Diploma in

Computer Science

Problem definition

Objectives

- Unpack formulating problems
- Recognise types of computing problems and their characteristics
- Recognise the relationship between Maths and Computer Science
- Explore real-world computation problems

Formulating problems

Problem formulation

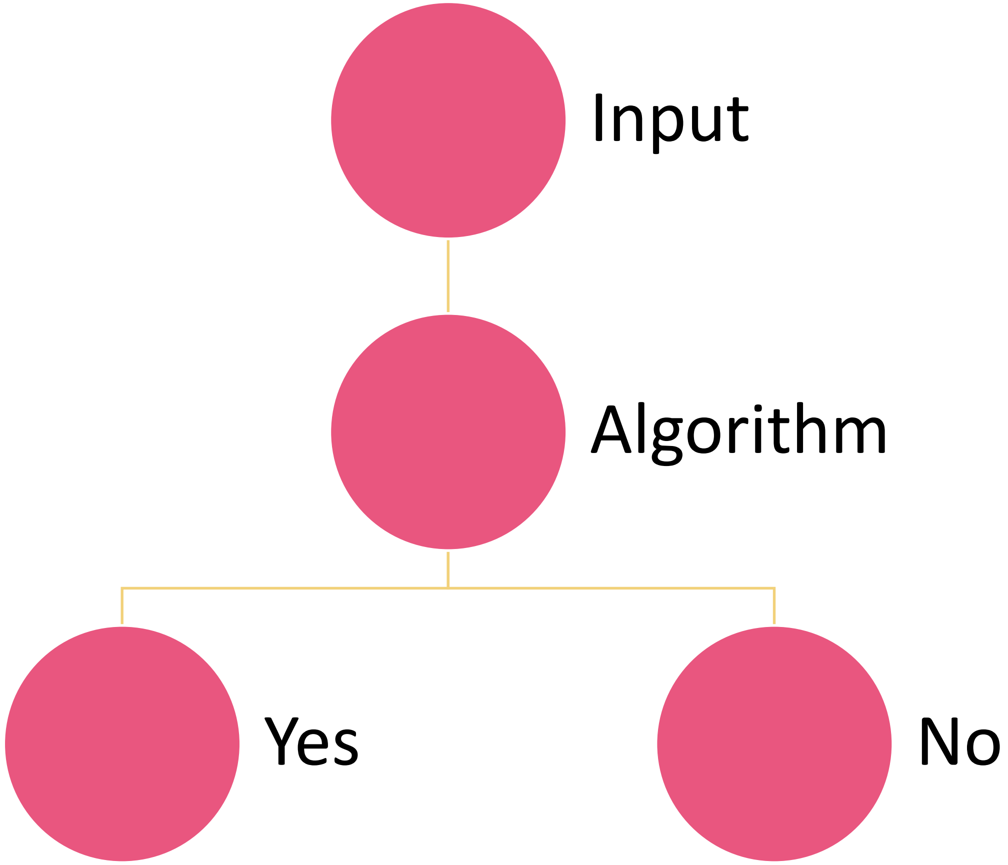
- Why are you writing the problem?
- Make sure problem addresses what it is supposed to
- View problem in a way that makes it easy to solve

Problem definition

- Many different types of problems
- Simple and complex problems
- Anything that you try to achieve computationally is a ‘problem’ or ‘set of related tasks’

Key steps in problem definition

- A problem should be described explicitly.
- A problem should be well stated.
- A computing problem is a collection of questions for the computer to solve.



Computing problems

Main types of computing problems

- Decision
- Search
- Counting
- Optimisation
- Function

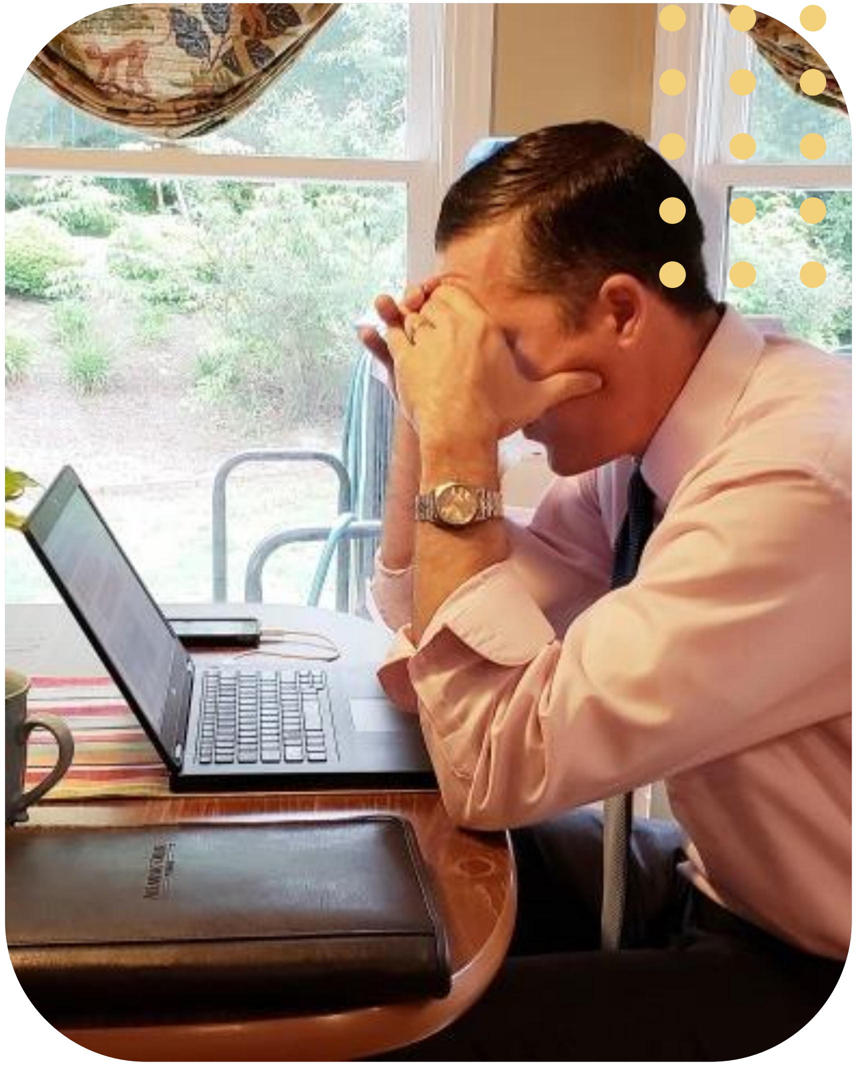
Decision problem

- Answer is either YES or NO
- Seeks a solution with a certain characteristic
- Provides a solution for every given input

Decision problem example

Check if sum of two numbers is equal to a predetermined value:

- Initialise z = 1
- Ask for x and y
- Add x and y
- Check if x + y = z
- If it is not, ask user for new input
- If it is equal, output z



Decision problem example

- Complex problems modelled using Turing machine
- Undecidable problem – never provides a solution

Search problem

- Algorithmic (step-by-step) way of verifying the answer
- Indicates whether search item is present or not
- Solves problem if at least one corresponding structure exists, and one of the instances of this structure is the output
- Defined by a set of states, a start state, a goal state, a Boolean function and a successor function

Search problem example

- Find the number 3 from the list of numbers: 7, 8, 5, 9, 2, 6, 3, 10, 4
- Simple search: read numbers one by one until get to value we want

Counting problem

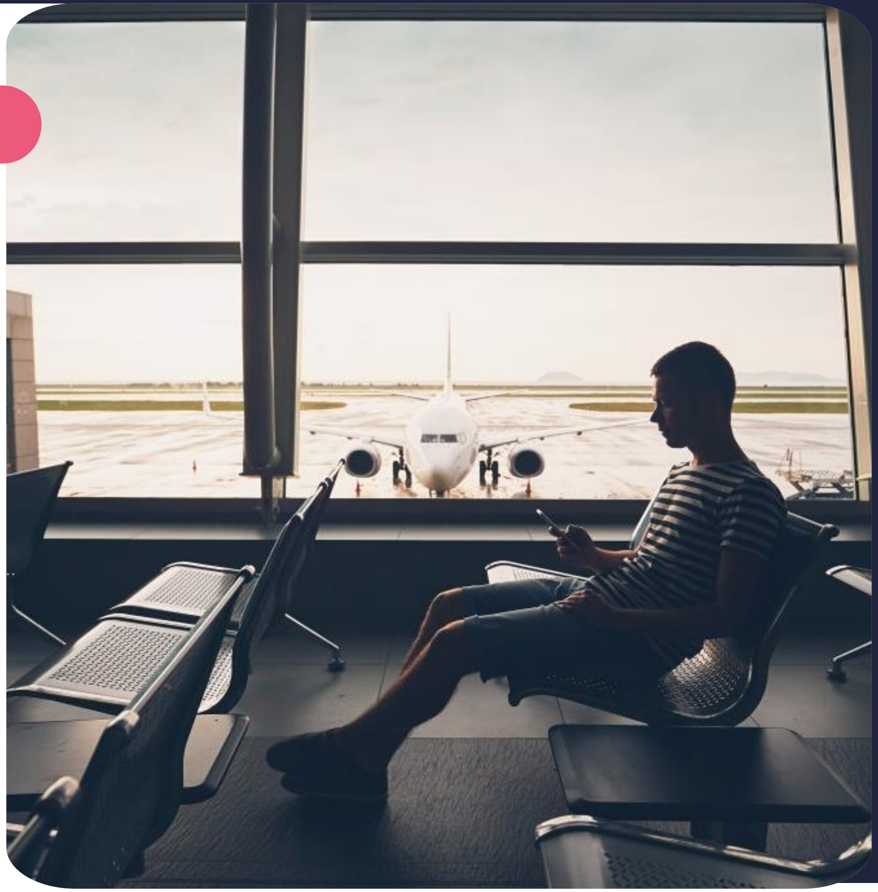
- Asks for the number of solutions of a given instance
- An object to check for certain characteristics
- Example: Edmond’s algorithm

Optimisation problem

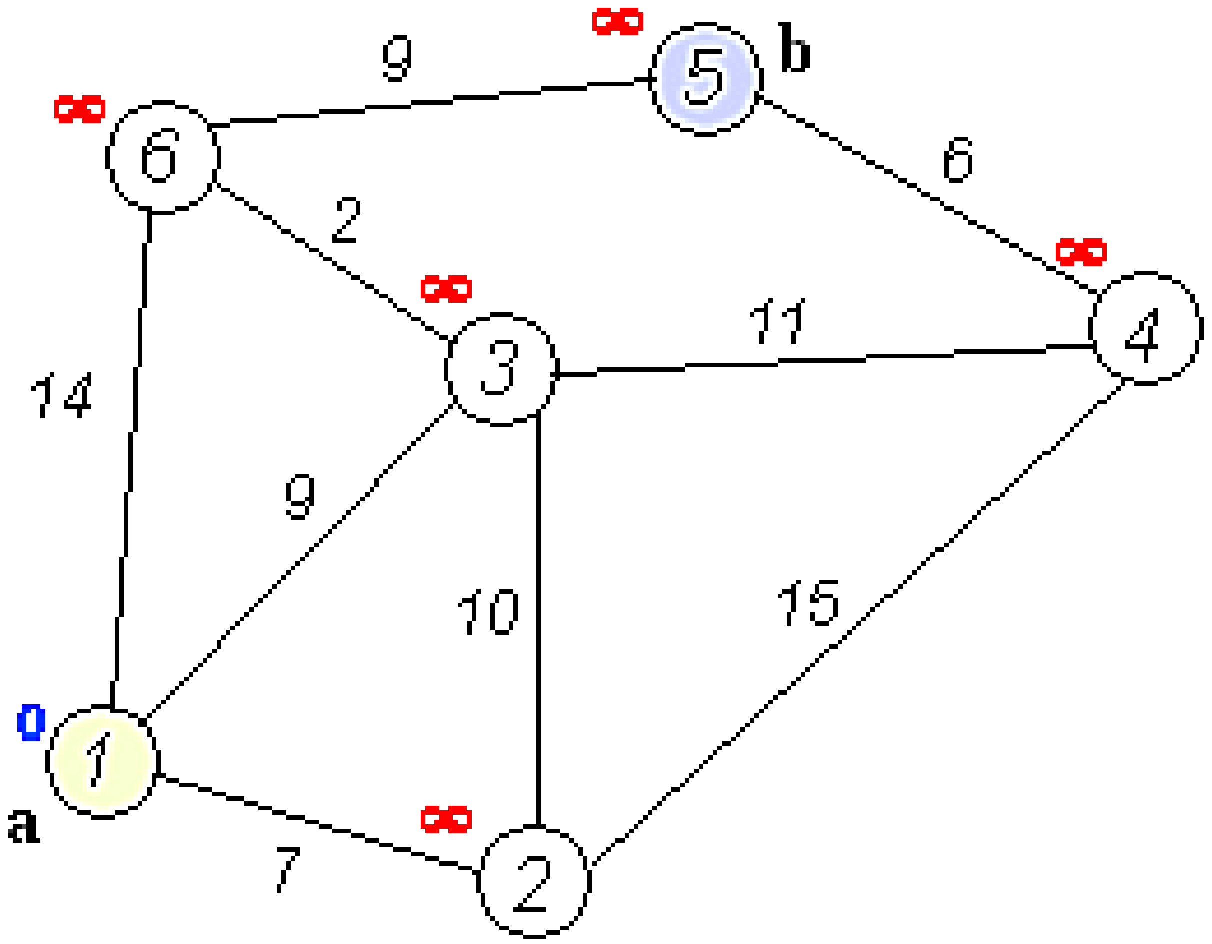
- Attempts to find best answer for a particular input
- Example: navigation app
- Goal is to output the solution that has the least ‘weight’
- Variable can be continuous or discrete

Optimisation problem example

- Flight scheduling system
- Algorithm finds most optimal routes
- Dijkstra's shortest path algorithm



Dijkstra’s shortest path algorithm



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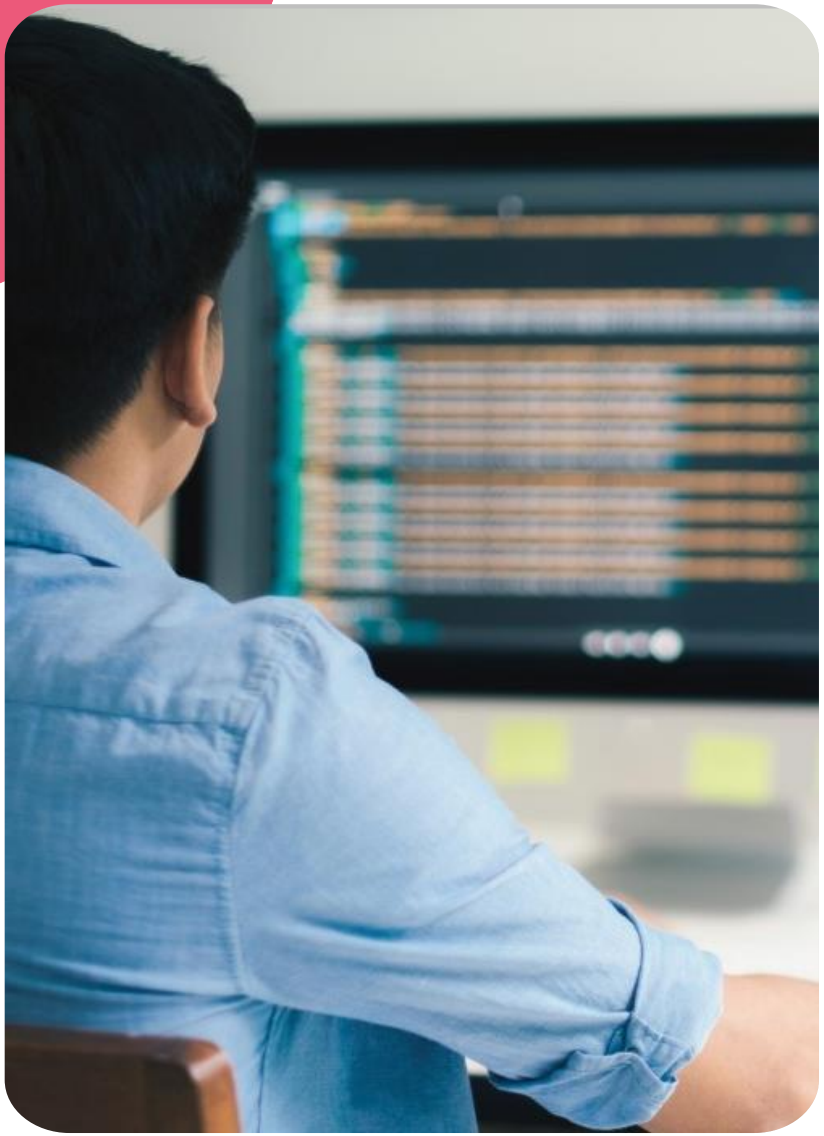
Function problem

- Similar to decision problems but output is not YES or NO
- Output more complex – numerical values and/or symbols



Function problem example

- Mathematical function: Given two numbers x and y, what is x divided by y?
- For every input there is a value x or y
- Every function problem can be turned into a decision problem – decision problem is a graph

Characteristics of computational problems

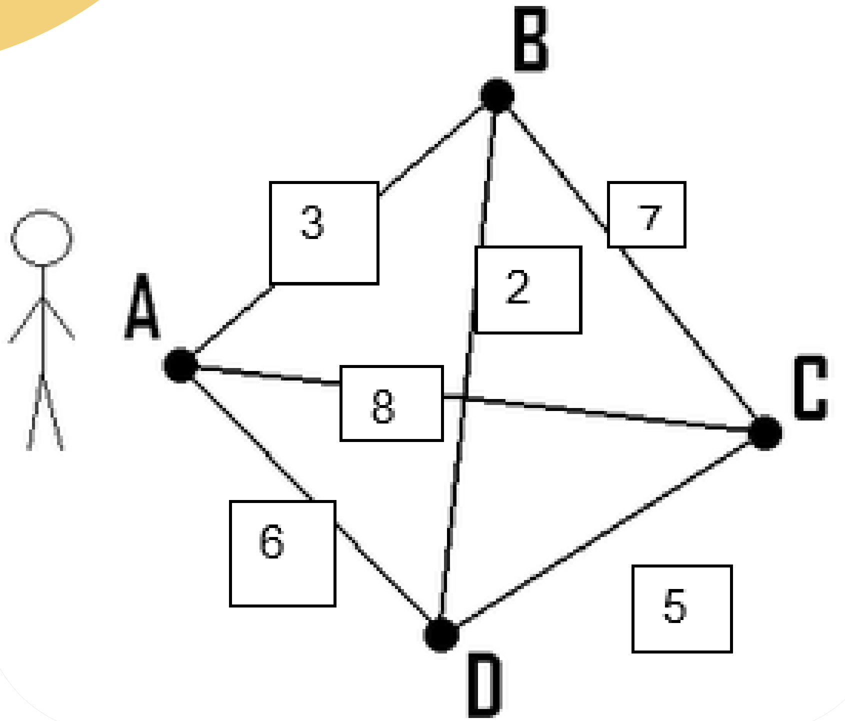
- Problem = task and input instances
- Identical tasks in set
- Different values for items to be operated on
- Input instance – instance of items to be acted on

Tractable problems

- **Algorithmic** – can be broken down into step-by-step instructions
- **Computable** – can be solved using a computer in a reasonable amount of time

Intractable problems

- Problems when execution time grows too quickly in relation to input to be solved in a reasonable time
- Examples: Travelling Salesman and scheduling a school timetable



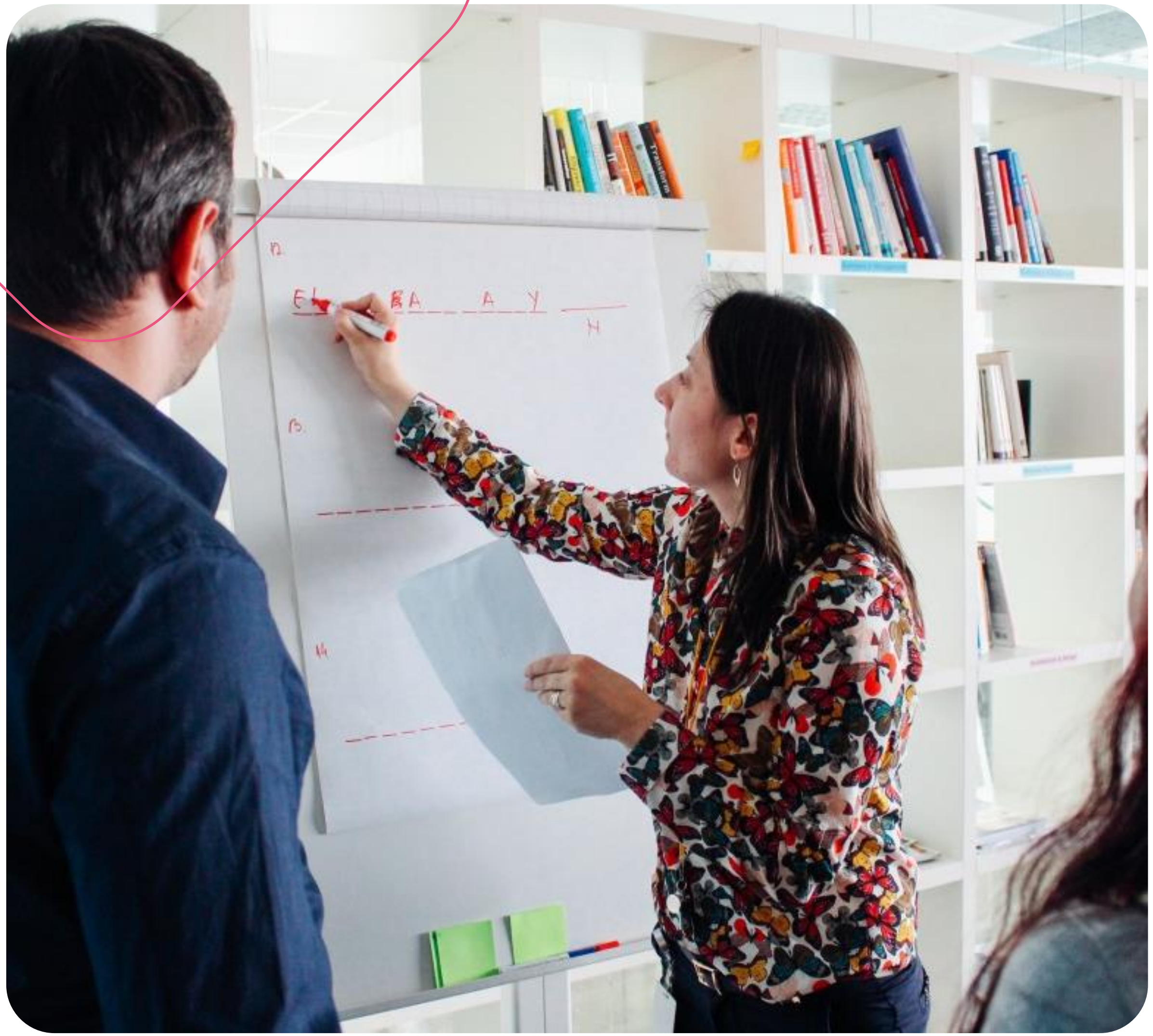
Intractable problem Example 1:

Travelling Salesman

- Find shortest route between set of cities that must be visited
- “Weight” = cost of moving from one city to another
- Only possible to solve the problem for a few ten thousands of cities – in 2006, 85 900!

Intractable problem Example 2:
School timetable

- Optimal solution required in exponential time
- Known as a ‘suboptimal’ or ‘approximate’ solution

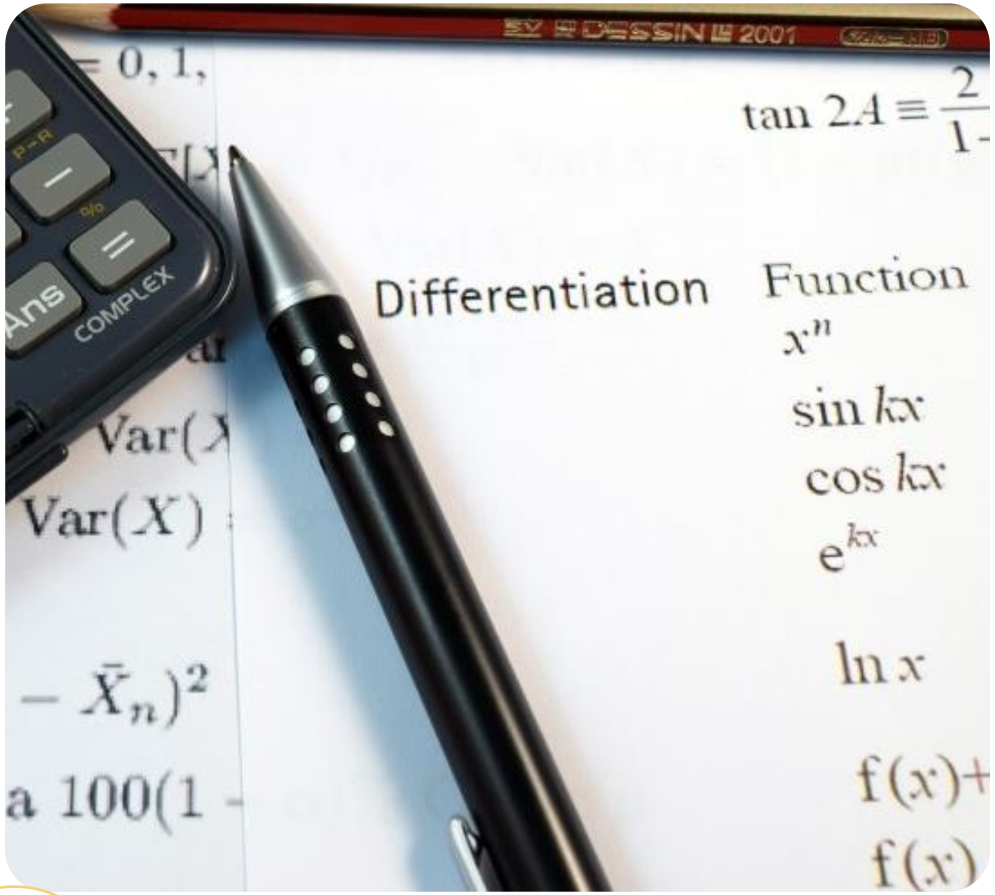


Heuristic problem-solving

- Output solutions to a problem in reasonable time
- Algorithms make decisions on how to approach problem
- Based on the value of knowledge, experience, and judgement in solving intractable problems

Reasonable time vs polynomial time

- Polynomial = algebraic expression consisting of variables and coefficients
- Polynomial time is FAST because its time cost is bound by some polynomial



Incomputable problem

- ‘Unsolvable’ or ‘undecidable’
- Computer gives wrong answer or runs forever

Incomputable problem Example: Halting problem

- Most famous of all undecidable problems
- Given a computer program and an input, will the program terminate or run forever?

Maths and computing

Maths and computational problems

- Coding in computers is mathematical expressions
- Computer Science – a subset of Mathematical Science

Did you know?

- Mathematics is a universal language – especially in Computer Science.
- The symbols and organisation to form equations are the same in every country of the world.

What is mathematical modelling?

- Translating problems from an application area into tractable mathematical formulations
- Aims to develop scientific understanding through quantitative expression of current knowledge of a system

Mathematical models

Models classified according to level of understanding

- Mechanistic models – more theoretical
- Empirical models – account for changes in conditions
- Stochastic models – predict possible outcomes based on stats
- Deterministic models – describe system inputs and outputs exactly

Examples of mathematical models

	Empirical	Mechanistic
Stochastic	Analysis of variance of variety yields over sites and years	Genetics of small populations based on Mendelian inheritance (probabilistic equations)
Deterministic	Predicting cattle growth from a regression relationship with feed intake	Planetary motion, based on Newtonian mechanics (differential equations)

A binary problem!

- Computers store text as a sequence of unique numbers representing characters
- Many standards and each standard assigns different numbers to the same character

For example:

"ä"is stored as 228 in the ISO-8859-1 standard but stored as the two-byte number 50084 in the UTF-8 standard. If a UTF-8 encoded "ä" is interpreted according to the IS0-8859-1 standard, it shows up as the character pair "Ã¤“.

- Need an algorithm that allows computers to agree on which encoding to use, for example, the HTTP header

Common Computing problems

Dining Philosophers' problem

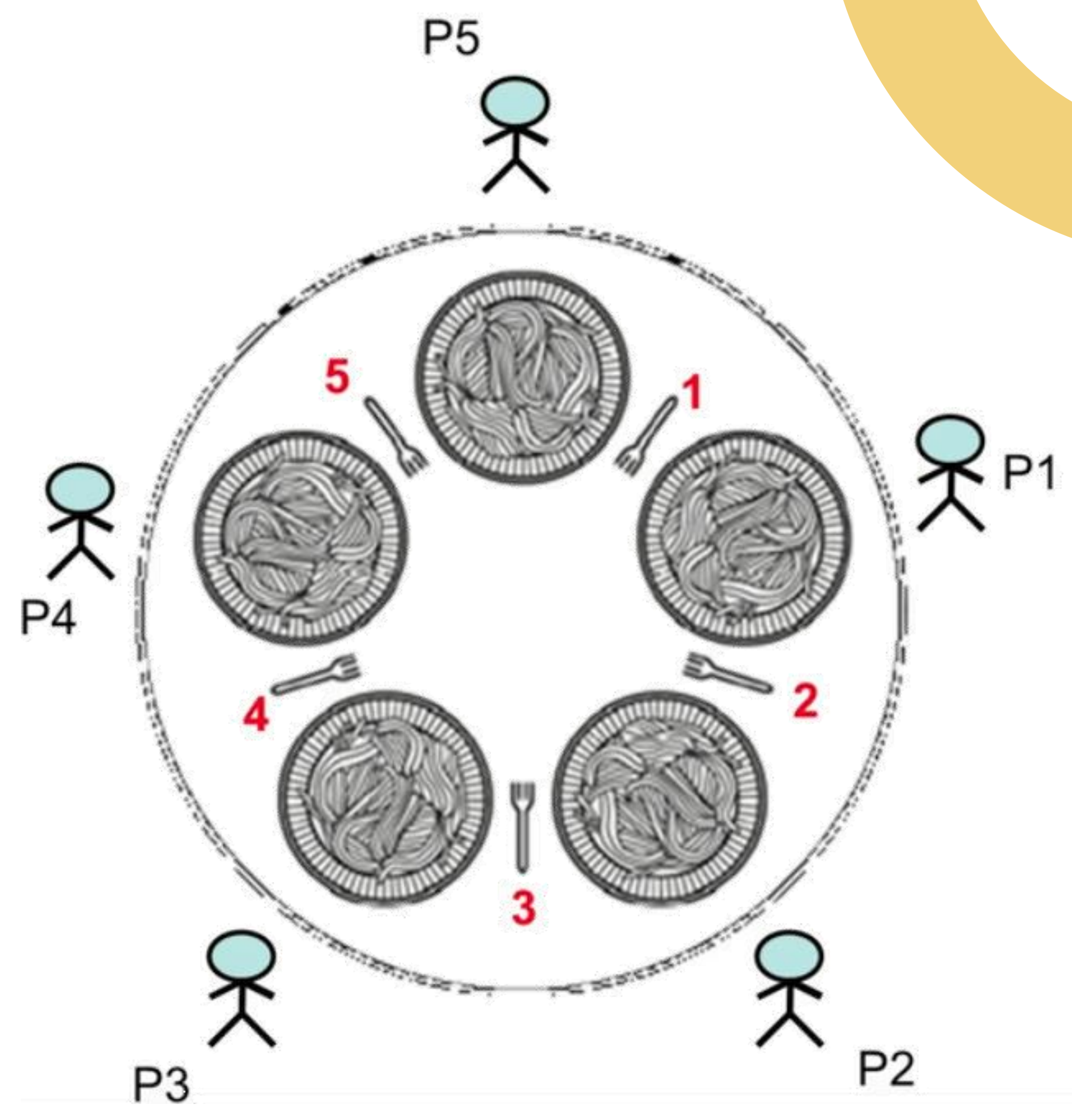
- Used in to illustrate a synchronisaton problem
- Five philosophers sitting around a circular table for a meal of spaghetti
- For any philosopher to eat, they must have one fork in the left hand and one in the right hand
- Just five forks on the table
- Philosophers agree to follow a strict pattern of thinking and eating

Dining Philosophers' problem

- Think → pick up left fork → think → pick up right fork → eat for 30 secs → put right fork down → put left fork down → repeat
- Make sure that every 30 seconds, at least one philosopher gets to eat

Solution to Dining Philosophers' problem

- Uses multi-threading
- Several programs run by the same processor
- Used when several devices want to access the same resource



Travelling Salesman problem

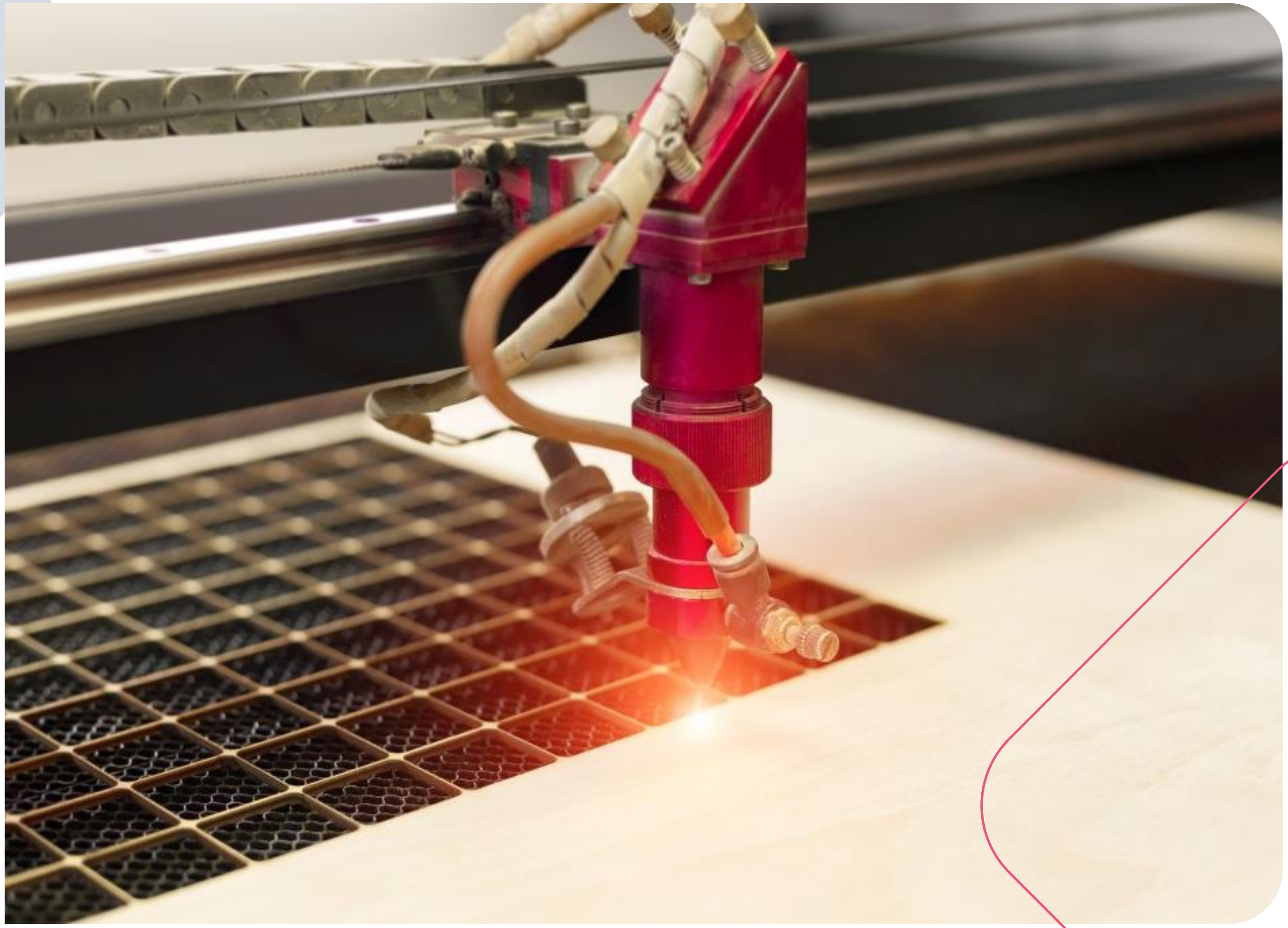
- State the problem as clearly as possible
- Look at solution
- Decide what algorithm needs to achieve

Solution to Travelling Salesman problem

- Calculate all possible permutations of routes
- Compare total distances
- Choose shortest route

DID YOU KNOW?

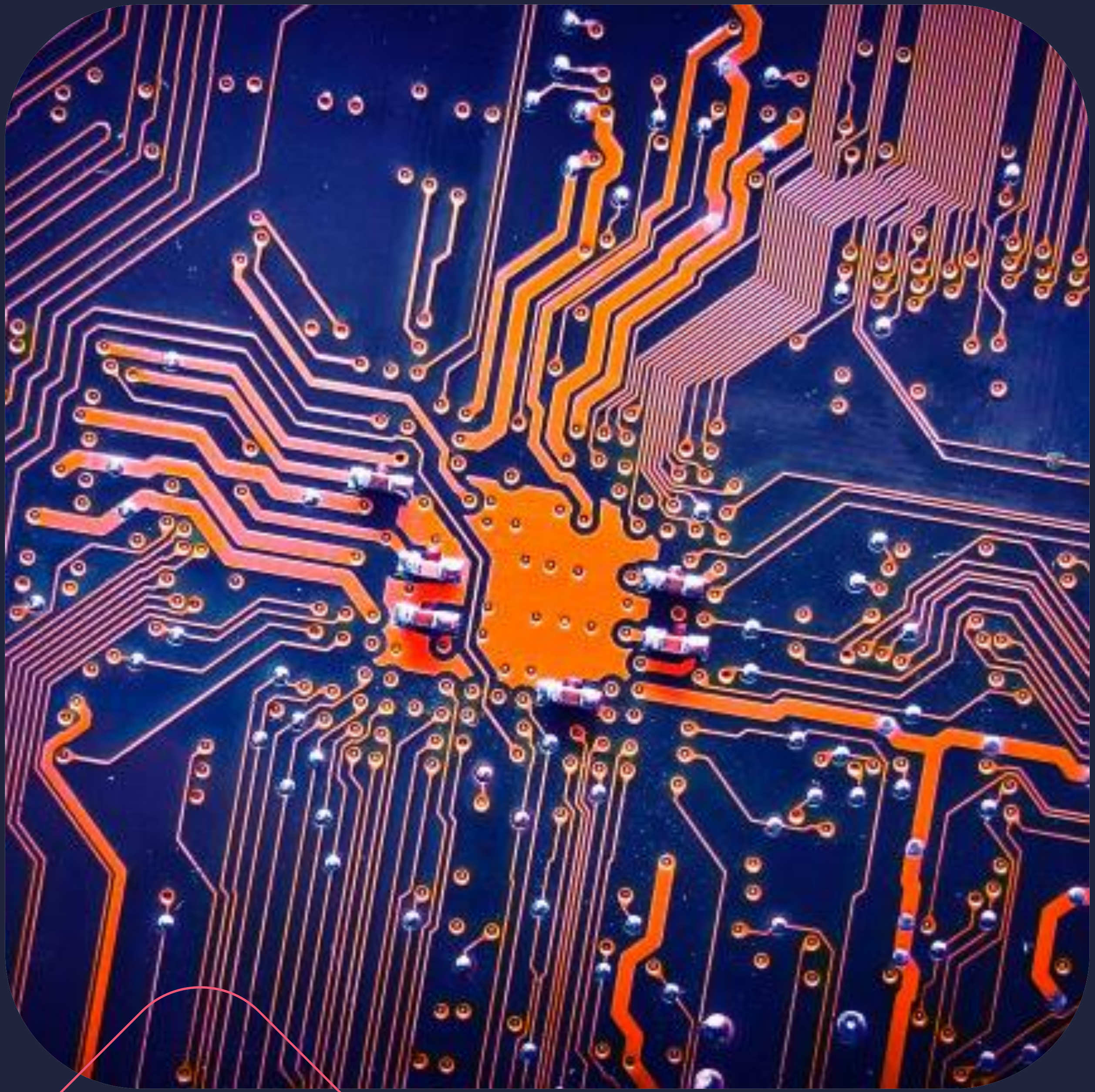
The Travelling Salesman algorithm is used in the manufacture of circuit boards to determine the order in which a laser can drill holes. It helps the robot to use as little time as possible on the drilling process, cutting production time.



Water jug measuring problem

How to measure 2 litres of water precisely in a 4-litre jug?

- Problem needs to be precise
- Have clear initial states and goal states
- Rules laid out
- All components mentioned



Recap

- Types of problems that are encountered in computing
- Approaching problems to come up with solutions