MTRN4030 Optimization Methods for Eng Systems

Lab exercise 3

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Task 1 – Simplex Method

1.a

Theory: This optimization problem is described as searching a product shipping model to satisfy supply and demand requirements at a minimum distribution cost. In Part A, this problem would be solved by the Northwest Corner method, with initial conditions shown in Figure 1:

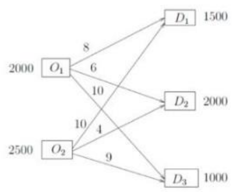

Figure 1

Practice:

1. Starting from an empty tableau;
2. Round 1: select the number or variable into the cell in upper left of the tableau.

Destination / Origins	D1	D2	D3
O1	1500	0	0
O2	0	0	0

3. Round 2:

Destination / Origins	D1	D2	D3
O1	1500	500	0
O2	0	0	0

4. Round 3:

Destination / Origins	D1	D2	D3
O1	1500	500	0
O2	0	0	0

5. Round 4:

Destination / Origins	D1	D2	D3
O1	1500	500	0
O2	0	0	0

6. Round 5:

Destination / Origins	D1	D2	D3
O1	1500	500	0
O2	0	15000	0

7. Round 6:

Destination / Origins	D1	D2	D3
O1	1500	500	0
O2	0	1500	1000

At the end of while loop execution, it is terminated at the round 6. Referring to conditions provided on the Figure 1, the final cost solution in Northwest Corner method can be determined:

$$Cost1 = (1500 \times 8 + 500 \times 6) + (1500 \times 4 + 1000 \times 9) = 30000 \quad \text{Equation 1}$$

1.b

Theory: In Part B, this optimization problem can be transferred into a combination of linear equations, and it can be solved by Simplex method. The total cost function will be constrained by the necessary and sufficient conditions.

Practice:

To determine the products transported along each routing from origins to destinations,

Destination / Origins	D1	D2	D3
O1	x_{11}	x_{12}	x_{13}

O2	x_{21}	x_{22}	x_{23}

The linear model expresses this optimization problem as six functions:

1. Minimize the cost function:

   $cost2 = 8 \times x_{11} + 6 \times x_{12} + 10 \times x_{13} + 10 \times x_{21} + 4 \times x_{22} + 9 \times x_{23}$ Equation 2

2. The constraints are generated from the description of problem, which are presented by six functions as below:

   $x_{11} + x_{12} + x_{13} = 2000;$

   $x_{21} + x_{22} + x_{23} = 2500;$

   $x_{11} + x_{21} = 1500;$

   $x_{12} + x_{22} = 2000;$

   $x_{13} + x_{23} = 1000;$

The function of "linprog" is available in MATLAB to solve the above linear model, the result is

Destination / Origins	D1	D2	D3
O1	1500	0	500
O2	0	2000	500

Substituting these these numbers of the delivered units into the Equ 2, the final cost using Simplex method is

$$cost2 = 8 \times 1500 + 6 \times 0 + 10 \times 500 + 10 \times 0 + 4 \times 2000 + 9 \times 500 = 29500$$

As a whole, the cost in Simplex method is 50 less than that in Northwest Corner method. Referring the comparable table, Table 1, it is apparent that the main reason behind the difference in cost is happened during the transportation from O2. Especially for the link between O2 and D2, this price of unit delivery is the cheapest one, and the Simplex method satisfies the sum of demand requirement of D2, unlike Northwest Corner method.

Method Name	From O1	From O2
Northwest Corner	15000	15000
Simplex	17000	12500

Table 1

Task 2

2.a

The problem can be stated as

$$find\ the\ extreme\ points\ of\ f(x,y) = xy(2+x)$$

$$Subject\ to\ g(x,y) = x^2 + y^2 = 2 \quad \text{Equation 3}$$

The langrage function is

$$L(x,y,\lambda) = \ xy(2+x) + \lambda \times (x^2 + y^2 - 2) \quad \text{Equation 4}$$

The necessary conditions for its extremum are given by

$$\frac{\partial L}{\partial x} = 2\lambda \cdot x + x \cdot y + y \cdot (x+2) = 0 \quad \text{Equation 5}$$

$$\frac{\partial L}{\partial y} = 2 \cdot \lambda \cdot y + x \cdot (x+2) = 0 \quad \text{Equation 6}$$

$$\frac{\partial L}{\partial \lambda} = x^2 + y^2 - 2 = 0 \quad \text{Equation 7}$$

The above three equations can be solved by "solve" function in MATLAB, to determine the bunch of solutions as

$$x = [-1.651, -1.6519, -0.7532, -0.7532, 1.0717, 1.0717]$$

$$y = [\ 0\ 0\ 1.1970\ \ -1.1970\ \ -0.9227\ 0.9227]$$

$$\lambda = [\ 0\ 0\ 0.3923\ \ -0.3923\ 1.7838\ \ -1.7838]$$

These can give the maximum and minimum value of $f(x,y)$ as

$$f_{maximum} = 3.0371 \text{ at } (1.0717, 0.9227)$$

$$f_{minimum} = -3.0371\ at\ (1.0717, -0.9227)$$

To illustrate the locations of the extremums, the 3-D plot is

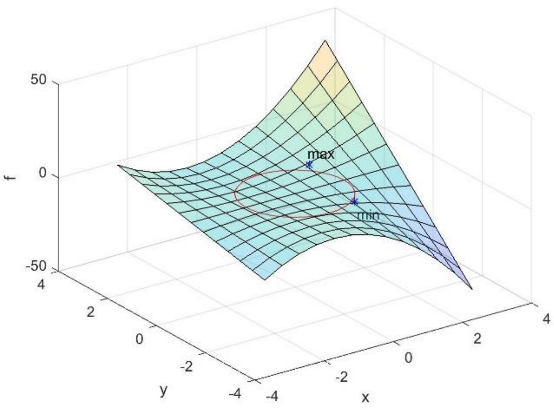

Figure 2

2.b

Assumption: The rectangular container is designed with a lid.

The problem can be represented as

$$minimize\ f(x,y) = x_1 \cdot x_2 \cdot x_3$$

$$Subject\ to\ g(x,y) = x_1 \cdot x_2 + x_1 \cdot x_3 + x_2 \cdot x_3 = 5 \quad \text{Equation 8}$$

The langrage function is

$$L(x,y,\lambda) = x_1 \cdot x_2 \cdot x_3 + \lambda \times (x_1 \cdot x_2 + x_1 \cdot x_3 + x_2 \cdot x_3 - 5) \quad \text{Equation 9}$$

The necessary conditions for its extremum are given by

$$\frac{\partial L}{\partial x_1} = \lambda \cdot (x_2 + x_3) + x_2 \cdot x_3 = 0 \quad \text{Equation 10}$$

$$\frac{\partial L}{\partial x_2} = \lambda \cdot (x_1 + x_3) + x_1 \cdot x_3 = 0 \quad \text{Equation 11}$$

$$\frac{\partial L}{\partial x_3} = \lambda \cdot (x_1 + x_2) + x_1 \cdot x_2 = 0 \quad \text{Equation 12}$$

$$\frac{\partial L}{\partial \lambda} = x_1 \cdot x_2 + x_1 \cdot x_3 + x_2 \cdot x_3 - 5 = 0 \quad \text{Equation 13}$$

The above three equations can be solved by "solve" function in MATLAB, to determine the solutions as

$$x_1 = [1.2910, -1.2910]$$

$$x_2 = [\ 1.2910, -1.2910]$$

$$x_3 = [1.2910, -1.2910]$$

These can give the maximum value of $f(x,y)$ as

$$f_{maximum} = 2.1517$$

2.c

The problem can be represented as

$$miximise\ P(x,y) = 8x_1x_2x_3^2 - 200(x_1 + x_2 + x_3)$$

$$Subject\ to\ g(x,y) = x_1 + x_2 + x_3 = 100 \quad \text{Equation 14}$$

The langrage function is

$$L(x,y,\lambda) = 8x_1x_2x_3^2 - 200(x_1 + x_2 + x_3) + \lambda \times (x_1 + x_2 + x_3 - 100) \quad \text{Equation 15}$$

The necessary conditions for its extremum are given by

$$\frac{\partial L}{\partial x_1} = 8x_2x_3^2 + \lambda - 200 = 0 \quad \text{Equation 16}$$

$$\frac{\partial L}{\partial x_2} = 8x_1x_3^2 + \lambda - 200 = 0 \quad \text{Equation 17}$$

$$\frac{\partial L}{\partial x_3} = 16x_1x_2x_3 + \lambda - 200 = 0 \quad \text{Equation 18}$$

$$\frac{\partial L}{\partial \lambda} = x_1 + x_2 + x_3 - 100 = 0 \quad \text{Equation 19}$$

The above three equations can be solved by "solve" function in MATLAB, to determine the solutions as

$$x_1 = [100, 0, 25]$$

$$x_2 = [\ 0, 0, 25]$$

$$x_3 = [0, 100, 50]$$

These can give the maximum value of $f(x, y)$ as

$$f_{maximum} = 12480000$$

Task 3

3.a

The problem can be expressed as

$$minimize\ f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 2)^2$$

$$subject\ to\ x_1 \geq 0, x_2 \geq 0, x_1 + x_2 \leq 2, x_2 - x_1 = 1 \quad \text{Equation 20}$$

The constraints can be rewritten as

$$g_1 = x1 \geq 0$$

$$g_2 = x_2 \geq 0$$

$$g_3 = -x_1 - x_2 + 2 \geq 0$$

$$h_1 = -x_1 + x_2 - 1$$

The KKT function is

$$L = (x_1 - 1)^2 + (x_2 - 2)^2 + \lambda_1 * x_1 + \lambda_2 * x_2 + \lambda_3 * (-x_1 - x_2 + 2) - \beta_1 * (-x_1 + x_2 - 1)$$

Equation 21

The KKT conditions are

$$\frac{\partial L}{\partial x_1} = \beta_1 + \lambda_1 - \lambda_3 + 2 * x_1 - 2 = 0 \quad \text{Equation 22}$$

$$\frac{\partial L}{\partial x_2} = \lambda_2 - \beta_1 - \lambda_3 + 2 * x_2 - 4 = 0 \text{ Equation 23}$$

And for $\lambda_j * g_j = 0, j = 1,2,3,$

$$\lambda_1 * x_1 = 0 \text{ Equation 24}$$

$$\lambda_2 * x_2 = 0 \text{ Equation 25}$$

$$\lambda_3 * (-x_1 - x_2 + 2) = 0 \text{ Equation 26}$$

Solving the Equations from 22 to 26 give

$$x_1 = [1, 0, 1, 0, \frac{1}{2}, 0, 2, -1]$$

$$x_2 = [2, 2, 0, 0, \frac{3}{2}, 1, 0, 0]$$

Substituting these values into Equation 20, this can provide the minimum result of $f(x_1, x_2)$ as

$$f_{minimum} = \frac{1}{2} \ at \ x_1 = \frac{1}{2}, x_2 = \frac{3}{2}$$

3.b

The problem can be expressed as

$$minimize \ f(x_1, x_2) = 3x_1 + \sqrt{3}x_2$$

$$subject \ to \ x_1 \geq 0, x_2 \geq 0, x_1 + x_2 \leq 2, x_2 - x_1 = 1 \text{ Equation 27}$$

The constraints can be rewritten as

$$g_1 = -x_1 + 5.73 \leq 0$$

$$g_2 = -x_2 + 7.17 \leq 0$$

$$g_3 = -3 + \frac{18}{x_1} + 6 * \frac{\sqrt{3}}{x_2} \leq 0$$

The KKT function is

$$L = 3x_1 + \sqrt{3}x_2 + \lambda_1 * (-x_1 + 5.73) + \lambda_2 * (-x_2 + 7.17) + \lambda_3 * \left(-3 + \frac{18}{x_1} + 6 * \frac{\sqrt{3}}{x_2}\right) \text{ Equation 28}$$

The KKT conditions are

$$\frac{\partial L}{\partial x_1} = 0 \text{ Equation 29}$$

$$\frac{\partial L}{\partial x_2} = 0 \text{ Equation 30}$$

And for $\lambda_j * g_j = 0, j = 1,2,3,$

$$\lambda_1 * (-x_1 + 5.73) = 0 \text{ Equation 31}$$

$$\lambda_2 * (-x_2 + 7.17) = 0 \text{ Equation 32}$$

$$\lambda_3 * \left(-3 + \frac{18}{x_1} + 6 * \frac{\sqrt{3}}{x_2}\right) = 0 \text{ Equation 33}$$

By considering $\lambda_j > 0$, Solving the Equations from 22 to 26 give

$$x_1 = [2.5359, 9.4641, 5.7300]$$

$$x_2 = [-2.5359, 9.4641, 7.1700]$$

Substituting these values into Equation 20 and constraints, this can provide the feasible minimum result of $f(x_1, x_2)$ as

$$f_{minimum} = 44.7846 \; at \; x_1 = 9.4641, x_2 = 9.4641$$

Task 4

4.a

The dynamical system model is given as

$$\dot{x} = ax + bu, a > 0, b > 0$$

The problem is purposed to determine the input, $u(t)$, such that the cost function to be minimized as follows

$$J = G\left(x(t_f)\right) + \int_0^{t_f} F(x,u)\,dx = \frac{1}{2}x^2(T) + \int_0^T \frac{1}{2}u^2(t)dt$$

$$with\ initial\ condition\ of\ x(0) = x_0 \text{ Equation 34}$$

The Hamiltonian function is defined as

$$H = F(x,u) + \lambda(t) * (a * x(t) + b * u(t)) \text{ Equation 35}$$

The necessary conditions for optimality are

$$\begin{cases} \frac{\partial H}{\partial u} = 0 \\ \frac{\partial H}{\partial x} = -\dot{\lambda} \text{ Equation 36} \\ \frac{\partial H}{\partial \lambda} = \dot{x} \end{cases}$$

Additionally, referring the boundary conditions, the relationship between the state equations and the arbitrary multiplier is defined as

$$\frac{\partial G}{\partial x} = \lambda(T) = x(T) \text{ Equation 37}$$

Due to $\frac{\partial H}{\partial x} = -\dot{\lambda}$, this equation can be rearranged as

$$\lambda' + a\lambda = 0 \quad \text{Equation 38}$$

By multiplying the both sides of Equation 38 by e^{at},

$$e^{at}\lambda' + ae^{at}\lambda = 0 \quad \text{Equation 39}$$

Integrating Equation 39 along the time, it is changed to

$$\frac{d(e^{at}\lambda)}{dt} = 0 \quad \text{Equation 40}$$

By simultaneously integrating with the both sides of Equation 40 to

$$\int \frac{d(e^{at}\lambda)}{dt} dt = \int 0dt$$

$$e^{at}\lambda = c$$

$$\lambda(t) = c * e^{-at} \quad \text{Equation 41}$$

It is clear that the arbitrary multiplier, λ, is the function of time. Referring to Equation 37,

$$c = x(T) \cdot e^{aT} \ at\ t = T \quad \text{Equation 42}$$

Due to $\frac{\partial \mathrm{H}}{\partial u} = 0$, this equation can be rearranged as

$$u(t) = -\lambda b \quad \text{Equation 43}$$

By substituting Equation 41 and Equation 42 into Equation 43, the input vector is figured out

$$\boldsymbol{u(t) = -b \cdot x(T) \cdot e^{a(T-t)}} \quad \textbf{\textit{Equation 44}}$$

Due to $\frac{\partial \mathrm{H}}{\partial \mathrm{x}} = -\dot{\lambda}$, this equation can be rearranged as

$$x' = ax + bu \quad \text{Equation 45}$$

By substituting Equation 44 into Equation 45

$$x' - ax = -b^2 \cdot x(T) \cdot e^{a(T-t)} \quad \text{Equation 46}$$

Multiplying Equation 46 by e^{-at} and integrating it

$$\int \frac{d(e^{-at})}{dt} dt = \int -b^2 x(T) e^{a(T-t)} dt \quad \text{Equation 47}$$

Developing Equation 48,

$$e^{-at}x = \frac{b^2 x(T) \cdot e^{aT}}{2a} e^{-2at} + c \quad \text{Equation 48}$$

Inserting the initial condition, $x(0) = x_0$, the constant is

$$c = x_0 - \frac{b^2 \cdot x(T) \cdot e^{aT}}{2a} \quad \text{Equation 49}$$

Taking Equation 49 back to Equation 48 and simplifying, the state equation is

$$\boldsymbol{x = \frac{b^2 \cdot x(T) \cdot e^{aT}}{2a}(e^{-at} - e^{at}) + x_0 \cdot e^{at}} \quad \textbf{Equation 50}$$

4.b

The problem is stated as

$$minimise\ the\ time\ taken, T = \int_0^T dt$$

$$and\ the\ boundary\ conditions\ are\ x(0) = 0, \dot{x}(0) = 0, x(T) = a, \dot{x}(T) = 0$$

The state equations are introduced as

$$\dot{x_1} = x_2, \dot{x_2} = u$$

Another control variable, v, is created to describe the acceleration and deceleration,

$$v^2 = (u + \alpha)(\beta - u)$$

By combining the above elements, the optimality function is written as

$$J = \int \{1 + \lambda_1(x_1' - x_2) + \lambda_2(x_2' - u) + \lambda_3(v^2 - (u + \alpha)(\beta - u))\}\, dt \quad \text{Equation 51}$$

It can be simplified as

$$J = \int_0^T F(x_1, x_2, u, v)\, dt \quad \text{Equation 52}$$

The conditions for optimality can be determined by applying the Euler equations,

$$\begin{cases} \dfrac{\partial F}{\partial x_1} - \dfrac{d}{dt}\left(\dfrac{\partial F}{\partial x_1'}\right) = 0 \\ \dfrac{\partial F}{\partial x_2} - \dfrac{d}{dt}\left(\dfrac{\partial F}{\partial x_2'}\right) = 0 \\ \dfrac{\partial F}{\partial u} = 0 \\ \dfrac{\partial F}{\partial v} = 0 \end{cases}$$

From the above work,

$$\begin{cases} \lambda_1' = 0 \\ -\lambda_1 - \lambda_2' = 0 \\ -\lambda_2 - \lambda_3(2u + \alpha - \beta) = 0 \\ 2\lambda_3 v = 0 \end{cases}$$

Two solution can be obtained, which are $\lambda_1 = \lambda_2 = \lambda_3 = 0$ or $v = 0$. Because the first one is trivial and make this problem meaningless, v set to 0.

Let the switching time to be τ, an expression for the control u is

$$x_2' = u = \begin{cases} \beta & t < \tau \\ \alpha & \tau < t < T \end{cases} \quad \text{Equation 53}$$

Integrating Equation 53 respect to time for velocity,

$$x_2 = \begin{cases} \beta t & t < \tau \\ \beta\tau - \alpha(t-\tau) & \tau < t < T \end{cases} \quad \text{Equation 54}$$

Due to $x_2(T) = 0$, the switching time is defined as

$$\tau = \frac{\beta}{\alpha+\beta}T \quad \text{Equation 55}$$

Integrating Equation 54 respect to time for displacement

$$x_1 = \begin{cases} \frac{\beta t^2}{2} & t < \tau \\ \frac{\beta\tau^2}{2} + \beta\tau(t-\tau) - \frac{\alpha}{2}(t^2 - \tau^2) + \alpha\tau(t-\tau) & \tau < t < T \end{cases} \quad \text{Equation 56}$$

Due to $x_1(T) = 0$, the final time is worked as

$$T = \sqrt{\frac{2A(\alpha+\beta)}{\beta^2+\alpha\beta-\alpha^2}} \quad \text{Equation 57}$$

By combining Equation 55 and Equation 57, the expression for switching is

$$\tau = \frac{\beta}{\alpha+\beta}\sqrt{\frac{2A(\alpha+\beta)}{\beta^2+\alpha\beta-\alpha^2}}$$

4.c

The expression of the system model is given by

$$\dot{x} = Ax + Bu, x(0) = x_0$$

The cost function is

$$J = \frac{1}{2}x(T)Px(T) + \frac{1}{2}\int_0^T (x^T Q_x x + u^T Q_u u)\, dt$$

The Hamiltonian function is

$$H = \frac{1}{2}x^T Q_x x + \frac{1}{2}u^T Q_u u + \lambda^T(Ax + Bu)$$

The optimal conditions based on Pontryagin's maximum principle are

$$\dot{x} = \frac{\partial x}{\partial \lambda} = Ax + Bu$$

$$-\dot{\lambda} = \frac{\partial H}{\partial x} = Q_x x + A^T\lambda$$

$$0 = \frac{\partial H}{\partial u} = Q_u u + \lambda^T B$$

When the u is equal to $-Q_u^{-1}B^T\lambda$, $\lambda = Px$ and $\dot{P} = 0$, the algebraic Riccati equation is defined as

$$PA + A^T P - PBQ_u^{-1}B^T P + Q_x = 0 \text{ Equation 58}$$

The following conditions are known

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Q_x = \begin{bmatrix} q^2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$Q_u = 1$$

By setting the $A, B, Q_x\ and\ Q_u$ into Equation 58, P is found as

$$P = \begin{bmatrix} q\sqrt{2q} & q \\ q & \sqrt{2q} \end{bmatrix}$$

Due to $K = -Q_u^{-1}B^T P$, the gain is defined as

$$K = [q \quad \sqrt{2q}]$$

The responses of system model in Simulink would be different as along with varying the value of q. The design layout of Simulink model is shown below

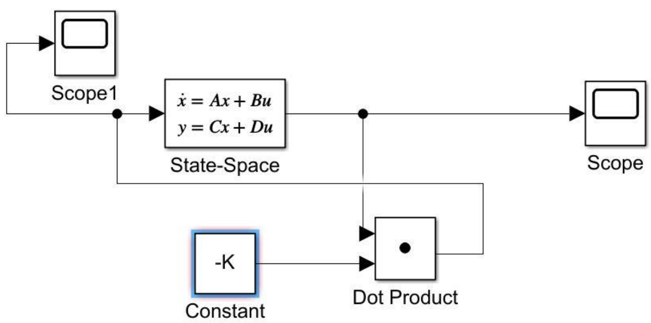

Figure 3

When the q is equal to 1,

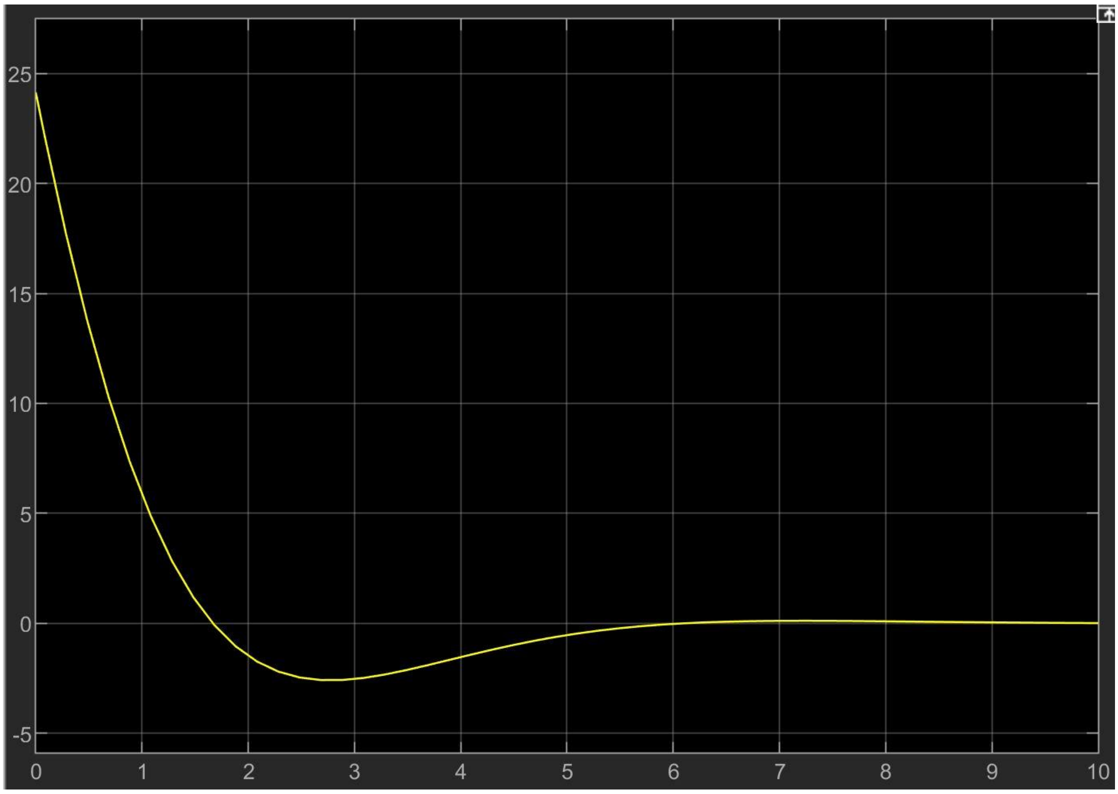

Figure 4 The input responses of $q = 1$

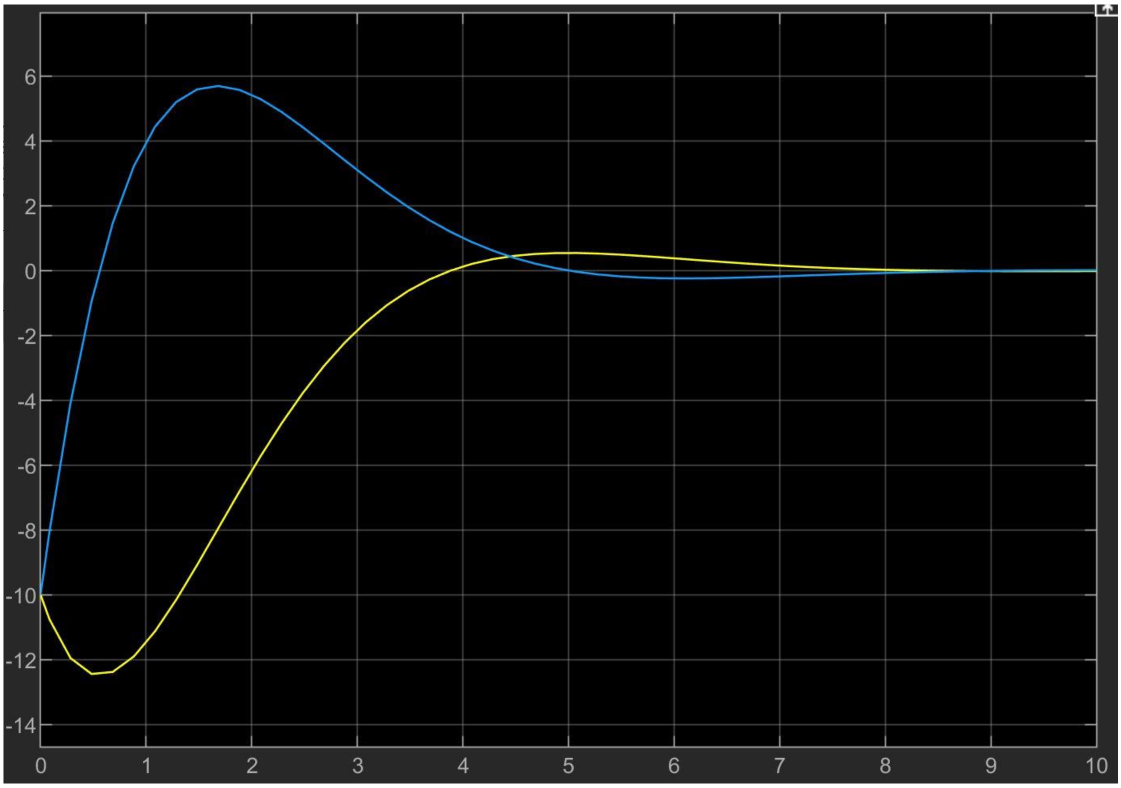

Figure 5 The state space responses of $q = 1$

When the q is equal to 2,

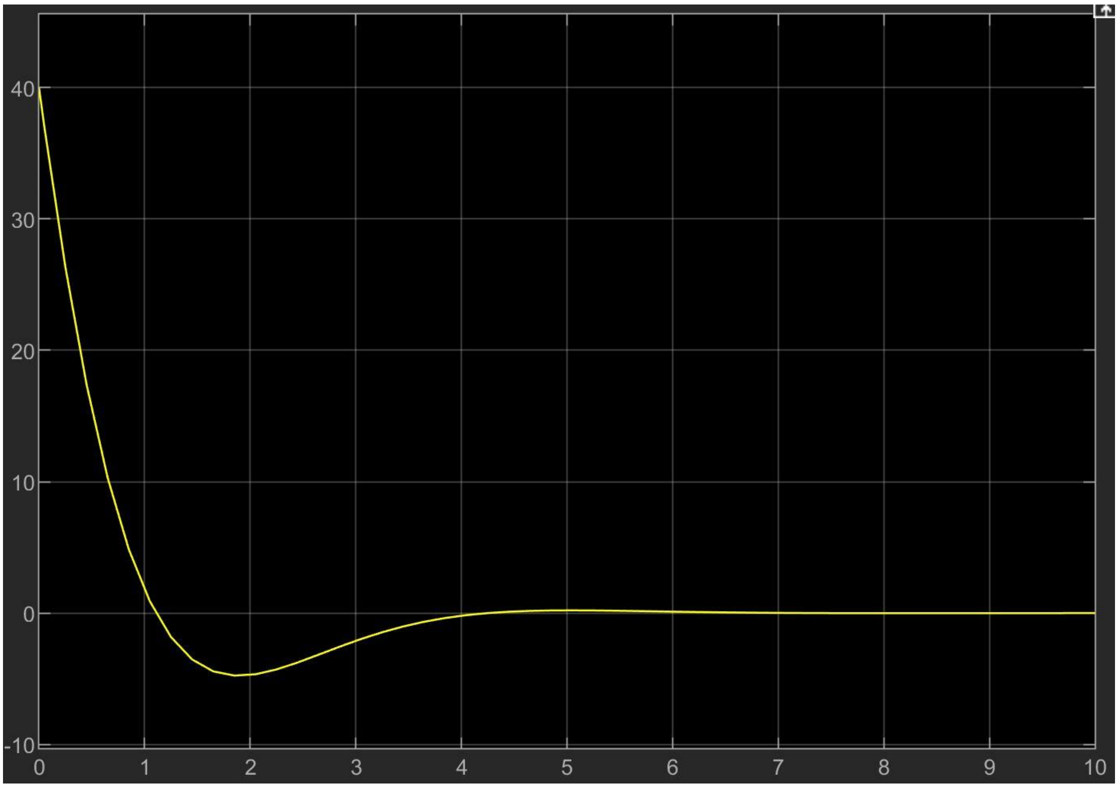

Figure 6 The input responses of $q = 2$

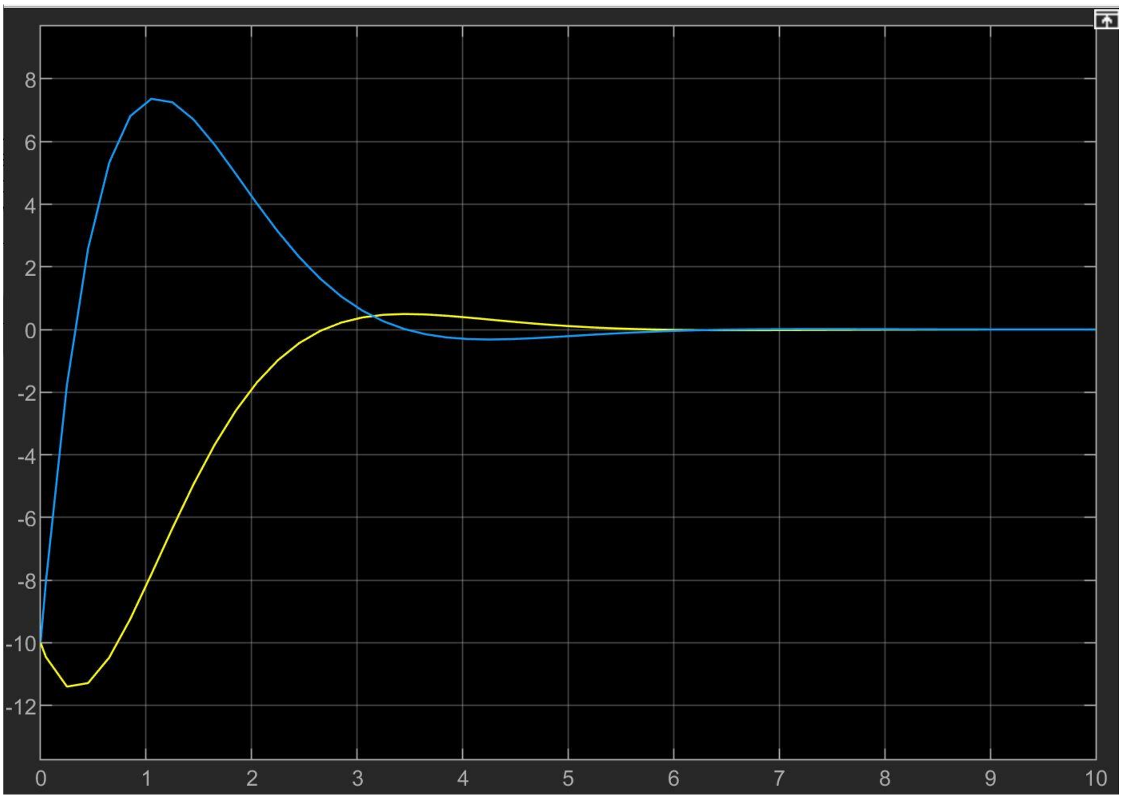

Figure 7 The state space responses of $q = 2$

When the q is equal to 5,

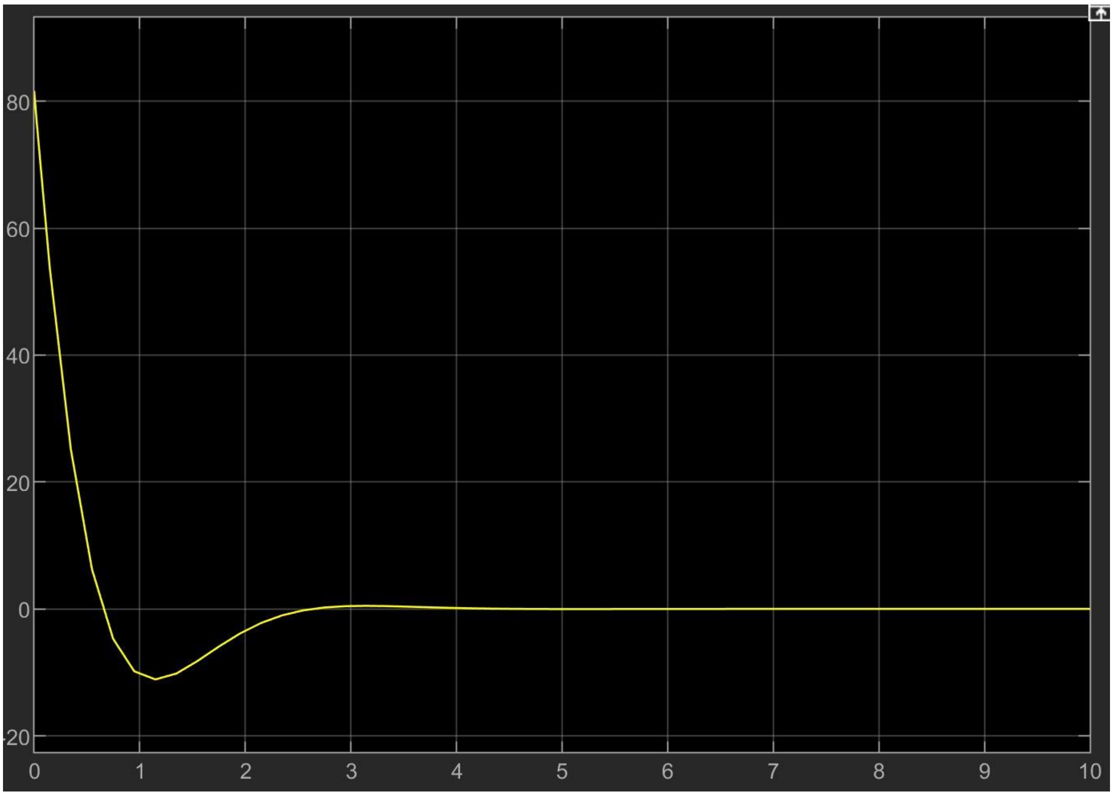

Figure 8 The input responses of $q = 5$

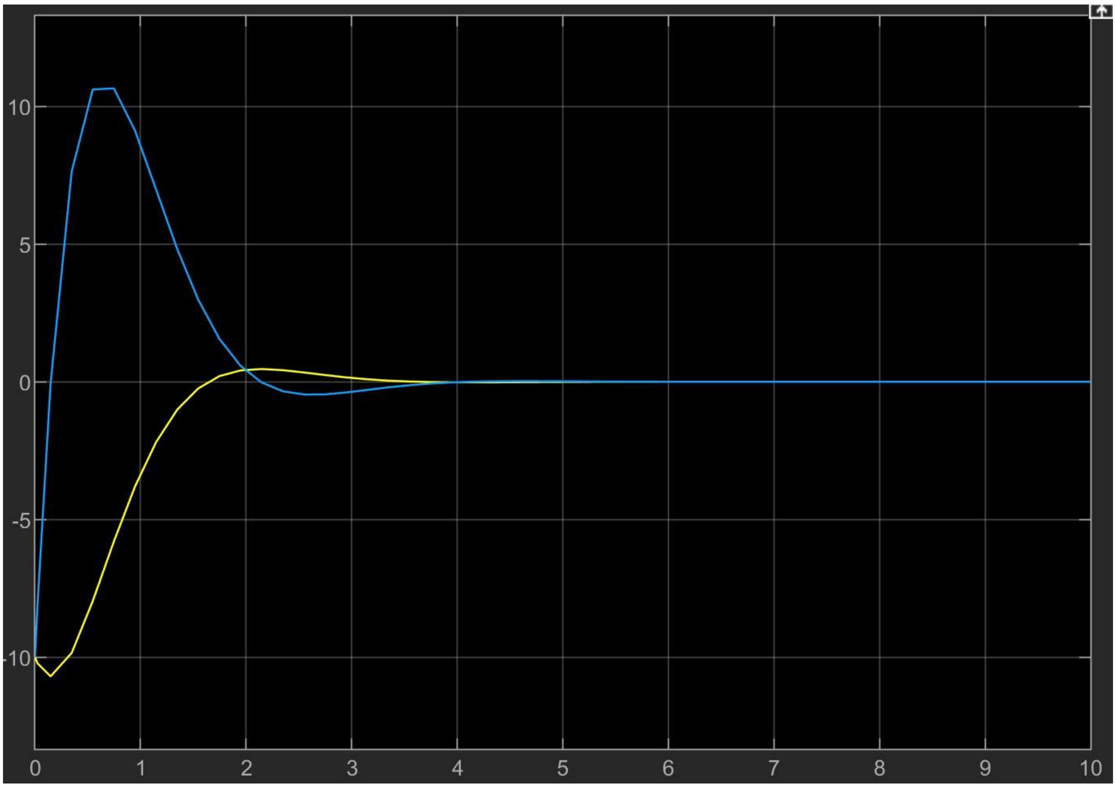

Figure 9 The state space responses of $q = 5$

All in all, by observing the above responses, it is clear that the settling time travelling to stable state is shrinking as the value of q increasing or the regulator gain factor going up. Besides, the overshoot of initial state would be increasing.

Appendix

Task1.1

```
Num_supply = 2;
Num_demand = 3;
soln = zeros(Num_supply, Num_demand);
Origin = [2000 2500];
Destination = [1500 2000 1000];

i = 1;
j = 1;

while i < 3
    while j < 4
        if Origin(i) > Destination(j)
            soln(i,j) = min(Origin(i), Destination(j));
            Origin(i) = Origin(i)-Destination(j);
            Destination(j)=0;
        elseif Origin(i) < Destination(j)
            soln(i,j) = min(Origin(i), Destination(j));
            Destination(j) = Destination(j)-Origin(i);
            Origin(i)=0;
        elseif Origin(i) == Destination(j)
            soln(i,j)=Origin(i);
            Destination(j) = Destination(j)-Origin(i);
            Origin(i)=0;
        end
%             disp('Origin: ');
%             disp(Origin);
%             disp('Destination: ')
%             disp(Destination);
        disp(soln);
        j = j+1;
    end
    j = 1;
    i = i+1;
end
```

Task1.2

```
Aeq = [1 1 1 0 0 0
       0 0 0 1 1 1
       1 0 0 1 0 0
       0 1 0 0 1 0
       0 0 1 0 0 1];
beq = [2000 2500 1500 2000 1000];
A =[];
b =[];
f = [8 6 10 10 4 9];
lb=[0 0 0 0 0 0];
ub=[];
x = linprog(f,A,b, Aeq, beq,lb,ub);
```

Task2.1

```
%% Question 2.a
clear;
syms x y lamda
[x, y] = meshgrid(-3:0.5:3,-3:0.5:3);
f = x.*y.*(2+x);
g = x.^2+y.^2-2;
figure(1);
surf(x,y,f,'FaceAlpha',0.3);grid on;hold on;
fp=fimplicit(@(x,y)x.^2+y.^2-2, 'Color', 'r');hold on;
syms x y lamda
f = x.*y.*(2+x);
g = x.^2+y.^2-2;
L = f+lamda*g;
vector_cond = jacobian(L, [x y lamda]) == 0;
soln = solve(vector_cond, [x y lamda]);

x = real(double(soln.x));
y = real(double(soln.y));
lamda = real(double(soln.lamda));

f = x.*y.*(2+x);

indexMax = find(f == max(f));
indexMin = find(f == min(f));
plot3(x(indexMax),y(indexMax),max(f), 'b*');hold on;
plot3(x(indexMin),y(indexMin),min(f), 'b*');

str={'max'};text(1.0717,1,9,str);
str={'min'};text(1.0717,-1,-9,str);
xlabel('x');ylabel('y'),zlabel('f');
```

Task2.2

```
%% Question 2.b
clear;
syms x1 x2 x3 lamda
f = x1.*x2.*x3;
g = x1.*x3+x1.*x2+x2.*x3-5;
L = f+lamda*g;
vector_cond = jacobian(L, [x1 x2 x3 lamda]) == 0;
soln = solve(vector_cond, [x1 x2 x3 lamda]);
x1 = real(double(soln.x1));
x2 = real(double(soln.x2));
x3 = real(double(soln.x3));
f = x1.*x2.*x3;
disp(max(f));
```

Task2.3

```
%% Question 2.c
clear;
syms x1 x2 x3 lamda
P = 8*x1*x2*x3^2-200*(x1+x2+x3);
g = x1+x2+x3-100;
L = P+lamda*g;
vector_cond = jacobian(L, [x1 x2 x3 lamda]) == 0;
soln = solve(vector_cond, [x1 x2 x3 lamda]);
x1 = real(double(soln.x1));
x2 = real(double(soln.x2));
x3 = real(double(soln.x3));
P = 8.*x1.*x2.*x3.^2-200.*(x1+x2+x3);
disp(max(P));
```

Task3.1

```
%% Question 3.a
syms x1 x2 l1 l2 l3 b1
f = (x1-1)^2+(x2-2)^2;
g1 = x1;
g2 = x2;
g3 = -x1-x2+2;
h1 = -x1+x2-1;

L = f+l1*g1+l2*g2+l3*g3-b1*h1;
L_diff = jacobian(L, [x1 x2]) == 0;
f1 = l1*g1 == 0;
f2 = l2*g2 == 0;
f3 = l3*g3 == 0;
f4 = b1*h1 == 0;

soln = solve(L_diff, f1, f2, f3, f4, [x1 x2 l1 l2 l3 b1]);
x1 = soln.x1;
x2 = soln.x2;
f = (x1-1).^2+(x2-1).^2;
f_mina = min(f);
indice = find(f == f_mina);
x1_a = x1(indice);
x2_a = x2(indice);
disp('the min of function:');disp(f_mina);disp(' x1:');disp(x1_a);disp(' x2:');disp(x2_a);
```

Task3.2

```
%% Question 3.b
clear;
syms x1 x2 l1 l2 l3
f = 3*x1+sqrt(3)*x2;
g1 = -x1+5.73;
g2 = -x2+7.17;
g3 = -3+18/x1+6*sqrt(3)/x2;

L = f + l1*g1+l2*g2+l3*g3;
L_diff = jacobian(L, [x1 x2]) == 0;
f1 = l1*g1 == 0;
f2 = l2*g2 == 0;
f3 = l3*g3 == 0;

soln = solve(L_diff, f1, f2, f3, [x1 x2 l1 l2 l3]);
x1 = double(soln.x1);
x2 = double(soln.x2);
% x1 = x1(4);
% x2 = x2(4);
f = 3.*x1+sqrt(3).*x2;
f_minb = min(f);
indice = find (f == f_minb);
x1_b = x1(indice);
x2_b = x2(indice);
disp('the min of function:');disp(f_minb);disp(' x1_b:');disp(x1_b);disp(' x2_b:');disp(x2_b);
```

Task4.3

```
%% Question 4.c

A = [0,1;0 0];
B = [0;1];
C = [1,0;0,1];
D = [0;0];
Qu = 1;
Q1 = [1 2 5];
K = zeros(3,2);
SYS = ss(A,B,C,D);


    Q = Q1(3);
    Qx = [Q^2,0;0,0];
    [K,S,E]=lqr(SYS,Qx,Qu);
```