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ការសិក្សាប្រៀបធៀប ការគណនាកម្រាលខណ្ឌពីរទិសដោយ វិធីមេគុណម៉ូម៉ង់ និងវិធីធាតុកំណត់

The Comparative of Design Two-ways slab by using Coefficient moment method and Finite element method

RATH RUNKAKADA

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Student's Name : **RATH RUNKAKADA**

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Thesis Evaluation Committee :

No.	Full Name	Field	Role in the committee	Signature
1	Mr. **Seun Sambath (PhD)**	Civil Engineering	Supervisor	
2	Mr. **Toch Sokheng**	Civil Engineering	1st Evaluator	
3	Mr. **Phon Try**	Civil Engineering	2nd Evaluator	
4	Mr. **Kaothon Panyabot**	Civil Engineering	3rd Evaluator	

Supervisor :...Dr. **Seun Sambath**

Vice President :... Dr. **Seun Sambath**

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NOTATIONS AND ABBREVIATIONS

ACI	American Concrete Institute
ASCE	American Society of Civil Engineers
Finite Element Method (FEM)	វិធីសាស្ត្រធាតុកំណត់
Coefficient Moment Method	វិធីសាស្ត្រមេគុណម៉ូម៉ង់
Direct design method	វិធីសាស្ត្រគណនាផ្ទាល់
Equivalent frame method	វិធីគ្រោងសមមូល
Software Design	កម្មវិធីកុំព្យូទ័រសម្រាប់គណនា
Hand Calculation	ការគណនាដោយដៃ
One-way slab	កម្រាលខណ្ឌធ្វើការមួយទិស
Two-way slab	កម្រាលខណ្ឌធ្វើការពីរទិស
Dead load	បន្ទុកអថេរ
Live load	បន្ទុកថេរ
Bending Moment	ម៉ូម៉ង់ពត់
Shear Force	កម្លាំងកាត់ទទឹង
Negative Moment	ម៉ូម៉ង់អវិជ្ជមាន
Positive Moment	ម៉ូម៉ង់វិជ្ជមាន
$M_{a.neg}$	ម៉ូម៉ង់អវិជ្ជមានតាមទិសខ្លី
$M_{b.neg}$	ម៉ូម៉ង់អវិជ្ជមានតាមទិសវែង
$M_{a.pos}$	ម៉ូម៉ង់វិជ្ជមានតាមទិសខ្លី
$M_{b.pos}$	ម៉ូម៉ង់វិជ្ជមានតាមទិសវែង
$V_{a.sup}$	កម្លាំងកាត់តាមទិសខ្លី

$V_{b.sup}$	កម្លាំងកាត់តាមទិសវែង
LL	Live load
DL	Dead load
មម	ខ្នាតជាមីលីម៉ែត្រ
mm	Millimeter
m	Meter
Perimeter	ប៉ារ៉ាម៉ែត្រ
Cover	ស្រទាប់ការពារបេតុង
Ceiling	ពិដាន
Partition	ជញ្ជាំងខណ្ឌ
MEP	Mechanic Electricity Plumbing
SDL	Super imposed dead load
SW	Self weight
RSA	Robot Structural Analysis Professional

Thesis Declaration

I hereby declare that this thesis entitled « **The Comparative of Analysis Two-ways slab by using Coefficient moment method and Finite element method**» is the result of my own study. The content of this thesis has not been submitted for any other academic degree at any institution. All sources of information used in this thesis have been properly acknowledged and referenced.

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I declare that:

- This thesis was prepared by myself under the guidance of my supervisor.
- The work presented in this thesis is my own effort, and all sources of information used have been clearly cited.
- This thesis has not been submitted to any other university or institution for any academic degree.
- I take full responsibility for any errors contained in this thesis.

Signature of the student:..Date:....22.03.2025.......

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I hereby certify that:

- This research study and thesis were conducted and prepared under my direct supervision.
- This thesis is the original research and written work of the candidate.
- All contributions made by me, or through guidance from the committee and other individuals, strictly comply with the general code of practice for thesis supervision.
- The thesis manuscript has been prepared in accordance with the thesis preparation guidelines of IU.

Signature of the supervisor:...Date:.....20.04.2025......

មូលន័យសង្ខេប

កម្រាលខណ្ឌបេតុងអារម៉េមានតួនាទីសំខាន់ក្នុងការទ្រទម្ងន់ និងចែកចាយបន្ទុកនៅសំណង់ដោយផ្អែកលើទិសធ្វើការ (មួយទិស ឬពីរទិស)។ វាត្រូវការវិភាគ និងគណនាអោយបានត្រឹមត្រូវ ដោយប្រើទ្រឹស្តីអេឡាស្ទិក ឬមេកានិកនៃរូបធាតុ។ ក្នុងការអនុវត្ត ពិសេសនៅកម្ពុជាបច្ចុប្បន្ន គេប្រើវិធីមេគុណ ACI និងកម្មវិធីកុំព្យូទ័រដើម្បីកំណត់ម៉ូម៉ង់ពត់និងកម្លាំងកាត់បានកាន់តែប្រសើរឡើង។ ភាពមិនទាន់ច្បាស់លាស់នៃលក្ខខណ្ឌប្រើប្រាស់វិធីវិភាគទាំងពីរ ព្រមទាំងសុក្រឹតភាពនៃលទ្ធផលវិភាគ និងការធ្វើការរបស់កម្រាលខណ្ឌធ្វើការពីរទិស ជំរុញឱ្យនិក្ខេបបទអំពីការប្រៀបធៀបការវិភាគកម្រាលខណ្ឌធ្វើការពីរទិសតាមវិធីមេគុណ ACI និងតាមវិធីធាតុកំណត់ កំណត់ចេញនូវគោលបំណងប្រៀបធៀបលទ្ធផលនៃការវិភាគ ការធ្វើការរបស់កម្រាលខណ្ឌ និងការផ្ទេរបន្ទុកទៅលើធ្នឹមទ្រ ហើយចុងក្រោយគឺលក្ខខណ្ឌប្រើប្រាស់នៃវិធីវិភាគទាំងពីរនេះ។ ការស្រាវជ្រាវនេះមានគោលបំណងសិក្សាប្រៀបធៀបលទ្ធផលការគណនាកម្រាលខណ្ឌធ្វើការពីរទិស ដោយប្រើវិធីសាស្រ្តមេគុណម៉ូម៉ង់ (hand calculation) និងវិធីសាស្រ្តធាតុកំណត់ (software design) ជាមួយកម្មវិធី Robot StructuralAnalysis។

តាមការសិក្សាស្រាវជ្រាវ បានអោយឃើញថា ភាពខុសគ្នារវាងកម្លាំងក្នុង (ម៉ូម៉ង់ពត់ និងកម្លាំងកាត់) ដែលទទួលបានតាមវិធីមេគុណ និងវិធីធាតុកំណត់ មានម៉ូម៉ង់ពត់នៅគល់ទម្រ ម៉ូម៉ង់ពត់នៅកណ្តាលប្រឡោះ និងកម្លាំងកាត់នៅគល់ទម្រតាមទិសខ្លី តម្លៃដែលទទួលបានតាមវិធីធាតុកំណត់តូចជាងតម្លៃដែលទទួលបានតាមវិធីមេគុណ។ ហើយសម្រាប់ម៉ូម៉ង់ពត់នៅគល់ទម្រ ម៉ូម៉ង់ពត់នៅកណ្តាលប្រឡោះ និងកម្លាំងកាត់នៅគល់ទម្រតាមទិសវែង តម្លៃដែលទទួលបានតាមវិធីធាតុកំណត់ ធំជាងតម្លៃដែលទទួលបានតាមវិធីមេគុណ ដោយលក្ខណៈធ្វើការពីរទិសរបស់កម្រាលខណ្ឌនៅតែបន្ត ទោះជាកម្រាលខណ្ឌមានប្រវែងជ្រុងខុសគ្នា ២ដងក៏ដោយ។

លក្ខណៈខាងលើ កើតឡើងដោយសារវិធីមេគុណ មានការសន្មត់ថា ទម្រទាំងបួនរបស់កម្រាលខណ្ឌជាទម្របង្កប់ ពោលគឺមិនមានសម្រុត ហើយទប់ម៉ូម៉ង់របស់កម្រាលខណ្ឌបានទាំងស្រុងដោយមិនអាស្រ័យនឹងភាពរឹងក្រាញរមួលទេ។ លើសពីនេះ នៅក្នុងវិធីមេគុណមានការចល័តបន្ទុកដោយតម្លៃម៉ូម៉ង់ពត់ ពីបន្ទុកអថេរកើនឡើង ធៀបនឹងតម្លៃម៉ូម៉ង់ពត់ពីបន្ទុកថេរ ។ រីឯវិធីធាតុកំណត់វិញ កម្រាលខណ្ឌនិងធ្នឹមទាំងពីរទិសដែលមានកម្រាស់និងវិមាត្រមុខកាត់ប្រែប្រួលទៅតាមទំហំកម្រាលខណ្ឌ ធ្វើការជាប្រព័ន្ធរួមមួយ ហើយពង្រាយកម្លាំងក្នុងទៅតាមទិសទាំងពីរ អាស្រ័យតាមភាពរឹងក្រាញធាតុ រវាងធាតុទាំងបីនេះ ជាពិសេស ធ្នឹមដែលទ្រកម្រាលខណ្ឌស្រុតក្នុងតម្លៃខុសគ្នា។

ដូច្នេះបើគណនាកម្រាលខណ្ឌតាមវិធីមេគុណ នោះគួរវិភាគធ្នឹម និងសសរជាគ្រោងក្នុងលំហ (ដោយគ្មានកម្រាលខណ្ឌ) ពោលគឺមិនត្រូវវិភាគនិងគណនាធ្នឹម និងសសរ ចេញពីគ្រឿងផ្គុំអគារទាំងមូលដែលជាប្រព័ន្ធនៃកម្រាលខណ្ឌធ្នឹម និងសសរនោះទេ។

ABSTRACT

Reinforced concrete slabs have a crucial role in bearing and distributing loads in structures based on their working direction (one-way or two-way). Accurate analysis and calculation are essential, often using elastic theory or mechanics of materials. In Cambodia, the ACI coefficient method and computer software are commonly employed to improve the accuracy of determining bending moments and shear forces. The ambiguity in the application conditions of these two analytical methods, along with the precision of results and the performance of two-way slabs, highlights the need for comparative studies. This research aims to compare the results of two-way slab analysis using the ACI coefficient method and finite element method (FEM), focusing on slab performance, load transfer to beams, and the specific application conditions of each method.

The study compares two-way slab calculations using hand calculations with moment coefficient methods and FEM software like Robot Structural Analysis. Findings reveal discrepancies in internal forces (bending moments and shear forces) between the two methods.

In the short direction, the bending moments at supports, mid-span bending moments, and shear forces at supports calculated by FEM are smaller than those derived using the ACI method. Conversely, in the long direction, these values calculated by FEM are larger than those from the ACI method.

These differences arise because the ACI coefficient method assumes all edges are fixed supports, resisting all bending moments without considering torsional stiffness, and the moving loads increase the bending moment more significantly than static loads. On the other hand, FEM integrates slabs, beams, and columns into a continuous framework, distributing internal forces based on the stiffness of the components, with beams deflecting differently under load.

Therefore, when using the ACI coefficient method, it is recommended to analyze beams and columns as a separate 3D framework without including slabs in the analysis. This approach avoids treating the entire system (slabs, beams, and columns) as a single integrated unit.

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My name is **Rath Runkakda**, a master's degree student at the **International University**, Faculty of Technology and Science, majoring in **Civil Engineering**. I would like to express my deepest gratitude to the following:

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CHAPTER 1
INTRODUCTION

1.1. Background of the Study

Reinforced concrete slabs are an essential component of building structures. They directly carry variable loads arising from building usage according to different functional purposes, as well as additional permanent distributed loads from architectural elements and construction components. Reinforced concrete slabs may be supported by beams, walls, columns, or foundations.

In structural behaviour, slabs transfer loads to supports in two primary ways:

1. Load transfer in one direction, known as **One-way slab**.
2. Load transfer in two directions, known as **Two-way slab**.

Because of the importance of slabs within the structural system, it is necessary to study their behaviour carefully, particularly through accurate analysis and calculation. In structural analysis theory, a one-way slab is commonly modelled as a beam (single-span or multi-span), while a two-way slab is modelled as a thin plate, which is studied within the theory of plate bending, an application of elasticity theory or mechanics of materials.

In plate bending theory, it is often difficult to obtain exact analytical solutions, and even approximate solutions are not easily derived. Therefore, in reinforced concrete design theory, various simplified or approximate methods have been developed to determine internal forces such as bending moments and shear forces in slabs. These include the load distribution method, coefficient method, strip method, direct design method, equivalent frame method, and others. With advancements in computer technology, the **Finite Element Method (FEM)** has also been widely applied to slab analysis. Each method possesses its own advantages, limitations, and applicable conditions.

In current engineering practice in Cambodia, the **ACI coefficient method** is commonly used for hand calculation of bending moments. At the same time, engineers also perform analytical modeling using structural analysis software to visualize and determine internal forces within slabs, typically using the finite element method for general applicability.

For this reason, this thesis selects the following topic for study:

"Comparison of Two-Way Slab Analysis Using the ACI Coefficient Method and the Finite Element Method."

1.2. Identification of problems

In general, structural analysis and design using computer software are considered more accurate and faster than manual calculations. However, determining bending moments and shear forces in two-way slabs using the ACI coefficient method by hand is simpler, more practical, and less time-consuming compared to finite element analysis using computer software. This is because the ACI method requires fewer parameters than computer-based modelling.

Differences in input parameters between the two methods lead to differences in analysis results. These differences affect reinforcement layout, slab behavior, and load transfer to supporting beams. As a result, each method may be suitable under different practical conditions.

1.3. Objectives of the Study

Due to uncertainties regarding the applicability and accuracy of the two methods, this research establishes the following objectives:

- To compare the results of two-way slab analysis using both methods.
- To evaluate slab behavior and load transfer to supporting beams based on reinforcement layouts derived from each method.
- To identify the appropriate conditions for applying each analysis method

1.4. Aim and scope

This research focuses on comparing two-way slab calculations using:

- The **moment coefficient method (hand calculation)**
- The **finite element method (software-based analysis)**

The software used for FEM analysis is **Robot Structural Analysis Professional**. The study emphasizes internal force results, including:

- Positive and negative bending moments in both short and long directions
- Shear forces at supports, representing distributed loads transferred from slabs to beams

Key aspects of the research methodology include:

- Studying **15 slab models** of reinforced concrete two-way slabs with varying slab dimensions and subjected to **two types of live loads** (residential and public buildings).
- Comparison conducted at sections taken through the mid-span in both directions.
- Manual analysis of bending moments and shear forces using **ACI 318M-63** moment coefficient provisions.
- Supporting beams are modeled based on typical Cambodian construction practice, generally shallow and slender beams.
- FEM analysis performed using **Robot Structural Analysis Professional 2025**.
- Conclusions and recommendations regarding the practical use of both methods.

1.5. Significance of the Study

The findings of this research provide useful insights for students and structural designers who intend to apply either the ACI coefficient method or the finite element method in practice. The study supports more informed decision-making in selecting appropriate methods according to different conditions, as discussed in Chapter 6.

In summary, this thesis contributes valuable reference material for future students, engineers, and researchers, serving as a foundation for improving analytical accuracy and validation of various design methods. It also supports further research on two-way slab analysis using both approaches, with the broader aim of contributing to national and societal development.

CHAPTER 2

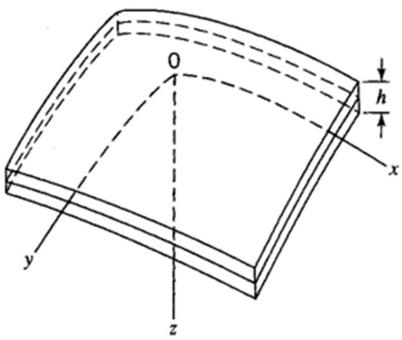
FOUNDATION REVIEWS

2.1. Plate Slabs and Flanged Slab Systems

A slab refers to a reinforced concrete structural element supported by reinforced concrete beams, composite beams, or directly by columns. A slab cast monolithically with reinforced concrete beams may behave in two different structural forms:

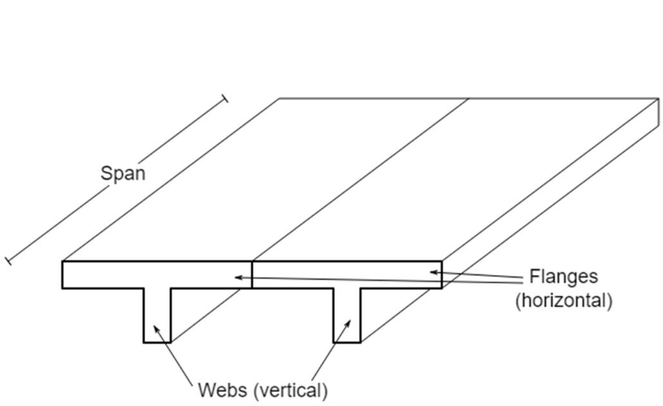
1. When the **shorter span governs the structural behavior**, the slab behaves as a **plate slab**, functioning primarily as a thin plate.
2. When the **longer span governs the behavior**, the slab behaves as a **flanged slab (beam–slab system)**, acting as the flange of reinforced concrete beams.

In a **plate slab system**, the slab acts as a thin plate subjected to distributed loads and transfers these loads to the supporting beams. In a **flanged slab system**, the slab acts as the flange of the beam, working integrally with the beam web in both directions. Structurally, this system consists of an assembly of interconnected **T-beams** in two orthogonal directions.



The internal forces in plate slabs may be determined using:

- Analytical methods that directly solve the plate equilibrium equations, or
- Numerical methods that solve equilibrium equations or minimize elastic strain energy.



In practical reinforced concrete slab design, internal forces (bending moments and shear forces) are commonly determined using:

- The **ACI coefficient method**, which distributes slab loads into short- and long-span components, and
- The **Finite Element Method (FEM)**, a numerical approach based on elastic strain energy theory, now fully implemented in modern structural analysis software.

For flanged slab systems, several methods have been developed for internal force determination, such as:

- The **Direct Design Method (DDM)**
- The **Equivalent Frame Method (EFM)**

These methods are also incorporated into structural design software to facilitate practical application.

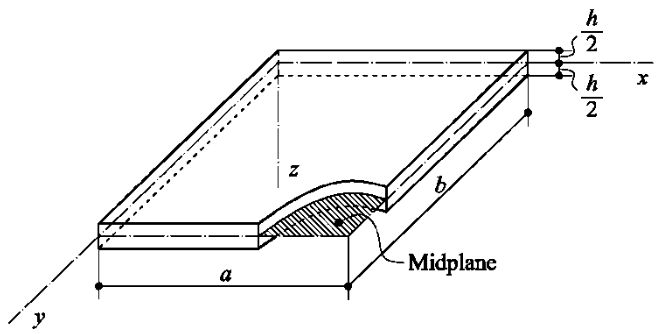
After determining internal forces in reinforced concrete slabs, cross-sectional design follows the principles of flexural member design. This includes:

- Design of sections at supports and mid-span in both short and long directions,
- Possible division into column strips and middle strips depending on the adopted analytical model.

Because reinforced concrete slabs have relatively thin cross-sections, concrete alone is generally sufficient to resist shear. Therefore, **shear reinforcement (stirrups)** is typically not provided in slabs.

2.2. Theory of Thin Plate Analysis

A thin plate is a structural element whose **thickness is small compared to its in-plane dimensions**. When a three-dimensional solid element becomes a thin plate, its thickness is denoted as *h*, while its in-plane dimensions are characterized by length and width (for rectangular plates) or diameter (for circular plates).



A thin plate subjected to **uniformly distributed transverse load** undergoes bending, and its behavior is governed by the **Sophie Germain–Lagrange plate equilibrium equation**.

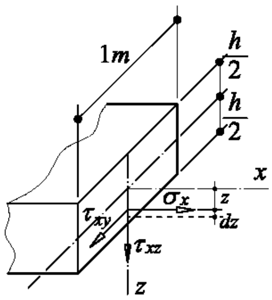
$$\nabla^4 w = \frac{q}{D}$$

and $w(x, y)$ represents the **deflection (displacement)** of the plate $D = \frac{Eh^3}{12(1-\nu^2)}$ represents the **flexural rigidity** of the plate. The flexural rigidity is expressed as:

$$\nabla^4 \cdots = \frac{\partial^4 \cdots}{\partial x^4} + 2\frac{\partial^4 \cdots}{\partial x^2 \partial y^2} + \frac{\partial^4 \cdots}{\partial y^4}$$

where:

- E is the **modulus of elasticity**,
- ν is the **Poisson's ratio** of the plate material.



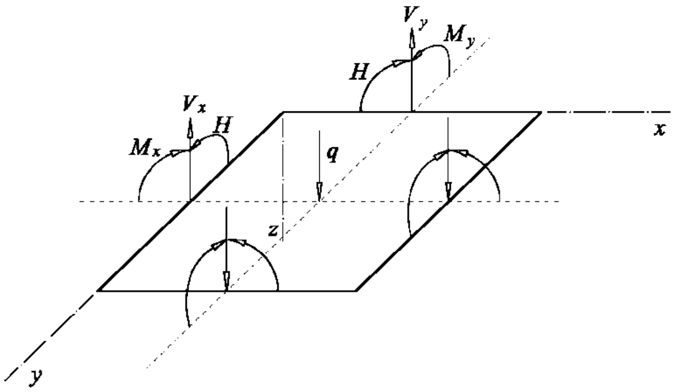
The deflection function must satisfy **boundary conditions** along the plate edges, which are commonly classified as:

Edge type	Edge $x = Constant$	Edge $y = Constant$
Fixed edge	$w = 0, \quad \frac{\partial w}{\partial x} = 0$	$w = 0, \quad \frac{\partial w}{\partial y} = 0$
Simply supported edge	$w = 0, \quad M_x = 0 \left(\frac{\partial^2 w}{\partial x^2} = 0\right)$	$w = 0, \quad M_y = 0 \left(\frac{\partial^2 w}{\partial y^2} = 0\right)$
Free edge	$M_x = 0, \quad V_x^* = 0$	$M_y = 0, \quad V_y^* = 0$

Once deflection is obtained, the **internal forces in the plate** are determined from established relationships, including:

Internal Force	Direction x	Direction y
Bending Moment	$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$	$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)$
Shear Force	$V_x = -D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$	$V_y = -D\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$

Shear Force (Combined)	$V_x^* = -D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + (2-\nu)\frac{\partial^2 w}{\partial y^2}\right)$	$V_y^* = -D\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial y^2} + (2-\nu)\frac{\partial^2 w}{\partial x^2}\right)$
Torsional Moment	$H = -(1-\nu)D\frac{\partial^2 w}{\partial x \partial y}$	



Stresses at a distance z from the neutral axis are determined based on bending stress relationships

- Internal Normal stress

$$\sigma_x = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_y = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)$$

$$\sigma_z = -\frac{E}{2(1-\nu^2)}\left(\frac{z^3}{3} - \frac{h^2}{4}z\right)\nabla^2\nabla^2 w + \frac{q_1 - q_2}{2} \approx 0$$

- Internal Shear stress

$$\tau_{xy} = -\frac{Ez}{1+\nu}\frac{\partial^2 w}{\partial x \partial y}$$

$$\tau_{zx} = \frac{E}{2(1-\nu^2)}\left(z^2 - \frac{h^2}{4}\right)\frac{\partial}{\partial x}\nabla^2 w \approx 0$$

$$\tau_{yz} = \frac{E}{2(1-\nu^2)}\left(z^2 - \frac{h^2}{4}\right)\frac{\partial}{\partial y}\nabla^2 w \approx 0$$

and deformation components (strains) in orthogonal directions are also defined accordingly.

- Normal Strain

$$\epsilon_x = -z\frac{\partial^2 w}{\partial x^2}, \qquad \epsilon_y = -z\frac{\partial^2 w}{\partial y^2}, \qquad \epsilon_z = 0$$

- Shear Strain

$$\gamma_{xy} = -2z\frac{\partial^2 w}{\partial x \partial y}, \qquad \gamma_{zx} = 0, \qquad \gamma_{yz} = 0$$

The total **elastic strain energy** of a thin plate is given by the integral of internal energy over the plate domain.

$$\Pi = U - W$$

$$U = \frac{1}{2}\iiint_V \left(\sigma_x\epsilon_x + \sigma_y\epsilon_y + \tau_{xy}\gamma_{xy}\right)dxdydz = -\frac{1}{2}\iint_A \left(M_x\kappa_x + M_y\kappa_y + 2H\kappa_{xy}\right)dxdy$$

$$U = \frac{1}{2}\iint_A \left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 + 2(1-\nu)\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right]\right\}dxdy$$

and

$$\kappa_x = \frac{\partial^2 w}{\partial x^2}, \qquad \kappa_y = \frac{\partial^2 w}{\partial y^2}, \qquad \kappa_{xy} = \frac{\partial^2 w}{\partial x \partial y}$$

The potential energy and the external work done by distributed loads are also defined

$$W = \iint_A q(x,y)w(x,y)dxdy$$

2.3. Thin Plate Analysis Using the Finite Element Method (FEM)

In the Finite Element Method, the entire plate is discretized into a system of small interconnected elements, which may be triangular or quadrilateral in shape. These elements are connected through shared **nodes**.

Each element contains a finite number of nodal points:

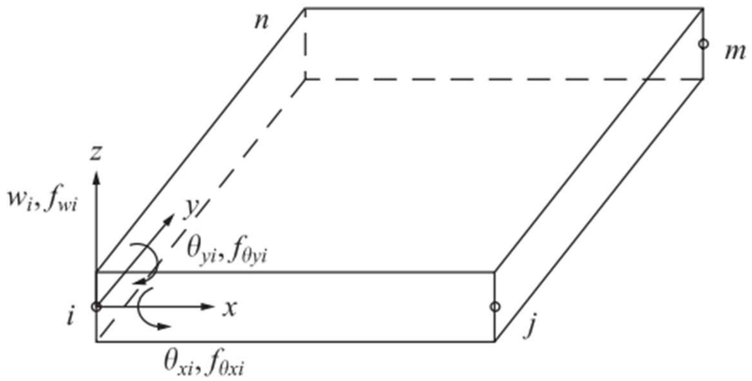
- A triangular element has **three nodes**,
- A quadrilateral element has **four nodes**.

For a plate subjected to bending, each node possesses degrees of freedom including:

- Vertical displacement
- Rotations about the two orthogonal axes

For a quadrilateral element with four nodes, the total number of nodal degrees of freedom is therefore twelve.

Based on these degrees of freedom, a polynomial displacement function is assumed. The unknown coefficients are determined using nodal compatibility conditions. The resulting formulation introduces the **shape function**, which defines displacement within the element.



$$\{d_i\} = \begin{bmatrix} w_i \\ \theta_{xi} \\ \theta_{yi} \end{bmatrix} = \begin{bmatrix} (w)_i \\ \left(\frac{\partial w}{\partial y}\right)_i \\ \left(-\frac{\partial w}{\partial x}\right)_i \end{bmatrix}$$

It is assumed that the displacement field within the element can be expressed as a function of four nodal points, denoted by i, j, m, n and each node possesses three degrees of freedom, resulting in a total of twelve nodal degrees of freedom.

$$\{d\} = \{\{d_i\} \quad \{d_j\} \quad \{d_m\} \quad \{d_n\}\}^T$$

This displacement vector represents the unknown nodal parameters, which are used to determine the coefficients of the assumed displacement function. These coefficients are then employed to construct the displacement field within the element.

Accordingly, the displacement vector at any point in the plate is expressed as:

$$\{\psi(x,y)\} = \begin{bmatrix} w \\ \frac{\partial w}{\partial y} \\ -\frac{\partial w}{\partial x} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & x & y & x^2 & xy & y^2 & x^3 & x^2y & xy^2 & y^3 & x^3y & xy^3 \\ 0 & 0 & 1 & 0 & x & 2y & 0 & x^2 & 2xy & 3y^2 & x^3 & 3xy^2 \\ 0 & -1 & 0 & -2x & -y & 0 & -3x^2 & -2xy & -y^2 & 0 & -3x^2y & -y^3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \cdots \\ a_{12} \end{bmatrix}$$

$$\{\psi\} = [P(x,y)]\{a\}$$

The vector of unknown coefficients $\{a\}$ can be determined from the nodal displacement conditions at the element nodes, as follows:

$$\{d\} = \begin{bmatrix} \psi(x_i, y_i) \\ \psi(x_j, y_j) \\ \psi(x_m, y_m) \\ \psi(x_n, y_n) \end{bmatrix} = \begin{bmatrix} [P(x_i, y_i)] \\ [P(x_j, y_j)] \\ [P(x_m, y_m)] \\ [P(x_n, y_n)] \end{bmatrix} \{a\} = [C]\{a\} \Rightarrow \{a\} = [C]^{-1}\{d\}$$

Substituting this into the displacement function gives:

$$\{\psi(x, y)\} = [P(x, y)][C]^{-1}\{d\} = [N(x, y)]\{d\}$$

where the **shape function matrix** is defined as:

$$[N(x, y)] = [P(x, y)][C]^{-1}$$

Curvature–Displacement Relationship

$$\{\kappa\} = \begin{bmatrix} \kappa_x \\ \kappa_y \\ 2\kappa_{xy} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial y^2} \\ 2\dfrac{\partial^2 w}{\partial x \partial y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 6x & 2y & 0 & 0 & 6xy & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2x & 6y & 0 & 6xy \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 4x & 4y & 0 & 6x^2 & 6y^2 \end{bmatrix} \{a\}$$

which can also be written as:

$$\{\kappa\} = [Q(x, y)]\{a\} = [B(x, y)]\{d\}$$

where

$$[B(x, y)] = [Q(x, y)][C]^{-1}$$

Bending Moment–Curvature Relationship

$$\{M\} = \begin{bmatrix} M_x \\ M_y \\ H \end{bmatrix} = [D]\{\kappa\} = [D][B(x, y)]\{d\}$$

Where the constitutive (rigidity) matrix is

$$[D] = -D \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix}$$

Strain Energy of the Plate Element

$$U = -\frac{1}{2} \iint_A \left(M_x \kappa_x + M_y \kappa_y + 2H\kappa_{xy} \right) dxdy = -\frac{1}{2} \iint_A \{\kappa\}^T \{M\} dxdy = \frac{1}{2} \{d\}^T [k] \{d\}$$

Element stiffness matrix

$$[k] = -\iint_A [B]^T[D][B]dxdy$$

Work done by external loads

$$W = \iint_A \{f\}^T\{\psi\}dxdy = \{d\}^T\{F_q\}$$

Where

$$\{F_q\}^T = \iint_A \{f\}^T[N]dxdy, \qquad \{f\} = \begin{bmatrix} q(x,y) \\ m_x(x,y) \\ m_y(x,y) \end{bmatrix}$$

After assembling all element matrices and applying the boundary conditions, the system equilibrium equation is obtained in terms of the nodal displacement vector $\{d\}$ as below:

$$[k]\{d\} = \{F_q\}$$

2.4. Analysis of Two-Way Slabs Using the coefficients Method ACI

According to **ACI 318-63**, bending moments and shear forces in two-way slabs can be determined using **moment coefficients** and **load distribution coefficients** in both the short and long directions. This method is based on the assumption that the uniformly distributed load acting on the slab is **distributed between the two orthogonal directions**. The resulting internal forces depend on the slab panel location, such as:

- Interior panels
- Edge panels
- Corner panels

The ACI method provides simplified expressions for calculating:

- Negative bending moments at supports
- Positive bending moments at midspan
- Shear forces in both directions

Coefficients Moment and Shear Force Equations

Internal force	Short direction (a)	Long direction (b)
Negative bending moment at supports	$M_{a,neg} = C_a w_u l_a^2$	$M_{b,neg} = C_b w_u l_b^2$
Positive bending moment at midspan	$M_{a,pos} = C_{a,dl} w_d l_a^2 + C_{a,ll} w_l l_a^2$	$M_{b,pos} = C_{b,dl} w_d l_b^2 + C_{b,ll} w_l l_b^2$

Shear force	$V_a = \frac{C_{a,v}W}{2l_b}$	$V_b = \frac{C_{b,v}W}{2l_a}$

Where:

- l_a, l_b, effective spans in the short and long directions of the slab panel
- w_d, w_l, dead load and live load per unit area of the slab
- $w_u = w_d + w_l$, total factored design load
- $W = w_u l_a l_b$, total factored load on the slab panel
- C_a, C_b, coefficients for negative bending moments in short and long directions
- $C_{a,dl}, C_{b,dl}$, coefficients for positive bending moments due to dead load
- $C_{a,ll}, C_{b,ll}$, coefficients for positive bending moments due to live load

TABLE 12.3

Coefficients for negative moments in slabs[a]

$M_{a,neg} = C_{a,neg} w l_a^2$
$M_{b,neg} = C_{b,neg} w l_b^2$
where w = total uniform dead plus live load

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,neg}$		0.045		0.050	0.075	0.071		0.033	0.061
	$C_{b,neg}$		0.045	0.076	0.050			0.071	0.061	0.033
0.95	$C_{a,neg}$		0.050		0.055	0.079	0.075		0.038	0.065
	$C_{b,neg}$		0.041	0.072	0.045			0.067	0.056	0.029
0.90	$C_{a,neg}$		0.055		0.060	0.080	0.079		0.043	0.068
	$C_{b,neg}$		0.037	0.070	0.040			0.062	0.052	0.025
0.85	$C_{a,neg}$		0.060		0.066	0.082	0.083		0.049	0.072
	$C_{b,neg}$		0.031	0.065	0.034			0.057	0.046	0.021
0.80	$C_{a,neg}$		0.065		0.071	0.083	0.086		0.055	0.075
	$C_{b,neg}$		0.027	0.061	0.029			0.051	0.041	0.017
0.75	$C_{a,neg}$		0.069		0.076	0.085	0.088		0.061	0.078
	$C_{b,neg}$		0.022	0.056	0.024			0.044	0.036	0.014
0.70	$C_{a,neg}$		0.074		0.081	0.086	0.091		0.068	0.081
	$C_{b,neg}$		0.017	0.050	0.019			0.038	0.029	0.011
0.65	$C_{a,neg}$		0.077		0.085	0.087	0.093		0.074	0.083
	$C_{b,neg}$		0.014	0.043	0.015			0.031	0.024	0.008
0.60	$C_{a,neg}$		0.081		0.089	0.088	0.095		0.080	0.085
	$C_{b,neg}$		0.010	0.035	0.011			0.024	0.018	0.006
0.55	$C_{a,neg}$		0.084		0.092	0.089	0.096		0.085	0.086
	$C_{b,neg}$		0.007	0.028	0.008			0.019	0.014	0.005
0.50	$C_{a,neg}$		0.086		0.094	0.090	0.097		0.089	0.088
	$C_{b,neg}$		0.006	0.022	0.006			0.014	0.010	0.003

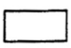
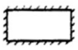
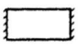
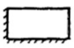
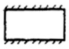
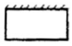
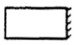
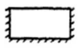
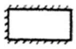
[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

- $C_{a,v}$, $C_{b,v}$, coefficients for shear force in short and long directions

TABLE 12.4

Coefficients for dead load positive moments in slabs[a]

$M_{a,pos,dl} = C_{a,dl} w l_a^2$
where w = total uniform dead load
$M_{b,pos,dl} = C_{b,dl} w l_b^2$

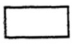
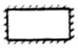
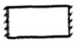
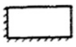
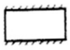
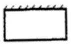
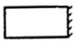
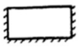
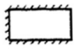
Ratio		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
$m = \frac{l_a}{l_b}$										
1.00	$C_{a,dl}$	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023
	$C_{b,dl}$	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020
0.95	$C_{a,dl}$	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
	$C_{b,dl}$	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
0.90	$C_{a,dl}$	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
	$C_{b,dl}$	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.85	$C_{a,dl}$	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
	$C_{b,dl}$	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.80	$C_{a,dl}$	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
	$C_{b,dl}$	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.75	$C_{a,dl}$	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
	$C_{b,dl}$	0.019	0.009	0.018	0.013	0.007	0.012	0.020	0.013	0.007
0.70	$C_{a,dl}$	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.033
	$C_{b,dl}$	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.65	$C_{a,dl}$	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.034
	$C_{b,dl}$	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.60	$C_{a,dl}$	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.036
	$C_{b,dl}$	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.55	$C_{a,dl}$	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.037
	$C_{b,dl}$	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.50	$C_{a,dl}$	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.038
	$C_{b,dl}$	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

TABLE 12.5

Coefficients for live load positive moments in slabs[a]

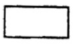
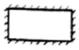
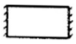
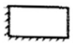
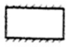
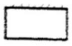
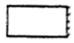
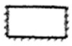
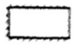
$M_{a,pos,ll} = C_{a,ll} w l_a^2$
where w = total uniform live load
$M_{b,pos,ll} = C_{b,ll} w l_b^2$

Ratio $m = \frac{l_a}{l_b}$		Case 1 	Case 2 	Case 3 	Case 4 	Case 5 	Case 6 	Case 7 	Case 8 	Case 9 
1.00	$C_{a,ll}$	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030
	$C_{b,ll}$	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.028
0.95	$C_{a,ll}$	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
	$C_{b,ll}$	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
0.90	$C_{a,ll}$	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
	$C_{b,ll}$	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.85	$C_{a,ll}$	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
	$C_{b,ll}$	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.80	$C_{a,ll}$	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
	$C_{b,ll}$	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.75	$C_{a,ll}$	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
	$C_{b,ll}$	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.70	$C_{a,ll}$	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
	$C_{b,ll}$	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.65	$C_{a,ll}$	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
	$C_{b,ll}$	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.60	$C_{a,ll}$	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
	$C_{b,ll}$	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.55	$C_{a,ll}$	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
	$C_{b,ll}$	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.50	$C_{a,ll}$	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
	$C_{b,ll}$	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.004

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

TABLE 12.6

Ratio of load W in l_a and l_b directions for shear in slab and load on supports[a]

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
										
1.00	W_a	0.50	0.50	0.17	0.50	0.83	0.71	0.29	0.33	0.67
	W_b	0.50	0.50	0.83	0.50	0.17	0.29	0.71	0.67	0.33
0.95	W_a	0.55	0.55	0.20	0.55	0.86	0.75	0.33	0.38	0.71
	W_b	0.45	0.45	0.80	0.45	0.14	0.25	0.67	0.62	0.29
0.90	W_a	0.60	0.60	0.23	0.60	0.88	0.79	0.38	0.43	0.75
	W_b	0.40	0.40	0.77	0.40	0.12	0.21	0.62	0.57	0.25
0.85	W_a	0.66	0.66	0.28	0.66	0.90	0.83	0.43	0.49	0.79
	W_b	0.34	0.34	0.72	0.34	0.10	0.17	0.57	0.51	0.21
0.80	W_a	0.71	0.71	0.33	0.71	0.92	0.86	0.49	0.55	0.83
	W_b	0.29	0.29	0.67	0.29	0.08	0.14	0.51	0.45	0.17
0.75	W_a	0.76	0.76	0.39	0.76	0.94	0.88	0.56	0.61	0.86
	W_b	0.24	0.24	0.61	0.24	0.06	0.12	0.44	0.39	0.14
0.70	W_a	0.81	0.81	0.45	0.81	0.95	0.91	0.62	0.68	0.89
	W_b	0.19	0.19	0.55	0.19	0.05	0.09	0.38	0.32	0.11
0.65	W_a	0.85	0.85	0.53	0.85	0.96	0.93	0.69	0.74	0.92
	W_b	0.15	0.15	0.47	0.15	0.04	0.07	0.31	0.26	0.08
0.60	W_a	0.89	0.89	0.61	0.89	0.97	0.95	0.76	0.80	0.94
	W_b	0.11	0.11	0.39	0.11	0.03	0.05	0.24	0.20	0.06
0.55	W_a	0.92	0.92	0.69	0.92	0.98	0.96	0.81	0.85	0.95
	W_b	0.08	0.08	0.31	0.08	0.02	0.04	0.19	0.15	0.05
0.50	W_a	0.94	0.94	0.76	0.94	0.99	0.97	0.86	0.89	0.97
	W_b	0.06	0.06	0.24	0.06	0.01	0.03	0.14	0.11	0.03

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

CHAPTER 3

Research Methodology

3.1. Selection of Slab Models for Analysis

In this study, a total of **15 two-way slab models** are selected for analysis. The slabs are subjected to two types of live loads:

$LL_1 = 2.00kN/m^2$ → representing **residential buildings**

$LL_2 = 5.00kN/m^2$ → representing **public buildings**

The slab dimensions are varied systematically in both directions, resulting in different aspect ratios, as shown in Table 3.1.

Table 3.1: Slab Models Based on Span Dimensions

Short span L_a	Long span L_b				
	$4m$	$5m$	$6m$	$7m$	$8m$
$4m$	Slab 1	Slab 2	Slab 3	Slab 4	Slab 5
$5m$		Slab 6	Slab 7	Slab 8	Slab 9
$6m$			Slab 10	Slab 11	Slab 12
$7m$				Slab 13	Slab 14
$8m$					Slab 15

Material Properties Used in the Study

The following material parameters are adopted for all slab models:

- Concrete compressive strength: $f_c' = 25\text{MPa}$
- Unit weight of concrete: $w_c = 24\ \text{kN/m}^2$
- Yield strength of main reinforcement steel: $f_y = 390MPa$
- Yield strength of secondary reinforcement steel: $f_y = 235MPa$

3.2. Determination of Slab Thickness

The minimum thickness of a reinforced concrete slab in Cambodia depends on the span length and the applied live load. For slabs with span up to **9.0 m**, the minimum thickness shall be determined using the appropriate coefficient based on the level of live load. If the live load is small, the denominator **180** is used, and if the live load is larger, the denominator **160** is used, as shown in the table below.

Live Load	Minimum slab thickness t_{min}
$LL = 2.00\ kN/m^2$	$\frac{Perimeter}{180} = \frac{2 \cdot (L_a + L_b)}{180}$
$LL = 5.00\ kN/m^2$	$\frac{Perimeter}{160} = \frac{2 \cdot (L_a + L_b)}{160}$

The following table presents the **minimum Two-way slab thicknesses** calculated for different slab panel dimensions when the live load is $LL = 2.00\ kN/m^2$:

Short span L_a	Long span L_b				
	$4m$	$5m$	$6m$	$7m$	$8m$
$4m$	90mm	100mm	120mm	130mm	140mm
$5m$		120mm	130mm	140mm	150mm
$6m$			140mm	150mm	160mm
$7m$				160mm	170mm
$8m$					180mm

The following table presents the **minimum Two-way slab thicknesses** calculated for different slab panel dimensions when the live load is $LL = 5.00\ kN/m^2$:

Short span L_a	Long span L_b				
	$4m$	$5m$	$6m$	$7m$	$8m$
$4m$	100mm	120mm	130mm	140mm	150mm
$5m$		130mm	140mm	150mm	170mm
$6m$			150mm	170mm	180mm
$7m$				180mm	190mm
$8m$					200mm

Beams in the long direction have a minimum cross-sectional height equal to 4 times the minimum slab thickness, and a minimum cross-sectional width equal to 2 times the minimum slab thickness. Beams in the short direction have cross-sectional dimensions identical to those in the long direction for floor panels where the square area size is equal to the length of the short-direction beam.

Cross-section of Long-Direction Beams, the minimum height and width are defined as follows:

Live Load	Minimum Beam Height h_b	Minimum Beam Width b_b
$LL = 2.00\ kN/m^2$	$\frac{Perimeter}{45} = \frac{2 \cdot (L_a + L_b)}{45}$	$\frac{Perimeter}{90} = \frac{2 \cdot (L_a + L_b)}{90}$
$LL = 5.00\ kN/m^2$	$\frac{Perimeter}{40} = \frac{2 \cdot (L_a + L_b)}{40}$	$\frac{Perimeter}{80} = \frac{2 \cdot (L_a + L_b)}{80}$

Cross-section of Short-Direction Beams, the minimum height and minimum width are determined as follows:

Live Load	Minimum Beam Height h_a	Minimum Beam Width b_a
$LL = 2.00\ kN/m^2$	$\frac{L_a}{11.25}$	$\frac{L_a}{22.5}$
$LL = 5.00\ kN/m^2$	$\frac{L_a}{10}$	$\frac{L_a}{20}$

The section of beam $b \times h$ (in $cm \times cm$) for case live load $LL = 2.00\ kN/m^2$

Short span L_a	Long span L_b				
	$4m$	$5m$	$6m$	$7m$	$8m$
$4m$	20x40	20x40	25x45	25x50	30x55
$5m$		25x45	25x50	30x55	30x60
$6m$			30x55	30x60	35x65
$7m$				35x65	35x70
$8m$					40x75

The section of beam $b \times h$ (in $cm \times cm$) for case live load $LL = 5.00\ kN/m^2$

Short span L_a	Long span L_b				
	$4m$	$5m$	$6m$	$7m$	$8m$
$4m$	20x40	25x45	25x50	30x55	30x60
$5m$		25x50	30x55	30x60	35x65

$6m$			30x60	35x65	35x70
$7m$				35x70	40x75
$8m$					40x80

For the cross-section of columns, we determine a square shape with dimensions equal to the width of the beam in the long direction:

$$b_c \times h_c = b_b \times b_b$$

3.3. Design Load on Slab

To ensure consistency between the **ACI coefficient method** and the **finite element method**, the same loading conditions are applied to both analytical approaches. For illustration, a representative slab panel with dimensions $4m \times 6m$ is selected, and the loads are calculated as follows:

Floor finishing (cover)	$Cover = 0.05m \cdot 20\ kN/m^3 = 1.00\ kN/m^2$
Ceiling and MEP services	$Ceiling = 0.50kN/m^2$
Partition load	$Partition = 1.00kN/m^2$
Superimposed dead load	$SDL = Cover + Ceiling + Partion$ $= 2.50kN/m^2$
Self-weight of slab	$SW = t \cdot 25\ kN/m^3 = 3.00\ kN/m^2$
Total dead load	$DL = SW + SDL = 5.50\ kN/m^2$
Live load	$LL = 2.00\ kN/m^2$
Factored design load	$w_u = 1.2DL + 1.6LL = 9.80\ kN/m^2$

3.4. Calculation of Internal Forces Using the ACI Coefficient Method

Effective spans of the slab	$l_a = 4m - 0.25m = 3.75m$ $l_b = 6m - 0.20m = 5.80m$
Span ratio of the slab	$\lambda = \frac{l_a}{l_b} = 0.647$
Moment coefficients (from ACI tables)	$C_a = 0.077, \quad C_b = 0.014$ $C_{a,dl} = 0.032, \quad C_{b,dl} = 0.006$ $C_{a,ll} = 0.053, \quad C_{b,ll} = 0.010$

Shear coefficients	$C_{a,v} = 0.853, \qquad C_{b,v} = 0.147$
Negative bending moment at supports	$M_{a,neg} = C_{a,neg} w_u l_a^2 = 10.65\ \frac{kNm}{1m}$ $M_{b,neg} = C_{b,neg} w_u l_b^2 = 4.524\ \frac{kNm}{1m}$
Positive bending moment at midspan	$M_{a,pos} = C_{a,dl} w_D l_a^2 + C_{a,ll} w_L l_a^2 = 5.383\ \frac{kNm}{1m}$ $M_{b,pos} = C_{b,dl} w_D l_b^2 + C_{b,ll} w_L l_b^2 = 2.356\ \frac{kNm}{1m}$
Total factored load on slab panel	$W = w_u l_a l_b = 213.15\ kN$ $V_a = \frac{C_{a,v} W}{2 l_b} = 15.669\ \frac{kN}{1m}$ $V_b = \frac{C_{a,v} W}{2 l_a} = 4.185\ \frac{kN}{1m}$

3.5 Analysis of Slabs Using the Finite Element Method (FEM)

In this study, the slab panels are analyzed using the **finite element method (FEM)** with the aid of the software **Robot Structural Analysis (RSA)**. The slab is modeled as a plate element, and the analysis is performed under the same loading conditions used in the ACI coefficient method in order to ensure consistency and enable a reliable comparison between the two approaches.

The modeling process includes defining the slab geometry, assigning material properties, specifying slab thickness, and applying boundary conditions and loads. The software then automatically discretizes the slab into finite elements (mesh generation) and performs numerical analysis to obtain the internal forces such as **bending moments and shear forces** in both directions.

Special attention is given to the correct assignment of supports and load combinations to ensure that the FEM model realistically represents the actual behavior of the slab. The results obtained from the FEM analysis are extracted and used for comparison with the results derived from the ACI coefficient method, as presented in the subsequent sections.

In the FEM modeling process, both **dead load (DL)** and **live load (LL)** are applied to the slab model. These loads are defined based on standard structural loading provisions. The applied loads are consistent with those used in the manual calculations to maintain equivalence between the two analysis methods. The load combinations adopted in the software follow commonly accepted design standards, including provisions consistent with **ASCE loading recommendations**, while the slab design and evaluation are aligned with the principles of the **ACI code**.

The finite element analysis results obtained from the software are extracted at critical locations of the slab, including at supports and at midspan, in both the short and long directions. These results are then compared with the values obtained using the ACI coefficient method in order to evaluate the differences between the two approaches. By Using meshing size of 0.05m, the comparison for the slab panel of size $4m \times 6m$ subjected to $LL = 2.00 kN/m^2$ is summarized in the table below:

Internal force	Short direction	Long direction
Negative bending moment at support $\left(\frac{kNm}{1m}\right)$	7.48	9.83
Positive bending moment at midspan $\left(\frac{kNm}{1m}\right)$	4.41	4.44
Shear force at support $\left(\frac{kN}{1m}\right)$	13.36	12.59

By changing the study parameters to two variable loads and 15 slabs area sizes, we will obtain the study results that will be presented in Chapter 4.

CHAPTER 4

The Result of study

Therefore, this study shows that the finite element method can be applied for slab analysis, and the results are consistent with those obtained using the ACI method.

4.1. The result of slab analysis by using Coefficient method ACI

- For case Live load: $LL = 2.00\ kN/m^2$

Size of Slab	Short Span			Long Span		
	$M_{neg}\left(\frac{kNm}{1m}\right)$	$M_{pos}\left(\frac{kNm}{1m}\right)$	$V_{sup}\left(\frac{kN}{1m}\right)$	$M_{neg}\left(\frac{kNm}{1m}\right)$	$M_{pos}\left(\frac{kNm}{1m}\right)$	$V_{sup}\left(\frac{kN}{1m}\right)$
$4m \times 4m$	5.783	2.729	8.455	5.783	2.729	8.455
$4m \times 5m$	8.724	4.207	12.556	5.546	2.691	6.219
$4m \times 6m$	10.65	5.383	15.669	4.524	2.356	4.185
$4m \times 7m$	11.918	6.178	17.406	3.31	1.859	2.778
$4m \times 8m$	12.39	6.729	18.283	3.472	1.231	2.018
$5m \times 5m$	9.95	4.63	11.638	9.95	4.63	11.638
$5m \times 6m$	14.218	6.695	16.405	9.713	4.537	9.178
$5m \times 7m$	17.052	8.28	19.869	7.95	4.00	6.565
$5m \times 8m$	19.023	9.644	22.249	6.758	3.338	4.775
$6m \times 6m$	15.205	7.018	14.82	15.205	7.018	14.82
$6m \times 7m$	20.833	9.683	20.099	14.933	6.786	12.219
$6m \times 8m$	24.799	11.863	24.121	13.289	6.395	9.477
$7m \times 7m$	21.89	10.03	18.288	21.89	10.03	18.288
$7m \times 8m$	29.019	13.393	23.929	22.031	9.829	15.696
$8m \times 8m$	30.151	13.724	22.04	30.151	13.724	22.04

- For case Live load: $LL = 5.00\ kN/m^2$

Size of Slab	Short Span			Long Span		
	$M_{neg}\left(\frac{kNm}{1m}\right)$	$M_{pos}\left(\frac{kNm}{1m}\right)$	$V_{sup}\left(\frac{kN}{1m}\right)$	$M_{neg}\left(\frac{kNm}{1m}\right)$	$M_{pos}\left(\frac{kNm}{1m}\right)$	$V_{sup}\left(\frac{kN}{1m}\right)$
$4m \times 4m$	9.097	4.679	13.3	9.097	4.679	13.3
$4m \times 5m$	13.653	7.264	19.95	8.452	4.485	9.505
$4m \times 6m$	16.192	9.12	23.824	6.879	3.996	6.362

$4m \times 7m$	17.528	10.315	25.937	4.837	3.092	4.013
$4m \times 8m$	18.466	11.357	27.249	5.175	2.126	3.007
$5m \times 5m$	15.128	7.676	17.694	15.128	7.676	17.694
$5m \times 6m$	21.241	11.024	24.74	14.268	7.382	13.433
$5m \times 7m$	25.413	13.706	29.612	11.849	6.687	9.784
$5m \times 8m$	28.198	15.988	33.315	9.67	5.31	6.863
$6m \times 6m$	22.662	11.404	22.088	22.662	11.404	22.088
$6m \times 7m$	31.182	15.861	30.324	22.016	11.041	17.976
$6m \times 8m$	36.973	19.506	35.963	19.813	10.491	14.129
$7m \times 7m$	32.636	16.239	27.265	32.636	16.239	27.265
$7m \times 8m$	42.72	21.529	35.53	31.792	15.623	22.695
$8m \times 8m$	44.186	21.833	32.3	44.186	21.833	32.3

4.2. The result of slab analysis by using Finite Element Method (FEM)

- For case Live load: $LL = 2.00\ kN/m^2$

Size of Slab	Short Span			Long Span		
	$M_{neg}\left(\frac{kNm}{1m}\right)$	$M_{pos}\left(\frac{kNm}{1m}\right)$	$V_{sup}\left(\frac{kN}{1m}\right)$	$M_{neg}\left(\frac{kNm}{1m}\right)$	$M_{pos}\left(\frac{kNm}{1m}\right)$	$V_{sup}\left(\frac{kN}{1m}\right)$
$4m \times 4m$	5.76	2.79	12.31	5.76	2.79	12.31
$4m \times 5m$	6.96	3.58	12.97	7.88	3.34	13.25
$4m \times 6m$	7.48	4.41	13.36	9.83	4.44	12.59
$4m \times 7m$	8.40	4.91	14.62	11.37	4.91	12.86
$4m \times 8m$	8.54	5.70	15.62	12.42	5.56	12.51
$5m \times 5m$	9.71	4.80	15.49	9.71	4.80	15.49
$5m \times 6m$	11.87	5.97	17.19	12.08	5.46	16.11
$5m \times 7m$	13.21	7.36	18.98	13.35	6.14	16.08
$5m \times 8m$	14.33	8.01	19.82	15.71	6.96	15.95
$6m \times 6m$	14.84	7.42	20.32	14.84	7.42	20.32
$6m \times 7m$	17.84	9.09	22.36	17.75	8.22	21.15
$6m \times 8m$	19.85	11.03	24.42	19.41	9.00	21.33
$7m \times 7m$	21.37	10.79	25.56	21.37	10.79	25.56
$7m \times 8m$	25.29	13.02	27.92	24.89	11.75	26.60
$8m \times 8m$	29.41	14.98	31.22	29.41	14.98	31.22

- For case Live load: $LL = 5.00\ kN/m^2$

Size of Slab	Short Span			Long Span		
	$M_{neg}\ \left(\frac{kNm}{1m}\right)$	$M_{pos}\ \left(\frac{kNm}{1m}\right)$	$V_{sup}\ \left(\frac{kN}{1m}\right)$	$M_{neg}\ \left(\frac{kNm}{1m}\right)$	$M_{pos}\ \left(\frac{kNm}{1m}\right)$	$V_{sup}\ \left(\frac{kN}{1m}\right)$
$4m \times 4m$	9.03	4.31	18.39	9.03	4.31	18.39
$4m \times 5m$	11.06	5.97	20.46	10.98	5.07	18.12
$4m \times 6m$	12.42	6.74	21.59	13.49	5.82	18.02
$4m \times 7m$	12.79	7.85	22.94	14.58	6.60	17.30
$4m \times 8m$	14.00	9.33	26.42	9.49	4.21	14.63
$5m \times 5m$	14.62	7.04	23.45	14.62	7.04	23.45
$5m \times 6m$	18.57	9.80	28.47	16.26	7.60	24.28
$5m \times 7m$	20.99	11.94	31.45	16.60	7.50	23.68
$5m \times 8m$	22.57	13.61	33.26	16.78	7.66	21.77
$6m \times 6m$	22.32	10.92	30.59	22.32	10.92	30.59
$6m \times 7m$	27.41	14.44	35.08	24.52	11.73	30.49
$6m \times 8m$	30.89	17.39	38.75	25.24	11.81	30.07
$7m \times 7m$	31.97	15.79	37.17	31.97	15.79	37.17
$7m \times 8m$	37.93	20.11	42.57	34.37	16.66	37.91
$8m \times 8m$	43.22	21.61	44.77	43.22	21.61	44.77

CHAPTER 5

Comparison of Results Between the Two Analysis Methods

5.1. Comparison of Negative Bending Moment at Support

- Negative bending moment for short span $(M_{a,neg})$

Size of Slab	For live load $LL = 2.00\ kN/m^2$			For live load $LL = 5.00\ kN/m^2$		
	Coefficient method ACI	Finite Element method	Difference (%)	Coefficient method ACI	Finite Element method	Difference (%)
4mx4m	5.783	5.76	−0.40%	9.097	9.03	−0.74%
4mx5m	8.724	6.96	−20.22%	13.653	11.06	−18.99%
4mx6m	10.65	7.48	−29.77%	16.192	12.42	−23.3%
4mx7m	11.918	8.40	−29.52%	17.528	12.79	−27.03%
4mx8m	12.39	8.54	−31.07%	18.466	14.00	−24.18%
5mx5m	9.95	9.71	−2.41%	15.128	14.62	−3.36%
5mx6m	14.218	11.87	−16.51%	21.241	18.57	−12.57%
5mx7m	17.052	13.21	−22.53%	25.413	20.99	−17.4%
5mx8m	19.023	14.33	−24.67%	28.198	22.57	−19.96%
6mx6m	15.205	14.84	−2.4%	22.662	22.32	−1.51%
6mx7m	20.833	17.84	−14.37%	31.182	27.41	−12.1%
6mx8m	24.799	19.85	−19.96%	36.973	30.89	−16.45%
7mx7m	21.89	21.37	−2.38%	32.636	31.97	−2.04%
7mx8m	29.019	25.29	−12.85%	42.72	37.93	−11.21%
8mx8m	30.151	29.41	−2.46%	44.186	43.22	−2.19%

From the comparison of negative bending moments at supports, it can be seen that the differences between the two methods are expressed as percentages (%), ranging approximately from **3% to 30%**, depending on the slab dimensions. In general, it can be observed that when the slab becomes more square (the two spans become closer in length), the percentage difference becomes smaller, and the results from both methods become closer to each other.

- Negative bending moment for long span $(M_{b,neg})$

Size of Slab	For live load $LL = 2.00\ kN/m^2$			For live load $LL = 5.00\ kN/m^2$		
	Coefficient method ACI	Finite Element method	Difference (%)	Coefficient method ACI	Finite Element method	Difference (%)
4mx4m	5.783	5.76	−0.40%	9.097	9.03	−0.74%
4mx5m	5.546	7.88	+42.08%	8.452	10.98	+29.91%
4mx6m	4.524	9.83	+117.29%	6.879	13.49	+96.1%
4mx7m	3.31	11.37	+243.5%	4.837	14.58	+201.43%
4mx8m	3.472	12.42	+257.72%	5.175	9.49	+83.38%
5mx5m	9.95	9.71	+2.41%	15.128	14.62	−3.36%
5mx6m	9.713	12.08	+24.37%	14.268	16.26	+13.96%
5mx7m	7.95	13.35	+67.92%	11.849	16.60	+40.1%
5mx8m	6.758	15.71	+132.47%	9.67	16.78	+73.53%
6mx6m	15.205	14.84	−2.40%	22.662	22.32	−1.51%
6mx7m	14.933	17.75	+18.86%	22.016	24.52	+11.37%
6mx8m	13.289	19.41	+46.06%	19.813	25.24	+27.39%
7mx7m	21.89	21.37	−2.38%	32.636	31.97	−2.04%
7mx8m	22.031	24.89	+12.98%	31.792	34.37	+8.11%
8mx8m	30.151	29.41	−2.46%	44.186	43.22	−2.19%

Regarding the negative bending moment at supports, the results obtained from the ACI method are generally higher than those obtained from the finite element method. The percentage difference varies according to slab dimensions, typically ranging from about **3% up to 200%**. This indicates that, in some cases, the negative bending moments predicted by the ACI method can be significantly greater than those obtained from the finite element analysis.

5.2. Comparison of Positive Bending Moment at mid span

- Positive bending moment for short span $(M_{a,pos})$

Size of Slab	For live load $LL = 2.00\ kN/m^2$			For live load $LL = 5.00\ kN/m^2$		
	Coefficient method ACI	Finite Element method	Difference (%)	Coefficient method ACI	Finite Element method	Difference (%)
4mx4m	2.729	2.79	+2.24%	4.679	4.31	−7.89%
4mx5m	4.207	3.58	−14.9%	7.264	5.97	−17.81%
4mx6m	5.383	4.41	−18.08%	9.12	6.74	−26.1%
4mx7m	6.178	4.91	−20.52%	10.315	7.85	−23.9%
4mx8m	6.729	5.70	−15.29%	11.357	9.33	−17.85%
5mx5m	4.63	4.80	+3.67%	7.676	7.04	−8.29%
5mx6m	6.695	5.97	−10.83%	11.024	9.80	−11.1%
5mx7m	8.28	7.36	−11.11%	13.706	11.94	−12.88%
5mx8m	9.644	8.01	−16.94%	15.988	13.61	−14.87%
6mx6m	7.018	7.42	+5.73%	11.404	10.92	−4.24%
6mx7m	9.683	9.09	−6.12%	15.861	14.44	−8.96%
6mx8m	11.863	11.03	−7.02%	19.506	17.39	−10.85%
7mx7m	10.03	10.79	+7.58%	16.239	15.79	−2.76%
7mx8m	13.393	13.02	−2.79%	21.529	20.11	−6.59%
8mx8m	13.724	14.98	+9.15%	21.833	21.61	−1.02%

For the positive bending moment at midspan in the short direction, the values obtained from the finite element method are smaller than those obtained from the coefficient method by up to about **20%** when the slab aspect ratio becomes more elongated in general. However, when the slab panel is nearly square and the live load is small, the opposite trend is observed. This difference does not vary significantly with the magnitude of the live load.

- Positive bending moment for short span $(M_{b,pos})$

Size of Slab	For live load $LL = 2.00\ kN/m^2$			For live load $LL = 5.00\ kN/m^2$		
	Coefficient method ACI	Finite Element method	Difference (%)	Coefficient method ACI	Finite Element method	Difference (%)
4mx4m	2.729	2.79	+2.24%	4.679	4.31	+7.89%

4mx5m	2.691	3.34	+24.12%	4.485	5.07	+13.04%
4mx6m	2.356	4.44	+88.46%	3.996	5.82	+45.65%
4mx7m	1.859	4.91	+164.12%	3.092	6.60	+113.45%
4mx8m	1.231	5.56	+351.67%	2.126	4.21	+98.02%
5mx5m	4.63	4.80	+3.67%	7.676	7.04	+8.29%
5mx6m	4.537	5.46	+20.34%	7.382	7.60	+2.95%
5mx7m	4.00	6.14	+53.5%	6.687	7.50	+12.16%
5mx8m	3.338	6.96	+108.51%	5.31	7.66	+44.26%
6mx6m	7.018	7.42	+5.73%	11.404	10.92	+4.24%
6mx7m	6.786	8.22	+21.13%	11.041	11.73	+6.24%
6mx8m	6.395	9.00	+40.73%	10.491	11.81	+12.57%
7mx7m	10.03	10.79	+7.58%	16.239	15.79	+2.76%
7mx8m	9.829	11.75	+19.54%	15.623	16.66	+6.64%
8mx8m	13.724	14.98	+9.15%	21.833	21.61	+1.02%

For the positive bending moment at midspan in the long direction, the values obtained from the finite element method are larger than those obtained from the coefficient method by approximately **3% to 300%**, depending on the aspect ratio of the slab panel. The difference does not vary significantly with the magnitude of the live load.

5.3. Comparison of Shear Force at Support

- Shear Force for short span $\left(V_{a,sup}\right)$

Size of Slab	For live load $LL = 2.00\ kN/m^2$			For live load $LL = 5.00\ kN/m^2$		
	Coefficient method ACI	Finite Element method	Difference (%)	Coefficient method ACI	Finite Element method	Difference (%)
4mx4m	8.455	12.31	+45.59%	13.3	18.39	+38.27%
4mx5m	12.556	12.97	+3.3%	19.95	20.46	+2.56%
4mx6m	15.669	13.36	−14.74%	23.824	21.59	−9.38%
4mx7m	17.406	14.62	−16.01%	25.937	22.94	−11.55%
4mx8m	18.283	15.62	−14.57%	27.249	26.42	−3.04%

5mx5m	11.638	15.49	+33.1%	17.694	23.45	+32.53%
5mx6m	16.405	17.19	+4.79%	24.74	28.47	+15.08%
5mx7m	19.869	18.98	−4.47%	29.612	31.45	+6.21%
5mx8m	22.249	19.82	−10.92%	33.315	33.26	−0.17%
6mx6m	14.82	20.32	+37.11%	22.088	30.59	+38.49%
6mx7m	20.099	22.36	+11.25%	30.324	35.08	+15.68%
6mx8m	24.121	24.42	+1.24%	35.963	38.75	+7.75%
7mx7m	18.288	25.56	+39.76%	27.265	37.17	+36.33%
7mx8m	23.929	27.92	+16.68%	35.53	42.57	+19.81%
8mx8m	22.04	31.22	+41.65%	32.3	44.77	+38.61%

For the shear force at the support in the short direction, the values obtained from the finite element method are greater than those obtained from the coefficient method by about **40%** for square slab panels, and conversely smaller by about **15%** when the aspect ratio is equal to 2. The difference in shear force between the two methods does not vary significantly with the magnitude of the live load.

- Shear Force for long span $(V_{b,sup})$

Size of Slab	For live load $LL = 2.00\ kN/m^2$			For live load $LL = 5.00\ kN/m^2$		
	Coefficient method ACI	Finite Element method	Difference (%)	Coefficient method ACI	Finite Element method	Difference (%)
4mx4m	8.455	12.31	+45.59%	13.3	18.39	+38.27%
4mx5m	6.219	13.25	+113.06%	9.505	18.12	+90.64%
4mx6m	4.185	12.59	+200.84%	6.362	18.02	+183.24%
4mx7m	2.778	12.86	+362.92%	4.013	17.30	+331.1%
4mx8m	2.018	12.51	+519.92%	3.007	14.63	+386.53%
5mx5m	11.638	15.49	+33.1%	17.694	23.45	+32.53%
5mx6m	9.178	16.11	+75.53%	13.433	24.28	+80.75%
5mx7m	6.565	16.08	+144.94%	9.784	23.68	+142.03%
5mx8m	4.775	15.95	+234.03%	6.863	21.77	+217.21%
6mx6m	14.82	20.32	+37.11%	22.088	30.59	+38.49%

6mx7m	12.219	21.15	+73.09%	17.976	30.49	+69.62%
6mx8m	9.477	21.33	+125.07%	14.129	30.07	+112.82%
7mx7m	18.288	25.56	+39.76%	27.265	37.17	+36.33%
7mx8m	15.696	26.60	+69.47%	22.695	37.91	+67.04%
8mx8m	22.04	31.22	+41.65%	32.3	44.77	+38.61%

For the shear force at the support in the long direction, the values obtained from the finite element method are always greater than those obtained from the coefficient method, by approximately **40% to 500%**, depending on the slab aspect ratio from 1 to 2. The difference in shear force between the two methods does not vary significantly with the magnitude of the live load. Based on the differences between the internal forces (bending moments and shear forces) obtained from the coefficient method and the finite element method, we observe that:

- For the bending moment at the support, the bending moment at midspan, and the shear force at the support in the short direction, the values obtained from the finite element method are smaller than those obtained from the coefficient method
- For the bending moment at the support, the bending moment at midspan, and the shear force at the support in the long direction, the values obtained from the finite element method are greater than those obtained from the coefficient method, because the two-way action behavior of the slab continues to exist even when the slab has an aspect ratio of 2.
- The above behavior occurs because the coefficient method assumes that all four supports of the slab are fixed supports, meaning that there is no settlement and that the slab moments are fully restrained regardless of torsional stiffness. In addition, within the coefficient method, the contribution of bending moments from live load is taken larger than that from dead load.
- In the finite element method, on the other hand, the slab and beams in both directions, with thickness and cross-sectional dimensions varying according to slab size, act together as a single system and distribute internal forces in both directions depending on the relative stiffness of these elements, especially because the supporting beams experience different magnitudes of deflection.

CHAPTER 6

Conclusions and Suggestion

6.1. Conclusions

Through the study of **15 two-way reinforced concrete slab models supported by beams**, with slab dimensions having aspect ratios varying from **1 to 2** and slab sizes ranging from **4 m × 4 m to 8 m × 8 m**, and subjected to **two cases of live load** for **residential buildings** and **public buildings**, using two analysis methods—namely the **ACI coefficient method** and the **finite element method**—we can observe the following characteristics:

- The two-way action behavior of the slab analyzed using the finite element method continues to exist even when the slab has an aspect ratio of 2, which is different from the ACI coefficient method, where the two-way action behavior of the slab may be neglected when the slab aspect ratio is equal to or greater than 2.
- The bending moment at the support, the bending moment at midspan, and the shear force at the support in the short direction obtained from the finite element method are generally smaller than those obtained from the ACI coefficient method.
- Conversely, the bending moment at the support, the bending moment at midspan, and the shear force at the support in the long direction obtained from the finite element method are generally greater than those obtained from the ACI coefficient method.
- The variation of the three characteristics above is almost independent of the live load on the slab, meaning that it changes only slightly with the live load.

6.2. Suggestion

Because the bending moments at the support and the bending moments at midspan obtained from the coefficient method and the finite element method have different values and proportions in the two directions, and because the shear forces at the support are also different between the two methods, therefore, if the slab is designed using the coefficient method, the beams and columns should be analyzed as a **space frame system (without the slab)**.

That is, the beams and columns should **not** be analyzed and designed as part of the whole building system consisting of slab–beam–column interaction.

At the same time, when analyzing the whole structural system that is connected with the slab, it is necessary to consider the **residual reactions** that occur in the beams supporting the slab under dead load and long-term live load.

In order to clearly observe the load transfer from the slab to the supporting beams, future researchers should analyze the beams and columns using two different analytical models: one **without the slab**, and the other **with the slab**.

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APPENDIX

APPENDIX A: REPORT OF CALCULATIONS BY COEFFICIENT METHOD.

Slab Analysis using Coefficient Method (ACI)

Live Load on Slab $LL := 5.00 \ \frac{kN}{m^2}$

Slab Dimension $L_a := 4 \ m \qquad L_b := 6 \ m$

Slab Thickness

$$F := \text{linterp}\left(\begin{bmatrix} 2 \\ 2.5 \\ 3 \\ 4 \\ 5 \\ 6 \\ 12 \end{bmatrix} \frac{kN}{m^2}, \begin{bmatrix} 180 \\ 175 \\ 170 \\ 165 \\ 160 \\ 155 \\ 150 \end{bmatrix}, LL\right) = 160$$

$$t_{min} := \frac{2 \cdot (L_a + L_b)}{F} = 125 \ mm$$

$$t := \text{Ceil}\left(t_{min}, 10 \ mm\right) = 130 \ mm$$

Beam Section (Long Span)

$$h_b := \max\left(\text{Ceil}\left(4 \cdot t_{min}, 50 \ mm\right), 300 \ mm\right) = 50 \ cm$$

$$b_b := \max\left(\text{Ceil}\left(2 \cdot t_{min}, 50 \ mm\right), 200 \ mm\right) = 25 \ cm$$

Beam Section (Short Span)

$$h_a := \max\left(\text{Ceil}\left(\frac{4 \cdot L_a}{F} \cdot 4, 50 \ mm\right), 300 \ mm\right) = 40 \ cm$$

$$b_a := \max\left(\text{Ceil}\left(\frac{4 \cdot L_a}{F} \cdot 2, 50 \ mm\right), 200 \ mm\right) = 20 \ cm$$

Load on Slab

$$Cover := 50 \ mm \cdot 20 \ \frac{kN}{m^3} = 1 \ \frac{kN}{m^2}$$

$$Ceiling := 0.50 \ \frac{kN}{m^2}$$

$$Partition := 1.00 \ \frac{kN}{m^2}$$

$$SDL := Cover + Ceiling + Partition = 2.5 \ \frac{kN}{m^2}$$

$$SW := t \cdot 25 \ \frac{kN}{m^3} = 3.25 \ \frac{kN}{m^2}$$

$$DL := SW + SDL = 5.75 \ \frac{kN}{m^2}$$

$$w_D := 1.2 \cdot DL = 6.9 \ \frac{kN}{m^2}$$

$$w_L := 1.6 \cdot LL = 8 \ \frac{kN}{m^2}$$

$$w_u := w_D + w_L = 14.9 \ \frac{kN}{m^2}$$

Shear&Moment Coefficient

$l_a := L_a - b_b = 3.75\ m \qquad l_b := L_b - b_a = 5.8\ m$

$\lambda := \frac{l_a}{l_b} = 0.647$

$$Case_2 := \begin{bmatrix} 0.50 & 0.086 & 0.006 & 0.037 & 0.002 & 0.066 & 0.004 & 0.94 & 0.06 \\ 0.55 & 0.084 & 0.007 & 0.035 & 0.003 & 0.062 & 0.006 & 0.92 & 0.08 \\ 0.60 & 0.081 & 0.010 & 0.034 & 0.004 & 0.058 & 0.007 & 0.89 & 0.11 \\ 0.65 & 0.077 & 0.014 & 0.032 & 0.006 & 0.053 & 0.010 & 0.85 & 0.15 \\ 0.70 & 0.074 & 0.017 & 0.030 & 0.007 & 0.049 & 0.012 & 0.81 & 0.19 \\ 0.75 & 0.069 & 0.022 & 0.028 & 0.009 & 0.045 & 0.014 & 0.76 & 0.24 \\ 0.80 & 0.065 & 0.027 & 0.026 & 0.011 & 0.041 & 0.017 & 0.71 & 0.29 \\ 0.85 & 0.060 & 0.031 & 0.024 & 0.012 & 0.037 & 0.019 & 0.66 & 0.34 \\ 0.90 & 0.055 & 0.037 & 0.022 & 0.014 & 0.034 & 0.022 & 0.60 & 0.40 \\ 0.95 & 0.050 & 0.041 & 0.020 & 0.016 & 0.030 & 0.025 & 0.55 & 0.45 \\ 1.00 & 0.045 & 0.045 & 0.018 & 0.018 & 0.027 & 0.027 & 0.50 & 0.50 \end{bmatrix}$$

$C_a := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 2 \rangle}, \lambda\right) = 0.077$

$C_b := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 3 \rangle}, \lambda\right) = 0.014$

$C_{a.dl} := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 4 \rangle}, \lambda\right) = 0.032$

$C_{b.dl} := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 5 \rangle}, \lambda\right) = 0.006$

$C_{a.ll} := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 6 \rangle}, \lambda\right) = 0.053$

$C_{b.ll} := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 7 \rangle}, \lambda\right) = 0.01$

$C_{a.v} := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 8 \rangle}, \lambda\right) = 0.853$

$C_{b.v} := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 9 \rangle}, \lambda\right) = 0.147$

Bending Moment

$M_{a.neg} := C_a \cdot w_u \cdot l_a^2 = 16.192\ \frac{kN \cdot m}{1\ m}$

$M_{b.neg} := C_b \cdot w_u \cdot l_b^2 = 6.879\ \frac{kN \cdot m}{1\ m}$

$M_{a.pos} := C_{a.dl} \cdot w_D \cdot l_a^2 + C_{a.ll} \cdot w_L \cdot l_a^2 = 9.12\ \frac{kN \cdot m}{1\ m}$

$M_{b.pos} := C_{b.dl} \cdot w_D \cdot l_b^2 + C_{b.ll} \cdot w_L \cdot l_b^2 = 3.996\ \frac{kN \cdot m}{1\ m}$

Shear Force

$W := w_u \cdot l_a \cdot l_b = 324.075\ kN$

$V_a := \frac{C_{a.v} \cdot W}{2 \cdot l_b} = 23.824\ \frac{kN}{1\ m}$

$V_b := \frac{C_{b.v} \cdot W}{2 \cdot l_a} = 6.362\ \frac{kN}{1\ m}$

Slab Analysis using Coefficient Method (ACI)

Live Load on Slab $LL := 2.00 \ \frac{kN}{m^2}$

Slab Dimension $L_a := 4 \ m \qquad L_b := 6 \ m$

Slab Thickness

$$F := \text{linterp}\left(\begin{bmatrix} 2 \\ 2.5 \\ 3 \\ 4 \\ 5 \\ 6 \\ 12 \end{bmatrix} \frac{kN}{m^2}, \begin{bmatrix} 180 \\ 175 \\ 170 \\ 165 \\ 160 \\ 155 \\ 150 \end{bmatrix}, LL\right) = 180$$

$$t_{min} := \frac{2 \cdot (L_a + L_b)}{F} = 111.111 \ mm$$

$$t := \text{Ceil}\left(t_{min}, 10 \ mm\right) = 120 \ mm$$

Beam Section (Long Span)

$$h_b := \max\left(\text{Ceil}\left(4 \cdot t_{min}, 50 \ mm\right), 300 \ mm\right) = 45 \ cm$$

$$b_b := \max\left(\text{Ceil}\left(2 \cdot t_{min}, 50 \ mm\right), 200 \ mm\right) = 25 \ cm$$

Beam Section (Short Span)

$$h_a := \max\left(\text{Ceil}\left(\frac{4 \cdot L_a}{F} \cdot 4, 50 \ mm\right), 300 \ mm\right) = 40 \ cm$$

$$b_a := \max\left(\text{Ceil}\left(\frac{4 \cdot L_a}{F} \cdot 2, 50 \ mm\right), 200 \ mm\right) = 20 \ cm$$

Load on Slab

$$Cover := 50 \ mm \cdot 20 \ \frac{kN}{m^3} = 1 \ \frac{kN}{m^2}$$

$$Ceiling := 0.50 \ \frac{kN}{m^2}$$

$$Partition := 1.00 \ \frac{kN}{m^2}$$

$$SDL := Cover + Ceiling + Partition = 2.5 \ \frac{kN}{m^2}$$

$$SW := t \cdot 25 \ \frac{kN}{m^3} = 3 \ \frac{kN}{m^2}$$

$$DL := SW + SDL = 5.5 \ \frac{kN}{m^2}$$

$$w_D := 1.2 \cdot DL = 6.6 \ \frac{kN}{m^2}$$

$$w_L := 1.6 \cdot LL = 3.2 \ \frac{kN}{m^2}$$

$$w_u := w_D + w_L = 9.8 \ \frac{kN}{m^2}$$

Shear&Moment Coefficient

$l_a := L_a - b_b = 3.75\ m$ $\qquad$ $l_b := L_b - b_a = 5.8\ m$

$$\lambda := \frac{l_a}{l_b} = 0.647$$

$$Case_2 := \begin{bmatrix} 0.50 & 0.086 & 0.006 & 0.037 & 0.002 & 0.066 & 0.004 & 0.94 & 0.06 \\ 0.55 & 0.084 & 0.007 & 0.035 & 0.003 & 0.062 & 0.006 & 0.92 & 0.08 \\ 0.60 & 0.081 & 0.010 & 0.034 & 0.004 & 0.058 & 0.007 & 0.89 & 0.11 \\ 0.65 & 0.077 & 0.014 & 0.032 & 0.006 & 0.053 & 0.010 & 0.85 & 0.15 \\ 0.70 & 0.074 & 0.017 & 0.030 & 0.007 & 0.049 & 0.012 & 0.81 & 0.19 \\ 0.75 & 0.069 & 0.022 & 0.028 & 0.009 & 0.045 & 0.014 & 0.76 & 0.24 \\ 0.80 & 0.065 & 0.027 & 0.026 & 0.011 & 0.041 & 0.017 & 0.71 & 0.29 \\ 0.85 & 0.060 & 0.031 & 0.024 & 0.012 & 0.037 & 0.019 & 0.66 & 0.34 \\ 0.90 & 0.055 & 0.037 & 0.022 & 0.014 & 0.034 & 0.022 & 0.60 & 0.40 \\ 0.95 & 0.050 & 0.041 & 0.020 & 0.016 & 0.030 & 0.025 & 0.55 & 0.45 \\ 1.00 & 0.045 & 0.045 & 0.018 & 0.018 & 0.027 & 0.027 & 0.50 & 0.50 \end{bmatrix}$$

$$C_a := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 2 \rangle}, \lambda\right) = 0.077$$

$$C_b := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 3 \rangle}, \lambda\right) = 0.014$$

$$C_{a.dl} := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 4 \rangle}, \lambda\right) = 0.032$$

$$C_{b.dl} := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 5 \rangle}, \lambda\right) = 0.006$$

$$C_{a.ll} := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 6 \rangle}, \lambda\right) = 0.053$$

$$C_{b.ll} := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 7 \rangle}, \lambda\right) = 0.01$$

$$C_{a.v} := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 8 \rangle}, \lambda\right) = 0.853$$

$$C_{b.v} := \text{linterp}\left(Case_2^{\langle 1 \rangle}, Case_2^{\langle 9 \rangle}, \lambda\right) = 0.147$$

Bending Moment

$$M_{a.neg} := C_a \cdot w_u \cdot l_a^{\ 2} = 10.65\ \frac{kN \cdot m}{1\ m}$$

$$M_{b.neg} := C_b \cdot w_u \cdot l_b^{\ 2} = 4.524\ \frac{kN \cdot m}{1\ m}$$

$$M_{a.pos} := C_{a.dl} \cdot w_D \cdot l_a^{\ 2} + C_{a.ll} \cdot w_L \cdot l_a^{\ 2} = 5.383\ \frac{kN \cdot m}{1\ m}$$

$$M_{b.pos} := C_{b.dl} \cdot w_D \cdot l_b^{\ 2} + C_{b.ll} \cdot w_L \cdot l_b^{\ 2} = 2.356\ \frac{kN \cdot m}{1\ m}$$

Shear Force

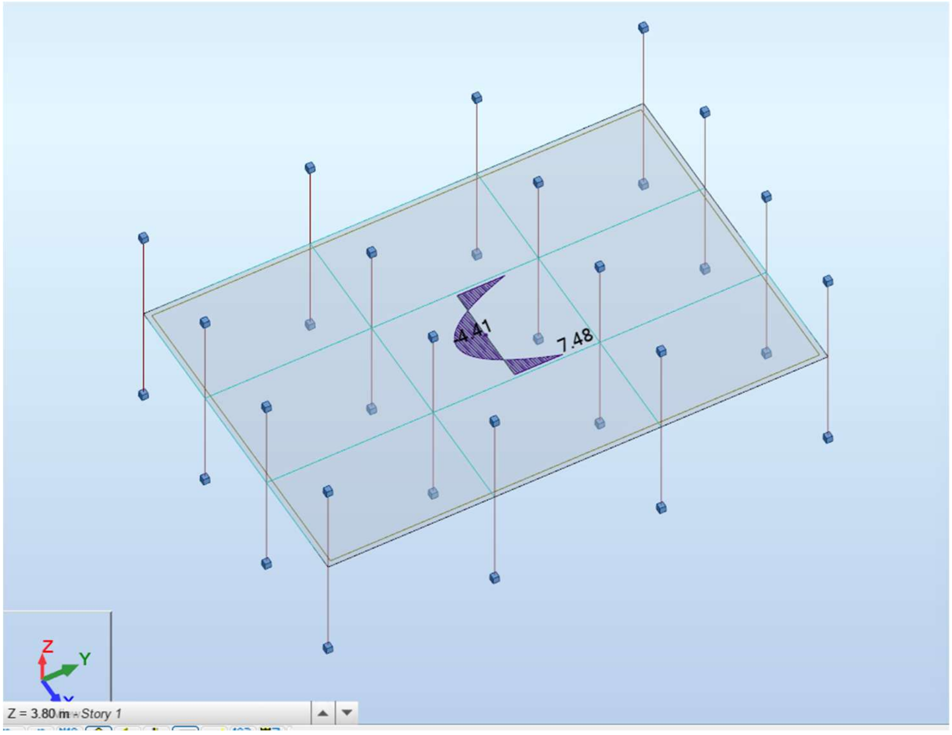
$$W := w_u \cdot l_a \cdot l_b = 213.15\ kN$$

$$V_a := \frac{C_{a.v} \cdot W}{2 \cdot l_b} = 15.669\ \frac{kN}{1\ m}$$

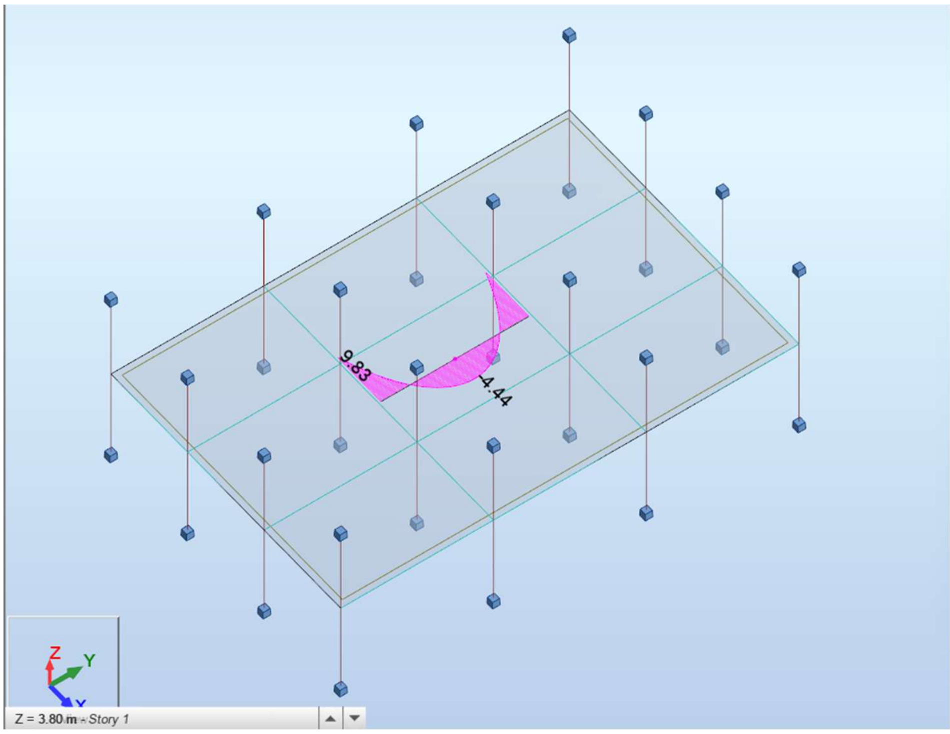
$$V_b := \frac{C_{b.v} \cdot W}{2 \cdot l_a} = 4.185\ \frac{kN}{1\ m}$$

APPENDIX B: SHEAR FORCE AND BENDING MOMENT DIAGRAM (FEM)

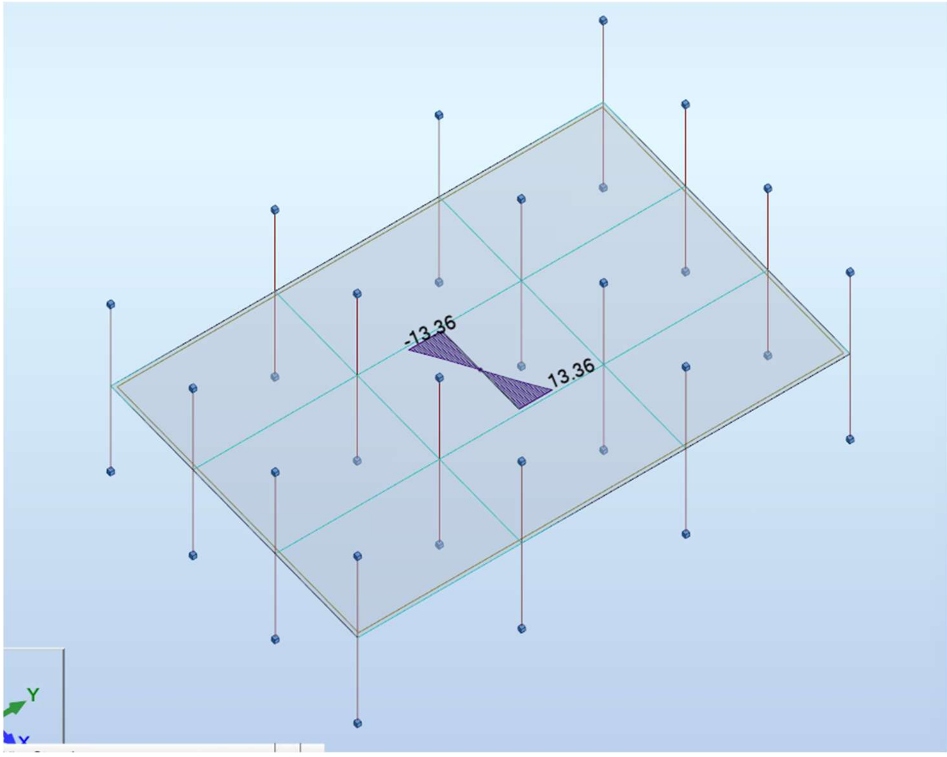
- For Slab 4x6 and Live load $2kN/m^2$.



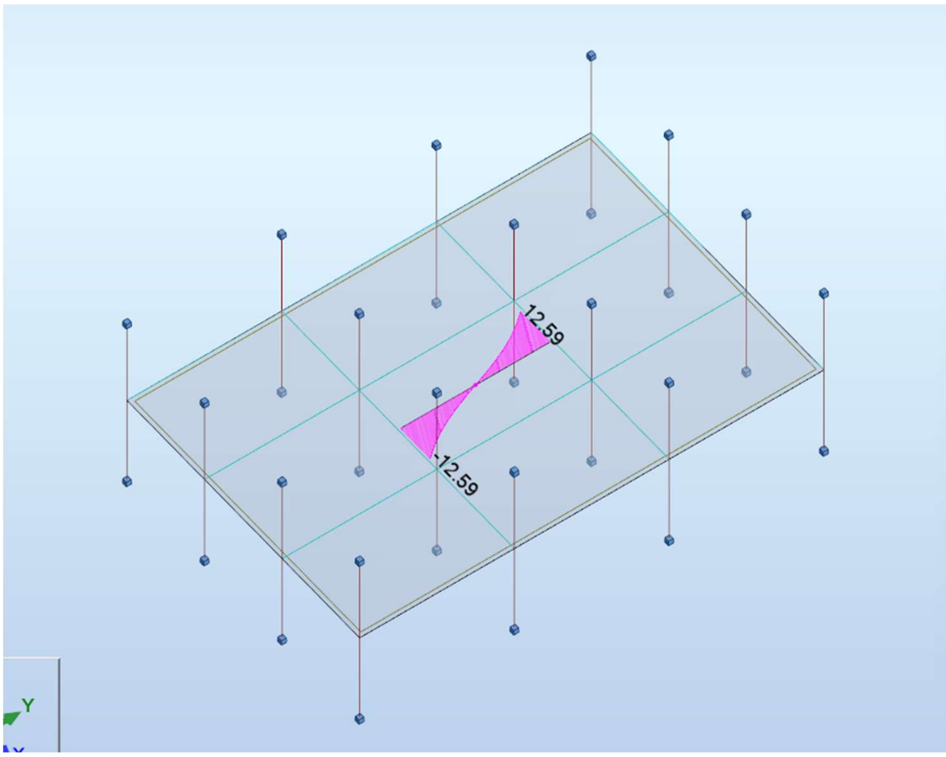
(Bending Moment Diagram for short span)



(Bending Moment Diagram for Long span)

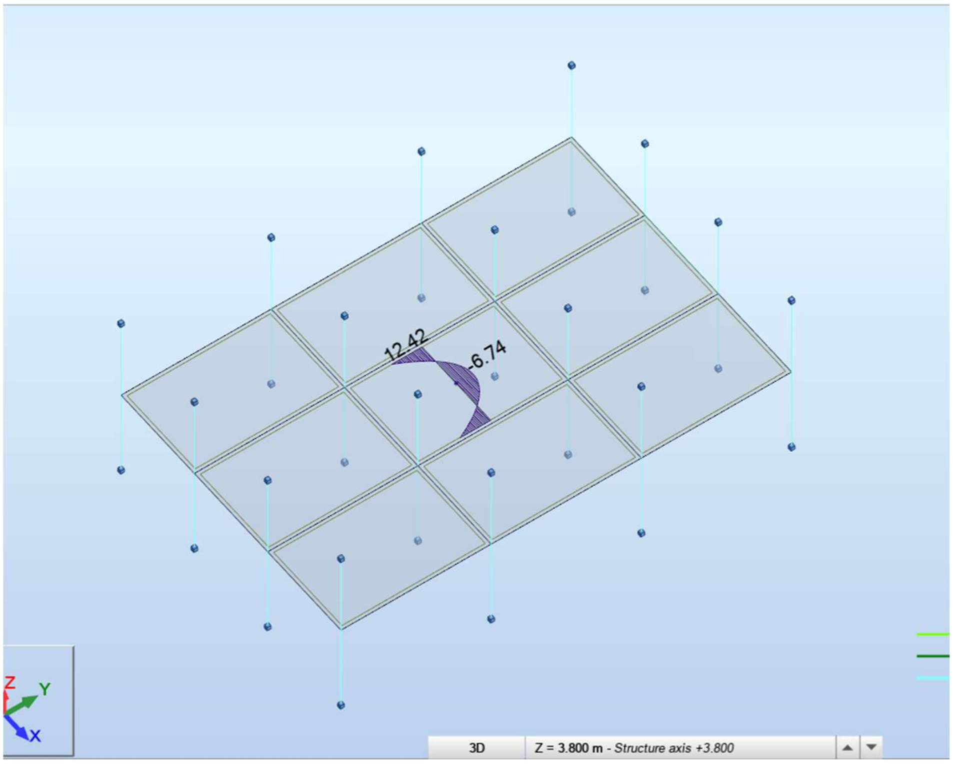


(Shear Force Diagram for short span)

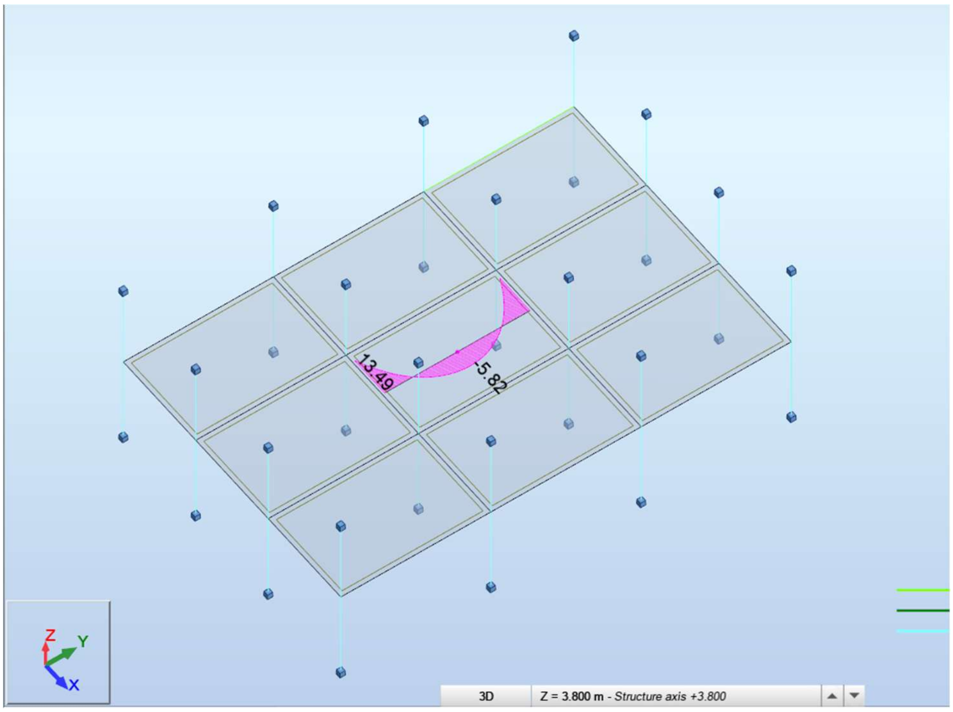


(Shear Force Diagram for Long span)

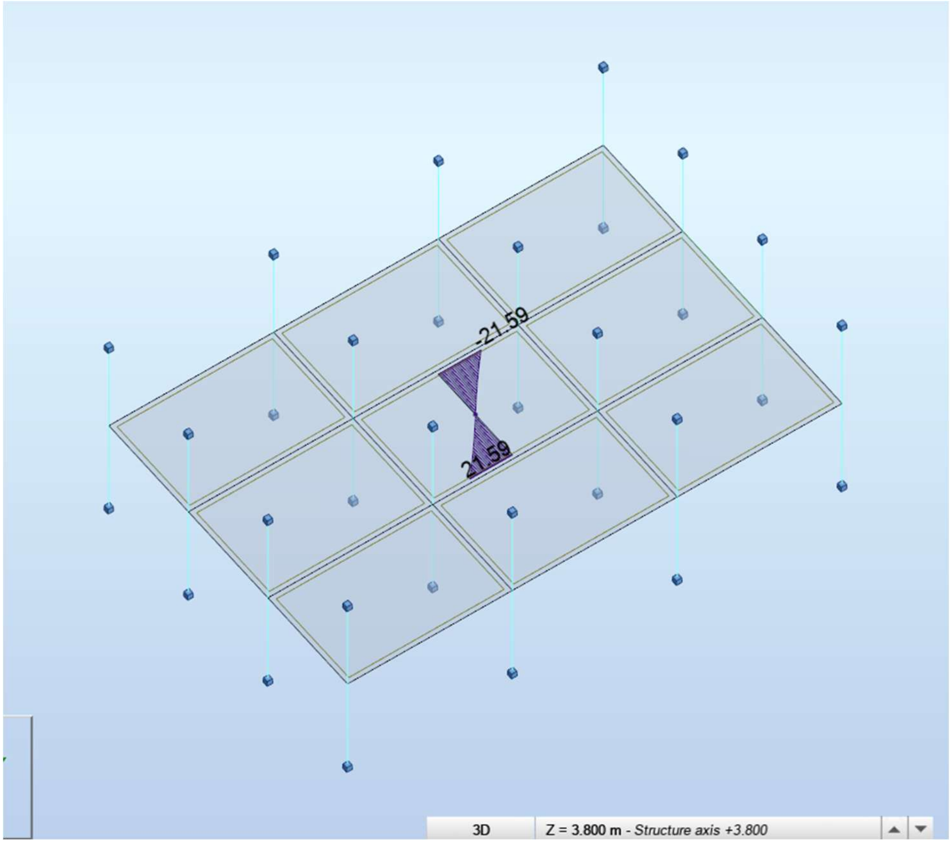
- For Slab 4x6 and Live load 5 kN/m^2



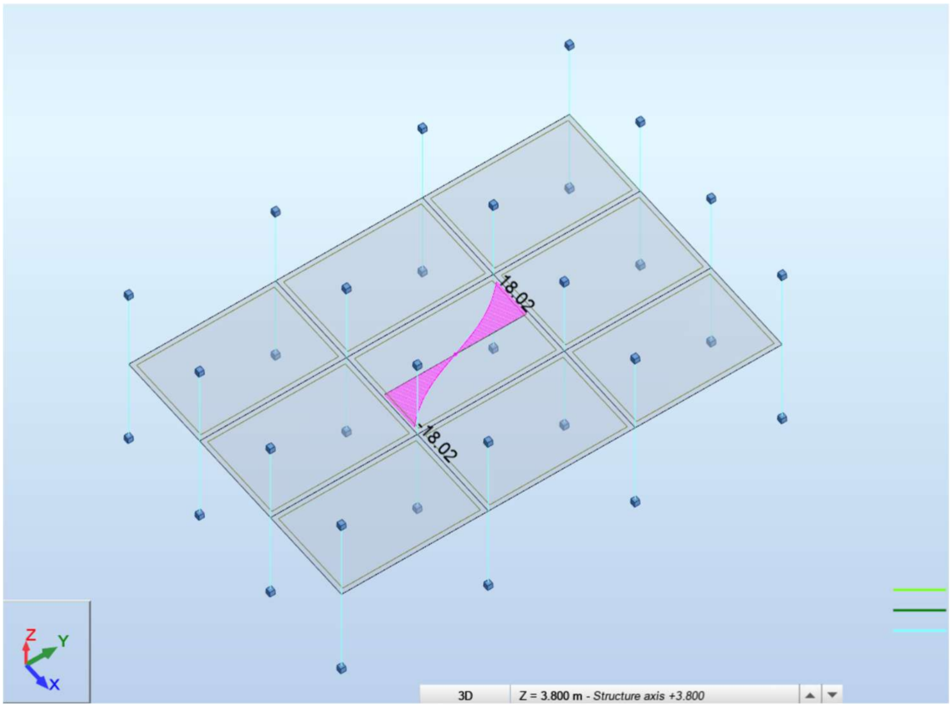
(Bending Moment Diagram for short span)



(Bending Moment Diagram for Long span)



(Shear Force Diagram for short span)



(Shear Force Diagram for Long span)