Problem Set 4

Due Wednesday, Nov 7, 2018

Cable equation

Figure 1 shows a single isolated junction of three semi-infinite cables. The father branch has radius a_1, and the two daughter branches have radii a_2 and a_3, and the branching point is at position $x = 0$. As a result, different branches would have different electrotonic length $\lambda_i = \sqrt{\frac{a_i r_m}{2\rho_L}}$. Now consider a current injection x_0 distance away from the branching point. Let's consider two cases. (1) The current is injected into the father branch. (2) The current is injected into one of the daughter branches. The current injection has the same functional form as what we discussed in the class:

$$i_e = -\frac{I_e}{2\pi a_i}\delta(x - x_0). \tag{1}$$

Derive the steady state solution for the spatial distribution of voltage along the father branch $v_1(x)$, and two daughter branches, $v_2(x)$ and $v_3(x)$. Note that at the branching point, the solution should satisfy the following boundary conditions

$$\begin{gathered} v_1(0) = v_2(0) = v_3(0). \\ a_1^2 \frac{\partial v_1}{\partial x}|_{0^-} = a_2^2 \frac{\partial v_2}{\partial x}|_{0^+} + a_3^2 \frac{\partial v_3}{\partial x}|_{0^+}. \end{gathered} \tag{2}$$

The second equation is derived from current conservation. Use $a_1 = 2$ μm, $a_2 = a_3 = 1$ μm, $\lambda_1 = 1$ mm, $\lambda_2 = \lambda_3 = 2^{-1/2}$ mm, $x_0 = 1$ mm and plot the normalized voltage distribution.

Note: Please read our lecture notes carefully on the derivation of cable equation. You could also refer the recommended textbook, chapter 6.3.

Maximum Entropy

The Entropy of the firing rate of a neuron r drawn from a distribution $p(r)$ is given by the following formula

$$h = -\int_{-\infty}^{\infty} p(r) \ln p(r) \tag{3}$$

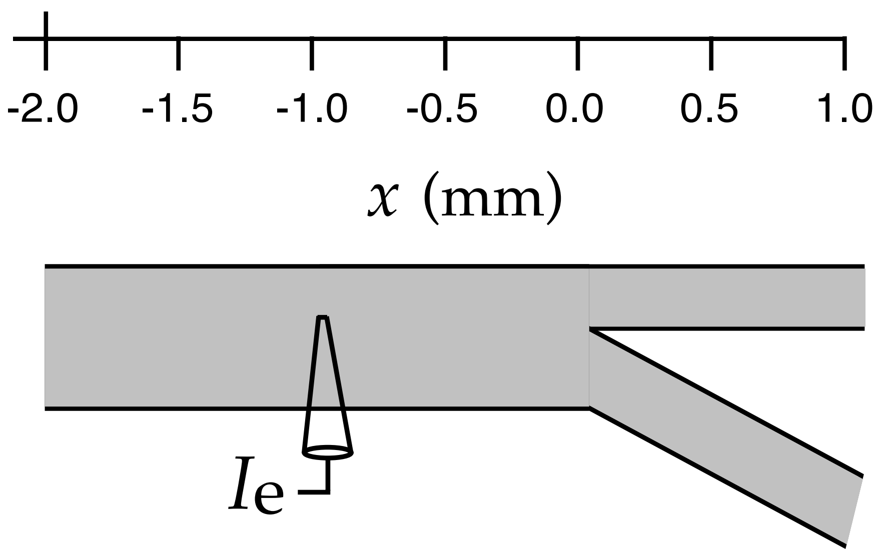

Figure 1: branching cable.

Use the Lagrange Multiplier method to evaluate the maximum entropy probability distribution $p(r)$ in the following cases:

1. The firing rate of a neuron r is a one dimensional continuous random variable, which takes only positive values and its mean is fixed. Hint: In addition to the mean, you should also take into account the constraint imposed by the normalization of p.

2. The firing rate of a neuron r is a one dimensional continuous random variable, which takes only positive values and its maximum is given by r_{max}.

3. There is no constraint on the range of r but its variance is given.