Project 8

MGMTMFE 405
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You will need to write codes for all the parts of the project. Make sure the codes work properly and understand the ideas behind each problem below. You may be asked to demonstrate how the codes work, by running them, and interpret the results. Code quality, speed, and accuracy will determine the grades.

1. Assume the dynamics of the short-term interest rate, under the risk-neutral measure, are given by the following SDE (**Vasicek Model**) :

$$dr(t) = \kappa(\bar{r} - r(t))dt + \sigma dW_t$$

with $r_0 = 5\%,\ \sigma = 18\%,\ \kappa = 0.82,\ \bar{r} = 5\%$.

(a) Use Monte Carlo Simulation (assume each time step is a day) to find the price of a Pure Discount Bond, with Face Value of \$1,000, maturing in $T = 0.5$ years (at time $t = 0$):

$$P(t,T) = \mathbb{E}_t^*\left[\$1{,}000 * exp\left(-\int_t^T r(s)ds\right)\right]$$

(b) Use Monte Carlo Simulation to find the price of a coupon paying bond, with Face Value of \$1,000, paying semiannual coupons of \$30 each, maturing in $T = 4$ years:

$$P(0,\boldsymbol{C},\boldsymbol{T}) = \mathbb{E}_0^*\left[\sum_{i=1}^{8} C_i * exp\left(-\int_0^{T_i} r(s)ds\right)\right]$$

where $\boldsymbol{C} = \{C_i = \$30 \text{ for } i = 1,2,\dots,7;\text{ and } C_8 = \$1{,}030\}$,
$\boldsymbol{T} = \{T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8\} = \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4\}$.

(c) Use Monte Carlo Simulation to find the price of a European Call option on the Pure Discount Bond in part (a). The option matures in 3 months and has a strike price of $K = \$980$. Use the explicit formula for the underlying bond price (only for the bond price).

(d) Use Monte Carlo Simulation to find the price of a European Call option on the coupon paying bond in part (b). The option matures in 3 months and has a strike price of $K = \$980$. Use Monte Carlo simulation for pricing the underlying bond.

2. Assume the dynamics of the short-term interest rate, under the risk-neutral measure, are given by the following SDE (**CIR model**):

$$dr_t = \kappa(\bar{r} - r_t)dt + \sigma\sqrt{r_t}dW_t$$

with $r_0 = 5\%, \sigma = 18\%,\ \kappa = 0.92,\ \bar{r} = 5.5\%$.

(a) Use Monte Carlo Simulation to find at time $t = 0$ the price $c(t,T,S)$ of a European Call option, with strike price of $K = \$980$, maturity of $T = 0.5$ years on a Pure Discount Bond with Face Value of \$1,000, that matures in $S = 1$ year:

$$c(t,T,S) = \mathbb{E}_t^*\left[exp\left(-\int_t^T r(u)du\right) * \max(P(T,S) - K, 0)\right]$$

(b) Compute the price $c(t,T,S)$ of the European Call option above using the explicit formula, and compare it to your findings in part (a) and comment on your findings.

3. Assume the dynamics of the short-term interest rate, under the risk-neutral measure, are given by the following system of SDEs (**G2++ model**):

$$\begin{cases} dx_t = -ax_t dt + \sigma dW_t^1 \\ dy_t = -by_t dt + \eta dW_t^2 \\ r_t = x_t + y_t + \phi_t \end{cases}$$

$x_0 = y_0 = 0,\ \phi_0 = r_0 = 3\%,\ dW_t^1 dW_t^2 = \rho dt,\ \rho = 0.7, a = 0.1,\ b = 0.3, \sigma = 3\%, \eta = 8\%.$
Assume $\phi_t = const = 3\%$ for any $t \geq 0$.

Use Monte Carlo Simulation to find at time $t = 0$ the price $p(t,T,S)$ of a European Put option, with strike price of $K = \$985$, maturity of $T = 0.5$ years on a Pure Discount Bond with Face value of \$1,000, that matures in $S = 1$ year.

4. ***[Optional. Not for grading]***
Assume the dynamics of the short-term interest rate, under the risk-neutral measure, are given by the following SDE (**CIR model**):

$$dr_t = \kappa(\bar{r} - r_t)dt + \sigma\sqrt{r_t}dW_t$$

with $r_0 = 5\%, \sigma = 18\%,\ \kappa = 0.92,\ \bar{r} = 5.5\%.$

Use a *Finite-Difference Method* to find at time $t = 0$ the price $c(t,T,S)$ of a European Call option, with strike price of $K = \$980$, maturity of $T = 0.5$ years on a Pure Discount Bond with Face Value of \$1,000, that matures in $S = 1$ year. The PDE is given as

$$\frac{\partial c}{\partial t} + \frac{1}{2}\sigma^2 r \frac{\partial^2 c}{\partial r^2} + \kappa(\bar{r} - r)\frac{\partial c}{\partial r} - rc = 0$$

with $c(T,T,S) = \max(P(T,S) - K, 0)$, and $P(T,S)$ is computed explicitly.

5. ***[Optional. Not for grading]***

Consider a European Put option, with strike price of $K = \$970$, maturity of $T = 0.5$ years on a Pure Discount Bond with Face Value of \$1,000, that matures in $S = 1.5$ years.
Which of the two models below would result in a more expensive price for the option?

(a) The Vasicek model $dr_t = \kappa(\bar{r} - r_t)dt + \sigma dW_t$ with $r_0 = 5\%,\ \sigma = 12\%,\ \kappa = 0.82,\ \bar{r} = 5\%.$
(b) The CIR model $dr_t = \kappa(\bar{r} - r_t)dt + \sigma\sqrt{r_t}dW_t$ with $r_0 = 5\%, \sigma = 54\%,\ \kappa = 0.82,\ \bar{r} = 5\%.$

Answer by using explicit formulas or by Monte Carlo simulations.

Is the answer consistent with your intuition?