1. We know the relationship between Miller indices [hkl] and hexagonal Miller-Bravais indices [uvtw] are follows:

$$\begin{cases} \frac{2h-k}{3} = u \\ \frac{2k-h}{3} = v \\ -(u+v) = t; l = w \end{cases} \quad \text{or} \quad \begin{cases} 2u+v = h \\ 2v+u = k \\ l = w \end{cases}$$

So, $[10\bar{1}0]$, $[01\bar{1}0]$, and $[11\bar{2}0]$ can be changed into [210], [120], and [110], respectively.

The transformation rules of crystal plane indices must be different, which is shown as follows:

$$(hkl) \leftrightarrow \left(hk\,\overline{h+k}\,l\right)$$

Therefore, the plane (111) and (032) can be transferred into $(11\bar{2}1)$ and $(03\bar{3}2)$, respectively.

How can we confirm these above?

Firstly, we should figure out the three axes' indices and [hkl] belong to the hexagonal axes.
In three axes, there are a1, a2 and z axes while the four axes system has one more a3 axis than three axes system, where the a1, a2, a3, and z are all vectors.

We can look at the **Figure 1**, a3 = a1 + a2, the four axes indices are [uvtw].

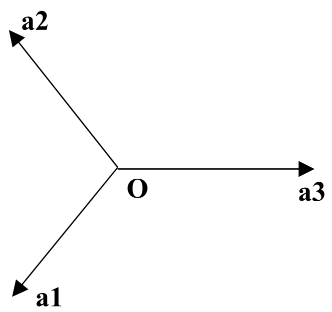

Figure 1. Four axes system.

[uvtw] = u a1 + v a2 + t a3 + w z
= (u-t)a1 + (v-t)a2 + w z = h a1 + k a2 + l z

Therefore, $\begin{cases} u-t = h \\ v-t = k \\ w = l; t = -(u+v) \end{cases}$, So we obtained $\begin{cases} 2u+v=h \\ 2v+u=k \\ w=l; t=-(u+v) \end{cases}$ or $\begin{cases} \frac{2h-k}{3}=u \\ \frac{2k-h}{3}=v \\ -(u+v)=t; l=w \end{cases}$.

The relationship between (hkl) and (uvtw) is obvious, where the t is also equal to –(u+v).

2. We get the two-dimensional rectangular counterclockwise rotation matrix:

$$\begin{pmatrix} cosa & -sina \\ sina & cosa \end{pmatrix}$$

Where the a is the angle of rotation.

On the other hand, the transformation of rectangular coordinate system to oblique coordinate system can be elucidated as **Figure 2**.

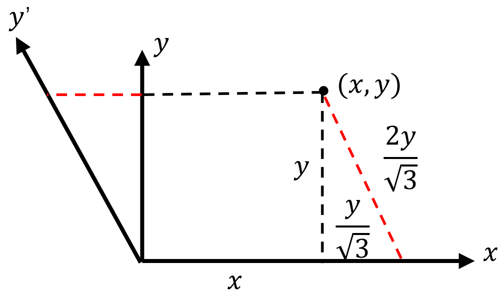

Figure 2. The transformation from rectangular coordinate system to oblique coordinate system

Look at the **Figure 2**, the point (x, y) in rectangular system, and the corresponding point $(x+\frac{1}{\sqrt{3}}y,\ \frac{2}{\sqrt{3}}y)$ of oblique system. Therefore, the transformation matrix of rectangular system to oblique system is $\begin{pmatrix} 1 & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{3}} \end{pmatrix}$.

Finally, we can firstly rotate (x, y) 120 degrees, and secondly transform it into oblique system. This transformation matrix is $\begin{pmatrix} 1 & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{3}} \end{pmatrix}\begin{pmatrix} cos120 & -sin120 \\ sin120 & cos120 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{2}{\sqrt{3}} \\ 1 & -\frac{1}{\sqrt{3}} \end{pmatrix}$.

Therefore, in the oblique system, the original point is $(t,w)=(x+\frac{1}{\sqrt{3}}y,\ \frac{2}{\sqrt{3}}y)$, while the finally transformed point is $(t',w')=\begin{pmatrix} 0 & -\frac{2}{\sqrt{3}} \\ 1 & -\frac{1}{\sqrt{3}} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\left(-\frac{2}{\sqrt{3}}y, x-\frac{1}{\sqrt{3}}y\right)=$

$(-w, t-w)$

Therefore, the oblique transformation matrix is $\begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$.

So, we get the transformation rules:

$$(x, y, z) \rightarrow \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = (\bar{y}, x - y, z) \;\; 3(00z)$$

$$\rightarrow \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$= (\bar{x} + y, \bar{x}, z) \;\; 3(00z).$$

Moreover, the 4(00z) transformation matrix is $\begin{pmatrix} cos90 & -sin90 & 0 \\ sin90 & cos90 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Therefore,

$$(x, y, z) \rightarrow \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = (\bar{y}, x, z) \;\; 4(00z) \rightarrow \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -y \\ x \\ z \end{pmatrix} = (\bar{x}, \; \bar{y}, \; z) \;\; 4(00z) \rightarrow \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -x \\ -y \\ z \end{pmatrix} = (y, \bar{x}, z) \;\; 4(00z).$$