# Quantum Hall Effect

Lianghong Mo

20220606

# Content

# 1.1 Classical Hall Effect and Drude Model

Newton's Law $$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E} - e\mathbf{v}\times\mathbf{B} - \frac{m\mathbf{v}}{\tau}$$

where $\tau$ is the scattering time.

$$\mathbf{J} = -ne\mathbf{v} \qquad \mathbf{J} = \sigma\mathbf{E}$$

thus we have

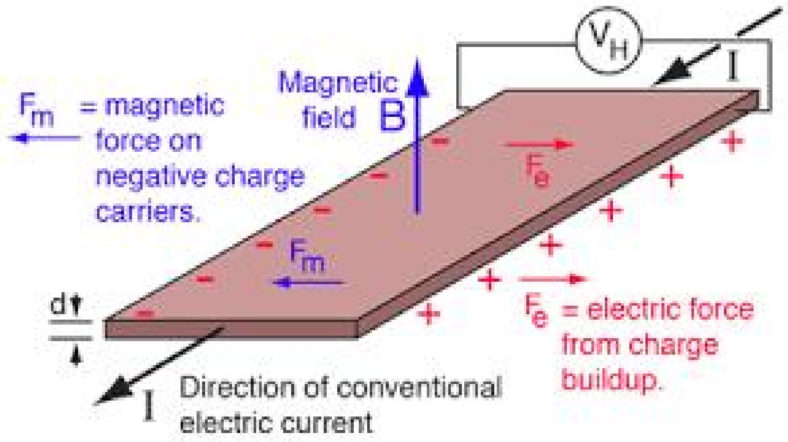
$$\sigma = \frac{\frac{ne^2\tau}{m}}{1+\omega_B^2\tau^2}\begin{pmatrix}1 & -\omega_B\tau\\ \omega_B\tau & 1\end{pmatrix}$$

$$\rho = \frac{m}{ne^2\tau}\begin{pmatrix}1 & \omega_B\tau\\ -\omega_B\tau & 1\end{pmatrix}$$

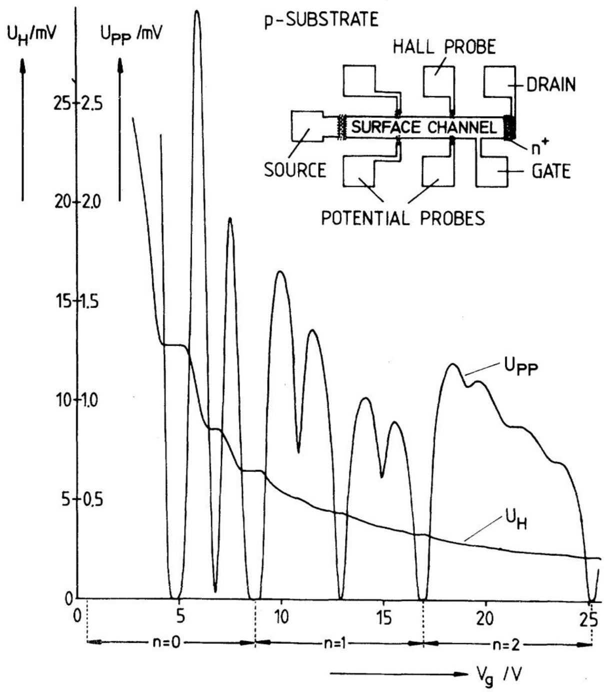
where $\omega_B = \dfrac{eB}{m}$

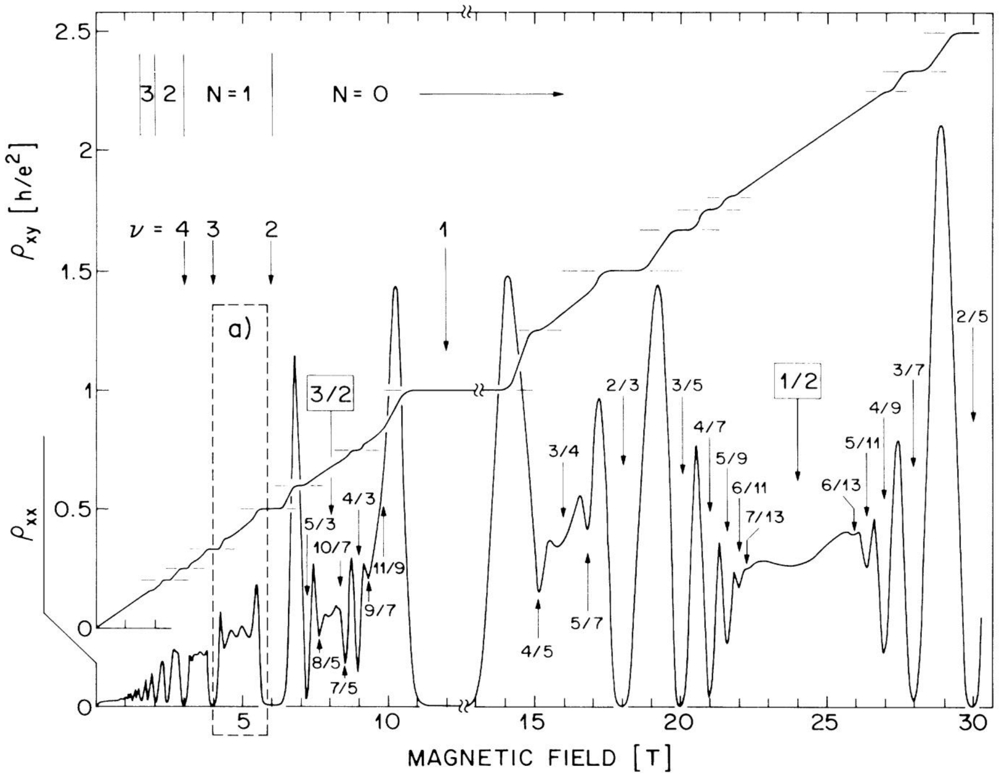
If $\rho_{xy}$ = 0 or equivalently, B=0, $\sigma_{xx} = \dfrac{1}{\rho_{xx}}$

If $B \neq 0, \tau \to \infty$, $\sigma_{xx} = 0, \rho_{xx} = 0$



# 1.2 IQHE & FQHE




$$\rho_{xy} = \frac{2\pi\hbar}{e^2}\frac{1}{\nu}$$

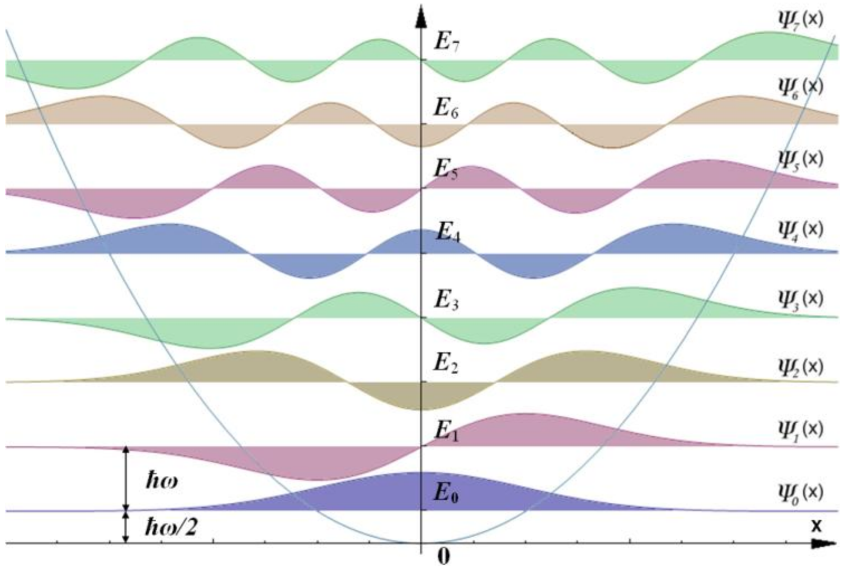
$$\rho_{xy} = \frac{B}{ne}$$

$$\nu = \frac{n}{\frac{B}{\Phi_0}}$$


[1]Klitzing K, Dorda G, Pepper M. New method for high-accuracy determination of the fine-structure constant based on quantized Hall resistance[J]. Physical review letters, 1980, 45(6): 494.
[2]Tsui D C, Stormer H L, Gossard A C. Two-dimensional magnetotransport in the extreme quantum limit[J]. Physical Review Letters, 1982, 48(22): 1559.
[3]Willett R, Eisenstein J P, Störmer H L, et al. Observation of an even-denominator quantum number in the fractional quantum Hall effect[J]. Physical review letters, 1987, 59(15): 1776.

# 2.Landau Levels

Consider a particle moving in the 2dEG, the Lagrangian is $L = \frac{1}{2}m\dot{\mathbf{x}}^2 - e\dot{\mathbf{x}}\cdot\mathbf{A}$




The canonical momentum is $\mathbf{p} = \frac{\partial L}{\partial \dot{\mathbf{x}}} = m\dot{\mathbf{x}} - e\mathbf{A}$

The Hamiltonian is $H = \dot{\mathbf{x}}\cdot\mathbf{p} - L = \frac{1}{2m}(\mathbf{p}+e\mathbf{A})^2$

Define the raising and lowering operator:

$$a = \frac{1}{\sqrt{2e\hbar B}}(\pi_x - i\pi_y) \quad \text{and} \quad a^\dagger = \frac{1}{\sqrt{2e\hbar B}}(\pi_x + i\pi_y)$$

The Hamiltonian takes the same form as the harmonic oscillator

$$H = \frac{1}{2m}\boldsymbol{\pi}\cdot\boldsymbol{\pi} = \hbar\omega_B\left(a^\dagger a + \frac{1}{2}\right)$$

If we choose Landau gauge $\mathbf{A} = xB\hat{\mathbf{y}}$

the wavefunction can be solved as $\psi_{n,k}(x,y) \sim e^{iky}H_n(x+kl_B^2)e^{-(x+kl_B^2)^2/2l_B^2}$ where $l_B = \sqrt{\frac{\hbar}{eB}}$

Tong D. Lectures on the quantum Hall effect[J]. arXiv preprint arXiv:1606.06687, 2016.

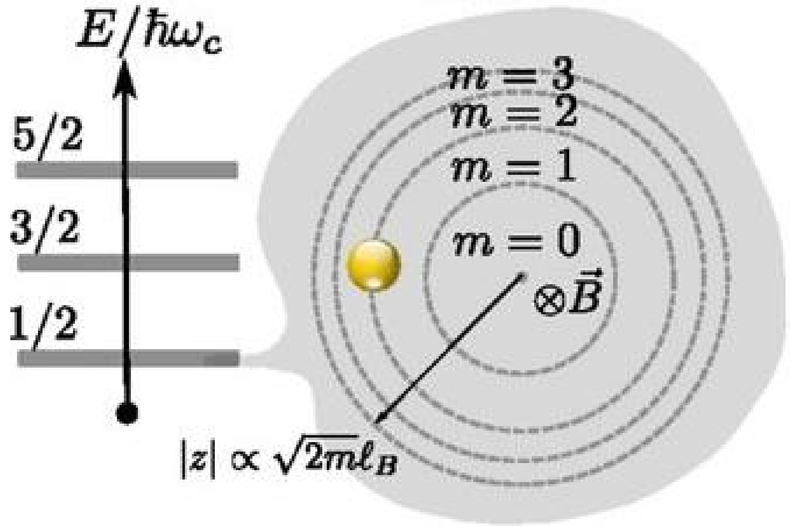
# 2.1 Symmetry Gauge $\mathbf{A} = -\frac{1}{2}\mathbf{r} \times \mathbf{B} = -\frac{yB}{2}\hat{\mathbf{x}} + \frac{xB}{2}\hat{\mathbf{y}}$

$$H = \frac{1}{2m}\boldsymbol{\pi} \cdot \boldsymbol{\pi} = \hbar\omega_B\left(a^\dagger a + \frac{1}{2}\right)$$

Another type of momentum $\tilde{\boldsymbol{\pi}} = \mathbf{p} - e\mathbf{A}$

which have a similar algebraic structure with canonical momentum,thus we an define

$$b = \frac{1}{\sqrt{2e\hbar B}}(\tilde{\pi}_x + i\tilde{\pi}_y) \quad \text{and} \quad b^\dagger = \frac{1}{\sqrt{2e\hbar B}}(\tilde{\pi}_x - i\tilde{\pi}_y)$$




$$|n, m\rangle = \frac{a^{\dagger\, n} b^{\dagger\, m}}{\sqrt{n!m!}}|0,0\rangle$$

Lowest Landau level

$$\psi_{LLL,m} \sim \left(\frac{z}{l_B}\right)^m e^{-|z|^2/4l_B^2}$$

The wavefunction with angular momentum m is peaked on a ring of radius $r = \sqrt{2m}l_B$,thus the degeneracy is $N = \frac{R^2}{2l_B^2} = \frac{BA}{\Phi_0}$



Tong D. Lectures on the quantum Hall effect[J]. arXiv preprint arXiv:1606.06687, 2016.

# 3.1 Theory of IQHE--Landauer Explanation

The total current is given by
$$\mathbf{I} = -\frac{e}{m}\sum_{\text{filled states}} \langle\psi| - i\hbar\nabla + e\mathbf{A}|\psi\rangle$$

Introduce an Electric field E in x-direction and the Hamiltonian is
$$H = \frac{1}{2m}\left(p_x^2 + (p_y + eBx)^2\right) + eEx$$

With the $\nu$ Landau levels filled, the current in the x and y-direction is

$$I_x = -\frac{e}{m}\sum_{n=1}^{\nu}\sum_{k}\langle\psi_{n,k}| - i\hbar\frac{\partial}{\partial x}|\psi_{n,k}\rangle = 0$$

$$I_y = -\frac{e}{m}\sum_{n=1}^{\nu}\sum_{k}\langle\psi_{n,k}| - i\hbar\frac{\partial}{\partial y} + exB|\psi_{n,k}\rangle = -\frac{e}{m}\sum_{n=1}^{\nu}\sum_{k}\langle\psi_{n,k}|\hbar k + eBx|\psi_{n,k}\rangle$$

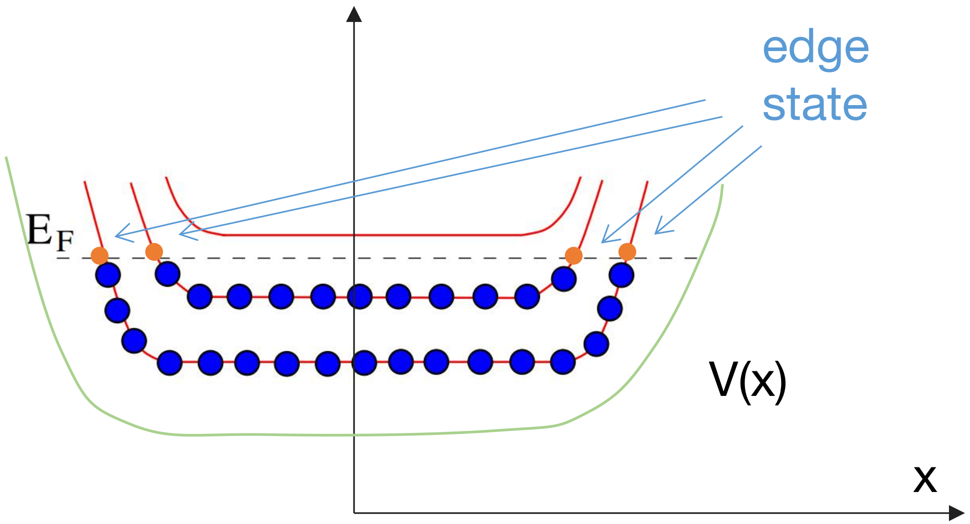
$$\boldsymbol{J} = \begin{pmatrix} 0 \\ e\nu E/\Phi_0 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{e\nu}{\Phi_0} \\ \frac{e\nu}{\Phi_0} & 0 \end{pmatrix}\begin{pmatrix} E \\ 0 \end{pmatrix}$$

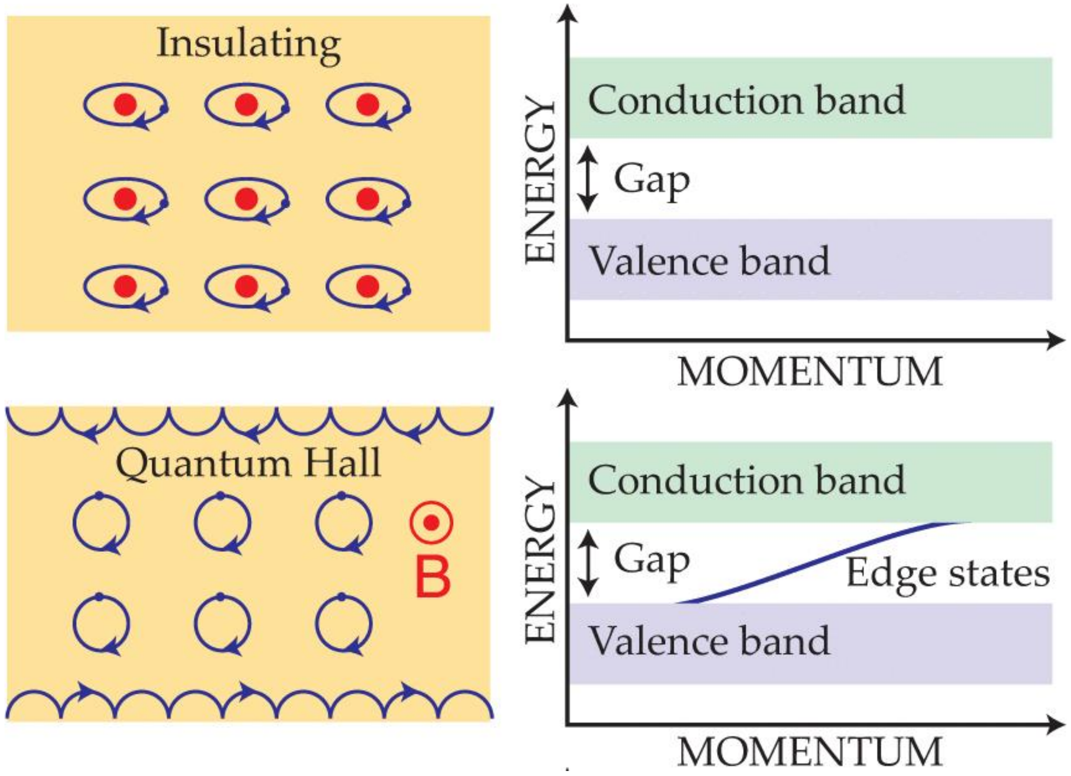
$$\boxed{\sigma_{xx} = 0 \quad \text{and} \quad \sigma_{xy} = \frac{e\nu}{\Phi_0} \quad \Rightarrow \quad \rho_{xx} = 0 \quad \text{and} \quad \rho_{xy} = -\frac{\Phi_0}{e\nu} = -\frac{2\pi\hbar}{e^2\nu}}$$



Tong D. Lectures on the quantum Hall effect[J]. arXiv preprint arXiv:1606.06687, 2016.

# 3.2Edge State




Interior is insulator but surface contains conducting states--Topological insulator.

In the absence of a potential:

$$[X,H]=[Y,H]=0$$

$$i\hbar\dot{X}=[X,H+V]=[X,V]=[X,Y]\frac{\partial V}{\partial Y}=il_B^2\frac{\partial V}{\partial Y}$$

The centre of mass drifts along equipotentials

$$i\hbar\dot{Y}=[Y,H+V]=[Y,V]=[Y,X]\frac{\partial V}{\partial X}=-il_B^2\frac{\partial V}{\partial X}$$

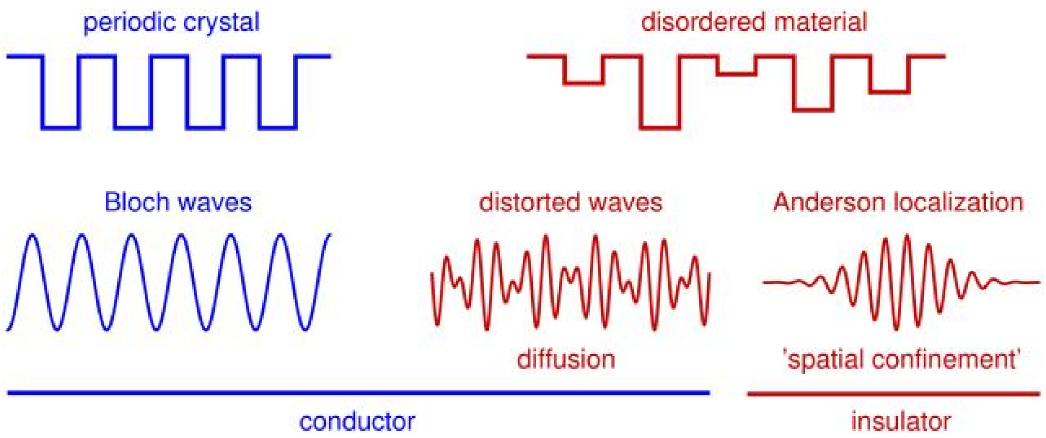
[1]Tong D. Lectures on the quantum Hall effect[J]. arXiv preprint arXiv:1606.06687, 2016.
[2]Day C. Quantum spin Hall effect shows up in a quantum well insulator, just as predicted[J]. Physics Today, 2008, 61(1): 19.

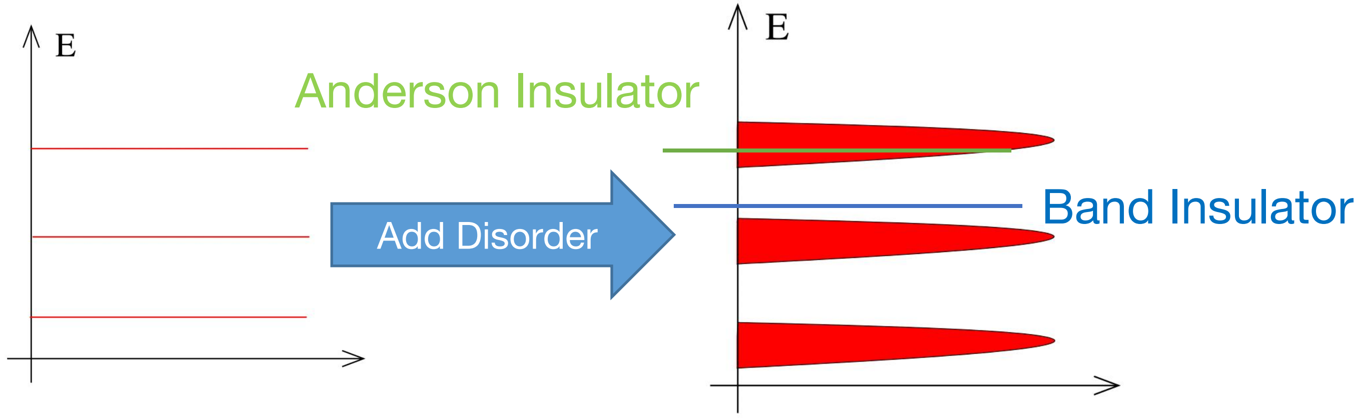
# 3.3 Quantum Phase Transition--Anderson Localization

$$\sigma = \frac{e^2 n\tau}{m} = \frac{e^2 n}{\frac{1}{2}mv_F^2}\frac{1}{2}v_F^2\tau = e^2\frac{n}{E_F}\frac{l^2}{2\tau} = e^2\rho D$$

state density

Diffusion Coefficient




| d | p(d) |
|---|---|
| 1 | 1 |
| 2 | 1 |
| 3 | 0.340537 |
| 4 | 0.193206 |
| 5 | 0.135178 |
| 6 | 0.104715 |
| 7 | 0.0858449 |
| 8 | 0.0729126 |


[1]OMBRELLARO E G. RANDOM WALKS AND THE PROBABILITY OF RETURNING HOME[J]. 2018.
[2]Tong D. Lectures on the quantum Hall effect[J]. arXiv preprint arXiv:1606.06687, 2016.

# 4.Theory of FQHE

$$H=\sum_j \frac{1}{2m_{\mathrm{b}}}\left[\frac{\hbar}{i}\nabla_j+\frac{e}{c}\boldsymbol{A}(\boldsymbol{r}_j)\right]^2+\frac{e^2}{\epsilon}\sum_{j<k}\frac{1}{|\boldsymbol{r}_j-\boldsymbol{r}_k|}+\sum_j U(\boldsymbol{r}_j)+g\mu\boldsymbol{B}\cdot\boldsymbol{S}$$

To get a feel for the relative importance of the various terms,we consider GaAs-AlGaAs heterostructure, on which most quantum hall experiment have been performed.

Cyclotron energy: $\hbar\omega_{\mathrm{c}} \approx 20B[\mathrm{T}]\,\mathrm{K}$

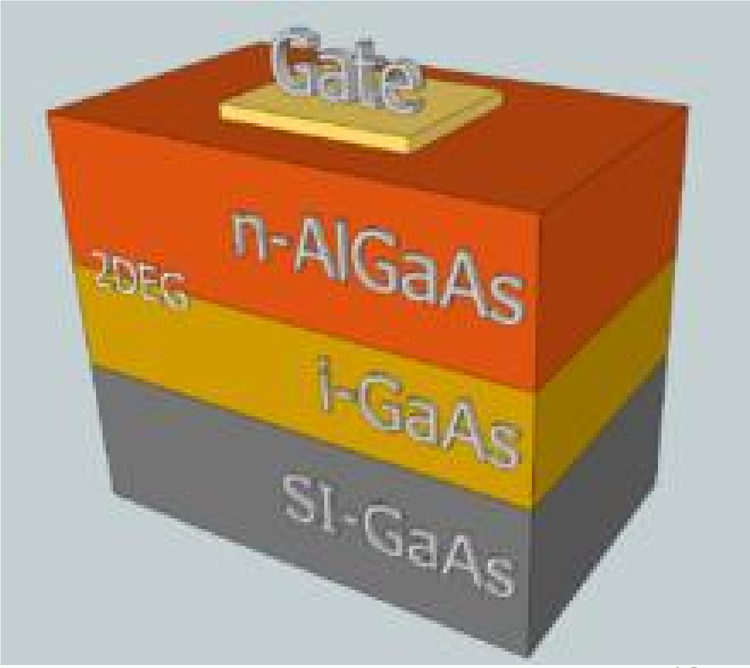
typical Coulomb energy: $V_{\mathrm{C}} \equiv \frac{e^2}{\epsilon\ell} \approx 50\sqrt{B[\mathrm{T}]}\,\mathrm{K}$

Zeeman Spliting: $E_{\mathrm{Z}} = 2g\mu_{\mathrm{B}}\boldsymbol{B}\cdot\boldsymbol{S} = \frac{g}{2}\frac{m_{\mathrm{b}}}{m_{\mathrm{e}}}\hbar\omega_{\mathrm{c}} \approx 0.3B[\mathrm{T}]\,\mathrm{K}$

magnetic length : $\ell = \left(\frac{\hbar c}{eB}\right)^{1/2} \approx \frac{25}{\sqrt{B[\mathrm{T}]}}\,\mathrm{nm}$

$\frac{e^2/\epsilon\ell}{\hbar\omega_{\mathrm{c}}} \to 0,$ Gap

$\frac{e^2/\epsilon\ell}{E_{\mathrm{Z}}} \to 0.$ Spin Polarized






Jain J K. Composite-fermion approach for the fractional quantum Hall effect[J]. Physical review letters, 1989, 63(2): 199.

# 4.Theory of FQHE--Laughlin Wave Function

How to guess ?

The simplest model

$$H = \mathcal{P}_{\rm LLL} \frac{e^2}{\epsilon} \sum_{j<k} \frac{1}{|\boldsymbol{r}_j - \boldsymbol{r}_k|} \mathcal{P}_{\rm LLL},$$

the wavefunction takes the form

$$\psi(z_1, \ldots, z_n) = f(z_1, \ldots, z_N) e^{-\sum_{i=1}^{N} |z_i|^2 / 4 l_B^2}$$

Consider the fullfiled landau level, the electrons are fermions, these states must be distinct. To build the many-particle wavefunction, we need to anti-symmetrise over all particles with slater determinant:

$$\psi(x_i) = \begin{vmatrix} \psi_1(x_1) & \psi_1(x_2) & \ldots & \psi_1(x_N) \\ \psi_2(x_1) & \psi_2(x_2) & \ldots & \psi_2(x_N) \\ \vdots & & & \vdots \\ \psi_N(x_1) & \psi_N(x_2) & \ldots & \psi_N(x_N) \end{vmatrix} \quad \Rightarrow \quad f(z_i) = \begin{vmatrix} z_1^0 & z_2^0 & \ldots & z_N^0 \\ z_1 & z_2 & \ldots & z_3 \\ \vdots & & & \vdots \\ z_1^{N-1} & z_2^{N-1} & \ldots & z_N^{N-1} \end{vmatrix} = \prod_{i<j} (z_i - z_j)$$

$$\psi_m(z) \sim z^{m-1} e^{-|z|^2 / 4 l_B^2} \qquad m = 1, \ldots, N$$

$$\Psi(z1, \ldots z_N) = \prod_{i<j} (z_i - z_j) \exp[-\frac{1}{4 l_B^2} \sum_i |z_i^2|] \quad \Rightarrow \quad \Psi(z1, \ldots z_N)_{1/m} = \prod_{i<j} (z_i - z_j)^m \exp[-\frac{1}{4 l_B^2} \sum_i |z_i^2|]$$

From the perspecive of the node to the wavefunction, we can see the incompressibility.

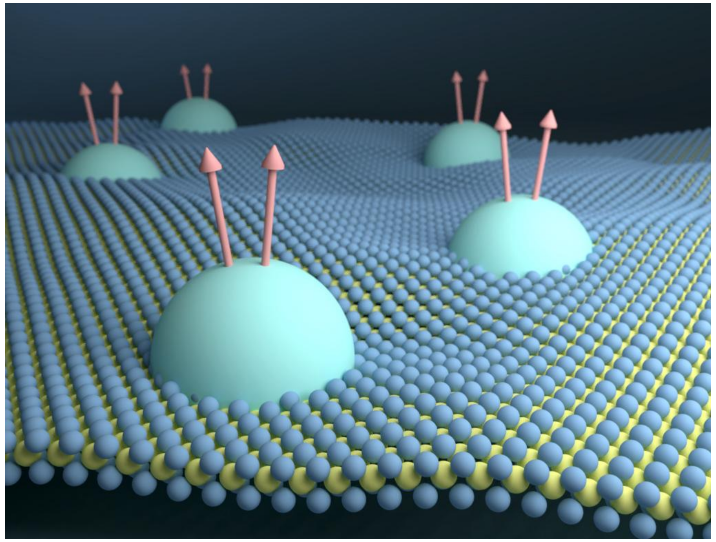
# 4.1 Theory of FQHE--Composite Fermion

**Definition** *A composite fermion is the bound state of an electron and an even number of quantized vortices.*

$$\Psi_\nu = \mathcal{P}_{\mathrm{LLL}} \prod_{j<k} (z_j - z_k)^{2p} \Phi_{\nu^*}$$

Filling fraction: $$\nu = \frac{\nu^*}{2p\nu^* \pm 1}$$ where $\nu^* = n, n \in Z$

$$B^* = B - 2p\rho\phi_0$$

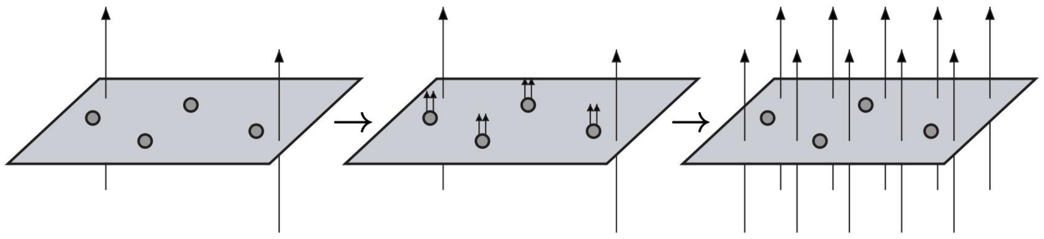


Each electron absorbes 2p flux quanta from the  external magnetic field to transform into a composite fermion. Specially, for 1/3 fqhe state, we have

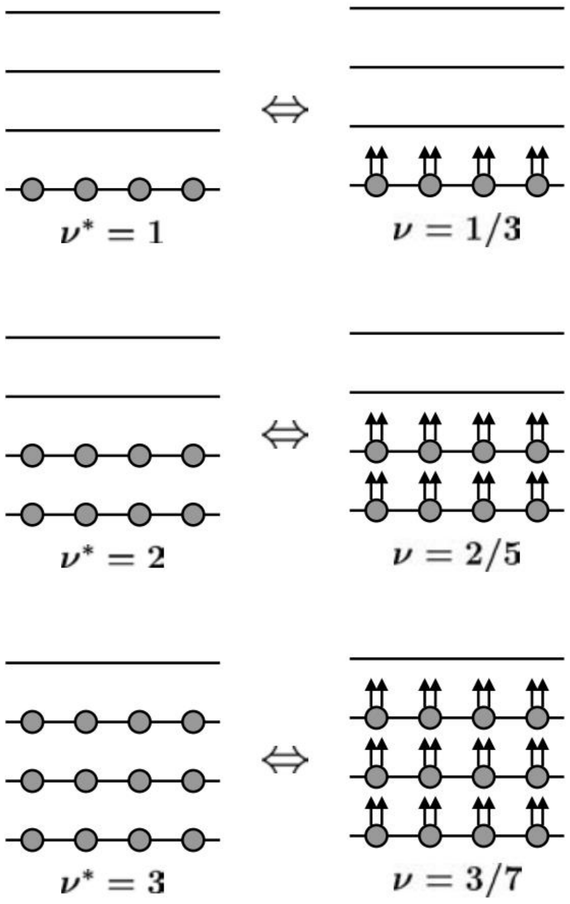
$$p = 1 \qquad \nu^* = 1$$

Jain J K. Composite-fermion approach for the fractional quantum Hall effect[J]. Physical review letters, 1989, 63(2): 199.

# 4.1Theory of FQHE--Composite Fermion & Boson



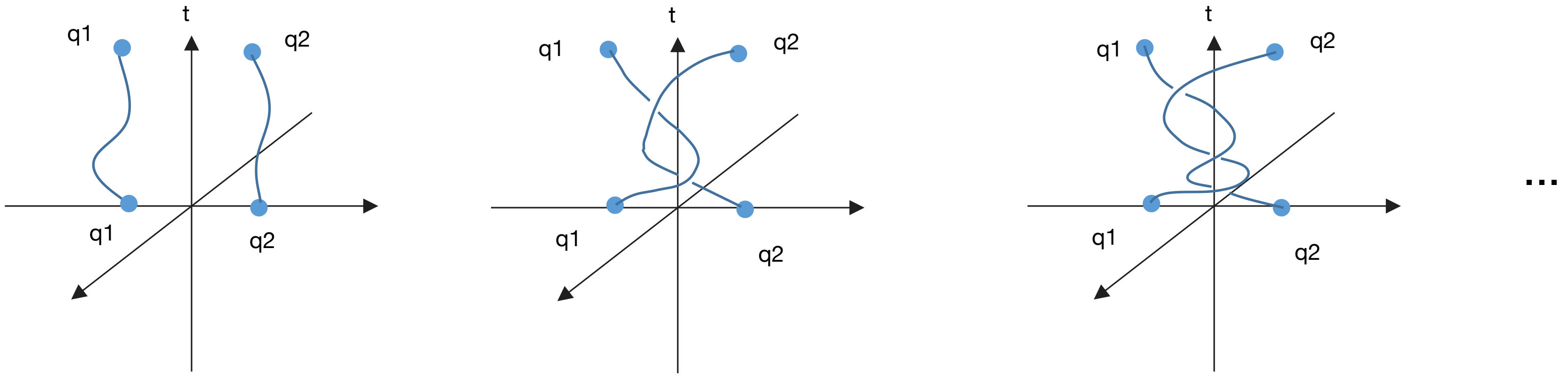

The goldenpath from the IQHE to the FQHE. We beginwith anIQHE state (a); attach to each electron two magnetic flux quanta to convert it into a composite fermion (b); and spread out the attached flux to obtainelectrons ina higher magnetic field, which is a FQHE state (c).




IQHE GS FQHE GS

[1]Jain J K. Composite-fermion approach for the fractional quantum Hall effect[J]. Physical review letters, 1989, 63(2): 199.
[2]Balram A C, Jain J K, Barkeshli M. Z n superconductivity of composite bosons and the 7/3 fractional quantum Hall effect[J]. Physical Review Research, 2020, 2(1): 013349.

# 4.2 Fractional Statistics and Anyons



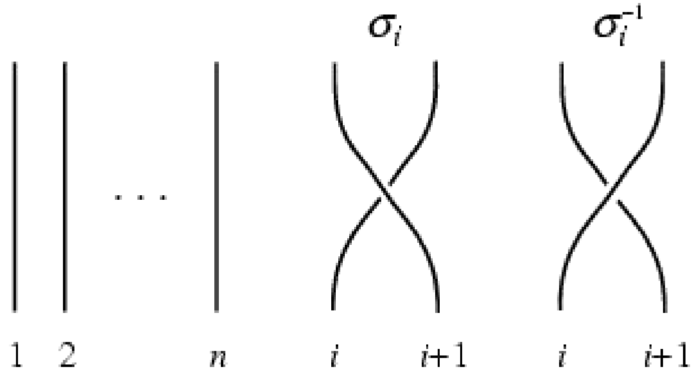

Two loops are considered equivalent if one can deform to the other by a continuous deformation. All homotopic loops are grouped into one class and the set of all such classes is called the fundamental group and denoted by $\pi_1$. Thus, we can organize the sum over the all loops into sum over homotopic classes $\alpha$ and assign different weights to different classes in the propagator:

$$K(q,t';q,t) = \sum_{\alpha\in\pi_1(M_N^d)} \chi(\alpha) \int_{q(t)=q;q(t')=q'} D_{q_\alpha} e^{\frac{i}{\hbar}\int_t^{t'} d\tau \mathscr{L}[q(\tau),\dot{q}(\tau)]}$$

[1]Wu Y S. Multiparticle quantum mechanics obeying fractional statistics[J]. Physical Review Letters, 1984, 53(2): 111.
[2]Lerda A. Anyons: Quantum mechanics of particles with fractional statistics[M]. Springer Science & Business Media, 2008.

# 4.2 Fractional Statistics and Anyons

Brief and Pictorial Review of Braid Group




$$\sigma_I \sigma_J = \sigma_J \sigma_I \quad \textit{for} \quad |I-J| \geq 2$$

$$\sigma_I \sigma_{I+1} \sigma_I = \sigma_{I+1} \sigma_I \sigma_{I+1}$$

$$\sigma_I^2 \neq 1$$

$$\chi(\sigma_K) = \exp[-i\nu \Delta\psi_{K,K+1}] = \exp[-i\nu \sum_{I<J} \Delta\psi_{IJ}^{(K)}] \quad \text{where} \quad \Delta_{IJ}^{(K)} \equiv \psi_{IJ}(t')^{(K)} - \psi_{IJ}(t)^{(K)} = \pi\delta_{I,K}\delta_{J,K+1}$$

where $\nu$ here refers to the statistics relating to specific system($\nu$ = 0 for boson and $\nu$ = 1 for Fermion(mod 2), or $\nu$ = $\mathrm{even}$ for boson and $\nu$ = $\mathrm{odd}$ for Fermion), we can generalised the formula as

$$\chi(\alpha) = \exp[-i\nu \sum_{I<J} \int_t^{t'} d\tau \frac{d}{d\tau} \psi_{IJ}^{(\alpha)}(\tau)] = \exp[-i\nu \sum_{I<J} \int_t^{t'} d\tau \frac{d}{d\tau} \tan^{-1}(\frac{y_I - y_J}{x_I - x_J})]$$


[1]Wu Y S. Multiparticle quantum mechanics obeying fractional statistics[J]. Physical Review Letters, 1984, 53(2): 111.
[2]Lerda A. Anyons: Quantum mechanics of particles with fractional statistics[M]. Springer Science & Business Media, 2008.

# 4.2 Fractional Statistics and Anyons

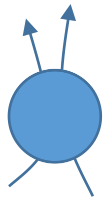
Consider a pure CS term with the least coupling $$\mathcal{L} = \frac{k}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda - A_\mu J^\mu$$ where $$J^0 = \sum_{\alpha=0}^{N}\delta(\mathbf{x}-\mathbf{x}_\alpha)$$ $$\mathbf{J} = \sum_{\alpha=0}^{N}\dot{\mathbf{x}}_\alpha\delta(\mathbf{x}-\mathbf{x}_\alpha)$$

Equation of motion in component

$$\rho = J^0 = \frac{k}{2\pi}B$$

$$J^i = \frac{k}{2\pi}\epsilon^{ij}E_j$$

$$\Phi = \oint_{C_I} d^2x\frac{2\pi}{k}e\sum_{a=1}^{N}\delta(\mathbf{x}-\mathbf{x_a(t)}) = \frac{2\pi e}{k}$$



Choose the gauge as follows

$$A_0 = 0$$
$$\nabla\cdot A = 0$$

and we can solve out the magnetic vetor potential of the system:

$$A_I^i(\vec{x}_1,\vec{x}_I,\ldots,\vec{x}_N) = \frac{e}{k}\sum_{a=1}^{N}\epsilon_{ij}\frac{x_I^j - x_J^j(t)}{|\vec{x}_I-\vec{x}_J|^2}$$

Total A-B phase factor $$\exp(iS_{int}) = \exp i[\int_t^{t'} d\tau\mathcal{L}_{int}] = \exp[-i\frac{e^2}{k}\sum_{I<J}\frac{d}{d\tau}\tan^{-1}(\frac{y_I-y_J}{x_I-x_J})]$$

which is in accordance with the phase factor of the propagator induced by different classes of loops from the perspective of braid group.

The statistics of the Anyons is $\frac{e^2}{k}$

[1]Wu Y S. Multiparticle quantum mechanics obeying fractional statistics[J]. Physical Review Letters, 1984, 53(2): 111.
[2]Lerda A. Anyons: Quantum mechanics of particles with fractional statistics[M]. Springer Science & Business Media, 2008.

# 4.3 K matrix Chern-Simons--Effective Theory of FQHE

Consider a charged boson or fermion on system in a magnetic field,

$$\mathcal{L} = -e\mathbf{A}\cdot\mathbf{J} + kinetic\ energy$$

The particle number current J has the following response to a change in electromagnetic fields

$$e\delta J_\nu = \frac{\nu e^2}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu \delta A_\lambda$$

Introduce a U(1) gauge field $a_\mu$ to describe the conserved particle number current and the current defined in this way satisfies the conservation law:

$$J_\mu = \frac{1}{2\pi}\partial_\nu a_\lambda \epsilon^{\mu\nu\lambda}$$

The effective lgg that produces the equation of motion above takes the following form:

$$\mathcal{L} = -\frac{m}{4\pi}a_\mu\partial_\nu a_\lambda \epsilon^{\mu\nu\lambda} + \frac{e}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda$$

It only describes the linear response of the ground state to the external electromagnetic field. We will introduce the boson/fermion excitation in the next slide.

# 4.3 K matrix Chern-Simons--Effective Theory of FQHE

Insert the source term,then the lgg becomes:

$$\mathcal{L} = -\frac{m}{4\pi} a_\mu \partial_\nu a_\lambda \epsilon^{\mu\nu\lambda} + \frac{e}{2\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda + q a_0 \delta(\mathbf{x} - \mathbf{x_0})$$

$$\frac{\delta \mathcal{L}}{\delta a_0} = -\frac{m}{2\pi} \partial_\nu a_\lambda \epsilon^{\nu\lambda} - \frac{e}{2\pi} \epsilon^{\mu\nu} \partial_\mu A_\nu + q\delta(\mathbf{x} - \mathbf{x_0}) = 0$$

$$J_0 = \frac{1}{2\pi} \partial_\nu a_\lambda \epsilon^{\mu\nu\lambda} = -\frac{e}{2\pi m} B + \frac{q}{m} \delta(x - x_0)$$

The first term indicates that the filling fraction is and the second term corresponds to the increase of the particle density associated with the excitation.

Wen X G. Topological orders and edge excitations in fractional quantum Hall states[J]. Advances in Physics, 1995, 44(5): 405-473.

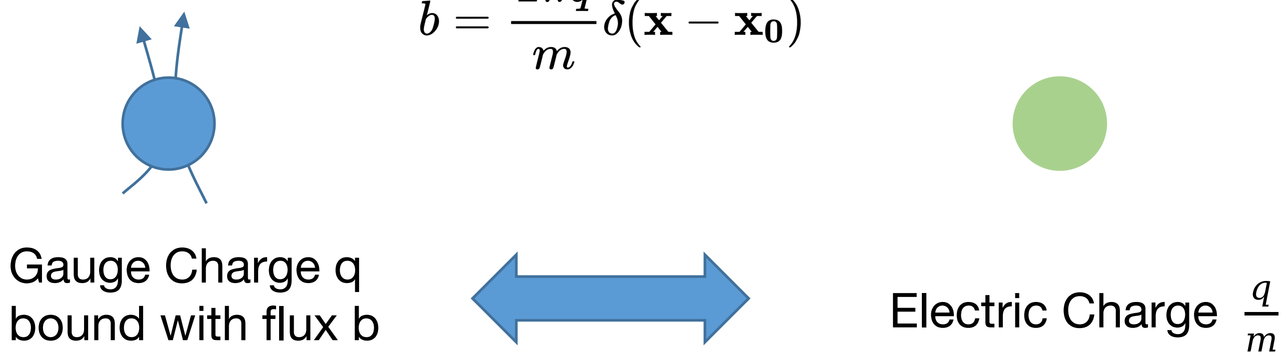
# 4.3 K matrix Chern-Simons--Effective Theory of FQHE

$$J_0 = \frac{1}{2\pi}\partial_\nu a_\lambda \epsilon^{\mu\nu\lambda} = -\frac{e}{2\pi m}B + \frac{q}{m}\delta(x - x_0)$$

The first term indicates that the filling fraction is $\nu = \frac{1}{m}$ ;The second term indicates that theincrease of the particle density associated with the excitation. If we set the background A=0, then we  obtain:

$$b = \frac{2\pi q}{m}\delta(\mathbf{x} - \mathbf{x_0})$$

Pictorically,




From the first picture perspective we can compute the statistics , we can obtain the quantization condition:

$$q \in \mathbb{Z}$$

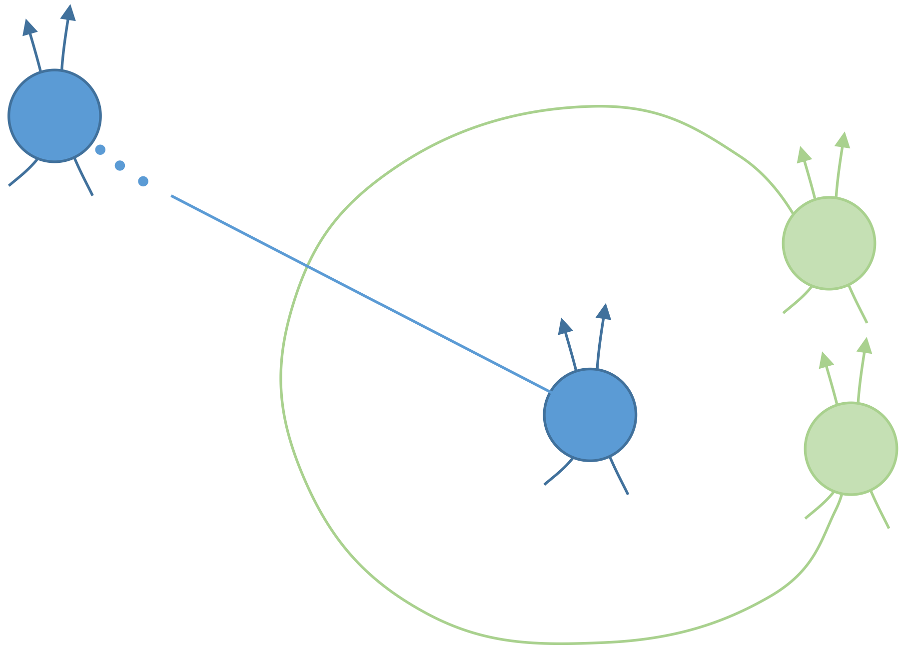
Wen X G. Topological orders and edge excitations in fractional quantum Hall states[J]. Advances in Physics, 1995, 44(5): 405-473.

# 4.3 K matrix Chern-Simons--Effective Theory of FQHE

The excitation created by the source term  is associated with $\frac{q}{m}$ unit of the $a_\mu$ flux. If we have two excitations carrying $a_\mu$  charge $q_1$ and $q_2$, moving one excitation around(different types) the other will induce a phase:

$$2\pi \times (number\ of\, a_\mu\ flux\ quanta) \times a_\mu charge \Rightarrow \frac{1}{m}q_1 q_2 2\pi$$

Interchange two excitation  both of which carry q charge $\frac{q^2}{m}\pi$ ,which is equivalent to  the Identical Anyons Statistics we obatined before.



Wen X G. Topological orders and edge excitations in fractional quantum Hall states[J]. Advances in Physics, 1995, 44(5): 405-473.

# 4.3 K matrix Chern-Simons--Effective Theory of FQHE

Generalize the effective field theory to other filling fractions:
Follow the same step as before, we introduce a new U(1) gauge field $\tilde{a}_\mu$ to construct the effective theory of the boson effective laughlin state. The total effctive theory (including the original condensate ) has the form:

$$\mathcal{L} =(-\frac{p_1}{4\pi}a_\mu\partial_\nu a_\lambda\epsilon^{\mu\nu\lambda} + \frac{e}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda) + (-\frac{p_2}{4\pi}\tilde{a}_\mu\partial_\nu\tilde{a}_\lambda\epsilon^{\mu\nu\lambda} + \frac{e}{2\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu\tilde{a}_\lambda)$$

$$\begin{aligned}\frac{\delta\mathcal{L}}{\delta\tilde{a}_0} &= -(\frac{p_2}{4\pi}\tilde{f}_{\nu\lambda} - \frac{1}{2\pi}\partial_\lambda a_\nu)\epsilon^{\mu\nu\lambda} \\ &= -\frac{1}{2\pi}(p_2\tilde{b}^\mu - b^\mu)\end{aligned}$$

$$\begin{aligned}\frac{\delta\mathcal{L}}{\delta a_0} &= -(\frac{p_1}{4\pi}f_{\nu\lambda} + \frac{e}{2\pi}\partial_\lambda A_\nu - \frac{1}{2\pi}\partial_\nu\tilde{a}_\lambda)\epsilon^{\mu\nu\lambda} \\ &= -\frac{1}{2\pi}(p_1 b^\mu + eB^\mu - \tilde{b}^\mu)\end{aligned}$$

both of which equal to zero :

$$p_1 b + eB - \tilde{b} = 0$$
$$p_2\tilde{b} - b = 0$$

Thus the filling fraction is:

$$\nu = \frac{b}{-eB} = \frac{1}{p_1 - \frac{1}{p_2}}$$

Wen X G. Topological orders and edge excitations in fractional quantum Hall states[J]. Advances in Physics, 1995, 44(5): 405-473.

# 4.3 K matrix Chern-Simons--Effective Theory of FQHE

The full effective theory with quasiparticle excitation is given by

$$\mathcal{L} = [-\frac{1}{4\pi}K^{IJ}a_{I\mu}\partial_\nu a_{J\lambda}\epsilon^{\mu\nu\lambda} + \frac{1}{2\pi}q^I A_\mu \partial_\nu a_{I\lambda}\epsilon^{\mu\nu\lambda}] + l_I a^I_\mu j^\mu$$

For fundamental quasiparticles the integer l=1 and for quasiholes l=-1.

Integrate out $a_\mu$, we obtain the effective action:

$$S_{eff} = \int d^3x \frac{q^I q^J K^{-1}_{IJ}}{4\pi} A_\mu \partial_\nu A_\lambda \epsilon^{\mu\nu\lambda} + l^I K^{-1}_{IJ} q^J A_\mu j^\mu$$

$$D = (det\ K)^g$$

$$\sigma_{xy} = q^I q^J K^{-1}_{IJ}$$

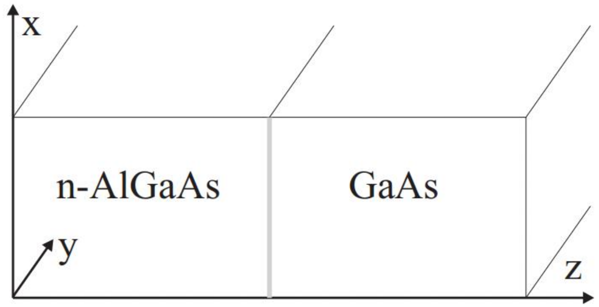
$$Q_q = l^I K^{-1}_{IJ} q^J$$

$$\theta = l_1^I K^{-1}_{IJ} l_2^J 2\pi$$

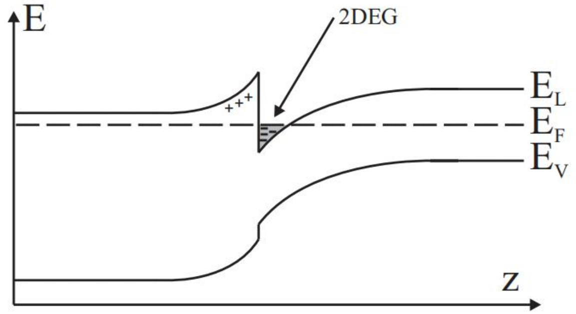
$$\nu = \mathbf{q}^{\mathbf{T}} K^{-1} \mathbf{q} = \frac{1}{m \pm \frac{1}{m_1 \pm \frac{1}{m_2 \pm \ldots}}}$$

Wen X G. Topological orders and edge excitations in fractional quantum Hall states[J]. Advances in Physics, 1995, 44(5): 405-473.

# 5.1Setup of QHE

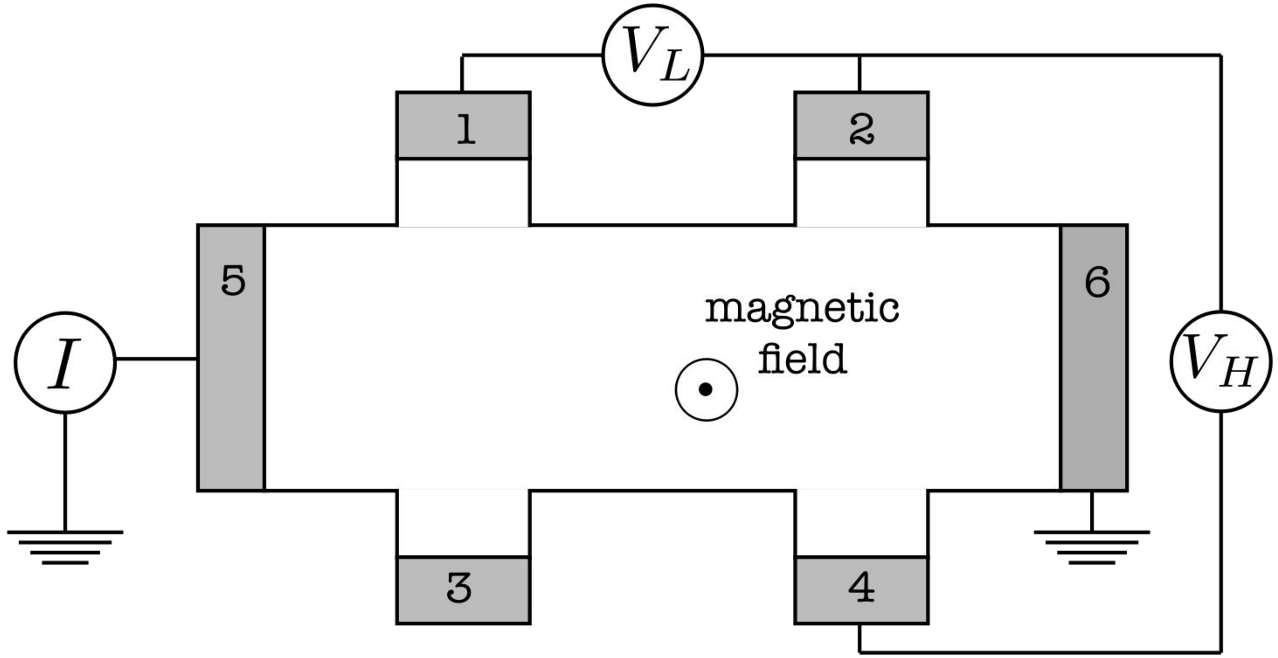



Heterostructure




Band Gap
GaAs:1.44eV
n-AlGaAs:2.12eV

high mobility




Current I along the x direction $j_x = \dfrac{I}{W}$

Measure the electric field using the Hall bar geometry from the voltage drops between the probes with voltages V1,2,3,4

x-component of the electric field $E_x \equiv \dfrac{V_1 + V_3 - V_2 - V_4}{2L}$

y -component of the electric field $E_y \equiv \dfrac{V_1 + V_2 - V_3 - V_4}{2W}$

Hall conductance $\sigma_{xy} = \dfrac{j_x E_y}{E_x^2 + E_y^2}$

[1]https://www.nano.physik.uni-muenchen.de/nanophotonics/_assets/pdf/f1/K1_QHE_instructions_english.pdf
[2]https://topocondmat.org/ocw/w3_pump_QHE/Laughlinargument_1.html?date=1539993600037

# 5.2 Observation of Fractional Charge

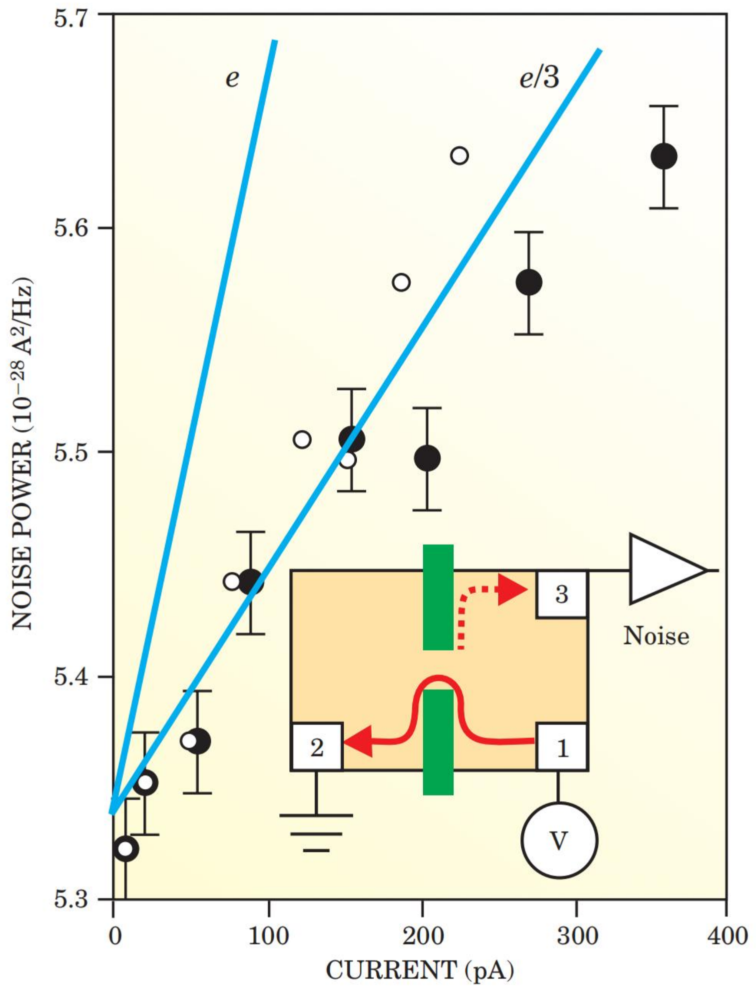
The shot noise technique focuses on the low-frequency fluctuations of the current and defined as

$$S = \int_{-\infty}^{\infty} dt \exp(i\omega t)\langle I(t)I(0) + I(0)I(t)\rangle.$$

The integral reduces to the mean square fluctuation of the total transmitted charge over a long time t and  the lowest quasiparticle charge dominates in the low-transmission limit

$$S = 2q_m I_T,$$

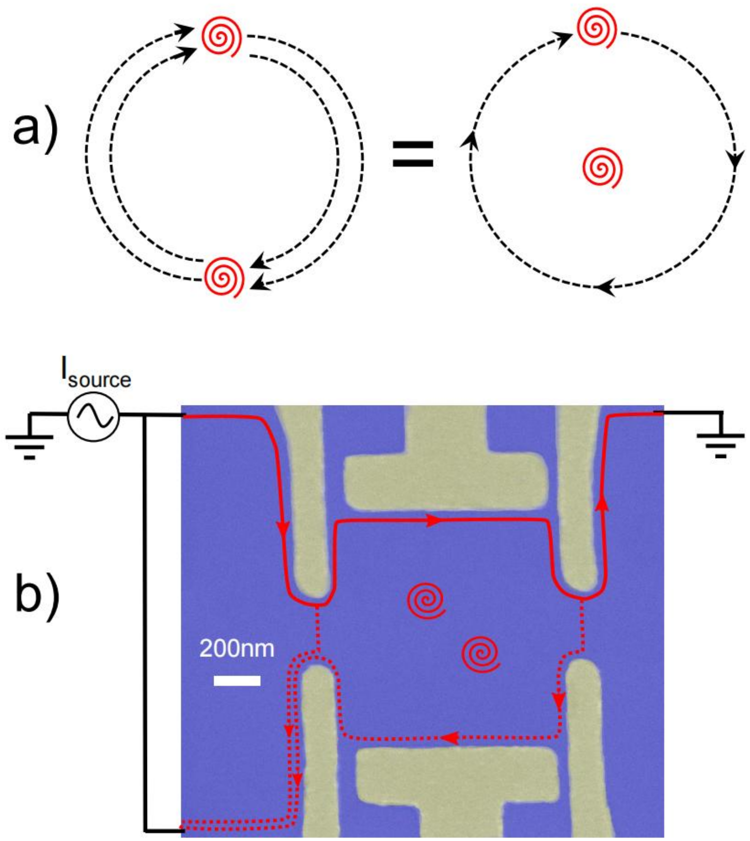
where $I_T$ is the average tunneling current.

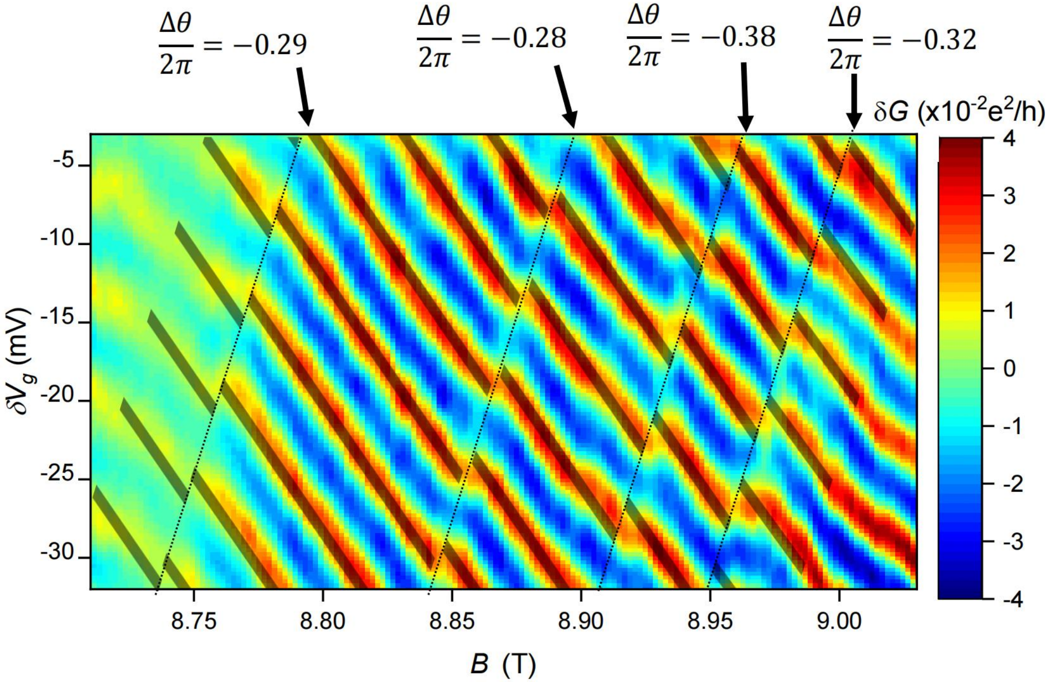



Current fluctuations can be measured at 2/3


[1]Feldman D E, Halperin B I. Fractional charge and fractional statistics in the quantum Hall effects[J]. Reports on Progress in Physics, 2021
[2]Beenakker C W J, Schonenberger C. Quantum shot noise[J]. arXiv preprint cond-mat/0605025, 2006..

# 5.3Experimental Probes of Fractional Statistics



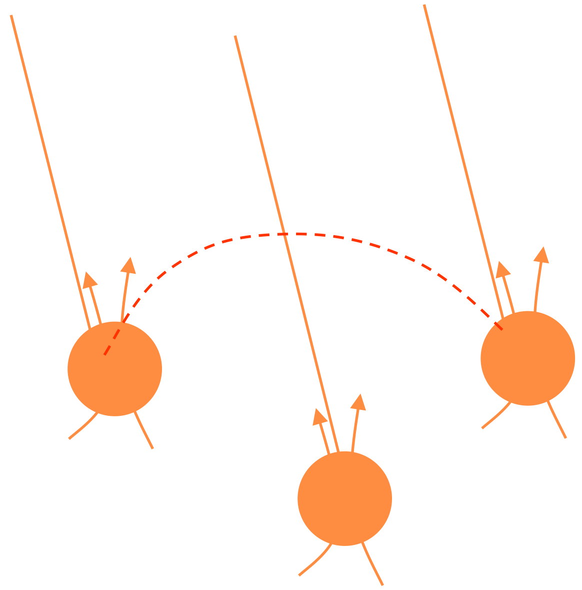




$$\theta = 2\pi \frac{e^*}{e} \frac{A_I B}{\Phi_0} + N_{qp}\theta_{anyon}$$

Nakamura J, Liang S, Gardner G C, et al. Direct observation of anyonic braiding statistics[J]. Nature Physics, 2020, 16(9): 931-936

# Thank You!

# Appendix



Identical Particle