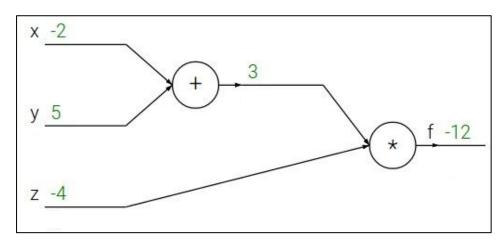
$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4

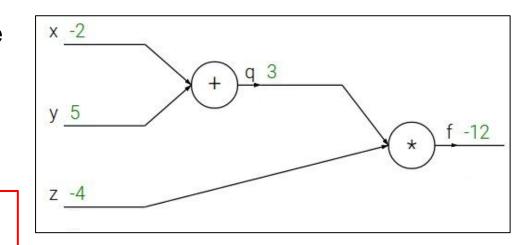


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 $rac{\partial f}{\partial q}=z, rac{\partial f}{\partial z}=q$

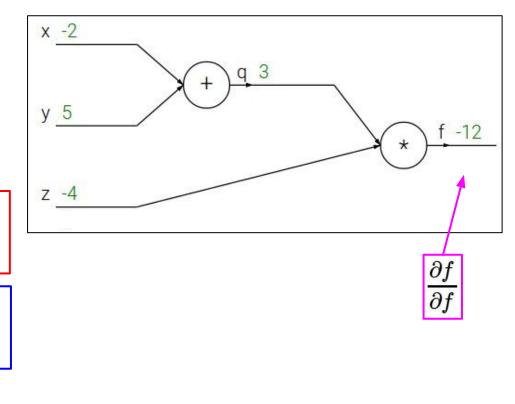


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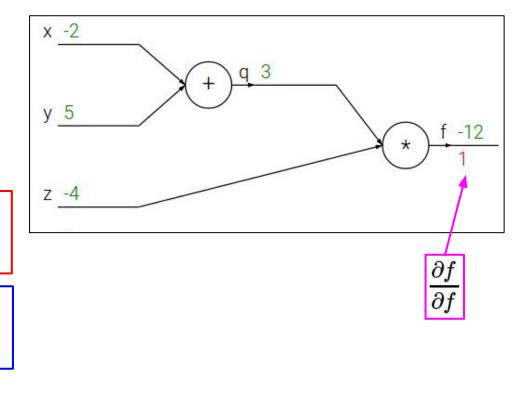


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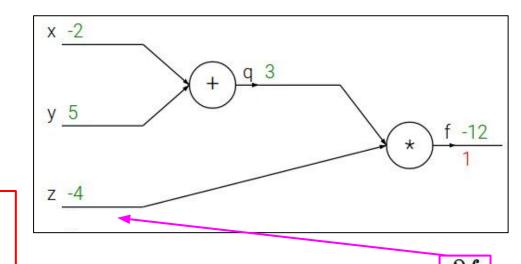
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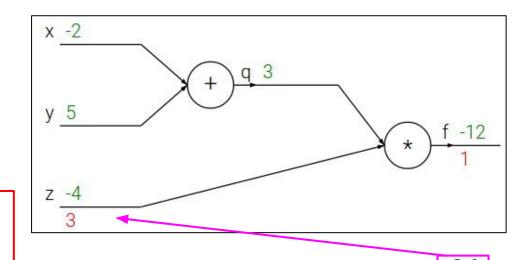


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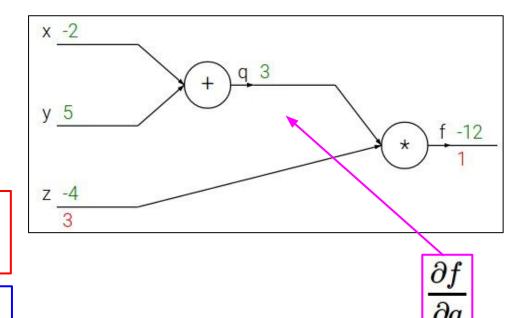


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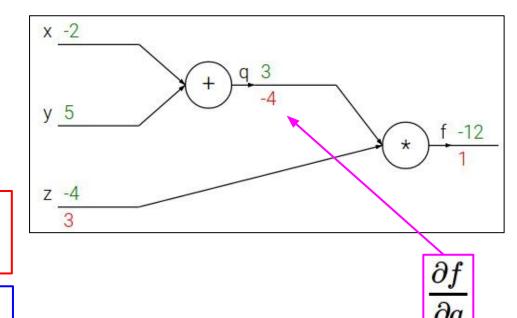


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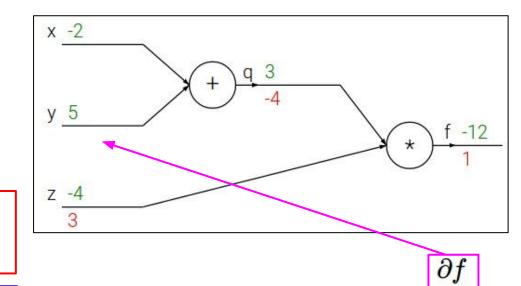


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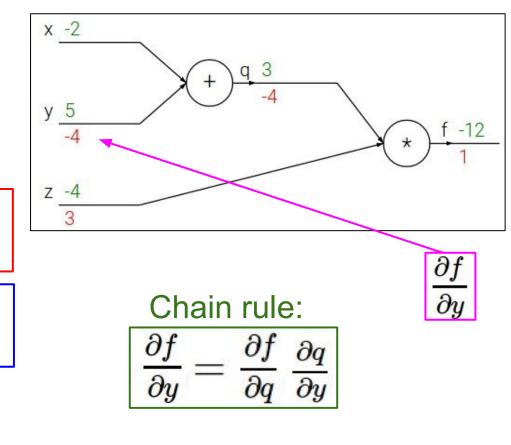


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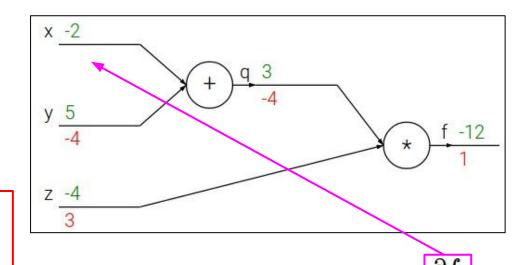
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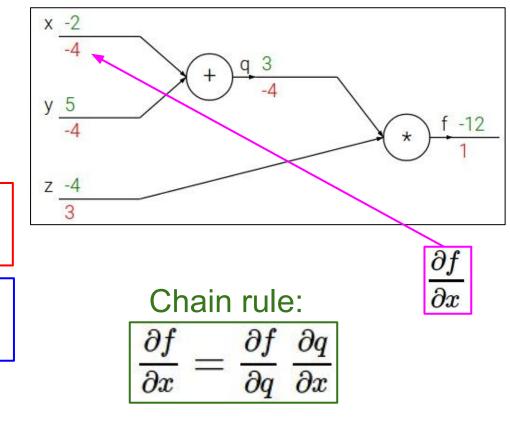


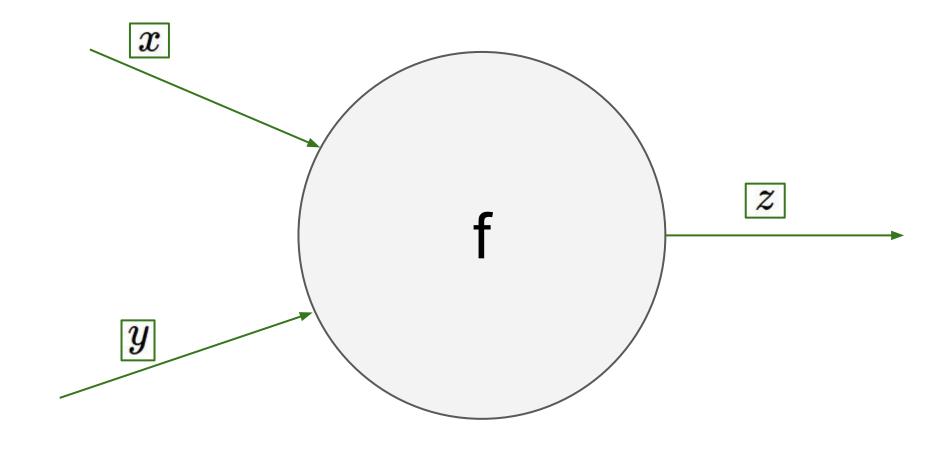
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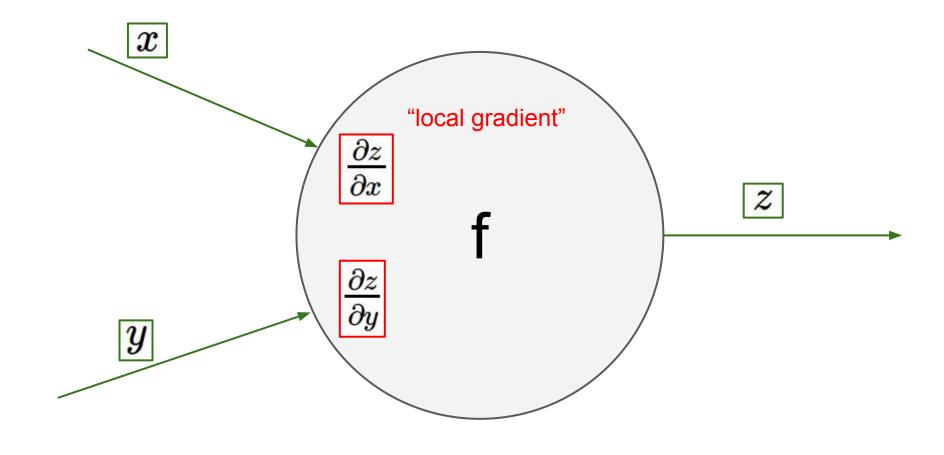
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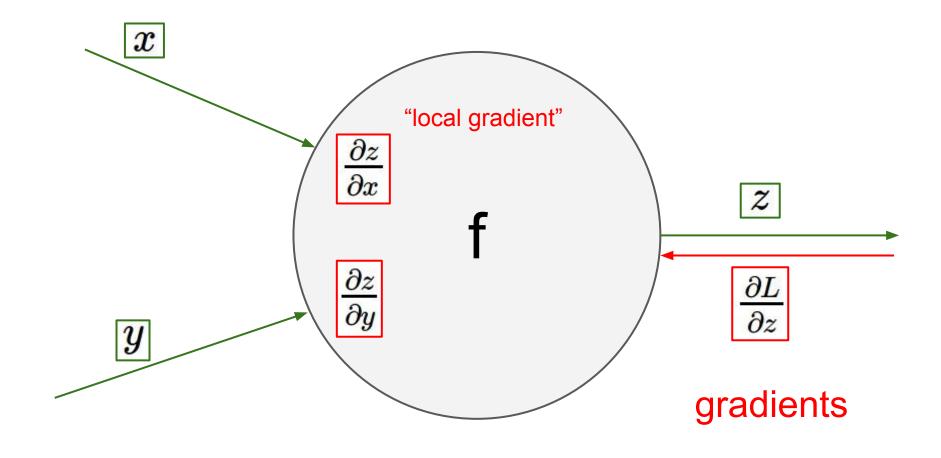
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 $\frac{\partial q}{\partial x}=1, \frac{\partial q}{\partial y}=1$

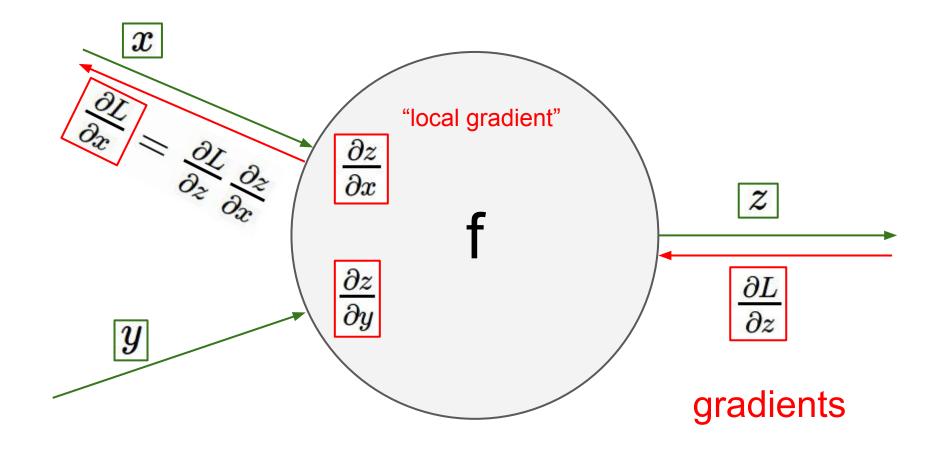
$$f=qz$$
 $rac{\partial f}{\partial q}=z, rac{\partial f}{\partial z}=q$

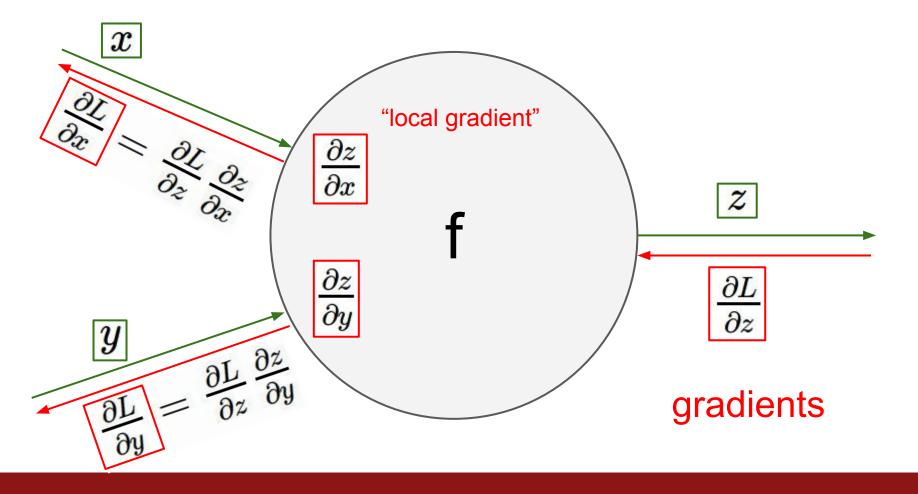


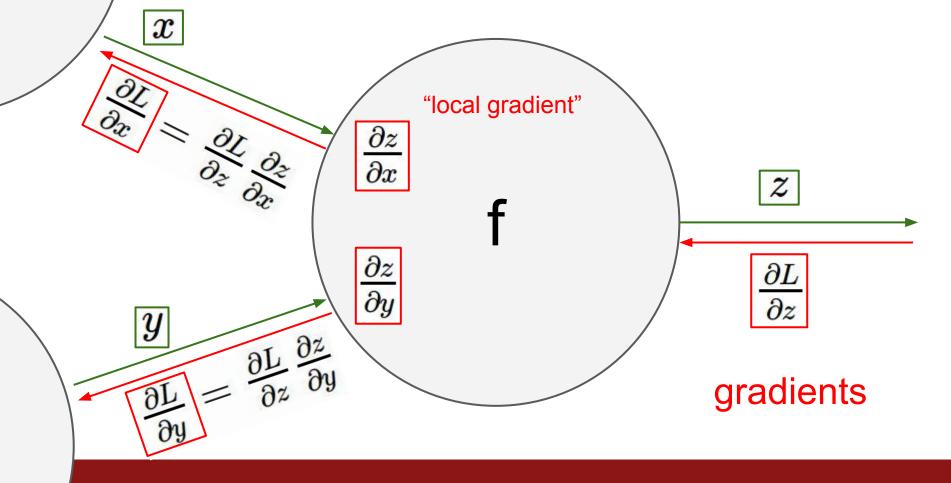




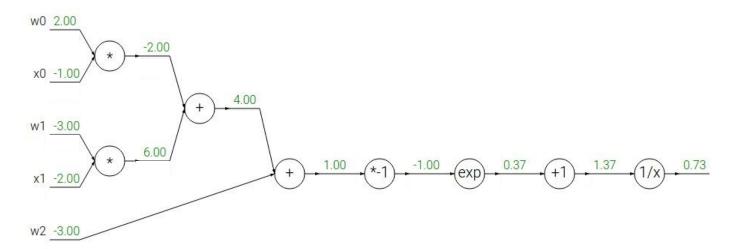




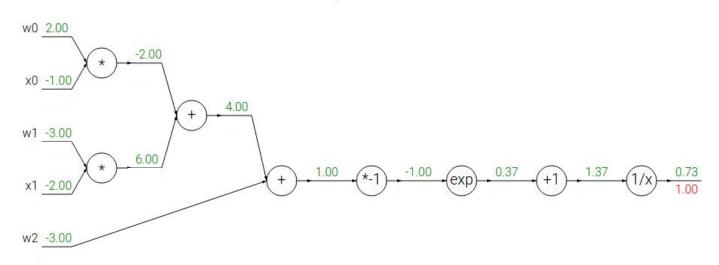




Another example: $f(w,x) = \frac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$

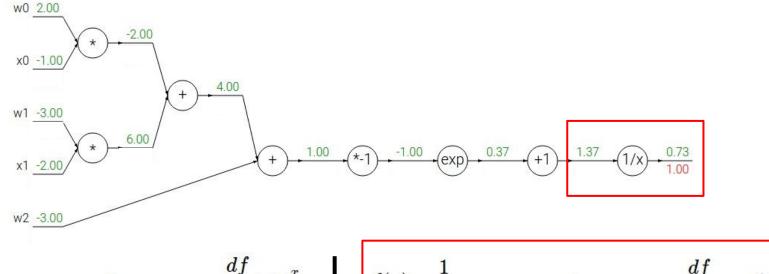


$$f(w,x) = rac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_1 x_2 + w_2 x_1 + w_2 x_2 + w_2 x_2$$



$$egin{aligned} f(x) = e^x &
ightarrow & rac{df}{dx} = e^x & f(x) = rac{1}{x} &
ightarrow & rac{df}{dx} = -1/x^2 \ f_a(x) = ax &
ightarrow & rac{df}{dx} = a & f_c(x) = c + x &
ightarrow & rac{df}{dx} = 1 \end{aligned}$$

Another example:
$$f(w,x) = \frac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$



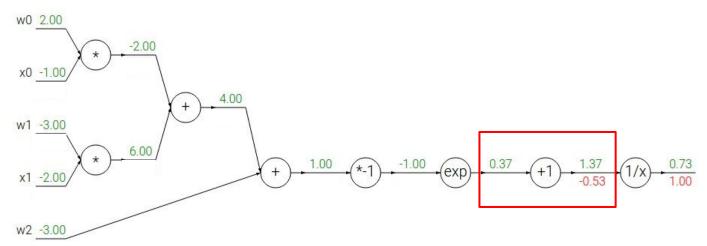
$$f(x) = e^x \hspace{1cm} o \hspace{1cm} rac{df}{dx} = e^x \hspace{1cm} f(x) = rac{1}{x} \hspace{1cm} o \hspace{1cm} rac{df}{dx} = -1/x \hspace{1cm} f_c(x) = c + x \hspace{1cm} o \hspace{1cm} rac{df}{dx} = 1$$

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 $f(x) = e^x$

 $f_a(x) = ax$

Another example:
$$f(w,x) = \frac{1}{1 + e^{-(w_0 x_0 +$$

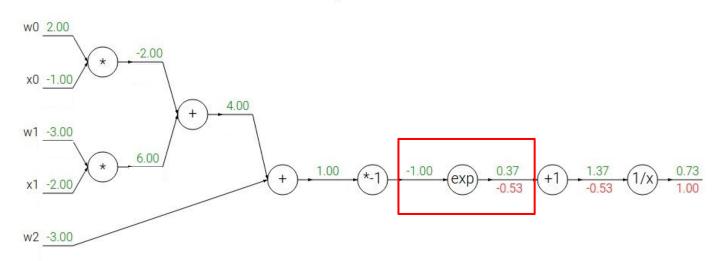


Another example:
$$f(w,x) = \frac{1}{1 + e^{-(w_0 x_0 + w_1 x_1)}}$$

$$\begin{array}{c} & & & & \\ & \times 0 & \underline{-1.00} \\ & \times 1 & \underline{-3.00} \\ & \times 1 & \underline{-2.00} \\ & \times 2 & \underline{-3.00} \\ \end{array}$$

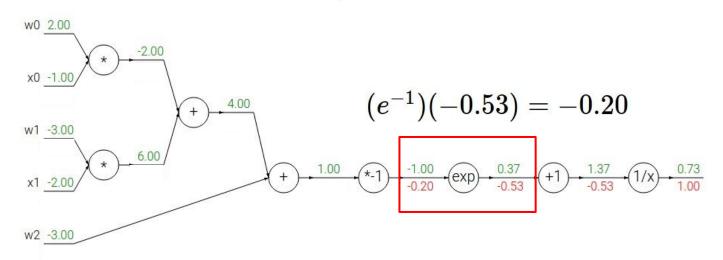
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$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



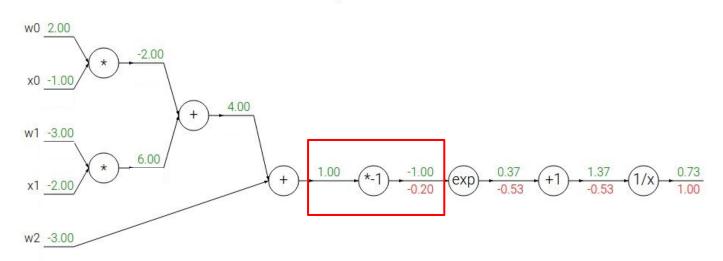
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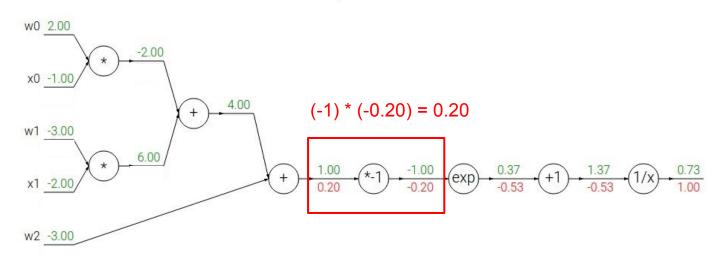
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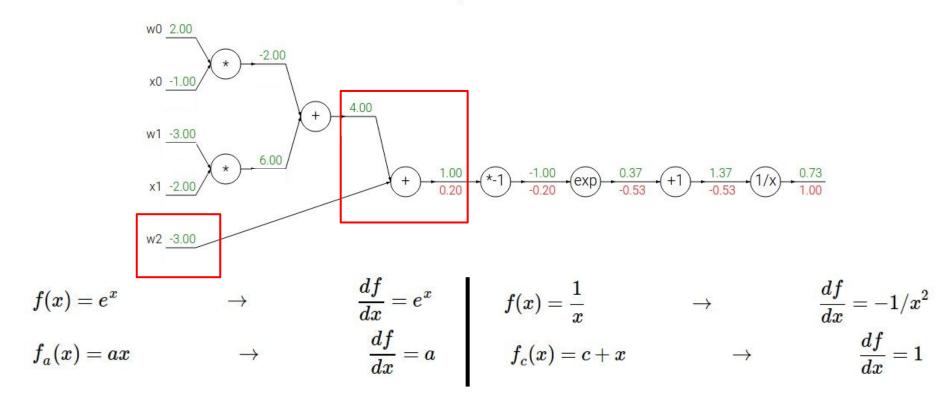
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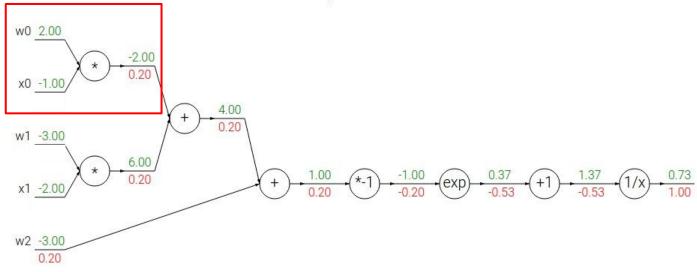


$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$f(x) = e^{x} \qquad \rightarrow \qquad \frac{df}{dx} = e^{x} \qquad f(x) = ax \qquad \rightarrow \qquad \frac{df}{dx} = a \qquad f_{c}(x) = c + x \qquad \rightarrow \qquad \frac{df}{dx} = 1$$
[local gradient] x [upstream gradient]
[1] x [0.2] = 0.2 (both inputs!)

$$f(x) = \frac{1}{x} \qquad \rightarrow \qquad \frac{df}{dx} = -1/x^{2}$$

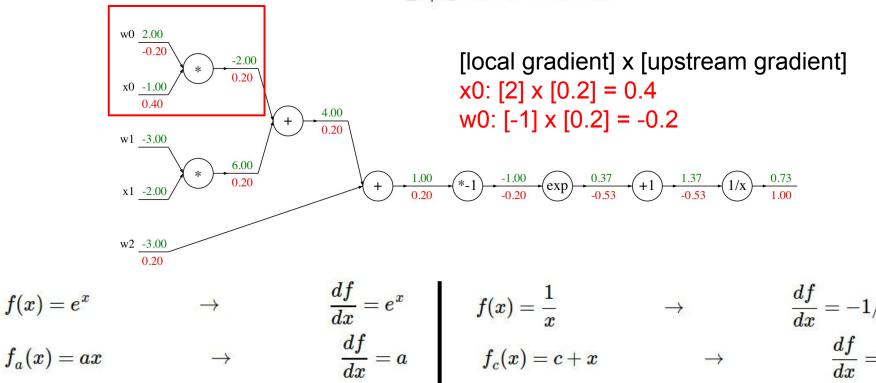
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 $f_a(x) = ax$

$$f(w,x) = rac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2 x_1 + w_2 x_2 + w_1 x_2 + w_2 x_2$$

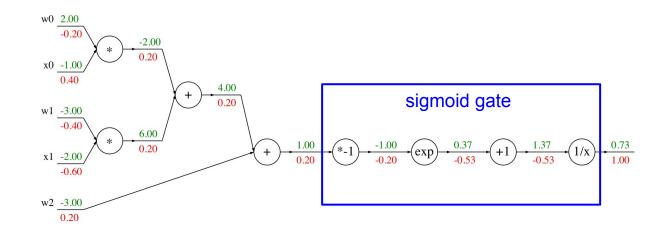


$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

sigmoid function

$$rac{d\sigma(x)}{dx} = rac{e^{-x}}{(1+e^{-x})^2} = \left(rac{1+e^{-x}-1}{1+e^{-x}}
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ight) = \left(1-\sigma(x)
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