Quantum Computing

- Lecture 8 (May 28, 2025)

- Today:
  - Entanglement
  - Pure states and mixed states
  - Exercises and Homework

Postulates of Quantum Computing

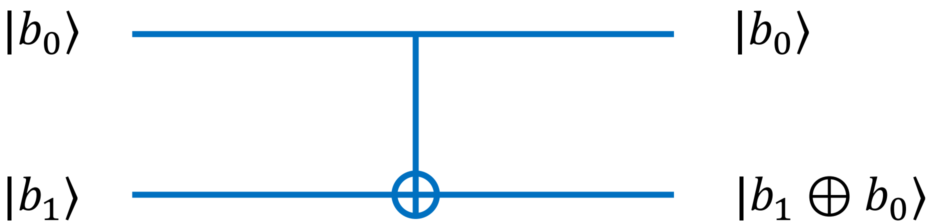
- Postulate 1: State space
- Postulate 2: Evolution and unitary transformation
- Postulate 3: Quantum Measurement
  - Projective measurement
- Postulate 4: Composite system

Postulates of Quantum Computing

- Postulate 1: State space **(isolated systems)**
- Postulate 2: Evolution and unitary transformation **(closed systems)**
- Postulate 3: Quantum Measurement
  - Projective measurement
- Postulate 4: Composite system

Controlled NOT Gate

- CNOT: If $b_0 = 0$, output b_1; Else, output $1 \oplus b_1$ (i.e., flip b_1 if $b_0 = 1$)



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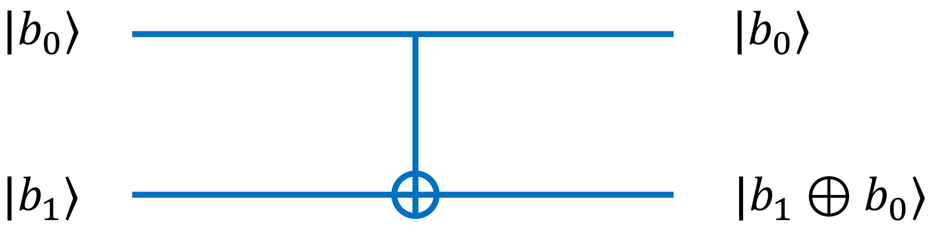
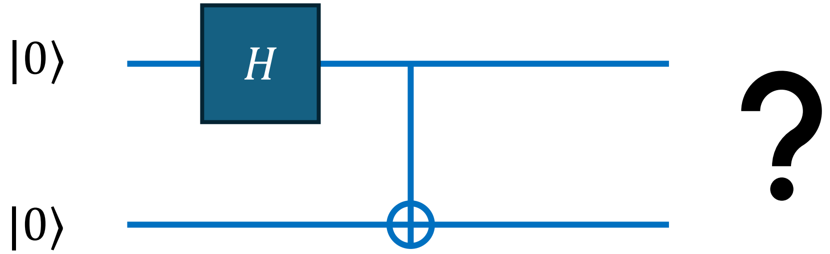


- $|00\rangle \rightarrow |00\rangle$
- $|01\rangle \rightarrow |01\rangle$
- $|10\rangle \rightarrow |11\rangle$
- $|11\rangle \rightarrow |10\rangle$
- Exercise (2min): Write the unitary of CNOT (in the computational basis)

Controlled NOT Gate



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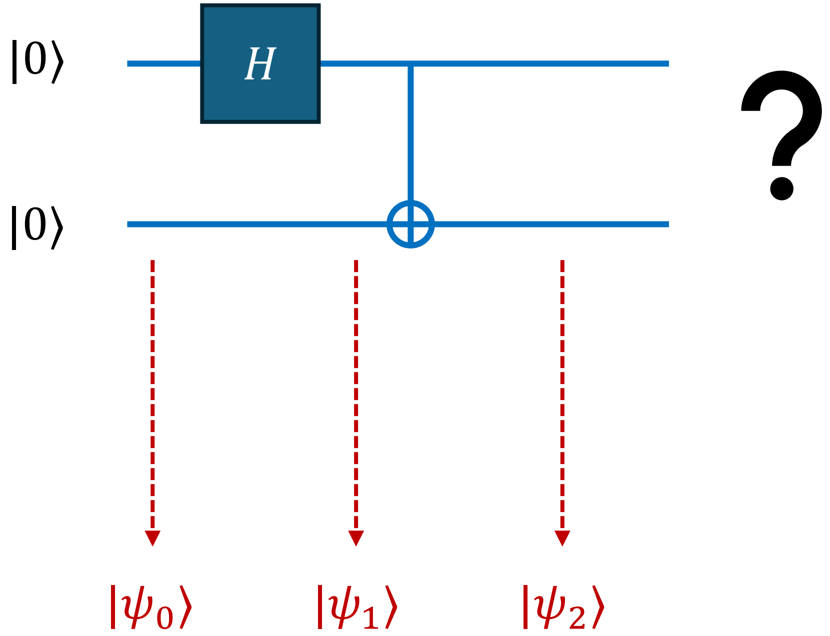

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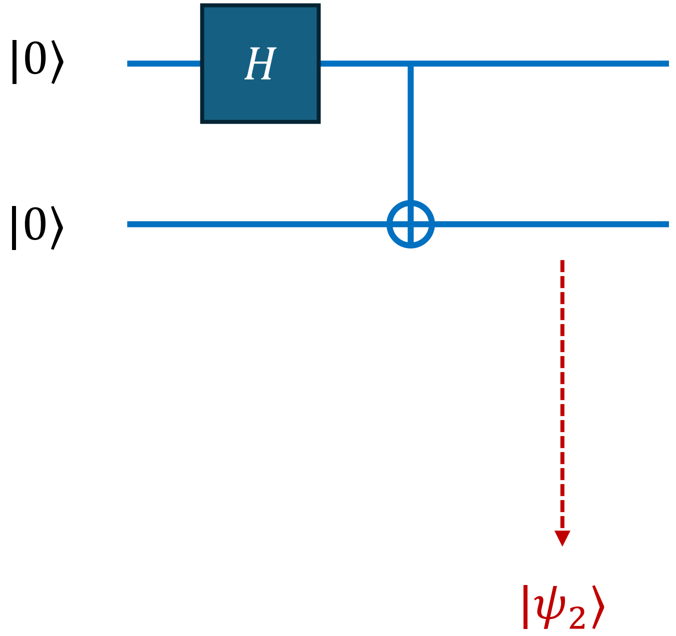

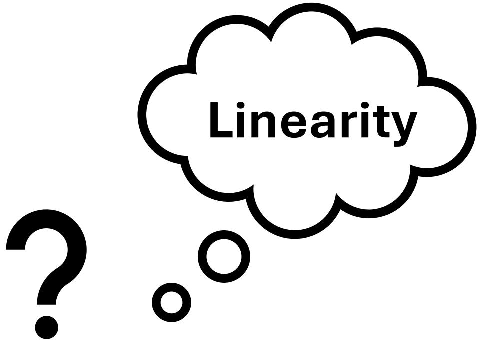

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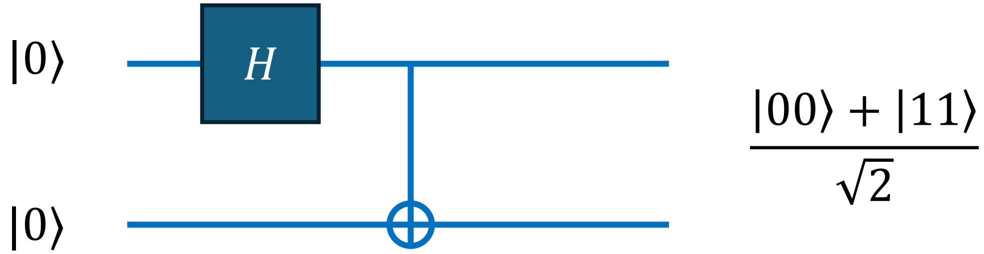

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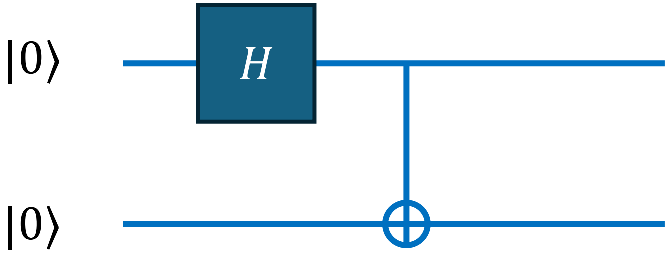
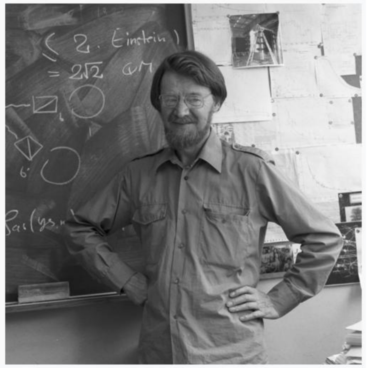
Controlled NOT Gate



$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

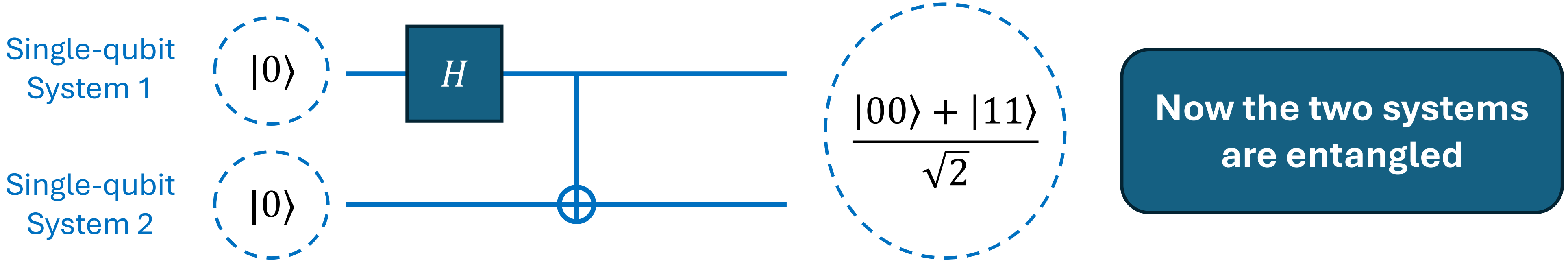
- $|00\rangle \rightarrow |00\rangle$
- $|01\rangle \rightarrow |01\rangle$
- $|10\rangle \rightarrow |11\rangle$
- $|11\rangle \rightarrow |10\rangle$



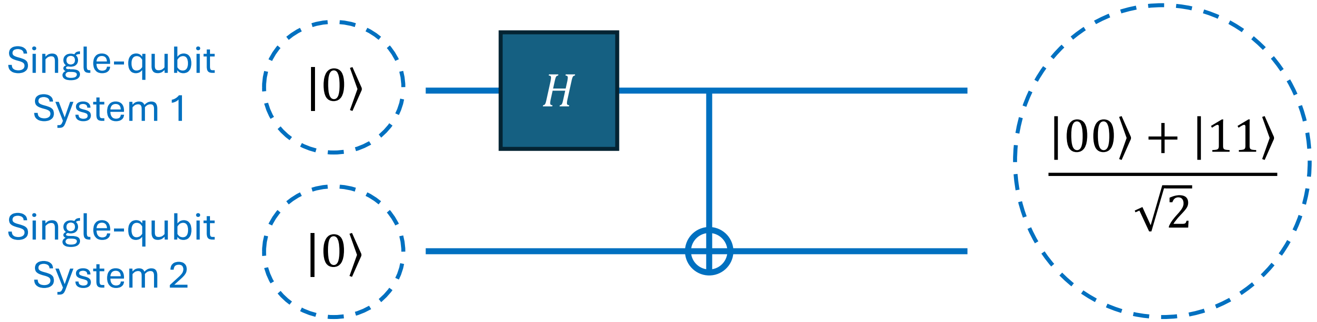
John Stewart Bell
(source: Wikipedia)

Bell state:
Impossible to be split into
a tensor product of two states
$|\varphi_1\rangle \otimes |\varphi_2\rangle$

Quantum Entanglement



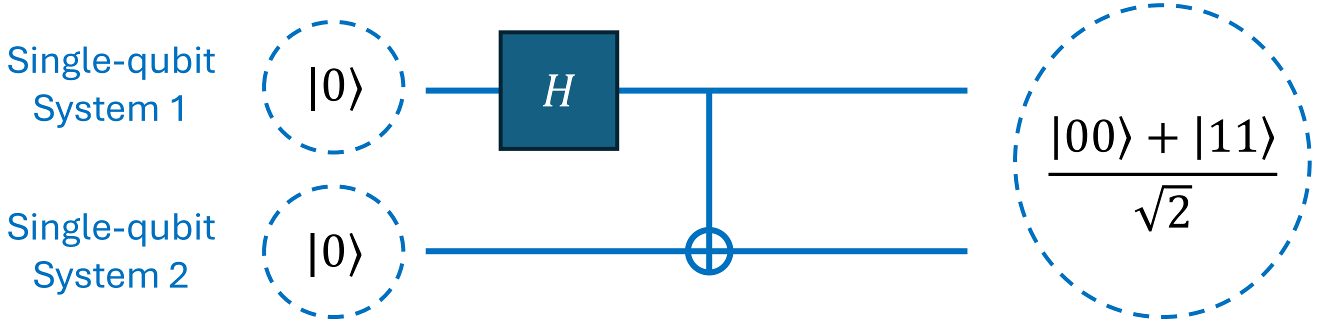
Quantum Entanglement



Now the two systems are entangled

Pure state: Can be described by a state vector
Mixed state: Can**not** ...

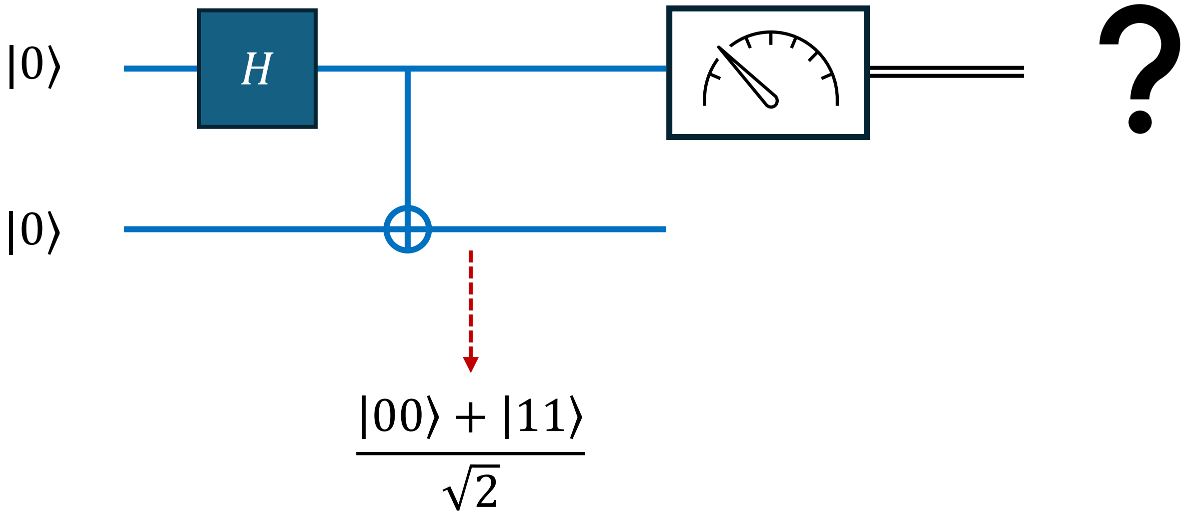
Quantum Entanglement



Small Exercise: (pure or mixed)

1. The initial state of system 1 is ____.
2. The states of system 1 and 2 (after H and CNOT) are ____.
3. The state of the total system (after H and CNOT) is ____.

Partial Measurement on Entangled States



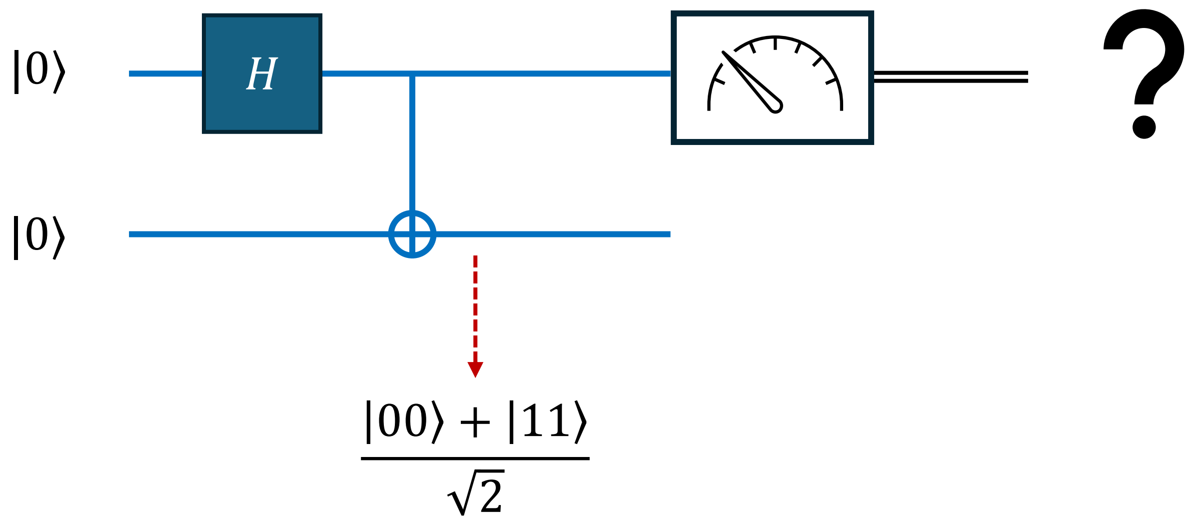
Partial Measurement on Entangled States

- Formalizing Partial Measurement (Do it on board)
  - Let's focus on the computational basis
  - General measurement: $\{M_m\}_m \to \{M_m \otimes I\}_m$
  - Projective measurement: $\boldsymbol{M} \otimes \boldsymbol{I} = (\sum_m m\boldsymbol{P}_m) \otimes \boldsymbol{I}$

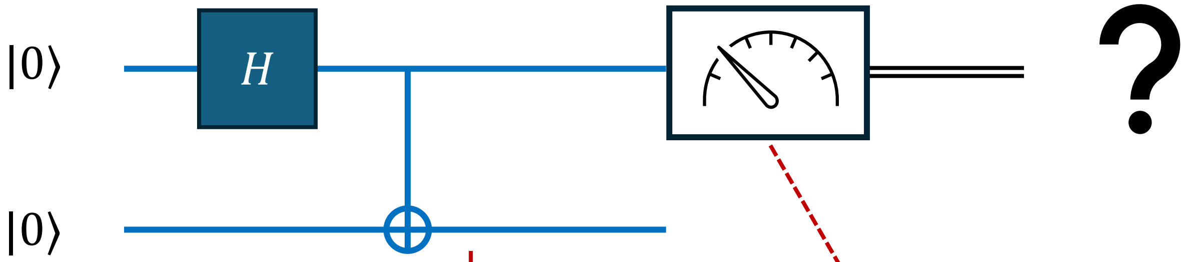
- Important notes:
  - $\{M_m \otimes I\}_m$ still satisfies the completeness equation
  - $\boldsymbol{M} \otimes \boldsymbol{I}$ is still an observable

- Example (Exercise):
  - Partial measurement on the state $\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$

Partial Measurement on Entangled States



Partial Measurement on Entangled States

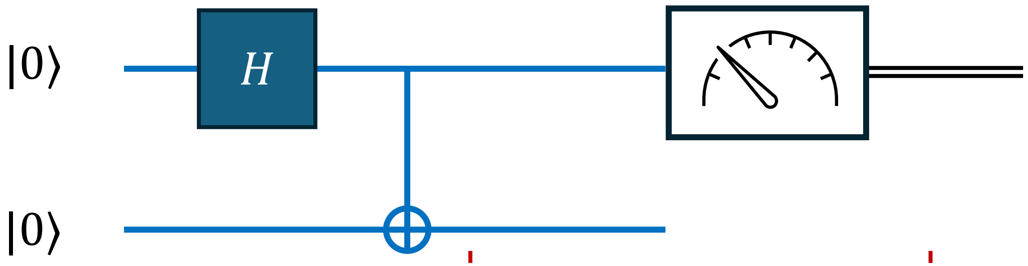


$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

The observable of the measurement is

$$\boldsymbol{M} = 0 \cdot |0\rangle\langle 0| + 1 \cdot |1\rangle\langle 1|$$

Partial Measurement on Entangled States



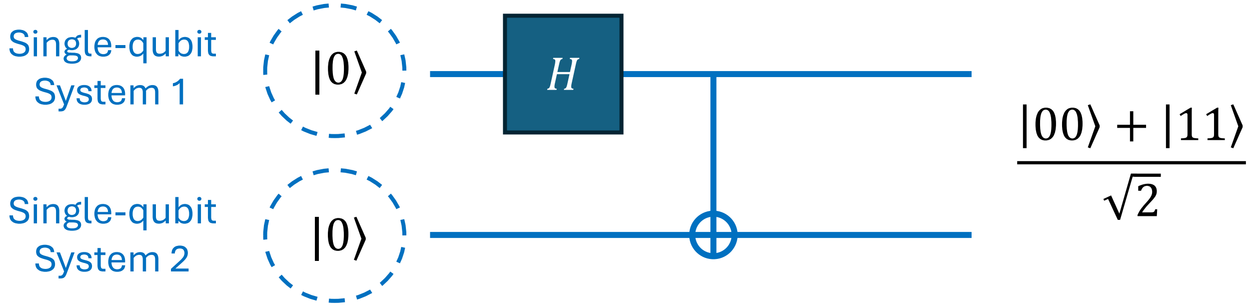
$$\begin{cases} 0. & \text{with prob. } ½ \\ 1. & \text{with prob. } ½ \end{cases}$$

For entangled states,
partial measurement leads to global collapse

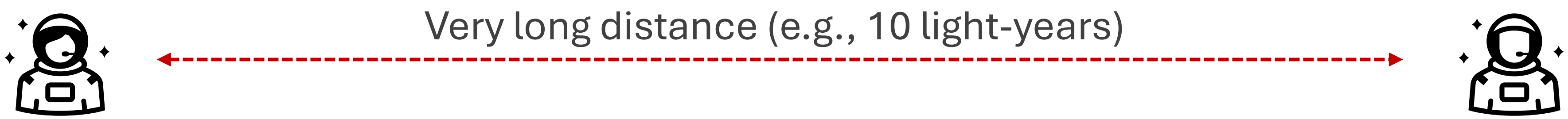
$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

State after measurement $\begin{cases} |00\rangle & \text{if the measurement outome is } 0 \\ |11\rangle & \text{if the measurement outome is } 0 \end{cases}$

Action at a Distance (Fernwirkung)



Action at a Distance

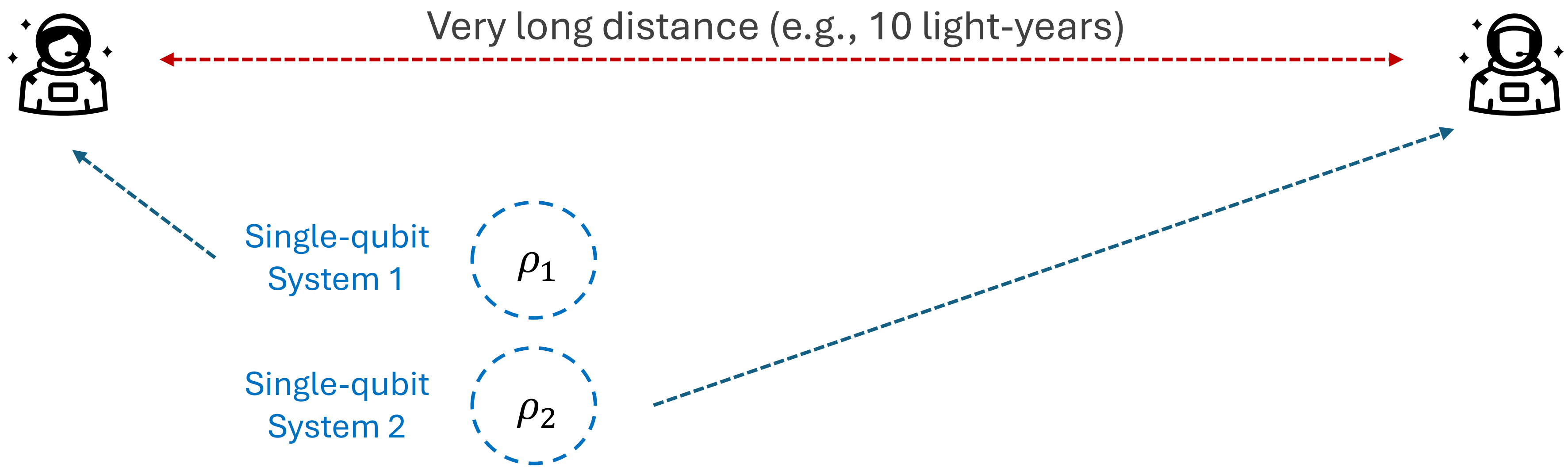


Single-qubit System 1 ρ_1

Single-qubit System 2 ρ_2

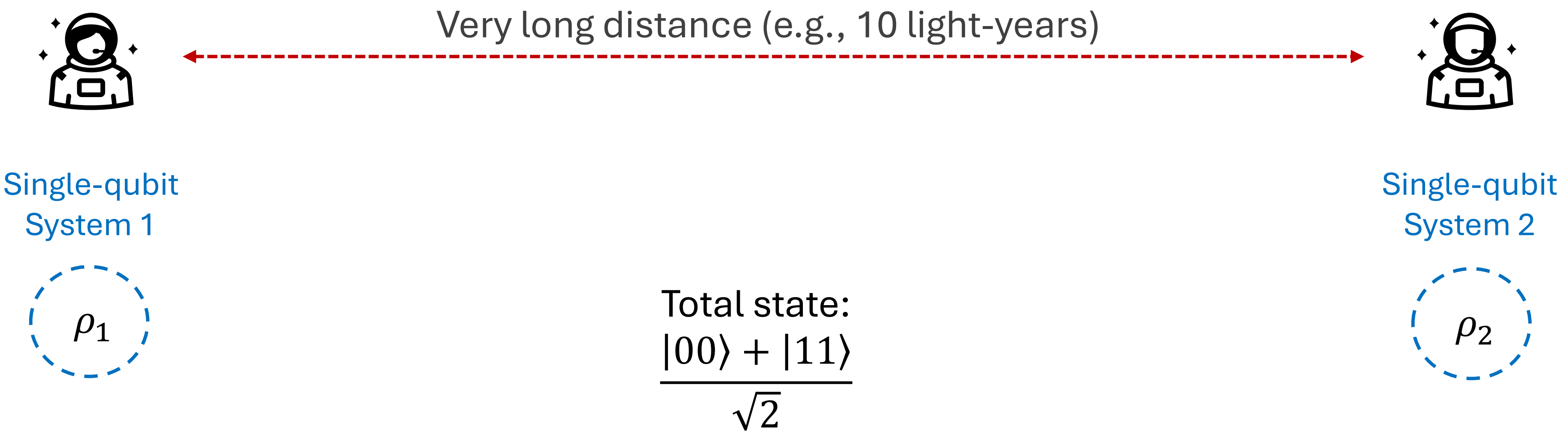
Total state: $\frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Action at a Distance



Total state: $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$

Action at a Distance

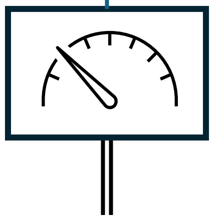


Action at a Distance

Very long distance (e.g., 10 light-years)

Single-qubit
System 1

ρ_1



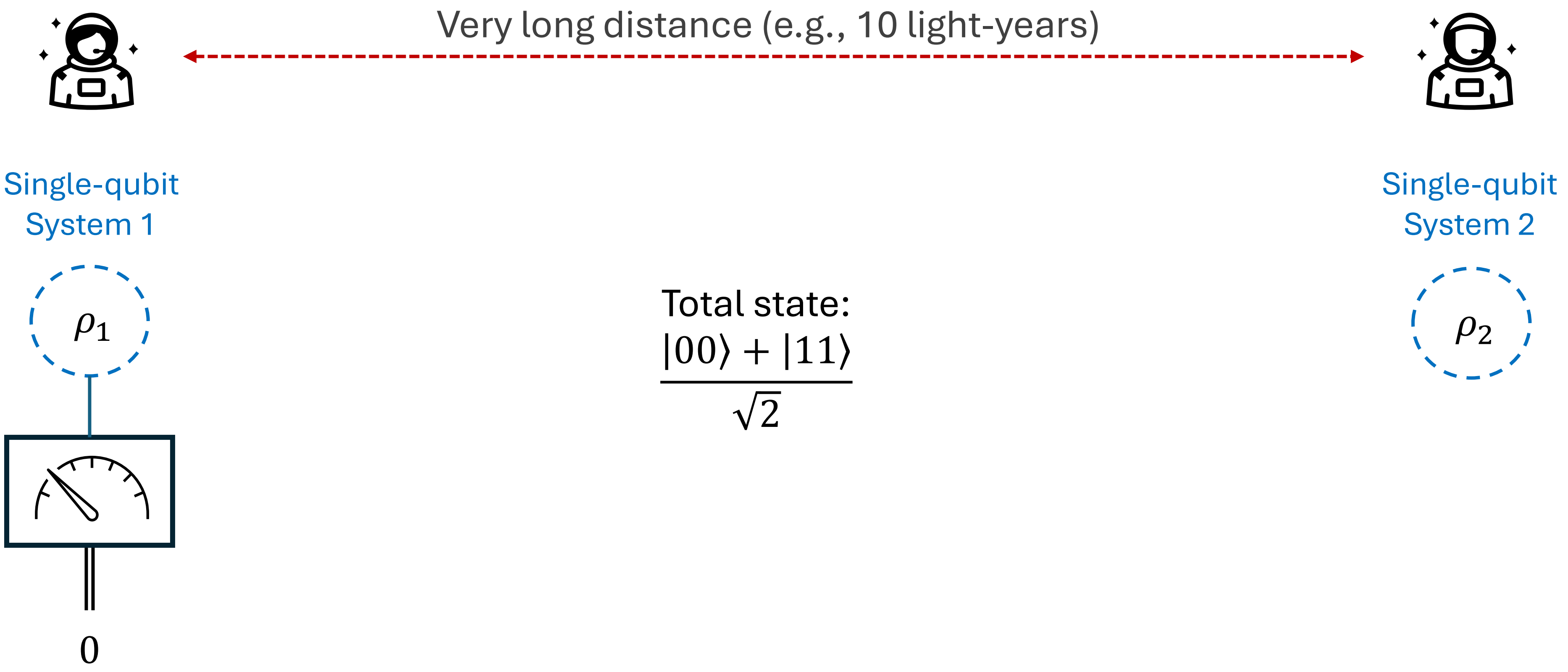
Total state:

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

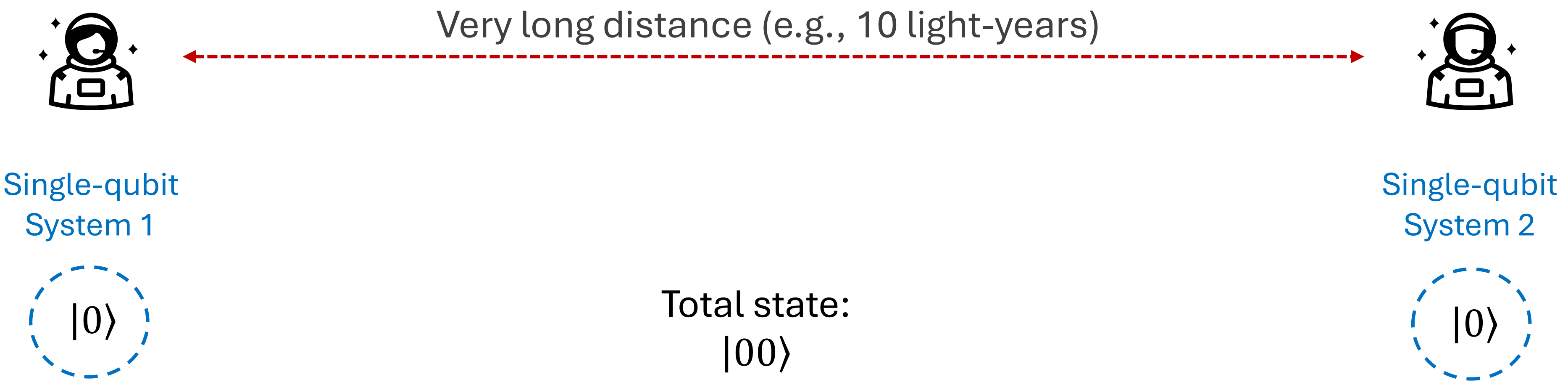
Single-qubit
System 2

ρ_2

Action at a Distance

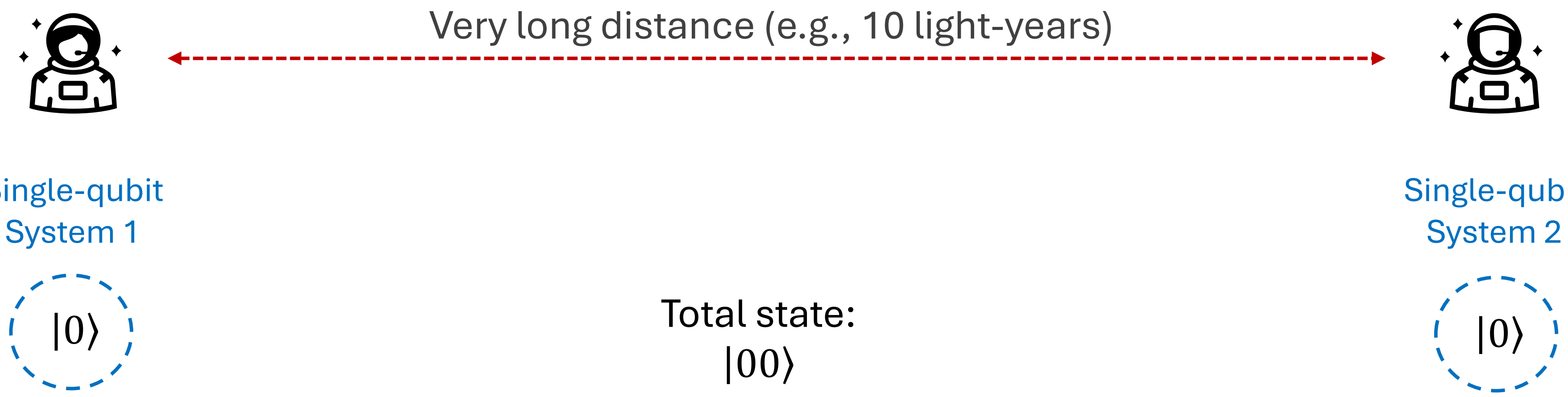


Action at a Distance



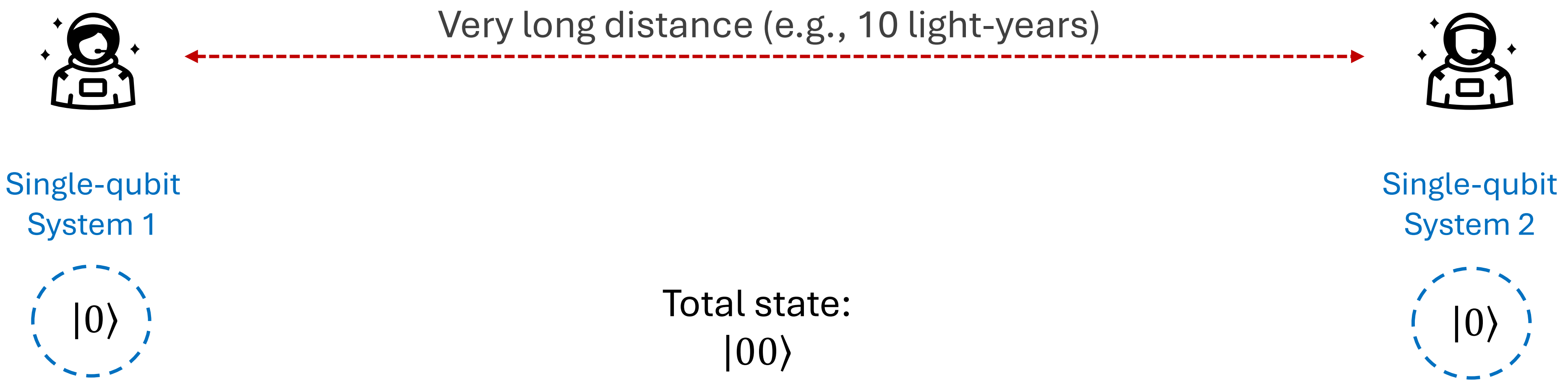
- “spukhafte Fernwirkung”

Action at a Distance



- “spukhafte Fernwirkung”
- A quick question: Is it a faster-than-light communication?

Action at a Distance



- “spukhafte Fernwirkung”
- A quick question: Is it a faster-than-light communication?
- Next topic: **Quantum transportation** (e.g., superdense coding)

Next Week

- Quantum transportation

- **No lecture tomorrow** (Ascension Day, May 29)
- **Homework 2** (about Simon algorithm, to be announced)