Project 5

MGMTMFE 405
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You will need to write codes for all the parts of the project. Make sure the codes work properly and understand the ideas behind each problem below. You may be asked to demonstrate how the codes work, by running them, and interpret the results. Code quality, speed, and accuracy will determine the grades.

1. Consider the following information on the stock of company XYZ: The current stock price is \$40, and the volatility of the stock price is $\sigma = 20\%$ per annum. Assume the prevailing risk-free rate is $r = 6\%$ per year. Use the following method to price the specified option:

(a) Use the **LSMC** method with 100,000 paths simulations (50,000 plus 50,000 antithetic) to price an American put option with strike price of $X = \$40$, maturity of 0.5-years, 1-year, 2-years, and current stock prices of \$36, \$40, \$44. Use the first k of the **Laguerre polynomials** for $k = 2, 3, 4$.

(b) Use the **LSMC** method with 100,000 paths simulations (50,000 plus 50,000 antithetic) to price an American put option with strike price of $X = \$40$, maturity of 0.5-years, 1-year, 2-years, and current stock prices of \$36, \$40, \$44. Use the first k of the **Hermite polynomials** for $k = 2, 3, 4$.

(c) Use the **LSMC** method with 100,000 paths simulations (50,000 plus 50,000 antithetic) to price an American put option with strike price of $X = \$40$, maturity of 0.5-years, 1-year, 2-years, and current stock prices of \$36, \$40, \$44. Use the first k of the **Simple Monomials** for $k = 2, 3, 4$.

(d) Compare all your findings above and comment.

Note: You will need to use weighted-polynomials as done by the authors of the method.

2. Forward-start options have strike prices to be determined at a future date. For example, a forward start put option payoff at maturity is

$$max(S_t - S_T, 0)$$

where the strike price of the put option is S_t. Here $0 \leq t \leq T$.

(a) Estimate the value of the forward-start European put option on a stock with these characteristics: $S_0 = \$65, K = \$60$, $\sigma = 20\%$ per annum, risk-free rate is $r = 6\%$ per year, $t = 0.2$ and $T = 1$.

(b) Estimate the value of the forward-start American put option on a stock with these characteristics: $S_0 = \$65, K = \$60$, $\sigma = 20\%$ per annum, risk-free rate is $r = 6\%$ per year, $t = 0.2$ and $T = 1$. The continuous exercise starts at time $t = 0.2$.

3. ***[Optional-NOT for grading]*** An unfair (biased) die is such that the number k has the probability $k/21$ to come up in every roll, where k is in $\{1, 2, 3, 4, 5, 6\}$. On average how many times does one have to roll the die until all six numbers come up? Answer by using simulations.

4. ***[Optional-NOT for grading]*** Start at 0 on a number line at time 0. Every second you move up by 1 unit with probability 0.75 or down by 1 unit with probability 0.25. If you ever get to -1, the game is over. What is the probability that the game will eventually be over?

5. ***[Optional-NOT for grading]*** Start at 0 on a number line, flip a fair coin, move +2 on heads and -1 on tails. What is the probability that you will eventually hit -1?

6. ***[Optional-NOT for grading]*** *Write a code to answer this question by simulation:*
Two gamblers A and B initially have $7 and $13, respectively. Each time, they throw a fair coin and if the outcome is a head, A will give $1 to B, otherwise, B will give $1 to A. The game is over only if A or B is ruined (the time is infinite). What is the probability that A will win?