QUASI-PERIODIC DYNAMICS
&
THE ONE DIMENSIONAL SCHRÖDINGER OPERATOR

The aim of this course is to give a broad introduction to many fundamental aspects of the modern theory of Dynamical Systems, yet focusing on a single problem related to physics : The analysis of the one-dimensional quasi-periodic Schrödinger operator.

The latter subject has known a fast growing interest in the last decade, to which the course will relate with references and motivations while staying at the basic knowledge level.

The analysis of the spectral theory of the one-dimensional quasi-periodic Schrödinger involves many fundamental aspects of the theory of dynamical systems : notions like *minimality, ergodicity, unique ergodicity, rotation number, uniform hyperbolicity, Lyapunov exponent, small divisors, KAM theory, reducibility or linearizability...* and relates as well to many different areas of mathematics such as complex analysis or spectral theory.

Studying these notions and phenomena at play in this particular context allows to weight their immediate applicability while providing insight into their signification and use in the broad theory of Dynamical Systems.

The course will be as much as possible self contained and relies only on basic real and complex analysis.

Each chapter will be accompanied by exercise sheets that aim to illustrate and complete the course notes and acquaint the students with the notions and tools that are introduced.

Plan of the course.

Chapter 1. Introduction to Dynamical Systems.
The definition of a dynamical system. Basic notions and properties.

Chapter 2. Some Ergodic theory.
Poincaré recurrence.
Birkhoff's ergodic theorem.
Ergodicity. Unique ergodicity. Mixing and weak mixing.
Kingman's subadditive ergodic theorem.

Chapter 3. Quasi-periodic dynamics on the torus.
Haar measure and unique ergodicity of minimal translations.
Rotation number of circle homeomorphisms.

Chapter 4. Arithmetics in Quasi-Periodic Dynamics. The Diophantine and Liouville phenomena.
Continued fraction algorithm. The sequence $n\alpha[1]$ on the circle.
The linear cohomolgical equation.
The Diophantine and Liouville phenomena in the simplest case : The study of skew products on $\mathbb{T}^2$.