Part 1 — Economist (theory & welfare)

1. Equilibrium Concept: Bayes–Nash Equilibrium (BNE)

- **Definition:** A strategy profile $s^* = (s_1^*, ..., s_n^*)$ constitutes a Bayes–Nash Equilibrium if, for every player $i \in N$, every type $\theta_i \in \Theta_i$, and given beliefs about other players' types induced by the common prior $p(\theta)$, the strategy $s_i^*(\theta_i)$ maximizes the expected utility of player i:

$$\mathbb{E}_{\theta_{-i}\sim p(\cdot|\theta_i)}\left[u_i(s_i^*(\theta_i), s_{-i}^*(\theta_{-i});\theta_i)\right] \geq \mathbb{E}_{\theta_{-i}\sim p(\cdot|\theta_i)}\left[u_i(s_i(\theta_i), s_{-i}^*(\theta_{-i});\theta_i)\right]$$

for all alternative strategies s_i.

Here:
$N = \{1,2\}$ (two bargainers).
Θ_i = type space of player i, e.g., risk aversion or fairness preference.
$S_i : \Theta_i \rightarrow A_i$, mapping from types to actions (proposals/acceptances).
u_i = payoff function (depends on final allocation and type).

- **Existence Theorem** (Harsanyi, 1967–68; Fudenberg & Tirole, 1991, Ch. 6):
If each player's type space Θ_i is finite, action sets A_i are finite, and utilities u_i are bounded, then at least one Bayes–Nash Equilibrium exists.

Brief proof idea:
1) Treat each player's type θ_i as a separate "agent" in an expanded normal-form game. Each agent chooses actions in A_i according to their type.
2) The expanded game is finite (finite types × finite actions). By Nash (1950), a mixed-strategy equilibrium exists for any finite normal-form game.
3) The mixed-strategy equilibrium of the expanded game corresponds to a type-dependent strategy $s_i^*(\theta_i)$ in the original incomplete information game.
4) Each player maximizes expected utility given beliefs over opponents' types; since the expanded game equilibrium accounts for all type contingencies, this ensures no profitable unilateral deviation in expectation (Osborne and Rubinstein 1994; Myerson 1991, ch. 5)..

2. Analytical solution

In our two-player bargaining game under incomplete information, each player $i \in \{1,2\}$ has a private type $\theta_i \in \Theta_i$ representing risk attitude or fairness preference, and chooses a proposal or acceptance $a_i \in A_i=[0, \text{cap}]$, where the cap is one of three experimental treatments: 10, 100, or 1000.

- Equilibrium Characterization:
  At BNE, each player's strategy $s_i^*(\theta_i)$ maximizes expected utility given beliefs over the opponent's type. For small cap (10), the equilibrium proposals tend to reflect relative gains: players with more "aggressive" types may demand a larger share, leading to finer disputes over each unit. For medium cap (100) and large cap (1000), absolute payoffs dominate, and players are more likely to offer and accept fair splits (approximately 50:50) to secure substantial absolute gains. If payoffs are heterogeneous (e.g., one player has concave utility), the equilibrium allocation adjusts to reflect marginal utility differences, with more risk-averse players accepting smaller shares earlier.

- Efficiency:
  Most equilibria are Pareto efficient in the sense that no player can improve their expected payoff without reducing the other player's payoff, because any feasible reallocation within the cap would reduce one player's utility. From a utilitarian perspective (maximizing sum of expected payoffs), efficiency increases with cap size: in larger-cap treatments, reaching agreement quickly maximizes absolute total payoffs, whereas in small-cap settings, strategic disputes can slightly reduce total expected payoffs due to delayed agreement or rejections.

- Fairness:
  Fairness improves as cap increases. In small-cap games, equilibrium allocations may be skewed toward types with stronger bargaining power, resulting in higher inequality and potential envy. In medium and large-cap games, equilibria tend to satisfy proportionality and are closer to envy-free, since players prefer to secure substantial absolute payoffs rather than insist on minor relative advantages. Heterogeneous payoff functions can reduce equality but still generally maintain proportionality, as players' strategies incorporate expected marginal utility differences.

Overall, the BNE analysis predicts that increasing the cap shifts equilibrium behavior from relative gain competition to more cooperative and fair splits, improving both utilitarian efficiency and fairness measures. This characterization provides a theoretical benchmark for interpreting the experimental outcomes under the three cap treatments.

3. Interpretation

While the Bayes–Nash Equilibrium provides a rigorous theoretical benchmark for our bargaining game, several considerations highlight its limitations and extensions in practice.

- Realism:
  The standard BNE assumes fully rational players with common knowledge of the type distribution and the ability to compute expected utilities over all possible type contingencies. In real-world bargaining, cognitive constraints, time pressure, and incomplete information about the opponent's preferences can make such calculations unrealistic. In our experimental design, for instance, players facing large caps (1000) may rely on heuristics or fairness norms rather than computing exact expected payoffs, reflecting bounded rationality.

- Multiplicity of Equilibria:
  Even in this simple two-player game, multiple BNE can exist, particularly under heterogeneous payoff functions or when the cap is discrete. For example, several splits may satisfy players' optimal responses depending on their types. This multiplicity can lead to coordination challenges and variability in observed outcomes, which is consistent with empirical observations in bargaining experiments.

- Refinements:
  Refinements such as Perfect Bayesian Equilibrium or Sequential Equilibrium are relevant if we consider dynamic alternating-offer protocols. These refinements incorporate belief updating and credible threats, narrowing down plausible equilibria and providing more predictive power. In our static one-shot proposals, BNE suffices, but dynamic versions or introducing time discounting could benefit from these refinements.

- Bounded Rationality and Computational Tractability:
  Computing BNE exactly can become intractable as the type space or cap increases, especially with continuous actions or multiple heterogenous types. Our experimental manipulations of cap (10, 100, 1000) illustrate this challenge: larger caps expand the strategy space, increasing computational complexity both theoretically and for human players. In practice, participants may adopt simplified strategies (e.g., anchoring on 50:50 splits or adjusting by a fixed percentage of cap), which are computationally feasible but only approximate the theoretical BNE. This demonstrates how bounded rationality and tractability constraints shape observed bargaining behavior and highlight the gap between formal equilibrium predictions and actual human play.