Lab Assignment 2

Scenario: Consider a target whose position is sampled at every $T = 1s$. This target is moving in a plane first with constant course and speed until $k = 40$ and then it completes a coordinated turn in 40 sampling periods with turn rate $\Omega = \frac{\pi}{40}/sample$. The target repeats the above pattern one more time; that is, after the above mentioned first coordinated turn, it moves with a constant course and speed for 40 sampling periods, which is followed by a coordinated turn in 40 sampling periods with a turn rate $\Omega = \frac{\pi}{40}/sample$. The target state is defined as $x = \left[\mathcal{E}\ \dot{\mathcal{E}}\ \eta\ \dot{\eta}\right]^T$ where $\mathcal{E}$ and η are the locations in x-y coordinate system and $\dot{\mathcal{E}}$ and $\dot{\eta}$ are the speeds in x and y directions, respectively. Assume that initial condition $x(0) = [2000m\ 0m/s\ 1000m - 15m/s]^T$

1) Using the nearly constant velocity model given by equation (1) , the coordinated turn model given in equation (2), and the measurement model given in equation (3) simulate the targets motion.

$$x(k+1) = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} \frac{1}{2}T^2 & 0 \\ T & 0 \\ 0 & \frac{1}{2}T^2 \\ 0 & T \end{bmatrix} v(k) \tag{1}$$

$$x(k+1) = \begin{bmatrix} 1 & \frac{\sin \Omega T}{\Omega} & 0 & -\frac{1-\cos \Omega T}{\Omega} \\ 0 & \cos \Omega T & 0 & -\sin \Omega T \\ 0 & \frac{1-\cos \Omega T}{\Omega} & 1 & \frac{\sin \Omega T}{\Omega} \\ 0 & \sin \Omega T & 0 & \cos \Omega T \end{bmatrix} x(k) + \begin{bmatrix} \frac{1}{2}T^2 & 0 \\ T & 0 \\ 0 & \frac{1}{2}T^2 \\ 0 & T \end{bmatrix} v(k) \tag{2}$$

$$z(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(k) + w(k) \tag{3}$$

In equations (1) and (2) $v(k)$ is a zero-mean white Gaussian noise with covariance $\Sigma_v = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$ and $\sigma^2 = 10^{-3}$

In equation (3), $w(k)$ is a zero-mean white Gaussian noise with covariance $\Sigma_w = \begin{bmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_w^2 \end{bmatrix}$ with $\sigma_w^2 = 10^{-1}$.

2) (a) Assuming that the target is always moving with a constant course and speed ; that is, assuming that the model in equation (1) is correct throughout the course of the target, to track the target, build a Kalman filter using the measurement model defined in equation (3).

2 (b) Assuming that the target is always moving with the model in equation (2) throughout the course of the target, to track the target, build a Kalman filter using the measurement model defined in equation (3).

3 Now using the models from equations (1), (2), and (3) build an interactive multiple model (IMM) estimator (Kalman filter) to track the target. While building the IMM Kalman filter assume that the Markov chain transition matrix between the models in equations (1) and (2) is

$$[p_{ij}] = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}$$

Provide the plot of the true and measured target courses for question 1, for questions 2 and 3 provide the true course, measured course and the tracked course together with the estimation error. Also for question 3, provide the plots of mixing probabilities as a function of k.