

# BEC304 - NETWORK ANALYSIS

## MODULE-1 BASIC CONCEPTS

→ Classification of Networks

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1) Linear, n/w & non linear network

Linear network: If the  $R, L, C$  offered by an element does not change linearly with the change in applied voltage or current in an element is called linear element.

non linear network: If the  $R, L, C$  offered by an element ~~not~~ change non linearly with the change in applied voltage or current in an element is called non linear element.

2) unilateral / Bilateral network:

unilateral n/w: Properties or characteristics change with direction of operation.

Ex: Diode, BJT, FET

Bilateral network: properties or characteristics remains same in either direction.

3) Active & Passive network:

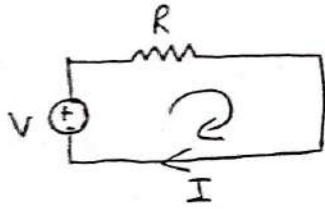
Active n/w: It is one which contains atleast one energy source either voltage or current.

Ex: BJT, power supply, opamp.

Passive n/w: It is one which does not contain any source to drive the circuit.

Ex:  $R, L, C$

$\rightarrow R$  in ohms (Resistor)

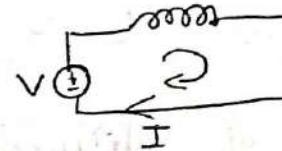


From Ohm's law

$$V = IR$$

$$I = \frac{V}{R}$$

Inductor ( $L$  in Henry)



From Ohm's law

$$V = I X_L$$

$$X_L = 2\pi f L$$

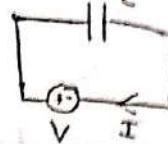
$$V = L \frac{dI}{dt}$$

$$\int V \cdot dt = L \int \frac{dI}{dt} dt$$

$$\int V \cdot dt = L I$$

$$I = \frac{1}{L} \int V \cdot dt$$

Capacitor ( $C$  in Farad)



From Ohm's law

$$V = I X_C$$

$$X_C = \frac{1}{2\pi f C}$$

$$V = \frac{1}{C} \int I dt$$

Apply diff

$$\frac{dV}{dt} = \frac{1}{C} \int dt \frac{dI}{dt}$$

$$\frac{dV}{dt} = \frac{1}{C} I$$

$$I = C \frac{dV}{dt}$$

#### 4. Lumped and distributed network.

circuit having elements  $R, L, C$  can be separated physically is called Lumped network

Ex: simple electrical n/w

#### Distributed network :

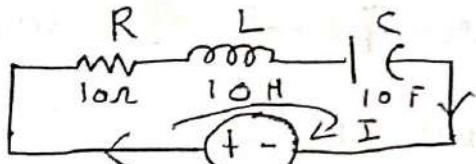
ckt's having elements  $R, L, C$  cannot be separated physically is called distributed network :

Ex: Transmission line, where  $R-L-C$  are distributed along the length & cannot be shown as separate element.

→ Impedance ( $Z$ ) ( $\Omega$ )

It is defined as the total opposition to the flow of electrons.

If a ckt consist of RLC



Voltage,  $f = 100 \text{ Hz}$ .

$$Z = R + jX_L - jX_C$$

$$Z = R + j(X_L - X_C) \dots \textcircled{1}$$

WKT

$$X_L = 2\pi f L$$

$$X_L = 2 \times 3.14 \times 100 \times 10$$

$$X_L = 6283.18$$

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{2 \times \pi \times 100 \times 10}$$

$$X_C = 1.591 \times 10^{-4}$$

$$Z = 10 + j6283.18 - j1.591 \times 10^{-4}$$

$$Z = 10 + j(6283.179)$$

$$|Z| = \sqrt{a^2 + b^2}$$

$$|Z| = \sqrt{10^2 + 6283.179}$$

$$|Z| = 6283.18$$

$$I = \frac{V}{R}$$

$$I = \frac{100}{6283.18}$$

$$I = 0.0159$$

$$I = 15.9 \times 10^{-3} \text{ A}$$

## → Admittance (Y)

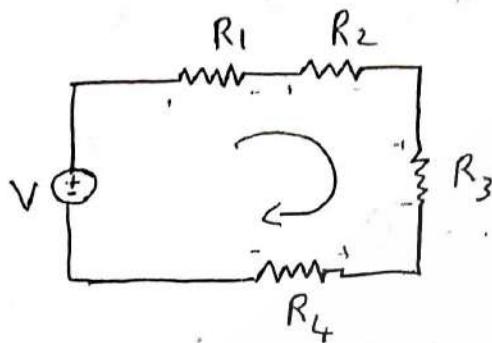
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unit is ( $\text{V}^{-1}$ ) mho

$$Y = \frac{1}{Z}$$

Admittance is the reciprocal of impedance

## → Kirchoff's Voltage law - (KVL)

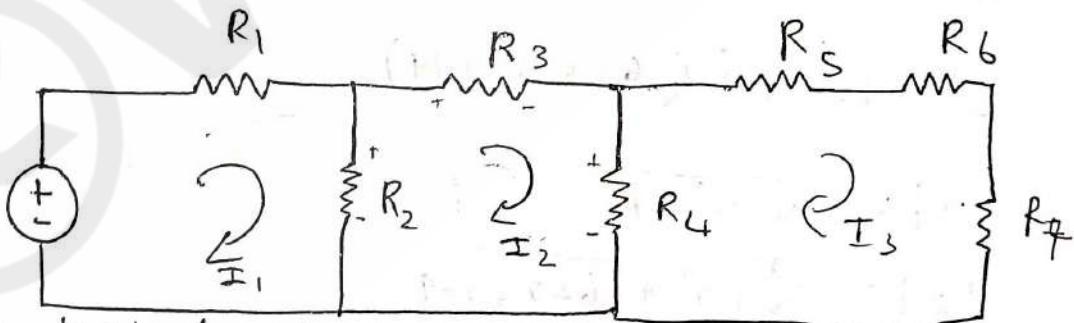


Applying KVL.

$$V - IR_1 - IR_2 - IR_3 - IR_4 = 0$$

$$V - I(R_1 + R_2 + R_3 + R_4) = 0$$

$$V = I(R_1 + R_2 + R_3 + R_4)$$



Apply KVL to loop-1

$$V - I_1 R_1 - (I_1 - I_2) R_2 = 0$$

$$V - I_1 R_1 - I_1 R_2 + I_2 R_2 = 0$$

$$V - I_1 (R_1 + R_2) + I_2 R_2$$

$$\boxed{V = I_1 (R_1 + R_2) - I_2 R_2}$$

$$-I_2 R_3 - I_2 R_4 + (I_2 - I_1) R_2 = 0$$

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$$-I_2 R_3 - I_2 R_4 - I_2 R_2 + I_1 R_2 = 0$$

$$-I_2 (R_3 + R_4 + R_2) + I_1 R_2 = 0$$

$$\boxed{I_1 R_2 = I_2 (R_3 + R_4 + R_2)}$$

loop 3 =

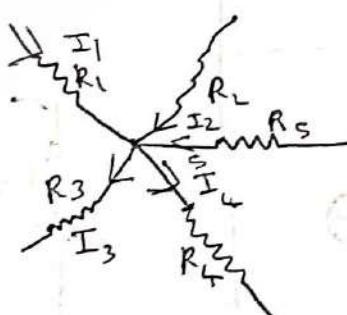
$$-I_3 R_5 - I_3 R_6 - I_3 R_7 - (I_3 - I_2) R_4 = 0$$

$$-I_3 (R_5 + R_6 + R_7 + R_4) + I_2 R_4 = 0$$

$$\boxed{I_2 R_4 = I_3 (R_5 + R_6 + R_7 + R_4)}$$

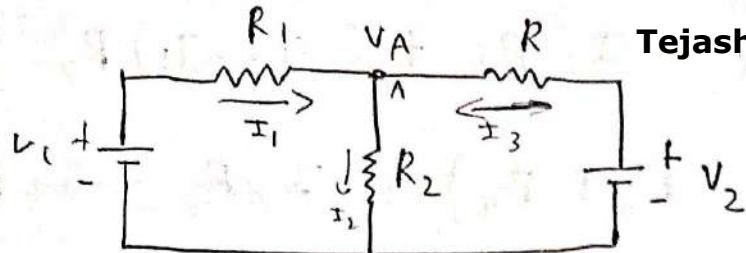
→ Kirchhoff's current law (KCL)

The sum of current entering into the node  
is equal sum of current leaving the node.



$$\boxed{I_1 + I_2 + I_3 = I_4}$$

Eg :



$$I_1 + I_3 = I_2$$

$$I_1 - I_2 + I_3 = 0$$

From ohm's law.

$$\frac{V_1 - V_A}{R_1} - \frac{V_A}{R_2} + \frac{V_2 - V_A}{R_3} = 0$$

### → Energy Sources:-

There are two basic energy source

1) Voltage

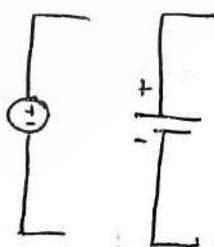
2) Current

It is broadly classified into two types

### → Independent source

- Voltage source can be ideal or practical
- Ideal voltage source is defined as the energy source which gives constant voltage with independent of current.

Ideal source



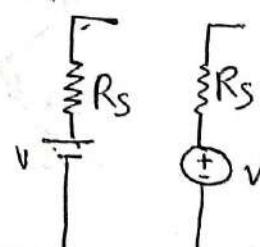
Ideal DC

Voltage source

Practical source



Ideal AC



Practical DC

Voltage source

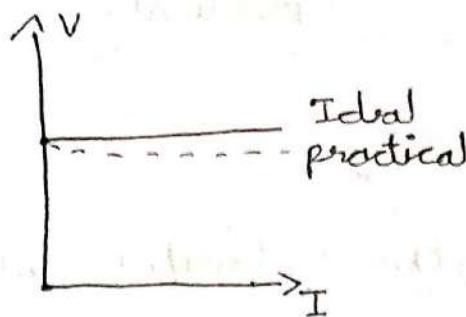


Practical AC

Voltage source

- All practical voltage source will have internal resistance called Source Resistance ( $R_S$ )

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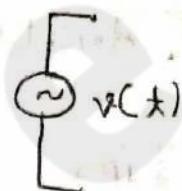
### Time Variant

\* Voltage source are again classified into :

→ Time variant AC Voltage source:

- Voltage varies w.r.t time

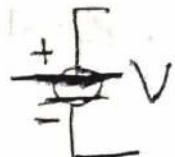
- It is denoted by small letters



→ Time invariant DC Voltage source:

- Voltage does not vary w.r.t time

- It is denoted by capital letters



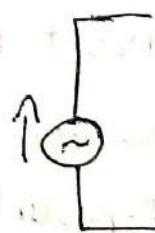
### Current source:

• current source can be ideal or practical

• In ideal current source current remains constant with independent of voltage



Ideal DC current  
Source

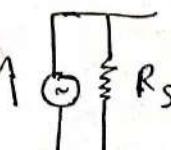


Ideal AC current  
Source

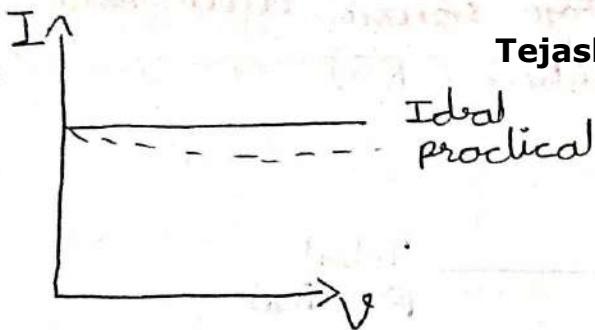
• In practical current source an internal resistance is present known as source resistance



Practical DC current source



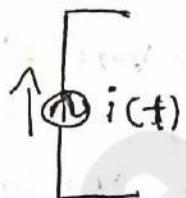
Practical AC current source



current sources are further classified into

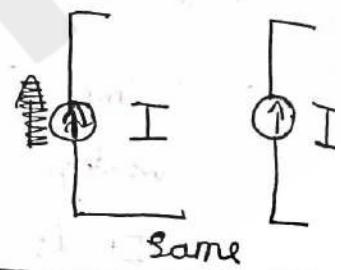
\* Time Variant AC current source.

- current changes w.r.t time
- denoted by small letter



\* Time invariant AC current source.

- current does not vary w.r.t time
- denoted by capital letter's



$\Rightarrow$  Dependent Sources

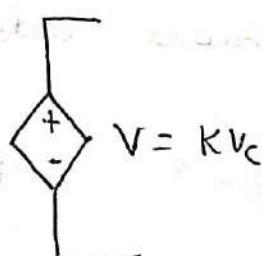


denoted by this

- Voltage Dependent Voltage source.
- ~~current~~ current Dependent Voltage source.
- Voltage Dependent current source.
- current Dependent current source.

1) Voltage Dependent Voltage source.

It produces Voltage as a function of Voltage



$K$  = scaling factor  
 $V_c$  = circuit voltage

## 2) current Dependent Voltage source:

It produces voltage as a function of current



$$V = K I_C$$

$K$  = scaling factor  
 $I_C$  = CKT current

## 3) Voltage Dependent current source :-

It produces current as a function of voltage



$$I = K V_C$$

$K$  = scaling factor  
 $V_C$  = CKT voltage

## 4) current Dependent current source :-

It produces current as a function of current

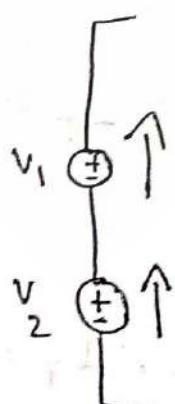


$$I = K I_C$$

$K$  = scaling factor  
 $I_C$  = CKT current

## → Addition & subtraction of voltage sources:-

1)



$$= \text{---} \quad V_T = V_1 + V_2$$

Voltage can be added or subtracted in series

2)

$$= \begin{cases} - & + \\ + & - \end{cases}$$

$$V_T = -V_1 - V_2$$

$$V_T = -(V_1 + V_2)$$

3)

$$= \begin{cases} + & - \\ - & + \end{cases}$$

*Case 1*

$$V_1 > V_2$$

$$\begin{cases} + & - \\ - & + \end{cases} V_T = V_1 - V_2$$

*Case 2*

$$V_2 > V_1$$

$$\begin{cases} - & + \\ + & - \end{cases} V_T = 1$$

4)

$$= \begin{cases} - & + \\ + & - \end{cases}$$

$V_1 > V_2$

$$\begin{cases} - & + \\ + & - \end{cases} V_T = -V_1 + V_2$$

$$V_T = V_2 - V_1$$

$V_2 > V_1$

$$\begin{cases} - & + \\ + & - \end{cases} V_T = -V_1 + V_2$$

$$V_T = V_2$$

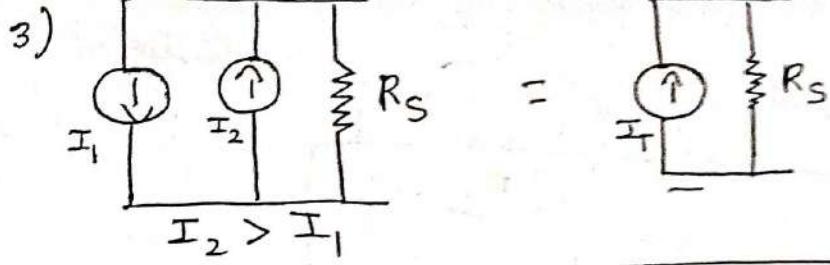
→ Addition & subtraction of current sources.

1)

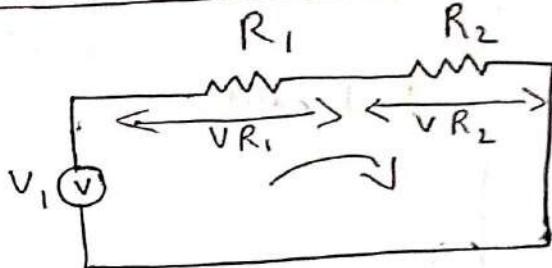
$$= \begin{cases} I_1 & I_2 \\ I_T & \end{cases} \quad \therefore I_T = I_1 + I_2$$

2)

$$= \begin{cases} I_1 & I_2 \\ I_T & \end{cases} \quad \therefore I_T = I_1 - I_2$$



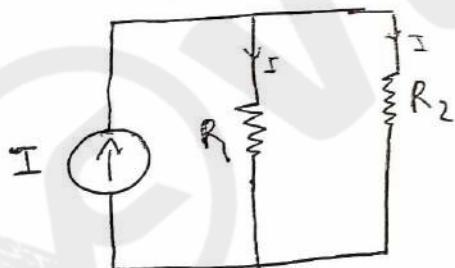
→ Voltage Divider Rule:



$$V_{R_1} = \frac{V \times R_2}{R_1 + R_2}$$

$$V_{R_2} = \frac{V \times R_1}{R_1 + R_2}$$

→ Current Divider Rule

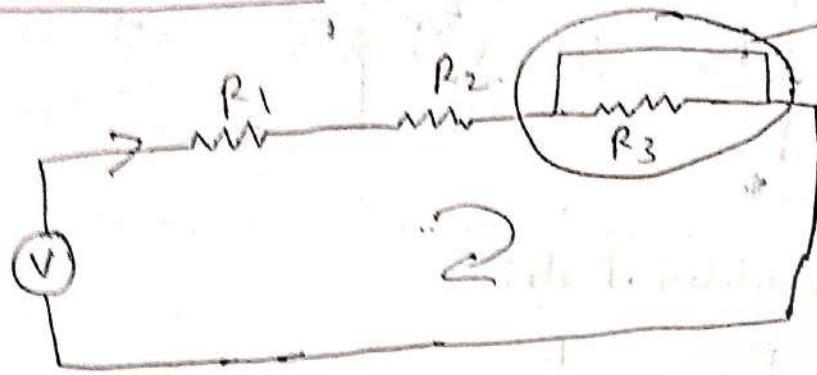


$$I_{R_1} = \frac{I \times R_2}{R_1 + R_2}$$

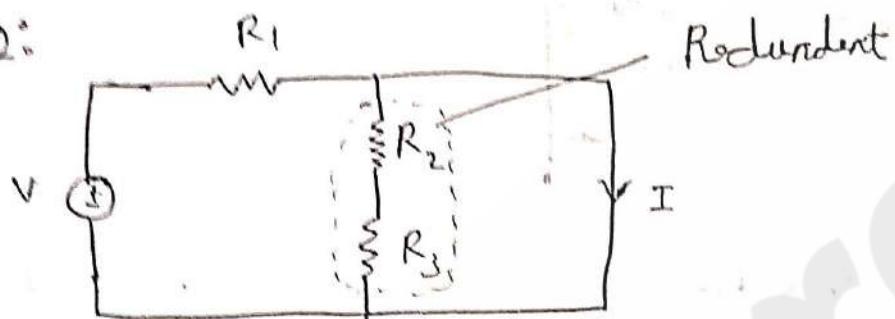
$$I_{R_2} = \frac{I \times R_1}{R_1 + R_2}$$

## → Redundant branches:

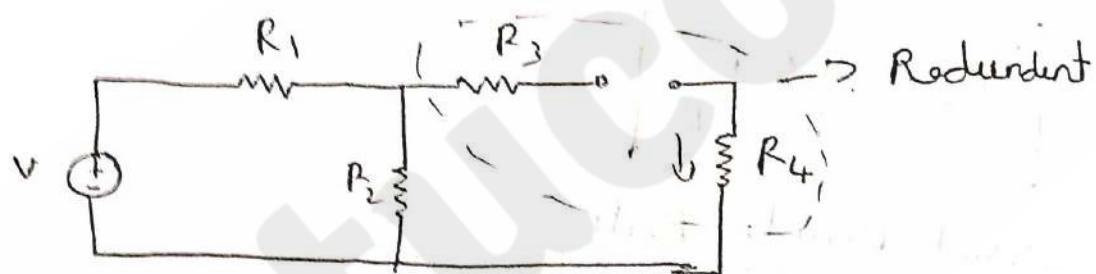
case①:



case②:

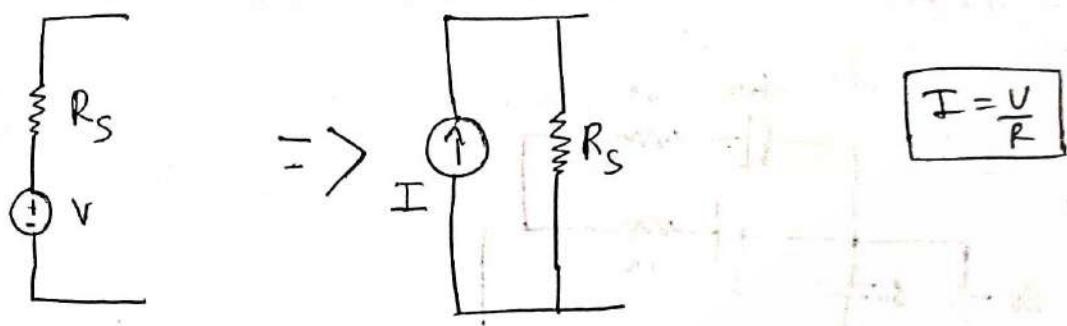


case③:

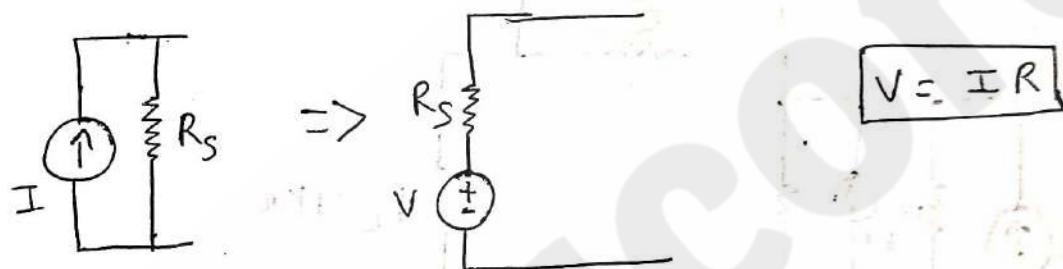


→ Source transformation:

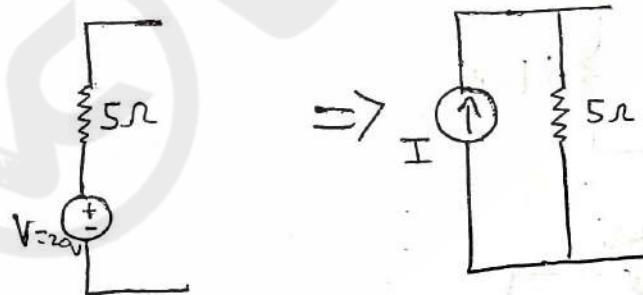
i) Voltage - current transformation ( $V - I$ )



ii) current - voltage transformation ( $I - V$ )



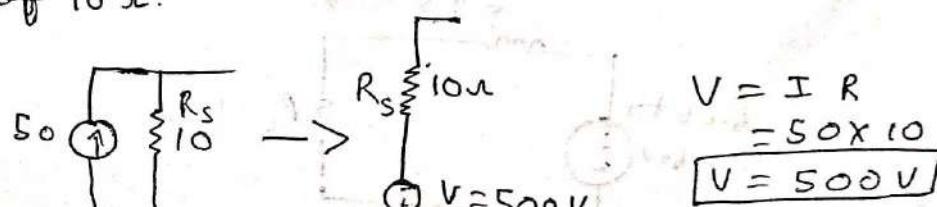
→ Transform a voltage source of 20V with an internal resistance of 5Ω to a current source



$$I = \frac{V}{R} = \frac{20}{5} = 4A$$

$$I = 4A$$

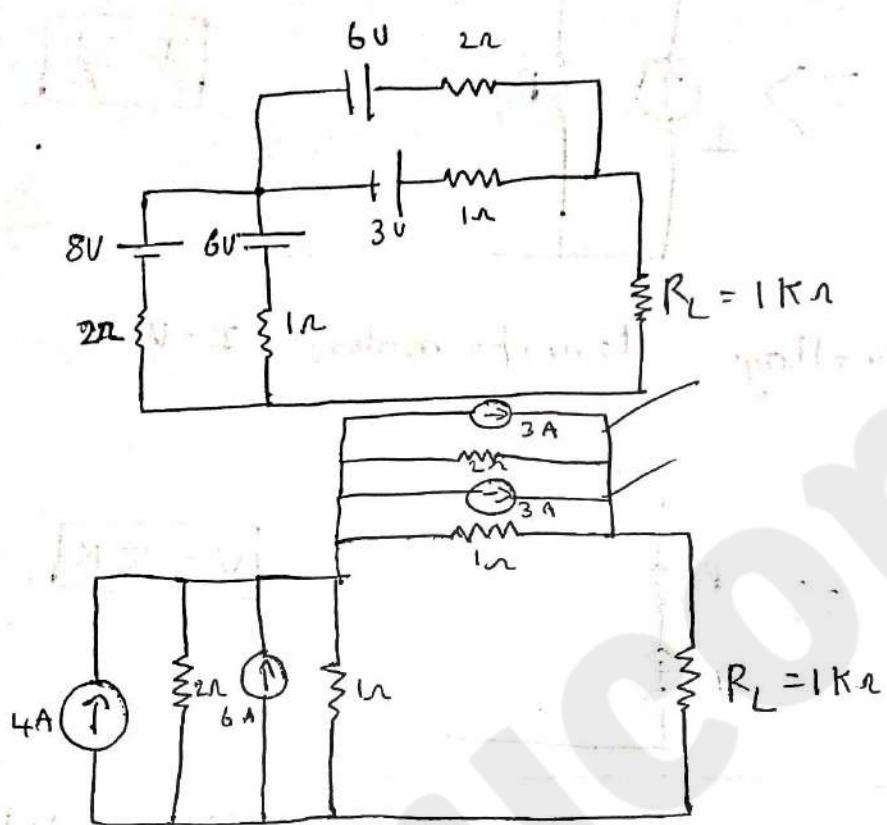
2) convert the given Isource of 50A with internal resistor of 10Ω.



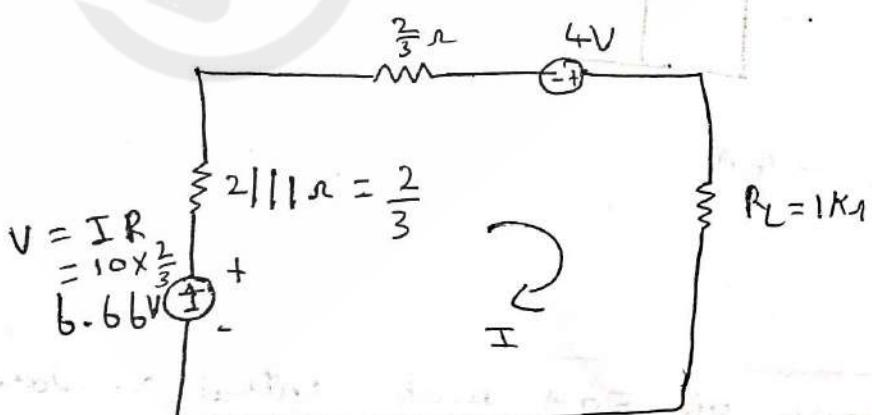
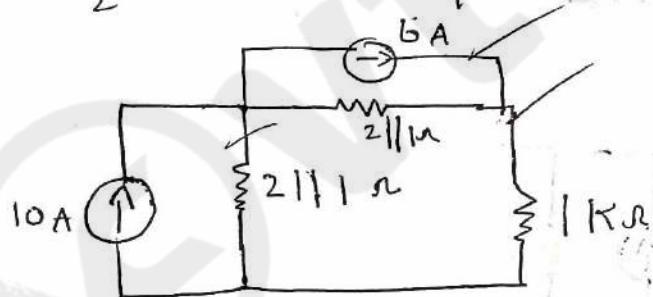
3) Using current transformation technique simplify the given network and find the current through

$$R_L = 1 \text{ k}\Omega$$

Soln:



$$I = \frac{8}{2} = 4 \text{ A} \quad I = \frac{6}{1} = 6 \text{ A} \quad I = \frac{6}{2} = 3 \text{ A} \quad I = \frac{3}{1} = 3 \text{ A}$$



$$\frac{2}{3} + \frac{2}{3} = \frac{4}{3} \Omega$$

$$6.66 + 4 = 10.66 \text{ V}$$

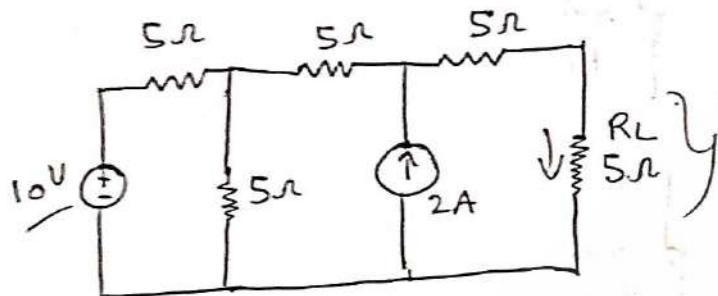
$$R_L = 1 \text{ k}\Omega$$

$$I = \frac{V}{R} = \frac{10.66}{4.73 + 1 \times 10^3}$$

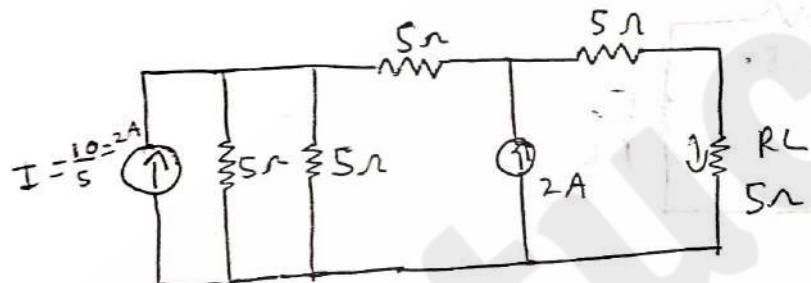
$$I = 10.65 \times 10^{-3}$$

$$\text{or } I = 10.65 \text{ mA}$$

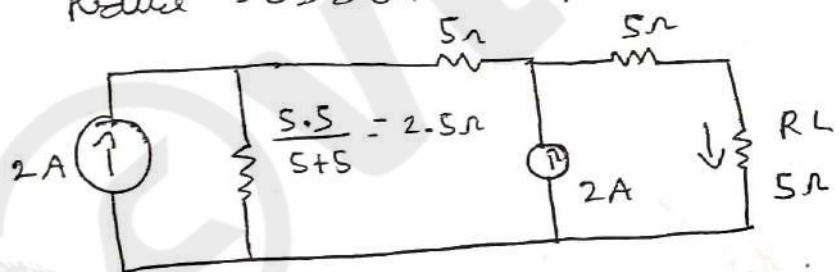
4) Using source transformation find the load current  $I_L$  in the following circuit shown below.



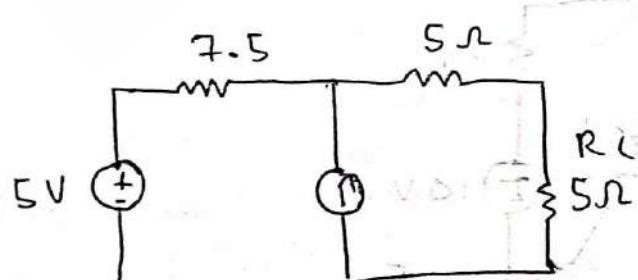
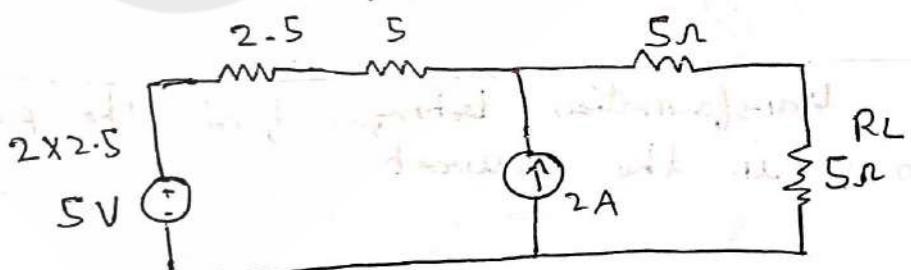
Soln: convert 10V 5Ω to current source.

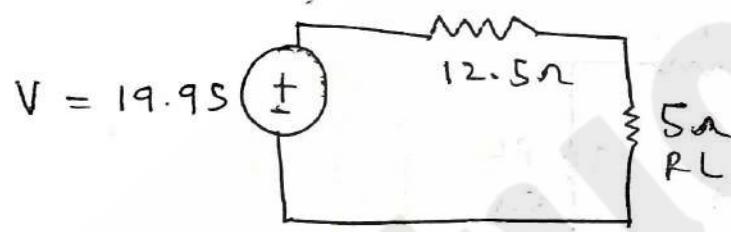
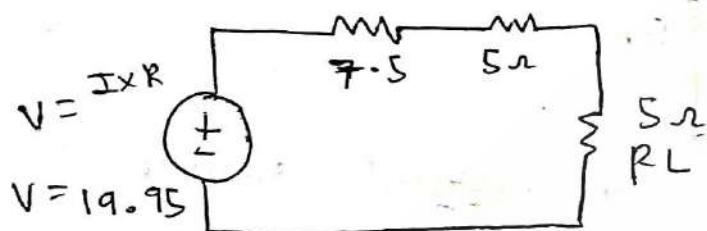
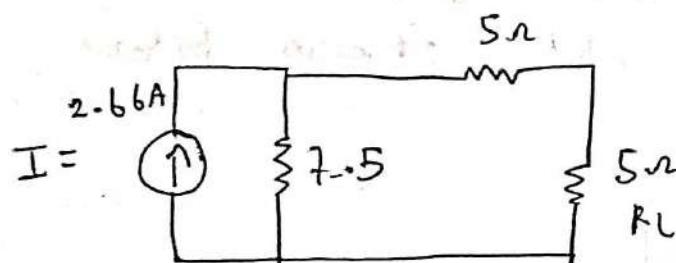
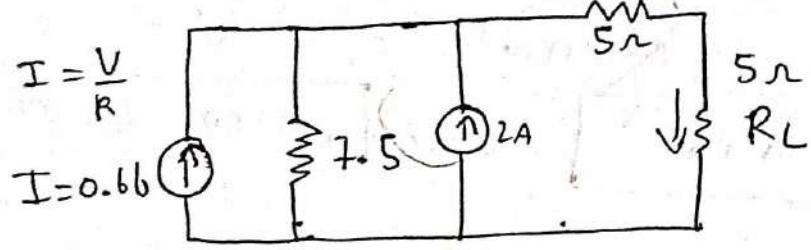


Reduce resistor in parallel.



convert A  $\&$  2.5 to voltage source.



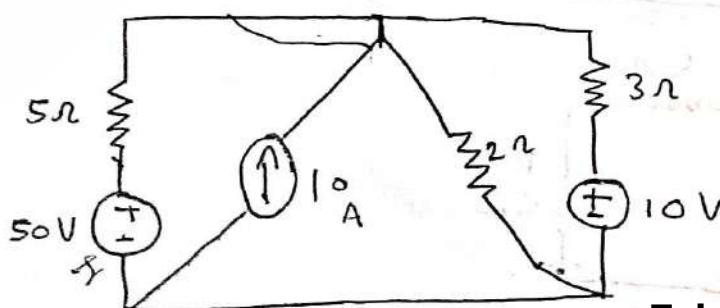


$$I = \frac{V}{R}$$

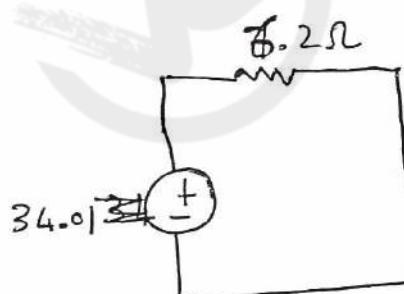
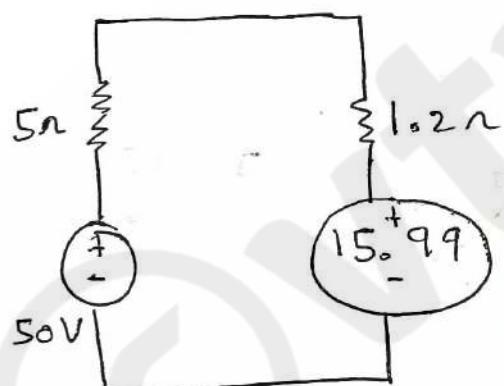
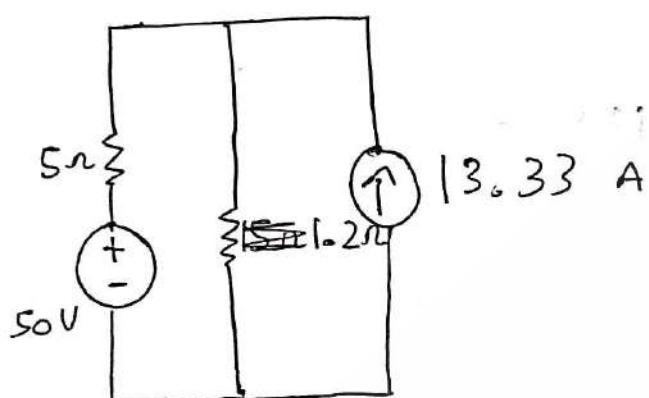
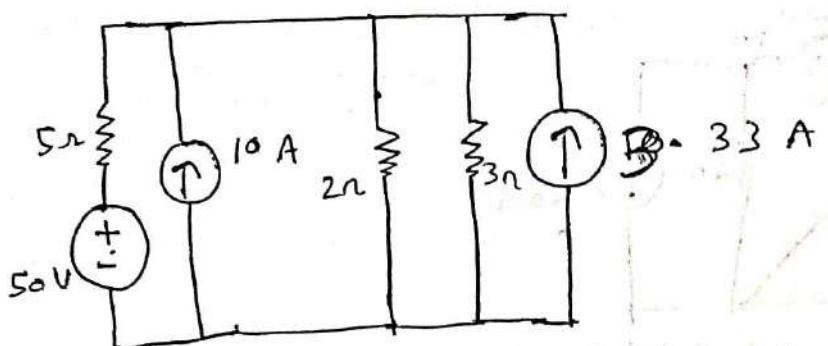
$$I = \frac{19.95}{12.5 + 5}$$

$$I_L = 1.14 A$$

~~5) Using source transformation technique find the power delivered by 50V in the network~~



$$P = V \cdot I$$



$$I = \frac{34.01}{0.2}$$

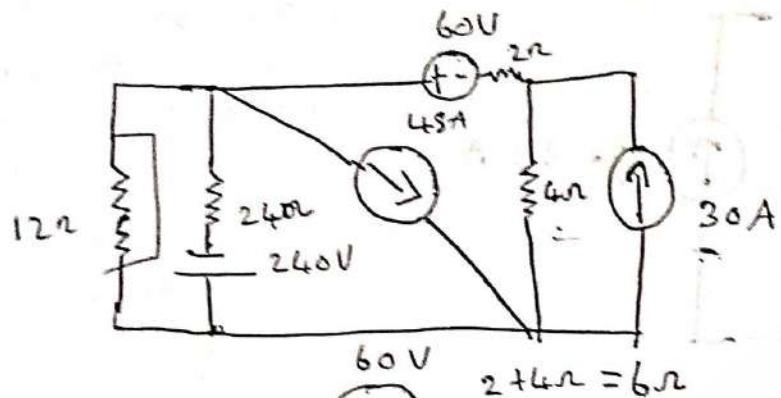
$$I = 5.48 \text{ A}$$

$$\text{Power} = V \times I$$

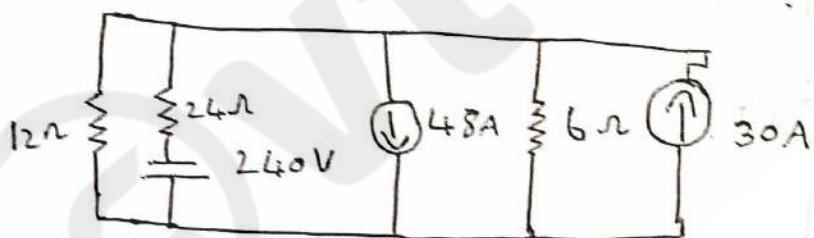
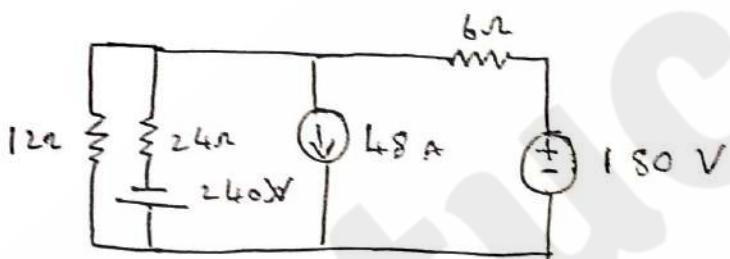
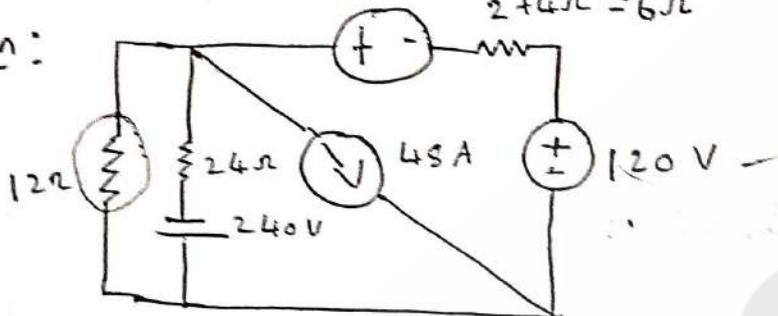
$$= 50 \times 5.48$$

$$P_{\text{out}} = 274.89 \text{ Watts}$$

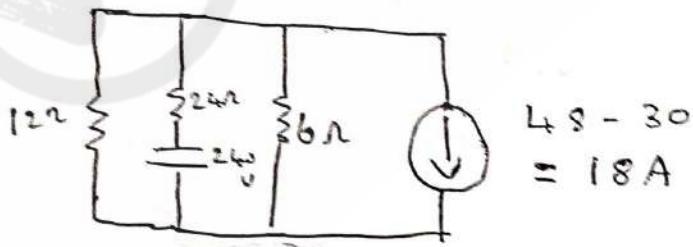
b) Find the current through  $12\Omega$  resistor in a network shown using source transformation.



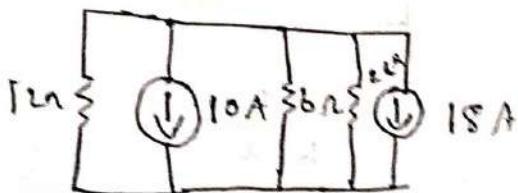
Soln:

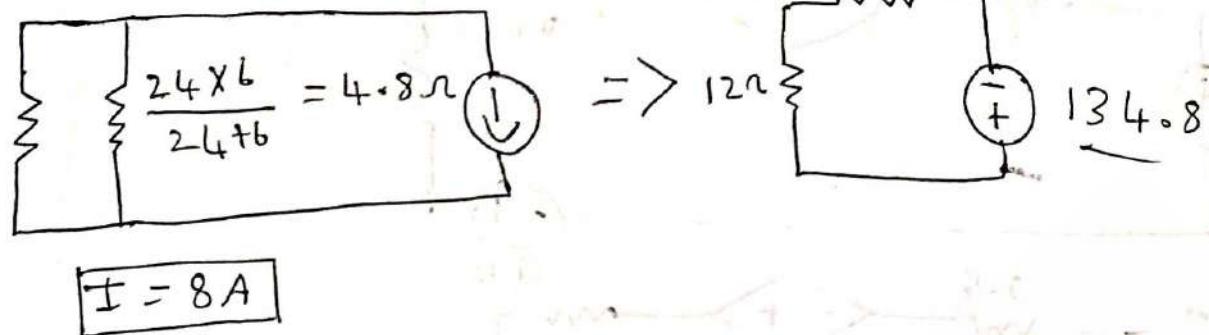


$$I = \frac{180}{6}$$

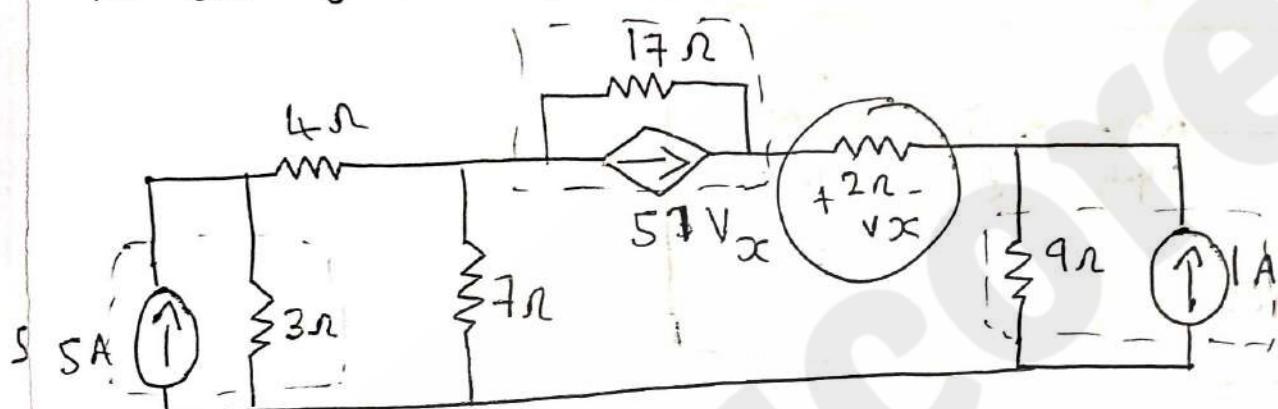


$$48 - 30 = 18A$$

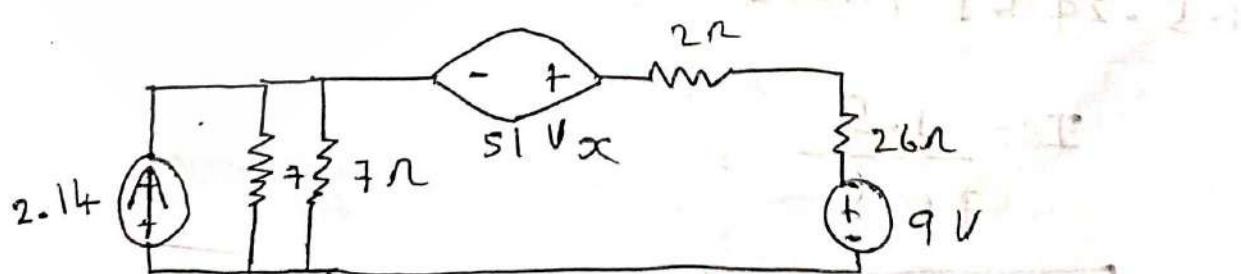
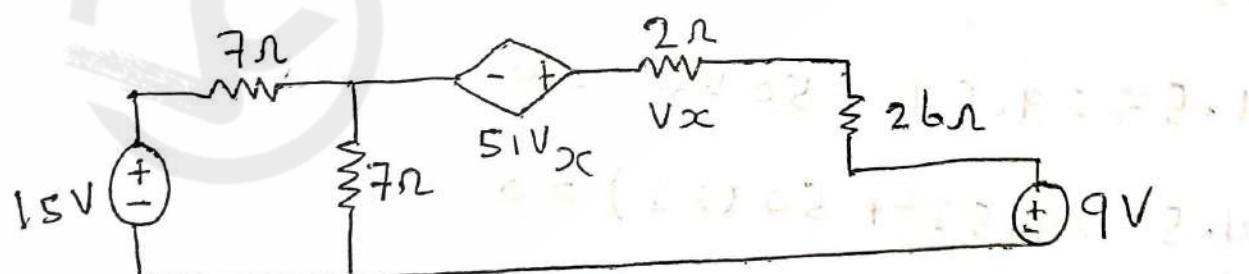
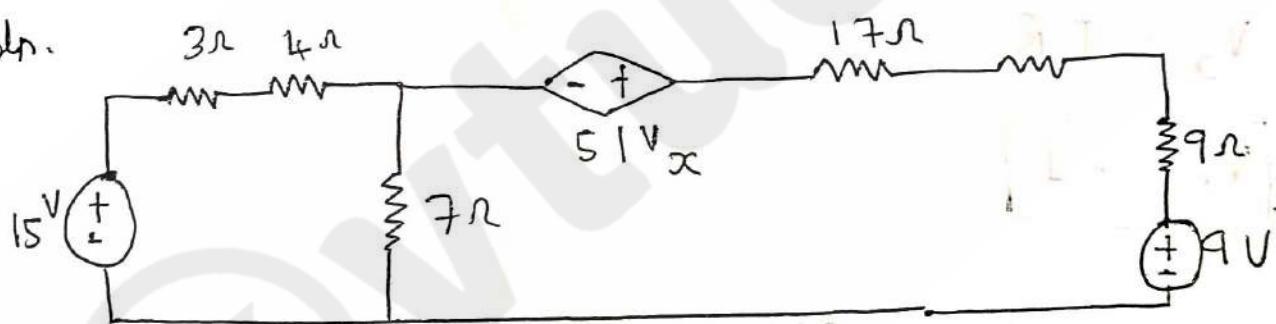




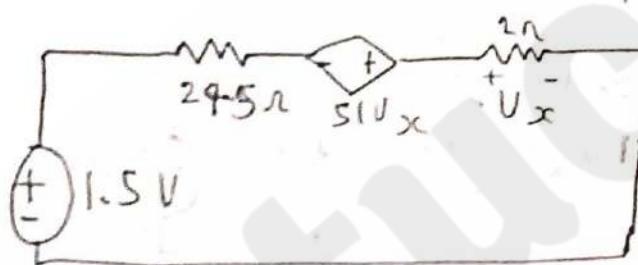
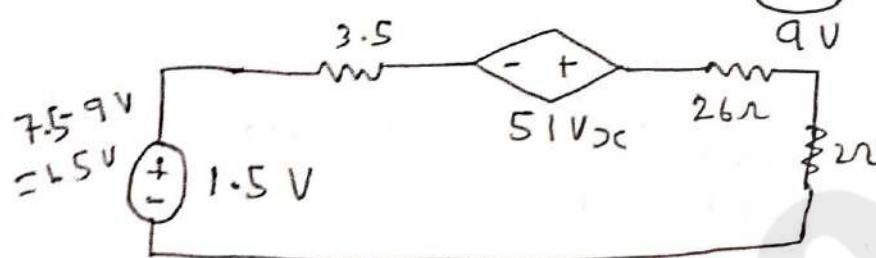
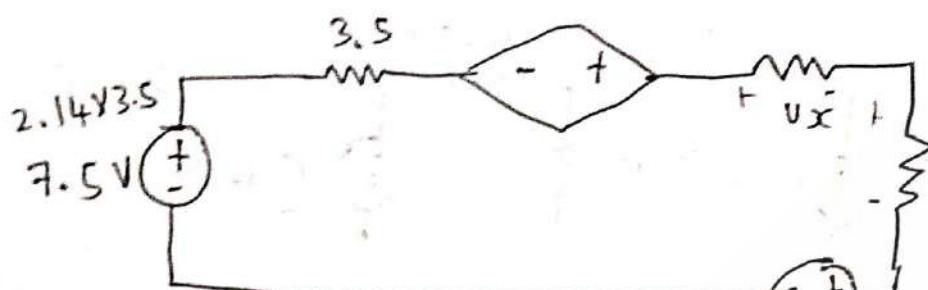
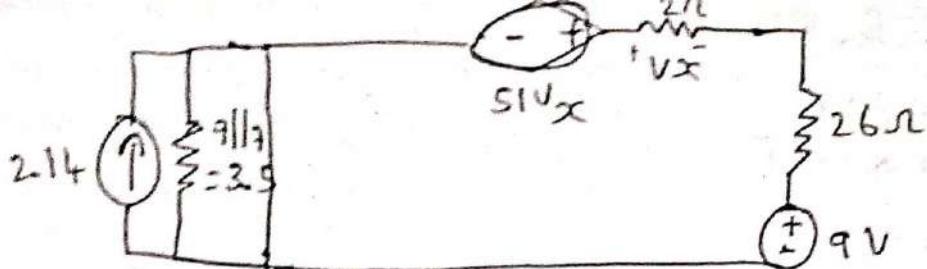
7) Using source transformation find the current  $I$  in the following ckt shown across  $2\Omega$



Soln.



$$A_m + S_m = I$$



$$V_x = IR$$

$$\boxed{V_x = 2I}$$

$$1.5 - 29.5I + 51V_x - V_x = 0$$

$$1.5 - 29.5I + 50V_x = 0$$

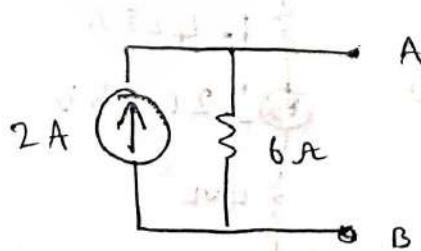
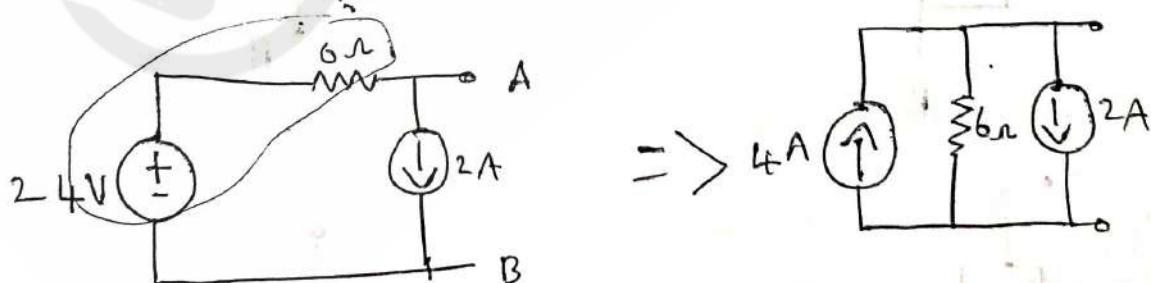
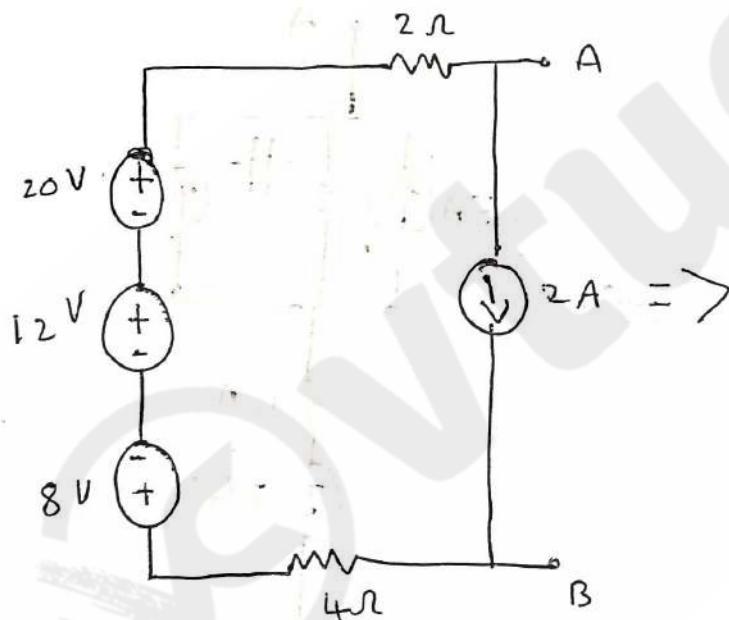
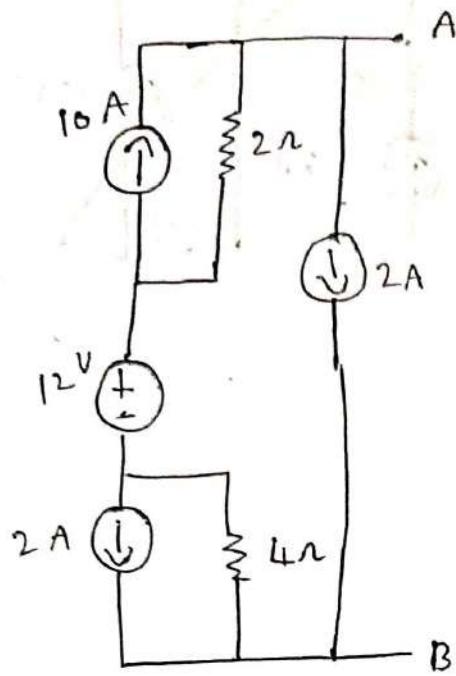
$$1.5 - 29.5I + 50(2I) = 0$$

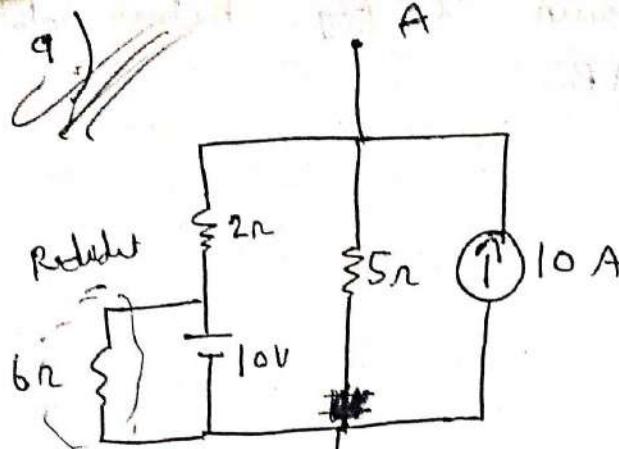
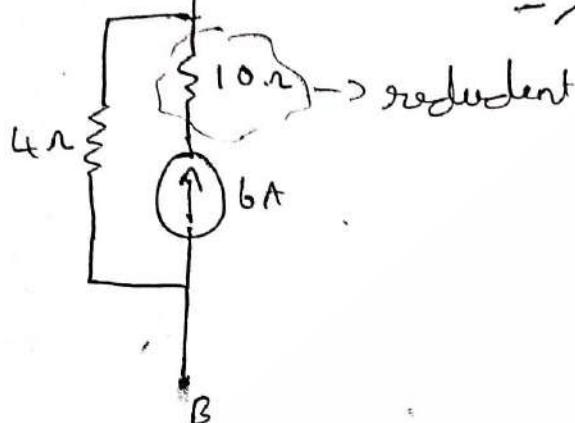
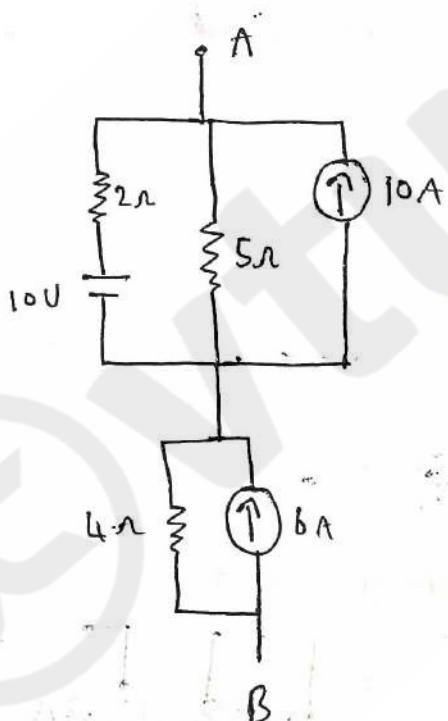
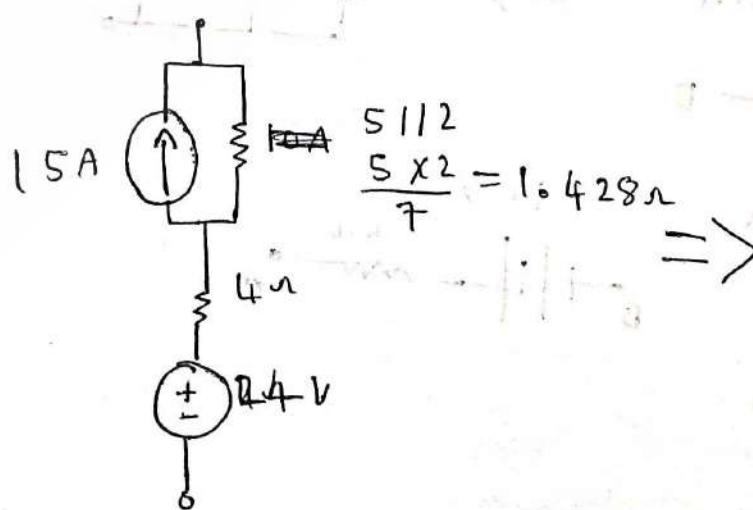
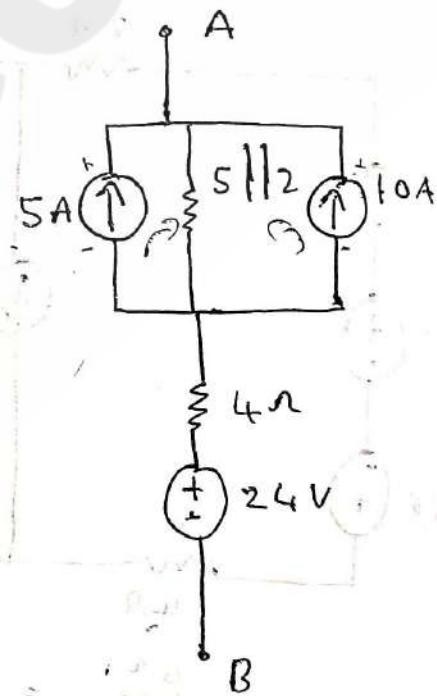
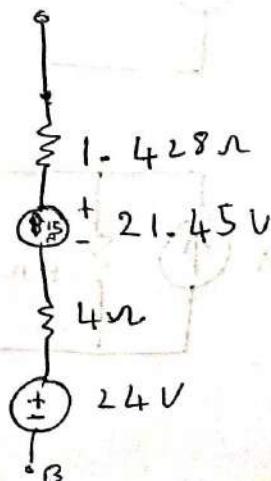
$$1.5 - 29.5I + 100I = 0$$

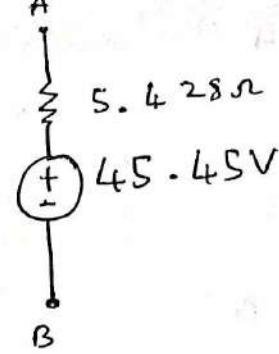
$$I = \frac{1.5}{70.5}$$

$$\boxed{I = 21.27 \text{ mA}}$$

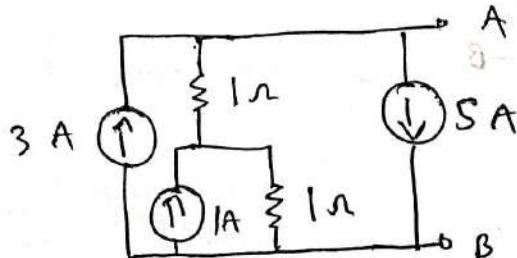
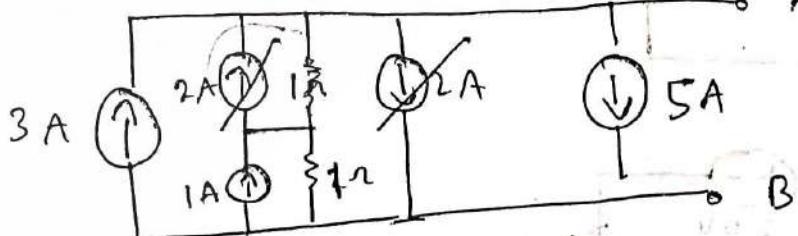
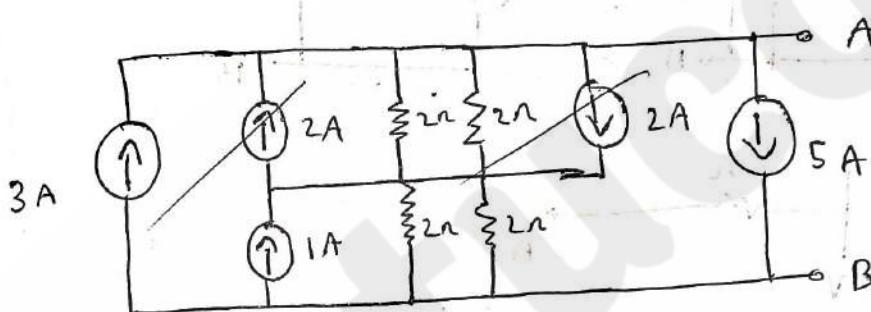
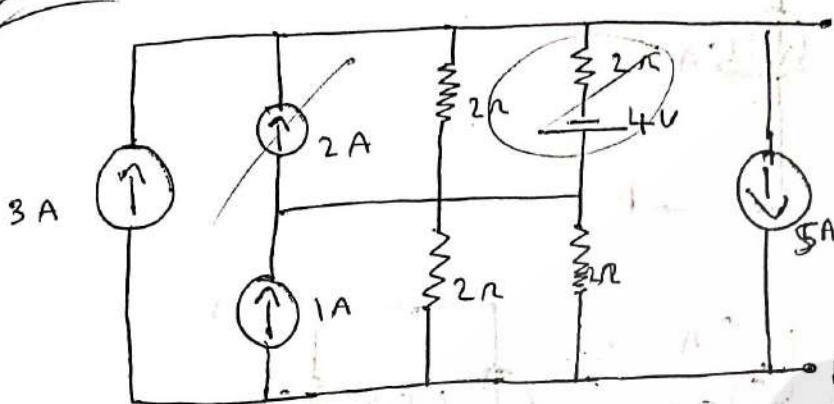
8) Reduce the Network shown in fig below into a voltage source between A B

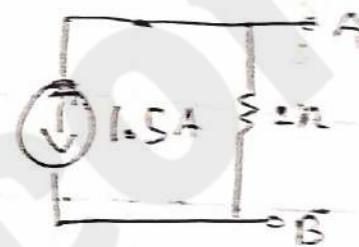
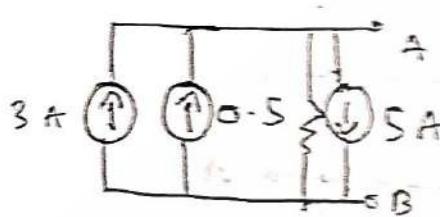
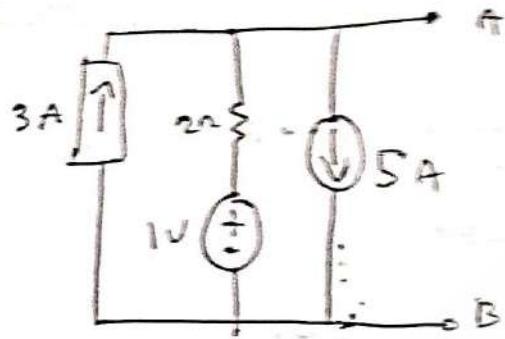
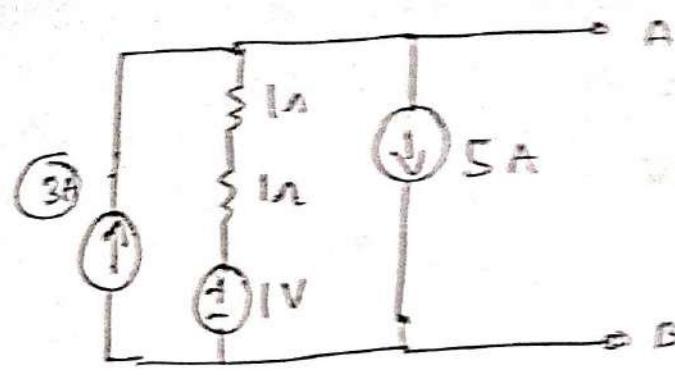


 $\Rightarrow$ Soln: $\Rightarrow$  $\Rightarrow$ 

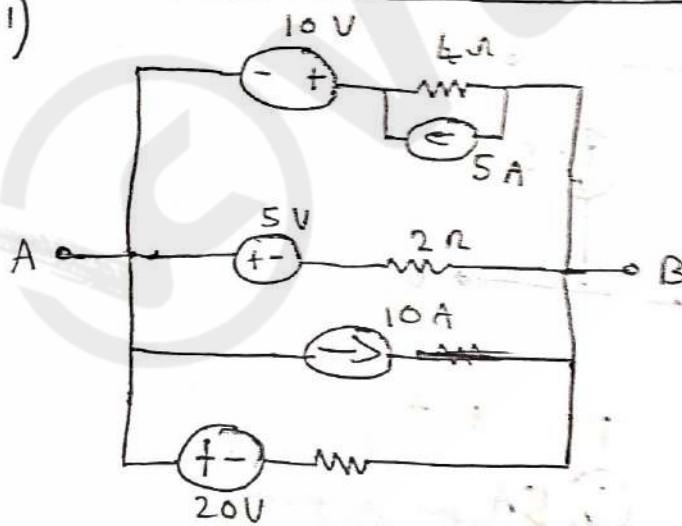


10)

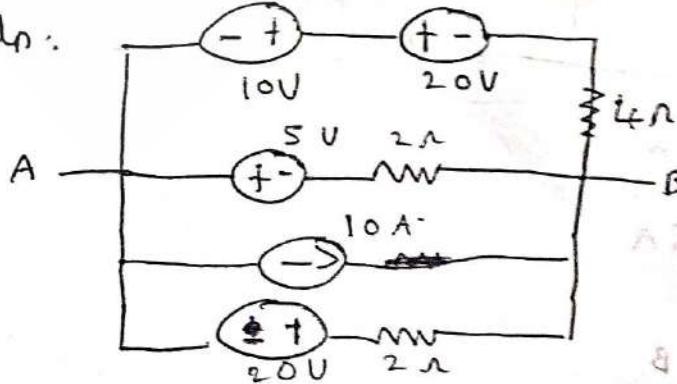


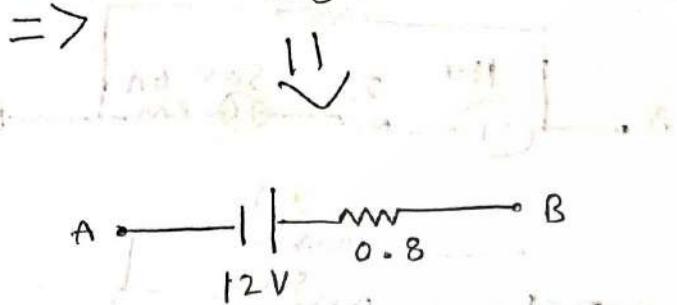
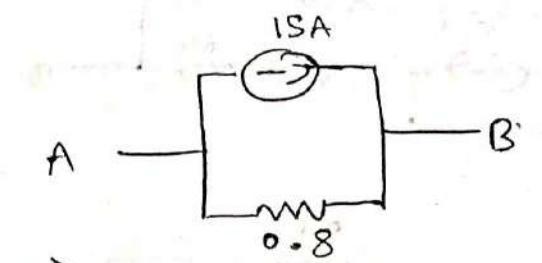
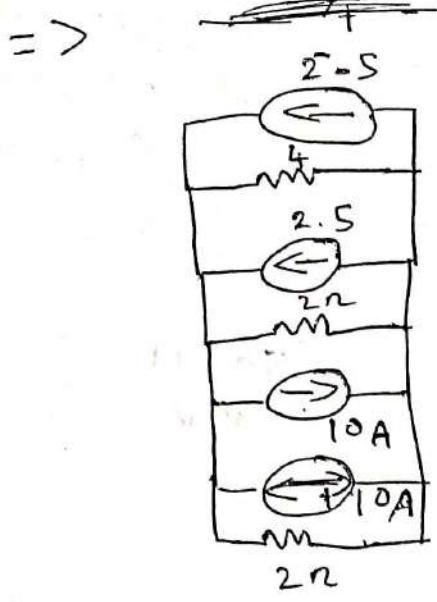


ii)

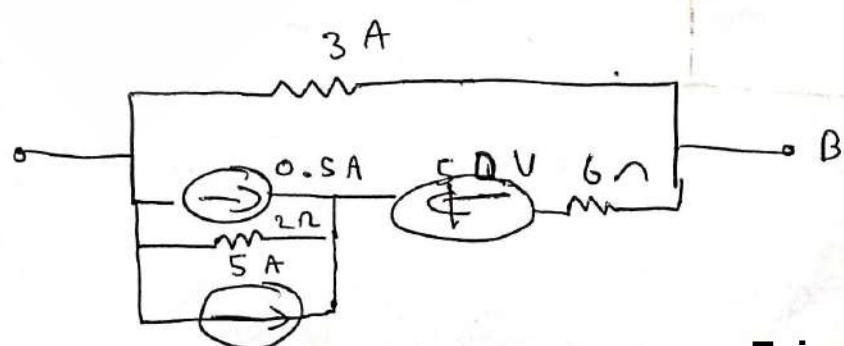
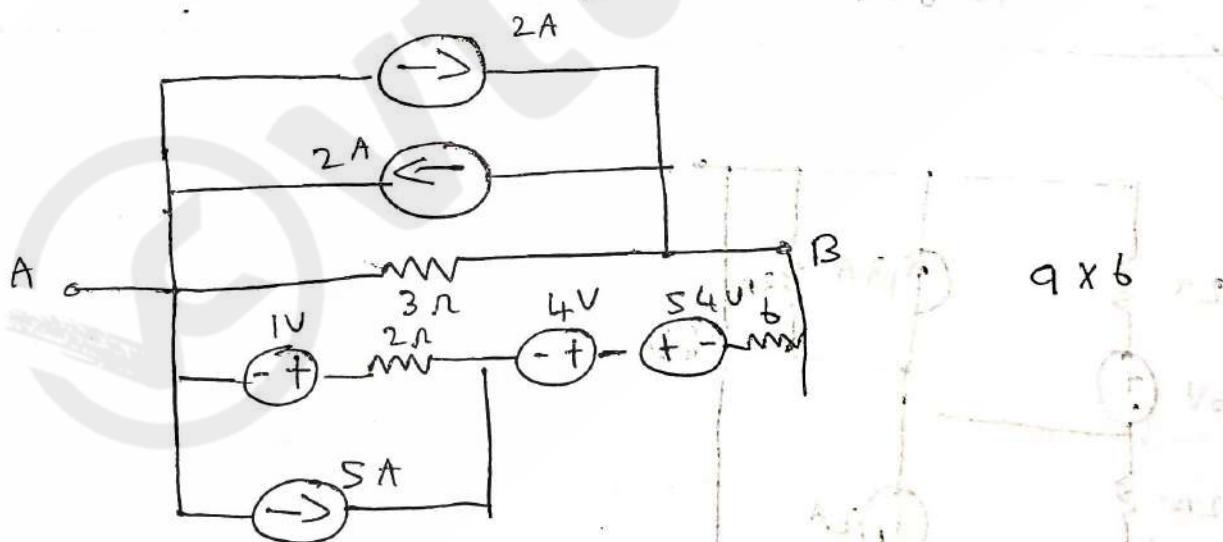
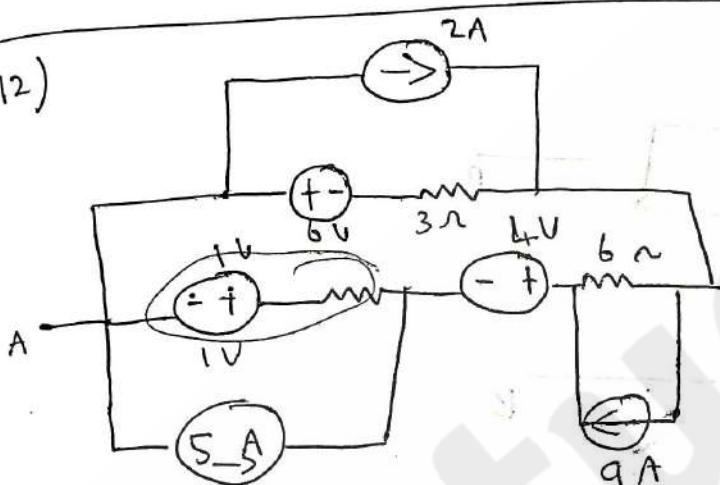


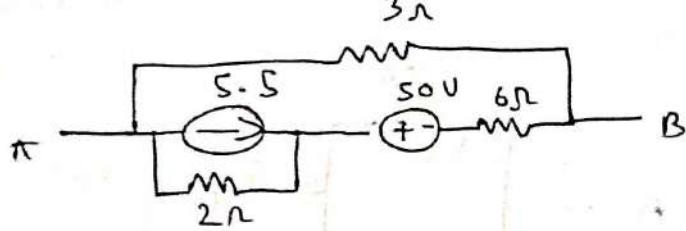
sols:



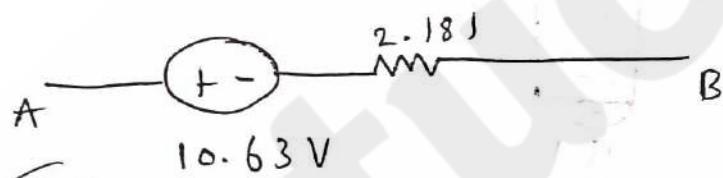
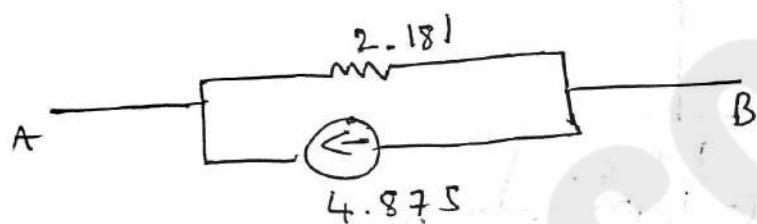
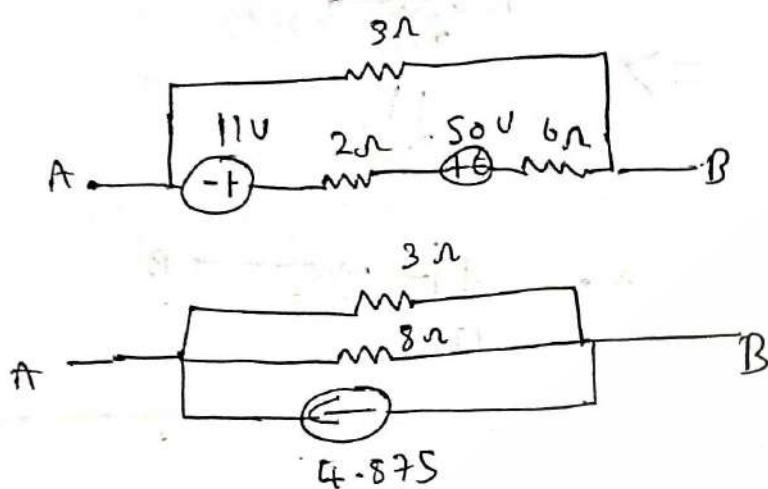


12)

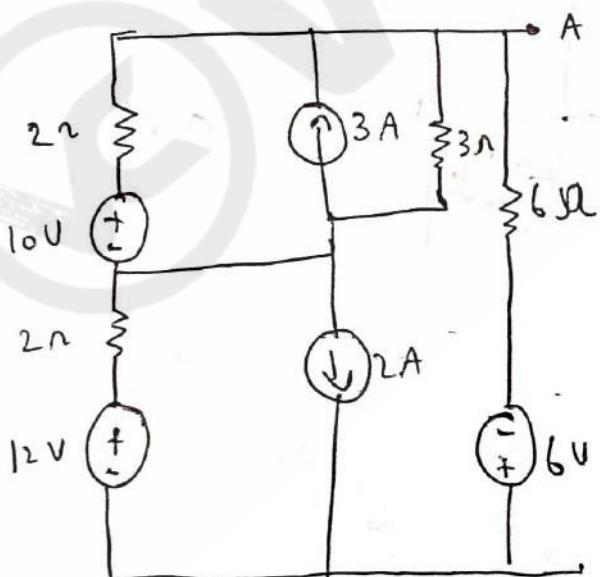


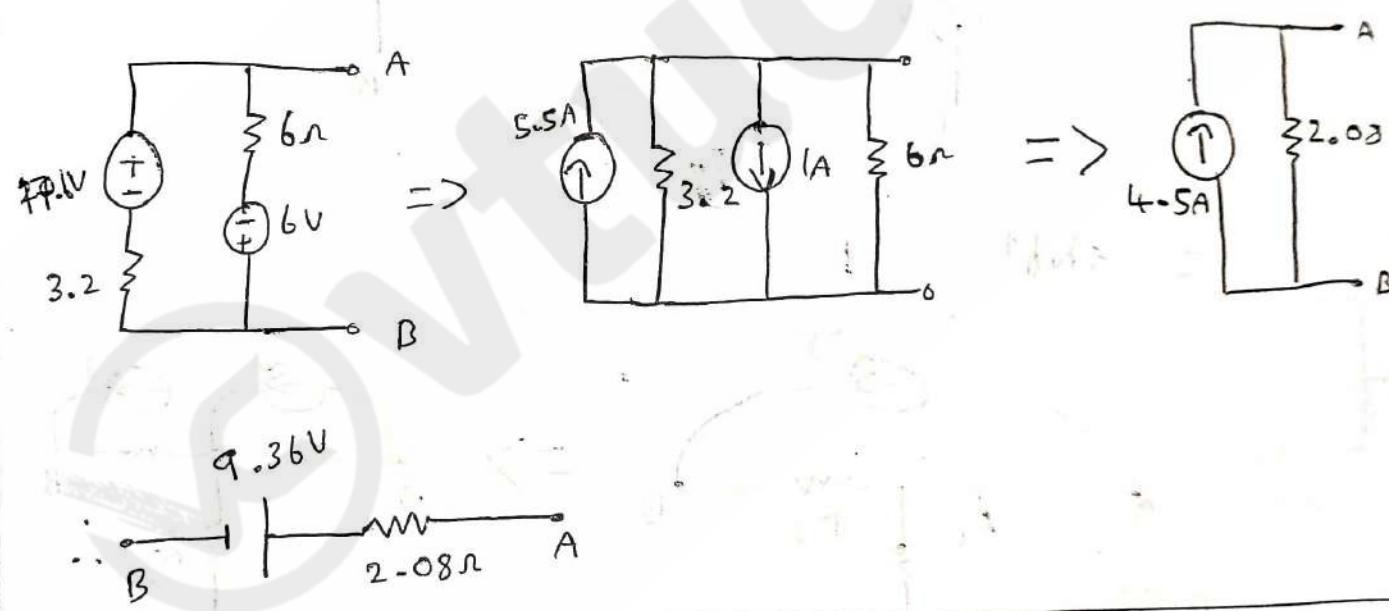
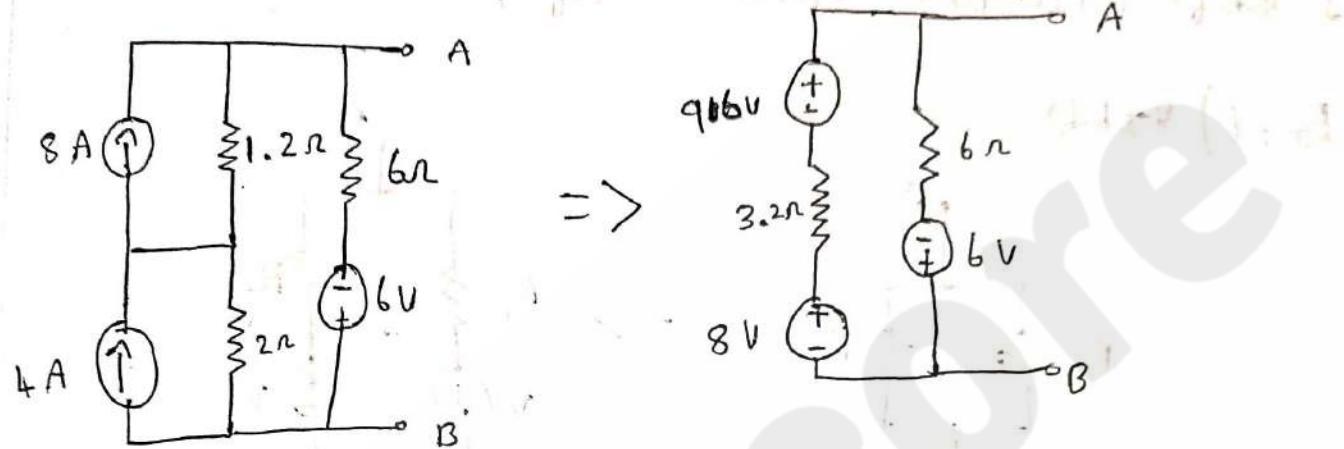
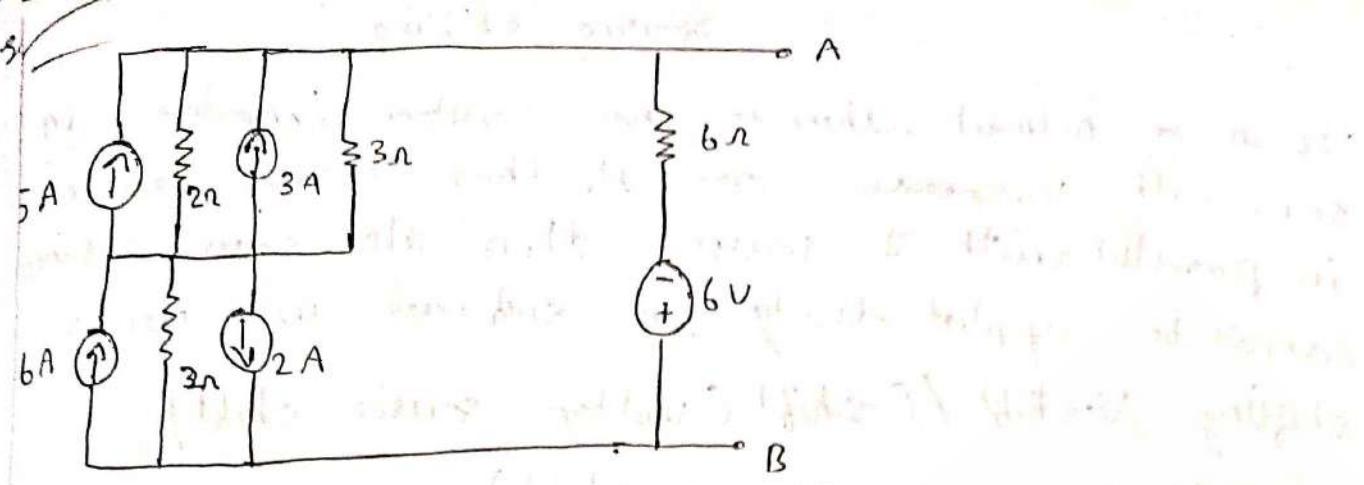


$$50 - 11 = 39 \text{ V}$$



(13)

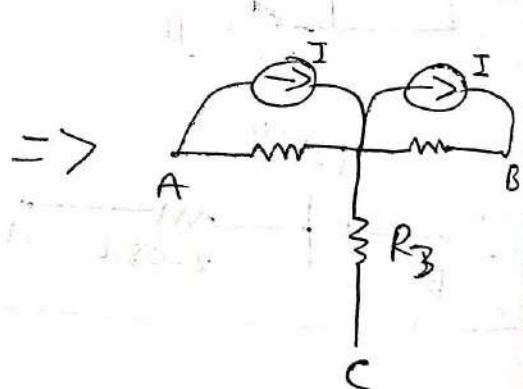
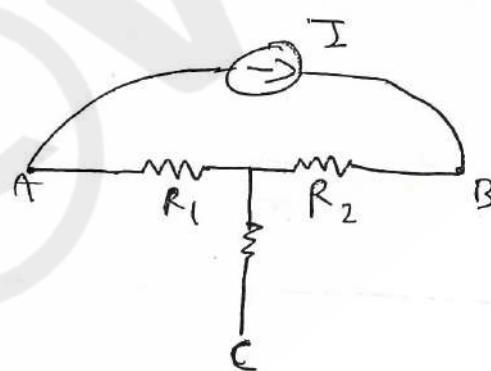
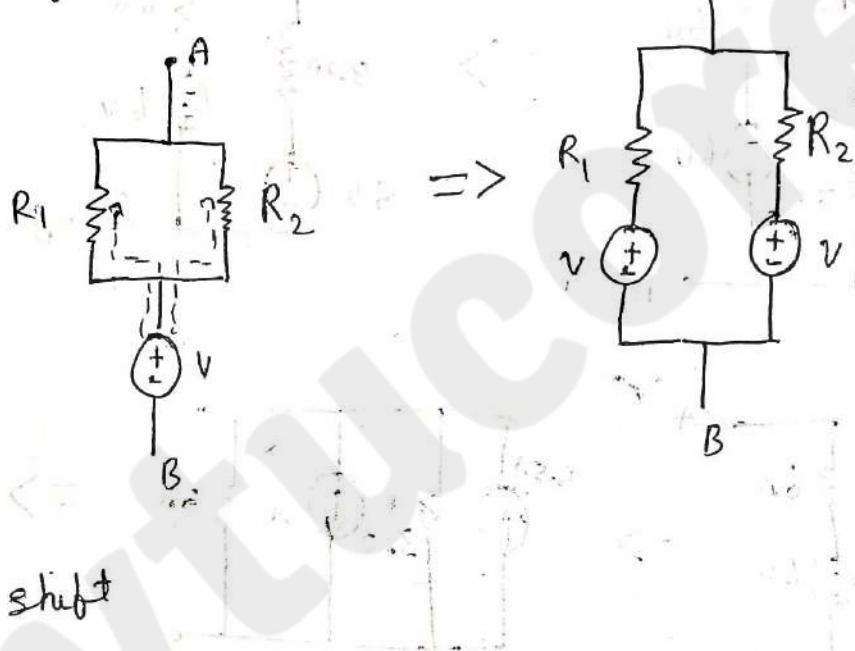




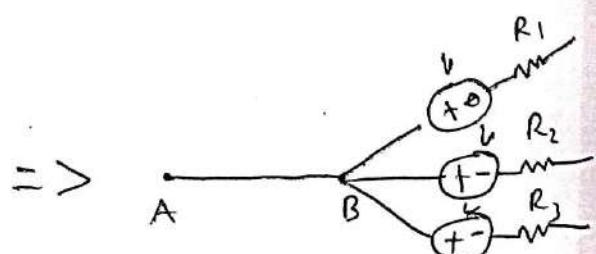
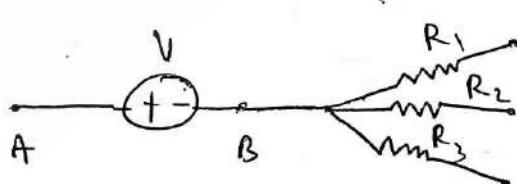
### Source shifting:

If in a network, there is no resistance / impedance in series with  $V_{source}$  or if there is no resistor in parallel with  $I_{source}$  then the source theorem cannot be applied directly in such cases we use source shifting V-shift / E-shift (voltage source shift) I-shift (current source shift)

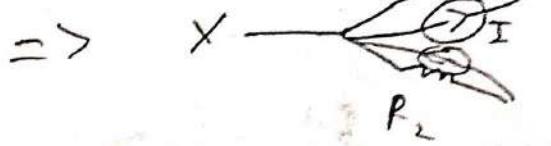
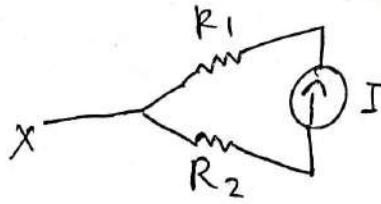
Ex: 1) V-shift



Ex: 2) V-shift



I shift:

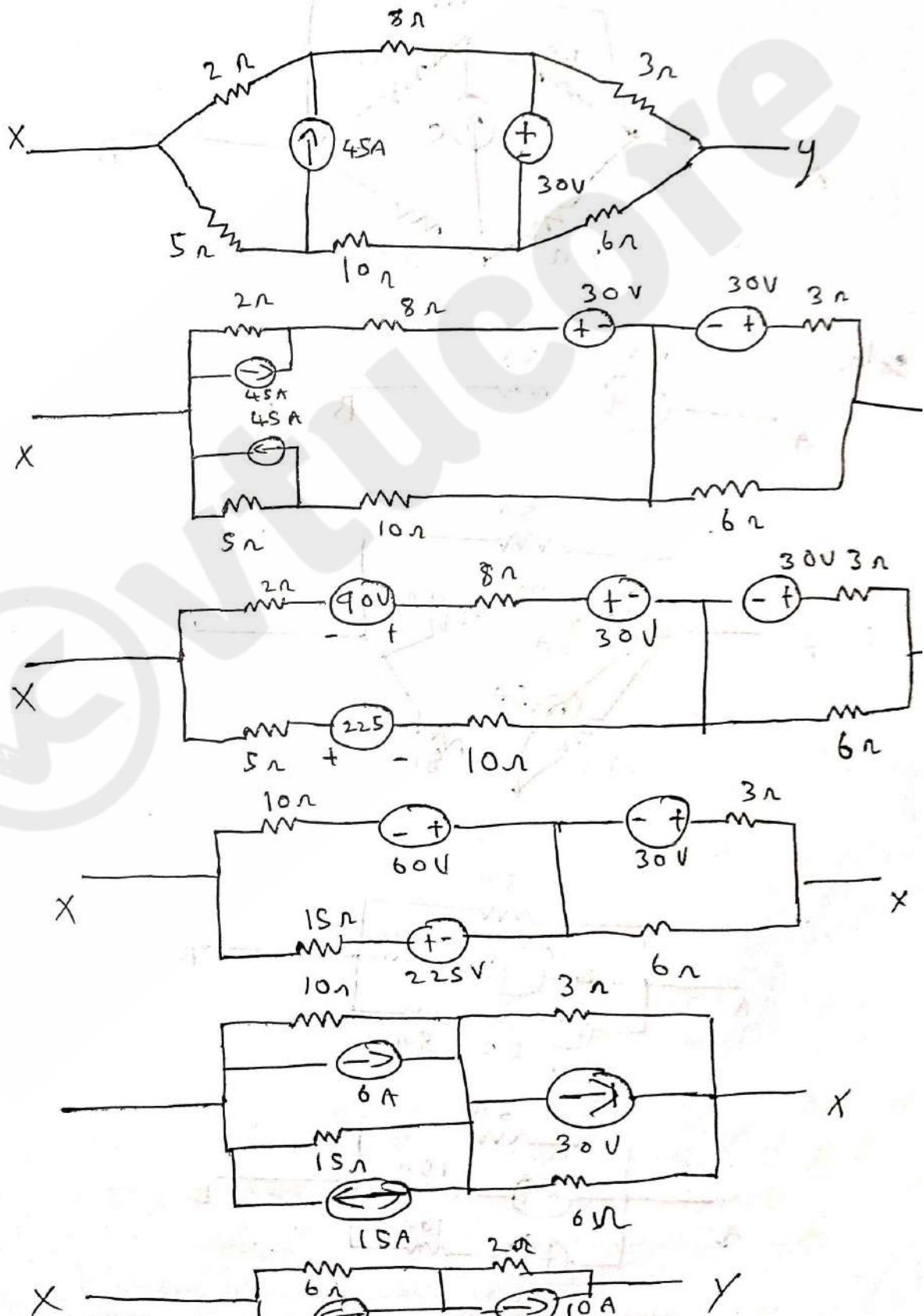


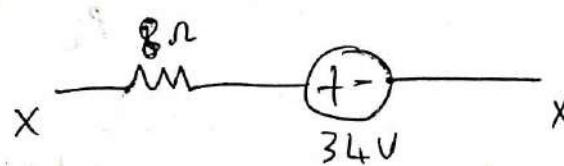
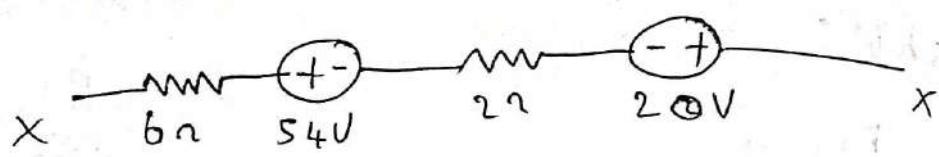
$R_1$

$R_2$

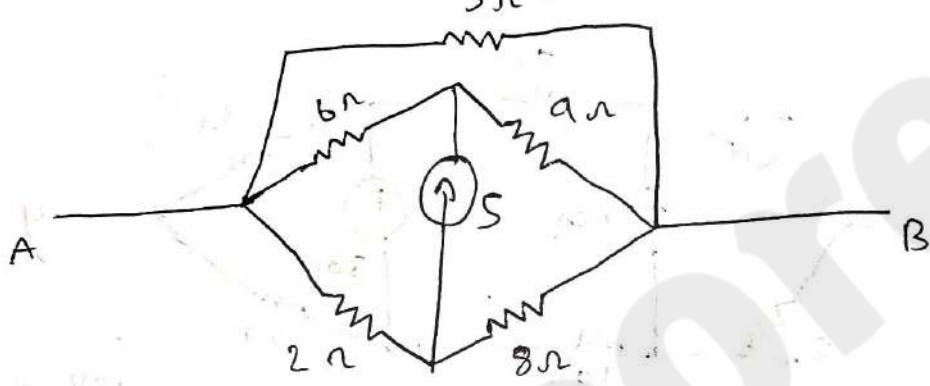
① Using source transformation and source shifting reduce the network into a single source across  $x \& y$ :

solt.

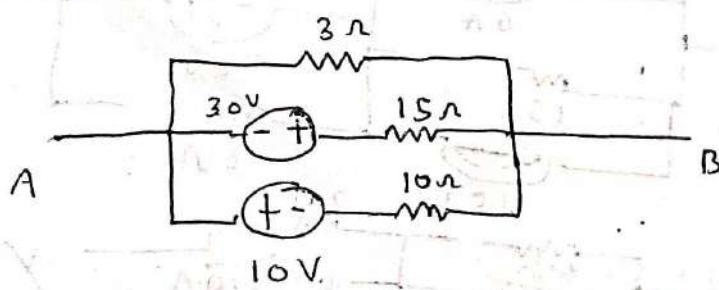
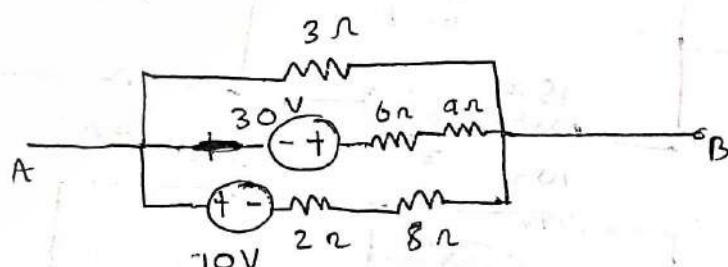
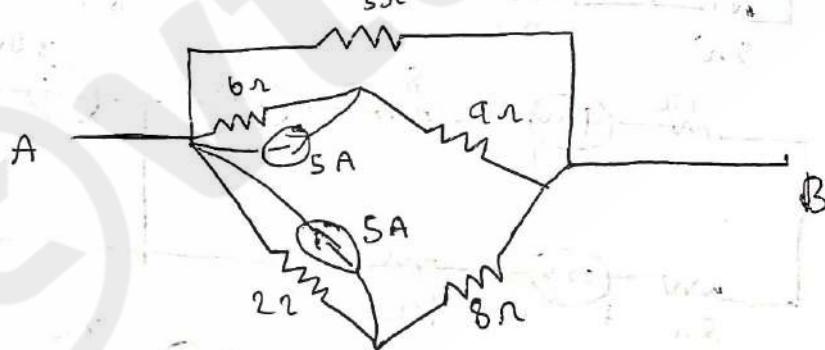


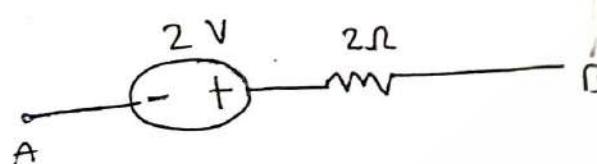
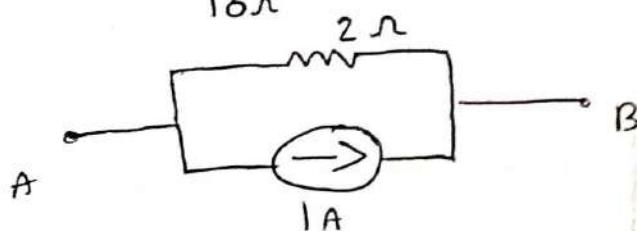
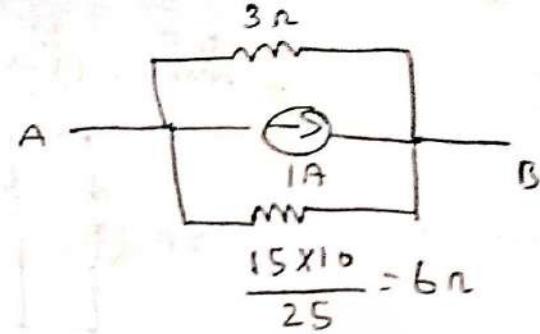
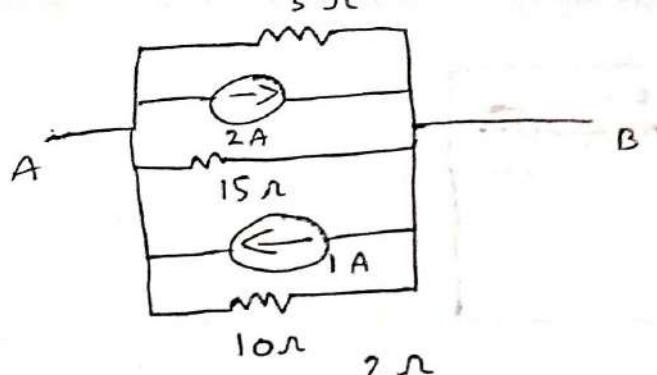


2)

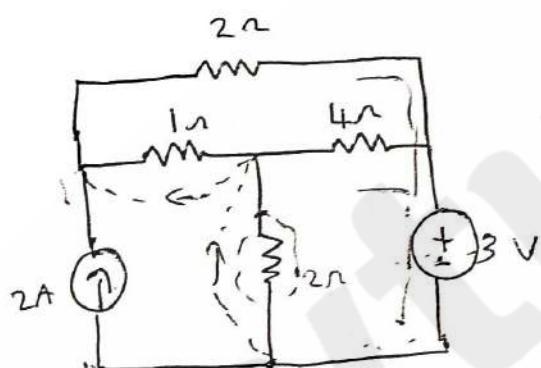


solt:



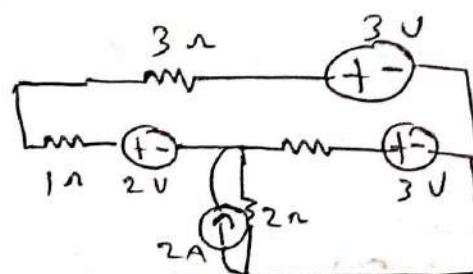
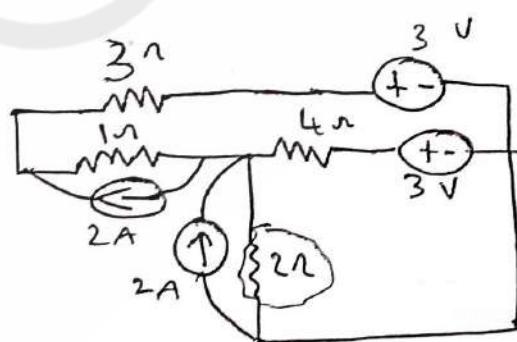


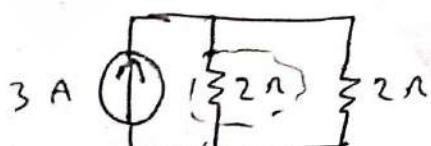
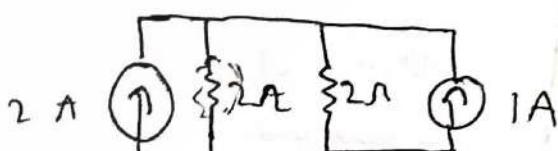
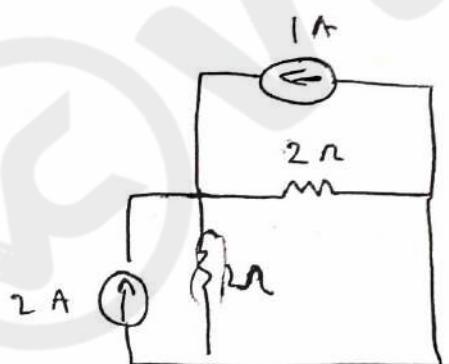
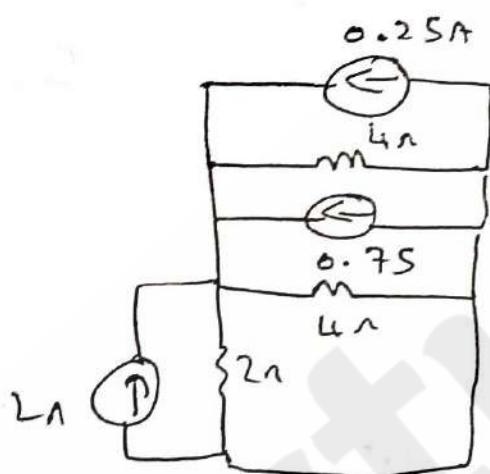
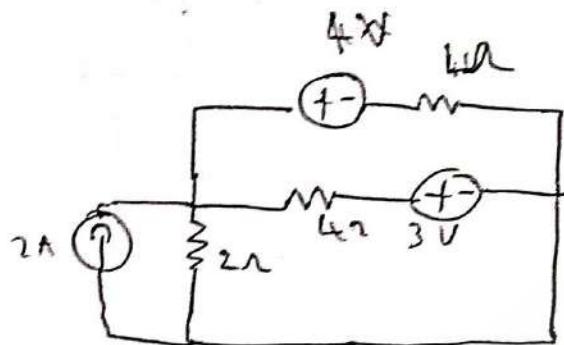
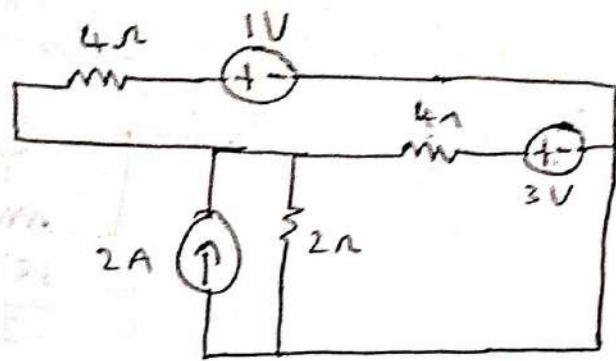
3)



$$V_{2\Omega} = I_{2\Omega} R$$

$$I_{2\Omega} = ?$$





current divider rule

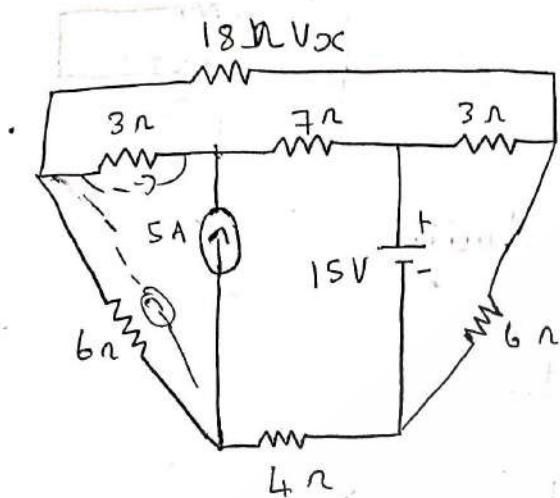
$$I_{2\Omega} = \frac{3 \times 2}{3+5} = \frac{6}{8} = 0.75 \text{ A}$$

$$V_{2n} = I_{2n} \times R$$

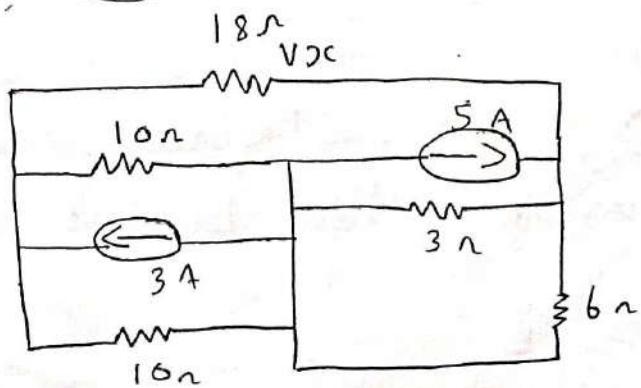
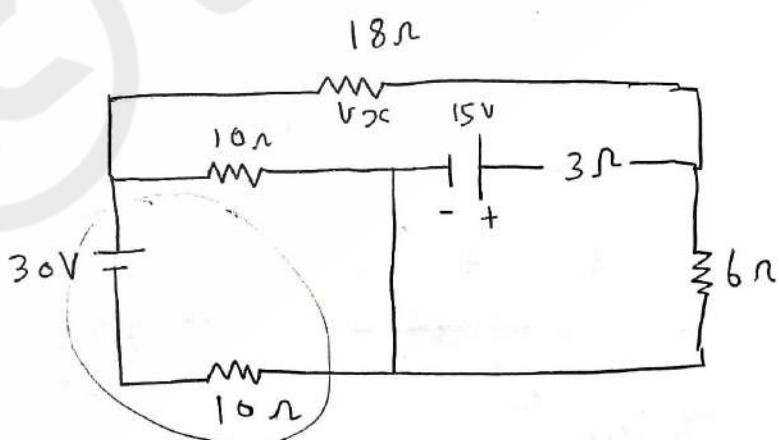
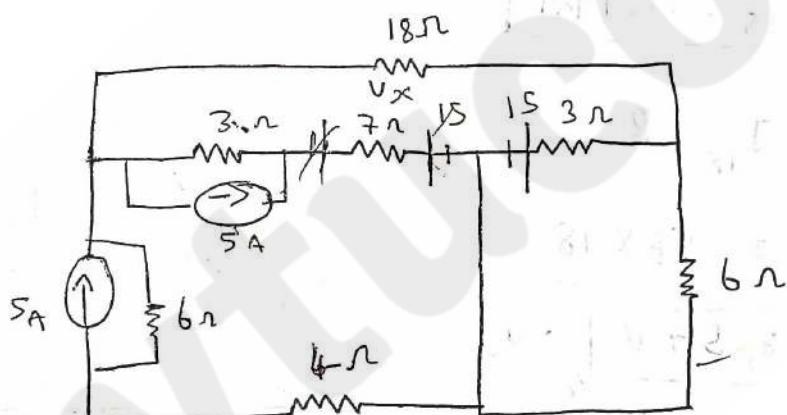
$$V_{2n} = 1.5 \times 2$$

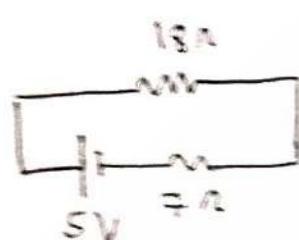
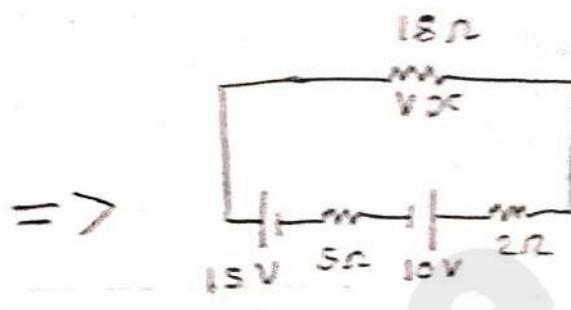
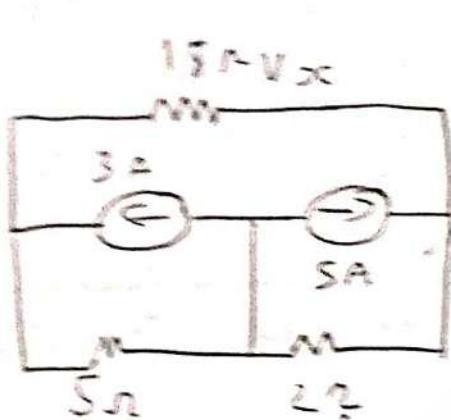
$$V_{2n} = 3 \text{ V}$$

4) Find the voltage ( $V_{2c}$ ) using source transformation and source shifting.



Soln:





applying current divider

$$I_{18} = \frac{0.71 \times 7}{18 + 7}$$

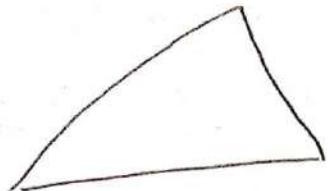
$$\boxed{I_{18} = 0.1984}$$

$$\therefore V_{18} = I_{18} \times R$$

$$V_{18} = 0.198 \times 18$$

$$\boxed{V_{18} = 3.57 \text{ V}} \quad // = v_x$$

→ star to delta and delta to star



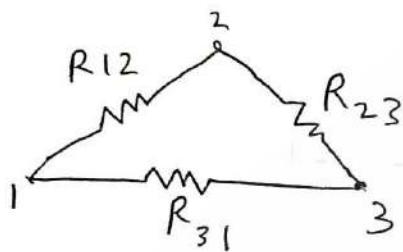
Delta



star

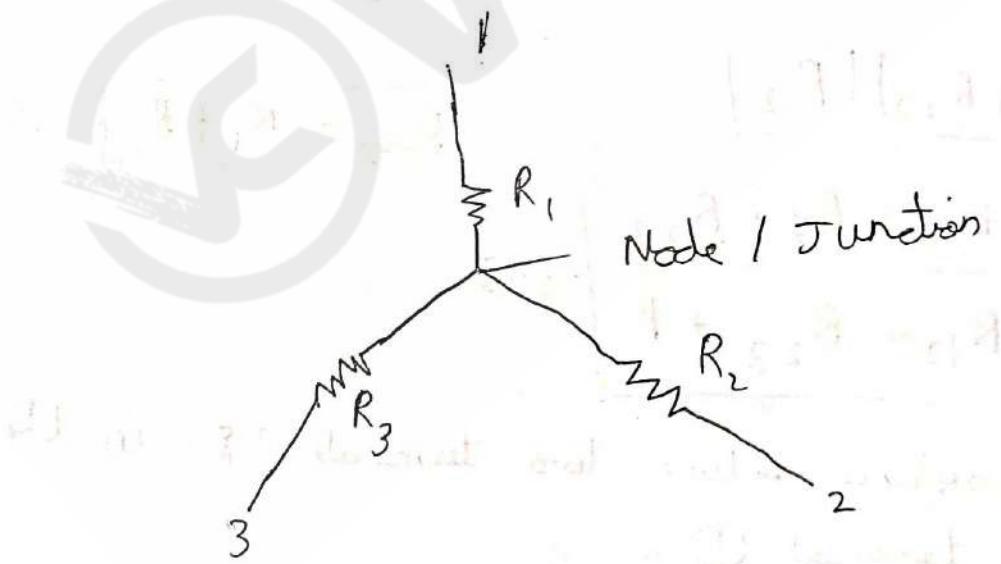
### Delta Network:

Three Resistors or impedances are said to be connected in Delta when they form a closed loop.



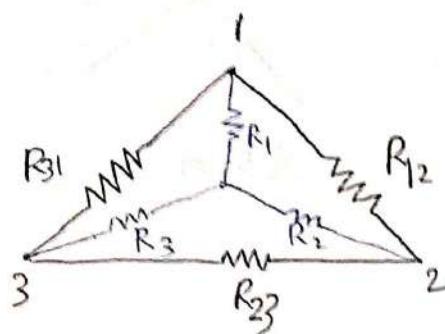
### Star Network:

Three Resistors or impedances are said to be connected in star when they form node or junction.

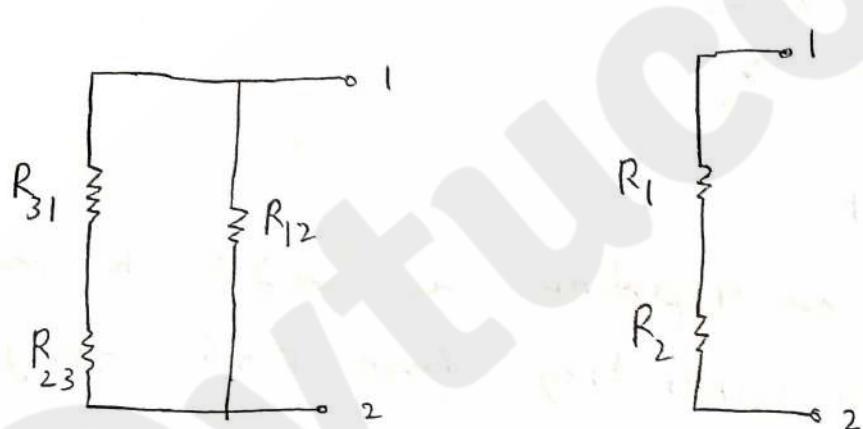


star and delta connections play a major role in network analysis to find voltage across or current through the given element.

→ Derivation (Delta to star Transformation)



consider the following network the equivalent resistance between two terminal 1, 2 in the absence of terminal 3 is given by.

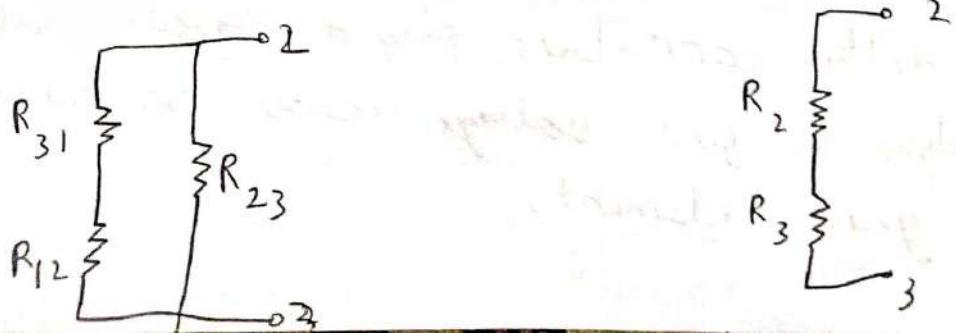


$$R_{eq} = (R_{31} + R_{23}) \parallel R_{12}$$

$$R_{eq} = R_1 + R_2 \quad \dots \textcircled{1}$$

$$R_{eq} = \frac{R_{12} R_{31} + R_{12} R_{23}}{R_{12} + R_{23} + R_{31}} \quad \dots \textcircled{2}$$

The equivalent resistance between two terminals 2 & 3 in the absence of the terminal 1



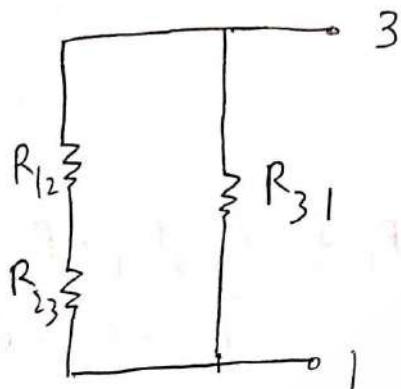
$$R_{eq} = (R_{31} + R_{12}) \parallel R_{23}$$

$$R_{eq} = R_2 + R_3$$

$$R_{eq} = \frac{R_{23}R_{31} + R_{23}R_{12}}{R_{23} + R_{12} + R_{31}} \quad \dots \textcircled{3}$$

(4)

The equivalent resistance between two terminals 1 & 3 in the absence of the terminal 2



$$R_{eq} = (R_{12} + R_{23}) \parallel R_{31}$$

$$R_{eq} = R_1 + R_3 \quad \dots \textcircled{6}$$

$$R_{eq} = \frac{R_{12}R_{31} + R_{31}R_{23}}{R_{12} + R_{23} + R_{31}} \quad \dots \textcircled{5}$$

equating eq (1) { (2), (3) { (4) } , 5 { (6)

$$\frac{R_{12}R_{31} + R_{12}R_{23}}{R_{12} + R_{23} + R_{31}} = R_1 + R_2 \quad \dots \textcircled{7}$$

$$\frac{R_{23}R_{31} + R_{23}R_{12}}{R_{23} + R_{12} + R_{31}} = R_2 + R_3 \quad \dots \textcircled{8}$$

$$\frac{R_{12}R_{31} + R_{31}R_{23}}{R_{12} + R_{31} + R_{23}} = R_1 + R_3 \quad \dots \textcircled{9}$$

adding eq ⑦ & ⑧

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$$\frac{R_{12}R_{31} + R_{12}R_{23}}{R_{12} + R_{23} + R_{31}} + \frac{R_{31}R_{12} + R_{31}R_{23}}{R_{12} + R_{23} + R_{31}} = R_1 + R_2 + R_3$$

$$\frac{2(R_{12}R_{31}) + R_{12}R_{23} + R_{31}R_{23}}{R_{12} + R_{23} + R_{31}} = 2R_1 + R_2 + R_3$$

sub eq ⑧ in above equation

$$\frac{2(R_{12}R_{31}) + R_{12}R_{23} + \cancel{R_{31}R_{23}}}{R_{12} + R_{23} + R_{31}} = 2R_1 + \frac{\cancel{R_{23}R_{31}} + R_{23}R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$\frac{2R_{12}R_{31} + R_{12}\cancel{R_{23}} + \cancel{R_{31}R_{23}} - \cancel{R_{23}R_{31}} - \cancel{R_{23}R_{12}}}{R_{12} + R_{23} + R_{31}} = 2R_1$$

$$\boxed{\frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}} = R_1} \quad ⑩$$

$$\boxed{R_2 = \frac{R_{12}R_{23}}{R_{12} + R_{23} + R_{31}}} \quad ⑪$$

$$\boxed{R_3 = \frac{R_{31}R_{23}}{R_{12} + R_{23} + R_{31}}} \quad ⑫$$

Star to delta transformation  
multiply eq 10 & 11 } 11 & 12, 12 & 10

$$\frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} \times \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}} = R_1 \times R_2$$

$$\frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}} \times \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} = R_2 \times R_3$$

$$\frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} \times \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} = R_3 \times R_1$$

$$R_1 \times R_2 = \frac{R_{12}^2 R_{31} R_{23}}{(R_{12} + R_{23} + R_{31})^2} \quad \dots \textcircled{13}$$

$$R_2 \times R_3 = \frac{R_{23}^2 R_{12} R_{31}}{(R_{12} + R_{23} + R_{31})^2} \quad \dots \textcircled{14}$$

$$R_3 \times R_1 = \frac{R_{31}^2 R_{23} R_{12}}{(R_{12} + R_{23} + R_{31})^2} \quad \dots \textcircled{15}$$

adding eq  $\textcircled{13}$  &  $\textcircled{14}$  &  $\textcircled{15}$

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12}^2 R_{31} R_{23} + R_{23}^2 R_{12} R_{31} + R_{31}^2 R_{23} R_{12}}{(R_{12} + R_{23} + R_{31})^2}$$

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12} R_{23} R_{31} (R_{12} + R_{23} + R_{31})}{(R_{12} + R_{23} + R_{31})^2}$$

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = R_{12} \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = R_{12} R_3$$

$$\boxed{R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}} \quad \dots \textcircled{16}$$

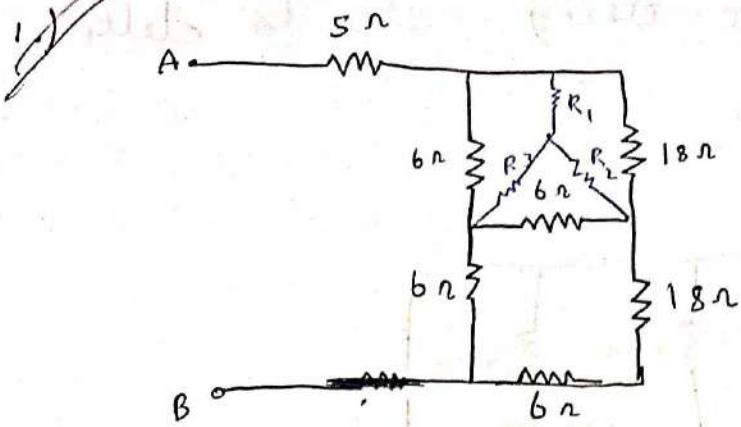
$$\boxed{R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}} \quad \dots \textcircled{17}$$

$$\boxed{R_{31} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}} \quad \dots \textcircled{18}$$

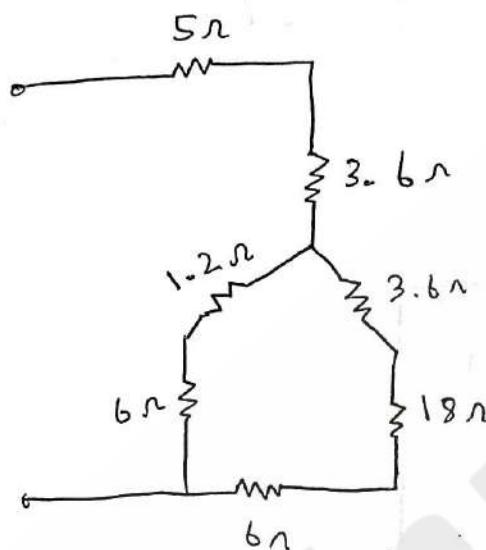
$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$$



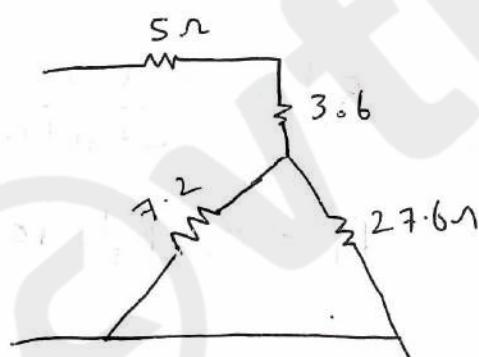
Find the equivalent  $R_{AB}$  using star to Delta transform.



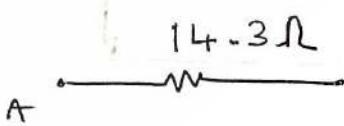
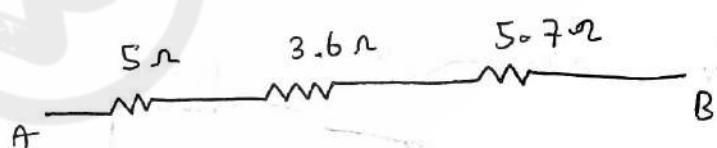
$$R_1 = \frac{6 \times 18}{6 + 18 + 6} = 3.6\Omega$$

$$R_2 = \frac{6 \times 18}{6 + 18 + 6} = 3.6\Omega$$

$$R_3 = \frac{6 \times 6}{6 + 6 + 18} = 1.2\Omega$$

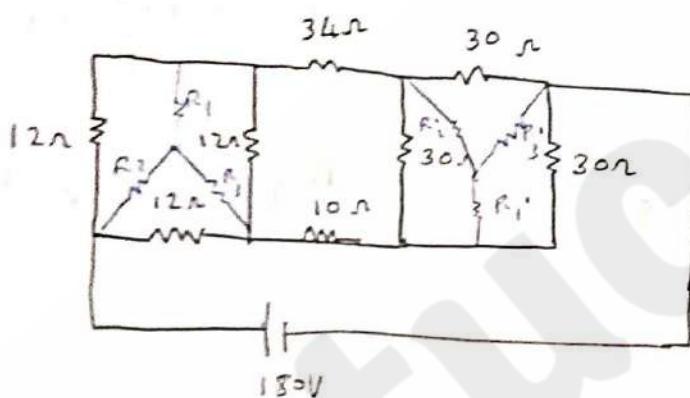
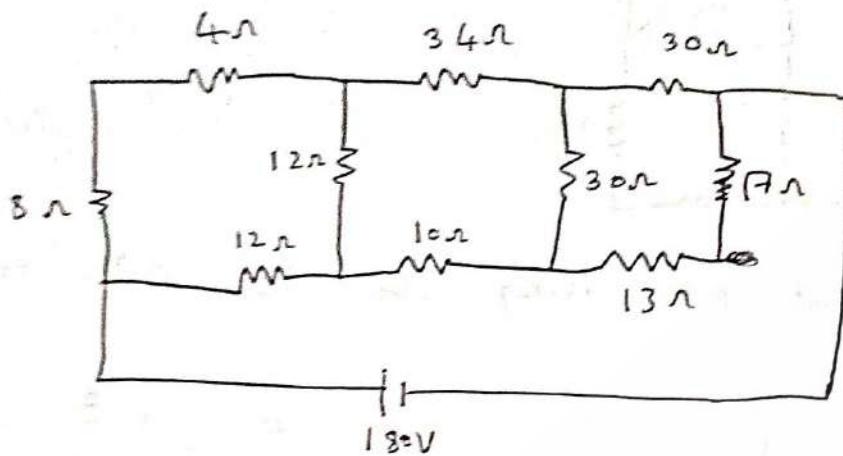


7.21) 27.6



$$\therefore R_{AB} = 14.3\Omega$$

2) Find the current I using star to delta transformation

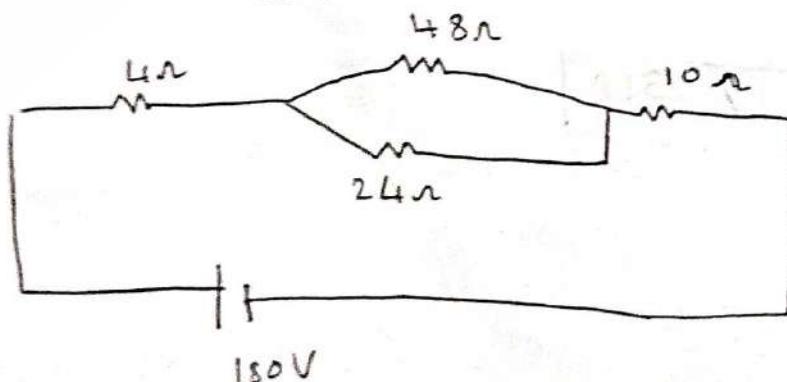
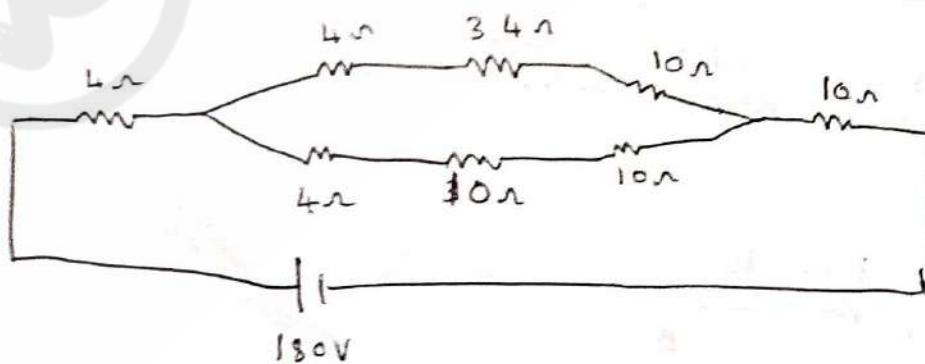


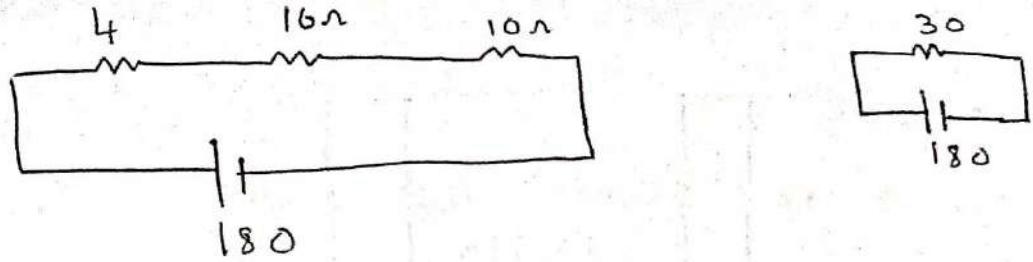
$$R_1 = \frac{12 \times 12}{12+12+12} = 4\Omega$$

$$R_1' = \frac{30 \times 30}{30+30+30}$$

$$R_1 = R_2 = R_3 = 4\Omega$$

$$R_1' = R_2' = R_3' = 10\Omega$$

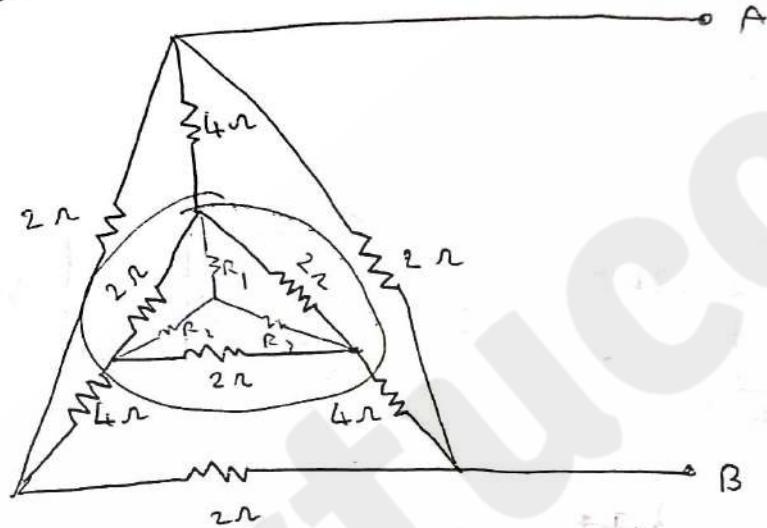




$$I = \frac{V}{R} = \frac{180}{30} = 6 \text{ A}$$

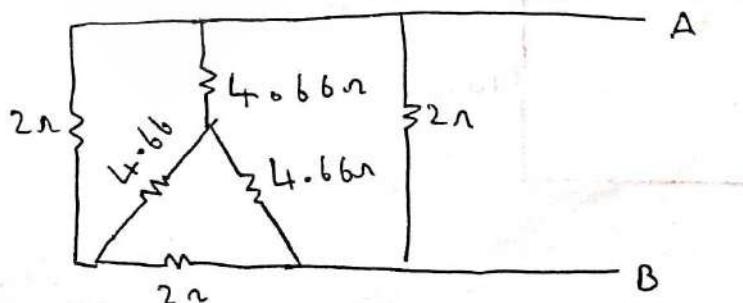
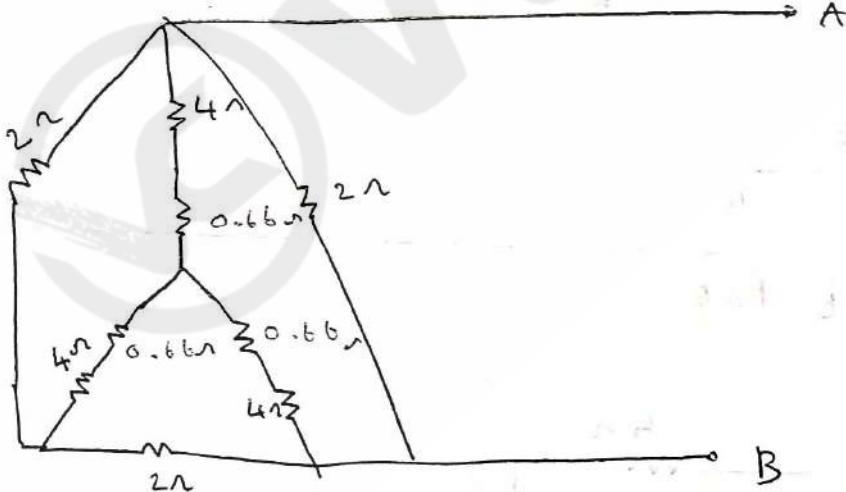
$$I = 6 \text{ A}$$

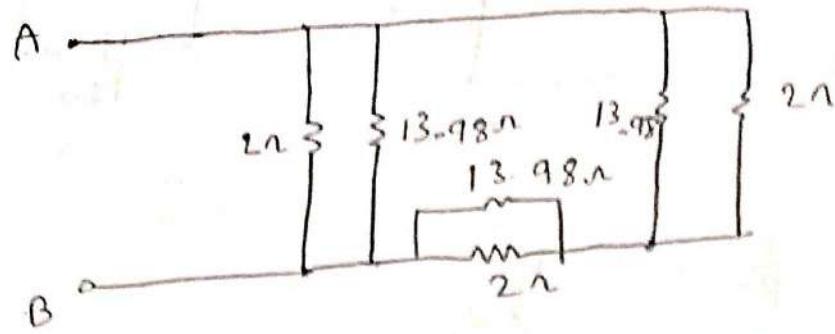
3) Find  $R_{AB}$



$$R_1 = R_2 = R_3$$

$$R_1 = \frac{2 \times 2}{2 + 2 + 2}$$

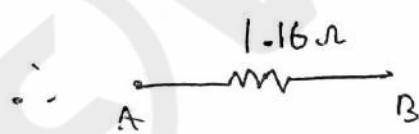




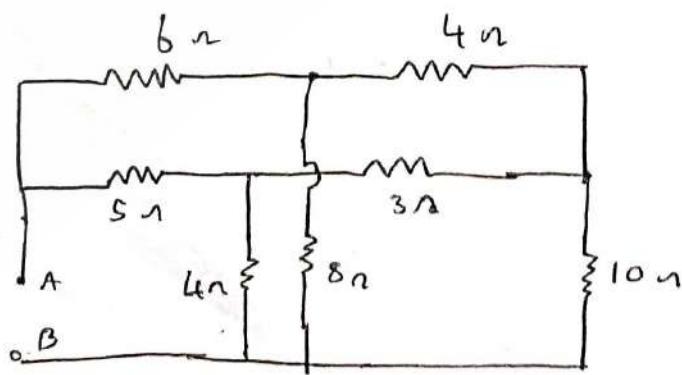
$$R_{AB} = R_1 + R_2 + \frac{R_1 R_2}{R_3} = 13.98\Omega$$

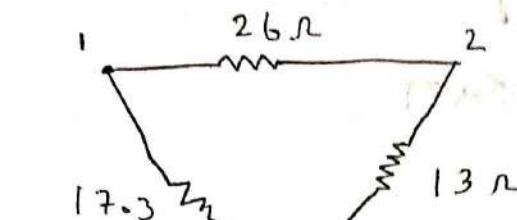
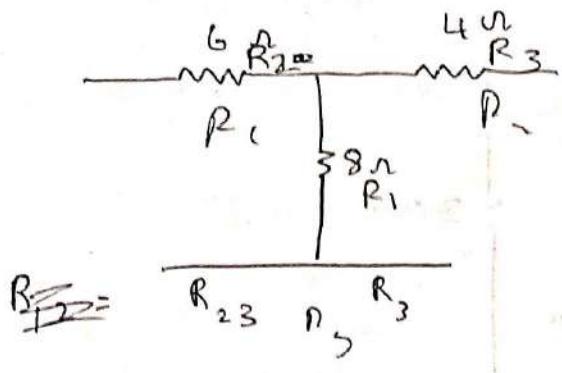
$$R_{23} = 13.98\Omega$$

$$R_{31} = 13.98\Omega$$



4) Find the value of  $R_{AB}$

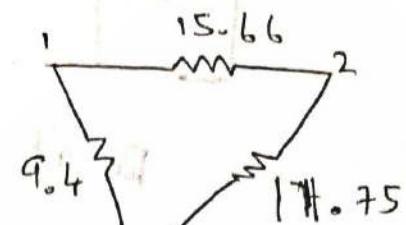
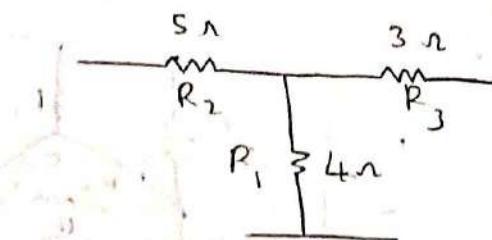




$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3} = 26 \Omega$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1} = 13 \Omega$$

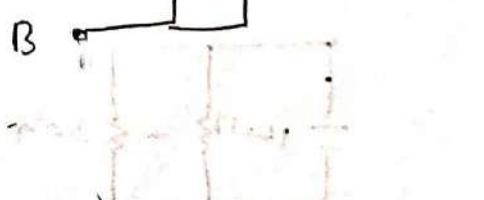
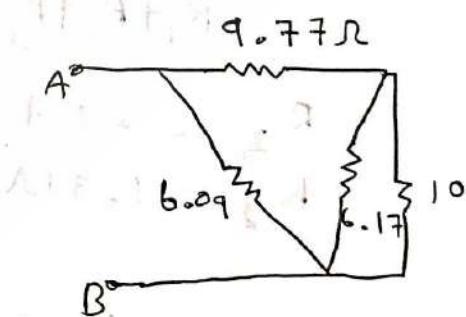
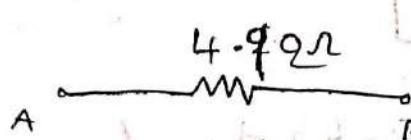
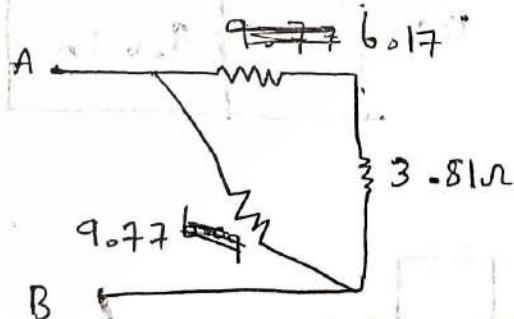
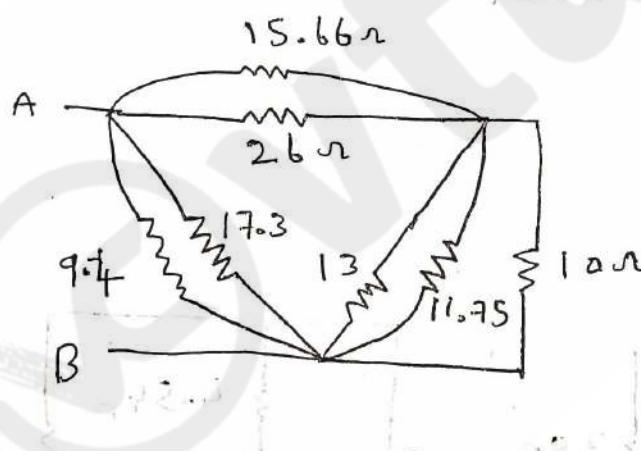
$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2} = 17.3 \Omega$$



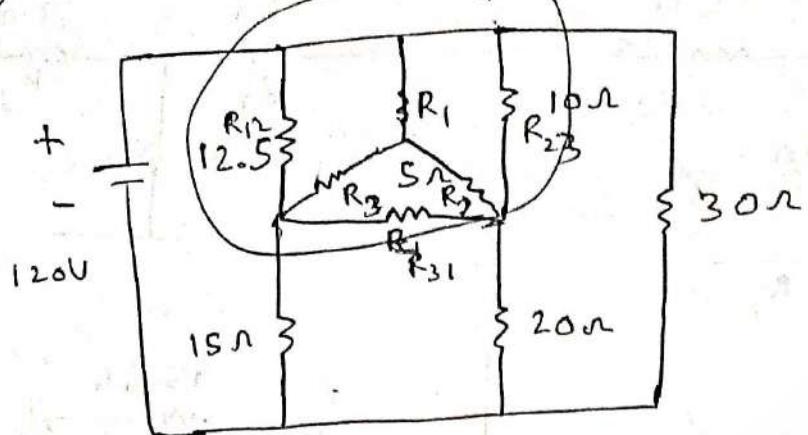
$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3} = 15.66 \Omega$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1} = 11.75 \Omega$$

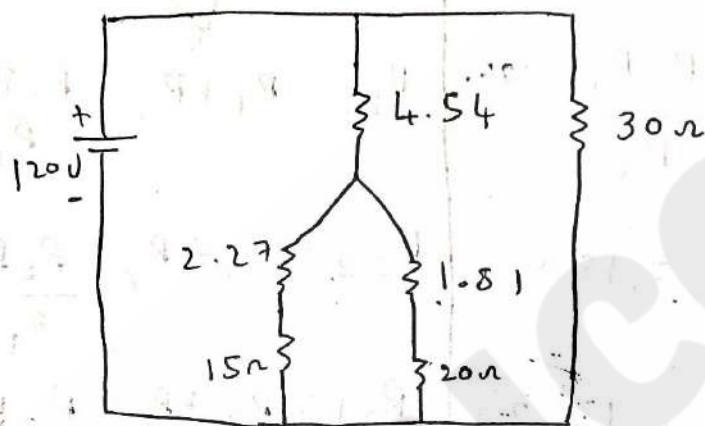
$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2} = 9.4 \Omega$$



5)



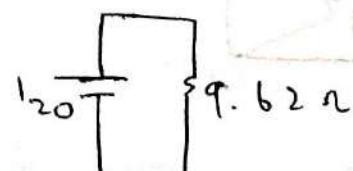
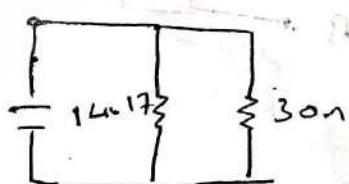
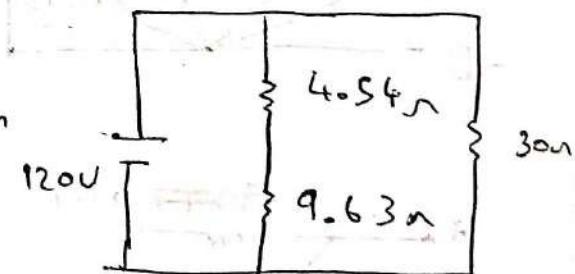
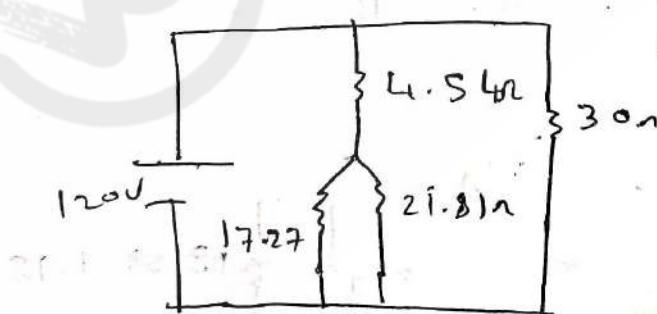
Partial circuit to start



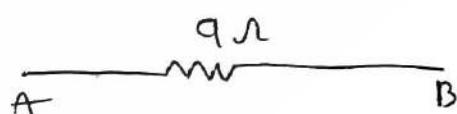
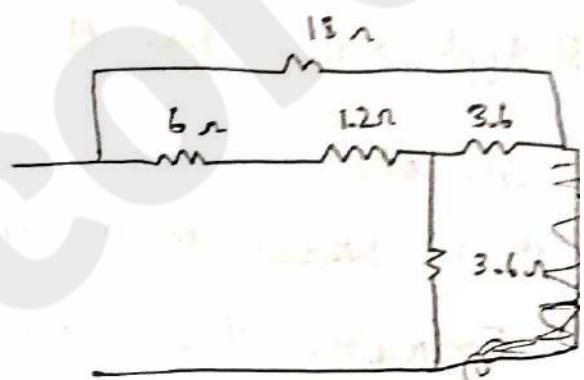
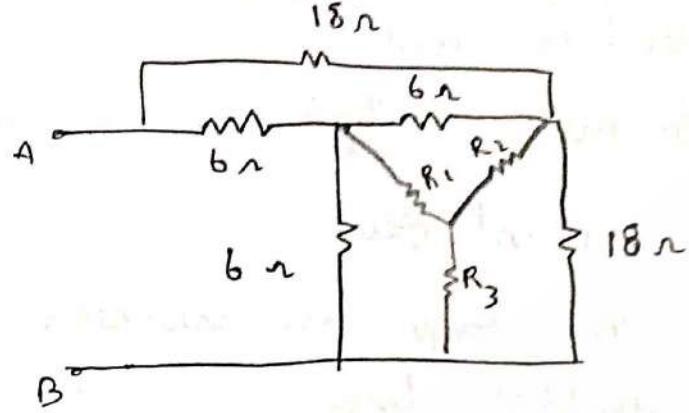
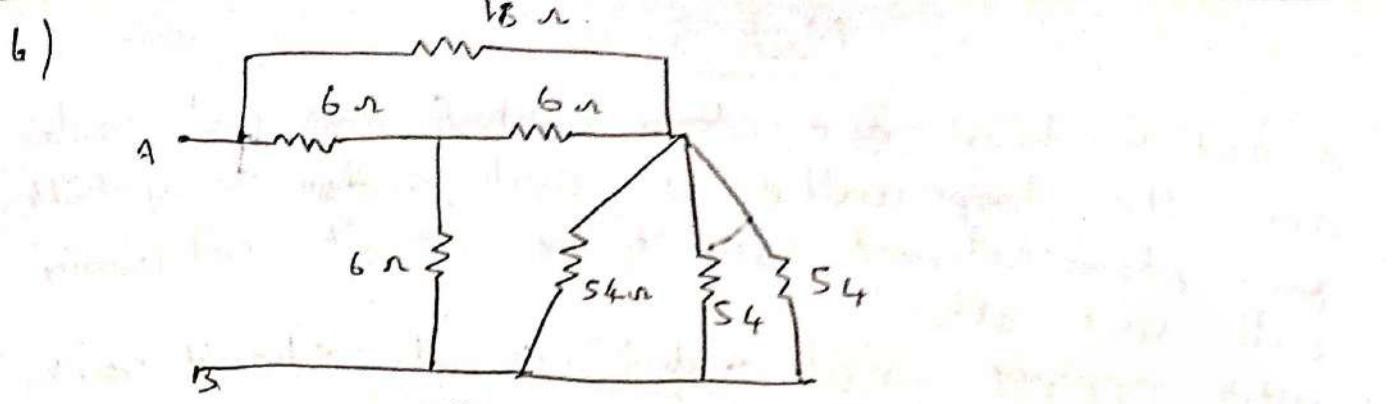
$$R_1 = \frac{R_{12} R_{23}}{R_1 + R_2 + R_3} = 4.54$$

$$R_{12} = 2.27\Omega$$

$$R_{23} = 1.81\Omega$$



$$I = \frac{V}{R} = \frac{120}{9.62} = 12.47A$$



## Mesh Analysis

- A mesh is defined as a loop which does not contain any other loops within it. Mesh analysis is applied for a planar network (elements of network not crossing with each other).
- While applying mesh analysis circuit should contain voltage sources & KVL is used.

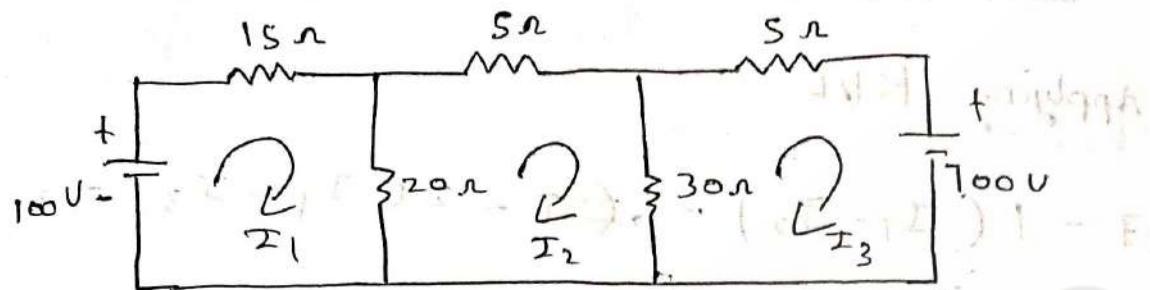
Steps to be followed in mesh analysis:

- 1) choose a conventional current flow
- 2) Identify and number the loops for convenience
- 3) Apply KVL for the identified loops.
- 4) use ohm's law to express branch voltages in terms of unknown mesh currents & the resistance
- 5) Formulate the ckt equations.
- 6) solve the simultaneous equations.

When the current source is in the perimeter (or) boundary of the ckt.

→ loop or mesh analysis:

- 1) Find the current through the  $30\Omega$  in the network shown using loop analysis



$$100 - 15I_1 - 20(I_1 - I_2) = 0$$

$$100 - 15I_1 - 20I_1 + 20I_2 = 0$$

$$100 - 35I_1 + 20I_2 = 0$$

$$\boxed{-35I_1 + 20I_2 = -100} \dots \textcircled{1}$$

$$-5I_2 - 30(I_2 - I_3) - 20(I_2 - I_1) = 0$$

$$-5I_2 - 30I_2 + 30I_3 - 20I_2 + 20I_1 = 0$$

$$20I_1 - 55I_2 + 30I_3 = 0$$

$$\boxed{20I_1 - 55I_2 = 0} \dots \textcircled{2}$$

$$-5I_3 - 100 - 30(-I_3 - I_2) = 0$$

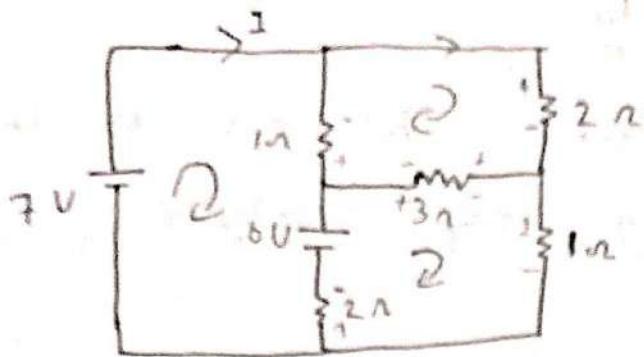
$$-5I_3 - 30I_3 + 30I_2 = 100$$

$$\boxed{30I_2 - 35I_3 = 100} \dots \textcircled{3}$$

$$I_{30\Omega} = (I_2 - I_3) = -1.6 + 4.22$$

$$\boxed{I_{30} = 2.62 \text{ A}}$$

2)



Applying KVL

$$7 - 1(I_1 - I_2) - 6 \cancel{I_1} - 2(I_1 - I_3) = 0$$

$$7 - I_1 + I_2 - 6 - 2I_1 + 2I_3 = 0$$

$$-3I_1 + I_2 + 2I_3 + 1 = 0$$

$$\boxed{-3I_1 + I_2 + 2I_3 = -1} \quad \textcircled{1}$$

Applying KVL to loop 2

$$-2(I_2) - 3(I_2 - I_3) - 1(I_2 - I_1) = 0$$

$$-2I_2 - 3I_2 + 3I_3 - I_2 + I_1 = 0$$

$$\boxed{I_1 - 6I_2 + 3I_3 = 0} \quad \textcircled{2}$$

Applying KVL to loop 3

$$-3(I_3 - I_2) - 1(I_3) - 2(I_3 - I_1) = -3$$

$$-3I_3 + 3I_2 - I_3 - 2I_3 + 2I_1 = -6$$

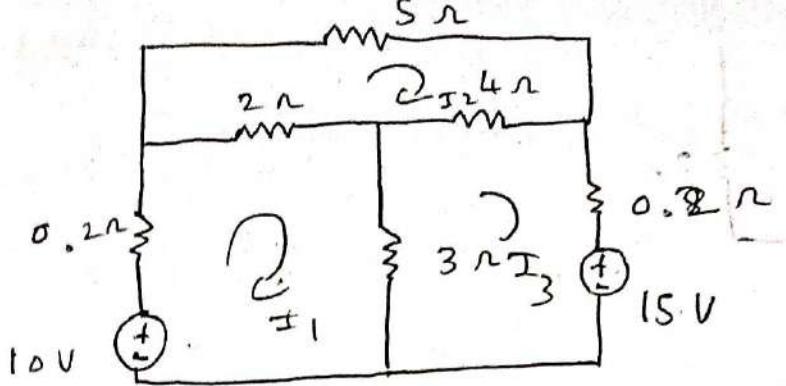
$$\boxed{2I_1 + 3I_2 - 6I_3 = -6}$$

$$\boxed{I_1 = 3}$$

$$\boxed{I_2 = 2A}$$

$$\boxed{I_3 = 3A}$$

3)



$$-0.2 I_1 - 2(I_1 - I_2) - 3(I_1 - I_3) = 0$$

$$-0.2 I_1 - 2 I_1 + 2 I_2 - 3 I_1 + 3 I_3 = 0$$

$$\boxed{-5.2 I_1 + 2 I_2 + 3 I_3 = -10} \quad \dots \textcircled{1}$$

$$-5 I_2 - 4 I_2 + 4 I_3 - 2 I_2 + 2 I_1 = 0$$

$$\boxed{2 I_1 - 11 I_2 + 4 I_3 = 0} \quad \dots \textcircled{2}$$

$$-0.2 I_3 - 15V - 3(I_3 - I_1) = -4(I_3 -$$

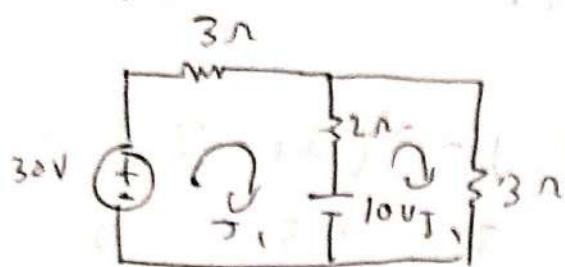
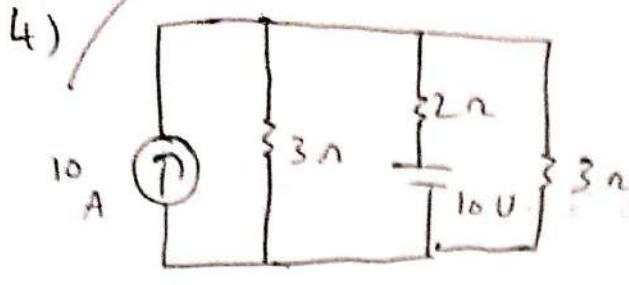
$$-0.2 I_3 - 3 I_3 + 3 I_1 - 4 I_3 + 4 I_2 = 15$$

$$\boxed{3 I_1 + 4 I_2 - 7.2 I_3 = 15} \quad \dots \textcircled{3}$$

$$\boxed{I_1 = 0.111}$$

$$\boxed{I_2 = -0.902}$$

$$\boxed{I_3 = -2.53}$$



Applying KVL

$$30 - 3I_1 - 2I_1 + 2I_2 - 10 = 0$$

$$-5I_1 + 2I_2 = -20 \quad \dots \textcircled{1}$$

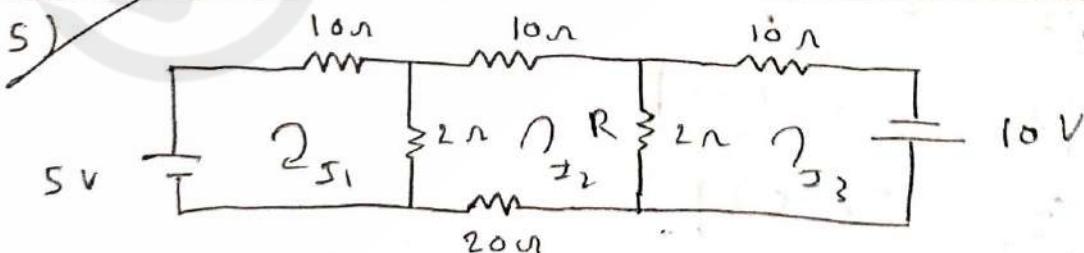
Applying KVL

$$10 - 2I_2 + 2I_1 - 3I_2 = 0$$

$$2I_1 - 5I_2 = -10 \quad \dots \textcircled{2}$$

$$I_1 = 5.714$$

$$I_2 = 4.285$$



Applying KVL to loop 1

$$5 - 10I_1 - 2I_1 + 2I_2 = 0$$

$$-12I_1 + 2I_2 = -5$$

Applying KVL to loop 2

$$-2(I_2 - I_1) - 10(I_2) - 2(I_2 - I_3) \stackrel{=} {0} (20V)$$

$$-2I_2 + 2I_1 - 30I_2 - 2I_2 + 2I_3 = 0$$

$$2I_1 - 34I_2 + 2I_3 = 0$$

Applying KVL to loop 3

$$-2I_3 + 2I_2 + 10I_3 + 10 = 0$$

$$2I_2 - 12I_3 = -10$$

$$I_1 = 0.429A$$

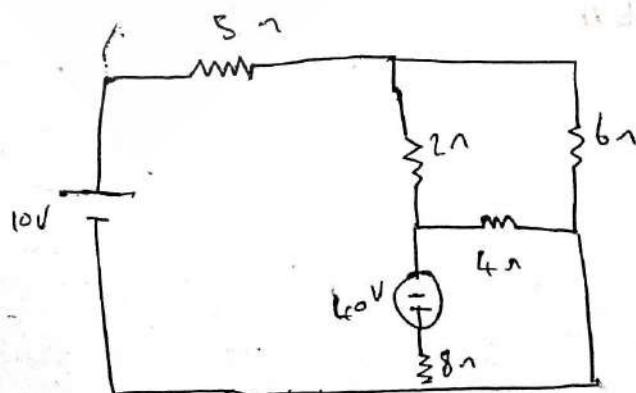
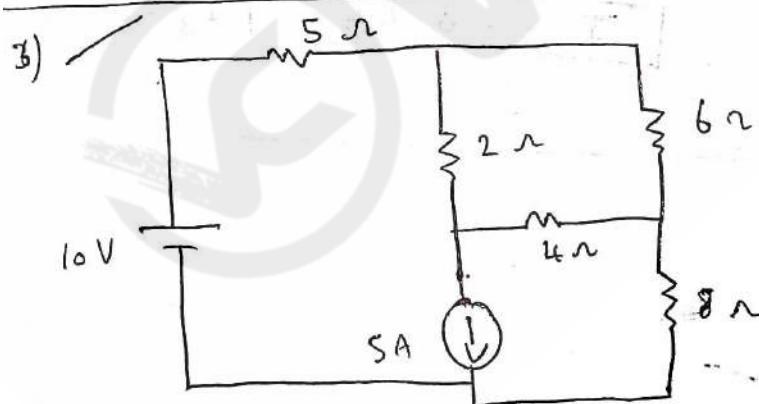
$$I_2 = 0.075A$$

$$I_3 = 0.8458A$$

$$V_R = 2(I_2 - I_3)$$

$$V_R = 2(0.075A - 0.8458)$$

$$V_R = -1.5416V$$



Apply KVL to loop 1

$$+10 - 5I_1 - 2I_1 + 2I_2 \neq -8I_1 + 8I_2$$

$$-15I_1 + 2I_2 + 8I_3 = -50$$

Apply KVL

$$-6I_2 - 4(I_2 - I_3) - 2(I_2 - I_1) = 0$$

$$-6I_2 - 4I_2 + 4I_3 - 2I_2 + 2I_1 = 0$$

$$2I_1 - 12I_2 + 4I_3 = 0$$

$$-40 - 4I_3 + 4I_2 - 8(I_3) + 8I_1 = 0$$

$$-40 - 8I_3 - 4I_3 + 4I_2 + 8I_2 = 0$$

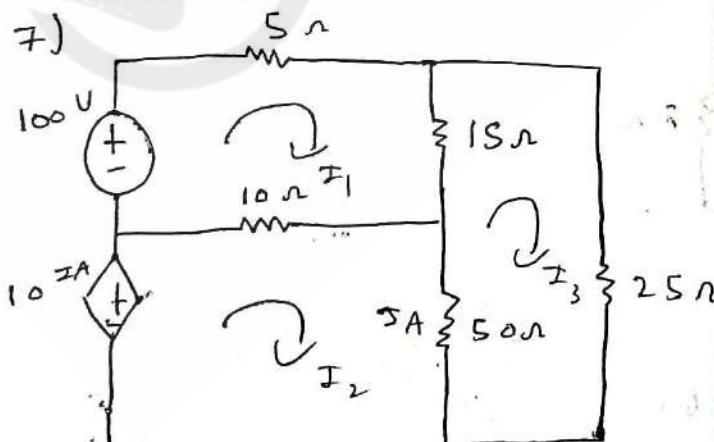
$$-40 - 12I_3 + 8I_1 + 4I_2 = 0$$

$$8I_1 + 4I_2 - 12I_3 = 40$$

$$I_1 = 2.29 A$$

$$I_2 = -0.24$$

$$I_3 = -1.88$$



$$IA = I_2 - I_3$$

Apply KVL to loop 1

$$100 - 5I_1 - 15I_1 + 15I_3 - 10I_1 + 10I_2 = 0$$
$$\boxed{-30I_1 + 10I_2 + 15I_3 = -100}$$

$$-10(I_2 - I_1) - 50(I_2 - I_3) + 10(I_2 - I_3) = 0$$

$$\boxed{10I_1 - 50I_2 + 40I_3 = 0 \dots (2)}$$

$$-15I_3 - 50I_3 + 50I_2 - 15I_3 + 25I_1 = 0$$

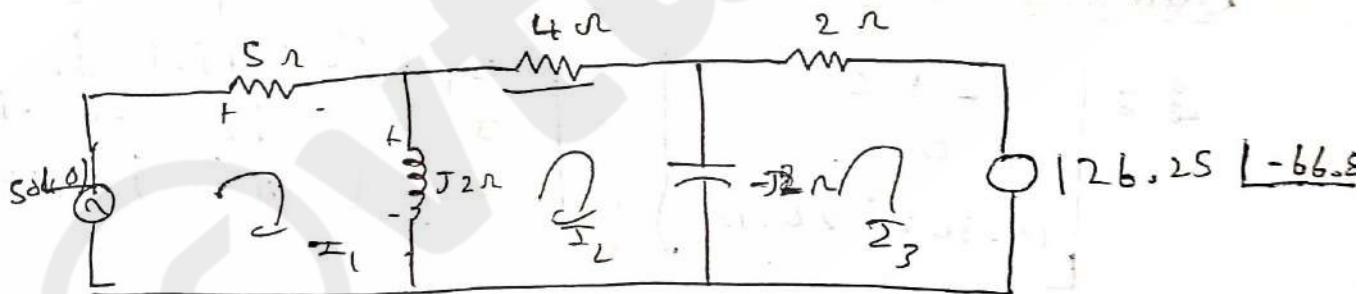
$$\boxed{25I_1 + 50I_2 - 90I_3 = 0 \dots (3)}$$

$$\boxed{I_1 = 6.06 \text{ A}}$$

$$\boxed{I_2 = 3.136 \text{ A}}$$

$$\boxed{I_3 = 3.030 \text{ A}}$$

8) Find the current through  $4\Omega$  resistance in the network shown using loop analysis.



$$50 - 5I_1 - J_2(I_1 - I_2) = 0$$

$$50 - 5I_1 - I_1 J_2 + J_2 I_2 = 0$$

$$50 - I_1(5 + J_2) + J_2 I_2 = 0$$

$$-I_1(5 + 2j) + j2I_2 = -50$$

$$\boxed{-(5 + 2j)I_1 + J_2 I_2 = -50}$$

$$-4I_2 - (-j_2)(I_2 - I_3) - j_2(j_2 - I_1) = 0$$

$$-4I_2 + j_2 I_2 - j_2 I_3 - \cancel{j_2 I_2} + j_2 I_1 = 0$$

$$+ j_2 I_1 - 4I_2 - j_2 I_3 = 0 \quad \text{②}$$

Applying KVL to loop 3

$$-2I_3 + (10.34 - j_2 4.12) - (-j_2)(j_3, j_1)$$

$$-2I_3 + (10.34 - j_2 4.12) + j_2 I_3 - j_2 I_2 = 0$$

$$\cancel{j_2} I_3 (-2 + j_3) - j_2 I_2 + (10.34 - j_2 4.12)$$

$$(10.34 - j_2 4.12) = j_2 I_2 - (-2 + j_2) I_3 \quad \text{③}$$

$$V = I \cdot R$$

$$\begin{bmatrix} -50 \\ 0 \\ 10.34 - j_2 4.12 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \begin{bmatrix} -(5+j_2) & j_2 & 0 \\ j_2 & -4 & -j_2 \\ 0 & j_2 & (2-j_2) \end{bmatrix}$$

$$= \begin{bmatrix} -5+j_2 & j_2 & 0 \\ j_2 & -4 & -j_2 \\ 0 & j_2 & (2-j_2) \end{bmatrix}$$

$$36 - 32j + 8 - j_2^2$$

$$\Delta = (-(5+2j)((-4(2-j_2) - j_2(j_2)) - j_2(j_2(2-j_2)))$$

$$= -5 - 2j(-8 + 8j + \cancel{j_2^2}) - j_2(4j_2 + 4)$$

$$\Delta = -(5 + 2j) \begin{bmatrix} -8 + 8j & -4 \\ -12 + j8 & 8 - j8 \end{bmatrix} - 5j^2 (4j + 4)$$

$$\boxed{\begin{aligned} j &= j \\ j^2 &= -1 \\ j^3 &= j^2 \cdot j \end{aligned}}$$

$$\Delta = -(5 + 2j)(-12 + j8) + 8 - j8$$

$$\Delta = (-5 - j2)(-12 + j8) + 8 - j8$$

$$\Delta = 60 - 40j + 24j + 16 + 8 - j8$$

$$\boxed{\Delta = 84 - 24j}$$

$$\boxed{\Delta = 87.36 \angle -15.94^\circ}$$

$$I_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta_1 = \begin{bmatrix} -50 & j2 & 0 \\ 0 & -4 & -j2 \\ 10 - 34 - 24j - 12j & j2 & (2 - j2) \end{bmatrix}$$

$$\left. \begin{aligned} \Delta_1 &= -50 \left( -4(2 - j2) - (-j2)(j2) \right) \\ &\quad - j2 \left( -j2(10 - 34 - 24j - 12j) \right) \end{aligned} \right]$$

$$\Delta_1 = -50 \left( -12 + 8j \right) + 41 \cdot 36 - 96 \cdot 48i$$

$$\Delta_1 = 600 - 400i + 41 \cdot 36 - 96 \cdot 48i$$

$$I_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta_1 = 641.36 - 496.48i$$

$$\Delta_1 = 811.03 \angle -37.74^\circ$$

$$I_1 = \frac{811.03 \angle -37.74^\circ}{87.36 \angle -15.94^\circ}$$

$$I_1 = 9.284 \angle -21.8^\circ$$

$$\Delta_2 = \begin{bmatrix} -(5+j2) & -50 & 0 \\ j2 & 0 & -j2 \\ 0 & 10.34-24.12j & (2-j2) \end{bmatrix}$$

$$\Delta_2 = -(5+j2)(-50 + j2(10.34-24.12j)) + 50(j2(2-j2) - (-j2(0)))$$

$$\Delta_2 = -5 - j2(48.24 + 20.68i) + 50(4+4i)$$

$$\Delta_2 = 36 - 36 - 96.48i + 200 + 200i$$

$$\Delta_2 = 236 - 36 + 103.52i$$

$$\Delta_2 = 258.03 \angle 23.65^\circ$$

$$I_2 = \frac{258.03 \angle 23.65^\circ}{87.36 \angle -15.94^\circ}$$

$$I_2 = 2.95 \angle 39.59^\circ$$

$$\Delta_2 = 243.24 + 218.4i$$

$$\Delta_2 = 327.08 \angle 41.95^\circ$$

$$I_2 = \frac{327.08 \angle 41.95^\circ}{87.36 \angle -15.94^\circ}$$

$$I_2 = 3.74 \angle 57.81^\circ$$

$$\Delta_3 = \begin{bmatrix} -(5+j2) & j2 & -50 \\ j2 & -4 & 0 \\ 0 & j2 & 10.34-24.12j \end{bmatrix}$$

$$\Delta_3 = -(5+j2)(-4(10.34-24.12j)-8) - j2(j2(10.34-24.12j)-0) - 50(j2(j2)+4)$$

$$\Delta_3 = -5 - j_2 (-41 \cdot 36 + 96 \cdot 48 i) - 41 \cdot 36 + 96 \cdot 48 i + 200$$

$$\Delta_3 = 187 \cdot 96 + 82 \cdot 72 i - 41 \cdot 36 + 96 \cdot 48 i + \cancel{200}$$

$$\boxed{\Delta_3 = 346 \cdot 6 + 179 \cdot 2 i}$$

$$\boxed{\Delta_3 = 390.18 \angle 27.33^\circ}$$

$$I_3 = \frac{\Delta_3}{\Delta} \quad I_3 = \frac{390.18 \angle 27.33^\circ}{87.36 \angle -15.94^\circ}$$

$$\boxed{I_3 = 4.46 \angle 43.27^\circ}$$

$$\Delta_2 = \begin{bmatrix} -(5+j_2) & -50 & 0 \\ j_2 & 0 & -j_2 \\ 0 & 10 \cdot 34 - 24 \cdot 12 i & (2-j_2) \end{bmatrix}$$

$$\Delta_2 = (-5-j_2)(j_2(10 \cdot 34 - 24 \cdot 12 i)) + 50(j_2(2-j_2))$$

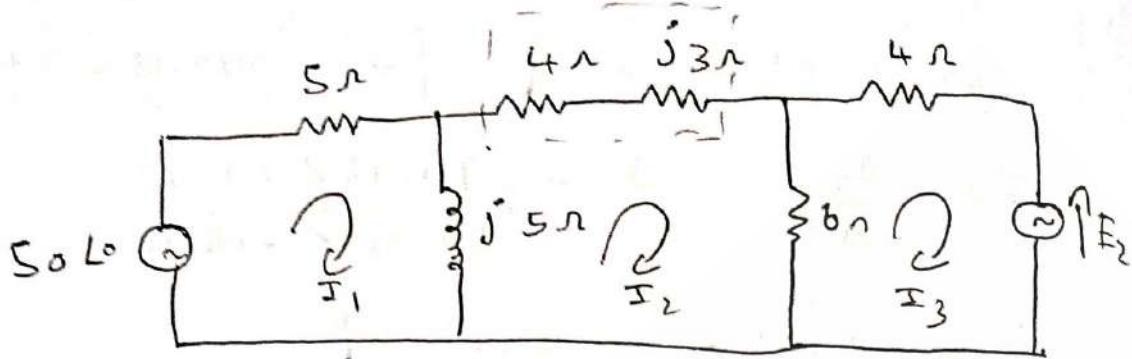
$$\Delta_2 = 0 \cdot 16 + 0 \cdot 12 j$$

$$\Delta_2 = 0 \cdot 2 \angle 36.86^\circ$$

$$I_2 = \frac{\Delta_2}{2} \quad \frac{0 \cdot 2 \angle 36.86^\circ}{87.36 \angle -15.94^\circ}$$

$$\boxed{I_2 = 2.28 \times 10^{-3} \angle +52.8^\circ}$$

9) Find the value of voltage source  $E_2$  in the network shown such that current through  $4 + j3\Omega$  is 0



$$E_2 = ?$$

$$I_{4+j3} = 0 = I_2 = 0$$

$$V = 50 \angle 0^\circ$$

$$V = 50$$

Apply KVL at loop 1

$$50 - 5I_1 - j5(I_1 - I_2) = 0$$

$$50 - 5I_1 - I_1 j5 + I_2 j5 = 0$$

$$50 - I_1(5 + j5) + I_2 j5 = 0 \quad \text{... } \textcircled{1}$$

$$50 - 5I_1(1 + j) + I_2 j5 = 0$$

$$50 = 5I_1(1 + j) - I_2 j5 \quad \text{... } \textcircled{1} \therefore I_2 =$$

$$I_1 = \frac{50}{5 + j5}$$

$$I_1 = 5 \angle 5^\circ$$

$$I_1 = 7.07 \angle 45^\circ A$$

Apply KVL to loop 2

$$-(4 + j3\pi)I_2 - 6(I_2 - I_3) - j5(I_2 - I_1) = 0$$
$$-6I_2 + 6I_3 - j5I_2 + j5I_1 = 0$$

$$6I_3 + j5I_1 = 0$$

Sub  $I_1$  in above eq

$$6I_3 + j5(5 - 5j) = 0$$

$$6I_3 = -j5(5 - j5)$$

$$I_3 = \frac{-j5(5 - j5)}{6}$$

$$I_3 = -(4 \cdot 16.6 + 4 \cdot 16j)$$

Apply KVL to loop 3

$$-4I_3 - E_2 - 6I_3 + j5I_2 = 0$$

$$-10I_3 = E_2$$

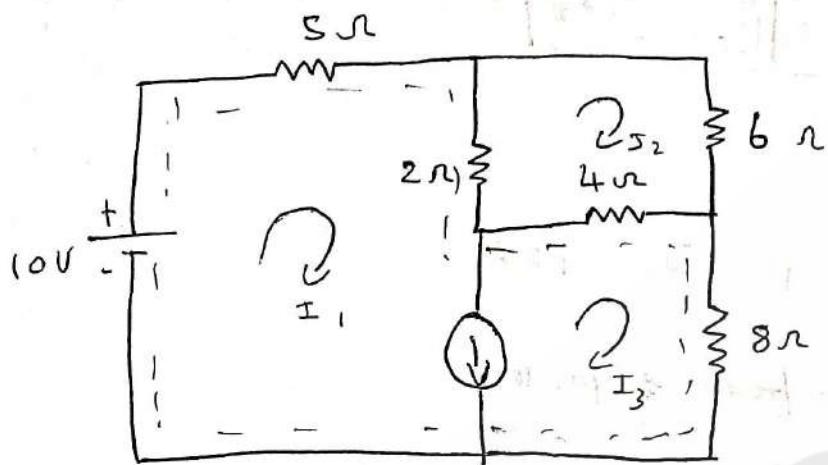
$$E_2 = -10(-4 \cdot 16.6 + 4 \cdot 16j)$$

$$E_2 = 41.66 + 41.66j$$

$$E_2 = 58.916 \angle 45^\circ$$

1) Find all the loop current using loop or mesh analysis.

Note ( If there is an ideal current source between any two loops, that loop we call it as superloop )



From super mesh.

$$I_1 - I_3 = 5 \quad \dots \quad (1)$$

Apply KVL to super mesh

$$10 - 5I_1 - 2(I_1 - I_2) - 4(I_3 - I_2) - 8I_3 = 0$$

$$10 - 5I_1 - 2I_1 + 2I_2 - 4I_3 + 4I_2 - 8I_3 = 0$$

$$-7I_1 + 6I_2 - 12I_3 = -10 \quad \dots \quad (2)$$

KVL loop (2)

$$-6I_2 - 4(I_2 - I_3) - 2(I_2 - I_1) = 0$$

$$-6I_2 - 4I_2 + 4I_3 - 2I_2 + 2I_1 = 0$$

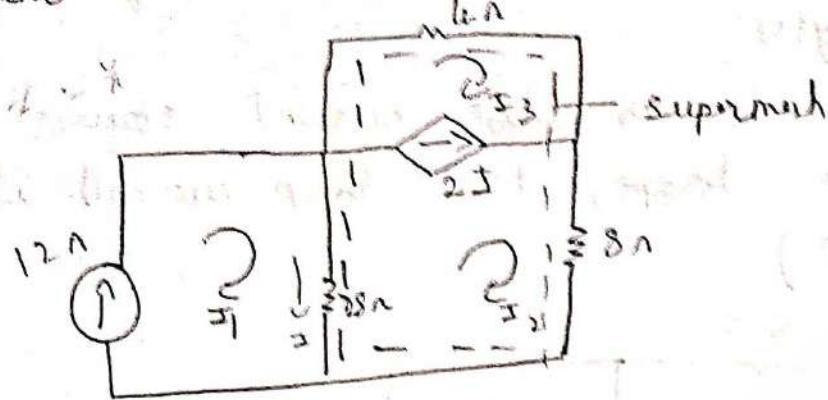
$$2I_2 - 12I_2 + 4I_3 = 0$$

$$I_1 = 3.75A$$

$$I_2 = 0.208A$$

$$I_3 = -1.25A$$

2) Find current  $I_2$  in  $28\Omega$  resistance by mesh method



$I_{2s}?$

Soln: from loop 1  $I_1 = 12A$

Loop 2 & 3 form super mesh

$$I_2 - I_3 = 2I$$

$$\text{from ckt } I = I_1 - I_2$$

$$I_2 - I_3 = 2I_1 - 2I_2$$

$$3I_2 - I_3 = 2I_1$$

$$-2I_1 + 3I_2 - I_3 = 0$$

$$-2(12) + 3I_2 - I_3 = 0$$

$$3I_2 - I_3 = 24 \quad \dots \quad ①$$

Apply KVL to super mesh

$$-4I_3 - 8I_2 - 28I_2 + 28I_1 = 0$$

$$28I_1 - 36I_2 - 4I_3 = 0$$

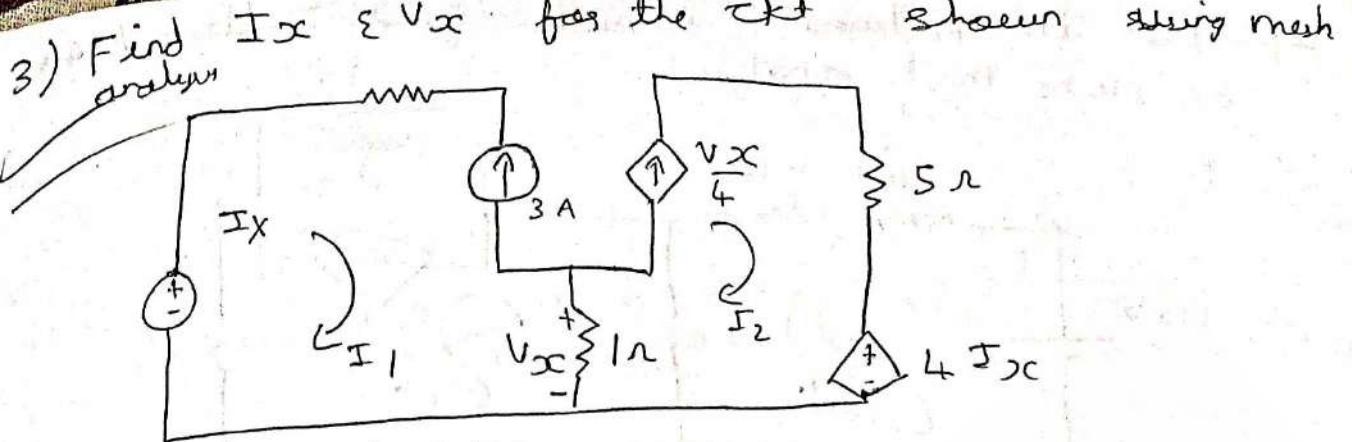
$$-36I_2 - 4I_3 = -336$$

$$I_2 = 9A$$

$$I_3 = 3A$$

$$I_{28\Omega} = I_1 - I_2$$

$$I_{28\Omega} = 3A$$



From loop 1

$$I_x = 3 \text{ A}$$

$$I_1 = -I_x = -3 \text{ A}$$

$$\boxed{I_1 = -3 \text{ A}}$$

from loop -2

$$I_2 = \frac{V_x}{4}$$

$$\begin{aligned} V_x &= IR \\ &= (I_1 - I_2) \times 1 \end{aligned}$$

$$V_x = I_1 - I_2$$

$$\therefore I_2 = \frac{I_1 - I_2}{4}$$

$$4I_2 = -3 - I_2$$

$$5I_2 = -3$$

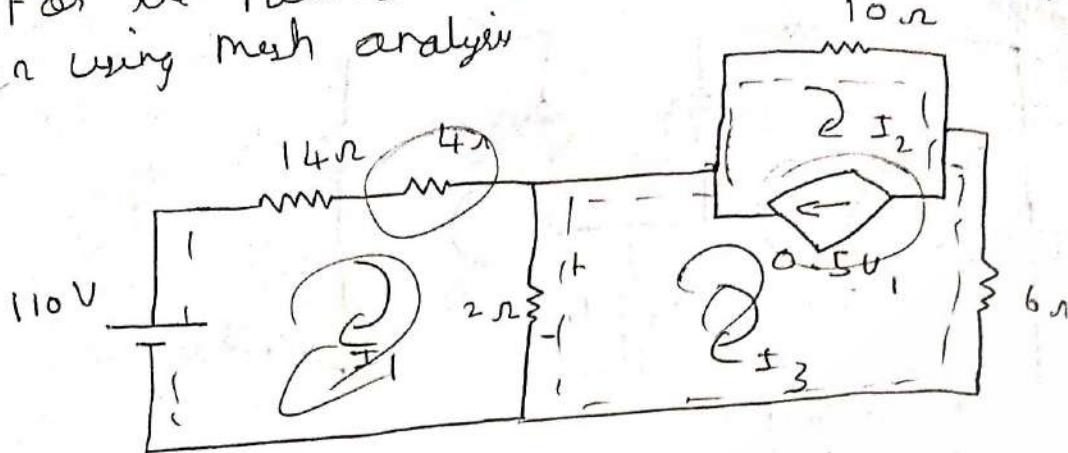
$$I_2 = \frac{-3}{5}$$

$$I_2 = -0.6 \text{ A}$$

$$V_{xc} = 4 \times (-0.6) \text{ A}$$

$$\boxed{V_{xc} = -2.4 \text{ V}}$$

4) For the network shown find  $I_1$  through  $I_6$  using mesh analysis



Loop 2 & 3 forms supermesh.

gives

$$I_{4n} = I_1$$

$$I_{6n} = I_3 =$$

$$I_2 - I_3 = 0.5V_1 \quad \dots \quad (1)$$

$$V_1 = IR$$

$$= (I_1 - I_3)2$$

$$V_1 = 2I_1 - 2I_3$$

$$I_2 - I_3 = 0.5(2I_1 - 2I_3)$$

$$I_2 - I_3 = I_1 - I_3$$

$$I_2 - I_3 - I_1 + I_3 = 0$$

$$I_2 - I_1 = 0 \dots (2)$$

$$\boxed{I_1 = I_2}$$

Apply KVL to super mesh.

$$-10I_2 - 6I_3 - 2I_3 + 2I_1 = 0$$

$$2I_1 - 10I_2 - 8I_3 = 0 \dots (3)$$

Apply KVL

$$110 - \cancel{18} I_1 - 2 I_1 + 2 I_3 = 0$$

$$- 20 I_1 + 2 I_3 = - 110 \quad \dots \quad (4)$$

$$I_1 = 5A$$

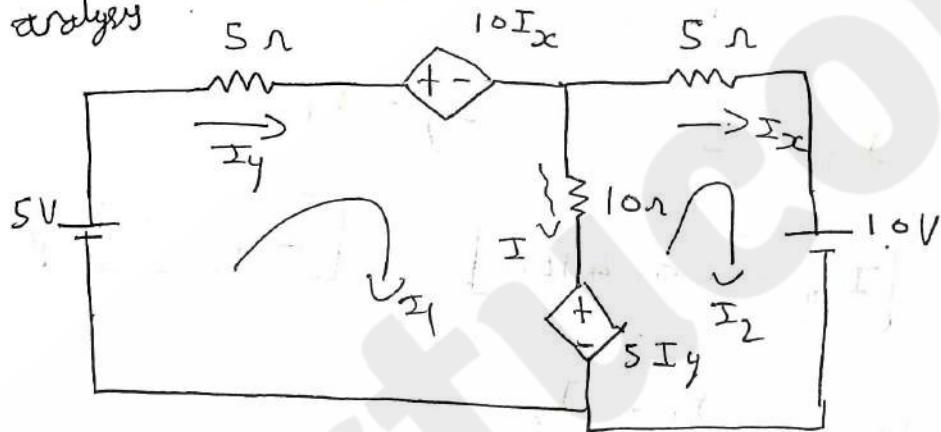
$$I_2 = 5A$$

$$I_3 = - 5A$$

$$I_{4n} = I_1 = 5A$$

$$I_{6n} = I_3 = - 5A$$

5) Find  $I$  through  $10\Omega$  Resistor using mesh analysis



$$\text{soln: } I_{10\Omega} = ?$$

$$I_1 = I_y, \quad I_2 = I_x$$

$$I = I_1 - I_2$$

Apply KVL to loop 1

$$5 - 5 I_1 - 10 I_2 - 10 I_1 + 10 I_2 = 5 I_y = 0$$

$$5 - 20 I_1 = 0$$

$$20 I_1 = 5$$

$$I_1 = \frac{5}{20}$$

$$I_1 = 0.25A$$

Apply KVL to loop 2

$$-5I_2 - 10 + 5I_1 - 10I_2 + 10I_1 = 0$$

$$15I_1 - 15I_2 = 10$$

$$15(0.25) - 15I_2 = 10$$

$$3.75 - 15I_2 = 10$$

$$-I_2 = \frac{10 - 3.75}{15}$$

$$I_2 = -0.416 \text{ A}$$

$$\therefore I_x = I_2 \quad I_y = I_1$$

$$I_{xc} = -0.416 \text{ A}$$

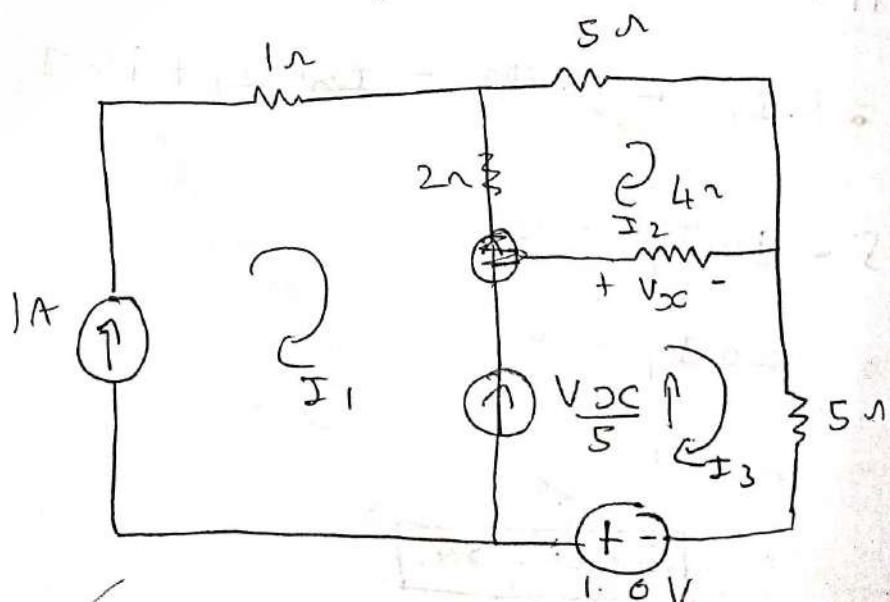
$$I_y = 0.25$$

$$I_{10n} = I_1 - I_2$$

$$I_{10n} = 0.25 + 0.416$$

$$I_{10n} = 0.66 \text{ A}$$

- b) For the network shown find power by 10V source using mesh current analysis



$$I_1 = 1A$$

loop 1 & 3 forms super mesh.

$$I_2 - I_1 = \frac{V_{DC}}{5}$$

$$V_{DC} = IR$$

$$V_{DC} = (I_3 - I_2) 4$$

$$V_{DC} = 4I_3 - 4I_2$$

$$(I_2 - I_1) = \frac{4I_3 - 4I_2}{5}$$

$$5I_2 - 5I_1 = 4I_3 - 4I_2$$

$$5I_2 - 5 = 4I_3 - 4I_2$$

$$\boxed{4I_2 + 1I_3 = 5} \dots \textcircled{2}$$

Apply KVL to loop 2

$$-5I_2 - 4I_2 + 4I_3 - 2I_2 + 2I_1 = 0$$

$$-11I_2 + 4I_3 = -2$$

$$\boxed{I_2 = 0.81 \text{ A}}$$

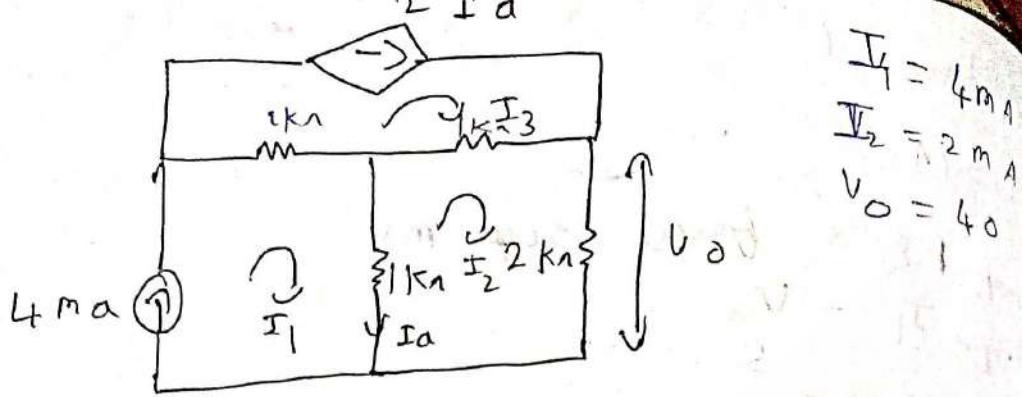
$$\boxed{I_3 = 1.74 \text{ A}}$$

$$P = V \times I$$

$$P = 10 \times 1.74$$

$$\boxed{P = 17.4 \text{ W}}$$

7)



$$\begin{aligned}I_1 &= 4 \text{ mA} \\I_2 &= 2 \text{ mA} \\V_o &= 4 \text{ V}\end{aligned}$$

from the above ckt

From loop 1

$$I_1 = 4 \text{ mA}$$

$$I_a = I_1 - I_2$$

From loop 3

$$I_3 = 2 I_a$$

$$I_3 = 2(I_1 - I_2)$$

$$I_3 = 2 I_1 - 2 I_2$$

$$2 I_2 + I_3 = 8 \quad \text{①}$$

Apply KVL to loop 2

$$-1 I_3 - 1 I_2 - 2 I_2 + \cancel{2 I_2} - I_1 = 0$$

$$I_1 - 4 I_2 + I_3 = 0$$

$$-4 I_2 + I_3 = -I_1$$

$$-4 I_2 + I_3 = -4 \quad | \times (-1)$$

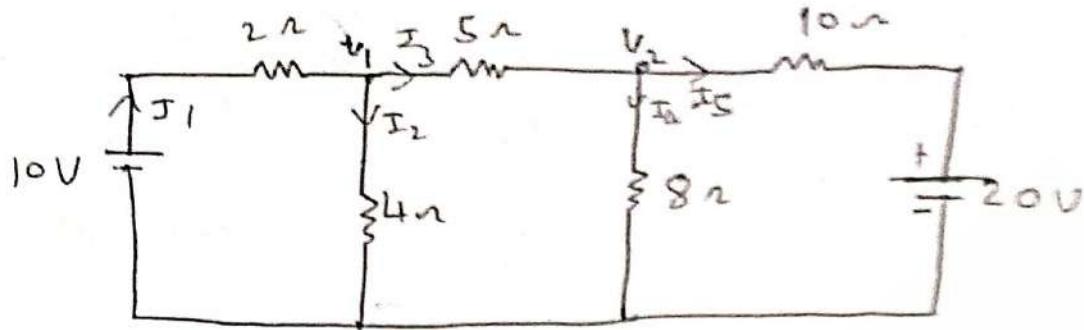
$$I_2 = 2 \text{ mA}$$

$$I_3 = 4 \text{ mA}$$

$$V_o = 2 \text{ mA} \times 2 \text{ k}\Omega$$

$$V_o = 4 \text{ V}$$

1) Find all branch currents using Node analysis



Apply KCL at  $V_1$

$$\frac{V_1 - 10}{2} + \frac{V_1}{4} + \frac{V_1 - V_2}{5} = 0$$

$$\frac{V_1}{2} - \frac{10}{2} + \frac{V_1}{4} + \frac{V_1 - V_2}{5} = 0$$

$$V_1 \left( \frac{1}{2} + \frac{1}{4} + \frac{1}{5} \right) - V_2 \left( \frac{1}{5} \right) = 5$$

$$0.95 V_1 - 0.2 V_2 = 5 \quad \dots \quad (1)$$

Apply KCL to  $V_2$

$$\frac{V_2 - V_1}{5} + \frac{V_2}{8} + \frac{V_2 - 20}{10} = 0$$

$$\frac{V_2}{5} - \frac{V_1}{5} + \frac{V_2}{8} + \frac{V_2 - 20}{10} = 0$$

$$V_2 \left( \frac{1}{5} + \frac{1}{8} + \frac{1}{10} \right) - V_1 (0.2) = 2$$

$$-0.2 V_1 + 0.425 V_2 = 2 \quad \dots \quad (2)$$

$$V_1 = 6.941 V$$

$$V_2 = 7.97 V$$

$$I_1 = \frac{V_1 - 10}{2}, \quad I_1 = \frac{6.9411 - 10}{2}$$

$$\boxed{I_1 = -1.53 A}$$

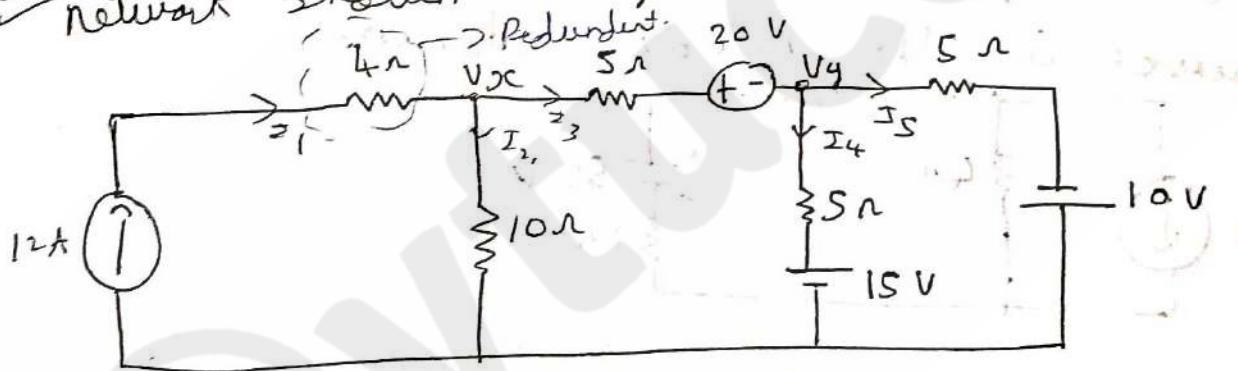
$$I_2 = \frac{V_1}{4} = \frac{6.941}{4} = 1.735 A$$

$$I_3 = \frac{V_2 - V_1}{5} = \frac{6.941 - 7.97}{5} = -0.206 A$$

$$I_4 = \frac{V_2}{8} = \frac{7.97}{8} = 0.996 A$$

$$I_5 = \frac{V_2 - 20}{10} = -1.203 A$$

Find the node voltage  $V_{DC}$  &  $V_y$  in the network shown using nodal analysis.



Apply KCL to  $V_{DC}$

$$\frac{V_{DC}}{10} + \frac{V_y - V_{DC}}{5} - \frac{20}{5} = 12$$

$$\frac{V_{DC}}{10} + \frac{V_y}{5} - \frac{V_{DC}}{5} = 12 + 4$$

$$V_{DC} \left( \frac{1}{10} + \frac{1}{5} \right) + 0.2 V_y = 16$$

$$\boxed{0.3 V_{DC} - 0.2 V_y = 16}$$

Applying KCL to node 4:

$$\frac{V_4 - V_x}{5} + \frac{V_y}{5} + \frac{V_y - 15}{5} + \frac{V_y + 10}{5} = 0$$

$$\frac{V_y}{10} - \frac{V_x}{5} + \frac{V_y}{5} + \frac{V_y - 15}{5} + \frac{V_y + 10}{5} = 0$$

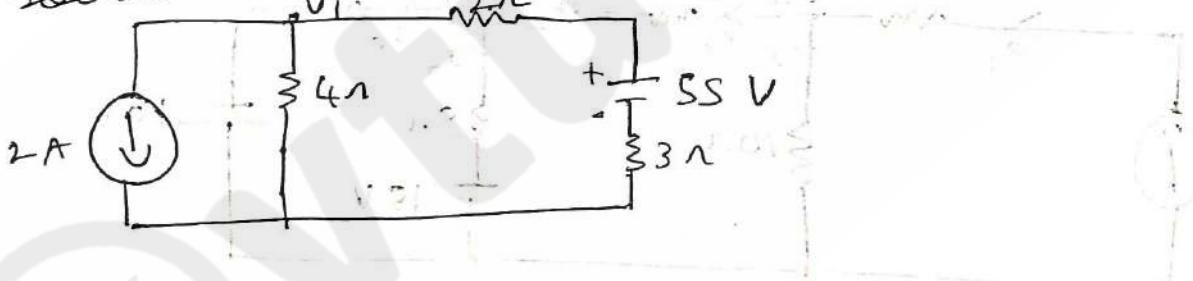
$$-0.2V_x + 0.6V_y = -3 \dots \textcircled{2}$$

solve \textcircled{1} \& \textcircled{2}

$$V_x = 64.28 \text{ V}$$

$$V_y = 16.42 \text{ V}$$

- 3) Find the power delivered by 2 A current source in the network shown.



$$\frac{V_1}{4} + \frac{V_1 - 55}{5} = -2$$

$$\frac{V_1}{4} + \frac{V_1}{5} - \frac{55}{5} = -2$$

$$V_1 \left( \frac{1}{4} + \frac{1}{5} \right) = -2 + 11$$

$$V_1 (0.45) = 9 \quad \left( \frac{1}{4} + \frac{1}{5} \right) = 0.45$$

$$V_1 = \frac{9}{0.45}$$

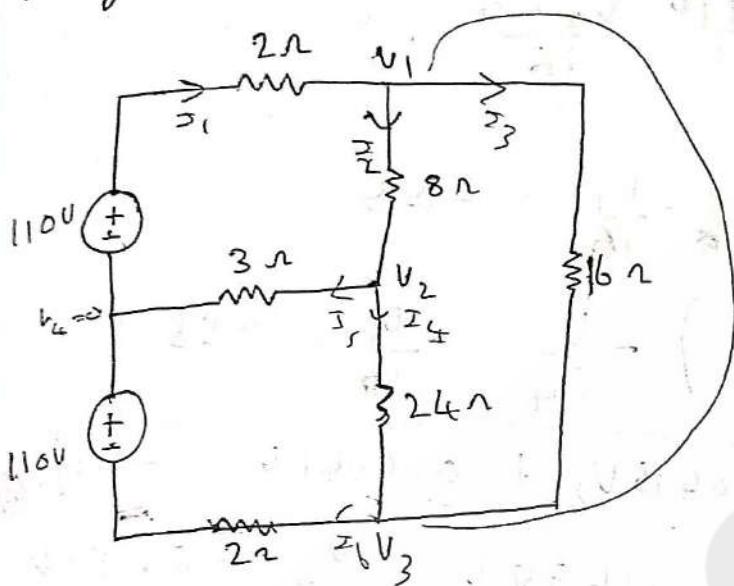
$$V_1 = 20 \text{ V}$$

Power delivered =  $V_1 \times I$

$$P = 20 \times 2$$

$$\boxed{P = 40 \text{ W}}$$

+1) Find all the branch currents using nodal analysis.



$$\frac{110}{2} \frac{V_1 - V_4}{2} + \frac{V_1 - V_2}{8} + \frac{V_1 - V_3}{16} = 55$$

$$\frac{V_1}{2} - \frac{V_4}{2} + \frac{V_1 - V_2}{8} + \frac{V_1 - V_3}{16} = 55$$

$$V_1 \left( \frac{1}{2} + \frac{1}{8} + \frac{1}{16} \right) - \frac{V_2}{8} - \frac{V_3}{16} = 55$$

$$0.6875 V_1 - 0.125 V_2 - 0.0625 V_3 = 55$$

KCL to Node 2

$$\frac{V_2 - V_1}{8} + \frac{V_2}{3} + \frac{V_2 - V_3}{24} = 0$$

$$\frac{V_2}{8} - \frac{V_1}{8} + \frac{V_2}{3} + \frac{V_2}{24} - \frac{V_3}{24} = 0$$

$$V_2 \left( \frac{1}{8} + \frac{1}{3} + \frac{1}{24} \right) - \frac{V_1}{8} - \frac{V_3}{24} = 0$$

$$-0.125 V_1 + 0.5 V_2 - 0.0416 V_3 = 0$$

Apply KCL to node 3

$$\frac{V_3 - V_2}{24} + \frac{V_3}{2} + \frac{110}{2} \frac{V_3 - V_1}{16} = 0$$

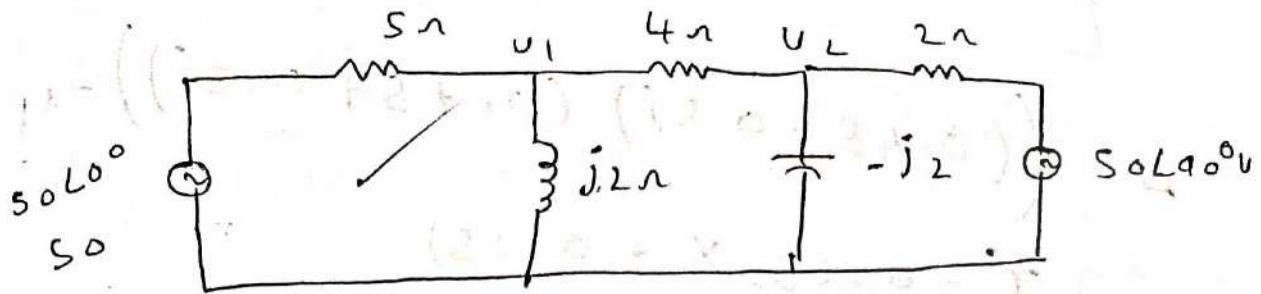
$$\frac{V_3}{24} - \frac{V_2}{24} + \frac{V_3}{2} + \frac{V_3 - V_1}{16} = -55$$

$$V_3 \left( \frac{1}{24} + \frac{1}{16} + \frac{1}{2} \right) - \frac{V_2}{24} - \frac{V_1}{16} = -55$$

$$-0.0625 V_1 - 0.0416 V_2 + 0.6041 V_3 = -55$$

$$V_1 = 74.64 V \quad V_2 = 11.79 V \quad V_3 = -82.50 V$$

→ Find the value of current  $I$  in the network shown using nodal analysis.



Apply KCL to node 1

$$\frac{V_1 - 50}{5} + \frac{V_1}{j2} + \frac{V_1 - V_2}{4} = 0$$

$$\frac{V_1 - 50}{5} + \frac{V_1}{j2} + \frac{V_1 - V_2}{4} = 0$$

$$V_1 \left( \frac{1}{5} + \frac{1}{j2} + \frac{1}{4} \right) - \frac{V_2}{4} = 10$$

$$(0.45 - 0.5j) - 0.25V_2 = 10 \quad \dots \textcircled{1}$$

Apply KCL to node 2

$$\frac{V_2 - V_1}{4} + \frac{V_2}{-j2} + \frac{V_2 - 50}{2} = 0$$

$$\frac{V_2}{4} - \frac{V_1}{4} + \frac{V_2}{-j2} + \frac{V_2 - 50}{2} = 0$$

$$V_2 \left( \frac{1}{4} + \frac{1}{-j2} + \frac{1}{2} \right) - 0.25V_1 = 25j$$

$$-0.25V_1 + (0.75 + 0.5j)V_2 = 25j \quad \dots \textcircled{2}$$

$$V = I \cdot R$$

$$\begin{bmatrix} \frac{V_1}{V_2} \end{bmatrix} = \begin{bmatrix} 1.0 \\ 2.5j \end{bmatrix} \begin{bmatrix} 0.45 - 0.5j \\ -0.25 \end{bmatrix}^{-1} \begin{bmatrix} 0.75 + 0.5j \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0.45 - 0.5j & -0.25 \\ -0.25 & 0.75 + 0.5j \end{bmatrix}$$

$$\Delta = ((0.45 - 0.5j)(0.75 + 0.5j)) - (-0.25 \times -0.25)$$

$$\boxed{\Delta = (0.525 - 0.15j)}$$

$$\Delta_1 = \begin{bmatrix} 10 & -0.25 \\ 25j & (0.75 + 0.5j) \end{bmatrix}$$

$$\Delta_1 = ((10)(0.75 + 0.5j)) - 0.25(25j)$$

$$\Delta_1 = 7.5 + 11.25j$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{7.5 + 11.25j}{0.525 - 0.15j}$$

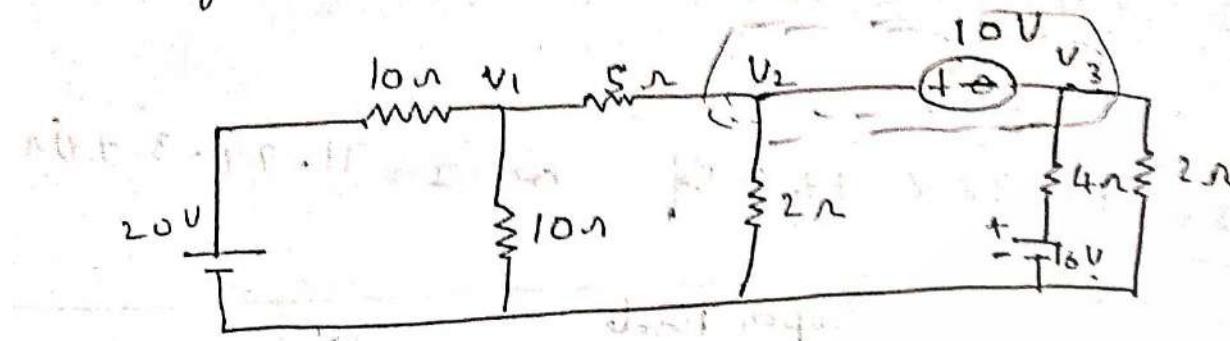
$$\boxed{V_1 = 24.76 \angle 72.25^\circ}$$

$$\Delta_2 = \begin{bmatrix} (0.45 - 0.5j) & 10 \\ -0.25 & 25j \end{bmatrix}$$

$$\Delta_2 = 15 + 11.25j$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{15 + 11.25j}{0.525 - 0.15j} \quad \therefore V_2 = 20.75 + 27.34 \angle 52.1^\circ$$

1) Find the value  $I_{4n}$  in the given network using nodal analysis.



node 2 & 3 forms super nod.

$$V_2 - V_3 = 10 \quad \dots \textcircled{1}$$

Apply KCL to node.

$$\frac{V_1 - 20}{10} + \frac{V_1}{10} + \frac{V_1 - V_2}{5} = 0$$

$$\cancel{\frac{V_1}{10}} + \frac{V_1}{10} + \frac{V_1}{5} - \frac{V_2}{5} = 2$$

$$V_1 \left( \frac{1}{10} + \frac{1}{10} + \frac{1}{5} \right) - \frac{V_2}{5} = 2$$

$$0.4V_1 - 0.2V_2 = 2 \quad \dots \textcircled{2}$$

Apply KCL to super nod.

$$\frac{V_2 - V_1}{5} + \frac{V_2}{2} + \frac{V_3 - 16}{4} + \frac{V_3}{2} = 0$$

$$\frac{V_2}{5} - \frac{V_1}{5} + \frac{V_2}{2} + \frac{V_3}{4} + \frac{V_3}{2} = 4$$

$$\left(\frac{V_1}{5}\right) + V_2 \left( \frac{1}{5} + \frac{1}{2} + \frac{1}{2} \right) + \frac{V_3}{4} + \frac{V_3}{2} = 4$$

$$-0.2V_1 + 0.7V_2 + 0.75V_3 = 4 \quad \dots \textcircled{3}$$

$$V_1 = 9.62$$

$$V_2 = 9.25$$

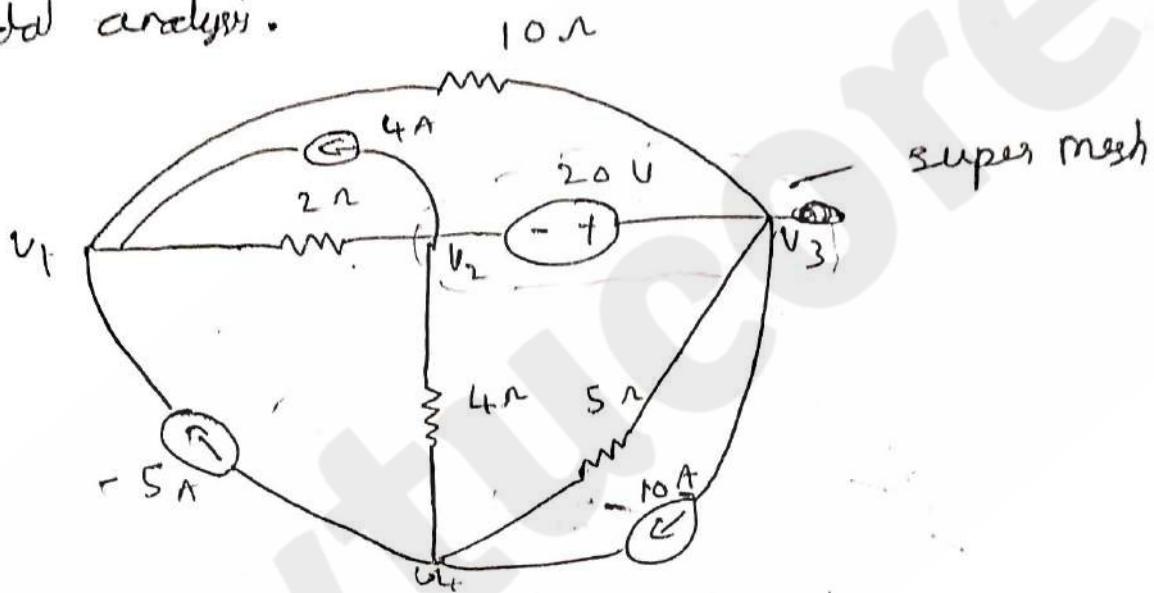
$$V_3 = -0.7407 \text{ V}$$

$$I_{4,n} = \frac{V_3 - 16}{4}$$

$$I_{4,n} = \frac{-0.7407 - 16}{4}$$

$$I_{4,n} = -4.18 \text{ A}$$

2) Find all node voltages  $V_1, V_2, V_3$  using nodal analysis.



$$V_3 - V_2 = 20 \quad \dots \quad (1)$$

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{10} = -4 - 5$$

$$\frac{V_1}{2} - \frac{V_2}{2} + \frac{V_1}{10} - \frac{V_3}{10} = -4 - 5$$

$$V_1 \left( \frac{1}{2} + \frac{1}{10} \right) - 0.5V_2 - 0.1V_3 = -9$$

$$0.6V_1 - 0.5V_2 - 0.1V_3 = -9$$

$$\frac{V_2}{4} + \frac{V_3}{5} + \frac{V_3 - V_1}{10} = 10$$

$$\frac{V_2}{4} + \frac{V_3}{5} + \frac{V_3}{10} - \frac{V_1}{10} = 10$$

$$-0.1V_1 + 0.25V_2 + 0.3V_3 = 10$$

③