

4.14

$$1.) \{ \beta \rightarrow \alpha \vee \gamma, \alpha \rightarrow \beta, \alpha, \gamma \rightarrow \beta, \neg \gamma \}$$

$$\beta \rightarrow \alpha \vee \gamma \equiv \neg \beta \vee \alpha \vee \gamma$$

$$\{ \neg \beta \vee \alpha \vee \gamma, \neg \alpha \vee \beta, \alpha, \neg \gamma \vee \beta, \neg \gamma \}$$

$$r_1 \mid \alpha = 1$$

$$\{ \beta, \neg \gamma \vee \beta, \neg \gamma \}$$

$$r_1 \mid \gamma = 0$$

$$\{ \beta \}$$

$$r_1 \mid \beta = 1$$

$\{ \} \rightarrow$  Satisficible  $\rightarrow$  No es tautología

$$2.) \{ \beta \rightarrow \neg \alpha, \neg \alpha \rightarrow \neg (\alpha \rightarrow \beta), \neg \alpha \}$$

$$\{ \neg \beta \vee \neg \alpha, \alpha \vee \neg (\neg \alpha \vee \beta), \neg \alpha \}$$

$$r_1 \mid \alpha = 0$$

$\{ \alpha \vee \neg (\neg \alpha \vee \beta) \} \rightarrow$  Insatisficible  $\rightarrow$  Es tautología

$$3.) \{ \alpha \rightarrow \beta, \beta \rightarrow \gamma, \alpha, \neg \gamma \}$$

$$\{ \neg \alpha \vee \beta, \neg \beta \vee \gamma, \alpha, \neg \gamma \}$$

$$r_1 \mid \alpha = 1$$

$$\{ \beta, \neg \beta \vee \gamma, \neg \gamma \}$$

$$r_1 \mid \beta = 1$$

$\{ \gamma, \neg \gamma \} \rightarrow$  Insatisficible  $\rightarrow$  Es tautología

4.)  $\{( \alpha \rightarrow \beta ) \rightarrow \alpha, \neg \alpha \}$   
 $\{ (\neg \alpha \vee \beta) \rightarrow \alpha, \neg \alpha \}$   
 $\{ \neg (\neg \alpha \vee \beta) \vee \alpha, \neg \alpha \}$

$r_1 \mid \alpha = 0$   
 $\{ \Box \} \rightarrow \text{Unsatisfiable} \rightarrow \text{Tautology}$

5.)  $\{ \beta \rightarrow \gamma, \neg (\alpha \rightarrow \gamma), \alpha \rightarrow \beta \}$   
 $\{ \neg \beta \vee \gamma, \neg (\neg \alpha \vee \gamma), \neg \alpha \vee \beta \}$

$\alpha = 0$   
 $\{ \neg \beta \vee \gamma, \Box \}$   
 $\downarrow$   
 Unsatisfiable

$\alpha = 1$   
 $\{ \neg \beta \vee \gamma, \neg \gamma, \beta \}$   
 $r_1 \mid \beta = 1$   
 $\{ \gamma, \neg \gamma \} \rightarrow \text{Unsatisfiable}$   
 Tautology

6.)  $\{ (\alpha \rightarrow \beta) \rightarrow \gamma, \beta, \neg \gamma \}$   
 $\{ \neg (\neg \alpha \vee \beta) \vee \gamma, \beta, \neg \gamma \}$

$r_1 \mid \beta = 1$   
 $\{ \gamma, \neg \gamma \} \rightarrow \text{Unsatisfiable} \rightarrow \text{Tautology}$

7.)  $\models (\neg \alpha \vee \beta) \wedge (\alpha \vee \neg \beta) \rightarrow (\alpha \leftrightarrow \beta) \quad \gamma \quad \models (\alpha \leftrightarrow \beta) \rightarrow (\neg \alpha \vee \beta) \wedge (\alpha \vee \neg \beta)$   
 $\neg \alpha \vee \beta, \alpha \vee \neg \beta \models \alpha \leftrightarrow \beta \quad \gamma \quad \alpha \leftrightarrow \beta \models (\neg \alpha \vee \beta) \wedge (\alpha \vee \neg \beta)$   
 $\neg \alpha \vee \beta, \alpha \vee \neg \beta \models \alpha \rightarrow \beta \quad \gamma \quad \neg \alpha \vee \beta, \alpha \vee \neg \beta \models \beta \rightarrow \alpha \quad \gamma \quad \alpha \leftrightarrow \beta \models \neg \alpha \vee \beta \quad \gamma$   
 $\alpha \leftrightarrow \beta \models \alpha \vee \neg \beta$

①  $\{ \neg \alpha \vee \beta, \alpha \vee \neg \beta, \neg (\neg \alpha \vee \beta) \}$

$\alpha = 0$        $\alpha = 1$

$\{ \neg \beta, \Box \}$

Unsatisfiable

$\{ \beta, \neg \beta \}$

Unsatisfiable

②  $\{ \neg \alpha \vee \beta, \alpha \vee \neg \beta, \neg (\neg \beta \vee \alpha) \}$

$\alpha = 0$        $\alpha = 1$

$\{ \neg \beta, \beta \}$

Unsatisfiable

$\{ \beta, \Box \}$

Unsatisfiable

Tautology

8.)  $\{ \neg(\alpha \rightarrow \beta), \neg\alpha, \beta \}$   $\rightarrow r1 \mid \beta=1$

$\{ \neg(\neg\alpha \vee \beta), \neg\alpha, \beta \}$   $\rightarrow \{ \square, \neg\alpha \} \rightarrow \text{Insatisficible} \rightarrow \text{Tautologis}$

9.)  $\{ \neg\alpha \rightarrow \neg\beta \}, \alpha \rightarrow \beta$

$\{ \alpha \vee \neg\beta, \neg\alpha \vee \beta \}$

$\alpha=0$

$\alpha=1$

$\{ \neg\beta \}$

$\{ \beta \}$

Insatisficibles, no es tautologis

10.)  $\{ \alpha \rightarrow \beta, \neg\alpha \rightarrow \beta, \neg\beta \}$

$\{ \neg\alpha \vee \beta, \alpha \vee \beta, \neg\beta \}$

$r1 \mid \beta=0$

$\{ \neg\alpha, \alpha \} \rightarrow \text{Insatisficible} \rightarrow \text{Tautologis}$

4.15

1.)  $\{ \neg(\alpha \wedge b), \neg c \vee a, b \} \models \neg\alpha \wedge \neg c$

$\{ \neg\alpha \vee \neg b, \neg c \vee a, b \} \models \neg\alpha \wedge \neg c$

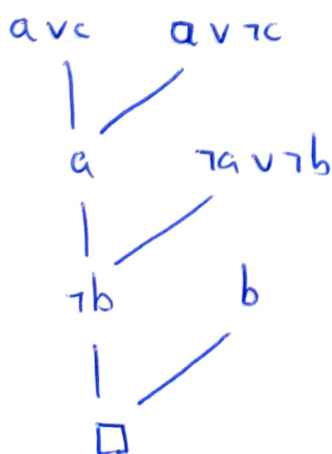
$\{ \neg\alpha \vee \neg b, \neg c \vee a, b, \alpha \vee c \}$

$r1 \mid b=1$

$\{ \neg\alpha, \neg c \vee a, \alpha \vee c \}$

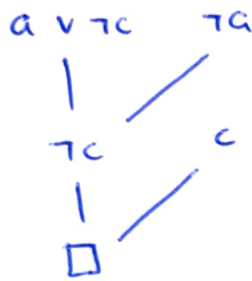
$r1 \mid a=0$

$\{ \neg c, c \} \rightarrow \text{Tautologis}$



2.)  $\{ \neg(a \wedge b), \neg c \vee a, b \} \models \neg a \rightarrow \neg c$

$\{ \neg a \vee b, \neg c \vee a, b, \text{~~negated~~ } \neg a, c \}$



$v_1 \mid b=1$   
 $\{ \neg c \vee a, \neg a, c \}$

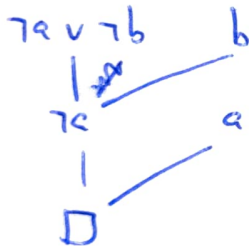
$v_1 \mid c=1$

$\{ a, \neg a \} \rightarrow \text{Unsatisfiable}$

3.)  $\{ \text{~~negated~~ } \neg a \vee b, \neg c \vee a, b, a \}$

$v_1 \mid a=1$

$\{ \neg b, b \} \text{ Unsatisfiable}$

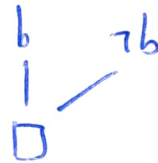


$\vee$

$\{ \neg a \vee b, \neg c \vee a, b, \neg b, \neg a \}$

$v_1 \mid b=1$

$\{ \neg c \vee a, \square, \neg a \} \text{ Unsatisfiable}$



4.)  $\{ \neg a \vee \neg b, \neg c \vee a, b, \neg c \}$

$v_1 \mid b=1$

$\{ \neg a, \neg c \vee a, \neg c \}$

$v_1 \mid a=0$

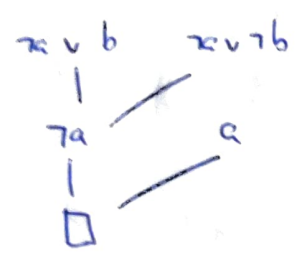
$\{ \neg c \} \rightarrow \text{Satisfiable}$

$b=1 \quad c=0=a$

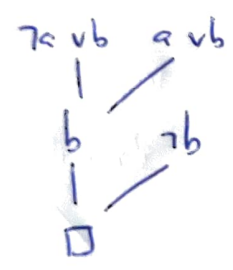
En este mundo sería verdad

4.16

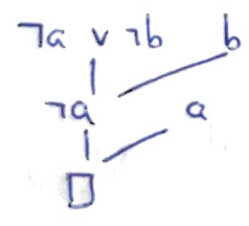
- 1.)  $\{a \rightarrow b, a \rightarrow \neg b\} \models \neg a$   $\rightarrow$   $r1 \mid a=1$   
 $\{\neg a \vee b, \neg a \vee \neg b, a\}$   $\rightarrow$   $\{b, \neg b\} \rightarrow$  Unsatisfiable



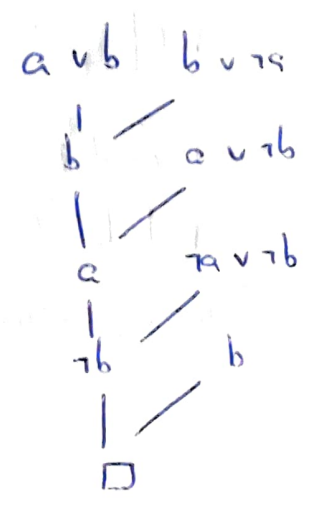
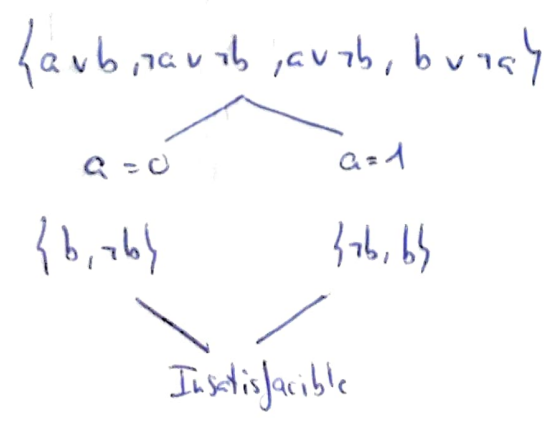
- 2.)  $\{a \rightarrow b, a \vee b\} \models b$   $\rightarrow$   $r1 \mid b=0$   
 $\{\neg a \vee b, a \vee b, \neg b\}$   $\rightarrow$   $\{\neg a, a\} \rightarrow$  Unsatisfiable



- 3.)  $\{a \rightarrow \neg b, a \wedge b\} \models c$   $\rightarrow$   $r1 \mid a=1$   
 $\{\neg a \vee \neg b, a, b, \neg c\}$   $\rightarrow$   $\{\neg b, b, \neg c\} \rightarrow$  Unsatisfiable



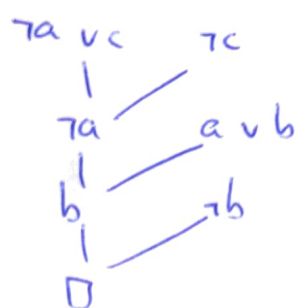
- 4.)  $\{a \vee b, \neg a \vee \neg b\} \models a \leftrightarrow \neg b$



5.)  $\{a \leftrightarrow \neg b, a \rightarrow c\} \models b \vee c$

$\{a \rightarrow \neg b, \neg b \rightarrow a, a \rightarrow c, \neg b, \neg c\}$

$\{\neg a \vee \neg b, b \vee a, \neg a \vee c, \neg b, \neg c\}$



$r1 \mid b=0$

$\{a, \neg a \vee c, \neg c\}$

$r1 \mid a=1$

$\{c, \neg c\} \rightarrow \text{Insatisfiable}$

6.)  $\{(a \wedge b) \leftrightarrow c, \neg c\} \models \neg a \wedge \neg b$

$\{(a \wedge b) \rightarrow c, c \rightarrow (a \wedge b), a \vee b\}$

$\{\neg a \vee \neg b \vee c, \neg c \vee (a \wedge b), a \vee b\}$

$\{\neg a \vee \neg b \vee c, \neg c \vee a, \neg c \vee b, a \vee b\}$

$c=1$

$\{a, b, a \vee b\}$

$r1 \mid a=1$

$\{b\}$

$c=0$

$\{\neg a \vee \neg b, a \vee b\}$

$a=0$

$a=1$

$\{b\}$

$\{\neg b\}$

Satisfiable

$a=1 \quad c=0=b$

En este mundo son falsas

7.)  $\{\neg(a \wedge b \wedge c), (a \wedge c) \vee (b \wedge c)\} \models a \rightarrow \neg b$

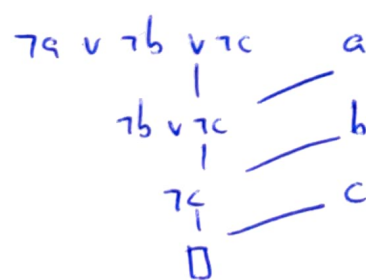
$\{\neg a \wedge \neg b \wedge \neg c, a \vee b, c, a, b\}$

$r1 \mid a=1$

$\{\neg b \wedge \neg c, c, b\}$

$r1 \mid b=1$

$\{\neg c, c\} \rightarrow \text{Insatisfiable}$



$$8.) \{ b \rightarrow (c \vee d), a \leftrightarrow \neg(b \wedge d) \} \models b \leftrightarrow (c \vee d)$$

$$\{ \neg b \vee c \vee a, a \rightarrow \neg(b \wedge d), \neg(b \wedge d) \rightarrow a, \neg(b \leftrightarrow (c \vee d)) \}$$

$$\{ \neg b \vee c \vee a, \neg a \vee \neg b \vee \neg d, (b \wedge d) \vee a, \neg((b \rightarrow (c \vee d)) \wedge ((c \vee d) \rightarrow b)) \}$$

$$\{ \neg b \vee c \vee a, \neg a \vee \neg b \vee \neg d, b \vee a, d \vee a, \neg((\neg b \vee c \vee d) \wedge ((\neg c \wedge \neg d) \vee b)) \}$$

$$\{ \neg b \vee c \vee a, \neg a \vee \neg b \vee \neg d, b \vee a, d \vee a, b, \neg c, \neg d, c \vee d \}$$

$$r1 \mid b = 1$$

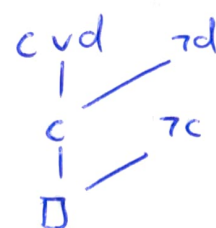
$$\{ c \vee a, \neg a \vee \neg d, d \vee a, \neg c, \neg d, c \vee d \}$$

$$r1 \mid c = 0$$

$$\{ a, \neg a \vee \neg d, d \vee a, \neg d, d \}$$

$$r1 \mid a = 1$$

$$\{ \neg d, d \} \rightarrow \text{Insatisfiable}$$



$$9.) \{ (a \wedge b) \rightarrow c, c \rightarrow (c \vee d) \} \models b \rightarrow (\neg a \rightarrow c)$$

$$\{ \neg a \vee b \vee c, \neg c \vee a \vee d, b, \neg a, \neg c \}$$

$$r1 \mid b = 1$$

$$\{ \neg c \vee c, \neg c \vee a \vee d, \neg a, \neg c \}$$

$$r1 \mid a = 0$$

$$\{ \neg c \vee d, \neg c \}$$

$$r1 \mid c = 0$$

$$\{ \} \rightarrow \text{Satisfiable}$$

$$\left( \begin{array}{l} b = 1 \\ a = c = d = 0 \end{array} \right)$$

En este mundo se cumple la fórmula.



10.)  $\{ (a \vee c) \rightarrow \neg a, c \rightarrow \neg a, b \rightarrow \neg a \} \models \neg a$

$\{ \underbrace{(\neg a \wedge \neg c) \vee \neg a}_{\neg a}, \neg c \vee \neg a, \neg b \vee \neg a, a \}$

$\neg a, \neg c, \neg b, a$

$\neg a \quad a$   
| /  
□

$\{ \square, \neg c, \neg b \} \rightarrow \text{Unsatisfiable}$

$r_1 \mid a=1$

11.)  $\{ (a \wedge b) \rightarrow c, c \rightarrow d, b \wedge \neg d \} \models \neg c$

$\{ \neg a \vee \neg b \vee c, \neg c \vee d, b, \neg d, a \}$

$\neg a \vee \neg b \vee c \quad a$   
| /  
 $\neg b \vee c \quad b$   
| /  
 $c \quad \neg c \vee d$   
| /  
 $d \quad \neg d$   
| /  
□

$\{ \neg b \vee c, \neg c \vee d, b, \neg d \}$

$r_1 \mid b=1$

$\{ c, \neg c \vee d, \neg d \}$

$r_1 \mid c=1$

Unsatisfiable  $\rightarrow \{ d, \neg d \}$

12.)  $\{ (a \rightarrow b) \vee (c \rightarrow d), \neg a \rightarrow a, \neg c \rightarrow c \} \models b \vee d$

$\{ \neg a \vee b \vee \neg c \vee d, a, c, \neg b, \neg d \}$

$r_1 \mid a=1$

$\{ b \vee \neg c \vee d, c, \neg b, \neg d \}$

$r_1 \mid c=1$

$\{ b \vee d, \neg b, \neg d \}$

$r_1 \mid b=0$

$\{ d, \neg d \} \rightarrow \text{Unsatisfiable}$

$\neg a \vee b \vee \neg c \vee d \quad a$   
| /  
 $b \vee \neg c \vee d \quad \neg b$   
| /  
 $\neg c \vee d \quad c$   
| /  
 $d \quad \neg d$   
| /  
□



13.)  $\{a \rightarrow (b \vee c, c \rightarrow d, \neg b \vee d\} \models \neg(a \wedge \neg d)$

$\{\neg a \vee b \vee c, \neg c \vee d, \neg b \vee d, a, \neg d\}$

$v_1 \mid a=1$

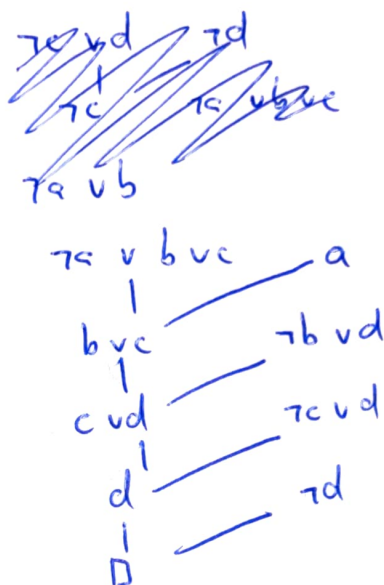
$\{b \vee c, \neg c \vee d, \neg b \vee d, \neg d\}$

$v_1 \mid d=0$

$\{b \vee c, \neg c, \neg b\}$

$v_1 \mid c=0$

$\{b, \neg b\} \rightarrow \text{Unsatisfiable}$



14.)  $\{(b \rightarrow a) \wedge b, c \rightarrow d, b \rightarrow c\} \models a \vee d$

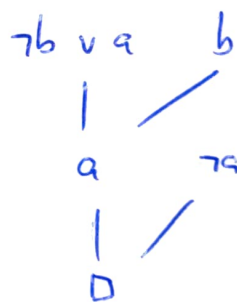
$\{\neg b \vee a, b, \neg c \vee d, \neg b \vee c, \neg a, \neg d\}$

$v_1 \mid b=1$

$\{a, \neg c \vee d, c, \neg d, \neg a\}$

$v_1 \mid a=1$

$\{\neg c \vee d, c, \neg d, \square\} \rightarrow \text{Unsatisfiable}$



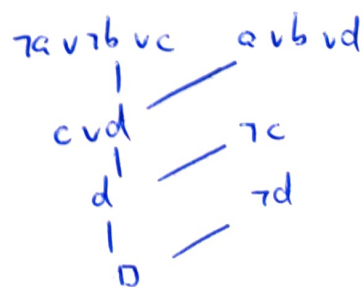
15.)  $\{(a \wedge b) \rightarrow c, (\neg a \wedge \neg b) \rightarrow d, a \leftrightarrow b\} \models c \vee d$

$\{\neg a \vee \neg b \vee c, a \vee b \vee d, a \vee b, \neg b \vee a, \neg c, \neg d\}$

$v_1 \mid c=0$

$\{\neg a \vee \neg b, a \vee b \vee d, \neg a \vee b, \neg b \vee a, \neg d\}$

$v_1 \mid d=0$



$a=0$  —  $\{\neg a \vee \neg b, a \vee b, \neg a \vee b, \neg b \vee a\}$  —  $a=1$   
 $\{b, \neg b\}$  —  $\{\neg b, b\}$   
 $\text{Unsatisfiable}$

$$16.) \{ a \rightarrow (b \vee c), d \vee \neg c, b \vee d \} \models a \rightarrow d$$

$$\{ \neg a \vee b \vee c, d \vee \neg c, b \vee d, a, \neg d \}$$

$$r1 \mid a=1$$

$$\{ b \vee c, d \vee \neg c, b \vee d, \neg d \}$$

$$r1 \mid d=0$$

$$\{ b \vee c, \neg c, b \}$$

$$r1 \mid b=1$$

$$\{ \neg c \} \rightarrow \text{Satisfiable}$$

$$\begin{array}{l} a=1=b \\ c=0=d \end{array}$$

Mundo en el  
que  $\models$  es falso

$$17.) \{ (\neg b \wedge \neg c) \rightarrow \neg a, a \rightarrow b, a \leftrightarrow c \} \models b \vee c$$

$$\{ b \vee c \vee \neg a, \neg a \vee b, \neg a \vee c, \neg c \vee a, \neg b, \neg c \}$$

$$r1 \mid b=0$$

$$\{ c \vee \neg a, \neg a, \neg a \vee c, \neg c \vee a, \neg c \}$$

$$r1 \mid a=0$$

$$\{ \neg c \} \rightarrow \text{Satisfiable}$$

$$a=b=c=0$$

Aquí  $\models$  es falso

$$18.) \{ a \rightarrow (c \rightarrow b), (b \vee c) \rightarrow a, c \rightarrow (a \vee b) \} \models b$$

$$\{ \neg a \vee \neg b, \neg b \vee a, \neg c \vee a, \neg c \vee a \vee b, \neg b \}$$

$$r1 \mid b=0$$

$$\{ \neg a, \neg c \vee a \}$$

$$r1 \mid a=0$$

$$\{ \neg c \} \rightarrow \text{Satisfiable}$$

$$a=b=c=0$$

Aquí  $\models$  es falso

19.)  $\{ (a \wedge b) \rightarrow c, (\neg a \wedge b) \rightarrow d, c \vee \neg b, e \rightarrow (a \wedge d) \} \models c$   
 $\{ \neg a \vee b \vee c, a \vee b \vee d, c \vee \neg b, c \wedge a, c \wedge d, e \}$

$$r_1 \mid e = 1$$

$$\{ c \vee b \vee c, a \vee b \vee d, c \vee \neg b, a, d \}$$

$$r_1 \mid a = 1$$

$$\{ b \vee c, \neg b, d \}$$

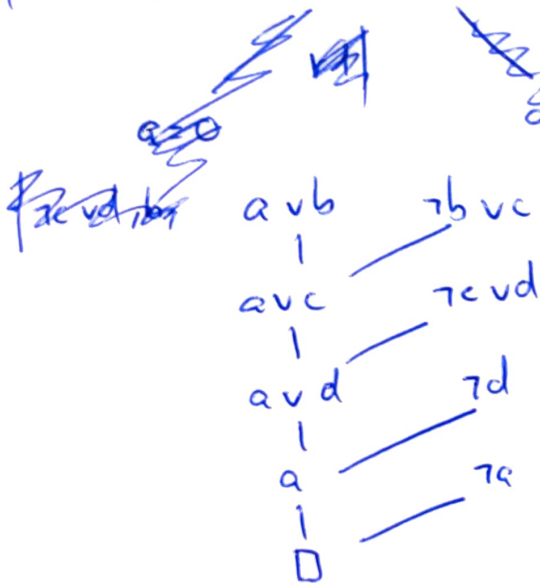
$$r_1 \mid b = 0$$

$$\{ c, d \} \rightarrow \text{Satisfiable}$$

$$\begin{array}{l} a = e = 1 \\ b = c = d = 0 \end{array}$$

20.)  $\{ c \rightarrow d, a \vee b, \neg(\neg a \rightarrow d), \neg e \rightarrow b \} \models b \wedge \neg c$

$$\{ c \vee d, a \vee b, \neg a, d, a \vee b, \neg b \vee c \} \rightarrow r_1 \mid a = 0$$



$$\{ c \vee d, b, d, \neg b \vee c \}$$

$$r_1 \mid b = 1$$

$$\{ c \vee d, d, c \}$$

$$r_1 \mid c = 1$$

$$\{ d, d \} \rightarrow \text{Unsatisfiable}$$