Project 5

MGMTMFE 405

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Question 1.
We calculate the America put option price using Least square Monte Carlo

a.
We first use Laguerre polynomials methods.

When X = 36,

	T = 0.5	T = 1	T = 2
K = 2	**3.99196**	**3.98558**	**3.98364**
K = 3	**3.99196**	**3.99107**	**3.95121**
K = 4	**3.99343**	**4.015**	**3.96203**

When X = 40,

	T = 0.5	T = 1	T = 2
K = 2	**1.35695**	**1.39966**	**1.71622**
K = 3	**1.26485**	**1.57019**	**1.80949**
K = 4	**1.24492**	**1.71622**	**1.90717**

When X = 44,

	T = 0.5	T = 1	T = 2
K = 2	**0.47344**	**0.565832**	**0.641593**
K = 3	**0.524406**	**0.733825**	**0.83147**
K = 4	**0.607336**	**0.821674**	**0.956385**

b.
We use Hermit Method
When X = 36,

	T = 0.5	T = 1	T = 2
K = 2	**4.16491**	**4.40598**	**4.73819**
K = 3	**3.99287**	**3.98286**	**4.07541**
K = 4	**3.99397**	**3.99612**	**4.21231**

When X = 40,

	T = 0.5	T = 1	T = 2
K = 2	**1.76303**	**2.27207**	**2.81626**
K = 3	**1.15315**	**1.62925**	**2.39495**
K = 4	**1.22911**	**1.58632**	**2.36915**

When X = 44,

	T = 0.5	T = 1	T = 2
K = 2	**0.620038**	**1.09052**	**1.64863**
K = 3	**0.416165**	**0.703358**	**1.56877**
K = 4	**0.500115**	**1.04984**	**1.51628**

c.

We first use Monomials methods.

When X = 36,

	T = 0.5	T = 1	T = 2
K = 2	**4.16491**	**4.40598**	**4.73819**
K = 3	**3.99381**	**3.98517**	**4.19602**
K = 4	**3.99397**	**3.98464**	**4.2053**

When X = 40,

	T = 0.5	T = 1	T = 2
K = 2	**1.76303**	**2.27207**	**2.81626**
K = 3	**1.22546**	**1.70502**	**2.59013**
K = 4	**1.22546**	**1.79132**	**2.5525**

When X = 44,

	T = 0.5	T = 1	T = 2
K = 2	**0.620038**	**1.09052**	**1.64863**
K = 3	**0.35898**	**0.946587**	**1.56357**
K = 4	**0.502354**	**0.86565**	**1.4867**

d.

In this exercise, we used least square Monte Carlo to calculate the price of the America Put option. We used 100,000 paths for each step with 200 exercise time steps for each path. We choose 200 exercise steps because it is a good compromise between computational time and accuracy.

In the least square regression, we used gaussian elimination method to calculate the regression result. One would expect the option price converges with high order of K, but we did not overserve in our calculation. While This may due to the numerical stability in the regression. For example, in the Laguerre polynomials methods, the number in the A matrix are in the order o 10^{-6} - 10^{-9} . This may lead to some numerical stability issue.

We also compared our LSMC result with the value from binomial approach, we found that monomial approach with K =2 is the best agreement. However, the higher order of the basis function does not give a better estimation of the option price, which may result from the instability of the matrix inverse calculation.

Question 2.

a. 3.15348.

b. 3.41439

We employed least square Monte Carlo Method to calculate the price of the forward-start option with continuous exercise time. We started from t = $T_{n-1,}$ we calculate the continuation value and the exercise value of each time step. We exercise the option if the exercise value is higher than the expected continuation value. We undergo this process recursively until we reach to the t.

In order to calculate the expected continuation value, we regress the realized payoff on the current exercise value. In the traditional least square Monte Carlo methods, we regress the realized payoff on the stock price. However, this approach does not apply to the forward start option pricing. It may because of that the “strike” price varies with each path, therefore the stock price do not have predictability to the future continuation price.

I also expect that the stock price should heavily depends on the accuracy of our prediction on the expected continuation value. If we come up with better model in predicting the future expected continuation value, we would have a higher pricing value for the option.