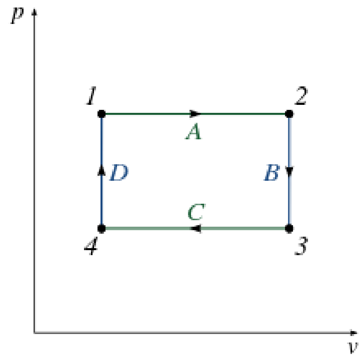
Question 1

For the P-V diagram below, answer the following:

(1) For each path (A-D) name the type of thermodynamic process

(2) Consider path C. If at point 3 the pressure was 130,000 Pa and volume 0.226923m^3, what is the work done on the system if at point 4 the volume is 0.15m^3?

(3) What is the work done on the system in path D?



Setup & solution

(1) For each path we associate one of 4 possible words with the thermodynamic process: Isobaric (Constant Pressure), Isochoric (Constant Volume), Isothermal (Constant Temperature), Adiabatic (No external heat added). Looking at our graph's x and y axes gets us:

- Path A: Isobaric
- Path B: Isochoric
- Path C: Isobaric
- Path D: Isochoric

(2) For a isochoric process we have that the work done for a change in volume is:

$$W = P\Delta V = P(V_f - V_o) = (130,000Pa)(0.15m^3 - 0.226923m^3) \approx \boxed{\text{-10,000 J}}$$

(3) Since there is no area under the graph of our P-V diagram on path D we have no work. so

$$\boxed{\text{W = 0J}}$$

Question 2 pt1

Light is an electromagnetic wave and travels at a speed (in space) of **exactly** 299,792,458 m/s. The human eye is most sensitive to yellow-green light, which has a wavelength of 5.45×10^{-7}m. What frequency of light is this?

Setup & solution

This is a direct example of how a wave's speed and wavelength are related. It's important to know that this relationship applies for all waves. We note that the speed of light so special so we give it the special character "c":

$$c = \lambda f \implies f = \frac{c}{\lambda} = \frac{299,792,458m/s}{5.45 \times 10^{-7}m} = \boxed{5.501\ \times 10^{14}\text{Hz}}$$

Question 2 pt2

The middle C string on a piano is under a tension of 944N. The period and wavelength of a wave on this string are 3.82ms and 1.26m, respectively. Find the linear density of the string.

Setup & solution

This is an application of the string with tension equation in addition to the wave speed relation:

$$v = \lambda f = \frac{\lambda}{T} = \sqrt{\frac{F}{m/L}} \implies m/L = F\left(\frac{T}{\lambda}\right)^2 = (944N)\left(\frac{0.00382s}{1.26m}\right)^2 = \boxed{0.00868 \text{ kg/m}}$$

Question 3

When one person shouts at a football game, the sound intensity level at the center of the field is 60.0 dB. When all the people shout together, the intensity level increases to 109 dB. Assuming that each person generates the same sound intensity at the center of the field:

(a) How many people are at the game?

(Note: dB are **not** on a linear scale i.e. 2 people scream at 2 dB does **not** imply 4 people scream at 4 dB)

(b) What is the total power emitted from the stadium 1.1 mile away in terms of the reference intensity I_o? (assuming negligible loss of power due to buildings, air etc)

1 mile = 1.609344 km

Setup & solution

(a) Since we cannot just do a direct division of 109dB/60dB because the dB scale is not linear, we must first convert our dB numbers into the linear scale of Intensity and then use those to determine the number of people.

$$\beta = (10dB)\log_{10}\left(\frac{\mathrm{I}}{\mathrm{I_o}}\right) \implies I = I_o * 10^{\left(\frac{\beta}{10}\right)}$$

1 person

$$I_1 = I_o * 10^{\left(\frac{\beta}{10}\right)} = I_o * 10^6$$

"N" people

$$I_N = I_o * 10^{\left(\frac{\beta}{10}\right)} = I_o * 10^{10.9}$$

So the number of people will be given by:

$$N = \frac{I_N}{I_1} = \frac{10^{10.9}}{10^6} = 10^{4.9} = \boxed{79{,}433 \text{ people}}$$

(b) The power output from an intensity is given by:

$$I = \frac{P}{A}$$

Since we assume the sound waves travel in all directions radially we can say:

$$A = 4\pi r^2$$

Thus:

$$I_N = \frac{P}{4\pi r^2} \implies P = I_N(4\pi r^2) = I_o * (10^{10.9})4\pi(1.1 \text{ miles } * 1609.344\frac{m}{\text{miles}})^2 \approx \boxed{(3.128 \times 10^{18})I_o}$$

Question 4

Sound exits a diffraction horn loudspeaker through a rectangular opening like a small doorway. Such a loudspeaker is mounted outside on a pole. In winter, when the temperature is $273°K$, the diffraction angle θ has a value $15.0°$. What is the diffraction angle for the same sound on summer day when the temperature is $311°K$.

Hint: Assume air to be an ideal gas and that frequency of the waves doesn't change.

Setup & solution

Since the diffraction opening is not a circular we can appeal to the regular diffraction equation:

$$\sin\theta = \frac{D}{\lambda}$$

Since we are testing this on the same doorway we know that D (the diffraction opening) is the same, but the wavelength λ and θ may not be between the two cases. In fact, as the air gets colder we expect the speed of sound to change as the air (an assumed ideal gas) temperature changes. We can then relate the speed of a sound wave at a temperature by:

$$v = \sqrt{\frac{\gamma k T}{m}} = T^{1/2}\sqrt{\frac{\gamma k}{m}}$$

Now because our diffraction equation has λ in in, we want to relate the speed of the wave to its wavelength. This can be achieved by:

$$v = f\lambda$$

Thus:

$$\lambda = \frac{T^{1/2}}{f}\sqrt{\frac{\gamma k}{m}}$$

Since we know the frequency of the waves emitted won't change we have:

$$\sin(\theta) = \frac{1}{T^{1/2}}\left(\frac{D}{f}\sqrt{\frac{m}{\gamma k}}\right)$$

Where we have put the variables that don't change between days in parenthesis. Of course this implies the assumption that the gas molecule average mass (m) and gamma constant ($\gamma = \frac{C_P}{C_V}$) don't change between days, but if we use the same air just at a different temperature this is admissible.
From here we just write out our cases

$$\begin{cases} \sin(\theta_1) = \frac{1}{T_1^{1/2}}\left(\frac{D}{f}\sqrt{\frac{m}{\gamma k}}\right), & (T_1 = 273°K) \\ \sin(\theta_2) = \frac{1}{T_2^{1/2}}\left(\frac{D}{f}\sqrt{\frac{m}{\gamma k}}\right), & (T_2 = 311°K) \end{cases}$$

We're looking for θ_2 so we can just take the ratio of these equations, cancel the alike terms and solve:

$$\frac{\sin(\theta_2)}{\sin(\theta_1)} = \left(\frac{T_1}{T_2}\right)^{1/2} \implies \theta_2 = \sin^{-1}\left(\sin(\theta_1)\left(\frac{T_1}{T_2}\right)^{1/2}\right) \approx \boxed{16°} \text{ or } \boxed{0.2799 \text{ rad}}$$

Question 5

The fundamental frequencies of two air columns are the same. Column A is open at both ends, while column B is open at only one end. The length of column A is 0.7m. What is the length of column B?

Setup & solution

For Longitudinal standing wave in an open pipe we have:

$$f_n = n(\frac{v}{2L_1})$$

If the frequency is the fundamental $\implies$ n=1. So

$$f_0 = \begin{cases} \frac{v}{2L_1}, & \text{(Open)} \\ \frac{v}{4L_2}, & \text{(Half closed)} \end{cases}$$

Since the speed of sound doesn't change between these pipes and they have the same fundamental we have:

$$\frac{v}{2L_1} = \frac{v}{4L_2} \implies L_2 = \frac{L_1}{2} = \boxed{0.35 \text{ m}}$$

(One could've also realized that the space for a standing wave is cut in two when using a half closed pipe which also implies $L_2 = \frac{L_1}{2}$)