Computational MRI (COMP0121) Coursework 1

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1 Spin excess

1.1

When a group of particle is placed in a magnetic field, the spin of each particle aligns parallel or anti-parallel to the external field. The Spin excess is given by Boltzmann distribution

$$\frac{N^-}{N^+} = e^{-E/kT} \tag{1.1}$$

where N^+ and N^- are the number of spins in lower energy alignment and higher energy alignment, respectively. E is the energy difference between the two spin energy levels, k, the Boltzmann constant, and T, the the absolute temperature. The split of spin energy level under external magnetic field is determined by Larmor frequency ω.

$$E = h\nu \tag{1.2}$$

where h is the Planck constant, and ω is given by

$$\omega = \gamma B \tag{1.3}$$

where γ is the gyromagnetic ratio, and B, the magnetic field strength. Combining Eq.1.1 to 1.3 yields

$$\frac{N^-}{N^+} = e^{-h\gamma B/kT} \tag{1.4}$$

1.2

Code for the computation of spin excess can be found in spin_excess.m file.

2 Forced precession with an on-resonance RF field

2.1

The translation matrix for rotating a 3-D vector about the z-axis by some angle θ is

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.1}$$

2.2

Similarly,

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \tag{2.2}$$

2.3

and

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \tag{2.3}$$

2.4

A video visualising the forced precession of magnetization $\overrightarrow{M} = [0\ 0\ 1]'$ in rotating frame is shown in the file video2.4.avi. A figure of that can be found in Fig. 1. Find the code producing these in Problem2.m. Note that video and script files are numbered after the corresponding problem number.

3 Free precession in the main static magnetic field

3.1

A video for the forced precession in laboratory frame can be find in video3.1.avi. A figure of that can be found in Fig. 2.

3.2

The free precession with the effect of T_1 and T_2 relaxation is governed by the Bloch equation,

$$\frac{d\overrightarrow{M}}{dt} = \gamma\overrightarrow{M} \times \overrightarrow{B} - \frac{\overrightarrow{M}_\perp}{T_2} + \frac{(M_0 - M_z)\hat{z}}{T_1} \tag{3.1}$$

This ordinary differential equation can be solve by Euler method directly. To make use of the rotation matrix developed in the last problem, the first term of Eq. 1.1 which merely results in rotation of $\overrightarrow{M}$ around $\overrightarrow{B}$ is solved analytically. This ensures that the choice of step size in Euler method will not otherwise affect the precision of the rotation. The reduced Bloch equation (in the on-resonance Rotating frame)

$$\frac{d\overrightarrow{M}^*}{dt} = -\frac{\overrightarrow{M}^*_\perp}{T_2} + \frac{(M_0 - M^*_z)\hat{z}}{T_1} \tag{3.2}$$

is solved instead, which governs only relaxation process, where $\overrightarrow{M}^*$ is magnetization in the on-resonance Rotating frame. The Euler's method is the following iteration,

$$\overrightarrow{M}^*_{k+1} = \overrightarrow{M}^*_k + \Delta t\left(-\frac{\overrightarrow{M}^*_{\perp,k}}{T_2} + \frac{(M_0 - M^*_{z,k})\hat{z}}{T_1}\right) \tag{3.3}$$

The magnetization in the laboratory frame can be calculated by translation, in this case, rotation around z axis.

$$\overrightarrow{M}_k = R_z(\theta_k)\overrightarrow{M}^*_k \tag{3.4}$$

where θ_k is the rotation angle with respect to the initial orientation of magnetization at step k. The parameters used in this report are (the same everywhere unless explicit specified): $T_1 = 10$ ms, $T_2 = 5$ ms, and $\omega_0 = 4\pi$ KHz. The results are in video3.2.avi and Fig. 3.

3.3

This is to solve Eq. 3.2. Find the results in in video3.3.avi and Fig. 4.

3.4

Find the results in in video3.4.avi and Fig. 5.

3.5

Find the results in in video3.5.avi and Fig. 6.

4 Free induction decay and inversion recovery

4.1

The voltage signal collected by a coil placed near the sample is given by

$$emf = -\frac{d}{dt}\int_{sample} d^3r\overrightarrow{M}(\overrightarrow{r},t)\cdot\overrightarrow{B}^{receive}(\overrightarrow{r}) \tag{4.1}$$

In the calculation, assume the sample to be small enough and a coil placed on the x-axis. Find the results in Fig. 7. RF pulses are applied every 25 ms.

4.2

The results for different Inversion Time are shown in Fig. 8. The $M_\perp(T_I)$ vs T_I is shown in Fig. 11.

5 Spin echo

5.1

A uniform distribution is not a good assumption for the isochromats frequencies at equilibrium. For presentation purpose, the relaxation are slowed down by using $T_1 = 20$ ms and $T_2 = 15$ ms only in Problem 5. The span of the frequencies are 0.05 (δ_ω/ω_0). The echo time $T_E = 15$ ms. The result is shown in Fig. 12.

5.2

Rejection method was used to generate random numbers following Lorentzian distribution. Same span was used, and $\Delta = 0.1$ for the Lorentzian distribution. Find the result in Fig. 13.

5.3

Figure 12 shows considerable side packet in the free precession. This is not the natural case when the material is approaching equilibrium under external magnetic field. Figure 13 shows the correct response with a good estimation of isochromats frequencies.

5.4

Find the result in Fig. 14.

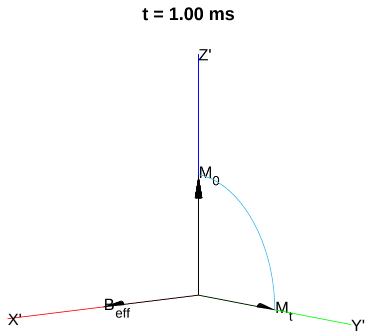

Figure 1: Forced precession in the rotating frame.

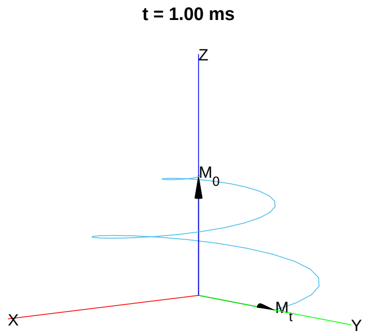

Figure 2: Forced precession in the laboratory frame.

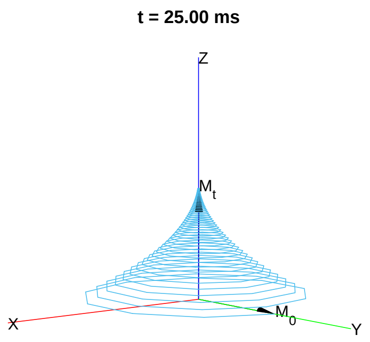

Figure 3: Free precession with relaxation in the laboratory frame.

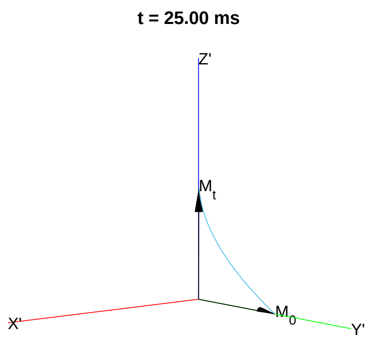

Figure 4: Free precession with relaxation in the laboratory frame.

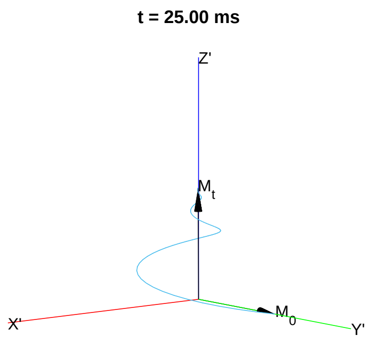

Figure 5: Free precession with relaxation in the laboratory frame, ahead of resonance frequency.

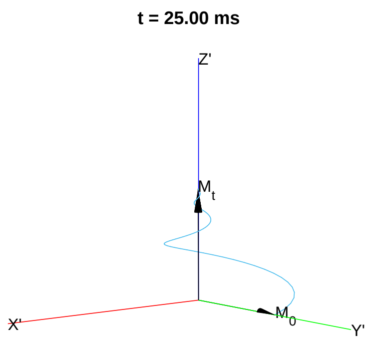

Figure 6: Free precession with relaxation in the laboratory frame, behind resonance frequency.

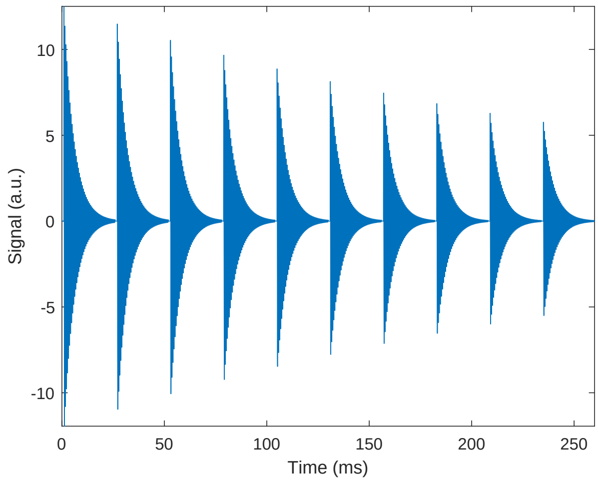

Figure 7: Free induction decay sequece.

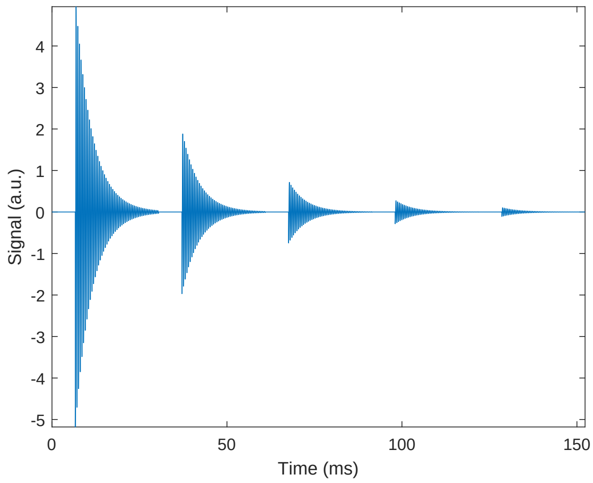

Figure 8: IR signal. $T_I = 0.5T_1 \ln 2$

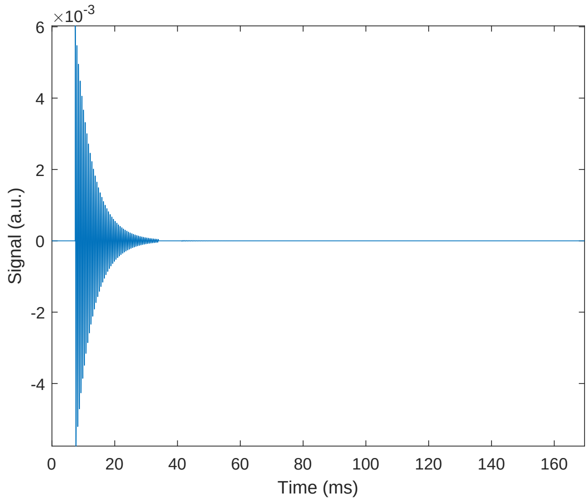

Figure 9: IR signal. $T_I = T_1 \ln 2$

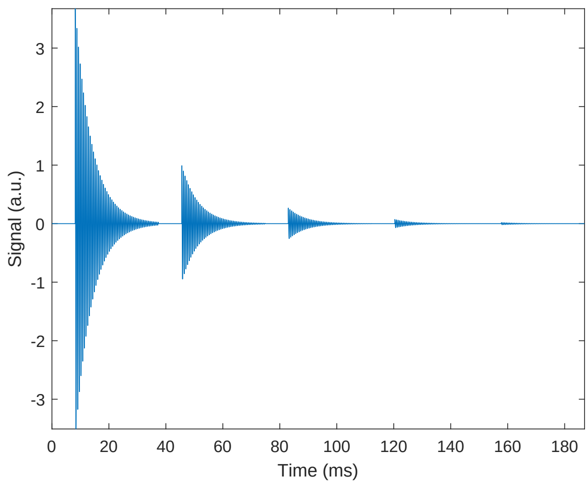

Figure 10: IR signal. $T_I = 1.5T_1 \ln 2$

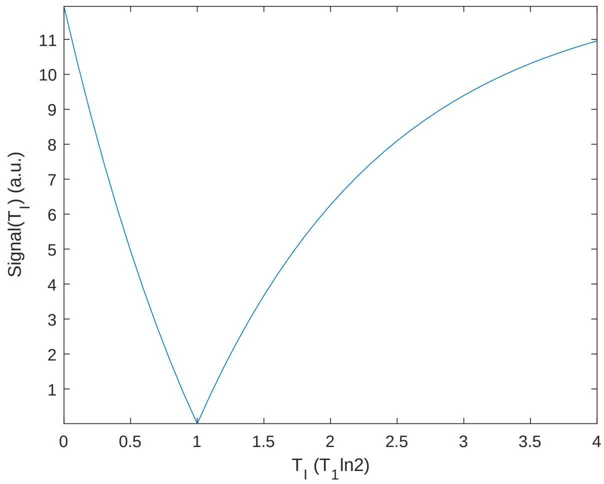

Figure 11: $M_\perp(T_I)$ vs T_I for T_1 estimation

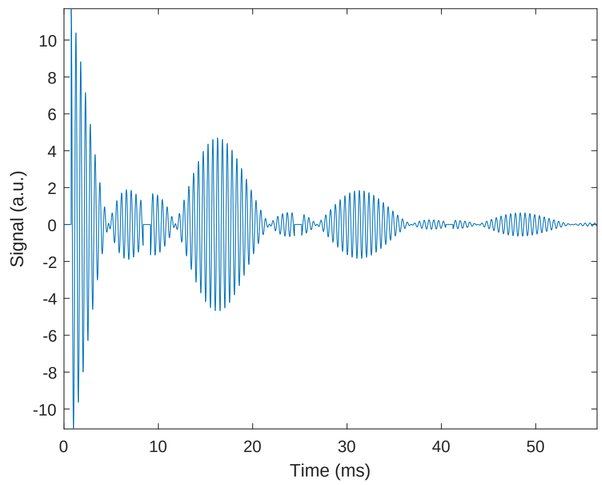

Figure 12: Spin echo, assuming unifor distribution.

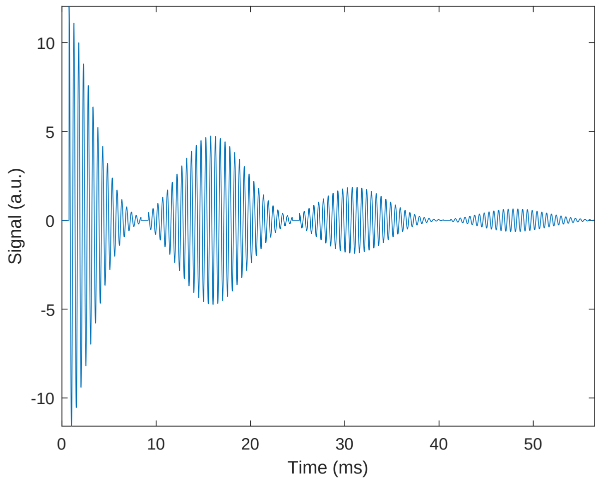

Figure 13: Spin echo, assuming unifor distribution.

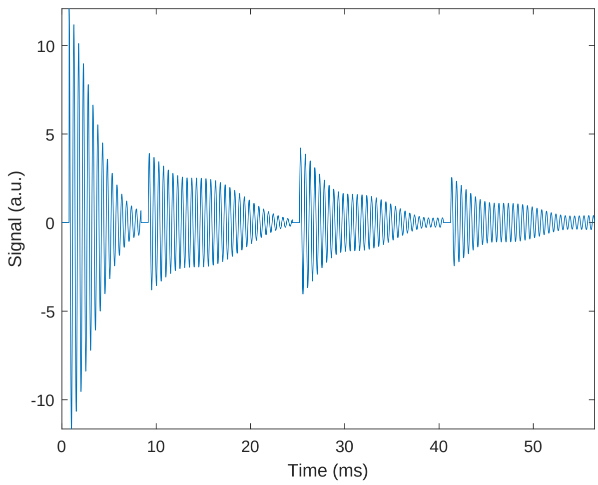

Figure 14: Hahn spin echo, assuming unifor distribution.