MTRN4030

Optimisation Methods for Engineering Systems

Laboratory Exercise 3 – weeks 6, 7, 8

Simplex, Constrained and Optimal Control Methods

Summary

This laboratory exercise is scheduled for three weeks (week 6, 7, 8). The first part of each meeting is a tutorial that tutors will provide a revision on the topic and guidelines to complete the exercises.

The exercises include the calculation and program-based implementation of constrained optimisation methods, Simplex method, Lagrangian, KKT, as well as the optimal control method. The implementation details of these methods will be investigated and their characteristics and/or limitations are to be identified and discussed. The second objective of the exercises is the development of Matlab codes to solve optimisation problems covered in this exercise. The last part is about the use of Matlab/Simulink to study the optimal control of a simple dynamic system.

An individual written report is required for this exercise, which carries 8%, in addition to the marks for the practical work that is 5%. Submission of the report is to be uploaded to Moodle. Details of the requirements are given in the Assessment section.

Simplex Method – Transportation Problem

The problem can be expressed by the formulation of a linear model, and it can be solved using the class of the simplex method. The problem deals with the transportation of some product from m origins, $O_1, \cdots, O_m$, to n destinations, $D_1, \cdots, D_n$, with the aim of minimizing the total distribution cost, where:

1. The origin O_i has a supply of a_i units, $i = 1, \cdots, m$.
2. The destination D_j has a demand for b_j units to be delivered, $j = 1, \cdots, n$.
3. c_{ij} is the cost per unit distributed from the origin O_i to the destination D_j.

The above problem can be expressed as finding a set of x_{ij}'s, to meet supply and demand requirements at a minimum distribution cost. The corresponding linear model is:

$$\text{Minimise } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} \leq a_i, \sum_{i=1}^{m} x_{ij} \leq b_j, x_{ij} \geq 0$$

Thus, the problem is to determine x_{ij}, the number of units to be transported from O_i to D_j, so that supplies will be consumed and demands satisfied at an overall minimum cost.

The first m constraints correspond to the supply limits, and they express that the supply of commodity units available at each origin must not be exceeded. The next n constraints ensure that the commodity unit requirements at destinations will be satisfied. The decision variables are defined positive since they represent the number of commodity units transported.

Thus, a transportation problem is specified by the number of origins, the number of destinations, the supplies, the demands and the per-unit transportation costs. All this information can be abbreviated in the form of a rectangular array.

The relevant data for any transportation problem can be summarized in a matrix format using a tableau called the transportation costs tableau (see below). The tableau displays the origins with their supply, the destinations with their demand and the transportation per-unit costs.

	D_1	D_2	$\cdots$	D_n	supply
O_1	c_{11}	c_{12}			a_1
$\vdots$			$\ddots$		$\vdots$
O_m				c_{mn}	a_m
demand	b_1	b_2	$\cdots$	b_n	

Theorem 1 The necessary and sufficient condition for a transportation problem to have a solution is that the total demand equals the total supply.

Definition 1 (Balanced problem) A transportation problem is said to be balanced if

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

If the transportation problem is unbalanced, we have to convert it into a balanced one before solving it. There are two possible cases:

Case 1. The demand exceeds the supply, $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$

It is not possible to satisfy the total demand with the existing supply. In this case, a dummy source or origin O_{m+1} is added to balance the model. Its corresponding supply and unit transportation cost are the following:

$$a_{m+1} = \sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i,\ c_{m+1,j} = 0$$

Case 2. The supply exceeds the demand, $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$

Being the total supply higher than the total demand, we add a dummy destination D_{n+1} to the problem, such that its demand and unit transportation costs are:

$$b_{n+1} = \sum_{i=1}^{n} a_i - \sum_{j=1}^{n} b_j,\ c_{i,n+1} = 0$$

The procedure of calculating an initial basic feasible solution is performed in a tableau of the same dimensions as the transportation costs tableau; the transportation solution tableau, where each position (i,j) is associated with the decision variable x_{ij}, that is, the number of units of product to be transported from origin O_i to destination D_j. Such positions (i,j) are called cells, and represent a solution. An empty cell denotes a value of zero.

The Northwest Corner method

Given a balanced transportation problem, and starting at a solution tableau with all the cells (i,j) empty, the following steps lead to an initial basic feasible solution.

Step 1: In the rows and columns under consideration, select the cell (i,j) in the upper left-hand corner (northwest corner) of the solution tableau (to begin, $i = 1, j = 1$).

Step 2: Assign to the variable x_{ij} the feasible amount consistent with the row and the column requirements of that cell, that is, the value:

$$x_{ij} = \min\{a_i, b_j\}$$

At least one of the requirements, the supply or the demand, will then be met. Adjust the supply a_i and the demand b_j as follows:

1. If a_i happens to be the minimum, then the supply of the origin O_i becomes zero, and the row i is eliminated from further consideration. The demand b_j is replaced by $b_j - a_i$.
2. If b_j happens to be the minimum, then the demand of the destination D_j becomes zero, and the column j is eliminated from further consideration. The supply a_i is replaced by $a_i - b_j$.
3. If $a_i = b_j$, then the adjusted values for the supply a_i and the demand b_j become both zero. The row i and the column j are eliminated from further consideration.

Step 3: Two cases may arise:

1. If only one row or only one column remains under consideration, then any remaining cells (i, j), that is, variables x_{ij} associated with these cells, are selected and the remaining supplies are assigned to them. Stop.
2. Otherwise, go to Step 1.

The above Northwest Corner method is a very simple procedure proposed to find a basic feasible solution for a transportation problem. However, the unit transportation costs play no role in this method, which simply selects the upper left-hand corner variable and assigns a value to it.

Lagrange Multiplier Method

Consider the equality constrained problem:

$$\text{minimize } f(\boldsymbol{x})$$
$$\text{subject to } h_j(\boldsymbol{x}) = 0,\ j = 1,2,\cdots,r < n.$$

Transform this constrained problem to an unconstrained problem via the introduction of so-called *Lagrange multipliers* $\lambda_j,\ j = 1,2,\cdots,r$, in the formulation of the *Lagrangian function:*

$$L(\boldsymbol{x}, \boldsymbol{\lambda}) = f(\boldsymbol{x}) + \textstyle\sum_{j=1}^{r} \lambda_j\, h_j(\boldsymbol{x}) = f(\boldsymbol{x}) + \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{h}(\boldsymbol{x}).$$

The *necessary conditions* for $\boldsymbol{x}^*$ to be a constrained internal *local minimum* of the equality constrained problem is that $\boldsymbol{x}^*$ corresponds to a stationary point $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*)$ of the Lagrangian function, i.e. that a vector exists such that

$$\frac{\partial L}{\partial x_i}(\boldsymbol{x}^*, \boldsymbol{\lambda}^*) = 0,\ i = 1,2,\cdots,n$$
$$\frac{\partial L}{\partial \lambda_j}(\boldsymbol{x}^*, \boldsymbol{\lambda}^*) = 0,\ j = 1,2,\cdots,r$$

Note that necessary conditions represent $n + r$ equations in the $n + r$ unknowns $x_1^*, x_2^*, \cdots, x_n^*, \lambda_1^*, \cdots, \lambda_r^*$. The solutions to these, in general non-linear equations, therefore give candidate solutions $\boldsymbol{x}^*$ to the problem. If $f(\boldsymbol{x})$ and the $h_j(\boldsymbol{x})$ are all convex functions, then these conditions indeed constitute sufficiency conditions. In this case the local constrained minimum is unique and represents the global minimum.

Karush and Kuhn and Tucker (KKT) Conditions

Consider the problem:

$$\text{minimize } f(\boldsymbol{x})$$
$$\text{subject to } g_j(\boldsymbol{x}) \leq 0,\ j = 1,2,\cdots,m$$

Define the Lagrangian:

$$L(\boldsymbol{x}, \boldsymbol{\lambda}) = \boldsymbol{f}(\boldsymbol{x}) + \sum_{j=1}^{m} \lambda_j\, g_j(\boldsymbol{x}).$$

At the point $\boldsymbol{x}^*$, corresponding to the solution of the primal problem, the following conditions must be satisfied:

$$\frac{\partial f}{\partial x_i}(\boldsymbol{x}^*) + \sum_{j=1}^{m} \frac{\partial g_j}{\partial x_i}(\boldsymbol{x}^*) = 0,\ i = 1,2,\cdots,n$$
$$g_j(\boldsymbol{x}^*) \leq 0,\ j = 1,2,\cdots,m$$
$$\lambda_j^* g_j(\boldsymbol{x}^*) = 0,\ j = 1,2,\cdots,m$$
$$\lambda_j^* \geq 0,\ j = 1,2,\cdots,m$$

KKT conditions also constitute *sufficient* conditions for $\boldsymbol{x}^*$ to be a constrained minimum, if $f(\boldsymbol{x})$ and the $g_j(\boldsymbol{x})$ are all convex functions.

Optimal Control

Consider the optimal control problem:

$$minimize \int_0^T L\,(x,u)dt + V(x(T))dt$$

subject to the constraint

$$\dot{x} = f(x,u), x \in R^n, u \in R^m$$

Abstractly, this is a constrained optimization problem where we seek a feasible trajectory $(x(t), u(t))$ that minimizes the cost function

$$J(x,u) = \int_0^T L\,(x,u)dt + V(x(T))$$

The term $L(x,u)$ is referred to as the integral cost and $V(x(T))$ is the final (or terminal) cost. There are many variations and special cases of the optimal control problem.

Infinite horizon optimal control. If we let $T = \infty$ and set $V = 0$, then we seek to optimize a cost function over all time. This is called the infinite horizon optimal control problem, versus the *finite horizon* problem with $T < \infty$. Note that if an infinite horizon problem has a solution with finite cost, then the integral cost term $L(x,u)$ must approach zero as $t \to \infty$.

Linear quadratic (LQ) optimal control. If the dynamical system is linear and the cost function is quadratic, we obtain the linear quadratic optimal control problem:

$$\dot{x} = Ax + Bu,\ J = \int_0^T (x^\top Qx + u^\top Ru)\,dt + x(T)^\top Px(T)$$

In this formulation, $Q \geq 0$ penalizes state error, $R > 0$ penalizes the input and $P > 0$ penalizes terminal state. This problem can be modified to track a desired trajectory (x_d, u_d) by rewriting the cost function in terms of $(x - x_d)$ and $(u - u_d)$.

Terminal constraints. It is often convenient to ask that the final value of the trajectory, denoted x_f, be specified. We can do this by requiring that $x(T) = x_f$ or by using a more general form of constraint: $\phi(x(T)) = 0,\ i = 1, \cdots, q$. The fully constrained case is obtained by setting $q = n$ and defining $\phi(x(T)) = x_i(T) - x_{i,f}$. For a control problem with a full set of terminal constraints, $V(x(T))$ can be omitted since its value is fixed.

Time optimal control. If we constrain the terminal condition to $x(T) = x_f$, let the terminal time T be free (so that we can optimize over it) and choose$L(x, u) = 1$, we can find the time-optimal trajectory between an initial and final condition. This problem is usually only well-posed if we additionally constrain the inputs u to be bounded.

A very general set of conditions are available for the optimal control problem that captures most of these special cases in a unifying framework. Consider a nonlinear system

$$\dot{x} = f(x, u), x \in R^n$$

where $x(0)$ given, $u \in R^p$, and$f(x, u) = (f_1(x, u), \cdots, f_n(x, u)): R^n \times R^p \to R^n$. We wish to minimize a cost function J with terminal constraints:

$$J = \int_0^T L\,(x, u)dt + V(x(T)),\ \phi(x(T)) = 0.$$

The function $\phi: R^n \to R^q$ gives a set of q terminal constraints.

Analogous to the case of optimizing a function subject to constraints, we construct the Hamiltonian:

$$H = L + \lambda^\top f = L + \sum_i \lambda_i f_i.$$

The variables λ_i are functions of time and are often referred to as the *costate* variables.

Pontryagin's Maximum Principle: If (x^*, u^*) is optimal, then there exists $\lambda^*(t) \in R^n$ and $v^*(t) \in R^n$ such that

$$\dot{x}_i = \frac{\partial H}{\partial \lambda_i},\ \dot{\lambda}_i = -\frac{\partial H}{\partial x_i},$$

$x(0)$ is given, and $\phi(x(T)) = 0$,

$$\lambda(T) = \frac{\partial V}{\partial x_i}(x(T)) + v^\top \frac{\partial \phi}{\partial x}$$

and

$$H(x^*(T), u^*(t), \lambda^*(t)) \leq H(x^*(t), u, \lambda^*(t)) \text{ for all } u \in \Omega.$$

The form of the optimal solution is given by the solution of a differential equation with boundary conditions. If $u = \operatorname{argmin} H(x, u, \lambda)$ exists, we can use this to choose the control law u and solve for the resulting feasible trajectory that minimizes the cost. The boundary conditions are given by the n initial states $x(0)$, the q terminal constraints on the state $\phi(x(T)) = 0$ and the $n - q$ final values for the Lagrange multipliers

$$\lambda(T) = \frac{\partial V}{\partial x}(x(T)) + v^\top \frac{\partial \phi}{\partial x}.$$

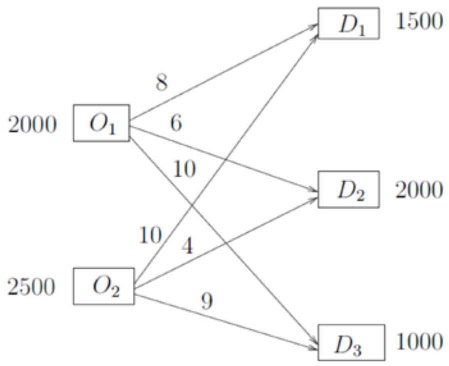
In this last equation, v is a free variable and so there are n equations in $n + q$ free variables, leaving $n - q$ constraints on $\lambda(T)$. In total, we thus have $2n$ boundary values. If H is differentiable, then a necessary condition for the optimal input is $\frac{\partial H}{\partial u} = 0$. We note that even though we are minimizing the cost, this is still usually called the maximum principle.

Exercises

The major objective of the followings tasks is to let you work out the exercises in order to gain a better understanding of the concepts. Although Matlab results (plots/graphs) are required to support your conclusions/discussions in the written report, programming codes are not needed (assessed as the practical work). Hence, a good programming style would be helpful to limit the bugs in your codes and arrive at the correct result.

Task 1 - Simplex Method

1.a. Consider the transportation problem described by the following graph:



Set up the optimisation problem. That is, express the minimisation cost equation, and the equations of the constraints.

Write the corresponding equations in vector/matrix form.

Draw the transportation tableau.

Write a Matlab program for the Northwest Corner method.

You have to show all intermediate results (the tableaus) in the report. But the Matlab code is not required (assessed as part of the practical work).

1.b. The function linprog is available in Matlab to solve linear programming problems. An extract from the help menu is given below.

```
X = LINPROG(f,A,b) attempts to solve the linear programming problem:

min f'*x    subject to:   A*x <= b

X = LINPROG(f,A,b,Aeq,beq) solves the problem above while additionally
satisfying the equality constraints Aeq*x = beq.

X = LINPROG(f,A,b,Aeq,beq,LB,UB) defines a set of lower and upper
bounds on the design variables, X, so that the solution is in the
range LB<=X<=UB. Use empty matrices for LB and UB, if no bounds exist.
Set LB(i)=-Inf if X(i) is unbounded below; set UB(i)=Inf if X(i) is
unbounded above.
```

Now, you have to convert the problems in Part 1.a to the appropriate function input format. Verify your results using Matlab with regard to the Northwest Corner method. Discuss the cause of discrepancies if any.

Show and discuss the results in the report.

Task 2 – Lagrange Multiplier Method

2.a. Given the function

$$f(x, y) = xy(2 + x),$$

subject to the equality constraint

$$g(x, y) = x^2 + y^2 = 2.$$

Obtain, using a Matlab program, the extreme points of $f(x, y)$. Identify the minimum (x_{min}, y_{min}) and maximum point (x_{max}, y_{max}). Illustrate your answer with 3-dimensinal plots of the function, constraint, and extreme points.

(Hint: use symbolics to represent the variables x, y, define the function f and constraint g. Then use the jacbian command to find the solution of the Lagrangian derivatives. Furthermore, solve for the condition that the constraint is satisfied, e.g. S=solve(L(1),L(2),g), where L(1), L(2) are the Lagrangian derivatives and 'g' is the constraint equation. Finally, determine the minimum and maximum points using the min and max commands.)

Show your resultant pots in the report.

2.b. Given a piece of metal sheet of surface area $S = 10m^2$, if it is manufactured as a rectangular container of sides $x_1,\ x_2,\ x_3$, what is the maximum volume V that can be enclosed.

Show your workings and results in the report.

2.c. A factory can produce 3 types of products $x_1,\ x_2,\ x_3$, but the facility can allow only a limited amount

$$g = x_1 + x_2 + x_3 = 100.$$

The profit that can be obtained is

$$P = 8x_1x_2x_3^2 - 200(x_1 + x2 + x_3).$$

What is the maximum profit that can be obtained and what are the optimum numbers of each item should be produced?

Show your workings and results in the report.

Task 3 - Karush and Kuhn and Tucker (KKT) Conditions

3.a. Use the KKT condition approach, find the minimum of the function

$$f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 2)^2$$

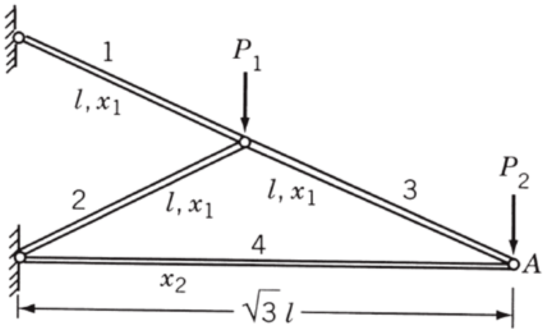
such that

$$x_1 \geq 0,\ x_2 \geq 0,\ x_1 + x_2 \leq 2,\ x_2 - x_1 = 1$$

What are the optimum values $f^*,\ x_1^*,\ x_2^*$?

Show your workings and results in the report.

3.b. Consider the four-bar truss shown below, in which members 1, 2, and 3 have the same cross-sectional area x_1 and the same length L, while member 4 has an area of cross section x_2 and length $\sqrt{3}L$.



The weight of the truss per unit value of L can be expressed as

$$f = 3x_1 + \sqrt{3}x_2.$$

The deflection of point A is constrained by

$$3 - 18x_1^{-1} - 6\sqrt{3}x_2^{-1} \geq 0.$$

Let the cross-sections bee constrained by $x_1 \geq 5.73$, $x_2 \geq 7.17$. Find the minimum weight (per L) and the cross-section areas.

Show your workings and results in the report.

Task 4 – Optimal Control

4.a. Given a dynamical system

$$\dot{x} = ax + bu, a > 0, b > 0,$$

and $x(0) = x_0$. Find the optimal trajectory $(x(t),\ u(t))$ such that the following cost is minimized

$$J = \frac{1}{2}\int_0^T u^2\, dt + \frac{1}{2}x^2(T).$$

Hint: find $\lambda = x(T)e^{a(T-t)}$, then $\frac{d\lambda}{dt} = x(T)\frac{de^{a(T-t)}}{d(T-t)}\frac{d(T-t)}{dt} = -x(T)ae^{(T-t)} = -a\lambda$

Show your workings and results in the report.

4.b. Suppose we wish to drive a car from a stationary position on a driveway into a stationary position in a garage, having moved some distance A. The available controls for the driver are the accelerator and the brake (for simplicity, we assume no gear changes), and we take the equation of motion of the car as

$$\frac{d^2x}{dt^2} = u$$

where $u = u(t)$ represents the applied acceleration or deceleration (braking) and x the distance travelled. The control u is subject to both a lower bound (maximum braking) and upper bound (maximum acceleration), that is,

$$-\alpha \leq u \leq \beta$$

where α, β are positive constants. The problem can now be stated: minimise the time taken, T, that is,

$$T = \int_0^T dt$$

and boundary conditions

$$x(0) = 0, \dot{x}(0) = 0, x(T) = a, \dot{x}(T) = 0$$

We can change the control constraint into an equality constraint by introducing another control variable, say v, where

$$v^2 = (u + \alpha)(\beta - u)$$

Since v^2 is positive, u must satisfy the constraint. Further introduce the usual state variable notation $x_1 = x$ so that

$$\dot{x}_1 = x_2,\ \dot{x}_2 = u$$

with boundary conditions

$$x_1(0) = \mathrm{x}_2(0) = 0, x_1(T\ ) = A, x_2(T) = 0$$

Now, formulate the augmented functional (containing three multipliers λ_1, λ_2, λ_3, for the two states $\dot{x}_1$ and $\dot{x}_2$ and control v^2). Then use the Euler equations to determine the conditions for optimality. One of the conditions obtained is

$$\frac{\partial F}{\partial v} - \frac{d}{dt}\frac{\partial F}{d\dot{v}} = 0 \Rightarrow 2v\lambda_3 = 0$$

Since λ_3 cannot be zero, then set $v = 0$. Hence, initially $u = \beta$ (driving forward) and then $u = -\alpha$ (stopping). Let the switching time of the control be τ, give an expression for the control u.

From, $\dot{x}_1 = x_2,\ \dot{x}_2 = u$, perform integrations to obtain x_1 and x_2.

Furthermore, x_1 and x_2 must be continuous at the switching time (where the control changes from β to $-\alpha$), find the switching time τ and the final time T.

Show your workings and results in the report.

4.c. This problem is concerned with the design of a linear quadratic regulator. Consider a system

$$\dot{x} = Ax + Bu,\ x(0) = x_0,$$

the cost function is

$$J = \frac{1}{2}\int_0^T (x^\top Q_x x + u^\top Q_u u)\, dt + \frac{1}{2}x^\top(T)Px(T)$$

The Hamiltonian is

$$H = \frac{1}{2}x^\top Q_x x + \frac{1}{2}u^\top Q_u u + \lambda^\top(Ax + Bu)$$

The optimal conditions based on Pontryagin's maximum principle are

$$\dot{x} = \frac{\partial H}{\partial \lambda} = Ax + Bu$$

$$-\dot{\lambda} = \frac{\partial H}{\partial x} = Q_x x + A^\top \lambda$$

$$0 = \frac{\partial H}{\partial u} = Q_u u + \lambda^\top B$$

From the last equation, we have $u = -Q_u^{-1}B^\top\lambda$. Put $\lambda = Px$, and set $\dot{P} = 0$, show that the algebraic Riccati equation can be given by

$$PA + A^T P - PBQ_u^{-1}B^T P + Q_x = 0.$$

Now, consider another system that is controlled only by the acceleration/deceleration, the system is given by

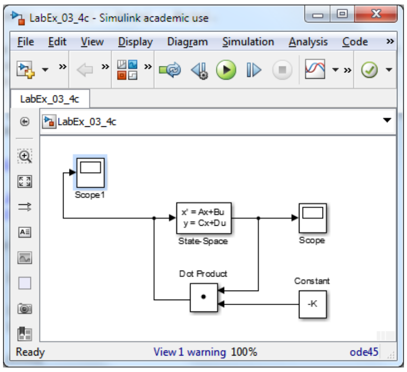
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Choose arbitrary q (e.g., 1, 2, 5) for $Q_x = \begin{bmatrix} q^2 & 0 \\ 0 & 0 \end{bmatrix}$, $Q_u = 1$. Show that the control gain is $K = [q \quad \sqrt{2q}]$. Select different combinations between Q_x and Q_u, comment on how they affect the system response. Verify your result with the Matlab **lqr** command and inspect the system response using Simulink.

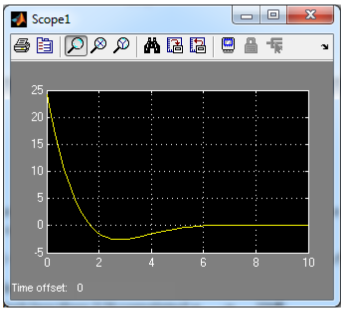
The Matlab command lqr provides the regulator gain factor $K = -Q_u^{-1}B^\top P$, $u = -Kx$. The command lqr can be called using:

> [K,S,E] = LQR(SYS,Q_x,Q_u) to calculate the optimal gain matrix K, such that: for a continuous-time state-space model SYS=ss(A,B,C,D), the state-feedback law $u = -Kx$ minimizes the cost function $J = \int (x^\top Q_x x + u^\top Q_u u)\, dt$, subject to the system dynamics $\dot{x} = Ax + Bu$.

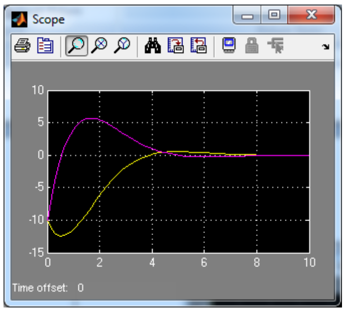
You have to define the system matrices A, B, C, D. Then construct a system structure using the command ss(A,B,C,D) before calling the lqr command. The gain K will be stored in the workspace. Furthermore, build a Simulink model and run, say, for 10 time-units (set the stop time parameter in the 'Simulation' menu). In the state-space model, you have to assign the appropriate system matrix values.



Simulink Model



Control



Response

Show your workings and results in the report.

Assessment

This laboratory exercise carries 13% of the overall marks for the course. The practical work carries 5% and the written report carries 8%.

The practical work (tasks 1 – 4) is assessed according to the following criteria:

Absent	0%
Work less than 1/5 completed	1%
Work less than 2/5 completed	2%
Work less than 3/5 completed	3%
Work less than 4/5 completed	4%
Work all correctly completed	5%

At the end of the scheduled laboratory period (week 8), the practical work (code and graphs) has to be marked-off by tutors. A signed copy by tutors of the mark sheet (download from Moodle) should be kept by the student and uploaded to Moodle within the week when marks are obtained.

Note: Time has to be allowed for tutors to mark the exercises. Each student will only be allocated 5 minutes for marking. Students are expected to register with the tutor for their order in the marking process.

The written report is assessed according to the following criteria:

Task 1, correctly illustrated the transportation problem solution procedures in the form of a tableau. 2%

Task 2, correctly demonstrate the Lagranian method calculation procedures (support your answer with calculation steps). 2%

Task 3, correctly show calculation procedures for the KKT method (support your answer with calculation steps). 2%

Task 4, correctly calculate the optimal control/trajectories and the use of Matlab lqr command. Discussion/comment on the choice of different weighting factors in the cost function, and how the system response is affected. 2%

In addition to the above requirements, the written report should be properly structured (use of section and sub-section headers, appropriate fonts – 12pt max, and proper layout of diagrams/graphs).

Submit your written report to Moodle on/before midnight Monday week 9. Note: when you use graphics/plots, press 'alt'+'prt sc' on your keyboard to capture a screen shot of the Matlab figures, then 'paste' on the MS Word file, and convert the whole document into a pdf file within 5MB for submission.