A State-space SIR model with Quarantine: Can intervention end the COVID-19 epidemic sooner?

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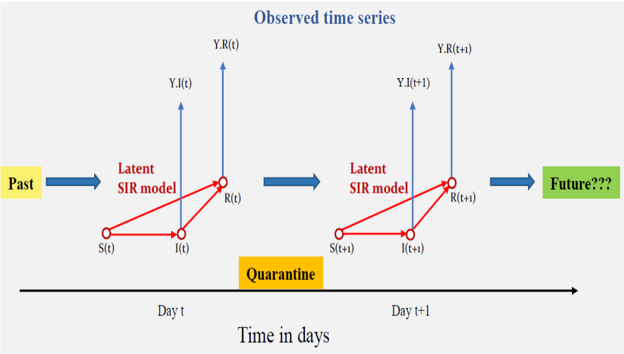
Overview of Analytic Tasks

- Model the processes of proportions of susceptible, infected and removed (recovered and death) compartments.
- Implement estimation and inference by Markov Chain Monte Carlo (MCMC) algorithm.
- Obtain posteriors of model parameters and predicted proportions with respective credible intervals.
- Create an R package eSIR.

State-Space SIR Model Formulation

Data: Observed the daily proportions of infected cases and removed (i.e. recovered and death) cases, respectively, Y_t^I and Y_t^R. We don't observe the proportion of susceptible cases.
Latent Processes: Assume the prevalence of susceptible, infected, and removed $\theta_t^S, \theta_t^I, \theta_t^R$ such that $\theta_t^S + \theta_t^I + \theta_t^R = 1$.



Different Models with Quarantine

- A Beta-Dirichlet state-space SIR model (BDSSM-SIR)
- BDSSM-SIR with a time-varying transmission rate
- BDSSM-SIR with a time-varying quarantine protocol
- Other extended BDSSM-SIR models

Overarching Goals

- Better understand the dynamics of the disease transmission of COVID-19
- Provide suggestions accounting for time-varying interventions via simulation-based exercises
- Develop a convenient toolbox to implement various DBSSM-SIR models

SIR Model: Latent Processes of Prevalence

$$\frac{d\theta_t^S}{dt} = -\beta\theta_t^S\theta_t^I, \quad \frac{d\theta_t^I}{dt} = \beta\theta_t^S\theta_t^I - \gamma\theta_t^I, \quad \frac{d\theta_t^R}{dt} = \gamma\theta_t^I,$$

- $\beta > 0$ is the disease transmission rate, $\gamma > 0$ is the remove rate
- $R_0 = \beta/\gamma$ is the reproduction number that indicates the expected number of cases generated by one infected case without intervention.

State-space SIR Model: Estimating SIR Using Data

Let Y_t^I and Y_t^R be the proportions of infection and removed state at time t. We assume Y_t^I and Y_t^R follows a Beta-Dirichlet state-space model(BDSSM), consisting of two observation processes:

$$\begin{aligned} Y_t^I | \boldsymbol{\theta}_t, \boldsymbol{\phi} &\sim \mathrm{Beta}(\lambda^I\theta_t^I, \lambda^I(1-\theta_t^I)), \\ Y_t^R | \boldsymbol{\theta}_t, \boldsymbol{\phi} &\sim \mathrm{Beta}(\lambda^R\theta_t^R, \lambda^R(1-\theta_t^R)), \end{aligned}$$

and the latent process

$$\boldsymbol{\theta}_t | \boldsymbol{\theta}_{t-1}, \boldsymbol{\phi} \sim \mathrm{Dirichlet}(\kappa f(\boldsymbol{\theta}_{t-1}, \beta, \gamma)),$$

where $\boldsymbol{\theta}_t = (\theta_t^S, \theta_t^I, \theta_t^R)^\top$ is the vector of the underlying prevalence of the susceptible, infectious and removed populations, and $\boldsymbol{\phi} = (\beta, \gamma, \boldsymbol{\theta}_0^\top, \lambda, \kappa)^\top$ with λ^I, λ^R and κ being parameters controlling respective variances for the observation and latent processes.

Runge-Kutta Solution of SIR Model

$f(\cdot)$ is be the solution to:

$$\frac{d\theta_t^S}{dt} = -\beta\theta_t^S\theta_t^I, \quad \frac{d\theta_t^I}{dt} = \beta\theta_t^S\theta_t^I - \gamma\theta_t^I, \frac{d\theta_t^R}{dt} = \gamma\theta_t^I$$

By the fourth order Runge-Kutta(RK4) approximation:

$$f(\boldsymbol{\theta}_{t-1}, \beta, \gamma) = \begin{pmatrix} \theta_{t-1}^S + 1/6[k_{t-1}^{S_1} + 2k_{t-1}^{S_2} + 2k_{t-1}^{S_3} + k_{t-1}^{S_4}] \\ \theta_{t-1}^I + 1/6[k_{t-1}^{I_1} + 2k_{t-1}^{I_2} + 2k_{t-1}^{I_3} + k_{t-1}^{I_4}] \\ \theta_{t-1}^R + 1/6[k_{t-1}^{R_1} + 2k_{t-1}^{R_2} + 2k_{t-1}^{R_3} + k_{t-1}^{R_4}] \end{pmatrix}.$$

Priors of the BDSSM-SIR model

In the Markov Chain Monte Carlo implementation, we specified the following priors:

$$\theta_0^I \sim \mathrm{Beta}(1, (Y_1^I)^{-1});$$
$$\theta_0^R \sim \mathrm{Beta}(1, (Y_1^R)^{-1});$$

???

$$\theta_0^S = 1 - \theta_0^R - \theta_0^I;$$
$$R_0 \sim \mathrm{LogN}(1.099, 0.096) \Rightarrow \mathrm{E}(R_0) = 3.15, \mathrm{SD}(R_0) = 1;$$
$$\gamma \sim \mathrm{LogN}(-2.955, 0.910) \Rightarrow \mathrm{E}(\gamma) = 0.0117, \mathrm{SD}(\gamma) = 0.1;$$
$$\beta = R_0\gamma;$$
$$\kappa \sim \mathrm{Gamma}(2, 0.0001);$$
$$\lambda^I \sim \mathrm{Gamma}(2, 0.0001);$$
$$\lambda^R \sim \mathrm{Gamma}(2, 0.0001).$$

Markov Chain Monte Carlo (MCMC) Algorithm

(1) Given M draws from $[\boldsymbol{\theta}_{1:t'}, \phi | Y_{1:t'}]$,

(2) For $m = 1, \ldots, M$:

(3) For $t = t' + 1, t' + 2, \ldots, T$:

(4) Draw $\boldsymbol{\theta}_t^{(m)}$ from $[\boldsymbol{\theta}_t | \boldsymbol{\theta}_{t-1}^{(m)}, \phi^{(m)}]$,

(5) Draw $Y_t^{(m)}$ from $[Y_t | \boldsymbol{\theta}_t^{(m)}, \phi^{(m)}]$,

(6) Then $Y_{(t'+1):T}^{(m)} = (Y_{t'+1}^{(m)}, Y_{t'+2}^{(m)}, \ldots, Y_T^{(m)})^{\mathrm{T}}$ constitutes a draw from $[Y_{(t'+1):T} | Y_{1:t'}]$.

Output: posterior $[\boldsymbol{\theta}_{1:T} | Y_{1:t'}]$ and predicted proportions $[Y_{(t'+1):T} | Y_{1:t'}]$
Note: ϕ includes γ, β and variance controllers.

MCMC algorithm

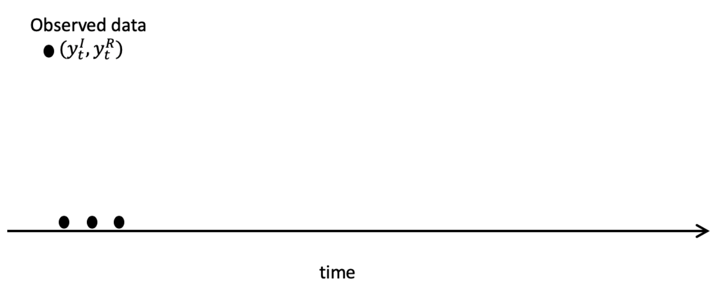


Figure: Data collection

MCMC algorithm

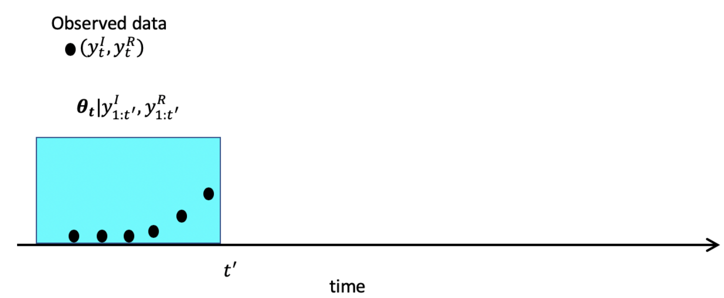


Figure: Posterior distributions of θ and parameters β,γ

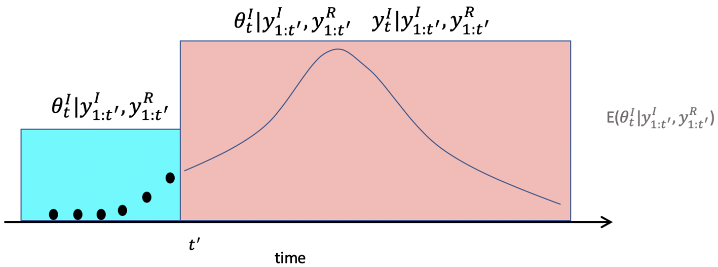


Figure: Add the forecast

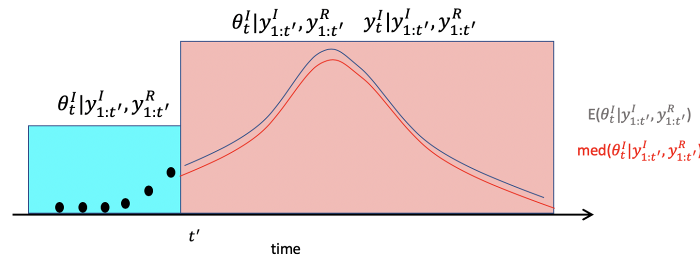
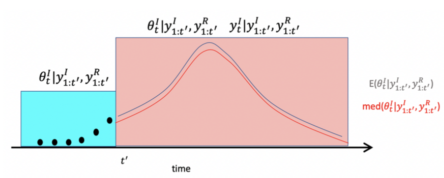
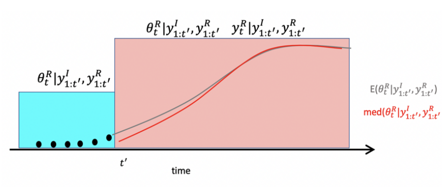


Figure: Add the forecast

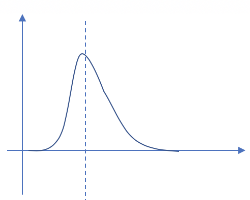
Infection



Removed



$\phi | y_{1:t'}^I, y_{1:t'}^R$



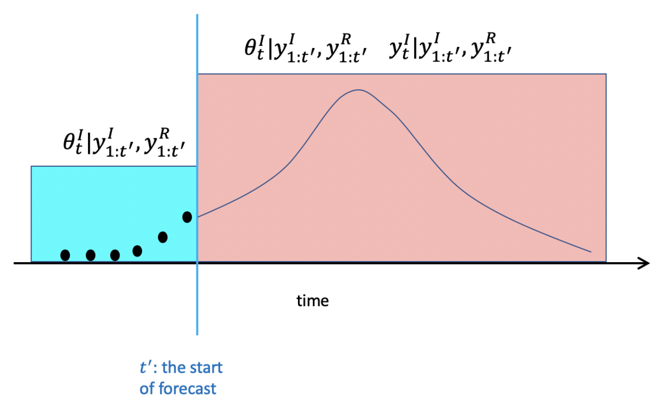
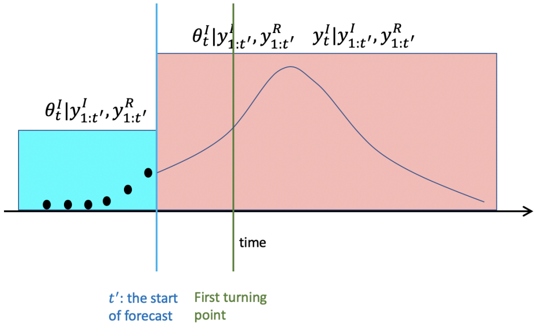
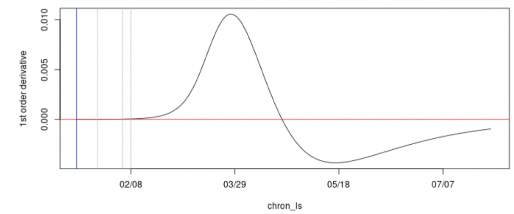


Figure: Key time points

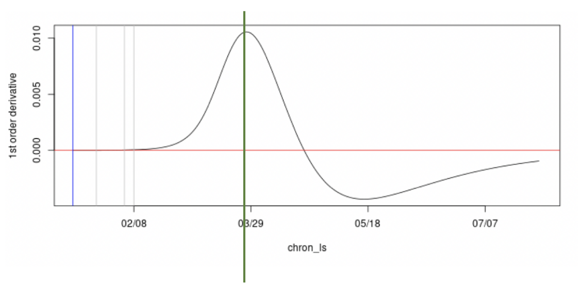


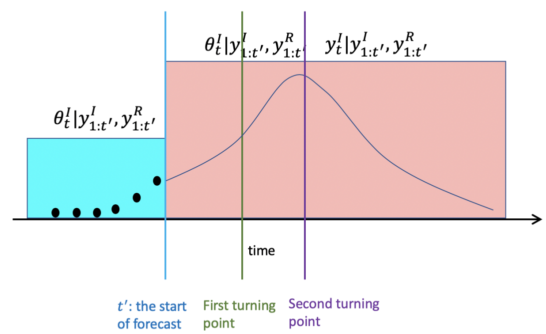
- First turning point: the turning point that the acceleration changes to deceleration $\Rightarrow$ LESS daily increase in infection prevalence later

$$\frac{d\theta_t^I}{dt} = \beta\theta_t^S\theta_t^I - \gamma\theta_t^I$$



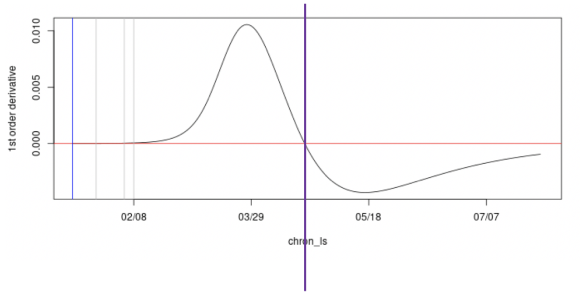
$$\frac{d\theta_t^I}{dt} = \beta\theta_t^S\theta_t^I - \gamma\theta_t^I \text{ is maximum}$$



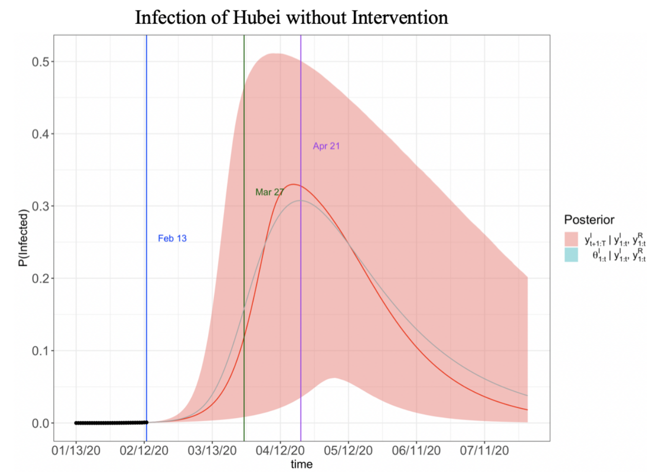


- First turning point: the turning point that the acceleration changes to deceleration $\Rightarrow$ LESS daily increase in infection prevalence later
- Second turning point: the turning point from increase to decrease $\Rightarrow$ see a STOP and turning back/down

$$\frac{d\theta_t^I}{dt} = \beta\theta_t^S\theta_t^I - \gamma\theta_t^I = 0$$

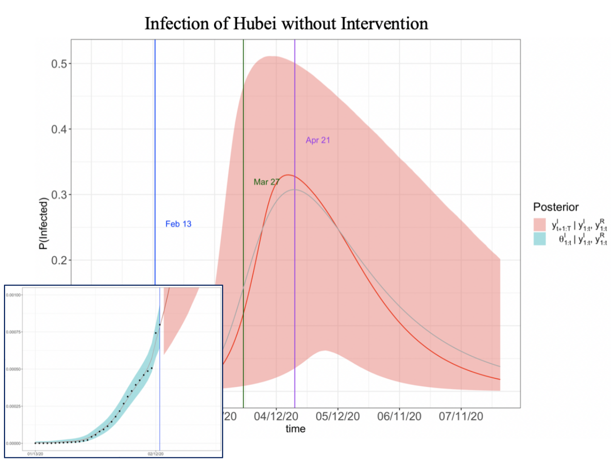


BDSSM-SIR of Hubei

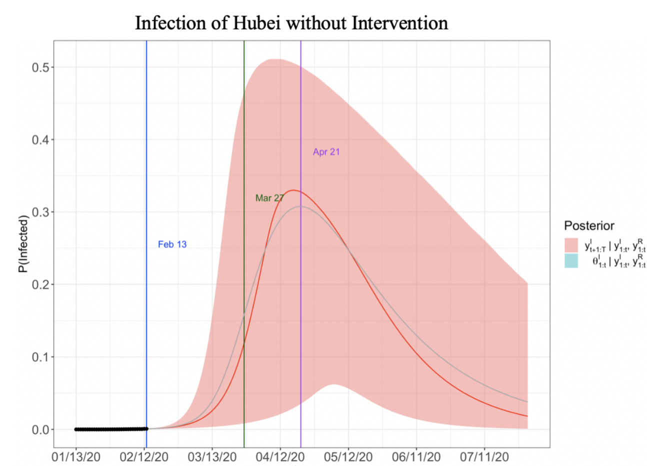


$[3/18, 05/25]$, $[3/30, 6/24]$

BDSSM-SIR of Hubei

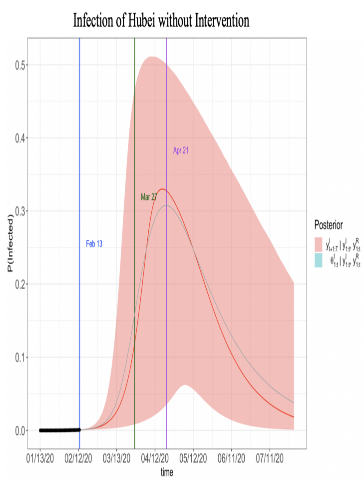


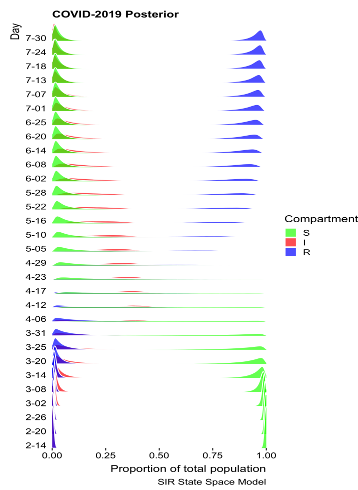
BDSSM-SIR of Hubei



- $R_0 = 4.06, \gamma = 0.042, \beta = 0.165$
- on average, **3 million population** "will be" infected on April 21

BDSSM-SIR of Hubei

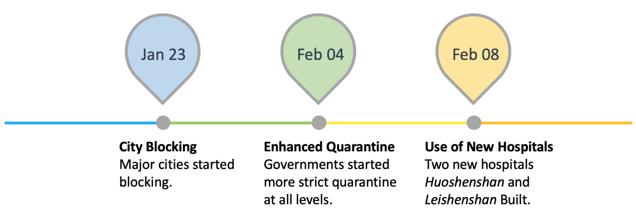




Limitations of BDSSM-SIR Model

- There are many forms of human interventions used to alter the transmission rate:
  1. Personal protective measures like wearing masks, safety glasses, hygiene, etc.
  2. Quarantine, hospitalization, city blockade, traffic control, limited activities, etc.
- Moreover, the virus can mutate to evolve
- In sum, β or the transmission rate may change by time.

Isolation Measures Taken in Hubei Province



Incorporating Quarantine in the Model

Suppose at a given time, $q^S(t)$ and $q^I(t)$ are the chance of a person being either in-home quarantine or in-hospital isolation, respectively. The modified chance of being infected is

$$\beta(1-q^S(t))\theta_t^S(1-q^I(t))\theta_t^I := \beta\pi(t)\theta_t^S\theta_t^I$$

with $\pi(t) = (1-q^S(t))(1-q^I(t))$.

Thus, $\pi(t)$ modifies the chance of a susceptible person meeting with an infected person or *vice versa*, which is termed as a *transmission modifier* due to quarantine.

BDSSM-SIR with Time-varying Transmission Rate Modifier

$$\frac{d\theta_t^S}{dt} = -\beta\pi(t)\theta_t^S\theta_t^I, \quad \frac{d\theta_t^I}{dt} = \beta\pi(t)\theta_t^S\theta_t^I - \gamma\theta_t^I, \quad \frac{d\theta_t^R}{dt} = \gamma\theta_t^I$$

The above model assumes primarily that the proportions of susceptible and infectious populations are not shrunken but the chance of a susceptible person meeting with an infected person is reduced by $\pi(t)$.

Time-varying Transmission Rate Modifier

- $\pi(t) \in [0, 1]$
- Step function reflecting the government-initiated macro isolation measures. For example,

$$\pi(t) = \begin{cases} \pi_{01} & \text{, if } t \leqslant \text{Jan 23,} \\ \pi_{02} & \text{, if } t \in (\text{Jan 23, Feb 4]city blockade} \\ \pi_{03} & \text{, if } t \in (\text{Feb 4, Feb 8] enhanced quarantine} \\ \pi_{04} & \text{, if } t > \text{Feb 8 use of new hospitals} \end{cases}$$

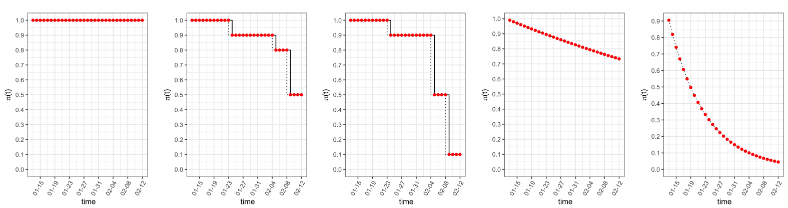
- Continuous function reflecting the gradually increased community-level awareness and responsibility of quarantine, or micro isolation measures. For example,

$$\pi(t) = \exp(-\lambda_0 t) \text{ or } \pi(t) = \exp(-(\lambda_0 t)^\nu)$$

Time-varying Transmission Rate Modifier

We choose $\exp(-\lambda_0 t)$ with $\lambda_0 = 0.01, 0.1$ and step functions with:

1. $\pi_{01} = 1, \pi_{02} = 1, \pi_{03} = 1, \pi_{04} = 1$
2. $\pi_{01} = 1, \pi_{02} = 0.9, \pi_{03} = 0.8, \pi_{04} = 0.5$
3. $\pi_{01} = 1, \pi_{02} = 0.9, \pi_{03} = 0.5, \pi_{04} = 0.1$



Note, the step function with [$\pi_{01} = 1, \pi_{02} = 1, \pi_{03} = 1, \pi_{04} = 1$] is equivalent to the continuous exponential one with $\lambda_0 = 0$

Constant $\pi(t)$: no intervention in Hubei

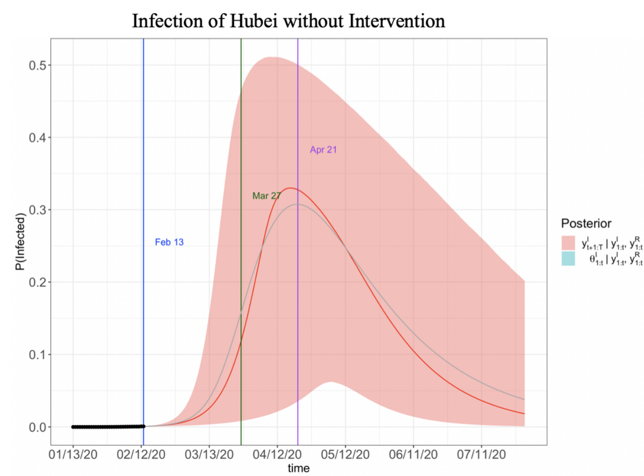


Figure: $\pi_{01} = \pi_{02} = \pi_{03} = \pi_{04} = 1$, 20M infected(30%)

Step function $\pi(t)$: intervention in Hubei

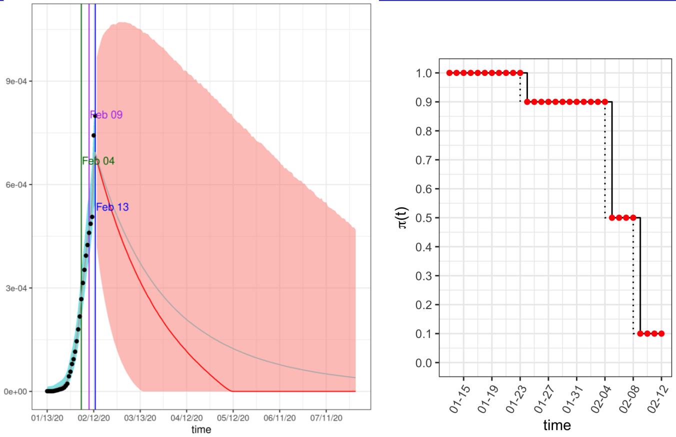


Figure: $\pi_{01} = 1, \pi_{02} = 0.9,\ \pi_{03} = 0.5,\ \pi_{04} = 0.1$

$R_0 = 4.06, \gamma = 0.042, \beta = 0.165$, [2/3, 2/4], [02/09, 02/09]

Step function $\pi(t)$: intervention in Hubei

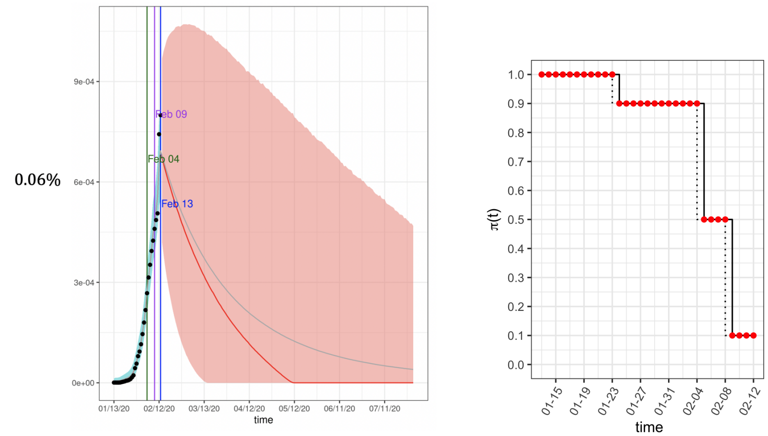


Figure: $\pi_{01} = 1, \pi_{02} = 0.9,\ \pi_{03} = 0.5,\ \pi_{04} = 0.1$, 36,000 infected

$R_0 = 4.06, \gamma = 0.042, \beta = 0.165$, [2/3, 2/4], [02/09, 02/09]

Hubei: 3 posterior proportions by time

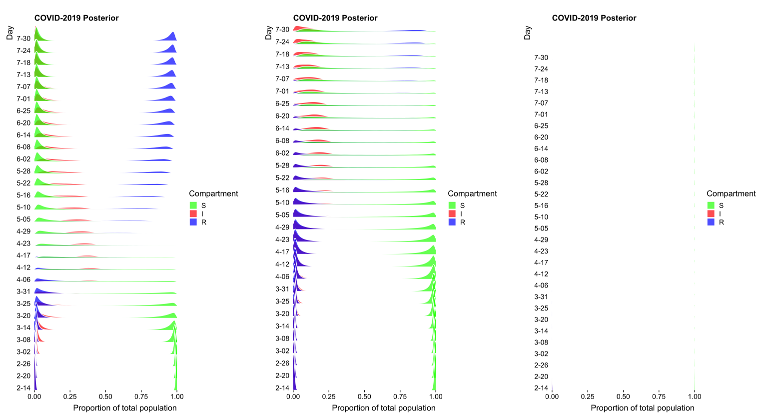
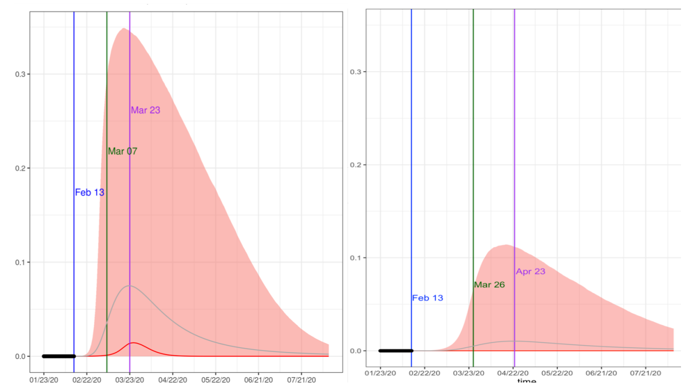


Figure: Posterior Proportions with increasing intervention

susceptible infected removed

Step function $\pi(t)$: outside Hubei



$\pi_{01} = \pi_{02} = \pi_{03} = \pi_{04} = 1$

$\pi_{01} = 1, \pi_{02} = 0.9,\ \pi_{03} = 0.8,$ $\pi_{04} = 0.5$

- $R_0 = 2.77,\ \gamma = 0.167,\ \beta = 0.431$ max median prevalence $= 1.5\%$
- $R_0 = 2.96,\ \gamma = 0.199,\ \beta = 0.56,$ max median prevalence$= 0.001\%$

Continuous $\pi(t)$: weak intervention in Hubei

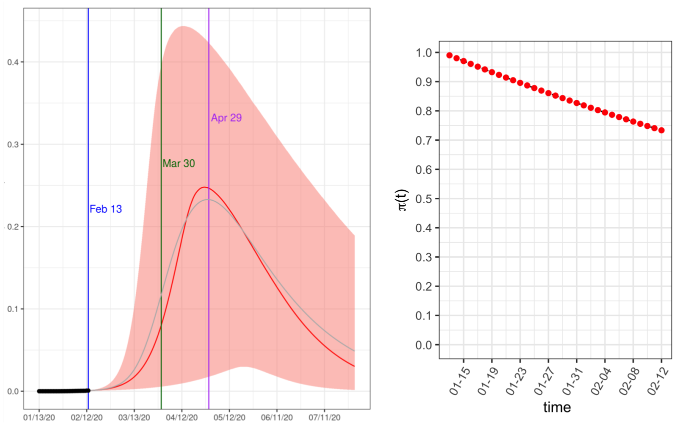


Figure: $\pi(t) = \exp(-0.01t)$, 15M infected

Continuous $\pi(t)$: strict intervention in Hubei

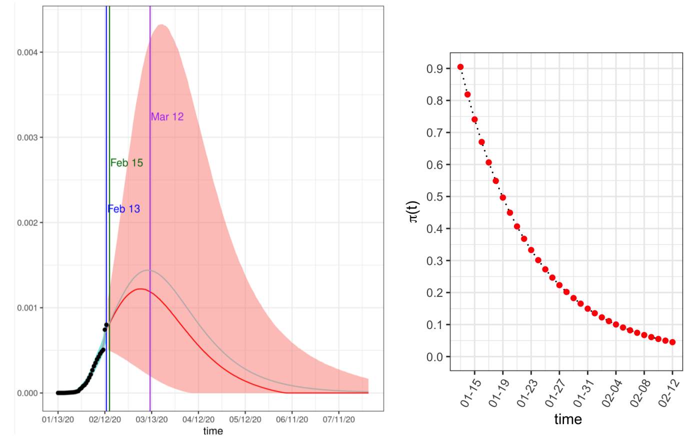
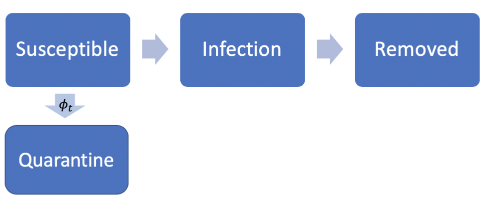


Figure: $\pi(t) = \exp(-0.1t)$, 72,000 infected

BDSSM-SIR with Time-varying Quarantine Protocols

Our second model allows to characterize time-varying proportions of susceptible due to government-enforced stringent in-home isolation. We expanded the SIR model by adding a quarantine compartment with a time-varying rate of quarantine ϕ_t, the chance of a susceptible person being willing to take in-home isolation at time t. The extended SIR is



$$\frac{d\theta_t^Q}{dt} = \phi_t \theta_t^S, \quad \frac{d\theta_t^S}{dt} = -\beta\theta_t^S\theta_t^I - \phi_t\theta_t^S,$$
$$\frac{d\theta_t^I}{dt} = \beta\theta_t^S\theta_t^I - \gamma\theta_t^I, \quad \frac{d\theta_t^R}{dt} = \gamma\theta_t^I.$$

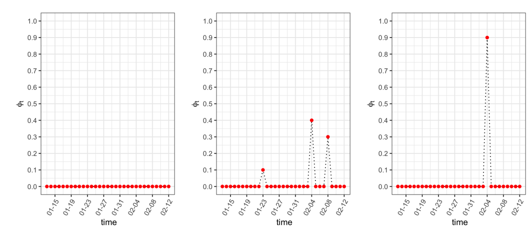
BDSSM-SIR with a Time-varying Quarantine Rate: ϕ_t

A pre-specified Dirac delta function:

$$\phi_t = \begin{cases} \phi_{01} & \text{, if } t = \text{Jan 23 city blockade} \\ \phi_{02} & \text{, if } t = \text{Feb 4 enhanced quarantine} \\ \phi_{03} & \text{, if } t = \text{Feb 8 use of new hospitals} \\ 0 & \text{, else} \end{cases}$$

Some examples of quarantine rates we considered:

- $\phi_{01} = 0, \phi_{02} = 0, \phi_{03} = 0$
- $\phi_{01} = 0.1, \phi_{02} = 0.4, \phi_{03} = 0.3$
- $\phi_{01} = 0, \phi_{02} = 0.9, \phi_{03} = 0$



ϕ_t: progressive quarantine in Hubei

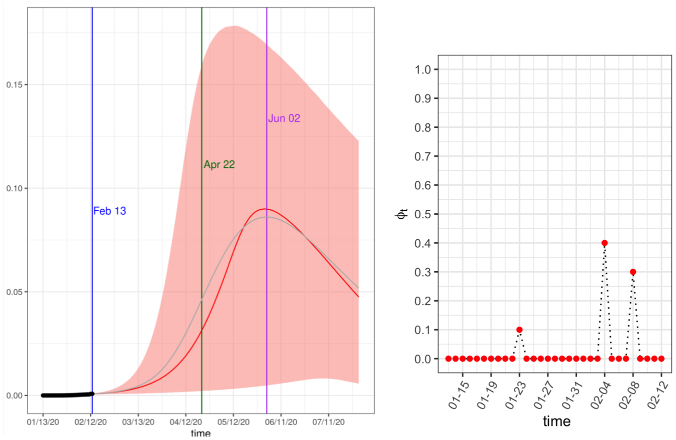


Figure: $\phi_{01} = 0.1, \phi_{02} = 0.4, \phi_{03} = 0.3$, 4.8M infected

ϕ_t: prompt strict quarantine in Hubei

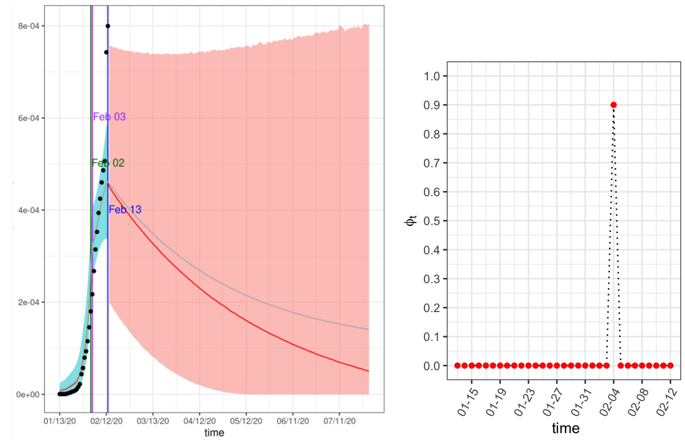


Figure: $\phi_{01} = 0, \phi_{02} = 0.9, \phi_{03} = 0$,, 30,000 infected

Add Hospitalization to BDSSM-SIR

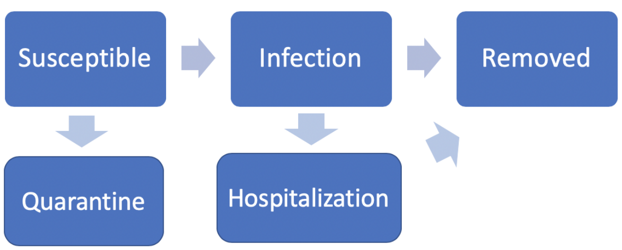


Figure: BDSSM-SIR with both quarantine and hospitalization

Add Hospitalization to BDSSM-SIR

$$\frac{d\theta_t^Q}{dt} = \phi_t \theta_t^S,$$
$$\frac{d\theta^S}{dt} = -\beta\theta^S\theta^I - \phi_t\theta_t^S$$
$$\frac{d\theta^I}{dt} = \beta\theta^S\theta^I - \gamma^R\theta^I - \gamma^H\theta^I,$$
$$\frac{d\theta^R}{dt} = \gamma^R\theta^I + \gamma^{RH}\theta_t^I,$$
$$\frac{d\theta^H}{dt} = \gamma^H\theta^I - \gamma^{RH}\theta_t^I$$

Add suspected compartment to BDSSM-SIR

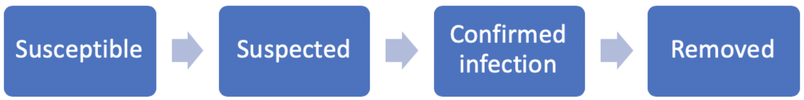


Figure: BDSSM-SIR with suspected cases

Add suspected compartment to BDSSM-SIR

$$
\begin{aligned}
\frac{d\theta^S}{dt} &= -\beta\theta^S\theta^U \\
\frac{d\theta^U}{dt} &= \beta\theta^S\theta^U - \gamma^C\theta^U, \\
\frac{d\theta^C}{dt} &= \gamma^C\theta^U - \gamma^R\theta^C_t, \\
\frac{d\theta^H}{dt} &= \gamma^R\theta^C
\end{aligned}
$$

Comprehensive BDSSM-SIR model

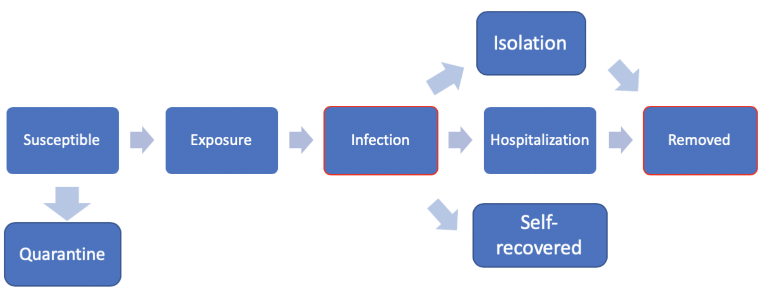


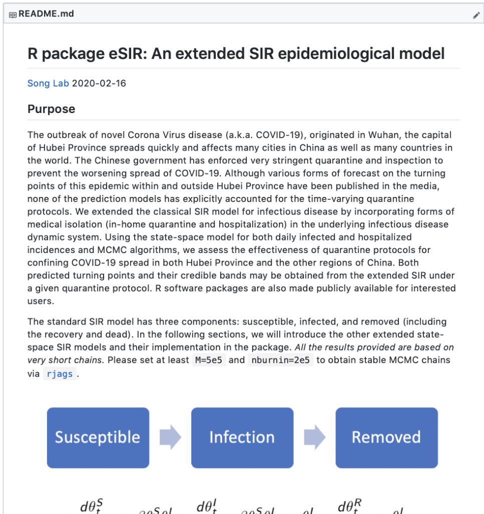
Figure: BDSSM-SIR with multiple compartments

Comprehensive BDSSM-SIR model

$$
\begin{aligned}
\frac{d\theta^Q}{dt} &= \phi_t\theta^S, \quad \frac{d\theta^S}{dt} = -\beta\theta^S\theta^I - \phi_t\theta^S, \\
\frac{d\theta^E}{dt} &= \beta\theta^S\theta^I - \alpha\theta^E, \\
\frac{d\theta^I}{dt} &= \alpha\theta^E - (1-\rho_t-\pi_t)\gamma^H\theta^I - \rho_t\gamma^S\theta^I - \pi_t\gamma^C\theta^I, \\
\frac{d\theta^H}{dt} &= (1-\rho_t-\pi_t)\gamma^H\theta^I - \eta^H\theta^H, \\
\frac{d\theta^S}{dt} &= \rho_t\gamma^S\theta^I - \eta^S\theta^S, \\
\frac{d\theta^C}{dt} &= \pi_t\gamma^C\theta^I, \\
\frac{d\theta^R}{dt} &= \eta^H\theta^H + \eta^S\theta^S,
\end{aligned}
$$

An R package: eSIR

`https://github.com/lilywang1988/eSIR`



README.md

R package eSIR: An extended SIR epidemiological model

Song Lab 2020-02-16

Purpose

The outbreak of novel Corona Virus disease (a.k.a. COVID-19), originated in Wuhan, the capital of Hubei Province spreads quickly and affects many cities in China as well as many countries in the world. The Chinese government has enforced very stringent quarantine and inspection to prevent the worsening spread of COVID-19. Although various forms of forecast on the turning points of this epidemic within and outside Hubei Province have been published in the media, none of the prediction models has explicitly accounted for the time-varying quarantine protocols. We extended the classical SIR model for infectious disease by incorporating forms of medical isolation (in-home quarantine and hospitalization) in the underlying infectious disease dynamic system. Using the state-space model for both daily infected and hospitalized incidences and MCMC algorithms, we assess the effectiveness of quarantine protocols for confining COVID-19 spread in both Hubei Province and the other regions of China. Both predicted turning points and their credible bands may be obtained from the extended SIR under a given quarantine protocol. R software packages are also made publicly available for interested users.

The standard SIR model has three components: susceptible, infected, and removed (including the recovery and dead). In the following sections, we will introduce the other extended state-space SIR models and their implementation in the package. *All the results provided are based on very short chains*. Please set at least `M=5e5` and `nburnin=2e5` to obtain stable MCMC chains via `rjags`.

Susceptible → Infection → Removed

Thank you!

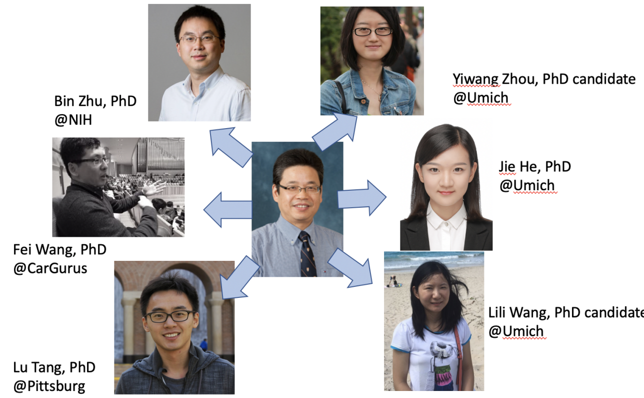


Figure: BDSSM-SIR in Song Lab: $R_0 = 6$

References:

1 Osthus, D., Hickmann, K. S., Caragea, P. C., Higdon, D., & Del Valle, S. Y. (2017). Forecasting seasonal influenza with a state-space SIR model. The annals of applied statistics, 11(1), 202.

2 Mkhatshwa, T., & Mummert, A. (2010). Modeling super-spreading events for infectious diseases: case study SARS. arXiv preprint arXiv:1007.0908.