Project 1

MGMTMFE 405

1. Use the Random Number generators discussed in the class to do the following:

(a) Using LGM method generate 10,000 Uniformly distributed random numbers on [0,1] and compute the empirical mean and the standard deviation of the sequence. (b) Now use built-in functions of whatever software you are using to do the same thing as in (a). (c) Compare your findings in (a) and (b) and comment (be short but precise).

```
Q1(a)
Mean = 0.495919
Std = 0.290521


Q1(b)
Mean = 0.501828
Std = 0.289213

Q1(c)
The theoretical mean and standard deviation of uniform distribution is
0.5 and 0.289. Both approaches give good estimations of the theoretical
values.
```

2. Use the numbers of part (a) of question 1 to do the following: (a) Generate 10,000 random numbers with the following distribution: $X = \{ -1 \; with \; probability \; 0.30 \; 0 \; with \; probability \; 0.35 \; 1 \; with \; probability \; 0.20 \; 2 \; with \; probability \; 0.15$ (b) Draw the histogram and compute the empirical mean and the standard deviation of the sequence of 10,000 numbers generated above in part (a).

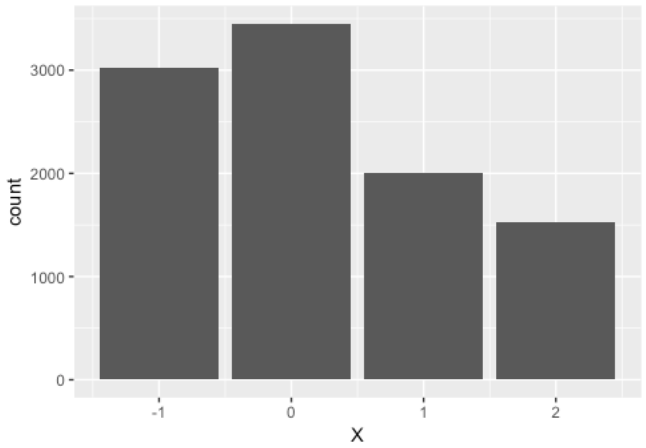

Figure 1. The histogram of the 10,000 random variables generated for Q2.

Q2(b)
Mean = 0.2007
Std = 1.05309

3. Use the idea of part (a) of Question 1 to do the following: (a) Generate 1,000 random numbers with Binomial distribution with $n = 44$ and $p = 0.64$. (Hint: A random variable with Binomial distribution (n, p) is a sum of n Bernoulli (p) distributed random variables, so you will need to generate 44,000 Uniformly distributed random numbers, to start with). (b) Draw the histogram. Compute the probability that the random variable X that has Binomial (44, 0.64) distribution, is at least 40: $P(X \geq 40)$. Use any statistics textbook or online resources for the exact number for the above probability and compare it with your finding and comment.

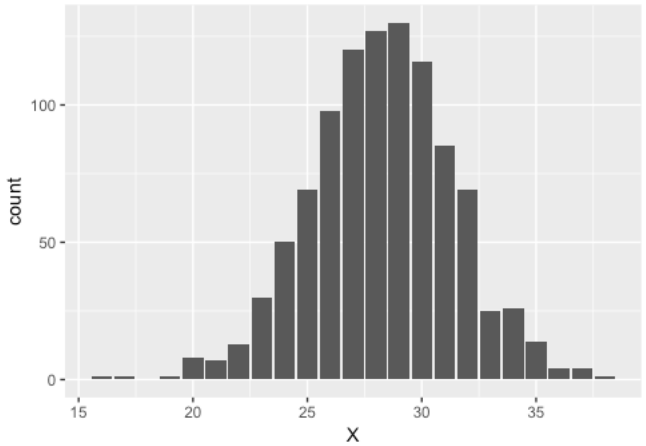

Figure 2. The histogram of the 1,000 random variables generated for Q2.

When sample 1000 times, we did not observe any incident that X > 40. So, I increase the sample size to 1 million, we observe 43 incidents where X > 40. The probability is approximately to be 4.3 *10^(-5).

We also calculate the probability of P(X>= 40)
P (X >= 40) = P(X = 40)+ P(X = 41)+ P(X = 42)+ P(X = 43)+ P(X = 44)
= C(44,40) * 0.64^40 * 0.36*4 + C(44,41) * 0.64^41 * 0.36*3 + C(44,42) * 0.64^42 * 0.36*2 + C(44,43) * 0.64^43 * 0.36*1 + C(44,44) * 0.64^44 * 0.36*0
= 4.028e-5 + 6.98e-6 + 8.87268463e-7+ 7.33658677e-8 + 2.96427748e-9
=0.00004822359

4. Use the numbers of part (a) of question 1 to do the following:(a) Generate 10,000 Exponentially distributed random numbers with parameter $\lambda = 1.5$. (b) Compute $P(X \geq 1)$ and $P(X \geq 4)$. c) Compute the empirical mean and the standard deviation of the sequence of 10,000 numbers generated above in part (a). Draw the histogram by using the 10,000 numbers of part (a).

Q4(b)
P($X \geq 1$) = 0.2206
P($X \geq 4$) = 0.0022

Q4(c)

Mean = 0.655419
Standard deviation = 0.659963

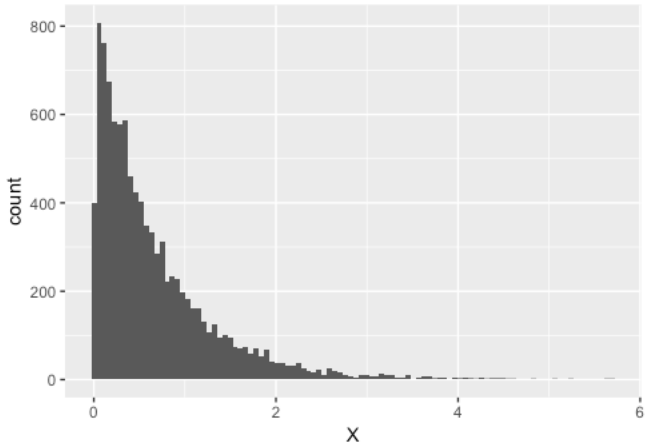

Figure 2. The histogram of the 10,000 random variables generated for Q4.

5. Use the idea of part (a) of Question 1 to do the following: (a) Generate 5,000 Uniformly distributed random numbers on [0,1]. (b) Generate 5,000 Normally distributed random numbers with mean 0 and variance 1, by BoxMuller Method. (c) Compute the empirical mean and the standard deviation of the sequence of numbers generated above of part (b). (d) Now use the Polar-Marsaglia method to do the same as in (b). Note: Here you will not have the same number of random variables as in (b). (e) Compute the empirical mean and the standard deviation of the sequence of numbers generated above of part (d). (f) Now compare the efficiencies of the two above-algorithms, by comparing the execution times to generate 5,000 normally distributed random numbers by the two methods. Which one is more efficient? If you do not see a clear difference, you need to increase the number of generated realizations of random variables to 10,000, 20,000, etc.

Q5 (c): The average value of normal from Box–Muller is -0.00519237
Q5 (c): The variance value of normal from Box–Muller is 0.998601
Q5 (d): The average value of normal from Polar–Marsaglia is 0.00263958
Q5 (d): The variance value of normal from Polar–Marsaglia is 0.969512

The time used for Box–Muller method is 0.000178

```
The time used for Polar-Marsaglia method is 0.000128

The Polar-Marsaglia method is faster than Box-Muller method.
```