Question 1:

Setup & solution

(a) We readily identify the equation for work as:

$$W = F * \Delta x * cos(\theta) \implies F = \frac{W}{\Delta x \cos(\theta)} = \frac{400J}{(5m)\cos(30°)} \approx \boxed{92.4 \text{ N}}$$

(b) The equation for power output is:

$$P = \frac{W}{\Delta t} = \frac{400J}{2-0} = \boxed{200\text{W}}$$

(c) Since we know that there's only one force being applied to the pallet in the x-direction that tells us we can use Newton's 2nd Law to get mass.

$$F = ma \implies m = \frac{F}{a}$$

The pallet is moving in one direction so we identify kinematics here to get acceleration:

X
$V_o = 0$
$V_f = 4.62m/s$
$a = ?$
$t = 2s$
$\Delta x = 5m$

Based on our chart:

$$V_f = V_o + at \implies a = \frac{V_f}{t} \approx 2.31m/s^2$$

Thus:

$$F = \frac{92.4N}{2.31m/s^2} = \boxed{40\text{kg}}$$

Question 2:

Idea:
We use the conservation of energy here. Energy is always conserved all throughout the path of any object (assuming no non-conservative forces like friction etc). So we can look at all the energy at two convenient points along the child's path. Additionally, potential energy is all relative, we can **choose** where to put potential energy =0 so long as we stay consistent throughout our work.

Setup & solution

(a) At the top of the slide, right before the child moves, the total energy of the child will be:

$$E_{total} = PE + KE = mgh + \frac{1}{2}mv^2$$

But since the child isn't moving v=0:

$$E_{total} = \boxed{\text{mgh}}$$

(b) At the bottom of the slide we let potential energy =0 be setting h=0 there. The child will have some speed right before they fly off so:

$$E_{total} = PE + KE = mgh + \frac{1}{2}mv^2 \implies E_{total} = \frac{1}{2}mv^2$$

(c) We know (assuming no non-conservative forces) the sum of all the energy must add up to be the same throughout the path so if we compare all the energy at the top of the slide to all the energy at the bottom of the slide we know they're equal:

$$E_{total} = PE_i = KE_f \implies mgh = \frac{1}{2}mv^2 \implies h = \frac{1}{2}\frac{v^2}{g} = \frac{(5m/s)^2}{2 * 9.81m/s^2} \approx \boxed{\text{1.3m}}$$

(d) Since we don't ignore friction we know that friction will steal some energy away from our total energy at the end. But if we sum up all the energy in the system before and afterwards it should still be the same so long as we include the energy loss due to friction. Writing this mathematically would be:

$$E_{total_i} = E_{total_f} = E_{child_f} + E_{friction}$$

So we see:

$$E_{friction} = E_{total_i} - E_{child_f} = mgh - \frac{1}{2}mv^2 = (20kg)(9.81)(1.3m) - \frac{1}{2}(20kg)(4m/s)^2 \approx 95J$$

Since Friction always does work in the opposite direction, it does **negative work** so

$$E_{friction} = \boxed{\text{-95J}}$$

Question 3:

Idea:
We can solve motion problems like this using kinematics, but you can also use conservation of energy! This problem uses both methods combined. We consider the two convenient points where the projectile has just left the barrel of the launcher and when the projectile is at the tip of its journey where it has no velocity.

Setup & solution

(a) Using h=0 at the point where the projectile is launched, we look at all the energy right after the ball is launched and right as it reaches the peak of its path.

$$E_{total_i} = mgh + \frac{1}{2}mv^2 = 0 + \frac{1}{2}mv^2$$

$$E_{total_f} = mgh + \frac{1}{2}mv^2 = mgh + 0$$

Conservation of energy tells us:

$$\frac{1}{2}mv^2 = E_{total_i} = E_{total_f} = mgh \implies h = \frac{1}{2}\frac{v^2}{g} = \frac{1}{2}\frac{(18.0m/s)}{(9.81m/s^2)} = \boxed{\text{16.5 m}}$$

(b) Air resistance is another non-conservative force which takes energy away from the projectile as it moves along its path. The sum total of all energy will still be the same but we'll have a new term in E_{total_f} which accounts for friction. Writing this mathematically:

$$E_{total_i} = E_{total_f} = E_{projectile_f} + E_{friction}$$

So:

$$E_{friction} = E_{total_i} - E_{projectile_f} = \frac{1}{2}mv^2 - mgh = \frac{1}{2}(0.75kg)(18.0m/s)^2 - (0.75kg)(9.81)(11.8m) \approx 35J$$

Since friction always opposites motion we say it does negative work:

$$W_{friction} = \boxed{\text{-35 J}}$$

Question 4:

Idea: Both test dummies will experience the same impulse when the car suddenly stops, but the amount of time it takes for the car to apply the force differs.

Setup & solution

(a) There are many ways of thinking about this, but I'll highlight two. The first way is to realize that airbags protect the person driving, and the damaging physical quantity is the amount of force applied, so airbags reduce the applied force. The "proper/mathematical" way would be to realize that the initial impulse on the driver is the same with or without an airbag. So adding an airbag means you increase the amount of time applied to the person for the same impulse. Thus:

$$J = F\Delta t$$

If J must stay constant and we increase Δt F must necessarily decrease. Thus the force is reduced.

(b) Both test dummies will have the same initial change in momentum but the time applied of the applied force will differ. We know:

$$\Delta t_A = 6\Delta t_B$$

We can find out the ratio by solving impulse for force:

$$J = F\Delta t \implies F = \frac{J}{\Delta t}$$

Taking the appropriate ratio:

$$\frac{F_B}{F_A} = \frac{\left(\frac{J}{\Delta t_B}\right)}{\left(\frac{J}{\Delta t_A}\right)} = \frac{\Delta t_A}{\Delta t_B} = \boxed{6}$$

(c) Yes! You need them to protect you otherwise you could take 6 times the force!

Question 5:

Idea:
This problem will require the use of conservation of momentum where the initial point is before the fuel is ejected and the final point is after the fuel is burned.

Setup & solution

(a) In the beginning before the fuel is ejected we look at the momentum for both **in the x-direction**:

$$\vec{p}_{rocket_i} = (m_{r_i} + m_{fuel})\vec{v}_{r_i} = (1.5 \times 10^5 kg + 2000kg)(15,000m/s) = \boxed{2.28{\times}10^9 kg\frac{m}{s}}$$

(b) After the fuel is expended it "pushes" off the rocket propelling the rocket forward while the fuel itself flies away This means the fuel the flies **backwards**! So it should have a negative momentum if we say the direction of the ship is positive.

$$\vec{p}_{fuel_f} = -m_{fuel}v_{fuel_f} = -(2000kg)(110,000m/s) \approx \boxed{\text{-}2.2{\times}10^8 kg\frac{m}{s}}$$

(c) We apply the conservation of momentum to find the final speed of the rocket:

$$\vec{p}_{total_i} = \vec{p}_{total_f} \implies \vec{p}_{r_i} = m_{r_f}\vec{v}_{r_f} + \vec{p}_{fuel_f} \implies \vec{v}_{r_f} = \frac{\vec{p}_{r_i} - \vec{p}_{fuel_f}}{m_r}$$

So:

$$\vec{v}_{r_f} = \frac{\vec{p}_{r_i} - \vec{p}_{fuel_f}}{m_r} = \frac{2.28 \times 10^9 - (-2.2 \times 10^8)}{1.5 \times 10^5 kg} \approx 16,666 \text{ m/s or } \boxed{16.7 \text{ km/s}}$$