Cryptography Engineering

- Lecture 3 (Nov 05, 2024)

- Today's notes:
  - Signed Diffie-Hellman Key Exchange (SigDH) Protocol
  - TLS handshake and HTTPS protocol
  - Case study: ECDSA

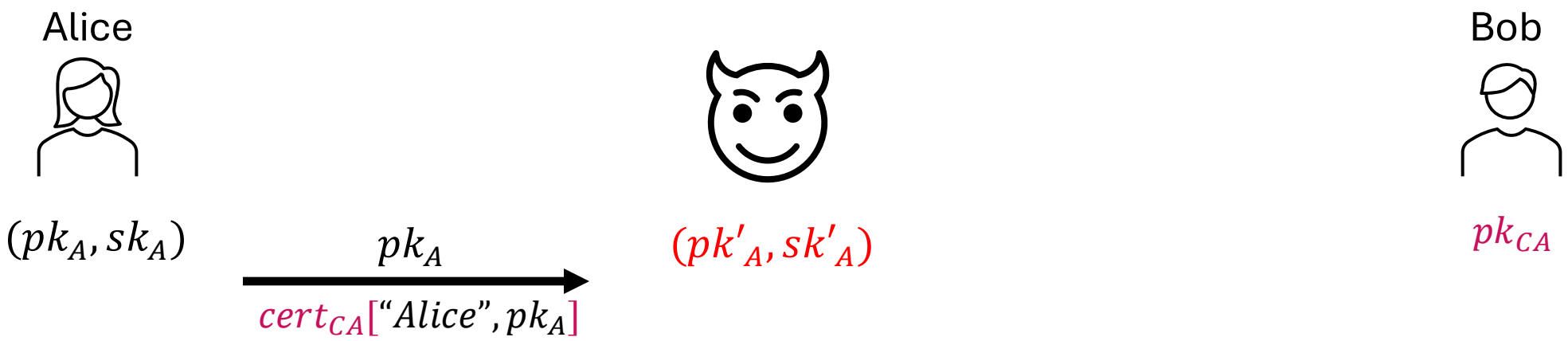
- Today's coding tasks:
  - Implement a toy example of TLS handshake
- First homework set

Code Review

- Some useful notes:
  - Crate x25519_dalek

Prevent MitM using signature/certificate

- Transporting **(malicious)** public keys **(with signature/certificate)**
  - (Note that a certificate binds a public key with the identity of owner)



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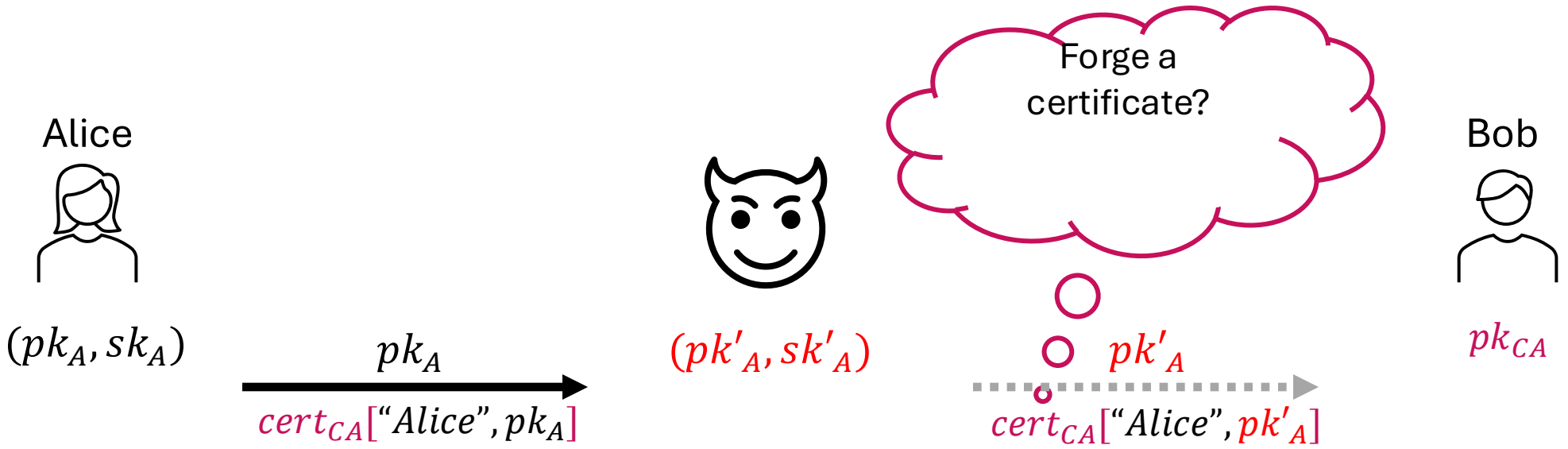

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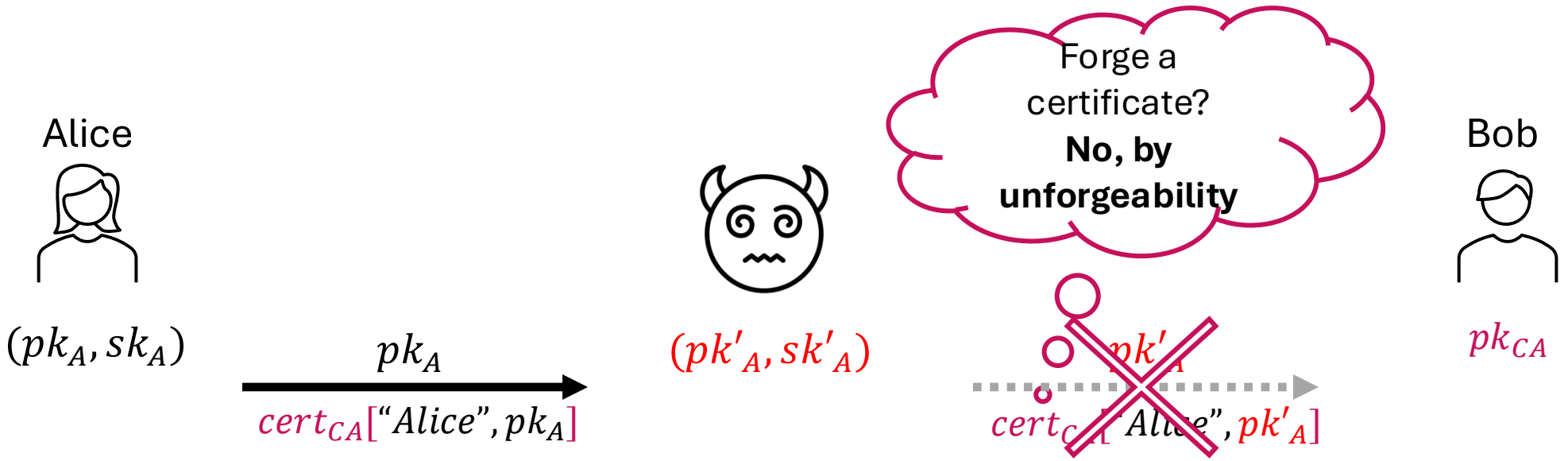
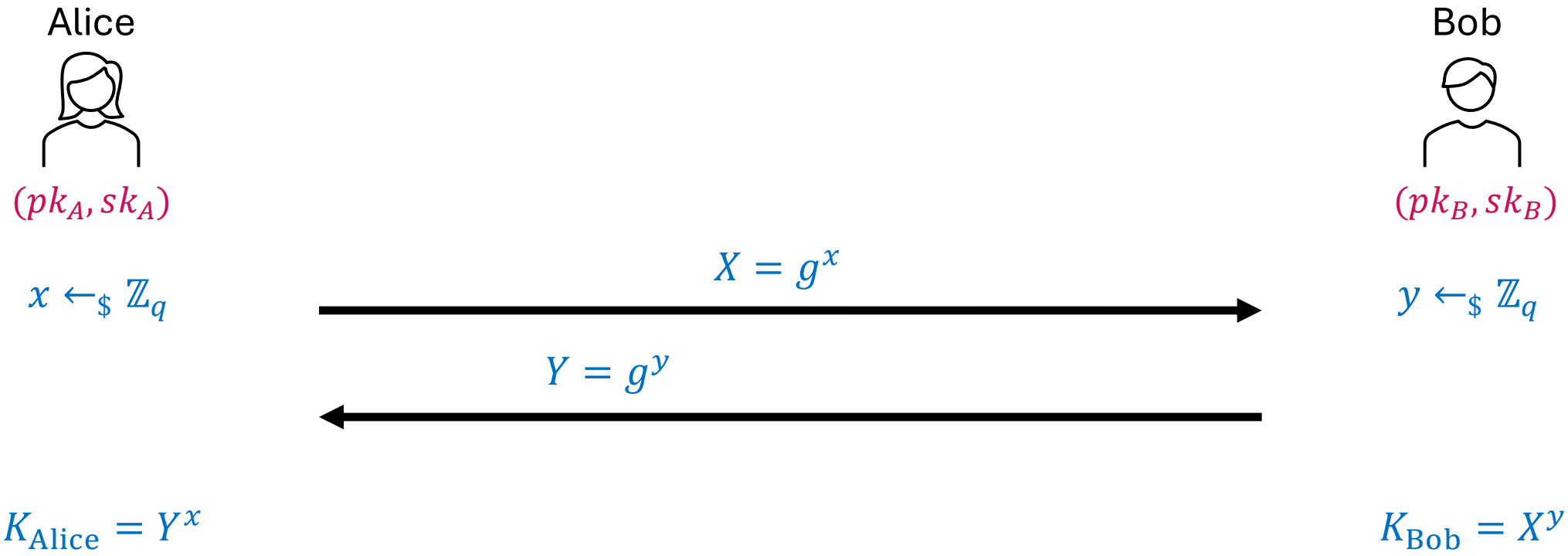


Signed Diffie-Hellman Protocol (Simplified)

- Use signature to avoid MitM attacks



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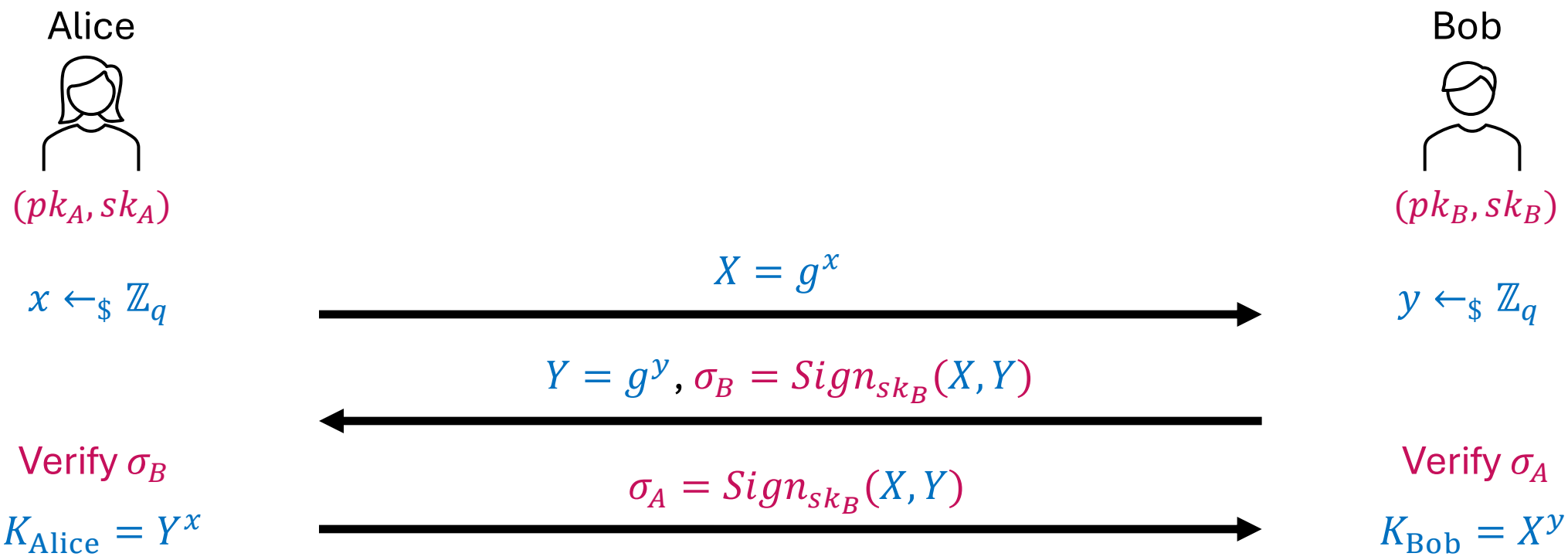
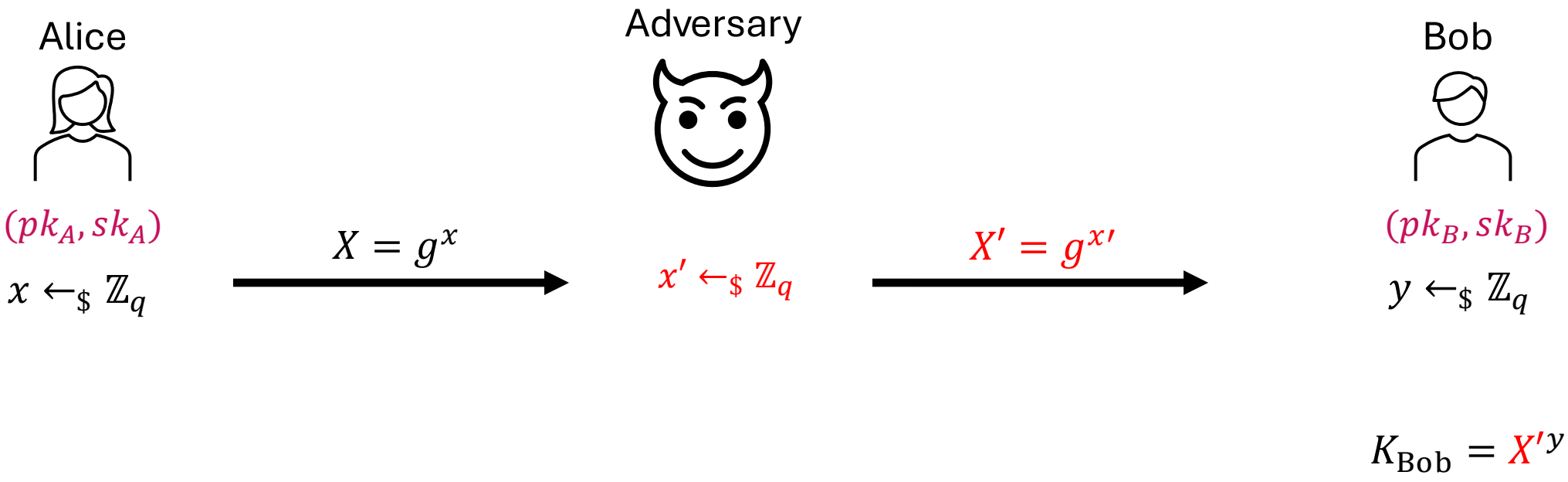


Signed Diffie-Hellman Protocol

- Can we launch a MitM attack on SigDH?



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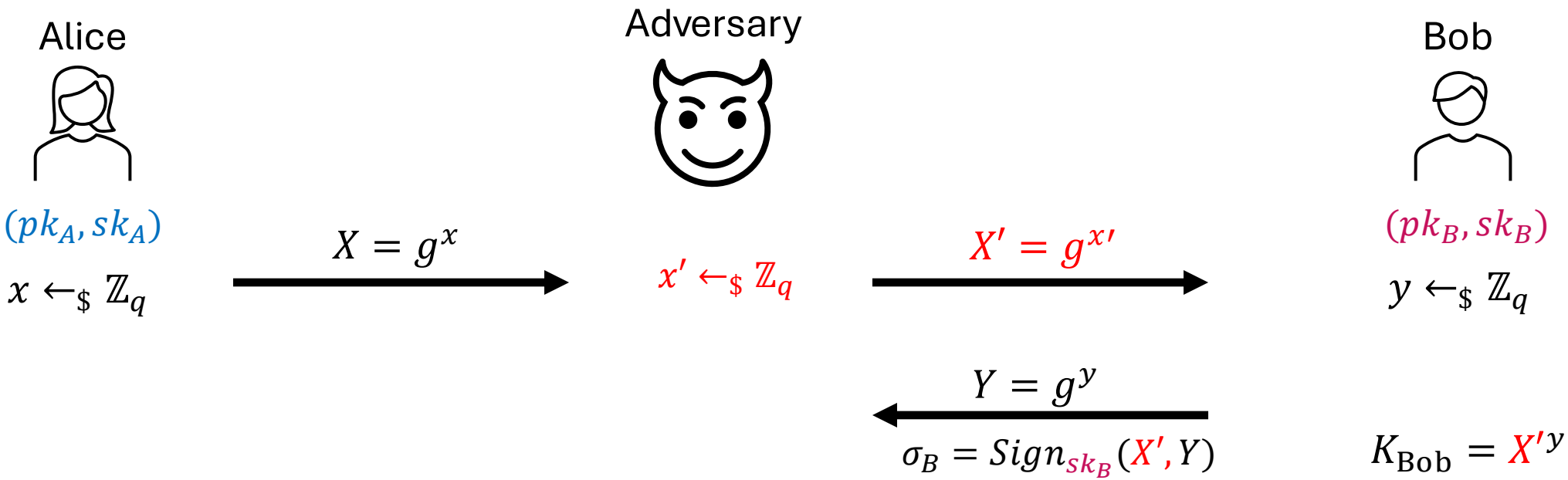

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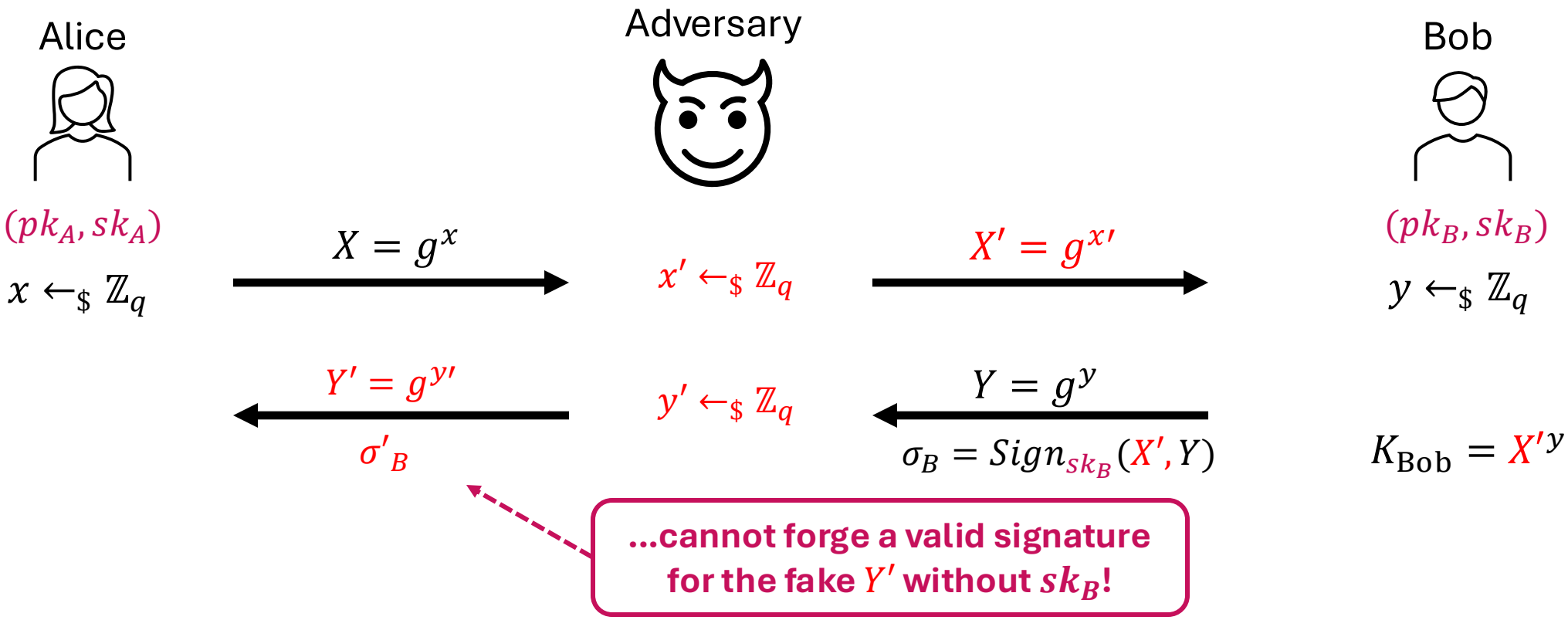

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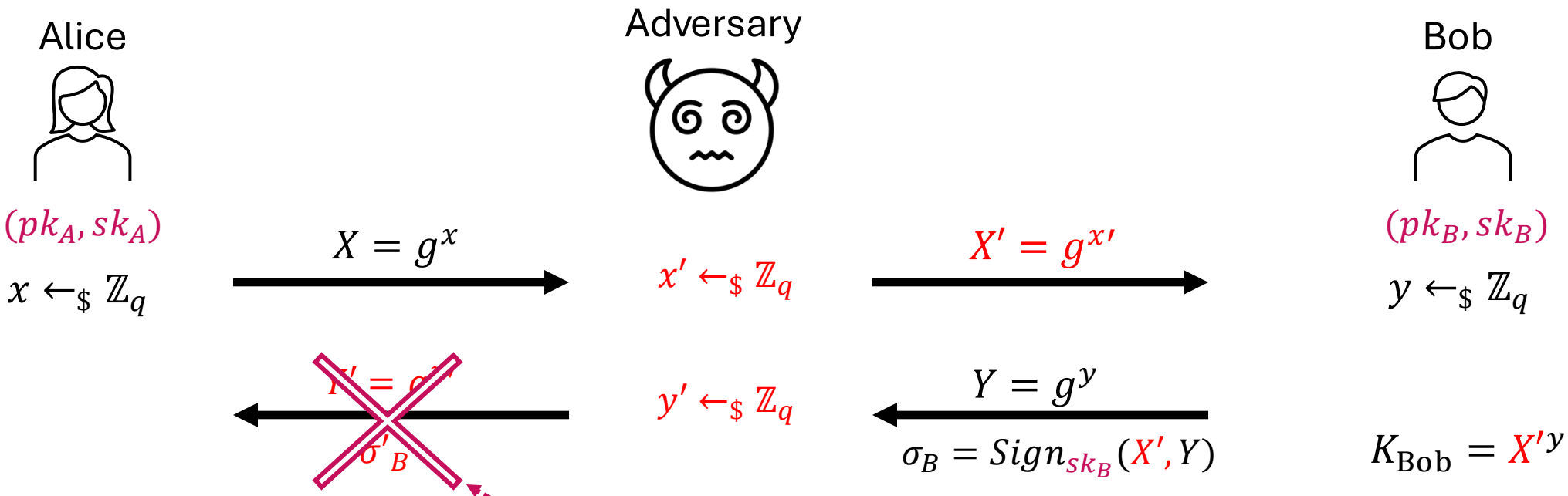
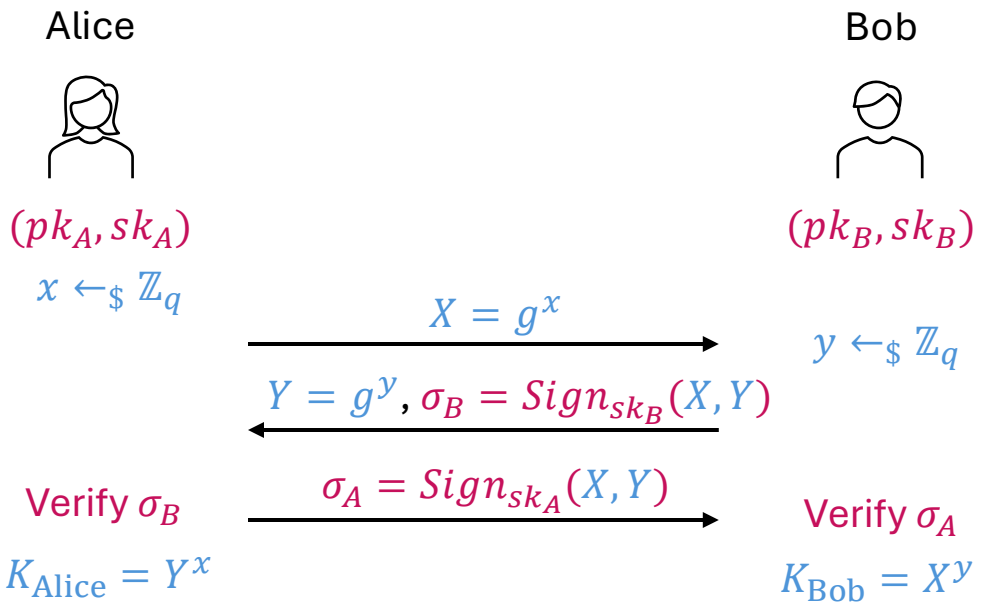


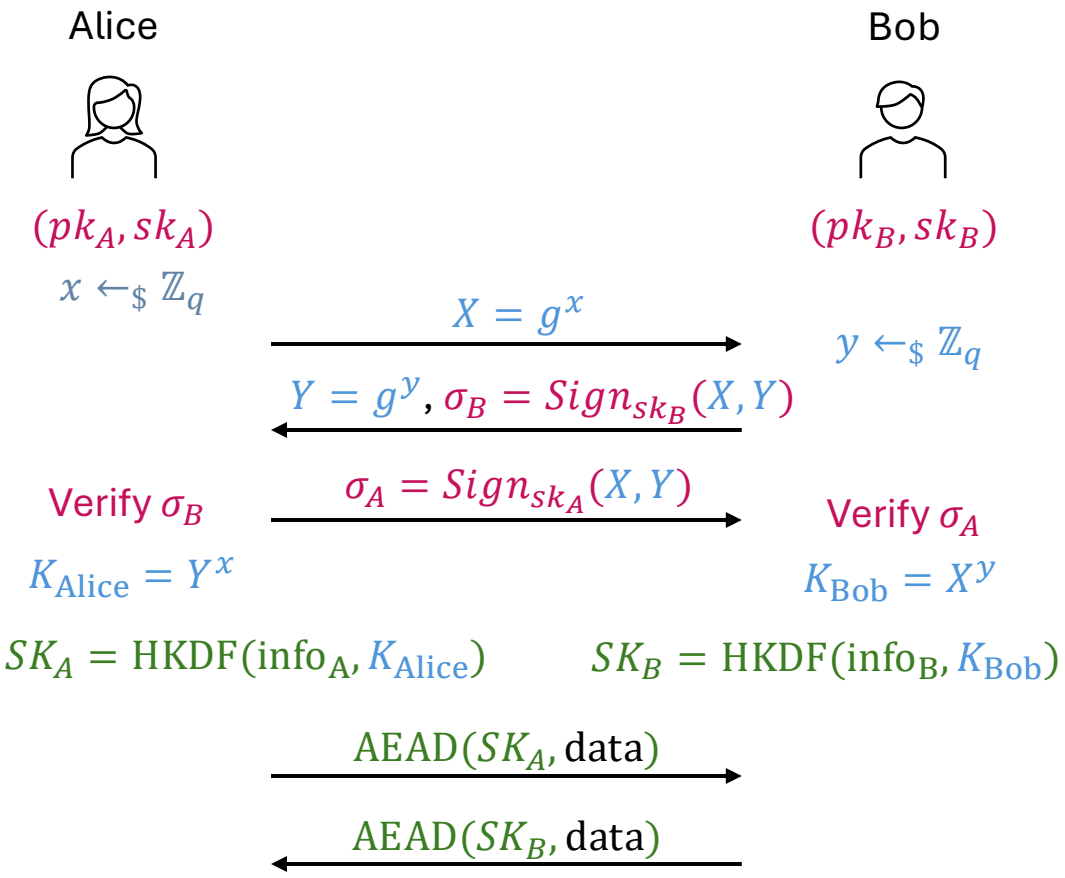
- **No, by unforgeability...**

Signed Diffie-Hellman Protocol



- SigDH
  - Add signature to avoid MitM
  - **Authenticated** Key Exchange

Signed Diffie-Hellman Protocol



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- In practice: ECDH + ECDSA + HKDF/HMAC + AEAD …

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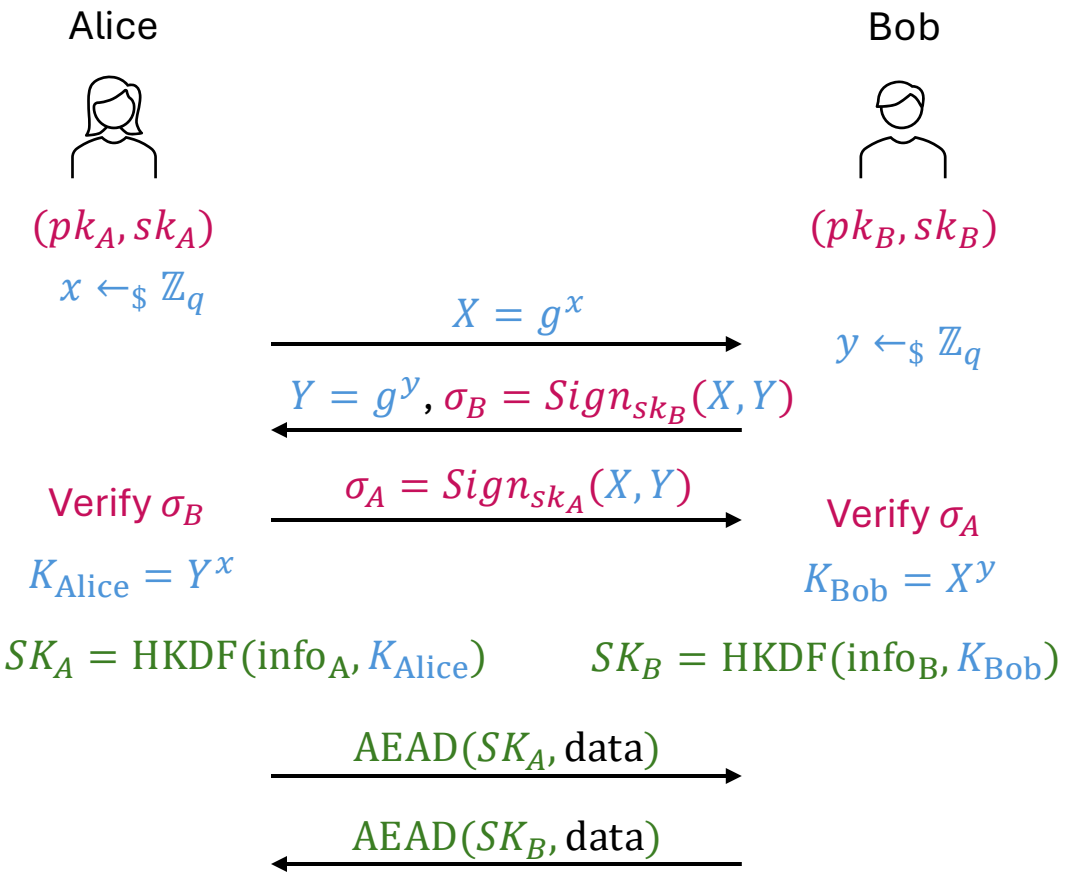
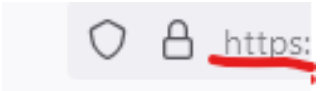


- SigDH
  - Add signature to avoid MitM
  - **Authenticated** Key Exchange
- In practice: ECDH + ECDSA + HKDF/HMAC + AEAD …
- Important Application: **TLS handshake protocol**…

TLS Handshake Protocol

- Transport Layer Security (TLS) Protocol
  - Designed to provide communications security over an open network
  - Used in HTTPS 

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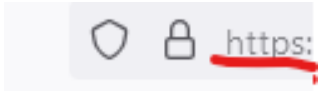
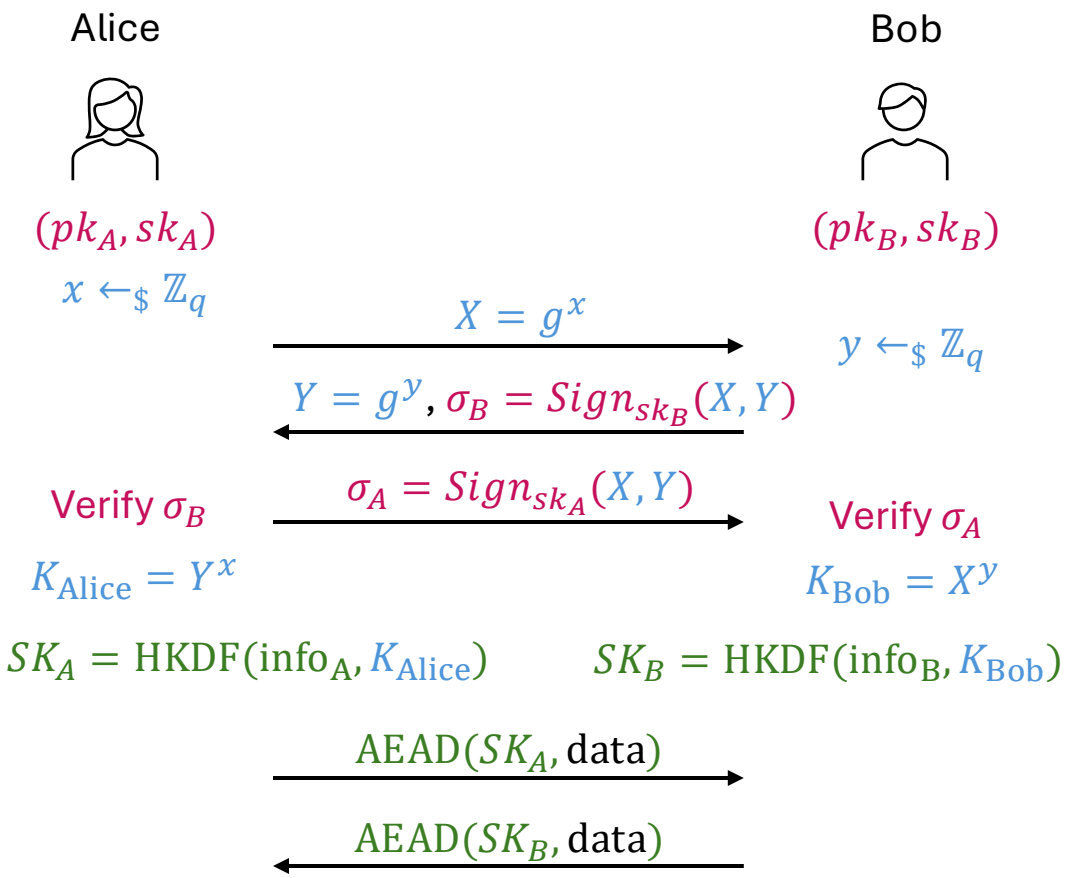
  - Used in HTTPS 

, Email protocols, OpenVPN, MySQL-over-TLS, …

  - TLS process (simplified):
    1. Session initialization (TCP/IP)
    2. **TLS handshake**
    3. **Encrypted Data Transfer**
    4. Session end

  - In this lecture, we mainly consider **the client-server setting**
    - **Server Authentication Only**: A client normally does not have static public-private key pair and certificates

SigDH in TLS (Simplified)



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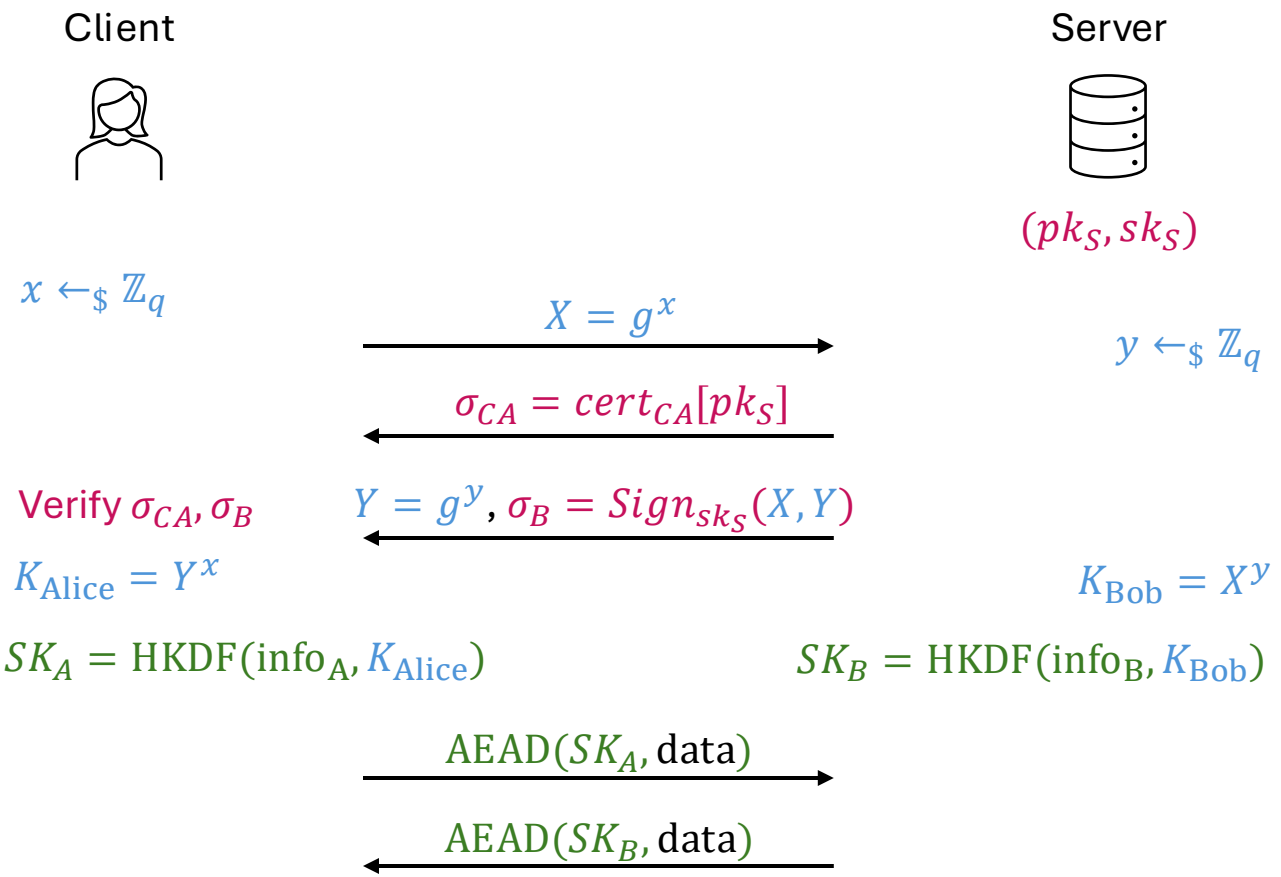

TLS Handshake Protocol

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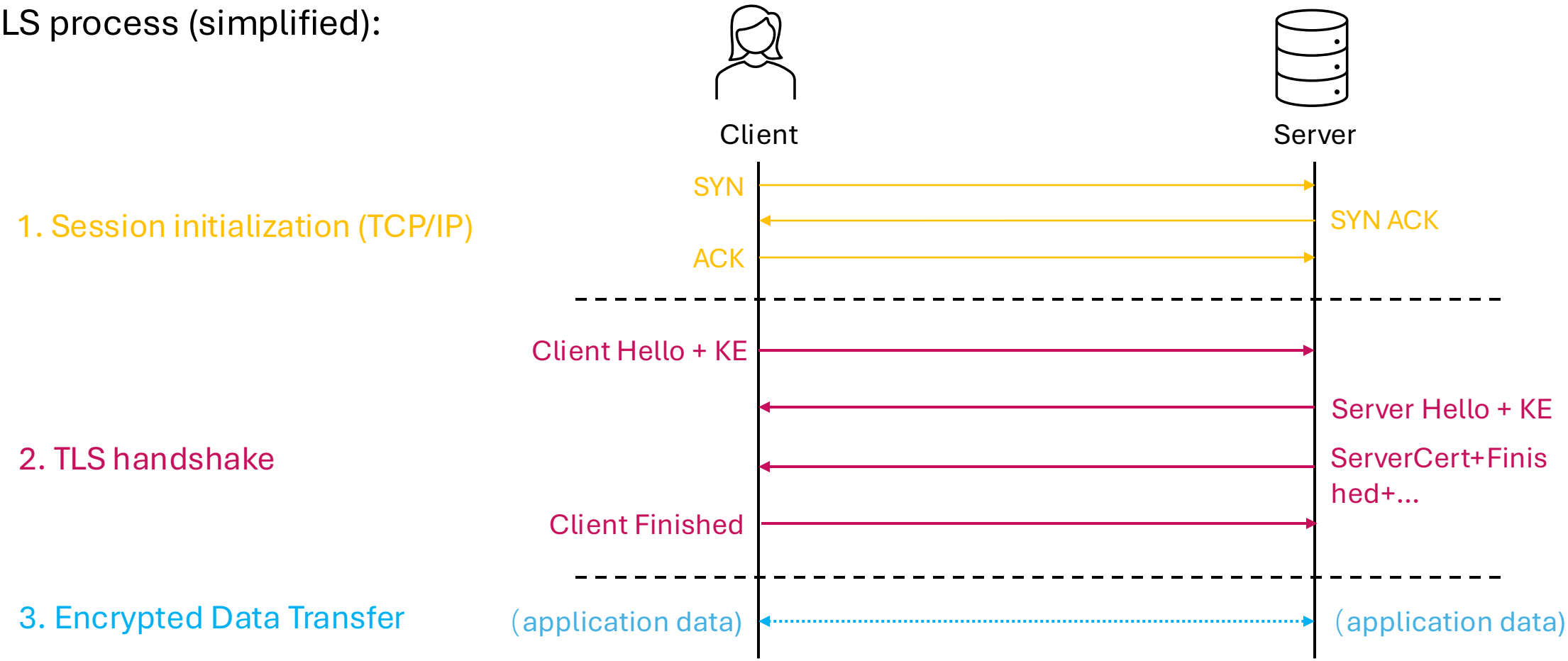

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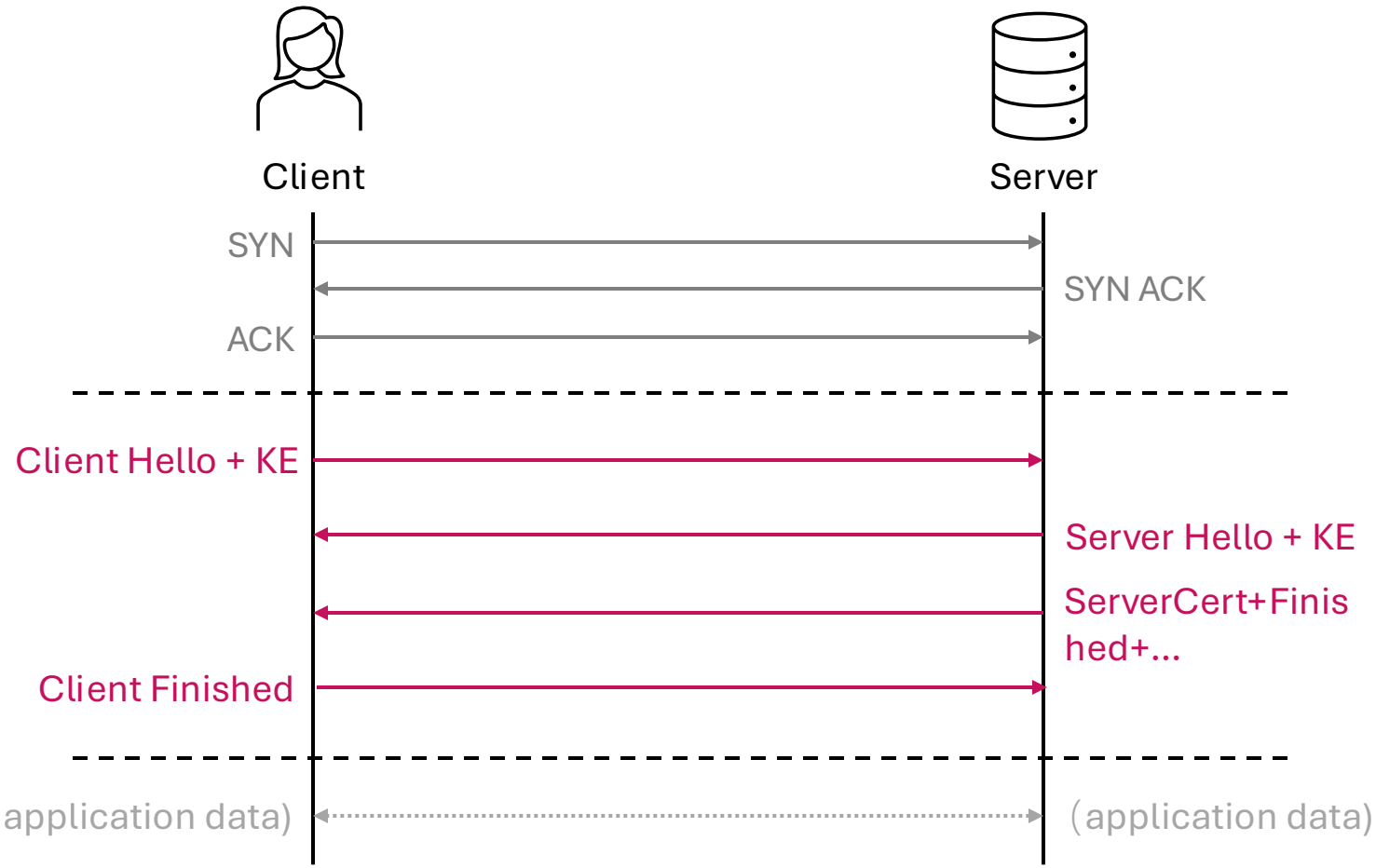
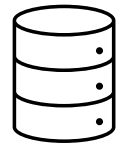


TLS Handshake Protocol

- TLS 1.3 handshake protocol (Simplified description, we ignore the TLS key schedule)

Client



Server

$(pk_S, sk_S), cert[pk_S]$

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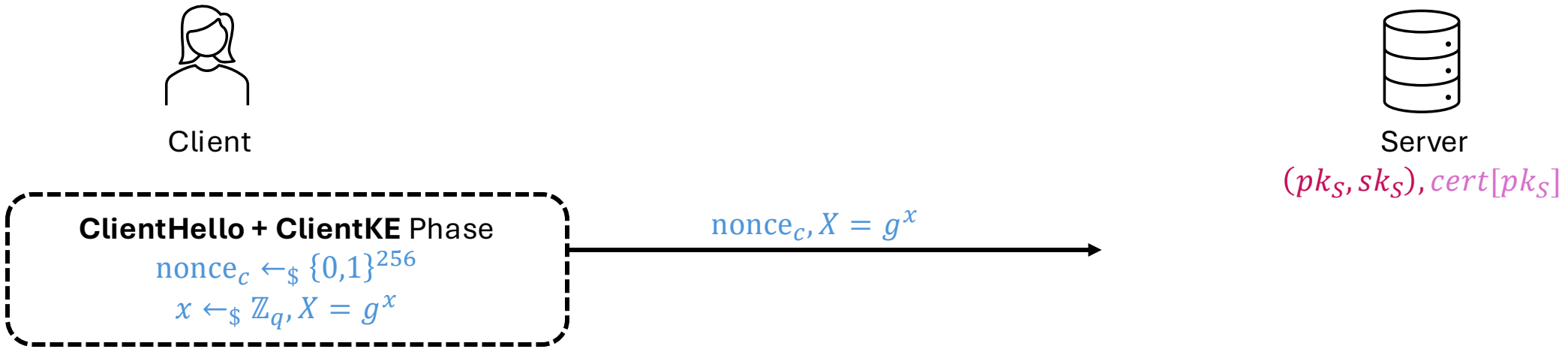

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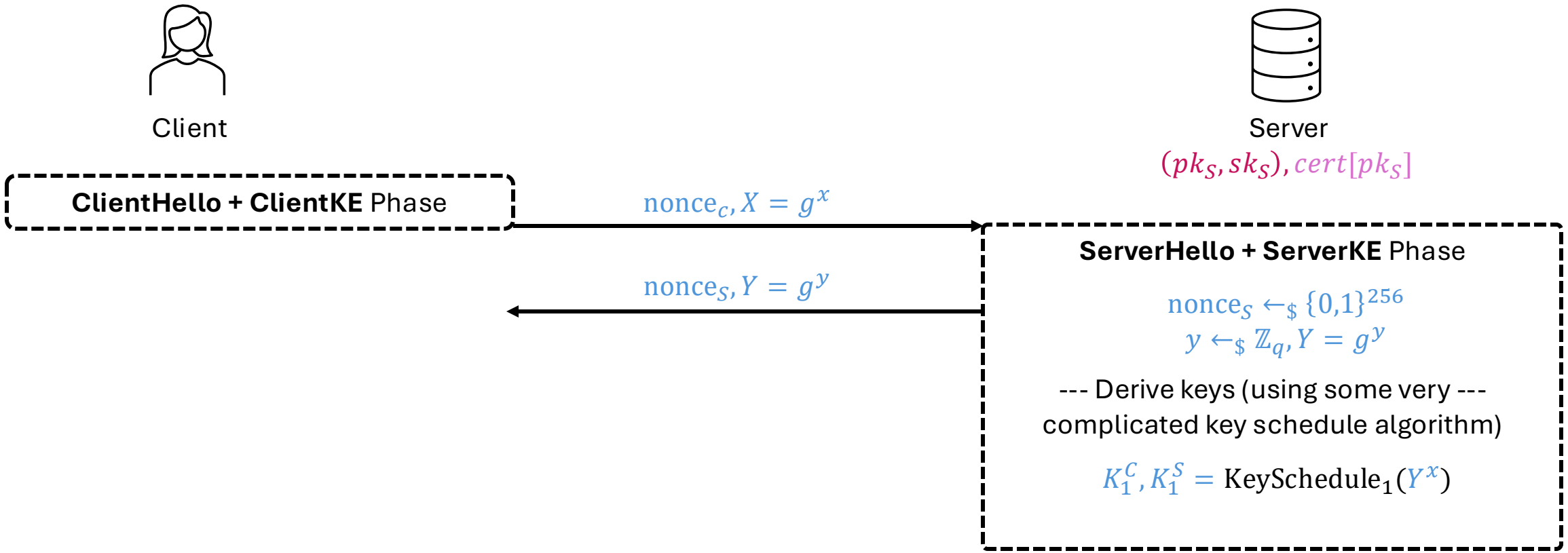

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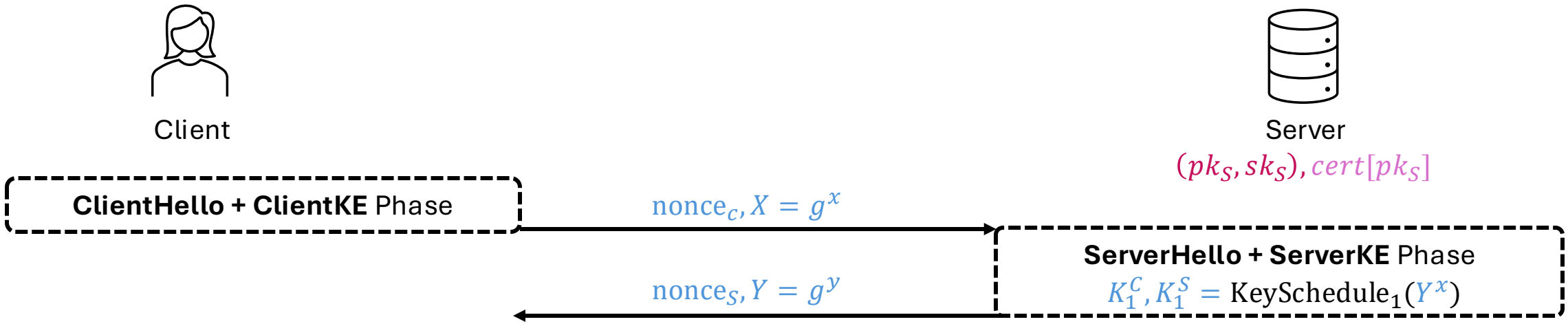

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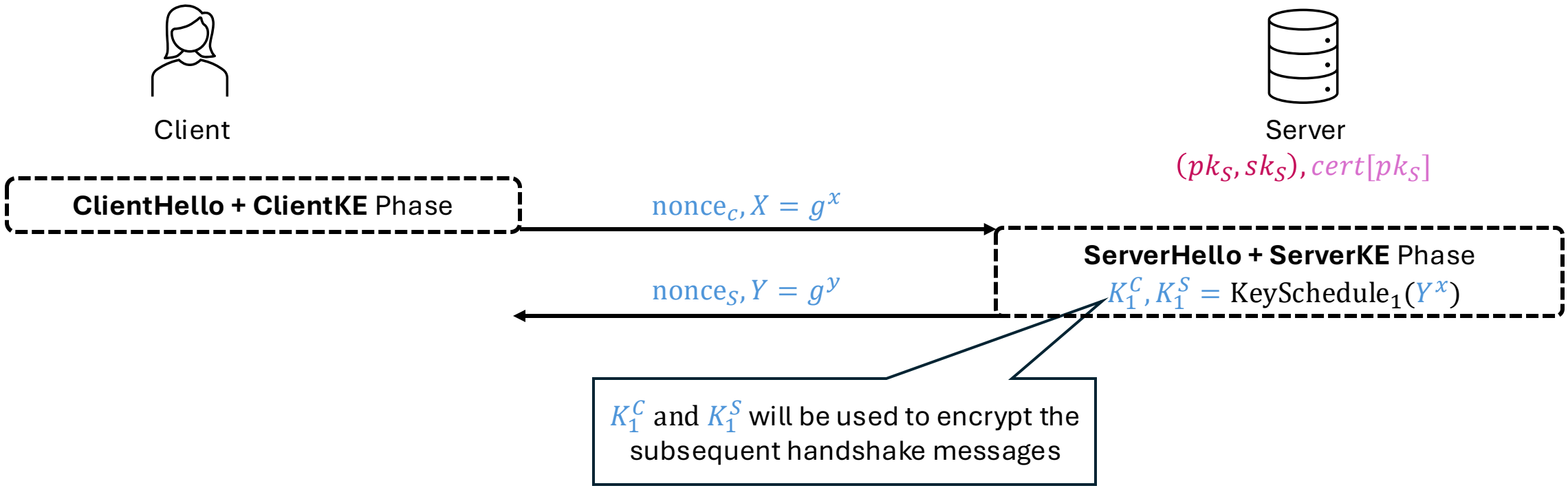

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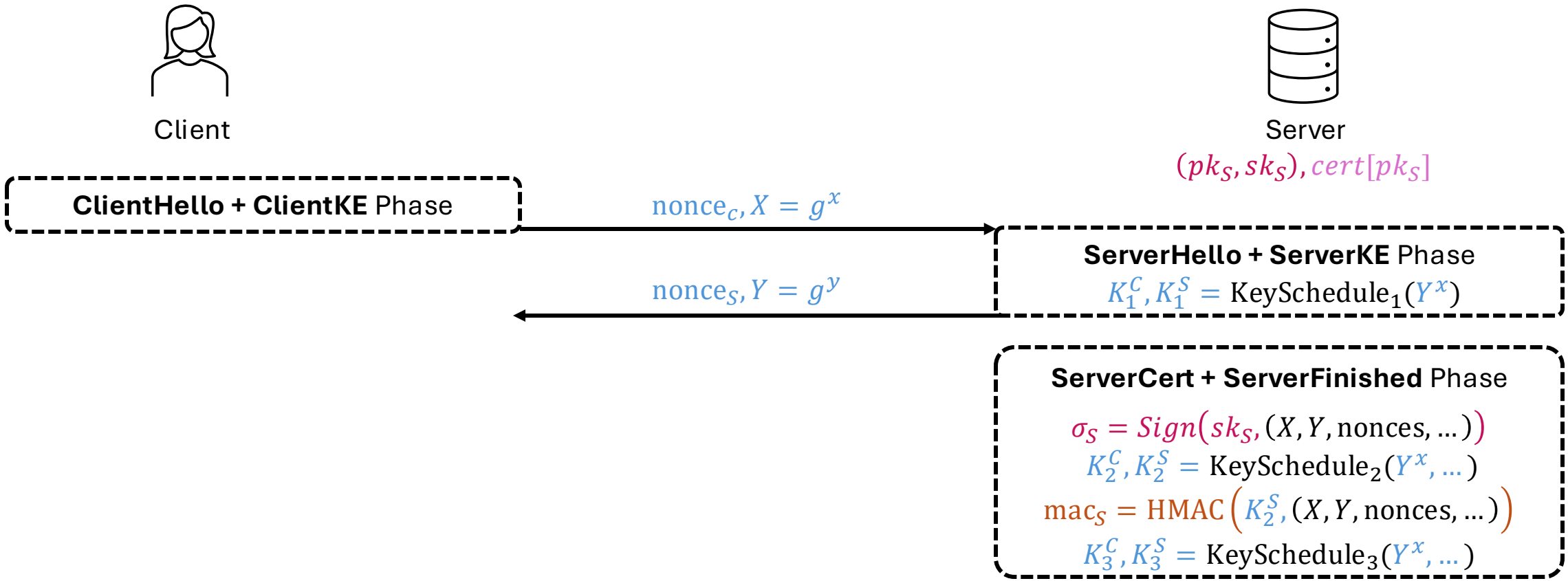

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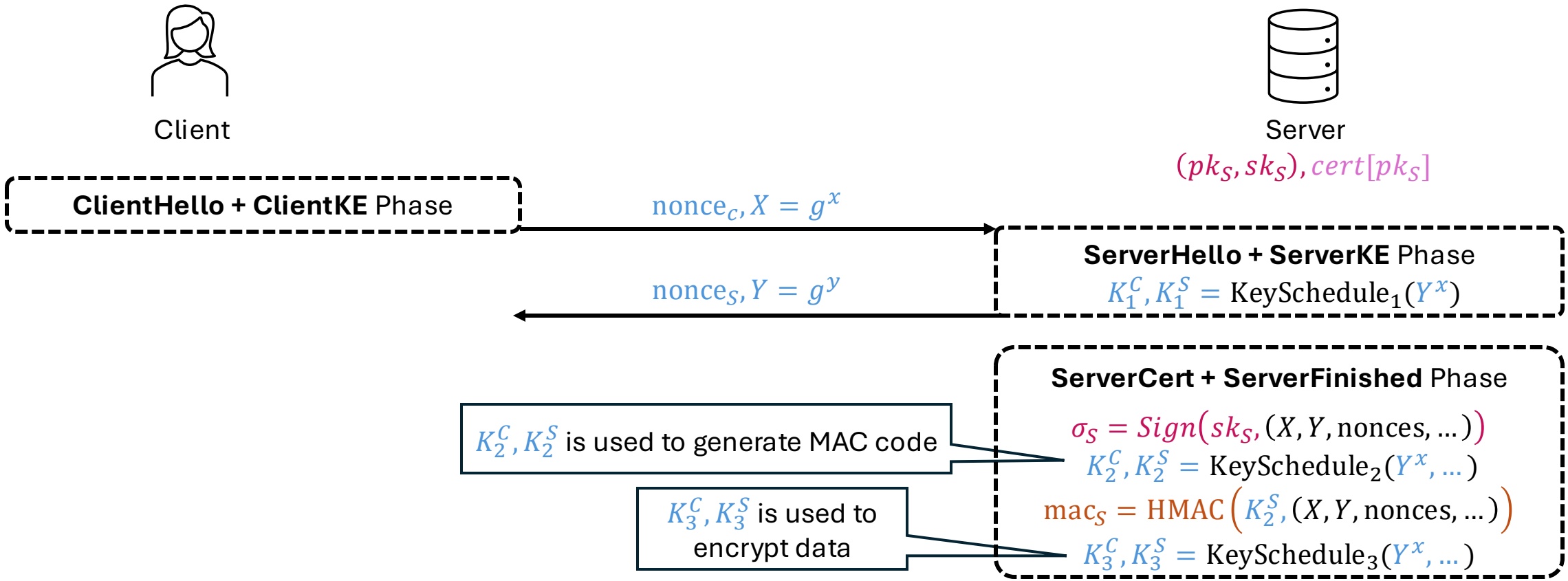

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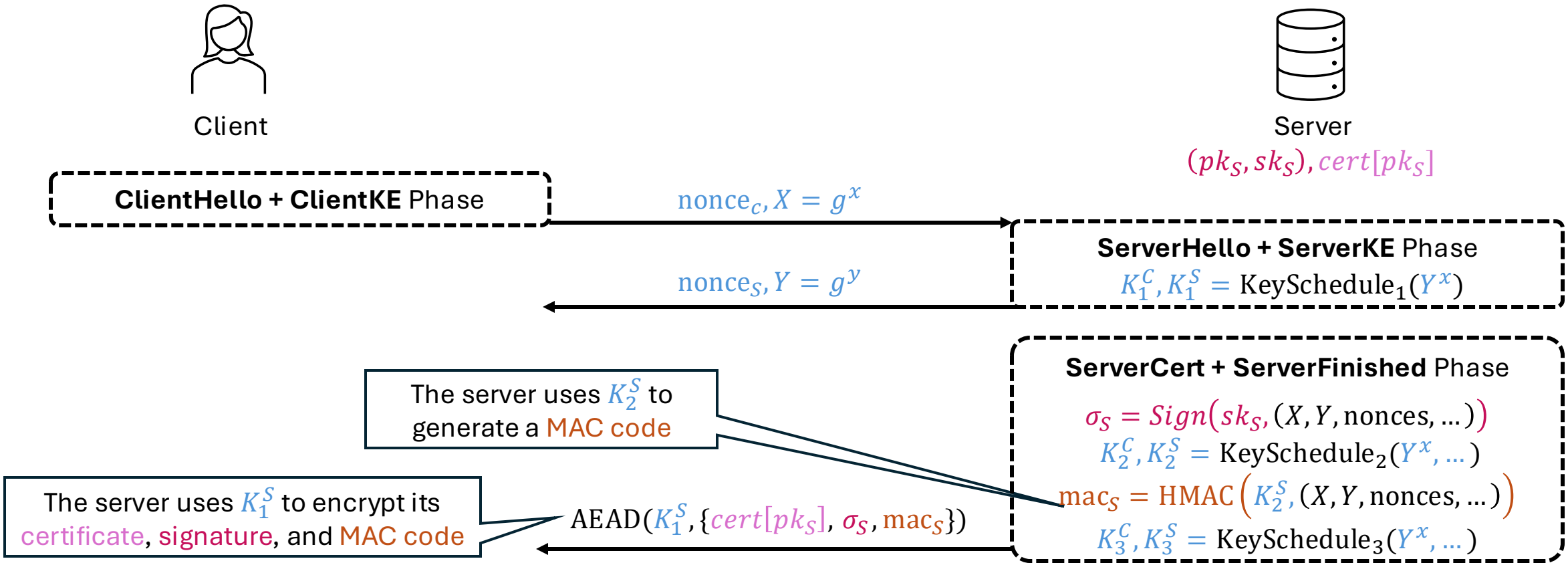

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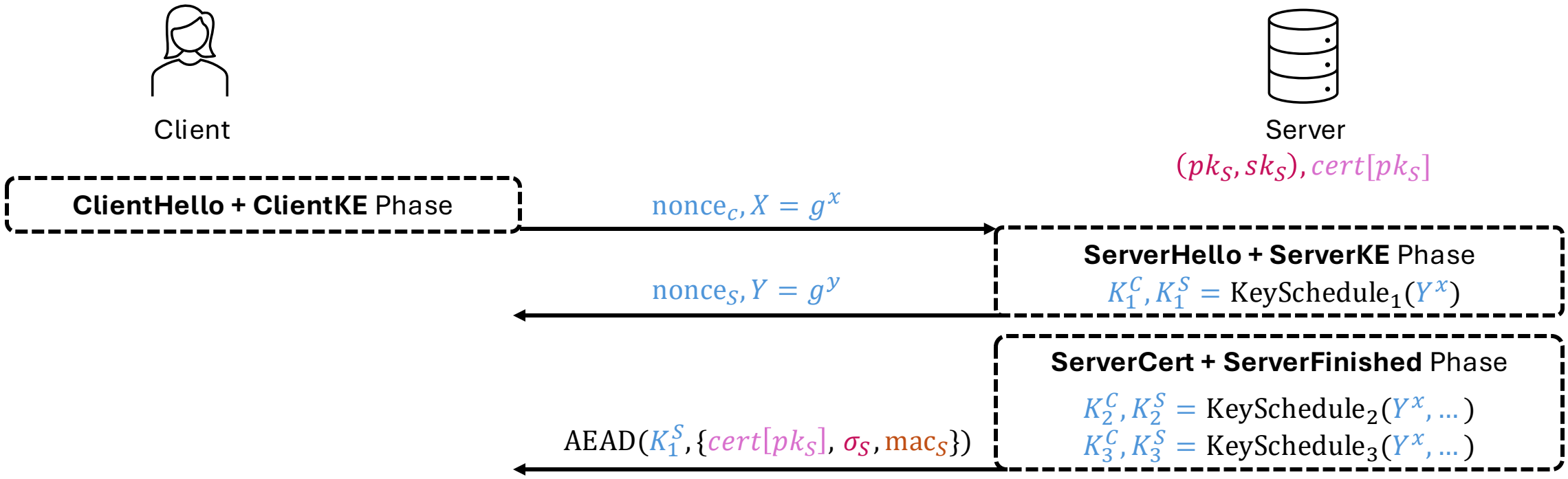

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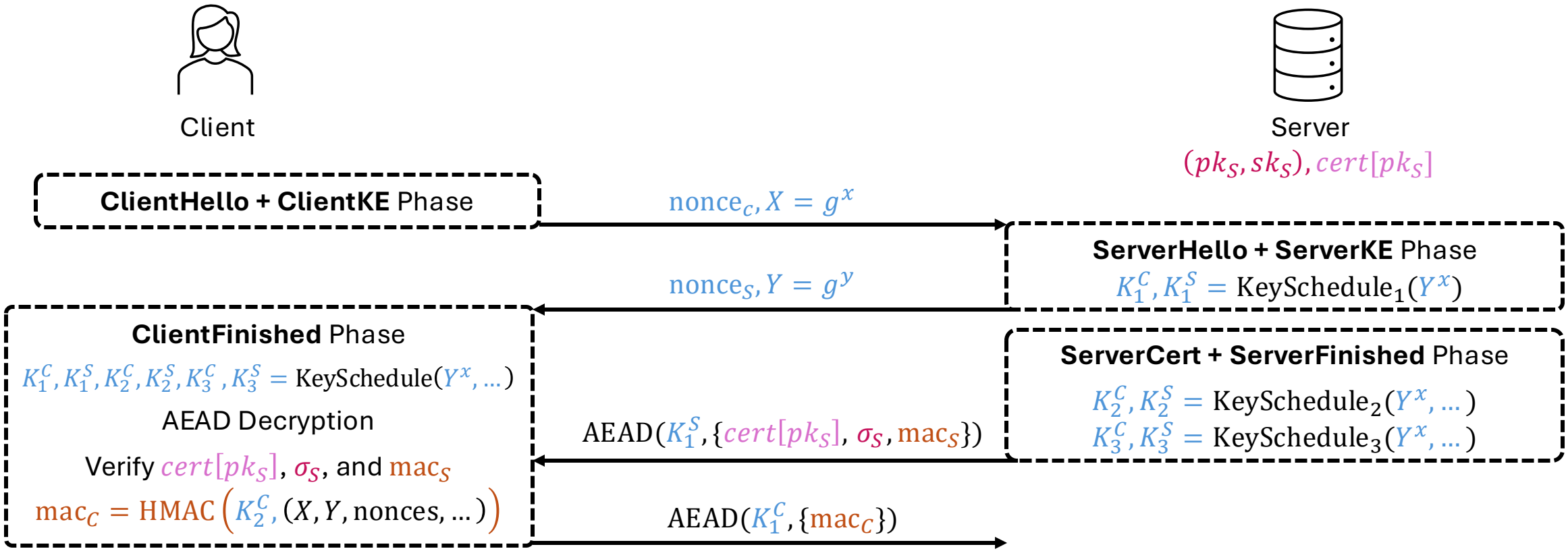

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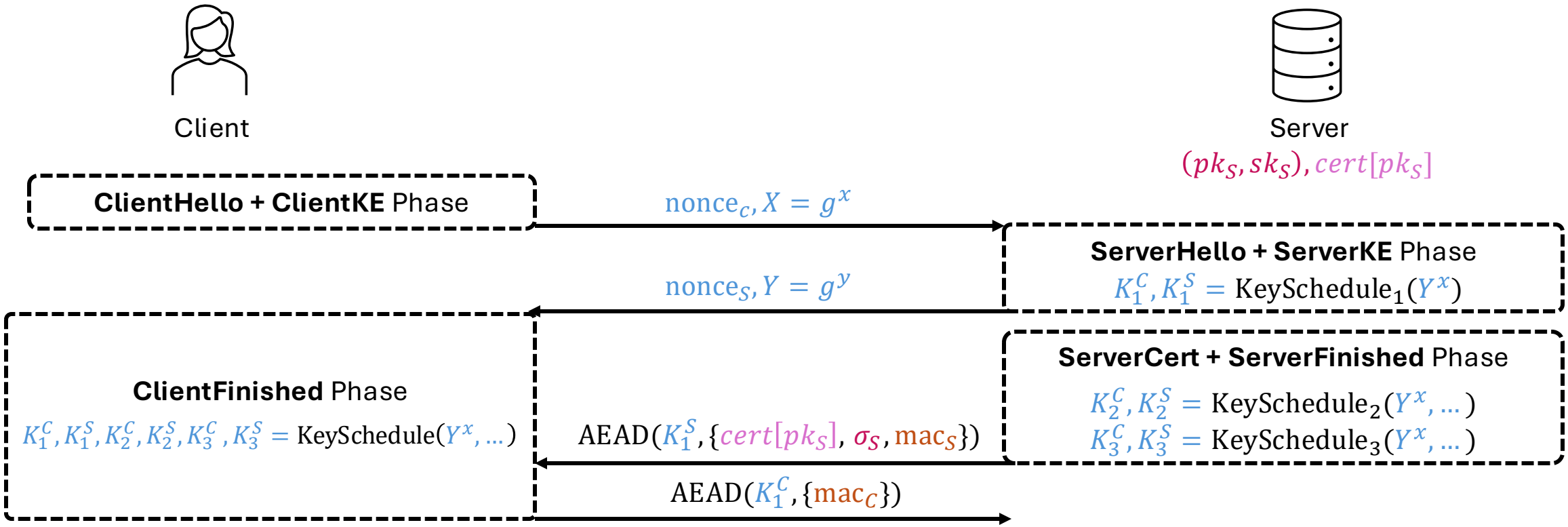

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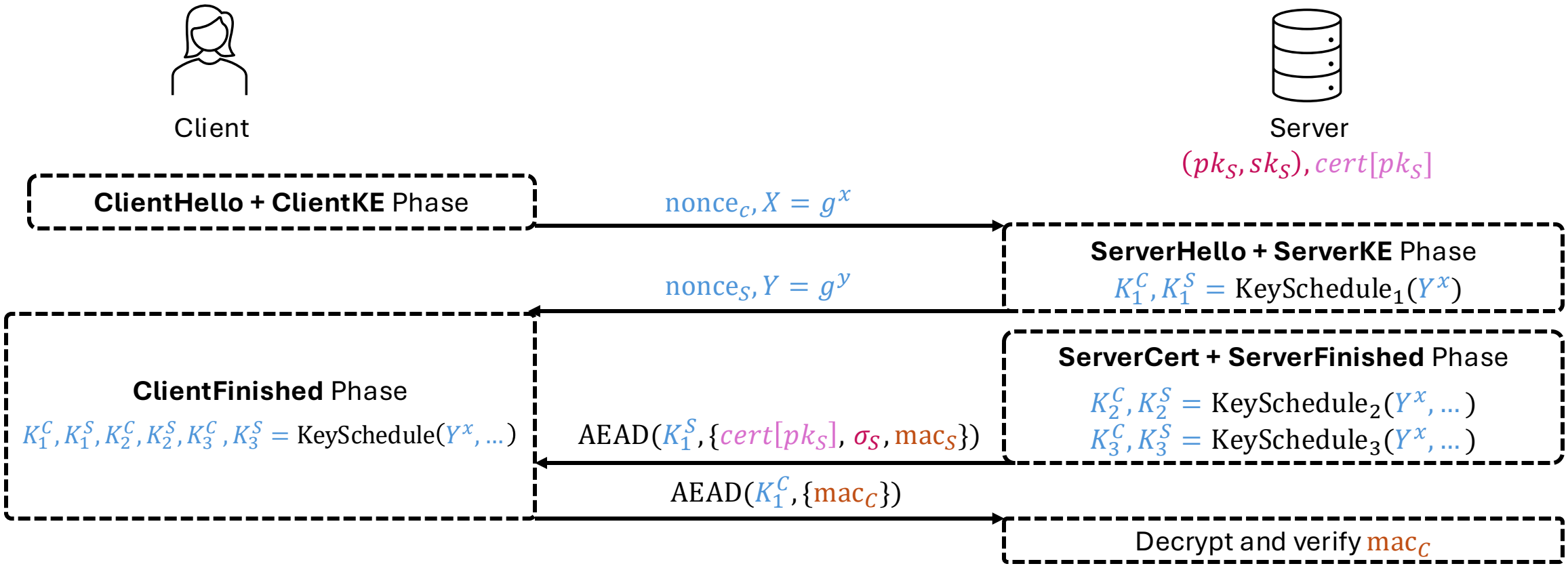

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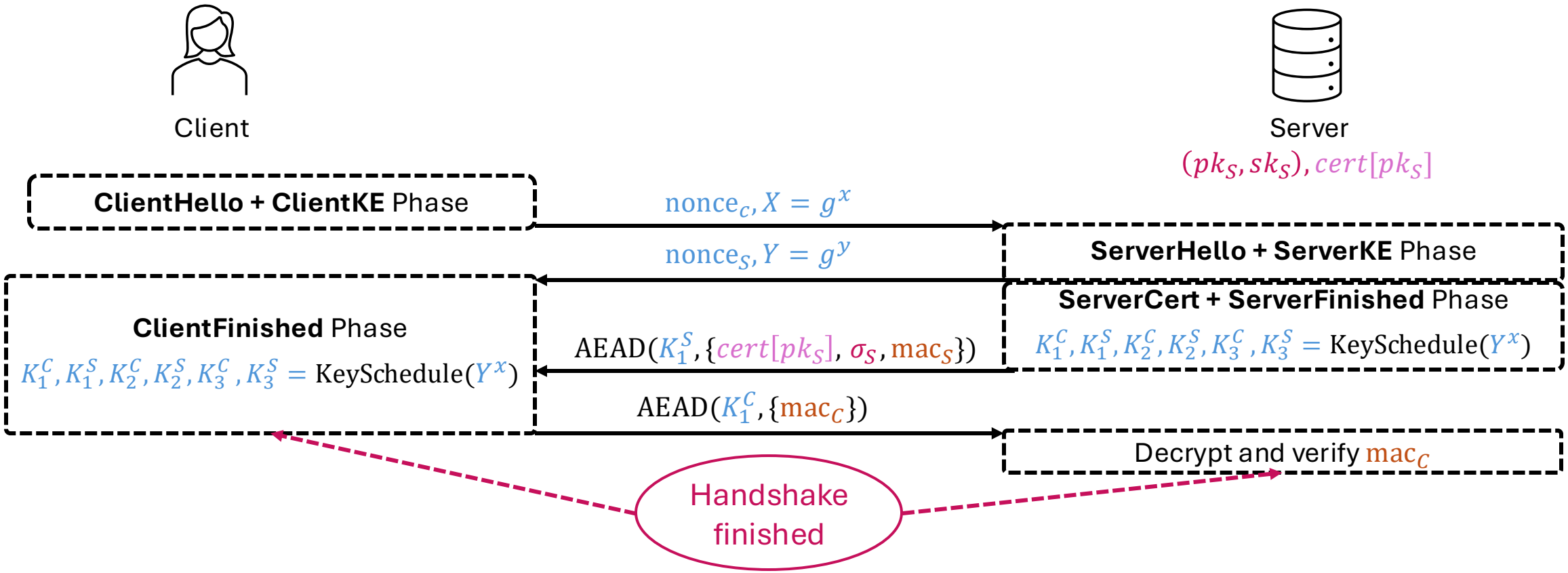

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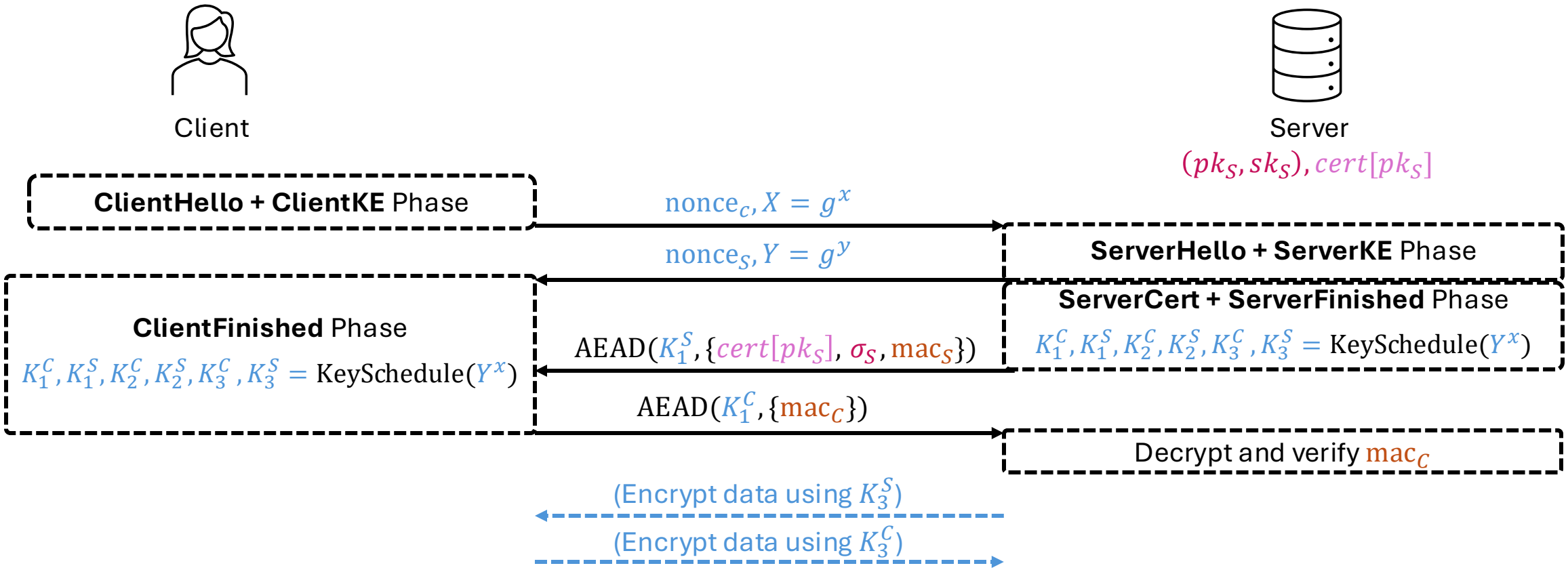
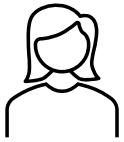


Case Study: HTTPs = HTTP over TLS

HTTP v.s. **HTTPs**



Client
(Browser)

http://www.google.com
Port: 80

Client
(Browser)

https://www.google.com
Port: 443

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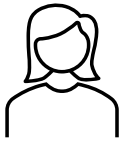

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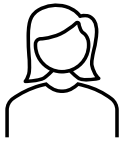
http://www.google.com
Port: 80

Just an example. You probably cannot access http://www.google.com because your browser or Google enforces HTTPs connections.



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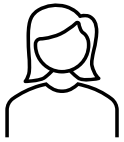

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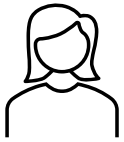

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HTTP request

HTTP response

Search “**How to look busy doing nothing**”

Search results

…

Case Study: HTTPs = HTTP over TLS

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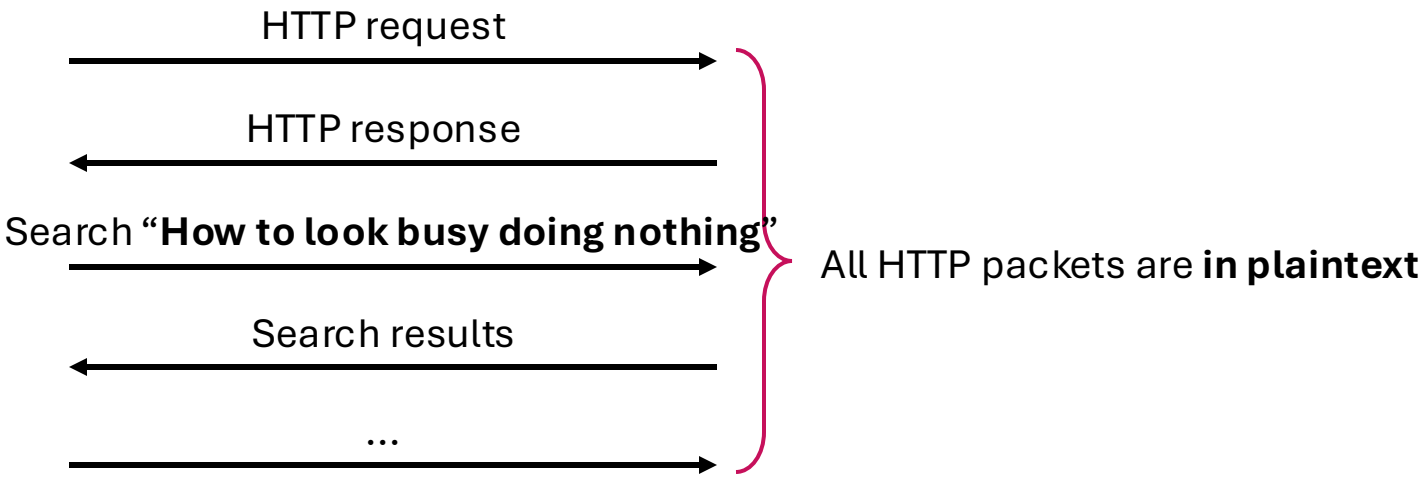

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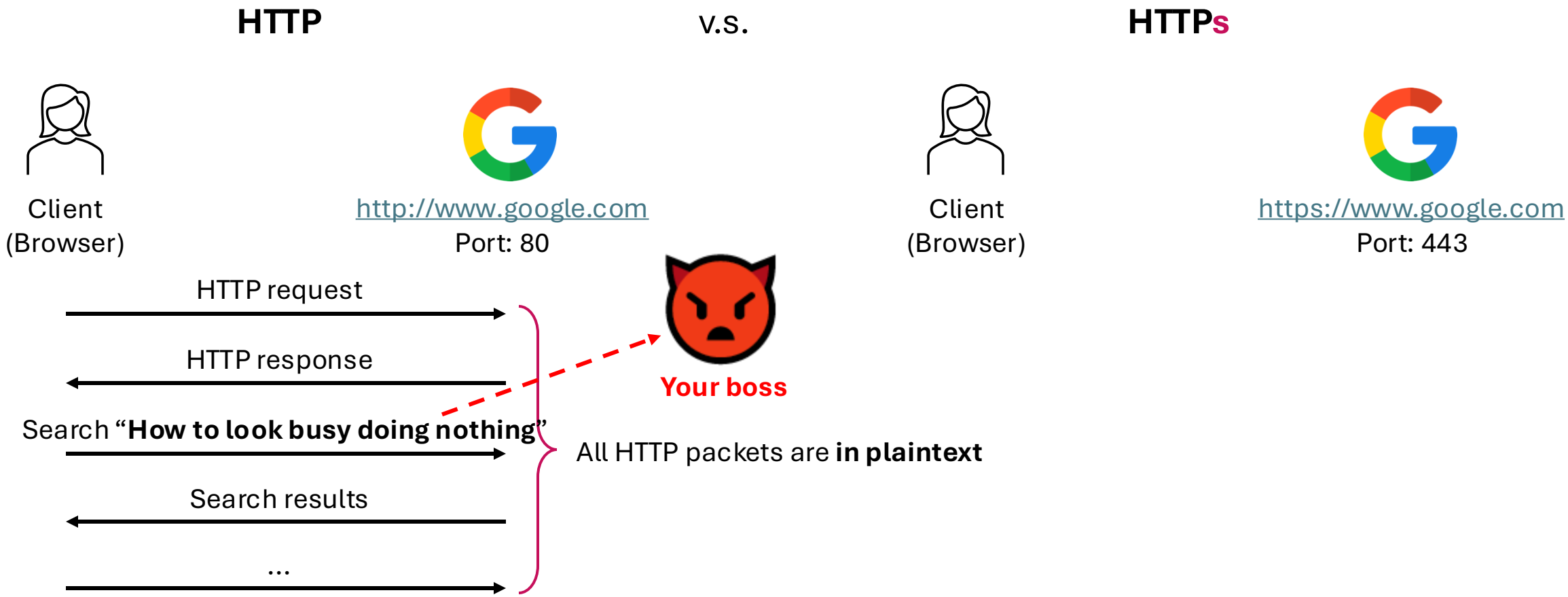

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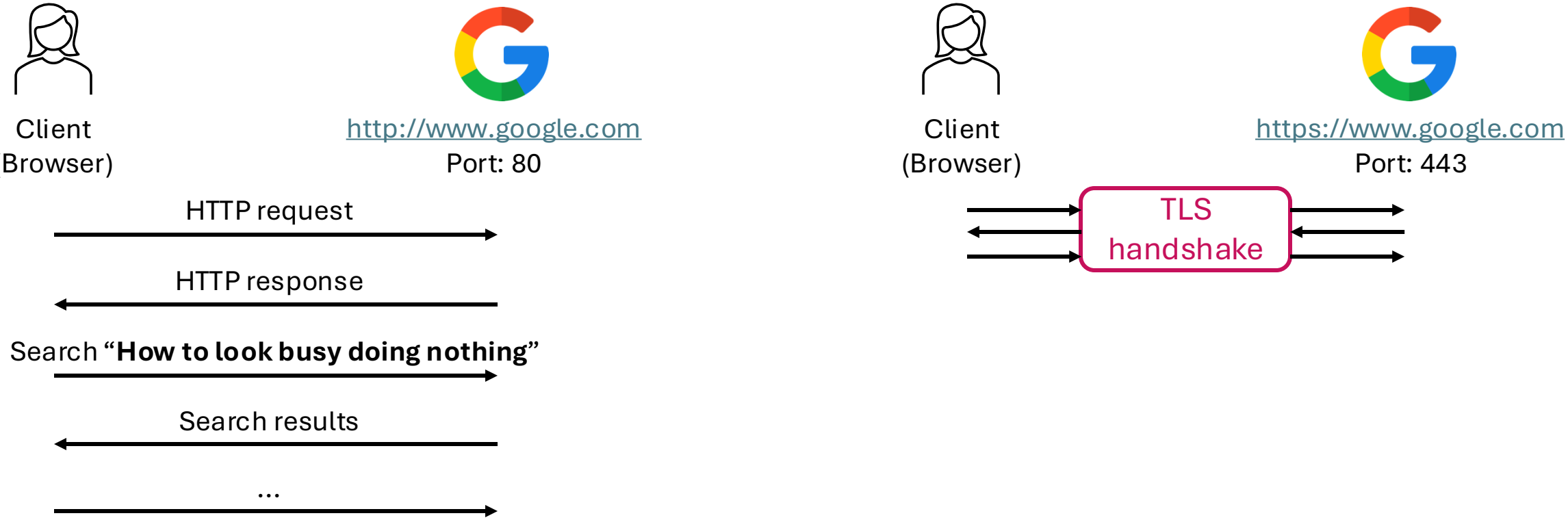

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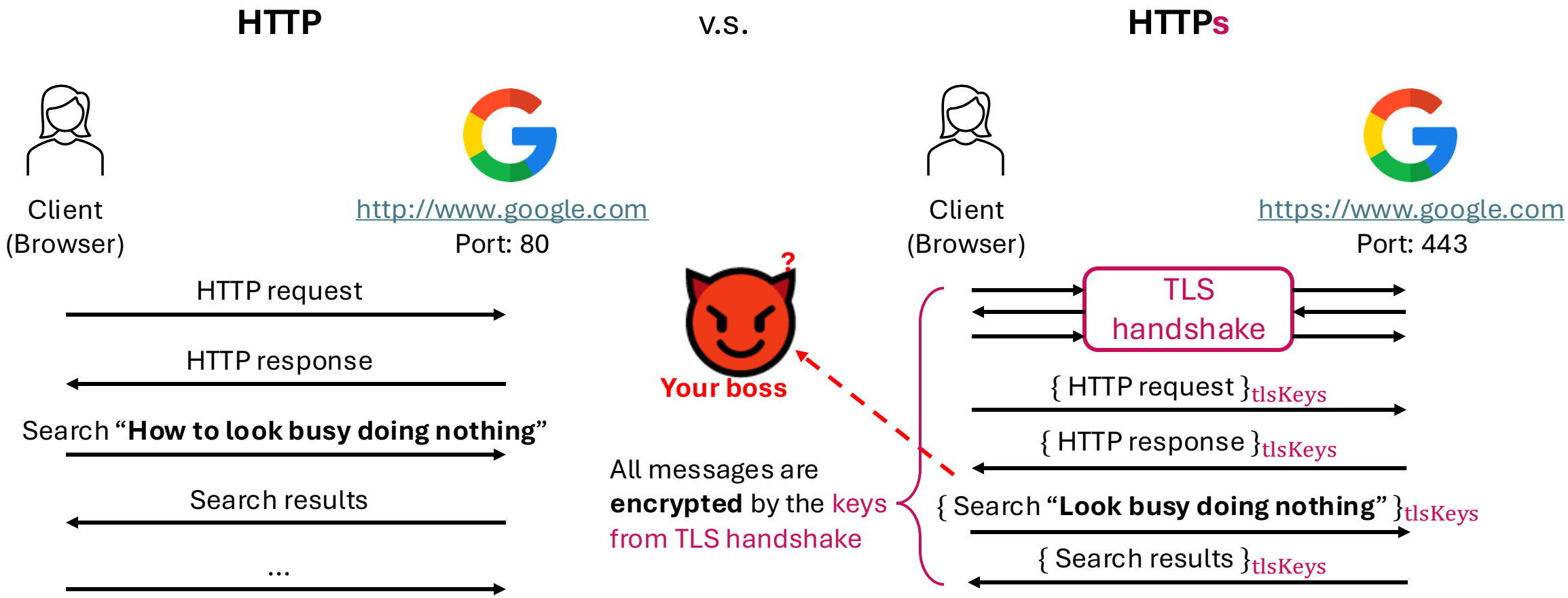


Homework

- Task 1: Implement the signed DH Hellman protocol.
- Task 2: Implement the tweaked TLS handshake protocol (using the KeySchedule algorithms in the next slide)
- Task 3: Implement the randomness reuse attack on ECDSA
- **Deadline: Nov 28th, 2025**

- Example code:
  - DHKE demo (from the second lecture)
  - Signature demo (using Ed25519)
  - HKDF-Extract-Expand demo
  - ECDSA demo

Homework

Warning: This key schedule scheme may not be secure. If you want to use TLS in real-world applications, please follow the TLS 1.3 standard

- Implement the tweaked TLS handshake protocol
  - Use the simplified key schedule algorithm:

$\mathbf{KeySchedule_1}(g^{xy})$:
1. HS = DeriveHS(g^{xy})
2. K_1^C = HKDF.Expand(HS, SHA256("ClientKE"))
3. K_1^S = HKDF.Expand(HS, SHA256("ServerKE"))
4. **return** K_1^C, K_1^S

$\mathbf{DeriveHS}(g^{xy})$:
1. ES = HKDF.Extract(0, 0) // 0 = zeros (bytes) of length 32
2. dES = HKDF.Expand(ES, SHA256("DerivedES"))
3. HS = HKDF.Extract(dES, SHA256(g^{xy}))
4. **return** HS

$\mathbf{KeySchedule_2}(\text{nonce}_C, X, \text{nonce}_S, Y, g^{xy})$:
1. HS = DeriveHS(g^{xy})
2. ClientKC = SHA256(nonce_C || X || nonce_S || Y || "ClientKC") // "||" is the concatenation operation
3. ServerKC = SHA256(nonce_C || X || nonce_S || Y || "ServerKC")
4. K_2^C = HKDF.Expand(HS, ClientKC)
5. K_2^S = HKDF.Expand(HS, ServerKC)
6. **return** K_2^C, K_2^S

  - … (next page)

Homework

Warning: This key schedule scheme may not be secure. If you want to use TLS in real-world applications, please follow the TLS 1.3 standard

$\mathbf{KeySchedule_3}(\text{nonce}_C, X, \text{nonce}_S, Y, g^{xy}, \sigma, cert[pk_S], \text{mac}_S)$:

1. HS = $\text{DeriveHS}(g^{xy})$
2. dHS = HKDF.Expand(HS, SHA256("DerivedHS"))
3. MS = HKDF.Extract(dHS, 0) // 0 = zeros (bytes) of length 32
2. ClientSKH = SHA256(nonce_C || X || nonce_S || Y || σ || $cert[pk_S]$ || mac_S|| "ClientEncK")
3. ServerSKH = SHA256(nonce_C || X || nonce_S || Y || σ || $cert[pk_S]$ || mac_S|| "ServerEncK")
2. K_3^C = HKDF.Expand(MS, ClientSKH)
3. K_3^S = HKDF.Expand(MS, ServerSKH)
4. **return** K_3^C, K_3^S

- **How to compute the signature/MAC code:**

  For server: $\sigma = Sign(sk_S, \text{SHA256}(\text{nonce}_C \,||\, X \,||\, \text{nonce}_S \,||\, Y \,||\, cert[pk_S]))$ // Use DSA with SHA256 and P256

  For server: $\text{mac}_S = \text{HMAC}\left(K_2^S, \text{SHA256}(\text{nonce}_C \,||\, X \,||\, \text{nonce}_S \,||\, Y \,||\, \sigma \,||\, cert[pk_S] \,||\, \text{"ServerMAC"})\right)$

  For client: $\text{mac}_C = \text{HMAC}\left(K_2^C, \text{SHA256}(\text{nonce}_C \,||\, X \,||\, \text{nonce}_S \,||\, Y \,||\, \sigma \,||\, cert[pk_S] \,||\, \text{"ClientMAC"})\right)$

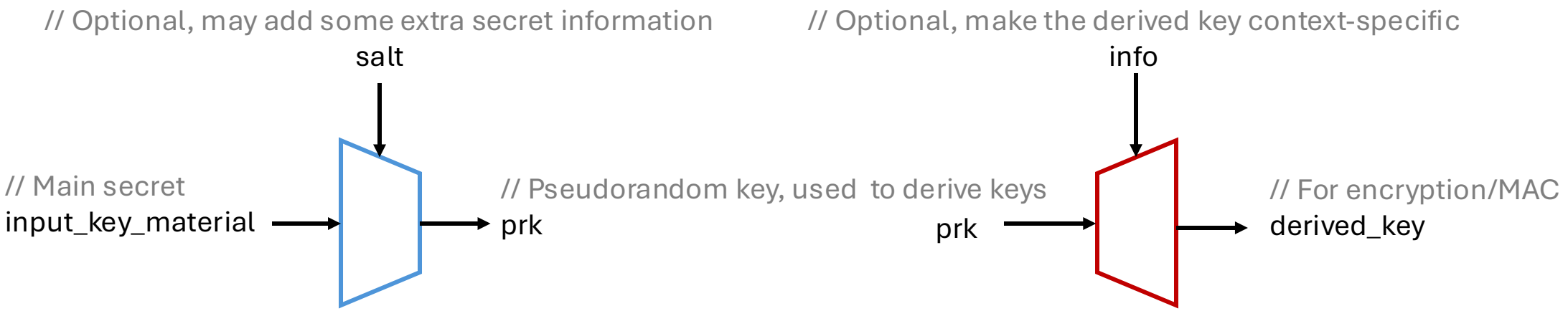
- **How to verify HMAC:** To verify if mac is the *valid* HMAC code of M with respect to the key K,

  Just check: mac =? HMAC(K, M)

Further Reading

- RFC 8446 - TLS 1.3: https://datatracker.ietf.org/doc/html/rfc8446
- RFC 2818 - HTTP over TLS: https://datatracker.ietf.org/doc/html/rfc2818
- Felix Günther's lecture notes on TLS 1.3: https://www.felixguenther.info/teaching/2019-tls-seminar/2019-tls-seminar_02-21_TLS13-intro-MSKE-MKC.pdf
- Cryptography analysis of TLS 1.3 handshake: https://eprint.iacr.org/2020/1044

Case study: HKDF.(Extract, Expand)



Case Study: ECDSA

- ECDSA (Elliptic Curve Digital Signature Algorithm): DSA based on Elliptic Curve
- ECDSA (based on EC) vs DSA (based on Module Integer Groups)
- Why do we prefer Elliptic Curve Groups over Module Integer Groups?
  - **Stronger:** For example, a 256-bit elliptic curve key offers comparable security to a 3072-bit RSA key…
  - **Shorter:** Smaller key size => shorter ciphertext/signature, reducing bandwidth usage…
  - **Faster:** Smaller key size => faster computations and lower computation overhead…

Case Study: ECDSA

- A quick background on Elliptic Curve Groups

- An elliptic curve E is a plane curve which consists of the points satisfying the equation:

$$E: y^2 = x^3 + ax + b$$

- In Elliptic-Curve Cryptography (ECC), we use ECs over finite fields.

Case Study: ECDSA

- ECDSA (Elliptic Curve Digital Signature Algorithm): DSA based on Elliptic Curve

- Public parameter (publicly known): $(\mathbb{CURVE}, \mathbb{G}, g, p)$
  - $\mathbb{CURVE}$: Tell the users what *the elliptic curve* and *equations* are being used.
  - $(\mathbb{G}, g, p)$: A subgroup $\mathbb{G}$ over $\mathbb{CURVE}$ with a **large prime** order p. The base point (generator) g generates $\mathbb{G}$.

- Key Generation:
  - $sk = d \leftarrow_{\$} \mathbb{Z}_p^*$ // (= {1, 2, …, p-1} , here "*" means that we exclude zero)
  - $pk = d \circ g$ // "$\circ$" is the "exponential operator" of Elliptic Curve, just like g^d.
    and you cannot recover d given $d \circ g$

Case Study: ECDSA

- Signing algorithm ($sk = d$: secret key, m: message):
  1) $e' = \mathrm{Hash}(m)$ // "Compress" the message and get its digest
  2) $e =$ $[\log_2 p]$ leftmost bits of e' // Truncate some bits to fit in the format
  3) $k \leftarrow_{\$} \mathbb{Z}_p^*$
  4) $(x, y) = k \circ g$ // g is the base point
  5) $r = x \mod p$ // Now r is an integer module p. Given x, y is determined.
  6) Assert $[x \mod p \neq 0]$ // Make sure we do not get a "trivial point"
  7) $s = k^{-1} \cdot (e + r \cdot d) \mod p$ // Signing
  8) return (r, s)

Case Study: ECDSA

- Verification algorithm (pk: public key, m: message, (r, s): signature):
  1) $e' = \mathrm{Hash}(m)$ // "Compress" the message and get its digest
  2) $e = [\log_2 p]$ leftmost bits of e' // Truncate some bits to fit in the format
  3) $u_1 = e \cdot s^{-1} \mod p$
  4) $u_2 = r \cdot s^{-1} \mod p$
  5) $(x, y) = u_1 \circ g + u_2 \circ pk$ // Recalculate the point
  6) Accept this signature if $x \equiv r \pmod p$. Otherwise, reject.

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- You can prove that the verification algorithm works correctly.
- ECDSA has unforgeability if the *Discrete Logarithm Problem* over the elliptic curve is hard.

Case Study: ECDSA

- **Do not reuse nonce** in DSA(/Schnorr/SM2/…) **!**
- In DSA, nonce (the k value) reuse => private key recovery => break the unforgeability

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Two DSA signatures of different messages: $m_1, \sigma_1 = (r, s_1)$ with nonce k; $m_2, \sigma_2 = (r, s_2)$ with nonce k (Same value r if k is the same)

By DSA construction:

$$s_1 = k^{-1} \cdot (H(m_1) + r \cdot d) \mod p$$
$$s_2 = k^{-1} \cdot (H(m_2) + r \cdot d) \mod p$$

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Two equations,
two unknowns k and d
Linear Algebra!

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By Linear Algebra:

$$\begin{bmatrix} s_1 & r \\ s_2 & r \end{bmatrix} \begin{bmatrix} k \\ d \end{bmatrix} = \begin{bmatrix} H(m_1) \\ H(m_2) \end{bmatrix} \mod p$$

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- Real-world event: Hacking the PlayStation 3 (2010-2011)…
  - A typical example of: Provable secure in the theoretical world, but wrong implementation in the real world.