PERGAMON

Computers in Biology and Medicine 28 (1998) 169–181

Butterworth equations for homomorphic filtering of images

Holger G. Adelmann *

Microscopy Workgroup, NWV Hagen, Alt-Steinbuecheler Weg 9, 51377 Leverkusen, Germany

Received 20 June 1997

Abstract

In digital image processing, the homomorphic filtering approach is derived from an illumination–reflectance model of the image. Homomorphic filtering can perform simultaneous dynamic range compression and contrast enhancement. Crucial for the success of the homomorphic approach is the selection of an appropriate frequency-domain filter function in order to modify the illumination and reflectance components of an image differently. The author found Butterworth type highpass equations far superior to other frequency-domain filter functions, including Gaussian equations, making the Butterworth highpass suitable for use with the homomorphic filtering approach. The program was written in Microsoft (MS) Visual C++ ™ (filter) as well as MS Visual Basic™ (user interface) to run as a module under the image processing software package Image-Pro Plus™.

Keywords: Homomorphic filter; Butterworth equations; Frequency domain filtering; Image processing

1. Introduction

The homomorphic filter is an approach based on an illumination–reflectance image model. It is said to be useful for image enhancement by simultaneous brightness range compression and contrast enhancement [1, 2]. The author could not find any commercially available software package that offers this filtering approach. Furthermore, search on this topic on the internet and in newsgroups dealing with computing and image processing was negative. Therefore the author decided to program an implementation of the homomorphic filter based on the theory given in Refs. [1, 2] including the design of an appropriate frequency-domain filter function and

---

Abbreviations: FFT, Fast Fourier transform, (x, y), Two-dimensional coordinates in the spatial domain, (u, v), Two-dimensional coordinates in the frequency domain.

* Corresponding author.

0010-4825/98/$19.00
PII: S0010-4825(98)00004-3

to evaluate the properties of the homomorphic filter compared to other advanced image enhancement methods, e.g. by frequency-domain Gaussian bandpass filtering [3] and local histogram equalization [4].

In order to understand an image as interpreted by the illumination–reflectance model, this model will be discussed briefly below. The non-zero and finite two-dimensional image function $f_{(x,y)}$ represents the usual notation of an image: the two spatial variables x and y denote any point in the image and the value of the function $f_{(x,y)}$ represents the amplitude (the brightness) of the given image pixel. The image of a certain object is usually formed by the light illuminating the object and the light reflected by the object. Illumination ($i_{(x,y)}$) and reflectance ($r_{(x,y)}$) components are forming the resulting image ($f_{(x,y)}$) of an object by a multiplicative relationship [2]:

$$f_{(x,y)} = i_{(x,y)} \cdot r_{(x,y)} \tag{1}$$

Where the nature of $i_{(x,y)}$ is determined by the properties of the light source and $r_{(x,y)}$ is determined by the optical properties of the given object (for the derivation of the theoretic bounds of both components see Refs. [1, 2]).

Providing ways to treat illumination and reflectance components of an image differently can offer new opportunities for image enhancement. Filtering of images in the frequency-domain is both versatile and powerful [3], but, unfortunately, a different treatment of illumination and reflectance components in the frequency domain is normally not possible, because the Fourier transform of the product the two components are forming is not separable [1], i.e.

$$F\{f_{(x,y)}\} \neq F\{i_{(x,y)}\} \cdot F\{r_{(x,y)}\} \tag{2}$$

A way to overcome this problem is to calculate the natural logarithm of the image first before applying the Fourier transformation

$$\ln(\mathrm{f}_{(x,y)}) = \ln(\mathrm{i}_{(x,y)}) + \ln(\mathrm{r}_{(x,y)}). \tag{3}$$

This will allow the separation of the illumination and reflectance components in Fourier space (where F, I and R are the Fourier transforms of $\ln f$, $\ln i$ and $\ln r$, respectively,

$$F_{(u,v)} = I_{(u,v)} + R_{(u,v)} \tag{4}$$

making them available for different treatment by frequency-domain filters. Filtering in the frequency-domain is done by multiplication of the Fourier transform of the image (F) with the Fourier transform of the filter (H)

$$G_{(u,v)} = F_{(u,v)} \cdot H_{(u,v)} = I_{(u,v)} \cdot H_{(u,v)} + R_{(u,v)} \cdot H_{(u,v)} \tag{5}$$

Because illumination is usually characterized by slow spatial variations thus being of low frequency, the illumination component is found near the center of the twodimensional Fourier transform. The reflectance component, however, represents the spatial variation amongst the object, thus being generally of higher frequency, depending on the amount of minute detail in the object. Therefore the reflectance component is located more in the outer area of the twodimensional Fourier spectrum. Although illumination and reflectance components are not *strictly* separated in Fourier space, the homomorphic approach is nevertheless useful and provides good image enhancement results as can be seen later. After applying the filter in the frequency-domain and subsequent inverse Fourier transformation of the resulting modified spectrum, the image is recalculated by an exponential transformation, undoing the logarithmic transform done previously. It should be noted that all calculations should be done on 32-bit representations of the image to avoid saturation and clipping due to the constrained range of values for 8-bit grayscale images.

Thus, the homomorphic approach can be summarized in Fig. 1 which illustrates the flow-chart that addresses the steps involved in this type of image processing.

Although the major points were covered here, a detailed derivation of the homomorphic approach is beyond the scope of this article and can be found in Refs. [1, 2].

2. The frequency-domain filter function

As discussed before, the illumination component is found near the center of the two-dimensional Fourier transform whereas the reflectance component is located more in the outer area of the twodimensional Fourier spectrum, although they are not *strictly* separated. In order to achieve simultaneous dynamic range compression and contrast enhancement, the transfer function of the frequency-domain filter should decrease the overall spectral energy of the illumination component while amplifying the spectral energy of the reflectance component of the image. In order to weighten the complex Fourier coefficients in the way described above, Gonzalez and Woods [1] stated, that a twodimensional filter function suitable for homomorphic filtering should produce a value of 0.5 for the central area of the Fourier transform (halving the spectral energy of the illumination component) and a value of 2.0 for the outer area of the frequency domain (doubling the spectral energy of the reflectance component) with a smooth transition between these two regions. However, in order to adjust the filter to the spatial demands of each object, the author found it necessary to provide a way to adjust the transition, the 'cut-off' of the filter response. Furthermore the transition slope should have a certain steepness for a good discrimination between the effects on illumination and reflectance components, but not too steep (or even rectangular) because this will introduce 'ringing' artifacts [1, 5] into the filtered image arising from the introduction of a harsh energy step in the frequency-domain. Of several possible transfer functions tested, a Gaussian highpass response with the universal equation given below (note that all filters are given here in their frequency-domain representation)

$$H_{(u,v)} = 1 - \mathrm{e}^{-a(u^2+v^2)} \tag{6}$$

as well as a modified Butterworth type highpass filter with the universal equation

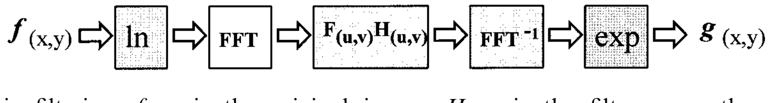

Fig. 1. Homomorphic filtering: $f_{(x,y)}$ is the original image, $H_{(u,v)}$ is the filter, $g_{(x,y)}$ the resulting image after homomorphic filtering (after Gonzalez and Woods, 1993, modified).

$$H_{(u,v)} = 1 - \frac{1}{1 + ((u^2 + v^2)/a)^n} \tag{7}$$

were chosen for closer evaluation with several test objects. Finally, the author found the modified Butterworth equation most suitable for use with the homomorphic filter approach, both in terms of allowing a steeper slope over the whole range than f.ex. the Gaussian equations as well as the ease of adjusting the transition (the cut-off) of the filter response. The Butterworth equations allow the setting of the transition point and the transition slope largely independent from each other, allowing a precise tuning of the filter to both the content and spatial dimensions of the image. In the modified Butterworth equation given in Eq. (7), the term

$$\frac{u^2 + v^2}{a} \tag{8}$$

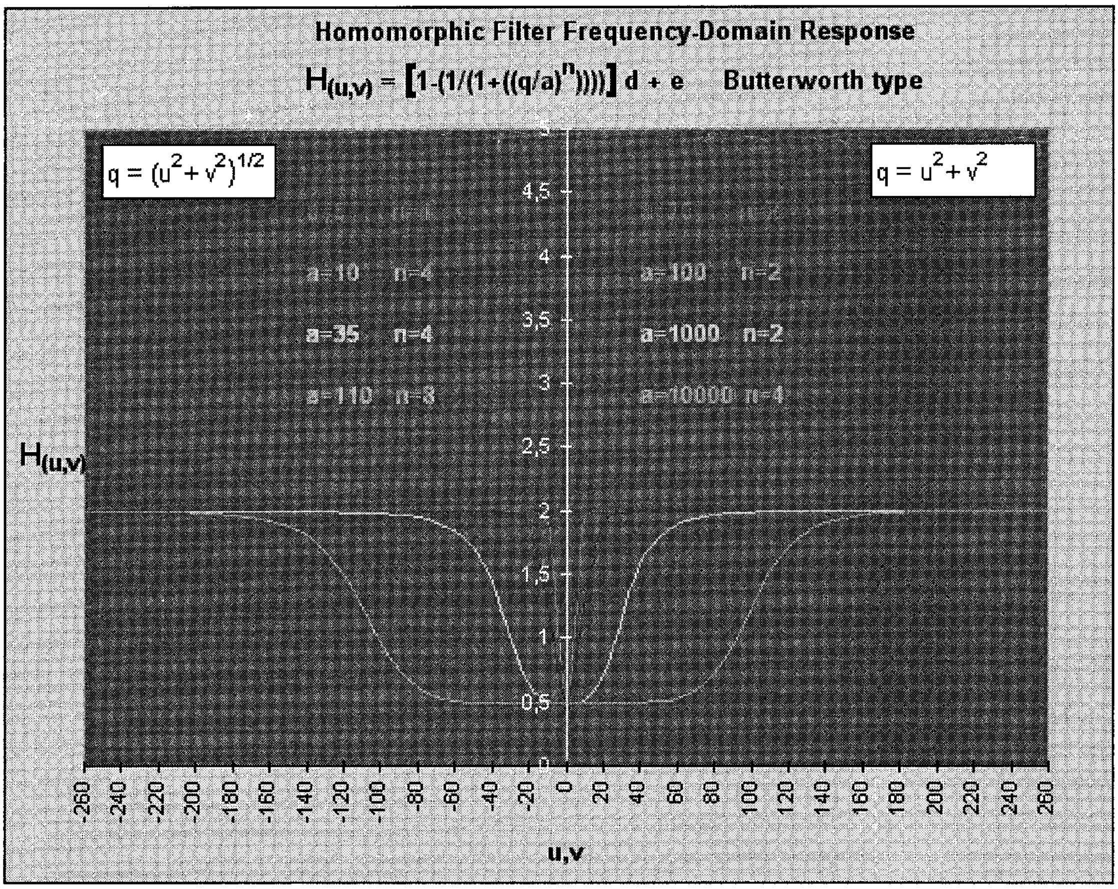

Fig. 2. The response produced by the modified Butterworth equation (right half) equals that of the original (left half) when different settings of coefficients a and n are used, saving computation time.

determines the transition point, whereas the Butterworth power coefficient n determines the steepness of the transition slope (the exponent n must be a positive power of two). The filter simulation in Fig. 2 shows different settings for a and n for a 256×256 pixel image spectrum, resulting in filter curves with different transition points and transition slopes. On the left side, curves for the traditional Butterworth equation are plotted with

$$q = \sqrt{u^2 + v^2} \tag{9}$$

whereas on the right side, curves for the modified equation are plotted with

$$q = u^2 + v^2 \tag{10}$$

A maximal function value of 2 and an offset of 0.5 as minimal value of the filter response as proposed by Gonzalez and Woods [1] can be easily achieved by multiplying the native filter response of Eq. (6) or Eq. (7) (yielding values between zero and unity) with an amplification constant d (1.5), plus the addition of an offset e (0.5)

$$H_{(u,v)}' = d \cdot H_{(u,v)} + e \tag{11}$$

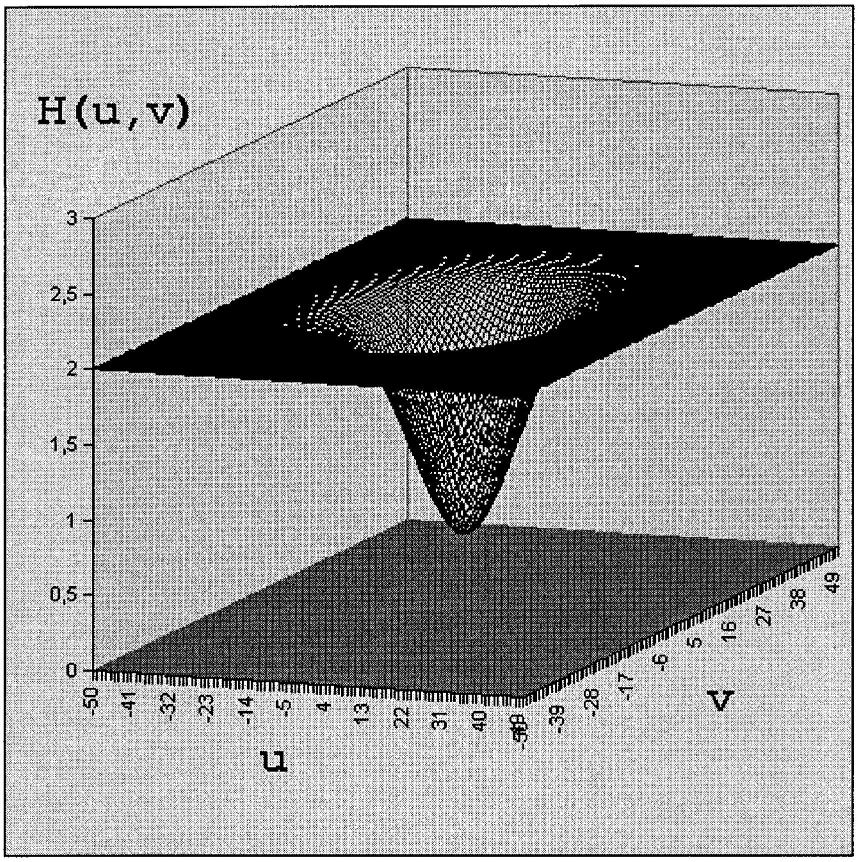

Fig. 3. Three-dimensional plot of a typical frequency-domain highpass filter function.

However, when applying the homomorphic filter onto different types of images, the author found a maximal amplification constant of 1.5 too much for several images, resulting in an 'overcontrasted' image as will be discussed along with sample images in the results section below.

Although displayed in a twodimensional plot above, it should be realized that the two-dimensional filter functions discussed are in fact rotationally symmetric around the $H_{(u,v)}$ axis. The threedimensional appearance of an example filter function is shown in Fig. 3 for better visualization.

3. The computer program

This chapter will not focus on the (trivial) Visual Basic user interface but will discuss the realization of the frequency-domain filter in C (implemented as Windows™DLL) in more detail (Fig. 4).

The modified Butterworth filter function is implemented in C as follows

$$(1-(1/(1+\text{pow}((\text{square\_q}/(\text{double})\text{coeff\_a}),(\text{double})\text{coeff\_n}))))^{*}\text{var\_d}+\text{var\_e}$$

where square_q represents the square of the radial distance of a given point from the center of the Fourier transform and is given by u^2+v^2. The variables coeff_a and coeff_n are identical to a and n in the Butterworth equation given in Eq. (7), whereas var_d is the amplification constant and var_e represents the offset as discussed above.

The floating point image was first loaded into a memory array and then all elements underwent logarithmic (ln) transformation (a very small offset was added prior to ln transformation to assure each element being non-zero). Fourier transformation of the image (both forward and inverse) was done by the build-in FFT functions of Image-Pro Plus, yielding an array of complex Fourier coefficients in floating point notation. Applying a frequency-domain filter generally means weighting the amplitude spectrum by multiplication of

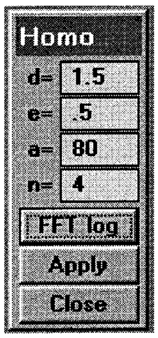

Fig. 4. The user interface, appearing as a module on the Image-Pro desktop, provides command buttons for forward transformation (ln plus FFT) as well as for applying the filter function (including inverse FFT plus exponential transformation). Furthermore it has numerical input fields for adjusting a, n, the amplification constant d and the offset e.

the current value of the amplitude spectrum with the corresponding local value of the filter function. This could be simplified in a way, that when multiplying both real and imaginary part of the complex Fourier coefficients with the filter function, only the amplitude is modified while the phase is not altered, thus there is no need to calculate the amplitude spectrum from the Fourier coefficients (and back), thus making this frequency-domain filter approach quite fast. After inverse Fourier transformation, the resulting array then underwent exponential transformation yielding the filtered image.

4. Results

Although the image model underlying the homomorphic approach is based on illumination and *reflection*, it also works well with images produced by *transillumination* (e.g. brightfield microscopy). This is not astonishing because the reflectance component is substituted here by an absorption component that also carries the spatial variations within the object. The illumination component is of course also present here. The first image example (Fig. 5) deals with Giemsa stained blood cells housing malaria parasites. The original image was taken with the brightfield microscope, it is a *high contrast–high dynamic range* image. Several image processing methods were used to enhance the perception of especially the structures in the dark areas, representing the parasites while trying to maintain the contrast balance of the structures

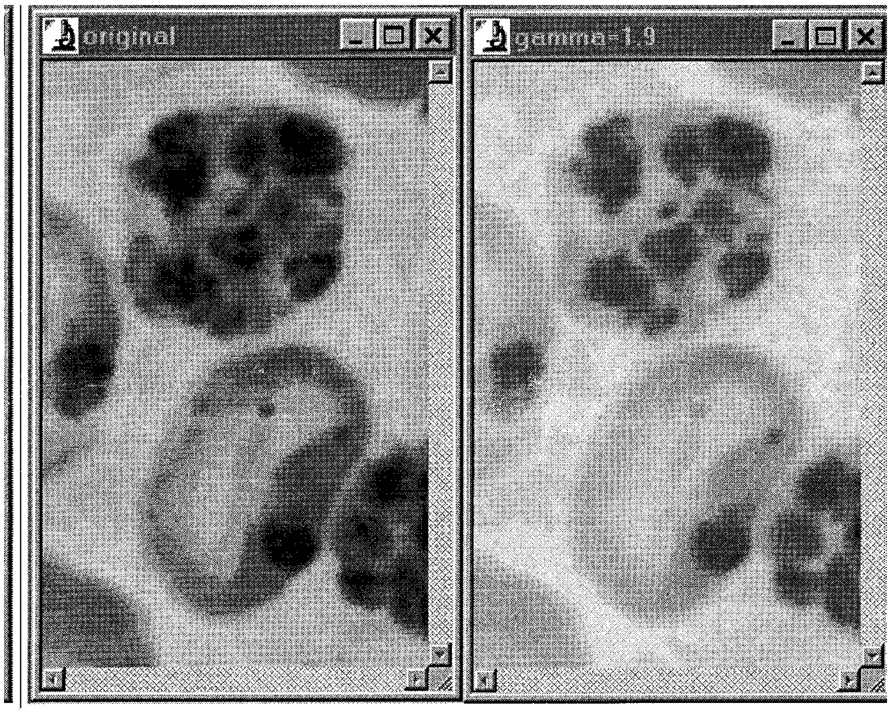

Figure 5(a1) *(continued overleaf)*.

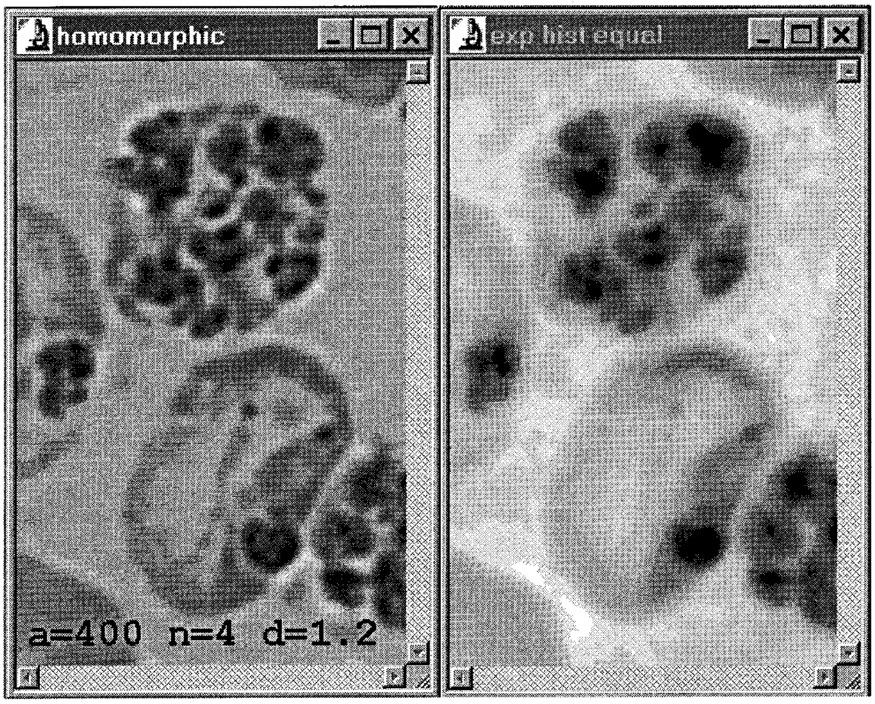

Figure 5(a2).

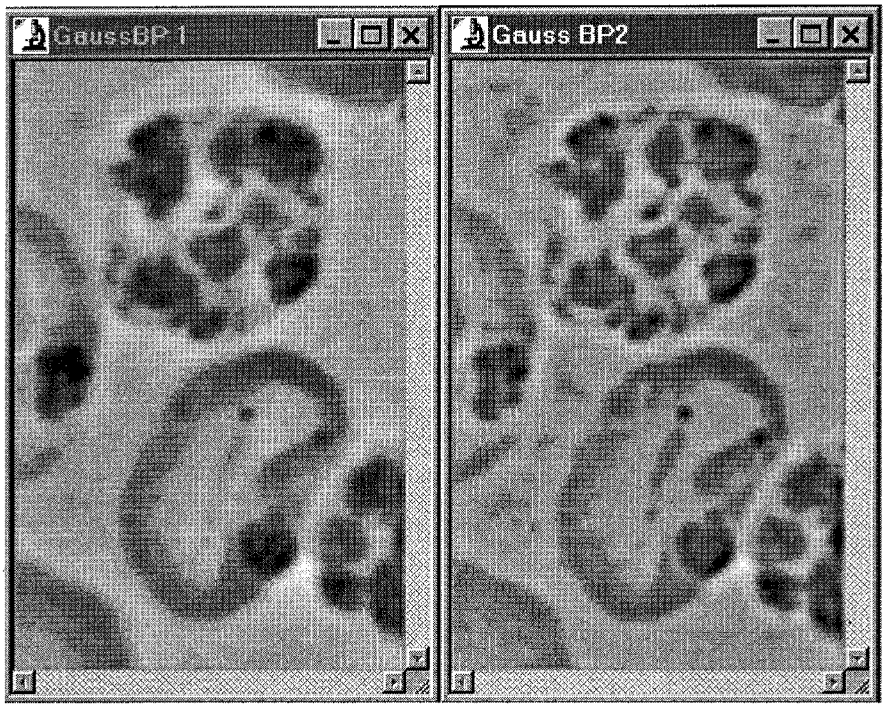

Fig. 5. (a1,a2,b) Giemsa stained blood cells housing malaria parasites (brightfield microscopy image).

in both bright and dark areas of the image (Fig. 5a and b).

As can be seen here, adjusting the *gamma* control (Fig. 5a1, right) results in somewhat clearer perception of the dark areas but bleaches the medium gray and bright areas completely. The *exponential histogram equalization* (Fig. 5a2, right) does not perform better on this example resulting in an unbalanced and overcontrasted image.

The *homomorphic filter* however performs a balanced equalization of both the bright and the dark areas (Fig. 5a2, left) with the parameter settings given in the image.

As can be seen in the next comparison (Fig. 5b), applying a frequency-domain *Gaussian bandpass* filter [3] performs quite good but removes too much low frequency from the image, before the perception of the dark structures is considerably improved, impairing object resemblance of the processed image.

In the left Fig. 5(b) subimage titled *GaussBP 1*, only a very small amount of low frequency around the zero-frequency-component (ZFC) was removed. This was obviously not enough to result in a good perceptibility of the structures in the dark image areas (the parasites). In the right Fig. 5(b) subimage titled *GaussBP 2*, a larger amount of low frequency was removed, making more structures in the dark regions discernible but resulting in a considerable contrast decrease in these areas.

The *homomorphic filter* works best here, decreasing the high dynamic range of the image and balancing the dark and bright areas while performing contrast enhancement in both areas. Although perception of detail in the dark areas is greatly enhanced, the contrast is balanced also for bright areas and the object resemblance of the processed image is largely maintained as compared to the other enhancement methods discussed.

Microscopic images obtained via brightfield transillumination are also sometimes difficult to enhance by *local histogram equalization*, a technique where the image histogram is optimized only in the context of the local neighbourhood of the area under processing.

The next example, again a malaria slide (Fig. 6), shows the comparison of *local histogram equalization* with *homomporphic filtering*.

Although there is some enhancement of the structures in the dark areas, these structures are disturbed by harsh and artificial borders introduced by the neighborhood processing. It is obvious, that maintaining object resemblance is not the domain of *local histogram equalization*.

The next example deals with an image captured from a bank video (Fig. 7, not a biological or medical application, but indeed an image generated by *reflected* light). This image was released from Media Cybernetics for demonstration purposes and comes along with their Image-Pro plus package. It is a *low contrast–high dynamic range* image. Here again, *homomorphic filtering* is compared to *gamma* adjustment and *local histogram equalization*.

Note how good the faces are enhanced with the *homomorphic filter*, applied once (HF, $a = 100$, $n = 4$, $d = 1.5$) or even twice (first HF, $a = 100$, $n = 4$, $d = 1.5$, then HF, $a = 20$, $n = 4$, $d = 1.5$). There is extensive processing needed in the given application in order to improve recognition of the faces that originally are hard to percept due to extremely low local contrast. Here again, the homomorphic filter has proven far superior to other enhancement methods.

The next example (Fig. 8) displays how the homomorphic filter can be used to reduce the inevitable halo effect in phase contrast microscopy. Note the bright halo both around the object (*Gregarina* sp.) as well as around the cytoplasmic inclusions within the object due to a

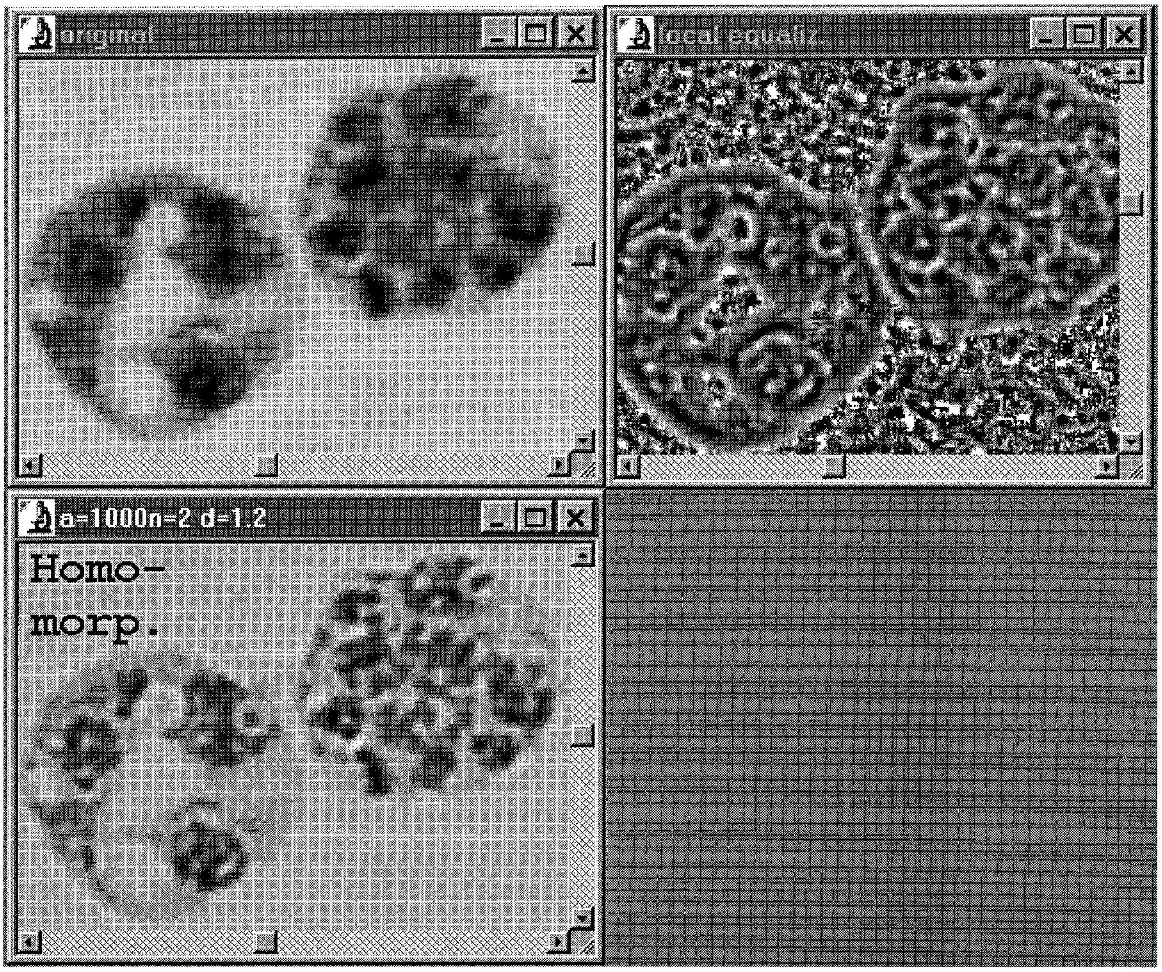

Fig. 6. Giemsa stained blood cells housing malaria parasites (brightfield microscopy image).

large phase retardation introduced by these relatively thick objects (Fig. 8a, original image). Note that despite the considerable reduction of the halos, the overall contrast of the filtered image (Fig. 8b) is maintained. In addition, features both in the very dark regions (nucleolus) as well as those in the very bright regions (granules within the object) became much more visible.

5. Discussion and conclusion

The homomorphic filter approach is mentioned only a few times in the literature, mostly in a theoretical fashion [1, 2]. As described in this article, it can be realized within the frame of a professional image processing package with an open programming interface. The

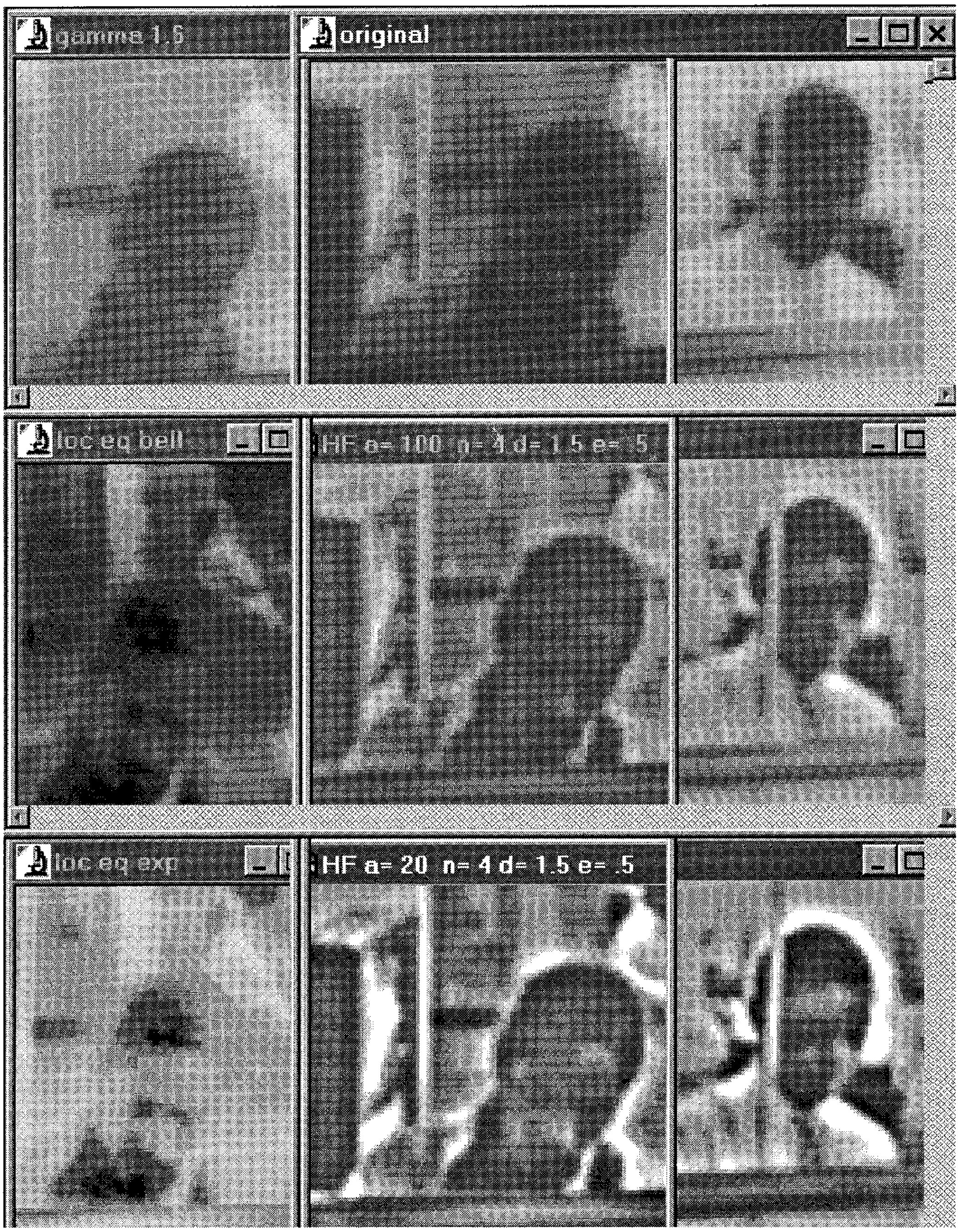

Fig. 7. Single frame from a bank video recording under low light condition.

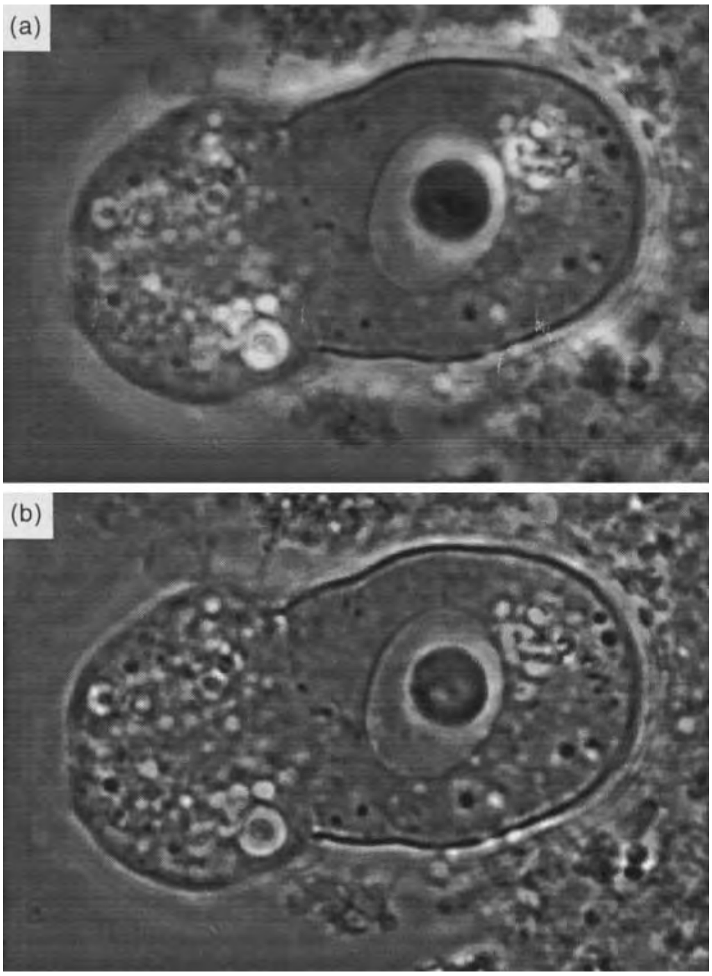

Fig. 8. (a) *Gregarina* sp. in phase contrast microscopy, original image with halos. (b) *Gregarina* sp. in phase contrast microscopy, after homomorphic filtering with settings $a = 300$, $n = 4$, $d = 1.2$ and $e = 0.5$.

implementation as stand-alone program is also possible if additionally the twodimensional Fourier transforms are programmed using well-known and effective algorithms. Butterworth type highpass filter functions were found to be most suitable for this approach. The homomorphic filter has proven his theoretical claim of simultaneous dynamic range compression and balanced contrast enhancement and was found a valuable tool for image processing. The homomorphic filter was often found to yield the best object resemblance amongst other enhancement methods. Although derived in theory from an image model based on illumination and reflectance, it can also be used with transillumination, e.g. brightfield and phase contrast microscopy. The author is currently investigating the use of homomorphic filtering with other microscopical methods such as differential interference contrast as well as with radiology and tomography images. The program code for non-profit personal use is available from the author upon request.

6. Summary

The design of the *homomorphic filter*, not a well known but valuable image processing tool performing simultaneous dynamic range compression and contrast enhancement, was described here along with the basics of its theoretical background. A suitable and adjustable filter function was derived for frequency-domain processing with the homomorphic filter approach. Example images were presented that display the valuable properties of this filter.

References

[1] R.C. Gonzales, R.E. Woods, Digital Image Processing, Addison-Wesley, Reading, Massachusetts, 1992.
[2] T.G. Stockham Jr.., Image processing in the context of a visual model, Proc. IEEE 60 (7) (1972) 828–842.
[3] H.G. Adelmann, A frequency-domain Gaussian filter module for quantitative and reproducible high-pass, low-pass and band-pass filtering of images, Am. Lab. 29 (6) (1979) 27–33.
[4] M. Sonka, V. Hlavac, R. Boyle, Image Processing, Analysis and Machine Vision, Chapman and Hall, London, 1995.
[5] K.R. Castleman, Digital Image Processing, Prentice-Hall, Englewood Cliffs, New Jersey, 1996.