Problem Set 2

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Derivation of the Goldman-Hodgkin-Katz formula for reversal potential

The reversal potential we discussed in the class only take into account one type of ion. However, some channels are not quite selective, and we need to combine the current flow from multiple ions, and the result is the Goldman-Hodgkin-Katz formula for reversal potential. I will write down the equation here, and it is your homework to provide the derivation of this formula.

$$E_m = \frac{k_B T}{e} \ln \left(\frac{\sum_{i=1}^{N} P_{M_i^+} [M_i^+]_{out} + \sum_{i=1}^{N} P_{A_j^-} [A_j^-]_{in}}{\sum_{i=1}^{N} P_{M_i^+} [M_i^+]_{in} + \sum_{i=1}^{N} P_{A_j^-} [A_j^-]_{out}} \right). \tag{1}$$

Here P denotes the permeability of a given ion.

Integrate and Fire Neuron

An integrate and fire neuron has a subthreshold membrane potential that obeys the equation

$$C \frac{dV}{dt} = -\frac{V}{R} + I(t) \tag{2}$$

Once the voltage crosses threshold, the neuron fires and V is reset to 0.

Consider the input current

$$I(t) = Q \sum_{k=-\infty}^{\infty} \delta(t - kT) \tag{3}$$

where a charge Q crosses the membrane periodically.

i. Derive the subthreshold membrane potential as a function of time for this input current. Describe the transient and steady state behavior of the potential. Illustrate the result by qualitative or (even better) numerically quantitative graphs.

ii. Under what conditions will the neuron fire spikes? Compute the firing frequency of the neuron when the conditions for firing are met. Plot the firing frequency as a function of the interesting variables and explain.

Hodgkin and Huxley Model

The Hodgkin-Huxley model for generation of an action potential is constructed by a summation of leaky current, a delayed-rectified K^+ current, and a transient Na^+ current:

$$\begin{aligned} C_m \frac{dV}{dt} &= -\bar{g}_{\mathrm{K}} n^4 (V - E_{\mathrm{K}}) - \bar{g}_{\mathrm{Na}} m^3 h (V - E_{\mathrm{Na}}) - \bar{g}_L (V - E_L) + I_e. \\ \frac{dn}{dt} &= \alpha_n (1-n) - \beta_n n. \\ \frac{dm}{dt} &= \alpha_m (1-m) - \beta_m m. \\ \frac{dh}{dt} &= \alpha_h (1-h) - \beta_h h. \end{aligned} \tag{4}$$

(a) Please simulate the dynamic equations and check whether it could generate action potentials. Below I will provide detailed parameter values used in Hodgkin and Huxley model.

$$\alpha_n = \frac{0.01(V+55)}{1-\exp(-0.1(V+55))}, \ \beta_n = 0.125 \exp(-0.0125(V+65)),$$

$$\alpha_m = \frac{0.1(V+40)}{1-\exp(-0.1(V+40))}, \ \beta_m = 4 \exp(-0.0556(V+65)),$$

$$\alpha_h = 0.07 \exp(-0.05(V+65)), \ \beta_h = \frac{1}{1+\exp(-0.1(V+35))}.$$

These rates have dimensions ms^{-1}. The maximal conductances and reversal potentials used in the model are $\bar{g}_{\mathrm{K}} = 0.36$ mS/mm^2, $\bar{g}_{\mathrm{Na}} = 1.2$ mS/mm^2, $\bar{g}_{\mathrm{L}} = 0.003$ mS/mm^2, $E_{\mathrm{L}} = -54.387$ mV, $E_{\mathrm{K}} = -77$ mV, $E_{\mathrm{Na}} = 50$ mV, $C_m = 10$ nF/mm^2.

(b) Show that there is a threshold current above which the system generates periodic pulses. Explore the frequency of the pulses as a function of current, just like what you did in the integrate-and-fire model.