to c \equiv (a \to c) \vee (b \to c) \Rightarrow (a+b+ab) \to c \equiv (1+a+ac) \vee (1+b+bc) \Rightarrow$

$\Rightarrow 1+a+b+ab+ac+bc+abc \equiv 1+\cancel{a}+\cancel{ac}+\cancel{1}+\cancel{b}+\cancel{bc}+\cancel{1}+\cancel{b}+\cancel{bc}+\cancel{a}+ab+abc+\cancel{ac}+\cancel{abc}+\cancel{abc}$

$\equiv 1+ab+abc \to$ No es cierto

d) $(a \vee b) \to c \equiv (a \to c) \wedge (b \to c) \Rightarrow 1+a+b+ab+ac+bc+abc \equiv (1+a+ac) \wedge (1+b+bc) \Rightarrow$

$\Rightarrow 1+a+b+ab+ac+bc+abc \equiv 1+b+bc+a+ab+abc+ac+\cancel{abc}+\cancel{abc} \to$ Sí es cierto

e) $a \to (b \vee c) \equiv (a \to b) \vee (a \to c) \Rightarrow a \to (b+c+bc) \equiv (1+a+ab) \vee (1+a+ac) \Rightarrow$

$\Rightarrow 1+a+ab+ac+abc \equiv 1+a+ab+\cancel{1}+\cancel{a}+ac+\cancel{1}+\cancel{a}+\cancel{ac}+\cancel{a}+\cancel{a}+\cancel{ac}+\cancel{ab}+\cancel{ab}+abc \to$ Sí es cierto

f) $a \to (b \to c) \equiv (a \wedge b) \to c \Rightarrow a \to (1+b+bc) \equiv ab \to c \Rightarrow$

$\Rightarrow 1+\cancel{a}+\cancel{a}+ab+abc \equiv 1+abc \to$ No es cierto

4.11

$$\Gamma = \{c \to (a \vee b),\ b \to (c \to a),\ d \wedge \neg(c \to a)\}$$

$$\Gamma'' = \{\neg c \vee a \vee b,\ \neg b \vee \neg c \vee a,\ d,\ c,\ \neg a\}$$

$\neg a$	$\in \Gamma$	$\neg b \vee \neg c$	res(1,2)	$\neg b$	Res(3,4)	$a \vee b$	res(,6)
$\neg b \vee \neg a \vee a$	$\in \Gamma$	c	$\in \Gamma$	$\neg c \vee a \vee b$	$\in \Gamma$	b	res(1,7)
						$\square$	res(5,8)

$$\{\neg c \vee b,\ \neg b \vee \neg c,\ d,\ c,\ \neg a\}$$

$$\mid \neg a = 1$$

$$\{\neg c \vee b,\ \neg b \vee \neg c,\ d,\ c\}$$

$$\mid c = 1$$

$$\{b = \neg b,\ d\}$$

$$\mid b = 1$$

$$\{\square, c\}$$

4.12

1.) $\beta \to \alpha \vee \gamma,\ \alpha \to \beta,\ \alpha,\ \gamma \to \beta \models \gamma$

$\gamma \to \beta$ ↳ $1 + \gamma + \gamma\beta$

$\alpha \to \beta$ ↳ $1 + \alpha + \alpha\beta$

$\beta \to \alpha \vee \gamma$ ↳ $1 + \beta + \alpha\beta + \beta\gamma + \alpha\beta\gamma$

$$(1+\beta+\alpha\beta+\beta\gamma+\alpha\beta\gamma)(1+\cancel{\alpha}+\alpha\beta)(1+\gamma+\gamma\beta)(1+\gamma)\cancel{\alpha} =$$

$$= (\cancel{1}+\cancel{\beta}+\alpha\beta+\cancel{\beta\gamma}+\cancel{\alpha\beta\gamma})\alpha\beta(1+\gamma+\gamma\beta)(1+\gamma) = \alpha\beta(1+\cancel{\gamma}+\cancel{\gamma\beta})(1+\gamma) = \alpha\beta\cdot(1+\gamma) =$$

$$= \alpha\beta + \alpha\beta\gamma \to \text{No es tautología}$$

2.) $(\beta \to \neg\alpha) \to ((\neg\alpha \to \neg(\alpha\to\beta)) \to \alpha)$

$\beta\to\neg\alpha,\ \neg\alpha\to\neg(\alpha\to\beta) \models \alpha$

↳ $1+\cancel{\beta}+\cancel{\beta}+\alpha\beta$ ↳ $(1+\alpha)\to \cancel{1+}\,\alpha+\alpha\beta = \cancel{1}+\cancel{1}+\alpha+\cancel{\alpha}+\cancel{\alpha\beta}+\cancel{\alpha}+\cancel{\alpha\beta}$

$(1+\alpha\beta)\cdot(\cancel{\alpha})\cdot(\cancel{1+\alpha}) = \cancel{1\alpha}(\alpha+\alpha\beta)\cdot(1+\alpha) = \cancel{\alpha}+\cancel{\alpha}+\cancel{\alpha\beta}+\cancel{\alpha\beta}=0$

3.) $(\alpha\to\beta)\to((\beta\to\gamma)\to(\alpha\to\gamma))$

$\alpha\to\beta,\ \beta\to\gamma,\ \alpha \models \gamma$

↳ $1+\alpha+\alpha\beta$ ↳ $1+\beta+\beta\gamma$

$(1+\alpha+\alpha\beta)\cdot(1+\beta+\beta\gamma)\cdot\alpha\cdot(1+\gamma) = (1+\alpha+\alpha\beta)\cdot(\alpha+\alpha\beta+\alpha\beta\gamma)(1+\gamma) =$

$= \cancel{\alpha}+\cancel{\alpha\beta}+\alpha\beta\gamma+\cancel{\alpha}+\cancel{\alpha\beta}+\cancel{\alpha\beta\gamma}+\cancel{\alpha\beta}+\cancel{\alpha\beta}+\cancel{\alpha\beta\gamma}\,(1+\gamma) = \cancel{\alpha\beta\gamma}+\cancel{\alpha\beta\gamma}=0$

4.) $((\alpha\to\beta)\to\alpha)\to\alpha$

$(\alpha\to\beta)\to\alpha, \models\alpha$

↓

$(1+\alpha+\alpha\beta)\to\alpha = \cancel{1}+\cancel{1}+\alpha+\cancel{\alpha\beta}+\cancel{\alpha}+\cancel{\alpha}+\cancel{\alpha\beta}$

$\cancel{1}\alpha\cdot(1+\alpha) = \cancel{\alpha}+\cancel{\alpha}=0$

5.) $(\beta\to\gamma)\to(\neg(\alpha\to\gamma)\to\neg(\alpha\to\beta))$

$\beta\to\gamma,\ \neg(\alpha\to\gamma), \models \neg(\alpha\to\beta)$

↓ $1+\beta+\beta\gamma$ ↳ $\cancel{1}+\cancel{1}+\alpha+\alpha\gamma$

$(1+\beta+\beta\gamma)(\alpha+\alpha\gamma)\cdot(1+\alpha+\alpha\beta) = (\alpha+\alpha\beta+\alpha\beta\gamma+\alpha\gamma+\cancel{\alpha\beta\gamma}+\cancel{\alpha\beta\gamma})\cdot(1+\alpha+\alpha\beta) =$

$= \cancel{\alpha}+\cancel{\alpha}+\cancel{\alpha\beta}+\cancel{\alpha\beta}+\cancel{\alpha\beta}+\cancel{\alpha\beta}+\cancel{\alpha\beta\gamma}+\cancel{\alpha\beta\gamma}+\cancel{\alpha\beta\gamma}+\cancel{\alpha\gamma}+\cancel{\alpha\gamma}+\cancel{\alpha\beta\gamma} = 0$

6.) $((\alpha\to\beta)\to\gamma)\to(\beta\to\gamma)$

$(\alpha\to\beta)\to\gamma,\ \beta\models\gamma$

$(\cancel{1}+\cancel{1}+\cancel{\alpha}+\alpha\beta+\gamma+\alpha\gamma+\alpha\beta\gamma)\cdot(\beta)\cdot(1+\gamma) = (\alpha\beta+\alpha\beta+\beta\gamma+\alpha\beta\gamma+\alpha\beta\gamma)\cdot(1+\gamma) = \cancel{\beta\gamma}+\cancel{\beta\gamma}=0$