Question 1:

Idea:
Since we have a collision, momentum will transfer from the bullet to the block to make the block have momentum. Since the bullet will be embedded into the block we know this mass addition needs to be accounted for.

Setup & solution

(a) Since we say the loss due to heating is **almost** zero that means we do lose some Kinetic Energy! Since $\Delta KE \neq 0$ we have an $\boxed{\text{Inelastic Collision}}$

(b) Before the bullet strikes the block is at rest and the bullet is moving, so:

$$\overrightarrow{P_{total_i}} = \vec{p_{bl}} + \vec{p_b} = 0 + \vec{p_b} = \boxed{\mathrm{m}_b \vec{v}_{b_i}}$$

(c) After the bullet strikes the two objects stick together and become one object. They have a combined final velocity

$$\overrightarrow{P_{total_f}} = \vec{p_{bl}} + \vec{p_b} = m_{bl}\vec{v}_f + m_b\vec{v}_f = \boxed{(\mathrm{m}_{bl} + m_b)\vec{v_f}}$$

(d) The total momentum is **always** conserved. That's true in general for **everything in the universe**! It can however transfer from object to object, and so in our case we ignore any momentum transferred to friction. Thus we can set (b) and (c) equal to each other.

$$\begin{aligned} \overrightarrow{P_{total_i}} &= \overrightarrow{P_{total_f}} \\ \implies m_b\overrightarrow{v_{b_i}} &= (m_{bl} + m_b)\vec{v_f} \\ \implies \vec{v_f} &= \frac{m_b\overrightarrow{v_{b_i}}}{(m_{bl} + m_b)} \end{aligned}$$

Since $\overrightarrow{v_{b_i}}$ and $\vec{v_f}$ point in the same direction we don't have to worry about signs. Thus

$$\vec{v_f} = \frac{m_b\overrightarrow{v_{b_i}}}{(m_{bl} + m_b)} = \frac{(0.0042kg)(762m/s)}{(0.0042kg + 10kg)} = \boxed{0.3199 \text{ m/s}}$$

Question 2

Idea:
The momentum of the cannon will contribute to the overall momentum of the truck. But we can only apply conservation of momentum in 1 dimension. So we should choose the x-direction for the conservation of momentum before and after the cannon fires

Setup & solution

(a) First the T-shirt doesn't move, so the only momentum is that of the tr/person/T-shirt mass.

$$\overrightarrow{P_{total_i}} = m_{total}\vec{v}_{tr_i} = \boxed{(\mathrm{m}_s + m_{tr})\vec{v}_{tr_i}}$$

(b) After the cannon fires we need to find out how much momentum the cannon contributes in the x-direction momentum of the shirt:

Using trig:

$$p_{s_x} = \cos(\theta)|p_s|$$

Thus (for +x in the direction of the truck)

$$\begin{aligned}\overrightarrow{P_{total_f}} &= \vec{p_s} + \overrightarrow{p_{tr_g}} \\ &= -\cos(\theta)|p_s| + m_{tr}v_{tr_f} \\ &= \boxed{\text{-}\cos(\theta)m_s|v_s| + m_{tr}v_{tr_f}}\end{aligned}$$

Notice the momentum of the T-shirt is negative as it points opposite to the truck's motion.

(c) Setting these momentum's equal and solving for the v_{tr_f}

$$\begin{aligned} &\overrightarrow{P_{total_i}} = \overrightarrow{P_{total_f}} \\ \Longrightarrow\ &(m_s + m_{tr})v_{tr_i} = -\cos(\theta)m_s|v_s| + m_{tr}v_{tr_f} \\ \Longrightarrow\ &(m_s + m_{tr})v_{tr_i} - m_{tr}v_{tr_f} = -\cos(\theta)m_s|v_s| \\ \Longrightarrow\ &m_s v_{tr_i} + m_{tr}v_{tr_i} - m_{tr}v_{tr_f} = -\cos(\theta)m_s|v_s| \\ \Longrightarrow\ &+m_{tr}v_{tr_i} - m_{tr}v_{tr_f} = -m_s v_{tr_i} - \cos(\theta)m_s|v_s| \\ \Longrightarrow\ &m_{tr}(v_{tr_i} - v_{tr_f}) = -m_s v_{tr_i} - \cos(\theta)m_s|v_s| \\ \Longrightarrow\ &m_{tr} = \frac{-m_s v_{tr_i} - \cos(\theta)m_s|v_s|}{(v_{tr_i} - v_{tr_f})}\end{aligned}$$

So

$$m_{tr} = \frac{-m_s v_{tr_i} - \cos(\theta)m_s|v_s|}{(v_{tr_i} - v_{tr_f})} = \frac{(-1.2kg)(2m/s) - \cos(45°)(1.2kg)(15m/s)}{(2m/s) - (2.5m/s)} \approx \boxed{30\text{kg}}$$

Question 3

Idea:
This is a direct application of the center of mass equation. We only need to write it out explicitly to see that we don't need the mass of the planks to solve this

Setup & solution

The center of mass equation for three objects is:

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} \implies = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \tag{1}$$

We since all the $m_i's$ are exactly the same, the equation reduces:

$$x_{cm} = \frac{m x_1 + m x_2 + m x_3}{3m} = \frac{m(x_1 + x_2 + x_3)}{3m} = \frac{(x_1 + x_2 + x_3)}{3}$$

Now we determine the center of masses for each of planks

$$x_1 = A + L_1/2 = 0.875m$$
$$x_2 = B + L_2/2 = 1.15m$$
$$x_3 = C + L_3/2 = 1.35m$$

So we have

$$x_{cm} = \frac{(x_1 + x_2 + x_3)}{3} = \frac{(0.875m) + (1.15m) + (1.35m)}{3} = \boxed{1.125\text{m}}$$

Question 4

Idea:
This problem will require us to use the rotational kinematics and the conversions between rotational and linear variables.

Setup & solution

(a) For each revolution the car travels 2π radians. That means we can figure the number of radians by

$$\Delta\theta = (227.4 \text{ rev})(2\pi\frac{\text{rad}}{\text{rev}}) = \boxed{1429 \ \ \text{rad}}$$

The linear distance is given by the **circumference** of the wheel (variable "s" for arc length). After all, each full rotation moves the car $2\pi r$ meters. Thus

$$\Delta x = S = \Delta\theta * r = (1429 \text{ rad})(0.7m) = \boxed{1000\text{m}}$$

(b) Using the linear/angular velocity connection:

$$v_f = \omega_f r \implies \omega_f = \frac{v_f}{r} = \frac{150m/s}{0.7m} \approx \boxed{214 \text{ rad/s}}$$

(c) Using one of the angular kinematic equations

$$\begin{aligned} \omega_f^2 &= \omega_i^2 + 2\alpha\Delta\theta \\ \implies \omega_f^2 &= 0 + 2\alpha\Delta\theta \\ \implies \alpha &= \frac{\omega_f^2}{2\Delta\theta} \end{aligned}$$

So

$$\alpha = \frac{(214\frac{\text{rad}}{s})^2}{2 * (1429 \text{ rad})} \approx \boxed{16 \text{ rad/s}^2}$$

(d) We can figure out the time it took the tire to spin all of its revolutions and that's be the exact same amount of time it takes for the car to move its entire path!

$$\begin{aligned} \omega_f &= \omega_i + \alpha t \\ \implies t &= \frac{\omega_f - \omega_i}{\alpha} \\ \implies t = \frac{\omega_f - 0}{\alpha} &= \frac{214 \text{ rad/s}}{16 \text{ rad}/s^2} \approx \boxed{13.4 \text{ s}} \end{aligned}$$

Question 5

Idea:
We use our understanding of Earth's rotation properties and some relative velocity ideas

Setup & solution

(a) The earth rotates it's entire circumference in roughly 24hr or 86,400 seconds. Thus the earth goes 2π radians in 24hrs so:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{86,400s} = \boxed{7.3\times10^{-5}\text{rad}/s}$$

(b) Using the linear to angular conversion:

$$v = \omega r = \omega R_E = (7.3 \times 10^{-5}\text{rad}/s)(6.38 \times 10^{6}m) \approx \boxed{464 \text{ m/s}}$$

(c) Because the people on earth are also rotating at the velocity! The relative velocity between the earth and humans is zero! The same goes for planes