MODULE 2

ANGLE MODULATION

Introduction:-

Angle Modulation, is a process of altering either frequency (or) phase of carrier signal in accordance with the instantaneous values of message signal, by keeping amplitude of carrier constant.

↳ General equation of Angle Modulated wave is given by

$$S(t) = A_c \cos \theta_i(t) \quad \text{——} (1)$$

where, A_c = Amplitude of carrier signal

$\theta_i(t)$ = Angle of the modulated signal.

Angle Modulation techniques are further divided into two types

- → Frequency Modulation (FM)
- → Phase Modulation (PM)

• **Frequency Modulation:-** It is a process of altering frequency of carrier signal in accordance with the instantaneous values of message signal by keeping amplitude, phase of carrier constant.

↳ The General equation of FM-signal is given by

$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int m(t)\,dt\right] \quad \text{——} (2)$$

where, K_f = Frequency Sensitivity parameter in Hz/volt.

$m(t)$ = Message signal

• **Phase Modulation:**

It is a process of altering phase of carrier signal in accordance with the instantaneous values of message signal.

↳ The General equation of PM signal is

$$S(t) = A_c \cos\left[2\pi f_c t + K_p m(t)\right] \quad \text{——} (3)$$

Where, K_p = Phase Sensitivity parameter.

1.1 Basic Definitions :-

The most commonly used angle modulation technique is "Frequency Modulation".

Some of the basic definitions with respect to frequency modulation are as follows.

(a) Instantaneous frequency $[f_i(t)]$:-

The instantaneous frequency of FM-signal is mathematically defined as, $\boxed{f_i(t) = f_c + K_f\, m(t)}$ as shown in figure 1.

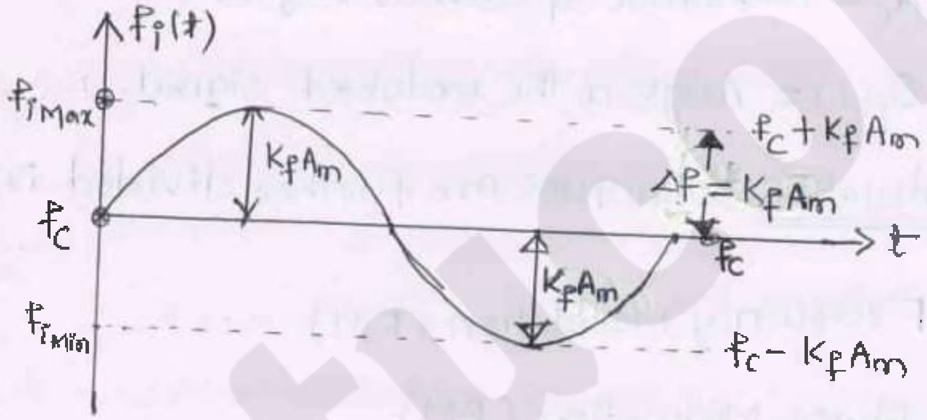

Figure 1: Instantaneous frequency $f_i(t)$ of FM signal for sinusoidal message signal $m(t)$

From figure 1, following observations can be made

↳ When $m(t) = 0$: $f_i(t) = f_c$ → Same as that of unmodulated carrier

↳ At peak value of $m(t)$ i.e., $|m(t)| = A_m$ $\Big\} \Rightarrow f_i(t)_{Max} = f_c + K_f A_m$

∴ Maximum frequency of FM signal is $\boxed{f_i(t)_{Max} = f_c + K_f A_m}$

(b) Angle of FM-signal :- $[\theta_i(t)]$

Instantaneous value, $\theta_i(t)$ of FM signal is related to its instantaneous frequency $f_i(t)$ as follows

$$\boxed{f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\theta_i(t)}$$

Since
$\omega_i(t) = \frac{d}{dt}\theta_i(t)$
⇓
$2\pi f_i(t) = \frac{d}{dt}\theta_i(t)$

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(c) Maximum Frequency deviation $[\Delta f_{max}]$:-

It is the difference between maximum frequency of FM signal to that of unmodulated carrier frequency.

It is denoted by Δf_{max}.

i.e., $\boxed{\Delta f_{max} = k_f A_m}$

Proof: From the definition of frequency deviation,

$$\Delta f_{max} = \text{Max. frequency of FM-signal} - \text{frequency of carrier signal}$$

$$\Delta f_{max} = f_i(t)_{max} - f_c$$

from figure 1, $f_i(t)_{max} = f_c + k_f A_m$

$$\therefore \Delta f_{max} = f_c + k_f A_m - f_c$$

$\boxed{\Delta f_{max} = k_f A_m}$ Hz $\Leftarrow$ Indicated in figure 1.

(d) Carrier Swing :-

It is the difference between maximum and minimum frequencies of FM signal as shown in figure 2.

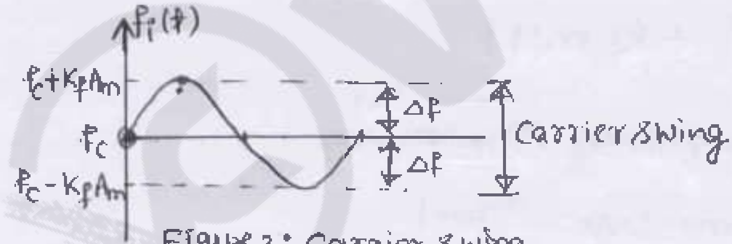


Figure 2: Carrier Swing

$\therefore$ Carrier swing (or) swing in carrier frequency is,

$$\text{Carrier Swing} = f_c + k_f A_m - (f_c - k_f A_m)$$
$$= \cancel{f_c} + k_f A_m - \cancel{f_c} + k_f A_m$$
$$= 2 k_f A_m$$
$$= 2\Delta f_{max}$$

$\therefore$ $\boxed{\text{The Carrier Swing} = f_i(t)_{max} - f_i(t)_{min} = 2 \times \Delta f_{max}}$

(e) Modulation Index : (β)

It is the ratio of maximum frequency deviation to that of frequency of message signal. It is denoted by symbol 'β'.

i.e., Modulation Index in FM $\Rightarrow$ $\boxed{\beta = \dfrac{\Delta f_{max}}{f_m}}$ No units

1.2. Frequency Modulation:-

Q) Define Frequency Modulation. Derive the time domain expression for frequency modulated wave & also sketch necessary waveforms.

→ Frequency Modulation is a process of altering the frequency of carrier signal in accordance with the instantaneous values of message signal by keeping amplitude & phase of carrier constant.

Time domain expression:-

- Let the instantaneous value of carrier signal is

$$c(t) = A_c \cos 2\pi f_c t \longrightarrow (1)$$

- Let the instantaneous value of message signal is

$$m(t) = A_m \cos 2\pi f_c t \longrightarrow (2)$$

- We know that the standard equation of Angle modulated wave is given by,

$$S(t) = A_c \cos \theta_i(t) \longrightarrow (3)$$

where $\theta_i(t)$ = Angle of FM wave (modulated wave)

- We know that the instantaneous frequency $f_i(t)$ of FM signal is given by

$$f_i(t) = f_c + k_f m(t) \quad \text{——} \ (4)$$

where, k_f = frequency sensitivity

$m(t)$ = message signal

- We know that the Angular frequency,

$$w_i(t) = \frac{d}{dt}\theta_i(t)$$

$$\Downarrow$$

$$2\pi f_i(t) = \frac{d}{dt}\theta_i(t)$$

$$\therefore f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\theta_i(t) \quad \text{——} \ (5)$$

Substitute $f_i(t) = f_c + k_f m(t)$ in equation (5) we get,

$$\therefore \quad f_c + K_f m(t) = \frac{1}{2\pi} \frac{d}{dt} \theta_i(t)$$

$$\therefore \frac{d}{dt}\theta_i(t) = 2\pi f_c + 2\pi K_f m(t) \quad \text{—— (6)}$$

⇒ Apply Integral on both sides of equation (6) we get

$$\int \frac{d}{dt}\theta_i(t) = \int [2\pi f_c + 2\pi K_f m(t)]\, dt$$

$$\Downarrow$$

$$\theta_i(t) = 2\pi f_c t + 2\pi K_f \int m(t)\, dt \quad \text{—— (7)}$$

∴ The General equation of FM signal is

$S(t) = A_c \cos \theta_i(t)$ using equation (7)

$$\boxed{S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int m(t)\, dt\right]} \quad \text{—— (8)}$$

Equation (8) is the general equation of FM signal for any message signal $m(t)$.

for, $m(t) = A_m \cos 2\pi f_m t$

$$\int m(t)\, dt = \int A_m \cos 2\pi f_m t\, dt \qquad \left(\because \int \cos mx\, dx = \frac{\sin mx}{m}\right)$$

$$= \frac{A_m}{2\pi f_m} \cdot \sin 2\pi f_m(t) \quad \text{—— (9)}$$

∴

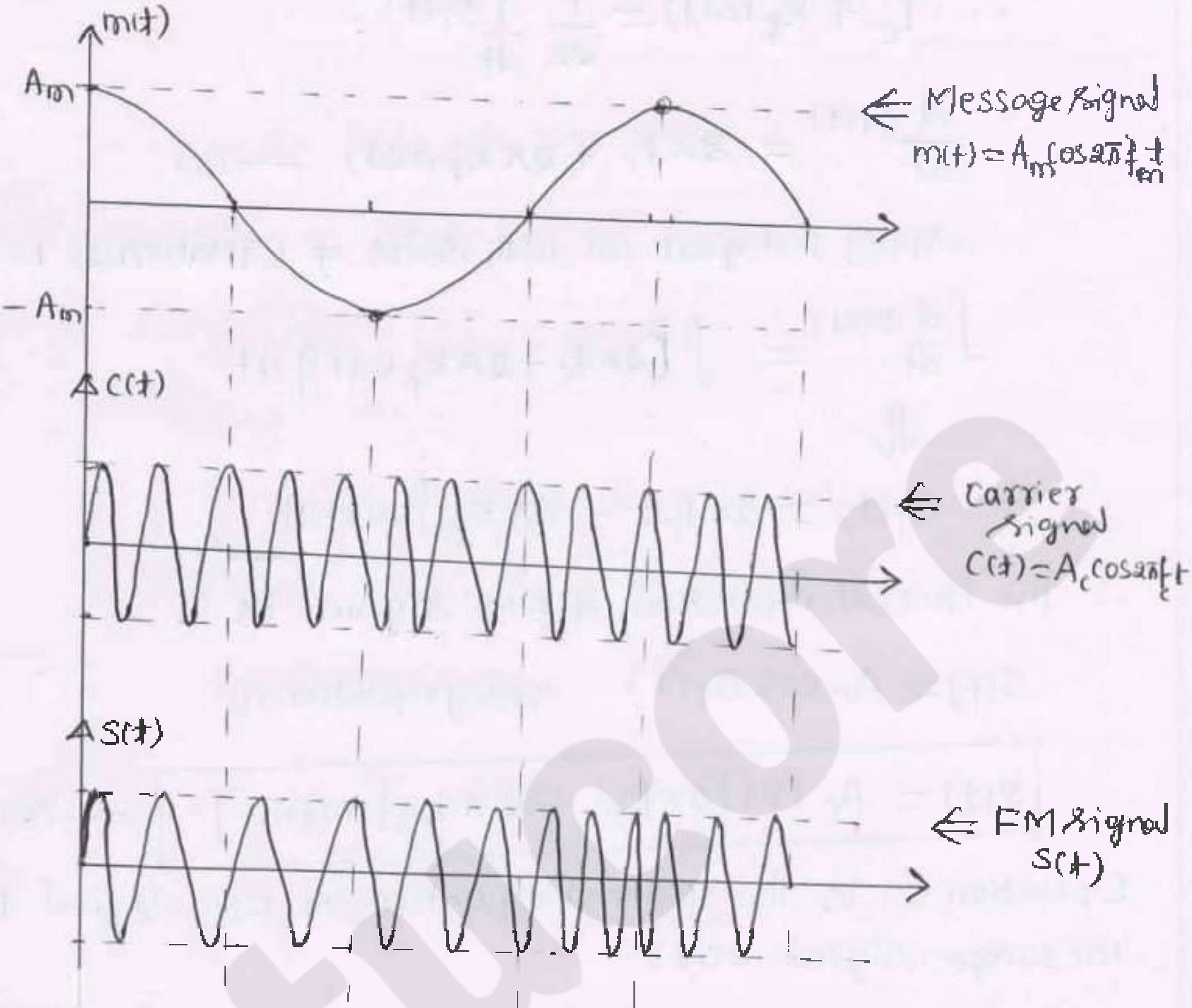
$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \times \frac{A_m}{2\pi f_m} \cdot \sin(2\pi f_m t)\right]$$

$$= A_c \cos\left[2\pi f_c t + \frac{K_f A_m}{f_m} \sin 2\pi f_m t\right]$$

$$\boxed{S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]} \quad \text{—— (10)}$$

Equation (10) is the standard equation of FM signal for $m(t) = A_m \cos 2\pi f_m t$; where $\beta = \frac{K_f A_m}{f_m} = \frac{\Delta f_{max}}{f_m} \leftarrow$ Modulation Index of FM signal.

The required waveforms of $m(t)$, $C(t)$ & $S(t)$ are shown in figure (2).



Figure(2): (a) Message signal m(t) (b) Carrier signal c(t)
(c) Frequency Modulated (FM) signal.

Figure 2(c) shows the time domain representation of FM-signal S(t) shown in equation (10). The frequency of S(t) linearly varies with respect to message signal m(t) i.e., $\left[f_i(t) = f_c + k_f m(t)\right]$.

1.2.1 Phase Modulation :-

- It is a process of altering phase of carrier signal in accordance with the instantaneous values of message signal m(t) by keeping amplitude & frequency of carrier constant.

Time Domain Expression of PM-signal is given by

$$S(t) = A_c \cos \theta_i(t) \quad ; \quad \theta_i(t) = 2\pi f_c t + k_p m(t)$$

$$\therefore \boxed{S(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]}$$ ← Phase Modulated Signal (PM-signal)

Where k_p = phase sensitivity parameter.

1.2.2 : Implementation of phase Modulator using Frequency Modulator and Frequency Modulator using phase Modulator

We know that the standard equation of FM signal for any message signal m(t) is given by

$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\,dt\right] \quad \text{—— (1)}$$

Similarly the standard equation of PM-signal for any message signal m(t) is given by,

$$S(t) = A_c \cos\left[2\pi f_c t + k_p\, m(t)\right] \quad \text{—— (2)}$$

By comparing equations (1) & (2), it is clear that there is a definite possibility of implementing FM-signal using phase Modulator and Vice-Versa.

Case(i) : Implementation of FM using PM :-

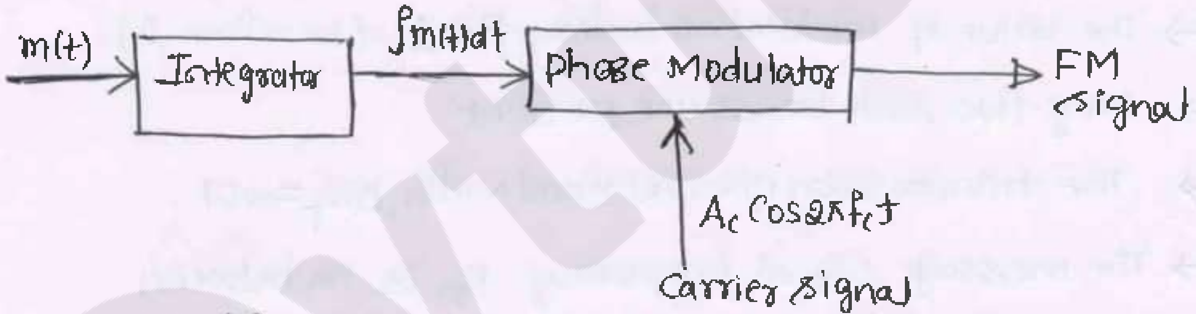


Figure 1: Generation of FM-signal using phase Modulator

↳ By changing phase Modulator input signal to $\int m(t)\,dt$ we can generate FM signal. as shown in figure 1.

Case(ii) : Implementation of PM using FM :-

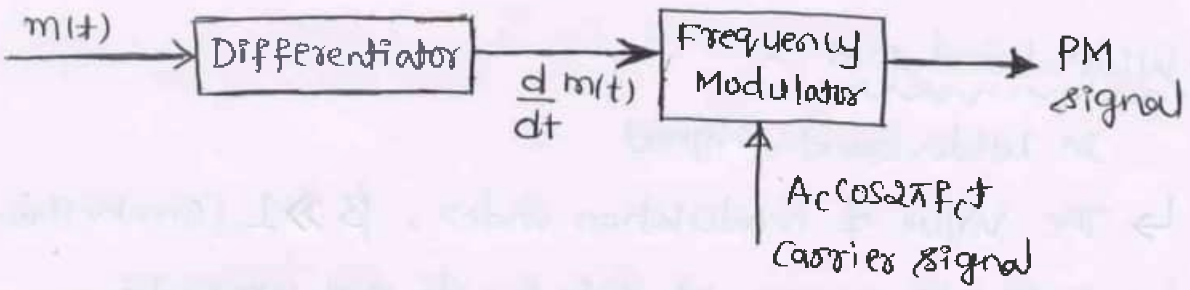


Figure 2 : Generation of PM-signal using FM :-

↳ By changing frequency modulator input signal to $\frac{d}{dt} m(t)$, we can generate PM-signal. as shown in figure(2).

∴ From Figure(1) & Figure(2) it is clear that both PM & FM signals are correlated to each other.

∴ In this module, only Frequency modulation is discussed in detail. If required PM-signal is generated by using Indirect method as shown in figure(2).

1.3. Classification of Frequency modulated Signals:-

Depending on the value of modulation index 'β' and channel Bandwidth FM-signals are classified into two two types

i) Narrow Band FM

ii) Wide Band FM

i) Narrow Band FM:

In Narrow band FM signal

↳ The value of modulation index, $\beta < 1$ (Less than 1)

↳ Only two side bands are present

↳ The transmission channel Bandwidth, $BW_T = 2W$

↳ The message signal frequency, f_m is in between 30HZ to 3KHZ.

↳ Maximum frequency deviation is 15KHZ

Application:- Narrow band FM-technique is mainly used for Speech signal transmission

Example: Mobile communication

ii) Wide-band FM:-

In Wide-band signal

↳ The Value of modulation index, $\beta \gg 1$. (Greater than 1)

↳ Infinite number of side bands are present.

**↳ The message signal frequency, f_m is in between 30HZ to 15KHZ.

*** The Bandwidth of Wide band FM signal can be calculated from Carson's Rule shown in equation (1)

$$\boxed{BW_T = 2f_m + 2\Delta f_{max}} \Leftarrow$$ CARSON's Rule to find Band width of wide band FM-Signal.

↳ The Maximum frequency deviation is 75 KHZ.

Applications of Wide band FM:-

* Wide-band FM technique is mainly used in High Quality Music signal transmission.

  Example: FM-channels

** Comparison between Narrow band FM and Wide band FM:

Parameter	Narrow Band FM	Wide Band FM
1. Modulation Index β	Less than 1	Greater than 1
2. Band width B_T	$2f_m$	$2f_m + 2\Delta f_{max}$
3. Number of side bands	Two	Infinity
4. frequency of Message s/l f_m	30 HZ to 3 KHZ	30 HZ to 15 KHZ
5. Maximum Frequency Deviation	15 KHZ	75 KHZ
6. Application	Speech signal Txn Ex: Mobile Communication	Music signal Txn Ex: FM-stations (or) channels.

3.4 Narrow band Frequency Modulation (In detail)

↳ Narrow band FM signals are characterized by modulation index, β less than 1.

↳ Narrow band FM signal equation can be derived from general FM equation for $m(t) = A_m \cos 2\pi f_m(t)$, as follows

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \quad \text{——} (1)$$

equation (1) is general FM equation for $m(t) = A_m \cos(2\pi f_m t)$ obtained in section 1.2.

$$\text{W.K.T } \cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\therefore (1) \Rightarrow \quad S(t) = A_c \cos 2\pi f_c t \cos(\beta \sin 2\pi f_m t) - A_c \sin(2\pi f_c t) \times \sin(\beta \sin 2\pi f_m t) \quad \longrightarrow (2)$$

For Narrow band FM-signals, $\beta < 1$

$\therefore$ The value of $\beta \sin 2\pi f_m t$ becomes less than 1-degree. and it approaches almost $0°$. Therefore

$$\cos(\beta \sin 2\pi f_m t) \simeq 1 \quad \text{——} (3) \qquad \left(\because \lim_{\theta \to 0°} \cos\theta \simeq 1\right)$$

$$\sin(\beta \sin 2\pi f_m t) \simeq \beta \sin 2\pi f_m t \quad \text{——} (4) \qquad \left(\because \lim_{\theta \to 0°} \sin\theta = \theta\right)$$

By substituting equations (3) & (4) in equation (2) we get Narrow band FM signal

$$\boxed{S(t) = A_c \cos 2\pi f_c t - A_c \beta \cdot \sin 2\pi f_c t \cdot \sin 2\pi f_m t} \quad \text{——} (5)$$

$\therefore$ Narrow band FM signal consists of 3-frequency components

↳ $f_c \Rightarrow$ Carrier signal
$f_c - f_m \Rightarrow$ Lower side band
$f_c + f_m \Rightarrow$ Upper side band

} Same as that of Standard AM signal.

$\therefore$ Total transmission Bandwidth of Narrow band FM $\Rightarrow \boxed{BW_T = 2f_m}$

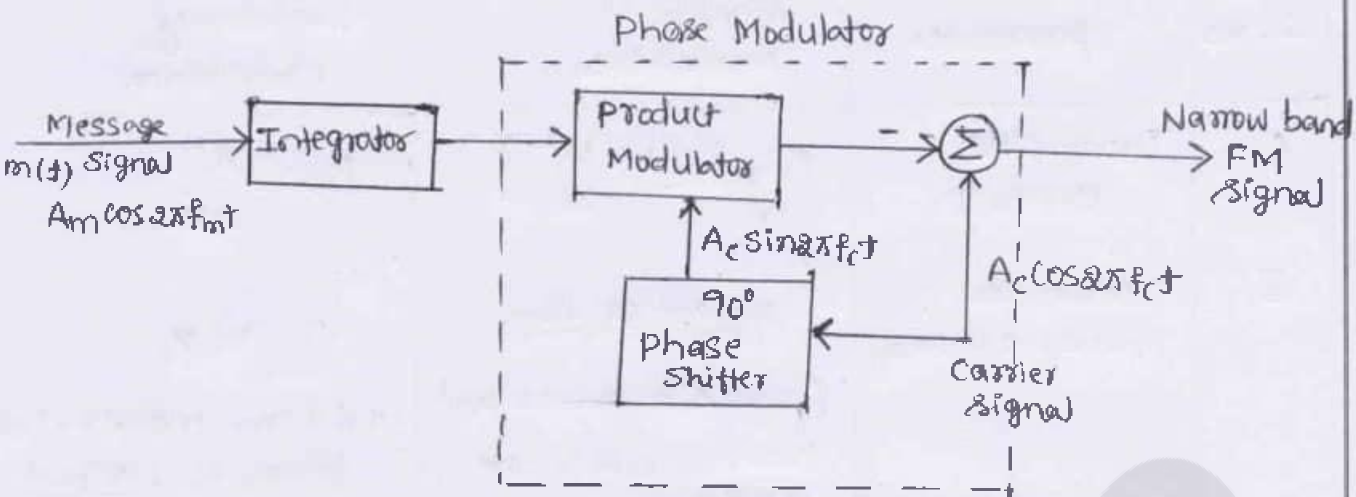


Figure 1: Indirect Method of Generating FM signal (Narrow band) Using phase modulator :-

Figure 1, shows the indirect method of generating Narrow band FM signal shown in figure ⑤, using phase modulator.

Note: The Band width required to transmit Narrow band FM signal is same as that of AM-Signal transmission channel bandwidth "$2f_m$".

* * *

* Comparison between AM and FM :-

Sl. No.	Parameter	Amplitude Modulation	Frequency Modulation
1.	Altering parameter of Carrier	Amplitude	Frequency.
2.	Constant parameter of Carrier	Frequency and Phase	Amplitude, phase
3.	Modulation Index	$\mu = K_a A_m < 1$	$\beta = \frac{\Delta f_{max}}{f_m}$ $\beta < 1$ (Narrow band) $\beta > 1$ (Wide band)

Sl. No	parameter	Amplitude Modulation	Frequency Modulation
4.	Transmitted power	$P_t = P_c\left[1+\frac{\mu_t^2}{2}\right]$	$P_t = \frac{A_c^2}{2R}$
5.	Maximum power efficiency	$\eta_{max} = 33.33\%$ (one side power and Carrier power are wasted)	$\eta_{max} = 100\%$ i.e. All the transmitted power is useful power
6.	Bandwidth	• $BW = 2f_m$ • Band width is Independent of Modu-lation Index	$BW = 2f_m$ Narrowband (or) $BW = 2f_m + 2\Delta f_{max}$ (wide band) • Band width depends on Modulation Index (wide band FM)
7.	Range of Communication	Covers Large Area Ex: Radio	Covers limited Area EX: FM-channels
8.	Complexity	Less Complex	More Complex
9.	Cost	Inexpensive	Expensive
10.	Noise Immunity	Affected by Noise	Immune to Noise.
11.	Types	- DSBSC • SSBSC	• Narrow band • Wide band
12.	Applications	Long distance Communication Ex: Radi	Short distance Communi-cation Ex: FM- stations.

MODULE-2

-: Angle Modulation :-

Numerical problems (VTU Q.P's & Additional problems)

List of Formulae :

1. General Equation of Angle Modulated wave:

$$S(t) = A_c \cos[\theta_i(t)] \quad ; \quad \boxed{P_t = \frac{A_c^2}{2R}}$$

$\Leftarrow$ Total power of FM signal (or) PM signal

2. General equation of FM wave:

$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\, dt\right]$$

for $m(t) = A_m \cos 2\pi f_m t$

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

3. Instantaneous frequency of FM signal :-

$$f_i(t) = \frac{1}{2\pi} \frac{d[\theta_i(t)]}{dt} \quad ; \quad \theta_i(t) = \text{Angle of FM signal.}$$

4. Maximum Frequency Deviation :-

$$\Delta f_{max} = k_f A_m = \beta \times f_m \quad ; \quad k_f = \text{Frequency Sensitivity Parameter.}$$

5. Modulation Index [β] :-

$$\beta = \frac{\Delta f_{max}}{f_m} \quad ; \quad \begin{array}{l} \beta < 1 \Rightarrow \text{Narrow band FM signal} \\ \beta > 1 \Rightarrow \text{Wideband FM signal.} \end{array}$$

6. Carrier Frequency Swing :-

$$\text{Carrier Swing} = 2\Delta f_{max}.$$

7. Band Width :-

- $BW = 2f_m + 2\Delta f_{max} \Longrightarrow$ Wide band FM signal CARSON'S Rule.
- $BW = 2f_m \Longrightarrow$ Narrow band FM signal.

1. The equation for a FM wave is,

$$S(t) = 10 \sin[5.7\times10^8 t + 5 \sin 12\times10^3 t]$$. calculate:

(i) carrier frequency — June/July - 2017

(ii) Modulating frequency

(iii) Modulation Index

(iv) Frequency deviation

(v) power dissipated in 100Ω.

Given data :-

Equation of FM signal,

$$S(t) = 10 \sin[5.7\times10^8 t + 5 \sin 12\times10^3 t] \quad \text{——— (1)}$$

W.K.T. Standard equation of FM signal,

$$S(t) = A_c \sin[2\pi f_c t + \beta \cdot \sin 2\pi f_m t] \quad \text{——— (2)}$$

By comparing (1) and (2) we get

$2\pi f_c = 5.7\times10^8$; $\beta = 5$; $2\pi f_m = 12\times10^3$; $A_c = 10V$.

$\therefore$

(i) Carrier frequency : $f_c = \frac{5.7\times10^8}{2\pi} = 90.718\times10^6$ HZ.

(ii) Modulating frequency : $f_m = \frac{12\times10^3}{2\pi} = 1.91\times10^3$ HZ.

(iii) Modulation Index : $\beta = 5$ $\therefore$ S(t) is Wide band FM s/g since $\beta > 1$.

(iv) Frequency deviation : $[\Delta f_{max}]$

W.K.T. $\beta = \frac{\Delta f_{max}}{f_m}$

$$\therefore \Delta f_{max} = \beta \times f_m = 5 \times 1.91\times10^3 = 9.55\times10^3 \text{ HZ}$$

(v) power dissipated in 100Ω :

$$P_t = \frac{A_c^2}{2R} = \frac{10^2}{2\times100} = \frac{100}{2\times100} = 0.5 \text{ W}$$

2. When a 50.4 MHZ Carrier is frequency modulated by a Sinusoidal AF modulating signal, the highest frequency reached is 50.405 MHZ. Calculate Dec 2016/Jan 2017
   (i) The Frequency deviation produced
   (ii) Carrier Swing of the wave.
   (iii) Lowest frequency reached

↳ Given data: $f_c = 50.4\,MHZ$.

highest frequency of FM: $f_i(t)_{max} = 50.405\,MHZ$

(i) The Frequency deviation produced:

W.K.T $f_i(t)_{Max} = f_c + \Delta f_{max}$

$$\therefore \Delta f_{max} = f_i(t)_{Max} - f_c = 50.405\times10^6 - 50.4\times10^6$$

$$\Delta f_{max} = 0.005\times10^6 = 5000 = 5\,KHZ$$

(ii) Carrier Swing: W.K.T Carrier Swing $= 2\times\Delta f_{max}$

$$= 2\times5\,KHZ$$

$$= 10\,KHZ$$

(iii) Lowest frequency reached:

$$f_i(t)_{min} = f_c - \Delta f_{max}$$

$$= 50.4\times10^6 - 5\times10^3$$

$$f_i(t)_{Min} = \underline{\underline{50.395\,MHZ}}$$

Note:

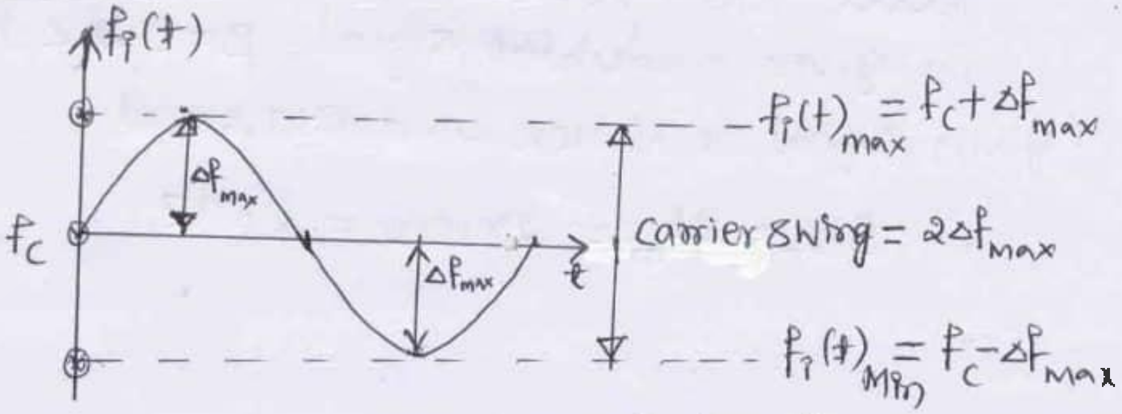


Figure: $f_i(t)$ of FM wave showing its Maximum, Minimum frequency & Carrier Swing

3) An angle modulated signal is defined by,

$s(t) = 10\cos[2\pi\times10^6 t + 0.2\sin 200\pi t]$ volts find the following:

June/July-2016

i) power in modulated signal

ii) The Frequency deviation, Δf

iii) The Approximate transmission bandwidth

↳ Given data: The Angle modulated signal (FM signal)

$$s(t) = 10\cos[2\pi\times10^6 t + 0.2\sin 2000\pi t] \quad \text{--- (1)}$$

W.K.T the standard equation of FM signal is

$$s(t) = A_c\cos[2\pi f_c t + \beta\sin 2\pi f_m t] \quad \text{--- (2)}$$

by Comparing (1) & (2) We get

$A_c = 10V$; $2\pi f_c = 2\pi\times10^6$; $\beta = 0.2$; $2\pi f_m = 2000\pi$

$\Rightarrow f_c = 1\times10^6$ HZ $\qquad \Rightarrow f_m = 1000$ HZ

i) Power in $s(t)$: $P_t = \frac{A_c^2}{2R}$; Take $R = 1\Omega$ ($\because$ Not given default value is $R = 1\Omega$)

$$P_t = \frac{10^2}{2\times1}$$

$$P_t = 50\,W$$

ii) Frequency deviation (Δf_{max}):-

$$\Delta f_{max} = \beta\times f_m = 0.2\times1000 = 200\,HZ.$$

iii) Transmission Bandwidth:-

for given modulated signal, $\beta = 0.2 < 1$ $\therefore$ the given signal is Narrow band FM signal.

$$\therefore BW_T = 2f_m = 2\times1000 = 2\,KHZ$$

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4. An FM wave is defined by $s(t) = 10\cos[2+\sin 6\pi t]$. Find the instantaneous frequency of $s(t)$

Dec-2014/ Jan-2015

↳ Given FM signal is

$$s(t) = 10\cos[2+\sin 6\pi t] \quad \text{--- (1)}$$

The instantaneous frequency of $s(t)$ is given by

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}[\theta_i(t)]$$

for given signal Angle, $\theta_i(t) = 2+\sin 6\pi t$

$$\therefore f_i(t) = \frac{1}{2\pi}\cdot\frac{d}{dt}[2+\sin 6\pi t]$$

$$= \frac{1}{2\pi}[0+6\pi\cos 6\pi t] \qquad \left(\because \frac{d}{dx}\sin mx = m\cdot\cos mx\right.$$

$$= \frac{6\pi\cdot\cos 6\pi t}{2\pi}$$

$$\boxed{f_i(t) = 3\cos 6\pi t}$$

5. A sinusoidal modulating wave of amplitude 5V and frequency 1KHZ is applied to a frequency modulator. The frequency sensitivity of the modulator is 50 Hz/volt. The carrier frequency is 100KHZ. Calculate (i) The frequency deviation (ii) Modulation Index.

↳ Given data: $A_m = 5V$; $f_m = 1KHZ$; $K_f = 50$ Hz/volt. $f_c = 100KHZ$.

(i) Frequency deviation: $\Delta f_{max} = K_f \times A_m = 50\times 5 = 250Hz$.

(ii) Modulation Index: $\beta = \frac{\Delta f_{max}}{f_m} = \frac{250}{1\times 10^3} = 0.25$

6) In an FM system, when the audio frequency is 500 HZ and modulating voltage 2.5V, the deviation produced is 5 KHZ. If the modulating voltage is increased to 7.5V, calculate the new value of frequency deviation. Calculate the modulation index in each case.

↳ Given data: $f_m = 500\,HZ$; $A_m = 2.5\,V$; $\Delta f = 5\,KHZ$.

$$\therefore \Delta f = k_f A_m$$

$$\Rightarrow k_f = \frac{\Delta f}{A_m} = \frac{5\times10^3}{2.5} = 2\times10^3 = 2\,KHZ$$

$$\Rightarrow \beta = \frac{\Delta f}{f_m} = \frac{5\times10^3}{500} = \underline{\underline{10}}$$

Case(ii): If $A_m = 7.5\,V$; $\Delta f = ?$; $\beta = ?$

$$\Delta f = k_f A_m = 2\times10^3 \times 7.5 = 15\times10^3 = 15\,KHZ.$$

$$\beta = \frac{\Delta f}{f_m} = \frac{15\times10^3}{500} = 30$$

7) The carrier swing of a frequency modulated signal is 70KHZ and the modulating signal is 7KHZ sine wave. Determine the modulation index of FM signal and Bandwidth.

↳ Given data: Carrier swing $= 70\,KHZ$; $f_m = 7\,KHZ$; $\beta = ?$

W.K.T the Modulation Index,

$$\beta = \frac{\Delta f_{max}}{f_m}$$

Carrier Swing $\Rightarrow 2\Delta f_{max} = 70\,KHZ$

$$\therefore \Delta f_{max} = \frac{70\,KHZ}{2} = 35\,KHZ$$

$$\therefore \boxed{\beta = \frac{35\,KHZ}{7\,KHZ} = 5} > 1$$ ∴ It is wide Band FM signal

∴ Using Carson's rule, $BW_T = 2f_m + 2\Delta f_{max} = 2\times7\,KHZ + 2\times35\,KHZ$
$= 14\,KHZ + 70\,KHZ$
$= 84\,KHZ$

* Transmission Bandwidth of FM-Signals:- [Carson's Rule]

we know that there are two-types of FM-signals,

⟨i⟩ Narrow-Band FM-signal ($\beta < 1$)

⟨ii⟩ Wide-band FM-signal. ($\beta \gg 1$)

The Approximate formula for finding transmission bandwidth ⟨B_T⟩ of FM-signals is given by "Carson's Rule"

According to Carson's formula (or rule) the B_T of FM-signal is given by

$$\langle i \rangle \quad B_T = 2f_m + 2\Delta f = 2f_m\left(1 + \frac{\Delta f}{f_m}\right) = 2f_m(1+D)$$

where D = deviation ratio $= \frac{\Delta f}{f_m}$

i.e., $B_T = 2f_m + 2\Delta f = 2f_m(1+D)$ for WBFM-signals ($\beta \gg 1$)

⟨ii⟩ if $\beta < 1$ for NBFM,

$$\boxed{B_T = 2f_m}$$

i.e., According to Carson's rule, BW of FM-signal is

$$BW_T = \begin{cases} 2f_m + 2\Delta f = 2f_m(1+D) & ; \beta \gg 1 \text{ (WBFM signals)} \\ 2f_m & ; \beta < 1 \text{ (NBFM signals)} \end{cases}$$

Note: The Deviation ratio is same as that of modulation index of FM

i.e., $\beta = \frac{\Delta f}{f_m} = D \hookrightarrow$ Deviation ration

3.5 Generation of FM-Waves :-

There are two basic methods of generating FM-waves.

<i> Direct Method.

<ii> Indirect Method <Armstrong Modulator>

*** IMP **

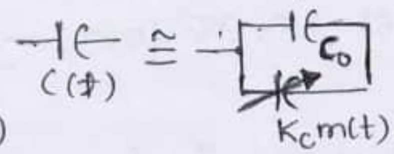
<i> Generation of frequency modulated signal using DIRECT-METHOD :-

<Q> Explain generation of frequency modulated signal using direct method. V.T.U June/July-2017 (5M)

↳ The Direct method uses a sinusoidal oscillator, with one of the reactive elements (example: Capacitive element) in the tank circuit of the oscillator being directly controlled by the message signal, m(t).

↳ In direct method of FM-signal generation, the instantaneous frequency of the carrier wave is varied directly in accordance with the message signal.

↳ Fig.1, shows a Hartly oscillator in which the Capacitive component of the tank circuit is,



$$C(t) = C_0 + K_c m(t) \quad \text{——— } (*)$$

where, C_0 = Total Capacitance in the absence of modulation.

K_c = Variable Capacitor Sensitivity to voltage change.

$m(t)$ = message signal = $A_m \cos(2\pi f_m t)$.

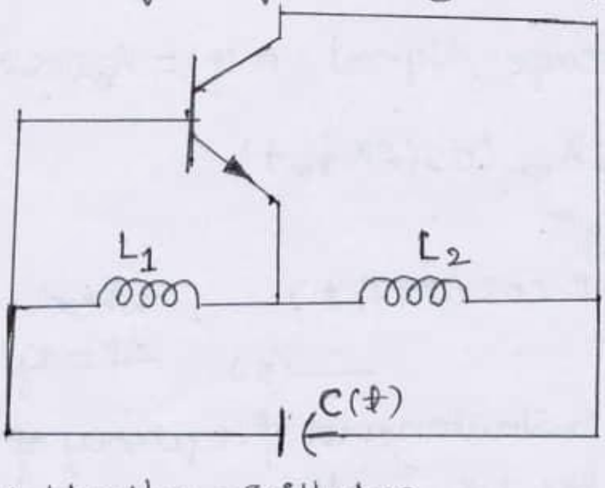


Fig.1: Hartley oscillator

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The frequency of the Hartley oscillator is given by

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,c(t)}} \quad : \text{where } c(t) = C_0 + K_c m(t)$$

$$\therefore \quad f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)[C_0 + K_c m(t)]}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_0\left[1+\frac{K_c}{C_0}m(t)\right]}}$$

$$f_i(t) = \frac{f_0}{\sqrt{1+\frac{K_c}{C_0}m(t)}} \quad : \text{where } f_0 = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_0}}$$

$$f_i(t) = f_0\left(1+\frac{K_c}{C_0}\cdot m(t)\right)^{-1/2} \qquad \text{--- (1)}$$

using Binomial theorem, $(1+x)^{-1/2} = \left(1-\frac{x}{2}\right)$

$$\therefore \quad \left(1+\frac{K_c}{C_0}m(t)\right)^{-1/2} = \left(1-\frac{K_c\,m(t)}{2C_0}\right) \qquad \text{--- (2)}$$

using equation (2) in (1) we get

$$f_i(t) = f_0\left(1-\frac{K_c\,m(t)}{2C_0}\right) \qquad \text{--- (3)}$$

Let us assume, $\frac{-K_c}{2C_0} = \frac{K_f}{f_0}$, where K_f = frequency Sensitivity Parameter.

$$\therefore \quad f_i(t) = f_0\left(1+\frac{K_f}{f_0}\cdot m(t)\right)$$

$$f_i(t) = f_0 + K_f\, m(t) \qquad \text{--- (4)}$$

for Sinusoidal message signal, $m(t) = A_m \cos 2\pi f_m t$

$$\therefore \quad f_i(t) = f_0 + \underbrace{K_f A_m}_{\Downarrow}\cdot\cos(2\pi f_m t)$$

$$\therefore \quad f_i(t) = f_0 + \Delta f\cos(2\pi f_m t) \qquad \text{--- (5)}$$

: where $\Delta f = K_f A_m$ = Maximum frequency deviation.

Equation (5) gives, the instantaneous frequency of FM-Wave generated by using direct method.

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↳ Therefore, the direct method is stright forward to implement and is capable of providing large frequency deviation (Δf).

↳ One of the major limitation of the direct method is," the carrier frequency is not obtained from a highly stable oscillator.

↳ To overcome, this limitation a closed loop feedback system for the carrier frequency stabilization is used to provide frequency stabilized FM wave. This arrangement is shown in Fig. 2.

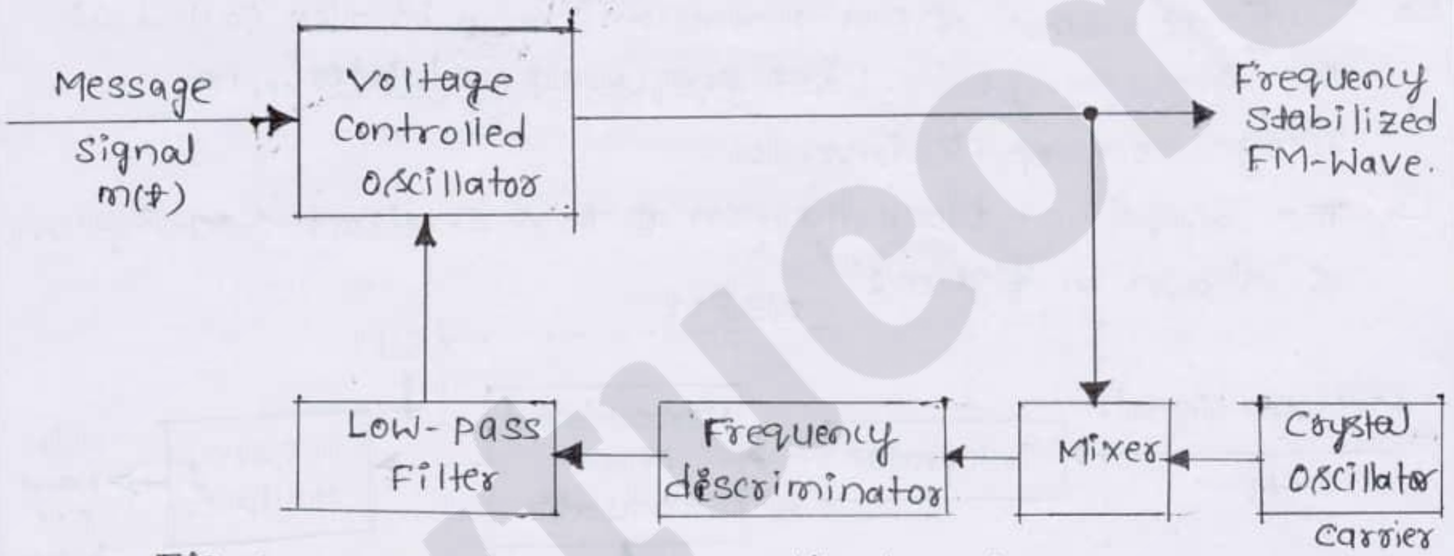

Fig. 2: Carrier frequency stabilization of Direct method FM-generation

↳ Fig. 2 consists of crystal oscillator, mixer, frequency descriminator, Low-pass filter and Voltage Controlled oscillator (VCO).

↳ This configuration provides

- Good frequency stability
- Required frequency deviation to generate WBFM.
- Constant proportionality between output frequency change to input voltage change.

∴ the required WBFM-Wave is obtained.

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<ii> Indirect Method (or) Armstrong Modulator :-

<Q> Explain the generation of FM-wave using Indirect (or) Armstrong method.

↳ In the indirect method, the message signal is first used to produce Narrow-band FM, which is followed by frequency multiplier to increase the frequency deviation to the desired level.

↳ The frequency multiplier produces Wide-band FM wave.

↳ Indirect method of FM-generation scheme is also called as the "Armstrong wide-band-frequency modulator", in recognition of its inventor.

↳ The simplified block diagram of this Indirect FM-System is shown in figure 1.

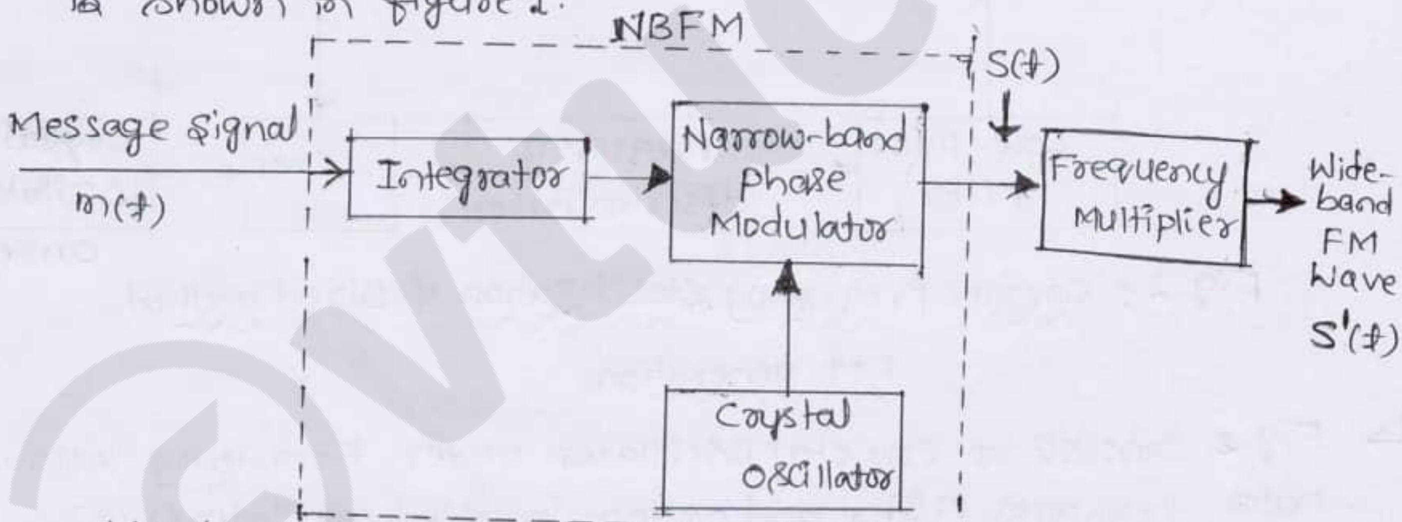


Fig. 1: Block diagram of the indirect method of generating a wide-band FM-wave

↳ In Fig. 1, the message signal m(t) is first integrated and then used to phase-modulate a carrier wave generated by crystal oscillator. which results in a NBFM (Narrow band FM-wave) S(t) with carrier frequency 'f_c' and modulation index '$\beta<1$', as shown in equation 1.

$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int m(t)\,dt\right] \quad (1)$$

↳ The use of crystal oscillator provides frequency stability.

↳ The Narrowband FM-wave is next multiplied in frequency by using Frequency multiplier.

↳ Frequency multiplier produces the required Wide band FM Wave, $s'(t)$ as shown in Fig.2.

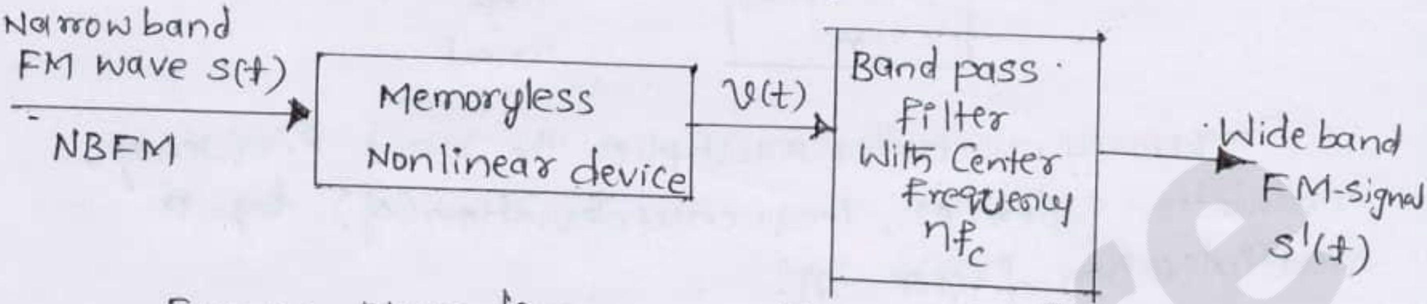


Fig.2: Block diagram of frequency multiplier

↳ A frequency multiplier consists of a memoryless non-linear device followed by a band pass filter having center frequency "$n \times f_c$".

↳ The output voltage of memoryless non-linear device is

$$v(t) = a_1 s(t) + a_2 s^2(t) + a_3 s^3(t) + \cdots + a_n s^n(t) \qquad (2)$$

where, $s(t)$ = Narrowband FM-signal with carrier frequency f_c and modulation index 'β'.

i.e., $s(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int m(t)dt]$ and its instantaneous frequency is,

$$f_i(t) = f_c + k_f m(t)$$

↳ when $v(t)$, the output of non-linear device is passed through a BPF having center frequency "$n \times f_c$", we get required wideband FM-signal $s'(t)$.

$$s'(t) = A_c \cos[2\pi f'_c t + 2\pi k'_f \int m(t)dt] \Rightarrow \text{required WBFM signal} \qquad (3)$$

where $f'_c = nf_c$ and $k'_f = n \times k_f$ & $\beta' = n \times \beta$

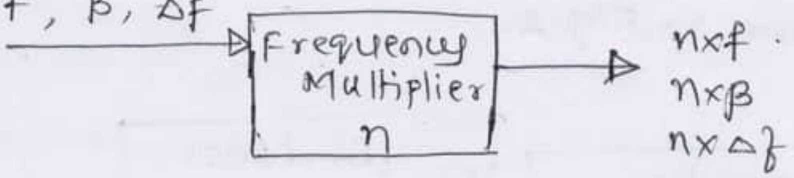
its instantaneous frequency is

$$f'_i(t) = f'_c + k'_f m(t).$$

∴ Required WBFM is obtained.

Problems on NBFM & WBFM with Frequency multipliers:- 24

Note:- Frequency multiplier,



∴ Frequency multiplier, multiplies the input frequency, modulation index (β), frequency deviation (Δf) by a multiplication factor "n".

1. A block diagram of FM transmitter is shown in Fig. 1. Compute the maximum frequency deviation Δf of the output of the FM transmitter and the carrier frequency, f_c. If $f_1 = 200\,KHZ$, $f_{LO} = 10.8\,MHZ$, $\Delta f_1 = 25\,HZ$, $n_1 = 64$ and $n_2 = 48$.

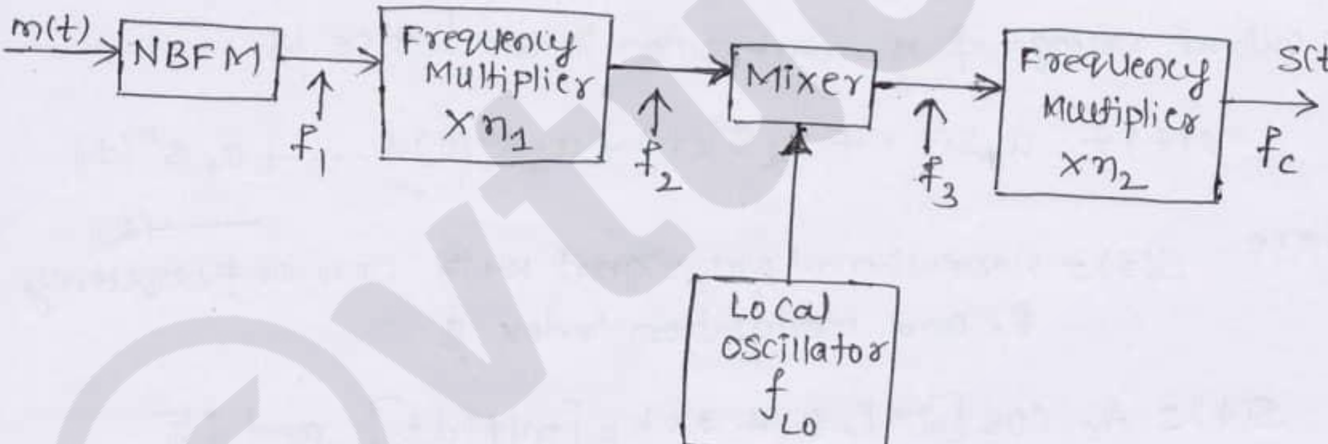


Fig1: FM-transmitter:-

↳ Solution:-

Given:-

$f_1 = 200\,kHZ$

$\Delta f_1 = 25\,HZ$.

$f_{LO} = 10.8\,MHZ$

$n_1 = 64$

$n_2 = 48$

$f_c = ?$

$\Delta f = ?$

↳ The Maximum frequency deviation 'Δf' at the output:-

$$\Delta f' = n_1 \times n_2 \times \Delta f_1 = 64 \times 48 \times 25 = \underline{76.8\,KHZ}$$

↳ The Carrier frequency "f_c" at the output:-

from the block diagram, the carrier frequency

$$f_c = n_2 \times f_3 \quad \text{---(1)}$$

$$f_3 = f_2 \pm f_{LO} = n_1 \times f_1 \pm f_{LO} = (64 \times 200 \times 10^3 \pm 10.8 \times 10^6)$$

$$f_3 = 23.6\,MHZ \text{ to } 2\,MHZ$$

$$\therefore f_c = n_2 \times f_3 = 48\,(23.6\,MHZ \text{ to } 2\,MHZ) = 1132.8\,MHZ \text{ to } 47.2\,MHZ$$

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2. For a Wideband Frequency modulator, if a narrowband carrier, $f_1 = 0.1$ MHZ, Second carrier $f_2 = 9.5$ MHZ, output carrier frequency = 100 MHZ and $\Delta f = 75$ KHZ. Calculate the multiplying factors n_1 and n_2 if NBFM frequency deviation is 20HZ. Draw the suitable block diagram of the modulator. (6-Marks) VTU Q.P.

Given:-

output carrier, $f_C = 100$ MHZ

overall frequency deviation

$\Delta f = 75$ KHZ

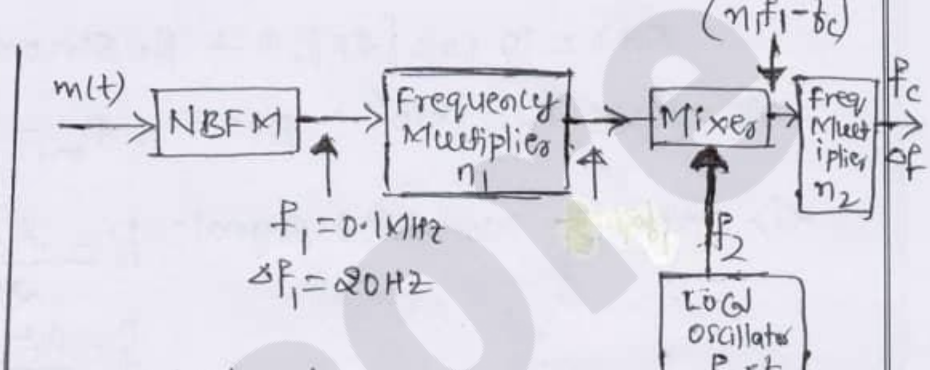
$f_1 = 0.1$ MHZ (carrier1)

$f_2 = 9.5$ MHZ ← Local oscillator frequency for Mixer

$n_1 = ?$

$n_2 = ?$

$\Delta f_1 = 20$ HZ ← NBFM freq. deviation



— : Block diagram :-

from given data the output frequency deviation is

$$\Delta f = n_1 \times n_2 \times \Delta f_1$$

$$\therefore n_1 \times n_2 = \frac{\Delta f}{\Delta f_1} = \frac{75\text{ KHZ}}{20\text{ HZ}} = 3.75 \times 10^3$$

$$\therefore \boxed{n_1 n_2 = 3750}$$

The o/p frequency of frequency Multiplier1 $\} = n_1 \times f_1$

The o/p frequency of Mixer is $\} = (n_1 f_1) - f_2$ [Assuming Lower frequency $\because f_{IF} = f_{RF} - f_{LO}$]

Frequency Multiplier2 o/p frequency $\} = f_C = n_2$ (output frequency of Mixer)

$$\therefore f_C = n_2 (n_1 f_1 - f_2)$$

$$f_C = n_1 n_2 f_1 - n_2 f_2 \quad \left(\text{W.K.T } n_1 n_2 = 3750\right)$$

$$(100 \times 10^6) = 3750 \times 0.1 \times 10^6 - n_2 \times 9.5 \times 10^6$$

$$\therefore n_2 = \frac{3750 \times 0.1 \times 10^6 - 100 \times 10^6}{9.5 \times 10^6} = \underline{28.95}$$

$$\therefore n_1 = \frac{3750}{n_2} = \frac{3750}{28.95} = 129.54$$

*** (3) An angle modulated signal is represented by 26.

$s(t) = 10\cos[2\pi\times10^6 t + 5\sin 2000\pi t + 10\sin 3000\pi t]$ Vol5

Find the following:

(i) power in the modulated signal (ii) Frequency deviation

(iii) Deviation Ratio (iv) phase deviation (v) Transmission Bandwidth.

Given: $s(t) = 10\cos[2\pi\times10^6 t + 5\sin 2000\pi t + 10\sin 3000\pi t]$

$s(t) = 10\cos[2\pi f_c t + \beta_1 \sin 2\pi f_{m_1} t + \beta_2 \sin 2\pi f_{m_2} t]$

$\therefore A_c = 10V ; f_c = 1\times10^6 ; \beta_1 = 5 ; f_{m_1} = 1000Hz ; \beta_2 = 10 ; f_{m_2} = 1500Hz$

(i) power in Modulated signal: $P_t = \frac{A_c^2}{2R}$ Let $\boxed{R = 1\Omega}$ ← Normalized Value.

$P_t = \frac{A_c^2}{2} = \frac{10^2}{2} = \frac{100}{2} = 50W$ //

(ii) Frequency deviation: "Δf"

W.K.T $\Delta f = \beta \times f_m$ ← for single message signal.

given modulated signal $s(t)$ consists of 2-message signals with $\beta_1 = 5 ; f_{m_1} = 1000Hz$ & $\beta_2 = 10 ; f_{m_2} = 1500Hz$.

$\therefore \Delta f = \beta_1 f_{m_1} + \beta_2 f_{m_2} = 5\times1000 + 10\times1500$

$\Delta f = 5000 + 15{,}000 = 20{,}000$ Hz.

$\boxed{\Delta f = 20KHz}$

(iii) Deviation Ratio (Modulation Index):-

$\beta = D = \frac{\Delta f}{f_m} = \frac{\Delta f}{W}$; where $W = \max(f_{m_1}, f_{m_2})$

$W = \max(1000, 1500)$

$W = 1500$

$\therefore \beta = D = \frac{20K}{1.5K} = 13.333$

(iv) phase Deviation:-

$\Delta\theta = |\theta_i(t) - \theta_c|_{max}$

$\Delta\theta = |5\sin 2000\pi t + 10\sin 3000\pi t|_{max}$

$\Delta\theta = 5 + 10 = 15$ radians

(v) Transmission Bandwidth:-

$BW = 2\Delta f + 2f_m = 2\Delta f + 2W = 2\times20K + 2(1.5K)$

$BW = 43KHz$ //

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3.6 FM-STEREO MULTIPLEXING:

Stereo Multiplexing is a form of frequency division multiplexing (FDM) designed to transmit two separate signals [$m_R(t)$ and $m_L(t)$] via the same carrier.

FM-stereo system consists of

a) FM-stereo transmitter

b) FM-stero Receiver.

FM-stereo transmitter:-

- Let $m_L(t)$ and $m_R(t)$ denote the two message signals picked up by Left hand and Right hand microphones at the transmitting end of the system, as shown in Fig. 1.
- It uses a pilot carrier frequency, $f_c = 19\,KHZ$. Frequency doubler produces, sub carrier, $\cos(4\pi f_c t)$.

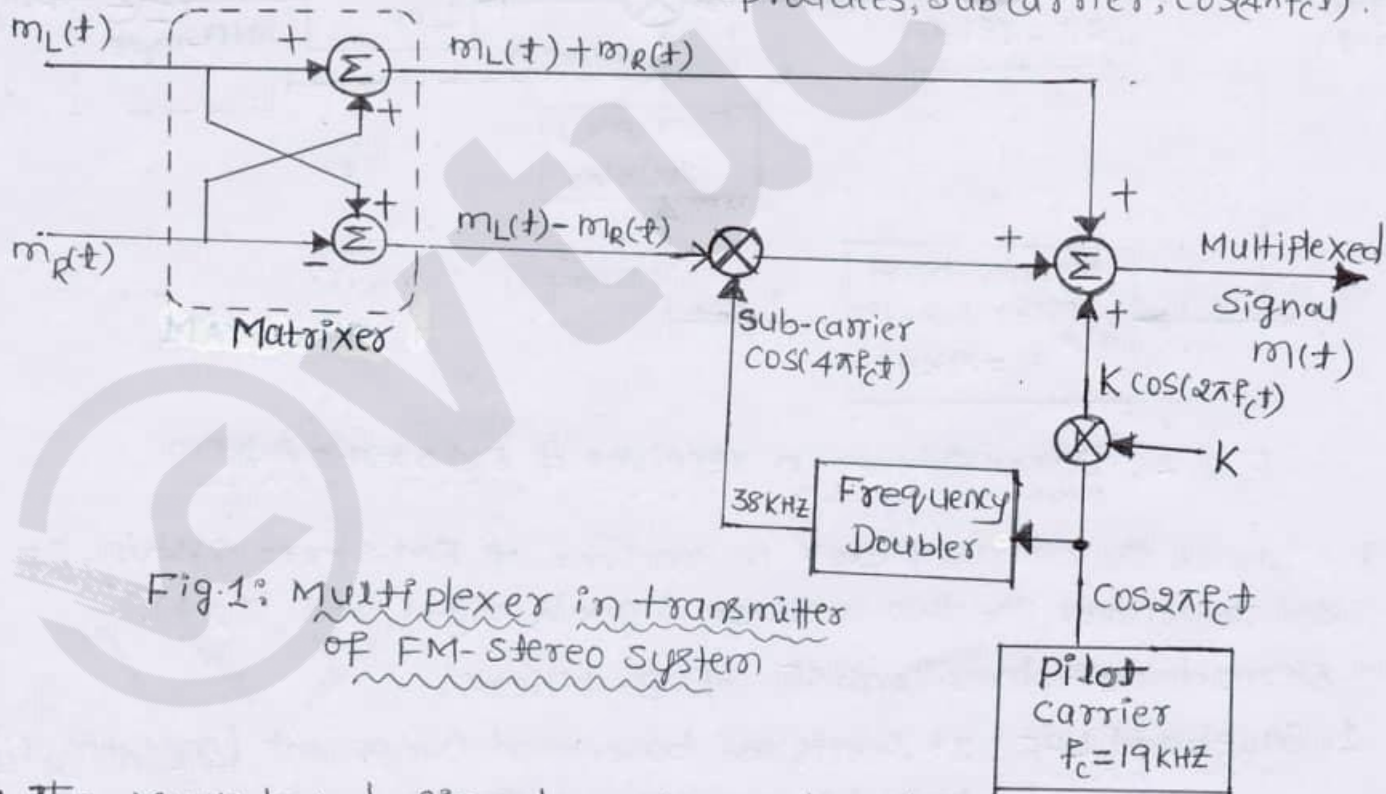

Fig 1: Multiplexer in transmitter of FM-stereo system

- The Multiplexed signal $m(t)$, produced at the output of multiplexer in transmitter of FM stereo system is,

$$m(t) = \underbrace{[m_L(t)+m_R(t)]}_{\substack{\text{Baseband}\\ \text{signal } ①}} + \underbrace{[m_L(t)-m_R(t)]\cos 4\pi f_c t}_{\substack{\text{DSBSC } ②\\ \text{signal}\\ 2f_c = 38KHZ}} + \underbrace{K\cos(2\pi f_c t)}_{\substack{\text{Pilot } ③\\ \text{carrier signal.}\\ f_c = 19KHZ}} \quad \text{--- (1)}$$

- Multiplexed signal, $m(t)$ consists of three different signals. (28)

1. $[m_L(t)+m_R(t)] \Rightarrow$ sum of $m_L(t)$, $m_R(t)$ generated by the simple matrixer. It is baseband signal.
2. $[m_L(t)-m_R(t)]\cos(4\pi f_c t) \Rightarrow$ DSBSC-signal produced by the product modulator.
3. $K.\cos(2\pi f_c t) \Rightarrow$ pilot carrier signal multiplied by a constant 'K'.

FM-Stereo Receiver :-

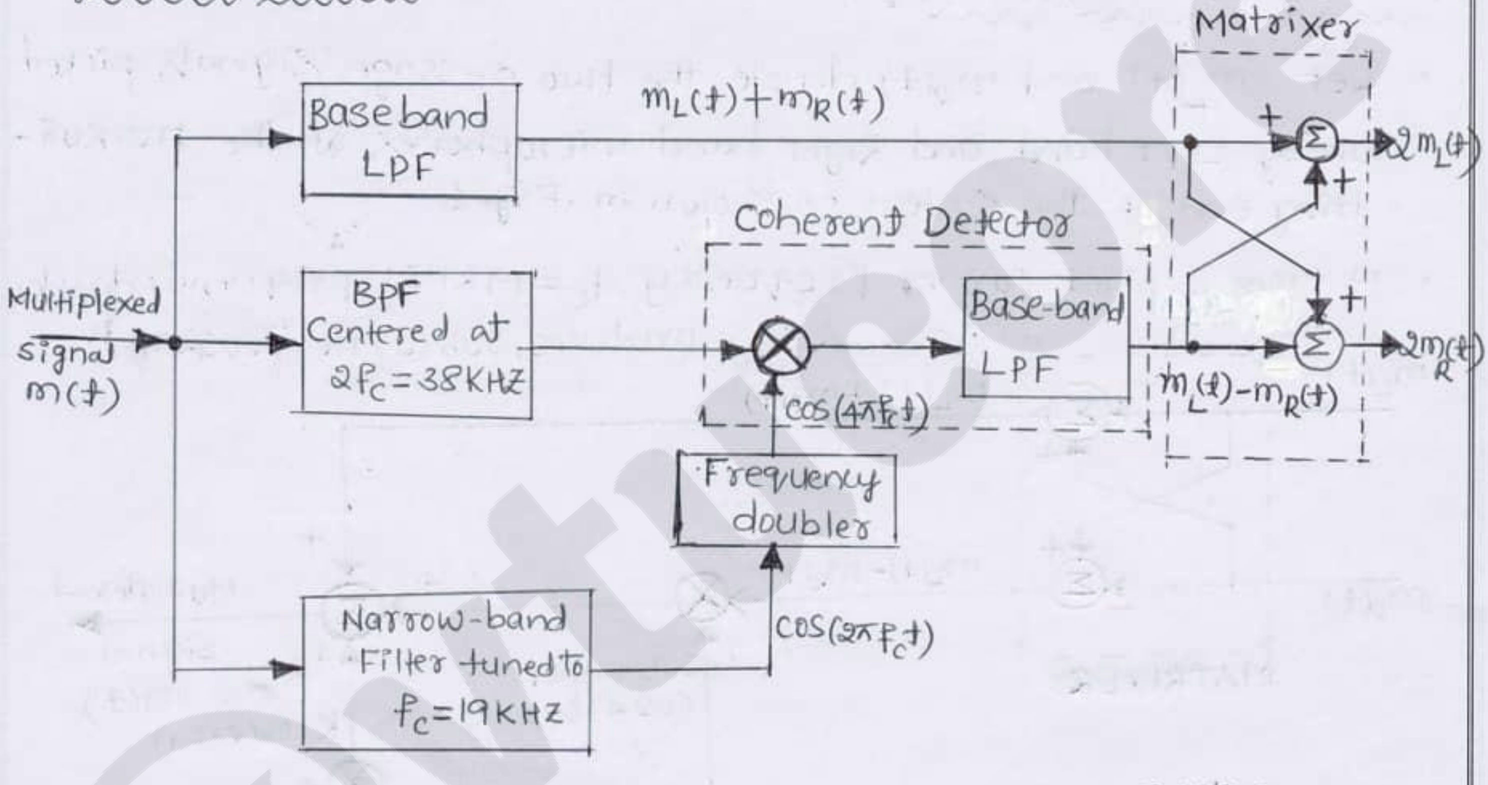

Fig. 2 : Demultiplexer in receiver of FM-stereo system

- Fig. 2. shows the demultiplexer in receiver of FM-stereo system. It is used to recover the two message signals $m_L(t)$ and $m_R(t)$.
- FM-stereo demultiplexer consists of 3-filters,

1. Baseband LPF : It selects the base-band component $[m_L(t)+m_R(t)]$ present in multiplexed signal $m(t)$.
2. BPF : (Bandpass filter) :- It selects the DSBSC-signal.
3. Narrowband filter:- It selects the pilot carrier signal, $\cos(2\pi f_c t)$.

- Frequency doubler produces the required subcarrier signal, $\cos(4\pi f_c t)$ for coherent detection of DSBSC-signal.
- Coherent Detector, recovers the difference signal $[m_L(t)-m_R(t)]$.
- Finally the Matrixer, produces the required signals $2m_L(t)$ and $2m_R(t)$.

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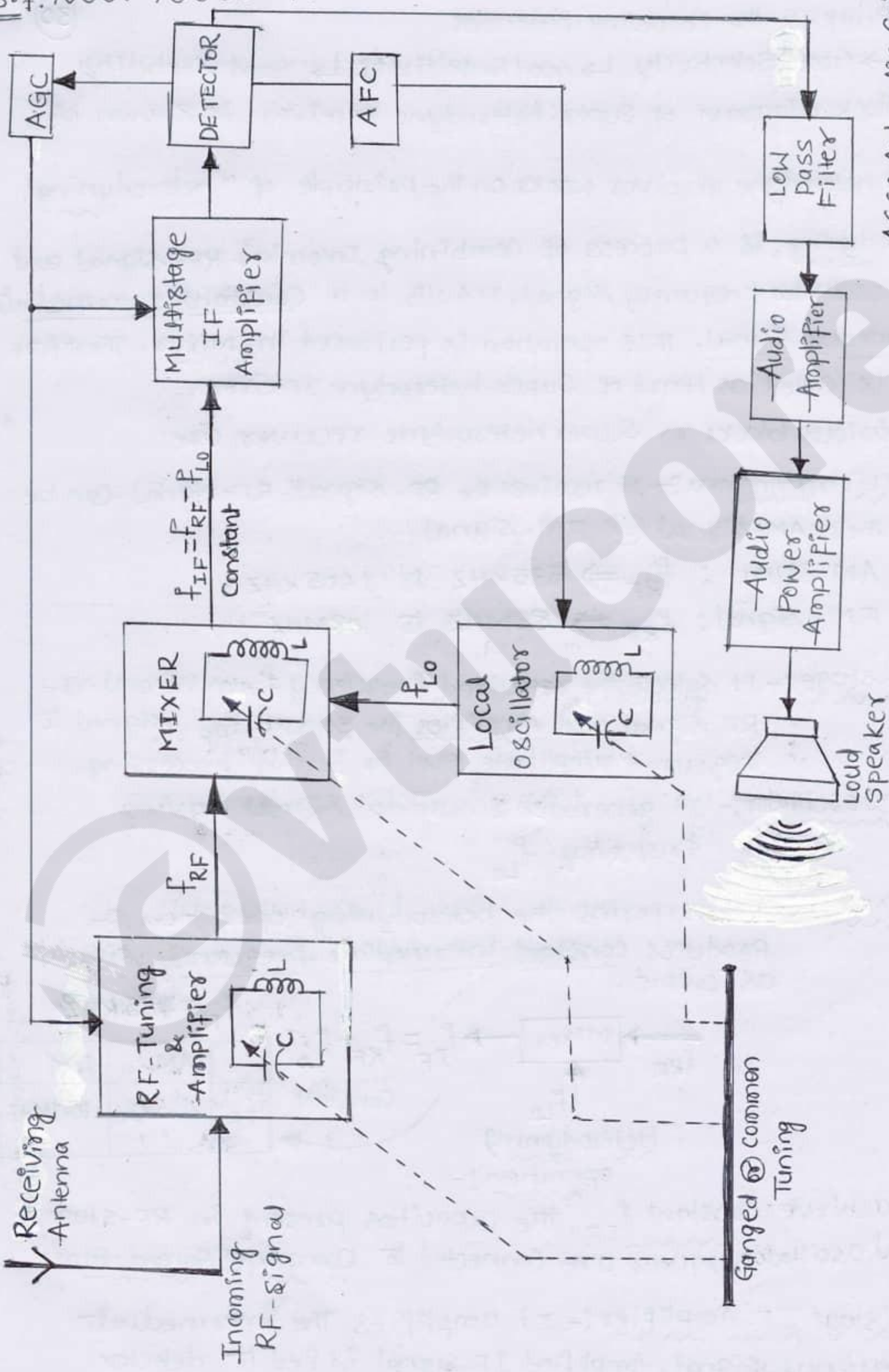
3.7: ** SUPER HETERODYNE RECEIVER: (29)



AGC: Automatic Gain Controller
AFC: Automatic Frequency control

Fig 1: Block diagram of Super-Heterodyne Receiver:

- The superheterodyne receiver, is a special type of receiver that fullfills the following features (30)
  ↳ Good Selectivity ↳ Good Sensitivity ↳ Good Stability.
- The block diagram of super-heterodyne receiver is shown in Fig.1.
- Super-heterodyne receiver works on the principle of "heterodyning".
- Heterodyning, is a process of combining incoming RF-signal and Local oscillator frequency signal, results in a constant Intermediate frequency signal. This operation is performed in Mixer. Therefore Mixer is called as Heart of Super-heterodyne receiver.
- The various blocks in Superheterodyne receiver are.

↳ Receiving Antenna:- It receives the RF-signal. RF-signal can be either AM-signal (or) FM-signal.

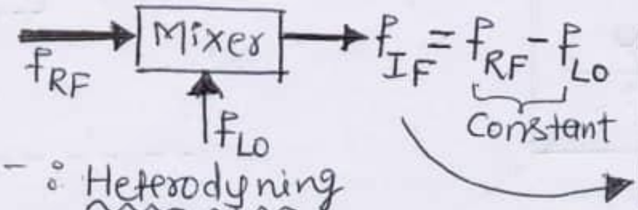
for AM-signal : $f_{RF} \Rightarrow$ 535 KHZ to 1605 KHZ

for FM-signal: $f_{RF} \Rightarrow$ 88 MHZ to 108 MHZ

↳ RF-stage:- It selects the required frequency from incoming RF-signal and Amplifies the selected f_{RF}, signal to required amplitude level for further processing.

↳ Local oscillator:- It generates sinusoidal signal having frequency "f_{LO}".

↳ * Mixer * : It performs the heterodyning operation. & produces constant intermediate frequency signal as output.



	AM	FM
f_{IF}	455 KHZ	10.7 MHZ

-: Heterodyning operation:-

* To achieve constant f_{IF}, the capacitors present in RF-stage, Local oscillator, Mixer are connected to common (or) Ganged tuner.

↳ Multistage IF-Amplifier:- It amplifies the intermediate frequency signal. Amplified IF-signal is fed to detector.

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↳ Detector :- It detects the message signal present in amplified IF-signal. (31)

↳ Low pass filter :- It eliminates any higher order harmonics present in detector output. It produces the required base-band signal at audio-frequency (AF)

↳ Audio Amplifier :- It amplifies the AF-signal to the required amplitude level.

↳ Audio power Amplifier :- It boosts the power level of amplified AF-signal to a power level suitable to drive the Loud-speaker.

↳ Loud-Speaker :- It converts electrical signal to physical sound signal.

Note : • In AM-superheterodyne receiver, $f_{IF} = f_{RF} - f_{LO} = 455\,KHZ$.
• In FM-superheterodyne receiver, $f_{IF} = f_{RF} - f_{LO} = 10.7\,MHZ$.

Numerical problems :-

1) Prove that the number of sidebands in Wide band FM-signal is Infinite.

↳ We know that the general expression of WBFM-signal is,

$$S(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right] \quad \text{——(1)}$$

Where β = Modulation Index = $\frac{\Delta f}{f_m} \gg 1$ for WBFM.

Equation (1) can be expressed as,

$$S(t) = Re\left[A_c e^{j\left[2\pi f_c t + \beta \sin 2\pi f_m(t)\right]}\right] \quad \left[\because \cos\theta = Re(e^{j\theta})\right] \quad \text{——(2)}$$

$$S(t) = Re\left[A_c \exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))\right] \quad \text{——(3)}$$

By using n^{th}-order Bessel function, $J_n(\beta)$ equation (3) can be re-arranged as (32)

$$S(t) = Re\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp\left[j2\pi(f_c + nf_m)t\right]\right] \quad (4)$$

The carrier amplitude A_c, is constant. therefore it can be taken outside the real-time operator $Re[\cdot]$. Take $Re[\cdot]$ inside the summation we get

$$S(t) = A_c \sum_{n=-\infty}^{\infty} Re\left[J_n(\beta)\exp[j2\pi(f_c+nf_m)t]\right]$$

$$Re\left[J_n(\beta)\exp[j2\pi(f_c+nf_m)t]\right] = J_n(\beta)\cos[2\pi(f_c+nf_m)t]$$

$$\therefore S(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c+nf_m)t] \quad (5)$$

By taking Fourier transform on both sides we get

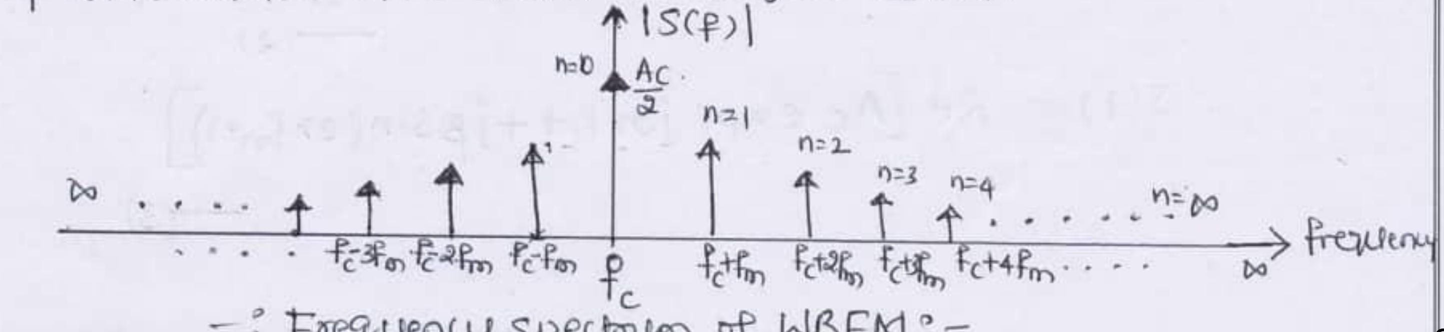
$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)\right] \quad (6)$$

$\therefore$ from equation (6), It is clear that the spectrum of wide band FM-signal consists of an infinite number of delta functions (side bands) spaced at $f = f_c \pm nf_m$ for $n = 0, \pm1, \pm2 \ldots \pm\infty$

In WBFM, $J_n(\beta)$ is finite for all values of 'n'. and

- $J_n(\beta) = J_{-n}(\beta)$ for n-even
- $J_n(\beta) = -J_{-n}(\beta)$ for n-odd
- As n, Increases $|J_n(\beta)|$ decreases and $\boxed{J_0(\beta) \simeq 1}$

$\therefore$ The spectrum of WBFM-signal consists of Infinite number of side bands as shown in figure below.



-: Frequency spectrum of WBFM :-

3.8 Demodulation of FM-Waves:-

Frequency Demodulation is the process of recovering original message signal from an incoming FM-wave.

There are two methods in frequency demodulation

<i> Frequency descriminator (OR) Balanced Slope detector

<ii> phase-Locked Loop.

<i> Frequency descriminator (OR) Balanced slope Detector :-

- The Balanced slope detector consists of two slope detector circuits. The Block diagram of Frequency descriminator (or) balanced slope detector is shown in Fig.1 and its equivalent circuit diagram is shown in Fig.2.

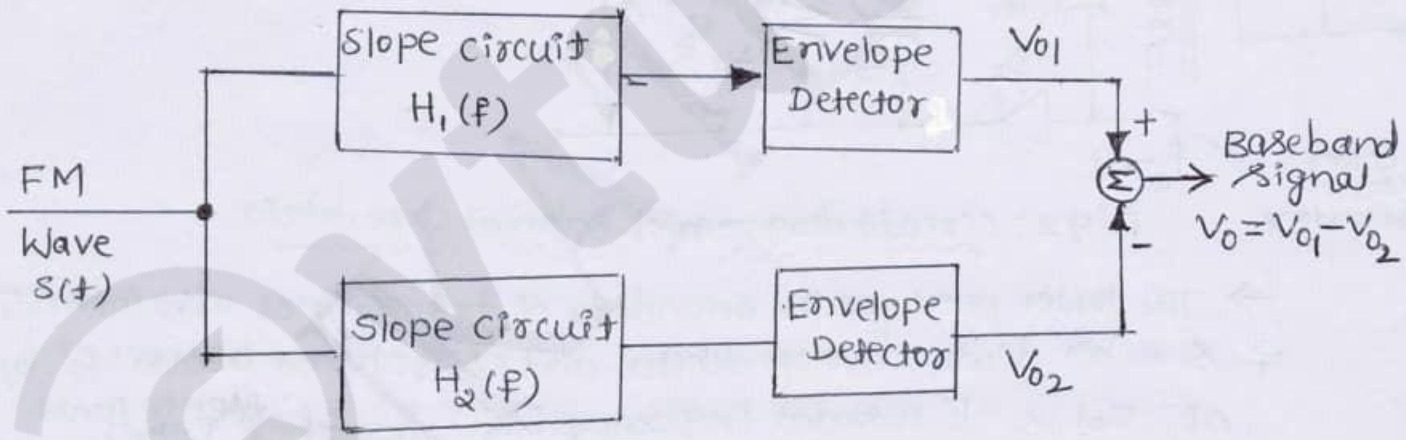


Fig1: Block diagram

The output voltage of frequency descriminator is, (or) (Balanced slope detector)

$$V_0 = V_{o1} - V_{o2} = \begin{cases} 0 & ; \ f_{in} = f_c \\ +ve & ; \ f_{in} > f_c \\ -ve & ; \ f_{in} < f_c \end{cases} \quad \text{——} (1)$$

where, f_{in} = Frequency of FM-wave

f_c = Carrier frequency (unmodulated carrier).

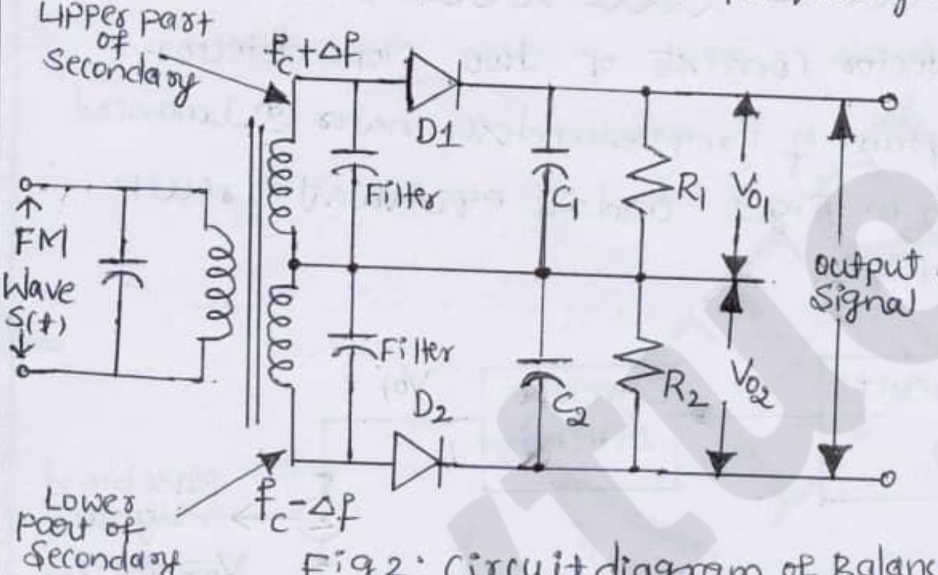
- This method is popular known as Balanced slope detector.

- The equivalent circuit diagram is shown in Fig.2. It consists of, (34)

↳ Center tapped transformer :• Its primary is tuned to frequency of FM-signal. "$f_c \pm \Delta f$". (Intermediate frequency)

• It produces 180° out-of-phase voltages at secondary windings.

↳ The upper part of the secondary of transformer, consists of Diode-Envelope detector and it is tuned above 'f_c' by Δf. That is its resonant frequency is "$f_c + \Delta f$" [upper tuned filter]



$$V_0 = V_{0_1} - V_{0_2} = \begin{cases} 0 & ; f_{in} = f_c \\ +ve & ; f_{in} > f_c \\ -ve & ; f_{in} < f_c \end{cases}$$

Fig2: Circuit diagram of Balanced slope detector

↳ The lower part of the secondary of transformer also consists of similar diode envelope detector and it is tuned below 'f_c' by Δf. That is its resonant frequency is "$f_c - \Delta f$". [Lower Tuned filter]

↳ It produces the required output voltage (Baseband message signal) $V_0 = m(t)$ as shown in Equation (1).

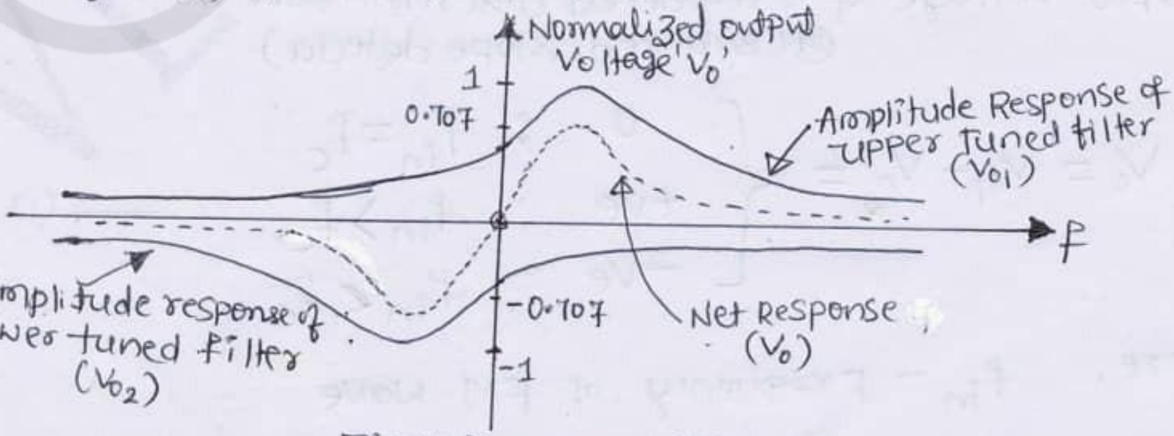


Fig3: Frequency Response

↳ The frequency response of upper & Lower tuned filters, Net response of the circuit is shown in Fig.3.

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FM-demodulation using phase Locked Loop:- (PLL)

phase Locked Loop (PLL) is a negative feedback system that consists of three major components

⟨i⟩ A Multiplier used as a phase detector (or) phase comparator.

⟨ii⟩ A - voltage controlled oscillator ⟨VCO⟩

⟨iii⟩ A - Loop filter, which is a Low pass filter ⟨LPF⟩

The Block diagram of PLL is shown in Fig.1.

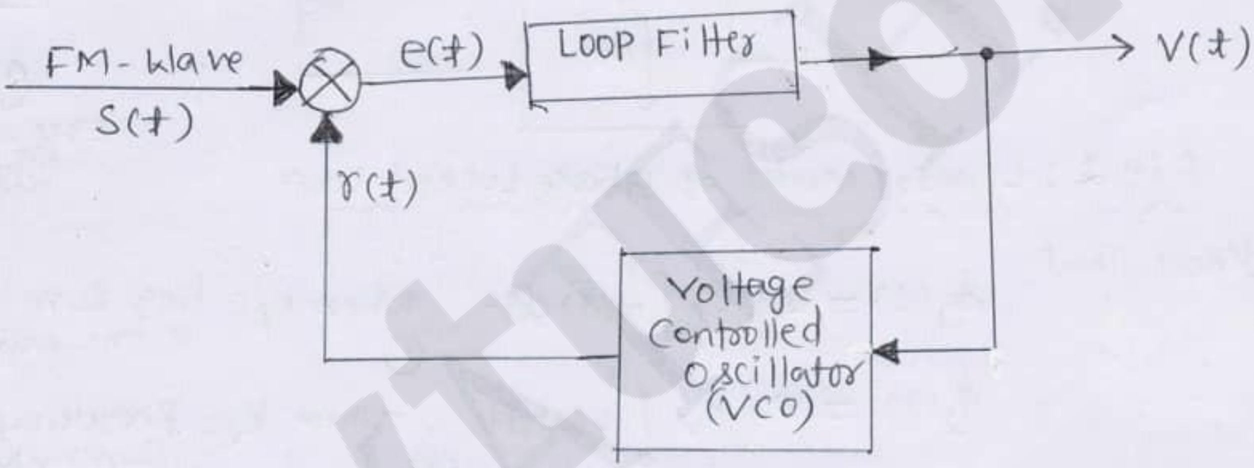

Fig.1: Block diagram of PLL

↳ The VCO output is defined as

$$r(t) = A_v \cos(2\pi f_c t + \phi_2(t)) \quad \text{—} ①$$

$$\text{Where } \phi_2(t) = 2\pi K_v \int_0^t v(t)\,dt.$$

↳ Then, the incoming signal (FM) (S(t)) and the VCO output r(t) are applied to the multiplier, then it gives error signal,

$$e(t) = r(t).S(t) \quad \text{—} ②$$

$$\text{Where } S(t) = A_c \sin[2\pi f_c t + \phi_1(t)] \quad \text{—} ③$$

$$\text{Where } \phi_1(t) = 2\pi K_f \int_0^t m(t)\,dt. \quad \text{—} ④$$

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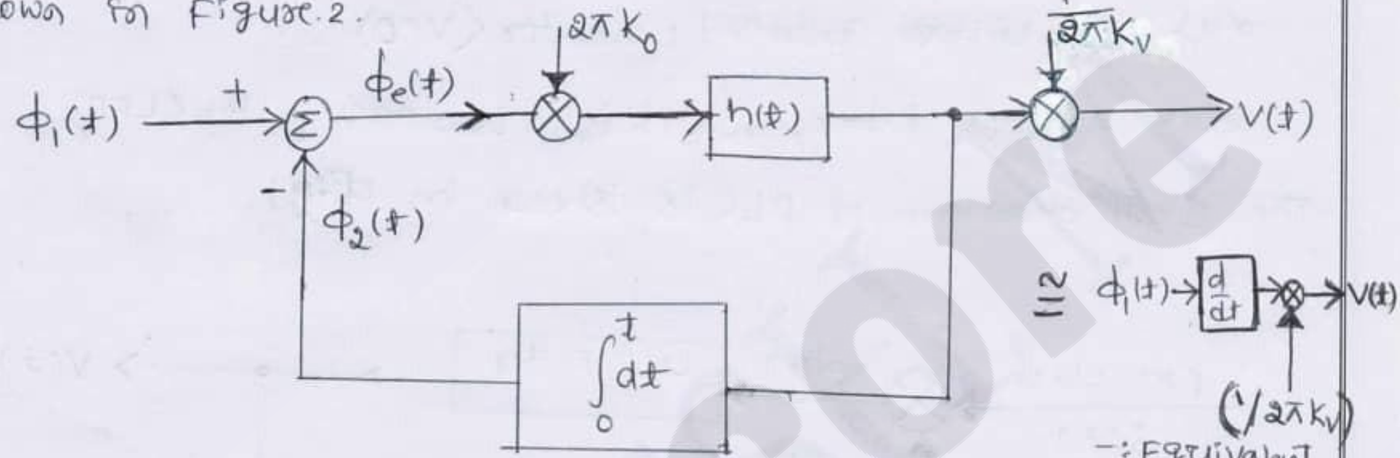
⟨i⟩ Linear Model of phase-Locked-Loop (Linear- PLL):-

V.T.U Q.P. 8 Marks

The phase locked loop (PLL) is said to be in phase-Lock, when the phase error $\boxed{\phi_e(t) = 0}$

The Linear model of PLL for the demodulation of FM-signal is shown in Figure.2.



-: Equivalent Model:-

Fig.2: Linear Model of Phase Locked Loop

We know that,

$$\phi_1(t) = 2\pi K_f \int_0^t m(t)\,dt \quad \text{— (1)}$$ where K_f = freq. sensitivity of FM-wave.

$$\phi_2(t) = 2\pi K_v \int_0^t v(t)\,dt \quad \text{— (2)}$$ where K_v = Frequency Sensitivity Constant of VCO

From fig.2, $$\phi_e(t) = \phi_1(t) - \phi_2(t) \quad \text{— (3)}$$

W.K.T. for phase-lock mode : $\phi_e(t) = 0$ (Assuming small error ≃ 0)

∴ Equation (3) ⇒ $$0 = \phi_1(t) - \phi_2(t)$$

$$\therefore \phi_1(t) = \phi_2(t)$$

Using equations (1) & (2) we get

$$2\pi K_f \int_0^t m(t)\,dt = 2\pi K_v \int_0^t v(t)\,dt$$

$$K_f \int_0^t m(t)\,dt = K_v \int_0^t v(t)\,dt \quad \text{— (4)}$$

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Differentiating both sides of equation (4), we get

$$k_f\, m(t) = k_v\, V(t).$$

$$\therefore \boxed{V(t) = \frac{k_f}{k_v}\cdot m(t) = K\, m(t)} \quad ; \text{ where } K = \frac{k_f}{k_v}$$

i.e., $V(t) \propto m(t)$

Thus, the output $V(t)$ of the Low pass-loop filter $[h(t)]$ is proportional to the original modulating signal.
i.e., The message signal present in FM-modulated wave $s(t)$ is recovered and it is produced at the output of loop filter.

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3.9: Non-Linear effects in FM-Wave :- <VTU Q.P>

Q) Write a short note on Non-linear effects in FM-system.

↳ Non-linear effects can be of two-types
(i) Strong (ii) Weak.

* Non-linearity is said to be strong, if it is intentionally introduced into the circuit in a controlled manner.
  Ex: square law devices.
* Non linearity is said to be weak, when it is inherently present in the circuit.

The effect such non-liniarities will limit $m(t)$ levels in the system.
In FM-generation system, weak non-linearity is present.
the effect of weak non-linearity in FM-systems can be by considering the input and output relation of the memoryless-non linear device used in the frequency multiplier.

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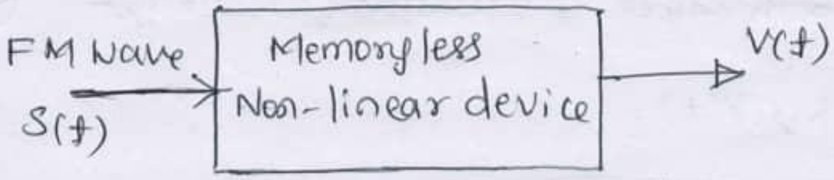


Fig 1: Non-linear device used in FM system:-

Consider a memoryless Non-linear device as shown in fig. 1.

W.K.T., the relation between input & output signal is

$$V(t) = a_1 S(t) + a_2 S^2(t) + a_3 S^3(t) + \cdots + a_n S^n(t) \quad — (1)$$

Let us consider upto 3rd order

i.e., $V(t) = a_1 S(t) + a_2 S^2(t) + a_3 S^3(t) \quad — (2)$

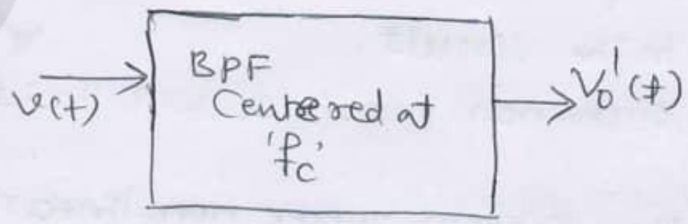
W.K.T the expression for FM-wave is

$$S(t) = A_c \cos[2\pi f_c t + \phi(t)] \quad \text{where } \phi(t) = 2\pi k_f \int_0^t m(t)\,dt$$

$$\therefore V(t) = a_1 A_c \cos[2\pi f_c t + \phi_1(t)] + a_2 A_c^2 \cos^2[2\pi f_c t + \phi_1(t)] + a_3 A_c^3 \cos^3[2\pi f_c t + \phi_1(t)] \quad — (3)$$

$\therefore$ The the output voltage consists of DC component and three FM-signals with carrier frequencies f_c, $2f_c$ & $3f_c$ having frequency deviations Δf, $2\Delta f$ and $3\Delta f$ respectively.

The Desired FM-signal can be separated using a BPF as shown in fig. 2.



We get the FM-system o/p after passing through BPF is

$$V_0'(t) = a_1 A_c \cos[2\pi f_c t + \phi(t)] \quad — (4)$$

equation (4) is same as that of FM-input signal

$S(t) = A_c \cos[2\pi f_c t + \phi(t)]$ Except for change in amplitude.

$\therefore$ Amplitude Non-linearities of the FM-S/m does not affect FM-signal.

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