Problem Set 3

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Due Wednesday, Nov 6, 2019

FitzHugh-Nagumo Model

The FitzHugh-Nagumo model

$$\dot{V} = V(a - V)(V - 1) - w + I, \tag{1}$$
$$\dot{w} = bV - cw, \tag{2}$$

imitates H-H models by having cubic (N-shaped) nullclines, where parameter a describes the shape of the cubic parabola $V(a - V)(V - 1)$, and $b > 0$, $c \leq 0$ describe the kinetics of the recovery variable w.

i. Determine nullclines of the model and draw the two-dimensional phase portrait of the model using MATLAB.

ii. Please analyze the stability of the equilibrium $(0, 0)$, and how they depends on the above parameters. plot the phase digram just like what we did in the class.

Bendixson's criterion

If the divergence of the vector field $\frac{\partial f(x,y)}{\partial x} + \frac{\partial g(x,y)}{\partial y}$ of a two-dimensional dynamical system is not identically zero, and does not change sign on the plane, then the dynamical system cannot have limit cycles. Use this criterion to show that the following I_K-model (a) cannot oscillate and (b) is always stable at least when $E_L > E_K$.

$$C\dot{V} = -g_L(V - E_L) - g_K m^4(V - E_K), \tag{3}$$
$$\dot{m} = (m_\infty(V) - m)/\tau(V) \tag{4}$$

(Hint: Look at the signs of the trace and the dterminant of the Jacobian matrix as well).

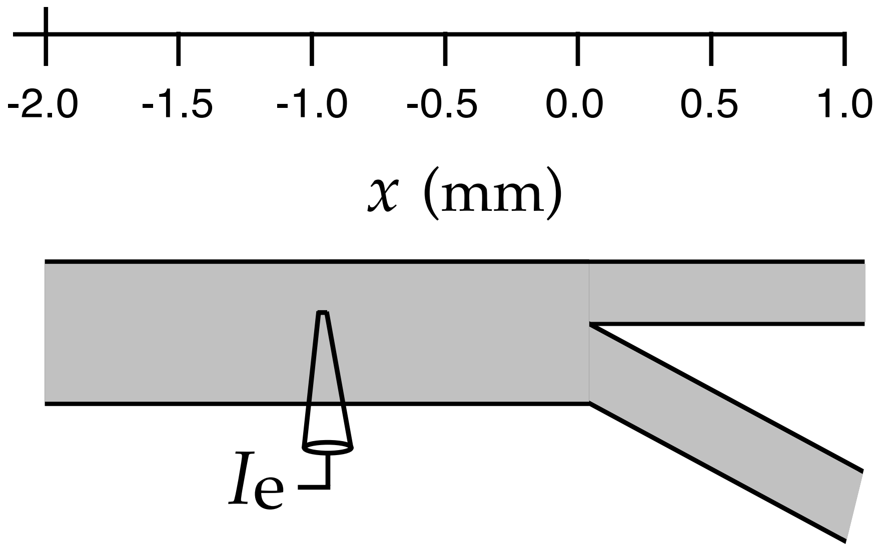

Figure 1: branching cable.

Cable equation

Figure 1 shows a single isolated junction of three semi-infinite cables. The father branch has radius a_1, and the two daughter branches have radii a_2 and a_3, and the branching point is at position $x = 0$. As a result, different branches would have different electrotonic length $\lambda_i = \sqrt{\frac{a_i r_m}{2\rho_L}}$. Now consider a current injection x_0 distance away from the branching point. Let's consider two cases. (1) The current is injected into the father branch. (2) The current is injected into one of the daughter branches. The current injection has the same functional form as what we discussed in the class:

$$i_e = -\frac{I_e}{2\pi a_i}\delta(x - x_0). \tag{5}$$

Derive the steady state solution for the spatial distribution of voltage along the father branch $v_1(x)$, and two daughter branches, $v_2(x)$ and $v_3(x)$. Note that at the branching point, the solution should satisfy the following boundary conditions

$$\begin{gathered} v_1(0) = v_2(0) = v_3(0). \\ a_1^2 \frac{\partial v_1}{\partial x}|_{0^-} = a_2^2 \frac{\partial v_2}{\partial x}|_{0^+} + a_3^2 \frac{\partial v_3}{\partial x}|_{0^+}. \end{gathered} \tag{6}$$

The second equation is derived from current conservation. Use $a_1 = 2$ μm, $a_2 = a_3 = 1$ μm, $\lambda_1 = 1$ mm, $\lambda_2 = \lambda_3 = 2^{-1/2}$ mm, $x_0 = 1$ mm and plot the normalized voltage distribution.

Note: Please read our lecture notes carefully on the derivation of cable equation. You could also refer the recommended textbook, chapter 6.3.