Lab 3 : Deterministic representation

Submission method: Zip the Matlab code and all the resulted files. Upload all those files to Blakboard.

This exercise explores deterministic signal representations through the application of spectral shearing interferometry. Spectral shearing interferometry is a technique used across a variety of fields, often for precise measurements and imaging of small particles, cells, and displacements.

For this exercise, we'll use data (see data.tar.gz) from a microscope that maps pulsed optical spectra onto a ~50 micron line in space. As objects move perpendicularly through the line, the optical spectrum is imprinted with amplitude and phase information and a 2D image can be built up by stacking the amplitude and phase information at each time instance. Spectral shearing interferometry provides the means to acquire both amplitude and quantitative phase images from the optical spectrum at extremely high rates.

The provided mat files contain data acquired with a conventional optical spectrum analyzer. The samples were yeast cells and red blood cells (labeled 'rbc') that were stepped through the microscope's 1D field of view in increments of 262 nm.

Provided are four mat files with two variables each. The first, called refSpec, is a spectrum (with wavelength in nm) acquired with nothing in the field of view. The second, specData, is a stack of power spectra as the object was moved incrementally through the field of view. The wavelengths measured are identical between the two variables.

I. Preliminaries

We would like to acquire amplitude and phase images. This is generally done in the following way. An incoming pulse gets split into two. One can be considered a reference pulse whereas the other pulse is spectrally sheared (frequency shifted), and delayed. These two pulses go through a process called sum-frequency generation by which they are recombined.

1. The power spectrum of a signal is $|E(\omega)|^2$, where $E(\omega)$ is frequency domain representation of the signal. Show that power spectrum resulting from the combination of a pulse with a delayed and spectrally sheared copy is the following.

$$S(\omega) = |E(\omega)|^2 + |E(\omega - \Omega)|^2 + 2\,|E(\omega)||E(\omega - \Omega)|\cos(\phi(\omega) - \phi(\omega - \Omega) + \omega\tau)$$

Where Ω is the frequency shift, and $\phi(\omega)$ is the spectral phase. The last term is the interference term and it is the term we will be most interested in.

2. Show that $S(\omega) = I(\omega)_{DC} + I(\omega)_{AC}\, e^{j\omega\tau} + I(\omega)_{AC}\, e^{-j\omega\tau}$ where $I(\omega)$ is spectral power. In other words show that the result from part 1. Can be broken into these three terms. What do these terms mean?

To process the data into amplitude and phase images, follow these steps:

1. Plot the refSpec variable. What you should get is an optical spectrum centered near 1560 nm with power values in dBm.
2. Use the refSpec to determine the region of the data to focus on. Determine the indices where the power is greater than 10 dB below the maximum (get rid of the fringes). Use these indices to slice

the specData variable, which is essentially a stack of spectral power measurements on a linear scale.

3. Slice the wavelengths corresponding to the region of interest using the same indices. Convert this to optical frequencies all in the range of 190+ THz. Remember that the speed of light is 3.0e8.
4. Interpolate the specData onto a linear frequency grid with the same limits as the original data (Use, e.g., 1024 or more points). Go ahead and center the frequency grid at zero.
5. Take the FFT of the specData along the frequency direction (across the rows). Plot one of the lines. What are the units of the x-axis of this Fourier transformed frequency data? Be careful to label the x-axis in terms of these units.
6. You should see three peaks in the FFT. Locate one of the peaks that is not at DC. What's its location (with units)?
7. Create a filter function centered at the sideband you located with a width appropriate to avoid aliasing.
8. Apply the filter function to all of the lines (e.g., bsxfun in Matlab) and inverse transform.
9. The absolute value of the result is our amplitude image (corresponding to received optical power).
10. The phase image (more accurately, the phase derivative computed across the shearing distance) is contained in the argument of the result.
    a. Unwrap the phase.
    b. Remove the linear and DC components from each line (you may want to process each line independently here, but it is possible to subtract only one reference since the interferometer is extremely stable) and plot the result.

Though the lines will naturally be correlated because of their proximity, we will process them independently here.