Question 1:

Idea:
The two children can be thought of as two masses applying a force (their weight) on a level beam. Each mass will provide a torque to the beam which will want to turn it one way or the other. Child 1 mass will make the beam turn counter-clockwise (positive torque) while child 2 mass will make it rotate clockwise (negative torque) we carrying out the analysis with this in mind:

Setup & solution

(a) We are pausing time and considering when the beam is parallel with the ground. Regardless, the children are equal distance apart on the beam since it balances on the center so the radius to each child is the same:

$$r = \frac{L}{2} = \boxed{1.5\text{m}}$$

(b) We use the definition of torque and the sign conventions discussed in the "idea" section above noting that the angle these children make at this moment in time is 90°:

$$\vec{\tau}_1 = +|r_1||F_1|\sin(\theta) = +|1.5m||m_1 g|\sin(90^\circ) = \boxed{+\ 294.3\text{ Nm}}$$
$$\vec{\tau}_2 = -|r_2||F_2|\sin(\theta) = -|1.5m||m_2 g|\sin(90^\circ) = \boxed{-\ 441.5\text{ Nm}}$$

(c)

$$\vec{\tau}_{net} = \sum \tau_i = \tau_1 + \tau_2 = (+294.3Nm) + (-441.5Nm) = -147.15\text{ Nm}$$

Thus the seesaw will rotate clockwise.

Challenge Question
In order for the seesaw to not move we need the net torque on the seesaw to be zero, i.e. we would have a **static** system. So the only trick here is to use algebra with the radius:

$$\vec{\tau}_{net} = \sum \tau_i = \tau_1 + \tau_2 = 0 \tag{1}$$

Now if we measure from the child 1 or the left side, we say child 1 is a distance "r" from the equilibrium fulcrum and child 2 is (L-r) = (3m-r) away from it. Plugging our expressions into (1):

$$\begin{aligned} 0 &= \vec{\tau}_1 + \vec{\tau}_2 \\ &= |r_1||F_1|\sin(90^\circ) - |r_2||F_2|\sin(90^\circ) \\ &= |r||m_1 g| - |(3-r)||m_2 g| \\ \implies 0 &= rm_1 - (3-r)m_2 \end{aligned}$$

Which using algebra we solve as

$$r = \frac{3m_2}{m_1 + m_2} = \boxed{2.25\text{m}}$$

Question 2

Idea:
Since the only thing that changes between these rolling objects is the moment of inertia I, we should expect that to be the main factor in determining which reaches the bottom first. Secondly, since both objects start at the same height, that means they both start with the same total energy, which is just potential energy. Thus all we need to do is see which object converts more energy into **linear** kinetic! Because that object will then reach the bottom faster (since they have the same mass too).

Setup & solution

(a) They're the same! Since they both start at the same initial height "h" and mass:

$$E_{total} = PE = mgh$$

(b) At the bottom of the path, the sphere will have three energies:

$$E_{sphere_f} = PE + KE + KE_R = mgh + \frac{1}{2}mv_{sph_f}^2 + \frac{1}{2}\mathrm{I}_{sph}\omega_f^2$$

since we set h=0 at the bottom we get our answer:

$$E_{sphere_f} = \frac{1}{2}mv_{sph_f}^2 + \frac{1}{2}\mathrm{I}_{\mathrm{sph}}\omega_f^2$$

(c) Just like in (b) the cylinder will have the same final energy except we use the cylinder moment of inertia:

$$E_{cylinder_f} = \frac{1}{2}mv_{cyl_f}^2 + \frac{1}{2}\mathrm{I}_{cyl}\omega_f^2$$

(d) The crucial thing to realize here is that the **total** energy of both objects is the same. So using our angular/linear velocity relationship we have:

$$E_{sphere_f} = \frac{1}{2}mv_{sph_f}^2 + \frac{1}{2}\mathrm{I}_{\mathrm{sph}}\omega_f^2 = \frac{1}{2}mv_{sph_f}^2 + \frac{1}{2}\mathrm{I}_{\mathrm{sph}}(\frac{v_{sph_f}}{R})^2 = \frac{1}{2}mv_{sph_f}^2 + \frac{1}{2}\frac{2}{5}mR^2(\frac{v_{sph_f}}{R})^2 = \frac{1}{2}mv_{sph_f}^2(1+\frac{2}{5})$$

$$E_{cylinder_f} = \frac{1}{2}mv_{cyl_f}^2 + \frac{1}{2}\mathrm{I}_{cyl}\omega_f^2 = \frac{1}{2}mv_{cyl_f}^2 + \frac{1}{2}\mathrm{I}_{cyl}(\frac{v_{cyl_f}}{R})^2 = \frac{1}{2}mv_{cyl_f}^2 + \frac{1}{2}\frac{1}{2}mR^2(\frac{v_{cyl_f}}{R})^2 = \frac{1}{2}mv_{cyl_f}^2(1+\frac{1}{2})$$

Whew, that's a lot. Now the very last step is to set equal these equations:

$$\frac{1}{2}mv_{sph_f}^2(\frac{7}{5}) = E_{sphere_f} = E_{cylinder_f} = \frac{1}{2}mv_{cyl_f}^2(\frac{3}{2})$$

This gives us:

$$\frac{v_{cyl}}{v_{sph}} = \sqrt{\frac{7}{5}\frac{2}{3}} = \sqrt{\frac{14}{15}}$$

(e) Thus since $\frac{v_{cyl}}{v_{sph}} < 1$ the cylinder will **always** roll slower than the sphere!

(f) Probably not!

Question 3

Idea:
The system is both the merry-go-round and the child. In this problem we just apply the **conservation of angular momentum** and use the angular momentum equation to get our answers:

Setup & solution

(a) For just the merry-go-round

$$\vec{L} = \mathrm{I}\vec{\omega_0} = -\frac{1}{2}MR^2\omega_0 = -\frac{1}{2}(120kg)(1.8m)^2(0.5rad/s) = \boxed{\text{-145.8 Js}}$$

(b) Applying conservation of angular momentum:

$$\vec{L}_{total_0} = \vec{L}_{total_f} \tag{1}$$
$$\implies L_{total_0} = \vec{L}_{merry} + \vec{L}_{child} = \vec{L}_{both} \tag{2}$$
$$\implies -\mathrm{I_{merry}}\omega_0 - \mathrm{I_{child}}\omega_0 = \mathrm{I_{m\&c}}\omega_f \tag{3}$$
$$\implies \omega_f = \frac{\omega_0(-\mathrm{I_{merry}} - \mathrm{I_{child}})}{\mathrm{I_{m\&c}}} \tag{4}$$
$$= -\frac{\omega_0(\frac{1}{2}M_{merry}R^2 + M_{child}R^2)}{\frac{1}{2}M_{both}R^2} \tag{5}$$
$$= -\frac{\omega_0(\frac{1}{2}M_{merry} + M_{child})}{\frac{1}{2}(M_{merry} + M_{child})} \tag{6}$$
$$= \boxed{\text{-0.6 rad/s}} \text{ or } \boxed{\text{0.6 rad/s clockwise}} \tag{7}$$

(c) If the child had run the other way our equations only change a negative sign:

$$\vec{L}_{total_0} = \vec{L}_{total_f} \tag{8}$$
$$\implies L_{total_0} = \vec{L}_{merry} + \vec{L}_{child} = \vec{L}_{both} \tag{9}$$
$$\implies -\mathrm{I_{merry}}\omega_0 + \mathrm{I_{child}}\omega_0 = \mathrm{I_{m\&c}}\omega_f \tag{10}$$
$$\implies \omega_f = \frac{\omega_0(-\mathrm{I_{merry}} - \mathrm{I_{child}})}{\mathrm{I_{m\&c}}} \tag{11}$$
$$= -\frac{\omega_0(\frac{1}{2}M_{merry}R^2 - M_{child}R^2)}{\frac{1}{2}M_{both}R^2} \tag{12}$$
$$= -\frac{\omega_0(\frac{1}{2}M_{merry} - M_{child})}{\frac{1}{2}(M_{merry} + M_{child})} \tag{13}$$
$$= \boxed{\text{-0.2 rad/s}} \text{ or } \boxed{\text{0.2 rad/s clockwise}} \tag{14}$$

Question 4

Idea:
This problem will require us to use the spring restoring force and the potential energy of a spring

Setup & solution

(1) We wish to determine the spring constant "k". Since we know the force F and the displacement of the spring, finding the constant is related by:

$$F = -kx \implies k = \frac{-F}{x} = -\frac{1.2N}{0.12m} = \boxed{10 \text{ N/m}}$$

(2) Since the spring is being pulled down by the gravitational force of the pen, know that we can use the spring equation with the same spring constant as in (1):

$$F = -kx \implies x = -\frac{F}{k} = -\frac{-mg}{k} = -\frac{-(0.042kh)(9.81)}{10} = \boxed{41\text{mm}}$$

(3) Using the work energy theorem and potential energy found in spring:

$$W = -\Delta PE = -PE_f + PE_0 = -PE_f + 0 = \frac{1}{2}kx^2 = -\frac{1}{2}(10N/m)(0.041m)^2 = \boxed{\text{-0.0085 J}}$$

Question 5

Idea:
This one is just a straightforward application of the stretch equation.

Setup & solution

$$F = Y\left(\frac{\Delta L}{L_o}\right)A \implies \Delta L = \frac{FL_o}{YA} = \frac{(890N)(9.1m)}{(2\times 10^{11})(\pi(0.005m)^2)} = \boxed{5.16\ \times 10^{-4}m}$$