# Problem Set 7

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## Poisson Spike-Train Statistics

1. Given the Homogeneous Poisson process (mean firing rate is independent of time),

$$P(n) = \frac{(rT)^n}{n!} \exp(-rT). \tag{1}$$

Calculate the mean $\langle n \rangle$ and variance $Var(n)$ of the spike count. Compute the Fano factor $Var(n)/\langle n \rangle$. Calculate the kurtosis of spike count defined as $k = \langle n^4 \rangle - 3\langle n^2 \rangle^2$ in the time interval $T$.

2. When the firing rate depends on time, we could also extend the homogeneous Poisson process to inhomogeneous Poisson process. When $n$ spikes occurs in an interval $T$ with $0 < t_1 < t_2 < ... < t_n < T$, Prove that the joint probability density is given by

$$p(t_1, t_2, ..., t_n) = \exp\left(-\int_0^T r(t)dt\right) \prod_{i=1}^{n} r(t_i) \tag{2}$$

Then, calculate the probability of seeing $n$ spikes $P(n)$ in the interval $T$. Check whether it has a similar expression as the homogeneous Poisson process and calculate the Fano Factor.

3. Generate a Poisson spike train with a time-dependent fire rate $r(t) = r_0[1 + cos(2\pi t/\tau)]$ where $r_0 = 100$ Hz and $\tau = 300$ ms. Generate a spike train for 20 s and plot it.

4*. Let's assume that the firing rate of a neuron has the following functional form: $r(t) = r_0 + r_1 \sin(\omega t + \theta)$, where the phase $\theta$ is drawn uniformly between 0 and $2\pi$ for each trial. Calculate the Fano Factor for the spike count in the time interval $T$ (as a function of $T$).

Note: Problems with * are optional. However, solving them will give you additional credits.