cs231n

lec3

선형분류(linear classification)

So far: Defined a (linear) score function $f(x,W) = Wx + b$

Example class scores for 3 images for some W:

How can we tell whether this W is good or bad?

airplane	-3.45	-0.51	3.42
automobile	-8.87	**6.04**	4.64
bird	0.09	5.31	2.65
cat	**2.9**	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	**-4.34**
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

- 이 모델의 분류(W 값)이 좋을까?
- 정도를 측정할 도구가 필요

→ loss function(손실함수)

Given a dataset of examples

$$\{(x_i, y_i)\}_{i=1}^{N}$$

Where x_i is image and y_i is (integer) label

Loss over the dataset is a sum of loss over examples:

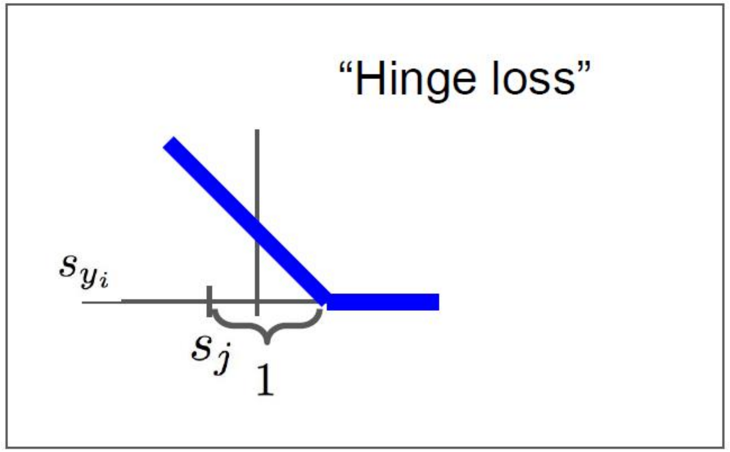
$$L = \frac{1}{N} \sum_i L_i(f(x_i, W), y_i)$$

손실 함수 (Loss function)

1. Multiclass SVM loss
2. Softmax Classifier

Multiclass SVM loss

Multiclass SVM loss:



$$L_i = \sum_{j \neq y_i} \begin{cases} 0 & \text{if } s_{y_i} \geq s_j + 1 \\ s_j - s_{y_i} + 1 & \text{otherwise} \end{cases}$$

$$= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Loss $L = \frac{(2.9 + 0 + 12.9)}{3}$ $= 5.7$	**2.9** $= \max(5.1 - 3.2 + 1, 0)$ $+ \max(-1.7 - 3.2 + 1,)$	**0**	**12.9**

Several question

- Q: What happens to loss if car scores change a bit?
- A: No change
- Q: what is the min/max possible loss?
- A: min = 0 , max = infinite
- Q: At initialization W is small so all s ≈ 0. What is the loss?
- A: $\frac{1}{N}\sum_i \sum_{j \neq y_i} \max(s_j - s_{y_i} + 1, 0) = \frac{1}{N}\sum_i (N-1) = \frac{N(N-1)}{N} = N - 1$
- Q: What if the sum was over all classes? (including j = y_i)
- A: loss value(손실값) +1
- Q: L_i값을 구할때 합대신 평균을 사용한다면?
- A: loss value는 작아지지만, 큰의미는 없다
- Q:$L_i = \sum_{j \neq y_i} \max(s_j - s_{y_i} + 1, 0)^2$ 으로 정의한다면
- A: 다른분류가 된다.

Softmax classifier

scores = unnormalized log probabilities of the classes.

$$\boxed{P(Y=k|X=x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}} \quad \text{where} \quad \boxed{s = f(x_i; W)}$$

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$\boxed{L_i = -\log P(Y = y_i | X = x_i)}$$

in summary: $$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

	unnormalized log probabilities	exp →	unnormalized probabilities	normalize →	probabilities
cat	**3.2**		**24.5**		**0.13** → L_i = -log(0.13) = **0.89**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

unnormalized log probabilities

probabilities

w→2w , loss is same!

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

			
cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

Before:

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

With W twice as large:

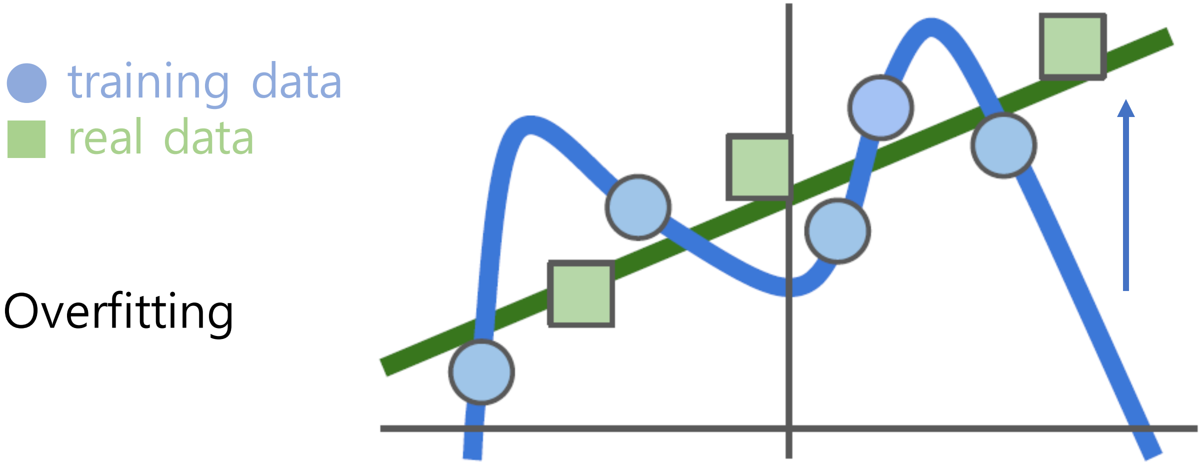
= max(0, 2.6 - 9.8 + 1)
+max(0, 4.0 - 9.8 + 1)
= max(0, -6.2) + max(0, -4.8)
= 0 + 0
= 0

→조정이 필요 = regularization

$$L(W) = \underbrace{\frac{1}{N}\sum_{i=1}^{N} L_i(f(x_i, W), y_i)} + \underbrace{\lambda R(W)}$$

Data loss: Model predictions should match training data

Regularization: Model should be “simple”, so it works on test data



Overfitting

Regularization

λ = regularization strength (hyperparameter)

$$L = \frac{1}{N} \sum_{i=1}^{N} \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1) + \boxed{\lambda R(W)}$$

In common use:

L2 regularization $R(W) = \sum_k \sum_l W_{k,l}^2$

L1 regularization $R(W) = \sum_k \sum_l |W_{k,l}|$

Elastic net (L1 + L2) $R(W) = \sum_k \sum_l \beta W_{k,l}^2 + |W_{k,l}|$

Max norm regularization (might see later)

Dropout (will see later)

Fancier: Batch normalization, stochastic depth

L2 Regularization (Weight Decay)

$$x = [1, 1, 1, 1]$$

$$R(W) = \sum_k \sum_l W_{k,l}^2$$

$$R(W) = \sum_k \sum_l |W_{k,l}|$$

$$w_1 = [1, 0, 0, 0]$$

$$w_2 = [0.25, 0.25, 0.25, 0.25]$$

(If you are a Bayesian: L2 regularization also corresponds MAP inference using a Gaussian prior on W)

$$w_1^T x = w_2^T x = 1$$

L1 Regularization 은 w1 선호

L2 Regularization 은 w2 선호

Recap

How do we find the best W?

- We have some dataset of (x,y)
- We have a **score function:** $s = f(x; W) \overset{\text{e.g.}}{=} Wx$
- We have a **loss function**:

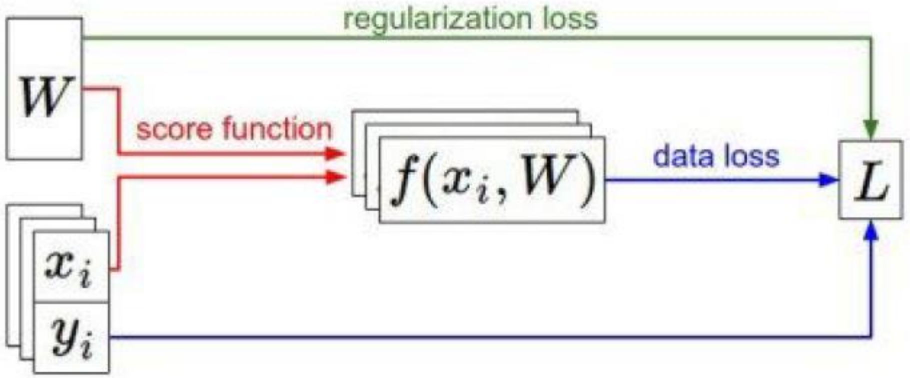
Softmax

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

SVM

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i + R(W)$$ Full loss



Optimization (적절한 w찾기)

- Random search
- → 시간낭비, 정답에대한 보장이 없음.

- Following Slope 기울기 따라가기

current W:	**W + h** (second dim)**:**	**gradient dW:**
[0.34,	[0.34,	[-2.5,
-1.11,	-1.11 + **0.0001**,	?,
0.78,	0.78,	?,
0.12,	0.12,	?,
0.55,	0.55,	?,
2.81,	2.81,	?,
-3.1,	-3.1,	?,
-1.5,	-1.5,	?,
0.33,…]	0.33,…]	?,…]
loss 1.25347	**loss 1.25353**	

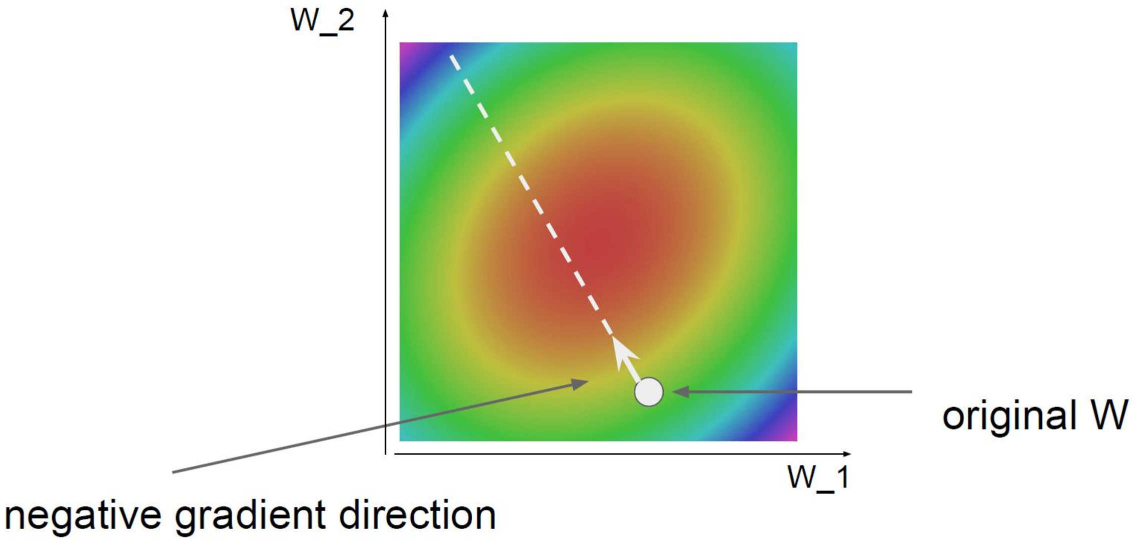
$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Using calculus

- Faster than numerical way

- But some time numerical gradient useful to debug

- Go plat area(red)

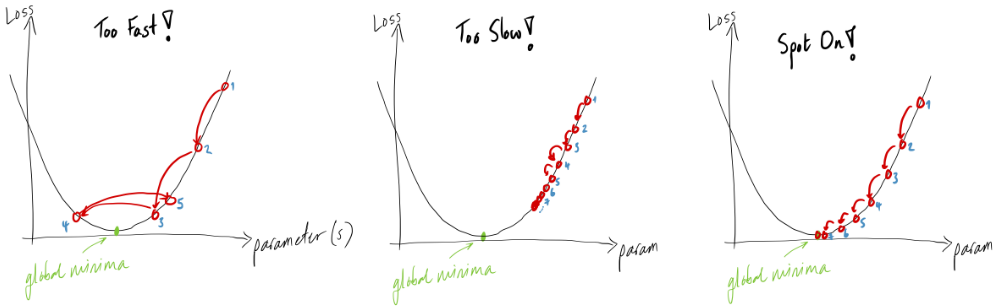


Gradient Descent

Step_size = learning rate.
→ Large size or small size?

```
# Vanilla Gradient Descent

while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```



Stochastic Gradient Descent (SGD)

$$L(W) = \frac{1}{N} \sum_{i=1}^{N} L_i(x_i, y_i, W) + \lambda R(W)$$

$$\nabla_W L(W) = \frac{1}{N} \sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W) + \lambda \nabla_W R(W)$$

Full sum expensive when N is large!

Approximate sum using a **minibatch** of examples
32 / 64 / 128 common

Thank you