# MTRN4230 Assignment 3

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**1.a A vector 𝑝𝐴 is rotated about 𝑍𝐴 axis by 𝜃 degrees and then rotated about 𝑋𝐴 axis by 𝜙 degrees. Give the rotation matrix considering the orders given.**

Answer:

First the vector is rotated about z-axis by degrees, the rotational matrix from frame A to frame B is,

Then, the vector is rotated about x-axis (rotation about fixed axes in frame A) by degrees, the rotational matrix from fame B to frame C is,

Hence, the rotation matrix from frame A to frame B is,

**1.b. Frame {B} initially coincident with frame {A}. Now rotate {B} about 𝑍𝐵 axis by 𝜃 degrees and rotate the resulting frame about 𝑋𝐵 𝑎𝑥𝑖𝑠 𝑏𝑦 𝜙 degrees. Find rotation matrix for vectors 𝑝B 𝑡𝑜 𝑝𝐴 .**

Answer:

Rotated about z-axis by degrees, the rotational matrix from frame A to frame C is,

Rotated about current x-axis by degrees, the rotational matrix from frame C to frame B is,

Hence, the rotation matrix from frame A to frame B is,

**1.c. Given below frames**



**Calculate when , and are given.**

Answer:

Define some vectors in frame A, B and C, so we can have the following equations,

Equate the above equations,

Due to,

Inserting this into the general equation and rearranging,

1. **1.d. Proof that inverse of a rotation matrix must be equal to its transpose and rotation matrix is orthonormal. Show it with the help of two vectors embedded in a rigid body so no matter how the body rotates, the geometric angel between them (two vectors) preserve.**

Answer:

First, there are two vectors embedded in a same coordinate frame or rigid body as shown below,

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Based on this example, the rotation matrix can be found as,

The matrix can be inverted as,

The transpose of the rotation matrix is,

Obviously, the inverse of the rotation matrix is equal to the transpose of it. Hence, it can be deduced that the rotation matrix is orthonormal by the mathematical rules.

1. **1.e. Show the link frames for the below manipulators schematically**
2. Answer:
3. A close up of a map

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4. **A close up of a map

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5. **1.f. A 2DOF positioning table is used to help welding (two rotary joints 𝜃1, 𝜃2). The forward kinematics from based (link 1) to the bed of the table (link 2) is**
6. ****

**Unit vector fixed in frame of link 2 is 2V. 𝐹𝑖𝑛𝑑 𝑖𝑛𝑣𝑒𝑟𝑠𝑒 − 𝑘𝑖𝑛𝑒𝑚𝑎𝑡𝑖𝑐 𝑠𝑜𝑙𝑢𝑡𝑖𝑜𝑛 𝑓𝑜𝑟 𝜃1, 𝜃2) when this unit vector is aligned with 𝑍 0𝑎𝑥𝑖𝑠. Are there multiple solutions and is there a singular condition?**

Answer:

When the unit vector is aligned with z-axis in frame 0, so we can have the equations as,

Then, we can derive the three sub-equations,

From equation 2, we can get the value,

Inserting this value into equation 1 and equation 2, we can obtain,

If both X and Y are equal to zero, then singular and is arbitrary.

**2. A manipulator shown below that is known as SCARA when d4= 0.1, a1 = 0.4 and a2 = 0.3.**

***2.a Use DH convention find the forward kinematics.***

Answer:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Link | Angle () | Offset () | length () | Twist () |
| 1 |  | 0 |  | 0 |
| 2 |  | 0 |  | 180 |
| 3 | 0 |  | 0 | 0 |
| 4 |  |  | 0 | 0 |

By referring the lecture slides, the homogeneous matrices can be generated,

Form a general matrix with respect to the base frame,

Where , , and

Then, the MATLAB codes and results are presented as following,

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where d4= 0.1, a1 = 0.4 and a2 = 0.3.

**2.b Find inverse kinematics.**

Answer: The corresponding homogeneous transform take the form as,

Compared with , we can see

So,

Simplifying the above equation in,

Base the triangle theorems, we can get,

Then, can be represented as,

Since,

And,

In summary, the inverse kinematics can be expressed as

where d4= 0.1, a1 = 0.4 and a2 = 0.3.

The MATLAB codes and results are presented as below,

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***2.c Create kinematics verification mechanism as below and show the error as defined below in SIMULINK (use the trajectory provided).***

Answer: The verification mechanism is to consider the provided trajectory data as inputs, then insert these inputs into forward kinematic model. After getting the forward kinematic matrix, implementing the inverse kinematic model. Compared the final outputs of with the given trajectory, the errors are calculated by (outputs - inputs). The error plots are shown as below.

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As the figures shown, all the error values are much closer to zero. So, the forward and inverse kinematic matrix are correct and reliable.

The MATLAB codes are presented as below,

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***2.d Plot workspace when* −180𝑜 ≤ 𝜃1, 𝜃2 ≤ 180𝑜 𝑎𝑛𝑑 0 ≤ 𝑑3 ≤ 0.1 *in MATLAB.***

The workspace is a cylinder shape as shown below,

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***2.e Calculate Jacobian and provide a function in MATLAB to calculate it.***

The calculation process of Jacobian matrix is shown as below,

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