Laboratory Experiment 1: Modelling and Control of an Inverted Pendulum on a Cart

MTRN3020 Modelling and Control of Mechatronic Systems

the statement: I verify that the contents of this report are my own work

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1. INTRODUCTION

The purpose of this experiment is to model and control an inverted pendulum mounted on a cart applied to a horizontal input force "u(t)". The controller should be designed to move the cart to the desired position while keeping the pendulum in an up top vertical position. A state feedback controller can be developed by the error dynamic representations and the proper poles of the system, which assures stable responses of the pendulum and the cart approaching the desired location. The derived controller is then applied to Quanser High Fidelity Linear Cart (HFLC) System.

2. AIM

The aims of these experiments can be formed in three aspects,

1. Learn to calculate the placement of the proper poles in order to satisfy the stability requirements in the time domain.
2. Learn to design the state feedback controller using the poles placement and error dynamics.
3. Learn to analyse the system responses and compare the experimental and simulation results.

3. PROCEDURE

The lab procedures can be demonstrated as below,

1. Using the time domain performance criteria to determine the poles of the system.
2. Using error dynamic equations to obtain the control gain vector of the state feedback system.
3. Loading these data calculated in step 1 & 2 into the computer system and initialize it.
4. After the computer system is initialized, trigger the system by moving the pendulum to the up top vertical position.
5. Observe and record the cart position and the pendulum angle data graphically.

4. DYNAMIC EQUATIONS AND STATE SPACE MODEL

The dynamic equations can be obtained by the New-Euler Approach, which needs to resolve the horizontal forces and moments generated by forces on the pendulum about pivot point. The equilibrium of the forces in horizontal direction can be stated as,

$$M\ddot{x} + m\frac{d^2}{dt^2}(x - lsin\theta) = u \tag{4.1}$$

Taking the moments about pivot point for the pendulum gives,

$$m\left\{\frac{d^2}{dt^2}(x - lsin\theta)\right\}lcos\theta + m\left\{\frac{d^2}{dt^2}(lcos\theta)\right\}lsin\theta = -mglsin\theta \tag{4.2}$$

After (4.1) and (4.2) solved and simplified, we can have the following equations,

$$(M + m)\ddot{x} + mlsin\theta\left(\dot{\theta}\right)^2 - mlcos\theta\ddot{\theta} = u \tag{4.3}$$

$$l\ddot{\theta} - cos\theta\ddot{x} - gsin\theta = 0 \tag{4.4}$$

Because it can be assumed that the pendulum is slightly deviated from the original position and the angular velocity of it is small, (4.3) and (4.4) are simplified as,

$$(M + m)\ddot{x} - ml\ddot{\theta} = u \tag{4.5}$$

$$l\ddot{\theta} - \ddot{x} - g\theta = 0 \tag{4.6}$$

Rearrange these two equations to,

$$\ddot{\theta} = \frac{(M + m)g}{Ml}\theta + \frac{1}{Ml}u \tag{4.7}$$

$$\ddot{x} = \frac{mg}{M}\theta + \frac{1}{M}u \tag{4.8}$$

When the input is equal to $u = \frac{\eta_m K_t}{r_{mp}} I_m$, the dynamic equations representing the inverted pendulum and cart system can be rewritten as,

$$\ddot{\theta} = \frac{(M + m)g}{Ml}\theta + \frac{\eta_m K_t}{Mlr_{mp}}I_m \tag{4.9}$$

$$\ddot{x} = \frac{mg}{M}\theta + \frac{\eta_m K_t}{Mr_{mp}}I_m \tag{4.10}$$

The corresponding state variables are introduced,

$$x_1 = x$$

$$x_2 = \theta$$

$$x_3 = \dot{x}$$

$$x_4 = \dot{\theta}$$

Hence, the state space representation of this system can be displayed as following,

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}I_m$$

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{pmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mg}{M} & 0 & 0 \\ 0 & \frac{(M+m)g}{Ml} & 0 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\eta_m K_t}{M r_{mp}} \\ \frac{\eta_m K_t}{M l r_{mp}} \end{bmatrix} I_m \tag{4.11}$$

The output equations is,

$$y = \boldsymbol{C}x$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \tag{4.12}$$

5. THE CONTROL GAIN

The control gain vector K' derived from error dynamic system is,

K' = [-0.1630, 36.6480, -0.0792, 0.5191, 0.0155]

The MATLAB script has been shown below,

```
% parameters initialization
g = 9.81; % acceleration of gravity
m = 0.127; % Pendulum mass
M = 3.22; % cart system mass
l = 0.1778; % distance from pivot point to pendulum's centre of mass
um = 0.8; % motor efficiency
Kt = 0.36; % motor torque constant
rmp = 1.11e-2; % motor pinion radius
Adash = [0 0 1 0 0; 0 0 0 1 0; 0 (m*g)/(M) 0 0 0; 0 (M+m)*g/(l*m) 0 0 0; -1 0 0 0 0]; % A' and B' matrix from error dynamic system
Bdash = [0 ; 0; (um*Kt)/(M*rmp) ; (um*Kt)/(M*l*rmp);0];
K = z5146927(Adash,Bdash);
```

```
% This function was done before the lab
function K = z5146927(Ai,Bi)
OS_perc = 6.247637661;
Tau_s = 1.155354157;
Fs_1 = 2.103091691;
Fs_2 = 2.165341033;

%Use τs and PO% given to you to calculate poles p1 and p2
Zeta = -log(OS_perc/100)/sqrt(pi^2+log(OS_perc/100)^2);
Natural_F = -log(0.02*sqrt(1-Zeta^2))/(Tau_s*Zeta);
Damping_F = sqrt(1-Zeta^2)*Natural_F;
Sigma = Zeta*Natural_F;

p1 = complex(-Sigma, -Damping_F);
p2 = complex(-Sigma, Damping_F);

%p3 = Real part of p1*fs1;
p3 = Fs_1*real(p1);

%p4 = Real part of p1 * fs2;
p4 = Fs_2*real(p1);

p5 = -0.1;

p=[p1, p2, p3, p4, p5];

K = place(Ai, Bi, p);

return
end
```

6. PLOT OF CART POSITION AND PENDULUM ANGLE

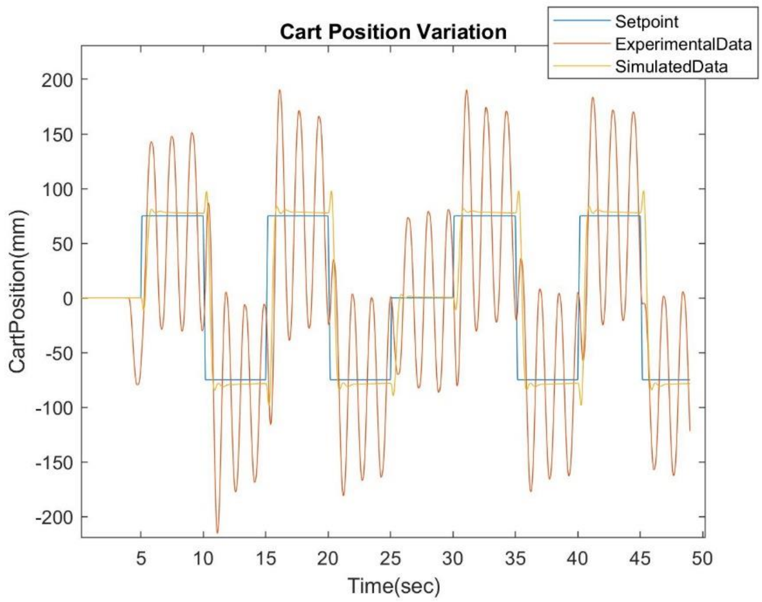

Figure 6.1 Cart Position Versus Time

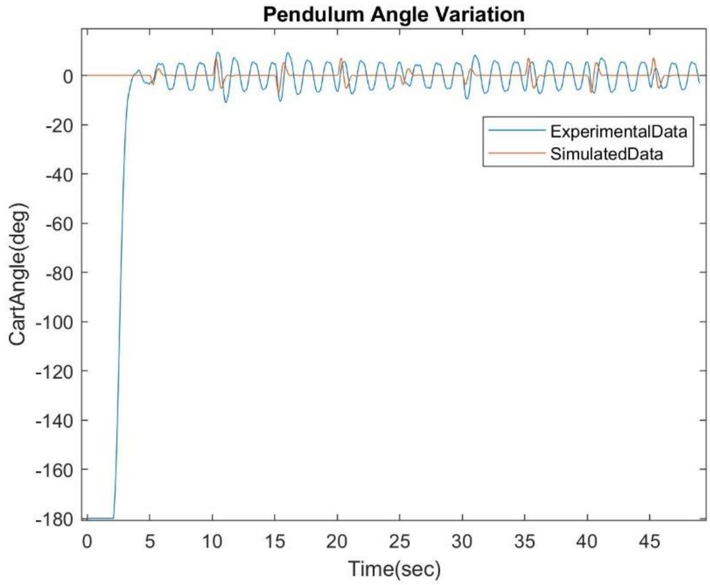

Figure 6.2 Pendulum Angle Versus Time

7. DISCUSSION

What can be clearly seen in Figure 6.1 and 6.2 is that the experimental results have more oscillations than the simulation results, this is because the two dominant poles of the system are a pair of complex conjugate poles. The complex poles indicate that the oscillations may occur in the output response of the system.

Another possible reason causing the discrepancies between the experimental and simulation results is from the encoder quantization errors. The encoder will sample the time-continuous measurements which may be the cart positions, into a discrete-time measurement. Then, Quantizing the series of the discrete number through the computer could generate a series of quantization errors. Therefore, this will impact on the behaviours of the system response.

Finally, the possible source of errors is from the external forces acting on the motion of the cart. These forces may be derived from the friction between the cart and rail or the pivot point between the inverted pendulum and the cart. Hence, the external elements should be considered and added into the control input, which will affect the dynamic equations and state-space model.