From equation (11d), (11e) and (11f), denoted as equations (1), (2) and (3)

$$\dot{\rho}_{\mu 1} = -[i(E_0^\mu - E_1^\mu) + \gamma_{\mu 1}]\rho_{\mu 1} - i\zeta_p(\rho_{11} - \rho_{\mu\mu}) + i\zeta_c\rho_{\mu\nu} \tag{1}$$

$$\dot{\rho}_{\nu 1} = -[i(E_0^\nu - E_1^\mu) + \gamma_{\nu 1}]\rho_{\nu 1} - i\zeta_c(\rho_{11} - \rho_{\nu\nu}) + i\zeta_p\rho_{\mu\nu}^* \tag{2}$$

$$\dot{\rho}_{\mu\nu} = -[i(E_0^\mu - E_0^\nu) + \gamma_{\mu\nu}]\rho_{\mu\nu} - i\zeta_p\rho_{\nu 1}^* + i\zeta_c^*\rho_{\mu 1} \tag{3}$$

Where ζ_p and ζ_c is given by

$$\zeta_p = -\Omega_p e^{-i\omega_p t}\cos\theta_1 \sin\theta_0 \tag{4}$$

$$\zeta_c = \Omega_c e^{-i\omega_c t}\cos\theta_1 \cos\theta_0 \tag{5}$$

Using perturbative expansion of density matrix elements to the first order, and the initial conditions which is given by $\rho_{\mu\mu}^{(0)} = 1$ and $\rho_{\nu\nu}^{(0)} = \rho_{11}^{(0)} = \rho_{\mu 1}^{(0)} = \rho_{\nu 1}^{(0)} = \rho_{\mu\nu}^{(0)} = 0$. Here we find that the first order expansion $\rho_{11}^{(1)} = \rho_{\nu\nu}^{(1)} = 0$ and the non-diagonal matrix elements(i.e.,$\rho_{\mu 1}^{(1)}$) should be nonzero. So, if we expand the matrix elements on both sides of equation (1) up to the first order, we get

$$\dot{\rho}_{\mu 1}^{(1)} = -[i(E_0^\mu - E_1^\mu) + \gamma_{\mu 1}]\rho_{\mu 1}^{(1)} + i\zeta_p + i\zeta_c\rho_{\mu\nu}^{(1)} \tag{6}$$

If we rotate our $\rho_{\mu 1}^{(1)}$ and $\rho_{\mu\nu}^{(1)}$ into $\rho_{\mu 1}^{(1)} e^{-i\omega_p t}$ and $\rho_{\mu\nu}^{(1)} e^{-i(\omega_p + E_0^\nu - E_1^\mu)t}$. Bring this term in to equation (6)

$$-i\omega_p\rho_{\mu 1}^{(1)} e^{-i\omega_p t} + \dot{\rho}_{\mu 1}^{(1)} e^{-i\omega_p t} = -[i(E_0^\mu - E_1^\mu) + \gamma_{\mu 1}]\rho_{\mu 1}^{(1)} e^{-i\omega_p t} + i\zeta_p + i\zeta_c\rho_{\mu\nu}^{(1)} e^{-i(\omega_p + E_0^\nu - E_1^\mu)t} \tag{7}$$

Bring Eq. (4) and (5) into (7), notice that $E_0^\nu - E_1^\mu = -\omega_c$

$$\begin{aligned}\dot{\rho}_{\mu 1}^{(1)} e^{-i\omega_p t} = &- [i(E_0^\mu - E_1^\mu - \omega_p) + \gamma_{\mu 1}]\rho_{\mu 1}^{(1)} e^{-i\omega_p t} - i\Omega_p e^{-i\omega_p t}\cos\theta_1 \sin\theta_0 \\ &+ i\Omega_c e^{-i\omega_c t}\cos\theta_1\cos\theta_0 \rho_{\mu\nu}^{(1)} e^{-i(\omega_p + E_0^\nu - E_1^\mu)t}\end{aligned} \tag{8}$$

Cancel out $e^{-i\omega_p t}$ both sides. the exponential part vanishes in last term on the RHS of (8). Then, we get the following equation with a sign difference comparing with target equation

$$\dot{\rho}_{\mu 1}^{(1)} = -[i(E_0^\mu - E_1^\mu - \omega_p) + \gamma_{\mu 1}]\rho_{\mu 1}^{(1)} - i\Omega_p\cos\theta_1\sin\theta_0 + i\Omega_c\cos\theta_1\cos\theta_0\rho_{\mu\nu}^{(1)} \tag{9}$$

The target equation is following

$$\dot{\rho}_{\mu 1}^{(1)} = -[i(E_1^\mu - E_0^\mu - \omega_p) + \gamma_{\mu 1}]\rho_{\mu 1}^{(1)} - i\Omega_p\cos\theta_1\sin\theta_0 + i\Omega_c\cos\theta_1\cos\theta_0\rho_{\mu\nu}^{(1)} \tag{10}$$

That is the problem!

The other equation is given by setting $\rho_{\nu 1}^{*(1)} = 0$ in equation (3) which could successfully derive out the second target equation with out using equation (2) (**Why????????**).