Cryptography Engineering

- Lecture 9 (Jan 16, 2024)
- Today's notes:
  - Recall previous contents
  - The OPAQUE protocol
  - Summary on password-based authentication
  - Notes on the final project

- Coding tasks/Homework:
  - Implement the OPAQUE protocol
  - **Bonus:** Implement OPAQUE using sockets

Previous lecture contents

- Welcome back from the Christmas holidays!

- L1: Recall some cryptographic primitives
- L2: Signature and Certificate
- L3: DHKE + Signature & Certificate = TLS handshake
- L4: Secure Messaging, E2EE, X3DH
- L5 & L6: Key chain, Double ratchet = Symmetric ratchet + DH ratchet
- L7: Passwords, Off/Online attacks, TLS + passwords, Salting
- L8: SCRAM (hashed+salted+iterated), Password-based AKE (EKE, SRP)

Previous Password-based Protocols

- TLS + hashed & salted passwords
- The SCRAM protocol
- The EKE protocol
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- Goal: Authentication via passwords; Resistance to offline attacks.

Previous Password-based Protocols

- **TLS + hashed & salted passwords**
  - Store $\left(\boldsymbol{r}, \boldsymbol{H}(\boldsymbol{pw}, \boldsymbol{r})\right)$ in the server, where $\boldsymbol{r}$ is the salt.
  - Transport $\boldsymbol{r}$ to the client, then the client prove its identity by responding $\boldsymbol{H}(\boldsymbol{pw}, \boldsymbol{r})$
  - Encrypted by TLS
- The SCRAM protocol
- The SRP protocol

Previous Password-based Protocols

- TLS + hashed & salted passwords
- **The TLS + SCRAM protocol**
  - Store $\left(\boldsymbol{r}, \boldsymbol{n}, \boldsymbol{H}^{\boldsymbol{n}}(\boldsymbol{pw}, \boldsymbol{r})\right)$ in the server, where $\boldsymbol{r}$ is the salt and $\boldsymbol{n}$ is the number of iterations.
  - Transport $\boldsymbol{r}$ and $\boldsymbol{n}$ to the client, then the client prove its identity by responding $\boldsymbol{H}^{\boldsymbol{n}}(\boldsymbol{pw}, \boldsymbol{r})$
  - Encrypted by TLS
- The SRP protocol

Previous Password-based Protocols

- TLS + hashed & salted passwords
- The TLS + SCRAM protocol
- **The SRP protocol**
  - Store $\left(\boldsymbol{r}, \boldsymbol{H}(\boldsymbol{pw}, \boldsymbol{r})\right)$ in the server, where $\boldsymbol{r}$ is the salt.
  - Password-based AKE:
    - Security guarantee even if the certificate is fake or the TLS connection is insecure.
  - Enhanced security via integrating with TLS

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- Advantage of storing hashed-salted passwords:
    1. Avoid cross-system leakage

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Plain pw	$\mathbf{O(1)}$
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  2. Increase the time **required to recover the password after leakage**.

This is also important in practice, e.g., notifying users to change their passwords after the leakage.

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- All protocols **reveal salt** (and the number of iterations) during the execution…

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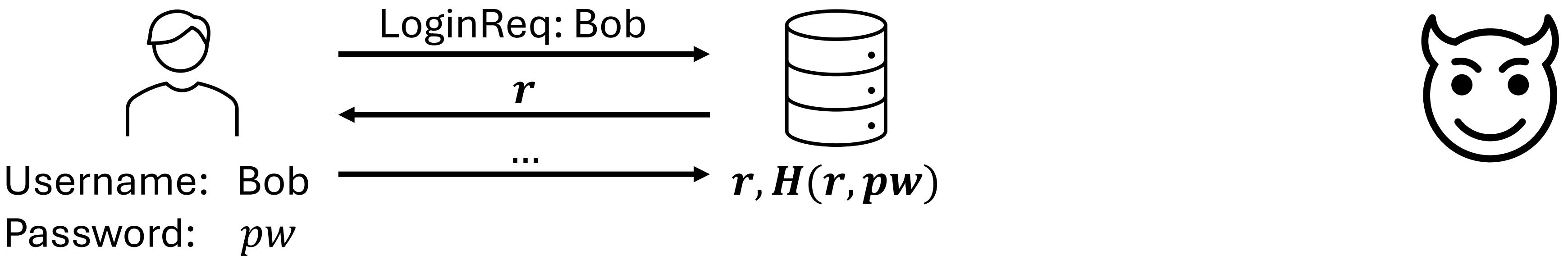
- All protocols **reveal salt** (and the number of iterations) during the execution...
    - May lead to **Precomputation Attacks**
    - $\mathbf{O}(|\boldsymbol{D}|) \rightarrow \mathbf{O}(\log|\boldsymbol{D}|)$ or even $\mathbf{O}(1)$

Precomputation Attacks on Passwords

- Suppose that the password is stored by hashing and salting.
  - **The adversary can learn the salt in some easy ways...**



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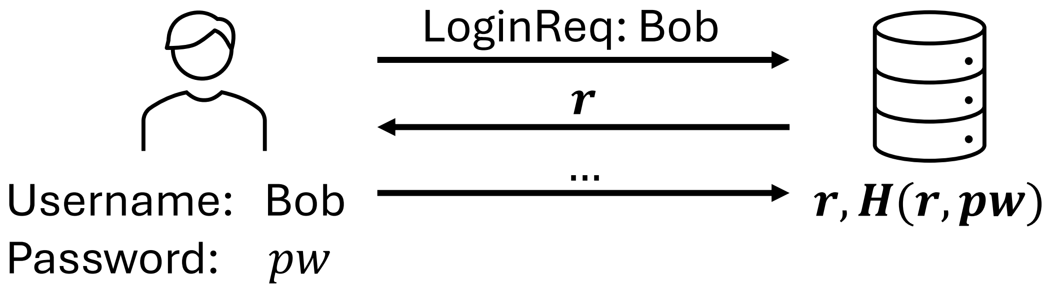

Suppose that the adversary knows the username…

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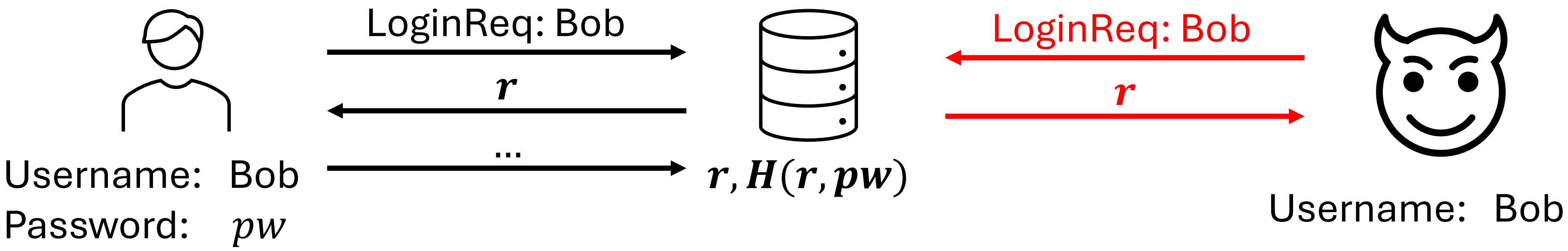

Suppose that the adversary knows the username...
Then it can get the salt...

Precomputation Attacks on Passwords

- Suppose that the password is stored by hashing and salting.
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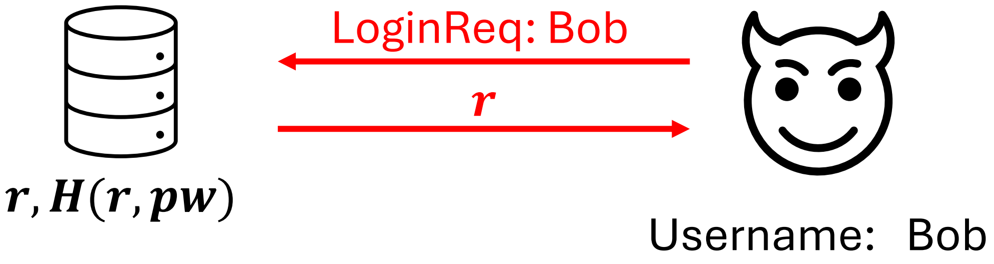
  - **Precompute a table containing all hashed passwords with the same salt**:



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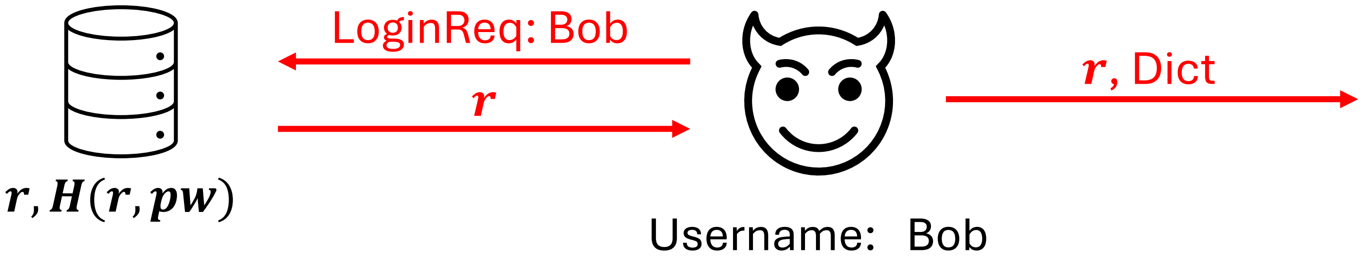

$pw \in$ Dict	The H(pw, r) values
pw_1	$\mathbf{H}(pw_1, r)$
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The table can be computed locally…

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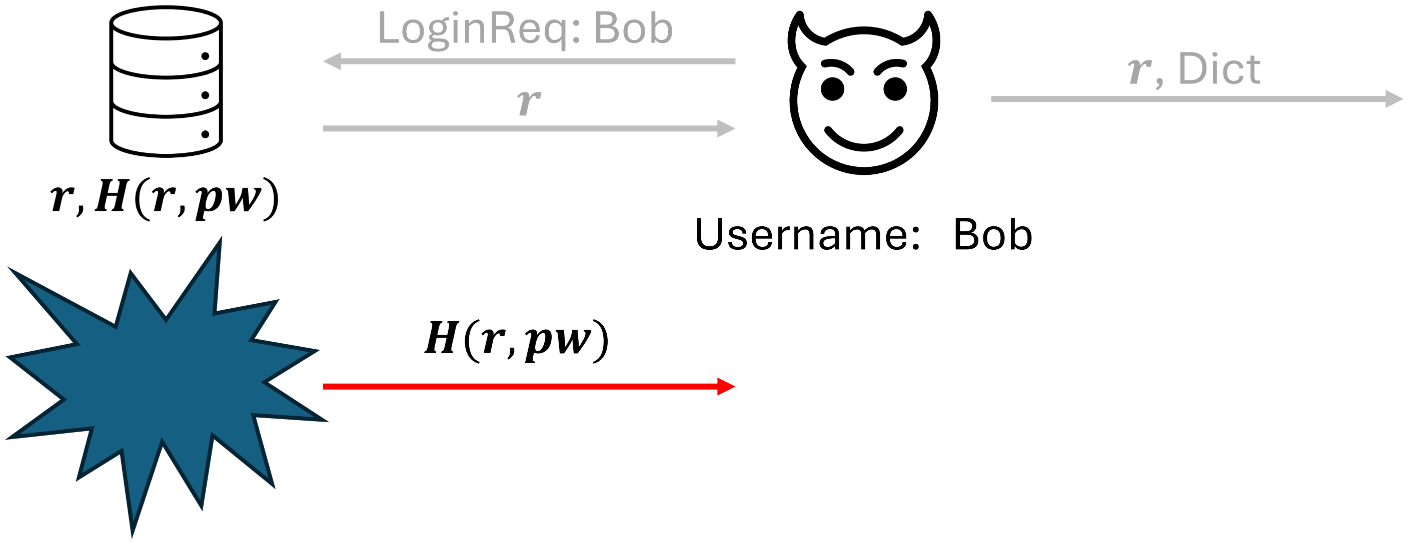

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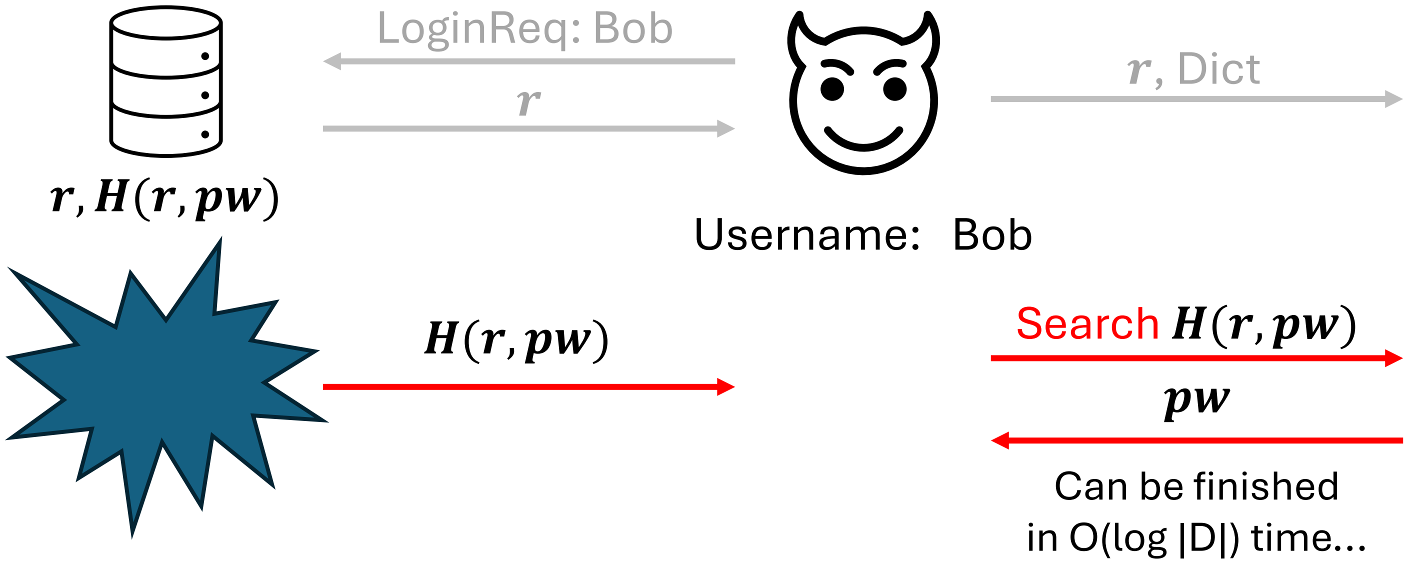

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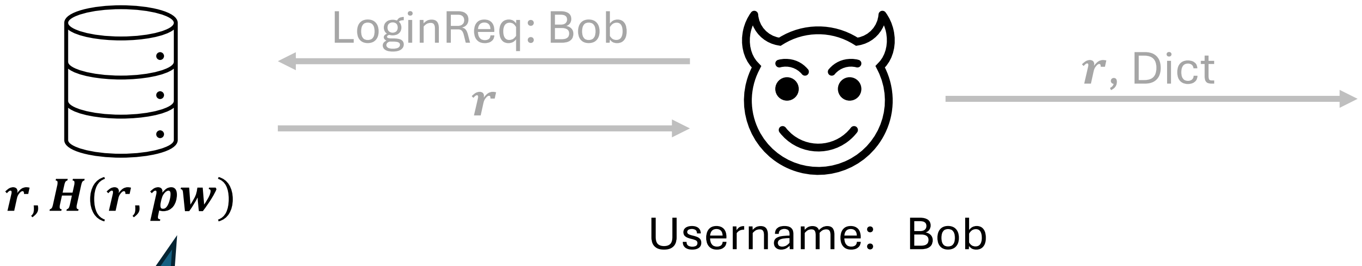

Why **O(log |D|)?**
Sort all H(pw, r) values **before** leakage, and then binary search…

Search $H(r, pw)$

pw

Can be finished in **O(log |D|)** time…

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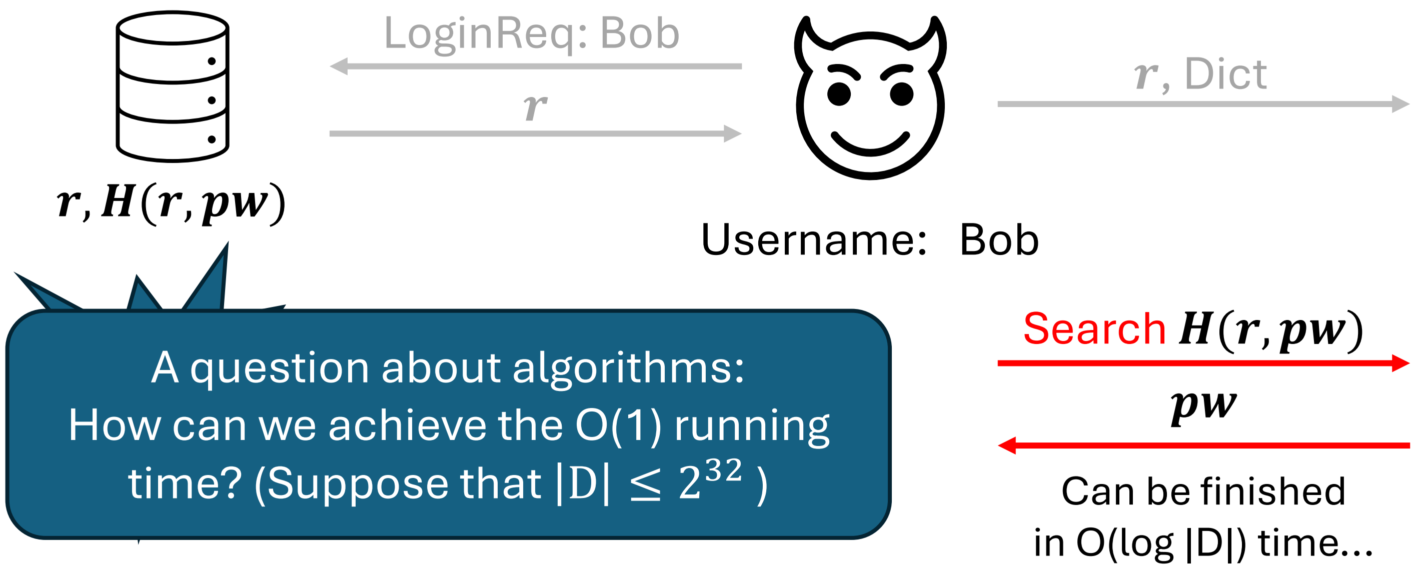

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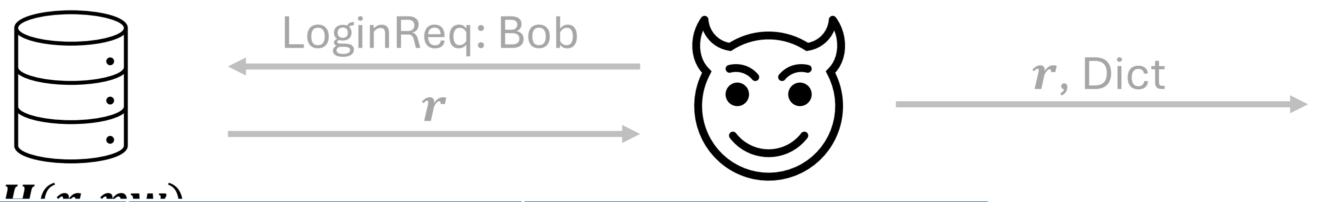

Storage	Required Time after leakage	...with precomputation...
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...arch $\boldsymbol{H(r, pw)}$

$\boldsymbol{pw}$

...an be finished

... O(log |D|) time...

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Precomputation Attacks on Passwords

- Comparison:

Attack Method to recover pw	Required Time *before* leakage	**Required Time *after* leakage**
Brute-force on Dictionary	-	$\mathbf{O}(\lvert\boldsymbol{D}\rvert)$
Precomputation	$\leq \mathbf{O}(\lvert\boldsymbol{D}\rvert \cdot \log\lvert\boldsymbol{D}\rvert)$	$\leq \mathbf{O}(\log\lvert\boldsymbol{D}\rvert)$

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- Reveal salt during the protocol => Precomputation attacks
- How can we protect the salt?
  - No straight-forward solutions that without using algebraic structures
  - Solution using algebraic structures: **Oblivious Pseudorandom Function** (OPRF)
- PAKE without revealing salt: **OPAQUE**

DH-based OPRF

- Classical PRF:
  - Pseudorandomness: If the PRF key is random, then the output of PRF is pseudorandom

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OPRF(k, input)

DH-based OPRF

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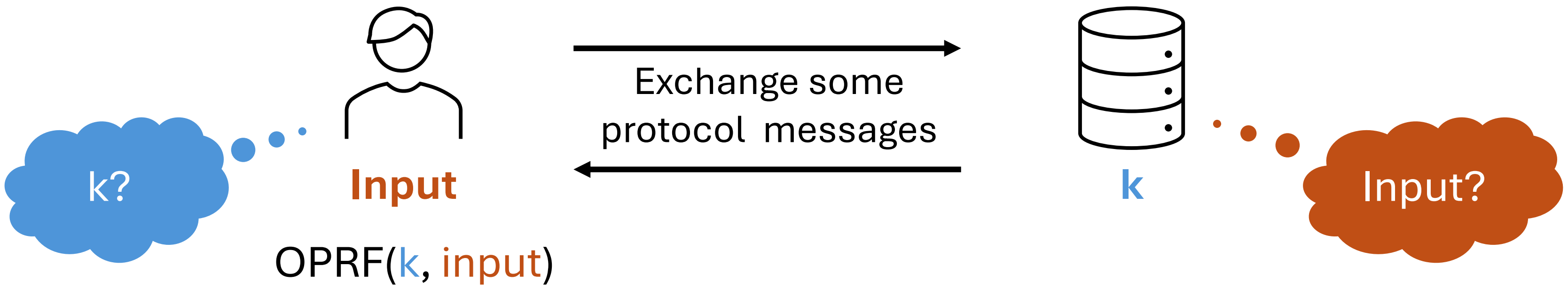
- Oblivious PRF:
  - Pseudorandomness
  - PRF in the two-party (client-server) computation setting
  - **Key privacy:** The client learns OPRF(k, input), **but it learns nothing about the key k**
  - **Input privacy:** The server knows the client has evaluated the ORRF, **but it does not know the input**



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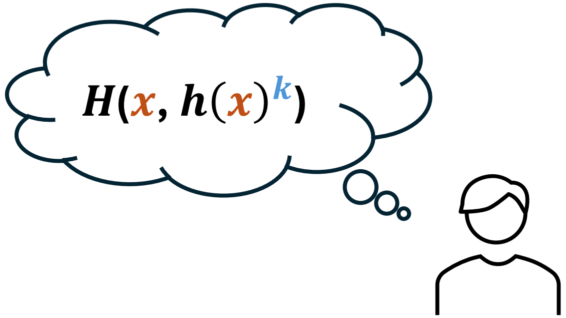


Input: x

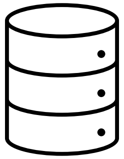
$(\mathbb{G}, g, q)$:

A q-order group $\mathbb{G}$ with a generator g

$h: \{0,1\}^* \rightarrow \mathbb{G}$

A hash function map the input into a group element

H: A normal hash function (e.g., SHA256,..)



k

DH-based OPRF

$\boldsymbol{H(x, h(x)^{k})}$

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$\boldsymbol{k}$

$\alpha \leftarrow_\$ \mathbb{Z}_q$

$$h(\boldsymbol{x})^{\alpha} \longrightarrow$$

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k

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$h(x)^\alpha$ →

← $h(x)^{\alpha \cdot k}$ $(= (h(x)^\alpha)^k)$

Compute $\alpha^{-1} \in \mathbb{Z}_q$

$h(x)^k = \left(h(x)^{\alpha \cdot k}\right)^{\alpha^{-1}}$

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DH-based OPRF

$H(x, h(x)^k)$

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k

$\alpha \leftarrow_\$ \mathbb{Z}_q$

$h(x)^\alpha$

Key Privacy: $h(x)^k$
=> k, solve dlog…

$h(x)^{\alpha \cdot k}$ $(= (h(x)^\alpha)^k)$

Input Privacy:
$h(x)^\alpha$ is “random”…

Compute $\alpha^{-1} \in \mathbb{Z}_q$

$h(x)^k = \left(h(x)^{\alpha \cdot k}\right)^{\alpha^{-1}}$

Compute $H(x, h(x)^k)$

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Compute $H(x, h(x)^{k})$

The OPRF here is
OPRF(*key:* k, *input:* x) = $H(x, h(x)^{k})$

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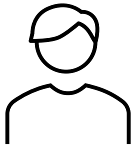
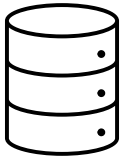


Input: $\boldsymbol{pw}$



Key: $\boldsymbol{s}$ (as the salt)

$\alpha \leftarrow_{\$} \mathbb{Z}_q$

$h(\boldsymbol{pw})^{\alpha} \longrightarrow$

$\longleftarrow h(\boldsymbol{pw})^{\alpha \cdot \boldsymbol{s}} \ (= (h(x)^{\alpha})^{k})$

Compute $\alpha^{-1} \in \mathbb{Z}_q$

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Compute $H(\boldsymbol{pw}, h(\boldsymbol{pw})^{\boldsymbol{s}})$

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$$\boldsymbol{rw} = H(\boldsymbol{pw}, h(\boldsymbol{pw})^{\boldsymbol{s}})$$

- Only the client knows the password
- Only the server knows the salt

DH-based OPRF

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- The $\boldsymbol{rw}$ value is pseudorandom by the pseudorandomness of OPRF, **but it can not be directly used as the session key!**
  - rw is always the same, but we expect that a new execution of the protocol produces a new session key...

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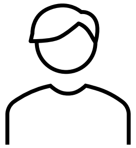
- **Solution: Use AKE protocol to share a session key, and use rw to protect the AKE messages...**

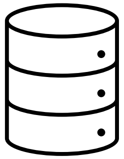
DH-based OPRF + AKE

- Brief introduction of AKE (Authenticated Key Exchange)
  - Two parties share an authenticated key using their long-term key pairs

DH-based OPRF + AKE

- Brief introduction of AKE (Authenticated Key Exchange)
  - Two parties share an authenticated key using their long-term key pairs
  - For example:





Client		Server
$(lpk_c, lsk_c), lpk_s$		$(lpk_s, lsk_s), lpk_c$
Generate (epk_c, esk_c)	$\xrightarrow{epk_c}$	
	$\xleftarrow{epk_s}$	Generate (epk_s, esk_s)
$SK = \text{KeyClient}(lsk_c, esk_c, lpk_s, epk_s, \ldots)$		$SK = \text{KeyServer}(lsk_s, esk_s, lpk_c, epk_c)$

  - Security Requirement: Pseudorandom session key, authentication, …

DH-based OPRF + AKE

pw

$s, rw = H(pw, h(pw)^s)$

Suppose that the server has the rw value

DH-based OPRF + AKE

pw

$s, rw = H(pw, h(pw)^s)$

$(lpk_c, lsk_c) \leftarrow \text{AKE.KeyGen}$
$(lpk_s, lsk_s) \leftarrow \text{AKE.KeyGen}$

Generate AKE key pairs

DH-based OPRF + AKE

pw

$s, rw = H(pw, h(pw)^s)$

$(lpk_c, lsk_c) \leftarrow$ AKE.KeyGen
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key_info = (lpk_c, lsk_c, lpk_s)

rw_key = KDF(rw)
enc_keys = AEAD(rw_key, key_info)

Encrypt generated keys using rw

DH-based OPRF + AKE

pw

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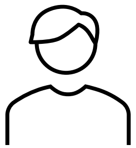

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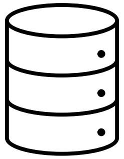

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$\alpha \leftarrow_{\$} \mathbb{Z}_q$

$$\xrightarrow{\quad h(pw)^{\alpha} \quad}$$

$$\xleftarrow{\quad h(pw)^{\alpha \cdot s} \quad}$$

$$h(pw)^{s} = \left(h(pw)^{\alpha \cdot s}\right)^{\alpha^{-1}}$$

$$rw = H(pw, h(pw)^{s})$$

DH-based OPRF + AKE

$\boldsymbol{pw}$

$\boldsymbol{s}$, $\boldsymbol{rw} = H(\boldsymbol{pw}, h(\boldsymbol{pw})^{\boldsymbol{s}})$

$(lpk_c, lsk_c), (lpk_s, lsk_s)$

key_info = (lpk_c, lsk_c, lpk_s)

rw_key = KDF($\boldsymbol{rw}$)

enc_keys = AEAD(rw_key, key_info)

$\alpha \leftarrow_\$ \mathbb{Z}_q$

$\xrightarrow{h(\boldsymbol{pw})^{\alpha}}$

$\xleftarrow{h(\boldsymbol{pw})^{\alpha \cdot \boldsymbol{s}}\text{, enc\_keys}}$

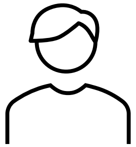
$h(\boldsymbol{pw})^{\boldsymbol{s}} = (h(\boldsymbol{pw})^{\alpha \cdot \boldsymbol{s}})^{\alpha^{-1}}$

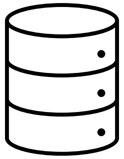
$\boldsymbol{rw} = H(\boldsymbol{pw}, h(\boldsymbol{pw})^{\boldsymbol{s}})$

rw_key = KDF($\boldsymbol{rw}$)

key_info= AEAD.Dec(rw_key, enc_keys) // Client gets (lpk_c, lsk_c, lpk_s)

DH-based OPRF + AKE





pw

s, $rw = H(pw, h(pw)^s)$

$(lpk_c, lsk_c), (lpk_s, lsk_s)$

key_info = (lpk_c, lsk_c, lpk_s)

rw_key = KDF(rw)

enc_keys = AEAD(rw_key, key_info)

$\alpha \leftarrow_\$ \mathbb{Z}_q$

$\xrightarrow{h(pw)^\alpha}$

$\xleftarrow{h(pw)^{\alpha \cdot s}\text{, enc\_keys}}$

$h(pw)^s = (h(pw)^{\alpha \cdot s})^{\alpha^{-1}}$

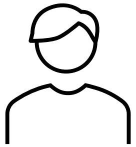
$rw = H(pw, h(pw)^s)$

rw_key = KDF(rw)

key_info= AEAD.Dec(rw_key, enc_keys) // Client gets (lpk_c, lsk_c, lpk_s)

Now the client can run the AKE protocol with Server

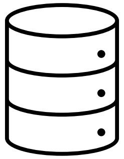
OPQAUE – Overview of Registration



Username
password: pw

(“Register”, Username, pw) $\longrightarrow$

Encrypted by TLS



$s \leftarrow_{\$} \mathbb{Z}_q$

$rw = H(pw, h(pw)^{s})$

rw_key = KDF(rw)

$(lpk_c, lsk_c) \leftarrow$ AKE.KeyGen, $(lpk_s, lsk_s) \leftarrow$ AKE.KeyGen

client_key_info = (lpk_c, lsk_c, lpk_s)

enc_client_keys = AEAD(rw_key, client_key_info)

OPQAUE – Overview of Registration

Username
password: pw

(“Register”, Username, pw)

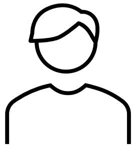
Encrypted by TLS

$s \leftarrow_{\$} \mathbb{Z}_q$

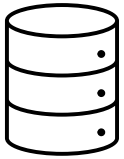
$(lpk_c,$

Then the server store {
user: **Username** // … as index
salt: $\boldsymbol{s}$
server_k_bundle: $\boldsymbol{lpk_c, lpk_s, lsk_s}$
client_enc_k_bundle: **enc_client_keys**
… // Auxiliary information
} in the password database

OPQAUE – Stage 1: OPRF



Username, password: $\boldsymbol{pw}$



$\alpha \leftarrow_{\$} \mathbb{Z}_q$

LoginRequest = (Username, $h(\boldsymbol{pw})^{\alpha}$) $\longrightarrow$

OPQAUE – Stage 1: OPRF

Username, password: $\boldsymbol{pw}$

$\alpha \leftarrow_{\$} \mathbb{Z}_q$

LoginRequest = (Username, $h(\boldsymbol{pw})^{\alpha}$) $\longrightarrow$

Retrieve ($\boldsymbol{s}$, server_k_bundle, client_enc_k_bundle)

// ...corresponds to the username

OPQAUE – Stage 1: OPRF

Username, password: $\boldsymbol{pw}$

$\alpha \leftarrow_{\$} \mathbb{Z}_q$

LoginRequest = (Username, $h(\boldsymbol{pw})^{\alpha}$)

Retrieve ($\boldsymbol{s}$, server_k_bundle, client_enc_k_bundle)

// ...corresponds to the username

$h(\boldsymbol{pw})^{\alpha \cdot \boldsymbol{s}}$, client_enc_k_bundle

$h(\boldsymbol{pw})^{\boldsymbol{s}} = (h(\boldsymbol{pw})^{\alpha \cdot \boldsymbol{s}})^{\alpha^{-1}}$

$\boldsymbol{rw} = H(\boldsymbol{pw}, h(\boldsymbol{pw})^{\boldsymbol{s}})$

rw_key = KDF($\boldsymbol{rw}$)

client_key_info= AEAD.Dec(rw_key, client_enc_k_bundle)

OPQAUE – Stage 1: OPRF

Username, password: pw

$\alpha \leftarrow_{\$} \mathbb{Z}_q$

LoginRequest = (Username, $h(pw)^{\alpha}$)

Retrieve (s, server_k_bundle, client_enc_k_bundle)

// ...corresponds to the username

$h(pw)^{\alpha \cdot s}$, client_enc_k_bundle

$h(pw)^{s} = (h(pw)^{\alpha \cdot s})^{\alpha^{-1}}$

$rw = H(pw, h(pw)^{s})$

rw_key = KDF(rw)

client_key_info= AEAD.Dec(rw_key, client_enc_k_bundle)

Parse client_key_info = (lpk_c, lsk_c, lpk_s)

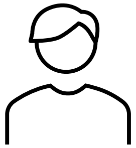
Parse server_k_bundle = (lpk_c, lpk_s, lsk_s)

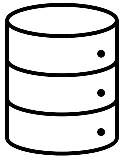
OPQAUE – Stage 2: AKE

Parse client_key_info = (lpk_c, lsk_c, lpk_s)

Parse server_k_bundle = (lpk_c, lpk_s, lsk_s)

OPQAUE – Stage 2: AKE





Username, password: pw

Parse client_key_info = (lpk_c, lsk_c, lpk_s)

Parse server_k_bundle = (lpk_c, lpk_s, lsk_s)

OPQAUE – Stage 2: AKE

Username, password: pw

OPRF stage

Parse client_key_info = (lpk_c, lsk_c, lpk_s)

Generate (epk_c, esk_c)

epk_c →

Parse server_k_bundle = (lpk_c, lpk_s, lsk_s)

Generate (epk_s, esk_s)

SK = KeyServer(…)

← epk_s

SK = KeyClient(…)

OPQAUE – Stage 3: Key Confirmation

Username, password: pw

OPRF stage

Parse client_key_info = (lpk_c, lsk_c, lpk_s)

Parse server_k_bundle = (lpk_c, lpk_s, lsk_s)

AKE stage (Homework)

SK = KeyClient(…)

SK = KeyServer(…)

OPQAUE – Stage 3: Key Confirmation

Username, password: pw

OPRF stage

Parse client_key_info = (lpk_c, lsk_c, lpk_s)

Parse server_k_bundle = (lpk_c, lpk_s, lsk_s)

AKE stage (Homework)

$SK =$ KeyClient(...)

$SK =$ KeyServer(...)

Key Confirmation (Homework)

OPQAUE – Summary

Username, password: *pw*

Registration:
Instead of storing (salt, H(salt pw)), we store
(salt, AEAD(rw, [AKE keys], ...)), where rw = DH-OPRF(salt, pw)
// This allows the future messages exchange to not reveal the salt (to prevent precomputation)

OPRF stage:
Allow the client to compute rw (to recover the AKE keys) without revealing the salt

AKE stage:
Use AKE protocol to share a fresh session key

Key Confirmation:
Confirm both parties share the same key

Summary on Password-based Authentication

- Use passwords to authenticate identities

- Storage of passwords & Protocols:
  - Plaintext (or hashed without salt) password: 👎
  - Hashed + salted password: 👍 **(SRP, ...)**
  - Hashed + salted + iterated password: 👍👍 **(SCRAM, ...)**
  - OPRF passwords: 👍👍👍 **(OPAQUE)**

- In Practice: Run over TLS

- Password-based AKE protocols: (secure guarantee even in an insecure TLS connection...)
  - SRP
  - OPAQUE (stronger)

Homework

- Implement the DH-OPRF protocol, and use it to implement the OPAQUE registration phase.
- Implement the HMQV AKE protocol

$(\mathbb{G}, g, q)$:

A q-order group $\mathbb{G}$ with a generator g

$lpk_c = A = g^a \in \mathbb{G}$
$lpk_s = B \in \mathbb{G}$
$lsk_c = a \in \mathbb{Z}_q$

HMQV-KG:

1. $lsk \leftarrow_\$ \mathbb{Z}_q$
2. $lpk = g^{lsk}$
3. Return (lpk, lsk)

$lpk_c = A \in \mathbb{G}$
$lpk_s = B = g^b \in \mathbb{G}$
$lsk_s = b \in \mathbb{Z}_q$

$x \leftarrow_\$ \mathbb{Z}_q$

$$\xrightarrow{\quad epk_c = x \quad}$$

$y \leftarrow_\$ \mathbb{Z}_q$

$SK =$ HMQV-KServer (b, y, A, X)

$$\xleftarrow{\quad epk_s = y \quad}$$

$SK =$ HMQV-KClient(a, x, B, Y)

(The HMQV-Kclient/KServer algorithms are given on the next page...)

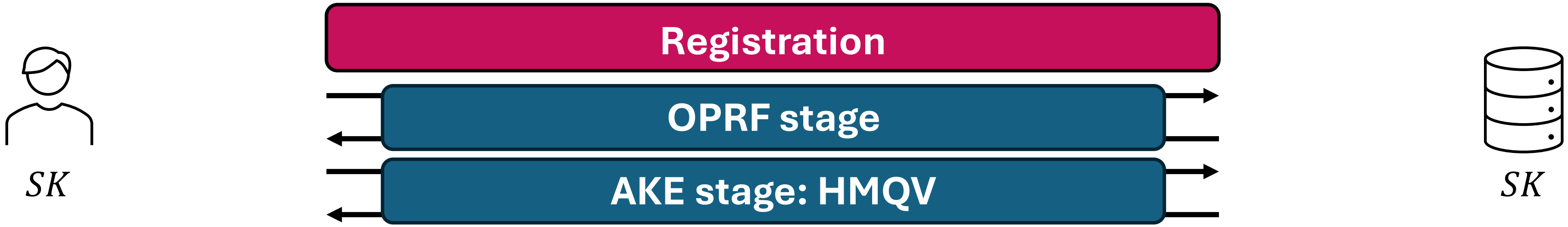
Homework

HMQV-KClient(a, x, B, Y)
1. d = SHA256(X, [Server's Name])
2. e = SHA256(Y, [Client's Name])
3. $ss = (YB^e)^{x+da \bmod q}$
4. SK = HKDF(ss)

HMQV-KServer(b, y, A, X)
1. d = SHA256(X, [Server's Name])
2. e = SHA256(Y, [Client's Name])
3. $ss = \left(XA^d\right)^{y+eb \bmod q}$
4. SK = HKDF(ss)

Homework

- Implement the OPAQUE protocol instantiating with the HMQV protocol, where the Key Confirmation works as follows:



Client:

$(K_c, K_s) =$ HKDF(SK, (“Key Confirmation”)

$mac_c =$ HMAC(K_c, “Client KC”)

$mac'_s =$ HMAC(K_s, “Server KC”)

Server:

$(K_c, K_s) =$ HKDF(SK, (“Key Confirmation”)

$mac'_c =$ HMAC(K_c, “Client KC”)

$mac_s =$ HMAC(K_s, “Server KC”)

$$\text{Client} \xrightarrow{mac_c} \text{Server}$$

$$\text{Client} \xleftarrow{mac_s} \text{Server}$$

Client:

Check $mac'_s =_? mac_s$

If valid, then accept SK

Server:

Check $mac'_c =_? mac_c$

If valid, then accept SK

Homework

- **(Bonus)** Implement the OPAQUE protocol (in the non-bonus homework) using sockets.
- **(Bonus)** What is the RTT of the OPAQUE protocol in the non-bonus homework? Can you improve it? If so, implement your improved version (can be without sockets)
  - One RTT = One “ $\rightleftarrows$ ” in the protocol…

- Lots of homework this lecture => No mandatory homework in the next lecture

Further Reading

- OPAQUE paper: https://eprint.iacr.org/2018/163
- OPAQUE IETF draft: https://www.ietf.org/archive/id/draft-irtf-cfrg-opaque-02.html
- HMQV paper: https://eprint.iacr.org/2005/176

Notes on Homework

- 1 *non-bonus* homework question = 1 point
- 1 *bonus* homework question = 2 points
- How to calculate the final grade of homework (≤ 40):

$$40 \times \left({}^{\text{points you obtain}} / {}_{\text{the number of questions}} \right)$$

// You need to get at least 40 × 60% = **24 points to qualify for the final exam.**

- You can submit **bonus homework before the final deadline: Feb 7th, 2025**
  - Please ensure that your code runs correctly, as you will not have an opportunity to resubmit it.

- If your code for **Homework Set 1 or 2 does not run correctly...**
  - You can resubmit it by the **extended deadline: January 21st, 2025.**

- Some suggestions:
  - **Include the sample input and its expected output** in the README file to help me verify your submission.