Computational MRI (COMP0121) Coursework 1

Gary Hui Zhang, PhD

Due: November 11th, 2020 (16:00)

General Notes:

1. Please read each problem carefully; make sure that you answer each one as completely as possible.
2. The problems have been phrased in the MR terms that you are expected to be familiar with at this point of the module; if you find any of them to be unfamiliar, look them up in the book chapters, the lecture slides, and the worksheets.
3. The problems are deliberately non-prescriptive with some of the key parameters. You are expected to know, or to be able to work out, the suitable choices of these, and can justify your choices.
4. For simulation/visualisation tasks, remember that the objective is to use these as a tool to better understand the underlying MR phenomena. So you are expected to describe what your simulation shows, explain if they make sense, and reflect on what you have learned from it.

Submission Guidance:

1. A written report: submit as a single pdf file; maximum three-page long; minimum font size 10; recommended to use Latex; include any figures at the end of the report, which will not be counted towards the page limit.
2. A code listing: submit as a single pdf file; no page limit
3. A collection of videos: submit as a single zip file; please label clearly which simulation visualisation each video corresponds to.

Problem 1: Spin excess

1. Write down the expression for computing spin excess; explain in plain words what factors spin excess depends on, and how.
2. Write a Matlab function to generalise this computation for arbitrary temperature, field strength, and gyromagnetic ratio.

Problem 2: Forced precession with an on-resonance RF field

1. Write down the 3-by-3 matrix for rotating a 3-D vector about the z-axis by some angle θ; explain its action on the unit vectors $\hat{x}, \hat{y}$ and $\hat{z}$ and use this assessment to verify that the results make sense. Implement the action of the rotation matrix as a Matlab function.
2. Repeat 2.1 but for the rotation about the x-axis.
3. Repeat 2.1 but for the rotation about the y-axis.

4. Choose the appropriate rotation matrix to simulate, and visualise, the forced precession of a magnetisation $\overrightarrow{M} = [0\,0\,1]'$ by 90^o, with an on-resonance RF field, along the $\hat{x}'$ axis in the rotating frame (of the Larmor frequency); assume that the duration of the RF pulse is 1 ms; consider carefully what quantities are particularly important to visualise.

Note: It is generally safe to ignore the effect of T_1/T_2 relaxation during RF pulses, as they are typically much shorter than T_1 and T_2 relaxation times. Unless noted explicitly, always ignore the effect of relaxation during RF pulses.

Problem 3: Free precession in the main static magnetic field

1. Ignoring the effect of T_1/T_2 relaxation to begin with, choose the appropriate rotation matrix from *Problem 2* to simulate, and visualise, the free precession of the magnetisation produced by the forced precession in *Problem 2*, in the laboratory frame; you will need to assume some fictitious Larmor frequency for the purpose of visualisation.
2. Now include the effect of T_1/T_2 relaxation in your simulation and visualisation; explain how this should be done mathematically.
3. Repeat 3.1 and 3.2 but now in the rotating frame of the Larmor frequency.
4. Repeat 3.3 but for an isochromat precessing slightly faster than the Larmor frequency.
5. Repeat 3.4 for an isochromat precessing slightly slower than the Larmor frequency.

Problem 4: **Free induction decay and Inversion Recovery**

Use the tools that you have developed in *Problems 2 and 3* to simulate, and visualise

1. the free induction decay sequence; assume a single isochromat that is precessing at the Larmor frequency.
2. Repeat 4.1 for the inversion recovery sequence.

Problem 5: Spin echo

Use the tools that you have developed in *Problems 2 and 3* to simulate, and visualise, the spin echo sequence.

To achieve this, one needs to simulate a collection of isochromats that precess at a range of frequencies $[\omega_0 - \delta\omega, \omega_0 + \delta\omega]$. By choosing the make-up of this collection appropriately, the combined magnetisation from the collection can give rise to the expected spin echo.

1. Start with assuming a uniform distribution, i.e. there are an equal proportion of isochromats at each frequency.
2. Repeat 5.1 but now assume a Lorentzian distribution (a.k.a a Cauchy distribution):

   $P(\omega; \omega_0, \Delta) = 1/(\pi\Delta + \pi(\omega - \omega_0)^2/\Delta)$, where ω_0 and Δ are two tunable constants.
3. Discuss your findings from 5.1 and 5.2; determine the right choice of the isochromat make-up.
4. With the appropriate choice of the isochromat frequency distribution, simulate the original Hahn echo: the second RF pulse having a flip angle of 90^o instead of 180^o.