MAA

Module-1

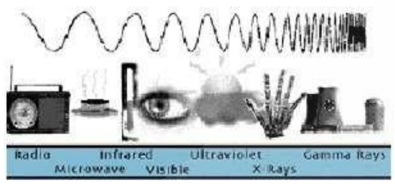
Microwave Engineering

- Microwave Networks
  - What are Microwaves?
  - S-parameters
  - Power Dividers
  - Couplers
  - Filters
  - Amplifiers

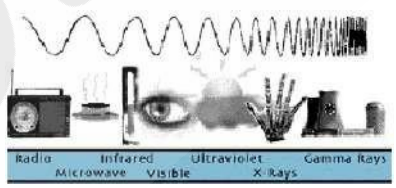
Microwave Engineering

Microwave engineering: Engineering and design of communication/navigation systems in the microwave frequency range.

Applications: Microwave oven, Radar, Satellite communication, direct broadcast satellite (DBS) television, personal communication systems (PCSs) etc.



What are Microwaves?



$$\text{frequency f}\left(Hz\right) = \frac{\text{velocity of light c}}{\text{wavelength }\lambda} = \frac{3\times10^{8}\left(m/s\right)}{\lambda\left(m\right)}$$

Microwaves: 30 cm – 1 cm (centimeter waves)

λ = 30 cm: $f = 3 \times 10^8 / 30 \times 10^{-2} = 1$ GHz

λ = 1 cm: $f = 3 \times 10^8 / 1 \times 10^{-2} = 30$ GHz

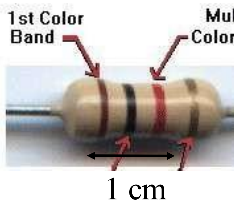
Note: 1 Giga = 10^9

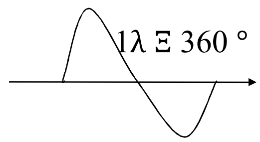
Millimeter waves: 10 mm – 1 mm

λ = 10 mm: $f = 3 \times 10^8 / 10 \times 10^{-3} = 30$ GHz

λ = 1 mm: $f = 3 \times 10^8 / 1 \times 10^{-3} = 300$ GHz

What are Microwaves?





Electrical length = Physical length/Wavelength (expressed in λ)

Phase delay = (2π or $360°$) x Physical length/Wavelength

RF

f =10 kHz, λ = c/f = 3 x 10^8/ 10 x 10^3 = 3000 m

Electrical length =1 cm/3000 m = $3.3 \times 10^{-6}\ \lambda$, Phase delay = 0.0012°

Microwave

f =10 GHz, λ = 3 x 10^8/ 10 x 10^9 = 3 cm

Electrical length = $0.33\ \lambda$, Phase delay = 118.8° !!!

Electrically long - The phase of a voltage or current changes significantly over the physical extent of the device

TABLE 0-1 U.S. MILITARY MICROWAVE B

Designation	Frequency range in gi
P band	0.225– 0.39
L band	0.390– 1.55
S band	1.550– 3.90
C band	3.900– 6.20
X band	6.200– 10.90
K band	10.900– 36.00
Q band	36.000– 46.00

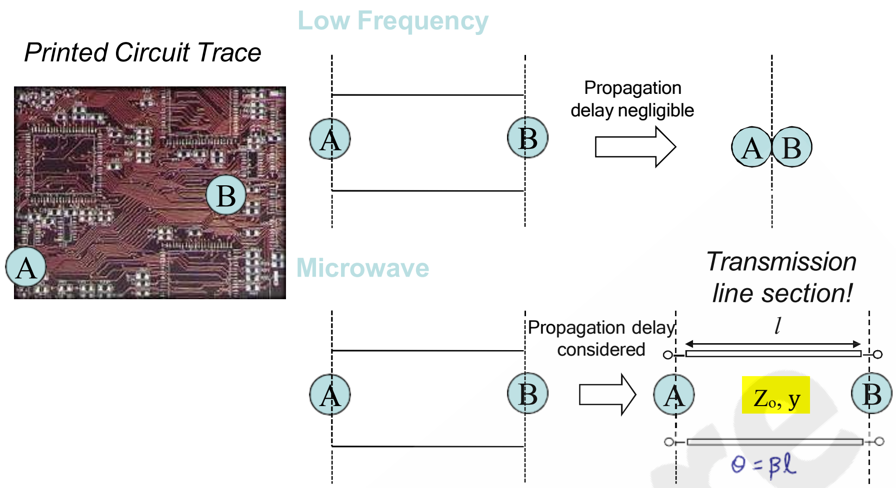
TABLE 0-2 U.S. NEW MILITARY MICROWAVE BANDS

Designation	Frequency range in gigahertz	Designation	Frequency r
A band	0.100–0.250	H band	6.00
B band	0.250–0.500	I band	8.00
C band	0.500–1.000	J band	10.00
D band	1.000–2.000	K band	20.00
E band	2.000–3.000	L band	40.00
F band	3.000–4.000	M band	60.00

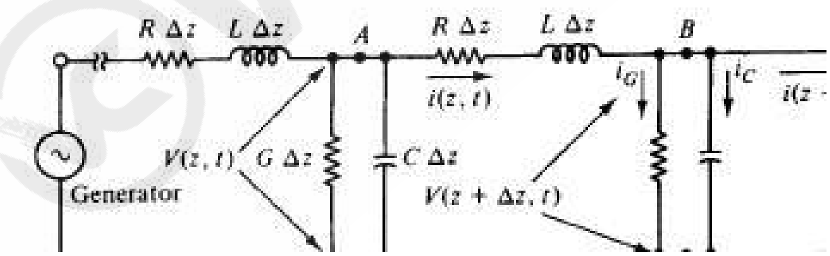
TABLE 0-3 IEEE MICROWAVE FREQUENCY B

Designation	Frequency range in gi
HF	0.003– 0.03
VHF	0.030– 0.30
UHF	0.300– 1.00
L band	1.000– 2.00
S band	2.000– 4.00
C band	4.000– 8.00
X band	8.000– 12.00
Ku band	12.000– 18.00
K band	18.000– 27.00
Ka band	27.000– 40.00

How to account for the phase delay?



Z_o: characteristic impedance
y (=α+jβ): Propagation constant



By Kirchoff's law,

$$v(z, t) = i(z, t)R\,\Delta z + L\,\Delta z\frac{\partial i(z, t)}{\partial t} + v(z, t) + \frac{\partial}{\;}$$ ----1

Rearrange Devide by Δz and omit(z,t)

$$-\frac{\partial v}{\partial z} = Ri + L\frac{\partial i}{\partial t}$$ ----2

Using KCL summation at B,in fig,

$$i(z,t) = v(z+\Delta z, t)G\,\Delta z + C\,\Delta z\,\frac{\partial v(z+\Delta z,t)}{\partial t} +$$

$$= \left[v(z,t) + \frac{\partial v(z,t)}{\partial z}\Delta z\right]G\,\Delta z \qquad \text{----3}$$

Rearrange Devide by Δz and omit(z,t) assume Δz=0,

$$-\frac{\partial i}{\partial z} = Gv + C\frac{\partial v}{\partial t} \qquad \text{----4}$$

Differentiate (2) wrt z and (4) wrt t, and combining,

$$\frac{\partial^2 v}{\partial z^2} = RGv + (RC + LG)\frac{\partial v}{\partial t} + L \qquad \text{----5}$$

Differentiate (2) wrt t and (4) wrt z, and combining

$$\frac{\partial^2 i}{\partial z^2} = RGi + (RC + LG)\frac{\partial i}{\partial t} + L \qquad \text{----6}$$

The instantaneous line voltage and current can be expressed as,

$$v(z,t) = \text{Re}\,\mathbf{V}(z) \qquad \text{----7}$$

$$i(z,t) = \text{Re}\,\mathbf{I}(z)e \qquad \text{----8}$$

$$\mathbf{V}(z) = \mathbf{V}_+ e^{-\gamma z} + \mathbf{V}_- e^{\gamma z} \qquad \text{----9}$$

$$\mathbf{I}(z) = \mathbf{I}_+ e^{-\gamma z} + \mathbf{I}_- e^{\gamma z} \qquad \text{---10}$$

$$\gamma = \alpha + j\beta \quad \text{(propagation c} \qquad \text{---11}$$

V_+ & I_+ complex amplitude in the posiive Z-direction.
V_- & I_- are complex ampliudes in he negative Z direction.
Iα=Attenuaion constant in nepers.
β=Phase constant in radians per second.
γ=Propagation constant

Substituting jω for δ/ δt in eqns. (2)(4) (5)&(6) and devide by e^{jwt} the Txn line Eqn in phasor form is given as,

$$\frac{d\mathbf{V}}{dz} = -\mathbf{ZI}$$ ---12

$$\frac{d\mathbf{I}}{dz} = -\mathbf{YV}$$ ---13

---14

$$\frac{d^2\mathbf{V}}{dz^2} = \gamma^2\mathbf{V}$$ ---15

Where,

$\mathbf{Z} = R + j\omega L$ (ohms per un ---16

$\mathbf{Y} = G + j\omega C$ (mhos per un ---17

---18

For lossless line R=G=0

$$\frac{d\mathbf{V}}{dz} = -j\omega L\mathbf{I}$$ ---19

$$\frac{d\mathbf{I}}{dz} = -j\omega C\mathbf{V}$$ ---20

---21

$$\frac{d^2\mathbf{V}}{dz^2} = -\omega^2 LC\mathbf{V}$$ ---22

Solutions of Txn lines equations

From 14,one possible solution is,

$$\mathbf{V} = \mathbf{V}_+ e^{-\gamma z} + \mathbf{V}_- e^{\gamma z} = \mathbf{V}_+ e^{-\alpha z} e^{-j\beta z} + \quad \text{---23}$$

$e^{-\beta z}$ implies wave travelling in positive z direction

$e^{\beta z}$ implies wave travelling in negative z direction

βz =Elecrical length of the line ,meassured iin radians.

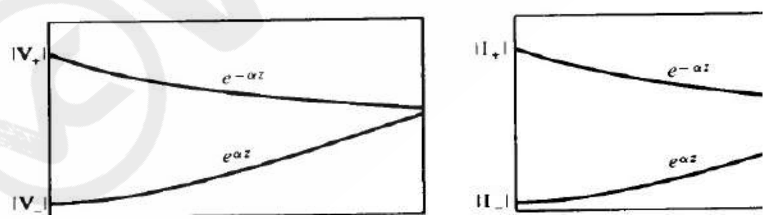
From 15, one possible solution is,

$$\mathbf{I} = \mathbf{Y}_0(\mathbf{V}_+ e^{-\gamma z} - \mathbf{V}_- e^{\gamma z}) = \mathbf{Y}_0(\mathbf{V}_+ e^{-\alpha z} e^{-j\beta z} - \quad \text{---24}$$

From 24, the charecteristic impedance of the line is defined as,

$$\mathbf{Z}_0 = \frac{1}{\mathbf{Y}} = \sqrt{\frac{\mathbf{Z}}{\mathbf{Y}}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 \quad \text{---25}$$

The magnitude of both voltage and current waves on the line i



At microwave frequencies it can be seen th ----26

$$R \ll \omega L \quad \text{and} \quad G$$

By using binomial expansion,the propagation constant is,

$$\begin{aligned}\gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \sqrt{(j\omega)^2 LC}\sqrt{\left(1 + \frac{R}{j\omega L}\right)\left(1 + \right.} \\ &\approx j\omega\sqrt{LC}\left[\left(1 + \frac{1}{2}\frac{R}{j\omega L}\right)\left(1 + \frac{1}{2}\right.\right. \\ &\approx j\omega\sqrt{LC}\left[1 + \frac{1}{2}\left(\frac{R}{j\omega L} + \frac{G}{j\omega C}\right)\right.\end{aligned} \quad \text{----27}$$

α&β are respectively given as,

$$\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) \qquad \text{----28}$$

$$\beta = \omega\sqrt{LC} \qquad \text{----29}$$

IIIrly, Z_0 is found to be,

$$\begin{aligned}\mathbf{Z}_0 &= \sqrt{\frac{R + j\omega L}{G + j\omega C}} \\ &= \sqrt{\frac{L}{C}}\left(1 + \frac{R}{j\omega L}\right)^{1/2}\left(1 + \frac{G}{j\omega C}\right. \\ &\approx \sqrt{\frac{L}{C}}\left(1 + \frac{1}{2}\frac{R}{j\omega L}\right)\left(1 - \frac{1}{2}\right. \\ &\approx \sqrt{\frac{L}{C}}\left[1 + \frac{1}{2}\left(\frac{R}{j\omega L} - \frac{G}{j\omega C}\right)\right]\end{aligned} \qquad \text{----30}$$

From (29),the Phase velocity is given as,

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \qquad \text{----31}$$

- The product of LC is independent of the size and separation of the conductors and depends only on the permeability μ and ε permitivity of the insulating medium.

The value of 1/Sqrt(LC) for insulated is approx equal to light in vaccum. Hence,

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c = 3 \times 1$$

In general phase velocity factor can be defined as,

$$\text{Velocity factor} = \frac{\text{actual phase velocity}}{\text{velocity of light in vacuu}}$$

$$v_r = \frac{v_\epsilon}{} = \frac{1}{}$$

For co-axial line the with solid dielectrics between conductors is given approx 0.65

Problem:

A transmission line has the following parameters:

$R = 2\ \Omega/\text{m}$ $\quad G = 0.5$ mmho/m $\quad f = 1$

$L = 8$ nH/m $\quad C = 0.23$ pF

wkt

$$\mathbf{Z}_0 = \sqrt{\frac{R + }{G + }}$$

$$= \sqrt{\frac{2 + j2\pi \times 10^9 \times 8 \times 10^{-9}}{0.5 \times 10^{-3} + j2\pi \times 10^9 \times 0.23}}$$

$$= \sqrt{\frac{50.31\angle 87.72^\circ}{15.29 \times 10^{-4}\angle 70.91^\circ}} = 181.39\angle 8.40^\circ = 179.44$$

$$\gamma = \sqrt{(R + j\omega L)(G +}$$

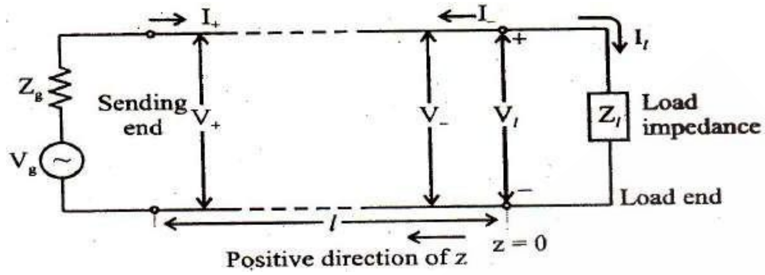
$$= \sqrt{(50.31\angle 87.72^\circ)(15.29 \times 10^{-4}\angle}$$

$$= \sqrt{769.24 \times 10^{-4}\angle 158.63}$$

$$= 0.2774\angle 79.31^\circ = 0.051 + j0.273$$

INPUT IMPEDANCE OF MICROWAVE TRANSMISSION LINE

Let us consider a microwave transmission line of length "l" **terminated in a load impedance** of Z_l Ω as shown in fig The expressions for voltage $V(z)$ and current $I(z)$ at any point on the line is given by equation



$$V(z) = V_l \cosh \gamma z + I_l Z_o \sinh \gamma z$$

$$I(z) = I_l \cosh \gamma z + \frac{V_l}{Z_o} \sinh \gamma z$$

The input impedance of a line of length "l" is given by

$$Z_S = \left.\frac{V(z)}{I(z)}\right|_{z=l} = \frac{V_l \cosh \gamma l + I_l Z_o \sinh \gamma l}{I_l \cosh \gamma l + \frac{V_l}{Z_o} \sinh \gamma l}$$

But, from fig.

$$V_l = I_l Z_l$$

Substituting for V_l in equation

$$Z_S = \frac{I_l Z_l \cosh \gamma l + I_l Z_o \sinh \gamma l}{I_l \cosh \gamma l + \frac{I_l Z_l}{Z_o} \sinh \gamma l}$$

Removing I_l in both numerator and denominator and simplifying, we get,

$$\mathbf{Z_S = Z_o \left[\frac{Z_l \cosh \gamma l + Z_o \sinh \gamma l}{Z_o \cosh \gamma l + Z_l \sinh \gamma l}\right]}$$

Line terminated in Z_o

When a microwave transmission line is terminated in its characteristic impedance Z_0, then

$$Z_l = Z_0$$

Substituting for $Z_l = Z_o$

$$Z_S = Z_o \left[\frac{Z_o \cosh \gamma l + Z_o \sinh \gamma l}{Z_o \cosh \gamma l + Z_l \sinh \gamma l}\right]$$

$$\therefore\ \mathbf{Z_S = Z_o}$$

i.e., when the line is terminated in Z_0, then the input impedance of that line is also equal to the characteristic impedance Z_0 irrespective of the length of the transmission line.

Given data $Z_0 = 710 \angle 14^\circ\ \Omega$; $\gamma = 0.007 + j\,0.028/\text{KM}$

$Z_R = 300\ \Omega$; $l = 100$ KM

$Z_s = ?$

$$Z_s = Z_0 \left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l}\right]$$

We have

$$\begin{aligned}
\cosh \gamma l &= \cosh (\alpha + j\beta)\, l \\
&= \cosh (\alpha l + j\beta l) \\
&= \cosh [0.007 \times 100 + j\, 0.28 \times 100] \\
&= \cosh [0.7 + j\, 2.8]
\end{aligned}$$

From Trignometry,

$$\cosh (A + j\,B) = \cosh A \cos B + j \sinh A \sin B$$

and $\sinh (A + j\,B) = \sinh A \cos B + j \cosh A \sin B$

$$\begin{aligned}
\therefore \quad \cosh \gamma l &= \cosh (0.7 + j\,2.8) \\
&= \cosh 0.7 \cos \frac{2.8 \times 180^0}{\pi} + j \sinh 0.7 \sin \frac{2.8 \times 180^0}{\pi} \\
&= (1.255)(-0.942) + j\,(0.759)(0.335)
\end{aligned}$$

$$\therefore \quad \mathbf{\cosh \gamma l = -1.182 + j\, 0.254}$$

And

$$\begin{aligned}
\sinh \gamma l &= \sinh (0.7 + j\,2.8) \\
&= \sinh 0.7 \cos \frac{2.8 \times 180^0}{\pi} + j \cosh 0.7 \sin \frac{2.8 \times 180^0}{\pi}
\end{aligned}$$

$$\therefore \quad \mathbf{\sinh \gamma l = -0.715 + j\, 0.446}$$

Given

$$Z_R = 300\ \Omega$$

$$Z_0 = 710 \angle 14^0 = 688.9 + j\,171.77\ \Omega$$

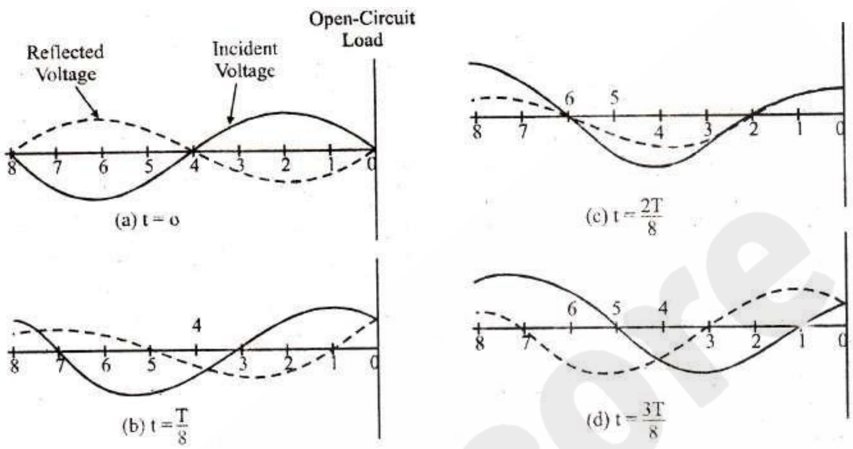
$$\begin{aligned}
Z_s &= 710 \angle 14^0 \left[\frac{300(-1.182 + j0.254) + (688.9 + j171.77)(-0.715 + j0.446)}{(688.9 + j171.77)(-1.182 + j0.254) + 300(-0.715 + j0.446)}\right] \\
&= 710 \angle 14^0 \left[\frac{-923.77 + j260.63}{-1072.41 + j105.75}\right] \\
&= 710 \angle 14^0 \left[\frac{959.83 \angle 164.24^0}{1077.61 \angle 174.37^0}\right] \\
&= \frac{(710)(959.83)}{(1077.61)} \angle 14^0 + 164.24^0 - 174.37^0
\end{aligned}$$

$$Z_s = \mathbf{632.4 \angle 3.87^0\ \Omega}$$

Reflection on a line not terminated in Z_O:

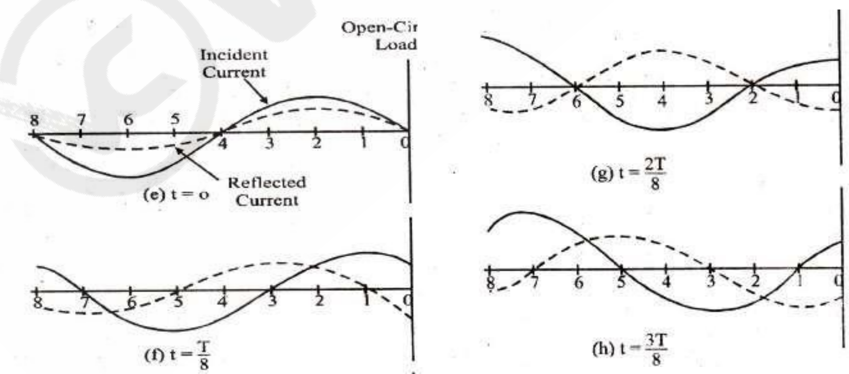
$$\mathbf{V}(z) = \mathbf{V}_+ e^{-\gamma z} + \mathbf{V}_- e^{\gamma z}$$
$$\mathbf{I}(z) = \mathbf{I}_+ e^{-\gamma z} + \mathbf{I}_- e^{\gamma z}$$
$$\gamma = \alpha + j\beta \quad \text{(propagation c}$$

$$V(z) = V_l \cosh \gamma z + I_l Z_o \sin$$
$$I(z) = I_l \cosh \gamma z + \frac{V_l}{Z} \sinh$$



(a) t = o
(c) $t = \frac{2T}{8}$
(b) $t = \frac{T}{8}$
(d) $t = \frac{3T}{8}$

Short circuited line:



(e) t = o
(g) $t = \frac{2T}{8}$
(f) $t = \frac{T}{8}$
(h) $t = \frac{3T}{8}$

$$V(z) = V_l \frac{e^{\gamma z} + e^{-\gamma z}}{2} + I_l Z_0 \frac{e^{\gamma z} - e^{-\gamma z}}{2} \ldots\ldots\ldots\ldots$$

$$I(z) = I_l \frac{e^{\gamma z} + e^{-\gamma z}}{2} + \frac{V_l}{Z} \frac{e^{\gamma z} - e^{-\gamma z}}{2} \ldots\ldots\ldots\ldots\ldots$$

$$V(z) = \frac{V_l}{2}\left(1 + \frac{I_l Z_o}{V_l}\right)e^{\gamma z} + \frac{V_l}{2}\left(1 - \frac{I_l Z_o}{V_l}\right)e^{-\gamma z}$$

$$I(z) = \frac{I_l}{2}\left(\frac{V_l}{I_l Z_o} + 1\right)e^{\gamma z} - \frac{I_l}{2}\left(\frac{V_l}{I_l Z_o} - 1\right)e^{-\gamma z}$$

Replacing V_l by $I_l Z_l$ inside all the parenthesis, we get from above equations

$$V(z) = \frac{V_l}{2}\left[\left(1 + \frac{Z_o}{Z_l}\right)e^{\gamma z} + \left(1 - \frac{Z_o}{Z_l}\right)e^{-\gamma z}\right]$$

$$I(z) = \frac{I_l}{2}\left[\left(1 + \frac{Z_l}{Z_o}\right)e^{\gamma z} + \left(1 - \frac{Z_l}{Z_o}\right)e^{-\gamma z}\right]$$

These equations can be rewritten as

$$V(z) = \frac{V_l(Z_l + Z_o)}{2Z_l}\left[e^{\gamma z} + \left(\frac{Z_l - Z_o}{Z_l + Z_o}\right)e^{-\gamma z}\right]$$

$$I(z) = \frac{I_l(Z_l + Z_o)}{2Z_o}\left[e^{\gamma z} - \left(\frac{Z_l - Z_o}{Z_l + Z_o}\right)e^{-\gamma z}\right]$$

Reflection Coefficient

It is defined as the ratio of the amplitudes of reflected voltage wave to the incident voltage wave at the receiving end, denoted by Γ given by

$$\Gamma = \frac{\text{reflected voltage at load}}{\text{incident voltage at load}} = \frac{Z_l - Z_o}{Z_l + Z_o}$$

Using equation , we get

$$V(z) = \frac{V_l(Z_l + Z_o)}{2Z_l}\left[e^{\gamma z} + \Gamma e^{-\gamma z}\right]$$

$$I(z) = \frac{I_l(Z_l + Z_o)}{2Z_o}\left[e^{\gamma z} - \Gamma e^{-\gamma z}\right]$$

The phase and magnitude of Z_l and Z_0 decide the sign of Γ and hence the polarity of the reflected wave. When $Z_l = Z_0$, then from equation it is clear that $\Gamma = 0$ and then there is no reflection.

The ratio of the voltage to the current at the load end is equal to load impedance Z_l

$$\therefore \quad Z_l = \frac{V_l}{I_l} = \frac{V_+ e^{\gamma l} + V_- e^{-\gamma l}}{\frac{V_+}{Z_o} e^{\gamma l} - \frac{V_-}{Z_o} e^{-\gamma l}}$$

$$\therefore \quad \frac{Z_l}{Z_o} = \frac{V_+ e^{\gamma l} + V_- e^{-\gamma l}}{V_+ e^{\gamma l} - V_- e^{-\gamma l}}$$

By componendo and dividendo, we get

$$\frac{Z_l - Z_o}{Z_l + Z_o} = \frac{V_+ e^{\gamma l} + V_- e^{-\gamma l} - V_+ e^{\gamma l} + V_- e^{-\gamma l}}{V_+ e^{\gamma l} + V_- e^{-\gamma l} + V_+ e^{\gamma l} - V_- e^{-\gamma l}} = \frac{2V_- e^{-\gamma l}}{2V_+ e^{\gamma l}}$$

$$\therefore \quad \frac{Z_l - Z_o}{Z_l + Z_o} = \frac{V_- e^{-\gamma l}}{V_+ e^{\gamma l}}$$

$$\therefore \quad \Gamma_l = \frac{Z_l - Z_o}{Z_l + Z_o}$$

Since Z_l is a complex quantity, Γ_l also becomes a complex quantity which can be expressed as

$$\Gamma_l = |\Gamma_l| e^{j\theta_l}$$

where $|\Gamma_l|$ = magnitude of reflected coefficient at load end which is always ≤ 1.

θ_l = Phase angle between incident and reflected voltages at the load end.

Transmission Coefficient (T)

The transmission coefficient "T" is defined as the ratio of transmitted voltage or current to the incident voltage or current given by

$$T = \frac{\text{transmitted voltage or current}}{\text{incident voltage or current}} = \frac{V_{tr}}{V_{inc}} = \frac{I_{tr}}{I_{inc}}$$

If P_{inc} = incident power

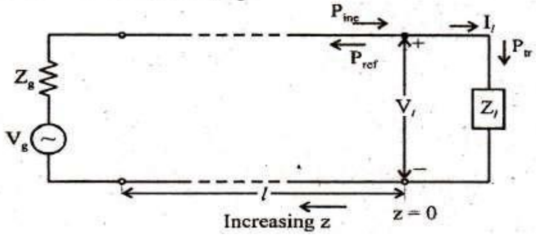
P_{tr} = transmitted power

P_{ref} = reflected power

then according to law of conservation of energy the incident power must be equal to the sum of transmitted and reflected power given by

$$P_{inc} = P_{tr} + P_{ref}$$

This power relation is illustrated in fig.



we have

$$\frac{V_- e^{-\gamma l}}{V_+ e^{\gamma l}} = \frac{Z_l - Z_o}{Z_l + Z_o} = \Gamma_l$$

we get

$$V_- e^{\gamma l} = V_+ e^{\gamma l}\left[\frac{Z_l - Z_o}{Z_l + Z_o}\right]$$

The voltage travelling wave at the load end in terms of V_{tr} can be written as

$$V_+ e^{\gamma l} + V_- e^{\gamma l} = V_{tr} e^{\gamma l}$$

Using equation , we get

$$V_+ e^{\gamma l} + V_+ e^{\gamma l}\left[\frac{Z_l - Z_o}{Z_l + Z_o}\right] = V_{tr} e^{\gamma l}$$

Removing $e^{\gamma l}$ throughout and simplifying, we get

$$V_+\left[1 + \frac{Z_l - Z_o}{Z_l + Z_o}\right] = V_{tr}$$

$$\therefore V_+\left[\frac{Z_l + Z_o + Z_l - Z_o}{Z_l + Z_o}\right] = V_{tr}$$

$$\therefore V_+\left[\frac{2Z_l}{Z_l + Z_o}\right] = V_{tr}$$

Using the definition of transmission coefficient of equation we get

$$\mathbf{T} = \frac{\mathbf{V_{tr}}}{\mathbf{V_{inc}}} = \frac{\mathbf{V_{tr}}}{\mathbf{V_+}} = \frac{\mathbf{2Z_l}}{\mathbf{Z_l + Z_o}}$$

The real power incident on the load is given by

$$P_{inc} = \left(\frac{V_+ e^{\alpha l}}{\sqrt{2}}\right)^2 \frac{1}{Z_o} = \frac{\left(V_+ e^{\alpha l}\right)^2}{2Z_o}$$

The real power reflected by the load (mismatched) is given by

$$P_{ref} = \left(\frac{V_- e^{-\alpha l}}{\sqrt{2}}\right)^2 \frac{1}{Z_o} = \frac{\left(V_- e^{\alpha l}\right)^2}{2Z_o}$$

The power carried to the load by the travelling waves is

$$P_{ref} = \left(\frac{V_{tr} e^{\alpha l}}{\sqrt{2}}\right)^2 \frac{1}{Z_l} = \frac{\left(V_{tr} e^{\alpha l}\right)^2}{2Z_l}$$

Using equation , we get

$$\frac{\left(V_+ e^{\alpha l}\right)^2}{2Z_o} = \frac{\left(V_{tr} e^{\alpha l}\right)^2}{2Z_l} + \frac{\left(V_- e^{-\alpha l}\right)^2}{2Z_o}$$

Dividing throughout by $\left(V_+ e^{\alpha l}\right)^2$, we get

$$\frac{1}{2Z_o} = \left(\frac{V_{tr} e^{\alpha l}}{V_+ e^{\alpha l}}\right)^2 \frac{1}{2Z_l} + \left(\frac{V_- e^{-\alpha l}}{V_+ e^{\alpha l}}\right)^2 \frac{1}{2Z_o}$$

$$\therefore \quad \frac{1}{Z_o} = \left(\frac{V_{tr}}{V_+}\right)^2 \frac{1}{Z_l} + \left(\frac{V_- e^{-\alpha l}}{V_+ e^{\alpha l}}\right)^2 \frac{1}{Z_o}$$

$$\therefore \quad \frac{1}{Z_o} = \frac{T^2}{Z_l} + \frac{\Gamma_l^2}{Z_o}$$

$$\therefore \quad \frac{T^2}{Z_l} = \frac{1}{Z_o}\left[1 - \Gamma_l^2\right]$$

$$\therefore \quad \mathbf{T^2 = \frac{Z_l}{Z_o}\left[1 - \Gamma_l^2\right]}$$

Another relationship between T and Γ_l

Consider $$1 + \Gamma_l = 1 + \frac{Z_l - Z_o}{Z_l + Z_l}$$

$$= \frac{Z_l + Z_o + Z_l - Z_o}{Z_l + Z_o}$$

$$\therefore 1 + \Gamma_l = \frac{2Z_l}{Z_l + Z_o} = T$$

Thus the transmission and reflection coefficients are related by another simple relationship.

$$T = 1 + \Gamma_l$$

Given $\quad Z_o = 100\angle 53.13^\circ\ \Omega = 60 + j\,80\ \Omega$

$$T = 1.09\angle 35.34^\circ = 0.889 + j\,0.6304$$

(i) the reflection coefficient

$$\Gamma_l = T - 1 = (0.889 + j\,0.6304) - 1$$

$$= -0.111 + j\,0.6304$$

$$\therefore\ \boldsymbol{\Gamma_l = -0.111 + j\,0.6304 = 0.64\angle 100^\circ}$$

(ii) we have

$$\Gamma_l = \frac{Z_l - Z_o}{Z_l + Z_o}$$

By Componendo and dividendo, we have

$$\frac{1+\Gamma_l}{1-\Gamma_l} = \frac{Z_l + Z_o + Z_l - Z_o}{Z_l + Z_o - Z_l + Z_o} = \frac{2Z_l}{2Z_o}$$

$$\therefore\ Z_l = Z_o\left[\frac{1+\Gamma_l}{1-\Gamma_l}\right]$$

$$= 100\angle 53.13^\circ\left[\frac{1+(-0.111 + j\,0.6304)}{1-(-0.111 + j\,0.6304)}\right]$$

$$\therefore\ \boldsymbol{Z_l = 85.36\angle 118.03^\circ = -40.11 + j\,75.35\ \Omega}$$

STANDING WAVE AND STANDING-WAVE RATIO

$$V(z) = V_+e^{\gamma z} + V_-e^{\gamma z}$$

$$= V_+e^{\alpha z}e^{+j\beta z} + V_-e^{-\alpha z}e^{-j\beta z}$$

$$= V_+e^{\alpha z}(\cos\beta z + j\sin\beta z) + V_-e^{-\alpha z}(\cos\beta z - j\sin\beta z)$$

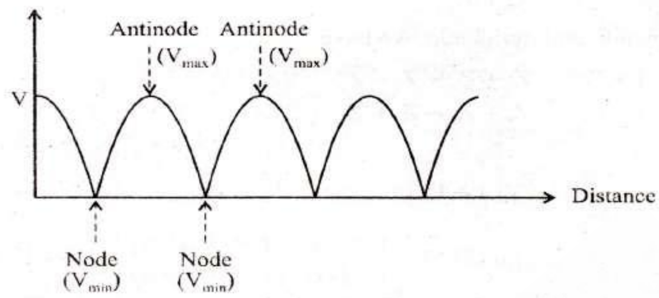
$$\therefore\ V(z) = \left(V_+e^{\alpha z} + V_-e^{-\alpha z}\right)\cos\beta z + j\left(V_+e^{\alpha z} - V_-e^{-\alpha z}\right)\sin\beta z$$

Let us assume that the quantities $V_+e^{\alpha z}$ and $V_-e^{-\alpha z}$ are real quantities so that the voltage wave equation can be represented as

$$V(z) = V_oe^{+j\theta}$$

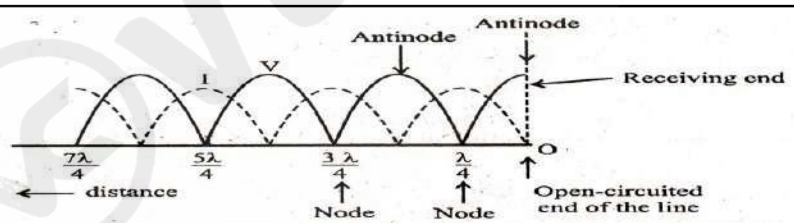
$$\text{where}\ \ V_o = \sqrt{\left(V_+e^{\alpha z} + V_-e^{-\alpha z}\right)^2\cos^2\beta z + \left(V_+e^{\alpha z} - V_-e^{-\alpha z}\right)^2\sin^2\beta z}$$

$$\text{and}\ \theta = \tan^{-1}\left[\frac{\left(V_+e^{\alpha z} - V_-e^{-\alpha z}\right)\sin\beta z}{\left(V_+e^{\alpha z} + V_-e^{-\alpha z}\right)\cos\beta z}\right]$$

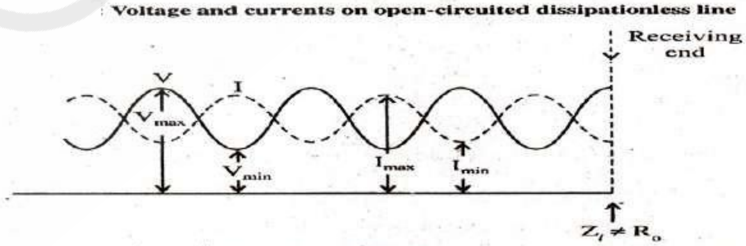


: **Illustrating standing waves on a line with open or short-circuit termination**

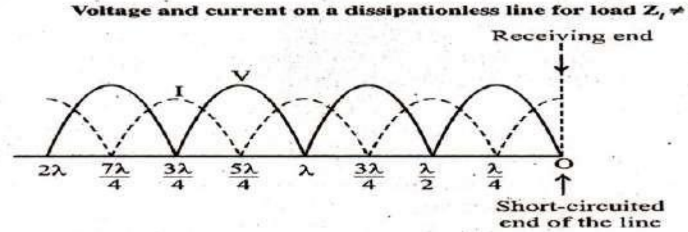
When a line is terminated in an open circuit, then $Z_l = \infty$. A considerable voltage exists across such an open-circuit. Thus an **"antinode"** is formed at the open-circuited end. Obviously, a **"node"** is created at a distance of $\left(\frac{\lambda}{4}\right)$ from the open-circuited end. Thus, nodes occur at distances $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ from the open end of line as shown in figure It is also seen that the current nodes occur at distances 0, $\frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$ ...... from the open-circuited end of the dissipationless line.



: **Voltage and currents on open-circuited dissipationless line**



Voltage and current on a dissipationless line for load $Z_l \neq R_o$



For short-circuited termination, the voltage and current standing waves appear as shown in figure. The voltage nodes occur at $0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \cdots\cdots$ and current nodes at $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \cdots\cdots$.

Standing-Wave Ratio (ρ)

The ratio of maximum to minimum magnitude of voltage or current on a line having standing waves is called as **"Standing-wave Ratio"** abbreviated as SWR and denoted by ρ given by

$$\rho = \left|\frac{V_{max}}{V_{min}}\right| = \left|\frac{I_{max}}{I_{min}}\right|$$

Relationship between ρ and Γ

voltage and current at any point on the line is given by

$$V(z) = \frac{V_l(Z_l + Z_o)}{2Z_l}\left[e^{\gamma z} + \Gamma e^{-\gamma z}\right]$$

$$I(z) = \frac{I_l(Z_l + Z_o)}{2Z_o}\left[e^{\gamma z} - \Gamma e^{-\gamma z}\right]$$

For a dissipationless line (lossless line), we have $\alpha = 0$

$$\therefore \qquad \gamma = j\beta$$

$$\text{Let} \quad \Gamma = |\Gamma| e^{j\theta}$$

where θ = phase angle of reflection coefficient

$$\text{Let} \quad \frac{V_l(Z_l + Z_o)}{2Z_l} = V'$$

With these substitutions,

$$V(z) = V'\left[e^{j\beta z} + |\Gamma| e^{j\theta} \; e^{-j\beta z}\right]$$

$$= V' e^{j\beta z} [1 + |\Gamma| e^{j\theta} \; e^{-j2\beta z}] = V' e^{j\beta z} [1 + |\Gamma| e^{-j(2\beta z - \theta)}]$$

Considering only the magnitude on both sides, we get

$$|V(z)| = |V'| [1 + |\Gamma| e^{-j(2\beta z - \theta)}]$$

This voltage $|V(z)|$ has maximum amplitude when the two components are in phase

i.e., at $z = Z_{max}$, $\mathbf{2\beta Z_{max} - \theta = 2\,n\pi}$

where $n = 0, 1, 2, 3, \ldots\ldots$ and $|V(z)| = |V_{max}|$

∴ At points where the incident and reflected voltages are in phase, the maximum absolute value of voltage is given by substituting equation

$$|V_{max}| = |V'| [1 + |\Gamma| e^{-j2n\pi}]$$

$$\therefore \quad |V_{max}| = |V'| [1 + |\Gamma|]$$

Since $e^{-j2n\pi} = 1$ for all n.

Similarly, the voltage has minimum amplitude when the two components (the incident and reflected voltages) are 180° out-of-phase

i.e., at $z = Z_{min}$, $\mathbf{2\beta Z_{min} - \theta = (2n + 1)\,\pi}$

where again $n = 0, 1, 2, 3 \cdots\cdots$ and $|V(z)| = |V_{min}|$. Substituting in equation

$$|V_{min}| = |V'| [1 + |\Gamma| e^{-j\,(2n+1)\,\pi}]$$

$$\therefore \quad |V_{min}| = |V'| [1 - |\Gamma|]$$

Since $e^{-j\,(2n-1)\,\pi} = -1$ for all n.

Substituting equations

$$\rho = \frac{|V_{max}|}{|V_{min}|} = \frac{|V'|[1+|\Gamma|]}{|V'|[1-|\Gamma|]}$$

$$\therefore \quad \rho = \frac{1+|\Gamma|}{1-|\Gamma|}$$

NOTE: Only the magnitude of reflection coefficient Γ is to be used while calculating ρ and not its phase angle. The SWR is only a real quantity and not a complex quantity.

Equation can be written as

$$\frac{\rho}{1} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

By componendo and dividendo,

$$\frac{\rho-1}{\rho+1} = \frac{1+|\Gamma|-[1-|\Gamma|]}{1+|\Gamma|+[1-|\Gamma|]} = \frac{2|\Gamma|}{2}$$

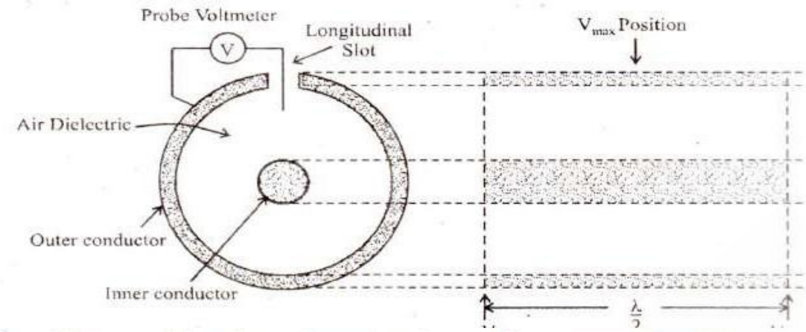
$$\therefore \qquad |\Gamma| = \frac{\rho-1}{\rho+1}$$

Using equation

$$|\Gamma| = \frac{\frac{|V_{max}|}{|V_{min}|}-1}{\frac{|V_{max}|}{|V_{min}|}+1}$$

$$\therefore \qquad |\Gamma| = \frac{|V_{max}|-|V_{min}|}{|V_{max}|+|V_{min}|}$$

If it is possible to measure $|V_{max}|$ and $|V_{min}|$, then by using equations $|\Gamma|$ and ρ it is possible to calculate the values of reflection coefficient and the standing wave ratio. For open-wire lines, these values can be readily calculated with measurements made for $|V_{max}|$ and $|V_{min}|$. But, for co-axial lines, a longitudinal slot of length more than half wavelength has to be cut as shown in fig. A voltage pick-up device such as wire probe is inserted into the slot and into the air dielectric of the line. A voltmeter can then be connected between the probe and the outer conductor as shown. By calibrating the meter, the value of SWR can be easily found out. The same arrangement can be used for measurement of wavelength λ. The distance between successive voltage minima (or voltage maxima) is equal to half-wavelength, from which 'λ' is calculated.



Slotted-line co-axial section and a probe voltmeter for measurement of V_{max}, V_{min}, G and r

reflection coefficient Γ is given by
$$\Gamma = \frac{Z_l - Z_o}{Z_l + Z_o}$$

For a dissipationless line $Z_0 = R_0$ and for a resistive load $Z_l = R_l$. With these substitutions,

$$\Gamma = |\Gamma| = \frac{R_l - R_o}{R_l + R_o}$$

Substituting this value of Γ in equation

$$\rho = \frac{1 + \frac{R_l - R_o}{R_l + R_o}}{1 - \frac{R_l - R_o}{R_l + R_o}}$$

$$= \frac{R_l + R_o + R_l - R_o}{R_l + R_o - R_l + R_o}$$

$$\rho = \frac{R_l}{R_o} \text{ (for } R_l > R_o)$$

$$\therefore \quad \text{and } \rho = \frac{R_o}{R_l} \text{ (for } R_l < R_o)$$

Find the reflection coefficient and voltage standing wave ratio of a line having $R_0 = 100\ \Omega$ and $Z_l = 100 - j\ 100\ \Omega$.

$$\Gamma = \frac{Z_l - Z_o}{Z_l + Z_o}$$

$$\Gamma = \frac{100 - j100 - 100}{100 - j100 + 100}$$

$$= \frac{-j100}{200 - j100}$$

$$= \frac{100 \angle -90^\circ}{223.61 \angle -26.57^\circ}$$

$$\mathbf{\Gamma = 0.4472 \angle -63.43^\circ}$$

$$\rho = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$= \frac{1 + 0.4472}{1 - 0.4472}$$

$$\mathbf{\rho = 2.618}$$

A certain low-loss line has a characteristic impedance of 400 Ω. Determine the SWR if the receiving end impedance is 650 – j 475Ω.

$$\Gamma = \frac{Z_l - Z_o}{Z_l + Z_o}$$

$$= \frac{650 - j475 - 400}{650 - j475 + 400}$$

$$= \frac{250 - j475}{1050 - j475}$$

Multiplying, we get $R_o^2 = (1333.2)(120)$

$$\therefore\ R_o = \sqrt{(1333.2)(120)}$$

$$\therefore\ R_o = 400\ \Omega$$

Substituting in the expression for R_l above, we get

$$R_l = 400\left[\frac{1 + 0.5385}{1 - 0.5385}\right]$$

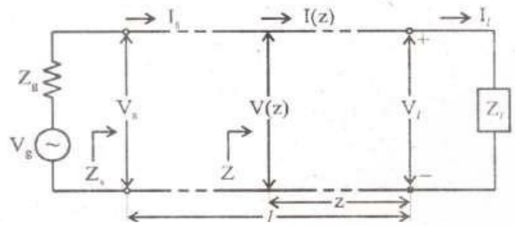
$$\therefore\ \mathbf{R_l = 1333.47\ \Omega}$$

LINE IMPEDANCE AND ADMITTANCE

The line impedance of a microwave transmission line is defined as the complex ratio of the voltage at any point on the line to the current at that point given by

$$Z \equiv \frac{V(z)}{I(z)}$$

This line impedance at any point "z" on the transmission line is illustrated in fig.



When "z" is replaced by the length "l" of the transmission line, then we shall reach the sending end on the line and "Z", the line impedance becomes equal to the input impedance Z_S of the line given by

$$Z_S = \left.\frac{V(z)}{I(z)}\right|_{z=l}$$

the line impedance Z is given by

$$Z = Z_o\left[\frac{Z_l \cosh \gamma z + Z_o \sinh \gamma z}{Z_o \cosh \gamma z + Z_l \sinh \gamma z}\right]$$

replacing "l" by "z"

For a lossless line (dissipationless line), we have ($\alpha = 0$)

$$\gamma = i\beta \text{ and } Z_o = R_o \text{ (pure resistance)}$$

$$\therefore \cosh \gamma z = \cosh j\beta z = \cos \beta z$$

$$\text{and } \sinh \gamma z = \sinh j\beta z = j\sin \beta z$$

Substituting, we get

$$Z = R_o\left[\frac{Z_l \cos\beta z + jR_o \sin\beta z}{R_o \cos\beta z + jZ_l \sin\beta z}\right]$$

$$\text{or } Z = R_o\left[\frac{Z_l + jR_o \tan\beta z}{R_o + jZ_l \tan\beta z}\right]$$

Line impedance in terms of reflection coefficient or standing wave ratio.

$$V(z) = \frac{V_l(Z_l + Z_o)}{2Z_l}\left[e^{\gamma z} + \Gamma e^{-\gamma z}\right]$$

$$I(z) = \frac{I_l(Z_l + Z_o)}{2Z_o}\left[e^{\gamma z} - \Gamma e^{-\gamma z}\right]$$

Since $V_l = I_l Z_l$,

$$V(z) = \frac{I_l Z_l(Z_l + Z_o)}{2Z_l}\left[e^{\gamma z} + \Gamma e^{-\gamma z}\right]$$

$$\therefore\ V(z) = \frac{I_l(Z_l + Z_o)}{2}\left[e^{\gamma z} + \Gamma e^{-\gamma z}\right]$$

Dividing equation $\frac{V(z)}{I(z)}$

$$Z = \frac{V(z)}{I(z)} = Z_o\left[\frac{e^{\gamma z} + \Gamma e^{-\gamma z}}{e^{\gamma z} - \Gamma e^{-\gamma z}}\right]$$

$$= Z_o\left[\frac{1 + \Gamma e^{-2\gamma z}}{1 - \Gamma e^{-2\gamma z}}\right]$$ by multiplying both numerator by denominator by $e^{-\gamma z}$

we have $\Gamma = |\Gamma| e^{j\theta}$

$$\left.\begin{aligned}\gamma &= j\beta \\ Z_o &= R_o\end{aligned}\right\}$$ for lossless line

Substituting,

$$Z = R_o\left[\frac{1 + |\Gamma| e^{j\theta} e^{-j2\beta z}}{1 - |\Gamma| e^{j\theta} e^{-j2\beta z}}\right]$$

$$\therefore\ Z = R_o\left[\frac{1 + |\Gamma| \underline{|\theta - 2\beta z}}{1 - |\Gamma| \underline{|\theta - 2\beta z}}\right]$$

Substituting for $|\Gamma|$

$$Z = R_o \left[\frac{1 + \left(\frac{\rho - 1}{\rho + 1} \right) \underline{|\theta - 2\beta z}}{1 - \left(\frac{\rho - 1}{\rho + 1} \right) \underline{|\theta - 2\beta z}} \right]$$

This expression for line impedance in terms of standing wave ratio ρ is a very useful expression since "ρ" can be easily measured using a simple detector or a standing wave meter.

Determination of characteristic impedance Z_0 and propagation constant γ

The characteristic impedance and propagation constant of a given transmission line can be determined with the help of two measurements.

(a) The input impedance of a line of length l is given by equation

$$Z_S = Z_0 \left[\frac{Z_l \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_l \sinh \gamma l} \right]$$

When the receiving end is short circuited, then

$$Z_l = 0$$

Substituting, we get the input impedance of short circuited line as

$$Z_{sc} = Z_0 \left[\frac{0 + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + 0} \right]$$

$$\therefore \ \mathbf{Z_{sc} = Z_0 \tanh \gamma l}$$

This impedance Z_{sc} is measured at the sending end by short circuiting the load end of the line.

(b) Now, the receiving end is open circuited so that,

$$Z_l = \infty$$

The expression for Z_S is now modified by dividing the numerator and denominator by Z_l to get

$$Z_S = Z_o\left[\frac{\cosh\gamma l + \left(\frac{Z_o}{Z_l}\right)\sinh\gamma l}{\left(\frac{Z_o}{Z_l}\right)\cosh\gamma l + \sinh\gamma l}\right]$$

As $Z_l \rightarrow \infty$, $\frac{1}{Z_l} \rightarrow 0$

$$\mathbf{Z_S = Z_{oc} = Z_o \coth \gamma l}$$

Multiplying equations , we get

$$Z_{sc}\, Z_{oc} = (Z_o \tanh \gamma l)\,(Z_o \coth \gamma l) = Z_0^2$$

$$\therefore \quad \mathbf{Z_o = \sqrt{Z_{sc} Z_{oc}}}$$

Dividing equation , we get

$$\frac{Z_{sc}}{Z_{oc}} = \frac{Z_o \tanh \gamma l}{Z_o \coth \gamma l} = \tanh^2 \gamma l$$

$$\therefore \quad \tanh \gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

$$\therefore \quad \gamma = \alpha + j\beta = \frac{1}{l}\tanh^{-1}\left[\sqrt{\frac{Z_{sc}}{Z_{oc}}}\right]$$

Line Admittance : The characteristic admittance and the generalized admittance at any point on the line are defined by

$$Y_o = \frac{1}{Z_o} = G_o \pm jB_o$$

$$Y = \frac{1}{Z} = G \pm jB$$

Normalized Impedance and Admittance

The normalized impedance of a transmission line is defined as the ratio of the line impedance Z to the characteristic impedance Z_o given by

$$z = \frac{Z}{Z_o} = r \pm jx$$

And the normalized admittance in defined as the ratio of the line admittance Y to the characteristic admittance Y_o given by

$$y = \frac{Y}{Y_o} = \frac{Z_o}{Z} = \frac{1}{z} = g \pm jb$$

Example The characteristic impedance of a transmission line is 100 Ω and the load impedance is 200 Ω. Calculate

(i) the voltage reflection coefficient

(ii) the voltage standing wave ratio

(iii) the reflected power in percent if the incident voltage is 90 volts.

$$\Gamma = \frac{Z_l - Z_o}{Z_l + Z_o} = \frac{200-100}{200+100}$$

$$\Gamma = 0.333$$

$$\rho = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.333}{1-0.333}$$

$$\rho = 2$$

$$\Gamma = \frac{\text{reflected voltage, } V_r}{\text{incident voltage, } V_i} = 0.333$$

$$\therefore \quad V_r = (0.333)(90)$$

$$\therefore \quad V_r = 30 \text{ volts}$$

$$\frac{\text{Reflected power}}{\text{Incident power}} = \frac{P_r}{P_i} = \left(\frac{V_r}{V_i}\right)^2 = \left(\frac{30}{90}\right)^2 = \Gamma^2 = 0.1111$$

$\therefore$ **Reflected power P_r = 11.11% of the incident power P_i**

Example A transmission line 2.413 wavelengths long is terminated in an impedance of 150 + j60 Ω. The line has negligible losses having a characteristic impedance of 75 Ω. Find the input impedance.

Solution :

$$\text{Given } l = 2.413\,\lambda$$

$$Z_l = 150 + j60\,\Omega$$

$$Z_o = 75\,\Omega = R_o$$

$$Z_s = ?$$

From trignometry,

$$\tanh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}} = \frac{0.7471 + j0.0654}{1}$$

By componendo and dividendo,

$$\frac{e^{\gamma l} + e^{-\gamma l} + e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l} - e^{\gamma l} + e^{-\gamma l}} = \frac{1 + 0.7471 + j0.0654}{1 - 0.7471 - j0.0654}$$

$$\therefore \quad \frac{2e^{\gamma l}}{2e^{-\gamma l}} = \frac{1.7471 + j0.0654}{0.2529 - j0.0654}$$

$$= \frac{1.7483 \angle 2.14^\circ}{0.2612 \angle -14.5^\circ}$$

$$\therefore \quad e^{2\gamma l} = 6.6933 \angle 16.64^\circ$$

Taking natural logarithm on both sides

$$2\gamma l = \log_e\left[6.6933 \angle 16.64^\circ\right]$$

$$\therefore \quad \gamma = \frac{1}{2l}\left[\log_e 6.6933 + j(16.64^\circ + 2n\pi)\right]$$

The above equation should hold good for all values of n = 0, 1, 2, ...

$$\therefore \quad \gamma = \frac{1}{2 \times 10}\left[1.9 + j\left(\frac{16.64^\circ \times \pi}{180^\circ} + 2n\pi\right)\right]$$

$$= 0.095 + j\,0.0145 + j\,0.3142\,n$$

Let us assume lowest value of n = 0. Then the propagation constant is given by

$$\boldsymbol{\gamma = \alpha + j\beta = 0.095 + j0.0145}$$

$\therefore$ Attenuation constant $\alpha = 0.095$ nepers/m

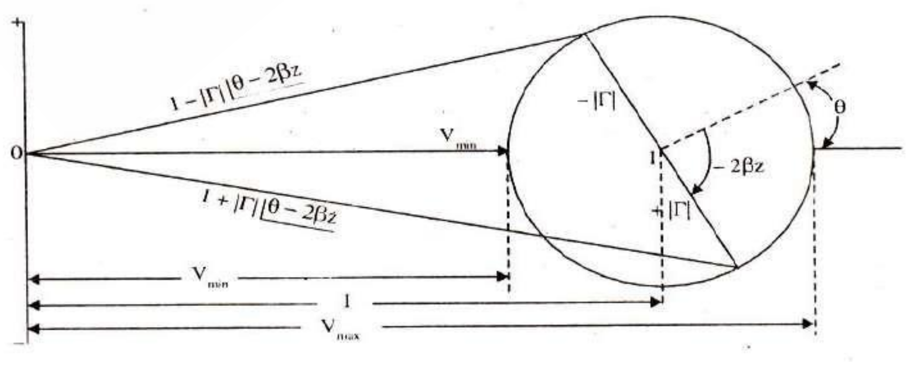
and phase constant $\beta = 0.0145$ rads/m

CALCULATION OF VOLTAGE MINIMA AND MAXIMA

The input impedance of a lossless line is given by equation

$$Z_S = R_o\left[\frac{1 + |\Gamma| \angle \theta - 2\beta z}{1 - |\Gamma| \angle \theta - 2\beta z}\right]$$

This expression can be represented by a phasor diagram as shown in fig



The numerator and denominator of equation are shown as two separate phasors, the results of adding unity to $|\Gamma|$ or $-|\Gamma|$.

It is evident from the phasor diagram that when $\theta - 2\beta z = \pm 2n\pi$ where n = 0, 1, 2 ......, the two phasors will coincide resulting in maximum input impedance and V_{max} positions.

$$\therefore \quad \theta - 2\beta z = \pm 2n\pi$$

$$\therefore \quad z = \frac{\theta}{2\beta} \pm \frac{2n\pi}{2\beta}$$

$$\therefore \quad z = \frac{1}{\beta}\left[\frac{\theta}{2} \pm n\pi\right]$$

From $\beta = \frac{2\pi}{\lambda}$

$$\therefore \quad z = \frac{\lambda}{2\pi}\left[\frac{\theta}{2} \pm n\pi\right]$$

$$\therefore \quad dV_{max} = \frac{\lambda\theta}{4\pi} \text{ when } n = 0$$

On the other hand, when $\theta - 2\beta z = \pm(2n-1)\pi$ where n = 0, 1, 2, ... the two phasors again coincide now resulting in minimum input impedance and V_{min} positions.

$$\therefore \quad \theta - 2\beta z = \pm(2n-1)\pi$$

$$\therefore \quad z = \frac{1}{2\beta}\left[\theta \pm (2n-1)\pi\right]$$

$$z = \frac{\lambda}{4\pi}\left[\theta \pm (2n-1)\pi\right]$$

Equation above gives varies V_{min} positions.

From equations it is clear that V_{max} and V_{min} points occur alternately at every quarter wavelength.

i.e., $z|_{V_{max}} - z|_{V_{min}} = \frac{\lambda}{4}$

Thus we can conclude that two successive V_{min} points (or V_{max} points) are separated by a distance of half wavelength.

(i) The first voltage minimum dV_{min} is located from the load end by putting n = 0 in equation

$$\therefore \quad dV_{min} = \frac{\lambda}{4\pi}[\theta \pm \pi]$$

(ii) The value of ρ is found using $\rho = \frac{|V_{max}|}{|V_{min}|}$

(iii) The distance between successive minima or maxima is calculated which is equal to $\frac{\lambda}{2}$.

Example:

A certain dissipationless air filled co-axial line is driven by an a.c. source operating at a frequency of 200 MHz. The characteristic impedance of the cable is $400\lfloor 0^\circ\ \Omega$ and is terminated in a load impedance of $50 + j\,80\,\Omega$. Determine (i) the magnitude and phase of reflection coefficient (ii) the standing wave ratio (iii) the location of first voltage minimum from load (iv) the location of first voltage maximum from load.

Given

$$f = 200 \text{ MHz}$$

$$Z_o = 400\lfloor 0^\circ\ \Omega$$

$$Z_l = 50 + j\,80\ \Omega$$

$$\Gamma\lfloor\theta = ?, \quad \rho = ?, \quad dV_{min} = ?, \quad dV_{max} = ?$$

ı, the reflection coefficient is given by

$$\Gamma = \frac{Z_l - Z_o}{Z_l + Z_o}$$

$$\therefore \quad \Gamma = \frac{50 + j80 - 400}{50 + j80 + 400} = \frac{-350 + j80}{450 + j80}$$

$$= \frac{359.03\lfloor 180 - 12.88^\circ}{457.06\lfloor 10.08^\circ}$$

$$\therefore \quad \Gamma = 0.7855\lfloor 157.04^\circ$$

(ii) From equation the standing wave ratio is given by

$$\rho = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.7855}{1 - 0.7855}$$

$$\therefore \quad \rho = 8.324$$

(iii) Since the line is dissipationless, the wavelength λ is give

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{200 \times 10^6} = 1.5 \text{ m}$$

and $\theta = 157.04^\circ$

From equation $dV_{min} = \frac{\lambda}{4\pi}[\theta \pm \pi]$

$$= \frac{1.5}{4\pi}\left[157.04^\circ \times \frac{\pi}{180^\circ} \pm \pi\right]$$

$$= 0.702 \text{ m considering positive sign}$$

[Negative sign yields negative value of dV_{min} which is not possible as it is outside the load impedance.] $\therefore$ $dV_{min} = 0.702$ **m from load end.**

(iv) From equation

$$z = \frac{\lambda}{2\pi}\left[\frac{\theta}{2} \pm n\pi\right]$$

For $n = 0$, $z = dV_{max}$

$$\therefore \quad dV_{max} = \frac{\lambda}{2\pi} \cdot \frac{\theta}{2}$$

$$= \frac{\lambda}{4\pi}\left[157.04^\circ \times \frac{\pi}{180^\circ}\right]$$

$\therefore$ $dV_{max} = 0.327$ **m from load end**

Note: The distance between dV_{min} and dV_{max} is always equal to $\lambda/4$.

Example : An antenna as load on a transmission line produces a SWR of 2.8 with a voltage minimum at a distance of 0.12 λ from the antenna terminals. Find the antenna impedance if R_o of the line is 300 Ω.

Solution :

Given

$$\text{VSWR} = \rho = 2.8$$

$$dV_{min} = 0.12\,\lambda$$

$$Z_o = R_o = 300\ \Omega$$

$$\text{Antenna impedance} = Z_l = ?$$

From equation $dV_{min} = \frac{\lambda}{4\pi}[\theta \pm \pi]$

$$\therefore \quad 0.12\,\lambda = \frac{\lambda}{4\pi}[\theta \pm \pi]$$

$$\therefore \quad \theta \pm \pi = 0.48\,\pi$$

$$\therefore \quad \theta = 0.48\,\pi \mp \pi$$

Taking negative value, the phase angle of the reflection coefficient is given by

$$\theta = -0.52\,\pi \text{ radians}$$

or

$$\theta = -0.52\,\pi \times \frac{180^\circ}{\pi} \text{ degrees}$$

$$\therefore \quad \boldsymbol{\theta = -93.6^\circ}$$

. the magnitude of reflection coefficient is given by

$$|\Gamma| = \frac{\rho - 1}{\rho + 1} = \frac{2.8 - 1}{2.8 + 1}$$

$$\therefore \quad |\Gamma| = 0.4737$$

$$\therefore \quad \boldsymbol{\Gamma = 0.4737 \angle -93.6^\circ}$$

From equation $\Gamma = \dfrac{Z_l - Z_o}{Z_l + Z_o}$

By componendo and dividendo

$$\frac{1+\Gamma}{1-\Gamma} = \frac{Z_l + Z_o + Z_l - Z_o}{Z_l + Z_o - Z_l + Z_o} = \frac{2Z_l}{2Z_o}$$

$$\therefore \quad Z_l = Z_o \frac{1+\Gamma}{1-\Gamma}$$

$$\therefore \quad Z_l = (300)\left[\frac{1 + 0.4737 \angle -93.6^\circ}{1 - 0.4737 \angle -93.6^\circ}\right]$$

$$= 300\left[\frac{1 - 0.0297 - j0.4728}{1 + 0.0297 + j0.4728}\right]$$

$$= 300\left[\frac{0.9703 - j\,0.4728}{1.0297 + j\,0.4728}\right] = 300\left[\frac{1.0794 \angle -25.98^\circ}{1.133 \angle 24.66^\circ}\right]$$

$$\therefore \quad \mathbf{Z_l = 285.8 \angle -50.64^\circ\ \Omega}$$

Line Admittance

When a transmission line is branched, it is better to solve the line equations for the line voltage, current, and transmitted power in terms of admittance rather than impedance. The characteristic admittance and the generalized admittance are defined as

$$\mathbf{Y}_0 = \frac{1}{\mathbf{Z}_0} = G_0 \pm jB_0$$

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = G \pm jB$$

Then the normalized admittance can be written

$$\mathbf{y} \equiv \frac{\mathbf{Y}}{\mathbf{Y}_0} = \frac{\mathbf{Z}_0}{\mathbf{Z}} = \frac{1}{\mathbf{z}} = g \pm jb$$

A lossless line has a characteristic impedance of 50 Ω and is terminated in a load resistance of 75 Ω. The line is energized by a generator which has an output impedance of 50 Ω and an open-circuit output voltage of 30 V (rms). The line is assumed to be 2.25 wavelengths long. Determine:

a. The input impedance

b. The magnitude of the instantaneous load voltage

c. The instantaneous power delivered to the load

$$\beta d = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

the input impedance is

$$\mathbf{Z}_{in} = \frac{R_0^2}{R_\ell} = \frac{(50)^5}{75} = 33.33\ \Omega$$

b. The reflection coefficient is

$$\Gamma_\ell = \frac{R_\ell - R_0}{R_\ell + R_0} = \frac{75 - 50}{75 + 50} = 0.20$$

Then the instantaneous voltage at the load is

$$\mathbf{V}_\ell = \mathbf{V}_+ e^{-j\beta\ell}(1 + \Gamma_\ell) = 30(1 + 0.20) = 36\ \text{V}$$

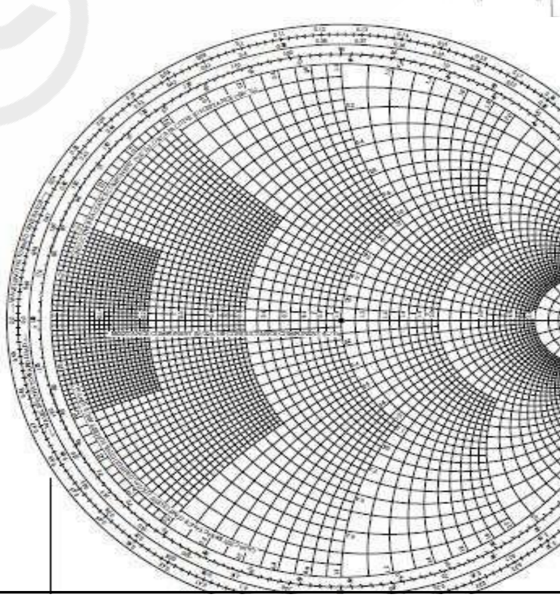
c. The instantaneous power delivered to the load is

$$\mathbf{P}_\ell = \frac{(36)^2}{75} = 17.28\ \text{W}$$

SMITH CHART

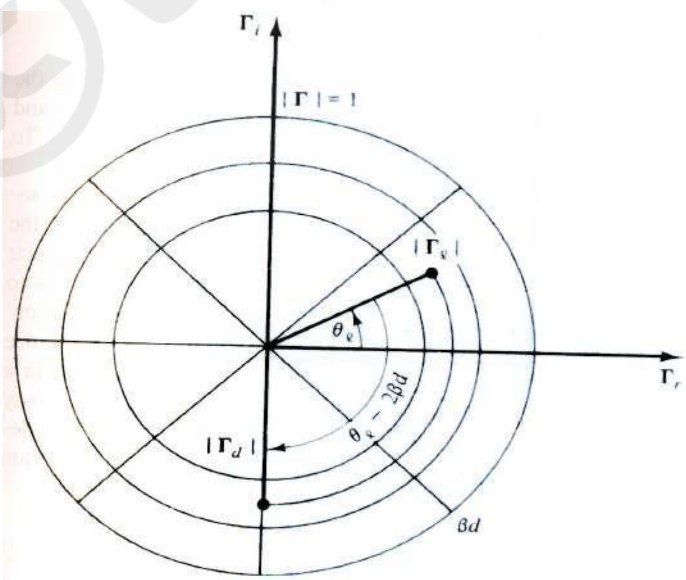
The input line impedance is given by

$$Z_s = Z_0 \left[\frac{Z_l + jR_o \tan\beta z}{Z_o + jZ_l \tan\beta z} \right]$$



- Consists of plot of normalized impednce or admitance with a angle and magnitude of a generalized complex reflection coefficient in a unity circle.
- Applicable to get analysis of Lossless and lossy line.
- By simple rotation of the chart the effect of the position on the line can be determined.

Substitution of Eq. $r = \dfrac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}$



Constant Γ circles and electrical-length radials βd.

Construction of Smith Chart

Smith chart is constructed on voltage reflection coefficient plane i.e., on the Γ-plane

we have

$$Z_s = Z_0 \left[\frac{1+\Gamma \,|\, \theta-2\beta z}{1-\Gamma \,|\, \theta-2\beta z} \right]$$

The input impedance Z_s when divided by Z_0 is called "***Normalized input impedance***" given by

$$\frac{Z_s}{Z_0} = \left[\frac{1+\Gamma \,|\, \theta-2\beta z}{1-\Gamma \,|\, \theta-2\beta z} \right]$$

Let the normalized input impedance (also called ***per unit impedance*** which is a complex quantity) be given by

$$\frac{Z_s}{Z_0} = r + j\,x$$

$$\Gamma_\ell = \frac{\mathbf{Z}_\ell - \mathbf{Z}_0}{\mathbf{Z}_\ell + \mathbf{Z}_0} = |\Gamma_\ell| e^{j\theta_\ell} = \Gamma_\gamma + j\Gamma_i$$

Since $|\Gamma_\ell| \le 1$, the value of Γ_ℓ must lie on or within the unity circle with a radius of 1. The reflection coefficient at any other location along a line

$$\Gamma_d = \Gamma_\ell e^{-2\alpha d} e^{-j2\beta d} = |\Gamma_\ell| e^{-2\alpha d} e^{j(\theta_\ell - 2\beta d)}$$

which is also on or within the unity circle. Figure shows circles for a constant reflection coefficient Γ and constant electrical-length radials βd.

the normalized impedance along a line is $\mathbf{z} = \frac{\mathbf{Z}}{\mathbf{Z}_0} = \frac{1 + \Gamma_\ell e^{-2\gamma d}}{1 - \Gamma_\ell e^{-2\gamma d}}$

With no loss in generality, it is assumed that $d = 0$; then

$$\mathbf{z} = \frac{1 + \Gamma_\ell}{1 - \Gamma_\ell} = \frac{\mathbf{Z}_\ell}{\mathbf{Z}_0} = \frac{R_\ell + jX_\ell}{\mathbf{Z}_0} = r + jx$$

and

$$\Gamma_\ell = \frac{\mathbf{z} - 1}{\mathbf{z} + 1} = \Gamma_r + j\Gamma_i$$

The quantity $|\Gamma| \underline{|\theta - 2\beta z}$ is also a complex quantity and let it be given by

$$|\Gamma| \underline{|\theta - 2\beta z} = u + j v$$

Using equations

$$r + j x = \frac{(1+u) + j v}{(1-u) - j v} \times \frac{[(1-u) + j v]}{[(1-u) + j v]} \text{ on rationalization}$$

$$= \frac{(1 + j v)^2 - u^2}{(1-u)^2 + v^2}$$

$$= \frac{(1 - v^2 - u^2)}{(1-u)^2 + v^2} + j \frac{2v}{(1-u)^2 + v^2}$$

Equating real and imaginary parts, we get

$$r = \frac{1 - v^2 - u^2}{(1 - u^2) + v^2}$$

and

$$x = \frac{2v}{(1-u)^2 + v^2}$$

Constant Resistance Circles

we have

$$r\,[1 - 2u + u^2 + v^2] = 1 - v^2 - u^2$$

$$\therefore \quad u^2 (1 + r) - 2\,u\,r + v^2 (1 + r) = (1 - r)$$

Dividing throughout by $(1 + r)$, we get

$$u^2 - 2u\left(\frac{r}{1+r}\right) + v^2 = \left(\frac{1-r}{1+r}\right)$$

Completing the square by adding $\left(\frac{r}{1+r}\right)^2$ to both sides, we get,

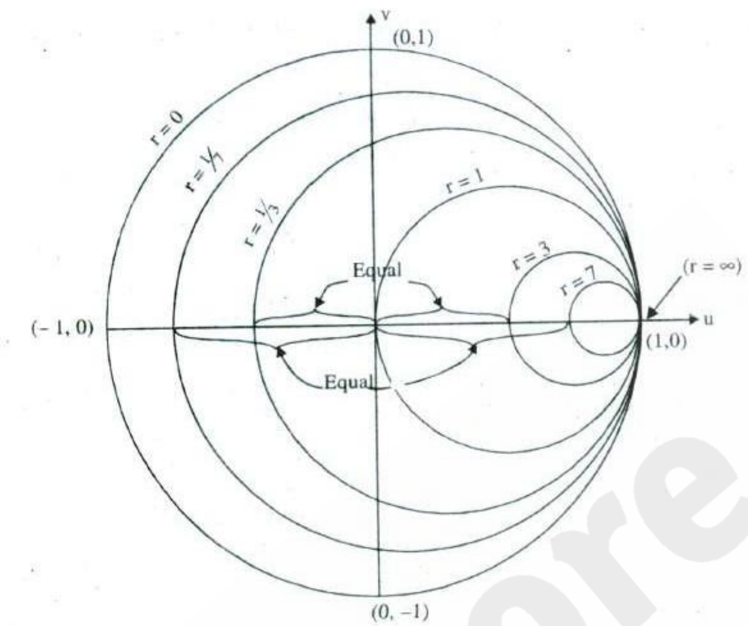
$$u^2 - 2u\left(\frac{r}{1+r}\right) + \left(\frac{r}{1+r}\right)^2 + v^2 = \frac{1-r}{1+r} + \left(\frac{r}{1+r}\right)^2$$

$$\left(u - \frac{r}{1+r}\right)^2 + v^2 = \frac{1 - r^2 + r^2}{(1+r)^2}$$

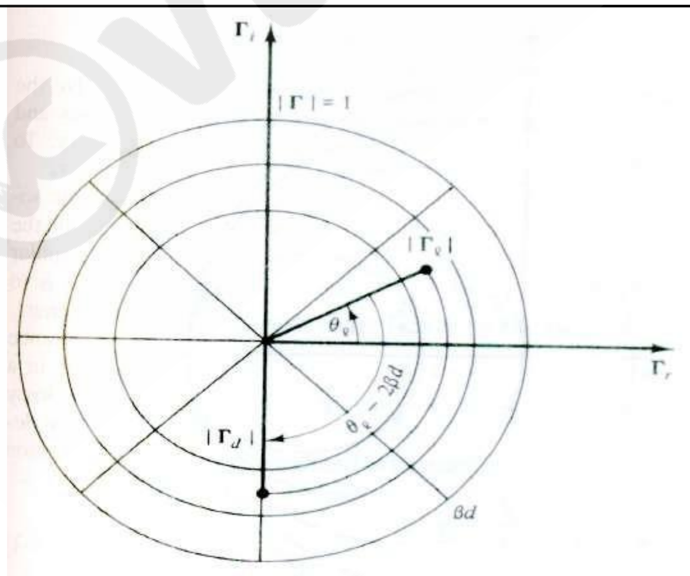
$$\therefore \quad \left(u - \frac{r}{1+r}\right)^2 + v^2 = \left(\frac{1}{1+r}\right)^2$$

Equation represents the equation of a circle of radius $\left(\frac{1}{1+r}\right)$ and centre at $\left(\frac{r}{1+r}, 0\right)$.

This equation actually represents normalized resistance circles, which are shown plotted in figure



Illustrating constant r circles



Constant Γ circles and electrical-length radials βd.

Constant Reactance Circles

From equation (1.156)

$$(u^2 + v^2 - 2u + 1)\,x = 2v$$

$$\therefore \quad (u^2 - 2u + 1)\,x + v^2x - 2v = 0$$

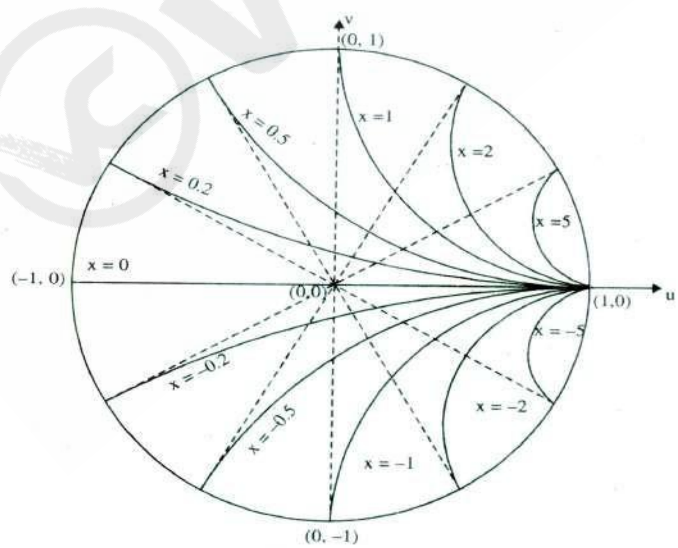
Dividing by x throughout, we get

$$(u-1)^2 + v^2 - \frac{2v}{x} = 0$$

Completing the square by adding $\frac{1}{x^2}$ to both sides, we get

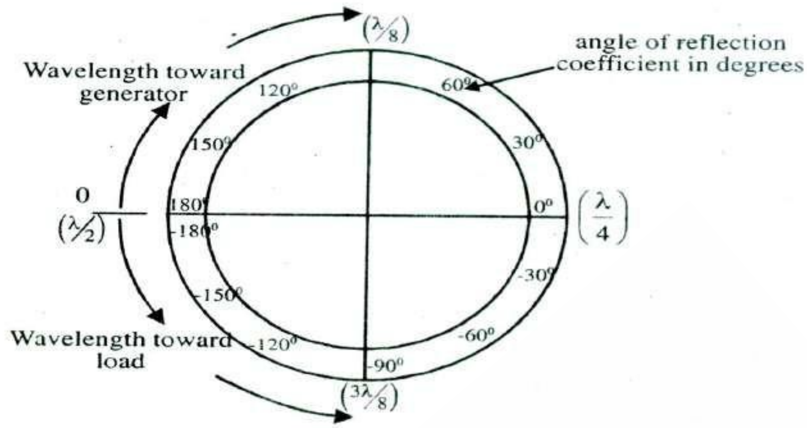
$$(u-1)^2 + \left(v - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

This equation is again the equation of a family of circles called "***constant reactance circles***" with radius equal to $\left(\frac{1}{x}\right)$ and centre at $\left(1, \frac{1}{x}\right)$ which are shown plotted in figure



: **Illustrating constant x-circles**

The superposition of these constant-resistance circles and constant reactance circles forms the ***SMITH CHART***.



: Illustrating outer circles of SMITH CHART

The outermost circle in figure represent the wavelength toward generator (clockwise) and the wavelength toward load (anti clock wise). The next inner circle shows the angle of reflection coefficient in degrees.

Because the incident and the reflected waves travel in the opposite directions, one degree on the line corresponds to two degrees phase difference between the incident and reflected waves.

Constant VSWR (ρ) Circles

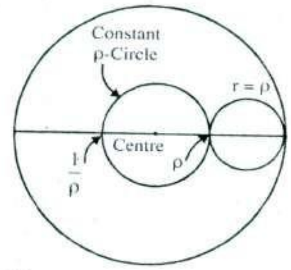
Using this in equation

$$r = \frac{1+u}{1-u}$$

$$r = \frac{1+|\Gamma|}{1-|\Gamma|} = \rho$$

$\therefore$ When $v = 0$, $r = \rho$

$\therefore$ To draw a constant ρ circle, a point $r = \rho$ is located on the central horizontal axis. With centre of Smith Chart as centre and radius equal to 'ρ', a circle is drawn cutting the central horizontal axis at ρ on the right hand side. The point on the left hand side of the chart where the circle cuts will always be at $(1/\rho)$ as illustrated in figure



: Illustrating Constant ρ–circle

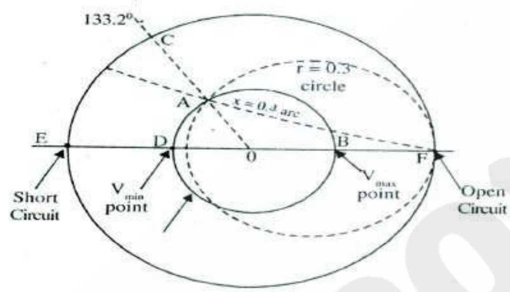
Applications and Properties of Smith Chart

1. Smith Chart consists of resistance and reactance circles (i.e., $r = \frac{R_l}{Z_o}$ and $x = \frac{X_l}{Z_o}$ circles where $Z_l = R_l + j X_l$). Hence the given load impedance Z_l has to be divided by Z_o before entering in the Smith Chart. This process is called "***normalization***" and the ratio $\left(\frac{Z_l}{Z_o}\right)$ is called "***per unit load impedance***" or "***normalized impedance***".
2. **Plotting of an impedance :** Any complex impedance can be shown by a single point on the Smith Chart. This point is the intersection of $r = \frac{R_l}{Z_o}$ circle and $x = j \frac{X_l}{Z_o}$ arc of a circle.

Example : Let $Z_l = 120 + j\, 160\ \Omega$ and $Z_o = 400\ \Omega$

Normalizing, we get $r = \frac{R_l}{Z_o} = \frac{120}{400} = 0.3$

and $x = \frac{X_l}{Z_o} = \frac{160}{400} = 0.4$



The point A in figure shows the point corresponding to a normalized impedance 0.3+ j 0.4 on the chart.

3. **VSWR Determination :** With 'O' as centre and 'OA' as radius, a circle is drawn which cuts the horizontal axis at a "r" value of 3.9 (shown as point B in figure 1.19). This gives the value of VSWR = ρ = 3.9.
4. **Determination of Γ in magnitude and direction** : The line OA in figure 1.19, is produced to meet the outer circle at point C. The angle at point C = θ = 133.2^0 is the phase angle of the reflection coefficient.

To find magnitude of Γ, the linear scale at the bottom of the chart is referred which is marked "**Voltage reflection coefficient**". With 0.0 as centre and radius exactly equal to OA, an arc is cut on the linear scale as shown in figure 1.20. The value of $|\Gamma|$ is read as 0.59 corresponding to point A.

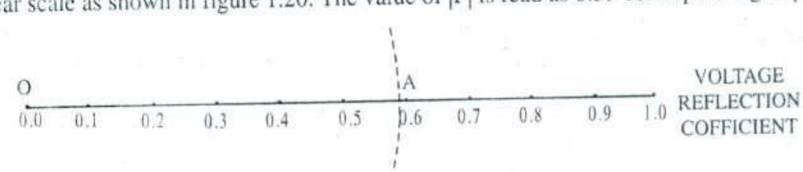


Fig. : Measurement of reflection coefficient

$\therefore$ The reflection coefficient $\Gamma = 0.59\ \underline{|133.2^0}$

5. **Location of V_{max} and V_{min}:** The constant ρ circle intersects the central horizontal axis at point D and B as shown in figure 1.19. The pint D corresponds to **Voltage minima** and B to **Voltage maxima.**

$\therefore$ Line impedance at $V_{max} = \frac{R_{max}}{Z_o} = r_{max} = 3.91$

and line impedance at $V_{min} = \frac{R_{min}}{Z_o} = r_{min} = 0.26$

(The reactive components are zero)

6. **Open and Short circuited line:** At point F on the horizontal axis, r = ∞ and x = ∞. This represents the open circuit termination of the line. At point E we have r = 0, x = 0. This represents short circuit termination.

7. **Movement along periphery of the chart :** Movement in clockwise direction from point E results in number of wavelengths toward generator and movement in anti-clockwise direction from point E in wavelengths toward load. One full rotation on the periphery corresponds to λ/2.

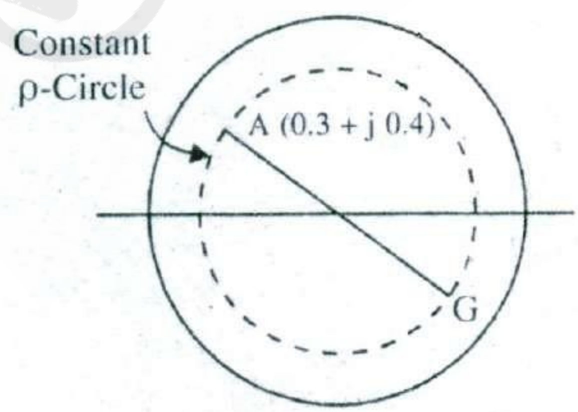
8. **Matched load:** Consider the circle r = 1.

$$\therefore \quad \frac{R_l}{Z_o} = 1 \quad \therefore \; R_l = Z_o$$

i.e., the resistive component of the load is equal to characteristic impedance of the line. This represents the condition of no reflection provided its reactive component is neutralized by equal but opposite reactance.

9. **Conversion of impedance to admittance**

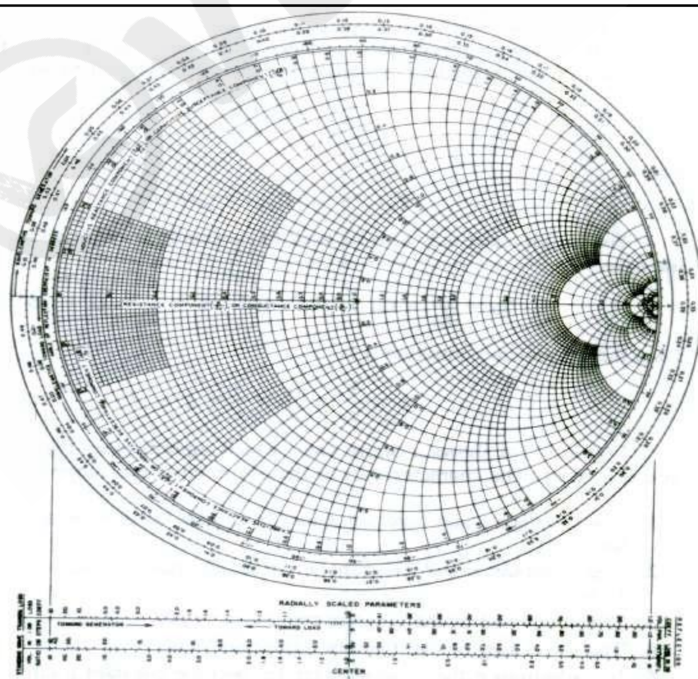
To find the admittance of an impedance at point A, the point A is rotated through constant S-circle by an amount λ/4 which is equivalent to 180^0. The point G diametrically opposite to A has a value 1.2 – j 1.6 which is the admittance corresponding to the impedance 0.3 + j 0.4.



Illustrating Conversion of impedance to admittance

The characteristics of the Smith chart are summarized as follows:

1. The constant r and constant x loci form two families of orthogonal circles in the chart.
2. The constant r and constant x circles all pass through the point ($\Gamma_r = 1$, $\Gamma_i = 0$).
3. The upper half of the diagram represents $+jx$.
4. The lower half of the diagram represents $-jx$.
5. For admittance the constant r circles become constant g circles, and the constant x circles become constant susceptance b circles.
6. The distance around the Smith chart once is one-half wavelength ($\lambda/2$).
7. At a point of $z_{\min} = 1/\rho$, there is a $V_{\min}$ on the line.
8. At a point of $z_{\max} = \rho$, there is a $V_{\max}$ on the line.
9. The horizontal radius to the right of the chart center corresponds to $V_{\max}$, $I_{\min}$, $z_{\max}$, and ρ (SWR).
10. The horizontal radius to the left of the chart center corresponds to $V_{\min}$, $I_{\max}$, $z_{\min}$, and $1/\rho$.
11. Since the normalized admittance **y** is a reciprocal of the normalized impedance **z**, the corresponding quantities in the admittance chart are 180° out of phase with those in the impedance chart.
12. The normalized impedance or admittance is repeated for every half wavelength of distance.
13. The distances are given in wavelengths toward the generator and also toward the load.

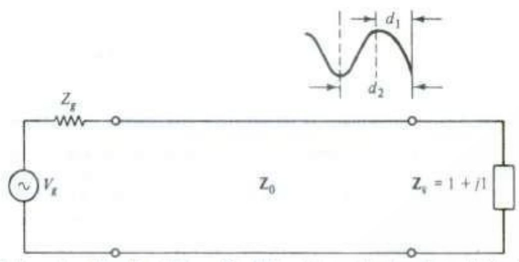


Smith chart.

A Smith chart or slotted line can be used to measure a standing-wave pattern directly and then the magnitudes of the reflection coefficient, reflected power, transmitted power, and the load impedance can be calculated from it.
is illustrated in the following examples.

Example Location Determination of Voltage Maximum and Minimum from Load

Given the normalized load impedance $z_\ell = 1 + j1$ and the operating wavelength



$\lambda = 5$ cm, determine the first V_{max}, first V_{min} from the load, and the VSWR ρ

1. Enter $z_\ell = 1 + j1$ on the chart as shown in Fig.
2. Read 0.162λ on the distance scale by drawing a dashed-straight line from the center of the chart through the load point and intersecting the distance scale.
3. Move a distance from the point at 0.162λ toward the generator and first stop at the voltage maximum on the right-hand real axis at 0.25λ. Then

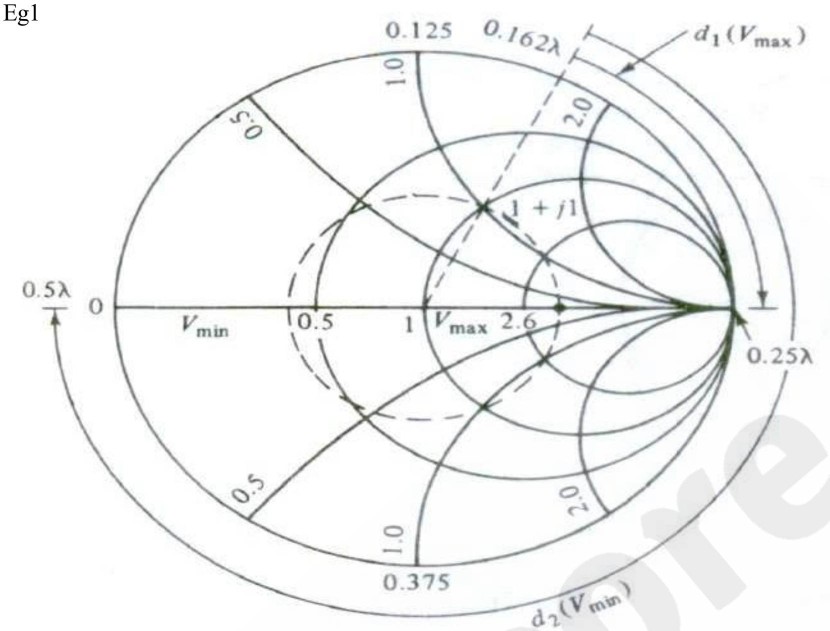
$$d_1(V_{max}) = (0.25 - 0.162)\lambda = (0.088)(5) = 0.44 \text{ cm}$$

4. Similarly, move a distance from the point of 0.162λ toward the generator and first stop at the voltage minimum on the left-hand real axis at 0.5λ. Then

$$d_2(V_{min}) = (0.5 - 0.162)\lambda = (0.338)(5) = 1.69 \text{ cm}$$

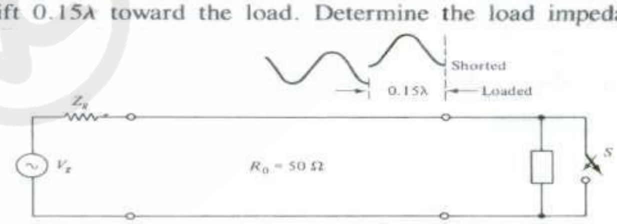
5. Make a standing-wave circle with the center at (1, 0) and pass the circle through the point of $1 + j1$. The location intersected by the circle at the right portion of the real axis indicates the SWR. This is $\rho = 2.6$.

Eg1



Impedance Determination with Short-Circuit Minima Shift

The location of a minimum instead of a maximum is usually specified because it can be determined more accurately. Suppose that the characteristic impedance of the line R_0 is 50 Ω, and the SWR $\rho = 2$ when the line is loaded. When the load is shorted, the minima shift 0.15λ toward the load. Determine the load impedance.



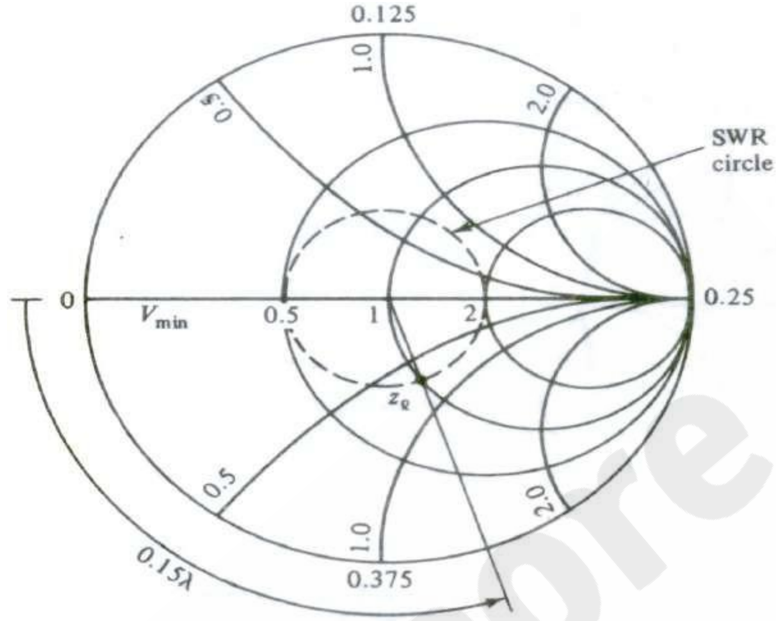
1. When the line is shorted, the first voltage minimum occurs at the place of the load as shown in Fig.
2. When the line is loaded, the first voltage minimum shifts 0.15λ from the load The distance between two successive minima is one-half wavelength.
3. Plot a SWR circle for $\rho = 2$.
4. Move a distance of 0.15λ from the minimum point along the distance scale toward the load and stop at 0.15λ.
5. Draw a line from this point to the center of the chart.
6. The intersection between the line and the SWR circle is

$$z_\ell = 1 - j0.65$$

7. The load impedance is

$$\mathbf{Z}_\ell = (1 - j0.65)(50) = 50 - j32.5\ \Omega$$

Eg:2



Example: A transmission line with a characteristic impedance $(50 + j\,0)\,\Omega$ is terminated in an impedance $(25 - j\,100\,\Omega)$. Determine the voltage reflection coefficient at the terminal load end of the line using Smith Chart.

Solution :

1. The given load impedance is first normalized.

$$\therefore \qquad z_l = \frac{Z_l}{Z_o} = \frac{25 - j100}{50} = 0.5 - j\,2 = r + j\,x$$

2. The point A on the Smith Chart is located at the intersection of r = 0.5 circle and x = – 2 arc [bottom half of Smith Chart is used for negative reactance] as shown in figure
3. With 'O' as centre, radius equal to OA, a circle is drawn. This is the constant-ρ circle.
4. This constant-ρ circle cuts the central horizontal axis at point B on the right side. At this point, the value of ρ (equal to VSWR) is read equal to the value of 'r' at point B. In figure the value of r at point B is equal to 10.

$$\therefore \qquad \boldsymbol{\rho = \text{VSWR} = 10.}$$

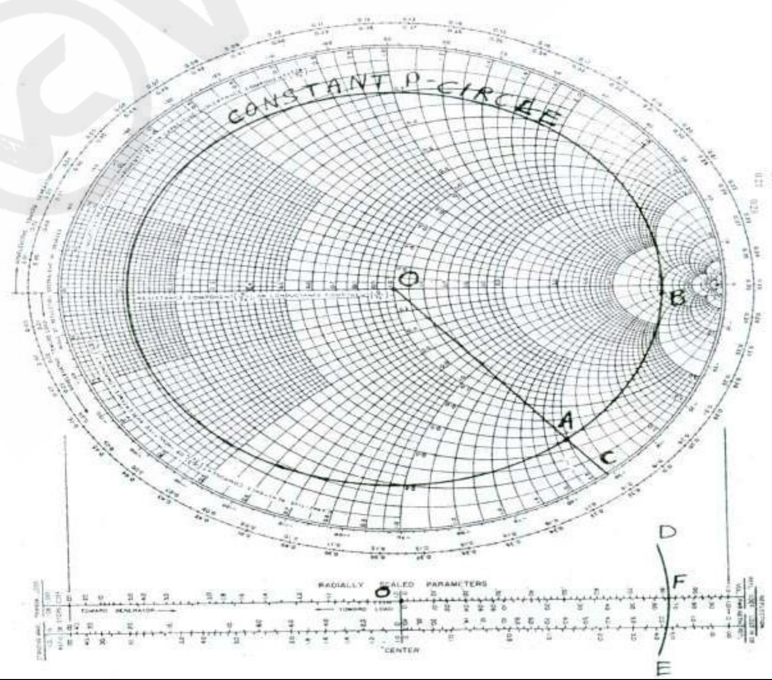
5. Below the Smith Chart, we have the radially scaled parameters showing 4 different quantities viz. Reflection coefficient and reflection loss in dB on the right hand side, Voltage standing wave and transmission loss on the left-hand side.

   With 'O' as centre and radius equal to OA and arc DE is cut on the reflection coefficient line as shown in figure . Let this intersection point be F. At point F, the value of $|\Gamma|$ is read as $|\Gamma| = 0.82$.
6. The line OA is extended upto point C. The angle corresponding to C gives the phase angle of reflection coefficient. At point C, the value of phase angle $\theta = -51^0$

   ∴ The voltage reflection coefficient at the terminal load end of the line is

$$\boldsymbol{\Gamma = 0.82 \,|\!-51^0}$$

EG 3



EG 4

Example: Determine the length of short circuited stub having characteristic impedance $Z_0 = 200\ \Omega$ and $Z_s = -j100\ \Omega$ using Smith Chart.

Solution :

1. The normalized value of input impedance is found using the relation

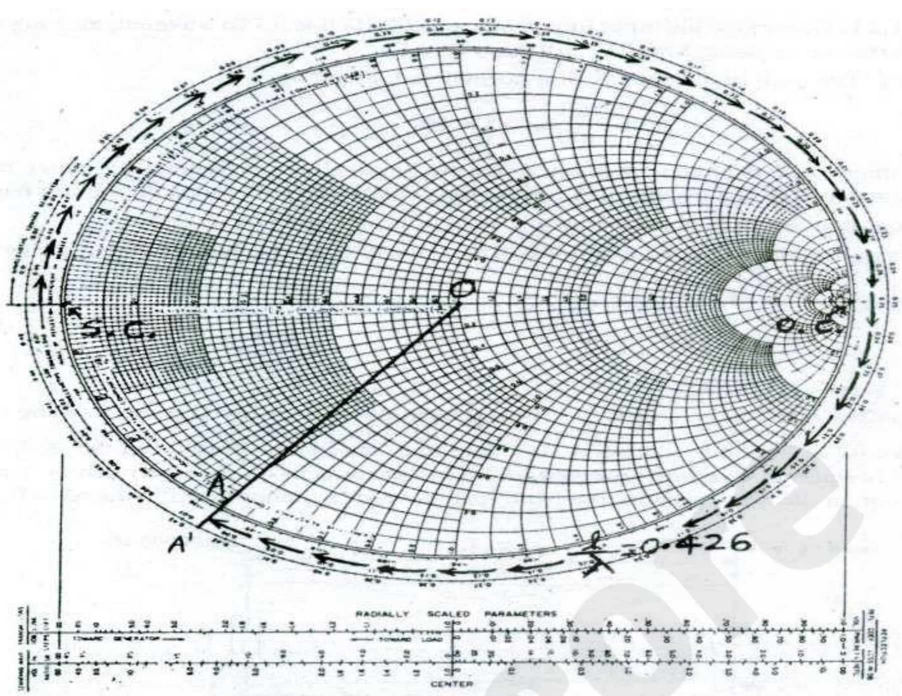
$$z_s = \frac{Z_s}{Z_o} = \frac{-j100}{200} = -j\ 0.5$$

2. The point A is located at the intersection of r = 0 circle and x = – j 0.5 arc
3. Let 'O' be the centre of the chart. Then OA is joined and extended to point A' as shown.
4. From the point of short circuit marked S.C. on the chart, the number of wavelengths upto point A' is measured moving in a clockwise direction as shown along the periphery of the chart. This length gives the value of (l/λ) where 'l' is the required length of the short circuited stub. From the chart, the distance of point A' from

$$\text{S.C.} = \frac{l}{\lambda} = 0.426$$

$\therefore$ Length of the line $= l = 0.426\ \lambda$

Knowing the value of λ, the exact length of the short-circuited line can be calculated.



Example : What length of transmission line with short-circuited termination of characteristic impedance of $Z_0 = 75 + j0\Omega$ will have a capacitive input susceptance of 0.025 ℧. Use Smith chart as an admittance chart.

Solution :

Solution :

1. Given $Z_o = 75\ \Omega$

$$\therefore\ Y_o = \frac{1}{Z_o} = \frac{1}{75} = 0.0133\ ℧$$

Given $Y_s = j\ 0.025\ ℧$

$$\therefore \text{ Normalized input admittance } y_s = \frac{Y_s}{Y_o}$$

$$= \frac{j0.025}{0.0133}$$

$$\therefore \qquad y_s = j1.875$$

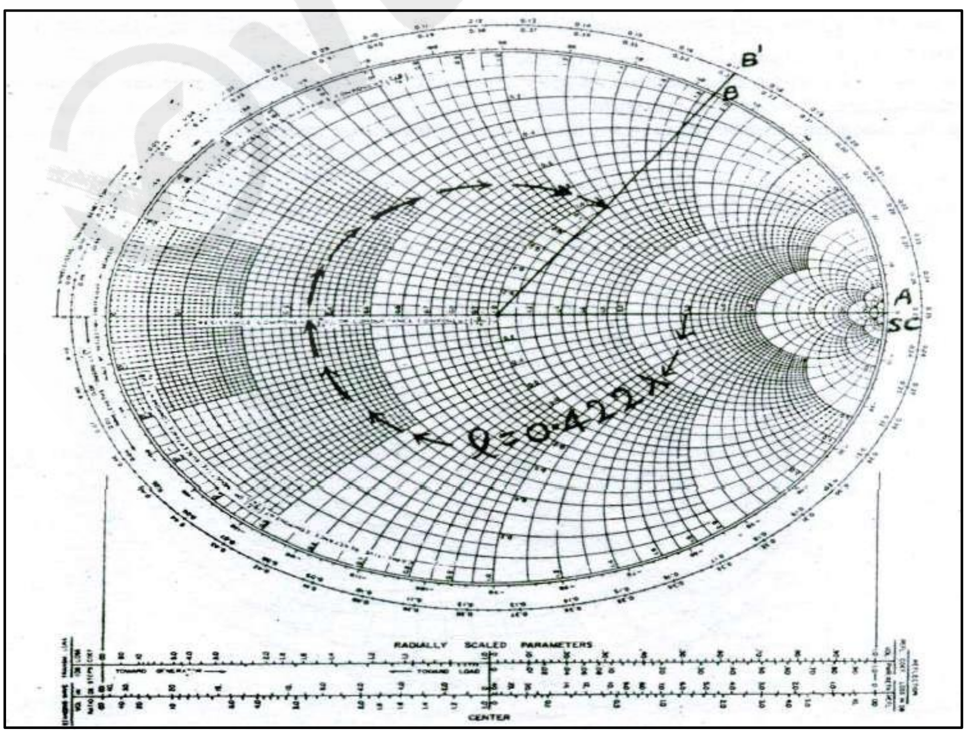
This input admittance point is located as point B in the Smith chart of figure

2. OB is joined and extended to B' on the periphery of the chart. The wavelength reading at point B' is 0.172 λ.

3. Point A on the Smith chart is the short-circuit point. The number of wavelengths from this short-circuit point A to B moving in the clockwise direction, gives the length 'l' of the short-circuited stub. From the chart,

$$l = 0.25\,\lambda + 0.172\,\lambda$$

or $$l = 0.422\,\lambda$$

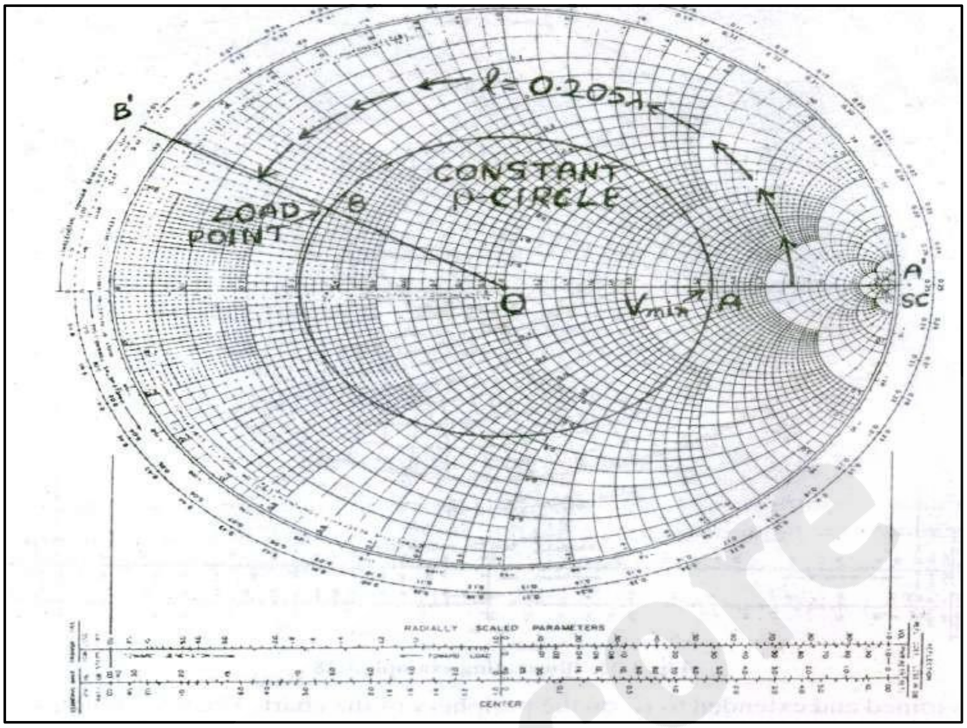


Example A VSWR of 3.25 (ρ) is observed on a line with a voltage minima at 0.205 λ from the terminal load end of the section. What is the value of the normalized load admittance at the terminals? Use Smith chart as admittance chart.

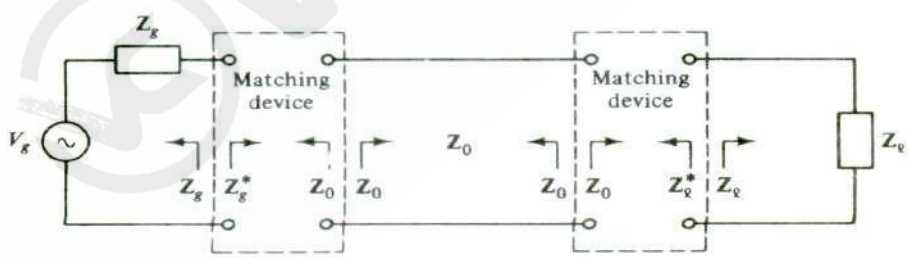
Solution :

1. With 'O' as centre and radius OA = r = 3.25, the constant-ρ circle is drawn as shown in figure The point A lying on the right side of the horizontal axis corresponds to V_{min} point since the Smith chart is being used as admittance chart.
2. OA is joined and extended to A'. A distance equal to 0.205 λ is moved on the periphery of the chart from point A' in anticlockwise direction to get point B'.
3. OB' is joined cutting the constant-ρ circle at point B. Point B is the required load admittance point.
4. From the chart, the normalized load admittance at point B is given by

$$y_l = 0.32 + j0.26$$



IMPEDANCE MATCHING



Matched transmission-line system.

- Matching for transmission lines is simply terminating the line by it's characteristic impedance.
- Usually required at RF frequencies.
- At audio frequencies iron core transformers are used.
- Usually antennas will have feeders
- As matching involves shunt connections of lines with transmisssion lines admittances have to be considerd.
- Convert impedance to admittance by rotating it by 180 degrees.

Important reasons for impedance-matching

1. Maximum power will be transmitted to the load, thus making the efficiency of transmission greatest.
2. A line terminated in R_0 has a standing wave ratio of unity and transmits a given power with a smaller peak voltage.
3. A line terminated in R_0 becomes non-resonant i.e., its input impedance remains at R_0 and will be independent of frequency.

Single-Stub Matching

good short circuit is easier to obtain than a good open circuit.

For a lossless line with $\mathbf{Y}_g = \mathbf{Y}_0$, maximum power transfer requires $\mathbf{Y}_{11} = \mathbf{Y}_0$, where $\mathbf{Y}_{11}$ is the total admittance of the line

The stub must be located at that point on the line where the real part of the admittance, looking toward the load, is $\mathbf{Y}_0$. In a normalized unit y_{11} must be in the form

$$y_{11} = y_d \pm y_s = 1$$

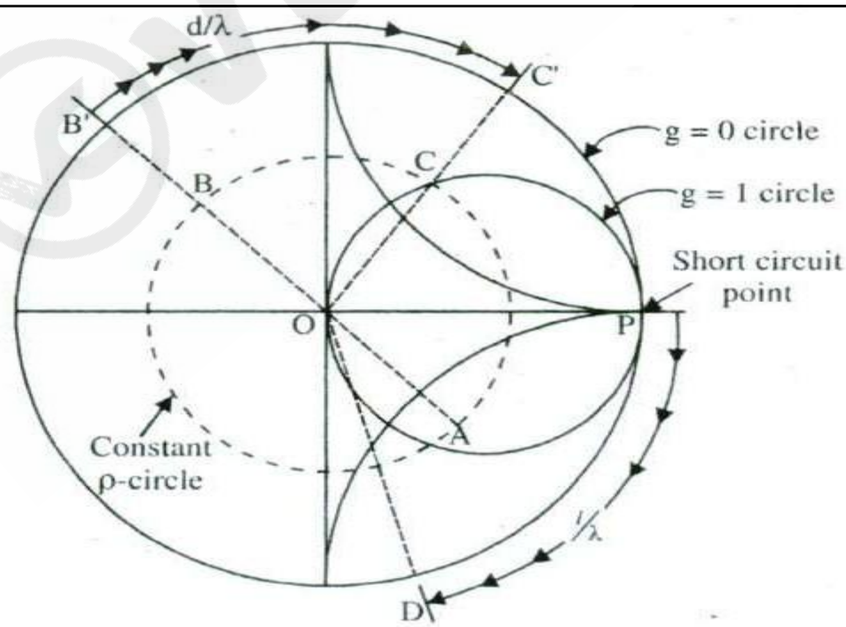
if the stub has the same characteristic impedance as that of the line. Otherwise

$$\mathbf{Y}_{11} = \mathbf{Y}_d \pm \mathbf{Y}_s = \mathbf{Y}_0$$

The stub length is then adjusted so that its susceptance just cancels out the susceptance of the line at the junction.

SINGLE-STUB MATCHING USING SMITH CHART

1. The normalized load impedance point is located at point A.
2. with 'O' as centre and OA as radius, the constant ρ-circle is drawn and the diametrically opposite point B is located on the constant–ρ circle as shown in figure
3. Location of the stub is found by moving towards generator (clockwise direction) from point B to point C which is the intersection of the constant-ρ circle with g = 1 circle. OB is extended to B' and OC to C'. The distance $\left(d/\lambda\right)$ corresponding to the arc B'C' gives the stub position.
4. The susceptance value of point C represents the susceptance of the line at the stub connection. Let this value be equal to "jb". This value of capacitive susceptance has to be neutralized by an inductive susceptance of – jb.
5. The point corresponding to a susceptance of – jb is plotted on g = 0 circle at point D as shown.
6. The point P in figure represents the short circuit point. The arc length from P to D moving clock-wise gives the value of $\left(l/\lambda\right)$ from which the length 'l' of the short circuited stub is found out.



: Illustrating Single Stub Matching using Smith Chart

Example: A load impedance of $Z_l = 60 - j\,80\ \Omega$ is required to be matched to a 50 ohm co-axial line, by using a short circuited stub of length 'l' located at a distance 'd' from the load. The wavelength of operation is 1 metre. Using Smith Chart, find 'd' and 'l'.

Solution :

Given load impedance $\quad Z_l = 60 - j\,80\ \Omega$

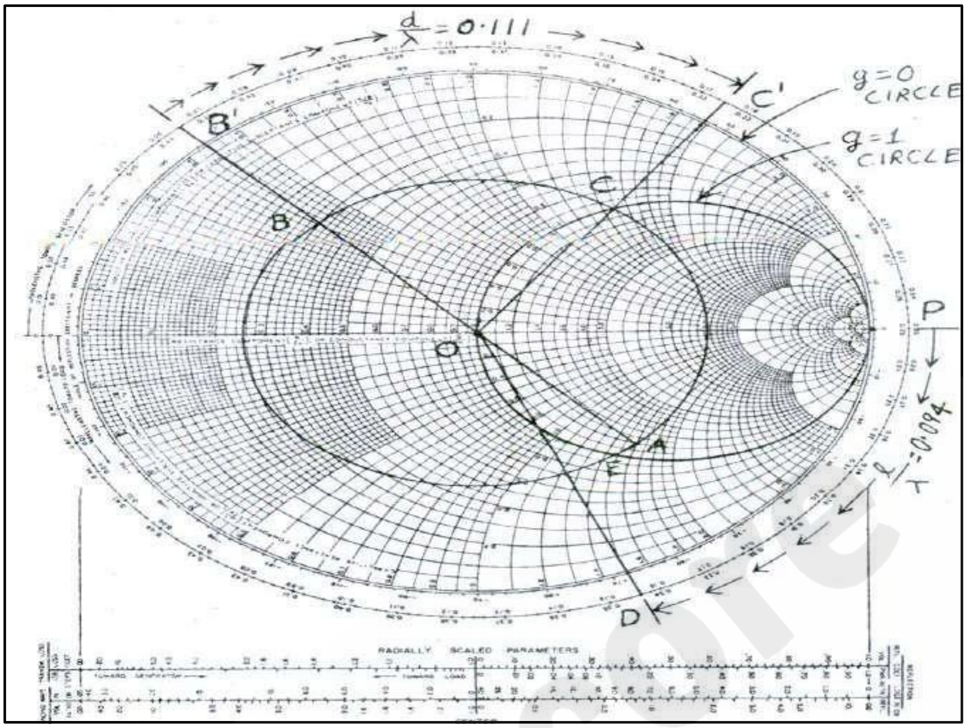
$Z_o = 50\ \Omega$

1. Normalized load impedance $\quad z_l = \dfrac{Z_l}{Z_o} = \dfrac{60 - j\,80}{50}$

$\therefore \quad z_l = 1.2 - j\,1.6$

This impedance is located in the Smith Chart as point A. Its normalized load admittance is found out which is diametrically opposite at point B. With O (centre of the Smith Chart) as centre and radius equal to OA, the constant-ρ-circle is drawn and point B is located, the value of VSWR = $\rho = 4$.

2. OB is joined and extended to B' on the periphery.
3. The circle $r = \dfrac{R}{R_0} = 1$ or $g = \dfrac{Y}{Y_0} = 1$ is the locus of all points for which the load impedance is equal to the characteristic impedance. The intersection of the ρ-circle with $g = 1$ circle as shown as point C in figure 1.37, will locate the stub.
4. In fact, there are two points C and E where constant-ρ-circle cuts $g = 1$ circle. But the point nearer to the load point B is always considered while going toward generator in clockwise direction. In this case it is point C.



5. OC is joined and extended to C' on the periphery. The number of wavelengths from B' to C' is measured. It is (0.176 – 0.065) λ = 0.111 λ = 0.111 × 1 m = 0.111 m = 11.1 cm

   ∴ **d = 11.1 cm**

6. The susceptance value of point C represents the susceptance of the line at the stub connection. It is equal to j1.5 indicating capacitive susceptance. This value of capacitive susceptance has to be neutralized by an inductive susceptance of – j1.5

7. The point corresponding to an admittance of 0 – j 1.5 is plotted on g = 0 circle and b = – 1.5 as shown in the chart indicated as point D.

8. The point P on the chart is the short-circuit point in admittance chart. The number of wavelengths from P, moving toward generator in clockwise direction, to E, is measured which gives the required inductive susceptance. From the chart

   Length of the stub = l = (0.344 – 0.25) λ

   ∴ l = (0.094) (1 m) = 0.094 m

   or **l = 9.4 cm**

Example : A line of $R_0 = 400\ \Omega$ is connected to a load of $200 + j\,300\ \Omega$, which is excited by a matched generator at 800 MHz. Find the location and length of a single stub nearest to the load to produce an impedance match.

Solution :

1. The normalized load impedance is given by

$$z_l = \frac{Z_l}{Z_o} = \frac{200 + j300}{400} = 0.5 + j\,0.75$$

This point is located on the chart as point A.

2. With 'O' as centre, radius OA, the constant-S circle is drawn and the diametrically opposite point 'B' is located as shown.

∴ Admittance of $z_l = y_l = 0.625 - j\,0.925$.

Point B on the chart is the load admittance point.

3. The constant-ρ circle cuts the g = 1 circle at two point C and D. Moving towards the generator in clockwise direction from point B, point C appears first. This point locates the position of the stub.

4. OC is joined and extended to C'. OB is also joined and extended to B'. The distance from B' to C' = (0.136 + 0.17) λ = 0.306 λ is the value of d

$$\therefore \quad d = 0.306\,\lambda = (0.306)\,(c/f)$$

$$= (0.306)\left(\frac{3\times10^{10}}{800\times10^{6}}\right)$$

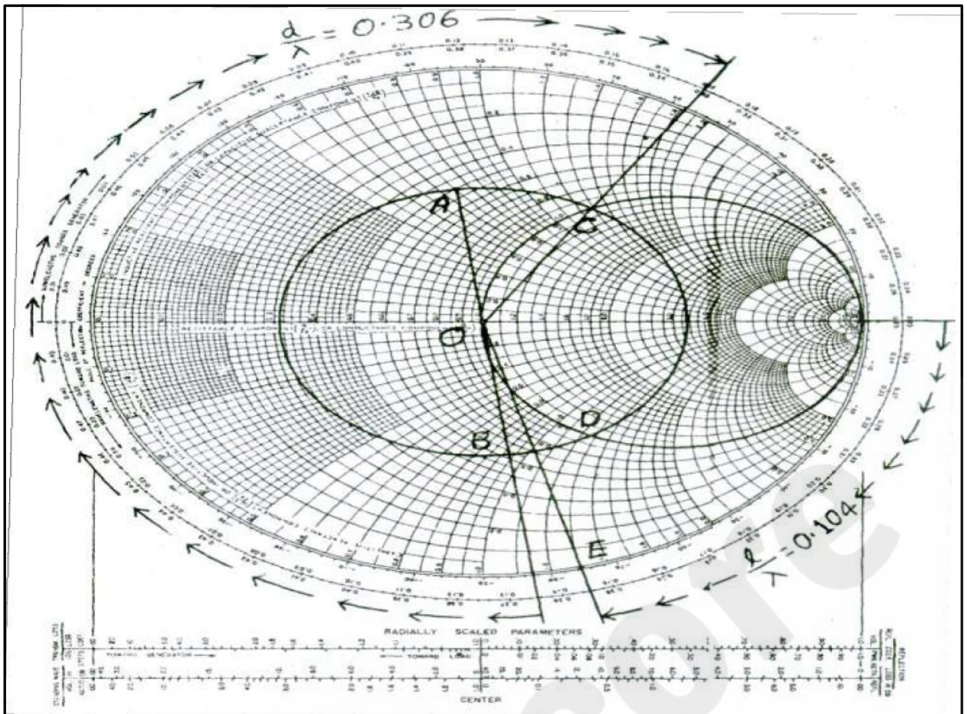
$$\therefore \quad \mathbf{d = 11.475\ cm}$$

5. The value of susceptance at point C is + j1.3 which is capacitive. The required inductive susceptance is marked as point E as shown in figure 1.38 on the chart. The point E determines the length of the short circuited stub. Since the piont P denotes the short circuit point, the distance from P to E in clockwise direction is the length of the stub 'l'.

∴ From the chart $l = (0.354 - 0.25)\,\lambda = 0.104\,\lambda$

$$= (0.104)\left(\frac{3\times10^{10}}{800\times10^{6}}\right)$$

$$\therefore \quad \mathbf{l = 3.9\ cm}$$



Example A 300 Ω line feeding an antenna has a standing wave ratio of 4 and the distance from load to the first voltage minimum is 28 cm. If the frequency is 150 MHz, design a single stub matching system to eliminate standing waves.

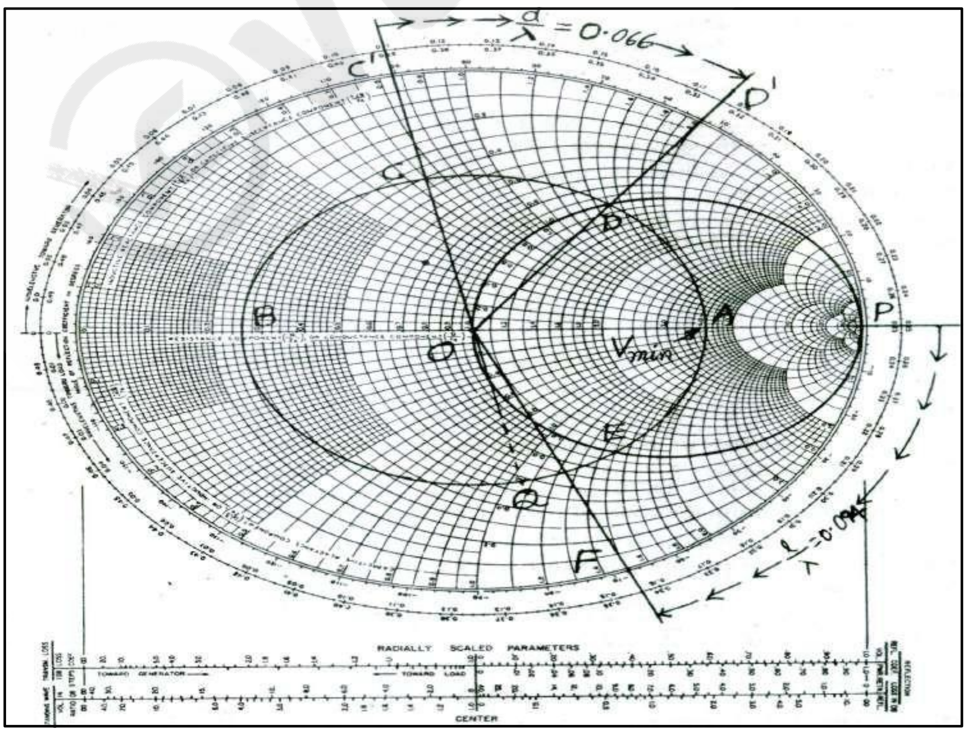
Solution :

1. The point A corresponding to VSWR = ρ = r = 4 is located on the right side of the horizontal axis as shown in figure
2. With 'O' as centre, radius OA, the constant ρ-circle is drawn cutting the left side of horizontal axis at B.
3. Since the Smith Chart is used as admittance chart, point A is the V_{min} point. OA is then joined and extended to P.
4. Given f = 150 MHz, dV_{min} = 28 cm

$$\therefore \qquad \lambda = \frac{c}{f} = \frac{3\times10^{10}}{150\times10^{6}}$$

$$= 200 \text{ cm}$$

$$\therefore \qquad \frac{dV_{min}}{\lambda} = \frac{28}{200} = 0.14$$



5. From point P, in order to locate the load amittance point, a distance equal to 0.14 λ is moved on the periphery, in anti-clockwise direction (wavelengths-toward-load direction), to get point C' as shown. OC' is joined to cut the constant ρ-circle at C. This is the load admitane point. From, the chart of figure 1.39, the normalized load admittance is

$$y_l = 0.4 + j\,0.75$$

The diametrically opposite point to C on the constant-ρ circle gives the normalized impedance at point Q as

$$z_l = 0.58 - j\,1.05$$

$\therefore$ The load impedance $Z_l = z_l R_o$

$$= (0.58 - j\,1.05)\,(300)$$

$$\therefore \quad \mathbf{Z_l = 174 - j\,315\ \Omega}$$

6. The constant-ρ circle cuts the g = 1 circle at two points D and E. While moving from load admittance point C in clockwise direction (toward generator), the point D is reached first. Therefore, the short circuited stub is to be connected at this point. OD is joined and extended to D'. The distance from C' to D' gives the location 'd' of the stub as

$$d = (0.176 - 0.11)\,\lambda = 0.066\,\lambda$$

$$d = (0.066)\,(200)$$

$$\therefore \quad \mathbf{d = 13.2\ cm}$$

7. The point P on the chart represents the short circuit point. The susceptance at point D is capacitive given by j 1.5. To neutralize this an inductive susceptance of −1.5 is required. The point F is located on the periphery of the chart corresponding to co-ordinates 0-j 1.5. The distance from P to F in clockwise direction gives the length 'l' of the short circuited stub as

$$l = (0.344 - 0.25)\,l$$

$$= (0.094)\,(200)$$

$$\therefore \quad \mathbf{l = 18.8\ cm}$$

A lossless line of characteristic impedance $R_0 = 50\ \Omega$ is to be matched to a load $\mathbf{Z}_\ell = 50/[2 + j(2 + \sqrt{3})]\ \Omega$ by means of a lossless short-circuited stub. The characteristic impedance of the stub is 100 Ω. Find the stub position (closest to the load) and length so that a match is obtained.

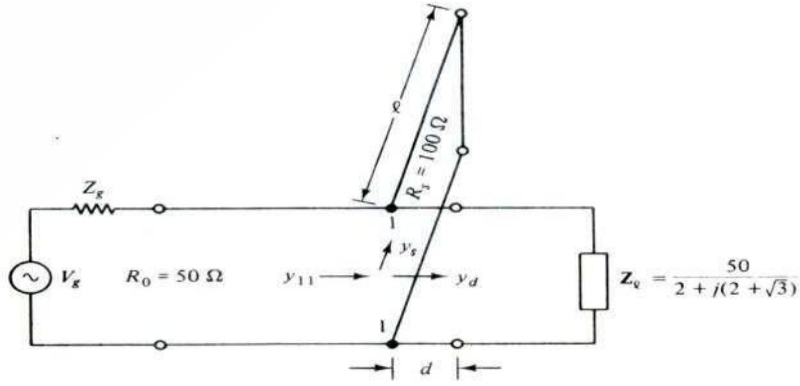


Figure Single-stub matching for Example

1. Compute the normalized load admittance and enter it on the Smith chart :

$$y_\ell = \frac{1}{z_\ell} = \frac{R_0}{\mathbf{Z}_\ell} = 2 + j(2 + \sqrt{3}) = 2 + j3.732$$

2. Draw a SWR circle through the point of y_ℓ so that the circle intersects the unity circle at the point y_d.

$$y_d = 1 - j2.6$$

Note that there are an infinite number of y_d. Take the one that allows the stub to be attached as closely as possible to the load.

3. Since the characteristic impedance of the stub is different from that of the line, the condition for impedance matching at the junction requires

$$\mathbf{Y}_{11} = \mathbf{Y}_d + \mathbf{Y}_s$$

where $\mathbf{Y}_s$ is the susceptance that the stub will contribute.

It is clear that the stub and the portion of the line from the load to the junction are in parallel, as seen by the main line extending to the generator. The admittances must be converted to normalized values for matching on the Smith

$$y_{11}\mathbf{Y}_0 = y_d\mathbf{Y}_0 + y_s\mathbf{Y}_{0s}$$

$$y_s = (y_{11} - y_d)\left(\frac{\mathbf{Y}_0}{\mathbf{Y}_{0s}}\right) = [1 - (1 - j2.6)]\frac{100}{50} = +j5.20$$

4. The distance between the load and the stub position can be calculated from the distance scale as

$$d = (0.302 - 0.215)\lambda = 0.087\lambda$$

5. Since the stub contributes a susceptance of $+j5.20$, enter $+j5.20$ on the chart and determine the required distance ℓ from the short-circuited end ($z = 0$, $y = \infty$), which corresponds to the right side of the real axis on the chart, by transversing the chart toward the generator until the point of $+j5.20$ is reached. Then

$$\ell = (0.50 - 0.031)\lambda = 0.469\lambda$$

When a line is matched at the junction, there will be no standing wave in the line from the stub to the generator.

6. If an inductive stub is required,

$$y'_d = 1 + j2.6$$

the susceptance of the stub will be

$$y'_s = -j5.2$$

7. The position of the stub from the load is

$$d' = [0.50 - (0.215 - 0.198)]\lambda = 0.483\lambda$$

and the length of the short-circuited stub is

$$\ell' = 0.031\lambda$$

Double-Stub Matching

Since single-stub matching is sometimes impractical because the stub cannot be placed physically in the ideal location, double-stub matching is needed.

consist of two short-circuited stubs connected in parallel with a fixed length

The length of the fixed section is usually one-eighth, three-eighths, or five-eighths of a wavelength

stub that is nearest the load is used to adjust the susceptance and is located at a fixed wavelength from the constant conductance unity circle ($g = 1$) on an appropriate constant-standing-wave-ratio circle. admittance of the line at the second stub as shown

$$y_{22} = y_{d2} \pm y_{s2} = 1$$

$$\mathbf{Y}_{22} = \mathbf{Y}_{d2} \pm \mathbf{Y}_{s2} = \mathbf{Y}_0$$

In these two equations it is assumed that the stubs and the main line have the same characteristic admittance. If the positions and lengths of the stubs are chosen properly, there will be no standing wave on the line to the left of the second stub measured from the load.

Example

The terminating impedance $\mathbf{Z}_\ell$ is $100 + j100\ \Omega$, and the characteristic impedance $\mathbf{Z}_0$ of the line and stub is $50\ \Omega$. The first stub is placed at 0.40λ away from the load. The spacing between the two stubs is $\frac{3}{8}\lambda$. Determine the length of the short-circuited stubs when the match is achieved. What terminations are forbidden for matching the line by the double-stub device?

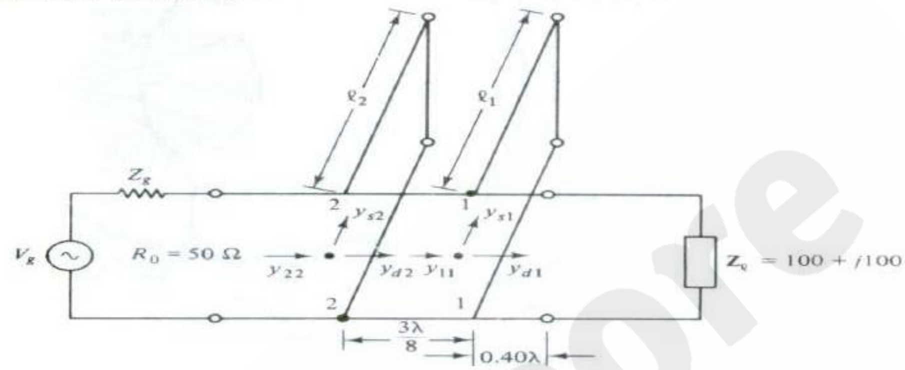


Figure Double-stub matching for Example

1. Compute the normalized load impedance z_ℓ and enter it on the chart as shown in

$$z_\ell = \frac{100 + j100}{50} = 2 + j2$$

2. Plot a SWR ρ circle and read the normalized load admittance 180° out of phase with z_ℓ on the SWR circle:

$$y_\ell = 0.25 - j0.25$$

3. Draw the spacing circle of $\frac{3}{8}\ell$ by rotating the constant-conductance unity circle

   ($g = 1$) through a phase angle of $2\beta d = 2\beta\frac{3}{8}\lambda = \frac{3}{2}\pi$ toward the load. Now y_{11} must be on this spacing circle, since y_{d2} will be on the $g = 1$ circle (y_{11} and y_{d2} are $\frac{3}{8}\lambda$ apart).

4. Move y_ℓ for a distance of 0.40λ from 0.458 to 0.358 along the SWR ρ circle toward the generator and read y_{d1} on the chart:

$$y_{d1} = 0.55 - j1.08$$

5. There are two possible solutions for y_{11}. They can be found by carrying y_{d1} along the constant-conductance ($g = 0.55$) circle that intersects the spacing circle at two points:

$$y_{11} = 0.55 - j0.11$$
$$y'_{11} = 0.55 - j1.88$$

6. At the junction 1-1,

$$y_{11} = y_{d1} + y_{s1}$$

Then

$$y_{s1} = y_{11} - y_{d1} = (0.55 - j0.11) - (0.55 - j1.08) = +j0.97$$

Similarly,

$$y'_{s1} = -j.080$$

7. The lengths of stub 1 are found as

$$\ell_1 = (0.25 + 0.123)\lambda = 0.373\lambda$$

$$\ell'_1 = (0.25 - 0.107)\lambda = 0.143\lambda$$

8. The $\frac{3}{8}\lambda$ section of line transforms y_{11} to y_{d2} and y_{11} to y'_{d2} along their constant standing-wave circles, respectively. That is,

$$y_{d2} = 1 - j0.61$$

$$y'_{d2} = 1 + j2.60$$

9. Then stub 2 must contribute

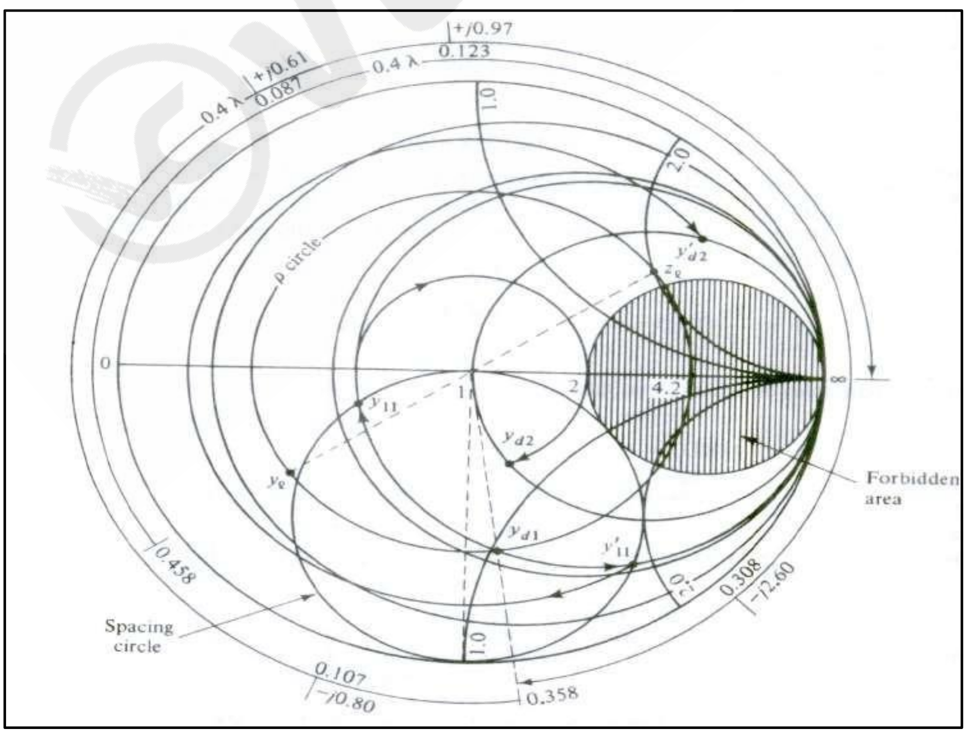
$$y_{s2} = +j0.61$$

$$y'_{s2} = -j2.60$$

10. The lengths of stub 2 are found as

$$\ell_2 = (0.25 + 0.087)\lambda = 0.337\lambda$$

$$\ell'_2 = (0.308 - 0.25)\lambda = 0.058\lambda$$



11. It can be seen from Fig. that a normalized admittance y_ℓ located inside the hatched area cannot be brought to lie on the locus of y_{11} or y'_{11} for a possible match by the parallel connection of any short-circuited stub because the spacing circle and $g = 2$ circle are mutually tangent. Thus the area of a $g = 2$ circle is called the *forbidden region* of the normalized load admittance for possible match.

Limitations of Double stub matching

Normally the solution of a double-stub-matching problem can be worked out backward from the load toward the generator, since the load is known and the distance of the first stub away from the load can be arbitrarily chosen. In quite a few practical matching problems, however, some stubs have a different $\mathbf{Z}_0$ from that of the line, the length of a stub may be fixed, and so on. So it is hard to describe a definite procedure for solving the double-matching problems.

COAXIAL CONNECTORS AND ADAPTORS

Interconnection between co-axial cables and microwave components is achieved with the help of shielded standard connectors. The average circumference of the co-axial cable, for high frequency operation must be limited to about one wavelength. This requirement is necessary to reduce propagation at higher modes and also to eliminate erratic reflection coefficients (VSWR close to unity), signal distortion and power losses.

(a) APC 3.5 (Amphenol Precision Connector – 3.5 mm)

HP (Hewlett - Packard) originally developed this connector, but it is now being manufactured by Amphenol. This connector can operate upto a frequency of 34 GHz and has a very low voltage standing wave ratio (VSWR). This connector provides repeatable connections and has 50 Ω characteristic impedance. The male or female of SMA connector can be connected to the opposite type of APC 3.5 connector.

APC-3.5 Male

BNC Female

(a) APC-3.5

(b) APC – 7 (Amphenol Precision connector – 7 mm)

This connector was also developed by HP but improved later by Amphenol. This connector provides repeatable connections and used for very accurate 50 ohm measurement applications. This connector provides a coupling mechanism without male or female distinction (i.e., sexless) and its VSWR is extremely low, less than 1.02 in the frequency range upto 18 GHz.

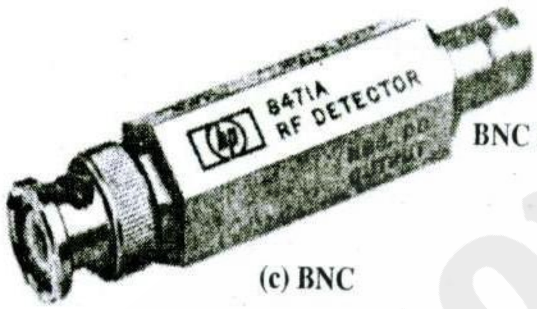
New coupling nut

Old coupling nut

(b) APC-7

(c) BNC (Bayonet Navy Connector)

This connector was developed during World War II and used for military applications. It has characteristic impedance 50 to 75 Ω and is connected to flexible co-axial cable with diameters upto 0.635 cm. It is extensively used in almost all electronic measuring equipments upto 1 GHz of frequencies. BNC can be used even upto 4 GHz frequency and beyond that it starts radiating electromagnetic energy.



BNC Female

(c) BNC

BNC Male

(e) SMC (Sub-Miniature C-type)

This connector is manufactured by Sealectro Corporation and its size is smaller than SMA connector. It is a 50 Ω connector that connects flexible cables upto a diameter of 0.317 cm and used upto a frequency of 7 GHz.

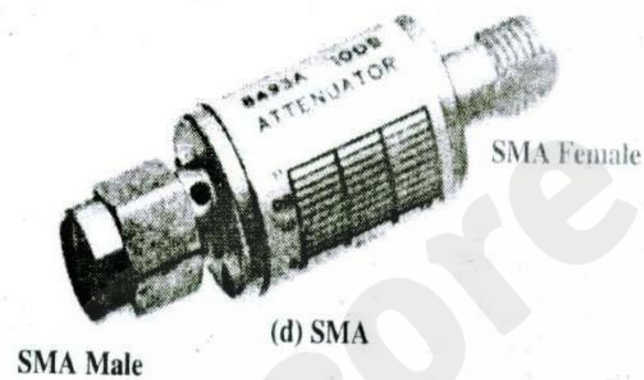
SMC Female

SMC Male

(e) SMC

(d) SMA (Sub-Miniature A type)

This type of connector is also called OSM connector as it is manufactured by Omni-Spectra Inc. SMA connectors are used on components for microwave systems. The disadvantage with these connectors is that at high frequencies greater than 24 GHz, it introduces higher order modes and hence not used above 24 GHz.



(d) SMA

(f) TNC (Threaded Navy Connector)

This connector is an improved version of BNC in the sense that it is threaded. This threading prevents radiation at high frequencies so that it can be used upto about 12 GHz frequency.

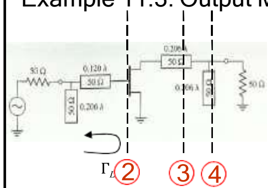
(g) Type-N (Type-Navy) connector

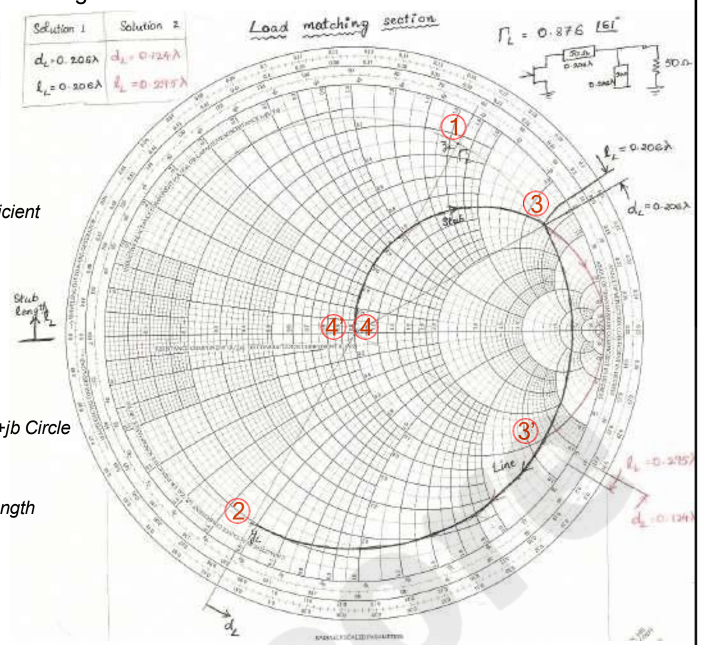
It is a 50 Ω or 75 Ω connector having a very low value of VSWR less than 1.02. This was developed during World War II and extensively used as a microwave measurement connector upto a frequency of 18 GHz.

(g) Type N

TNC Plug

Example 11.3: Output Matching Network





Step 1: Plot the reflection coefficient

$\Gamma_L = 0.876\angle 61°$

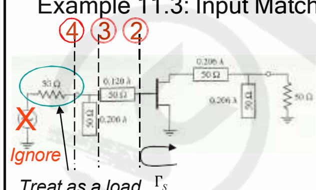
Shunt Stub Problem: Admittance Calculation

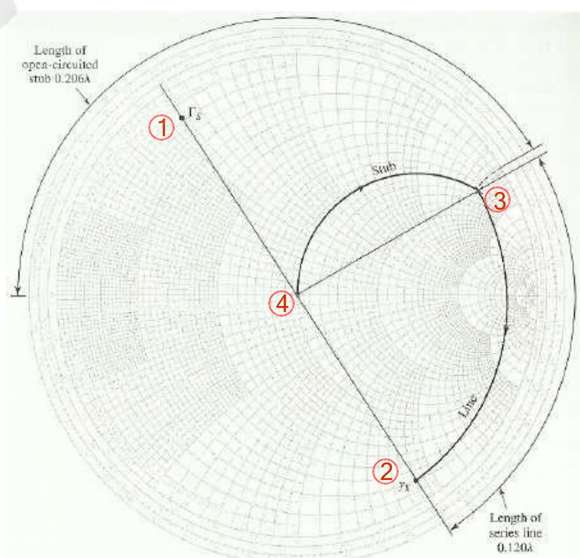
Step 2: Find the Admittance y_L

Step 3:Intersection points on 1+jb Circle

Step 4:Open-Circuited Stub Length

Example 11.3: Input Matching Network





Step 1: Plot the reflection coefficient

$\Gamma_s = 0.872\angle 123°$

Shunt Stub Problem: Admittance Calculation

Step 2: Find the Admittance y_s

Step 3:Intersection points on 1+jb Circle

$y_1 = 1 + j3.5$

$d_1 = 0.120\lambda$

Step 4:Open-Circuited Stub Length

$l_1 = 0.206\lambda$

and is called the *propagation constant*. α is called the *attenuation constant* and β, the *phase constant*. At very high frequencies when the losses in the line are small, Eq. 3.7 gives

$$\alpha = 1/2\left(R\sqrt{C/L} + G\sqrt{L/C}\right) \quad (3.8)$$

$$\beta = \omega\sqrt{LC} \quad (3.9)$$

The solutions of Eqs 3.5 and 3.6 give the voltage and current at any point z and are given by

$$V(z) = V_1 e^{-\gamma z} + V_2 e^{+\gamma z} \quad (3.10)$$

$$I(z) = I_1 e^{-\gamma z} + I_2 e^{+\gamma z} \quad (3.11)$$

The first term in the right-hand side represents the *incident wave* and the second term represents the *reflected wave*. The values of V_1, V_2, I_1, I_2, are to be determined.

Here, V_1 = generator-end voltage amplitude
I_1 = generator-end current amplitude
V_2 = load-end voltage amplitude
I_2 = load-end current amplitude (3.12)

3.3 CHARACTERISTIC AND INPUT IMPEDANCES

When the reflected wave in the line is zero, the ratio $V(z)/I(z)$ is called the *characteristic impedance* of the line and is defined by

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (3.13)$$

When $R << \omega L$ and $G << \omega C$ for low loss lines and also for microwave frequencies, $Z_0 = \sqrt{L/C}$. The impedance of the line z looking towards the load is called the *input impedance* of the line.

$$Z_{in} = \frac{V(z)}{I(z)} = \frac{V_1 e^{-\gamma z} + V_2 e^{\gamma z}}{(V_1 e^{-\gamma z} - V_2 e^{\gamma z})/Z_0} \quad (3.14)$$

When the line of length l is terminated by a load Z_L, as shown in Fig. 3.2, the input impedance becomes

$$Z_{in} = \frac{(Z_L \cosh \gamma l + Z_0 \sinh \gamma l)}{(Z_0 \cosh \gamma l + Z_L \sinh \gamma l)} Z_0 \quad (3.15)$$

(i) If the line is short-circuited, $Z_L = 0$ and $V_L = 0$, therefore,

$$Z_{inSC} = Z_0 \tanh \gamma l \quad (3.16)$$

(ii) For an open-circuited line, $Z_L = \infty$, $I_L = 0$, and

$$Z_{inOC} = Z_0 \coth \gamma l \quad (3.17)$$

(iii) For a lossless line, $\alpha = 0$, $\gamma = j\beta$ and Eqs 3.15–3.17 reduce to

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \quad (3.18)$$

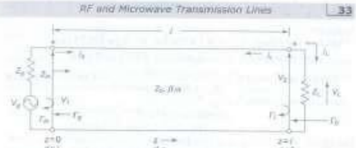


Fig. 3.2 A uniform transmission line of length l

$$Z_{inSC} = j Z_0 \tan \beta l \quad (3.19)$$

$$Z_{inOC} = -j Z_0 \cot \beta l \quad (3.20)$$

Example 3.1 A telephone line has R = 6 ohms/km, L = 2.2 mh/km, C = 0.005 μf/km, and G = 0.05 μmho/km. Determine Z_0, α, β at 1 kHz. If the line length is 100 km, determine the attenuation and phase shift of the signal. Calculate the phase velocity of the signal.

Solution

$$\omega = 2\pi * 1000 = 6280 \text{ rad/sec}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{6 + j6280 \times 2.2 \times 10^{-3}}{0.05 \times 10^{-6} + j6280 \times 0.005 \times 10^{-6}}}$$

$$= 678.23 - j140.35 \text{ ohm}$$

$$\gamma = \sqrt{(R + j\omega L)(R + j\omega C)}$$

$$= \sqrt{\left(6 + j6280 \times 2.2 \times 10^{-3}\right)\left(0.05 \times 10^{-6} + j6280 \times 0.005 \times 10^{-6}\right)}$$

$$= 0.0045 + j0.0213 = \alpha + j\beta$$

Therefore, α = 0.0045 Np/km and
β = 0.0213 rad/km

For a 100 km length,

attenuation = 0.45 Np = 8.686 × 0.45 = 3.91 dB

phase shift = 2.13 rad

$$\text{Phase velocity} = \frac{\omega}{\beta} = \frac{6280 \text{ (rad/s)}}{0.0213 \text{ (rad/km)}} = 294.84 \times 10^3 \text{ km/s}$$

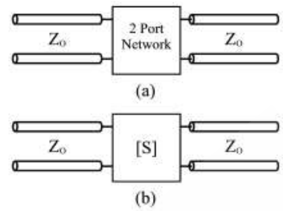
Example 3.2 In Example 3.1, if a signal of 1 V supplies power to the line terminated in its characteristic impedance, find the power delivered at the receiving end.

Scattering Parameters (S-Parameters)

Consider a circuit or device inserted into a T-Line as shown in the Figure. We can refer to this circuit or device as a two-port network.

The behavior of the network can be completely characterized by its scattering parameters (S-parameters), or its scattering matrix, [S].

Scattering matrices are frequently used to characterize multiport networks, especially at high frequencies. They are used to represent microwave devices, such as amplifiers and circulators, and are easily related to concepts of gain, loss and reflection.

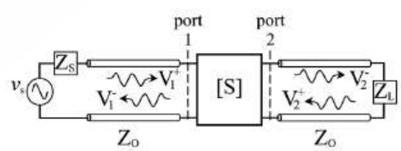


Scattering matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

Scattering Parameters (S-Parameters)

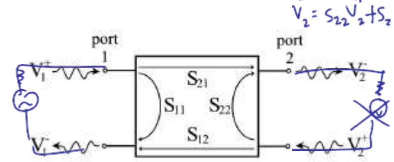
The scattering parameters represent ratios of voltage waves entering and leaving the ports (If the same characteristic impedance, jjjkl;j, at all ports in the network are the same).



$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+.$$

$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+.$$

$V_2^- = S_{22}V_2^+ + S_{21}V_1^+$



In matrix form this is written

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}, \quad [V]^- = [S][V]^+.$$

o/p @ ① $V_1^- = S_{11}V_1^+ + S_{12}V_2^+$

Reflected signal

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0}$$: Reflection coefficient $0 \le |S_{11}| \le 1$

input signal

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{V_1^+=0}$$ Transmission coefficient from Port ② to ①

i/p @ ②

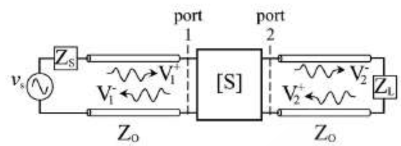
$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$: Transmission coefficient from Port ① to ②

$$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+=0}$$: Reflection coefficient at Port ②

Scattering Parameters (S-Parameters)

Properties:

1) Reciprocity



The two-port network is reciprocal if the transmission characteristics are the same in both directions (i.e. $S_{21} = S_{12}$).

It is a property of passive circuits (circuits with no active devices or ferrites) that they form reciprocal networks.

A network is reciprocal if it is equal to its transpose. Stated mathematically, for a reciprocal network

$$[S] = [S]^t,$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^t = \begin{bmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{bmatrix}.$$

Condition for Reciprocity: $S_{12} = S_{21}$

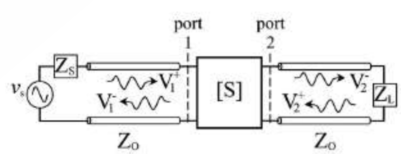
By inspection :

If the network is symmetrical $\Rightarrow$ Reciprocal

Scattering Parameters (S-Parameters)

Properties:

2) Lossless Networks



A lossless network does not contain any resistive elements and there is no attenuation of the signal. No real power is delivered to the network. Consequently, for any passive lossless network, what goes in must come out!

In terms of scattering parameters, a network is lossless if

$$[S]^t [S]^* = [U],$$ where $[U]$ is the unitary matrix $[U] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$

For a 2-port network, the product of the transpose matrix and the complex conjugate matrix yields

$$[S]^t [S]^* = \begin{bmatrix} \left(|S_{11}|^2 + |S_{21}|^2\right) & \left(S_{11}S_{12}^* + S_{21}S_{22}^*\right) \\ \left(S_{12}S_{11}^* + S_{22}S_{21}^*\right) & \left(|S_{12}|^2 + |S_{22}|^2\right) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

If the network is reciprocal and lossless

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

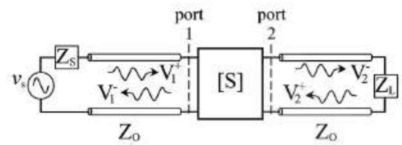
$$S_{11}S_{12}^* + S_{21}S_{22}^* = 0$$

Scattering Parameters (S-Parameters)

Return Loss and Insertion Loss

Two port networks are commonly described by their return loss and insertion loss. The return loss, RL, at the ith port of a network is defined as

$$\boxed{RL_i = -20\log\left|\frac{V_i^-}{V_i^+}\right| = -20\log\left|\Gamma_i\right|}.$$

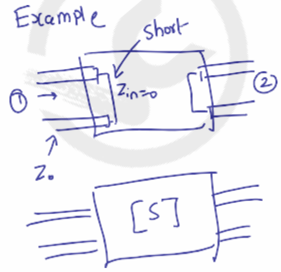


The insertion loss, IL, defines how much of a signal is lost as it goes from a jth port to an ith port. In other words, it is a measure of the attenuation resulting from insertion of a network between a source and a load.

$$\boxed{IL_{ij} = -20\log\left|\frac{V_i^-}{V_j^+}\right|}.$$

Scattering Parameters (S-Parameters)

Example



$S_{12} = S_{21} = 0$ (Reciprocal) ✓

$$S_{11} = \Gamma_{in,1} = \frac{Z_{in}-Z_o}{Z_{in}+Z_o} = \frac{0-Z_o}{0+Z_o} = -1$$

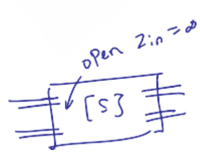
$$S_{22} = \Gamma_{in,2} = -1 = 1\angle 180^\circ$$

$$[S] = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Lossless condition check ✓

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

$$|-1|^2 \quad 0 \quad = 1$$

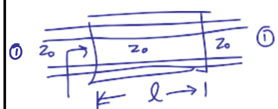


$$[S] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$S_{11} = \Gamma_{in,1} = \frac{Z_{in}-Z_o}{Z_{in}+Z_o} = \frac{1-\frac{Z_o}{Z_{in}}}{1+\frac{Z_o}{Z_{in}}} = \frac{1-\frac{Z_o}{\infty}}{1+\frac{Z_o}{\infty}} = 1\angle 0^\circ$$

Scattering Parameters (S-Parameters)

Thruline



$Z_{in} = Z_0$ $\quad \theta = \beta\ell$

(rad/m) (m)

Reflection Coefficient

$$S_{11} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = 0, \quad S_{22} = 0$$

Transmission Coefficients

$$S_{21} = |S_{21}| \angle S_{21} = 1 \angle -\theta = 1\, e^{-j\theta} = e^{-j\beta\ell},$$

$$S_{21} = S_{12}$$

$$[S] = \begin{bmatrix} 0 & e^{-j\beta\ell} \\ e^{-j\beta\ell} & 0 \end{bmatrix}$$

Example

$$[S] = \begin{bmatrix} +\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & +\frac{1}{\sqrt{2}} \end{bmatrix}$$

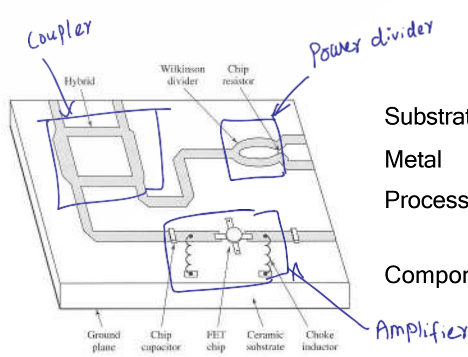
Microwave Integrated Circuits

Microwave Integrated Circuits (MIC):

Traces: transmission lines,

Passive components: resistors, capacitors, and inductors

Active devices: diodes and transistors.



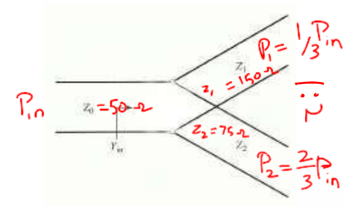
Substrate	Teflon fiber, alumina, quartz etc.
Metal	Copper, Gold etc.
Process	Conventional printed circuit (Photolithography and etching)
Components	Soldering and wire bonding

Power Divider

PD 1.1

A T-junction power divider consists of one input port and two output ports.

Lossless T-junction Power Divider



Input Power: $P_{in} = \frac{1}{2}\frac{V_o^2}{Z_o}$

Output Power: $P_1 = \frac{1}{2}\frac{V_o^2}{Z_1}$ $\quad P_2 = \frac{1}{2}\frac{V_o^2}{Z_2}$

$P_1 = \frac{1}{3}P_{in}$

$P_2 = \frac{2}{3}P_{in}$

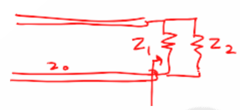
Design Example

1:2 Power Divider $Z_o = 50\,\Omega$

Solution

$Z_1 = 3Z_o = 150\,\Omega$

$Z_2 = \frac{3}{2}Z_o = 75\,\Omega$



Input Impedance

Input Port: $Z_{in} = Z_1 \parallel Z_2 = 50\,\Omega$

Output Port 1: $Z_{in1} = Z_o \parallel Z_2 = 30\,\Omega$

Output Port 2: $Z_{in2} = Z_o \parallel Z_1 = 37.5\,\Omega$

Reflection Coefficient

$\Gamma = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = 0$

Input Port is matched

$\Gamma_1 = \frac{Z_{in1} - Z_1}{Z_{in1} + Z_1} = -0.666$

$\Gamma_2 = \frac{Z_{in2} - Z_2}{Z_{in2} + Z_2} = -0.333$

Output Ports are not matched

$Z_{in} = Z_1 \parallel Z_2 = 50\,\Omega$

$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{50-50}{50+50} = 0$

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

$S_{11} = 0$, $S_{21} =$

COUPLERS

Couplers

A reciprocal, lossless, matched four-port network behaves as a directional coupler

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

Symmetric Coupler

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

Antisymmetric Coupler

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

For a lossless network: $\alpha^2 + \beta^2 = 1$

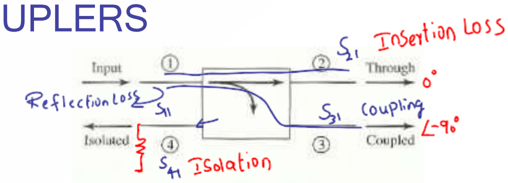
Insertion Loss: $IL = -20\log(|S_{21}|)$.

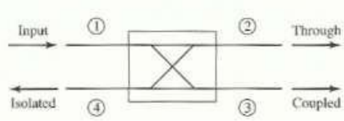
Coupling coefficient: $C = -20\log(|S_{31}|)$.

Isolation: $I = -20\log(|S_{41}|)$,

Directivity: $D = 20\log\left(\frac{|S_{31}|}{|S_{41}|}\right)$,

$D = I - C$ (dB)





A coupler will transmit half or more of its power from its input (port 1) to its through port (port 2).

A portion of the power will be drawn off to the coupled port (port 3), and ideally none will go to the isolated port (port 4).

If the isolated port is internally terminated in a matched load, the coupler is most often referred to as a directional coupler.

COUPLERS

Design Example

Example 10.10: Suppose an antisymmetrical coupler has the following characteristics:

Given

C = 10.0 dB
D = 15.0 dB
IL = 2.00 dB
VSWR = 1.30

Voltage standwing wave ratio

VSWR = 1.30 $\Rightarrow$ $|S_{11}| = \dfrac{VSWR - 1}{VSWR + 1} = 0.130.$ $|S_{11}| = |S_{22}| = |S_{33}| = |S_{44}|$

Insertion Loss: $IL = -20\log\left(|S_{21}|\right).$ $\Rightarrow$ $|S_{21}| = 10^{-2/20} = 0.794.$

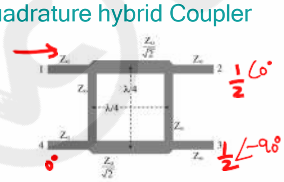
Coupling coefficient: $C = -20\log\left(|S_{31}|\right).$ $\Rightarrow$ $|S_{31}| = 10^{-10/20} = 0.316.$

$I = D + C = 25dB,$ $\Rightarrow$ $|S_{41}| = 10^{-25/20} = 0.056.$

$$[S] = \begin{bmatrix} 0.130 & 0.794 & 0.316 & 0.056 \\ 0.794 & 0.130 & 0.056 & -0.316 \\ 0.316 & 0.056 & 0.130 & 0.794 \\ 0.056 & -0.316 & 0.794 & 0.130 \end{bmatrix}$$

COUPLERS

Quadrature hybrid Coupler

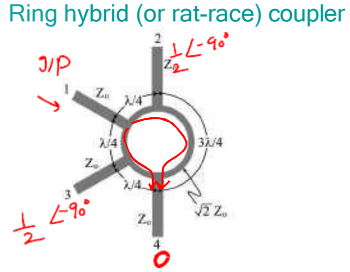


$$[S] = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix},$$

The quadrature hybrid (or branch-line hybrid) is a 3 dB coupler. The quadrature term comes from the 90 deg phase difference between the outputs at ports 2 and 3.

The coupling and insertion loss are both equal to 3 dB.

Ring hybrid (or rat-race) coupler



$$[S] = \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}.$$

A microwave signal fed at port 1 will split evenly in both directions, giving identical signals out of ports 2 and 3. But the split signals are 180 deg out of phase at port 4, the isolated port, so they cancel and no power exits port 4.

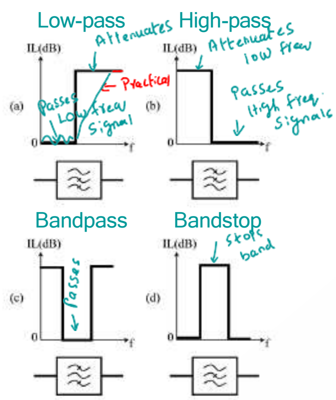
The insertion loss and coupling are both equal to 3 dB. Not only can the ring hybrid split power to two ports, but it can add and subtract a pair of signals.

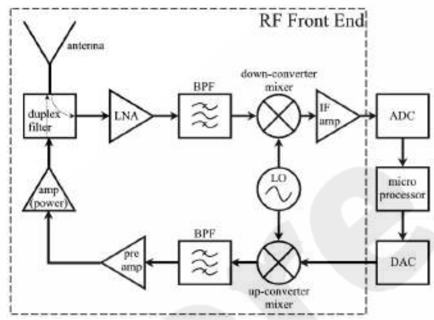
Filters

Filters are two-port networks used to attenuate undesirable frequencies.

Microwave filters are commonly used in transceiver circuits.

The four basic filter types are low-pass, high-pass, bandpass and bandstop.





Filters

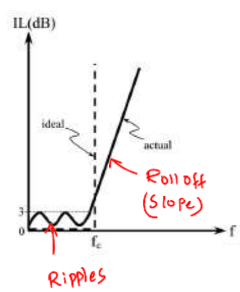
Low-pass Filters

A low-pass filter is characterized by the insertion loss versus frequency plot in Figure. Notice that there may be ripple in the passband (the frequency range desired to pass through the filter), and a roll off in transmission above the cutoff or corner frequency, fc.

Simple filters (like series inductors or shunt capacitors) feature 20 dB/decade roll off. Sharper roll off is available using active filters or multisection filters.

Active filters employ operational amplifiers that are limited by performance to the lower RF frequencies. Multisection filters use passive components (inductors and capacitors), to achieve filtering.

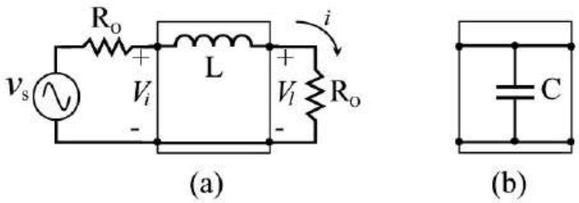
The two primary types are the Butterworth and the Chebyshev. A Butterworth filter has no ripple in the passband, while the Chebyshev filter features sharper roll off.



Lumped Element Filters

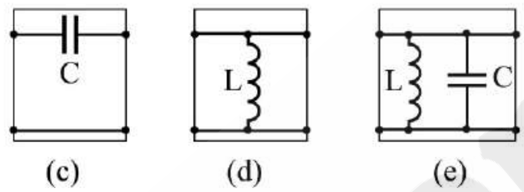
Some simple lumped element filter circuits are shown below.

Low-pass Filters



High-pass Filters

Band-pass Filters

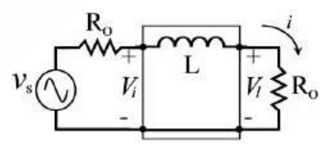


Lumped Element Filters

Low-pass Filter Example

Power delivered to the load

$$P_L = \frac{v_l^2}{R_o}, \qquad v_l = \frac{R_o}{2R_o + jxL} v_s .$$



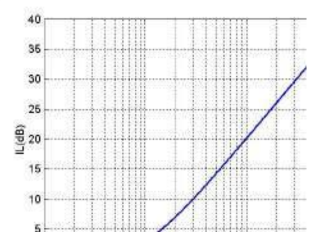
Maximum available Power:

$$P_A = \frac{v_L^2}{R_o} = \frac{\left(\frac{v_s}{2}\right)^2}{R_o} = \frac{v_s^2}{4R_o}.$$

Insertion Loss

$$IL = 10\log\left(\frac{P_L}{P_A}\right)$$

$$IL = 20\log\left(\left|1 + \frac{jxL}{2R_o}\right|\right).$$



The 3 dB cutoff frequency, also termed the corner frequency, occurs where insertion loss reaches 3 dB.

$$20\log\left(\left|1 + \frac{jxL}{2R_o}\right|\right) = 3 \;\Rightarrow\; \left|1 + \frac{jxL}{2R_o}\right| = 10^{\frac{3}{20}} = \sqrt{2} \;\Rightarrow\; \frac{xL}{2R_o} = 1, \;\Rightarrow\; f_c = \frac{R_o}{\pi L}.$$

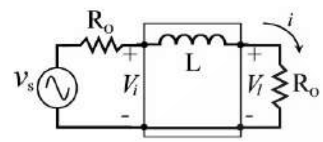
Lumped Element Filters

Low-pass Filter Example

Example 10.12: Let us design a low-pass filter for a 50.0 Ω system using a series inductor. The 3 dB cutoff frequency is specified as 1.00 GHz.

The 3 dB cutoff frequency is given by

$$f_c = \frac{R_o}{\pi L}.$$



Therefore, the required inductance value is

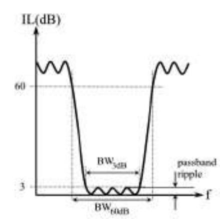
$$L = \frac{R_o}{\pi f} = \frac{50\Omega}{\pi\left(1x10^9\, 1/s\right)}\left(\frac{H}{\Omega s}\right) = 15.9\ nH.$$

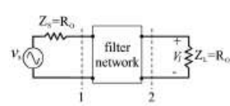
Filters

Band-pass Filters

The insertion loss for a bandpass filter is shown in Figure. Here the passband ripple is desired small. The sharpness of the filter response is given by the shape factor, SF, related to the filter bandwidth at 3dB and 60dB by

$$SF = \frac{BW_{60dB}}{BW_{3dB}}.$$





A filter's insertion loss relates the power delivered to the load without the filter in place (PL) to the power delivered with the filter in place (PLf):

$$IL = 10\log\left(\frac{P_L}{P_{Lf}}\right).$$

Amplifier Design

Microwave amplifiers are a common and crucial component of wireless transceivers. They are constructed around a microwave transistor from the field effect transistor (FET) or bipolar junction transistor (BJT) families.

A general microwave amplifier can be represented by the 2-port S-matrix network between a pair of impedance-matching networks as shown in the Figure below. The matching networks are necessary to minimize reflections seen by the source and to maximize power to the output.