Question 1

The surface temperature of the sun is roughly $6000K$. The sun has a radius of 695,508 km and can be approximated as a perfect black body ($\implies e = ?$).

(a) What is the amount of energy radiated from the sun in 2 second?

(b) What is the amount of net power radiated from the sun if the surrounding space has a temperature of roughly $3K$?

$\sigma = 5.67 \times 10^{-8} \mathrm{J}/(\mathrm{s} m^2 K^4\ )$

Setup & solution

(a) For the first part we just apply the radiative energy law from Chapter 13 where we note that the sun is approximately a blackbody so $e \sim 1$:

$$\begin{aligned} Q &= e\sigma T^4 At \\ &= (1)(5.67 \times 10^{-8})(6000)^4(4\pi(695508 \times 10^3)^2)(2) \\ &\approx \boxed{8.93 \times 10^{26} J} \text{ or } \boxed{8.93 \times 10^{14} TJ} \end{aligned}$$

(b) For net power we just subtract the power going into the star from the power going out of the star. The surrounding space has a radiative temperature of 3K so it will produce some tiny amount of power going into the sun. So

$$\begin{aligned} P_{net} &= e\sigma A(T^4 - T_o^4) \\ &= (1)(5.67 \times 10^{-8})(4\pi(695508 \times 10^3)^2)((6000)^4 - 3^4) \\ &\approx \boxed{4.47 \times 10^{26} W} \text{ or } \boxed{4.47 \times 10^{14} TW} \end{aligned}$$

(c) The amount of energy output by the sun in 1 second is $4.47 \times 10^{14} J$! Comparing that to the total amount of energy ever produced by nuclear bombs we have:

$$\frac{E_{sun}}{E_{nuke}} = \frac{4.47 \times 10^{14} TJ}{2,135,000 TJ} = 2.09 \times 10^8$$

So in 1 second the sun outputs 200 million times more energy than all the bombs ever detonated on earth since 1996!

Question 2

A container holds 2.0 moles of gas. The total average potential energy of the gas molecules in the container is equal to the kinetic energy of an 8×10^{-3}kg bullet with a speed of 770 m/s. What is the temperature of the gas in Kelvin?

R = 8.31446261815324 J/(mol K)
$N_A = 6.022 \times 10^{23} \text{mol}^{-1}$
k = $1.38 \times 10^{-23} J/K$

Setup & solution

First we find the energy of the bullet:

$$KE_{bullet} = \frac{1}{2}mv^2 = \frac{1}{2}(8 \times 10^{-3}kg)(770m/s)^2 = 2371.6J$$

Then we relate temperature to energy using the average kinetic energy **per particle** in an ideal gas:

$$\bar{KE} = \frac{3}{2}kT$$

Since we have "N" particles in this gas we have:

$$KE_{tot} = \frac{3}{2}NkT$$

where KE_{tot} will be equal to the amount of energy of the bullet.

Since we do not know how many particles we have we use the definition of moles:

$$n = \frac{N}{N_A} \implies N = N_A n$$

where N_A is Avogadro's number:
Thus:

$$KE_{bullet} = \bar{KE}_{tot} = \frac{3}{2}N_A nkT \implies T = \frac{2(KE_{bullet})}{3N_A nk}$$

Plugging in:

$$T = \frac{2 * (2371.6J)}{3(6.022 \times 10^{23}\text{mol}^{-1})(2 \text{ mol})(1.38 \times 10^{-23}J/K)} = \boxed{95.1\text{K}}$$

(Alternatively you could've set the internal energy of the gas equal to that of the bullet energy, solve for temperature and you'll get the same answer)

Question 3

A tube has a length of 0.015m and a cross-sectional area of 7.0×10^{-4}m. The tube is filled with a solution of sucrose in water. The diffusion constant of sucrose in water is $5.0 \times 10^{-10}\ m^2/s$. A difference in concentration of 3.0×10^{-3} kg$/m^3$ is maintained between the ends of the tube. How much time is required for the 8.0×10^{-13}kg of sucrose to be transported through the tube?

Setup & solution

This is basically a plug and chug using the Chapter 14 diffusion equation:

$$m = \frac{DA(\Delta Concentration)t}{L} \implies t = \frac{mL}{DA(\Delta Concentration)}$$

Plugging in:

$$t = \frac{mL}{DA(\Delta Concentration)} = \frac{(8 \times 10^{-13}kg)(0.015m)}{(5 \times 10^{-10}m^2/s)(7 \times 10^{-4}\ \mathrm{m}^2)(3 \times 10^{-3}kg/m^3)} \approx \boxed{11.4\ \ \mathrm{sec}}$$

Question 4

A monatomic ideal expands at a **constant pressure** supplied by an input energy Q. What percentage of the heat being supplied to the gas is used to increase the internal energy of the gas? (This can be solved symbolically)
Hint: 2nd Law of Thermodynamics and find particular ratio **Hint Hint:** When a gas undergoes a constant

temperature change with added heat, the final temperature is less than the initial temperature

Setup & solution

The physics understanding is that since we are adding heat to a gas and we see the gas maintaining a constant pressure this must imply the gas is expanding and the internal energy of the gas must be changing. Since the gas is expanding work must be being done so we can relate $W, \Delta U$ and Q by the 2nd Law of Thermodynamics:

$$\Delta U = Q - W$$

Now, the question is asking for the percentage of heat being supplied to the gas that changes its internal energy ΔU, this is an equivalent statement to "what is $\Delta U/Q$?".

So dividing by Q:

$$\frac{\Delta U}{Q} = 1 - \frac{W}{Q}$$

For any monatomic ideal gas at **constant pressure** we have:

$$W = -\frac{3}{2}nR\Delta T, \quad C_P = \frac{5}{2}R, \quad , \quad Q = C_P n\Delta T$$

Now, when a gas expands adiabatically (no heat transfers to the system) this means we do **positive** work. You can justify this by saying that since the gas is expanding, the final temperature must cool down to keep the pressure constant i.e. $T_f{<}T_o$ so $\Delta T \implies -\Delta T$. We can plug these in:

$$\frac{\Delta U}{Q} = 1 - \frac{W}{Q} = 1 - \frac{\frac{3}{2}nR\Delta T}{C_P n\Delta T} = 1 - \frac{3}{2}\frac{R}{C_P} = 1 - \frac{3}{5} = \boxed{2/5} \text{ or } \boxed{40\%}$$

Question 5

The wattage of a commercial ice maker is 225W and is the rate at which it does work. The ice maker operates just like a refrigerator or an air conditioner and has a coefficient of performance of 3.6. The water going into the unit has a temperature of $15.0°C$, and the ice maker produces ice cubes at $0.0°C$. Ignoring the work needed to keep stored ice from melting, find the maximum amount (in kg) of ice that the unit can produce in one day of continuous operation.

$c_w = 4180$ J/(kg K)
$L_f = 3.33 \times 10^5$J/kg (latent heat of fusion water)

Setup & solution

We look at the refrigerator equation and see what we need:

$$\frac{|Q_C|}{|W|} = \text{refrigerator coefficient} := \text{r.c.}$$

If a fridge is turning $15.0°C$ water to $0.0°C$ ice we can treat this part just like a phase change problem. So we will need to drop the water down to $0.0°C$ water then bring it to $0.0°C$ ice:

$$Q_{water} = mc_w\Delta T, \qquad Q = mL_f$$

So the total amount of energy we need to **remove** in order to turn a mass "m" of water into ice will be (careful of signs):

$$Q_{total} = mc_w\Delta T - mL_f = m(c_w\Delta T - L_f)$$

Now this is amount of heat we need to **remove** from the water to turn it to ice. Thus Q_{total} is the rejected heat out of the system (Q_C) because we need do work to pull the heat and force it into a higher temperature.

The last piece of the puzzle is figuring out how much work we can do in a day. This is done by realizing the ice maker can do 225W in 24hrs so by the definition of power we can find the number of Joules of work that can be done.

$$P = \frac{W}{t} \implies W = Pt$$

Thus we plug in for Q_C and W

$$\begin{aligned}
r.c. &= \frac{|Q_C|}{|W|} \\
\implies |Q_C| &= |W| * (r.c.) \\
\implies m|(c_w\Delta T + L_f)| &= |Pt| * (r.c.) \\
\implies m &= \left|\frac{Pt(r.c.)}{(c_w\Delta T - L_f)}\right|
\end{aligned}$$

Plugging in gets us:

$$m = \left|\frac{Pt(r.c.)}{(c_w\Delta T + L_f)}\right| = \left|\frac{(225W)(86400 \text{ sec})(3.6)}{(4180\text{J/kg K})(-15.0°C) - (3.33 \times 10^5 \text{ J/kg})}\right| \approx \boxed{200 \text{ kg}}$$