Cryptography Engineering

- Lecture 6 (Nov 26, 2025)
- Today's notes:
  - Fujisaki-Okamoto transformation – 2
  - Authentication via KEM
  - PQ-TLS: KEM + Sign
  - KEM-TLS

Fujisaki-Okamoto Transformation

- Let PKE = (KG, Enc, Dec) be a public-key encryption scheme
- Let H, G be two hash functions (Quick question: How to instantiate them using SHA256)
- We construct an FOKEM scheme based on PKE

KeyGen:
1. $(pk, sk) \leftarrow$ KG
2. $prk \leftarrow \{0,1\}^L$
3. $pk := pk$
4. $sk := (prk, sk)$

Encaps$(pk = pk)$:
1. $m \leftarrow_{\$}$ MsgSpace
2. $r := G(pk, m)$
   // randomness for PKE
3. $c :=$ Enc$(pk, m; r)$
4. $K := H(pk, m, c)$
5. $c := c$
6. return (c, K)

Decaps$(sk = (prk, sk), c = c)$:
1. $m =$ Dec(sk, c)
2. $r := G(pk, m)$
   // Recover randomness
3. $c' :=$ Enc$(pk, m; r)$
   // Re-encryption check
4. If $c == c'$: return $H(pk, m, c)$
5. Else: return $H(pk, prk, c)$

Fujisaki-Okamoto Transformation

- **Implicit rejection:** Return a pseudorandom key (rather than returning a rejection symbol).

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  - Quick question: Why the returning key (if the re-enc check fails) is pseudorandom?

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Fujisaki-Okamoto Transformation

- **Implicit rejection:** Return a pseudorandom key (rather than returning a rejection symbol).
  - Quick question: Why the returning key (if the re-enc check fails) is pseudorandom?
  - (1) prk is random and secret => H(pk, prk, c) also looks like random
  - (2) The same input to Decaps => The same output

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Fujisaki-Okamoto Transformation

- **Explicit rejection:** Return a rejection symbol (or throw an exception)

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2. ~~$prk \leftarrow \{0,1\}^L$~~
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5. Else: return **REJECT**

Fujisaki-Okamoto Transformation

- Constant-time comparison
  - Regular equality checks may leak timing information
  - E.g., early-return behavior can reveal where the first differing bit occurs

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Fujisaki-Okamoto Transformation

- Constant-time comparison
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CT-Decaps$(sk = (prk, sk), c = c)$:

1. $m = \text{Dec}(sk, c)$
2. $r := G(pk, m)$
3. $c' := \text{Enc}(pk, m; r)$
4. $K_0 = H(pk, m, c)$
5. $K_1 = H(pk, prk, c)$
6. $b =$ **constant-time-eq**(c', c)
7. $K =$ **constant-time-select**(b, K_0, K_1)
   // $K = (1 - b) \cdot K_0 \oplus b \cdot K_1$
8. return K

Decaps$(sk = (prk, sk), c = c)$:

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4. If $c == c'$: return $H(pk, m, c)$
5. Else: return $H(pk, prk, c)$

Post-quantum Cryptography

- Most widely deployed cryptosystems rely on the hardness of Diffie–Hellman and Factoring (RSA).
- However, these problems are no longer considered hard in the presence of large-scale quantum computers.

- **Post-quantum cryptography:**
  - Cryptographic algorithms run in classical computers
  - Security against adversaries with quantum computers

- NIST PQC standardized algorithms:
  - KEM schemes: ML-KEM (Crystal-Kyber)
  - Signature schemes: ML-DSA (Crystal-Dilithium)

Post-quantum Cryptography

- Post-quantum secure KEM
  - Design a IND-CPA PKE scheme first
  - Use the FO transform (or its variants) to get an IND-CCA KEM scheme

- Post-quantum secure Signature
  - More complex structures…

Post-quantum TLS

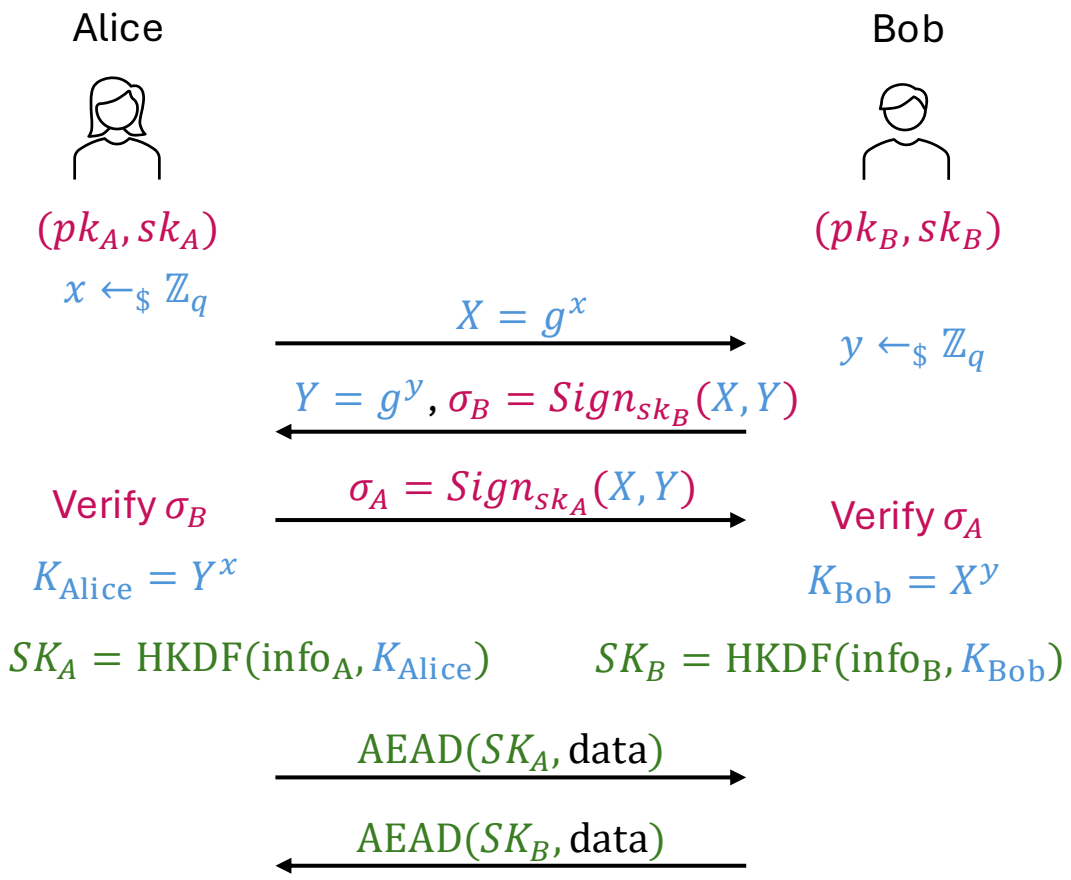
- Post-quantum secure KEM
  - Design a IND-CPA PKE scheme first
  - Use the FO transform (or its variants) to get an IND-CCA KEM scheme

- Post-quantum secure Signature
  - More complex structures…

- TLS 1.3 is based on DH problems, so it is not post-quantum secure.
  - **Can we have post-quantum TLS?**

Post-quantum TLS

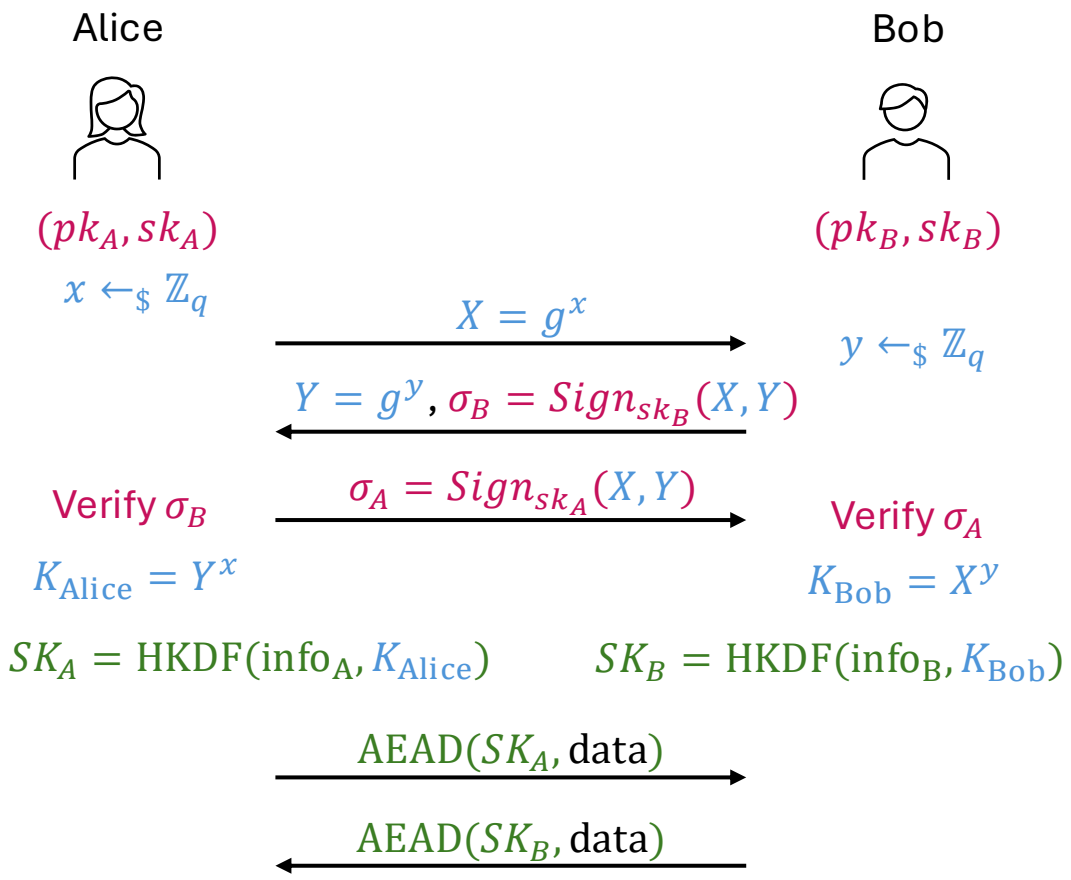
- The signed DH protocol



Post-quantum TLS

- The signed DH protocol

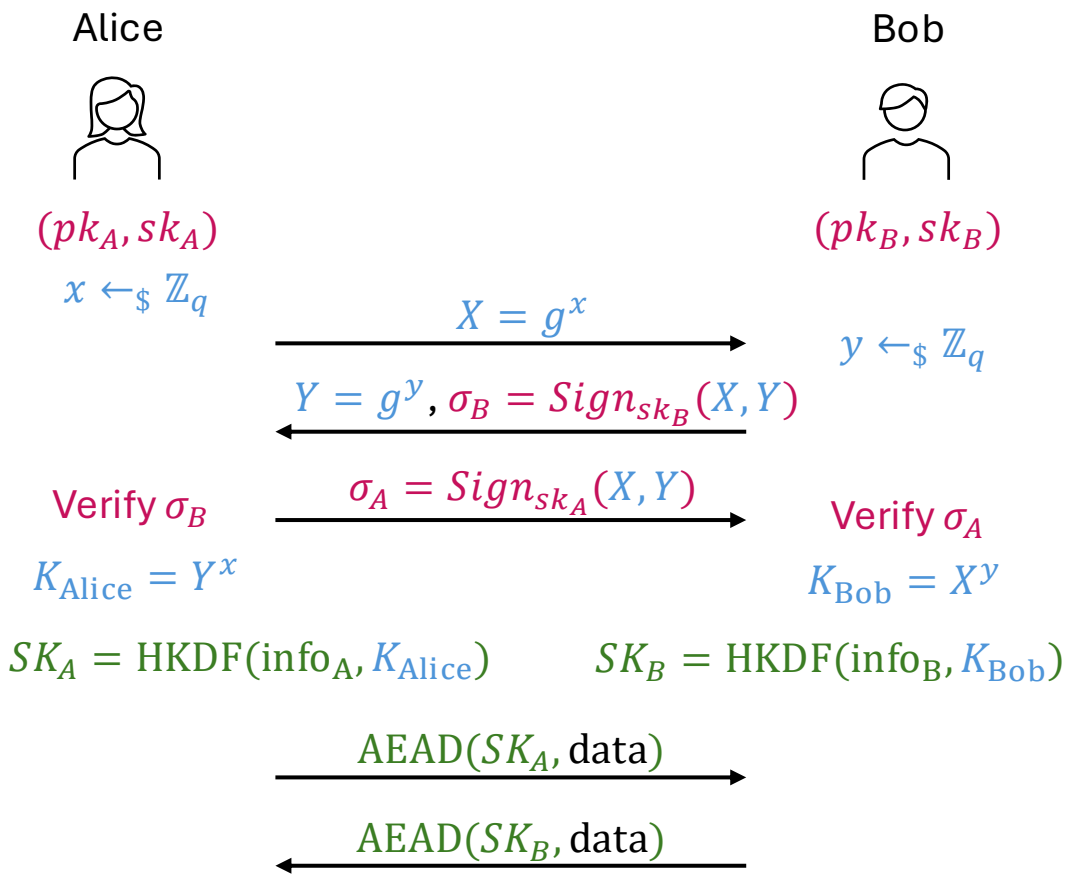
- To make it post-quantum secure, which parts should we change?



Post-quantum TLS

- The signed DH protocol

- To make it post-quantum secure, which parts should we change?
  - Symmetric algorithms remain secure
  - **The DHKE and the signature scheme (e.g., ECDSA, RSA) are not post-quantum secure**



Post-quantum TLS

- The signed DH protocol
- To make it post-quantum secure, which parts should we change?
  - Symmetric algorithms remain secure
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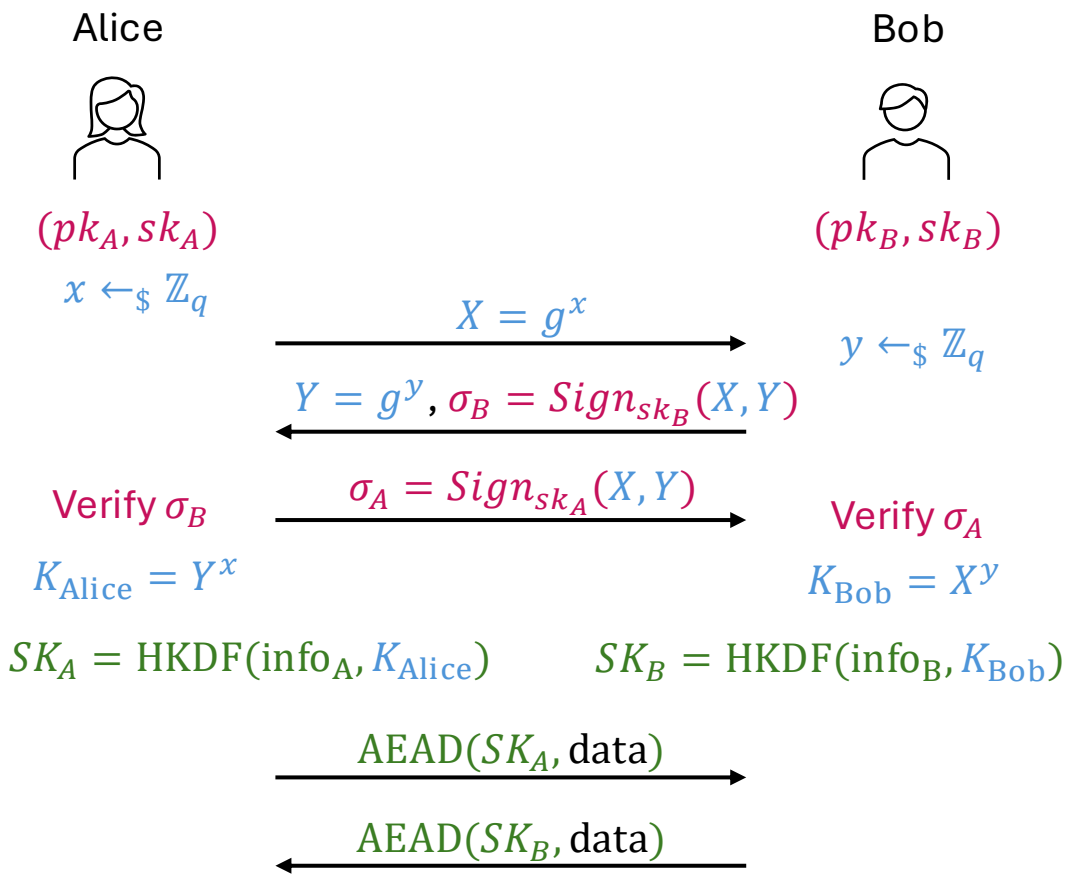
- We first replace the signature with ML-DSA (or other post-quantum secure signature schemes)



Post-quantum TLS

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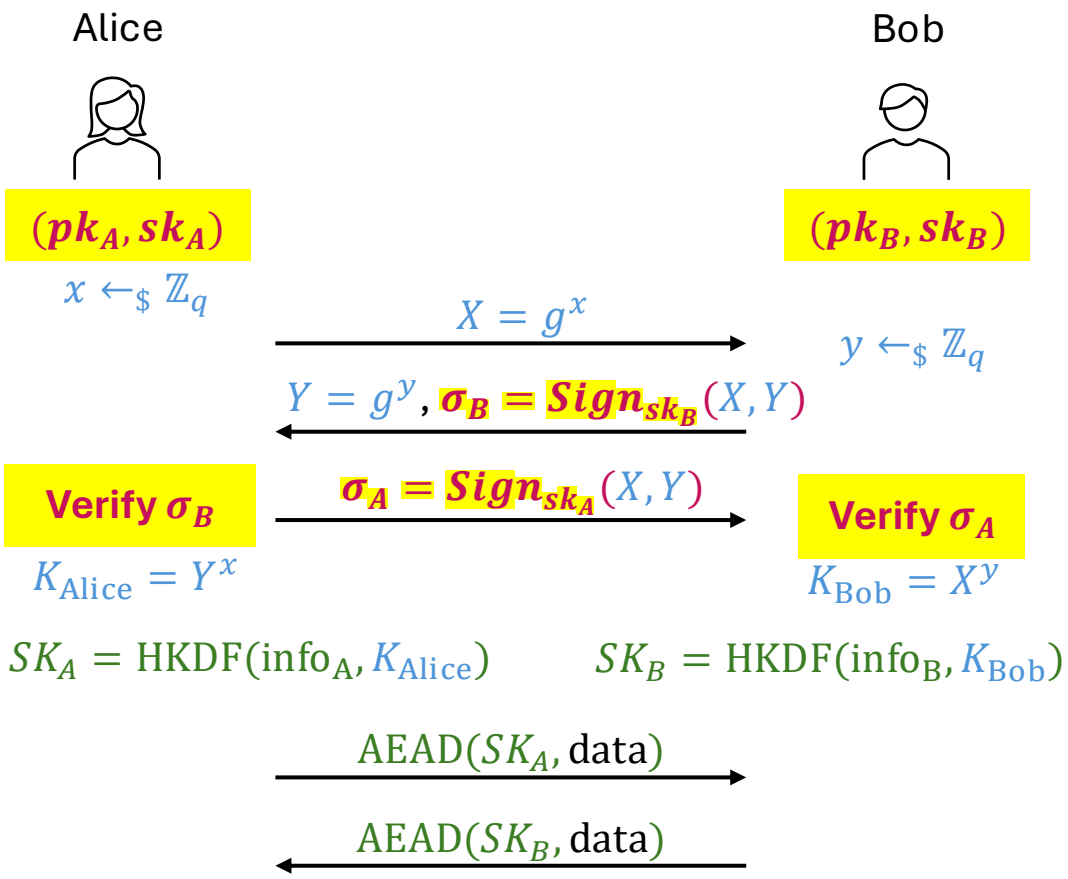
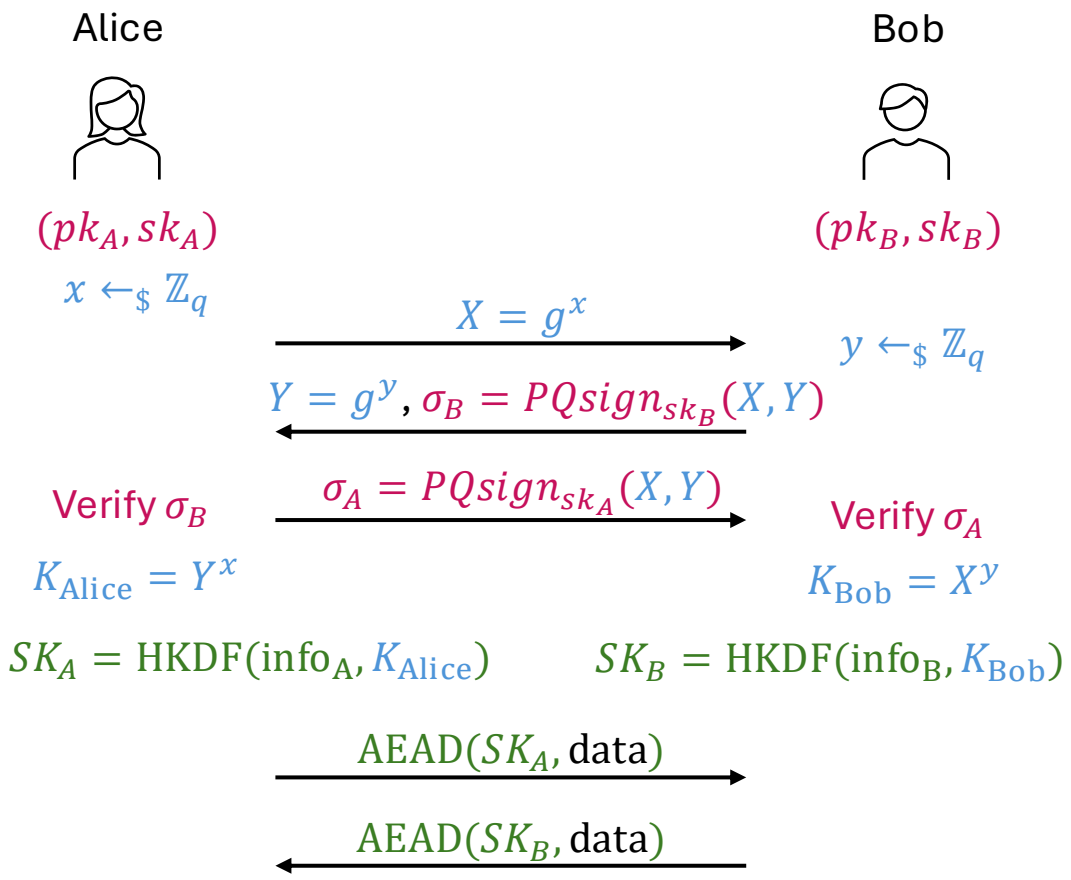


Post-quantum TLS

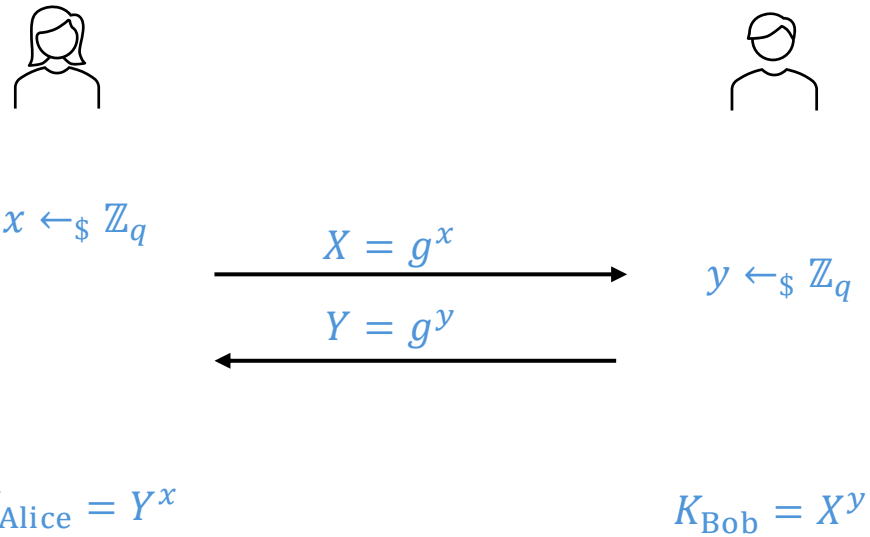
- The DH + PQsign protocol

- How can we replace the DH part?



Post-quantum TLS

- DH key exchange



Post-quantum TLS

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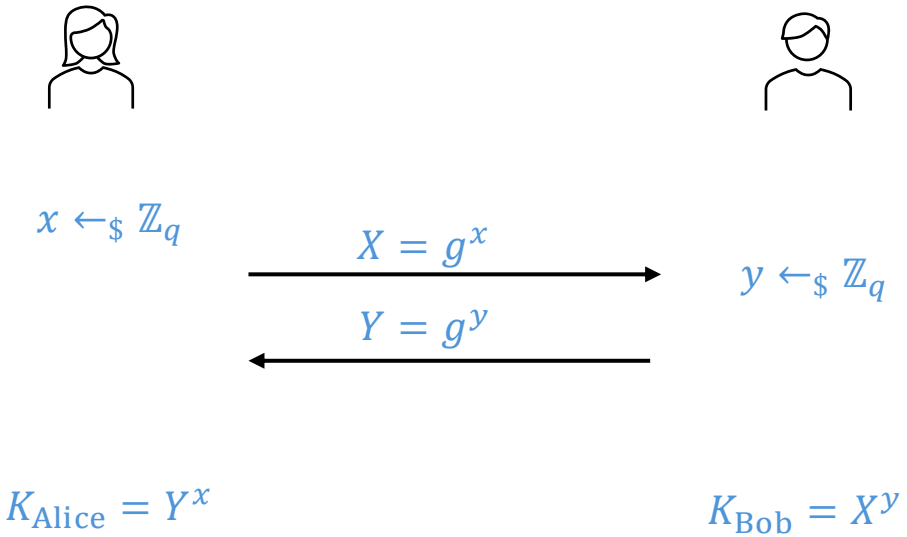
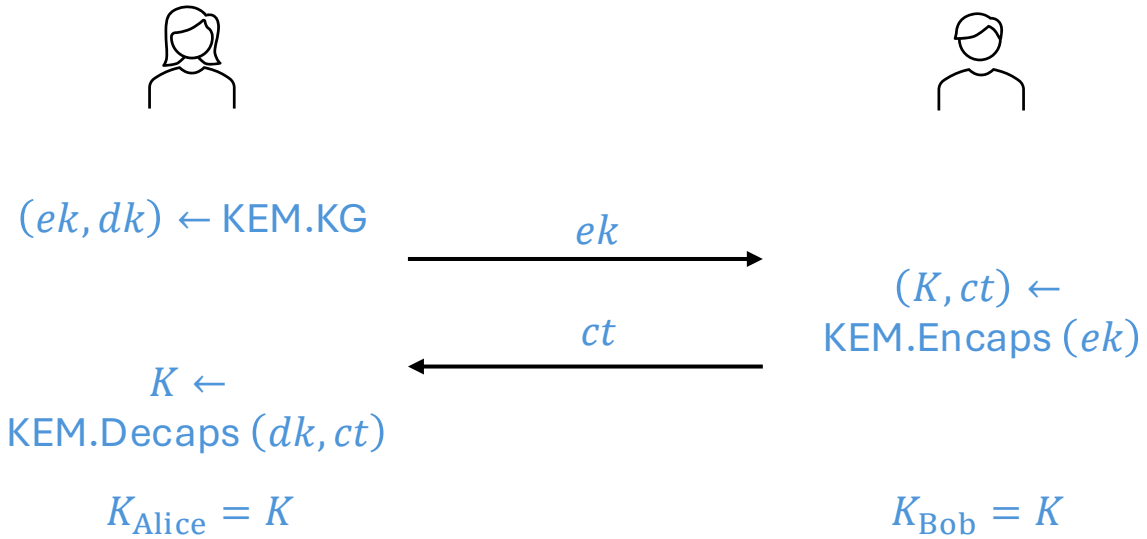
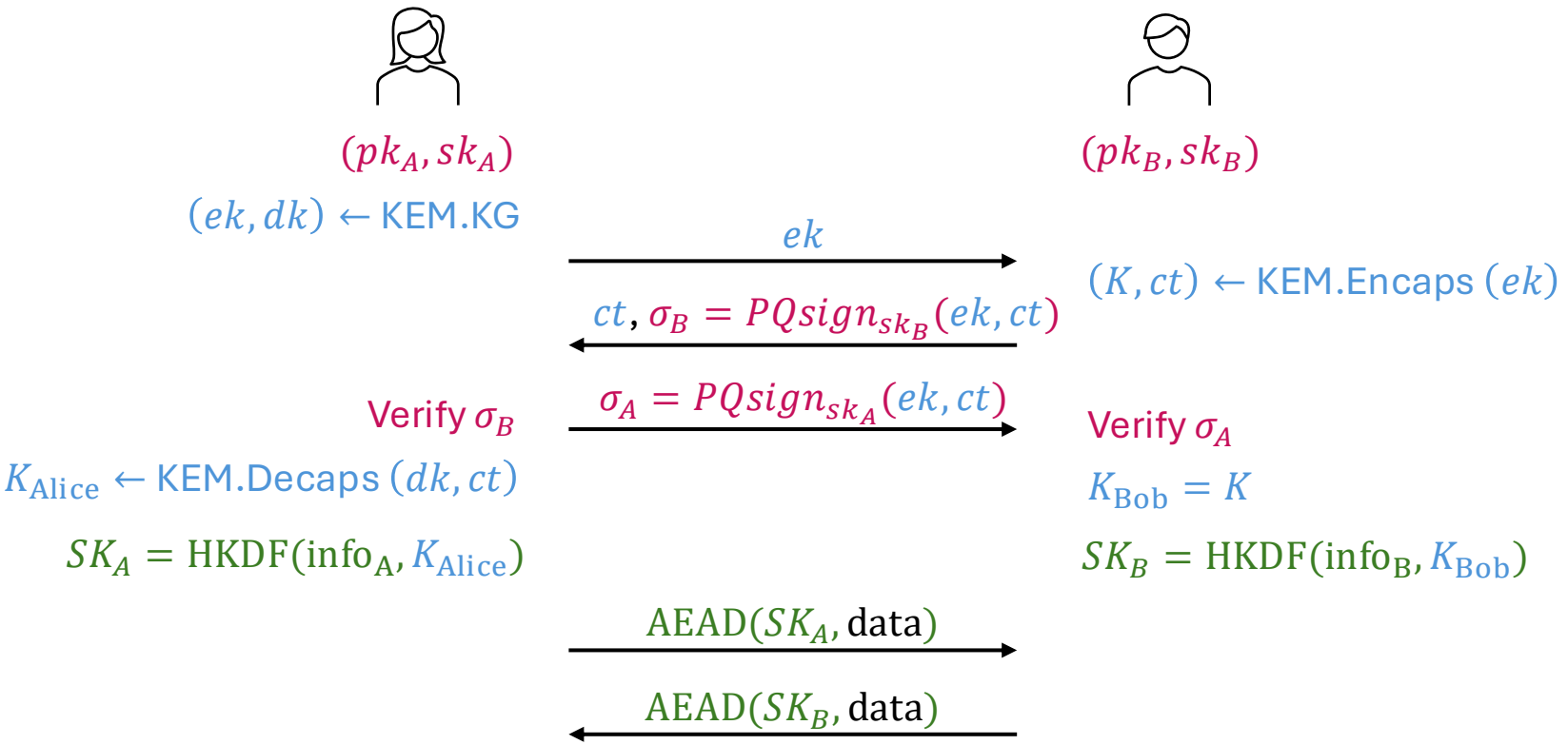


- KEM-based key exchange



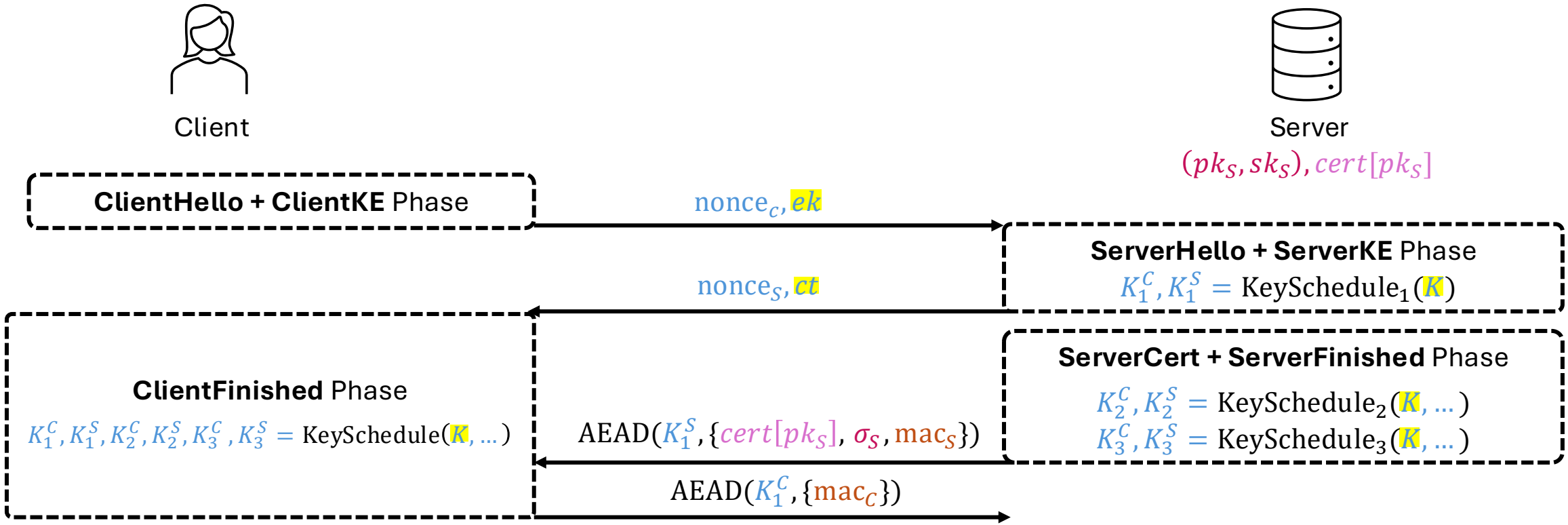
Post-quantum TLS

- PQKEM + PQSign = PQTLS (demo)



Post-quantum TLS

- Integrate the PQTLS into the TLS 1.3 framework (Homework 1)



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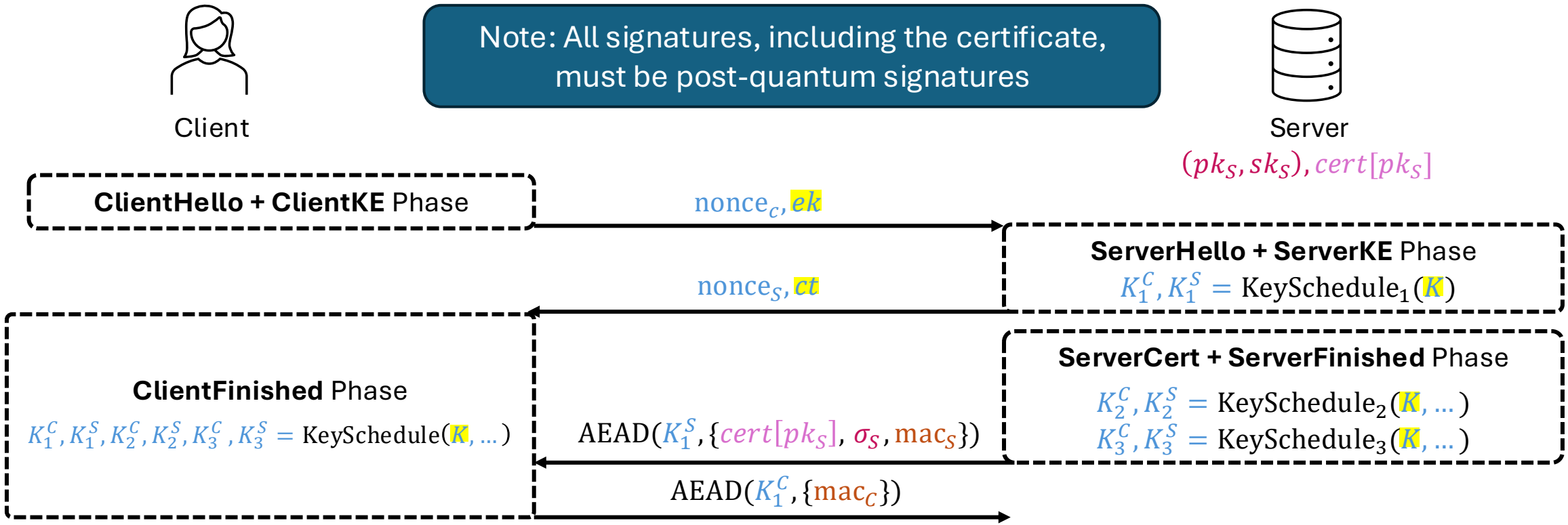

Authentication via KEM

- In post-quantum cryptography, signature schemes are generally less efficient than KEM

- In TLS 1.3, we use signature to
  - (1) Certify the server's public key (signed by the CA)
  - (2) Authenticate the serverKE message (signed by the server)

Authentication via KEM

- In post-quantum cryptography, signature schemes are generally less efficient than KEM

- In TLS 1.3, we use signature to
  - (1) Certify the server's long-term public key (signed by the CA) …one-time price
  - (2) **Authenticate the serverKE message (signed by the server)** **…needed in every session**

Authentication via KEM

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- In TLS 1.3, we use signature to
  - (1) Certify the server's public key (signed by the CA) …one-time price
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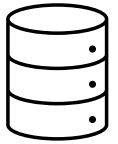
- **Can we replace the second part with KEM?**

Authentication via KEM

- Use KEM to authenticate messages



$(pk_S, sk_S), cert[pk_S]$

$cert[pk_S],$
$m, \sigma_S = Sign_{sk_S}(m)$

Verify $cert[pk_S]$
Verify σ_S

Only the sk owner can generate the signature
=> If σ_S verifies successfully, then m is from the server

Authentication via KEM

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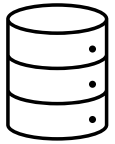

$(pk_{KEM}, sk_{KEM}), cert[pk_{KEM}]$

When using KEM:
having the sk ⇔ Decrypt ciphertexts

Authentication via KEM

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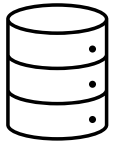

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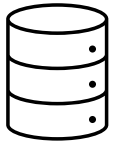

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$(ct, K) \leftarrow$ Encaps(pk_{KEM})

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$ct, nonce$

$K \leftarrow$ Decaps(sk_{KEM}, ct)

Authentication via KEM

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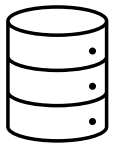

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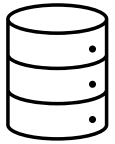
$\tau =$ HMAC$(K, ct, nonce)$

Verify τ

Authentication via KEM

- Use KEM to authenticate messages

Having sk ⇔ Decrypt ct and get K ⇔ Generate a valid HMAC



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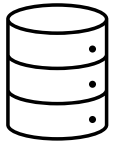

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Verify τ

More message flows…

KEM-TLS

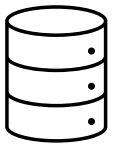
- **KEM-TLS (demo):** Integrate the KEM authentication into the TLS framework

$(ek, dk) \leftarrow$ KEM.KG

Random nonce_c

ClientHello + ClientKE

nonce_c, ek



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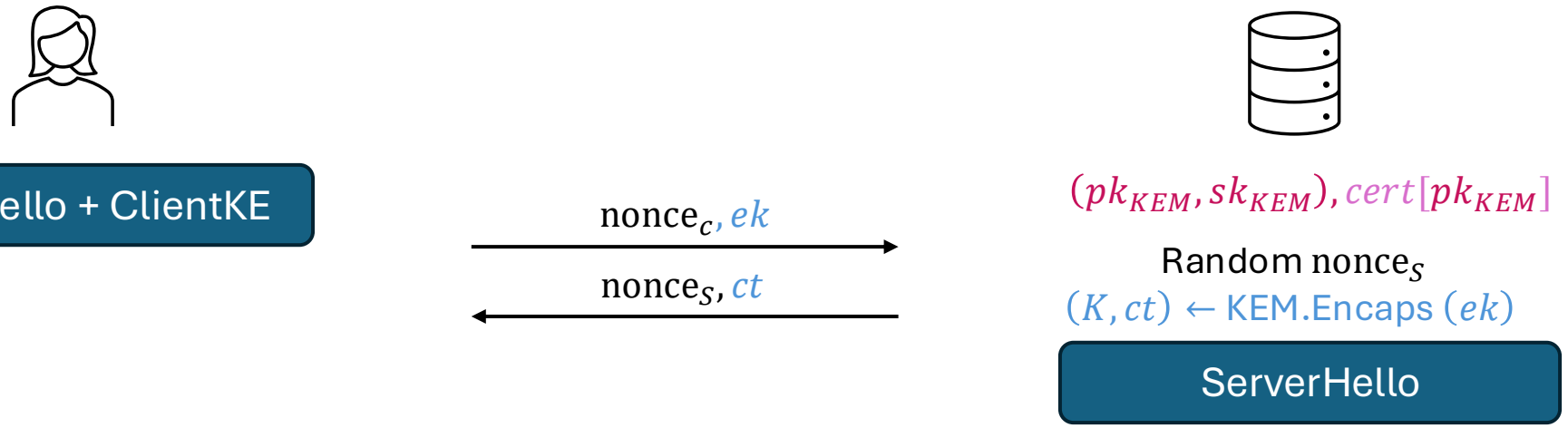


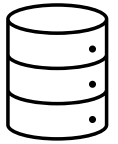
KEM-TLS

- **KEM-TLS (demo):** Integrate the KEM authentication into the TLS framework

ClientHello + ClientKE

nonce_c, ek →

← nonce_S, ct



$(pk_{KEM}, sk_{KEM}), cert[pk_{KEM}]$

ServerHello

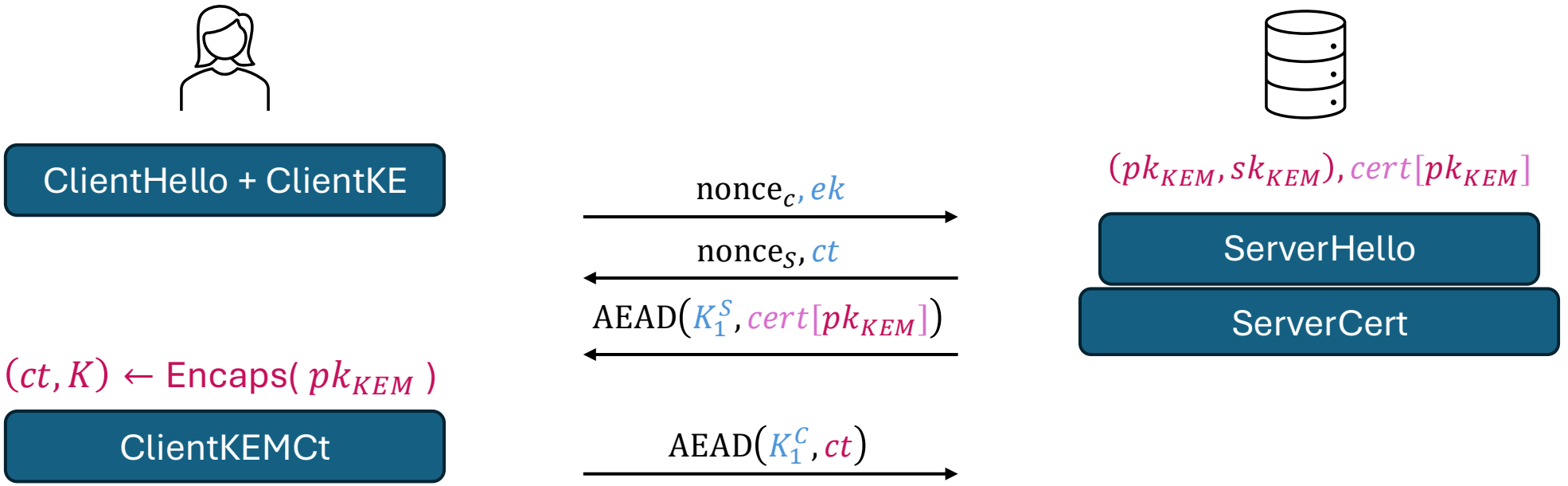
$K_1^C, K_1^S = \text{KeySchedule}_1(K)$

← $\text{AEAD}(K_1^S, cert[pk_{KEM}])$

ServerCert

KEM-TLS

- **KEM-TLS (demo):** Integrate the KEM authentication into the TLS framework



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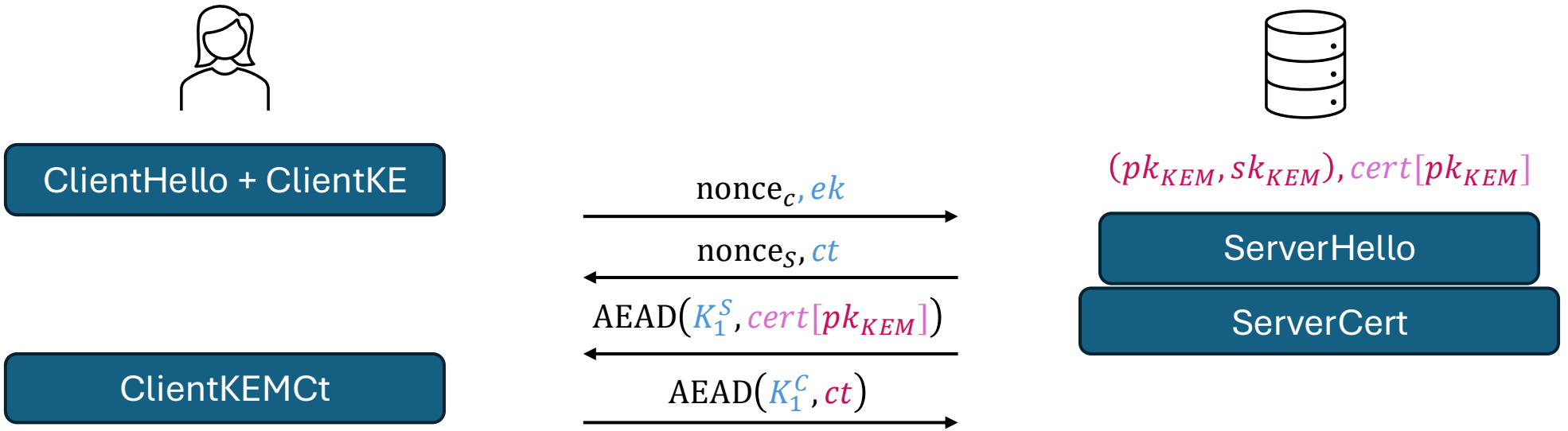
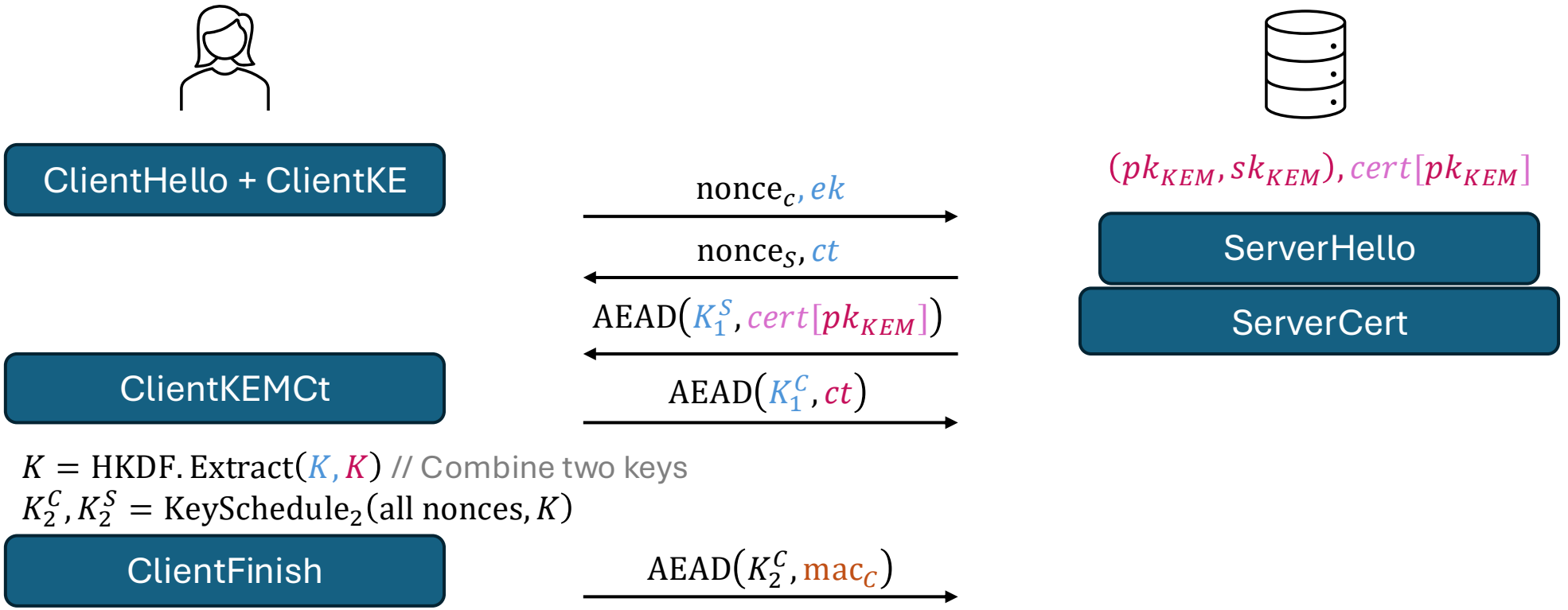


$K = \text{HKDF.Extract}(K, K)$ // Combine two keys

$K_2^C, K_2^S = \text{KeySchedule}_2(\text{all nonces}, K)$

KEM-TLS

- **KEM-TLS (demo):** Integrate the KEM authentication into the TLS framework



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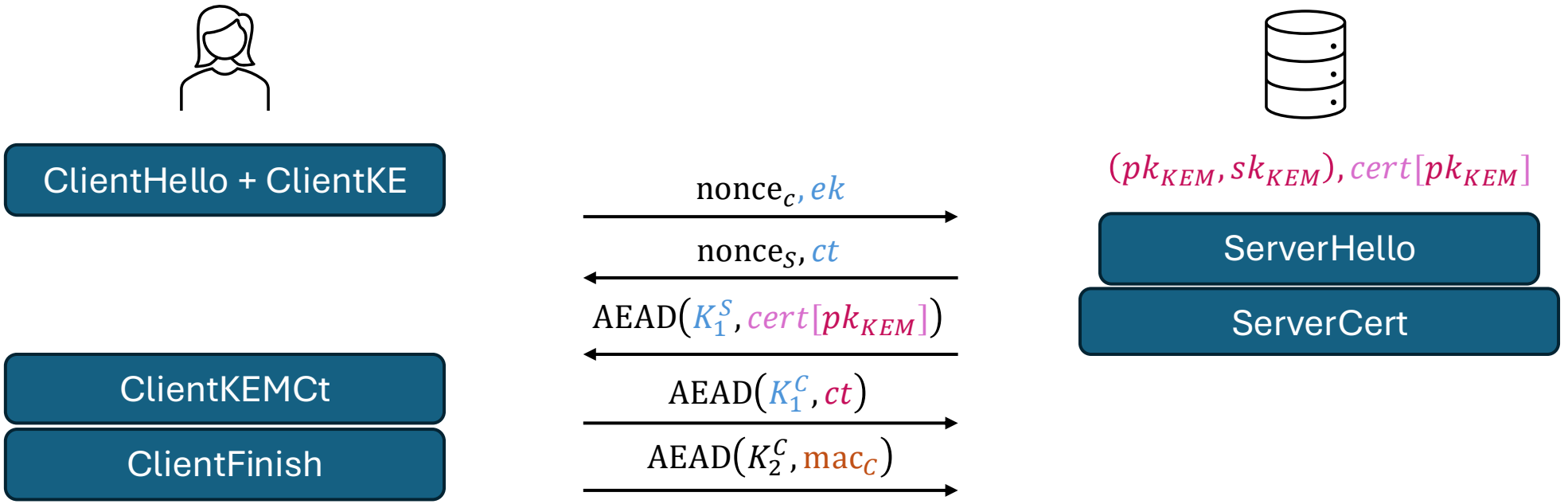

KEM-TLS

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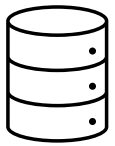
ClientHello + ClientKE

nonce_C, ek →

← nonce_S, ct

← $\text{AEAD}(K_1^S, cert[pk_{KEM}])$



$(pk_{KEM}, sk_{KEM}), cert[pk_{KEM}]$

ServerHello

ServerCert

ClientKEMCt

ClientFinish

→ $\text{AEAD}(K_1^C, ct)$

→ $\text{AEAD}(K_2^C, \text{mac}_C)$

Decrypt ct

$K = \text{HKDF.Extract}(K, K)$ // Combine two keys

$K_2^C, K_2^S = \text{KeySchedule}_2(\text{all nonces}, K)$

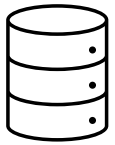
KEM-TLS

ClientHello + ClientKE

nonce_C, ek →

← nonce_S, ct

← $\text{AEAD}(K_1^S, cert[pk_{KEM}])$



$(pk_{KEM}, sk_{KEM}), cert[pk_{KEM}]$

ServerHello

ServerCert

ClientKEMCt

ClientFinish

→ $\text{AEAD}(K_1^C, ct)$

→ $\text{AEAD}(K_2^C, \text{mac}_C)$

← $\text{AEAD}(K_2^S, \text{mac}_S)$

ServerFinish

← (Encrypt data using K_3^S)

→ (Encrypt data using K_3^C)

Homework

- 1. Find Kyber (or ML-KEM) and Dilithium (or ML-DSA) implementations in your programming language. Then implement the PQ-TLS protocol.
- 2. Find Kyber (or ML-KEM) implementations in your programming language, then implement the KEM-TLS protocol.
- 3. Compare the efficiency between your PQ-TLS and KEM-TLS implementations.
- **DDL: Jan 09th, 2026 at 23:59**
- Rust:
  - https://docs.rs/ml-kem/latest/ml_kem/
  - https://docs.rs/ml-dsa/latest/ml_dsa/
- Python:
  - https://github.com/GiacomoPope/kyber-py
  - https://github.com/GiacomoPope/dilithium-py

Reference

- PQ-TLS: https://cic.iacr.org/p/1/2/6
- KEM-TLS: https://kemtls.org/
- Crystal-Kyber and ML-KEM: https://nvlpubs.nist.gov/nistpubs/FIPS/NIST.FIPS.203.pdf
- Crystal-Dilithium and ML-DSA: https://nvlpubs.nist.gov/nistpubs/FIPS/NIST.FIPS.204.pdf