Quantum Computing

- Lecture 1 (April 23, 2025)

- Today:
  - Admin.
  - Overview of this module

Contact Information

- Course coordinator: Prof. Jiaxin Pan
- Lecturer & TA: **Runzhi Zeng**
- Email:
  - jiaxin.pan@uni-kassel.de
  - runzhi.zeng@uni-kassel.de

- Office hours
  - Office: Room 2628
  - 2 pm – 2:30 pm, Wednesday
  - (Please send an email in advance)

- All information is available on:
  - https://runzhizeng.github.io/QC-s25/

Time

- Summer semester 2025: 23.04.2025 – 24.07.2025

- 14 Weeks: Wednesday and Thursday every week

- Lecture dates:
  - April: 23, 24, 30
  - May: ~~01~~(Labor Day), 7, 8, ~~14-15~~ (Travel), 21, 22, 28-~~29~~ (Ascension)
  - June: 4, 5, 11, 12, 18, ~~19~~(Corpus Christi), 25, 26.
  - July: 2, 3, 9, 10, 16,17, 23, 24.

Format

- Wednesday 12:00 – 13:30:
  - Two lectures (~40min each) + 10min break

- Thursday 10:00 – 12:00:
  - One lecture (~45min)
  - **Exercise and Q&A (~45min-1h)**
  - Explanation of selected exercise questions (~15min-30min)
    - I may ask you to present your solutions

- This module involves a large amounts of calculations
  - Please bring your **pen and paper (especially on Thursday!)**
  - You can also bring your laptop/iPad to check the lecture notes at any time

Resources

- Lecture notes: Will be updated at https://runzhizeng.github.io/QC-s25
- Calculation Manuscripts: Would be updated at the Moodle.

- Textbooks:
  - ***Quantum Computation and Quantum Information*** by Michael Nielsen and Isaac Chuang
  - *Linear Algebra and Learning from Data* by Gilbert Strang
  - *An Introduction to Quantum Computing* by Phillip Kaye, Raymond Laflamme, and Michele Mosca.
  - *Quantum Computing: A Gentle Introduction* by Eleanor Rieffel and Wolfgang Polak
  - ...

Resources

- Resources of other QC courses:

  (Parts of this module are based on these external course materials)

  - Quantum Computation and Information (Videos) by Prof. Ryan O'Donnell (Carnegie Mellon University)
  - Quantum Cryptography by Prof. Qipeng Liu (UC San Diego)
  - Quantum Cryptography by Prof. Mark Zhandry (Princeton University)
  - Introduction to Quantum Computing by Prof. Dakshita Khurana and Prof. Makrand Sinha (University of Illinois)
  - Introduction to Quantum Computing by Prof. Henry Yuen (Columbia University)
  - Lecture Notes of Quantum Information Science by Prof. Scott Aaronson (UT Austin)

- Miscellaneous:
  - Qubit Zoo: “Zoo” of interesting qubits and quantum gates
  - Quantum Programming (Simulated): Q# and Qiskit

Homework and Exam

- Homework: Some problem sets (notice time: 1~2 weeks).

- Exam type (Oral or written?): To be decided
- When? To be decided

What is Quantum Computing?

- Computation based on **quantum mechanics**, rather than classical physics

- **Quantum mechanics:**
  - Classical physics does not work in some cases
  - -> Quantization, introduced/explained by Planck, Einstein, ...
  - -> Quantum theory, formalized by Schrödinger, Heisenberg, Dirac...

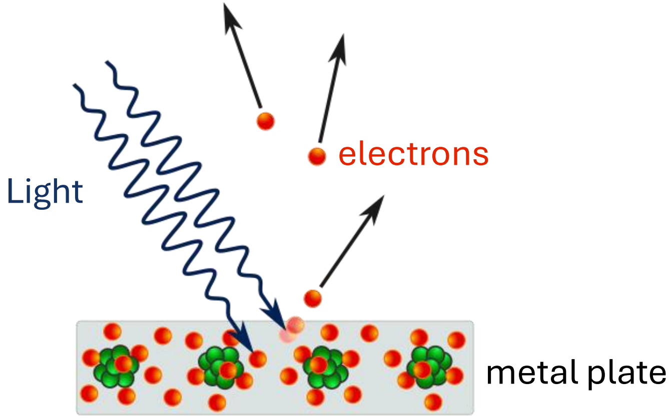
Quantum Mechanics

- Computation based on **quantum mechanics**, rather than classical physics

- **Quantum mechanics:**
  - Classical physics does not work in some cases

    Classical physics:
    "Light is **continuous wave** (with energy)
    $\Longrightarrow$ Shine light on the plate for a long time
    $\Longrightarrow$ Electrons should be emitted eventually"

Example: Photoelectric effect



(Source: Wikipedia)

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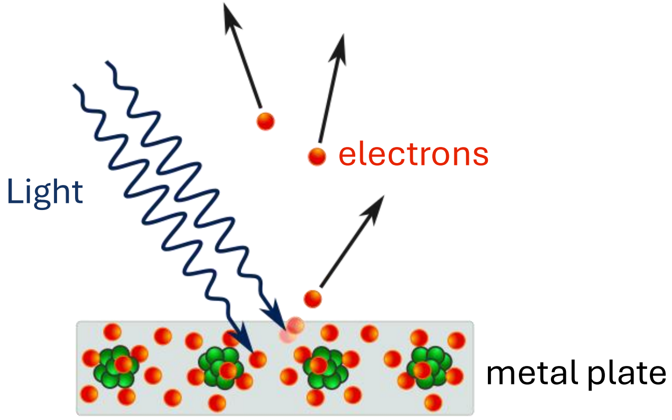
Classical physics:
"Light is **continuous wave** (with energy)
⟹ Shine light on the plate for a long time
⟹ Electrons should be emitted eventually"

Double slit experiment:
Light is a wave,
or at least it behaves like a wave
https://en.wikipedia.org/wiki/Double-slit_experiment

Example: Photoelectric effect



(Source: Wikipedia)

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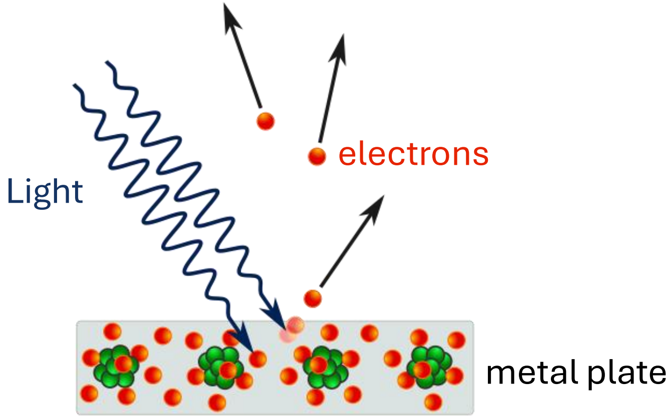
Classical physics:
"Light is **continuous wave** (with energy)
$\Rightarrow$ Shine light on the plate for a long time
$\Rightarrow$ Electrons should be emitted eventually"

Reality (Experiments):
1. There is a *threshold frequency.*
   (Electrons are emitted **only if** the light's frequency is high enough)
2. The emission of electrons is "immediately", regardless of light's intensity

Example: Photoelectric effect



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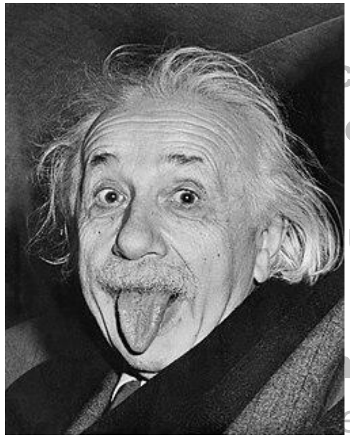

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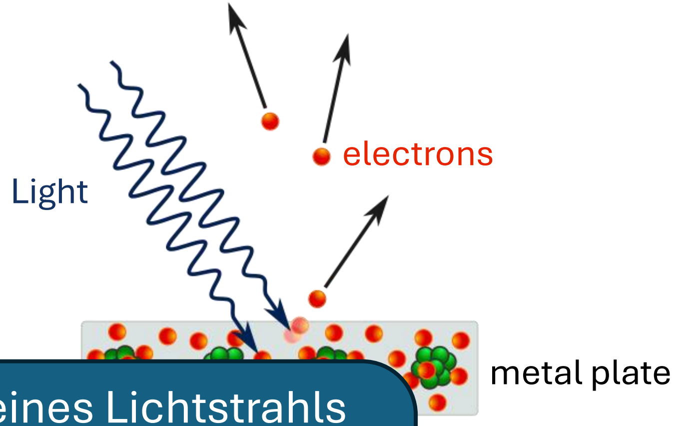

Wenn sich nämlich bei der Ausbreitung eines Lichtstrahls die Energie nicht kontinuierlich im ganzen Raum verteilt, sondern aus einzelnen, **im Raum lokalisierten Quanten besteht**, dann erklärt das diemerkwürdigen Eigenschaften der Photoelektrizität...

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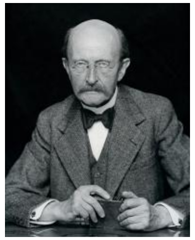
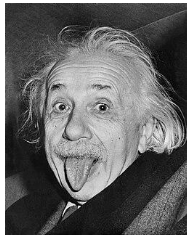
Example: $E = h \cdot v$
E: Energy of the photon
h: Planck's constant
v: Frequency of the photon





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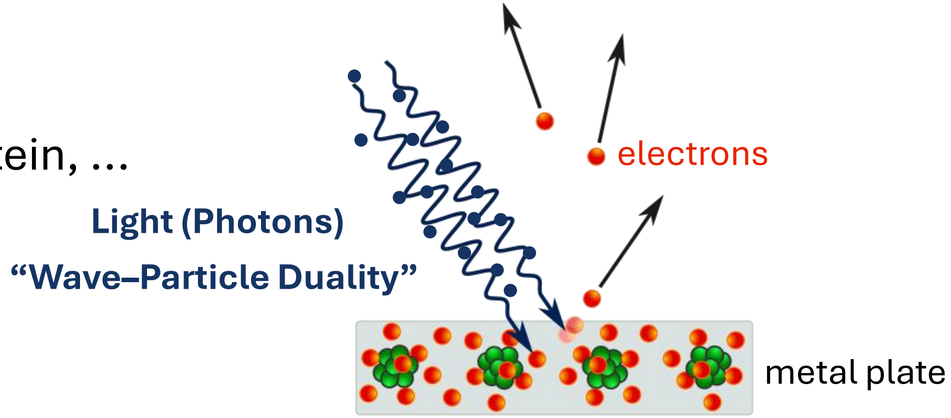

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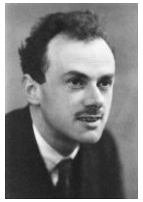
  - -> Quantum theory, formalized by Schrödinger, Heisenberg, **Dirac**, …



$$U|\psi\rangle\langle\phi||\psi\rangle = \langle\phi|\psi\rangle U|\psi\rangle$$

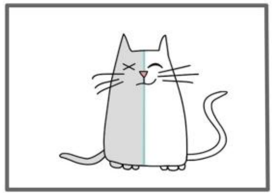
(Dirac's notation)

(Source: Wikipedia)

$$i\hbar\frac{d}{dt}|\Psi(t)\rangle = \hat{H}(\Psi(t))$$

(Schrödinger equation)

(Source: Wikipedia)



Schrödinger's Cat

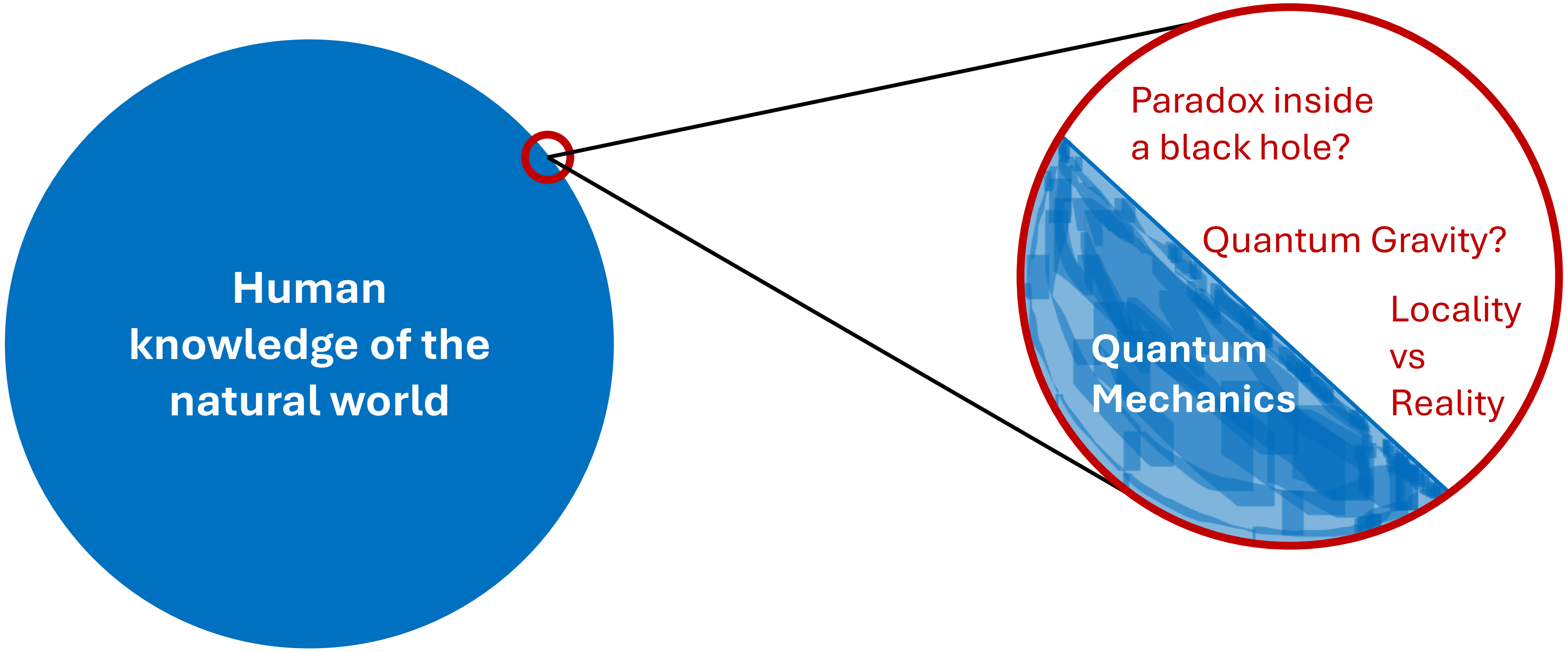
(picture from Medium)

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

(Heisenberg Uncertainty Principle)

(Source: Wikipedia)

Quantum Mechanics



Quantum Computing

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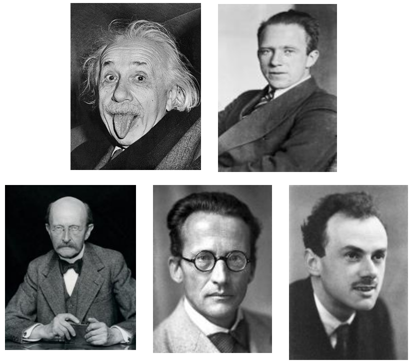
Quantum Mechanics



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Information Theory
+ Quantum Mechanics
= Quantum Computing

(Source of pictures: Wikipedia)

Quantum Computing

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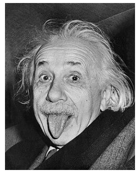

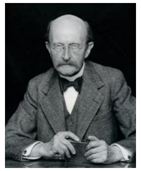

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Information Theory
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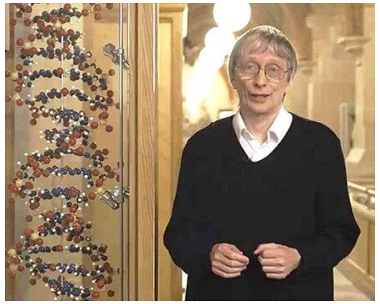
Richard Feynman

- Simulating quantum systems with classical computers is *inefficient*
- **Quantum Systems/Computers are required**



David Deutsch

- Deutsch's algorithm, Deutsch-Jozsa algorithm
- **Quantum Turing Machine**

(Source of pictures: Wikipedia)

Quantum Computing

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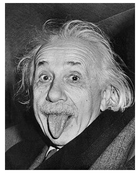
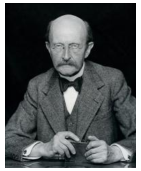
Quantum Mechanics





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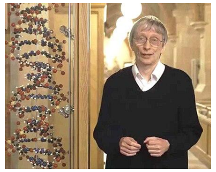
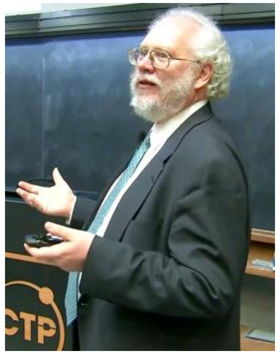
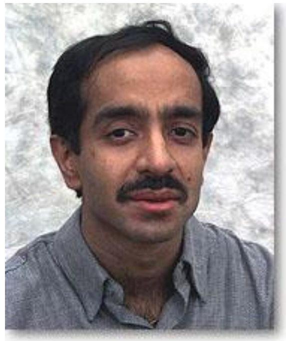




Peter Williston Shor

- **Breakthrough: Shor's algorithm**
- **Break most of existing public-key cryptosystems**
- … which motivates "post-quantum cryptography"



Lov K. Grover

- **Grover search: A Quantum search algorithm**
- Significant impacts on information theory, computation complexity, cryptography, …

(Source of pictures: Wikipedia)

Quantum Computing

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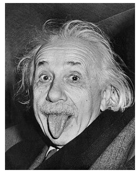
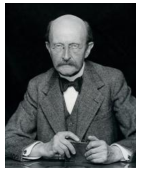
Quantum Mechanics





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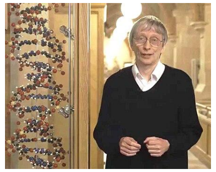
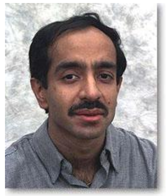
Information Theory + Quantum Mechanics = Quantum Computing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Advances in quantum computing

Google IBM Microsoft D:Wave intel®

(Source of pictures: Wikipedia)

Quantum Computing

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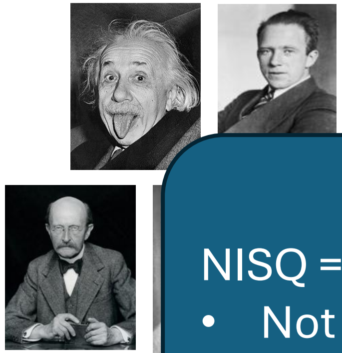
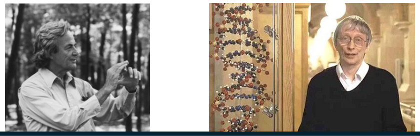
Quantum Mechanics



Information Theory + Quantum Mechanics = Quantum Computing



Advances in quantum computing

We are now in the NISQ era!

NISQ = Noisy Intermediate-Scale Quantum

- Not yet powerful enough to run Shor’s or Grover’s algorithms at scale
- But quantum hardware is **scaling up!**
- **Quantum error correction** is still needed for fault-tolerant computing

(Source of pictures: Wikipedia)

Quantum Computer vs Classical Computer

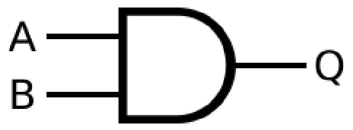
(Classical World)

(Quantum World)

Quantum Computer vs Classical Computer

Classical bit(s): **00101**
01011
10110

- **0** = Low voltage (e.g., 0V)
- **1** = High voltage (e.g., 3.3V – 5V)



Logic gate

(Classical World)

(Quantum World)

(Source of pictures: Wikipedia)

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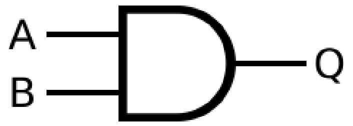
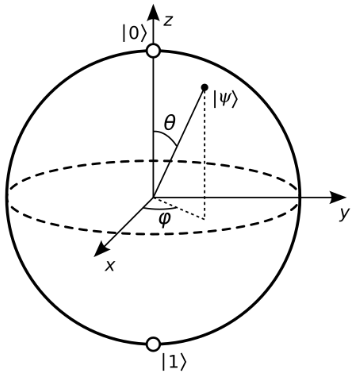


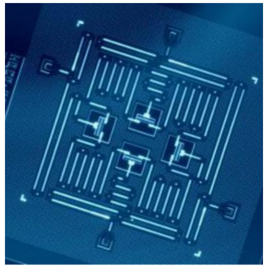
Logo gate

(Classical World)

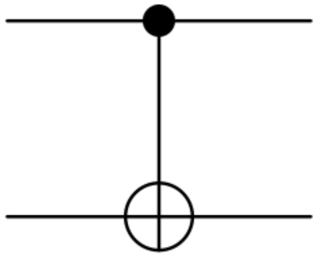
(Quantum World)



Single quantum bit **(qubit)** represented by Bloch sphere
Superposition of 0 and 1!



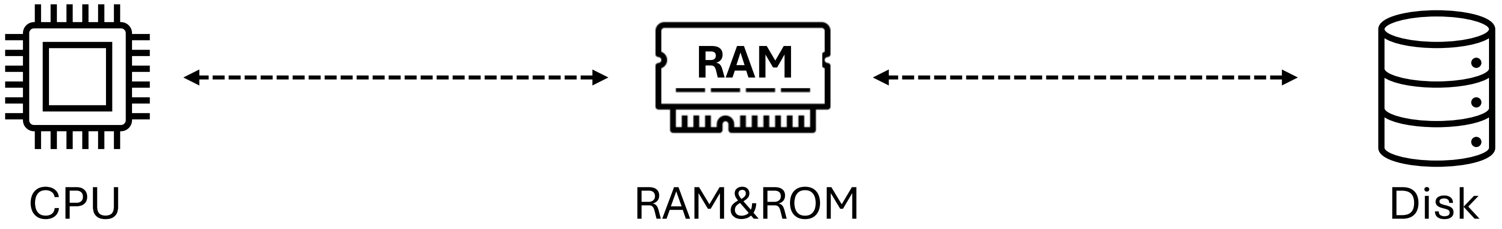
superconducting qubits (IBM)



Quantum logic gate

(Source of pictures: Wikipedia)

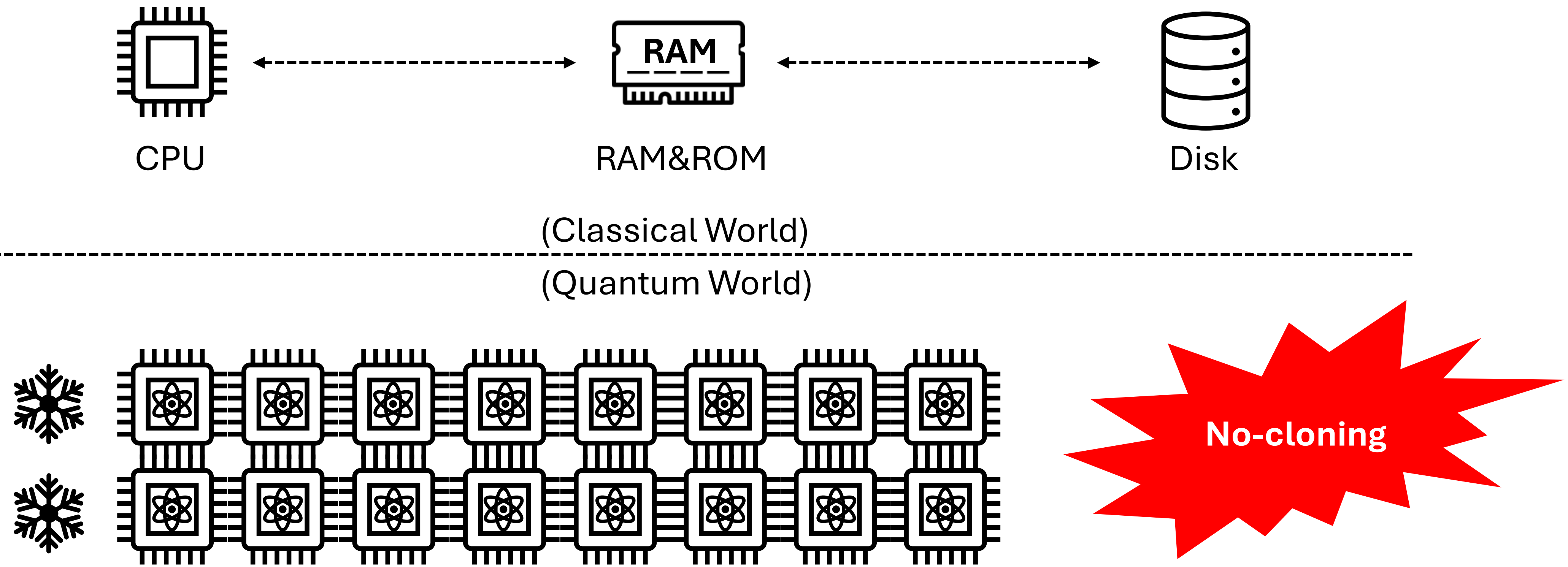
Quantum Computer vs Classical Computer



(Classical World)

(Quantum World)

Quantum Computer vs Classical Computer



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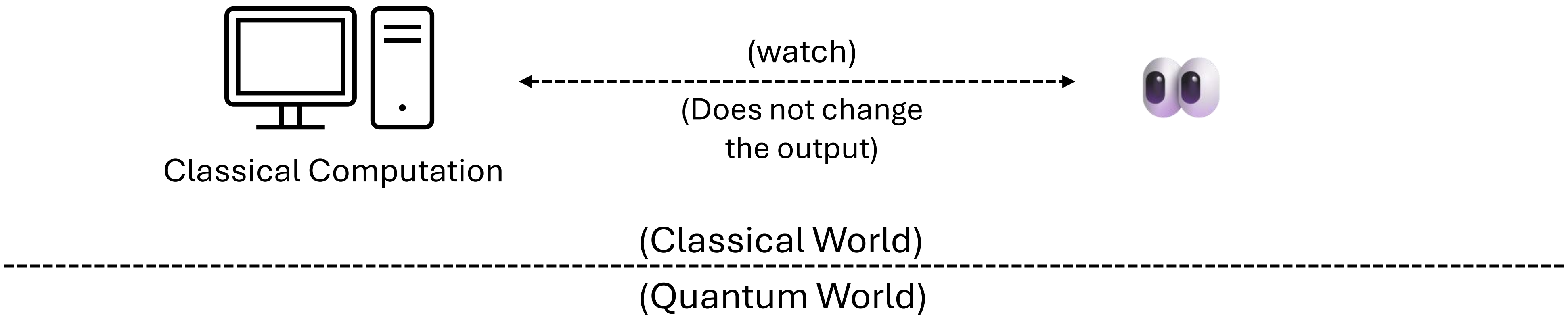

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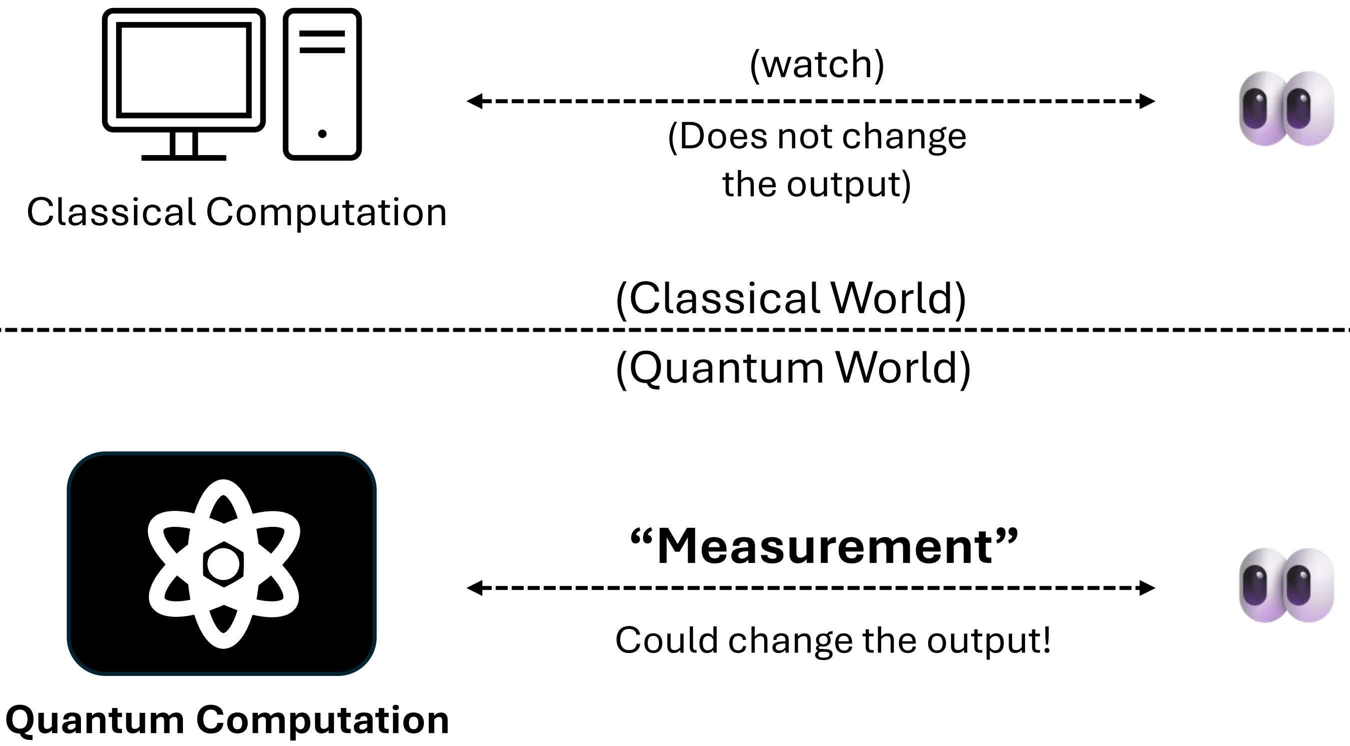

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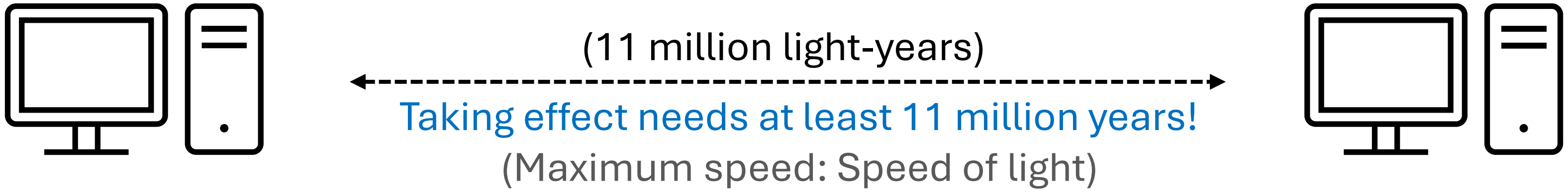
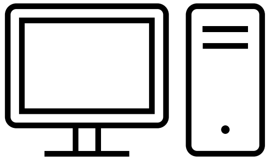


(Classical World)

(Quantum World)

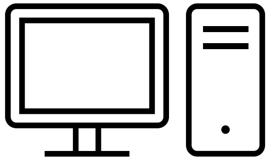
Quantum Computer vs Classical Computer



(11 million light-years)

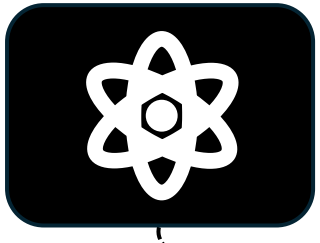
Taking effect needs at least 11 million years!

(Maximum speed: Speed of light)



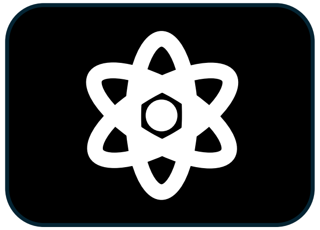
(Classical World)

(Quantum World)



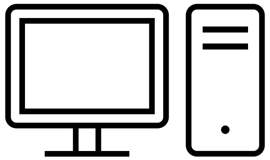
(11 million light-years)

Could “take effect” **immediately**
if they have prior **entanglement**



Entanglement

Quantum Computer vs Classical Computer

$N = pq$ $(\approx 2^{1024})$ $\longrightarrow$  $\xrightarrow{?}$ p, q

$p \neq q$ are big primes

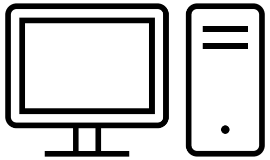
Unknown!

(Classical World)

(Quantum World)

Quantum Computer vs Classical Computer

$\boldsymbol{N = pq}$
$(\approx 2^{1024})$
$\boldsymbol{p \neq q}$ are big primes



Unknown!

$\boldsymbol{p, q}$

(Classical World)

(Quantum World)

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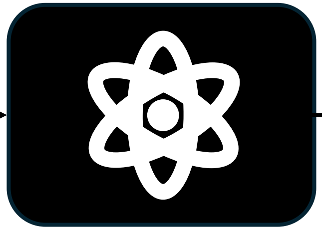

$\boldsymbol{p, q}$

Using Shor's algorithm

(Though no existing quantum computer can run this yet.)

Quantum Computer vs Classical Computer

- What makes Quantum Computing powerful?
  - Quantum **Superposition – Qubits**
  - **Unitary quantum gates** instead of logic gates
  - Quantum **Entanglement**
  - Quantum **Measurement**
  - **Quantum algorithms** utilizing quantum properties…

Impact on Computational Complexity

- **Exponential speedups for some specific problems**
  - Factoring, discrete logarithm, or more generally, hidden (finite abelian) subgroup problem
- **Polynomial speedups for generic search problems**
  - Grover search
  - Improve some lower bounds

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- **Quantum Computers $\neq$ More "Computable"**
  - They **cannot solve uncomputable** problems (e.g., the halting problem)

- **Quantum Computers $\neq$ Always more efficient**
  - No known advantage in many problems (e.g., Traveling Salesman Problem)

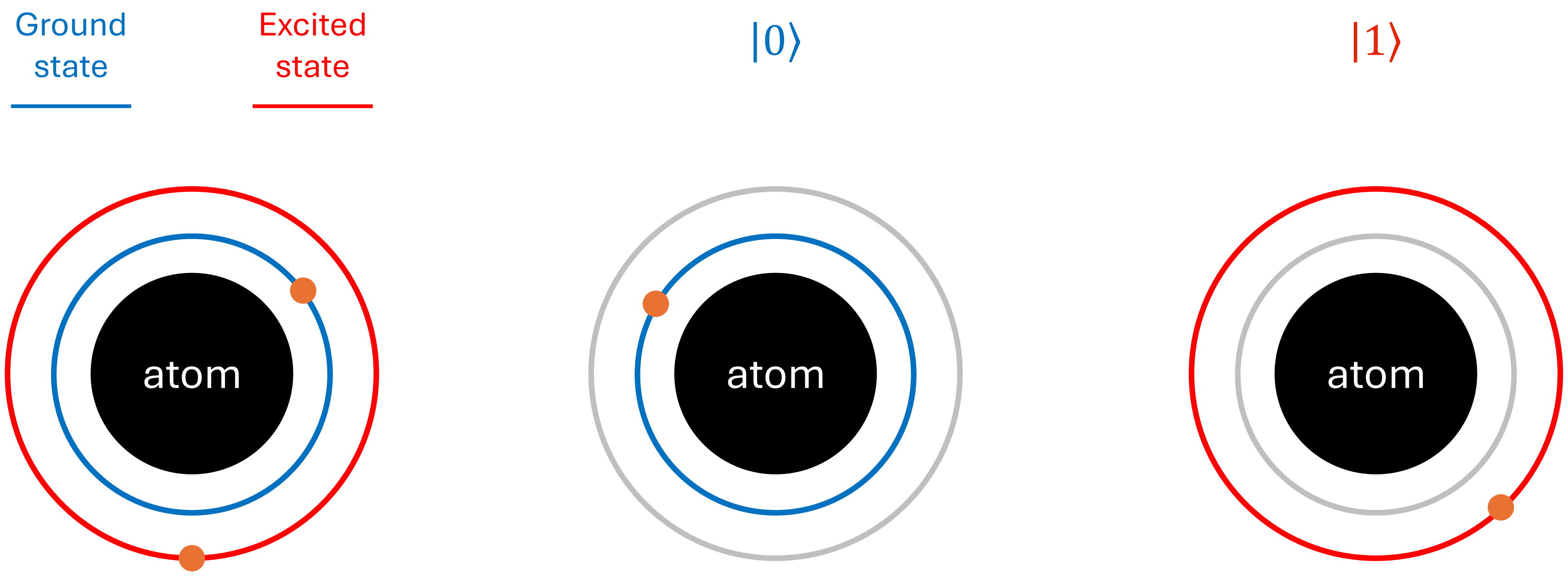
Overall Goals

- Main topics:
  - **Quantum mechanics** and its **linear algebra formulation**
  - **Entanglement and Measurement**
  - **Quantum Algorithms:**
    - Described by **quantum gates/circuits, unitaries**
    - Quantum “parallelism” – **evaluation on superposition**
    - Applications of quantum algorithms – QKD, QFT, search, …
  - **Quantum Information**
  - Quantum Programming (TBD)?

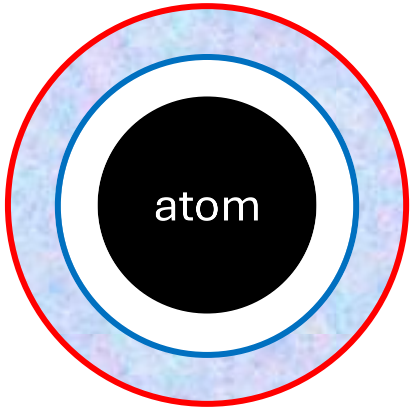
Overall Goals

- After completing this module, you should be able to:
  - **Explain** the fundamental principles of quantum computing (QC) and basic quantum mechanics.
  - **Use** the relevant linear algebra (including qubit representations and quantum gates) to formalize quantum computing notions and perform **basic calculations**.
  - **Describe and apply** quantum algorithms such as the Quantum Fourier Transform and Grover's search algorithm.
  - **Design** some simple quantum circuits/algorithms based on the algorithms you learned
  - **Read and understand** introductory research papers on quantum computing and cryptography.

Qubit and Superposition



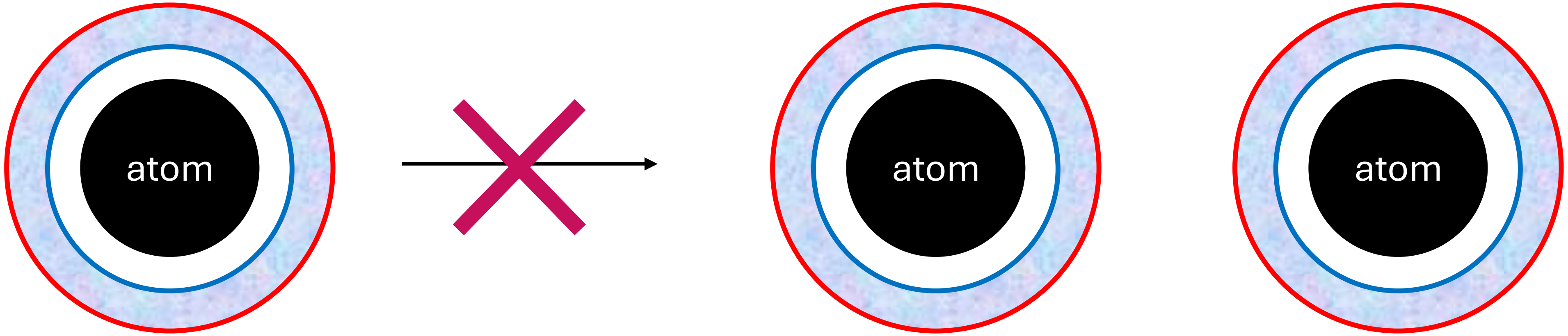
Qubit and Superposition



$$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$$

We do not know where ● is...
Or, ● is in “superposition”...

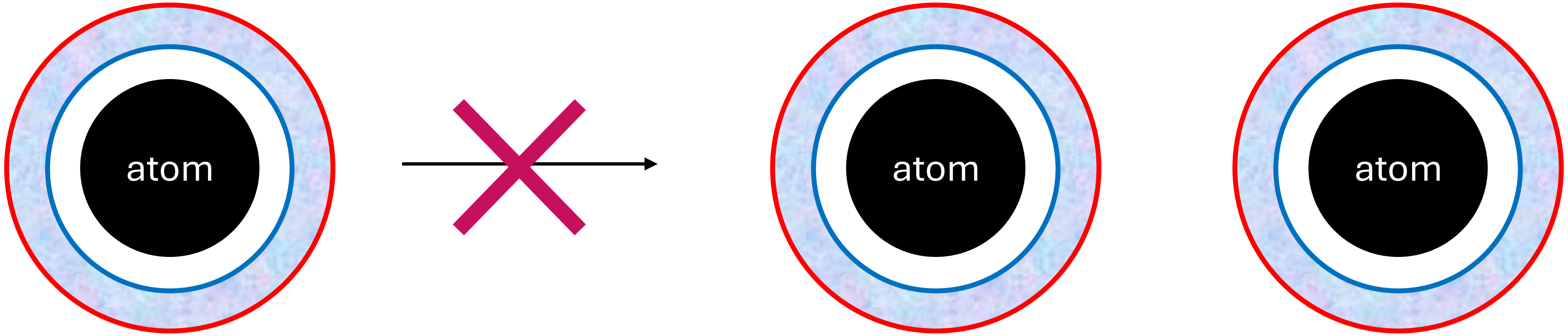
Qubit and Superposition



$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$ $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$ $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$

No-cloning

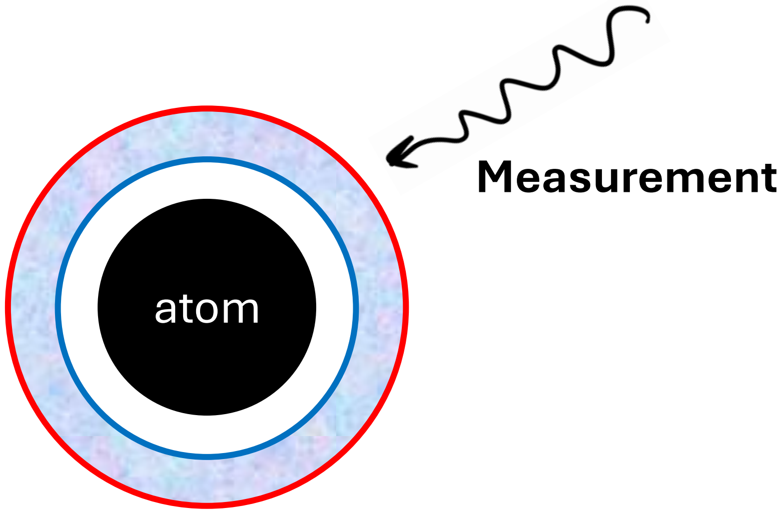
Qubit and Superposition



$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$ $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$ $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$

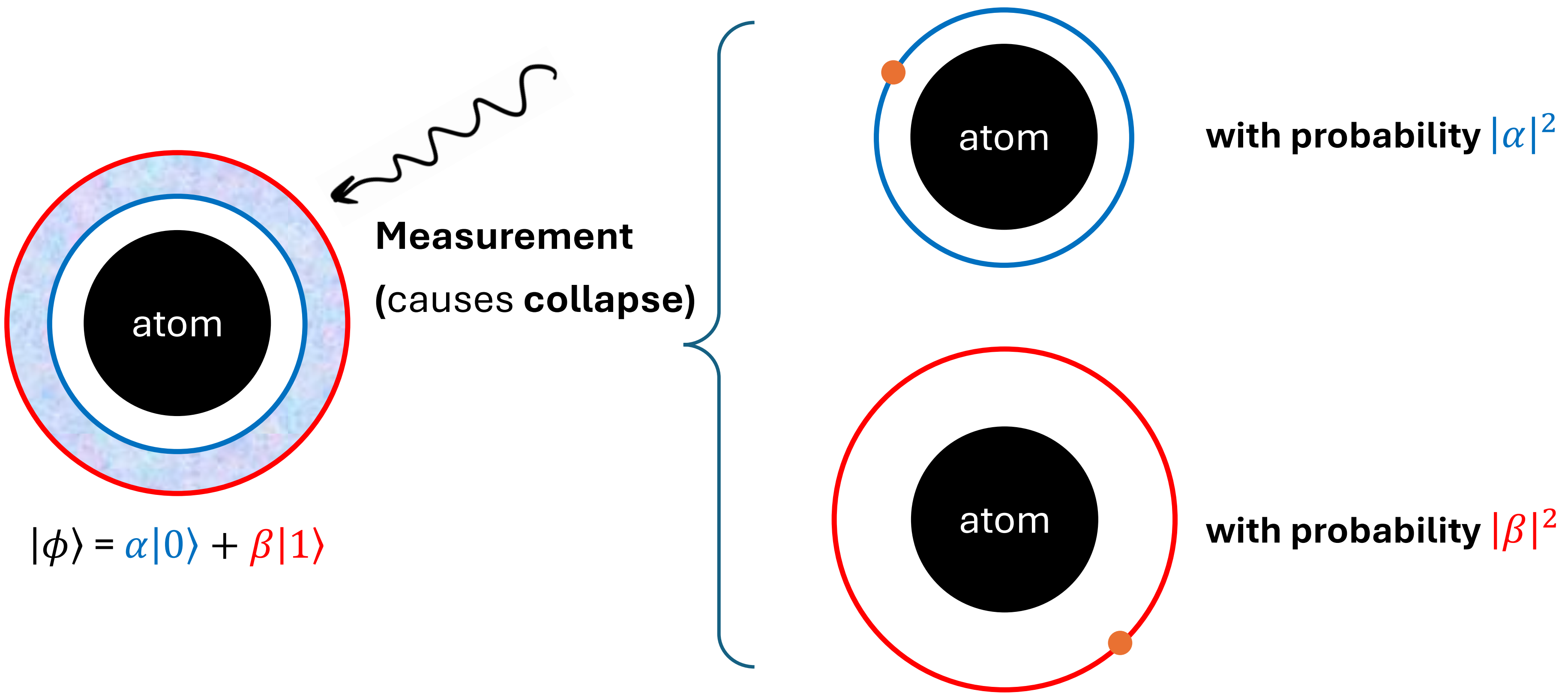
No-cloning → Quantum key distribution, quantum money, ...

Measurement

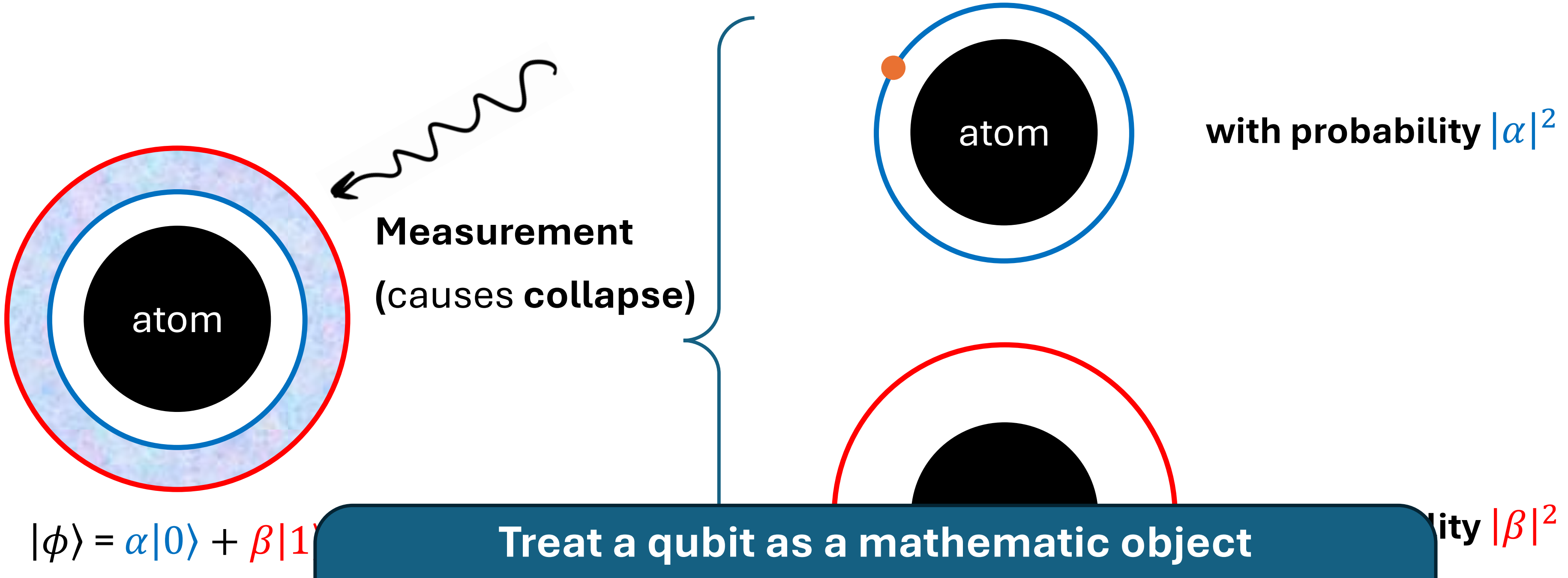


$$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Measurement

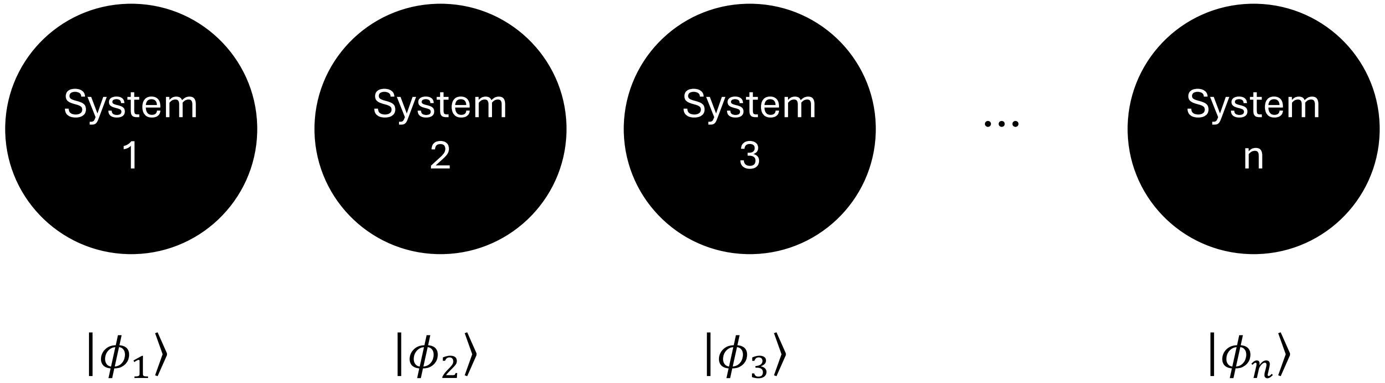


Measurement

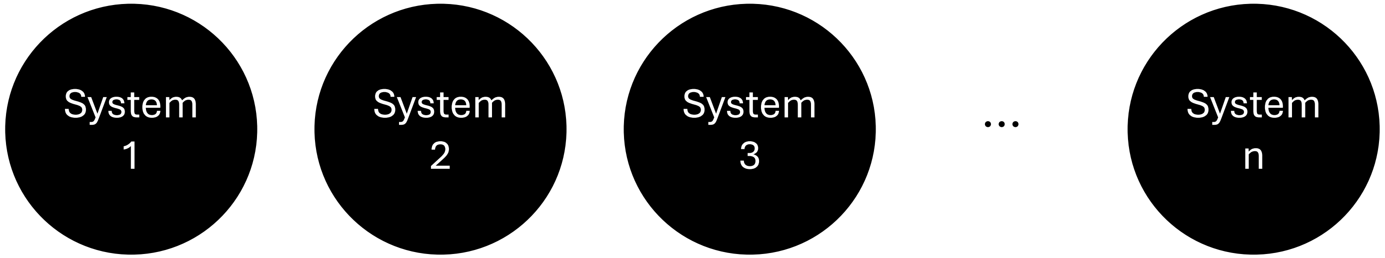


Treat a qubit as a mathematic object
(regardless of its physical meaning)
so we can focus on quantum computing
without diving into physics, philosophy, …

Multiple Qubits



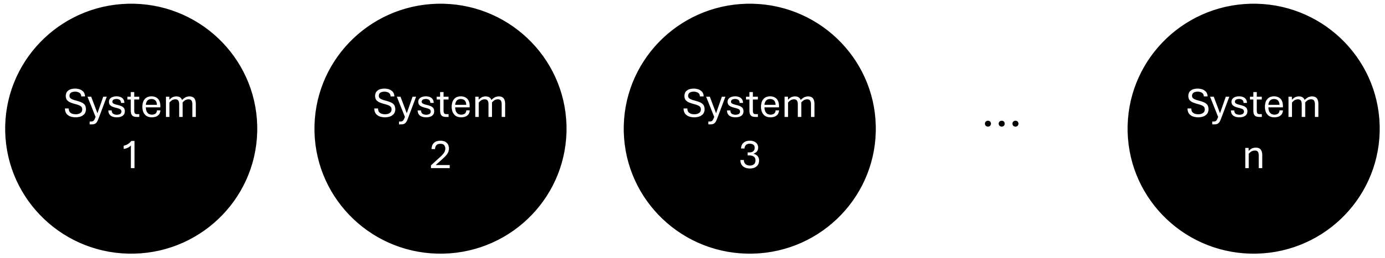
Multiple Qubits



$|\phi_1\rangle$ $|\phi_2\rangle$ $|\phi_3\rangle$ $|\phi_n\rangle$

The state of the composite system: $|\psi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \otimes |\phi_3\rangle \otimes \cdots \otimes |\phi_n\rangle$, $\otimes$: Tensor product

Multiple Qubits



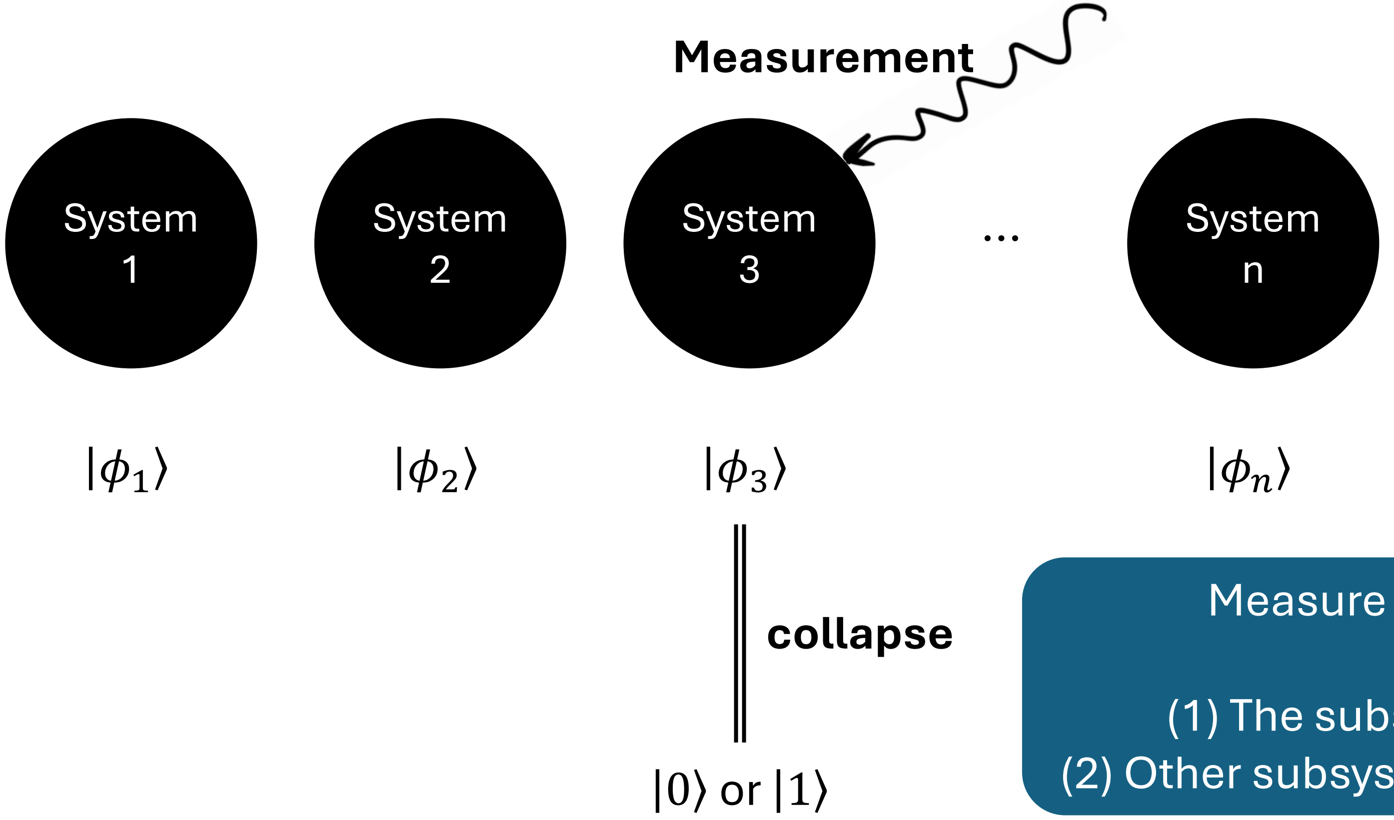
$|\phi_1\rangle$ $|\phi_2\rangle$ $|\phi_3\rangle$ $|\phi_n\rangle$

The state of the composite system: $|\psi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \otimes |\phi_3\rangle \otimes \cdots \otimes |\phi_n\rangle$, $\otimes$: Tensor product

Examples: $|0\rangle \otimes |1\rangle \otimes |1\rangle \otimes |1\rangle = |0111\rangle,\ \ |1\rangle \otimes |0\rangle \otimes |1\rangle \otimes |0\rangle \otimes |1\rangle = |10101\rangle$

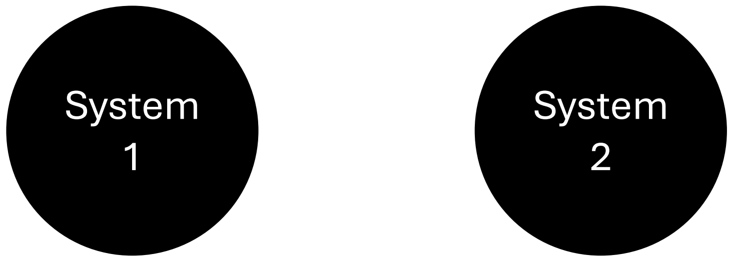
$|0\rangle \otimes |1\rangle \otimes (\alpha|0\rangle + \beta|1\rangle) \otimes |1\rangle$, $|0\rangle \otimes |1\rangle \otimes \frac{|0\rangle+|1\rangle}{\sqrt{2}} \otimes |1\rangle$

Multiple Qubits



Measure one subsystem
=>
(1) The subsystem collapses
(2) Other subsystems remain the same

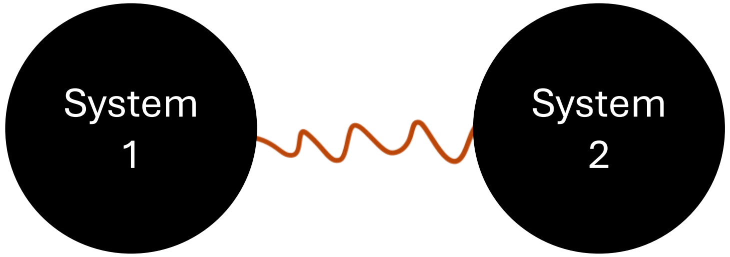
Entanglement



$|\phi_1\rangle$ $\quad\quad$ $|\phi_2\rangle$

$$|\psi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle$$

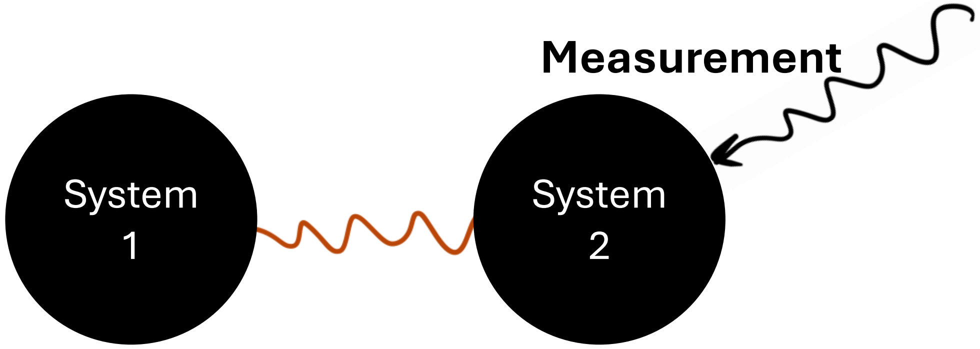
Entanglement



$$|\boldsymbol{\psi}\rangle$$

$$\cancel{|\psi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle}$$

Entanglement



$|\psi\rangle$

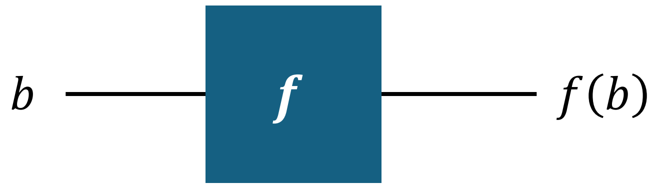
collapse

some basic state

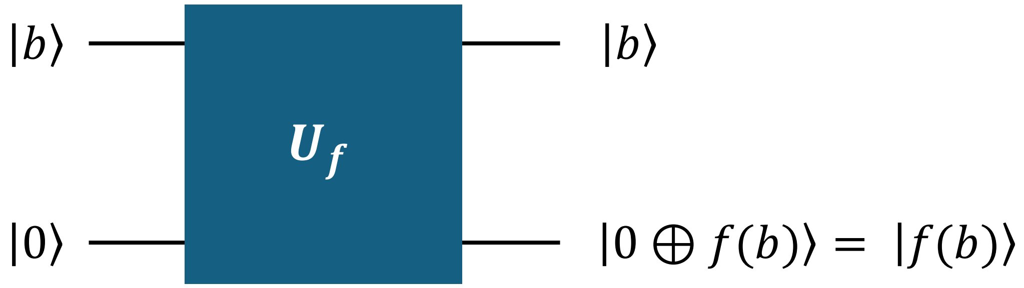
Measure one subsystem
=>
The whole system collapses

Unitaries and Superposition Evaluation

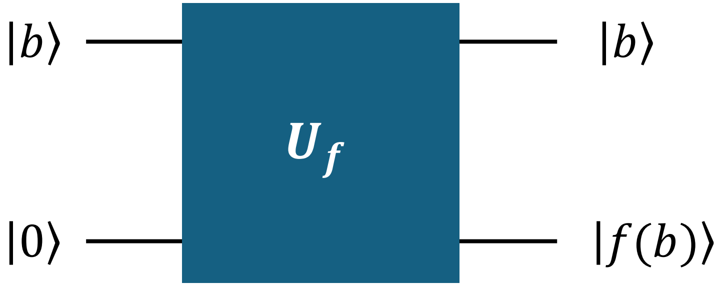
Let $f:\{0,1\} \to \{0,1\}$ be a classical bit function:



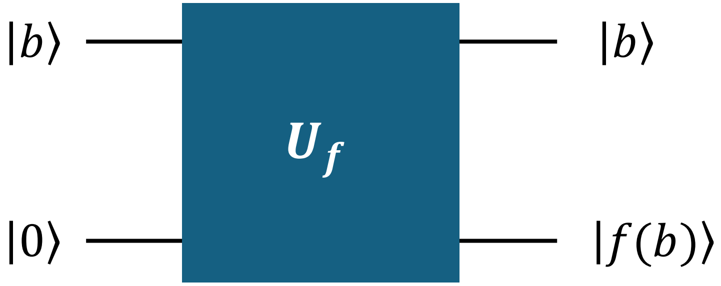
The "quantum version" of f:

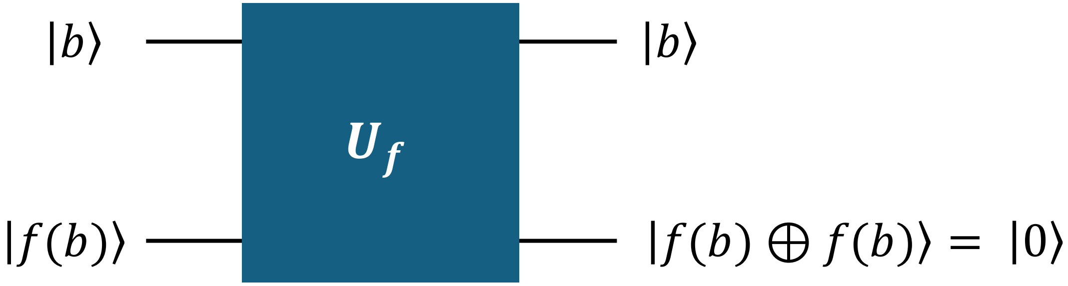


Unitaries and Superposition Evaluation



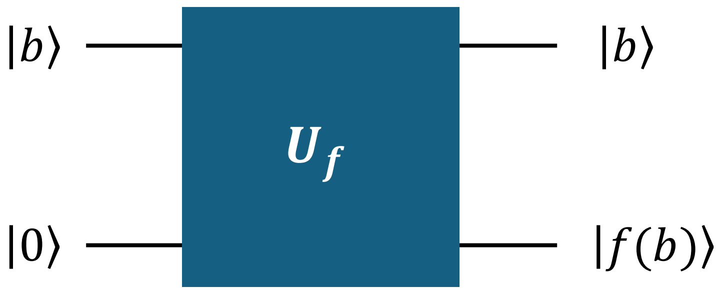
Unitaries and Superposition Evaluation

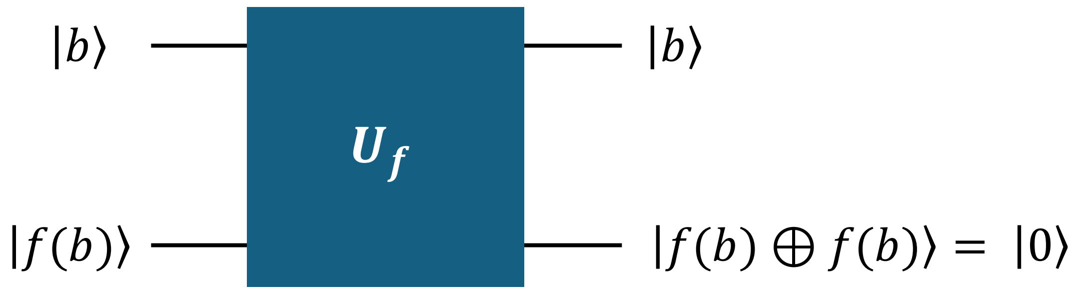


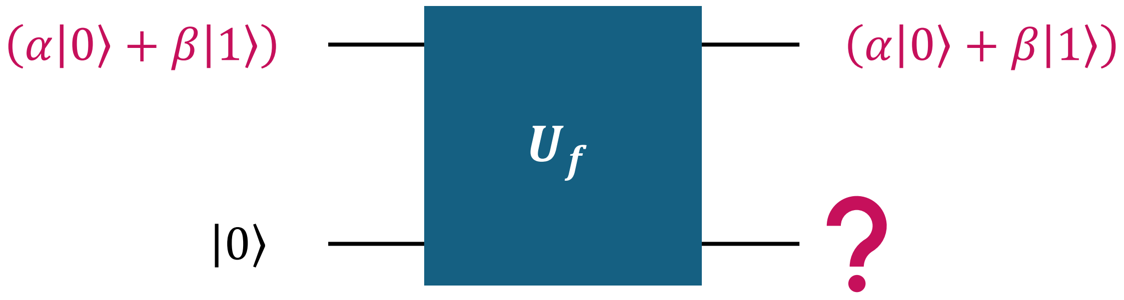


Reversible Computation

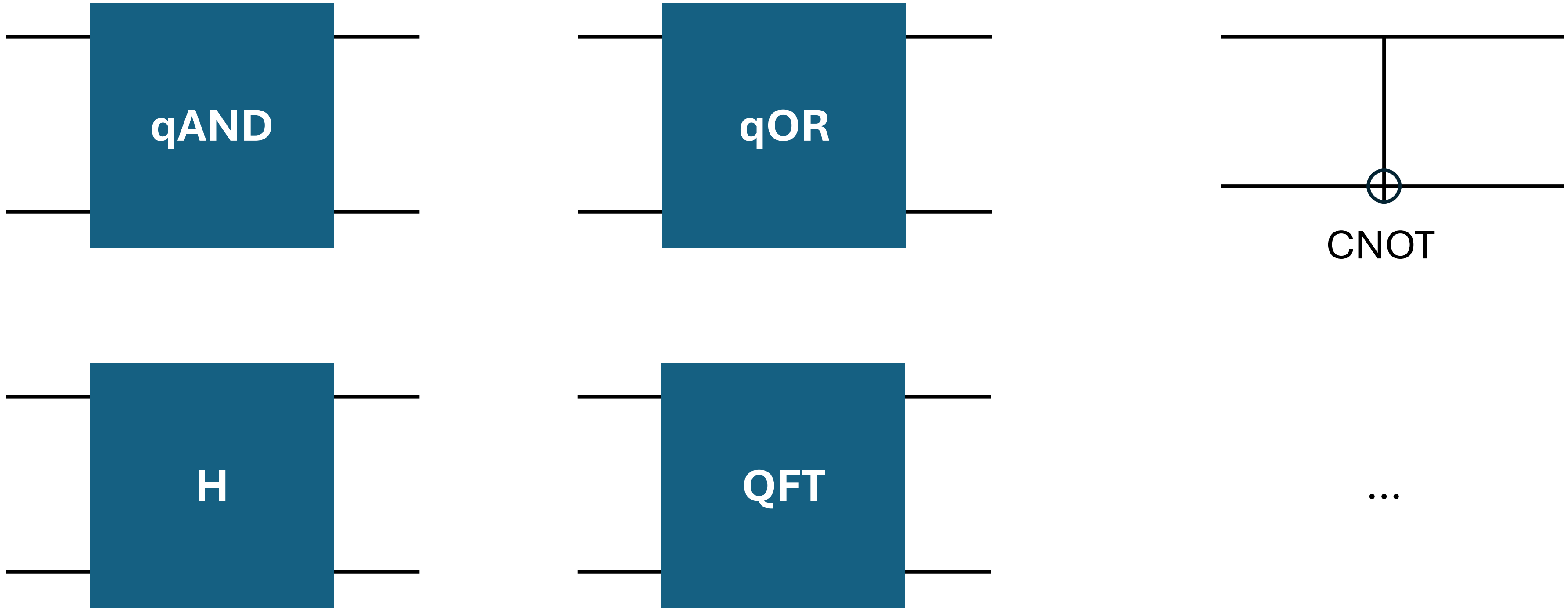
Unitaries and Superposition Evaluation



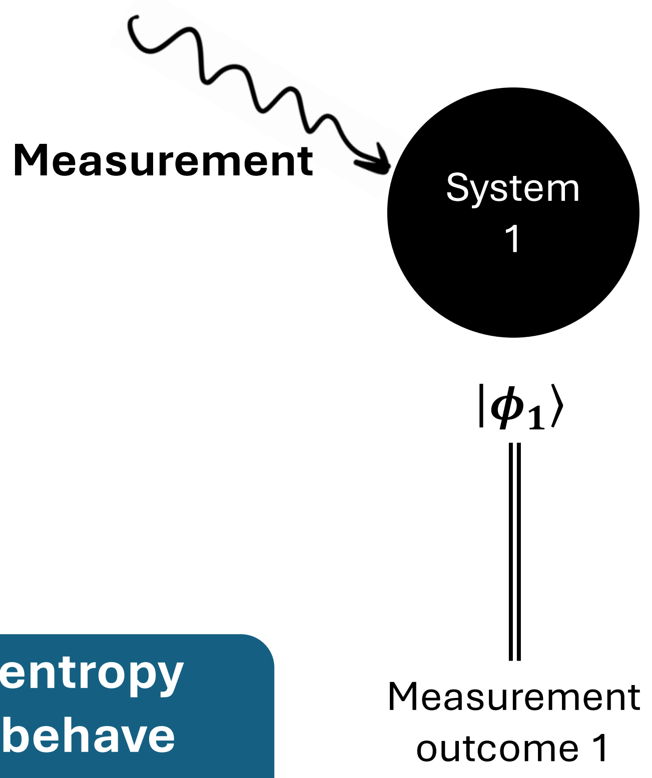




Quantum Gates and Algorithms



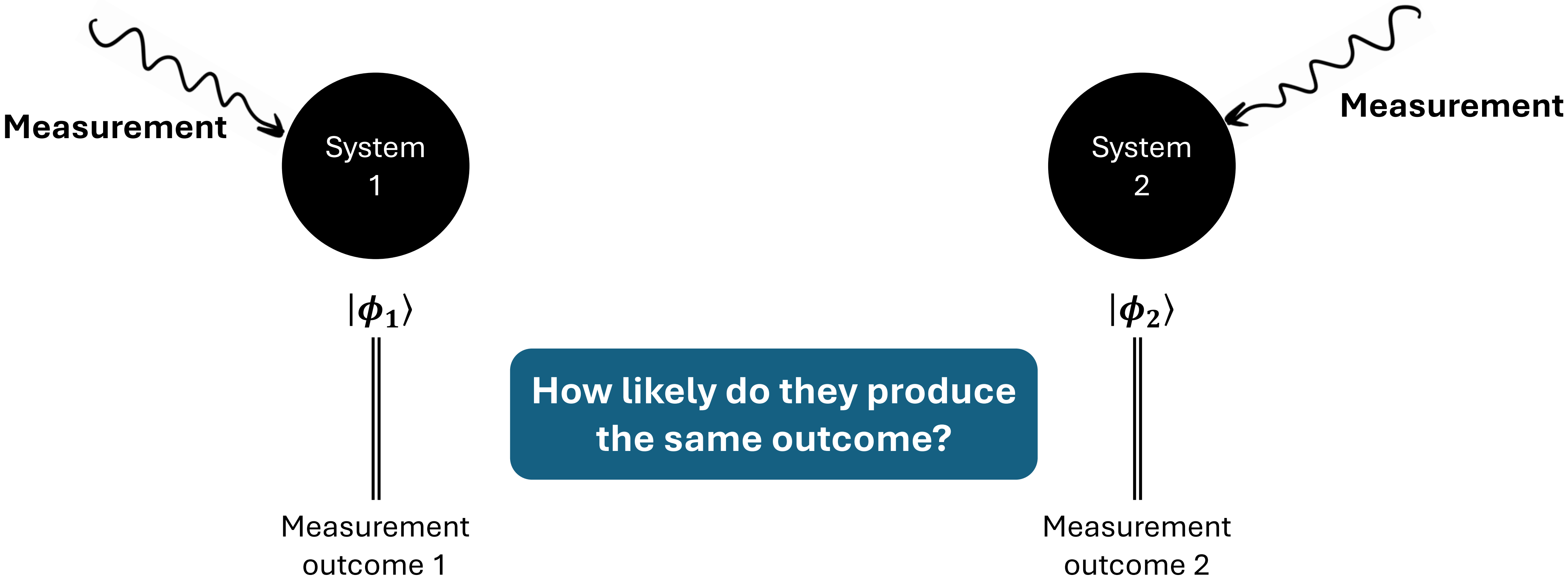
Quantum Information – Entropy and Randomness



How does the entropy of the system behave after measurement?

How much "randomness" does it provide?

Quantum Information - Distinguishability



Thursday’s Topic

- Quantum state, qubit, and their linear algebra formulation

- Bring your **pen** and **paper**