Problem Set 2

Due Wednesday, Sep 26, 2018

How granule cells sample inputs

In the last lecture, we discussed the synaptic organization of the cerebellum, Figure 1. Mossy fibers, which are long range projections from various brain regions, make connections with granule cells, the most numerous ($\sim 10^{11}$) neurons in the whole brain. These granule cells, has a very small convergence: each of which only receives inputs from a handful mossy fibers. The output of granule cells, called parallel fibers, travel along the cerebellar cortex for a few millimeters and synapse onto Purkinje dendrites. Purkinje cells have the highest convergence in the whole brain, each of which receives inputs from more than 10^5 synapses from parallel fibers. The granule-to-Purkinje projections are what we called in the class "Connecting dense array to sparse array with extreme convergence and divergence".

Why do we need so many granule cells? What can we say about the number of granule cells N, the number of mossy fiber inputs M, and the convergence of a granule cell K? Perhaps each granule cell is sampling a different combinations of mossy fiber inputs. The higher the functional diversity, the more powerful computation downstream circuits (e.g., Purkinje dendrites) could perform, such as classification.

Assume each granule cell can choose K inputs out of all M mossy fibers, the number of possibilities is simply a binomial coefficient $\binom{M}{K}$. Now we ask the following questions.

- What is the probability p that granule cells *all* receive *different* combinations of inputs?
- For given N and M, plot p as a function of K, and show when p reaches its maximum.
- Using $N = 21000$, $M = 7000$, compute K when p approaches 95 percent of its maximum.
- Discuss whether it is beneficial to have small K when N is very large.

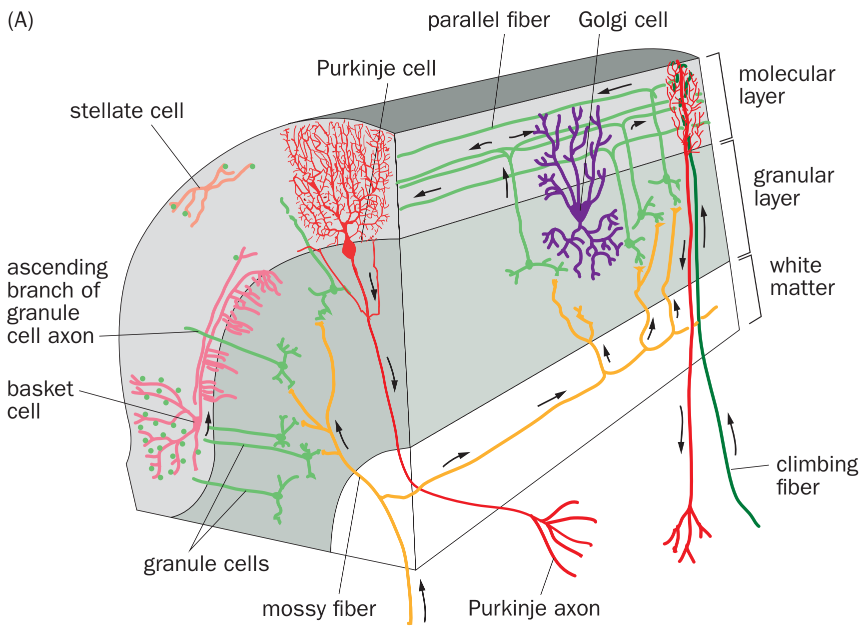

Figure 1: Synaptic organization of the cerebellum