Cryptography Engineering

- Lecture 6 (Nov 27, 2024)

- Today's notes:
  - Double Ratchet Algorithm
  - Signal Secure Messaging Protocol
  - Introduction to Password Login

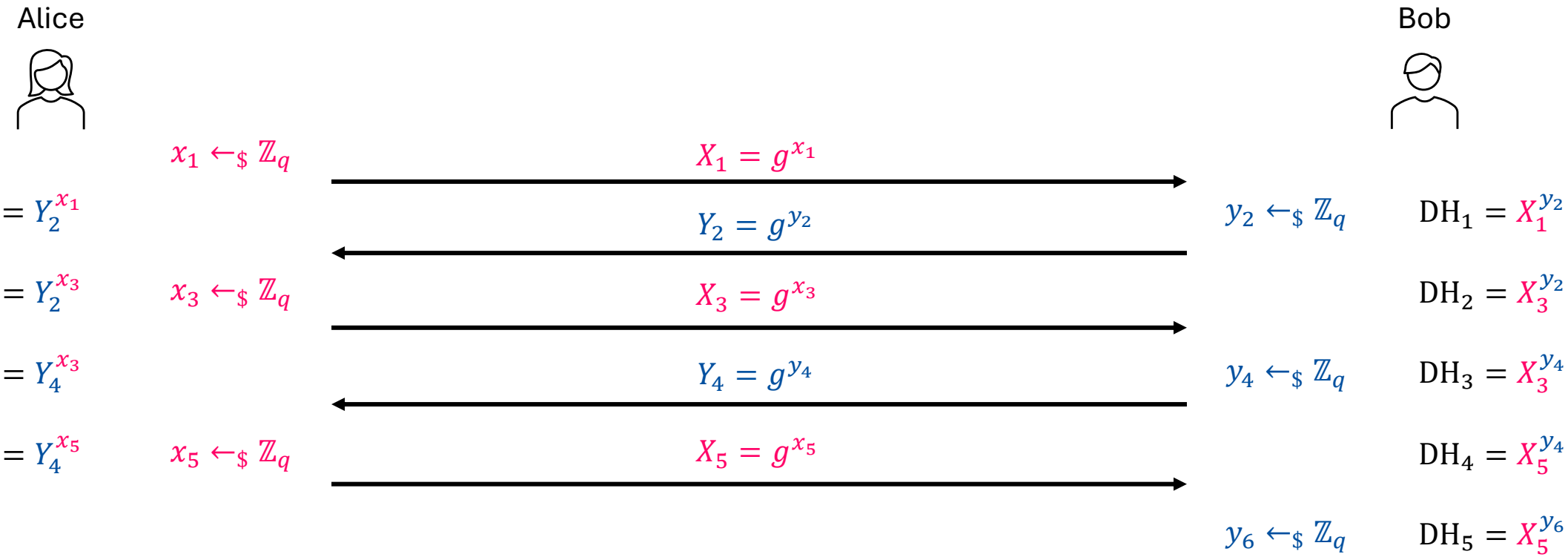
- No homework

Double Ratchet

- The main idea: Symmetric-key Ratchet + **Diffie-Hellman Ratchet**

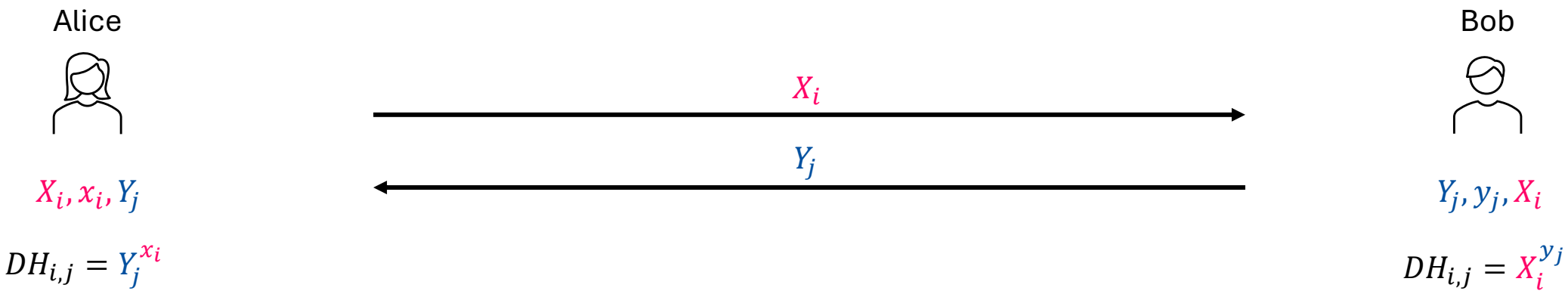
Double Ratchet – DH Ratchet

- Main idea of DH Ratchet: Running DHKE continuously with *rotating ephemeral keys*…



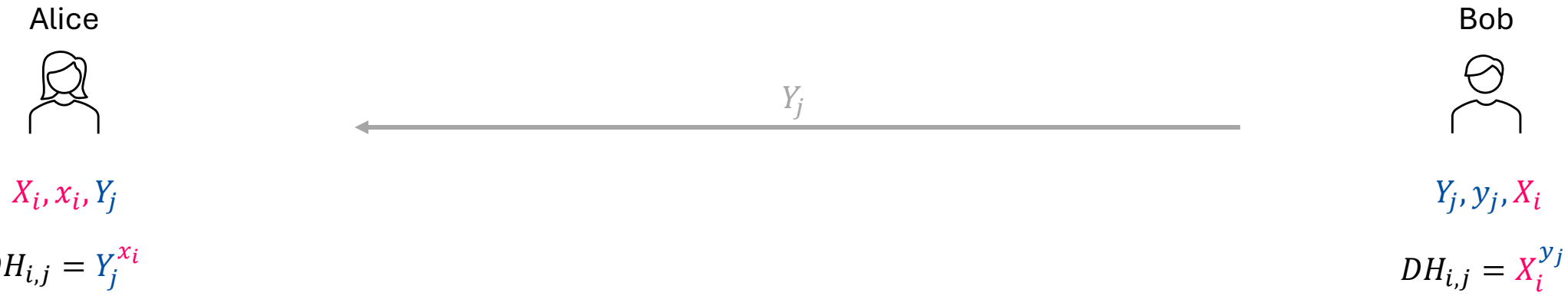
Double Ratchet – DH Ratchet

- Main idea of DH Ratchet: Running DHKE continuously with *rotating ephemeral keys*…



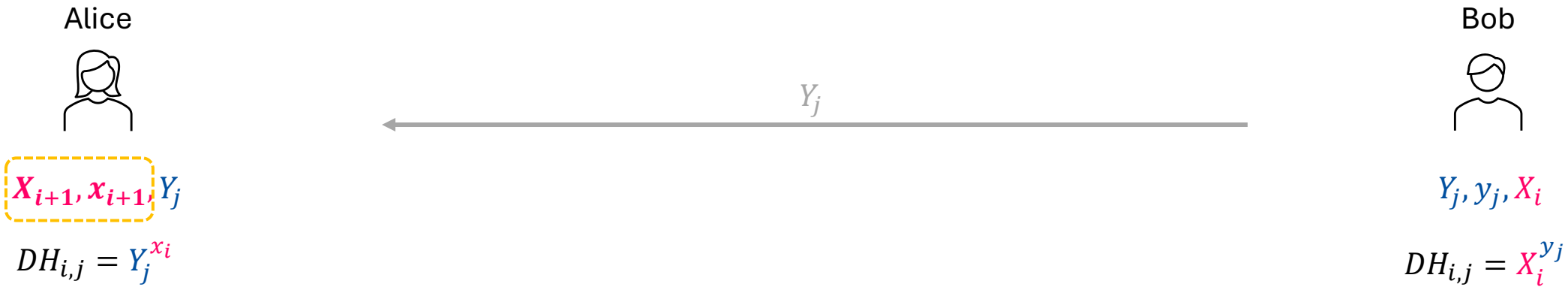
Double Ratchet – DH Ratchet

- Main idea of DH Ratchet: Running DHKE continuously with *rotating ephemeral keys*…



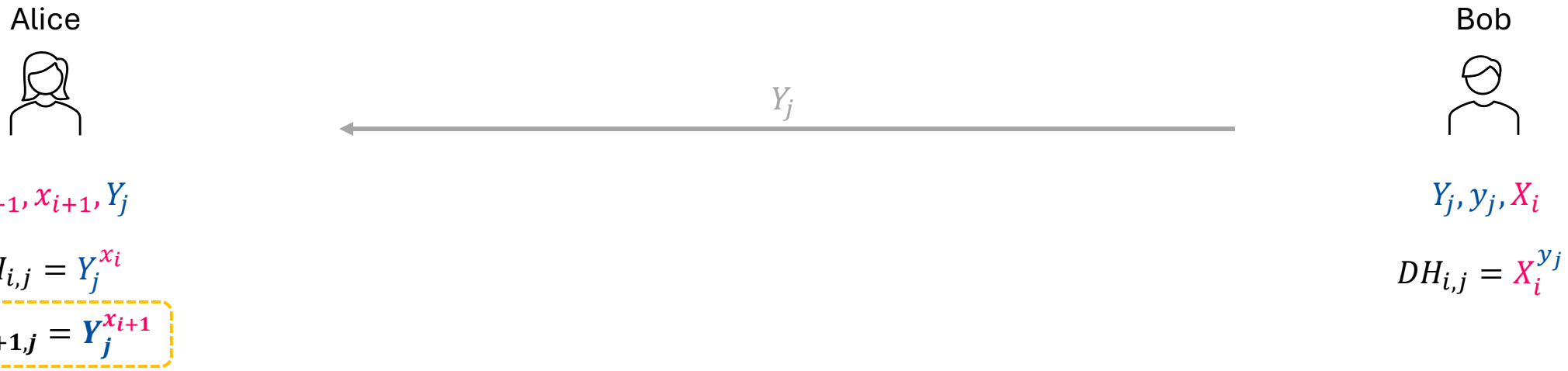
Double Ratchet – DH Ratchet

- Main idea of DH Ratchet: Running DHKE continuously with *rotating ephemeral keys*…



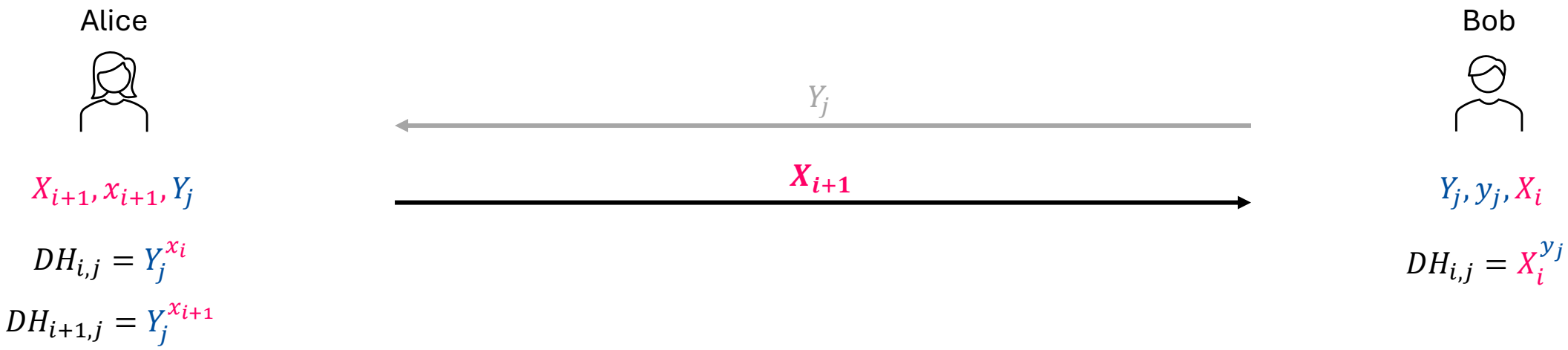
Double Ratchet – DH Ratchet

- Main idea of DH Ratchet: Running DHKE continuously with *rotating ephemeral keys*…



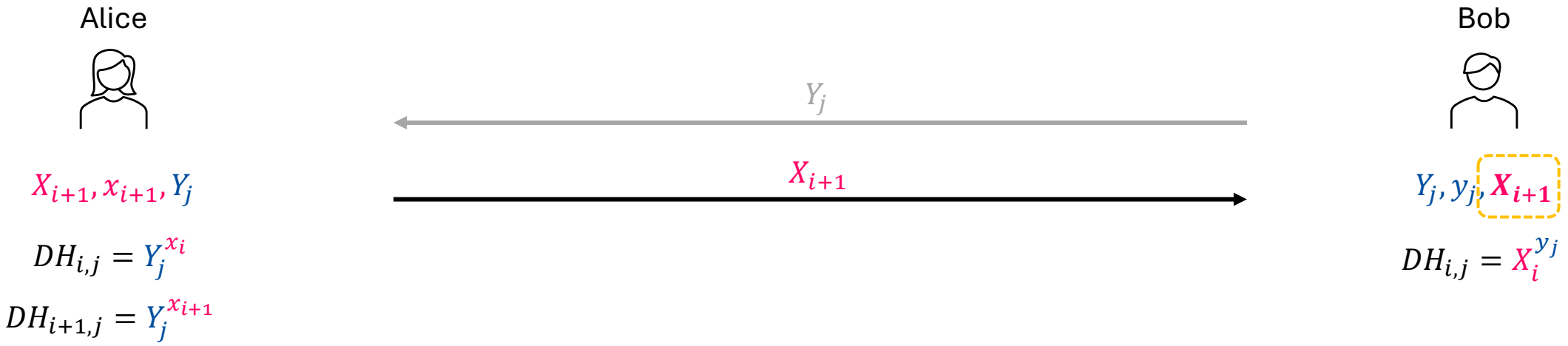
Double Ratchet – DH Ratchet

- Main idea of DH Ratchet: Running DHKE continuously with *rotating ephemeral keys*…



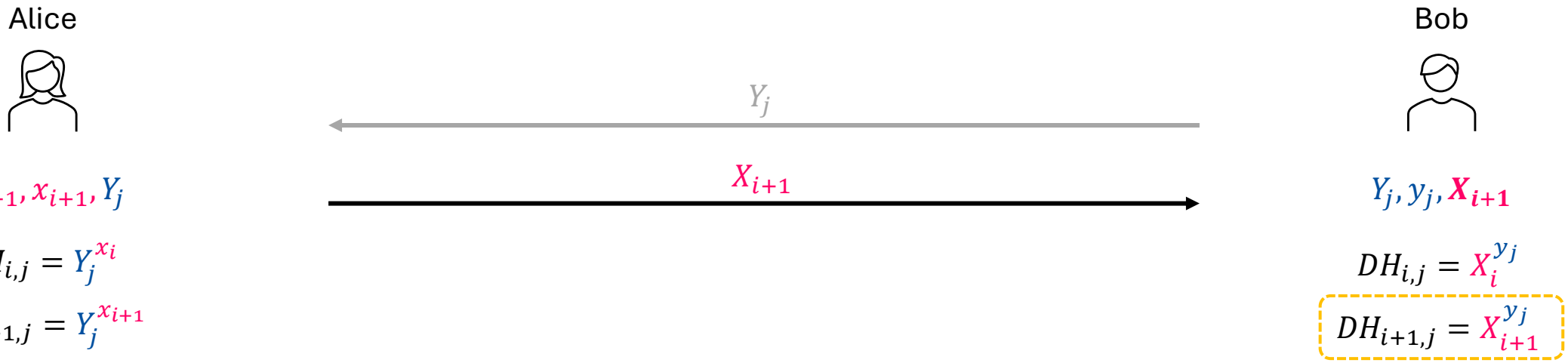
Double Ratchet – DH Ratchet

- Main idea of DH Ratchet: Running DHKE continuously with *rotating ephemeral keys*…



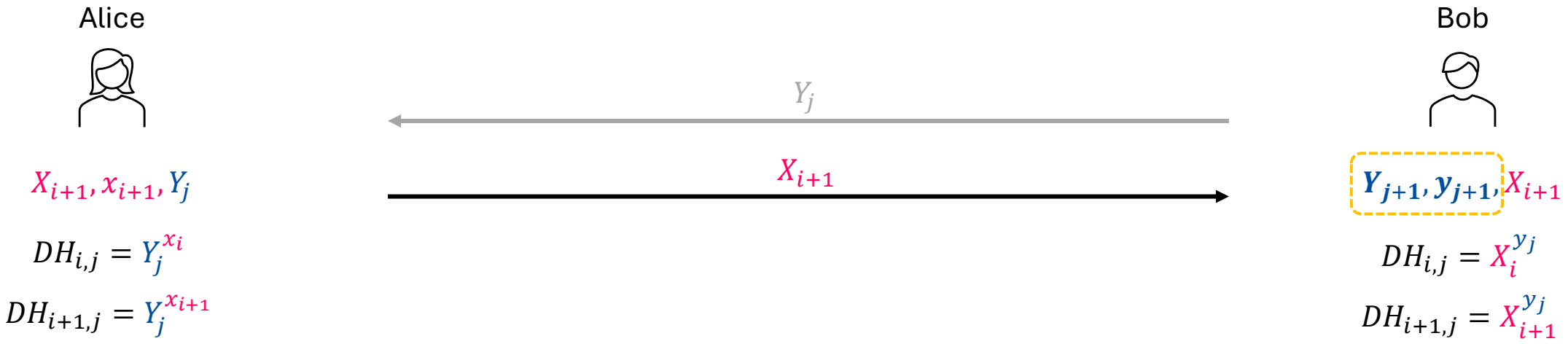
Double Ratchet – DH Ratchet

- Main idea of DH Ratchet: Running DHKE continuously with *rotating ephemeral keys*…



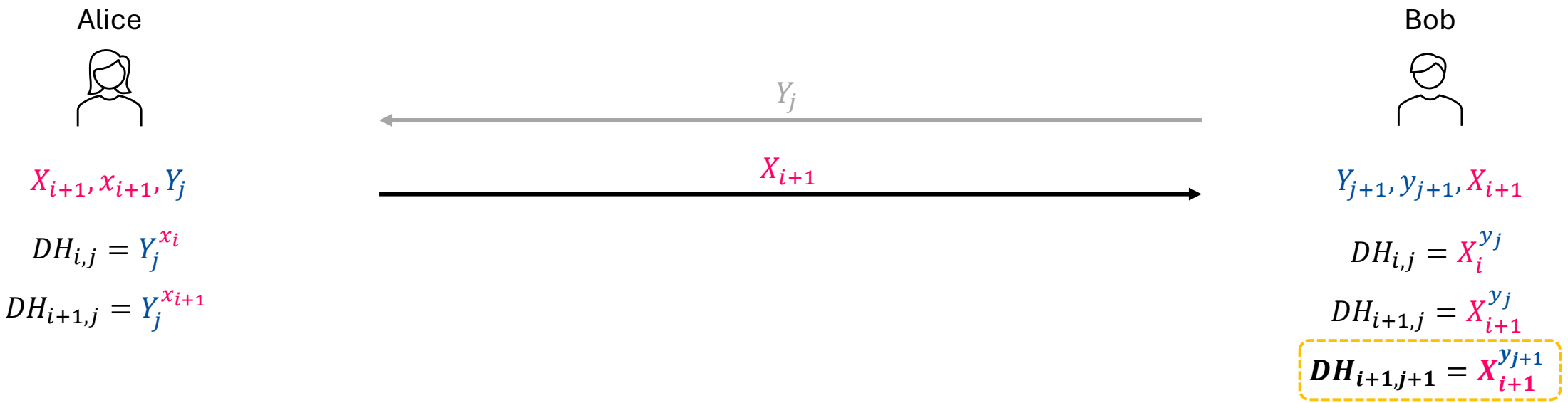
Double Ratchet – DH Ratchet

- Main idea of DH Ratchet: Running DHKE continuously with *rotating ephemeral keys*…



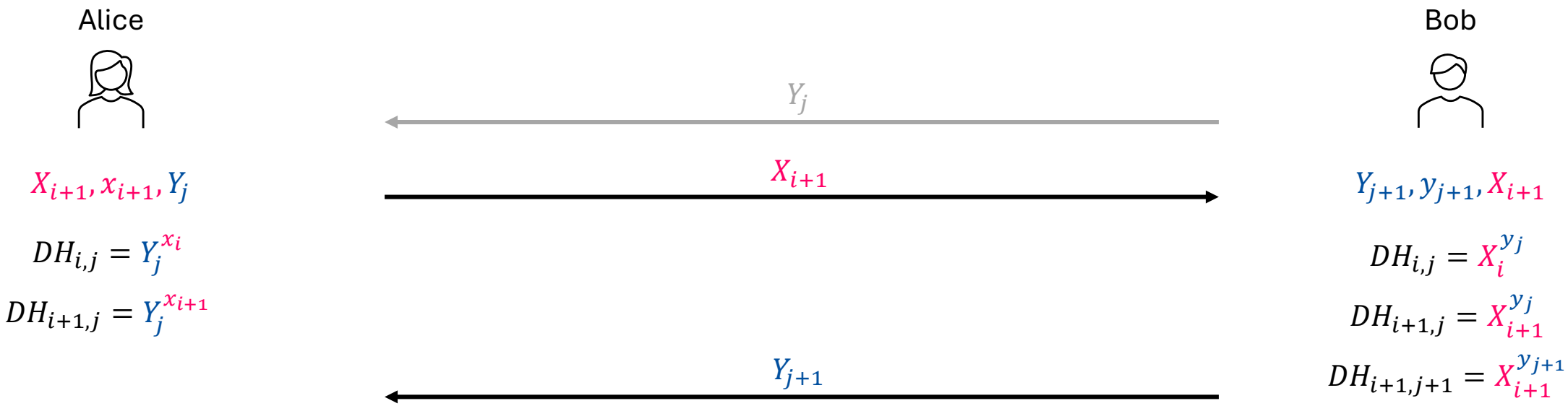
Double Ratchet – DH Ratchet

- Main idea of DH Ratchet: Running DHKE continuously with *rotating ephemeral keys*…



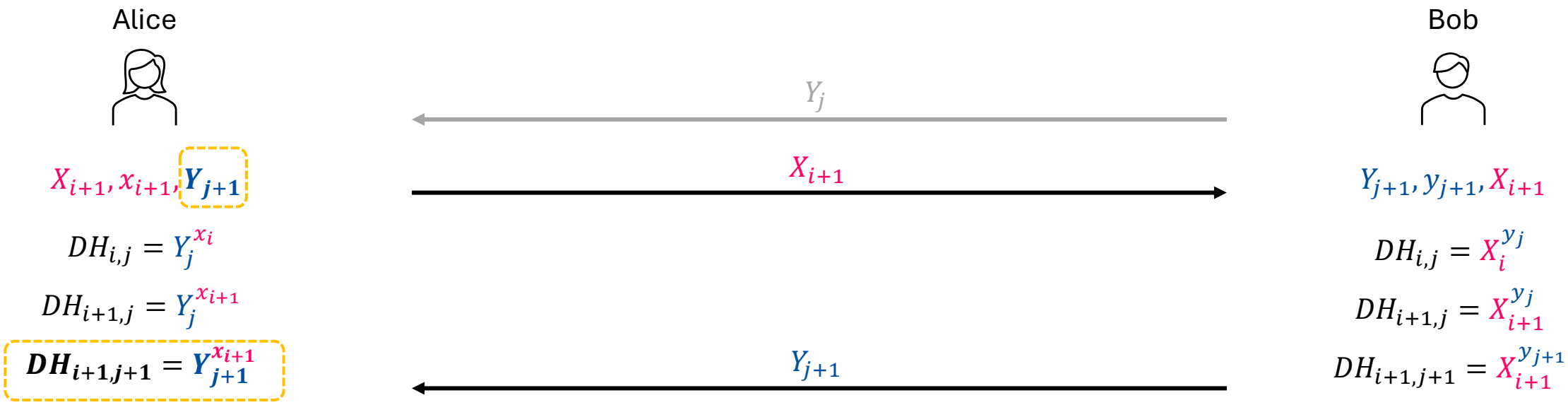
Double Ratchet – DH Ratchet

- Main idea of DH Ratchet: Running DHKE continuously with *rotating ephemeral keys*…



Double Ratchet – DH Ratchet

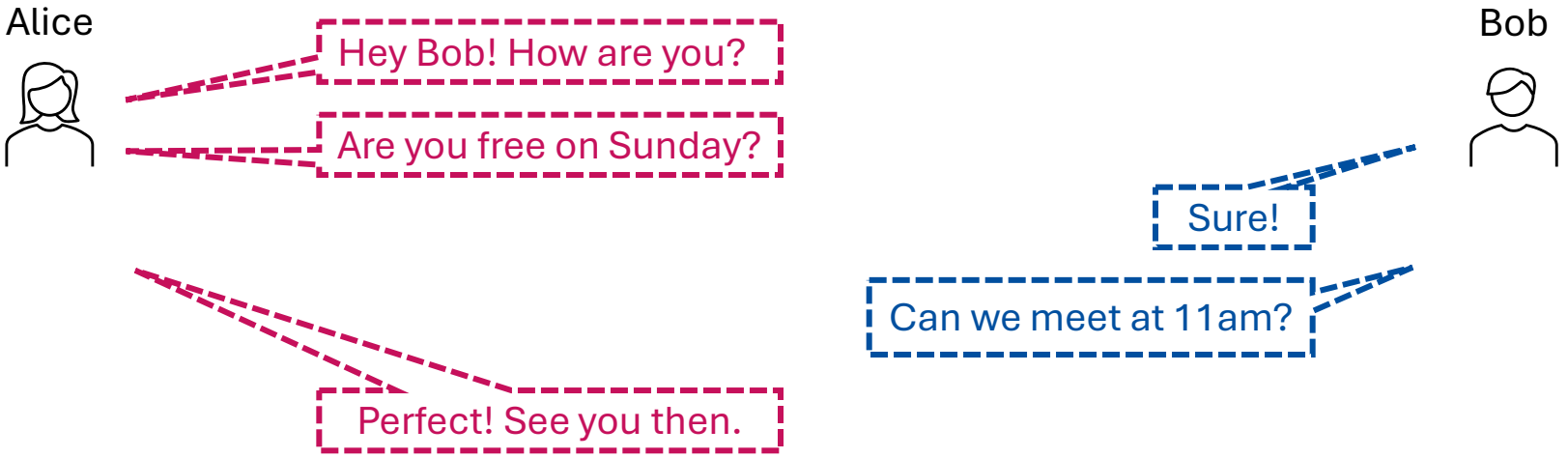
- Main idea of DH Ratchet: Running DHKE continuously with *rotating ephemeral keys*…



Double Ratchet

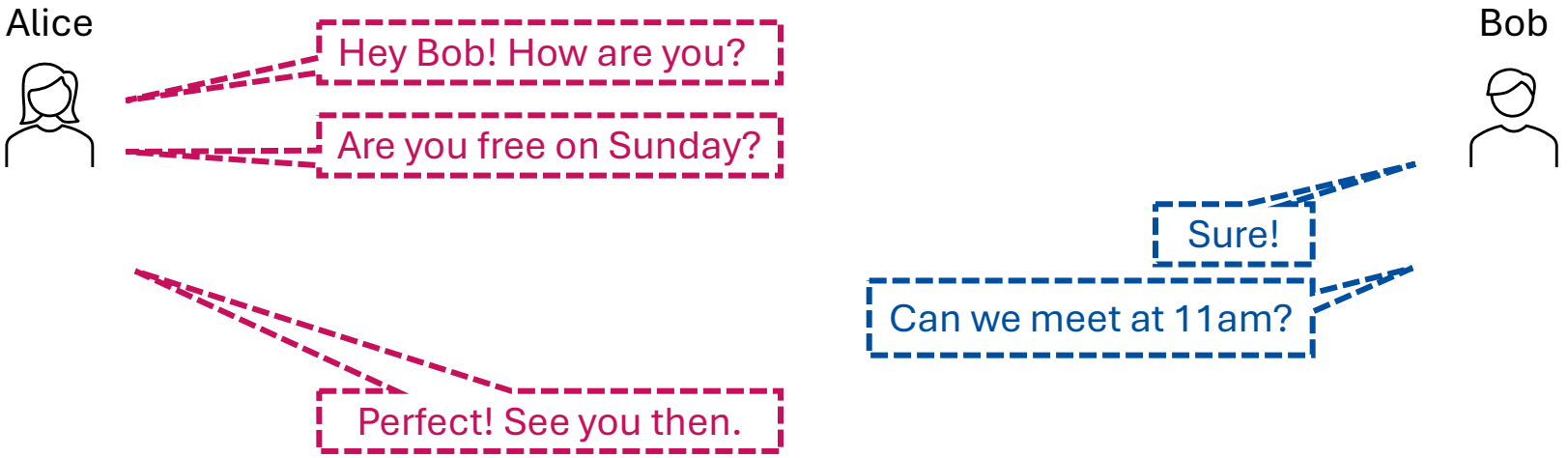
- The main idea: Symmetric-key Ratchet + Diffie-Hellman Ratchet
  - When a party sends messages (**before** its peer party replies): Use Symmetric-key Ratchet…
  - When the peer party replies: Use Diffie-Hellman Ratchet to update the key…

- Example:



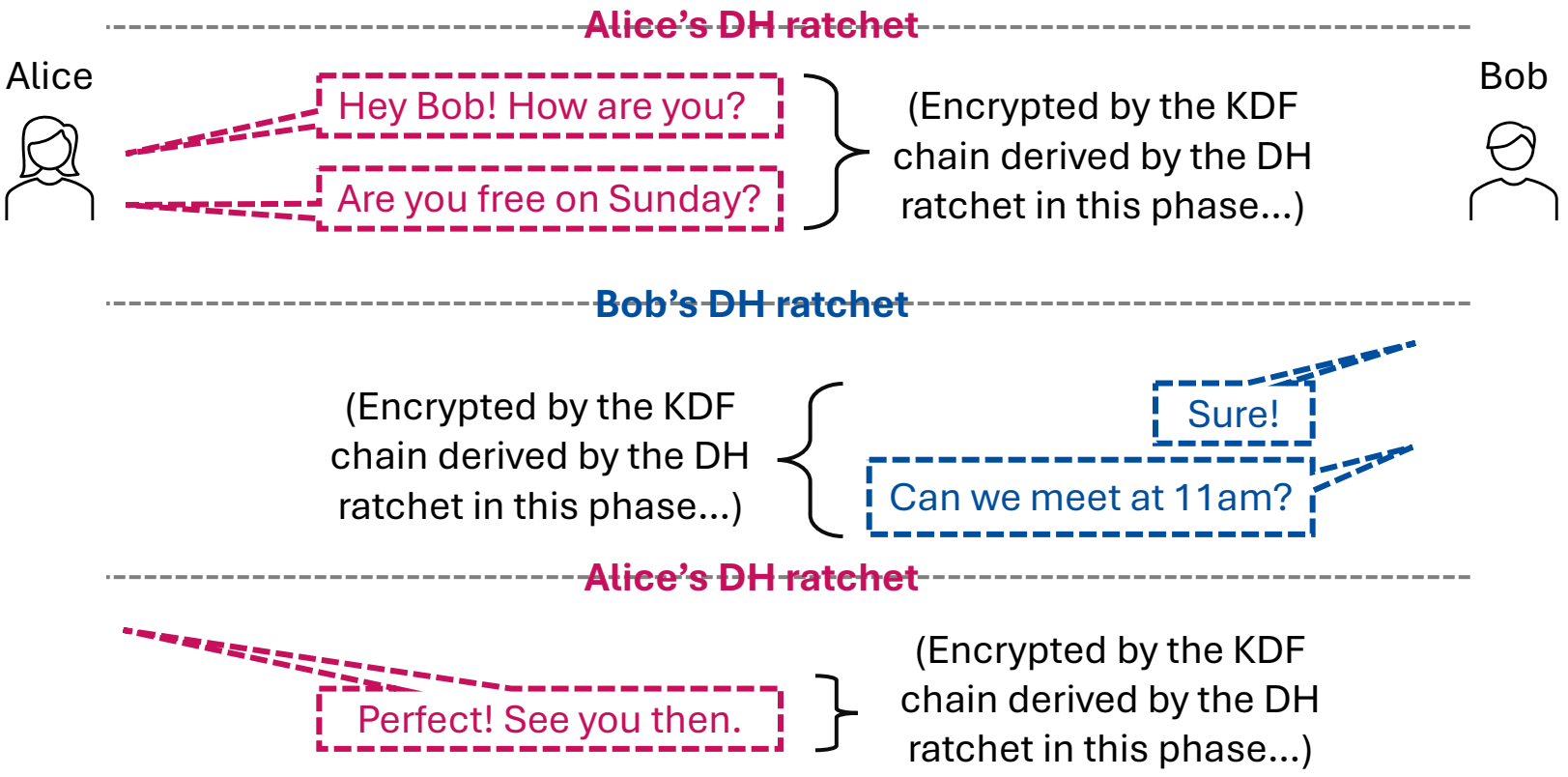
Double Ratchet

- The main idea: Symmetric-key Ratchet + Diffie-Hellman Ratchet



Double Ratchet

- The main idea: Symmetric-key Ratchet + Diffie-Hellman Ratchet

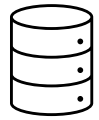


Double Ratchet

Alice

X_i, x_i, Y_j

Root key
(from previous stage)



All messages
are relayed by
the server

Bob

Y_j, y_j, X_i

Root key

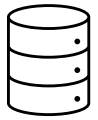
Double Ratchet

Alice

X_{i+1}, x_{i+1}, Y_j

Root key
(from previous stage)

$DH_{i+1,j} = Y_j^{x_{i+1}}$
(as auxiliary
input of KDF)



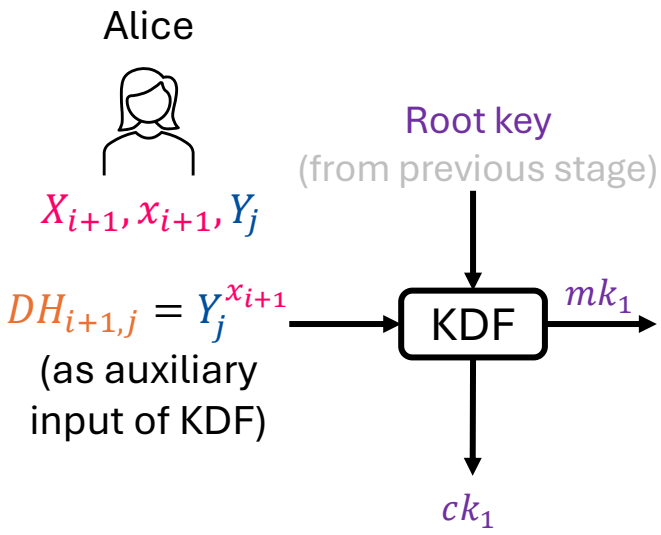
All messages
are relayed by
the server

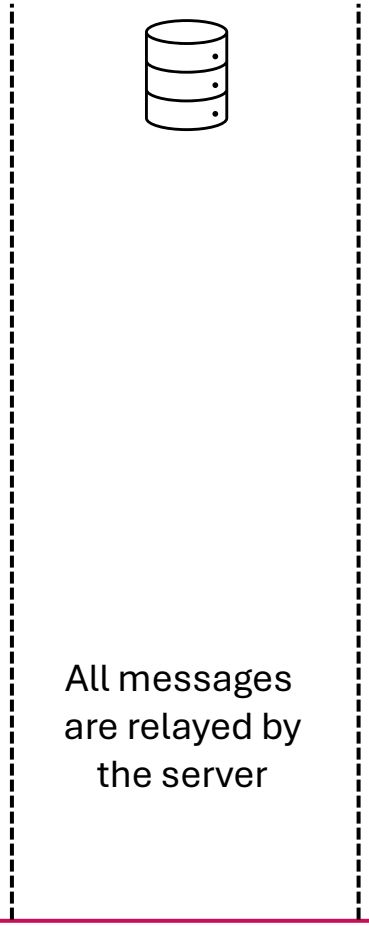
Bob

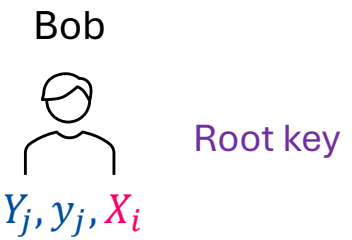
Y_j, y_j, X_i

Root key

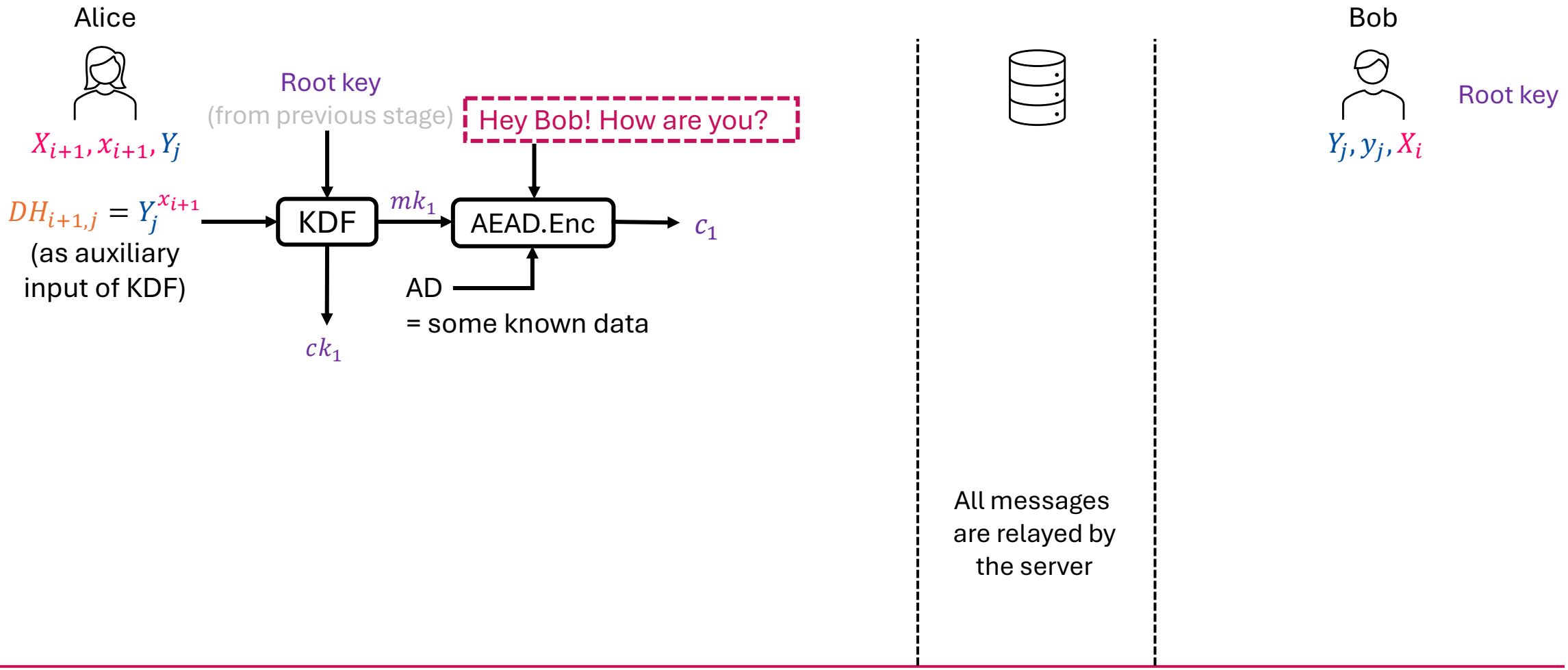
Double Ratchet



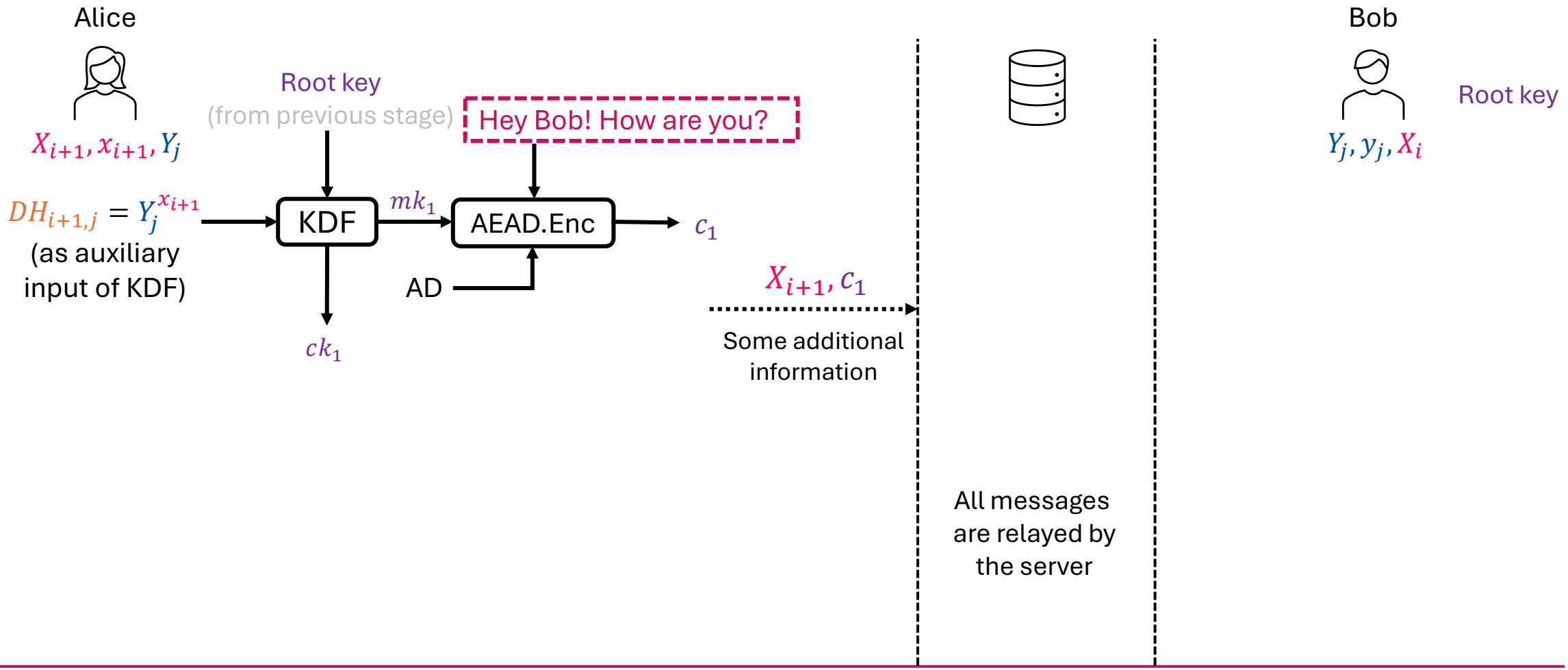




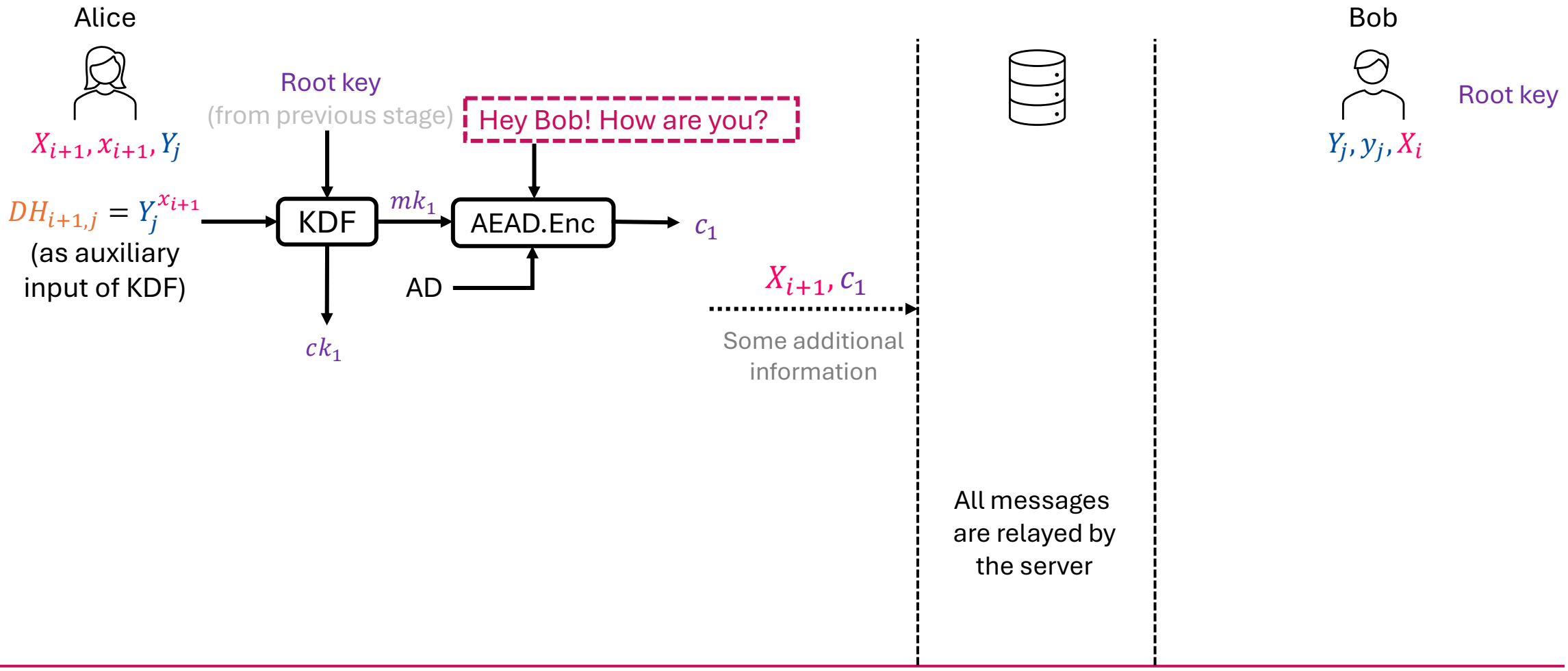
Double Ratchet



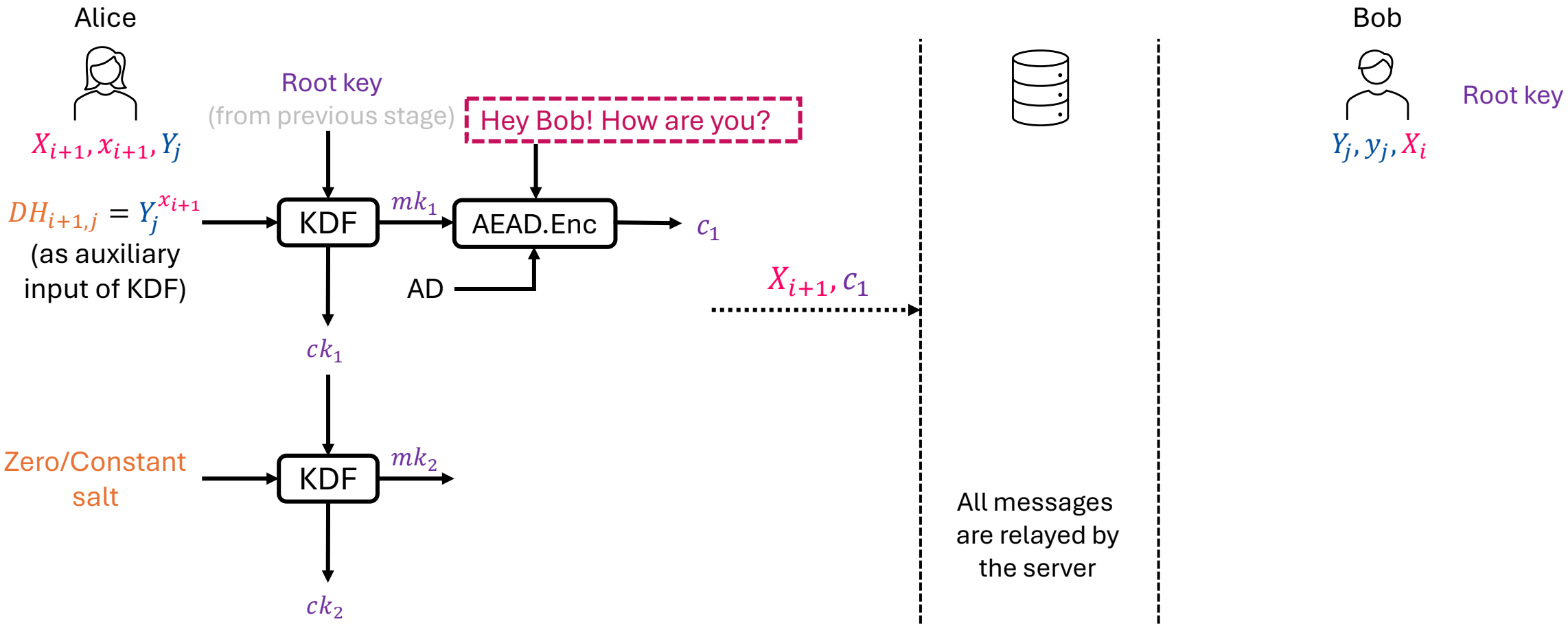
Double Ratchet



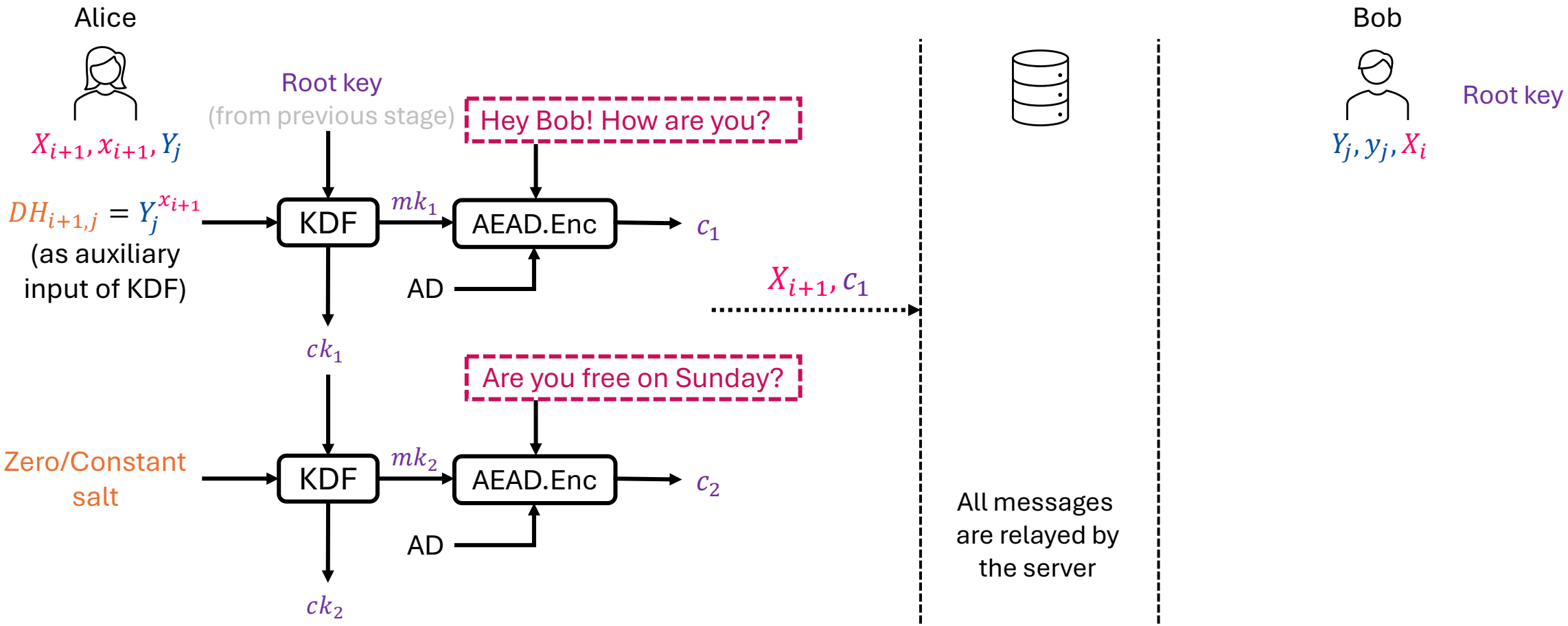
Double Ratchet



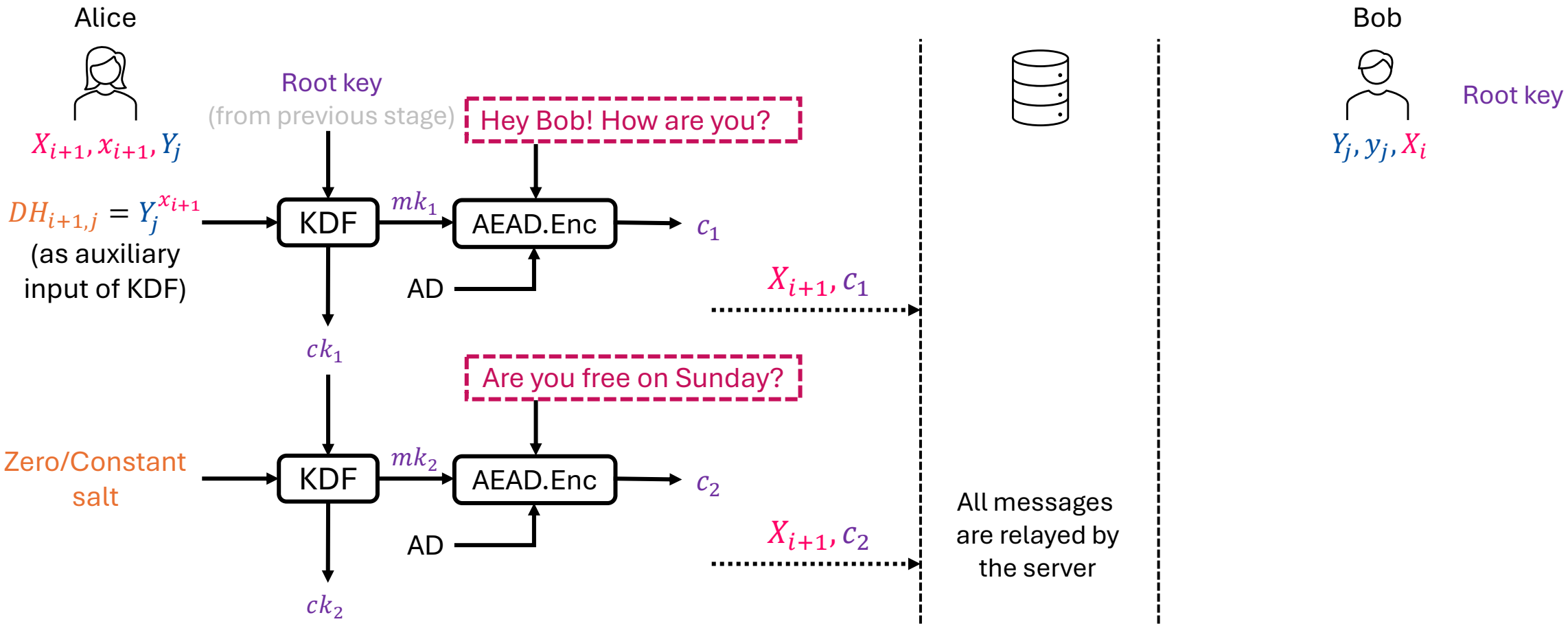
Double Ratchet



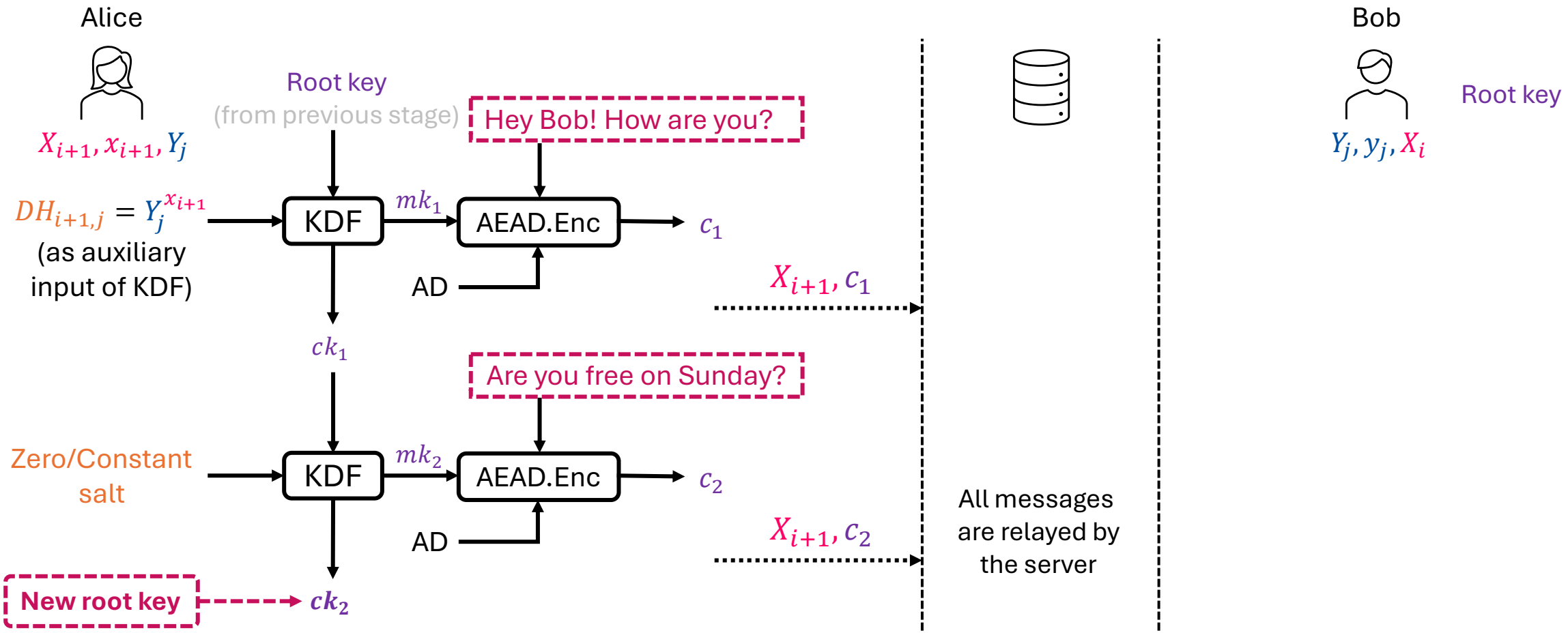
Double Ratchet



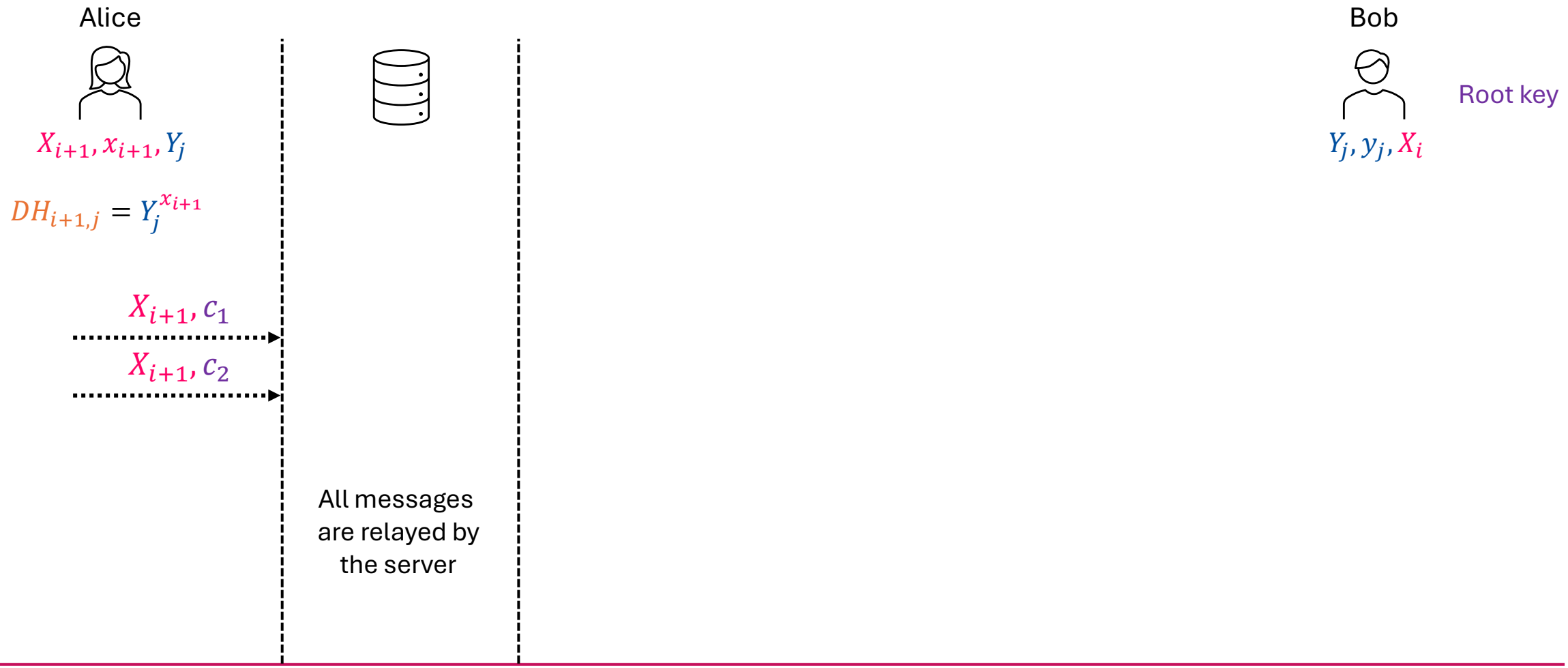
Double Ratchet



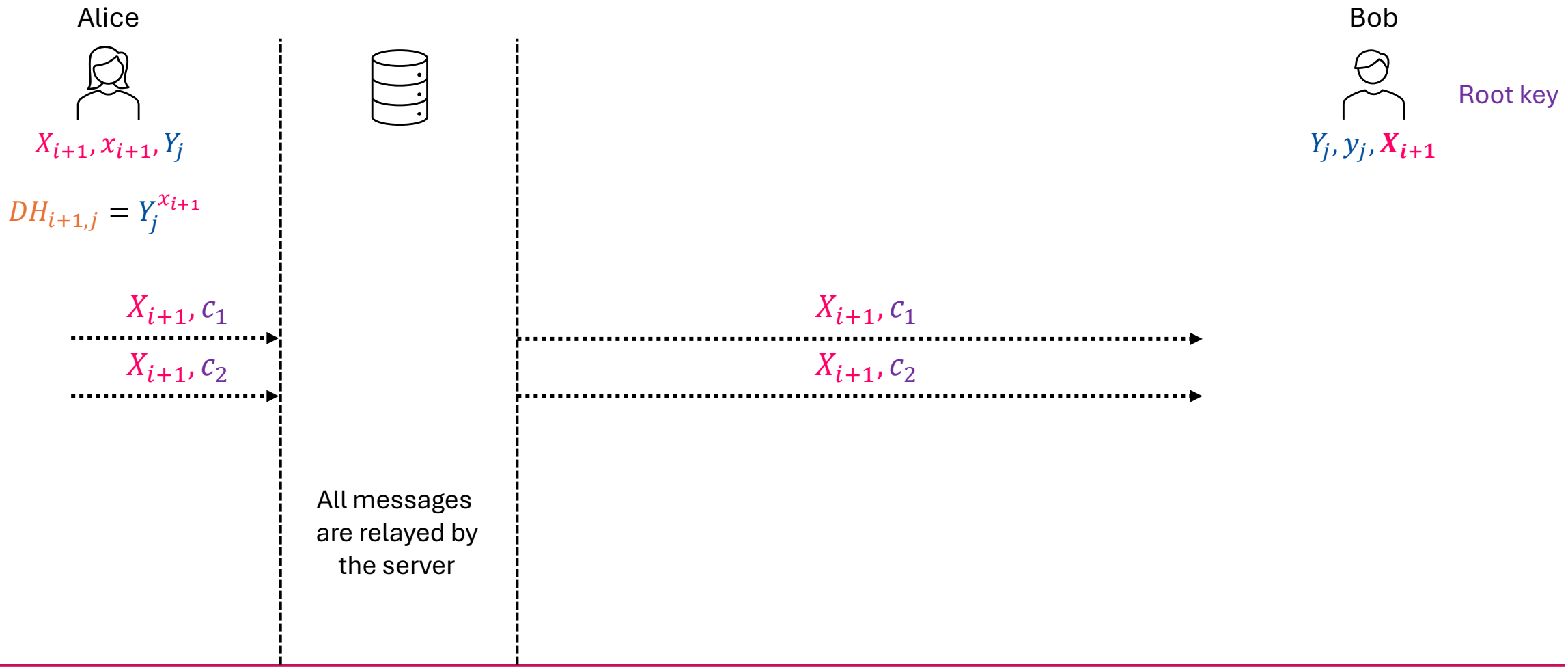
Double Ratchet



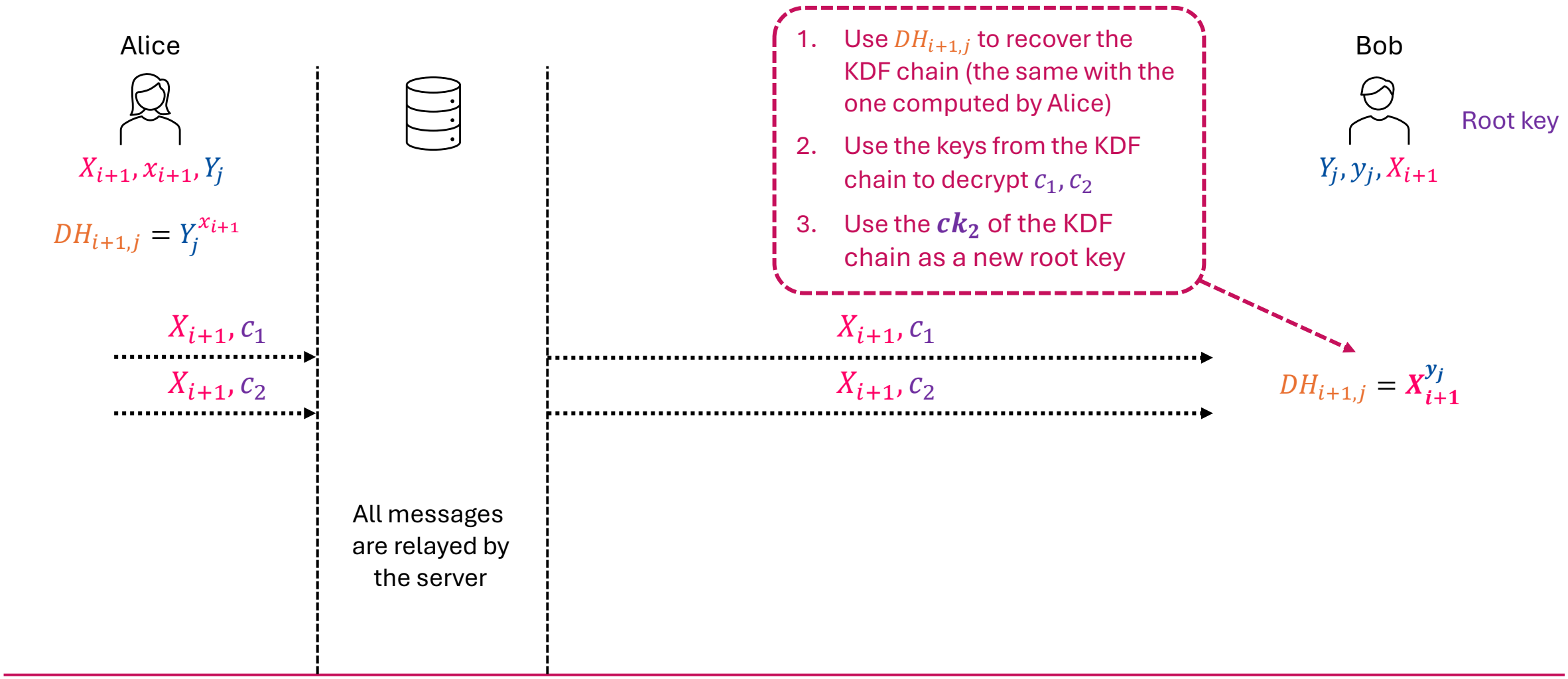
Double Ratchet



Double Ratchet



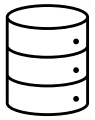
Double Ratchet



Double Ratchet

Alice

X_{i+1}, x_{i+1}, Y_j

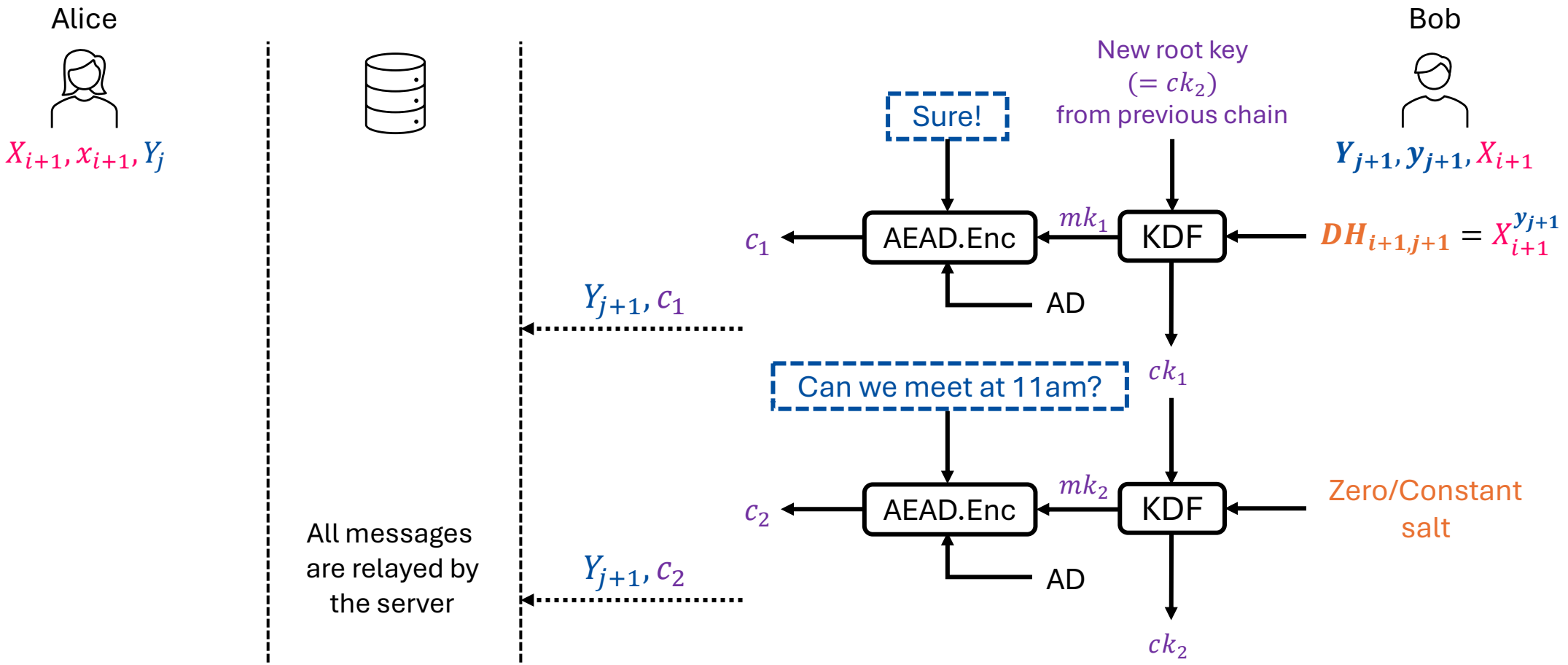


All messages are relayed by the server

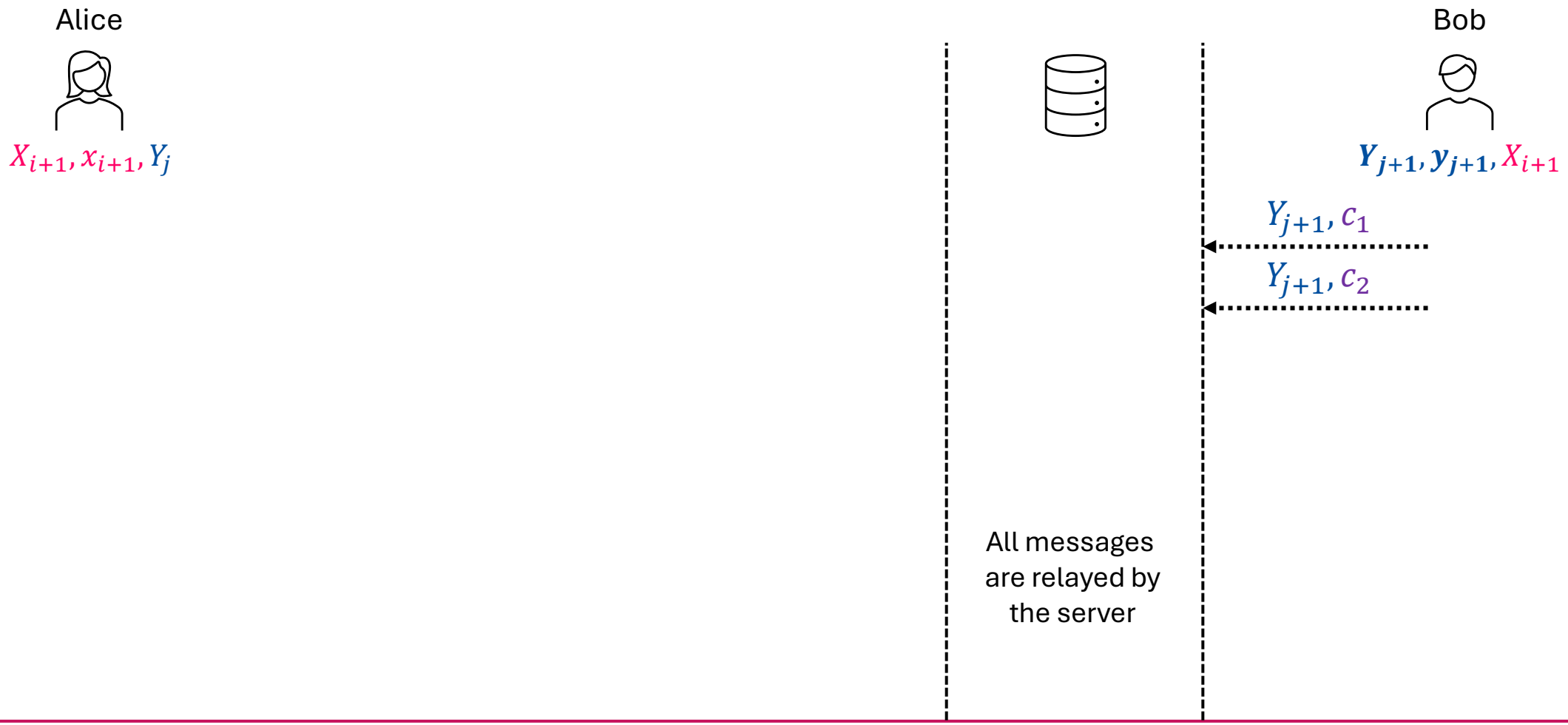
Bob

$Y_{j+1}, y_{j+1}, X_{i+1}$

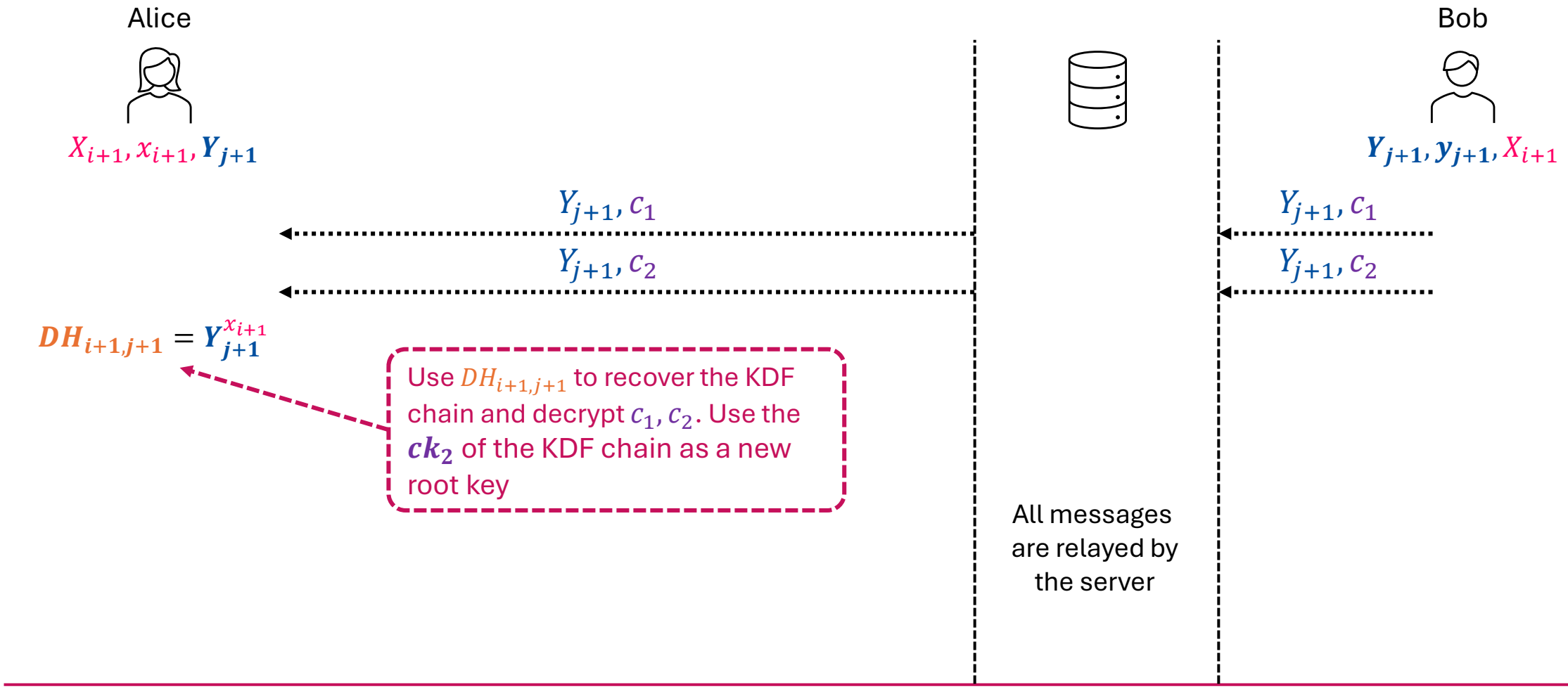
Double Ratchet



Double Ratchet



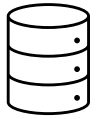
Double Ratchet



Double Ratchet

Alice

$X_{i+2}, x_{i+2}, Y_{j+1}$

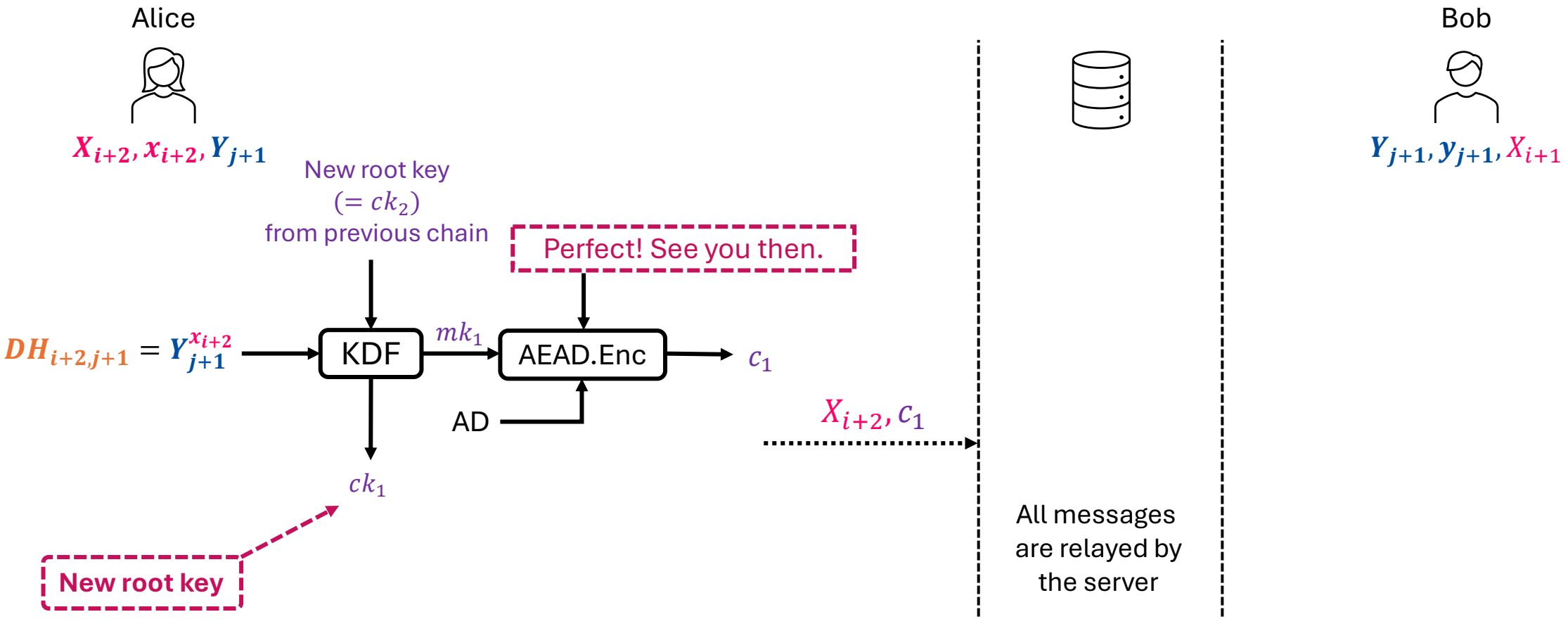


All messages are relayed by the server

Bob

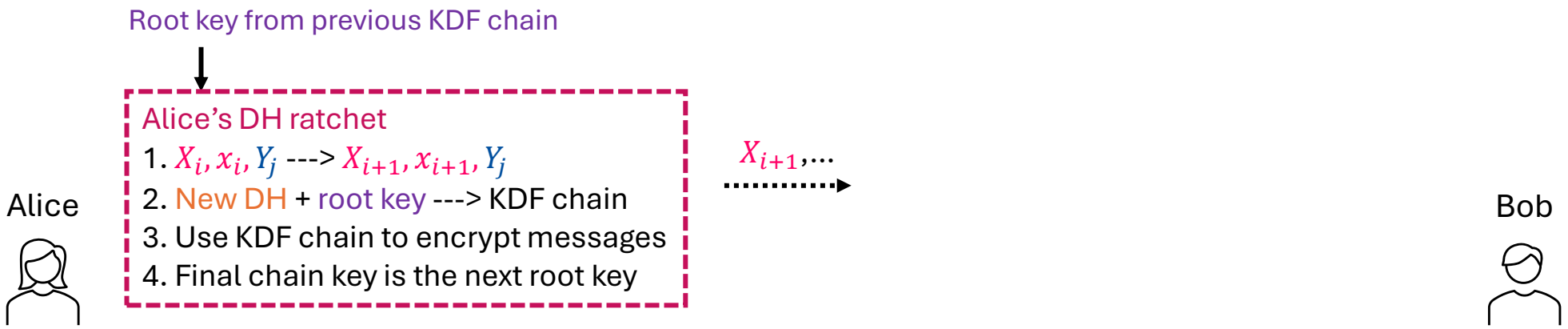
$Y_{j+1}, y_{j+1}, X_{i+1}$

Double Ratchet



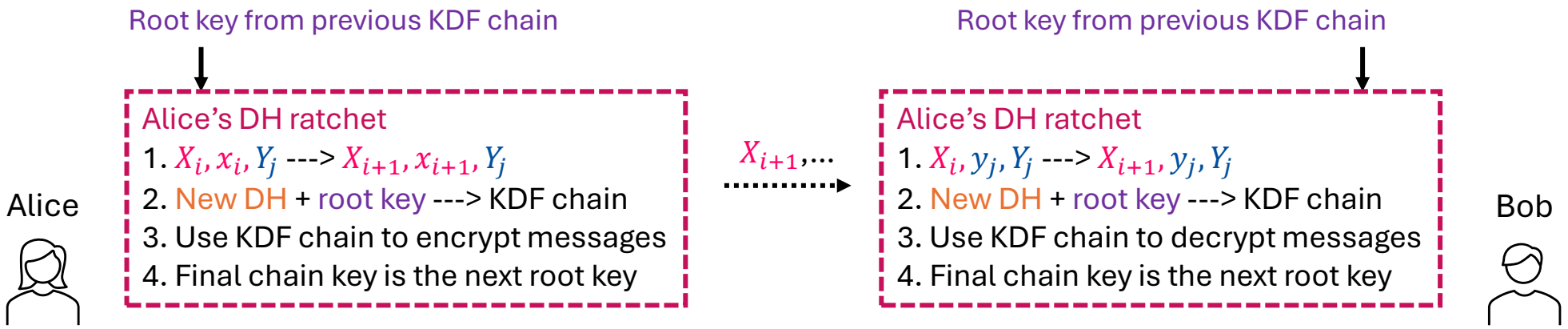
Double Ratchet

- The main idea: Symmetric-key Ratchet + Diffie-Hellman Ratchet



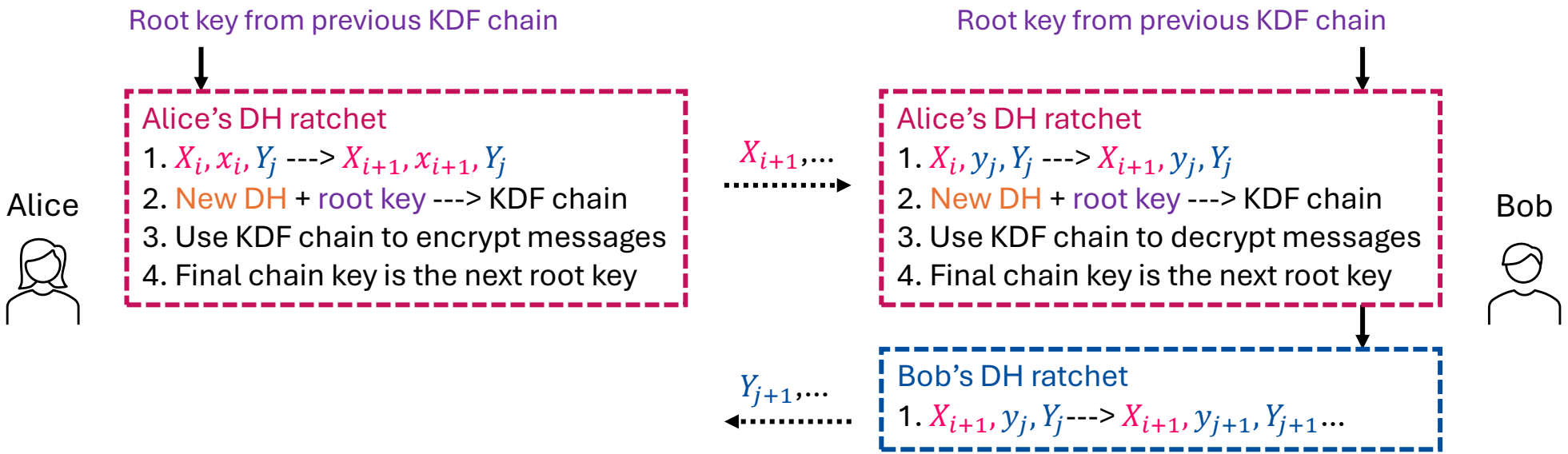
Double Ratchet

- The main idea: Symmetric-key Ratchet + Diffie-Hellman Ratchet



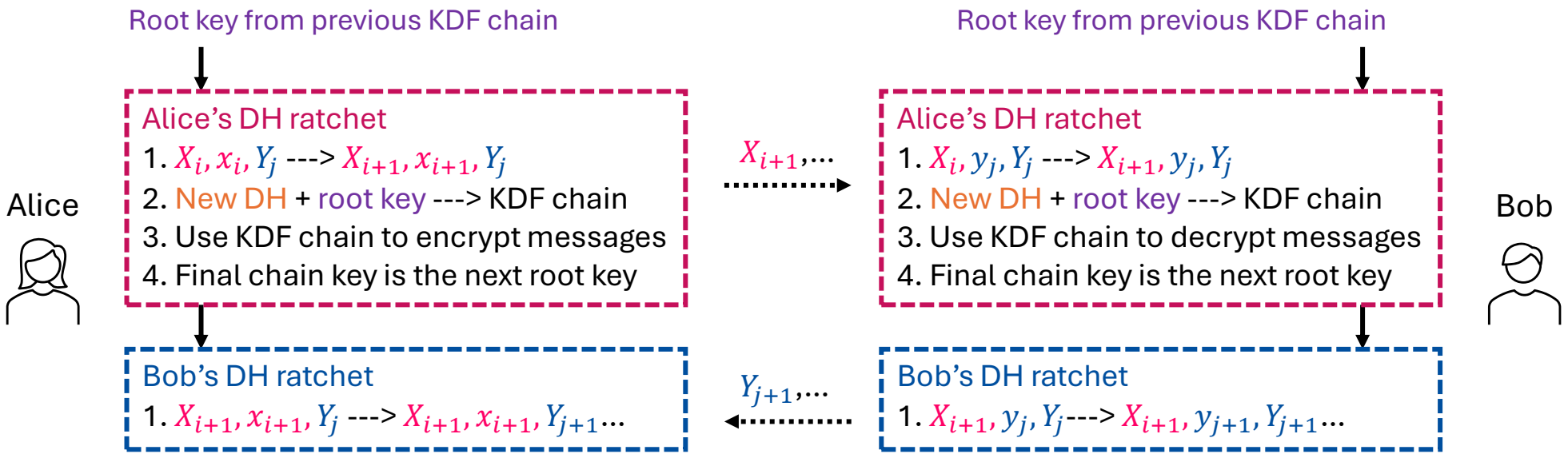
Double Ratchet

- The main idea: Symmetric-key Ratchet + Diffie-Hellman Ratchet



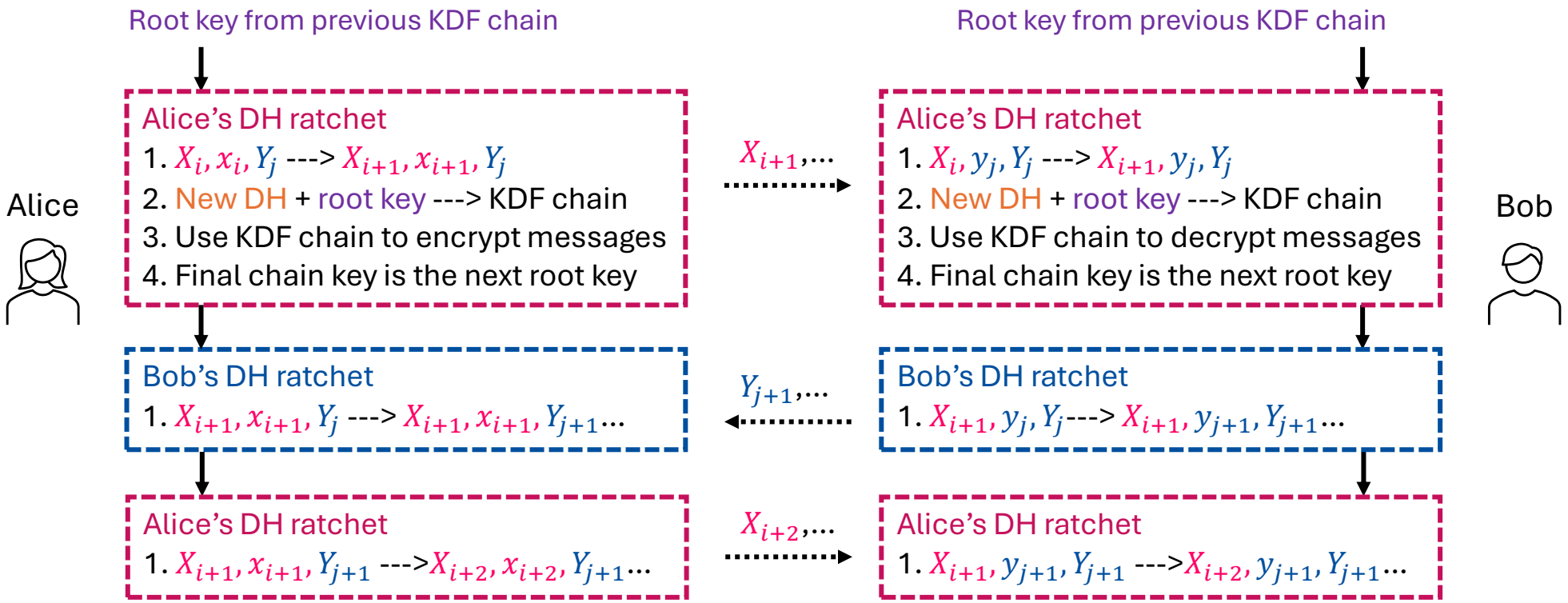
Double Ratchet

- The main idea: Symmetric-key Ratchet + Diffie-Hellman Ratchet



Double Ratchet

- The main idea: Symmetric-key Ratchet + Diffie-Hellman Ratchet



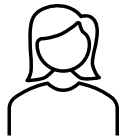
X3DH + Double Ratchet

- Integrate Double Ratchet algorithm with X3DH
  - Use X3DH to bootstrap Double Ratchet
  - The Double Ratchet plays the role of a ‘post-X3DH’ protocol…

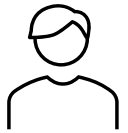
X3DH + Double Ratchet

- Recall of X3DH:

Public parameters: $(\mathbb{G}, g, q)$:
A q-order EC group $\mathbb{G}$ with a generator g



Alice

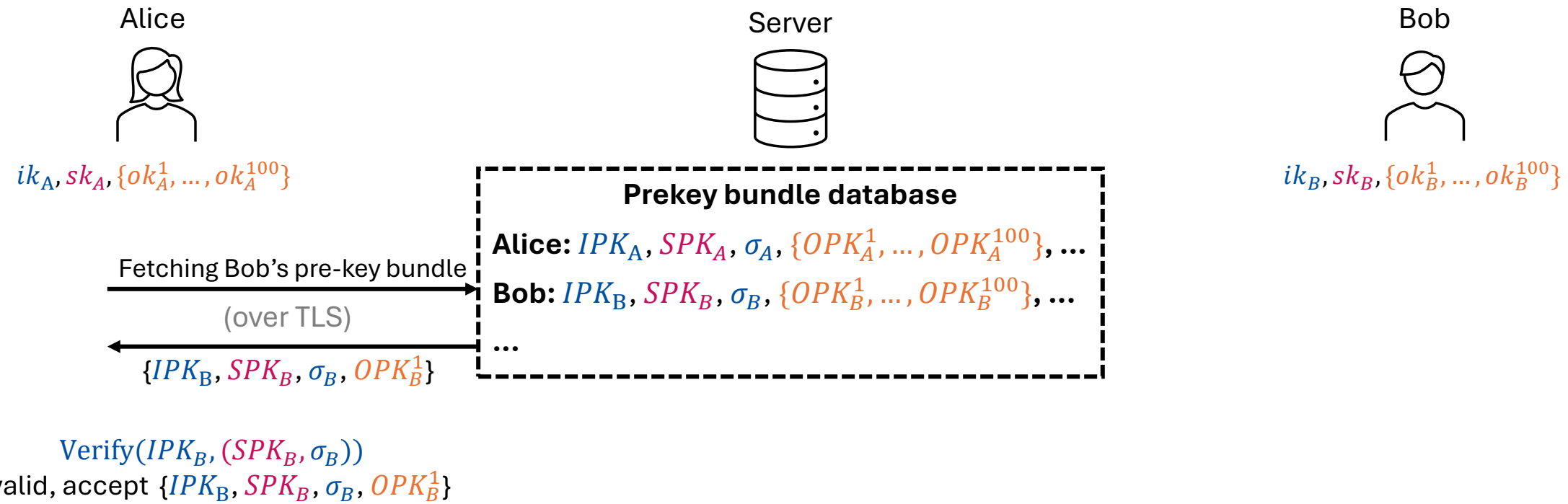


Bob

		Alice	Bob
Long-term secret (static)	Identity secret key (IK)	$ik_{\mathrm{A}} \in_{\$} \mathbb{Z}_q$	$ik_{\mathrm{B}} \in_{\$} \mathbb{Z}_q$
	Identity public key (IPK)	$IPK_{\mathrm{A}} (= g^{ik_A})$	IPK_{B}
Mid-term secret (updated periodically)	**Signing secret pre-key (SK)**	$\boldsymbol{sk_{\mathrm{A}}} \in_{\$} \mathbb{Z}_{\boldsymbol{q}}$	$\boldsymbol{sk_{\mathrm{B}}} \in_{\$} \mathbb{Z}_{\boldsymbol{q}}$
	Signing public pre-key (SPK)	$\boldsymbol{SPK_{\mathrm{A}}}$	$\boldsymbol{SPK_{\mathrm{B}}}$
Short-term secret (used once)	One-time secret pre-keys (OK)	$\{ok_{\mathrm{A}}^1, ok_A^2, \ldots\} \subseteq_{\$} \mathbb{Z}_q$	$\{ok_{\mathrm{B}}^1, ok_B^2, \ldots\} \subseteq_{\$} \mathbb{Z}_q$
	One-time public pre-keys (OPK)	$(OPK_{\mathrm{A}}^1, OPK_{\mathrm{A}}^2, \ldots)$	$(OPK_{\mathrm{B}}^1, OPK_{\mathrm{B}}^2, \ldots)$

X3DH + Double Ratchet

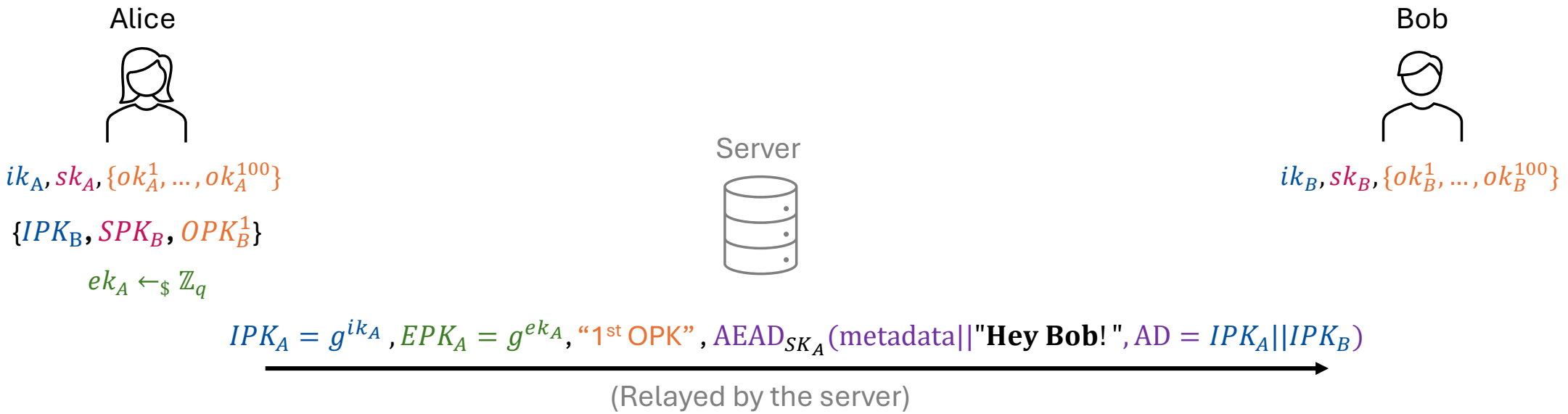
- Recall of X3DH:



$\text{Verify}(IPK_B, (SPK_B, \sigma_B))$
if valid, accept $\{IPK_B, SPK_B, \sigma_B, OPK_B^1\}$

X3DH + Double Ratchet

- Recall of X3DH:



$SK_A = \text{X3DH\_Key\_Alice}(ik_A, ek_A, IPK_B, SPK_B, OPK_B^1)$

X3DH + Double Ratchet

- Initialize Double Ratchet using the SK from X3DH

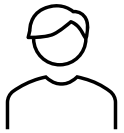
Alice

$ik_A, sk_A, \{ok_A^1, \dots, ok_A^{100}\}$

$\{IPK_B, SPK_B, OPK_B^1\}$

$SK_A = \text{X3DH\_Key\_Alice}(ik_A, ek_A, IPK_B, SPK_B, OPK_B^1)$

Bob



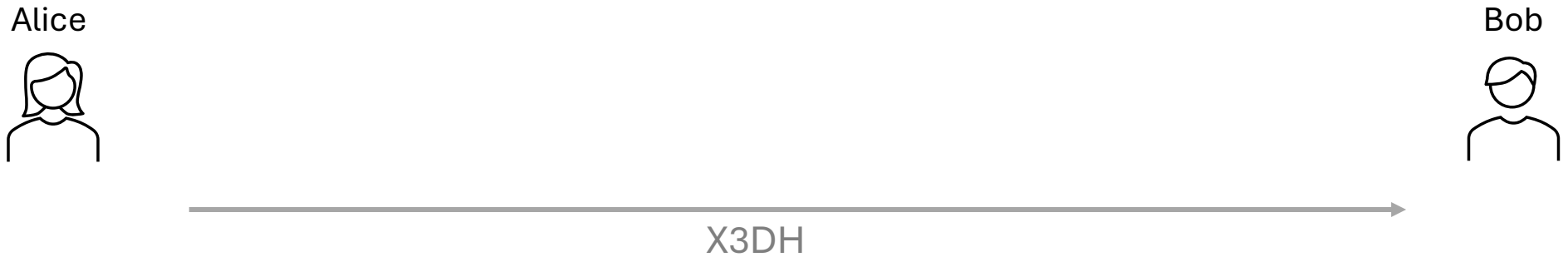
$ik_B, sk_B, \{ok_B^1, \dots, ok_B^{100}\}$

Root key $= SK_A$

$X_0 = \perp, x_0 = \perp, Y_0 = SPK_B$

X3DH + Double Ratchet

- Initialize Double Ratchet using the SK from X3DH



SK_A = X3DH_Key_Alice(...)

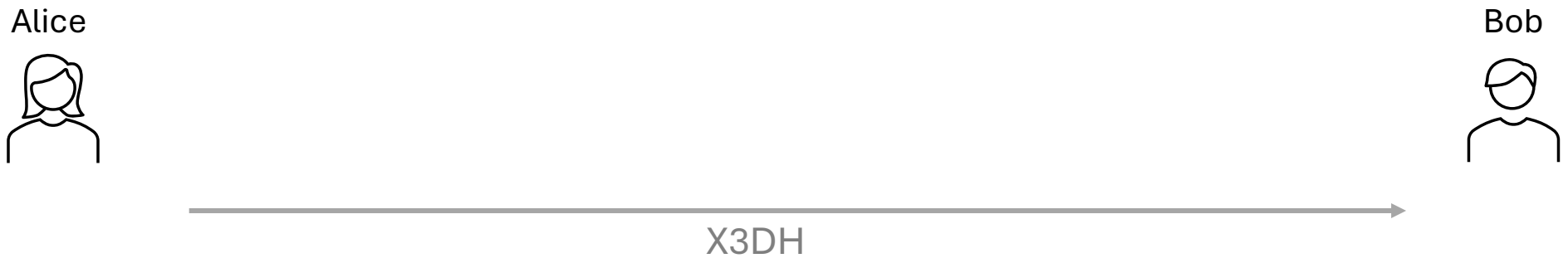
~~Alice's DH ratchet~~

Root key = SK_A

$X_0 = \perp, x_0 = \perp, Y_0 = SPK_B$ (Signing public pre-key of Bob)

X3DH + Double Ratchet

- Initialize Double Ratchet using the SK from X3DH



SK_A = X3DH_Key_Alice(...)

~~Alice's DH ratchet~~

Root key = SK_A

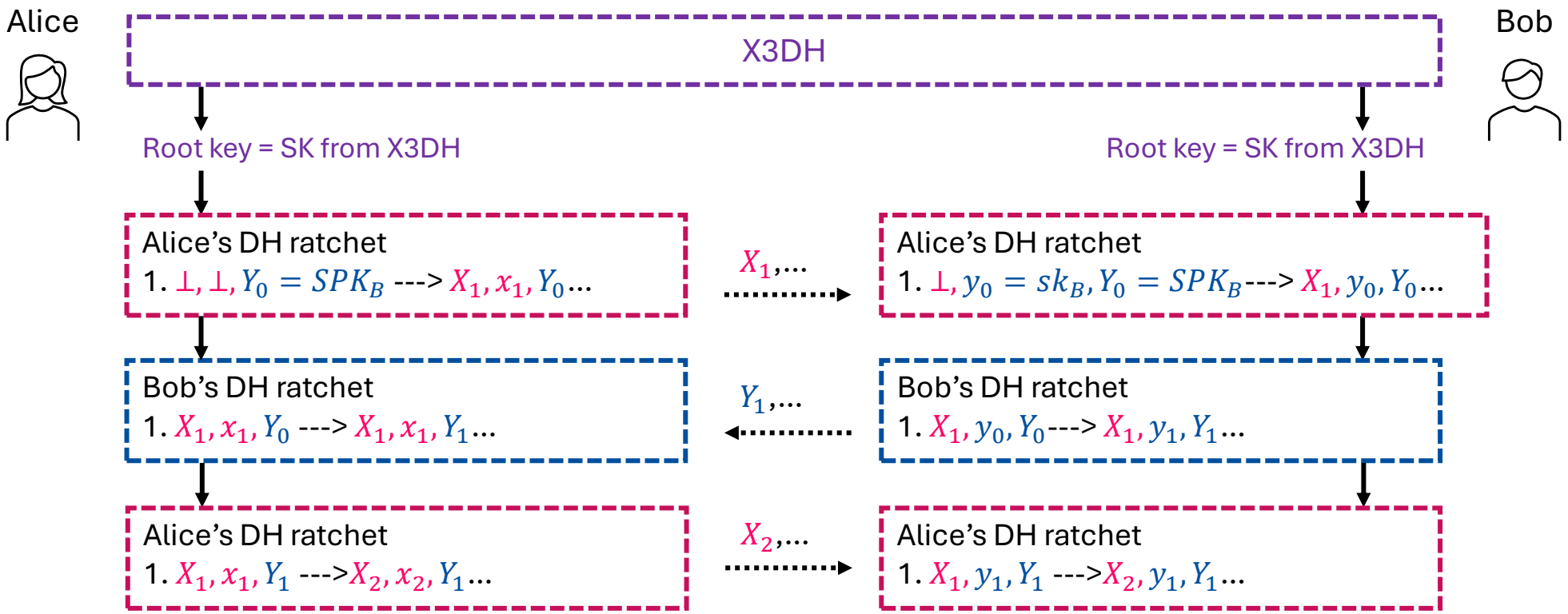
$X_0 = \perp, x_0 = \perp, Y_0 = SPK_B$ (Signing public pre-key of Bob)

$X_1 = g^{x_1}, x_1 \leftarrow_{\$} \mathbb{Z}_q, DH_{1,0} = Y_0^{x_1}$

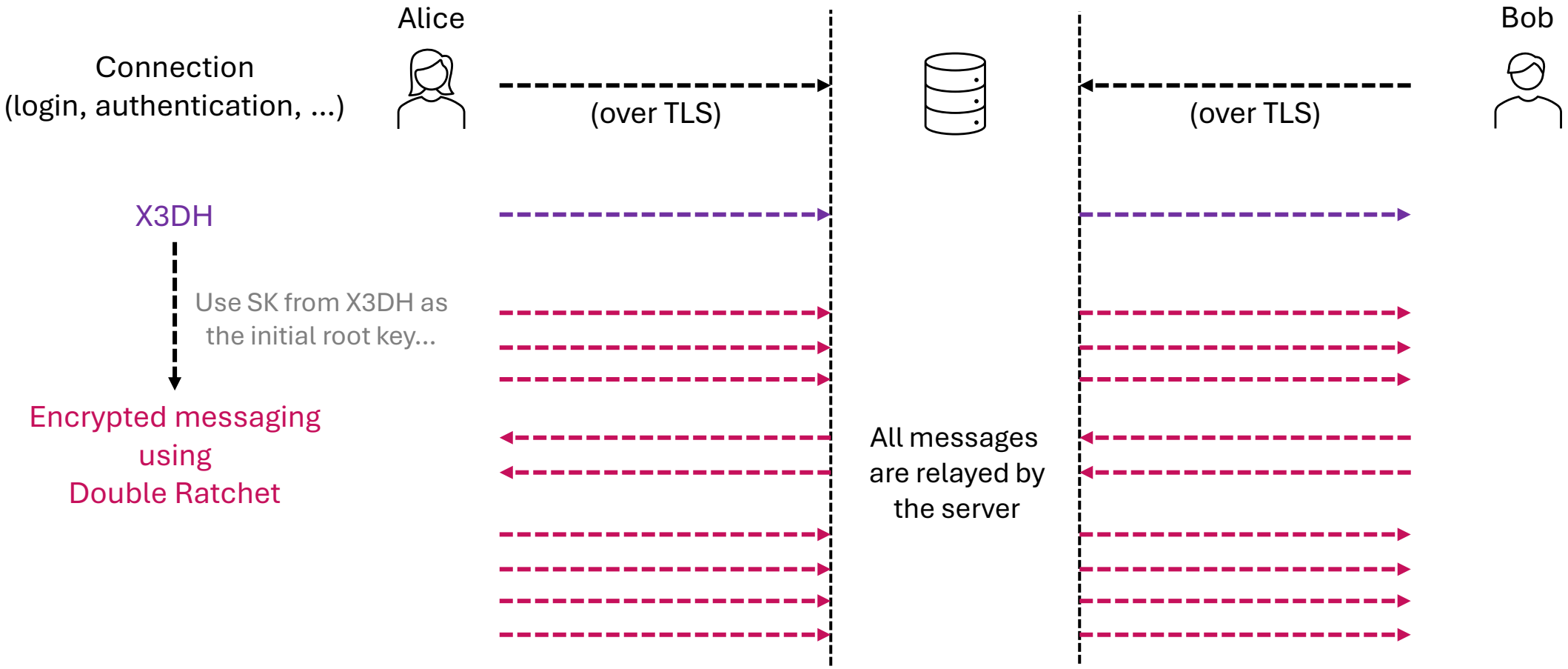
Use $DH_{1,0}$ to derive a KDF chain to encrypt messages...

Double Ratcheting

- Initialize Double Ratchet using the SK from X3DH



Signal Secure Messaging Protocol

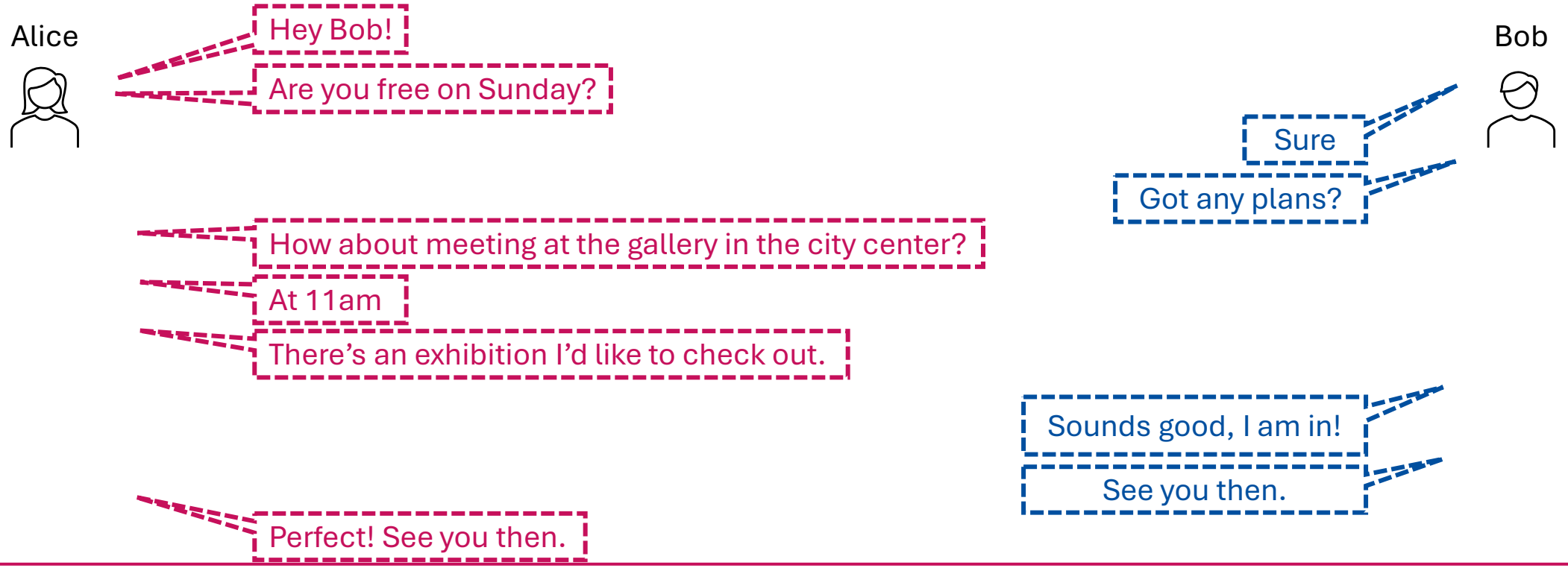


Signal Secure Messaging Protocol

- Some technical details we do not cover:
  - XEdDSA and VXEdDSA:
    - ➢ DH key pairs for key exchange and signature…
  - Header encryption:
    - ➢ Cannot tell which messages belong to which sessions, or the ordering of messages within a session…
  - Out-of-order messages:
  - Session management and asynchronous settings

Coding tasks

- (Without sockets) Use X3DH and Double Ratchet to encrypt this conversation (or you can choose other conversations):



Further Reading

- Technical Documentations of Signal: https://signal.org/docs/
- Some research papers of analyzing security of Ratchet algorithms:
  - Bellare et al's work on formalizing ratcheted encryption/key exchange: https://eprint.iacr.org/2016/1028
  - Alwen et al's work on formalizing Double Ratchet: https://eprint.iacr.org/2018/1037
  - Collins et al's work on Tight security of Double Ratchet: https://eprint.iacr.org/2024/1625
  - ...