MODULE - 01

REVISION OF VECTOR CALCULUS.

• Vector Quantity:

Which has both magnitude and direction in space is called Vector Quantity.

Ex: Displacement, Velocity, force, acceleration.

• Scalar Quantity:

Which has only magnitude and no direction in space is called Scalar Quantity.

Ex: Mass, density, pressure, Volume, time.

> Scalar Notation

A or A [Italic or plain]

> Vector Notation

A or $\vec{A}$ [Bold or plain with arrow]

• THE VECTOR ALGEBRA.

* Addition of 2 Vectors

$$A+B = B+A$$

It obeys commutative law.

* Addition of 3 Vectors

$$A+(B+C) = (A+B)+C$$

It obeys Associative law.

* Subtraction of Vector Quantity.

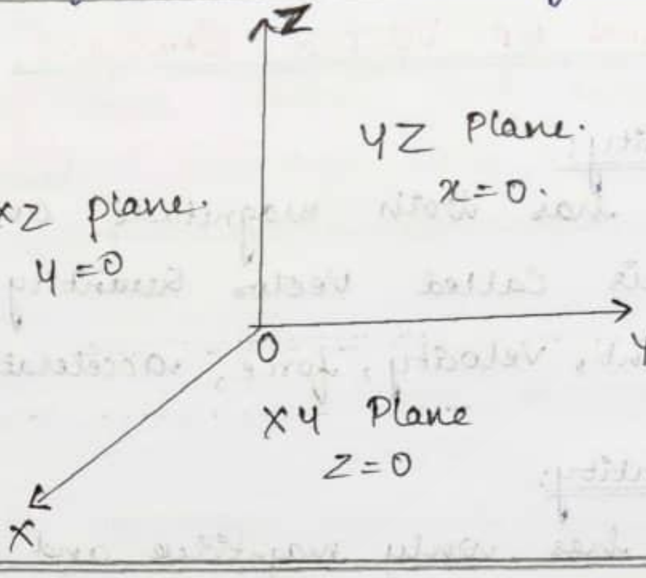
$$A-B = A+(-B)$$

The direction of Vector $\vec{B}$ is reversed.

$$A-B = 0$$

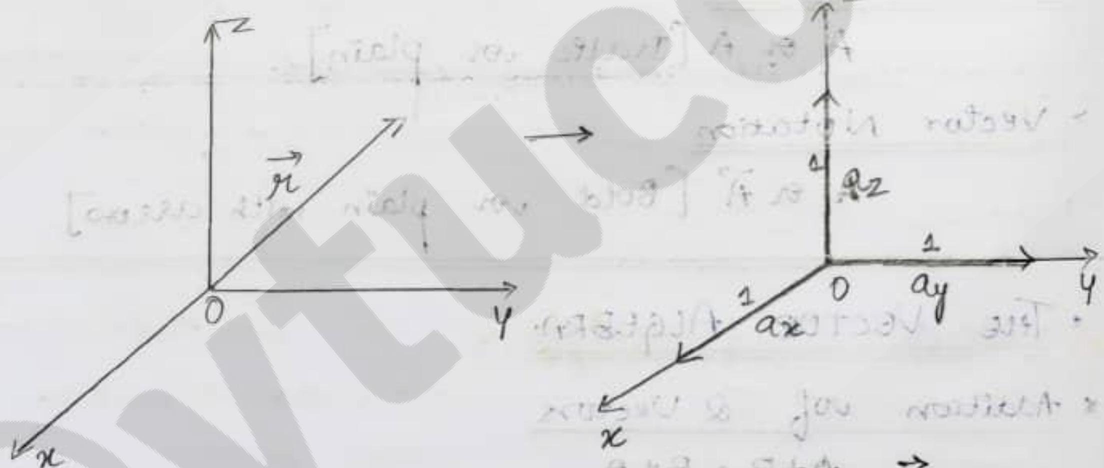
$$\Rightarrow A = B$$

• The Rectangular Co-ordinate System.



• Vector Components and Unit Vectors

› Expressing a vector in rectangular Co-ordinate system.

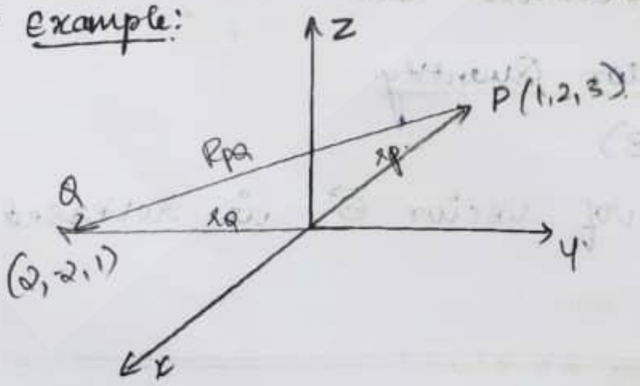


→ ax, ay and az are unit vectors of $\vec{r}$.

→ r is a Vector extending Outward from the origin

→ xyz are component Vectors of Vector 'r'.

→ Example:



$r_P = a_x + 2a_y + 3a_z$

$r_Q = 2a_x - 2a_y + a_z$

as r_p and R_{PQ} in same direction.

$$r_p + R_{PQ} = r_Q.$$

$$R_{PQ} = r_Q - r_p.$$

$$R_{PQ} = 2a_x - 2a_y + a_z - a_x - 2a_y - 3a_z$$

$$\boxed{R_{PQ} = a_x - 4a_y - 2a_z}$$

> **To find magnitude of Vector and unit Vector.**

Let $B = B_x a_x + B_y a_y + B_z a_z$

$$|B| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

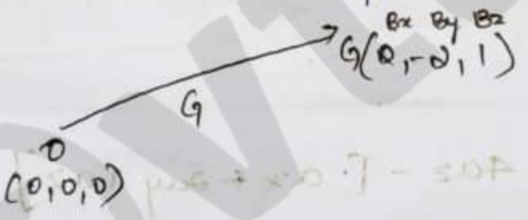
Unit Vector in the direction of $\vec{B}$

$$a_B = \frac{B}{|B|}$$

• **Problem:**

1. Specify the unit vector extending from origin towards a point $G(2,-2,1)$

Sol



$$G = 2a_x - 2a_y + a_z$$

$$a_G = \frac{G}{|G|}$$

$$|G| = \sqrt{(2)^2 + (-2)^2 + 1^2}$$

$$= \sqrt{9}$$

$$\boxed{|G| = 3}$$

$$a_G = \frac{2a_x - 2a_y + a_z}{3}$$

$$a_G = \frac{2}{3}a_x - \frac{2}{3}a_y + \frac{1}{3}a_z$$

$$\boxed{a_G = 0.66a_x - 0.66a_y + 0.33a_z}$$

2. For a given points $M(-1,2,1)$, $N(3,-3,0)$, $P(-2,-3,-4)$ find

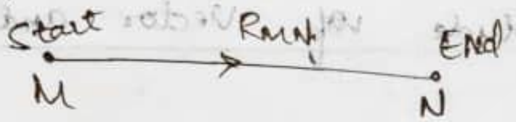
i) R_{MN}

ii) $R_{MN} + R_{MP}$

iii) $|r_m|$

iv) a_{mp}.

Sol To find R_{MN}



$R_M = -a_x + 2a_y + a_z$

$R_N = 3a_x - 3a_y$

$R_P = -2a_x - 3a_y - 4a_z$

$\therefore R_{MN} = R_N - R_M$

$= 3a_x - 3a_y - (-a_x + 2a_y + a_z)$

$= 3a_x - 3a_y + a_x - 2a_y - a_z$

i) $\boxed{R_{MN} = 4a_x - 5a_y - a_z}$

$\therefore R_{MN} + R_{MP} =$

$R_{MP} = R_P - R_M$

$= -2a_x - 3a_y - 4a_z - [-a_x + 2a_y + a_z]$

$= -2a_x - 3a_y - 4a_z + a_x - 2a_y - a_z$

$\boxed{R_{MP} = -a_x - 5a_y - 5a_z}$

ii) $\therefore R_{MN} + R_{MP} = 4a_x - 5a_y - a_z - a_x - 5a_y - 5a_z$

ii) $\boxed{R_{MN} + R_{MP} = 3a_x - 10a_y - 6a_z}$

(iii) $r_M = -a_x + 2a_y + a_z$

$|r_m| = \sqrt{(-1)^2 + 2^2 + 1^2}$

$= \sqrt{6}$

iii) $\boxed{|r_m| = 2.44}$

iv) $a_{mp} = a_{mp} = \dfrac{R_{MP}}{|R_{MP}|}$

$|R_{mp}| = \sqrt{(-1)^2 + (-5)^2 + (-5)^2}$

$= \sqrt{51}$

$= \dfrac{R_P - R_M}{7.14}$

$= \dfrac{-a_x - 5a_y - 5a_z}{7.14}$

$a_{mp} = \dfrac{1}{7.14} a$

$a_{mp} = \dfrac{-1}{7.14} a_x - \dfrac{5}{7.14} a_y - \dfrac{5}{7.14} a_z$

iv) $\boxed{a_{mp} = -0.14\, a_x - 0.70\, a_y - 0.70\, a_z}$

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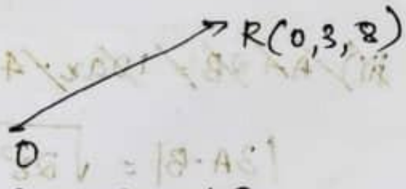
3. For a given points $P(1,-3,5)$, $Q(2,4,6)$, $R(0,3,8)$. Find i) position of Vector P and R.

ii) Distance Vector R_{QR}

iii) Distance Between Q and R.

Sol: i) Position of Vector P and R



$r_p = a_x - 3a_y + 5a_z$

$r_R = 3a_y + 8a_z$

ii) $r_{QR} = r_R - r_Q$

$r_Q = 2a_x + 4a_y + 6a_z$

$r_{QR} = 3a_y + 8a_z - 2a_x - 4a_y - 6a_z$

$$\boxed{r_{QR} = -2a_x - a_y + 2a_z}$$

iii) $R_{QR} = R_R - R_Q$

$R_{QR} = -2a_x - a_y + 2a_z$

To find distance,

$|R_{QR}| = \sqrt{(-2)^2 + (-1)^2 + (2)^2}$

$= \sqrt{9}$

$= \underline{\underline{3}}$

The distance btw Q and R is $\underline{\underline{3}}$

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4. If $A = 10a_x - 4a_y + 6a_z$ and $B = 2a_x + a_y$ find
i) the component of vector A along a_y.
ii) Magnitude of $3A - B$.
iii) Find unit vector along $A + 2B$.

Sol i) The vector component along a_y is -4

ii) $3A - B = 3(10a_x - 4a_y + 6a_z) - (2a_x + a_y)$

$= 30a_x - 12a_y + 18a_z - 2a_x - a_y$

$$\boxed{3A - B = 28a_x - 13a_y + 18a_z}$$

~~iii) A + 2B = 10ax - 4ay + 6az~~

$|3A - B| = \sqrt{28^2 + (-13)^2 + 18^2}$

$$\boxed{|3A - B| = 35.73}$$

iii) $A + 2B = 10a_x - 4a_y + 6a_z + 2[2a_x + a_y]$

$= 10a_x - 4a_y + 6a_z + 4a_x + 2a_y$

$A + 2B = 14a_x - 2a_y + 6a_z$

$$a_{A+2B} = \frac{R_{A+2B}}{|A+2B|}$$

$|A+2B| = \sqrt{14^2 + (-2)^2 + 6^2}$

$|A+2B| = 15.36$

$$a_{A+2B} = \frac{14a_x - 2a_y + 6a_z}{15.36}$$

$$\boxed{a_{A+2B} = 0.91a_x - 0.13a_y + 0.39a_z}$$

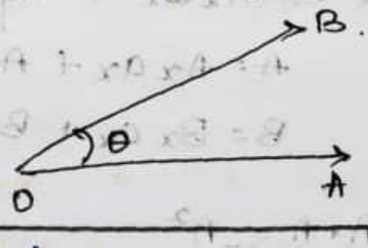
★ **Vector Multiplication methods.**

1. Dot product or Scalar product
2. Cross product or Vector product.

• **Dot product**

The Dot product of 2 Vectors A & B is given by

$A \cdot B = |A|\,|B| \cos\theta$



> **Characteristics of Dot product.**

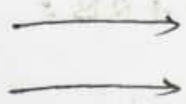
1. When 2 vectors A & B are perpendicular to each other their dot product is zero.

$A \perp B$ $\boxed{A \cdot B = 0}$

2. when 2 vector A & B are parallel to each other their dot product is $|A||B|$

$A \parallel B \qquad A \cdot B = |A||B| \cos\theta$

$= |A||B|(1)$



$\boxed{A \cdot B = |A||B|}$

3. when 2 vector A & B are opposite to each other their dot product is $\rightarrow$ $|A||B|$

$A \cdot B = |A||B| \cos\theta$

$= |A||B| \cos 180^\circ$

$A \cdot B = -|A||B|$

4. Dot product of identical unit vectors is always 1

$a_x \cdot a_x = 1 \qquad a_y \cdot a_y = 1 \qquad a_z \cdot a_z = 1$

5. Dot product of non-identical unit vectors is always zero

$a_x \cdot a_y = 0 \qquad a_x \cdot a_z = 0 \qquad a_y \cdot a_z = 0$.

6. $A \cdot B = B \cdot A$, Dot product obeys commutative law.

7. $A \cdot B = A_x B_x + A_y B_y + A_z B_z$

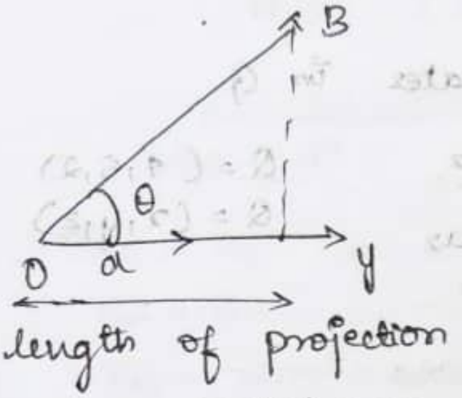
$A = A_x a_x + A_y a_y + A_z a_z$

$B = B_x a_x + B_y a_y + B_z a_z$

8. $A \cdot A = A^2$

> Applications of the dot product.

1.



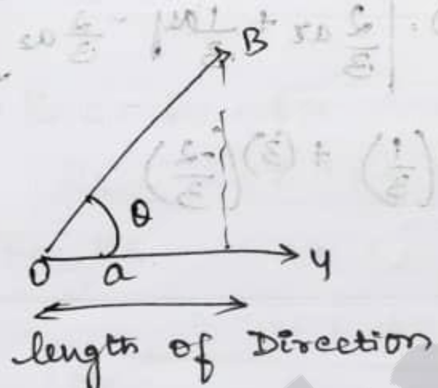
$|a| = 1$

Because it is a Unit Vector

$B \cdot a = |B||a| \cos\theta$

$B \cdot a = |B| \cos\theta$.

2.



$(B \cdot a) \cdot a$

• PROBLEMS!

1. A Vector field $G = y a_x - 2.5 x a_y + 3 a_z$ and the point $Q(4, 5, 2)$. Find

i) G at Q

ii) Scalar component of G at Q in the direction of $a_N = \frac{1}{3}(2a_x + a_y - 2a_z)$

iii) Vector Component of G at Q in direction of a_N.

Sol i) $G = y a_x - 2.5 x a_y + 3 a_z$

G at $Q = ?$

Substitute Q co-ordinates in G

$$G = y a_x - 2.5 x a_y + 3 a_z \qquad Q = (4, 5, 2)$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad Q = (x, y, z)$$
$$= 5 a_x - 2.5(4) a_y + 3 a_z$$

$$\boxed{G \text{ at } Q = 5 a_x - 10 a_y + 3 a_z}$$

ii) Scalar Component of G in direction of a_N.

$$G \cdot a_N = (5 a_x - 10 a_y + 3 a_z) \cdot \frac{1}{3}(2 a_x + a_y - 2 a_z)$$
$$= (5 a_x - 10 a_y + 3 a_z) \cdot \left[\frac{2}{3} a_x + \frac{1}{3} a_y - \frac{2}{3} a_z\right]$$
$$= \left(5 \times \frac{2}{3}\right) + (-10)\left(\frac{1}{3}\right) + (3)\left(\frac{-2}{3}\right)$$

$$\boxed{G \cdot a_N = -2}$$

iii) $(G \cdot a_N) \cdot a_N$

$$= (-2) \cdot \frac{1}{3}\left[2 a_x + a_y - 2 a_z\right]$$
$$= (-2)\left[\frac{2}{3} a_x + \frac{a_y}{3} - \frac{2}{3} a_z\right]$$

$$\boxed{(G \cdot a_N) \cdot a_N = -\frac{4}{3} a_x - \frac{2}{3} a_y + \frac{4}{3} a_z}$$

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2. The vertices of Δ^{le} are located at $A(6, -1, 2)$ $B(-2, 3, -4)$, $C(-3, 1, 5)$, find,

i) R_{AB}

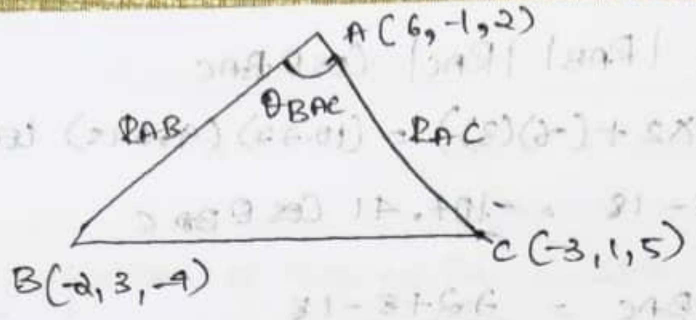
ii) R_{AC}

iii) angle θ_{BAC} at vertex A

iv) vector projection of R_{AB} on R_{AC}

v) length of projection of R_{AB} on R_{AC}.

8)



i) $R_{AB} = R_B - R_A$

$R_A = 6a_x - a_y + 2a_z$

$R_B = -2a_x + 3a_y - 4a_z$

$R_B - R_A = (-2a_x + 3a_y - 4a_z) - (6a_x - a_y + 2a_z)$

$= -2a_x + 3a_y - 4a_z - 6a_x + a_y - 2a_z$

$$\boxed{R_{AB} = -8a_x + 4a_y - 6a_z}$$

ii) $R_{AC} = R_C - R_A$

$R_C = -3a_x + a_y + 5a_z$

$R_C - R_A = -3a_x + a_y + 5a_z - (6a_x - a_y + 2a_z)$

$= -3a_x + a_y + 5a_z - 6a_x + a_y - 2a_z$

$$\boxed{R_{AC} = -9a_x + 2a_y + 3a_z}$$

iii) $R_{AB} \cdot R_{AC} = |R_{AB}|\,|R_{AC}| \cos\theta_{BAC}$

$|R_{AB}| = \sqrt{(-8)^2 + (4)^2 + (-6)^2}$

$= \sqrt{64 + 16 + 36}$

$= \sqrt{116}$

$$\boxed{|R_{AB}| = 10.77}$$

$|R_{AC}| = \sqrt{(-9)^2 + (2)^2 + (3)^2}$

$= \sqrt{81 + 4 + 9}$

$= \sqrt{94}$

$$\boxed{|R_{AC}| = 9.695}$$

$R_{AB} \cdot R_{AC} = |R_{AB}|\,|R_{AC}| \cos\theta_{BAC}$

$(-8\times 9) + 4\times 2 + (-6)(3)) = (10.77)(9.698) \cos\theta_{BAC}$

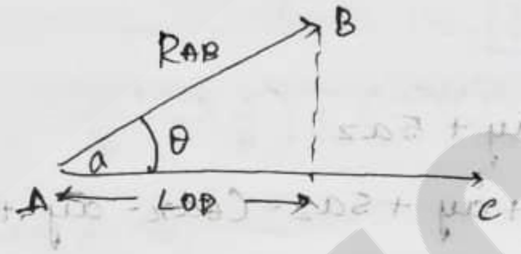
$+72 + 8 - 18 = 104.41 \cos\theta_{BAC}$

$\cos\theta_{BAC} = \frac{72+8-18}{104.41}$

$\theta_{BAC} = \cos^{-1}\left(\frac{62}{104.41}\right)$

$\boxed{\theta_{BAC} = 53.588}$

v) length of projection of R_{AB} on R_{AC}

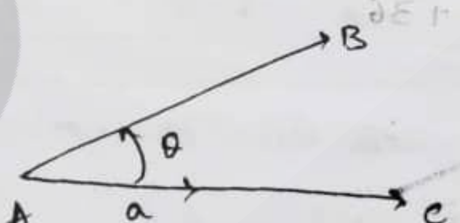


$R_{AB} \cdot a_{AB} = |R_{AB}|(1)\cos\theta$

$= 10.77 \cos(53.58)$

$\boxed{R_{AB} \cdot a_{AB} = 6.39 \text{ meters.}}$

iv) Vector projection of R_{AB} on R_{AC}.



$a_{AC} = \frac{R_{AC}}{|R_{AC}|}$

$|R_{AC}| = \sqrt{-9^2 + 2^2 + 3^2}$

$\boxed{|R_{AC}| = 9.69}$

$$a_{AC} = \frac{-9a_x + 2a_y + 3a_z}{9.69}$$

Vector projection of R_{AB} on $R_{AC} = 6.39\,mt \times \frac{-9a_x + 2a_y + 3a_z}{9.69}$

$$= 0.659(-9a_x + 2a_y + 3a_z)$$

- Coulomb's Law

"The force of attraction or the repulsion between any two point charges is directly proportional to product of the charges and inversely proportional to square of distance between the charges."

$$F \propto \frac{Q_1 Q_2}{r^2} \longrightarrow (1).$$

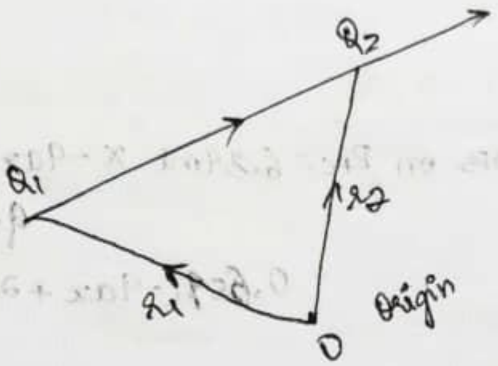
Point charge $\rightarrow Q_1 \bullet \xleftrightarrow{\;r\;} \circ Q_2 \leftarrow$ point charge

The expression (1) can be re-written as $F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r^2}$

where, $\varepsilon_0 \rightarrow$ permittivity of free space $= 8.854 \times 10^{-12}$

$$\boxed{F = 9 \times 10^9\, m/F \;\frac{Q_1 Q_2}{r^2}} \rightarrow (2)$$

> Vector form of coulomb's law:



In this figure the force acts along the line joining the two charges

- The force between Q_1 and Q_2 is repulsive if both of them are like charges.
- The force between Q_1 and Q_2 is attractive if both of them are unlike charges.
- The vector $R_{12} = r_2 - r_1$
- The vector force on Q_2 by $Q_1 = F_2$.

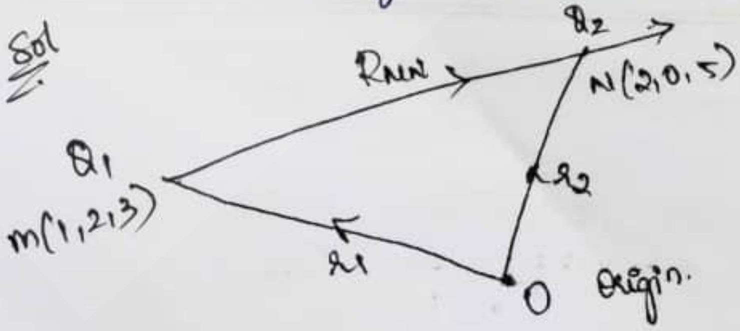
$$F_2 = \frac{Q_1 Q_2}{4\pi \varepsilon_0 (R_{12})^2} \times a_{12}$$

$$a_{12} = \frac{R_{12}}{|R_{12}|}$$

• PROBLEM:

1. A charge $Q_1 = 3\times10^{-4}$ Coulombs at $m(1,2,3)$ and the charge $Q_2 = -10^{-4}$ Coulombs at $N(2,0,5)$ in vaccum. Find force exerted on Q_2 by Q_1

Sol



$Q_1 = 3\times10^{-4}C$ at $m(1,2,3)$

$Q_2 = -10^{-4}C$ at $N(2,0,5)$

$$R_{MN} = R_N - R_M$$

$$R_N = 2a_x + 5a_z$$

$$R_M = a_x + 2a_y + 3a_z$$

$$R_{MN} = 2a_x + 5a_z - a_x - 2a_y - 3a_z$$

$$\boxed{R_{MN} = a_x - 2a_y + 2a_z}$$

$$|R_{MN}| = \sqrt{1^2 + (-2)^2 + (2)^2}$$

$$= \sqrt{9}$$

$$|R_{MN}| = 3$$

$$F_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 |(R_{MN})^2|} \times a_{MN}$$

$$= \frac{3\times10^{-4} \times (-10)^{-4}}{4\pi\, 8.854\times10^{-12} \times (3)^2} \times a_{MN}$$

$$F_2 = 30 \times a_{MN}$$

$$a_{MN} = \frac{R_{MN}}{|R_{MN}|} = \frac{a_x - 2a_y + 2a_z}{3}$$

$$\therefore F_2 = \cancel{30}^{10} \left(\frac{a_x - 2a_y + 2a_z}{\cancel{3}} \right)$$

$$= (a_x - 2a_y + 2a_z)\,10$$

$$\boxed{F_2 = 10a_x - 20a_y + 20a_z} \text{ N/mtr}$$

- **Electric field Intensity (E):**

$$E = \frac{F_1}{Q_t}$$

$$F_1 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_t}{r^2} a_r$$

$$\therefore \boxed{E = \frac{F_1}{Q_t} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{r^2} a_r} \; N/C$$

i) Case(i): Electric field intensity E due to the charge Q_1 located at the centre of spherical Co-ordinate System.

$$E = \frac{Q_1}{4\pi\varepsilon_0 (r)^2} a_r = \frac{Q_1}{4\pi\varepsilon_0 r^2}$$

where, $r \to$ radius
$a_r \to$ ~~that~~ radial Unit Vector

ii) Case (ii): Electric field intensity E due to two point charges Q_1 at r_1 and Q_2 at r_2

To find the electric field intensity due to charge Q_1 and Q_2 first find the force due the Q_1 and the force due to Q_2 on the test charge

$$F_1 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_t}{|r-r_1|^2} a_{r-r_1}$$

$$F_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_t}{|r-r_2|^2} a_{r-r_2}$$

$$E_1 = \frac{F_1}{Q_t} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{|r-r_1|^2} a_{r-r_1}$$

$$E_2 = \frac{F_2}{Q_t} = \frac{1}{4\pi\varepsilon_0} \frac{Q_2}{|r-r_2|^2} a_{r-r_2}$$

$E = E_1 + E_2$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{|r-r_1|^2} a_{r-r_1} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{|r-r_2|^2} a_{r-r_2}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{|r-r_1|^2} a_{r-r_1} + \frac{Q_2}{|r-r_2|^2} a_{r-r_2}\right]}$$

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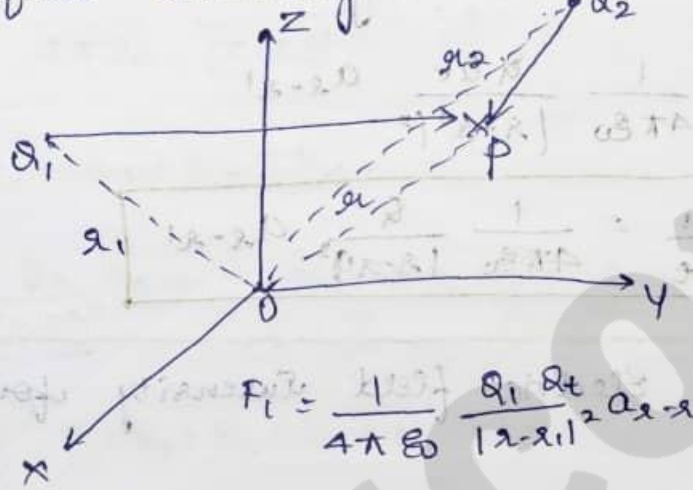
1. For a given figure, find the resultant electric field intensity.



$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_t}{|r-r_1|^2} a_{r-r_1}$$

Sol

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_t}{|r-r_1|^2} a_{r-r_1}$$

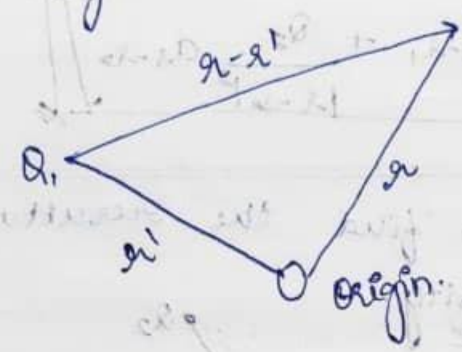
$$E_1 = \frac{F_1}{Q_t} = \frac{Q_1}{4\pi\varepsilon_0 |r-r_1|^2} a_{r-r_1}$$

$$E_2 = \frac{F_2}{Q_t} = \frac{Q_2}{4\pi\varepsilon_0 |r-r_2|^2} a_{r-r_2}$$

$E(r) = E_1 + E_2$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{|r-r_1|^2} a_{r-r_1} + \frac{Q_2}{|r-r_2|^2} a_{r-r_2}\right]}$$

2. For a given figure. Find the electric field due to the charge Q, which is located not at the Origin

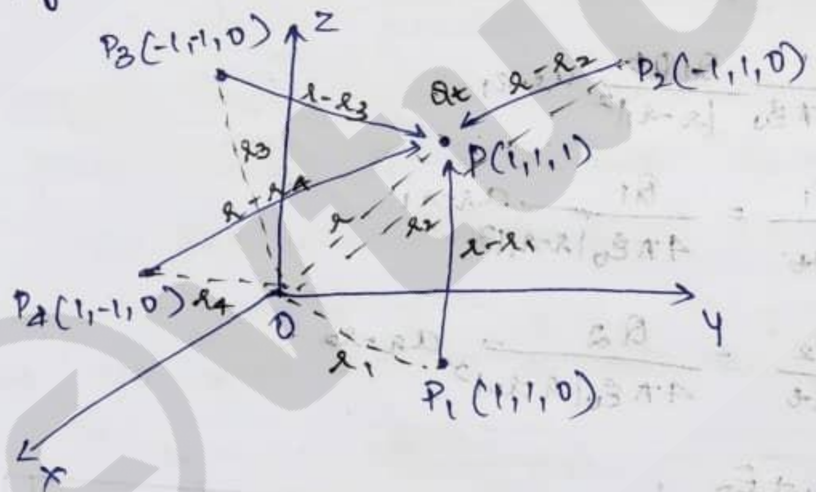


Sol

$$F_1 = \frac{1}{4\pi\epsilon_0} \frac{Q Q_t}{|r-r'|^2} a_{r-r'}$$

$$\boxed{E = \frac{F_1}{Q_t} = \frac{1}{4\pi\epsilon_0} \frac{Q}{|r-r'|^2} a_{r-r'}}$$

3. Find the Electric field intensity for the given figure.



Sol.

$$F_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_t}{(r-r_1)^2} a_{r-r_1}$$

$$F_2 = \frac{1}{4\pi\epsilon_0} \frac{Q_2 Q_t}{(r-r_2)^2} a_{r-r_2}$$

$$F_3 = \frac{1}{4\pi\epsilon_0} \frac{Q_3 Q_t}{(r-r_3)^2} a_{r-r_3}$$

$$F_4 = \frac{1}{4\pi\epsilon_0} \frac{Q_4 Q_t}{(r-r_4)^2} a_{r-r_4}$$

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{(r-r_1)^2}a_{r-r_1}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{(r-r_2)^2}a_{r-r_2}$$

$$E_3 = \frac{1}{4\pi\varepsilon_0}\frac{Q_3}{(r-r_3)^2}a_{r-r_3}$$

$$E_4 = \frac{1}{4\pi\varepsilon_0}\frac{Q_4}{(r-r_4)^2}a_{r-r_4}$$

$$E = E_1 + E_2 + E_3 + E_4.$$

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{(r-r_1)^2}a_{r-r_1} + \frac{Q_2}{(r-r_2)^2}a_{r-r_2} + \frac{Q_3}{(r-r_3)^2}a_{r-r_3} + \frac{Q_4}{(r-r_4)^2}a_{r-r_4}\right]$$

If $Q_1 = Q_2 = Q_3 = Q_4 = Q.$

$$E = \frac{Q}{4\pi\varepsilon_0}\left[\frac{a_{r-r_1}}{(r-r_1)^2} + \frac{a_{r-r_2}}{(r-r_2)^2} + \frac{a_{r-r_3}}{(r-r_3)^2} + \frac{a_{r-r_4}}{(r-r_4)^2}\right]$$

The charges at P_1, P_2, P_3, P_4 is 3 nano-Coulombs.

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(r-r_1)^2}a_{r-r_1}$$

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{3\times10^{-9}}{(r-r_1)^2}\cdot\frac{r-r_1}{|r-r_1|}$$

$r = (1,1,1) = a_x + a_y + a_z$

$r_1 = (1,1,0) = a_x + a_y.$

$r - r_1 = a_x + a_y + a_z - a_x - a_y$

$$\boxed{r - r_1 = a_z}$$

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{3\times10^{-9}}{1}\times\frac{a_z}{1}$$

$$\boxed{E_1 = \frac{3\times10^{-9}}{4\pi\varepsilon_0} a_z}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q}{(r-r_1)^2} a_{r-r_1}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0} \frac{3\times10^{-9}}{(r-r_2)^2} a_{r-r_1}$$

$$r-r_2 = a_x+\cancel{a_y}+\cancel{a_z}+a_x-\cancel{a_z}$$

$$\boxed{r-r_2 = 2a_x + a_y}$$

$$E_2 = \frac{3\times10^{-9}}{4\pi\varepsilon_0} \frac{1}{(\sqrt{2^2+1^2})^2} \frac{r-r_2}{|r-r_2|}$$

$$\boxed{E_2 = \frac{3\times10^{-9}}{4\pi\varepsilon_0} \frac{(2a_x+a_z)}{5\sqrt{5}}}$$

$$E_3 = \frac{1}{4\pi\varepsilon_0} \frac{3\times10^{-9}}{(r-r_3)^2} a_{r-r_3}$$

$$r-r_3 = a_x+a_y+a_z+a_x+a_y$$

$$\boxed{r-r_3 = 2a_x + 2a_y + a_z}$$

$$E_3 = \frac{3\times10^{-9}}{4\pi\varepsilon_0} \frac{1}{(\sqrt{2^2+2^2+1^2})^2} \frac{2a_x+2a_y+a_z}{\sqrt{2^2+2^2+1^2}}$$

$$= \frac{3\times10^{-9}}{4\pi\varepsilon_0} \frac{1}{9} \frac{2a_x+2a_y+a_z}{3}$$

$$\boxed{E_3 = \frac{3\times10^{-9}}{4\pi\varepsilon_0} \frac{2a_x+2a_y+a_z}{27}}$$

$$E_4 = \frac{1}{4\pi\epsilon_0}\frac{3\times10^{-9}}{(r-r_4)^2} a_{r-r_4}$$

$$r-r_4 = \cancel{a_x} + a_y + a_z - \cancel{a_x} + a_y.$$

$$\boxed{r-r_4 = 2a_y + a_z}$$

$$E_4 = \frac{3\times10^{-9}}{4\pi\epsilon_0}\frac{1}{(\sqrt{2^2+1^2})^2}\frac{2a_y+a_z}{\sqrt{2^2+1^2}}$$

$$\boxed{E_4 = \frac{3\times10^{-9}}{4\pi\epsilon_0}\frac{(2a_y+a_z)}{5\sqrt5}}$$

$$E = \frac{3\times10^{-9}}{4\pi\epsilon_0}\left[a_z + \frac{2a_x+a_z}{5\sqrt5} + \frac{2a_x+2a_y+a_z}{27} + \frac{2a_y+a_z}{5\sqrt5}\right]$$

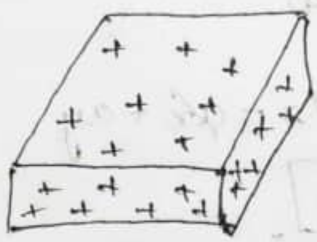
$$= 9\times10^{9}\times3\times10^{-9}\left[a_z + \frac{2a_x+2a_y+a_z}{27} + \frac{2a_x+2a_y+2a_z}{5\sqrt5}\right]$$

$$\boxed{E = 27\left[a_z + \frac{2a_x+2a_y+a_z}{27} + \frac{4a_x+4a_y+2a_z}{5\sqrt5}\right]}$$

- Types of Charge Distributions.

1. Point charge distribution
2. Line Charge distribution
3. Surface charge distribution
4. Volume charge distribution.

• Volume charge distribution.



The total charge Q distributed uniformly over a volume V. The volume charge density

$$\rho_V = \frac{\text{Total charge distributed}}{\text{Total volume}}$$

$$\boxed{\rho_V = \frac{Q}{V}\ C/m^3}$$

The total charge Q is given by

$$\boxed{Q = \int \rho_V\, dv}$$

• **Note:** For spherical co-ordinate system,

$dv = r^2 \sin\theta \,.\, d\theta \,.\, d\phi\, dr$. [Spherical]

$dv = \rho\, d\rho\, dz\, d\phi$ [Cylindrical]

$dv = dx \,.\, dy \,.\, dz$ [x, y, z plane]

---

1. Find the total charge inside the volume, having volume charge density as $\rho_V = 10z^2 e^{-0.1x} \sin \pi y$ C/m³. The volume is defined as $-2 \le x \le 2$, $0 \le y \le 1.3$, $3 \le z \le 4$.

Sol $$Q = \int_{x=-2}^{2} \int_{y=0}^{1.3} \int_{z=3}^{4} 10z^2 e^{-0.1x} \sin \pi y \, dx\, dy\, dz$$

$$Q = 10\int_{-2}^{2} e^{-0.1x}\,dx \int_{0}^{1.3} \sin \pi y\,dy \int_{0}^{4} z^2\,dz$$

$$= 10\left[\frac{e^{-0.1x}}{-0.1}\right]_{-2}^{2}\left[\frac{-\cos\pi y}{\pi}\right]_{0}^{1.3}\left[\frac{z^3}{3}\right]_{0}^{4}$$

$$= 10\left[\frac{e^{-0.2} - e^{0.2}}{-0.1}\right]\left[\frac{-\cos\pi(1.3) + \cos\pi(0)}{\pi}\right]\left[\frac{4^3 - 0^3}{3}\right]$$

$$\boxed{Q = 316.163\,C}$$

---

2. Find charge in a volume defined by Radius R $1mt \le r \le 2mt$ in a spherical Co-ordinate system where Volume charge density $\rho_v = \frac{5\cos^2\phi}{r^4}$ c/m3.

Soln $Q = \int \rho_v\,dv$

$$= \int \frac{5\cos^2\phi}{r^4} r^2 \sin\theta\, d\theta\, d\phi\, dr.$$

$$= \int_{\theta=0}^{\pi}\int_{\phi=0}^{\phi=2\pi}\int_{r=1mt}^{r=2mt} \frac{5\cos^2\phi}{r^4} r^2 \sin\theta\, d\theta\, d\phi\, dr.$$

$$= 5\int_{r=1mt}^{2mt} \frac{r^2}{r^4}\,dr \int_{\theta=0}^{\pi} \sin\theta\, d\theta \int_{\phi=0}^{\phi=2\pi} \cos^2\phi\, d\phi$$

$$= 5\int_{1}^{2} \frac{1}{r^2}\,dr\,(-\cos\theta)_0^{\pi} \int_{\phi=0}^{2\pi} \frac{1+\cos 2\phi}{2}\,d\phi$$

$$= 5\left[\left(\frac{-1}{r}\right)_1^2 (-\cos\theta)_0^{\pi}\ \frac{1}{2}\left[[\phi]_0^{2\pi} + \frac{\sin 2\phi}{2}\right]_{\phi=0}^{2\pi}\right]$$

$$= \frac{5}{2}\left[\left(\frac{-1}{2}\right)_1^2 \left(-(\cos\pi - \cos 0)\right)(2\pi - 0) + \frac{1}{2}\left[\left(\cancel{\sin 2(2\pi)}^{\,0} - \sin 2(0)\right)\right]\right]$$

$$= \frac{5}{2}\left[\frac{-1}{2} + \frac{1}{1}\right](+2)(+2\pi) + \frac{1}{2}(0)\Big]$$

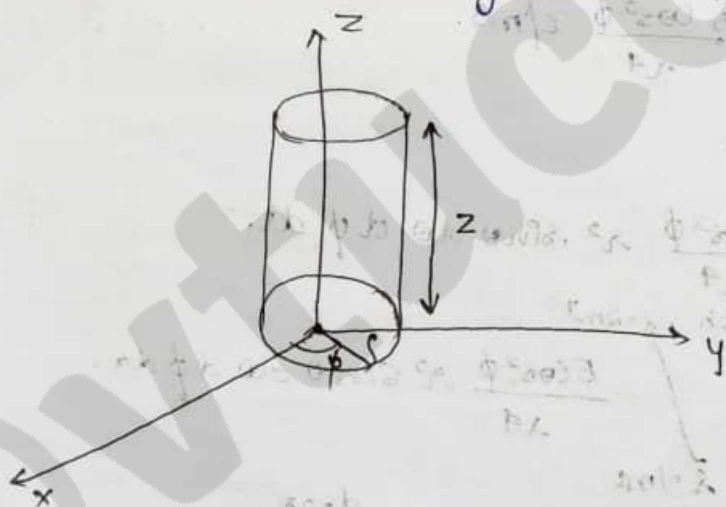
$$= \frac{5}{2}\left(\frac{1}{2}\times 2\right)2\pi$$

$$= 5\pi$$

$$\boxed{Q = 15.70\ c/m^3}$$

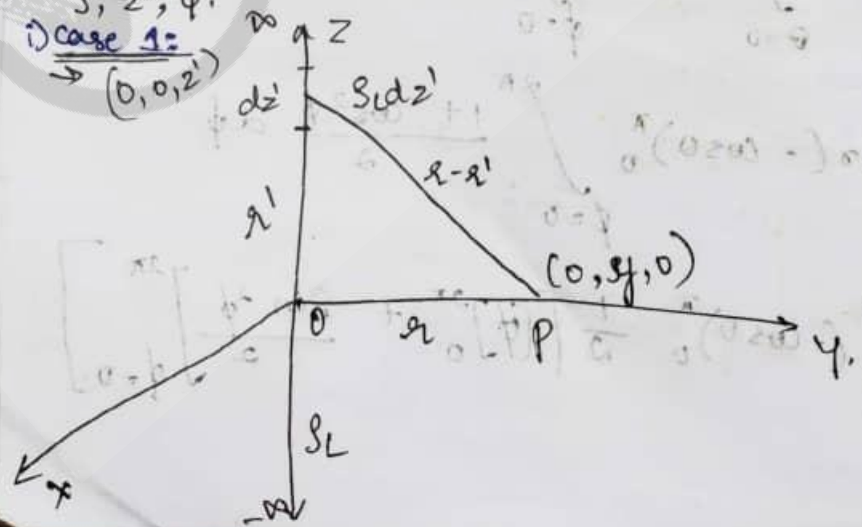
- Electric field intensity due to a line charge

8/9m Cylindrical Co-ordinate System.



The Co-ordinates for Cylindrical System are ρ, z, ϕ.

i) Case 1:
→ $(0,0,z')$



$\phi = 0$
$z = 0$
ρ = present

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{\rho_L dz'}{|r-r'|^2} a_{r-r'}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\rho_L dz'}{|r-r'|^2} \frac{r-r'}{|r-r'|}$$

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{\rho_L dz'}{|r-r'|^3} (r-r') \rightarrow (1)$$

$r = y a_y = \rho a_\rho$ Let Co-ordinates of $P = (0, y, 0)$
Point charge has $(0, 0, z')$

$$\boxed{r' = a_z z'}$$

$r - r' = \rho a_\rho - a_z z'$

$|r-r'| = \sqrt{\rho^2 + (-z')^2}$

$$\boxed{|r-r'| = (\rho^2 + (z')^2)^{1/2}}$$

$$(1) \Rightarrow dE = \frac{1}{4\pi\varepsilon_0} \frac{\rho_L dz'}{[\rho^2 + (z')^2]^{3/2}} [\rho a_\rho - a_z z']$$

$$dE_\rho = \frac{1}{4\pi\varepsilon_0} \frac{\rho_L dz'}{[\rho^2 + (z')^2]^{3/2}} \times 1$$

The electric field intensity due to an infinite line length charge is given by,

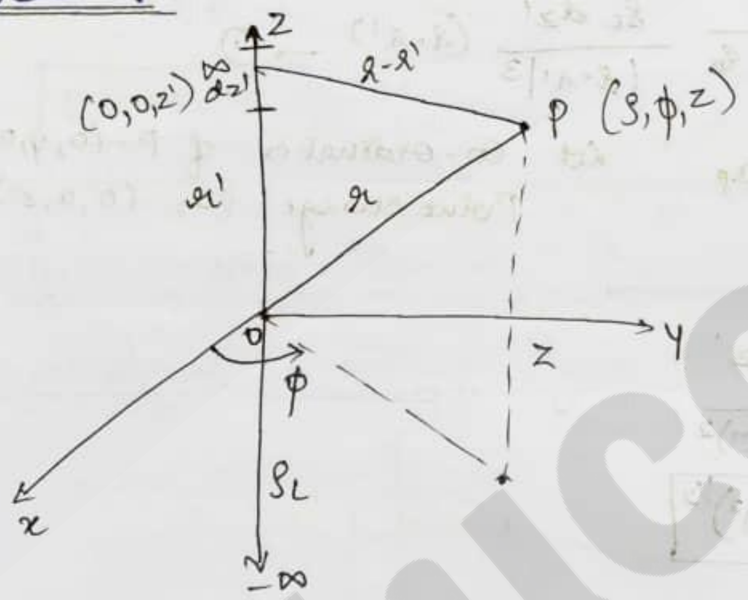
$$dE_\rho = \int_{-\infty}^{\infty} \frac{1}{4\pi\varepsilon_0} \frac{\rho_L dz'}{[\rho^2 + (z')^2]^{3/2}}$$

$$\boxed{E_\rho = \frac{\rho_L}{2\pi\varepsilon_0 \rho}}$$

Note: For a line charge the electric field is inversely proportional to the distance between the line charge and the point. where as for a point charge. The electric field is inversely proportional to square of the distance between the point charge and the point

---

ii) Case 2 :



$$dE = \frac{1}{4\pi\epsilon_0}\frac{\rho_L dz'}{|r-r'|^2} a_{r-r'}$$

$$r = \rho a_\rho + \cancel{\phi a_\phi}^{0} + z a_z$$

$$\boxed{r = \rho a_\rho + z a_z}$$

$$r' = a_z z'$$

$$r - r' = \rho a_\rho + z a_z - a_z z'$$

$$r - r' = \rho a_\rho + a_z[z-z']$$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{\rho_L dz'}{|\rho a_\rho + a_z[z-z']|}\ \frac{\rho a_\rho + a_z[z-z']}{|\rho a_\rho + a_z[z-z']|}$$

$$|r-r'| = \sqrt{\rho^2 + (z-z')^2}$$

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{\rho_L \, dz'}{[\rho^2 + (z-z')^2]^{3/2}} \times \rho a_\rho + a_z [z-z'].$$

Electric field intensity due to infinite line length charge

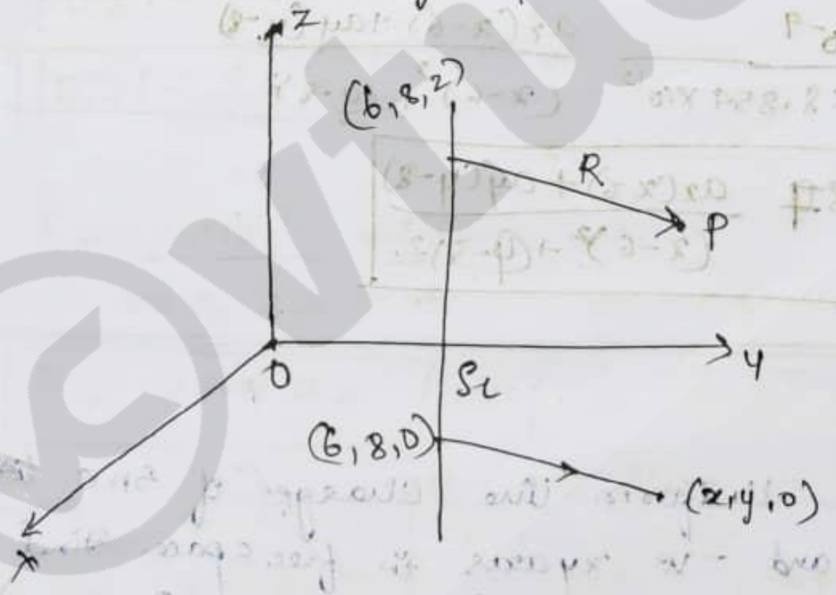
$$dE_\rho = \int_{-\infty}^{\infty} \frac{1}{4\pi\varepsilon_0} \frac{\rho_L \, dz'}{[\rho^2 + (z-z')^2]^{3/2}} \times \rho a_\rho + a_z (z-z')$$

$$\boxed{E = \frac{\rho_L \, a_\rho}{2\pi\varepsilon_0 \rho}}$$

$$\boxed{E = \frac{\rho_L}{2\pi\varepsilon_0 \rho}}$$

iii) Case 3 :

The line charge parallel to z-axis.



An infinite line charge parallel to z-axis at $x=6$, $y=8$, is located. The distance between the point P and the line charge is r.

$$E = \frac{\rho_L}{2\pi\varepsilon_0 \rho} a_\rho$$

$$E = \frac{\rho_L}{2\pi\varepsilon_0 |R|} a_R$$

$$|R| = x a_x + y a_y + \cancel{z a_z} - (6a_x + 8a_y + \cancel{z a_z})$$

$$= x a_x + y a_y - 6a_x - 8a_y$$

$$\underline{R = a_x(x-6) + a_y(y-8)}$$

$$|R| = \sqrt{(x-6)^2 + (y-8)^2}$$

$$E = \frac{\rho_L}{2\pi\varepsilon_0 [(x-6)^2 + (y-8)^2]^{1/2}} \frac{a_x(x-6) + a_y(y-8)}{[(x-6)^2 + (y-8)^2]^{1/2}}$$

$$E = \frac{\rho_L \; a_x(x-6) + a_y(y-8)}{2\pi\varepsilon_0 [(x-6)^2 + (y-8)^2]}$$

If $\rho_L = 5nC$

$$E = \frac{5\times10^{-9}}{2\times\pi\times8.854\times10^{-12}} \frac{a_x(x-6) + a_y(y-8)}{(x-6)^2 + (y-8)^2}$$

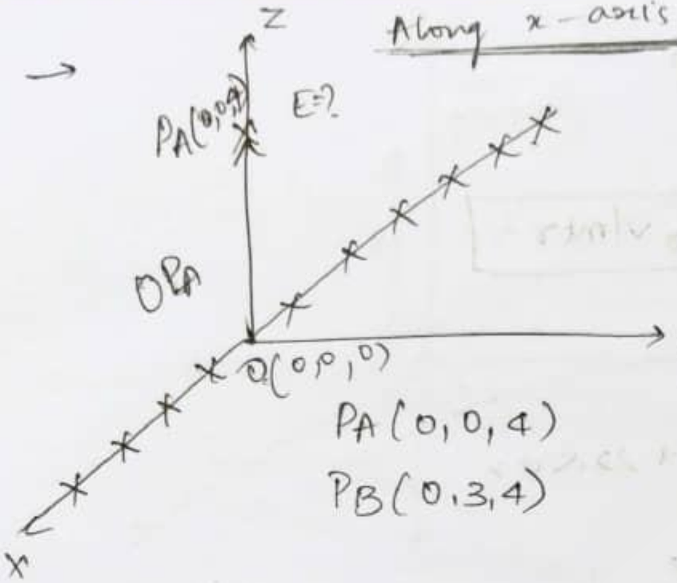
$$\boxed{E = 89.87 \frac{a_x(x-6) + a_y(y-8)}{(x-6)^2 + (y-8)^2}}$$

- Problems:

1. An infinite uniform line charge of 5nC lies along +ve and -ve xy axis in free space. Find electric field intensity at $P_A(0, 0, 4)$ & $P_B(0, 3, 4)$

Q1 →

Along x-axis



$P_A(0,0,4)$

$P_B(0,3,4)$

$$E = \frac{\rho_L}{2\pi \varepsilon_0 \rho} a_\rho$$

$$E = \frac{\rho_L}{2\pi \varepsilon_0 |OP_A|} a_{OP_A}$$

$OP_A = 4a_z$

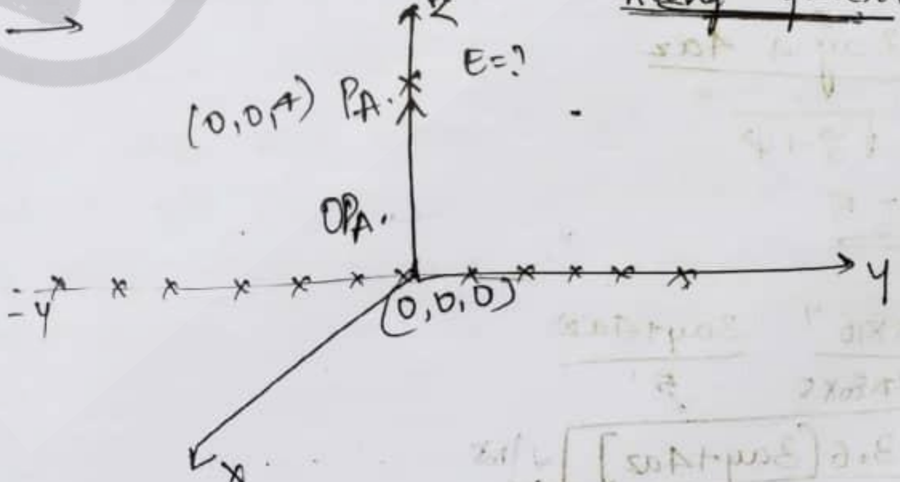
$|OP_A| = \sqrt{4^2}$

$$\boxed{|OP_A| = 4\,mt}$$

$$E = \frac{5\times10^{-9}}{2\pi \varepsilon_0\, 4m} \cdot \frac{4a_z}{4}$$

$$\boxed{E_x = 22.5\, a_z} \text{ Volts/mtr}$$

→ Along y-axis



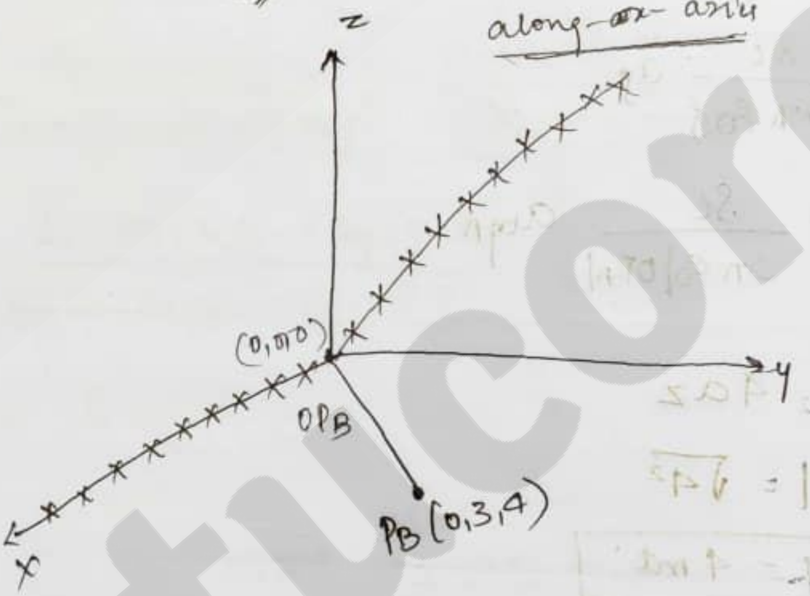
$$E = \frac{\rho_L}{2\pi\varepsilon_0 \rho} a_\rho$$

$$\boxed{E_y = 22.50\, a_z \text{ V/mtr}}$$

$$E_{P_A} = E_x + E_y$$

$$= 22.5 a_z + 22.5 a_z$$

$$\boxed{E_{P_A} = 45 a_z}$$



$$E = \frac{\rho_L}{2\pi\varepsilon_0 \rho} a_\rho$$

$$E = \frac{\rho_L}{2\pi\varepsilon_0 |OP_B|} a_{OP_B}$$

$$OP_B = 3a_y + 4a_z$$

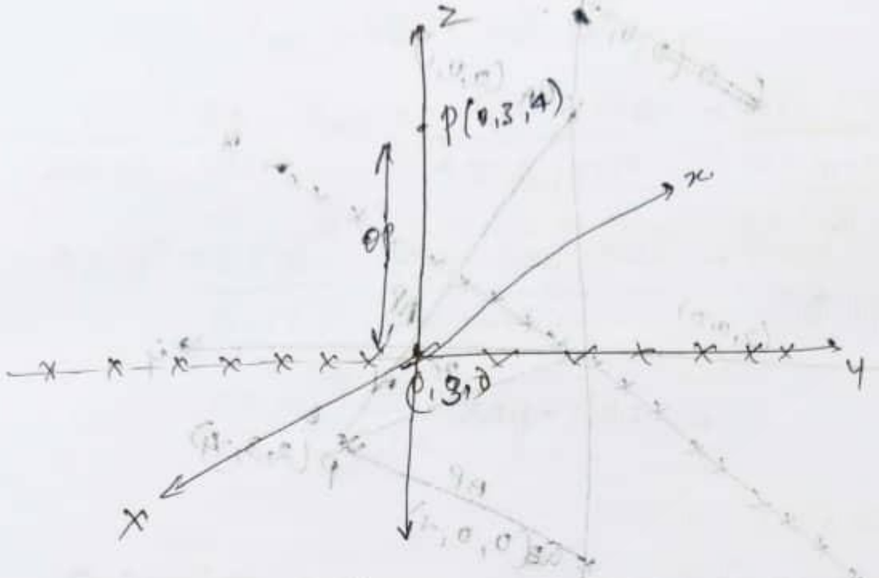
$$|OP_B| = \sqrt{3^2 + 4^2}$$

$$|OP_B| = 5$$

$$E = \frac{5 \times 10^{-9}}{2\pi\varepsilon_0 \times 5} \cdot \frac{3a_y + 4a_z}{5}$$

$$\boxed{E = 3.6\,[3a_y + 4a_z]} \text{ V/mtr}$$

→



$$E_p = \frac{\rho_L}{2\pi \varepsilon_0 |OP|} a_{op}$$

$$Op = \cancel{3a_y} + 4a_z - \cancel{3a_y} = 4a_z$$

$$|OP| = \sqrt{4^2} = \underline{4mt}$$

$$E_p = \frac{5\times10^{-9}}{2\pi\varepsilon_0\times4} \frac{a_p}{|OP|}$$

$$= \frac{5\times10^{-9}}{2\pi\varepsilon_0\times4} \frac{\cancel{4}a_z}{\cancel{4}}$$

$$\boxed{E_p = 22.46\ a_z} \ N/mt.$$

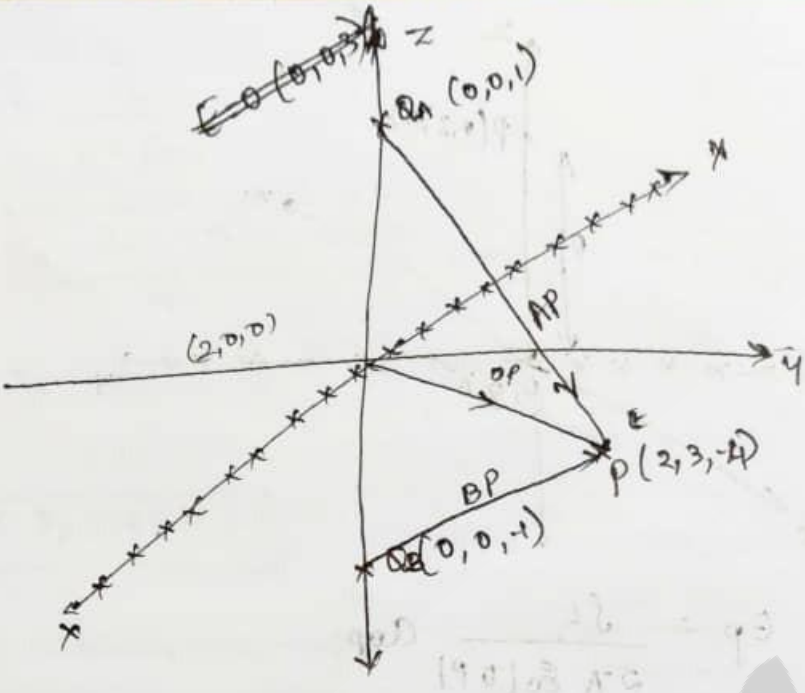
---

2. An infinite uniform line charge with charge density $\rho_L = 2nC/m$. lies along x-axis in free space while the point charges of 8nC each are located at (0,0,1) and (0,0,-1).

i) Find E at (2,3,-4)

ii) To what value should ρ_L be changed to cause the electric field to be zero at (0,0,3)

Sol i)



i) $AP = 2a_x + 3a_y - 4a_z - a_z$

$\underline{AP = 2a_x + 3a_y - 5a_z}$

$|AP| = \sqrt{2^2 + 3^2 + -5^2}$

$= \sqrt{4 + 9 + 25}$

$\underline{|AP| = \sqrt{38} = 6.16 \text{ mtr}}$

$BP = 2a_x + 3a_y - 4a_z + a_z$

$\underline{BP = 2a_x + 3a_y - 3a_z}$

$|BP| = \sqrt{2^2 + 3^2 + -3^2}$

$= \sqrt{4 + 9 + 9}$

$\underline{|BP| = \sqrt{22} = 4.69 \text{ mtr}}$

$OP = \cancel{2a_x} + 3a_y - 4a_z - \cancel{2a_x}$

$\underline{OP = 3a_y - 4a_z}$

$|OP| = \sqrt{3^2 + -4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$\underline{|OP| = 5 \text{ mtr}}$

$$E_P = E_{QA} + E_{QB} + E_L$$

$$E_P = \frac{1}{4\pi\epsilon_0}\frac{Q_A}{|AP|^2}a_{AP} + \frac{Q_B}{4\pi\epsilon_0|BP|^2}a_{BP} + \frac{\rho_L}{2\pi\epsilon_0|OP|}a_{op}.$$

$$= \frac{9\times10^9 \times 8\times10^{-9}}{(6.16)^2}\frac{2a_x+3a_y-5a_z}{(6.16)} + \frac{9\times10^9\times8\times10^{-9}}{(4.69)^3}2a_x+3a_y-3a_z$$

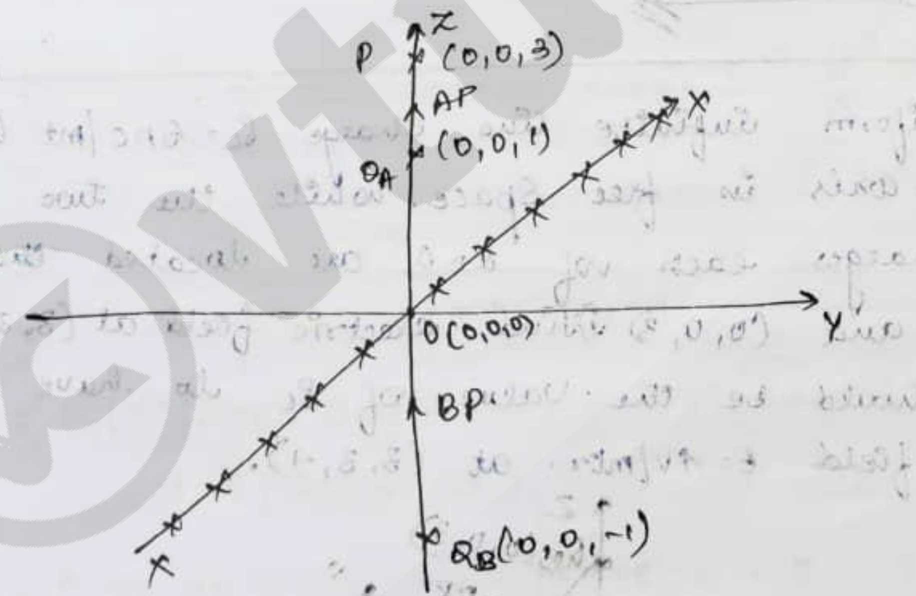
$$+ \frac{2\times10^{-9}}{2\pi\epsilon_0(5)^2}3a_y-4a_z$$

$$E_P = 0.308(2a_x+3a_y-5a_z) + 0.697(2a_x+3a_y-3a_z) + 1.43(3a_y-4a_z)$$

$$= 0.616a_x + 0.924a_y - 1.54a_z + 1.394a_x + 2.091a_y - 2.091a_z + 4.29a_y - 5.72a_z$$

$$\boxed{E_P = 2.01a_x + 7.305a_y + 9.351a_z}$$

ii)



$AP = 3a_z - a_z$
$\underline{AP = 2a_z}$

$|AP| = \sqrt{2^2}$
$\underline{|AP| = 2\ mtr}$

$OP = 3a_z - 0$
$\underline{OP = 3a_z}$

$|OP| = \sqrt{3^2}$
$\underline{|OP| = 3\ mtr.}$

$BP = 3a_z + a_z$
$\underline{BP = 4a_z}$

$|BP| = \sqrt{4^2}$
$\underline{|BP| = 4\ mtr}$

$$E = \frac{Q_A}{4\pi\varepsilon_0|AP|^2} a_{AP} + \frac{Q_B}{4\pi\varepsilon_0|BP|^2} a_{BP} + \frac{\rho_L}{2\pi\varepsilon_0|OP|} a_{OP}$$

$$0 = \frac{8\times10^{-9}\times9\times10^{9}}{(2)^3}(2a_z) + \frac{8\times10^{-9}\times9\times10^{9}}{(4)^3}(4a_z) + \frac{\rho_L}{2\pi\times8.854\times10^{-12}(8)}(3a_z)$$

$$0 = 9(2a_z) + 1.125(4a_z) + \frac{x\,(3a_z)}{1.66893\times10^{-10}}$$

$$0 = 18a_z + 4.5a_z + \frac{\rho_L\,3a_z}{1.66\times10^{-10}}$$

$$\frac{\rho_L\,3a_z}{1.66\times10^{-10}} = -18a_z - 4.5a_z$$

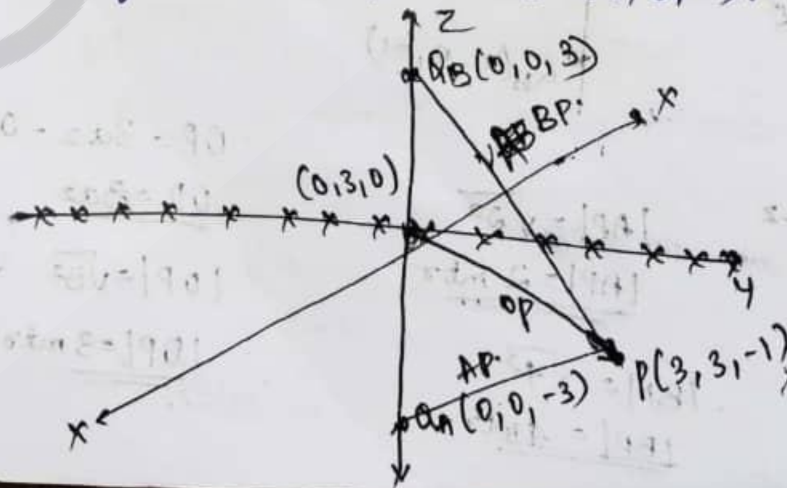
$$\rho_L = \frac{-22.5\cancel{a_z}\times1.66\times10^{-10}}{3\cancel{a_z}}$$

$$\boxed{\rho_L = -3.75\,nC}$$

---

3. An uniform infinite line charge $\rho_L = 5nC/mt$ lie along y-axis in free space. while the two point charges each of $2nC$ are located ~~that~~ $(0,0,-3)$ and $(0,0,3)$ i) Find electric field at $(3,3,-1)$ ii) what should be the value of ρ_L to have electric field $E = 4V/mtr.$ at $(3,3,-1)$.

Sol



8) $E_P = E_{QA} + E_{QB} + E_l$.

$AP = 3a_x + 3a_y - a_z + 3a_z$

$AP = 3a_x + 3a_y + 2a_z$

$|AP| = \sqrt{3^2+3^2+2^2} = \sqrt{9+9+4} = \sqrt{22} = 4.69\text{mt}$

$BP = 3a_x + 3a_y - a_z - 3a_z$

$BP = 3a_x + 3a_y - 4a_z$

$|BP| = \sqrt{3^2+3^2+(-4)^2} = \sqrt{9+9+16} = \sqrt{34} = 5.83$

$OP = 3a_x + \cancel{3a_y} - a_z - \cancel{3a_y}$

$OP = 3a_x - a_z$

$|OP| = \sqrt{3^2+(-1)^2} = \sqrt{10} = 3.16$.

$$E_P = \frac{1}{4\pi\varepsilon_0}\frac{Q_A}{|AP|^2} a_{AP} + \frac{Q_B}{4\pi\varepsilon_0 |BP|^2} a_{BP} + \frac{\rho_l}{2\pi\varepsilon_0 |OP|} a_{OP}$$

$$= \frac{9\times10^9 \times 2\times10^{-9}}{(4.69)^3}(3a_x+3a_y+2a_z) + \frac{9\times10^9\times2\times10^{-9}}{(5.83)^3}(3a_x+3a_y-4a_z)$$

$$+ \frac{5\times10^{-9}}{2\pi\times8.854\times10^{-12}(3.16)^2}(3a_x - a_z)$$

$$= 0.174(3a_x + 3a_y + 2a_z) + 0.09(3a_x+3a_y-4a_z) + 9(3a_x - a_z)$$

$$= 0.522a_x + 0.522a_y + 0.348a_z + 0.27a_x + 0.27a_y - 0.36a_z + 27a_x - 9a_z$$

$$\boxed{E_P = 27.79a_x + 0.79a_y - 9a_z}$$

ii) $E = 4\,V/mtr$; $\rho_L = ?$

$$4 = \frac{1}{4\pi\varepsilon_0}\frac{Q_A}{|AP|^2}a_{AP} + \frac{Q_B}{4\pi\varepsilon_0|BP|^2}a_{BP} + \frac{\rho_L}{2\pi\varepsilon_0|OP|}a_{OP}$$

$$\cancel{4 = 9\times10^9 \times 2\times10^{-9}}$$

$$4 = 0.174(3a_x + 3a_y + 2a_z) + 0.09(3a_x + 3a_y - 4a_z) + \frac{\rho_L}{2\pi\times8.854\times(3.16)^2}(3a_x - a_z)$$

$$4 - 0.522a_x - 0.522a_y - 0.348a_z - 0.27a_x - 0.27a_y + 0.36a_z = \frac{\rho_L}{5.55\times10^{-10}}(3a_x - a_z)$$

$$\left(\frac{4 - 0.79a_x - 0.79a_y - 0.7a_z}{3a_x - a_z}\right)5.55\times10^{-10} = \rho_L$$

$$\cancel{\rho_L = \frac{4}{3a_x - a_z}}$$

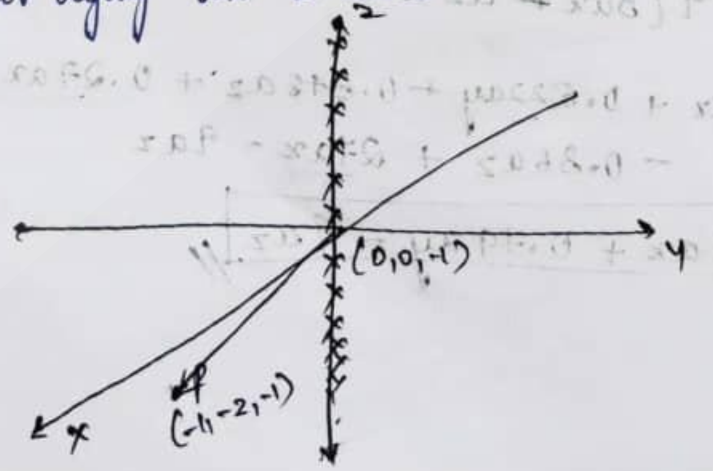
$$\left(\frac{2\times10^{-9} - 3.95\times10^{-10}a_x - 3.95\times10^{-10}a_y - 3.5\times10^{-10}a_z}{3a_x - a_z}\right) = \rho_L$$

$$\boxed{\rho_L = \frac{2n - 0.39n a_x - 0.39n a_y - 0.85n a_z}{3a_x - a_z}}$$

---

4. Find the Electric field at a point P of P(-1, -2, -1) while line charge with charge density $\rho_L = 5nc/mtr$ lying on z-axis

Sol



$OP = -a_x - 2a_y - \cancel{a_z} + \cancel{a_z}$

$OP = -a_x - 2a_y$

$|OP| = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5} = 2.23$

$$E_p = \frac{\rho_L}{2\pi \varepsilon_0 |OP|} a_\phi$$

$$= \frac{5 \times 10^{-9}}{2 \times \pi \times 8.85 \times 10^{-12} (2.23)^2} \left(\cancel{-a_x - 2a_y}\right)$$

$$= 18.07(-a_x - 2a_y)$$

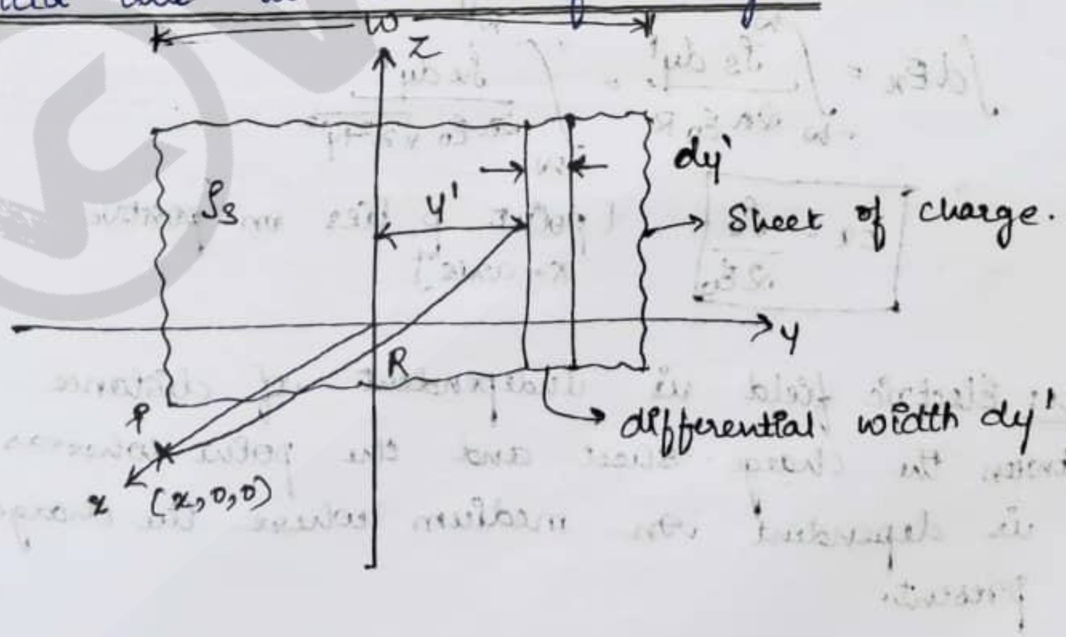
$$\boxed{E_p = -18.07 a_x - 36.14 a_y}$$

$$E_p = \frac{\rho_L}{2\pi \varepsilon_0 \rho}$$

$$= \frac{5 \times 10^{-9}}{2 \times \pi \times 8.854 \times 10^{-12} \times 2.23}$$

$$\boxed{E_p = 40.3}$$

- **Field due to Sheet of charge**



A sheet of charge is placed in y-z plane. To find the electric field at point p which lies on x-axis consider a differential width sheet dy'.

The line charge density ρ_L is given by the surface charge density ρ_S into differential ~~charge~~ width dy'

$$\rho_L = \rho_S \, dy'$$

The distance between the line charge to the point p which is on x-axis is given by

$$R = \sqrt{x^2+y'^2}$$

Therefore, the electric field at a point p due to a differential width sheet dy' is given by

$$E = \frac{\rho_L}{2\pi \varepsilon_0 \rho}$$

∴ The electric field at a point p on x-axis due to entire sheet charge is given by.

$$\int dE_x = \int_{-\infty}^{\infty} \frac{\rho_S \, dy'}{2\pi \varepsilon_0 R} = \int_{-\infty}^{\infty} \frac{\rho_x \, dy'}{2\pi \varepsilon_0 \sqrt{x^2+y'^2}}$$

$$\boxed{E_x = \frac{\rho_S}{2\varepsilon_0}}$$ [point p lies on positive x-axis]

• **Note:** Electric field is independent of distance between the charge sheet and the point whereas it is dependent on medium where the charge is present.

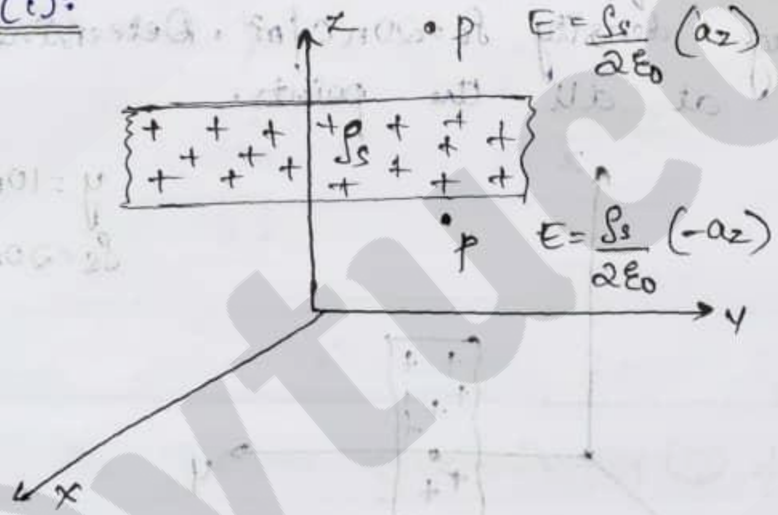
$$\boxed{E_x = \frac{-\rho_s}{2\varepsilon_0}}$$ [point p lies on negative x-axis]

To come out of the sign problem, the electric field due to a sheet of charge is represented as

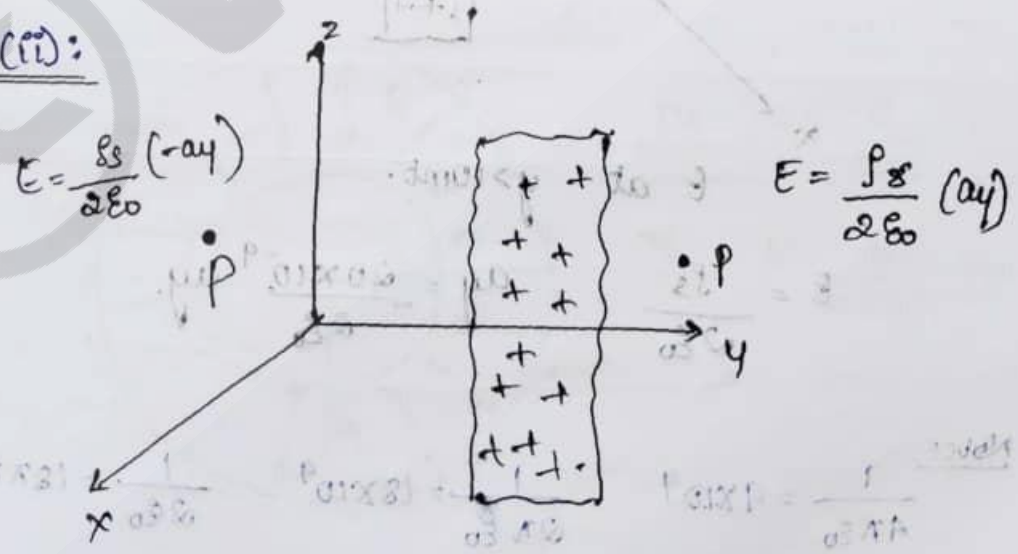
$$E = \frac{\rho_s}{2\varepsilon_0} a_n$$

The unit vector an represents that the direction of the electric field is outward as well as perpendicular to point p.

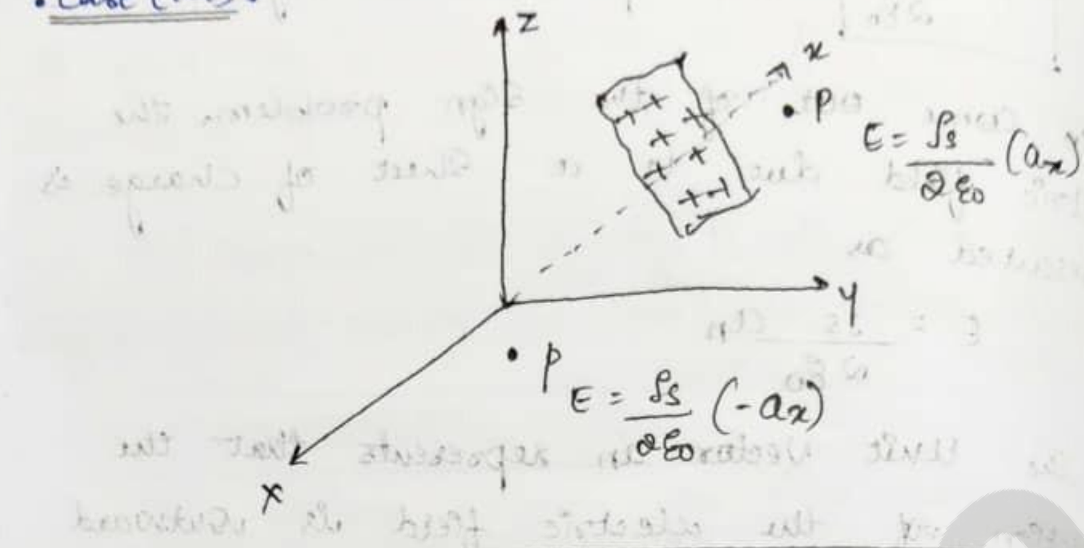
- Case (i):



- Case (ii):

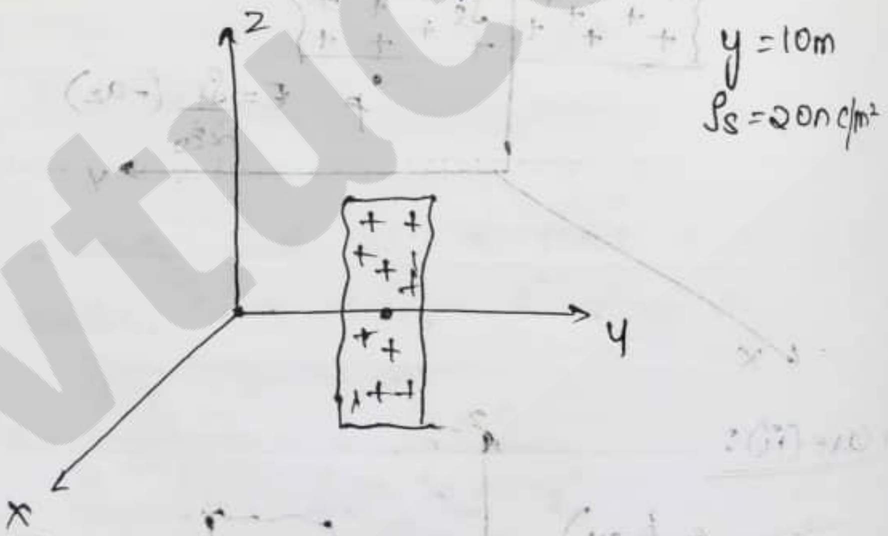


• Case (iii):



1. A sheet of charge lies on y at y = 10mtr in the form of infinite square sheet with surface charge density $\rho_s = 20 nC/m^2$. Determine electric field at all the points.

Sol



E at y>10mt.

$$E = \frac{\rho_s}{2\varepsilon_0} a_y = \frac{20\times10^{-9}}{2\varepsilon_0} a_y.$$

Note:
$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \quad \frac{1}{2\pi\varepsilon_0} = 18\times10^9 \quad \frac{1}{2\varepsilon_0} = 18\pi\times10^9$$

$a_y = \frac{20\times10^{-9}}{2\varepsilon_0} a_y$

$= 18\pi \times 10^{9} \times 20 \times 10^{-9} a_y$

$\boxed{E = 360\pi\ a_y\ V/mtr}$

E at $y < 10$mt.

$E = \frac{\rho_s}{2\varepsilon_0}(-a_y)$

$= 20\times10^{-9} \times 18\pi \times 10^{9}(-a_y)$

$\boxed{E = -360\pi\ a_y\ V/mtr}$

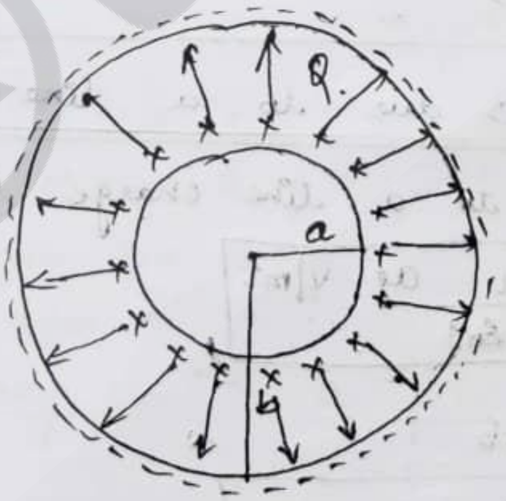
E at $y = 10$.

$E = \frac{\rho_s}{2\varepsilon_0}(0)$ $\because a_y = 0$

$\boxed{E = 0}$

- Electric flux Density (D)

The Electric flux Density (D) due to point charge.



$D_{r=a} = \frac{Q}{4\pi a^2} a_r$

$D_{r=b} = \frac{Q}{4\pi b^2} a_r$

If $a=0$ the inner circle functions as the point charge Q then the electric flux density D due to a point charge Q at any radial distance r is given by,

$$\boxed{D = \frac{Q}{4\pi r^2} a_r}$$

> The relationship E and D:

$$E = \frac{1}{4\pi \epsilon_0} \frac{Q}{r^2} a_r \longrightarrow (1)$$

$$D = \frac{Q}{4\pi r^2} a_r \longrightarrow (2)$$

Divide (2) by (1)

$$\frac{D}{E} = \frac{\frac{Q}{4\pi r^2} a_r}{\frac{1}{4\pi \epsilon_0} \frac{Q}{r^2} a_r} = \epsilon_0$$

$$\boxed{D = \epsilon_0 E}$$

For a given Electric flux density is $\epsilon_0 E$ (ϵ_0 times of EF)

> The flux density D due to a line charge

The E.F due to a line charge is given by,

$$\boxed{E = \frac{\rho_L}{2\pi \epsilon_0 \rho} a_\rho \ \text{V/mt}}$$

$$D = \epsilon_0 E$$

$$\boxed{D = \epsilon_0 \left[\frac{\rho_L}{2\pi \epsilon_0 \rho} a_\rho \right]}$$

$$\boxed{D = \frac{\rho_L}{2\pi\rho} a_\rho}$$

- **The flux density D due to a surface charge density.**

$$E = \frac{\rho_s}{2\epsilon_0} a_n$$

$$D = \epsilon_0 E$$

$$\boxed{D = \frac{\rho_s}{2} a_n}$$

1. The flux due to point charge

$$D = \frac{Q}{4\pi r^2} a_r$$

2. Due to line charge

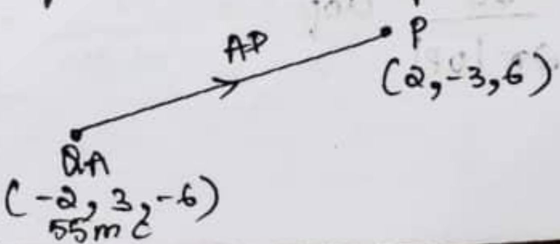
$$D = \frac{\rho_L}{2\pi\rho} a_\rho$$

3. Due to surface charge

$$D = \frac{\rho_s}{2} a_n$$

1. Calculate D in rectangular co-ordinates at point p(2,-3,6) produced by
a) a point charge $Q_A = 55mC$ at Q(-2, 3,-6)
b) a uniform line charge with a charge density $\rho_L = 20 mC/m$ on x-axis.

Sol. a) Density due to a point charge



$$D = \frac{Q}{4\pi r^2} a_r$$

$$D = \frac{Q}{4\pi |AP|^2} a_{AP}$$

$AP = 2a_x - 3a_y + 6a_z - (-2a_x + 3a_y - 6a_z)$

$AP = 4a_x - 6a_y + 12a_z$

$|AP| = \sqrt{16+36+144}$

$\boxed{|AP| = 14mt}$

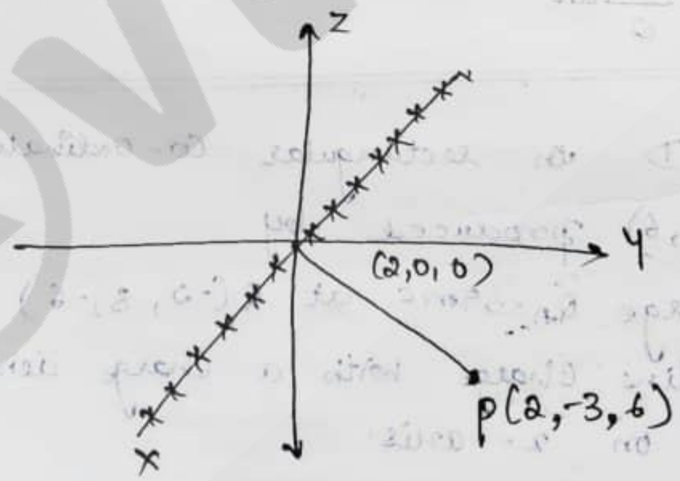
$$D = \frac{55\times10^{-3}}{4\pi (14)^2 \times 14}(4a_x - 6a_y + 12a_z)$$

$D = 1.595 \times 10^{-6}(4a_x - 6a_y + 12a_z)$

$\boxed{D = 1.6(4a_x - 6a_y + 12a_z)\ \mu C}$

b) Due to a line charge

$$D = \frac{\rho_L}{2\pi \rho} a_\rho \qquad \rho_L = 20mc$$



$$D = \frac{\rho_L}{2\pi |OP|} a_{op}$$

$OP = \cancel{2a_x} - 3a_y + 6a_z - (\cancel{2a_x})$

$$\boxed{OP = -3a_y + 6a_z}$$

$|OP| = \sqrt{9 + 36} = \sqrt{45} = \underline{6.708\, mt}$

$$D = \frac{20 \times 10^{-3}}{2\pi (6.708)^2} (-3a_y + 6a_z)$$

$$\boxed{D = 70.7 (-3a_y + 6a_z) \mu C}$$

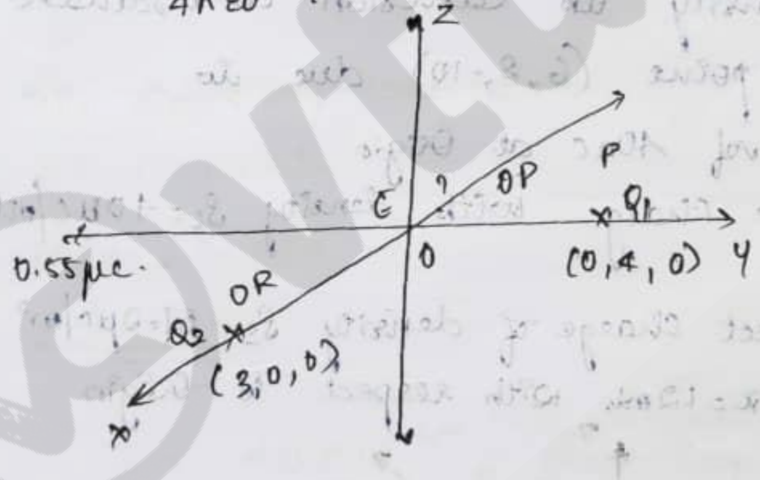
---

2. Find the E.F Intensity at E and electric flux density D at the Origin due to $Q_1 = 0.35\mu C$ at $(0,4,0)$ and $Q_2 = 0.55\mu C$ at $(3,0,0)$

Sol:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{r^2} a_r$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{|OP|^2} a_{op}$$



$OP = 4a_y$

$|OP| = \sqrt{4^2} = \underline{4mt}$

$$E_1 = \frac{1}{4\pi\varepsilon_0} \frac{0.35 \times 10^{-6}}{(4mt)^2} \frac{4a_y}{4}$$

$$\boxed{E_1 = 196.87 a_y \ V/mt}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0} \frac{0.55\times10^{-6}}{(3mt)^2} \frac{3a_x}{3}$$

$$OR = 3a_x$$
$$|OR| = \sqrt{3^2} = 3mt$$

$$\boxed{E_2 = 550\,a_x \; V/mt}$$

$$E = E_1 + E_2$$
$$\boxed{E = 196.87a_y + 550\,a_x}$$

$$D = \varepsilon_0 E$$
$$\boxed{D = 8.854\times10^{-12}(196.87a_y + 550\,a_x)}$$

~~Q. Calculate the flux density in rectangular~~

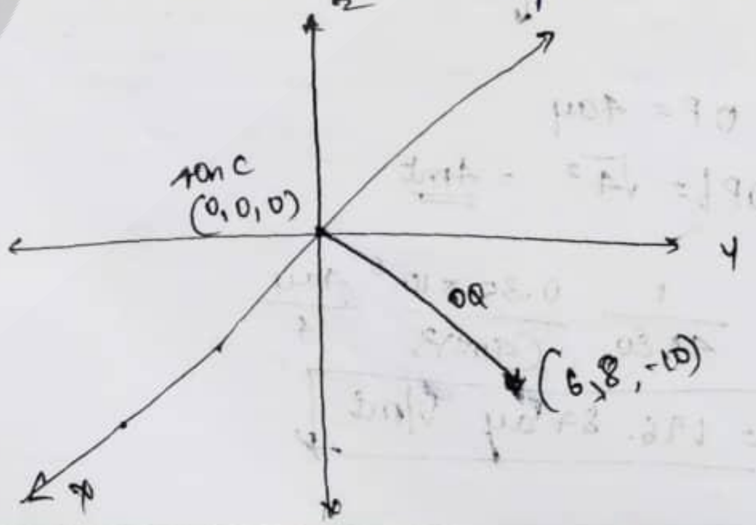
3. Find flux density in Cartesian Co-ordinate System at a point (6, 8, -10) due to

i) Point charge of 40nC at Origin

ii) A uniform line charge with density $\rho_L = 40\mu C/mt$ on z-axis

iii) A uniform Sheet charge of density $\rho_S = 57.2\mu C/m^2$ on the plane at $x = 12$mt. with respect to Origin

Sol



i) $D = \frac{1}{4\pi} \frac{Q}{|OQ|^2} a_{OP}$

$OP = 6a_x + 8a_y - 10a_z$

$|OP| = \sqrt{36+64+100}$

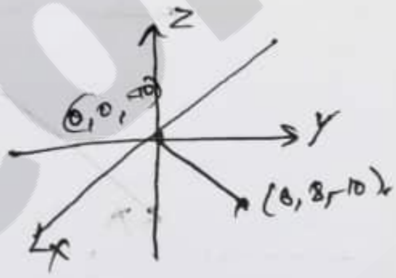
$|OP| = 14.14 mt$

$D = 9\times10^{9} \times \frac{40\times10^{-9}}{(14.14)^3}(6a_x + 8a_y - 10a_z)$

$= \cancel{0.127(6a_x}$

$$\boxed{D = 1.125\times10^{-12}(6a_x + 8a_y - 10a_z}$$

Sr :(ii) $D = \frac{\rho_L}{2\pi\rho} a_\rho$ c/m.



$D = \frac{40\times10^{-6}}{2\pi |OP|} a_\rho$

$OP = 6a_x + 8a_y - 10a_z + 10a_z$

$OP = 6a_x + 8a_y$

$|OP| = \sqrt{36+64}$

$|OP| = 10mt$

$D = \frac{40\times10^{-6}}{2\times\pi\times10} \times \frac{6a_x + 8a_y}{10}$

$$\boxed{D = 6.366\times10^{-8}(6a_x + 8a_y}$$

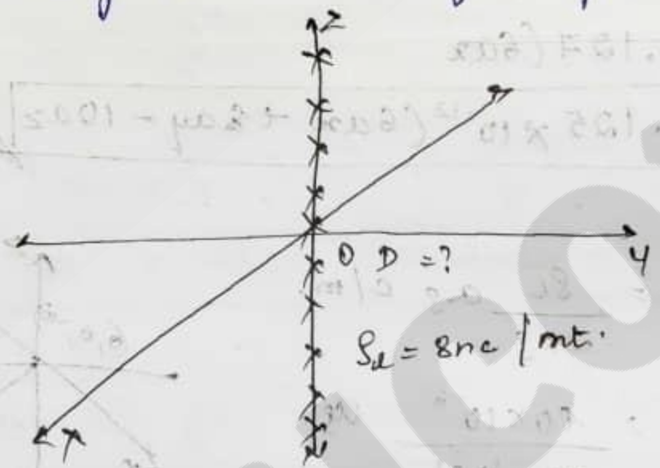
iii) $D = \frac{\rho_s}{2} a_n$

$= \frac{57.2 \times 10^{-6} \, C/m^2}{2} \times (-a_x)$

$$\boxed{D = -28.6 \, a_x \ \mu C/m^2}$$

4. Find the flux density in the Origin due to an Uniform line charge of 8nC lying along z-axis in free space.

Sol



Let us take $\rho = 3mt$

E.F E due to a line charge at a distance $\rho = 3mt$.

$E = \frac{\rho_l}{2\pi\epsilon_0 \rho} a_\rho$

$= \frac{8 \times 10^{-9}}{2\pi \times (8.854 \times 10^{-12})(3)} (a_\rho)$

$$\boxed{E = 47.93 a_\rho}$$

$D = \epsilon_0 E$

$D = 8.854 \times 10^{-12} \times 47.93 \, a_\rho$

$$\boxed{D = 4.2 \times 10^{-10} a_\rho}$$

5. Given Electric flux density $D = 0.3r^2 a_r$ nC/m² in free Space. find E at $p(r = 2mt, \theta = 25°, \phi = 90°)$

Sol

$$D = \varepsilon_0 E$$

$$E = \frac{D}{\varepsilon_0}$$

$$= \frac{0.13\ r^2\ a_r\ nC/m^2}{8.854 \times 10^{-12}}$$

$$= \frac{(0.3)(2mtr)^2\ a_r \times 10^{-9}/m^2}{8.854 \times 10^{-12}}$$

$$\boxed{E = 185.53\ a_r\ V/m}$$