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In[2]:= SingularType = {"E12", "E13", "E14b", "Z11", "Z12", "Z13b",
    "W12", "W13b", "Q10", "Q11", "Q12b", "S11", "S12", "U12", "E14", "Z13",
    "W12b", "W13", "Q12", "U12b", "U12c", "Z13T", "Q12T", "U12bT"};

DefiningEquation22[TT_List, TXYZ_List] := Module[
    {tt = TT, aa, bb, cc, txyz = TXYZ, x, y, z}, x = txyz[[1]]; y = txyz[[2]]; z = txyz[[3]];
    aa = tt[[1]]; bb = tt[[2]]; cc = tt[[3]]; {bb * y^7 + aa * x^3 + cc * z^2,
    aa * x * y^5 + bb * x^3 + cc * z^2, bb * y^8 + aa * x^3 + cc * z^2,
    bb * y^5 + aa * y * x^3 + cc * z^2, bb * x * y^4 + aa * y * x^3 + cc * z^2,
    bb * y^6 + aa * y * x^3 + cc * z^2,
    bb * y^5 + aa * x^4 + cc * z^2,
    aa * y * x^4 + bb * y^4 + cc * z^2,
    bb * y^4 + cc * z^3 + aa * y * x^2,
    bb * y^3 * z + cc * z^3 + aa * y * x^2, bb * y^5 + cc * z^3 + aa * y * x^2,
    cc * z^4 + aa * x^2 * y + bb * y^2 * z,
    cc * z^3 * x + aa * x^2 * y + bb * y^2 * z,
    cc * z^4 + bb * y^3 + aa * x^3,
    cc * z^3 + bb * y^4 * x + aa * x^2, cc * z^3 * y + bb * x * y^3 + aa * x^2,
    aa * x^2 * y + cc * z^5 + bb * y^2, bb * y^2 * x + cc * y * z^4 + aa * x^2,
    aa * x^2 * y + bb * x * y^3 + cc * z^3, aa * x^2 * y + bb * y^3 + cc * z^4,
    aa * x^2 * y + bb * x * y^2 + cc * z^4, aa * x^3 + bb * x * y^6 + cc * z^2,
    aa * x^2 + bb * x * y^5 + cc * z^3, aa * x^2 + bb * x * y^3 + cc * z^4}];

JacWWBasis[TXYZ_List] :=
    Module[{txyz = TXYZ, x, y, z, res}, x = txyz[[1]]; y = txyz[[2]]; z = txyz[[3]];
    res = {{1, y, y^2, x, y^3, xy, y^4, xy^2, y^5, xy^3, xy^4, xy^5}, {1, y, y^2, x, y^3, xy,
    y^4, xy^2, x^2, xy^3, x^2y, x^2y^2, x^2y^3}, {1, y, y^2, x, y^3, xy, y^4, xy^2, y^5,
    xy^3, y^6, xy^4, xy^5, xy^6}, {1, y, x, y^2, xy, x^2, y^3, xy^2, y^4, xy^3, xy^4},
    {1, y, x, y^2, xy, x^2, y^3, xy^2, x^2y, xy^3, x^2y^2, x^2y^3}, {1, y, x, y^2, xy, y^3, x^2,
    xy^2, y^4, xy^3, y^5, xy^4, xy^5}, {1, y, x, y^2, xy, x^2, y^3, xy^2, x^2y, xy^3, x^2y^2, x^2y^3},
    {1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^2y^2, xy^3, x^2y^3},
    {1, y, z, x, y^2, yz, xz, y^3, y^2z, y^3z}, {1, y, z, x, y^2, yz, z^2, xz, y^2z, yz^2, y^2z^2},
    {1, y, z, x, y^2, yz, y^3, xz, y^2z, y^4, y^3z, y^4z},
    {1, z, x, y, z^2, xz, yz, z^3, xz^2, yz^2, yz^3}, {1, z, x, y, z^2, xz, yz, xy,
    xz^2, yz^2, xyz, xyz^2}, {1, z, x, y, z^2, xz, yz, xy, xz^2, yz^2, xyz, xyz^2},
    {1, y, x, y^2, xy, y^3, xy^2, z, yz, xz, y^2z, xyz, y^3z, xy^2z},
    {1, y, z, y^2, yz, x, z^2, y^2z, xy, xz, xy^2, xyz, xy^2z},
    {1, x, y, z, xz, yz, z^2, xz^2, yz^2, z^3, xz^3, yz^3},
    {1, z, y, z^2, yz, x, z^3, yz^2, xz, xy, xz^2, xyz, xyz^2},
    {1, x, y, xy, y^2, xy^2, z, xz, yz, xyz, y^2z, xy^2z},
    {1, x, y, y^2, z, xz, yz, y^2z, z^2, xz^2, yz^2, y^2z^2},
    {1, x, y, xy, z, xz, yz, xyz, z^2, xz^2, yz^2, xyz^2},
    {1, y, y^2, y^3, y^4, y^5, x, xy, xy^2, xy^3, xy^4, x^2, x^2y, x^2y^2, x^2y^3, x^2y^4},
    {1, y, y^2, y^3, y^4, x, xy, xy^2, xy^3, z, yz, y^2z, y^3z, y^4z, xz, xyz, xy^2z, xy^3z},
    {1, y, y^2, x, xy, z, yz, y^2z, xz, xyz, z^2, yz^2, y^2z^2, xz^2, xyz^2}}]; res];

DegtxyzOne[EQN_, TXYZ_List] := Module[{eqn = EQN, txyz = TXYZ, tab, len, mat, vv, i},
    tab = CoefficientRules[eqn, txyz]; len = Length[tab];
    mat = Table[tab[[i, 1]], {i, 1, len}]; vv = Table[1, {i, 1, len}]; Inverse[mat].vv];

(*Define a partial order among vectors.
    Input: uu, vv.
    Output: if uu ≥ vv, then 1; otherwise, 0.*)

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PartialOrder[UU_List, VV_List] :=
Module[{uu = UU, vv = VV, len, res = 0, i}, len = Length[uu];
If[Table[Sign[uu[[i]] - vv[[i]] + 1], {i, 1, len}] == Table[1, {i, 1, len}], res = 1];
res];

(*Truncate monomials in variable varthat are of order larger than MM*)
PolyTruncate[FF_, VARTAB_, MM_] :=
Module[{ff = FF, var = VARTAB, mm = MM, i, oldcoef, newcoef, len},
oldcoef = CoefficientRules[ff, var];
newcoef = CoefficientRules[ff, var]; len = Length[oldcoef];
For[i = 1, i ≤ len, newcoef[[i, 2]] = Which[Total[oldcoef[[i, 1]]] ≤ mm,
oldcoef[[i, 2]], True, 0]; i++]; FromCoefficientRules[newcoef, var]];

(*Pick up monomials in variable varthat are of order larger than MM*)
PolyPickup[FF_, VARTAB_, MM_] :=
Module[{ff = FF, var = VARTAB, mm = MM, i, oldcoef, newcoef, len},
oldcoef = CoefficientRules[ff, var];
newcoef = CoefficientRules[ff, var]; len = Length[oldcoef];
For[i = 1, i ≤ len, newcoef[[i, 2]] = Which[Total[oldcoef[[i, 1]]] == mm,
oldcoef[[i, 2]], True, 0]; i++]; FromCoefficientRules[newcoef, var]];

(*Input: defining equation "EQN" and list "TXYZ" of variables.
Output: the list of Jacobian ideal*)

JacIdeal[EQN_, TXYZ_List] :=
Module[{eqn = EQN, i, txyz = TXYZ, len, res}, len = Length[txyz];
res = Table[D[eqn, txyz[[i]]], {i, len}]; res];

(*The next two commands come from
http://forums.wolfram.com/mathgroup/archive/2011/Mar/msg00362.html*)

moduleGroebnerBasis[polys_, vars_, cvars_, opts___] :=
Module[{newpolys, rels, len = Length[cvars], gb, j, k, rul},
rels = Flatten[Table[cvars[[j]] * cvars[[k]], {j, len}, {k, j, len}]];
newpolys = Join[polys, rels];
gb = GroebnerBasis[newpolys, Join[cvars, vars], opts];
rul = Map[{# → {}} &, rels];
gb = Flatten[gb /. rul];
Collect[gb, cvars]]

(*Input: Jacobian ideal "polys" and list "vars" of variables
Output: {Groebner basis, conversion matrix that
express Groebner basis in terms of the original generators*)
conversionMatrix[polys_, vars_] := Module[{aa, coords, pmat, len = Length[polys],
newpolys, mgb, gb, convmat, fvar, rvars}, coords = Array[aa, len + 1];
fvar = First[coords];
rvars = Rest[coords];
pmat = Transpose[Join[{polys}, IdentityMatrix[len]]];
newpolys = pmat.coords;
mgb = moduleGroebnerBasis[newpolys, vars, coords];
gb = mgb /. Join[{fvar → 1}, Thread[rvars → 0]] /. 0 → Sequence[];
convmat = Select[mgb, !FreeQ[#, fvar] &] /. fvar → 0;
{gb, convmat /. Thread[rvars → Table[UnitVector[len, j], {j, len}]]}]

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(*)
(*The next command produces {list of degrees, list of a monomial basis},
when the number of variables is equal to 3, 4,
or 5. Such command is not necessary in the current calculations*)

JacWWOneBasis[EQN_, TXYZ_List] :=
Module[{eqn=EQN,txyz=TXYZ,tab,len,monotab,i,tab22,tmax,s,j,k,
  max4=0,max5=0,res={},tab33={},tab44,degtab,t1,r,q,d1,len22,len44},
  tab= conversionMatrix[JacIdeal[eqn,txyz],txyz][[1]];
  len=Length[tab];degtab=DegtxyzOne[eqn,txyz];
  monotab=Table[Exponent[MonomialList[tab[[i]],txyz][[1]],txyz],{i,len}];
  tmax=Map[Max,Transpose[monotab]];len22=Length[txyz];
  If[len22>4,max4=tmax[[4]];If[len22>5,max5=tmax[[5]];
  For[i=0,i<tmax[[1]],For[j=0,j<tmax[[2]],For[k=0,k<tmax[[3]],
    For[r=0,r<max4,For[q=0,q<max5,
      If[len22==3,t1={i,j,k},If[len22==4,t1={i,j,k,r},t1={i,j,k,r,q}]];
      d1=Total[Table[PartialOrder[t1,monotab[[s]]],{s,len}]];
      If[d1==0,AppendTo[tab33,{degtab.t1,t1}];q++;r++;k++;j++;i++;
      tab44=Transpose[Sort[tab33]];len44=Length[tab44[[2]]];
      {tab44[[1]],Table[FromCoefficientRules[{tab44[[2]][[i]]->1},txyz],{i,len44}]}];
  ]]]]]];

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(*Input: defining equation EQN, variables TXYZ,
a given monomial basis JAC, a Groebner basis GROBAS.
Output: {the matrix of residue pairing  $(\phi_i, \phi_j)$ , dual basis  $\{\phi^1, \phi^2, \dots, \phi^\mu\}$ *)
Metric[EQN_, TXYZ_List, JAC_List, GROBAS_List] :=
Module[{eqn = EQN, txyz = TXYZ, jac = JAC, i, j, monocw, Groebner = GROBAS, degtab,
  degxyz, len, mat, cw, jacGB}, degxyz = DegtxyzOne[eqn, txyz]; len = Length[jac];
  degtab = Table[Exponent[jac[[i]], txyz].degxyz, {i, len}]; cw = degtab[[len]];
  monocw = (PolynomialReduce[jac[[len]], Groebner, txyz])[[2]];
  mat = Table[0, {i, len}, {j, len}]; For[i = 1, i <= len,
    For[j = 1, j <= len, If[degtab[[i]] + degtab[[j]] == cw, mat[[i, j]] =
      Coefficient[(PolynomialReduce[jac[[i]] * jac[[j]], Groebner, txyz])[[2]],
        monocw]; j++; i++]; {mat, Inverse[mat].jac}];

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(*Input: a polynomial FF, variables TXYZ,
a given monomial basis JAC= $\{\phi_1, \phi_2, \dots, \phi_\mu\}$  of Jac(W), a Groebner basis GROBAS,
the dual basis  $\{\phi^1, \phi^2, \dots, \phi^\mu\}$ , and the dimension  $\mu$  of Jac(W).
Output: *)
FindRemainer[FF_, TXYZ_List, JAC_List, GROBAS_List, DUALJAC_List, LEN_] :=
Module[{f = FF, txyz = TXYZ, jac = JAC, Groebner = GROBAS,
  dualjac = DUALJAC, len = LEN, monocw, tab, i},
  monocw = (PolynomialReduce[jac[[len]], Groebner, txyz])[[2]];
  tab = Table[Coefficient[(PolynomialReduce[dualjac[[i]] * f, Groebner, txyz])[[2]],
    monocw], {i, len}]; tab.jac];

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(* Input: a polynomial FF, together with other data.
Output: reduction of FF in the cohomology *)

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CohomologyReduction[FF_, TXYZ_List, JAC_List,

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GROBAS_List, DUALJAC_List, LEN_, CONV_List, LENXYZ_] :=
Module[{f = FF, txyz = TXYZ, jac = JAC, Groebner = GROBAS, dualjac = DUALJAC,
  lenxyz = LENXYZ, conv = CONV, monocw, tab, i, len = LEN, mm = 0,
  tab11 = {}, polyrr, poly, red11}, poly = f; While[mm == 0, polyrr =
  FindRemainer[poly, txyz, jac, Groebner, dualjac, len]; AppendTo[tab11, polyrr];
  red11 = (PolynomialReduce[poly - polyrr, Groebner, txyz][[1]]).conv;
  poly = Expand[-t * Total[Table[D[red11[[i]], txyz[[i]]], {i, lenxyz}]]];
  If[ToString[poly] == ToString[0], mm = 1]; Expand[Total[tab11]]];

(* Output the primitive form up to order MM.
*)
PrimitiveForm[EQN_, TXYZ_List, JAC_List, MM_, VAR_] :=
Module[{eqn = EQN, txyz = TXYZ, mm = MM, var = VAR, jac = JAC, len, lenxyz,
  tab, Groebner, conv, para, sca, k, psi, expotab, polyrow, red11, dualjac,
  psirow, len22, pos22, i, eone, vv, res11 = {}, primitive}, len = Length[jac];
  lenxyz = Length[txyz]; tab = conversionMatrix[JacIdeal[eqn, txyz], txyz];
  Groebner = tab[[1]]; conv = tab[[2]]; para = Table[var[[j]], {j, 1, len}];
  dualjac = Metric[eqn, txyz, jac, Groebner][[2]];
  sca = Sum[(para.jac)^k / (k! t^k), {k, 1, mm}];
  psi = Table[0, {i, 1, len}, {j, 1, len}];
  expotab = Table[Exponent[jac[[i]], txyz], {i, 1, len}]; For[i = 1, i ≤ len,
  polyrow = jac[[i]] * sca; red11 = Coefficient[t^mm * CohomologyReduction[
    polyrow, txyz, jac, Groebner, dualjac, len, conv, lenxyz], t^mm];
  psirow = CoefficientRules[red11, txyz]; len22 = Length[psirow];
  For[k = 1, k ≤ len22, pos22 = Position[expotab, psirow[[k]][[1]]][[1, 1]];
  psi[[i, pos22]] = psirow[[k]][[2]]; k++; i++; eone = IdentityMatrix[len][[1]];
  vv = eone; AppendTo[res11, 1]; For[i = 1, i ≤ mm, vv = -(vv.psi);
  AppendTo[res11, PolyTruncate[vv.jac, para, mm]]; i++; primitive = Total[res11]];

(*Output the 4-pt function*)
FourPointFunction[EQN_, TXYZ_List, JAC_List, MM_, VAR_, VARVV_] :=
Module[{eqn = EQN, txyz = TXYZ, jac = JAC, mm = MM, var = VAR,
  varvv = VARVV, len, para, paravv, expotab, sca, k, lenxyz, GrConv,
  Groebner, conv, MatDual, basismat, res11, dualjac, red, flatvar,
  coe22, flatvartab, uutab, vvtab, pos22, i, j, coe33, tab33, res},
  len = Length[jac]; para = Table[var[[j]], {j, 1, len}]; paravv = para /. var → varvv;
  expotab = Table[Exponent[jac[[i]], txyz], {i, 1, len}];
  sca = Sum[(para.jac)^k / (k! t^k), {k, mm}];
  lenxyz = Length[txyz]; GrConv = conversionMatrix[JacIdeal[eqn, txyz], txyz];
  Groebner = GrConv[[1]]; conv = GrConv[[2]];
  MatDual = Metric[eqn, txyz, jac, Groebner];
  dualjac = MatDual[[2]]; basismat = MatDual[[1]];
  red = CohomologyReduction[sca, txyz, jac, Groebner, dualjac, len, conv, lenxyz];
  flatvar = PolyTruncate[Coefficient[t^mm * red, t^(mm - 1)], para, 2];
  coe22 = PolyTruncate[Coefficient[t^mm * red, t^(mm - 2)], para, 3];
  flatvartab = CoefficientRules[flatvar, txyz];
  uutab = Table[0, {i, 1, len}];
  For[i = 1, i ≤ len, pos22 = Position[expotab, flatvartab[[i, 1]]][[1, 1]];
  uutab[[pos22]] = flatvartab[[i, 2]]; i++; vvtab = (2 para - uutab) /. var → varvv;
  res11 = CoefficientRules[PolyTruncate[
    Expand[coe22 /. Table[para[[i]] → vvtab[[i]], {i, 1, len}], paravv, 3], txyz];
  tab33 = Table[0, {i, 1, len}]; For[i = 1, i ≤ len, coe33 = res11[[i]];
  pos22 = Position[expotab, coe33[[1]]][[1, 1]];

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tab33[[pos22]] = PolyPickup[coe33[[2]], paravv, 3]; i++;
res = Expand[-1 * (paravv.basismat).tab33 / 4]; res];

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(*Print out final ouput. SCAL is used to take a multiplication of the 4-point function*)

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Output[TT_List, POS_, TXYZ_List, MM_, VAR_, VARVV_] :=
Module[{mm = MM, tt = TT, k, pos = POS, var = VAR, varvv = VARVV, mat, i, j,
  len, para, txyz = TXYZ, jac, eqn, deg}, jac = JacWWBasis[txyz][[pos]];
eqn = DefiningEquation22[tt, txyz][[pos]]; len = Length[jac];
deg = DegtxyzOne[eqn, txyz]; para = Table[var, {j, 1, len}]; i = pos;
Print["Type: ", SingularType[[i]]]; Print["Defining Equation is: ", eqn];
Print["Degree of variables", txyz, " : ", deg];
Print["Monomial Basis= ", jac]; Print["Degree of the monomial basis: ",
  Table[Exponent[jac[[i]], txyz].deg, {i, len}]];
Print["Parameters= ", para]; Print["deltaWW= ", para.jac];
Print["Up to order ", mm, ", the primitive form is: "]; Print[" "];
Print[PrimitiveForm[eqn, txyz, jac, mm, var]]; Print[" "];
Print["The four point function multiplying with (-1) is given by: "];
Print[" "]; Print[Expand[FourPointFunction[eqn, txyz, jac, mm, var, varvv]]];

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In[18]:= (*tt={a, b, c} denotes the normalization. t

is used to generate the flat coordinates here *)

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tt = {1, 1, 1}; txyz = {x, y, z};
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For[pos = 1, pos ≤ 24,
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  Output[tt, pos, txyz, 3, u, t]; pos++];
```

Type: E12

Defining Equation is: $x^3 + y^7 + z^2$

Degree of variables{x, y, z} : $\left\{\frac{1}{3}, \frac{1}{7}, \frac{1}{2}\right\}$

Monomial Basis= $\{1, y, y^2, x, y^3, xy, y^4, xy^2, y^5, xy^3, xy^4, xy^5\}$

Degree of the monomial basis: $\left\{0, \frac{1}{7}, \frac{2}{7}, \frac{1}{3}, \frac{3}{7}, \frac{10}{21}, \frac{4}{7}, \frac{13}{21}, \frac{5}{7}, \frac{16}{21}, \frac{19}{21}, \frac{22}{21}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}$

deltaWW= $u_1 + y u_2 + y^2 u_3 + x u_4 + y^3 u_5 + x y u_6 + y^4 u_7 + x y^2 u_8 + y^5 u_9 + x y^3 u_{10} + x y^4 u_{11} + x y^5 u_{12}$

Up to order 3, the primitive form is:

$$1 + \frac{4}{147} u_{11} u_{12}^2 + \frac{y u_{12}^3}{49}$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned}
& \frac{1}{14} t_5 t_6 t_7^2 + \frac{1}{18} t_6^3 t_8 + \frac{1}{7} t_5^2 t_7 t_8 + \frac{1}{7} t_3 t_7^2 t_8 + \frac{1}{6} t_4 t_6 t_8^2 + \frac{1}{14} t_5^2 t_6 t_9 + \frac{1}{7} t_3 t_6 t_7 t_9 + \\
& \frac{1}{7} t_3 t_5 t_8 t_9 + \frac{1}{7} t_2 t_7 t_8 t_9 + \frac{1}{14} t_2 t_6 t_9^2 + \frac{1}{14} t_5^3 t_{10} + \frac{1}{6} t_4 t_6^2 t_{10} + \frac{2}{7} t_3 t_5 t_7 t_{10} + \frac{1}{14} t_2 t_7^2 t_{10} + \\
& \frac{1}{6} t_4^2 t_8 t_{10} + \frac{1}{14} t_3^2 t_9 t_{10} + \frac{1}{7} t_2 t_5 t_9 t_{10} + \frac{1}{7} t_3 t_5^2 t_{11} + \frac{1}{6} t_4^2 t_6 t_{11} + \frac{1}{7} t_3^2 t_7 t_{11} + \frac{1}{7} t_2 t_5 t_7 t_{11} + \\
& \frac{1}{7} t_2 t_3 t_9 t_{11} + \frac{1}{18} t_4^3 t_{12} + \frac{1}{14} t_3^2 t_5 t_{12} + \frac{1}{14} t_2 t_5^2 t_{12} + \frac{1}{7} t_2 t_3 t_7 t_{12} + \frac{1}{14} t_2^2 t_9 t_{12}
\end{aligned}$$

Type: E13

Defining Equation is: $x^3 + xy^5 + z^2$

Degree of variables $\{x, y, z\}$: $\left\{\frac{1}{3}, \frac{2}{15}, \frac{1}{2}\right\}$

Monomial Basis= $\{1, y, y^2, x, y^3, xy, y^4, xy^2, x^2, xy^3, x^2y, x^2y^2, x^2y^3\}$

Degree of the monomial basis: $\left\{0, \frac{2}{15}, \frac{4}{15}, \frac{1}{3}, \frac{2}{5}, \frac{7}{15}, \frac{8}{15}, \frac{3}{5}, \frac{2}{3}, \frac{11}{15}, \frac{4}{5}, \frac{14}{15}, \frac{16}{15}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}\}$

deltaWW=

$$u_1 + y u_2 + y^2 u_3 + x u_4 + y^3 u_5 + x y u_6 + y^4 u_7 + x y^2 u_8 + x^2 u_9 + x y^3 u_{10} + x^2 y u_{11} + x^2 y^2 u_{12} + x^2 y^3 u_{13}$$

Up to order 3, the primitive form is:

$$1 - \frac{4 u_{12} u_{13}}{75} - \frac{y u_{13}^2}{25}$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned}
& -\frac{3}{10} t_6 t_7^3 - \frac{3}{5} t_5 t_7^2 t_8 + \frac{1}{10} t_5 t_6 t_8^2 + \frac{1}{15} t_3 t_8^3 + \frac{1}{90} t_6^3 t_9 + \frac{3}{5} t_5 t_6 t_7 t_9 + \frac{2}{5} t_4 t_7^2 t_9 + \frac{1}{5} t_5^2 t_8 t_9 + \\
& \frac{1}{15} t_4 t_6 t_8 t_9 + \frac{2}{5} t_3 t_7 t_8 t_9 - \frac{1}{10} t_4 t_5 t_9^2 - \frac{1}{15} t_3 t_6 t_9^2 - \frac{1}{30} t_2 t_8 t_9^2 + \frac{1}{10} t_5 t_6^2 t_{10} - \\
& \frac{3}{10} t_5^2 t_7 t_{10} - \frac{3}{10} t_3 t_7^2 t_{10} + \frac{1}{5} t_3 t_6 t_8 t_{10} + \frac{1}{10} t_2 t_8^2 t_{10} + \frac{1}{30} t_4^2 t_9 t_{10} + \frac{1}{5} t_3 t_5 t_9 t_{10} + \\
& \frac{1}{5} t_2 t_7 t_9 t_{10} + \frac{1}{10} t_2 t_6 t_{10}^2 + \frac{3}{10} t_5^2 t_6 t_{11} + \frac{1}{15} t_4 t_6^2 t_{11} + \frac{3}{5} t_4 t_5 t_7 t_{11} + \frac{2}{5} t_3 t_6 t_7 t_{11} + \\
& \frac{1}{15} t_4^2 t_8 t_{11} + \frac{2}{5} t_3 t_5 t_8 t_{11} + \frac{1}{5} t_2 t_7 t_8 t_{11} - \frac{2}{15} t_3 t_4 t_9 t_{11} - \frac{1}{15} t_2 t_6 t_9 t_{11} + \frac{1}{10} t_3^2 t_{10} t_{11} + \\
& \frac{1}{5} t_2 t_5 t_{10} t_{11} - \frac{1}{30} t_2 t_4 t_{11}^2 + \frac{1}{5} t_4 t_5^2 t_{12} + \frac{1}{10} t_4^2 t_6 t_{12} + \frac{2}{5} t_3 t_5 t_6 t_{12} + \frac{2}{5} t_3 t_4 t_7 t_{12} + \\
& \frac{1}{5} t_2 t_6 t_7 t_{12} + \frac{1}{5} t_3^2 t_8 t_{12} + \frac{1}{5} t_2 t_5 t_8 t_{12} - \frac{1}{15} t_2 t_4 t_9 t_{12} + \frac{1}{5} t_2 t_3 t_{10} t_{12} + \frac{2}{45} t_4^3 t_{13} + \\
& \frac{1}{5} t_3 t_4 t_5 t_{13} + \frac{1}{10} t_3^2 t_6 t_{13} + \frac{1}{5} t_2 t_5 t_6 t_{13} + \frac{1}{5} t_2 t_4 t_7 t_{13} + \frac{1}{5} t_2 t_3 t_8 t_{13} + \frac{1}{10} t_2^2 t_{10} t_{13}
\end{aligned}$$

Type: E14b

Defining Equation is: $x^3 + y^8 + z^2$

Degree of variables $\{x, y, z\}$: $\left\{\frac{1}{3}, \frac{1}{8}, \frac{1}{2}\right\}$

Monomial Basis= $\{1, y, y^2, x, y^3, xy, y^4, xy^2, y^5, xy^3, y^6, xy^4, xy^5, xy^6\}$

Degree of the monomial basis: $\left\{0, \frac{1}{8}, \frac{1}{4}, \frac{1}{3}, \frac{3}{8}, \frac{11}{24}, \frac{1}{2}, \frac{7}{12}, \frac{5}{8}, \frac{17}{24}, \frac{3}{4}, \frac{5}{6}, \frac{23}{24}, \frac{13}{12}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}\}$

deltaWW= $u_1 + y u_2 + y^2 u_3 + x u_4 + y^3 u_5 + x y u_6 + y^4 u_7 + x y^2 u_8 + y^5 u_9 + x y^3 u_{10} + y^6 u_{11} + x y^4 u_{12} + x y^5 u_{13} + x y^6 u_{14}$

Up to order 3, the primitive form is:

$$1 + \frac{3}{128} u_{13}^2 u_{14} + \frac{3}{128} u_{12} u_{14}^2 + \frac{5}{96} y u_{13} u_{14}^2 + \frac{11}{384} y^2 u_{14}^3$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned} & \frac{1}{24} t_7^3 t_8 + \frac{1}{12} t_6^2 t_8^2 + \frac{1}{18} t_4 t_8^3 + \frac{1}{16} t_6 t_7^2 t_9 + \frac{1}{4} t_5 t_7 t_8 t_9 + \frac{1}{16} t_5 t_6 t_9^2 + \frac{1}{8} t_3 t_8 t_9^2 + \\ & \frac{1}{18} t_6^3 t_{10} + \frac{3}{16} t_5 t_7^2 t_{10} + \frac{1}{3} t_4 t_6 t_8 t_{10} + \frac{1}{8} t_5^2 t_9 t_{10} + \frac{1}{4} t_3 t_7 t_9 t_{10} + \frac{1}{16} t_2 t_9^2 t_{10} + \\ & \frac{1}{12} t_4^2 t_{10}^2 + \frac{1}{8} t_5 t_6 t_7 t_{11} + \frac{1}{16} t_5^2 t_8 t_{11} + \frac{1}{8} t_3 t_7 t_8 t_{11} + \frac{1}{8} t_3 t_6 t_9 t_{11} + \frac{1}{8} t_2 t_8 t_9 t_{11} + \\ & \frac{1}{8} t_3 t_5 t_{10} t_{11} + \frac{1}{8} t_2 t_7 t_{10} t_{11} + \frac{1}{16} t_2 t_6 t_{11}^2 + \frac{1}{6} t_4 t_6^2 t_{12} + \frac{3}{16} t_5^2 t_7 t_{12} + \frac{1}{8} t_3 t_7^2 t_{12} + \\ & \frac{1}{6} t_4^2 t_8 t_{12} + \frac{1}{4} t_3 t_5 t_9 t_{12} + \frac{1}{8} t_2 t_7 t_9 t_{12} + \frac{1}{16} t_3^2 t_{11} t_{12} + \frac{1}{8} t_2 t_5 t_{11} t_{12} + \frac{1}{24} t_5^3 t_{13} + \\ & \frac{1}{6} t_4^2 t_6 t_{13} + \frac{1}{4} t_3 t_5 t_7 t_{13} + \frac{1}{16} t_2 t_7^2 t_{13} + \frac{1}{8} t_3^2 t_9 t_{13} + \frac{1}{8} t_2 t_5 t_9 t_{13} + \frac{1}{8} t_2 t_3 t_{11} t_{13} + \\ & \frac{1}{18} t_4^3 t_{14} + \frac{1}{16} t_3 t_5^2 t_{14} + \frac{1}{16} t_3^2 t_7 t_{14} + \frac{1}{8} t_2 t_5 t_7 t_{14} + \frac{1}{8} t_2 t_3 t_9 t_{14} + \frac{1}{16} t_2^2 t_{11} t_{14} \end{aligned}$$

Type: Z11

Defining Equation is: $x^3 y + y^5 + z^2$

Degree of variables $\{x, y, z\}$: $\left\{\frac{4}{15}, \frac{1}{5}, \frac{1}{2}\right\}$

Monomial Basis= $\{1, y, x, y^2, xy, x^2, y^3, xy^2, y^4, xy^3, xy^4\}$

Degree of the monomial basis: $\left\{0, \frac{1}{5}, \frac{4}{15}, \frac{2}{5}, \frac{7}{15}, \frac{8}{15}, \frac{3}{5}, \frac{2}{3}, \frac{4}{5}, \frac{13}{15}, \frac{16}{15}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}\}$

deltaWW= $u_1 + y u_2 + x u_3 + y^2 u_4 + x y u_5 + x^2 u_6 + y^3 u_7 + x y^2 u_8 + y^4 u_9 + x y^3 u_{10} + x y^4 u_{11}$

Up to order 3, the primitive form is:

$$1 + \frac{17}{675} u_{10} u_{11}^2 + \frac{2 y u_{11}^3}{81}$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned}
& -\frac{5}{18} t_5 t_6^3 + \frac{1}{3} t_4 t_6^2 t_7 + \frac{1}{15} t_4 t_5 t_7^2 - \frac{1}{90} t_3 t_7^3 + \frac{1}{18} t_5^3 t_8 + \frac{2}{15} t_4^2 t_7 t_8 + \frac{1}{3} t_3 t_6 t_7 t_8 + \frac{1}{10} t_2 t_7^2 t_8 + \\
& \frac{1}{6} t_3 t_5 t_8^2 + \frac{1}{30} t_4^2 t_5 t_9 + \frac{1}{3} t_3 t_5 t_6 t_9 + \frac{1}{3} t_2 t_6^2 t_9 - \frac{1}{15} t_3 t_4 t_7 t_9 + \frac{1}{15} t_2 t_5 t_7 t_9 + \\
& \frac{1}{15} t_2 t_4 t_8 t_9 - \frac{1}{30} t_2 t_3 t_9^2 + \frac{1}{18} t_4^3 t_{10} + \frac{1}{6} t_3 t_5^2 t_{10} + \frac{1}{3} t_3 t_4 t_6 t_{10} + \frac{1}{5} t_2 t_4 t_7 t_{10} + \\
& \frac{1}{6} t_3^2 t_8 t_{10} + \frac{1}{30} t_2^2 t_9 t_{10} + \frac{1}{15} t_2 t_4^2 t_{11} + \frac{1}{6} t_3^2 t_5 t_{11} + \frac{1}{3} t_2 t_3 t_6 t_{11} + \frac{1}{15} t_2^2 t_7 t_{11}
\end{aligned}$$

Type: Z12

Defining Equation is: $x^3 y + x y^4 + z^2$

Degree of variables $\{x, y, z\}$: $\left\{\frac{3}{11}, \frac{2}{11}, \frac{1}{2}\right\}$

Monomial Basis= $\{1, y, x, y^2, x y, x^2, y^3, x y^2, x^2 y, x y^3, x^2 y^2, x^2 y^3\}$

Degree of the monomial basis: $\left\{0, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}, \frac{5}{11}, \frac{6}{11}, \frac{6}{11}, \frac{7}{11}, \frac{8}{11}, \frac{9}{11}, \frac{10}{11}, \frac{12}{11}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}$

deltaWW=

$$u_1 + y u_2 + x u_3 + y^2 u_4 + x y u_5 + x^2 u_6 + y^3 u_7 + x y^2 u_8 + x^2 y u_9 + x y^3 u_{10} + x^2 y^2 u_{11} + x^2 y^3 u_{12}$$

Up to order 3, the primitive form is:

$$1 - \frac{6 u_{11} u_{12}}{121} - \frac{5 y u_{12}^2}{121} + \frac{29 u_{10} u_{12}^2}{1331} + \frac{9 x u_{12}^3}{1331}$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned}
& -\frac{10}{33} t_5 t_6^3 + \frac{5}{22} t_5 t_6^2 t_7 + \frac{7}{22} t_5 t_6 t_7^2 - \frac{7}{22} t_5 t_7^3 + \frac{3}{11} t_4 t_6^2 t_8 + \frac{4}{11} t_4 t_6 t_7 t_8 - \\
& \frac{6}{11} t_4 t_7^2 t_8 + \frac{1}{11} t_4 t_5 t_8^2 + \frac{2}{11} t_3 t_6 t_8^2 - \frac{1}{22} t_3 t_7 t_8^2 + \frac{1}{22} t_2 t_8^3 + \frac{1}{66} t_5^3 t_9 - \frac{2}{11} t_4 t_5 t_6 t_9 - \\
& \frac{2}{11} t_3 t_6^2 t_9 + \frac{6}{11} t_4 t_5 t_7 t_9 + \frac{1}{11} t_3 t_6 t_7 t_9 + \frac{4}{11} t_3 t_7^2 t_9 + \frac{2}{11} t_4^2 t_8 t_9 + \frac{1}{11} t_3 t_5 t_8 t_9 - \\
& \frac{1}{11} t_2 t_6 t_8 t_9 + \frac{3}{11} t_2 t_7 t_8 t_9 - \frac{1}{11} t_3 t_4 t_9^2 - \frac{1}{22} t_2 t_5 t_9^2 + \frac{1}{22} t_4 t_5^2 t_{10} + \frac{1}{22} t_4^2 t_6 t_{10} + \\
& \frac{4}{11} t_3 t_5 t_6 t_{10} + \frac{7}{22} t_2 t_6^2 t_{10} - \frac{3}{22} t_4^2 t_7 t_{10} - \frac{1}{11} t_3 t_5 t_7 t_{10} + \frac{1}{11} t_2 t_6 t_7 t_{10} - \frac{3}{22} t_2 t_7^2 t_{10} - \\
& \frac{1}{11} t_3 t_4 t_8 t_{10} + \frac{1}{11} t_2 t_5 t_8 t_{10} + \frac{1}{22} t_3^2 t_9 t_{10} + \frac{1}{11} t_2 t_4 t_9 t_{10} - \frac{1}{22} t_2 t_3 t_{10}^2 + \\
& \frac{5}{22} t_4^2 t_5 t_{11} + \frac{1}{11} t_3 t_5^2 t_{11} + \frac{2}{11} t_3 t_4 t_6 t_{11} - \frac{1}{11} t_2 t_5 t_6 t_{11} + \frac{5}{11} t_3 t_4 t_7 t_{11} + \\
& \frac{3}{11} t_2 t_5 t_7 t_{11} + \frac{1}{11} t_3^2 t_8 t_{11} + \frac{3}{11} t_2 t_4 t_8 t_{11} - \frac{1}{11} t_2 t_3 t_9 t_{11} + \frac{1}{22} t_2^2 t_{10} t_{11} + \\
& \frac{1}{11} t_3 t_4^2 t_{12} + \frac{3}{22} t_3^2 t_5 t_{12} + \frac{2}{11} t_2 t_4 t_5 t_{12} + \frac{3}{11} t_2 t_3 t_6 t_{12} + \frac{2}{11} t_2 t_3 t_7 t_{12} + \frac{1}{11} t_2^2 t_8 t_{12}
\end{aligned}$$

Type: Z13b

Defining Equation is: $x^3 y + y^6 + z^2$

Degree of variables{x, y, z} : $\left\{\frac{5}{18}, \frac{1}{6}, \frac{1}{2}\right\}$

Monomial Basis= $\{1, y, x, y^2, xy, y^3, x^2, xy^2, y^4, xy^3, y^5, xy^4, xy^5\}$

Degree of the monomial basis: $\left\{0, \frac{1}{6}, \frac{5}{18}, \frac{1}{3}, \frac{4}{9}, \frac{1}{2}, \frac{5}{9}, \frac{11}{18}, \frac{2}{3}, \frac{7}{9}, \frac{5}{6}, \frac{17}{18}, \frac{10}{9}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}\}$

deltaWW=

$$u_1 + y u_2 + x u_3 + y^2 u_4 + x y u_5 + y^3 u_6 + x^2 u_7 + x y^2 u_8 + y^4 u_9 + x y^3 u_{10} + y^5 u_{11} + x y^4 u_{12} + x y^5 u_{13}$$

Up to order 3, the primitive form is:

$$1 + \frac{5}{243} u_{12}^2 u_{13} + \frac{5}{243} u_{10} u_{13}^2 + \frac{115 y u_{12} u_{13}^2}{1944} + \frac{26}{729} y^2 u_{13}^3$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned} & \frac{1}{6} t_6^2 t_7^2 - \frac{1}{3} t_5 t_7^3 + \frac{5}{108} t_6^3 t_8 + \frac{1}{12} t_5^2 t_8^2 + \frac{1}{18} t_3 t_8^3 + \frac{1}{18} t_5 t_6^2 t_9 + \frac{1}{3} t_4 t_7^2 t_9 + \\ & \frac{2}{9} t_4 t_6 t_8 t_9 + \frac{1}{3} t_3 t_7 t_8 t_9 + \frac{1}{18} t_4 t_5 t_9^2 - \frac{1}{36} t_3 t_6 t_9^2 + \frac{1}{12} t_2 t_8 t_9^2 + \frac{1}{18} t_5^2 t_{10} + \\ & \frac{1}{6} t_4 t_6^2 t_{10} + \frac{1}{3} t_3 t_6 t_7 t_{10} + \frac{1}{3} t_3 t_5 t_8 t_{10} + \frac{1}{9} t_4^2 t_9 t_{10} + \frac{1}{6} t_2 t_6 t_9 t_{10} + \frac{1}{12} t_3^2 t_{10}^2 + \\ & \frac{1}{18} t_4 t_5 t_6 t_{11} - \frac{1}{36} t_3 t_6^2 t_{11} + \frac{1}{3} t_3 t_5 t_7 t_{11} + \frac{1}{3} t_2 t_7^2 t_{11} + \frac{1}{36} t_4^2 t_8 t_{11} + \\ & \frac{1}{18} t_2 t_6 t_8 t_{11} - \frac{1}{18} t_3 t_4 t_9 t_{11} + \frac{1}{18} t_2 t_5 t_9 t_{11} + \frac{1}{18} t_2 t_4 t_{10} t_{11} - \frac{1}{36} t_2 t_3 t_{11}^2 + \\ & \frac{1}{6} t_3 t_5^2 t_{12} + \frac{5}{36} t_4^2 t_6 t_{12} + \frac{1}{12} t_2 t_6^2 t_{12} + \frac{1}{3} t_3 t_4 t_7 t_{12} + \frac{1}{6} t_3^2 t_8 t_{12} + \frac{1}{6} t_2 t_4 t_9 t_{12} + \\ & \frac{1}{36} t_2^2 t_{11} t_{12} + \frac{1}{54} t_4^3 t_{13} + \frac{1}{6} t_3^2 t_5 t_{13} + \frac{1}{9} t_2 t_4 t_6 t_{13} + \frac{1}{3} t_2 t_3 t_7 t_{13} + \frac{1}{18} t_2^2 t_9 t_{13} \end{aligned}$$

Type: W12

Defining Equation is: $x^4 + y^5 + z^2$

Degree of variables{x, y, z} : $\left\{\frac{1}{4}, \frac{1}{5}, \frac{1}{2}\right\}$

Monomial Basis= $\{1, y, x, y^2, xy, x^2, y^3, xy^2, x^2y, xy^3, x^2y^2, x^2y^3\}$

Degree of the monomial basis: $\left\{0, \frac{1}{5}, \frac{1}{4}, \frac{2}{5}, \frac{9}{20}, \frac{1}{2}, \frac{3}{5}, \frac{13}{20}, \frac{7}{10}, \frac{17}{20}, \frac{9}{10}, \frac{11}{10}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}$

deltaWW=

$$u_1 + y u_2 + x u_3 + y^2 u_4 + x y u_5 + x^2 u_6 + y^3 u_7 + x y^2 u_8 + x^2 y u_9 + x y^3 u_{10} + x^2 y^2 u_{11} + x^2 y^3 u_{12}$$

Up to order 3, the primitive form is:

$$1 - \frac{u_{11} u_{12}}{20} - \frac{y u_{12}^2}{20}$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned} & \frac{1}{20} t_5^2 t_7^2 + \frac{1}{8} t_5 t_6^2 t_8 + \frac{1}{5} t_4 t_5 t_7 t_8 + \frac{1}{10} t_4^2 t_8^2 + \frac{1}{10} t_2 t_7 t_8^2 + \frac{1}{8} t_5^2 t_6 t_9 + \frac{1}{10} t_4^2 t_7 t_9 + \frac{1}{10} t_2 t_7^2 t_9 + \\ & \frac{1}{4} t_3 t_6 t_8 t_9 + \frac{1}{8} t_3 t_5 t_9^2 + \frac{1}{10} t_4^2 t_5 t_{10} + \frac{1}{8} t_3 t_6^2 t_{10} + \frac{1}{5} t_2 t_5 t_7 t_{10} + \frac{1}{5} t_2 t_4 t_8 t_{10} + \frac{1}{20} t_2^2 t_{10}^2 + \\ & \frac{1}{15} t_4^3 t_{11} + \frac{1}{4} t_3 t_5 t_6 t_{11} + \frac{1}{5} t_2 t_4 t_7 t_{11} + \frac{1}{8} t_3^2 t_9 t_{11} + \frac{1}{10} t_2 t_4^2 t_{12} + \frac{1}{8} t_3^2 t_6 t_{12} + \frac{1}{10} t_2^2 t_7 t_{12} \end{aligned}$$

Type: W13b

Defining Equation is: $x^4 y + y^4 + z^2$

Degree of variables $\{x, y, z\}$: $\left\{\frac{3}{16}, \frac{1}{4}, \frac{1}{2}\right\}$

Monomial Basis= $\{1, x, y, x^2, xy, y^2, x^3, x^2 y, xy^2, y^3, x^2 y^2, xy^3, x^2 y^3\}$

Degree of the monomial basis: $\left\{0, \frac{3}{16}, \frac{1}{4}, \frac{3}{8}, \frac{7}{16}, \frac{1}{2}, \frac{9}{16}, \frac{5}{8}, \frac{11}{16}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{9}{8}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}\}$

deltaWW=

$$u_1 + x u_2 + y u_3 + x^2 u_4 + x y u_5 + y^2 u_6 + x^3 u_7 + x^2 y u_8 + x y^2 u_9 + y^3 u_{10} + x^2 y^2 u_{11} + x y^3 u_{12} + x^2 y^3 u_{13}$$

Up to order 3, the primitive form is:

$$1 - \frac{3 u_{11} u_{13}}{64} + \frac{57 u_{12}^2 u_{13}}{2048} - \frac{7 y u_{13}^2}{128} + \frac{9}{512} u_{10} u_{13}^2 + \frac{29 x u_{12} u_{13}^2}{1024} + \frac{5 x^2 u_{13}^3}{1024}$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned} & \frac{3}{16} t_6^2 t_7^2 - \frac{1}{3} t_5 t_7^3 + \frac{1}{48} t_6^3 t_8 - \frac{1}{2} t_4 t_7^2 t_8 + \frac{1}{16} t_5^2 t_8^2 + \frac{3}{32} t_5 t_6^2 t_9 + \frac{1}{2} t_4 t_6 t_7 t_9 + \\ & \frac{1}{4} t_4 t_5 t_8 t_9 + \frac{1}{8} t_2 t_8^2 t_9 + \frac{1}{8} t_4^2 t_9^2 + \frac{1}{8} t_3 t_6 t_9^2 + \frac{1}{8} t_2 t_7 t_9^2 + \frac{1}{32} t_5^2 t_6 t_{10} - \frac{1}{16} t_4 t_6^2 t_{10} + \\ & \frac{1}{2} t_4 t_5 t_7 t_{10} + \frac{3}{8} t_3 t_7^2 t_{10} + \frac{1}{8} t_4^2 t_8 t_{10} + \frac{1}{16} t_3 t_6 t_8 t_{10} + \frac{1}{4} t_2 t_7 t_8 t_{10} + \frac{1}{16} t_3 t_5 t_9 t_{10} - \\ & \frac{1}{16} t_2 t_6 t_9 t_{10} - \frac{1}{16} t_3 t_4 t_{10}^2 - \frac{1}{32} t_2 t_5 t_{10}^2 + \frac{1}{8} t_4 t_5^2 t_{11} + \frac{1}{8} t_4^2 t_6 t_{11} + \frac{1}{8} t_3 t_6^2 t_{11} + \\ & \frac{1}{4} t_2 t_6 t_7 t_{11} + \frac{1}{4} t_2 t_5 t_8 t_{11} + \frac{1}{4} t_2 t_4 t_9 t_{11} + \frac{1}{32} t_3^2 t_{10} t_{11} + \frac{1}{16} t_2^2 t_{11}^2 + \frac{1}{4} t_4^2 t_5 t_{12} + \\ & \frac{1}{8} t_3 t_5 t_6 t_{12} - \frac{1}{32} t_2 t_6^2 t_{12} + \frac{1}{2} t_3 t_4 t_7 t_{12} + \frac{1}{4} t_2 t_5 t_7 t_{12} + \frac{1}{4} t_2 t_4 t_8 t_{12} + \frac{1}{16} t_3^2 t_9 t_{12} - \\ & \frac{1}{16} t_2 t_3 t_{10} t_{12} + \frac{1}{8} t_3 t_4^2 t_{13} + \frac{1}{4} t_2 t_4 t_5 t_{13} + \frac{3}{32} t_3^2 t_6 t_{13} + \frac{1}{4} t_2 t_3 t_7 t_{13} + \frac{1}{8} t_2^2 t_8 t_{13} \end{aligned}$$

Type: Q10

Defining Equation is: $x^2 y + y^4 + z^3$

Degree of variables $\{x, y, z\}$: $\left\{\frac{3}{8}, \frac{1}{4}, \frac{1}{3}\right\}$

Monomial Basis= $\{1, y, z, x, y^2, yz, xz, y^3, y^2 z, y^3 z\}$

Degree of the monomial basis: $\left\{0, \frac{1}{4}, \frac{1}{3}, \frac{3}{8}, \frac{1}{2}, \frac{7}{12}, \frac{17}{24}, \frac{3}{4}, \frac{5}{6}, \frac{13}{12}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$

deltaWW= $u_1 + y u_2 + z u_3 + x u_4 + y^2 u_5 + y z u_6 + x z u_7 + y^3 u_8 + y^2 z u_9 + y^3 z u_{10}$

Up to order 3, the primitive form is:

$$1 + \frac{3}{128} u_9 u_{10}^2 + \frac{11 y u_{10}^3}{384}$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned} & \frac{1}{24} t_5^3 t_6 + \frac{1}{18} t_3 t_6^3 + \frac{1}{4} t_4 t_5^2 t_7 - \frac{1}{3} t_3^2 t_7^2 + \frac{1}{4} t_4^2 t_6 t_8 + \frac{1}{8} t_2 t_5 t_6 t_8 + \frac{1}{2} t_2 t_4 t_7 t_8 + \\ & \frac{1}{4} t_4^2 t_5 t_9 + \frac{1}{8} t_2 t_5^2 t_9 + \frac{1}{6} t_3^2 t_6 t_9 + \frac{1}{16} t_2^2 t_8 t_9 + \frac{1}{18} t_3^3 t_{10} + \frac{1}{4} t_2 t_4^2 t_{10} + \frac{1}{16} t_2^2 t_5 t_{10} \end{aligned}$$

Type: Q11

Defining Equation is: $x^2 y + y^3 z + z^3$

Degree of variables $\{x, y, z\}$: $\left\{\frac{7}{18}, \frac{2}{9}, \frac{1}{3}\right\}$

Monomial Basis= $\{1, y, z, x, y^2, y z, z^2, x z, y^2 z, y z^2, y^2 z^2\}$

Degree of the monomial basis: $\left\{0, \frac{2}{9}, \frac{1}{3}, \frac{7}{18}, \frac{4}{9}, \frac{5}{9}, \frac{2}{3}, \frac{13}{18}, \frac{7}{9}, \frac{8}{9}, \frac{10}{9}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}\}$

deltaWW= $u_1 + y u_2 + z u_3 + x u_4 + y^2 u_5 + y z u_6 + z^2 u_7 + x z u_8 + y^2 z u_9 + y z^2 u_{10} + y^2 z^2 u_{11}$

Up to order 3, the primitive form is:

$$1 - \frac{5 u_{10} u_{11}}{108} - \frac{y u_{11}^2}{24} + \frac{13}{648} u_9 u_{11}^2 + \frac{25 z u_{11}^3}{1944}$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned} & \frac{1}{36} t_5 t_6^3 + \frac{1}{4} t_5^2 t_6 t_7 + \frac{1}{36} t_3 t_6^2 t_7 - \frac{1}{24} t_4^2 t_7^2 - \frac{1}{9} t_3 t_5 t_7^2 - \frac{1}{18} t_2 t_6 t_7^2 + \\ & \frac{1}{2} t_4 t_5 t_6 t_8 - \frac{1}{6} t_3 t_4 t_7 t_8 - \frac{1}{4} t_3^2 t_8^2 - \frac{1}{12} t_5^3 t_9 + \frac{1}{4} t_4^2 t_6 t_9 + \frac{1}{12} t_2 t_6^2 t_9 + \frac{1}{36} t_3^2 t_7 t_9 + \\ & \frac{1}{6} t_2 t_5 t_7 t_9 + \frac{1}{2} t_2 t_4 t_8 t_9 + \frac{1}{4} t_4^2 t_5 t_{10} + \frac{1}{4} t_3 t_5^2 t_{10} + \frac{1}{12} t_3^2 t_6 t_{10} + \frac{1}{3} t_2 t_5 t_6 t_{10} - \\ & \frac{1}{9} t_2 t_3 t_7 t_{10} + \frac{1}{12} t_2^2 t_9 t_{10} + \frac{5}{108} t_3^3 t_{11} + \frac{1}{4} t_2 t_4^2 t_{11} + \frac{1}{6} t_2 t_3 t_5 t_{11} + \frac{1}{12} t_2^2 t_6 t_{11} \end{aligned}$$

Type: Q12b

Defining Equation is: $x^2 y + y^5 + z^3$

Degree of variables{x, y, z} : $\left\{\frac{2}{5}, \frac{1}{5}, \frac{1}{3}\right\}$

Monomial Basis= $\{1, y, z, x, y^2, yz, y^3, xz, y^2z, y^4, y^3z, y^4z\}$

Degree of the monomial basis: $\left\{0, \frac{1}{5}, \frac{1}{3}, \frac{2}{5}, \frac{2}{5}, \frac{8}{15}, \frac{3}{5}, \frac{11}{15}, \frac{11}{15}, \frac{4}{5}, \frac{14}{15}, \frac{17}{15}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}$

deltaWW= $u_1 + y u_2 + z u_3 + x u_4 + y^2 u_5 + y z u_6 + y^3 u_7 + x z u_8 + y^2 z u_9 + y^4 u_{10} + y^3 z u_{11} + y^4 z u_{12}$

Up to order 3, the primitive form is:

$$1 + \frac{11}{600} u_{11}^2 u_{12} + \frac{11}{600} u_9 u_{12}^2 + \frac{13}{200} y u_{11} u_{12}^2 + \frac{1}{24} y^2 u_{12}^3$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned} & \frac{t_6^4}{72} + \frac{1}{10} t_5 t_6 t_7^2 + \frac{1}{2} t_4 t_5 t_7 t_8 - \frac{5}{12} t_3^2 t_8^2 + \frac{1}{6} t_3 t_6^2 t_9 + \frac{1}{4} t_4^2 t_7 t_9 + \\ & \frac{3}{20} t_5^2 t_7 t_9 + \frac{1}{10} t_2 t_7^2 t_9 + \frac{1}{12} t_3^2 t_9^2 + \frac{1}{4} t_4^2 t_6 t_{10} + \frac{1}{20} t_5^2 t_6 t_{10} + \frac{1}{10} t_2 t_6 t_7 t_{10} + \\ & \frac{1}{2} t_2 t_4 t_8 t_{10} + \frac{1}{10} t_2 t_5 t_9 t_{10} + \frac{1}{4} t_4^2 t_5 t_{11} + \frac{1}{20} t_5^3 t_{11} + \frac{1}{6} t_3^2 t_6 t_{11} + \\ & \frac{1}{5} t_2 t_5 t_7 t_{11} + \frac{1}{20} t_2^2 t_{10} t_{11} + \frac{1}{18} t_3^3 t_{12} + \frac{1}{4} t_2 t_4^2 t_{12} + \frac{1}{20} t_2 t_5^2 t_{12} + \frac{1}{20} t_2^2 t_7 t_{12} \end{aligned}$$

Type: S11

Defining Equation is: $x^2 y + y^2 z + z^4$

Degree of variables{x, y, z} : $\left\{\frac{5}{16}, \frac{3}{8}, \frac{1}{4}\right\}$

Monomial Basis= $\{1, z, x, y, z^2, xz, yz, z^3, xz^2, yz^2, yz^3\}$

Degree of the monomial basis: $\left\{0, \frac{1}{4}, \frac{5}{16}, \frac{3}{8}, \frac{1}{2}, \frac{9}{16}, \frac{5}{8}, \frac{3}{4}, \frac{13}{16}, \frac{7}{8}, \frac{9}{8}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}\}$

deltaWW= $u_1 + z u_2 + x u_3 + y u_4 + z^2 u_5 + x z u_6 + y z u_7 + z^3 u_8 + x z^2 u_9 + y z^2 u_{10} + y z^3 u_{11}$

Up to order 3, the primitive form is:

$$1 - \frac{3 u_{10} u_{11}}{64} - \frac{7 z u_{11}^2}{128} + \frac{9}{512} u_8 u_{11}^2 + \frac{5 y u_{11}^3}{1024}$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned}
& -\frac{5}{32} t_5^2 t_6^2 + \frac{1}{48} t_5^3 t_7 + \frac{1}{4} t_4 t_6^2 t_7 + \frac{1}{4} t_3 t_6 t_7^2 - \frac{1}{16} t_4 t_5^2 t_8 - \frac{1}{8} t_3 t_5 t_6 t_8 - \\
& \frac{1}{16} t_2 t_6^2 t_8 + \frac{1}{8} t_4^2 t_7 t_8 + \frac{1}{16} t_2 t_5 t_7 t_8 - \frac{1}{32} t_3^2 t_8^2 - \frac{1}{16} t_2 t_4 t_8^2 - \frac{1}{16} t_3 t_5^2 t_9 - \\
& \frac{1}{2} t_2 t_5 t_6 t_9 + \frac{1}{2} t_3 t_4 t_7 t_9 - \frac{1}{8} t_2 t_3 t_8 t_9 - \frac{1}{8} t_2^2 t_9^2 + \frac{1}{8} t_4^2 t_5 t_{10} + \frac{1}{8} t_2 t_5^2 t_{10} + \\
& \frac{1}{2} t_3 t_4 t_6 t_{10} + \frac{1}{4} t_3^2 t_7 t_{10} + \frac{1}{32} t_2^2 t_8 t_{10} + \frac{1}{4} t_3^2 t_4 t_{11} + \frac{1}{8} t_2 t_4^2 t_{11} + \frac{3}{32} t_2^2 t_5 t_{11}
\end{aligned}$$

Type: S12

Defining Equation is: $x^2 y + y^2 z + x z^3$

Degree of variables $\{x, y, z\}$: $\left\{\frac{4}{13}, \frac{5}{13}, \frac{3}{13}\right\}$

Monomial Basis= $\{1, z, x, y, z^2, x z, y z, x y, x z^2, y z^2, x y z, x y z^2\}$

Degree of the monomial basis: $\left\{0, \frac{3}{13}, \frac{4}{13}, \frac{5}{13}, \frac{6}{13}, \frac{7}{13}, \frac{8}{13}, \frac{9}{13}, \frac{10}{13}, \frac{11}{13}, \frac{12}{13}, \frac{15}{13}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}$

deltaWW=

$$u_1 + z u_2 + x u_3 + y u_4 + z^2 u_5 + x z u_6 + y z u_7 + x y u_8 + x z^2 u_9 + y z^2 u_{10} + x y z u_{11} + x y z^2 u_{12}$$

Up to order 3, the primitive form is:

$$1 - \frac{12 u_{10} u_{12}}{169} + \frac{30 u_{11}^2 u_{12}}{2197} - \frac{2 x u_{12}^2}{169} + \frac{20 u_8 u_{12}^2}{2197} + \frac{93 z u_{11} u_{12}^2}{4394} + \frac{9 z^2 u_{12}^3}{2197}$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned}
& -\frac{5 t_4^4}{156} + \frac{1}{13} t_5 t_6^2 t_7 - \frac{1}{13} t_5^2 t_7^2 - \frac{1}{13} t_4 t_6 t_7^2 - \frac{1}{26} t_3 t_7^3 + \frac{5}{26} t_5^2 t_6 t_8 + \frac{1}{26} t_4 t_6^2 t_8 + \frac{2}{13} t_4 t_5 t_7 t_8 + \\
& \frac{1}{13} t_3 t_6 t_7 t_8 + \frac{1}{26} t_2 t_7^2 t_8 - \frac{3}{52} t_4^2 t_8^2 - \frac{2}{13} t_3 t_5 t_8^2 - \frac{1}{13} t_2 t_6 t_8^2 - \frac{1}{39} t_5^3 t_9 - \frac{2}{13} t_4 t_5 t_6 t_9 - \\
& \frac{1}{13} t_3 t_6^2 t_9 + \frac{1}{13} t_4^2 t_7 t_9 + \frac{1}{13} t_3 t_5 t_7 t_9 + \frac{1}{13} t_2 t_6 t_7 t_9 + \frac{1}{13} t_3 t_4 t_8 t_9 + \frac{1}{13} t_2 t_5 t_8 t_9 - \\
& \frac{1}{26} t_3^2 t_9^2 - \frac{1}{13} t_2 t_4 t_9^2 - \frac{1}{13} t_4 t_5^2 t_{10} + \frac{1}{26} t_4^2 t_6 t_{10} + \frac{1}{13} t_3 t_5 t_6 t_{10} + \frac{2}{13} t_2 t_6^2 t_{10} - \\
& \frac{3}{13} t_3 t_4 t_7 t_{10} - \frac{2}{13} t_2 t_5 t_7 t_{10} + \frac{1}{26} t_3^2 t_8 t_{10} + \frac{1}{13} t_2 t_4 t_8 t_{10} + \frac{1}{13} t_2 t_3 t_9 t_{10} - \frac{1}{26} t_2^2 t_{10}^2 + \\
& \frac{5}{26} t_4^2 t_5 t_{11} + \frac{7}{26} t_3 t_5^2 t_{11} + \frac{3}{13} t_3 t_4 t_6 t_{11} + \frac{4}{13} t_2 t_5 t_6 t_{11} + \frac{3}{26} t_3^2 t_7 t_{11} + \frac{1}{13} t_2 t_4 t_7 t_{11} - \\
& \frac{2}{13} t_2 t_3 t_8 t_{11} + \frac{1}{26} t_2^2 t_9 t_{11} + \frac{5}{26} t_3^2 t_4 t_{12} + \frac{2}{13} t_2 t_4^2 t_{12} + \frac{3}{13} t_2 t_3 t_5 t_{12} + \frac{3}{26} t_2^2 t_6 t_{12}
\end{aligned}$$

Type: U12

Defining Equation is: $x^3 + y^3 + z^4$

Degree of variables $\{x, y, z\}$: $\left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{4}\right\}$

Monomial Basis= $\{1, z, x, y, z^2, x z, y z, x y, x z^2, y z^2, x y z, x y z^2\}$

Degree of the monomial basis: $\left\{0, \frac{1}{4}, \frac{1}{3}, \frac{1}{3}, \frac{1}{2}, \frac{7}{12}, \frac{7}{12}, \frac{2}{3}, \frac{5}{6}, \frac{5}{6}, \frac{11}{12}, \frac{7}{6}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}$

deltaWW=

$$u_1 + z u_2 + x u_3 + y u_4 + z^2 u_5 + x z u_6 + y z u_7 + x y u_8 + x z^2 u_9 + y z^2 u_{10} + x y z u_{11} + x y z^2 u_{12}$$

Up to order 3, the primitive form is:

$$1 + \frac{1}{72} u_{11}^2 u_{12} + \frac{1}{72} u_8 u_{12}^2 + \frac{1}{36} z u_{11} u_{12}^2 + \frac{1}{72} z^2 u_{12}^3$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned} & \frac{1}{8} t_5^2 t_6 t_7 + \frac{1}{6} t_3 t_6^2 t_8 + \frac{1}{6} t_4 t_7^2 t_8 + \frac{1}{4} t_2 t_5 t_7 t_9 + \frac{1}{6} t_3^2 t_8 t_9 + \frac{1}{4} t_2 t_5 t_6 t_{10} + \frac{1}{6} t_4^2 t_8 t_{10} + \\ & \frac{1}{8} t_2^2 t_9 t_{10} + \frac{1}{8} t_2 t_5^2 t_{11} + \frac{1}{6} t_3^2 t_6 t_{11} + \frac{1}{6} t_4^2 t_7 t_{11} + \frac{1}{18} t_3^3 t_{12} + \frac{1}{18} t_4^3 t_{12} + \frac{1}{8} t_2^2 t_5 t_{12} \end{aligned}$$

Type: E14

Defining Equation is: $x^2 + x y^4 + z^3$

Degree of variables $\{x, y, z\}$: $\left\{\frac{1}{2}, \frac{1}{8}, \frac{1}{3}\right\}$

Monomial Basis= $\{1, y, x, y^2, x y, y^3, x y^2, z, y z, x z, y^2 z, x y z, y^3 z, x y^2 z\}$

Degree of the monomial basis: $\left\{0, \frac{1}{8}, \frac{1}{2}, \frac{1}{4}, \frac{5}{8}, \frac{3}{8}, \frac{3}{4}, \frac{1}{3}, \frac{11}{24}, \frac{5}{6}, \frac{7}{12}, \frac{23}{24}, \frac{17}{24}, \frac{13}{12}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}\}$

deltaWW= $u_1 + y u_2 + x u_3 + y^2 u_4 + x y u_5 + y^3 u_6 +$

$$x y^2 u_7 + z u_8 + y z u_9 + x z u_{10} + y^2 z u_{11} + x y z u_{12} + y^3 z u_{13} + x y^2 z u_{14}$$

Up to order 3, the primitive form is:

$$1 + \frac{1}{64} u_{12}^2 u_{14} + \frac{1}{64} u_{10} u_{14}^2 + \frac{1}{48} y u_{12} u_{14}^2 + \frac{1}{192} y^2 u_{14}^3$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned}
& -\frac{1}{16} t_3^2 t_5 t_9 + \frac{1}{8} t_5^2 t_6 t_9 + \frac{1}{4} t_4 t_5 t_7 t_9 + \frac{1}{4} t_3 t_6 t_7 t_9 + \frac{1}{8} t_2 t_7^2 t_9 - \frac{1}{8} t_3^2 t_4 t_{10} - \\
& \frac{1}{8} t_2 t_3 t_5 t_{10} + \frac{1}{2} t_4 t_5 t_6 t_{10} + \frac{3}{8} t_3 t_6^2 t_{10} + \frac{1}{8} t_4^2 t_7 t_{10} + \frac{1}{4} t_2 t_6 t_7 t_{10} + \frac{1}{6} t_8 t_9^2 t_{10} - \\
& \frac{1}{24} t_3^3 t_{11} + \frac{1}{4} t_4 t_5^2 t_{11} + \frac{1}{2} t_3 t_5 t_6 t_{11} + \frac{1}{4} t_3 t_4 t_7 t_{11} + \frac{1}{4} t_2 t_5 t_7 t_{11} - \frac{1}{4} t_6^2 t_7 t_{11} + \\
& \frac{1}{6} t_8^2 t_{10} t_{11} - \frac{1}{6} t_9^2 t_{11}^2 - \frac{1}{9} t_8 t_{11}^3 - \frac{1}{16} t_2 t_3^2 t_{12} + \frac{1}{4} t_4^2 t_5 t_{12} + \frac{1}{2} t_3 t_4 t_6 t_{12} + \\
& \frac{1}{4} t_2 t_5 t_6 t_{12} - \frac{1}{6} t_6^3 t_{12} + \frac{1}{4} t_2 t_4 t_7 t_{12} + \frac{1}{6} t_8^2 t_9 t_{12} + \frac{1}{2} t_3 t_4 t_5 t_{13} + \frac{1}{8} t_2 t_5^2 t_{13} + \\
& \frac{3}{8} t_3^2 t_6 t_{13} - \frac{1}{2} t_5 t_6^2 t_{13} + \frac{1}{4} t_2 t_3 t_7 t_{13} - \frac{1}{2} t_4 t_6 t_7 t_{13} - \frac{1}{9} t_9^3 t_{13} - \frac{2}{3} t_8 t_9 t_{11} t_{13} - \\
& \frac{1}{6} t_8^2 t_{13}^2 + \frac{1}{8} t_3 t_4^2 t_{14} + \frac{1}{4} t_2 t_4 t_5 t_{14} + \frac{1}{4} t_2 t_3 t_6 t_{14} - \frac{1}{4} t_4 t_6^2 t_{14} + \frac{1}{8} t_2^2 t_7 t_{14} + \frac{1}{18} t_8^3 t_{14}
\end{aligned}$$

Type: Z13

Defining Equation is: $x^2 + x y^3 + y z^3$

Degree of variables $\{x, y, z\}$: $\left\{\frac{1}{2}, \frac{1}{6}, \frac{5}{18}\right\}$

Monomial Basis= $\{1, y, z, y^2, y z, x, z^2, y^2 z, x y, x z, x y^2, x y z, x y^2 z\}$

Degree of the monomial basis: $\left\{0, \frac{1}{6}, \frac{5}{18}, \frac{1}{3}, \frac{4}{9}, \frac{1}{2}, \frac{5}{9}, \frac{11}{18}, \frac{2}{3}, \frac{7}{9}, \frac{5}{6}, \frac{17}{18}, \frac{10}{9}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}\}$

deltaWW=

$$u_1 + y u_2 + z u_3 + y^2 u_4 + y z u_5 + x u_6 + z^2 u_7 + y^2 z u_8 + x y u_9 + x z u_{10} + x y^2 u_{11} + x y z u_{12} + x y^2 z u_{13}$$

Up to order 3, the primitive form is:

$$1 + \frac{7}{486} u_{12}^2 u_{13} + \frac{7}{486} u_{10} u_{13}^2 + \frac{5}{243} y u_{12} u_{13}^2 + \frac{2}{729} y^2 u_{13}^3$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned}
& -\frac{1}{12} t_6^2 t_7^2 - \frac{1}{6} t_5 t_7^3 - \frac{5}{108} t_6^3 t_8 - \frac{1}{6} t_5^2 t_8^2 - \frac{1}{9} t_3 t_8^3 - \frac{1}{18} t_5 t_6^2 t_9 + \frac{1}{3} t_4 t_7^2 t_9 + \\
& \frac{4}{9} t_4 t_6 t_8 t_9 + \frac{1}{3} t_3 t_7 t_8 t_9 + \frac{1}{9} t_4 t_5 t_9^2 + \frac{1}{36} t_3 t_6 t_9^2 + \frac{1}{6} t_2 t_8 t_9^2 + \frac{1}{18} t_5^3 t_{10} - \\
& \frac{1}{6} t_4 t_6^2 t_{10} - \frac{1}{6} t_3 t_6 t_7 t_{10} + \frac{1}{3} t_3 t_5 t_8 t_{10} + \frac{2}{9} t_4^2 t_9 t_{10} - \frac{1}{6} t_2 t_6 t_9 t_{10} - \\
& \frac{1}{24} t_3^2 t_{10}^2 + \frac{1}{9} t_4 t_5 t_6 t_{11} + \frac{1}{36} t_3 t_6^2 t_{11} + \frac{1}{3} t_3 t_5 t_7 t_{11} + \frac{1}{3} t_2 t_7^2 t_{11} - \frac{1}{9} t_4^2 t_8 t_{11} + \\
& \frac{1}{9} t_2 t_6 t_8 t_{11} - \frac{1}{9} t_3 t_4 t_9 t_{11} + \frac{1}{9} t_2 t_5 t_9 t_{11} + \frac{1}{9} t_2 t_4 t_{10} t_{11} - \frac{1}{18} t_2 t_3 t_{11}^2 + \\
& \frac{1}{6} t_3 t_5^2 t_{12} + \frac{5}{18} t_4^2 t_6 t_{12} - \frac{1}{12} t_2 t_6^2 t_{12} + \frac{1}{3} t_3 t_4 t_7 t_{12} + \frac{1}{6} t_3^2 t_8 t_{12} + \frac{1}{3} t_2 t_4 t_9 t_{12} + \\
& \frac{1}{18} t_2^2 t_{11} t_{12} - \frac{2}{27} t_4^3 t_{13} + \frac{1}{6} t_3^2 t_5 t_{13} + \frac{2}{9} t_2 t_4 t_6 t_{13} + \frac{1}{3} t_2 t_3 t_7 t_{13} + \frac{1}{9} t_2^2 t_9 t_{13}
\end{aligned}$$

Type: W12b

Defining Equation is: $x^2 y + y^2 + z^5$

Degree of variables $\{x, y, z\}$: $\left\{\frac{1}{4}, \frac{1}{2}, \frac{1}{5}\right\}$

Monomial Basis= $\{1, x, y, z, xz, yz, z^2, xz^2, yz^2, z^3, xz^3, yz^3\}$

Degree of the monomial basis: $\left\{0, \frac{1}{4}, \frac{1}{2}, \frac{1}{5}, \frac{9}{20}, \frac{7}{10}, \frac{2}{5}, \frac{13}{20}, \frac{9}{10}, \frac{3}{5}, \frac{17}{20}, \frac{11}{10}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}$

deltaWW= $u_1 + x u_2 + y u_3 + z u_4 + x z u_5 + y z u_6 + z^2 u_7 + x z^2 u_8 + y z^2 u_9 + z^3 u_{10} + x z^3 u_{11} + y z^3 u_{12}$

Up to order 3, the primitive form is:

$$1 - \frac{u_9 u_{12}}{20} - \frac{z u_{12}^2}{20}$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned} & \frac{1}{4} t_3 t_5^2 t_6 + \frac{1}{4} t_2 t_5 t_6^2 + \frac{1}{4} t_3^2 t_5 t_8 + \frac{1}{2} t_2 t_3 t_6 t_8 - \frac{1}{5} t_7^2 t_8^2 + \frac{1}{2} t_2 t_3 t_5 t_9 + \frac{1}{4} t_2^2 t_6 t_9 + \frac{1}{15} t_7^3 t_9 + \\ & \frac{1}{10} t_6 t_7^2 t_{10} - \frac{2}{5} t_5 t_7 t_8 t_{10} - \frac{1}{5} t_4 t_8^2 t_{10} + \frac{1}{5} t_4 t_7 t_9 t_{10} - \frac{1}{10} t_5^2 t_{10}^2 + \frac{1}{10} t_4 t_6 t_{10}^2 + \frac{1}{4} t_2 t_3^2 t_{11} - \\ & \frac{1}{5} t_5 t_7^2 t_{11} - \frac{2}{5} t_4 t_7 t_8 t_{11} - \frac{2}{5} t_4 t_5 t_{10} t_{11} - \frac{1}{10} t_4^2 t_{11}^2 + \frac{1}{4} t_2^2 t_3 t_{12} + \frac{1}{10} t_4 t_7^2 t_{12} + \frac{1}{10} t_4^2 t_{10} t_{12} \end{aligned}$$

Type: W13

Defining Equation is: $x^2 + x y^2 + y z^4$

Degree of variables $\{x, y, z\}$: $\left\{\frac{1}{2}, \frac{1}{4}, \frac{3}{16}\right\}$

Monomial Basis= $\{1, z, y, z^2, yz, x, z^3, yz^2, xz, xy, xz^2, xyz, xyz^2\}$

Degree of the monomial basis: $\left\{0, \frac{3}{16}, \frac{1}{4}, \frac{3}{8}, \frac{7}{16}, \frac{1}{2}, \frac{9}{16}, \frac{5}{8}, \frac{11}{16}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{9}{8}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}\}$

deltaWW=

$$u_1 + z u_2 + y u_3 + z^2 u_4 + y z u_5 + x u_6 + z^3 u_7 + y z^2 u_8 + x z u_9 + x y u_{10} + x z^2 u_{11} + x y z u_{12} + x y z^2 u_{13}$$

Up to order 3, the primitive form is:

$$1 - \frac{5 u_{11} u_{13}}{64} + \frac{15 u_{12}^2 u_{13}}{1024} - \frac{y u_{13}^2}{128} + \frac{3}{256} u_{10} u_{13}^2 + \frac{11}{512} z u_{12} u_{13}^2 + \frac{3}{512} z^2 u_{13}^3$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned}
& -\frac{3}{32} t_6^2 t_7^2 - \frac{1}{6} t_5 t_7^3 - \frac{1}{48} t_6^3 t_8 - \frac{1}{4} t_4 t_7^2 t_8 - \frac{1}{8} t_5^2 t_8^2 - \frac{3}{32} t_5 t_6^2 t_9 - \frac{1}{4} t_4 t_6 t_7 t_9 + \\
& \frac{1}{4} t_4 t_5 t_8 t_9 + \frac{1}{8} t_2 t_8^2 t_9 - \frac{1}{16} t_4^2 t_9^2 - \frac{1}{8} t_3 t_6 t_8^2 - \frac{1}{16} t_2 t_7 t_8^2 + \frac{1}{16} t_5^2 t_6 t_{10} + \frac{1}{16} t_4 t_6^2 t_{10} + \\
& \frac{1}{2} t_4 t_5 t_7 t_{10} + \frac{3}{8} t_3 t_7^2 t_{10} + \frac{1}{8} t_4^2 t_8 t_{10} + \frac{1}{8} t_3 t_6 t_8 t_{10} + \frac{1}{4} t_2 t_7 t_8 t_{10} + \frac{1}{8} t_3 t_5 t_9 t_{10} + \\
& \frac{1}{16} t_2 t_6 t_9 t_{10} - \frac{1}{8} t_3 t_4 t_{10}^2 - \frac{1}{16} t_2 t_5 t_{10}^2 + \frac{1}{8} t_4 t_5^2 t_{11} - \frac{1}{16} t_4^2 t_6 t_{11} - \frac{1}{8} t_3 t_6^2 t_{11} - \\
& \frac{1}{8} t_2 t_6 t_7 t_{11} + \frac{1}{4} t_2 t_5 t_8 t_{11} - \frac{1}{8} t_2 t_4 t_9 t_{11} + \frac{1}{16} t_3^2 t_{10} t_{11} - \frac{1}{32} t_2^2 t_{11}^2 + \frac{1}{4} t_4^2 t_5 t_{12} + \\
& \frac{1}{4} t_3 t_5 t_6 t_{12} + \frac{1}{32} t_2 t_6^2 t_{12} + \frac{1}{2} t_3 t_4 t_7 t_{12} + \frac{1}{4} t_2 t_5 t_7 t_{12} + \frac{1}{4} t_2 t_4 t_8 t_{12} + \frac{1}{8} t_3^2 t_9 t_{12} - \\
& \frac{1}{8} t_2 t_3 t_{10} t_{12} + \frac{1}{8} t_3 t_4^2 t_{13} + \frac{1}{4} t_2 t_4 t_5 t_{13} + \frac{3}{16} t_3^2 t_6 t_{13} + \frac{1}{4} t_2 t_3 t_7 t_{13} + \frac{1}{8} t_2^2 t_8 t_{13}
\end{aligned}$$

Type: Q12

Defining Equation is: $x^2 y + x y^3 + z^3$

Degree of variables $\{x, y, z\}$: $\left\{\frac{2}{5}, \frac{1}{5}, \frac{1}{3}\right\}$

Monomial Basis= $\{1, x, y, xy, y^2, xy^2, z, xz, yz, xyz, y^2z, xy^2z\}$

Degree of the monomial basis: $\left\{0, \frac{2}{5}, \frac{1}{5}, \frac{3}{5}, \frac{2}{5}, \frac{4}{5}, \frac{1}{3}, \frac{11}{15}, \frac{8}{15}, \frac{14}{15}, \frac{11}{15}, \frac{17}{15}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}$

deltaWW=

$$u_1 + x u_2 + y u_3 + x y u_4 + y^2 u_5 + x y^2 u_6 + z u_7 + x z u_8 + y z u_9 + x y z u_{10} + y^2 z u_{11} + x y^2 z u_{12}$$

Up to order 3, the primitive form is:

$$1 + \frac{1}{75} u_{10}^2 u_{12} + \frac{1}{75} u_8 u_{12}^2 + \frac{1}{50} y u_{10} u_{12}^2 + \frac{1}{25} u_{11} u_{12}^2$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned}
& -\frac{3}{10} t_2^2 t_4 t_8 - \frac{1}{10} t_3 t_4^2 t_8 + \frac{1}{5} t_2 t_4 t_5 t_8 + \frac{3}{10} t_4 t_5^2 t_8 + \frac{2}{5} t_2 t_3 t_6 t_8 + \frac{1}{5} t_3 t_5 t_6 t_8 - \\
& \frac{1}{4} t_7^2 t_8^2 - \frac{1}{10} t_2 t_4^2 t_9 + \frac{1}{5} t_4^2 t_5 t_9 + \frac{1}{5} t_2^2 t_6 t_9 + \frac{1}{5} t_3 t_4 t_6 t_9 + \frac{1}{5} t_2 t_5 t_6 t_9 - \\
& \frac{1}{5} t_5^2 t_6 t_9 + \frac{1}{6} t_7 t_8 t_9^2 - \frac{t_9^4}{36} - \frac{1}{10} t_2^3 t_{10} - \frac{1}{5} t_2 t_3 t_4 t_{10} + \frac{1}{10} t_2^2 t_5 t_{10} + \frac{2}{5} t_3 t_4 t_5 t_{10} + \\
& \frac{3}{10} t_2 t_5^2 t_{10} - \frac{1}{5} t_5^3 t_{10} + \frac{1}{10} t_3^2 t_6 t_{10} + \frac{1}{6} t_7^2 t_9 t_{10} + \frac{1}{10} t_2^2 t_4 t_{11} + \frac{1}{5} t_3 t_4^2 t_{11} + \\
& \frac{3}{5} t_2 t_4 t_5 t_{11} - \frac{3}{5} t_4 t_5^2 t_{11} + \frac{1}{5} t_2 t_3 t_6 t_{11} - \frac{2}{5} t_3 t_5 t_6 t_{11} + \frac{1}{6} t_7^2 t_8 t_{11} - \frac{1}{3} t_7 t_9^2 t_{11} - \\
& \frac{1}{6} t_7^2 t_{11}^2 + \frac{1}{5} t_2^2 t_3 t_{12} + \frac{1}{10} t_3^2 t_4 t_{12} + \frac{1}{5} t_2 t_3 t_5 t_{12} - \frac{1}{5} t_3 t_5^2 t_{12} + \frac{1}{18} t_7^3 t_{12}
\end{aligned}$$

Type: U12b

Defining Equation is: $x^2 y + y^3 + z^4$

Degree of variables{x, y, z} : $\left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{4}\right\}$

Monomial Basis= $\{1, x, y, y^2, z, xz, yz, y^2z, z^2, xz^2, yz^2, y^2z^2\}$

Degree of the monomial basis: $\left\{0, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{7}{12}, \frac{7}{12}, \frac{11}{12}, \frac{1}{2}, \frac{5}{6}, \frac{5}{6}, \frac{7}{6}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}$

deltaWW= $u_1 + x u_2 + y u_3 + y^2 u_4 + z u_5 + x z u_6 + y z u_7 + y^2 z u_8 + z^2 u_9 + x z^2 u_{10} + y z^2 u_{11} + y^2 z^2 u_{12}$

Up to order 3, the primitive form is:

$$1 + \frac{7}{288} u_8^2 u_{12} - \frac{u_{11} u_{12}}{24} - \frac{y u_{12}^2}{16} + \frac{1}{72} u_4 u_{12}^2 + \frac{11}{288} z u_8 u_{12}^2 + \frac{1}{72} z^2 u_{12}^3$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned} & \frac{1}{4} t_3 t_4 t_6^2 + \frac{1}{2} t_2 t_4 t_6 t_7 + \frac{1}{12} t_3 t_4 t_7^2 + \frac{1}{2} t_2 t_3 t_6 t_8 + \frac{1}{4} t_2^2 t_7 t_8 + \frac{1}{12} t_3^2 t_7 t_8 - \\ & \frac{3}{16} t_6^2 t_9^2 + \frac{1}{16} t_7^2 t_9^2 + \frac{1}{8} t_5 t_8 t_9^2 + \frac{1}{2} t_2 t_3 t_4 t_{10} - \frac{3}{4} t_5 t_6 t_9 t_{10} - \frac{3}{16} t_5^2 t_{10}^2 + \frac{1}{4} t_2^2 t_4 t_{11} + \\ & \frac{1}{12} t_3^2 t_4 t_{11} + \frac{1}{4} t_5 t_7 t_9 t_{11} + \frac{1}{16} t_5^2 t_{11}^2 + \frac{1}{4} t_2^2 t_3 t_{12} + \frac{1}{36} t_3^3 t_{12} + \frac{1}{8} t_5^2 t_9 t_{12} \end{aligned}$$

Type: U12c

Defining Equation is: $x^2 y + x y^2 + z^4$

Degree of variables{x, y, z} : $\left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{4}\right\}$

Monomial Basis= $\{1, x, y, x y, z, x z, y z, x y z, z^2, x z^2, y z^2, x y z^2\}$

Degree of the monomial basis: $\left\{0, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{7}{12}, \frac{7}{12}, \frac{11}{12}, \frac{1}{2}, \frac{5}{6}, \frac{5}{6}, \frac{7}{6}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}$

deltaWW=

$u_1 + x u_2 + y u_3 + x y u_4 + z u_5 + x z u_6 + y z u_7 + x y z u_8 + z^2 u_9 + x z^2 u_{10} + y z^2 u_{11} + x y z^2 u_{12}$

Up to order 3, the primitive form is:

$$1 + \frac{1}{72} u_8^2 u_{12} - \frac{u_{10} u_{12}}{12} - \frac{u_{11} u_{12}}{12} + \frac{1}{72} u_4 u_{12}^2 + \frac{1}{36} z u_8 u_{12}^2 + \frac{1}{72} z^2 u_{12}^3$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned}
& -\frac{1}{3} t_2 t_4 t_6^2 + \frac{1}{6} t_3 t_4 t_6^2 + \frac{1}{3} t_2 t_4 t_6 t_7 + \frac{1}{3} t_3 t_4 t_6 t_7 + \frac{1}{6} t_2 t_4 t_7^2 - \frac{1}{3} t_3 t_4 t_7^2 - \frac{1}{3} t_2^2 t_6 t_8 + \\
& \frac{1}{3} t_2 t_3 t_6 t_8 + \frac{1}{6} t_3^2 t_6 t_8 + \frac{1}{6} t_2^2 t_7 t_8 + \frac{1}{3} t_2 t_3 t_7 t_8 - \frac{1}{3} t_3^2 t_7 t_8 - \frac{1}{8} t_6^2 t_9^2 + \frac{1}{8} t_6 t_7 t_9^2 - \\
& \frac{1}{8} t_7^2 t_9^2 + \frac{1}{8} t_5 t_8 t_9^2 - \frac{1}{3} t_2^2 t_4 t_{10} + \frac{1}{3} t_2 t_3 t_4 t_{10} + \frac{1}{6} t_3^2 t_4 t_{10} - \frac{1}{2} t_5 t_6 t_9 t_{10} + \frac{1}{4} t_5 t_7 t_9 t_{10} - \\
& \frac{1}{8} t_5^2 t_{10}^2 + \frac{1}{6} t_2^2 t_4 t_{11} + \frac{1}{3} t_2 t_3 t_4 t_{11} - \frac{1}{3} t_3^2 t_4 t_{11} + \frac{1}{4} t_5 t_6 t_9 t_{11} - \frac{1}{2} t_5 t_7 t_9 t_{11} + \\
& \frac{1}{8} t_5^2 t_{10} t_{11} - \frac{1}{8} t_5^2 t_{11}^2 - \frac{1}{9} t_2^3 t_{12} + \frac{1}{6} t_2^2 t_3 t_{12} + \frac{1}{6} t_2 t_3^2 t_{12} - \frac{1}{9} t_3^3 t_{12} + \frac{1}{8} t_5^2 t_9 t_{12}
\end{aligned}$$

Type: Z13T

Defining Equation is: $x^3 + x y^6 + z^2$

Degree of variables $\{x, y, z\}$: $\left\{\frac{1}{3}, \frac{1}{9}, \frac{1}{2}\right\}$

Monomial Basis= $\{1, y, y^2, y^3, y^4, y^5, x, x y, x y^2, x y^3, x y^4, x^2, x^2 y, x^2 y^2, x^2 y^3, x^2 y^4\}$

Degree of the monomial basis: $\left\{0, \frac{1}{9}, \frac{2}{9}, \frac{1}{3}, \frac{4}{9}, \frac{5}{9}, \frac{1}{3}, \frac{4}{9}, \frac{5}{9}, \frac{2}{3}, \frac{7}{9}, \frac{2}{3}, \frac{7}{9}, \frac{8}{9}, 1, \frac{10}{9}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}, u_{16}\}$

deltaWW= $u_1 + y u_2 + y^2 u_3 + y^3 u_4 + y^4 u_5 + y^5 u_6 + x u_7 + x y u_8 +$
 $x y^2 u_9 + x y^3 u_{10} + x y^4 u_{11} + x^2 u_{12} + x^2 y u_{13} + x^2 y^2 u_{14} + x^2 y^3 u_{15} + x^2 y^4 u_{16}$

Up to order 3, the primitive form is:

$$1 - \frac{5 u_{15}^2}{216} - \frac{5 u_{14} u_{16}}{108} - \frac{2}{27} y u_{15} u_{16} - \frac{1}{24} y^2 u_{16}^2 + \frac{13}{648} u_{11} u_{16}^2 + \frac{25 x u_{16}^3}{1944}$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned}
& -\frac{1}{3} t_6^3 t_8 - \frac{3}{4} t_5 t_6^2 t_9 + \frac{1}{36} t_5 t_9^3 - \frac{1}{2} t_5^2 t_6 t_{10} - \frac{1}{2} t_4 t_6^2 t_{10} + \frac{1}{6} t_5 t_8 t_9 t_{10} + \frac{1}{6} t_4 t_9^2 t_{10} + \\
& \frac{1}{12} t_4 t_8 t_{10}^2 + \frac{1}{6} t_3 t_9 t_{10}^2 + \frac{1}{36} t_2 t_{10}^3 - \frac{1}{12} t_5^3 t_{11} - \frac{1}{2} t_4 t_5 t_6 t_{11} - \frac{1}{4} t_3 t_6^2 t_{11} + \frac{1}{12} t_5 t_8^2 t_{11} + \\
& \frac{1}{6} t_4 t_8 t_9 t_{11} + \frac{1}{12} t_3 t_9^2 t_{11} + \frac{1}{6} t_3 t_8 t_{10} t_{11} + \frac{1}{6} t_2 t_9 t_{10} t_{11} + \frac{1}{12} t_2 t_8 t_{11}^2 + \frac{5}{12} t_6^2 t_7 t_{12} + \\
& -\frac{2}{3} t_5 t_6 t_8 t_{12} + \frac{1}{4} t_5^2 t_9 t_{12} + \frac{1}{2} t_4 t_6 t_9 t_{12} + \frac{1}{36} t_8^2 t_9 t_{12} + \frac{1}{36} t_7 t_9^2 t_{12} + \frac{1}{3} t_4 t_5 t_{10} t_{12} + \\
& \frac{1}{3} t_3 t_6 t_{10} t_{12} + \frac{1}{18} t_7 t_8 t_{10} t_{12} + \frac{1}{12} t_4^2 t_{11} t_{12} + \frac{1}{6} t_3 t_5 t_{11} t_{12} + \frac{1}{6} t_2 t_6 t_{11} t_{12} + \frac{1}{36} t_7^2 t_{11} t_{12} - \\
& \frac{1}{9} t_5 t_7 t_{12}^2 - \frac{1}{12} t_4 t_8 t_{12}^2 - \frac{1}{18} t_3 t_9 t_{12}^2 - \frac{1}{36} t_2 t_{10} t_{12}^2 + \frac{2}{3} t_5 t_6 t_7 t_{13} + \frac{1}{3} t_5^2 t_8 t_{13} + \\
& \frac{1}{2} t_4 t_6 t_8 t_{13} + \frac{1}{54} t_8^3 t_{13} + \frac{1}{2} t_4 t_5 t_9 t_{13} + \frac{1}{3} t_3 t_6 t_9 t_{13} + \frac{1}{9} t_7 t_8 t_9 t_{13} + \frac{1}{6} t_4^2 t_{10} t_{13} + \\
& \frac{1}{3} t_3 t_5 t_{10} t_{13} + \frac{1}{6} t_2 t_6 t_{10} t_{13} + \frac{1}{18} t_7^2 t_{10} t_{13} + \frac{1}{6} t_3 t_4 t_{11} t_{13} + \frac{1}{6} t_2 t_5 t_{11} t_{13} - \frac{1}{6} t_4 t_7 t_{12} t_{13} - \\
& \frac{1}{9} t_3 t_8 t_{12} t_{13} - \frac{1}{18} t_2 t_9 t_{12} t_{13} - \frac{1}{18} t_3 t_7 t_{13}^2 - \frac{1}{36} t_2 t_8 t_{13}^2 + \frac{1}{4} t_5^2 t_7 t_{14} + \frac{1}{2} t_4 t_6 t_7 t_{14} + \\
& \frac{1}{2} t_4 t_5 t_8 t_{14} + \frac{1}{3} t_3 t_6 t_8 t_{14} + \frac{1}{12} t_7 t_8^2 t_{14} + \frac{1}{4} t_4^2 t_9 t_{14} + \frac{1}{3} t_3 t_5 t_9 t_{14} + \frac{1}{6} t_2 t_6 t_9 t_{14} + \\
& \frac{1}{12} t_7^2 t_9 t_{14} + \frac{1}{3} t_3 t_4 t_{10} t_{14} + \frac{1}{6} t_2 t_5 t_{10} t_{14} + \frac{1}{12} t_3^2 t_{11} t_{14} + \frac{1}{6} t_2 t_4 t_{11} t_{14} - \frac{1}{9} t_3 t_7 t_{12} t_{14} - \\
& \frac{1}{18} t_2 t_8 t_{12} t_{14} - \frac{1}{18} t_2 t_7 t_{13} t_{14} + \frac{1}{3} t_4 t_5 t_7 t_{15} + \frac{1}{3} t_3 t_6 t_7 t_{15} + \frac{1}{6} t_4^2 t_8 t_{15} + \frac{1}{3} t_3 t_5 t_8 t_{15} + \\
& \frac{1}{6} t_2 t_6 t_8 t_{15} + \frac{1}{9} t_7^2 t_8 t_{15} + \frac{1}{3} t_3 t_4 t_9 t_{15} + \frac{1}{6} t_2 t_5 t_9 t_{15} + \frac{1}{6} t_3^2 t_{10} t_{15} + \frac{1}{6} t_2 t_4 t_{10} t_{15} + \\
& \frac{1}{6} t_2 t_3 t_{11} t_{15} - \frac{1}{18} t_2 t_7 t_{12} t_{15} + \frac{1}{12} t_4^2 t_7 t_{16} + \frac{1}{6} t_3 t_5 t_7 t_{16} + \frac{1}{6} t_2 t_6 t_7 t_{16} + \frac{5}{108} t_7^3 t_{16} + \\
& \frac{1}{6} t_3 t_4 t_8 t_{16} + \frac{1}{6} t_2 t_5 t_8 t_{16} + \frac{1}{12} t_3^2 t_9 t_{16} + \frac{1}{6} t_2 t_4 t_9 t_{16} + \frac{1}{6} t_2 t_3 t_{10} t_{16} + \frac{1}{12} t_2^2 t_{11} t_{16}
\end{aligned}$$

Type: Q12T

Defining Equation is: $x^2 + xy^5 + z^3$

Degree of variables $\{x, y, z\}$: $\left\{\frac{1}{2}, \frac{1}{10}, \frac{1}{3}\right\}$

Monomial Basis=

$$\{1, y, y^2, y^3, y^4, x, xy, xy^2, xy^3, z, yz, y^2z, y^3z, y^4z, xz, xyz, xy^2z, xy^3z\}$$

Degree of the monomial basis:

$$\left\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{1}{3}, \frac{13}{30}, \frac{8}{15}, \frac{19}{30}, \frac{11}{15}, \frac{5}{6}, \frac{14}{15}, \frac{31}{30}, \frac{17}{15}\right\}$$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}, u_{16}, u_{17}, u_{18}\}$

$$\begin{aligned}
\text{deltaWW} = & u_1 + y u_2 + y^2 u_3 + y^3 u_4 + y^4 u_5 + x u_6 + x y u_7 + x y^2 u_8 + x y^3 u_9 + z u_{10} + \\
& y z u_{11} + y^2 z u_{12} + y^3 z u_{13} + y^4 z u_{14} + x z u_{15} + x y z u_{16} + x y^2 z u_{17} + x y^3 z u_{18}
\end{aligned}$$

Up to order 3, the primitive form is:

$$1 + \frac{1}{75} u_{16} u_{17}^2 + \frac{y u_{17}^3}{150} + \frac{1}{75} u_{16}^2 u_{18} + \frac{2}{75} u_{15} u_{17} u_{18} + \frac{1}{25} y u_{16} u_{17} u_{18} +$$

$$\frac{1}{50} y^2 u_{17}^2 u_{18} + \frac{1}{25} u_{14} u_{18}^2 + \frac{1}{50} y u_{15} u_{18}^2 + \frac{1}{50} y^2 u_{16} u_{18}^2 + \frac{1}{75} y^3 u_{17} u_{18}^2$$

The four point function multiplying with (-1) is given by:

$$-\frac{1}{20} t_6 t_7^2 t_{11} - \frac{1}{20} t_6^2 t_8 t_{11} + \frac{1}{5} t_5 t_7 t_8 t_{11} + \frac{1}{10} t_4 t_8^2 t_{11} + \frac{1}{5} t_5 t_6 t_9 t_{11} + \frac{1}{5} t_4 t_7 t_9 t_{11} +$$

$$\frac{1}{5} t_3 t_8 t_9 t_{11} + \frac{1}{10} t_2 t_9^2 t_{11} - \frac{1}{10} t_6^2 t_7 t_{12} + \frac{1}{5} t_5 t_7^2 t_{12} + \frac{2}{5} t_5 t_6 t_8 t_{12} + \frac{2}{5} t_4 t_7 t_8 t_{12} +$$

$$\frac{1}{5} t_3 t_8^2 t_{12} - \frac{1}{5} t_5^2 t_9 t_{12} + \frac{1}{5} t_4 t_6 t_9 t_{12} + \frac{1}{5} t_3 t_7 t_9 t_{12} + \frac{1}{5} t_2 t_8 t_9 t_{12} - \frac{t_{12}^4}{36} -$$

$$\frac{1}{20} t_6^3 t_{13} + \frac{3}{5} t_5 t_6 t_7 t_{13} + \frac{3}{10} t_4 t_7^2 t_{13} - \frac{2}{5} t_5^2 t_8 t_{13} + \frac{2}{5} t_4 t_6 t_8 t_{13} + \frac{2}{5} t_3 t_7 t_8 t_{13} +$$

$$\frac{1}{10} t_2 t_8^2 t_{13} - \frac{2}{5} t_4 t_5 t_9 t_{13} + \frac{1}{5} t_3 t_6 t_9 t_{13} + \frac{1}{5} t_2 t_7 t_9 t_{13} - \frac{1}{3} t_{11} t_{12}^2 t_{13} - \frac{1}{6} t_{11}^2 t_{13}^2 -$$

$$\frac{1}{3} t_{10} t_{12} t_{13}^2 + \frac{2}{5} t_5 t_6^2 t_{14} - \frac{3}{5} t_5^2 t_7 t_{14} + \frac{3}{5} t_4 t_6 t_7 t_{14} + \frac{1}{5} t_3 t_7^2 t_{14} - \frac{4}{5} t_4 t_5 t_8 t_{14} +$$

$$\frac{2}{5} t_3 t_6 t_8 t_{14} + \frac{1}{5} t_2 t_7 t_8 t_{14} - \frac{1}{5} t_4^2 t_9 t_{14} - \frac{2}{5} t_3 t_5 t_9 t_{14} + \frac{1}{5} t_2 t_6 t_9 t_{14} - \frac{1}{3} t_{11}^2 t_{12} t_{14} -$$

$$\frac{1}{3} t_{10} t_{12}^2 t_{14} - \frac{2}{3} t_{10} t_{11} t_{13} t_{14} - \frac{1}{6} t_{10}^2 t_{14}^2 + \frac{2}{5} t_5^2 t_6 t_{15} - \frac{3}{20} t_4 t_6^2 t_{15} + \frac{3}{5} t_4 t_5 t_7 t_{15} -$$

$$\frac{1}{5} t_3 t_6 t_7 t_{15} - \frac{1}{20} t_2 t_7^2 t_{15} + \frac{1}{5} t_4^2 t_8 t_{15} + \frac{2}{5} t_3 t_5 t_8 t_{15} - \frac{1}{10} t_2 t_6 t_8 t_{15} + \frac{1}{5} t_3 t_4 t_9 t_{15} +$$

$$\frac{1}{5} t_2 t_5 t_9 t_{15} + \frac{1}{18} t_{11}^3 t_{15} + \frac{1}{3} t_{10} t_{11} t_{12} t_{15} + \frac{1}{6} t_{10}^2 t_{13} t_{15} - \frac{1}{5} t_5^3 t_{16} + \frac{3}{5} t_4 t_5 t_6 t_{16} -$$

$$\frac{1}{10} t_3 t_6^2 t_{16} + \frac{3}{10} t_4^2 t_7 t_{16} + \frac{2}{5} t_3 t_5 t_7 t_{16} - \frac{1}{10} t_2 t_6 t_7 t_{16} + \frac{2}{5} t_3 t_4 t_8 t_{16} +$$

$$\frac{1}{5} t_2 t_5 t_8 t_{16} + \frac{1}{10} t_3^2 t_9 t_{16} + \frac{1}{5} t_2 t_4 t_9 t_{16} + \frac{1}{6} t_{10} t_{11}^2 t_{16} + \frac{1}{6} t_{10}^2 t_{12} t_{16} - \frac{2}{5} t_4 t_5^2 t_{17} +$$

$$\frac{1}{5} t_4^2 t_6 t_{17} + \frac{2}{5} t_3 t_5 t_6 t_{17} - \frac{1}{20} t_2 t_6^2 t_{17} + \frac{2}{5} t_3 t_4 t_7 t_{17} + \frac{1}{5} t_2 t_5 t_7 t_{17} + \frac{1}{5} t_3^2 t_8 t_{17} +$$

$$\frac{1}{5} t_2 t_4 t_8 t_{17} + \frac{1}{5} t_2 t_3 t_9 t_{17} + \frac{1}{6} t_{10}^2 t_{11} t_{17} - \frac{1}{5} t_4^2 t_5 t_{18} - \frac{1}{5} t_3 t_5^2 t_{18} + \frac{1}{5} t_3 t_4 t_6 t_{18} +$$

$$\frac{1}{5} t_2 t_5 t_6 t_{18} + \frac{1}{10} t_3^2 t_7 t_{18} + \frac{1}{5} t_2 t_4 t_7 t_{18} + \frac{1}{5} t_2 t_3 t_8 t_{18} + \frac{1}{10} t_2^2 t_9 t_{18} + \frac{1}{18} t_{10}^3 t_{18}$$

Type: U12bT

Defining Equation is: $x^2 + x y^3 + z^4$

Degree of variables $\{x, y, z\}$: $\left\{\frac{1}{2}, \frac{1}{6}, \frac{1}{4}\right\}$

Monomial Basis= $\{1, y, y^2, x, x y, z, y z, y^2 z, x z, x y z, z^2, y z^2, y^2 z^2, x z^2, x y z^2\}$

Degree of the monomial basis: $\left\{0, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{12}, \frac{7}{12}, \frac{3}{4}, \frac{11}{12}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1, \frac{7}{6}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}\}$

$$\text{deltaWW} = u_1 + y u_2 + y^2 u_3 + x u_4 + x y u_5 + z u_6 + y z u_7 + \\ y^2 z u_8 + x z u_9 + x y z u_{10} + z^2 u_{11} + y^2 z^2 u_{12} + x z^2 u_{13} + x y z^2 u_{14} + x y z^2 u_{15}$$

Up to order 3, the primitive form is:

$$1 - \frac{u_{14}^2}{24} + \frac{1}{72} u_{10}^2 u_{15} - \frac{u_{13} u_{15}}{12} - \frac{1}{24} y u_{14} u_{15} + \frac{1}{72} u_5 u_{15}^2 + \frac{1}{36} z u_{10} u_{15}^2 + \frac{1}{72} z^2 u_{15}^3$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned} & \frac{1}{12} t_5^2 t_7^2 + \frac{1}{3} t_4 t_5 t_7 t_8 + \frac{1}{6} t_4^2 t_8^2 - \frac{1}{3} t_3 t_5 t_8^2 - \frac{1}{12} t_4^2 t_7 t_9 + \frac{1}{3} t_3 t_5 t_7 t_9 + \\ & \frac{2}{3} t_3 t_4 t_8 t_9 + \frac{1}{3} t_2 t_5 t_8 t_9 + \frac{1}{6} t_3^2 t_9^2 - \frac{1}{12} t_2 t_4 t_9^2 + \frac{1}{3} t_3 t_4 t_7 t_{10} + \frac{1}{3} t_2 t_5 t_7 t_{10} - \\ & \frac{1}{3} t_3^2 t_8 t_{10} + \frac{1}{3} t_2 t_4 t_8 t_{10} + \frac{1}{3} t_2 t_3 t_9 t_{10} + \frac{1}{12} t_2^2 t_{10}^2 - \frac{1}{8} t_8^2 t_{11}^2 + \frac{1}{8} t_7 t_9 t_{11}^2 + \\ & \frac{1}{8} t_6 t_{10} t_{11}^2 - \frac{1}{36} t_4^3 t_{12} + \frac{1}{3} t_3 t_4 t_5 t_{12} + \frac{1}{6} t_2 t_5^2 t_{12} - \frac{1}{2} t_7 t_8 t_{11} t_{12} + \frac{1}{4} t_6 t_9 t_{11} t_{12} - \\ & \frac{1}{8} t_7^2 t_{12}^2 - \frac{1}{4} t_6 t_8 t_{12}^2 + \frac{1}{3} t_3 t_4^2 t_{13} - \frac{1}{3} t_3^2 t_5 t_{13} + \frac{1}{3} t_2 t_4 t_5 t_{13} - \frac{1}{4} t_7^2 t_{11} t_{13} - \\ & \frac{1}{2} t_6 t_8 t_{11} t_{13} - \frac{1}{2} t_6 t_7 t_{12} t_{13} - \frac{1}{8} t_6^2 t_{13}^2 + \frac{1}{3} t_3^2 t_4 t_{14} - \frac{1}{12} t_2 t_4^2 t_{14} + \frac{1}{3} t_2 t_3 t_5 t_{14} + \\ & \frac{1}{4} t_6 t_7 t_{11} t_{14} + \frac{1}{8} t_6^2 t_{12} t_{14} - \frac{1}{9} t_3^3 t_{15} + \frac{1}{3} t_2 t_3 t_4 t_{15} + \frac{1}{6} t_2^2 t_5 t_{15} + \frac{1}{8} t_6^2 t_{11} t_{15} \\ & , \end{aligned}$$

(*Output the 5-pt function for U12c*)

```
FivePointFunction[EQN_, TXYZ_List, JAC_List, MM_, VAR_, VARVV_, PRIM_] :=
Module[{eqn = EQN, txyz = TXYZ, jac = JAC, mm = MM, var = VAR, varvv = VARVV,
  prim = PRIM, len, para, paravv, expotab, sca, k, lenxyz, GrConv, Groebner, pr22,
  conv, MatDual, basismat, res11, dualjac, red, flatvar, coe22, flatvartab,
  uutab, vvtab, pos22, i, j, coe33, tab33, res, uut22, uut33, uv22, uv33, uv25},
  len = Length[jac]; para = Table[var, {j, 1, len}]; paravv = para /. var -> varvv;
  expotab = Table[Exponent[jac[[i]], txyz], {i, 1, len}]; pr22 =
    PolyPickup[prim, para, 2] + 1; sca = Sum[(para.jac)^k * pr22 / (k! t^k), {k, mm}];
  lenxyz = Length[txyz]; GrConv = conversionMatrix[JacIdeal[eqn, txyz], txyz];
  Groebner = GrConv[[1]]; conv = GrConv[[2]];
  MatDual = Metric[eqn, txyz, jac, Groebner];
  dualjac = MatDual[[2]]; basismat = MatDual[[1]];
  red = CohomologyReduction[sca, txyz, jac, Groebner, dualjac, len, conv, lenxyz];
  flatvar = PolyTruncate[Coefficient[t^mm * red, t^(mm - 1)], para, 3];
  coe22 = PolyTruncate[Coefficient[t^mm * red, t^(mm - 2)], para, 4];
  flatvartab = CoefficientRules[flatvar, txyz];
  uutab = Table[0, {i, 1, len}];
  For[i = 1, i <= len, pos22 = Position[expotab, flatvartab[[i, 1]]][[1, 1]];
    uutab[[pos22]] = flatvartab[[i, 2]]; i++];
  uut22 = Table[PolyPickup[uutab[[i]], para, 2], {i, 1, len}];
  uut33 = Table[PolyPickup[uutab[[i]], para, 3], {i, 1, len}];
```

```

vvtab = (para - uut22) /. var → varvv;
uv22 = uut22; uv25 = Expand[uv22 /. Table[para[[i]] → vvtab[[i]], {i, 1, len}]];
uv33 = Table[PolyPickup[uv25[[i]], paravv, 3], {i, 1, len}];
vvtab = ((para - uut22 - uut33) /. var → varvv) - uv33;
res11 = CoefficientRules[PolyTruncate[
  Expand[coe22 /. Table[para[[i]] → vvtab[[i]], {i, 1, len}]], paravv, 4], txyz];
tab33 = Table[0, {i, 1, len}]; For[i = 1, i ≤ len, coe33 = res11[[i]];
  pos22 = Position[expotab, coe33[[1]]][[1, 1]];
  tab33[[pos22]] = PolyPickup[coe33[[2]], paravv, 4]; i++];
res = Expand[(paravv.basismat).tab33 / 5]; res];

```

(*Print out final output. SCAL is used to take a multiplication of the 4-point function*)

```

Output55[TT_List, POS_, TXYZ_List, MM_, VAR_, VARVV_] :=
Module[{mm = MM, tt = TT, k, pos = POS, var = VAR, varvv = VARVV, mat, i, j, len,
  para, txyz = TXYZ, jac, eqn, deg, prim}, jac = JacWWBasis[txyz][[pos]];
eqn = DefiningEquation22[tt, txyz][[pos]]; len = Length[jac];
deg = DegtxyzOne[eqn, txyz]; para = Table[var, {j, 1, len}]; i = pos;
Print["Type: ", SingularType[[i]]]; Print["Defining Equation is: ", eqn];
Print["Degree of variables", txyz, " : ", deg];
Print["Monomial Basis= ", jac]; Print["Degree of the monomial basis: ",
  Table[Exponent[jac[[i]], txyz].deg, {i, len}]];
Print["Parameters= ", para]; Print["deltaWW= ", para.jac];
Print["Up to order ", 3, ", the primitive form is: "]; Print[" "];
prim = PrimitiveForm[eqn, txyz, jac, 3, var]; Print[prim]; Print[" "];
Print["The four point function multiplying with (-1) is given by: "];
Print[" "]; Print[Expand[FourPointFunction[eqn, txyz, jac, 3, var, varvv]]];
Print["The five point function is given by: "]; Print[" "];
Print[Expand[FivePointFunction[eqn, txyz, jac, mm, var, varvv, prim]]];

```

(*tt={a, b, c} denotes the normalization. t
is used to generate the flat coordinates here *)

```

tt = {1, 1, 1}; txyz = {x, y, z};
For[pos = 21, pos ≤ 21,
  Output55[tt, pos, txyz, 4, u, t]; pos++];

```

Type: U12c

Defining Equation is: $x^2 y + x y^2 + z^4$

Degree of variables{x, y, z} : $\left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{4}\right\}$

Monomial Basis= $\{1, x, y, xy, z, xz, yz, xyz, z^2, xz^2, yz^2, xyz^2\}$

Degree of the monomial basis: $\left\{0, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{7}{12}, \frac{7}{12}, \frac{11}{12}, \frac{1}{2}, \frac{5}{6}, \frac{5}{6}, \frac{7}{6}\right\}$

Parameters= $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}$

deltaWW=

$u_1 + x u_2 + y u_3 + x y u_4 + z u_5 + x z u_6 + y z u_7 + x y z u_8 + z^2 u_9 + x z^2 u_{10} + y z^2 u_{11} + x y z^2 u_{12}$

Up to order 3, the primitive form is:

$$1 + \frac{1}{72} u_8^2 u_{12} - \frac{u_{10} u_{12}}{12} - \frac{u_{11} u_{12}}{12} + \frac{1}{72} u_4 u_{12}^2 + \frac{1}{36} z u_8 u_{12}^2 + \frac{1}{72} z^2 u_{12}^3$$

The four point function multiplying with (-1) is given by:

$$\begin{aligned} & -\frac{1}{3} t_2 t_4 t_6^2 + \frac{1}{6} t_3 t_4 t_6^2 + \frac{1}{3} t_2 t_4 t_6 t_7 + \frac{1}{3} t_3 t_4 t_6 t_7 + \frac{1}{6} t_2 t_4 t_7^2 - \frac{1}{3} t_3 t_4 t_7^2 - \frac{1}{3} t_2^2 t_6 t_8 + \\ & \frac{1}{3} t_2 t_3 t_6 t_8 + \frac{1}{6} t_3^2 t_6 t_8 + \frac{1}{6} t_2^2 t_7 t_8 + \frac{1}{3} t_2 t_3 t_7 t_8 - \frac{1}{3} t_3^2 t_7 t_8 - \frac{1}{8} t_6^2 t_9^2 + \frac{1}{8} t_6 t_7 t_9^2 - \\ & \frac{1}{8} t_7^2 t_9^2 + \frac{1}{8} t_5 t_8 t_9^2 - \frac{1}{3} t_2^2 t_4 t_{10} + \frac{1}{3} t_2 t_3 t_4 t_{10} + \frac{1}{6} t_3^2 t_4 t_{10} - \frac{1}{2} t_5 t_6 t_9 t_{10} + \frac{1}{4} t_5 t_7 t_9 t_{10} - \\ & \frac{1}{8} t_5^2 t_{10}^2 + \frac{1}{6} t_2^2 t_4 t_{11} + \frac{1}{3} t_2 t_3 t_4 t_{11} - \frac{1}{3} t_3^2 t_4 t_{11} + \frac{1}{4} t_5 t_6 t_9 t_{11} - \frac{1}{2} t_5 t_7 t_9 t_{11} + \\ & \frac{1}{8} t_5^2 t_{10} t_{11} - \frac{1}{8} t_5^2 t_{11}^2 - \frac{1}{9} t_2^3 t_{12} + \frac{1}{6} t_2^2 t_3 t_{12} + \frac{1}{6} t_2 t_3^2 t_{12} - \frac{1}{9} t_3^3 t_{12} + \frac{1}{8} t_5^2 t_9 t_{12} \end{aligned}$$

The five point function is given by:

$$\begin{aligned}
& \frac{1}{54} t_4^3 t_6^2 - \frac{1}{54} t_4^3 t_6 t_7 + \frac{1}{54} t_4^3 t_7^2 + \frac{1}{9} t_2 t_4^2 t_6 t_8 - \frac{1}{18} t_3 t_4^2 t_6 t_8 - \frac{1}{18} t_2 t_4^2 t_7 t_8 + \frac{1}{9} t_3 t_4^2 t_7 t_8 + \\
& \frac{1}{18} t_2^2 t_4 t_8^2 - \frac{1}{18} t_2 t_3 t_4 t_8^2 + \frac{1}{18} t_3^2 t_4 t_8^2 - \frac{1}{18} t_6^3 t_8 t_9 + \frac{1}{12} t_6^2 t_7 t_8 t_9 + \frac{1}{12} t_6 t_7^2 t_8 t_9 - \\
& \frac{1}{18} t_7^3 t_8 t_9 + \frac{1}{27} t_2 t_4^3 t_{10} - \frac{1}{54} t_3 t_4^3 t_{10} + \frac{1}{24} t_6^4 t_{10} - \frac{1}{12} t_6^3 t_7 t_{10} + \frac{1}{24} t_6 t_7^3 t_{10} - \frac{1}{48} t_7^4 t_{10} - \\
& \frac{1}{12} t_5 t_6^2 t_8 t_{10} + \frac{1}{12} t_5 t_6 t_7 t_8 t_{10} + \frac{1}{24} t_5 t_7^2 t_8 t_{10} - \frac{1}{12} t_4 t_6^2 t_9 t_{10} + \frac{1}{12} t_4 t_6 t_7 t_9 t_{10} + \\
& \frac{1}{24} t_4 t_7^2 t_9 t_{10} - \frac{1}{6} t_2 t_6 t_8 t_9 t_{10} + \frac{1}{12} t_3 t_6 t_8 t_9 t_{10} + \frac{1}{12} t_2 t_7 t_8 t_9 t_{10} + \frac{1}{12} t_3 t_7 t_8 t_9 t_{10} - \\
& \frac{1}{12} t_4 t_5 t_6 t_{10}^2 + \frac{1}{6} t_2 t_6^2 t_{10}^2 - \frac{1}{12} t_3 t_6^2 t_{10}^2 + \frac{1}{24} t_4 t_5 t_7 t_{10}^2 - \frac{1}{6} t_2 t_6 t_7 t_{10}^2 - \frac{1}{24} t_3 t_6 t_7 t_{10}^2 + \\
& \frac{1}{24} t_2 t_7^2 t_{10}^2 + \frac{1}{24} t_3 t_7^2 t_{10}^2 - \frac{1}{12} t_2 t_5 t_8 t_{10}^2 + \frac{1}{24} t_3 t_5 t_8 t_{10}^2 - \frac{1}{48} t_9^3 t_{10}^2 - \frac{1}{54} t_2 t_4^3 t_{11} + \\
& \frac{1}{27} t_3 t_4^3 t_{11} - \frac{1}{48} t_6^4 t_{11} + \frac{1}{24} t_6^3 t_7 t_{11} - \frac{1}{12} t_6 t_7^3 t_{11} + \frac{1}{24} t_7^4 t_{11} + \frac{1}{24} t_5 t_6^2 t_8 t_{11} + \\
& \frac{1}{12} t_5 t_6 t_7 t_8 t_{11} - \frac{1}{12} t_5 t_7^2 t_8 t_{11} + \frac{1}{24} t_4 t_6^2 t_9 t_{11} + \frac{1}{12} t_4 t_6 t_7 t_9 t_{11} - \frac{1}{12} t_4 t_7^2 t_9 t_{11} + \\
& \frac{1}{12} t_2 t_6 t_8 t_9 t_{11} + \frac{1}{12} t_3 t_6 t_8 t_9 t_{11} + \frac{1}{12} t_2 t_7 t_8 t_9 t_{11} - \frac{1}{6} t_3 t_7 t_8 t_9 t_{11} + \frac{1}{12} t_4 t_5 t_6 t_{10} t_{11} - \\
& \frac{1}{6} t_2 t_6^2 t_{10} t_{11} - \frac{1}{24} t_3 t_6^2 t_{10} t_{11} + \frac{1}{12} t_4 t_5 t_7 t_{10} t_{11} + \frac{1}{6} t_2 t_6 t_7 t_{10} t_{11} + \frac{1}{6} t_3 t_6 t_7 t_{10} t_{11} - \\
& \frac{1}{24} t_2 t_7^2 t_{10} t_{11} - \frac{1}{6} t_3 t_7^2 t_{10} t_{11} + \frac{1}{12} t_2 t_5 t_8 t_{10} t_{11} + \frac{1}{12} t_3 t_5 t_8 t_{10} t_{11} + \frac{1}{48} t_9^3 t_{10} t_{11} + \\
& \frac{1}{24} t_4 t_5 t_6 t_{11}^2 + \frac{1}{24} t_2 t_6^2 t_{11}^2 + \frac{1}{24} t_3 t_6^2 t_{11}^2 - \frac{1}{12} t_4 t_5 t_7 t_{11}^2 - \frac{1}{24} t_2 t_6 t_7 t_{11}^2 - \frac{1}{6} t_3 t_6 t_7 t_{11}^2 - \\
& \frac{1}{12} t_2 t_7^2 t_{11}^2 + \frac{1}{6} t_3 t_7^2 t_{11}^2 + \frac{1}{24} t_2 t_5 t_8 t_{11}^2 - \frac{1}{12} t_3 t_5 t_8 t_{11}^2 - \frac{1}{48} t_9^3 t_{11}^2 + \frac{1}{18} t_2^2 t_4 t_{12} - \\
& \frac{1}{18} t_2 t_3 t_4^2 t_{12} + \frac{1}{18} t_3^2 t_4^2 t_{12} - \frac{1}{36} t_5 t_6^3 t_{12} + \frac{1}{24} t_5 t_6^2 t_7 t_{12} + \frac{1}{24} t_5 t_6 t_7^2 t_{12} - \frac{1}{36} t_5 t_7^3 t_{12} - \\
& \frac{1}{12} t_2 t_6^2 t_9 t_{12} + \frac{1}{24} t_3 t_6^2 t_9 t_{12} + \frac{1}{12} t_2 t_6 t_7 t_9 t_{12} + \frac{1}{12} t_3 t_6 t_7 t_9 t_{12} + \frac{1}{24} t_2 t_7^2 t_9 t_{12} - \\
& \frac{1}{12} t_3 t_7^2 t_9 t_{12} + \frac{1}{192} t_9^4 t_{12} - \frac{1}{6} t_2 t_5 t_6 t_{10} t_{12} + \frac{1}{12} t_3 t_5 t_6 t_{10} t_{12} + \frac{1}{12} t_2 t_5 t_7 t_{10} t_{12} + \\
& \frac{1}{12} t_3 t_5 t_7 t_{10} t_{12} + \frac{1}{12} t_2 t_5 t_6 t_{11} t_{12} + \frac{1}{12} t_3 t_5 t_6 t_{11} t_{12} + \frac{1}{12} t_2 t_5 t_7 t_{11} t_{12} - \frac{1}{6} t_3 t_5 t_7 t_{11} t_{12}
\end{aligned}$$