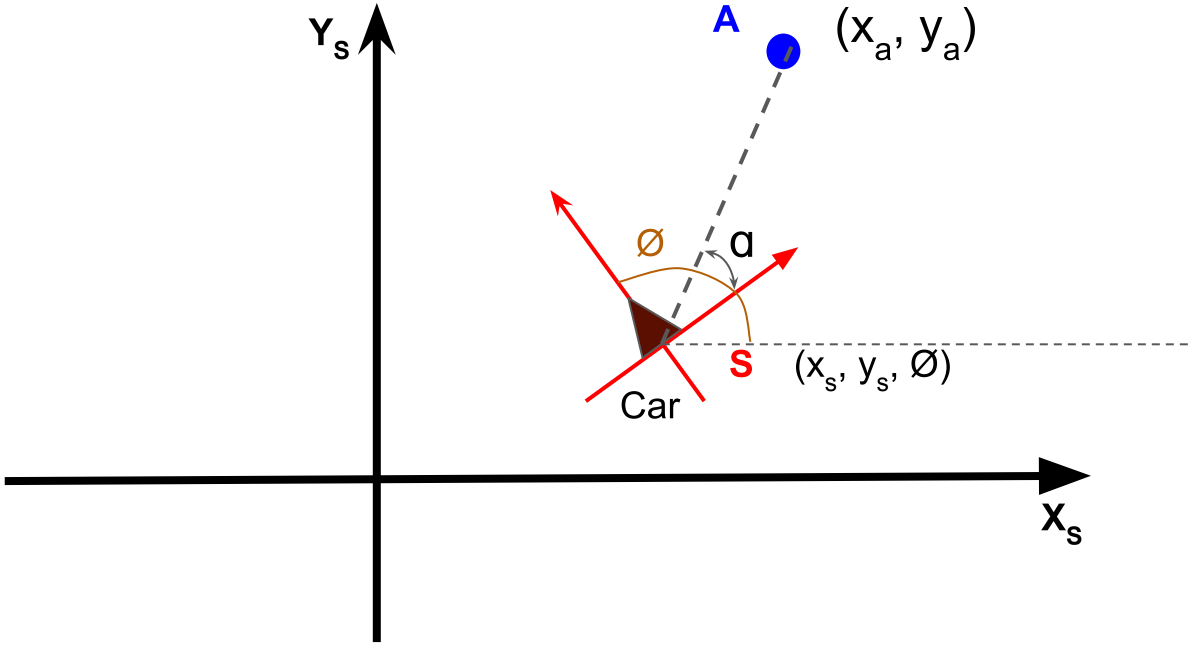
Part 1 (a)



$$h(\mathbf{X}) = \begin{bmatrix} h_1(x,y) \\ h_2(x,y) \end{bmatrix} = \begin{bmatrix} \sqrt{(x_a - x_s)^2 + (y_a - y_s)^2} \\ \tan^{-1}(y_a - y_s, x_a - x_s) - \phi + \pi/2 \end{bmatrix} = \begin{bmatrix} r \\ \alpha \end{bmatrix}$$

Part 1 (a)

(measurement)

(predicted expected value)

$$\mathbf{z}(k+1) = \mathbf{y}_{measurement}(k+1) - \mathbf{H}\cdot\hat{\mathbf{X}}(k+1|k)$$

(just intermediate calculations)

$$\mathbf{S} = \mathbf{H}\cdot\mathbf{P}(k+1|k)\cdot\mathbf{H}^T + \mathbf{R}(k+1)$$

$$\mathbf{K}(k+1) = \mathbf{P}(k+1|k)\cdot\mathbf{H}^T\cdot\mathbf{S}^{-1}$$

(E5)

(Updated Expected Value)

(Updated covariance)

$$\hat{\mathbf{X}}(k+1|k+1) = \hat{\mathbf{X}}(k+1|k) + \mathbf{K}(k+1)\cdot\mathbf{z}(k+1)$$

$$\mathbf{P}(k+1|k+1) = \mathbf{P}(k+1|k) - \mathbf{P}(k+1|k)\cdot\mathbf{H}^T\cdot\mathbf{S}^{-1}\cdot\mathbf{H}\cdot\mathbf{P}(k+1|k)$$

$$\mathbf{H} = \left.\frac{\partial h(\mathbf{X})}{\partial \mathbf{X}}\right|_{\mathbf{X}=\hat{\mathbf{X}}^-} = \left.\begin{bmatrix} \frac{\partial h_1}{\partial x} & \frac{\partial h_1}{\partial y} & \frac{\partial h_1}{\partial \phi} \\ \frac{\partial h_2}{\partial x} & \frac{\partial h_2}{\partial y} & \frac{\partial h_2}{\partial \phi} \end{bmatrix}\right|_{\mathbf{X}=\hat{\mathbf{X}}^-} = \left.\begin{bmatrix} -\frac{(x_a - x)}{\sqrt{(x_a-x)^2+(y_a-y)^2}} & -\frac{(y_a - y)}{\sqrt{(x_a-x)^2+(y_a-y)^2}} & 0 \\ \frac{(y_a - y)}{(x_a-x)^2+(y_a-y)^2} & \frac{-(x_a - x)}{(x_a-x)^2+(y_a-y)^2} & -1 \end{bmatrix}\right|_{\mathbf{X}=\hat{\mathbf{X}}^-}$$

[AAS2020_lecture04][Estimation_GaussianFilter].pdf [AAS]_L05_Localization Problem Using EKF[2020].pdf

Part 1 (b)

$$\mathbf{X}(k+1) = f\left(\mathbf{X}(k), \mathbf{u}(k)\right)$$

$$\Downarrow$$

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ \phi(k+1) \end{bmatrix} = \begin{bmatrix} x(k) \\ y(k) \\ \phi(k) \end{bmatrix} + T \cdot \begin{bmatrix} v(k) \cdot \cos\left(\phi(k)\right) \\ v(k) \cdot \sin\left(\phi(k)\right) \\ \omega(k) \end{bmatrix}$$

Our inputs are polluted with noise.

$$\mathbf{X}(k+1) = f\left(\mathbf{X}(k), \mathbf{u}(k)\right) = f\left(\mathbf{X}(k), \mathbf{u}_m(k) + \delta\mathbf{u}(k)\right) = f\left(\mathbf{X}(k), \mathbf{u}_m(k)\right) + \delta f$$

$$\delta f = \left(f\left(\mathbf{X}(k), \mathbf{u}(k)\right) - f\left(\mathbf{X}(k), \mathbf{u}_m(k)\right)\right)$$

Linearization with first order Taylor

$$\delta f = \left.\frac{\partial f}{\partial \mathbf{u}}\right|_{\mathbf{u}=\mathbf{u}_{measured}} \cdot \delta\mathbf{u} = \mathbf{F}_{\mathbf{u}} \cdot \delta\mathbf{u}$$

[AAS]_L05_ProcessModelUGV [2020].pdf

Part 1 (b)

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ \phi(k+1) \end{bmatrix} = \begin{bmatrix} x(k) \\ y(k) \\ \phi(k) \end{bmatrix} + T \cdot \begin{bmatrix} v(k) \cdot \cos(\phi(k)) \\ v(k) \cdot \sin(\phi(k)) \\ \omega(k) \end{bmatrix}$$

$$\mathbf{F_u} = \left.\frac{\partial f}{\partial(v,\omega)}\right|_{\substack{\mathbf{X}=\hat{\mathbf{X}}, \\ v=v_m, \omega=\omega_m}} = \left.\begin{bmatrix} T \cdot \cos(\phi(k)) & 0 \\ T \cdot \sin(\phi(k)) & 0 \\ 0 & T \end{bmatrix}\right|_{\phi(k)=\hat{\phi}(k)}$$

P(t+1|t) = J*P(t|t)*J' + F_u*P_u*F'_u + Q

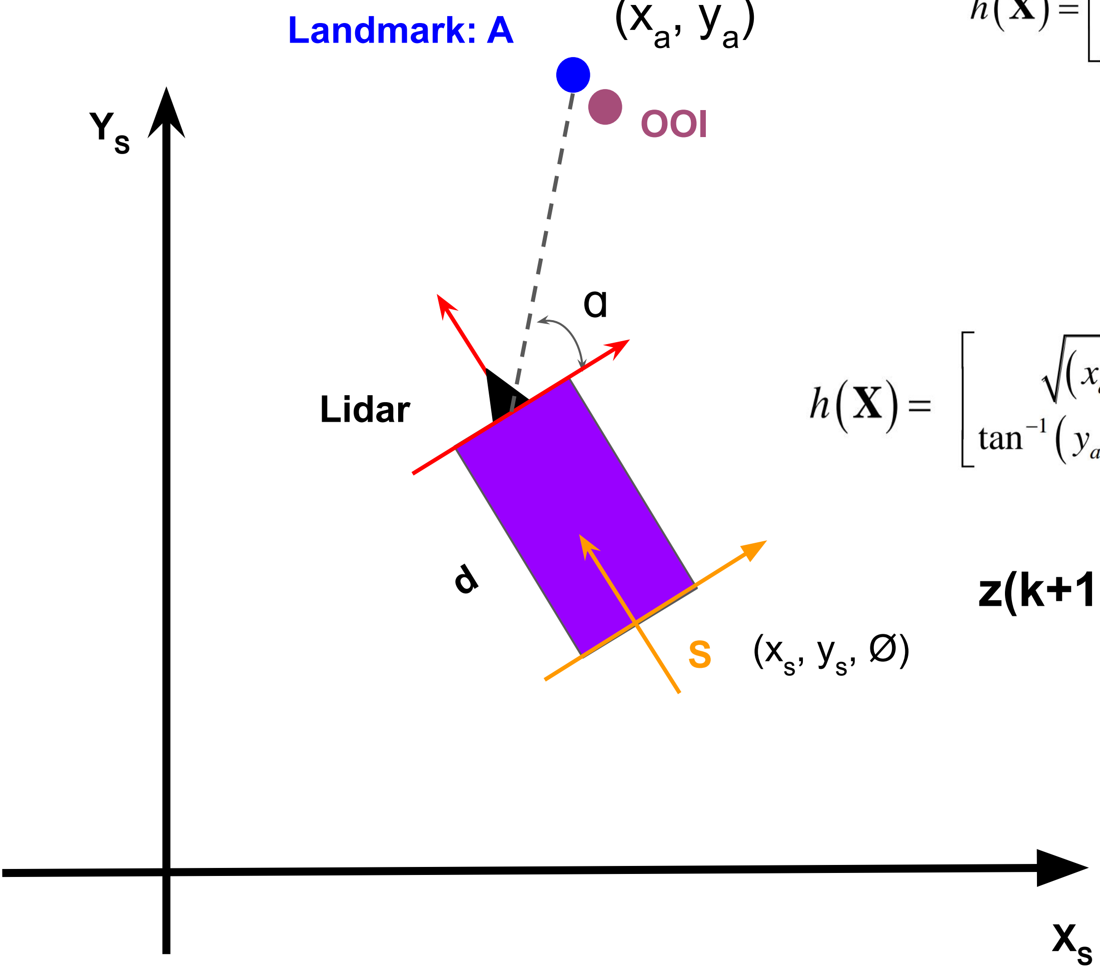
Input noise

Model approximation noise

[AAS]_L05_ProcessModelUGV [2020].pdf

Part 2

First method



$$h(\mathbf{X}) = \begin{bmatrix} h_1(x,y) \\ h_2(x,y) \end{bmatrix} = \begin{bmatrix} \sqrt{(x_a - x_s)^2 + (y_a - y_s)^2} \\ \tan^{-1}(y_a - y_s, x_a - x_s) - \phi + \pi/2 \end{bmatrix} = \begin{bmatrix} r \\ \alpha \end{bmatrix}$$

$$x_s = x + d \cdot \cos(\phi)$$
$$y_s = y + d \cdot \sin(\phi)$$
$$\Downarrow$$

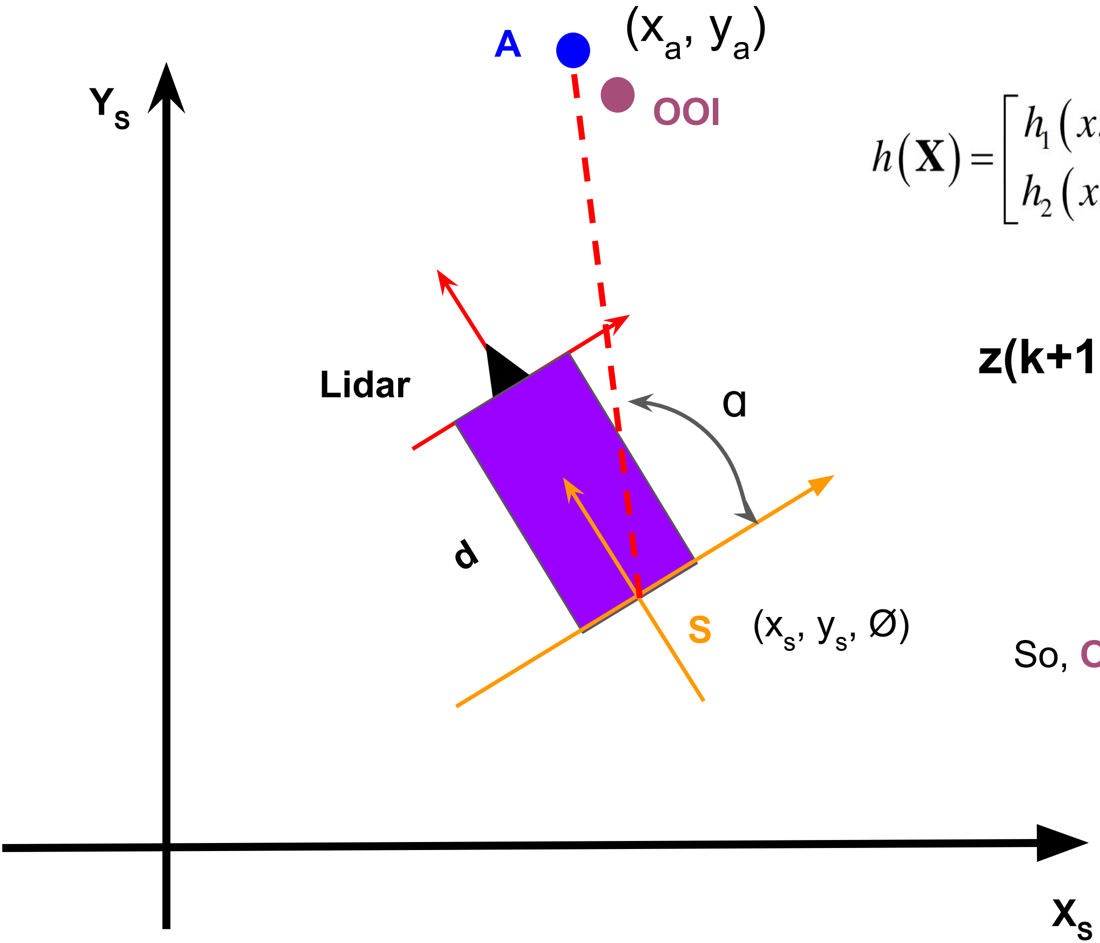
$$h(\mathbf{X}) = \begin{bmatrix} \sqrt{(x_a - x - d \cdot \cos(\phi))^2 + (y_a - y - d \cdot \sin(\phi))^2} \\ \tan^{-1}(y_a - y - d \cdot \sin(\phi), x_a - x - d \cdot \cos(\phi)) - \phi + \pi/2 \end{bmatrix} = \begin{bmatrix} r \\ \alpha \end{bmatrix}$$

$$\mathbf{z(k+1) = y_{measurement}(k+1) - h(X(k+1|k))}$$

OOI in local coordinate (red one)

Part 2

Second method



$$h(\mathbf{X})=\begin{bmatrix} h_1(x,y) \\ h_2(x,y) \end{bmatrix}=\begin{bmatrix} \sqrt{(x_a-x_s)^2+(y_a-y_s)^2} \\ \tan^{-1}(y_a-y_s, x_a-x_s)-\phi+\pi/2 \end{bmatrix}=\begin{bmatrix} r \\ \alpha \end{bmatrix}$$

$$\mathbf{z(k+1) = y_{measurement}(k+1) - h(X(k+1|k))}$$

OOI in local coordinate (orange one)

So, OOIs should move from **red one** to **orange one**

x = x

y = y + d