Project 2

MGMTMFE 405

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1.

Seed = 523
ρ = -0.680412

2.

Seed = 32958
E = 1.54865

3.
(a)
E(a1) = 4.87354
E(a2) = 1.00596
E(a3) = 1.01891
E(a4) = 1.10454

(b)
For the last three simulation, the expected values are very close to 1. Moreover, we tried ito's lemma on the $e$xp(-t/2)*$cos(Wt)$, and we found it is a martingale. Therefore, the expected value of the function equals to the value at t0, which is 1. Our simulation gives a very reliable approximation of the theoretical value. Moreover, we calculated the variance of the random variables, we found that the variance of random variable increases dramatically with t.

(c) For the variance reduction, we choose the control variates approach. For the first one, we choose the $\boldsymbol{Y = W_t^2}$, where $\boldsymbol{\delta = t}$**.** For the last three, we choose the $\boldsymbol{Y = sinW_t}$, wehre $\boldsymbol{\delta = 0}$. After implemented the control variate methods, the variance reduced dramatically. The result is shown as follow:

Q3.c c1 The VAR Before Reduction is 46.0725

Q3.c c1 The VAR for Q3 c1 now is 0.50218

Q3.c c2 The VAR Before Reduction is 0.120647

Q3.c c2 The VAR now is 0.12082

Q3.c c3 The VAR Before Reduction is 11.3457

Q3.c c3 The VAR now is 0.125195

Q3.c c4 The VAR Before Reduction is 328.888

Q3.c c4 The VAR now is 0.128223

4.
(a)

Q4. The option price is 9.56801

Q4. The variance of random variables is 738.487

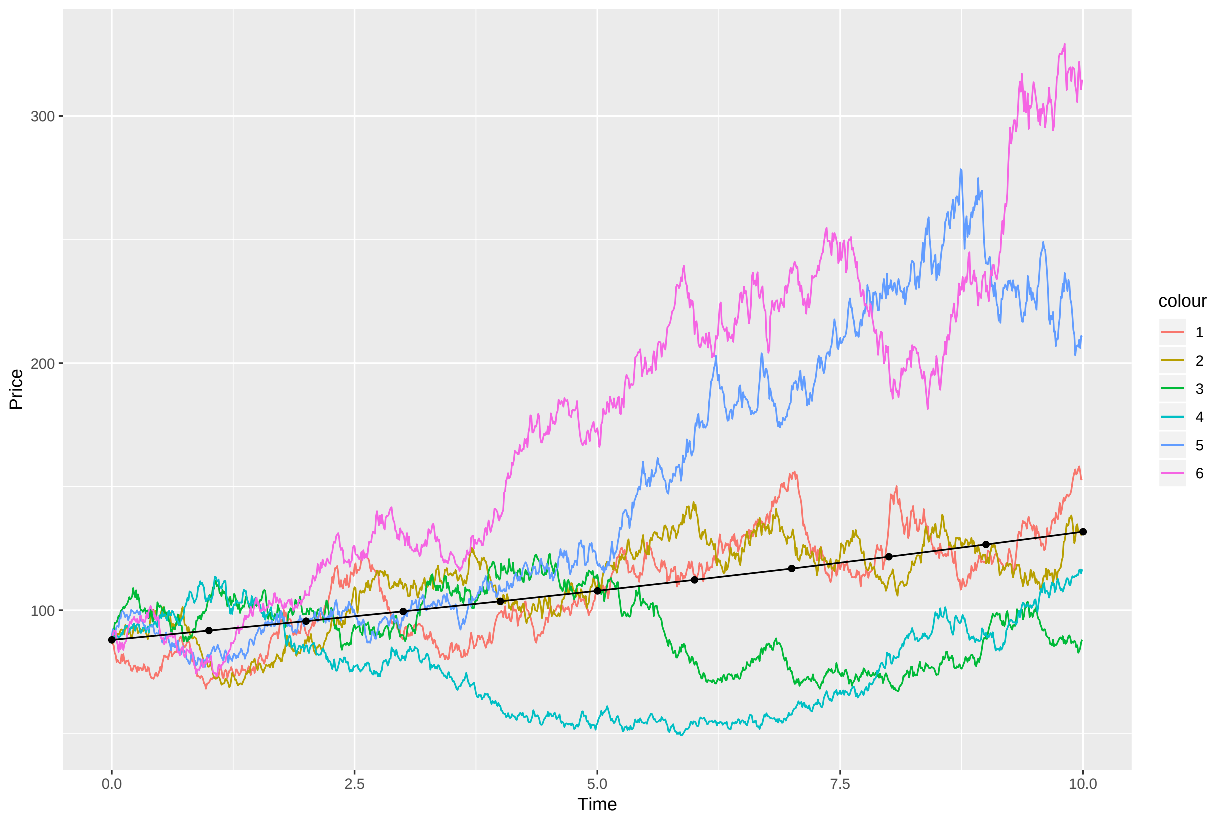
(b)

According the Black-scholes formula, we calculated the price of the call option to be 9.986969. After we applied the antithetic methods:

Q4b. The option price is 9.61165

Q4b. The VAR of random variables is 332.667

5.
(a) The black point line shows the result from part a (E(Sn)). And the colored lines show the results from part b.



(d) if we consider σ from 18% to 35%, the $E(Sn)$ plot theoretically won't change according to the law of large number. However, the six plots will show a higher volatility during the same time period. These plots are expected to be more widely distributed from the $E(Sn)$ plot.

6.

(a) 3.14169

(b) 3.12905

(c) We implemented the importance sampling to estimate of Pi. We choose a function of t(x) as following:

$$t(x) = \begin{cases} \dfrac{1 - a^x}{1 - a/3}, & when\ 0 < \mathrm{x} < 1 \\ 0 & \end{cases}$$

To use the importance sampling, we need to calculate the $\int \frac{f(x)g(x)}{t(x)} t(x)dx$, which is equivalent to calculate the $\frac{f(x)g(x)}{t(x)}$ under the measure of t(x). So, we need to generate the random variables with pdf of t(x). First, I tried to find the inverse function of the CDF of t(x) but I cannot get the inverse function of the CDF of the t(x) distribution, So, I used the acceptance-rejection methods to generate the random variables. Then, we use the new random variables to calculate the integral.

The result is shown as below:

Q6.c the integral for Q6b is: 3.14255

Q6.c the variance is: 0.0726703