**Question 1:**

In 0.75s, a 7.00kg block is pulled through a distance of 4.0m on a frictionless horizontal surface, starting from rest. The block has a constant acceleration and is pulled by means of a horizontal spring that is attached to the block. The spring constant of the spring is 415 N/m.By how much does the spring stretch? How much work is done by the spring to stretch?

**Idea:**
The rock has only one force acting on it, the spring. And since the rock is accelerating form rest to some distance, it has an acceleration on it coming from the spring (We imagine that someone is pulling the spring here also but that's not important for the problem). So, we have information about time, distance and initial velocity, which should hint that we use kinematics to find the acceleration, then use that acceleration in Newton's 2nd Law to find the force the spring exerts, which should then tell us the distance.

**Setup & solution**

**(1)** To find how much the spring has stretched we need the spring force equation:

$$\vec{F}_{spring} = kx$$

But we don't know the force. Luckily we know that a force is pulling on the spring (because the rock must be pulled by the spring) so we can use a force body diagram on the rock (assuming no friction):
Rock:

$$\vec{F}_{net_{rock}} = \vec{F}_{spring} = m_{rock}a_{rock}$$

Setting these equations equal and solving for x gets us:

$$x = \frac{m_{rock}a_{rock}}{k}$$

Now we look for a way to find the acceleration of the rock. Since we know its intial speed, time and distance we use kinematics to solve for a:

$$\Delta x = v_o t + \frac{1}{2}at^2 \implies a = \frac{2\Delta x}{t^2}$$

Inserting into our expression:

$$x = \frac{m_{rock}2\Delta x}{kt^2} = \frac{2(7kg)(4.0m)}{(415N/m)(0.75s)^2} = \boxed{0.24 \text{ m}}$$

**(2)** To find the work done by a spring that has stretched some distance we use:

$$W = \frac{1}{2}kx^2 = \frac{1}{2}(415N/m)(0.26m)^2 = \boxed{11.9\text{J}}$$

**Question 2**

One end of a piano wire is wrapped around a cylindrical tuning peg and the other end is fixed in place. The tuning peg is turned so as to stretch the wire. The piano wire is made from steel (Y = $2.0\times10^{11}$N/m$^2$). It has a radius of 0.8mm and an unstrained length of 0.76m. The radius of the tuning peg is 1.8 mm. Initially there is no tension in the wire. Find the tension in the wire when the peg is turned through two revolutions.

**Idea:**
Here we have a linear object (the wire) stretching some distance $\Delta L$ as a force F (tension) is pulling on it. This is a direct example of the stretching equation in 1D but we have to think carefully about $\Delta L$

**Setup & solution**

Since we have the young's modulus we identify the 1D stretching equation as the relevant one to use here:

$$F = Y\left(\frac{\Delta L}{L_o}\right)A$$

We see F is the tension, Y is given, A is found using the cross-sectional area of a wire, which is a circle. $L_o$ is the initial length and finally $\Delta L$ is the circumference of the peg times 2. So:

$$T = Y\left(\frac{2*2\pi r_p}{L_o}\right)(\pi r_w^2) = (2.0\times10^{11}N/m)\left(\frac{2*2\pi*(1.8\times10^{-3}m)}{0.76m}\right)(\pi(0.8\times10^{-3}m)^2) \approx \boxed{\text{12,000 N}}$$

**Question 3**

United States currency is printed using intaglio presses that generate a printing pressure of $8 \times 10^4$ lbs/in$^2$. A \$20 bill is 6.1in by 2.6in. Calculate the magnitude (in Newtons) of force that the printing press applies to one of the sides of the bill.
(1 N $\approx$ 4.448 lbs) *Note: lb $\neq$ lbs

(1 meter = 39.3701 inches)

**Idea:**
This is just a direct example of the pressure definition equation for a given area and pressure. The only tricky aspect is using units.

**Setup & solution**

We start by converting everything into S.I. units:

$$P = 8 \times 10^4 \frac{\text{lbs}}{\text{in}^2} \cdot \frac{1 \text{ N}}{4.448 \text{ lbs}} \cdot \frac{(39.3701)^2 \text{in}^2}{\text{m}^2} = 2.79 \times 10^7 \text{N/m}^2$$

$$A = 6.1 \text{ in } \times 2.6 \text{ in } = 15.89 \text{ in}^2 \cdot \frac{\text{m}^2}{(39.3701)^2 \text{in}^2} = 0.010252 \text{m}^2$$

Using our pressure equation:

$$\vec{P} = \frac{\vec{F}}{A} \implies \vec{F} = \vec{P} A = (2.79 \times 10^7 \text{N/m}^2)(0.010252 \text{m}^2) \approx \boxed{2.86 \times 10^5 N}$$

**Question 4**

Tom hanks is lost at sea in the movie "Castaway" and wants to travel with whatever belongings he has. He decides to build a raft that has dimensions 3m wide by 4m long by 0.5m tall. The raft is square, made entirely of wood with density of roughly 400 kg/$m^3$. Answer the following:

- (a) What is the mass of the raft?
- (b) If the raft was put into water and it floats, what volume of water is displaced? ($\rho_w \approx 1000kg/m^3$)
- (c) How much of the raft is submerged under water? i.e. how far does the water come up on the raft's 0.5m sides?

**Idea:**
This problem emphasizes the idea that buoyant force is proportional to the amount of volume of liquid displaced and density. The raft will displace a certain amount of water and the water will exert a force back onto the raft (Archimedes principle). But at a certain amount of water displacement the buoyant force will be great enough to keep the raft afloat.

**Setup & solution**

**(a)** Since we desire the mass and know the volume of the raft we can use the density equation:

$$\rho = \frac{m}{V} \implies m = \rho * (L * W * H) = (400)(3 * 4 * 0.5) = \boxed{2400 \text{ kg}}$$

**(b)** The raft floats so that means the net force is zero! Thus we can use the sum of the forces in the y-direction to solve for the buoyant force which will get us the amount of water displaced:

$$\overrightarrow{F_{net}} = \sum \vec{F}_i = F_b - W = 0 \implies F_b = W \implies \rho_w g V = mg$$

Canceling g and solving for V gets us:

$$V = \frac{m}{\rho_w} = \frac{1600 \text{ kg}}{1000 \text{ kg/m}^3} = \boxed{2.4 \ \ \text{m}^3}$$

**(c)** This one is slightly trickier. You first realize that as you lower the raft the area on the bottom of the raft displaces water, then the rest of the displacement happens in the direction of the sides of the raft. We'll call the height it reaches up on the side of the raft "z". So to do this:

$$V_{displaced} = 1.6\text{m}^3 = A_{raft} \times z \implies z = \frac{1.6\text{m}^3}{A} = \frac{1.6\text{m}^3}{3m * 4m} \approx \boxed{0.2\text{m}}$$

**Question 5**

A patient in a hospital has been diagnosed with Aortic Stenosis (when the aortic valve does not close properly). Doctors need to know how fast blood travels through the valve when it should be closed. An echo-cardiogram was performed showing the diameter of heart before the valve to be 3cm with velocity 0.34 m/s. The patient will become critical if the aortic velocity passes 4.0 m/s. What must be the diameter of the aortic valve if the doctors should fear the patient going critical?

**Idea:**
This problem incorporates real medical procedures to determine a semi-realistic scenario. It is an application of the continuity equation knowing all but one of the variables. $A_1$ and $v_1$ are the cross sectional area and velocity in the tube before the valve and $A_2$ and $v_2$ are in the valve itself.

**Setup & solution**
Using the continuity equation and solving for $A_2$:

$$A_1 v_1 = A_2 v_2 \implies A_2 = \frac{A_1 v_1}{v_2} = \frac{\pi (d_1/2)^2 (0.34 m/s)}{v_1} = \frac{\pi (0.015 m)^2 (0.34 m/s)}{(4 m/s)} \approx 6 \times 10^{-5} \text{m}^2$$

Solving for the diameter of the valve:

$$A_2 = \pi \left(\frac{d}{2}\right)^2 \implies d = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{4(6 \times 10^{-5} \text{ m}^2)}{\pi}} = 0.00875\text{m} = \boxed{8.75 \text{ mm}}$$