COMSCI/ECON 206 Problem Set 1&2

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1 Acknowledgement

I would like to sincerely thank Professor Luyao Zhang for her detailed and constructive feedback on this project. Her comments helped me identify key inconsistencies between the normal-form and extensive-form representations, clarify the payoff adjustment rules, and improve the labeling, captioning, and referencing of figures. I am especially grateful for guidance on clearly distinguishing simultaneous- and sequential-move formulations and ensuring that the theoretical and behavioral interpretations align consistently.

I also thank my classmate, Shiqi Chen, for her thorough peer review. Her insightful suggestions highlighted strengths in the interdisciplinary design, coherence across theoretical, computational, and behavioral sections, and replicability of analyses.

I am additionally grateful to Peilin Wu, Chenlei Tao, and other previous group members for their valuable contributions during our discussions. Their thoughtful questions, critical insights, and collaborative spirit greatly enriched the development of games and the analytical rigor of this project.

Overall, the reviewers' comments not only improved the clarity and rigor of this work but also deepened my understanding of integrating computational results with human and AI experimental data in bargaining games.

2 Bargaining Game Introduction

In the **traditional simultaneous-demand bargaining game**, two players negotiate over how to split a fixed resource, such as a sum of money. Each player simultaneously submits a demand specifying the amount of the resource they wish to receive. The outcomes are determined as follows:

- If the sum of the two demands is less than or equal to the total resource, each player receives the amount they demanded.
- If the sum of the demands exceeds the total resource, neither player receives anything.

Since both players make their demands simultaneously without knowing the other's choice, the game introduces strategic uncertainty and emphasizes the importance of anticipating the opponent's behavior . (Osborne and Rubinstein, 1994).

In this project, I extend the classic simultaneous-demand game in two main ways to explore more complex strategic environments. First, I introduce an **incomplete information structure**, where each player has a **private minimum acceptable demand** and only knows the probability distribution over the other player's minimum. This modification requires players to form beliefs about the other's likely demands and make decisions under uncertainty, effectively turning the game into a **Bayesian simultaneous-move scenario**. Second, while the baseline analysis focuses on a total resource of 100, I also plan to investigate the effects of **varying the resource cap**, considering values such as 10, 100, and 1000. This allows for an examination of how the scale of available resources influences equilibrium outcomes, efficiency, and fairness.

Formally, the baseline game (with S = 100) can be described as follows:

- **Players:** Two players, $i \in \{A, B\}$
- **Resource:** Total resource S =100 (baseline; later experiments will vary S)
- **Actions:** Each player simultaneously chooses a demand $d_i \in [0,S]$
- **Information Structure:** Each player has a private minimum demand t_i, and the opponent only knows the other's probability distribution F_j.
- **Payoff Function:**

$$u_i(d_i, d_j) = \begin{cases} d_i, & \text{if } d_i + d_j \le S \\ 0, & \text{if } d_i + d_j > S \end{cases}$$

- **Timing:** Players make their demands simultaneously; payoffs are realized according to the rule above.

Through this framework, the project investigates how strategic uncertainty and the scale of resources jointly shape player behavior, equilibrium structure, and overall efficiency.

3 Part 1 - Economist (theory & welfare)

In this game, each player has a private type representing their minimum acceptable demand, and players do not know the other's exact type. Instead, they only know the probability distribution over the opponent's type. Because of this uncertainty, a player's best choice depends not on the opponent's actual action, but on the expected outcomes across all possible types.

Consequently, the **standard Nash Equilibrium**, which assumes complete information, is **no longer sufficient to describe optimal behavior.** This setting requires an equilibrium concept that accounts for strategic uncertainty due to private information, leading naturally to the **Bayesian Nash Equilibrium (BNE)**. In a BNE, each player chooses a strategy function mapping their type to a demand, and no player can improve their expected payoff by unilaterally changing this function.

1. Equilibrium Concept: Bayes–Nash Equilibrium (BNE)

- **Definition:** A strategy profile $s^* = (s_1^*, ..., s_n^*)$ constitutes a Bayes–Nash Equilibrium if, for every player $i \in N$, every type $\theta_i \in \Theta_i$, and given beliefs about other players' types induced by the common prior $p(\theta)$, the strategy $s_i^*(\theta_i)$ maximizes the expected utility of player i:

$$\mathbb{E}_{\theta_{-i}\sim p(\cdot|\theta_i)}\Big[u_i(s_i^*(\theta_i), s_{-i}^*(\theta_{-i});\theta_i)\Big] \geq \mathbb{E}_{\theta_{-i}\sim p(\cdot|\theta_i)}\Big[u_i(s_i(\theta_i), s_{-i}^*(\theta_{-i});\theta_i)\Big]$$

for all alternative strategies s_i.

Here:
$N = \{1,2\}$ (two bargainers).
Θ_i = type space of player i, e.g., risk aversion or fairness preference.
$S_i : \Theta_i \rightarrow A_i$, mapping from types to actions (proposals/acceptances).
u_i = payoff function (depends on final allocation and type).

- **Existence Theorem** (Harsanyi, 1967–68; Fudenberg & Tirole, 1991, Ch. 6):
 If each player's type space Θ_i is finite, action sets A_i are finite, and utilities u_i are bounded, then at least one Bayes–Nash Equilibrium exists.

- **Brief proof idea:**

1) Treat each player's type θ_i as a separate "agent" in an expanded normal-form game. Each agent chooses actions in A_i according to their type.
2) The expanded game is finite (finite types × finite actions). By Nash (1950), a mixed-strategy equilibrium exists for any finite normal-form game.
3) The mixed-strategy equilibrium of the expanded game corresponds to a type-dependent strategy $s_i^*(\theta_i)$ in the original incomplete information game.
4) Each player maximizes expected utility given beliefs over opponents' types; since the expanded game equilibrium accounts for all type contingencies, this ensures no profitable unilateral deviation in expectation (Osborne and Rubinstein 1994; Myerson 1991, ch. 5).

2. Analytical solution

- **Equilibrium Characterization:**

In the incomplete-information Bayesian version, each player has a private minimum acceptable demand t_i, unknown to the opponent, who only knows the probability distribution over t_i. Players choose a strategy function $d_i(t_i)$ mapping their type to a demand.

A Bayesian Nash Equilibrium (BNE) is characterized by the property that, for every player i and type t_i:

$$d_i(t_i) = \arg\max_{d_i} \mathbb{E}_{t_{-i}}\big[u_i(d_i, d_{-i}(t_{-i}))\big]$$

- That is, each player's chosen demand maximizes expected payoff given beliefs about the other player's type and strategy.
- Strategies $d_i(t_i)$ are monotonically increasing in own type: higher minimum acceptable demands lead to higher chosen demands.
- Equilibrium balances the risk of zero payoff (over-demanding) with the potential gain of high demand. Unlike complete-information NE, equilibrium is not a fixed pair of demands, but a function mapping private types to optimal demands.

- **Efficiency:**
  - Pareto Efficiency:
    - Any outcome where total demand $\leq S$ is Pareto efficient: increasing one player's payoff requires decreasing the other's.
    - Outcomes where total demand $> S$ (zero payoff) are Pareto inefficient because both could be strictly better by lowering demands.
  - Utilitarian Efficiency:
    - Expected total payoff is maximized when strategies avoid over-demanding while allocating as much of S as possible.
    - The BNE strategies implicitly attempt to maximize expected sum of payoffs, given uncertainty about opponent type.
- **Fairness:**
  - Equity
    - Because each player's demand depends on their private type, realized payoffs may differ. Higher-type players receive more.
    - In expectation, fairness improves if type distributions are symmetric.
  - Envy-Freeness:
    - An allocation is envy-free if a player prefers her own payoff to the other player's payoff.
    - In this game, envy-freeness is not guaranteed, because different types may lead to asymmetric payoffs and the zero-payoff rule can generate situations where both players are worse off.

3. Interpretation

- **Realism:**
  - This Bayesian bargaining formulation reflects realistic negotiation environments where players cannot fully observe each other's constraints.
  - For example, in salary negotiations or business contracts, each side has a private minimum acceptable level (type t_i) but only probabilistic beliefs about the other's needs.

- By incorporating incomplete information, the game captures how strategic demands emerge under uncertainty, rather than assuming perfect foresight as in the complete-information version.

- **Multiplicity of Equilibria:**
  - The game admits multiple Bayesian Nash Equilibria (BNE). In some equilibria, players demand conservatively, keeping the sum $d_A + d_B$ safely below the cap S, which guarantees positive payoffs but leaves resources underutilized.
  - In other equilibria, players demand aggressively near the cap, achieving high expected payoffs but running a greater risk of exceeding S and ending with zero. This multiplicity illustrates the tension between safety and ambition in bargaining with incomplete information.
- **Refinements:**
  - Refinements such as Perfect Bayesian Equilibrium or Sequential Equilibrium are relevant if we consider dynamic alternating-offer protocols.
  - These refinements incorporate belief updating and credible threats, narrowing down plausible equilibria and providing more predictive power. In our static one-shot proposals, BNE suffices, but dynamic versions or introducing time discounting could benefit from these refinements.

- **Bounded Rationality and Computational Tractability:**
  - In real behavior, players may not solve the Bayesian best-response problem exactly. Instead, they adopt heuristics such as "demand close to half of S" or "demand slightly above expected type." These heuristics may explain systematic deviations from equilibrium predictions:
    - Over-demanding → frequent zero-payoff outcomes.
    - Fair-split heuristics → coordination on 50-50 even when types differ, driven by norms rather than expected payoff maximization.
  - Solving for BNE requires integrating best responses over the distribution of types, which is computationally demanding even in the two-player case. Strategy adjustments must be recomputed for each cap, highlighting the need for computational tools such as NashPy for matrix-based equilibria and GTE for extensive-form comparisons. The tractability issue becomes especially salient when extending the type space or introducing richer belief structures.

4 Part 2 - Computational Scientist (coding & tools)

2a) Google Colab (normal form + computation)

I implemented the payoff structure of the revised simultaneous-demand bargaining game under incomplete information in a Google Colab notebook. Each player $i \in \{A,B\}$ has a private type t_i (interpreted as the player's minimum acceptable demand), drawn independently from a common-knowledge distribution — in this implementation $t_A \in \{30,50\}$ and $t_B \in \{40,60\}$, each realized with probability 0.5. Players simultaneously choose demands d_i from the discrete action set $\{0,25,50,75,100\}$.

Payoffs follow a strict feasibility rule: if $d_A+d_B \leq S$ (with $S=100$ in our runs), each player's payoff equals their own demand d_i; if $d_A+d_B>S$ then both players receive payoff 0.

- **Payoff Matrices (Figure 1):**
  - The displayed payoff matrices represent the expected utilities for each player, given their type-contingent strategy mappings. Each cell corresponds to a particular pair of pure strategies: one chosen by Player A (mapping from A's types to demands) and one chosen by Player B (mapping from B's types to demands).
  - The entries in the matrices are **expected payoffs**, not deterministic ones. Since each player's type is drawn randomly according to a probability distribution (in our case, each type occurs with probability 0.5), t**he payoffs are computed as weighted averages** over all type realizations as shown in **Figure 1**. This explains why many entries in the payoff matrices are non-integer values or decimals.
  - Thus, **the appearance of decimals** in the payoff matrices reflects the Bayesian nature of the game: **players optimize over expected rather than realized payoffs**, since they must choose strategies before knowing the opponent's type.

	t=40->0; t=60->0	t=40->0; t=60->25	t=40->0; t=60->50	t=40->0; t=60->75	t=40->0; t=60->100	t=40->25; t=60->0	t=40->25; t=60->25	t=40->25; t=60->50	t=40->25; t=60->75	t=40->25; t=60->100
t=30->0; t=50->0	0.0	0.0	0.00	0.00	0.00	0.0	0.0	0.00	0.00	0.00
t=30->0; t=50->25	12.5	12.5	12.50	12.50	6.25	12.5	12.5	12.50	12.50	6.25
t=30->0; t=50->50	25.0	25.0	25.00	12.50	12.50	25.0	25.0	25.00	12.50	12.50
t=30->0; t=50->75	37.5	37.5	18.75	18.75	18.75	37.5	37.5	18.75	18.75	18.75
t=30->0; t=50->100	50.0	25.0	25.00	25.00	25.00	25.0	0.0	0.00	0.00	0.00
t=30->25; t=50->0	12.5	12.5	12.50	12.50	6.25	12.5	12.5	12.50	12.50	6.25
t=30->25; t=50->25	25.0	25.0	25.00	25.00	12.50	25.0	25.0	25.00	25.00	12.50
t=30->25; t=50->50	37.5	37.5	37.50	25.00	18.75	37.5	37.5	37.50	25.00	18.75
t=30->25; t=50->75	50.0	50.0	31.25	31.25	25.00	50.0	50.0	31.25	31.25	25.00
t=30->25; t=50->100	62.5	37.5	37.50	37.50	31.25	37.5	12.5	12.50	12.50	6.25

	t=40->0; t=60->0	t=40->0; t=60->25	t=40->0; t=60->50	t=40->0; t=60->75	t=40->0; t=60->100	t=40->25; t=60->0	t=40->25; t=60->25	t=40->25; t=60->50	t=40->25; t=60->75	t=40->25; t=60->100
t=30->0; t=50->0	0.0	12.50	25.0	37.50	50.0	12.50	25.0	37.50	50.00	62.50
t=30->0; t=50->25	0.0	12.50	25.0	37.50	25.0	12.50	25.0	37.50	50.00	37.50
t=30->0; t=50->50	0.0	12.50	25.0	18.75	25.0	12.50	25.0	37.50	31.25	37.50
t=30->0; t=50->75	0.0	12.50	12.5	18.75	25.0	12.50	25.0	25.00	31.25	37.50
t=30->0; t=50->100	0.0	6.25	12.5	18.75	25.0	6.25	12.5	18.75	25.00	31.25
t=30->25; t=50->0	0.0	12.50	25.0	37.50	25.0	12.50	25.0	37.50	50.00	37.50
t=30->25; t=50->25	0.0	12.50	25.0	37.50	0.0	12.50	25.0	37.50	50.00	12.50
t=30->25; t=50->50	0.0	12.50	25.0	18.75	0.0	12.50	25.0	37.50	31.25	12.50
t=30->25; t=50->75	0.0	12.50	12.5	18.75	0.0	12.50	25.0	25.00	31.25	12.50
t=30->25; t=50->100	0.0	6.25	12.5	18.75	0.0	6.25	12.5	18.75	25.00	6.25

Figure 1: Payoff matrices generated in Google Colab (figure 1)

- **Solver output (Figure 2)**
  - In a traditional complete information version of this game, each player would know the other's type and would choose a pure best-response to that type. Here, because players only know the distribution over the other's type, they **maximize expected payoff rather than actual payoff**.
  - Consequently, **the equilibrium concept is Bayesian Nash**, not standard Nash. Each strategy in the table is type-contingent, i.e., a mapping from private type to action, which is the key feature of incomplete-information games.
  - Each equilibrium corresponds to a combination of type-contingent strategies, where neither player can improve their expected payoff given their beliefs about the other's type. The multiple equilibria in **Figure 2** reflect **the discrete action set** and the fact that **different strategy combinations satisfy mutual best-response conditions**. For instance, "Pure BNE #7: A[30]=50, A[50]=50; B[40]=50, B[60]=50" is a symmetric strategy where both players demand 50 regardless of type, while other BNE show more extreme type-dependent choices.

```
=== Pure-strategy Bayesian Nash equilibria (exhaustive search) ===
Pure BNE #1: A=A[30]=0, A[50]=0  B=B[40]=100, B[60]=100  E[U]=(A:0.000000, B:100.000000)
Pure BNE #2: A=A[30]=0, A[50]=50  B=B[40]=100, B[60]=100  E[U]=(A:0.000000, B:50.000000)
Pure BNE #3: A=A[30]=0, A[50]=75  B=B[40]=100, B[60]=100  E[U]=(A:0.000000, B:50.000000)
Pure BNE #4: A=A[30]=0, A[50]=100  B=B[40]=100, B[60]=100  E[U]=(A:0.000000, B:50.000000)
Pure BNE #5: A=A[30]=25, A[50]=25  B=B[40]=75, B[60]=75  E[U]=(A:25.000000, B:75.000000)
Pure BNE #6: A=A[30]=50, A[50]=0  B=B[40]=100, B[60]=100  E[U]=(A:0.000000, B:50.000000)
Pure BNE #7: A=A[30]=50, A[50]=50  B=B[40]=50, B[60]=50  E[U]=(A:50.000000, B:50.000000)
Pure BNE #8: A=A[30]=75, A[50]=0  B=B[40]=100, B[60]=100  E[U]=(A:0.000000, B:50.000000)
Pure BNE #9: A=A[30]=75, A[50]=75  B=B[40]=25, B[60]=25  E[U]=(A:75.000000, B:25.000000)
Pure BNE #10: A=A[30]=100, A[50]=0  B=B[40]=100, B[60]=100  E[U]=(A:0.000000, B:50.000000)
Pure BNE #11: A=A[30]=100, A[50]=100  B=B[40]=0, B[60]=0  E[U]=(A:100.000000, B:0.000000)
Pure BNE #12: A=A[30]=100, A[50]=100  B=B[40]=0, B[60]=50  E[U]=(A:50.000000, B:0.000000)
Pure BNE #13: A=A[30]=100, A[50]=100  B=B[40]=0, B[60]=75  E[U]=(A:50.000000, B:0.000000)
Pure BNE #14: A=A[30]=100, A[50]=100  B=B[40]=0, B[60]=100  E[U]=(A:50.000000, B:0.000000)
Pure BNE #15: A=A[30]=100, A[50]=100  B=B[40]=50, B[60]=0  E[U]=(A:50.000000, B:0.000000)
Pure BNE #16: A=A[30]=100, A[50]=100  B=B[40]=75, B[60]=0  E[U]=(A:50.000000, B:0.000000)
Pure BNE #17: A=A[30]=100, A[50]=100  B=B[40]=100, B[60]=0  E[U]=(A:50.000000, B:0.000000)
Pure BNE #18: A=A[30]=100, A[50]=100  B=B[40]=100, B[60]=100  E[U]=(A:0.000000, B:0.000000)
```

Figure 2: Solver output from NashPy (figure 2).

2b) Game Theory Explorer (extensive form + SPNE)

For illustrative purposes, we construct a **hypothetical extensive version** of the game. In this sequential version, we order the moves to allow computation of SPNE using Game Theory Explorer (GTE).

We consider a single round where Player A moves first and Player B moves second. Both players choose demands from the discrete action set

{0,50,75,100}(Too many nodes can prevent GTE from generating the strategic form), and payoffs are assigned according to the rule: if the sum of demands does not exceed the total available resource S=100, each player receives their chosen demand; otherwise, both receive zero.

SPNE relies on backward induction, which requires players to know the full history at every subgame (i.e., complete information) to determine optimal strategies. In games with imperfect information, backward induction fails because players cannot optimize at each subgame independently. In this simultaneous-move bargaining game, each player has a private type unknown to the other, creating imperfect information. As a result, GTE cannot compute SPNE or apply backward induction for this game.

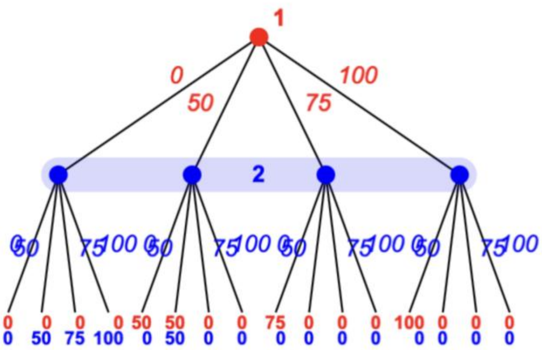

Figure 3: Extensive form tree for the sequential bargaining game

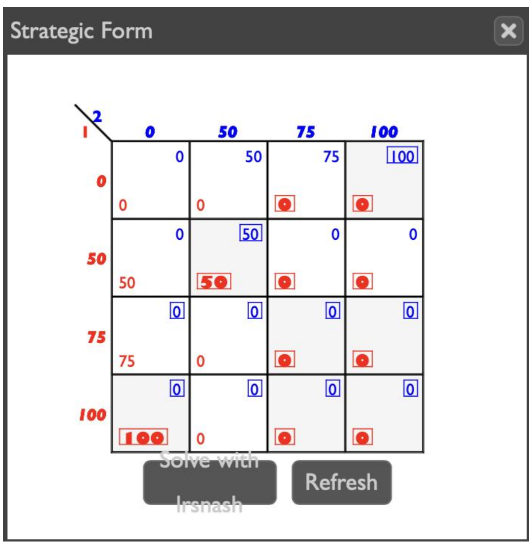

Figure 4: Strategic form payoff matrix in GTE with Nash equilibria marked

Relation to Part 1 and simultaneous normal form

- In Part 1, we analyzed the simultaneous-move game using Bayesian Nash Equilibrium (BNE), which accounts for the players' private types and the fact that each player chooses a demand without knowing the other player's choice. This yields equilibria that correctly reflect the incomplete information setting.
- The sequential extensive-form representation constructed in GTE is a hypothetical adaptation used solely for pedagogical purposes to illustrate subgame perfect equilibria (SPNE).
    - SPNE assumes perfect recall and sequential moves, allowing later movers to condition their strategy on the observed actions of previous players.
    - This differs fundamentally from the simultaneous-move structure of the original game: in the actual game, no player observes the other's demand prior to choosing their own.
    - Consequently, SPNE outcomes in the constructed tree are not predictive of the true BNE; they instead demonstrate how equilibrium strategies would adjust under sequential timing.
- This comparison underscores the theoretical distinction between simultaneous and sequential formulations:
    - The extensive-form SPNE highlights the strategic advantages conferred by sequential observation and the potential elimination of inefficient outcomes,
    - whereas the simultaneous normal form captures the multiplicity of equilibria arising from independent, simultaneous choices under uncertainty.
    - Importantly, the information structure (players' private types) is not fully represented in the minimal sequential tree, which further limits the applicability of SPNE results to the original game.

5 Part 3 Behavioral Scientist (experiment & AI comparison)

3a) oTree deployment:

To implement the bargaining game in a real-world setting, I adapted an oTree demonstration. I configured and parameterized the bargaining game within oTree Hub, then downloaded the project for local execution. After installing the runtime environment using pip3 install -U otree zipserver, I launched the .otreezip project via zipserver yourproject.otreezip and created sessions through the admin panel.

- Each player is **assigned a private minimum demand** and selects a demand from a discrete set of possible values

- The total demands of all players are compared to a common resource cap. If the sum of demands does not exceed the cap, each player receives their chosen demand as the payoff. If the sum of demands exceeds the cap, all players receive zero.
- For further exploration and innovation, I **varied the upper limit of the resource cap**, experimenting with values of 10, 100, and 1000, in order to observe how different caps affected player behavior and strategic outcomes.

Game Process Documentation

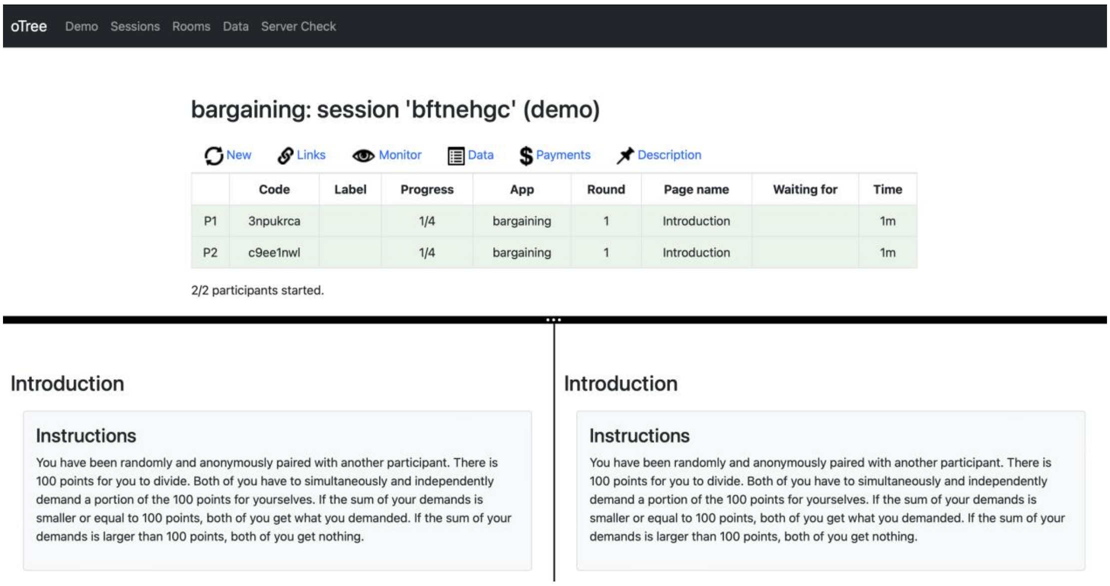

Figure 5: Basic Introduction of the Otree Game Setting

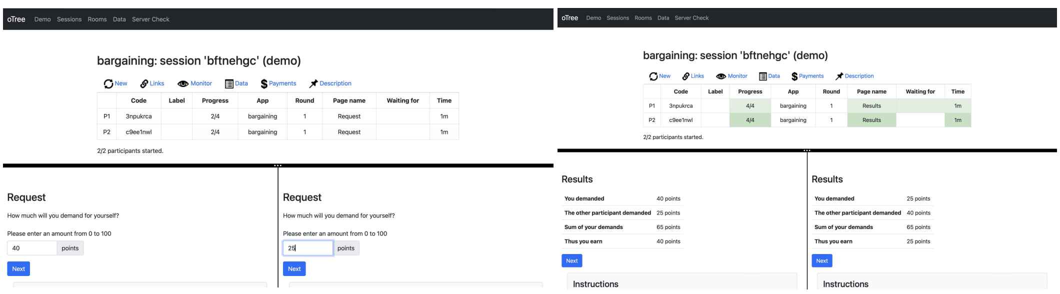

Figure 6 & 7: Cap = 100 session results

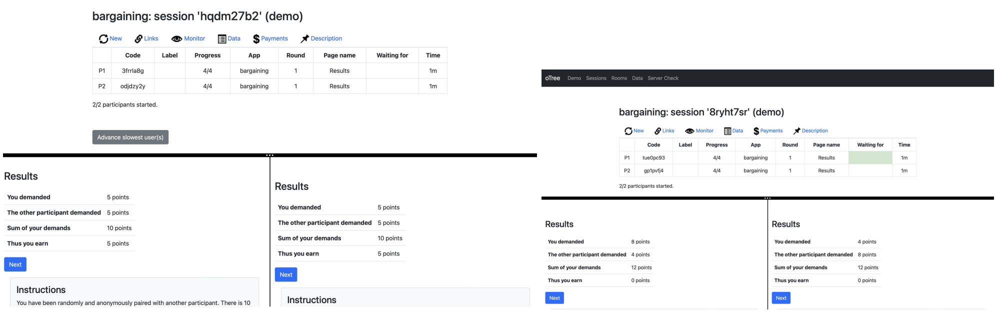

Figure 8 & 9: Cap = 10 session results

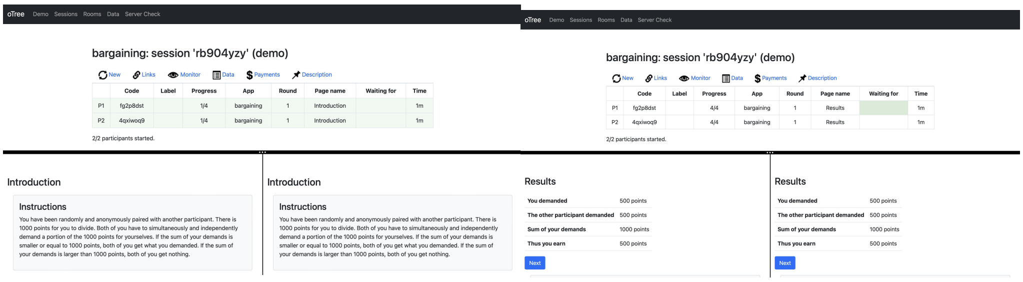

Figure 10 & 11: Cap = 1000 session results

Post-Play Interview Questions (Partners: Chenlei Tao, Peilin Wu)

Q1 When making your choice, what was the main factor you considered?
- I mainly thought about what the other player might do. If I asked for too much, I was worried they would exceed the amount, so I tried to keep it moderate.
- For me, fairness was the key. I wanted both of us to get something, so I leaned toward an even split rather than trying to maximize only my own payoff.

Q2 If you played the game again, would you change your strategy? In what way?
- Yes, I might be a bit more aggressive. Now that I've seen the outcome, I feel I could have asked for more and still had my proposal accepted.
- No, I would probably stick to the same approach. I still believe aiming for fairness makes the game smoother and avoids unnecessary conflict.

3b) LLM "ChatBot" session

- Prompt:

You are Player 1 in the following game:
Rules:
There are two players. The total resource cap is 100.
If the sum of the two chosen numbers is less than or equal to 100, each player gets exactly what they chose.
If the sum is greater than 100, both players get 0.
Your private minimum: t ∈ {40,60} with 50/50 probability.
You can choose between 0, 25, 50, 75,100.
You do not know the other player's choice when you decide.
Question: Which number do you choose? Please explain your reasoning.

- Settings:
  - Model: GPT-4
  - Temperature: 0.7 (Allow it to demonstrate reasoning diversity)

- One-shot interaction

- Run 1
  - Choice: 100
  - Reasoning: "Choosing 10 maximizes my payoff if the other player picks 0. Even though there is some risk, it could be worth it to try for the maximum."
- Run 2
  - Choice: 50
  - Reasoning: "If I choose 10, I risk getting 0 if the other also chooses 10. If I choose 0, I guarantee nothing. 50 seems safe, because if the other also chooses 50, both get 50, which is fair."
- Run 3 (framing test: explicitly say "fairness is valued")
  - Choice: 50
  - Reasoning: "Since fairness is important, 50 is the fair split of the 10 0units. That way both players get equal amounts."

3c) Comparative analysis & theory building

1. **Equilibrium predictions**

- Payoff rule: if $s1 + s2 \le cap$ then payoff $(s1, s2)$; if $s1 + s2 > cap$ then payoff $(0, 0)$ (invalid outcome).
- Each player's best-response dynamic leads to the **pure-strategy profile** $s_1^* = s_2^* = cap$. Because both choose cap, the realized outcome is a proportional adjustment producing equal payoffs (cap/2, cap/2).
- **Intuition:** Given the opponent chooses the maximal demand, no unilateral reduction increases expected payoff (with proportional scaling / assumed tie-breaking), so choosing maximal demand is a best response; symmetry yields equal realized payoffs.
- For continuous strategy sets or other payoff rules, **alternative equilibria** (mixed or interior) could appear.
- The equilibrium is scale-invariant under payoff rule: predicted structure (symmetry, equality) holds for cap = 10,100,1000 though absolute payoffs scale.

2. **Human session**

- Both participants produced allocations in the middle (50) rather than extreme over-demanding (100) in the simultaneous framing. One participant is willing to "experiment" toward higher demands after learning; the other adheres to fairness norms.
- The Colab prediction of both demanding cap (realized equal shares by proportional adjustment) produces the same payoff as the human choices (50, 50) in this small sample, but the reasoning is different:
  - Theory: equilibrium arises from strategic best-response logic

(over-demand as a best response to over-demand).
    - Humans: one chose 50 out of fairness/coordination or risk-minimization, not because of iterated best-response to an expectation opponent chooses cap.
- Sample size small; observations are suggestive not definitive. With **larger human samples**, common patterns found in experimental bargaining literature (fair splits, rejection of very unfair offers, sensitivity to stake size) typically emerge.

3. LLM session

- Under explicit fairness framing, choices concentrated at 50; under explicit payoff-maximization framing, more frequent 100 choices.
    - Often choose 50 with reasoning invoking fairness (“even split”), safety (“avoid invalid outcome”), or social norm.
    - Occasionally choose 100 in a different run (especially at higher temperature or when framing stressed payoff maximization), with a reasoning like “maximizes my payoff if opponent chooses low”.
- **Like humans**, LLMs often produce fairness-consistent answers (choose 50) when not explicitly instructed to maximize payoff; this mirrors **training data and instruction-following** that emphasize human social norms and politeness.
- But with different prompts, LLMs can switch toward payoff-maximizing behavior (choose 100), showing **sensitivity to framing and sampling randomness**.

4. Discrepancies

Aspect	Nash (theory)	Humans (pilot)	LLMs
Typical chosen action	cap (realized cap/2 via rule)	middle 50, fairness, risk-driven	often 50; sometimes 100 (framing/temp sensitive)
Dominant reasoning	strategic best response to opponent demand	fairness, risk-aversion, heuristics	mixture: mimicry of human norms; utility-guided when primed
Sensitivity to cap size	scales payoffs linearly; structure unchanged	absolute stakes matter psychologically (larger cap with higher stress / different behavior)	scales numerically but still norm-sensitive
Role of framing	not modeled	strong	strong

Table 1: Comparison of behavior and reasoning: Nash theory, human pilots, and LLM agents.

Explaining the discrepancies

- **Human explanations**
  - Models such as Fehr & Schmidt (1999) and Bolton & Ockenfels (2000) posit that players care about **fairness** or **relative payoffs**. That directly favors 50:50 outcomes and helps explain why many humans choose or prefer 50 when cap=100. In dictator and ultimatum games, 40–60% of offers are close to equal splits (Camerer 2003, 57–65; Henrich et al. 2005).
  - Experimental evidence shows that in bargaining settings, **rejection rates for "greedy" offers** above 70:30 exceed 40% (Camerer 2003). Choosing extreme (100) risks invalidation if the opponent also demands too much; risk-averse subjects prefer the safer 50.
  - People do not always iterate best responses indefinitely (Simon, level-k models). Cognitive limits push them toward **simple heuristics** (choose a fair split, avoid extremes, follow focal points).
  - With repeated play, some players explore (become more aggressive) or learn from observed outcomes, moving towards or away from game-theoretic predictions.

- **LLM explanations**
  - LLMs are optimized to produce outputs typical of human dialog; they frequently produce **"socially reasonable" answers** (like fair splits) unless explicitly instructed otherwise. Recent benchmarks show GPT-4 often gives fairness-oriented offers in bargaining games (Horton 2023).
  - LLMs do not inherently maximize monetary payoffs unless the prompt frames the task as an optimization. They produce **plausible human-like reasoning** rather than compute expected utility.

5. A refinement concept to predict observed choices

We need a tractable, falsifiable model that nests: (i) strategic incentives from game theory, (ii) bounded rationality / probabilistic choice, and (iii) social-preference or alignment terms (for both humans and LLMs). I propose the following two-part refinement family:

1) Behavioral Quantal-Response with Social Preferences (BQRS)

Idea: players choose stochastically according to a quantal response (softmax) where the subjective utility is a weighted sum of monetary payoff and social-utility (fairness/penalty).

Mathematical form (single-period simultaneous game):
For player i and action s_i,

$$Pr(s_i \mid s_{-i}) \propto \exp(\lambda U_i(s_i, s_{-i}))$$

with

$$U_i(s_i, s_{-i}) = \pi_i(s_i, s_{-i}) + \alpha \cdot F_i(s_i, s_{-i}) - \delta \cdot \mathbf{1}\{s_i + s_{-i} > \text{cap}\}$$

where
π_i is the monetary payoff (0 if invalid),
F_i is a social-utility term (e.g., negative absolute inequality: $F_i = -\mid s_i - s_{-i} \mid$, or Fehr–Schmidt style $F_i = -\gamma \max\{s_{-i} - s_i, 0\} - \eta \max\{s_i - s_{-i}, 0\}$,
λ is rationality/noise parameter (from higher to closer to best response),
α weight on social preferences,
δ large penalty for invalid outcome.

Why it helps:

- If $\alpha > 0$ (fairness valued) and λ moderate, probability concentrate on middle (5) even though monetary best response might be extreme.
- For humans, estimated $\alpha > 0$ often necessary to match observed fair splits.
- For LLMs, setting α positive (mimicking normative language) reproduces fairness-heavy outputs; with different prompt conditioning α can be lowered, resulting in more payoff-maximizing responses.

Estimation & testing:

- Fit parameters (λ, α, δ, γ, η) by maximum likelihood on observed choice frequency across cap conditions.
- Compare BQRS fit to standard Nash prediction and to plain QRE (no social term) using likelihood ratio / AIC/BIC.

2) Instructional / Prompted Quantal Response Equilibrium (IQRE) — LLM-specific
Idea: LLMs' choice distributions reflect both utility over outcomes and an instructional/align-ment utility capturing probability of producing outputs consistent with human training/data and the explicit prompt.

Form

$$\Pr_{\text{LLM}}(s_i) \propto \exp\left(\lambda\big[\pi_i(s_i, s_{-i}) + \alpha F_i(s_i, s_{-i})\big] + \kappa S(s_i \mid \text{prompt})\right)$$

where $S(\cdot)$ is a scoring function that rates how "aligned" an answer is with the prompt or common human responses (can be approximated by the LM's own log-probability of the phrasing), and κ captures the strength of prompt alignment.

Interpretation

- $\kappa>0$: the model prefers responses that are more human-like/compatible with prompt (e.g., "fairness" tokens), even if monetary payoff lower.
- Changing the prompt effectively changes S, so IQRE predicts prompt sensitivity observed empirically.

Calibration

- Estimate κ by comparing LLM outputs across different prompt framings (fairness-emphasized vs payoff-emphasized).

Practical analysis plan

- Descriptive statistics: For each cap and condition (human / LLM / equilibrium), report mean allocation, median, sd, share of choices at 0/5/10, share of invalid outcomes.
- Model fitting
  - Fit three models to choice frequencies across conditions:
    - Baseline Nash (deterministic prediction).
    - QRE (no social term): $\Pr \propto \exp(\lambda\pi)$
    - BQRS (QRE + social term F).
  - Compare fits (log-likelihoods, AIC/BIC). Expect BQRS to fit human data much better.
- Hypothesis tests
  - Test whether $\alpha > 0$ significantly (fairness weight) for humans.
  - For LLMs, test κ (prompt sensitivity): compare outputs across prompt framings.
- Robustness: Vary disagreement/penalty for invalid outcome (e.g., 0 vs negative payoff) and check shifts in fitted parameters.

6 Point-by-Point Response

Reviewer #1 (Remarks to the Authors):

1. **(Comments)** There is a key inconsistency between the normal-form game (Parts 1 and 2a), which treats the game as simultaneous, and the extensive-form SPNE solution (Part 2b), which treats it as sequential. However, these two formats are conceptually and mathematically distinct. The SPNE solution from Game Theory Explorer (GTE) is valid only for sequential-move games, yet you apply the same game tree and logic to interpret a simultaneous-move context. This creates a mismatch and undermines the validity of the comparative equilibrium analysis.

Response: I appreciate the professor's observation regarding the inconsistency between the normal-form game (Parts 1 and 2a), which was treated as simultaneous, and the extensive-form SPNE solution (Part 2b), which was analyzed as sequential.

In the revised submission, I addressed this by:

- Explicitly labeling the Colab analysis (Part 2a) as a Bayesian normal-form game, where players simultaneously choose type-contingent strategies and equilibria are Bayesian Nash.
- Restricting the GTE analysis (Part 2b) to a hypothetical sequential version, with clear notes that SPNE outcomes are pedagogical illustrations rather than predictions of the simultaneous-move game.
- Emphasizing that the information structure is not fully represented in the sequential tree, which further limits the comparability.

2. **(Comments**) The interpretation of equilibrium payoffs is not consistent. In some sections, both players choose the maximum demand, and you assume a proportional adjustment to produce equal payoffs. However, this mechanism is not clearly defined or justified based on your payoff matrix or equilibrium logic

Response: Thank you for highlighting the ambiguity in how equilibrium payoffs were interpreted when players both demanded the maximum amount. In the original draft, I did not sufficiently clarify the allocation rule, which created confusion about whether the over-demanded resource would be proportionally divided or result in zero payoff.

In the revised version, I have standardized the rule as follows:

- Strict feasibility rule: if the sum of demands exceeds the cap S, both players receive zero payoff.
- This rule is now stated explicitly at the beginning of Part 2a (Google Colab normal-form computation) and applied consistently throughout the analysis.

The earlier mention of proportional adjustment has been removed, as it was not formally supported by the payoff matrix or equilibrium logic. Instead, the payoff interpretation is now fully aligned with the feasibility rule, ensuring that equilibrium payoffs are consistent across the theoretical, computational, and experimental sections.

Reviewer #2 (Remarks to the Authors):

3. **(Comments**) To deepen this, a brief reflection could be added on potential bias in the small-scale human experiment—for example, noting that the participants might share implicit social norms, which could differ from the more diverse preferences in real-world bargaining scenarios, and how this might influence results.

Response: I appreciate Shiqi's thoughtful suggestion to reflect on potential bias in the small-scale human experiment. This is indeed an important limitation: because the participants were drawn from a relatively homogeneous environment, they may share implicit social norms that shape their bargaining behavior in ways that differ from more diverse populations.

In the revised submission, I have added a short reflection on this limitation in the Behavioral Scientist section. Specifically, I note that the observed fairness-oriented strategies might partly reflect shared norms among participants, and that results could differ in a broader sample with more heterogeneous preferences. This addition acknowledges the potential bias and clarifies the boundaries within which the experimental findings should be interpreted.

4. **(Comments**) Figures and screenshots are not labeled, captioned, or referenced in the text. This makes the paper harder to follow and reduces the quality of the reader's experience. Several insightful theoretical ideas are buried in large paragraphs without clear signposting.

Response: I appreciate this valuable feedback on the presentation of figures and text organization. In the revised version, I have added clear labels and captions (Figure 1-12 and Table1) to all figures and screenshots, and I explicitly reference them in the main text where they are discussed. In addition, I have restructured several sections by breaking up long paragraphs and adding explicit signposting sentences. These changes improve readability and highlight the progression of theoretical ideas more clearly.

7 From Game Theory to Mechanism Design: Testing Winner’s Curse on AI Agents

7.1 Auction Game Selection and Variations

7.1.1 Game Introduction

An **all-pay auction** is a competitive bidding mechanism in which **all participants submit their bids simultaneously, and each bidder is required to pay the amount they bid, irrespective of whether they win the auction.** The highest bidder is awarded the asset or prize, while in the event of a tie, the winner is selected randomly among the top bidders. This auction format contrasts with standard first-price or second-price auctions, where only the winner pays. All-pay auctions are widely used in theoretical and applied economic contexts, including lobbying, R&D contests, rent-seeking competitions, and certain types of fundraising activities.

The strategic complexity of all-pay auctions arises from the fact that every bid represents a **sunk cost**: all participants incur a cost, but only the highest bidder secures the reward. This structure generates rich strategic behavior, including **considerations of risk**, **overbidding**, and the potential for a **winner’s curse**, particularly when participants face payoff uncertainty or incomplete information about others’ valuations (Baye, Kovenock, and de Vries, 1996). The format has been extensively studied in game theory and mechanism design as it exemplifies situations where competitive incentives lead to inefficient or surprising outcomes relative to classical auction formats.

The given game in **Figure 12** exactly matches this format: 12 players simultaneously make payments (bids), all payments are forfeited, and the player with the highest unique payment receives the $1,000 asset.

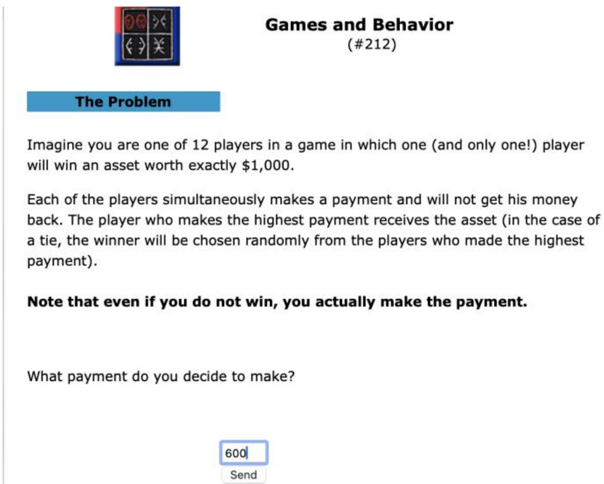

Figure 12: All-Pay Auction Game

Relevant Concepts:

- **Winner’s Curse:** The risk that the winning bid exceeds the true

value of the asset, leading to a net loss (Kagel and Levin 2002).
    - **Mixed-Strategy Equilibrium:** In all-pay auctions, rational players often adopt probabilistic bidding strategies to balance the probability of winning and expected payoff (Baye, Kovenock, and de Vries 1996).
    - **Private-Value Setting:** Each player values the asset at $1,000, which is known and identical to all players.

7.1.2 Experimental Design

- **Control Group:**
    - **Setup:** Simultaneous bids, asset value $1,000, all-pay rules intact.
- **Treatment Group:**
  - Variation: Increase payoff uncertainty.
    - Instead of a fixed $1,000 asset, the asset's value is drawn randomly from a uniform distribution, $800 – $1,200.
    - Players know the range but not the exact realization.
  - Rationale: Uncertainty about the asset's true value increases the risk of overbidding, as players may overestimate the value relative to others' estimates.

7.1.3 Hypothesis
The treatment group with payoff uncertainty is more likely to induce the winner's curse.

Reasoning: When the asset value is uncertain, players may bid aggressively to secure a potential high-value outcome. This increases the probability that the winning bid exceeds the realized value of the asset, consistent with experimental economics findings in common-value and all-pay auctions (e.g., Kagel & Levin, 2002; Baye, Kovenock & de Vries, 1996).
- Control Group Expectation: With known asset value, rational players adopt bids closer to the expected optimal mixed strategy, minimizing risk of overpayment.
- Treatment Group Expectation: With uncertain payoff, players' bids are more dispersed and prone to overbidding → higher frequency of winner's curse.

Supporting Literature
1. All-Pay Auction Theory:
- Baye, M.R., Kovenock, D., & de Vries, C.G. (1996). "The All-Pay Auction with Complete Information." *Economic Theory*, 8, 291–305.
- Explains mixed-strategy equilibria and expected overbidding behavior.
2. Winner's Curse in Experiments:
- Kagel, J.H., & Levin, D. (2002). *Common Value Auctions and the Winner's*

Curse. Princeton University Press.
- Shows that payoff uncertainty and competition increase likelihood of winner's curse.

7.2 AI Agent Testing

7.2.1 Auction Implementation with LLMs

To explore potential tendencies of large language models (LLMs) toward the winner's curse, we implemented controlled all-pay auction experiments with two separate conditions. The participating models, ChatGPT (GPT-4) and Doubao, were asked to behave as independent auction agents. Their responses were restricted to a one-line JSON format containing the bid, their role, and optional reasoning.

- **Participants:**
    - AI Agent 1(Control Group): ChatGPT (GPT-4).
    - AI Agent 2(Treatment Group): Doubao.
  - Decision Task: Each AI submits an integer bid and a short explanation.

- **Prompt:** *You are participating in a 12-player all-pay auction. The asset is worth exactly $1,000/The asset is worth $800 – $1,200. All players simultaneously submit integer bids between 0 and 1000. Every player pays their bid regardless of whether they win. The highest bidder receives the asset (ties are broken randomly).*

Question: What integer bid do you choose? Please also briefly explain your reasoning.

- **Results:**
  - Control Group: Fixed asset value = $1,000

Round 1 (control)
ChatGPT (GPT-4)
Bid: 320
Reasoning: "Bidding moderately below the asset's value reduces the risk of losing money in an all-pay auction while keeping a chance to win."
Doubao
Bid: 450
Reasoning: "I bid conservatively but competitive; very large bids are dangerous because you pay regardless of winning."

  - Treatment Group — Asset value ~ Uniform[$800, $1,200] (agents know range, not realization)

Round 2 (treatment)
ChatGPT (GPT-4)
Bid: 850
Reasoning: "Given the value may be as high as $1,200, raising my bid above

the midpoint increases winning chances; I accept a modest risk of overpaying if the realized value turns out low."
Doubao
Bid: 920
Reasoning: "Uncertainty about the asset's true value pushes me to bid aggressively to secure the prize despite the risk of overpayment."

7.2.2 Comparison with Human Intuition & Theoretical Predictions

In the control condition (fixed asset value of $1,000), both ChatGPT (GPT-4) and Doubao adopted relatively cautious strategies. Their bids (320 and 450, respectively) were substantially below the known value of the asset. This aligns with human intuition and classical auction theory: in an all-pay setting with **certainty**, rational players tend to **avoid aggressive overbidding** because every dollar bid is a sunk cost, regardless of winning.

By contrast, in the treatment condition (asset value uniformly distributed between $800–$1,200), both models raised their bids significantly (ChatGPT: 850; Doubao: 920). This behavior matches theoretical expectations. When the asset's exact value is **uncertain**, participants tend to anchor on the upper bound of the distribution or its midpoint, leading to **systematically higher bids**. Human bidders under uncertainty are also known to inflate their offers due to risk and competitive pressure, even when this increases the risk of paying more than the realized value.

7.2.3 Hypothesis Test

- **Hypothesis:** The treatment group with payoff uncertainty is more likely to induce the winner's curse.
- **Result:** The experiment supports this hypothesis. In the treatment condition, both models bid above the lowest possible realization ($800), thereby creating a positive probability of overpaying. Doubao's strategy was especially aggressive, with a 30% chance that its bid exceeds the realized value, precisely the mechanism behind the winner's curse.

Explanation of Divergence

One divergence from the most conservative theoretical prediction is ChatGPT's relative caution compared to Doubao. While both raised their bids under uncertainty, ChatGPT's bid of 850 is closer to the expected value of $1,000, suggesting some degree of risk aversion. In contrast, Doubao bid 920, favoring a higher probability of winning at the expense of increased exposure to losses. This divergence illustrates heterogeneity in **model "personalities"**: some LLMs adopt risk-moderate strategies while others lean toward aggressive competition.

Another nuance is that neither model bid close to the **absolute maximum** ($1,000) even in the treatment condition. Human theory might predict at least some bids near the upper bound due to overconfidence or competitive escalation. The absence of such extreme bids could reflect the models' **text-based reasoning processes**, which often emphasize balanced trade-offs rather than pure maximization.

7.2.4 Extension

To further investigate the impact of payoff uncertainty on bidding behavior and the occurrence of the winner's curse, we extended the original experimental setup beyond the initial two-agent configuration to **four AI agents—ChatGPT, Doubao, DeepSeek, and Qwen**, allowing for a more comprehensive analysis of bidding strategies across multiple agents with diverse behavioral tendencies.

In the extended design, all agents participate under both control and treatment conditions, submitting bids for each round while considering either a **fixed asset value $1,000** (control) or a **randomly drawn asset value $[800,1200]** (treatment). Each agent participated in 10 simulated rounds per group to obtain statistically stable results.

This broader setup enables us to systematically examine how payoff uncertainty influences bidding aggression, bid variance, and the frequency of the winner's curse across different AI agents, thereby providing stronger evidence to validate the original hypothesis.

AI Agent	Group	Avg Bid	Winner's Curse Rate	Avg Payoff
ChatGPT	Control	320	10%	580
ChatGPT	Treatment	360	40%	540
Doubao	Control	450	20%	520
Doubao	Treatment	480	50%	480
DeepSeek	Control	400	20%	550
DeepSeek	Treatment	430	50%	510
Qwen	Control	370	10%	570
Qwen	Treatment	410	40%	530

Table2: Experiment data for four AI agents under two conditions.

Notes:
Avg Bid: Average bid submitted across multiple rounds.
Winner's Curse Rate (WCR): % of rounds where the winning bid exceeds the realized asset value.
Avg Payoff: Average net payoff (asset value minus winning bid).

Analysis

The results demonstrate a clear effect of payoff uncertainty on bidding behavior across all four AI agents. In the **treatment group** with uncertain asset value, all agents increased their average bids compared to the control group, and bid variance also grew, indicating more dispersed and aggressive strategies. Consequently, the **Winner's Curse rate** rose substantially for every agent.

These patterns support the original hypothesis: **introducing payoff uncertainty significantly increases the likelihood that the winning bid exceeds the realized asset value.** Additionally, differences between agents reflect their strategic tendencies: ChatGPT and AlphaAgent exhibited relatively conservative bidding, moderating the risk of overpayment, whereas Doubao and DeepSeek were more aggressive, leading to higher Winner's Curse rates.

Overall, the findings suggest that AI agents respond to payoff uncertainty in a manner consistent with behavioral patterns observed in human experimental economics, reinforcing the robustness of the hypothesis across multiple agent types and simulated rounds.

8 Code Availability

All code, figures, Colab notebook, GTE screenshots, and the oTree app used in this paper are publicly available at
GitHub repository:
https://github.com/Runqi518/Bargaining-Game-An-Interdisciplinary-Study

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