

Cryptography Engineering

- Lecture 6 (Nov 26, 2025)
- Today's notes:
 - Fujisaki-Okamoto transformation – 2
 - Authentication via KEM
 - PQ-TLS: KEM + Sign
 - KEM-TLS

Fujisaki-Okamoto Transformation

- Let $\text{PKE} = (\text{KG}, \text{Enc}, \text{Dec})$ be a public-key encryption scheme
- Let H, G be two hash functions (Quick question: How to instantiate them using SHA256)
- We construct an **FOKEM** scheme based on **PKE**

KeyGen:

1. $(pk, sk) \leftarrow \text{KG}$
2. $prk \leftarrow \{0,1\}^L$
3. $pk := pk$
4. $sk := (prk, sk)$

Encaps($pk = pk$):

1. $m \leftarrow_{\$} \text{MsgSpace}$
2. $r := G(pk, m)$
// randomness for PKE
3. $c := \text{Enc}(pk, m; r)$
4. $K := H(pk, m, c)$
5. $c := c$
6. return (c, K)

Decaps($sk = (prk, sk), c = c$):

1. $m = \text{Dec}(sk, c)$
2. $r := G(pk, m)$
// Recover randomness
3. $c' := \text{Enc}(pk, m; r)$
// Re-encryption check
4. If $c == c'$: return $H(pk, m, c)$
5. Else: return $H(pk, prk, c)$

Fujisaki-Okamoto Transformation

- **Implicit rejection:** Return a pseudorandom key (rather than returning a rejection symbol).

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 - Quick question: Why the returning key (if the re-enc check fails) is pseudorandom?

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Fujisaki-Okamoto Transformation

- **Implicit rejection:** Return a pseudorandom key (rather than returning a rejection symbol).
 - Quick question: Why the returning key (if the re-enc check fails) is pseudorandom?
 - (1) prk is random and secret $\Rightarrow H(pk, prk, c)$ also looks like random
 - (2) The same input to Decaps \Rightarrow The same output

KeyGen:

1. $(pk, sk) \leftarrow KG$
2. $prk \leftarrow \{0,1\}^L$
3. $pk := pk$
4. $sk := (prk, sk)$

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1. $m \leftarrow_{\$} \text{MsgSpace}$
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// Re-encryption check
4. If $c == c'$: return $H(pk, m, c)$
5. Else: return $H(pk, prk, c)$

Fujisaki-Okamoto Transformation

- **Explicit rejection:** Return a rejection symbol (or throw an exception)

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- ~~2. $prk \leftarrow \{0,1\}^L$~~
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4. $sk := (prk, sk)$

Encaps($pk = pk$):

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// Recover randomness
3. $c' := \text{Enc}(pk, m; r)$
// Re-encryption check
4. If $c == c'$: return $H(pk, m, c)$
5. Else: return **REJECT**

Fujisaki-Okamoto Transformation

- Constant-time comparison
 - Regular equality checks may leak timing information
 - E.g., early-return behavior can reveal where the first differing bit occurs

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Fujisaki-Okamoto Transformation

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4. If $c == c'$: return $H(pk, m, c)$

5. Else: return $H(pk, prk, c)$

Fujisaki-Okamoto Transformation

- Constant-time comparison
 - Regular equality checks may leak timing information
 - E.g., early-return behavior can reveal where the first differing bit occurs

CT-Decaps($sk = (prk, sk), c = c$):

1. $m = \text{Dec}(sk, c)$
2. $r := G(pk, m)$
3. $c' := \text{Enc}(pk, m; r)$
4. $K_0 = H(pk, m, c)$
5. $K_1 = H(pk, prk, c)$
6. $b = \text{constant-time-eq}(c', c)$
7. $K = \text{constant-time-select}(b, K_0, K_1)$
// $K = (1 - b) \cdot K_0 \oplus b \cdot K_1$
8. return K

Decaps($sk = (prk, sk), c = c$):

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Post-quantum Cryptography

- Most widely deployed cryptosystems rely on the hardness of Diffie–Hellman and Factoring (RSA).
- However, these problems are no longer considered hard in the presence of large-scale quantum computers.
- **Post-quantum cryptography:**
 - Cryptographic algorithms run in classical computers
 - Security against adversaries with quantum computers
- NIST PQC standardized algorithms:
 - KEM schemes: ML-KEM (Crystal-Kyber)
 - Signature schemes: ML-DSA (Crystal-Dilithium)

Post-quantum Cryptography

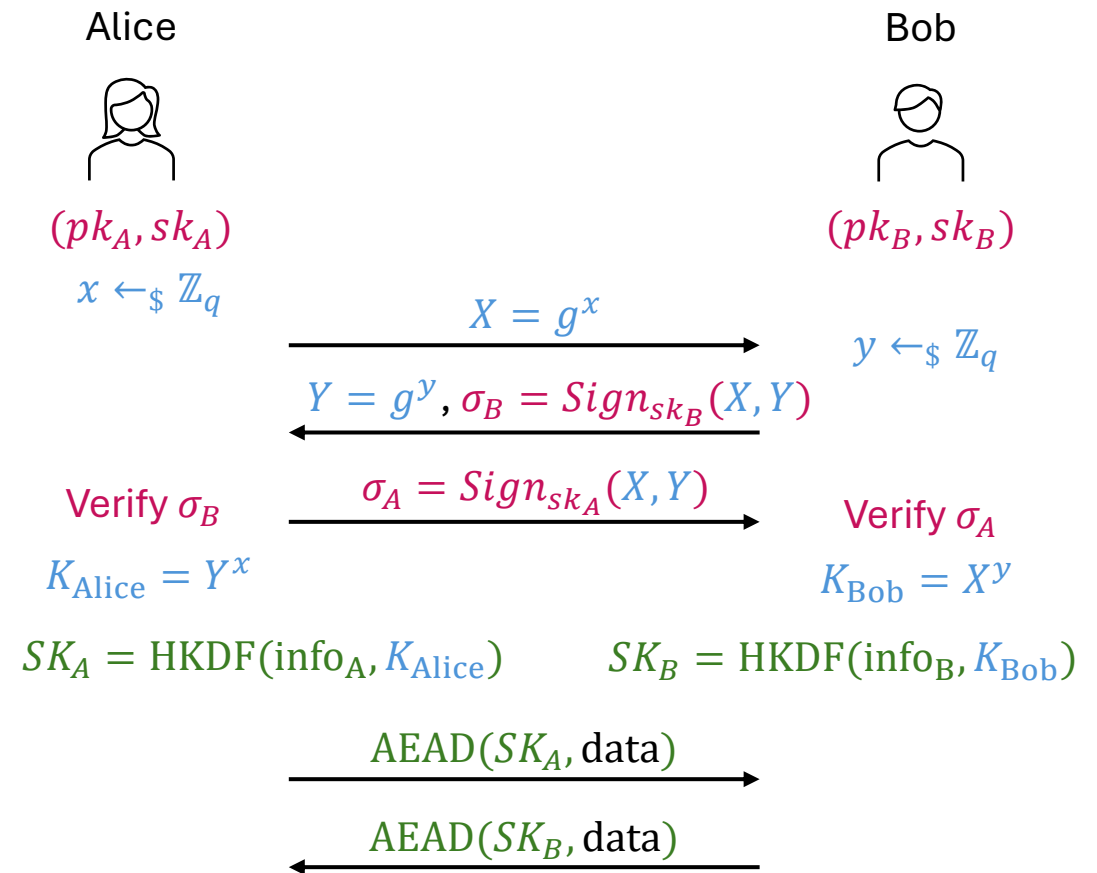
- Post-quantum secure KEM
 - Design a IND-CPA PKE scheme first
 - Use the FO transform (or its variants) to get an IND-CCA KEM scheme
- Post-quantum secure Signature
 - More complex structures...

Post-quantum TLS

- Post-quantum secure KEM
 - Design a IND-CPA PKE scheme first
 - Use the FO transform (or its variants) to get an IND-CCA KEM scheme
- Post-quantum secure Signature
 - More complex structures...
- TLS 1.3 is based on DH problems, so it is not post-quantum secure.
 - **Can we have post-quantum TLS?**

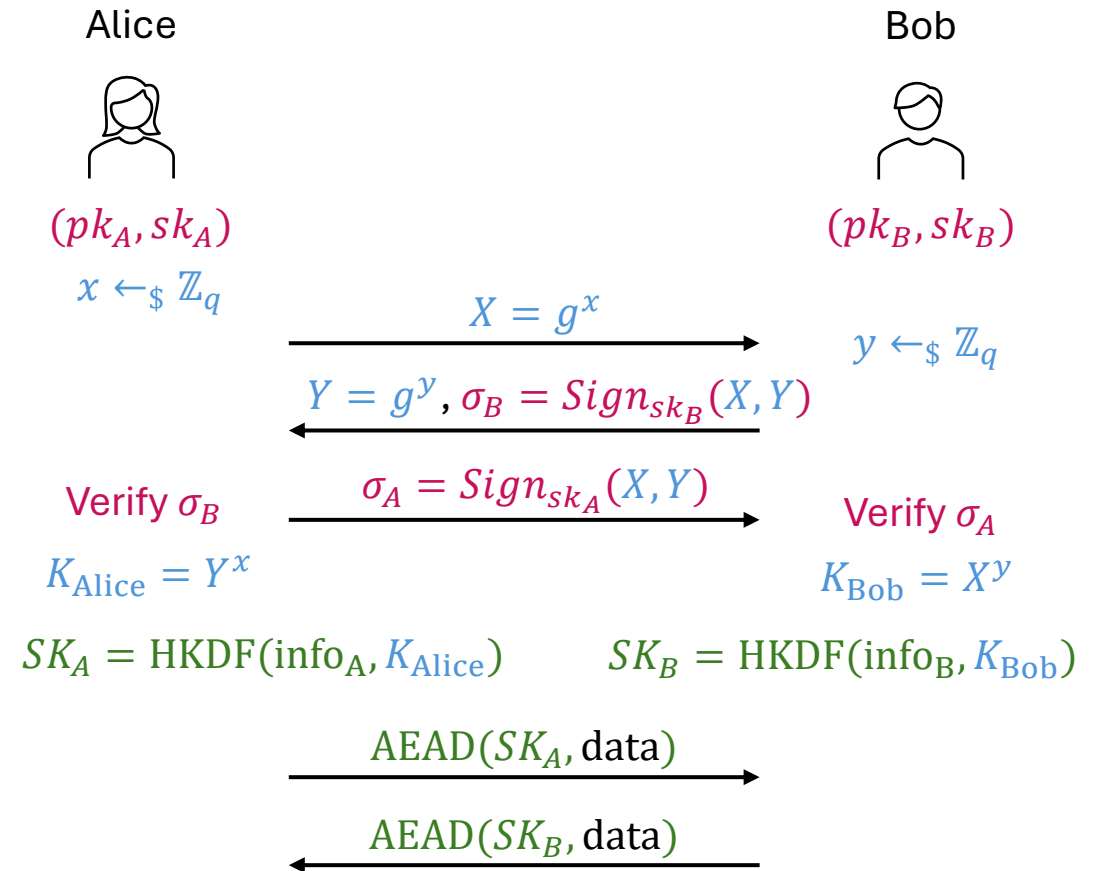
Post-quantum TLS

- The signed DH protocol



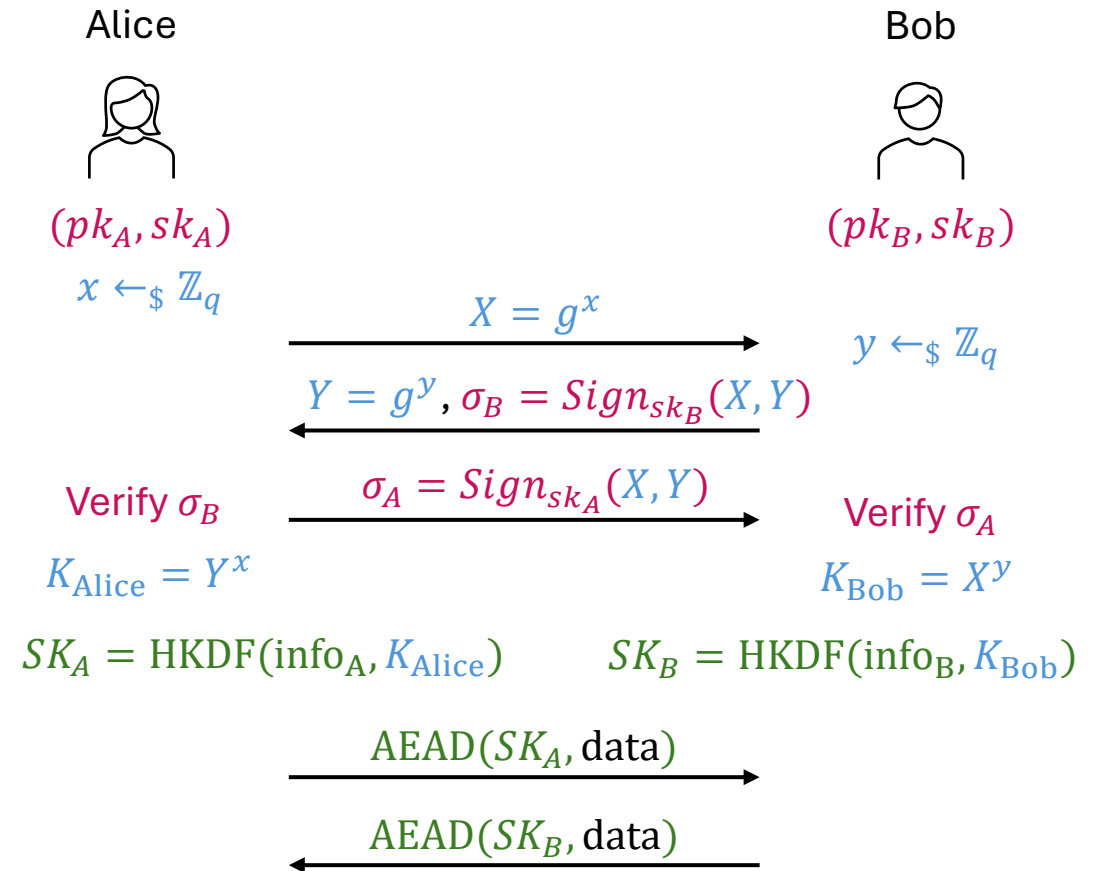
Post-quantum TLS

- The signed DH protocol
- To make it post-quantum secure, which parts should we change?



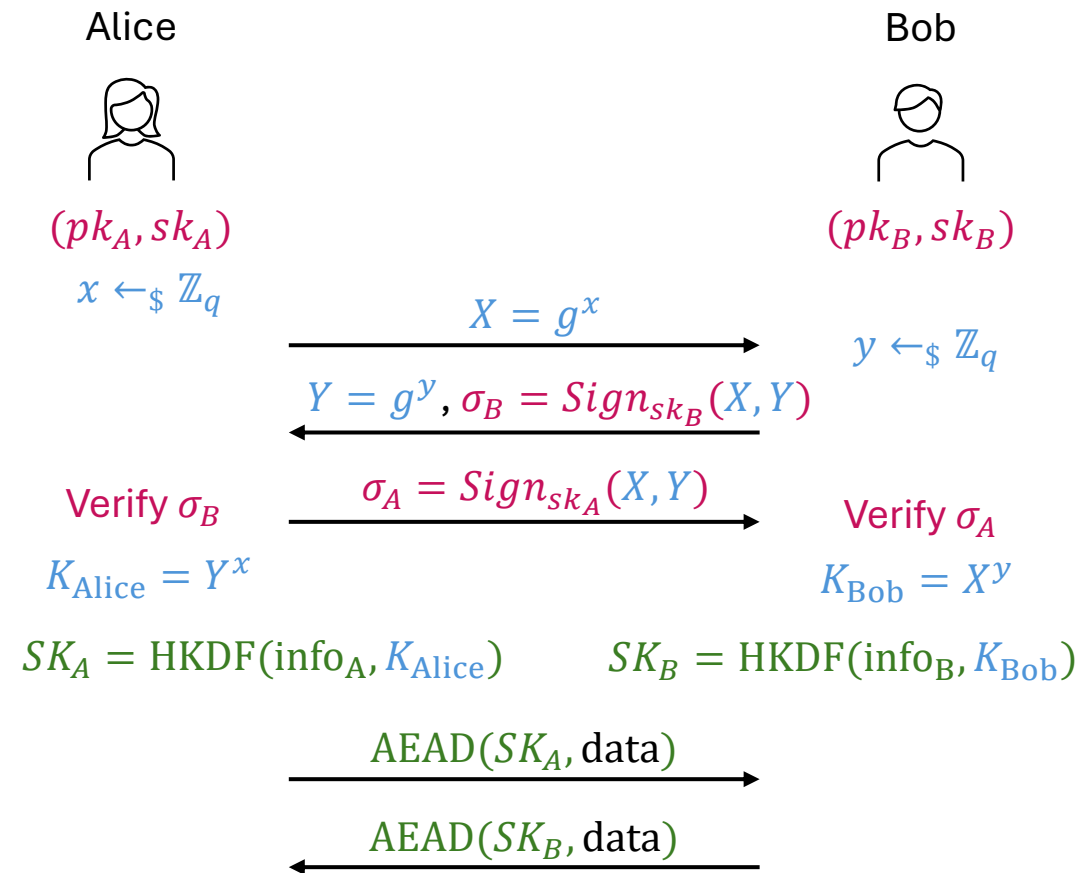
Post-quantum TLS

- The signed DH protocol
- To make it post-quantum secure, which parts should we change?
 - Symmetric algorithms remain secure
 - **The DHKE and the signature scheme (e.g., ECDSA, RSA) are not post-quantum secure**



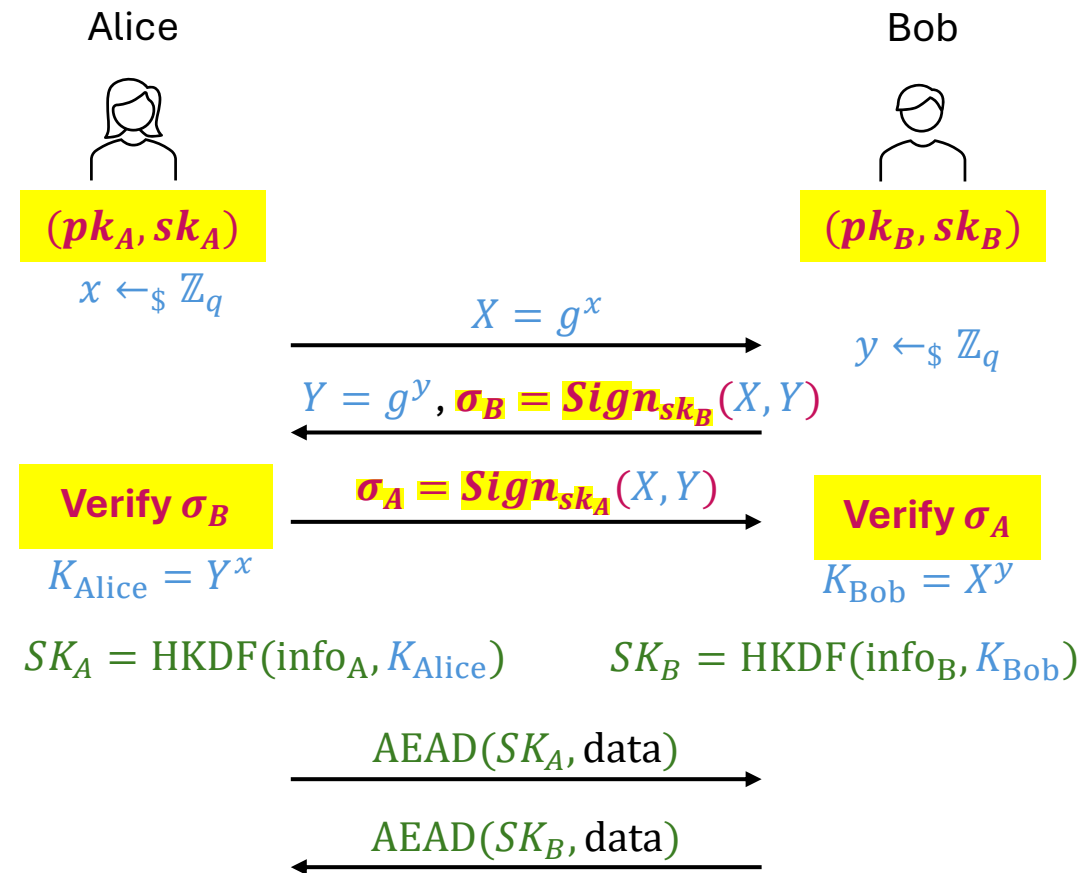
Post-quantum TLS

- The signed DH protocol
- To make it post-quantum secure, which parts should we change?
 - Symmetric algorithms remain secure
 - The DHKE and the signature scheme (e.g., ECDSA, RSA) are not post-quantum secure
- We first replace the signature with ML-DSA (or other post-quantum secure signature schemes)



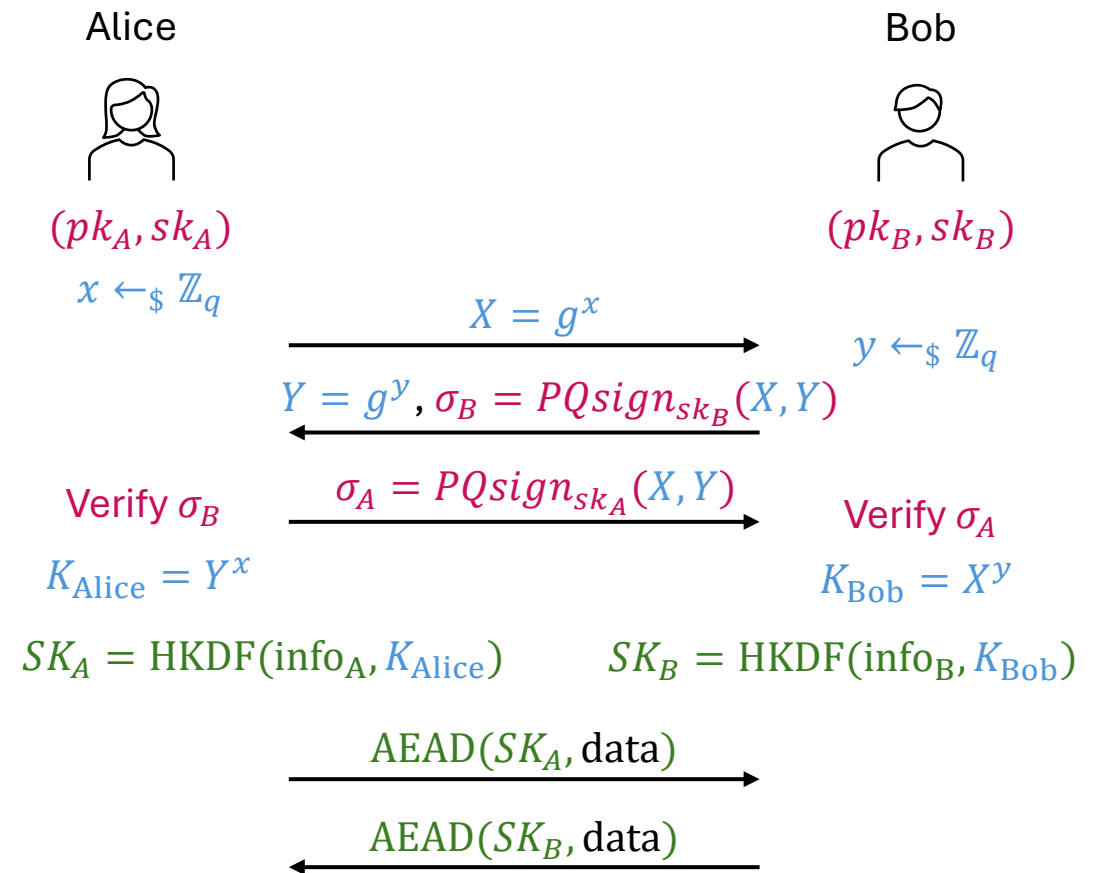
Post-quantum TLS

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- To make it post-quantum secure, which parts should we change?
 - Symmetric algorithms remain secure
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- We first replace the signature part with **ML-DSA** (or other post-quantum secure signature schemes)



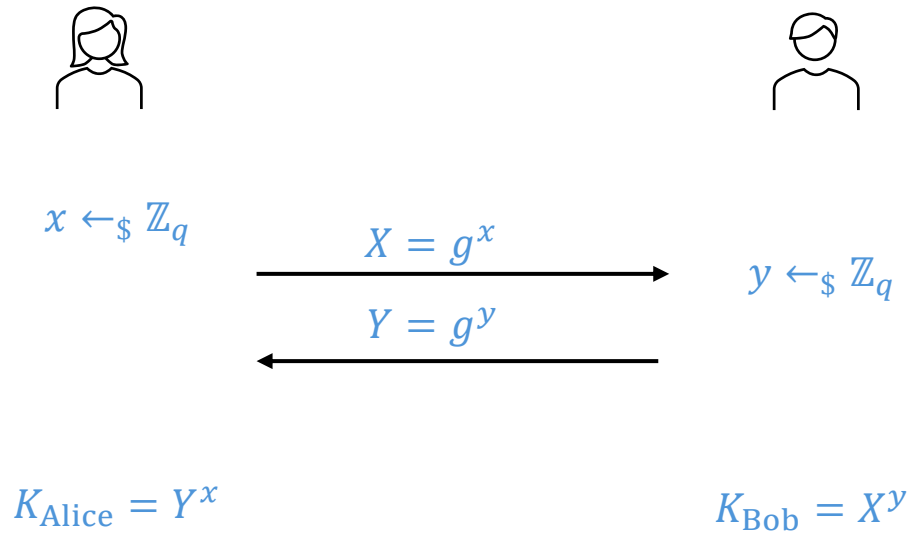
Post-quantum TLS

- The DH + PQsign protocol
- How can we replace the DH part?



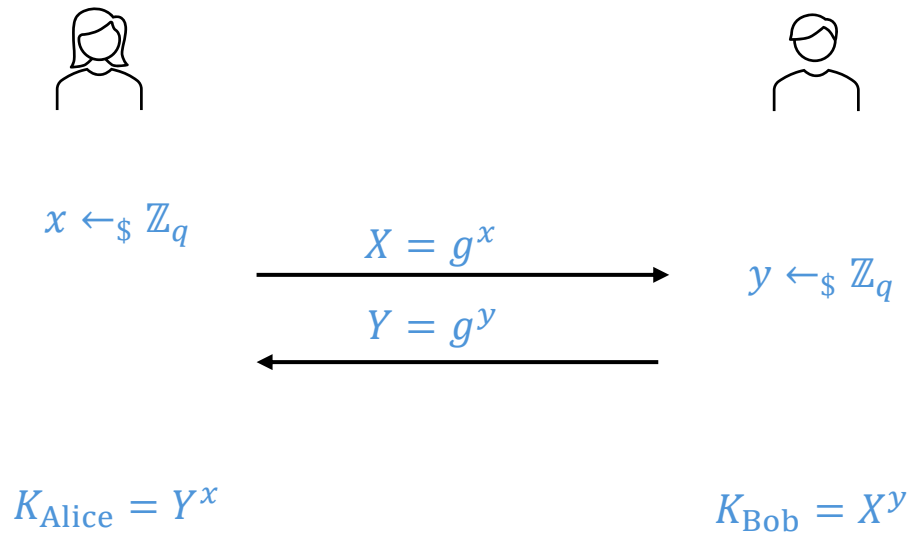
Post-quantum TLS

- DH key exchange

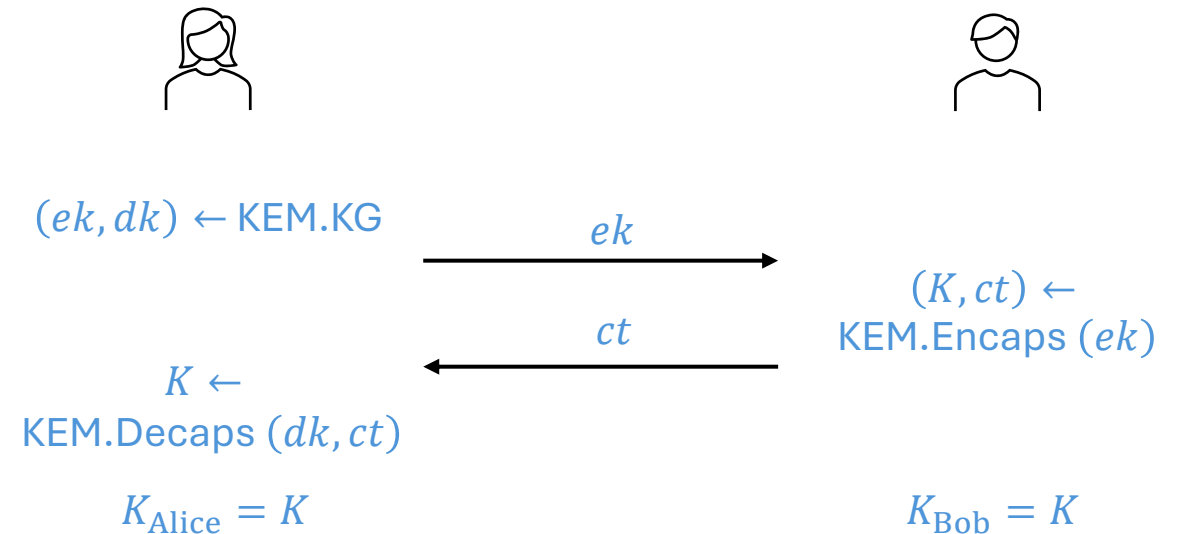


Post-quantum TLS

- DH key exchange

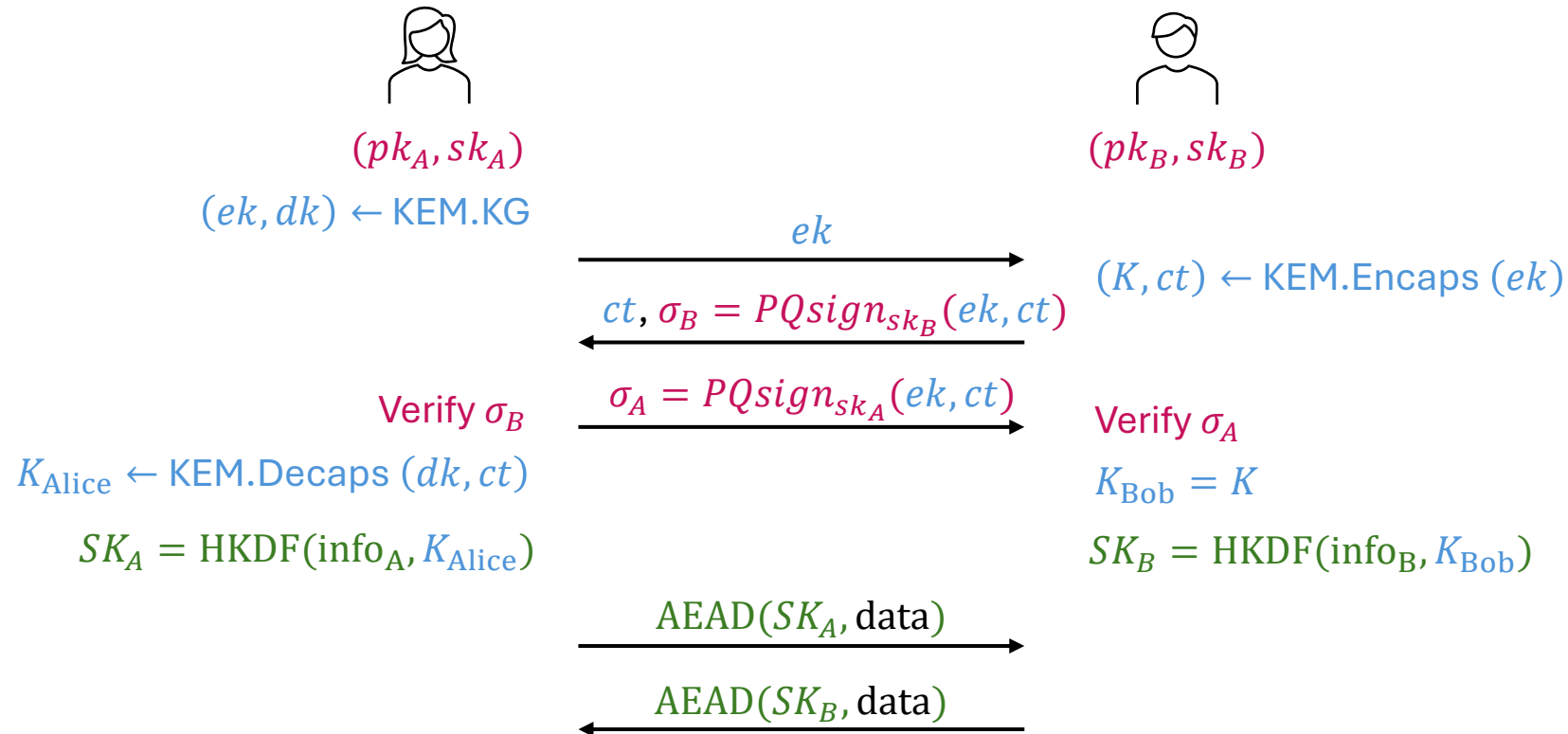


- KEM-based key exchange



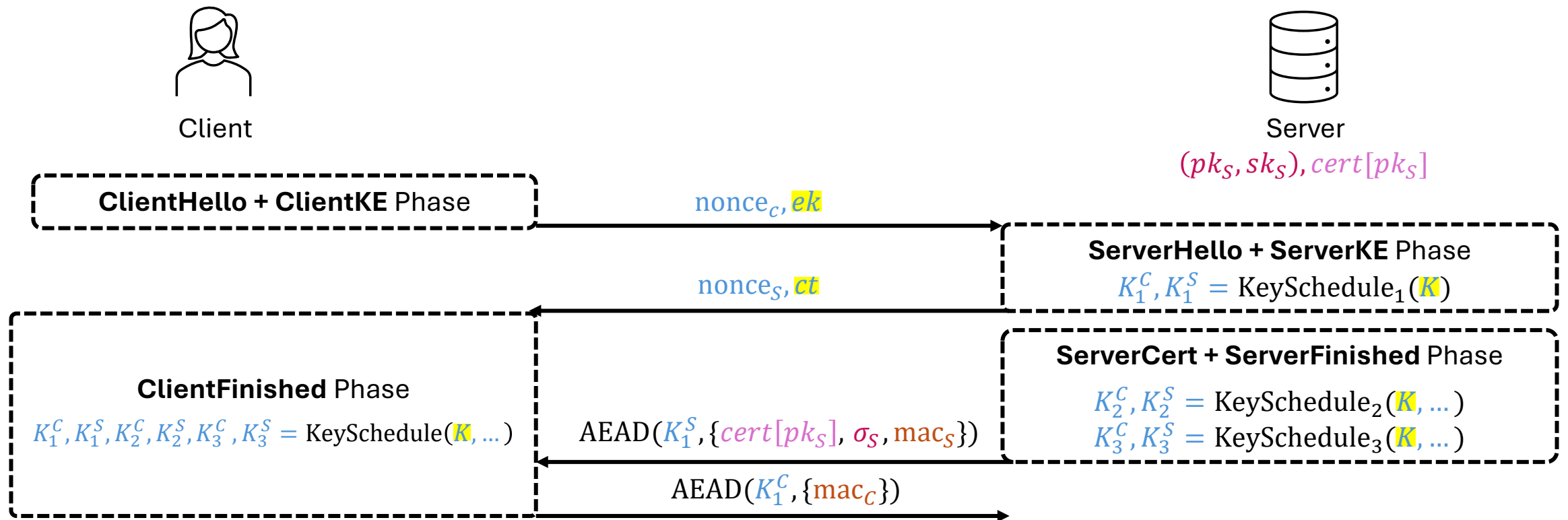
Post-quantum TLS

- PQKEM + PQSign = PQTLS (demo)



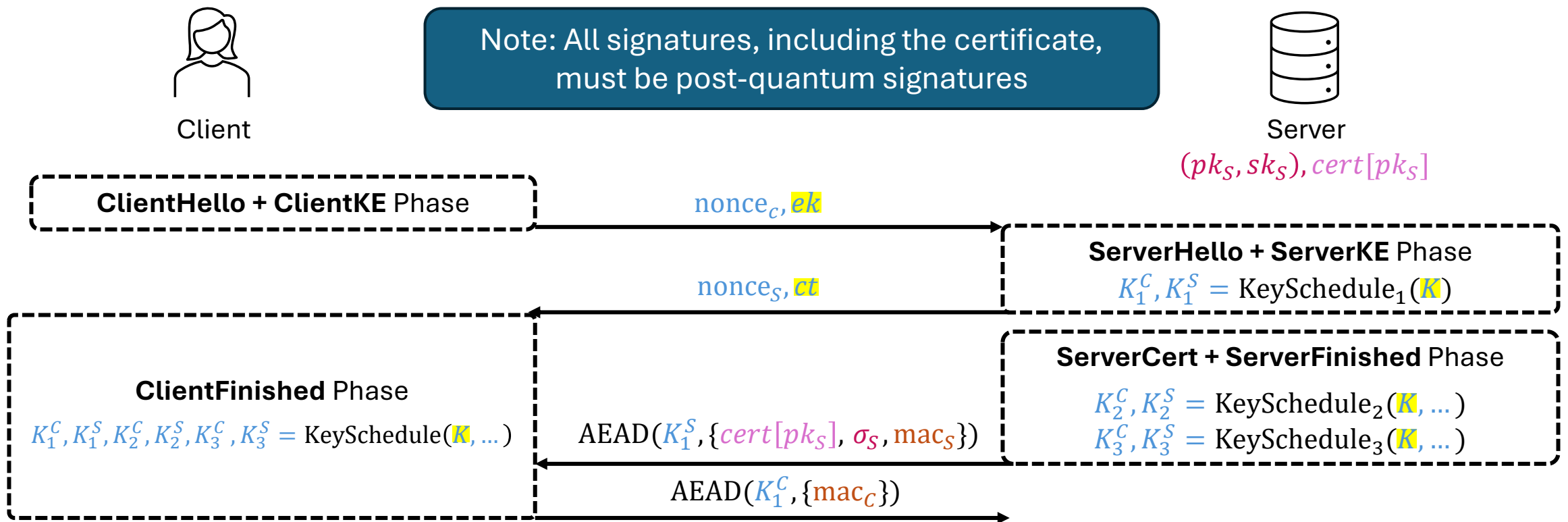
Post-quantum TLS

- Integrate the PQTLS into the TLS 1.3 framework (Homework 1)



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Post-quantum TLS based on KEM

- In post-quantum cryptography, signature schemes are generally less efficient than KEM
- In TLS 1.3, we use signature to
 - (1) Certify the server's public key (signed by the CA)
 - (2) Authenticate the serverKE message (signed by the server)

Post-quantum TLS based on KEM

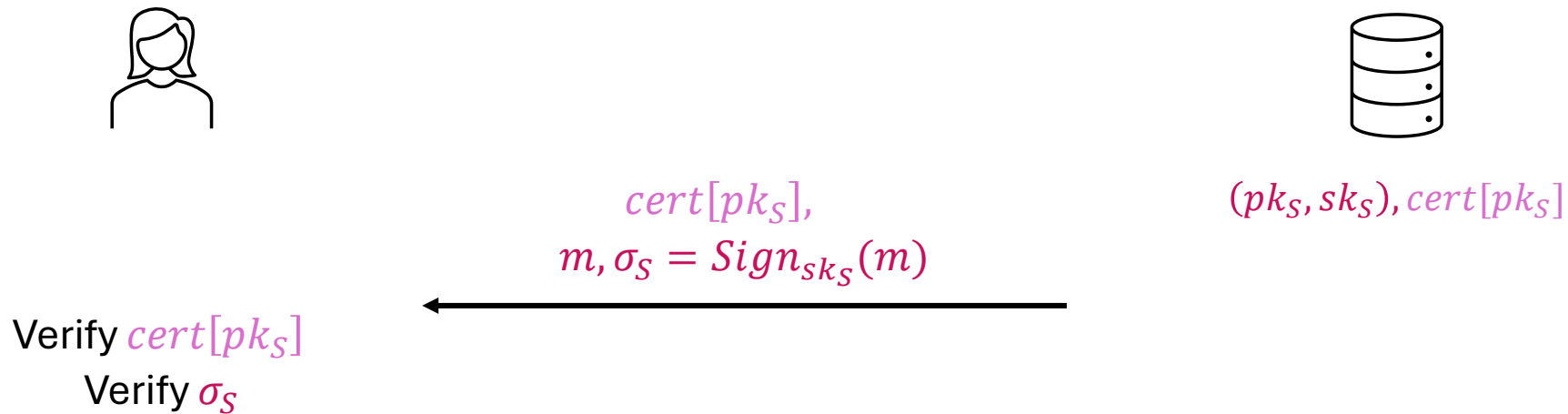
- In post-quantum cryptography, signature schemes are generally less efficient than KEM
- In TLS 1.3, we use signature to
 - (1) Certify the server's long-term public key (signed by the CA) ...one-time price
 - (2) **Authenticate the serverKE message (signed by the server)** ...needed in every session

Post-quantum TLS based on KEM

- In post-quantum cryptography, signature schemes are generally less efficient than KEM
- In TLS 1.3, we use signature to
 - (1) Certify the server's public key (signed by the CA) ...one-time price
 - (2) Authenticate the serverKE message (signed by the server) ...needed in every session
- Can we replace the second part with KEM?

Post-quantum TLS based on KEM

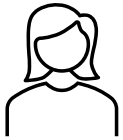
- Use KEM to authenticate messages



Only the sk owner can generate the signature
=> If σ_S verifies successfully, then m is from the server

Post-quantum TLS based on KEM

- Use KEM to authenticate messages

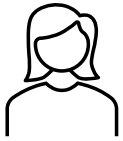


$(pk_{KEM}, sk_{KEM}), cert[pk_{KEM}]$

When using KEM:
having the $sk \Leftrightarrow$ Decrypt ciphertexts

Post-quantum TLS based on KEM

- Use KEM to authenticate messages



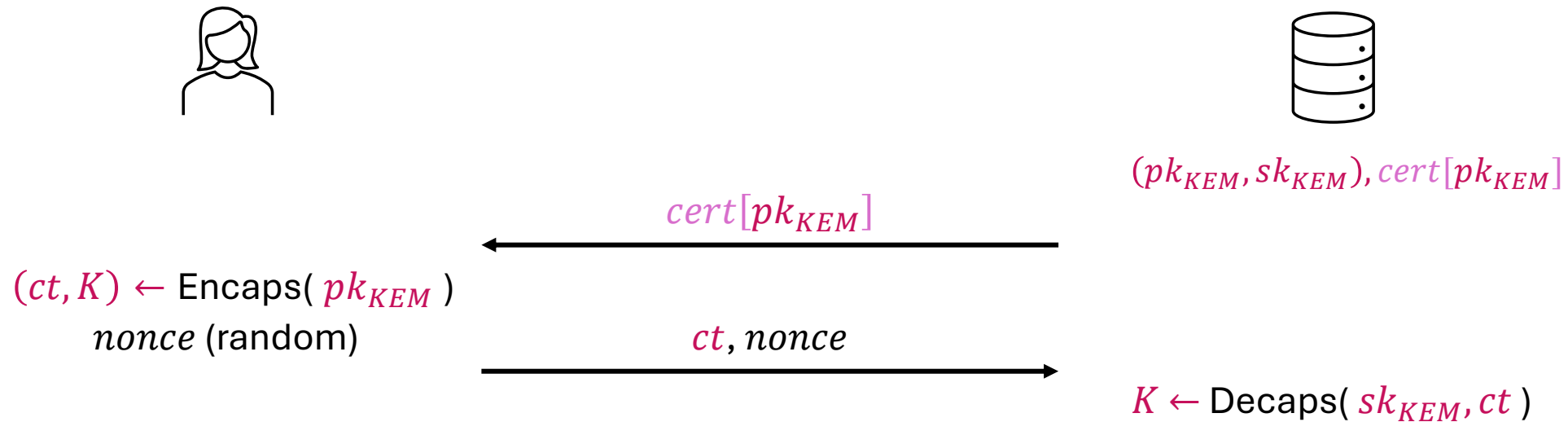
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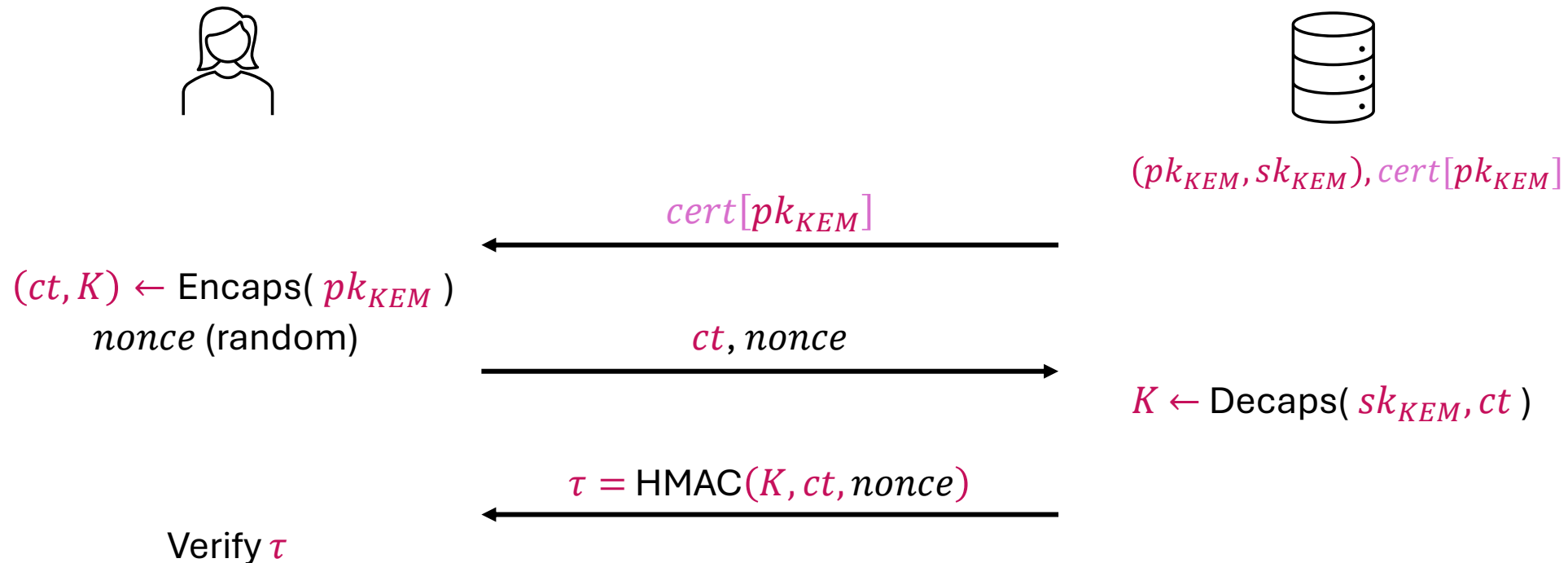
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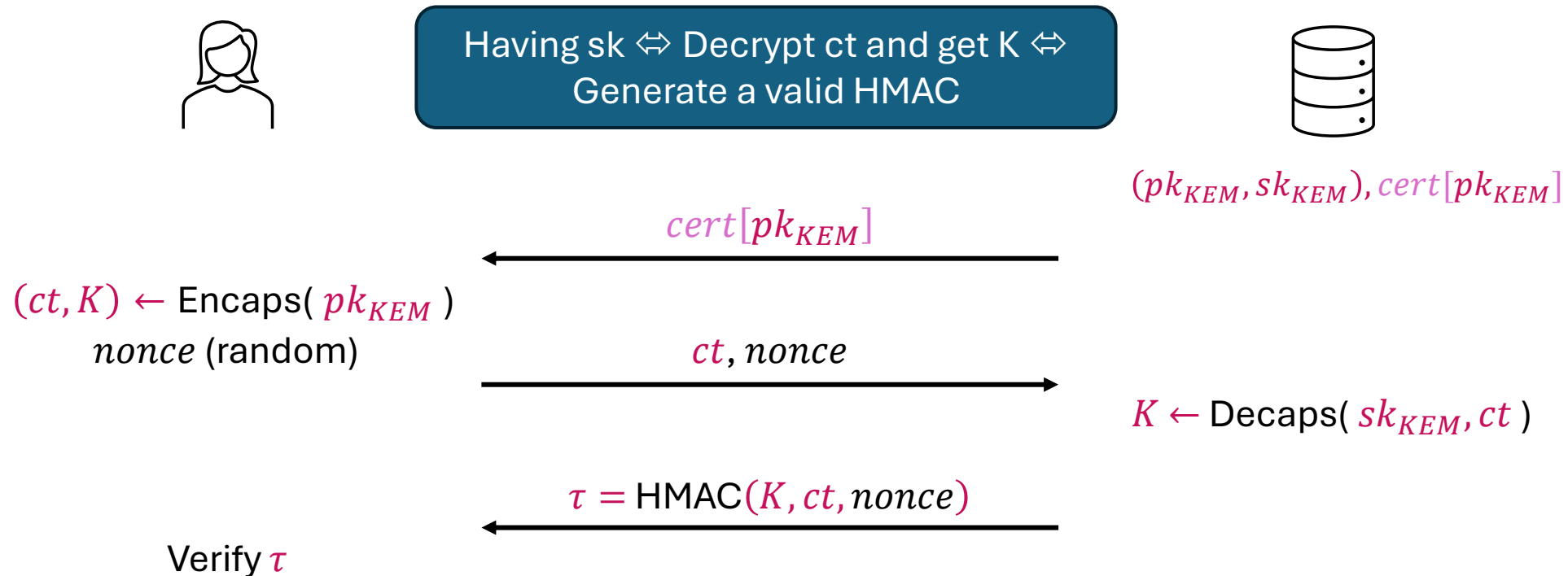
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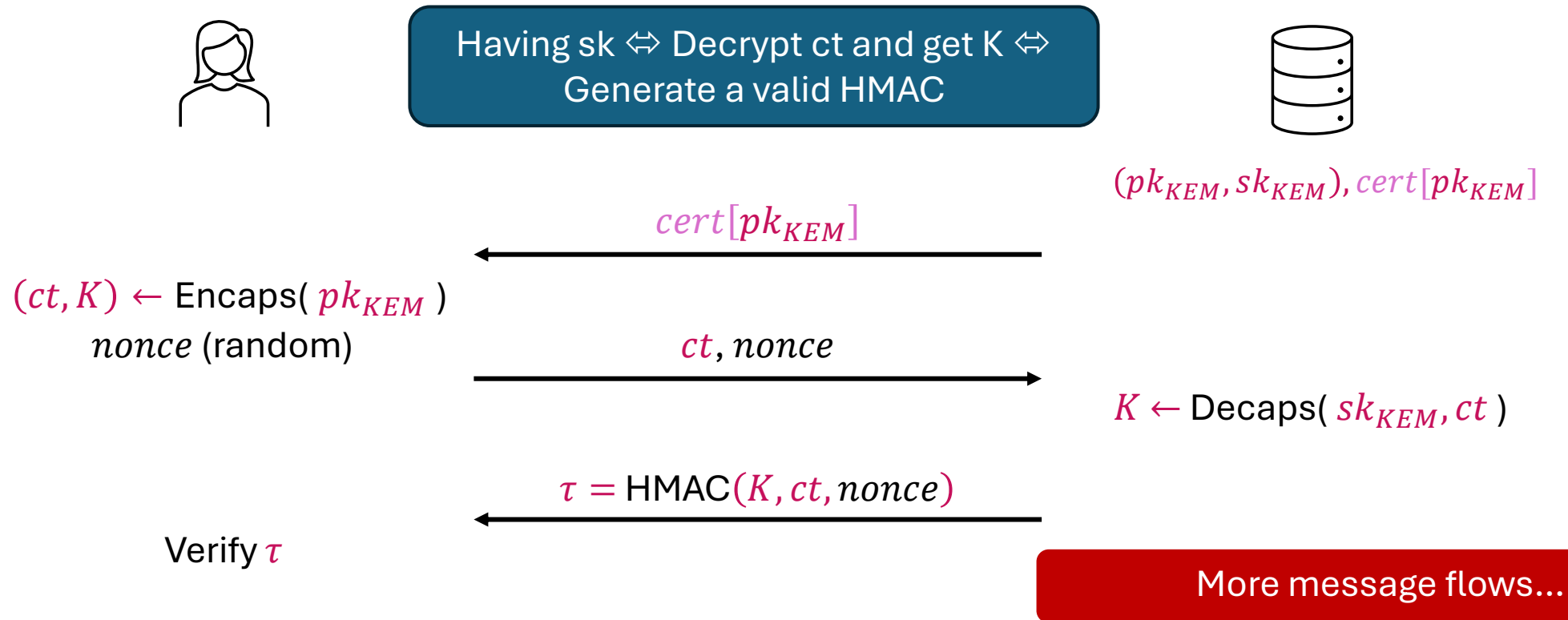
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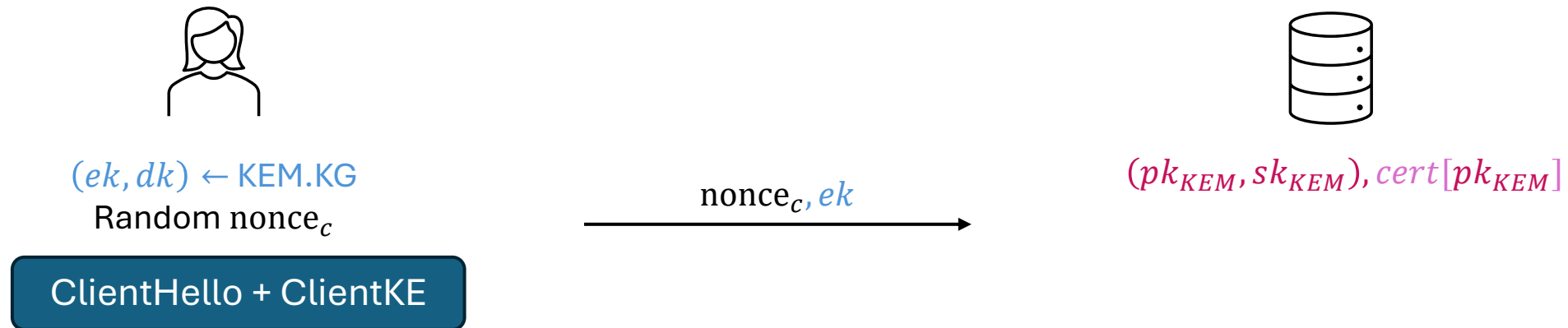
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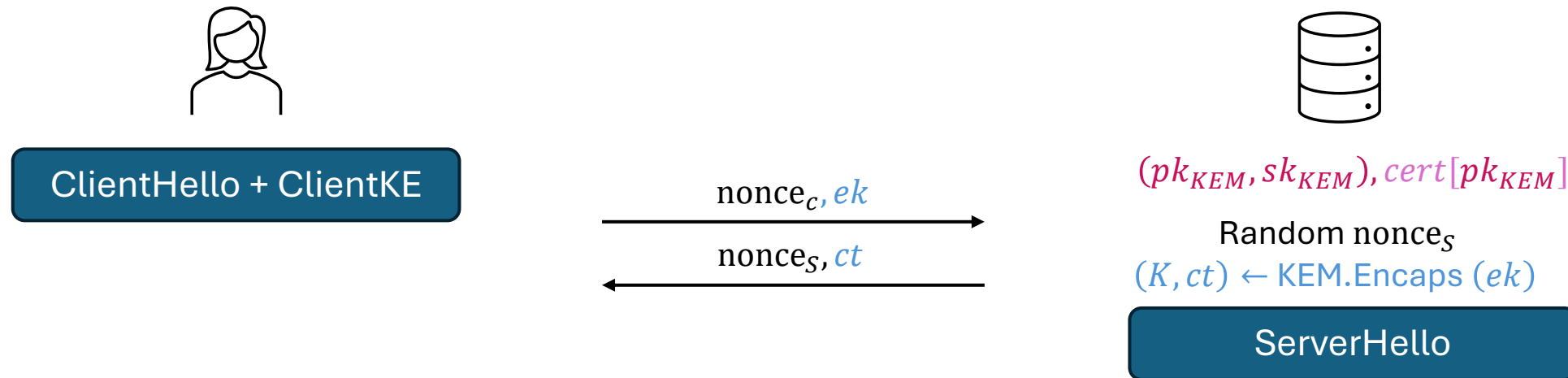
KEM-TLS

- **KEM-TLS (demo):** Integrate the KEM authentication into the TLS framework



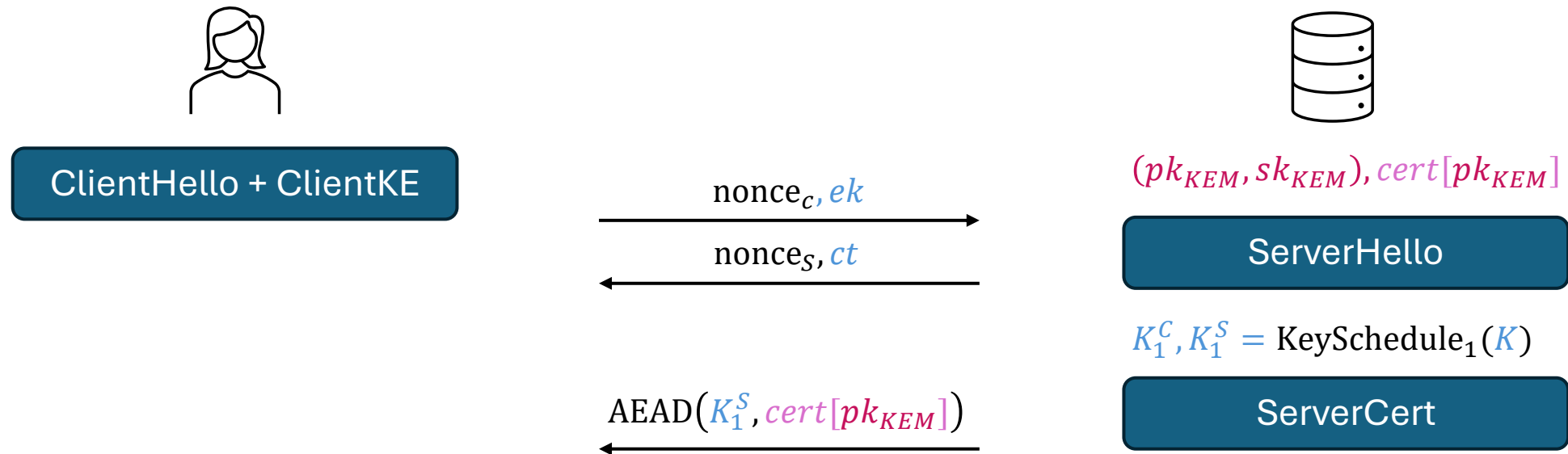
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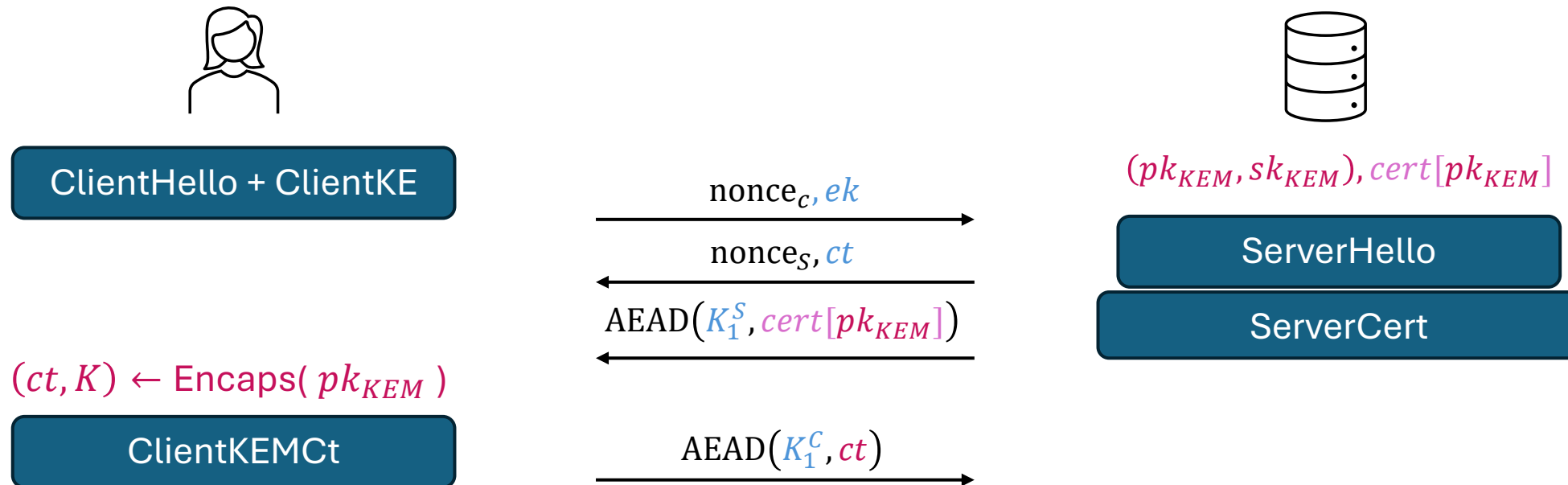
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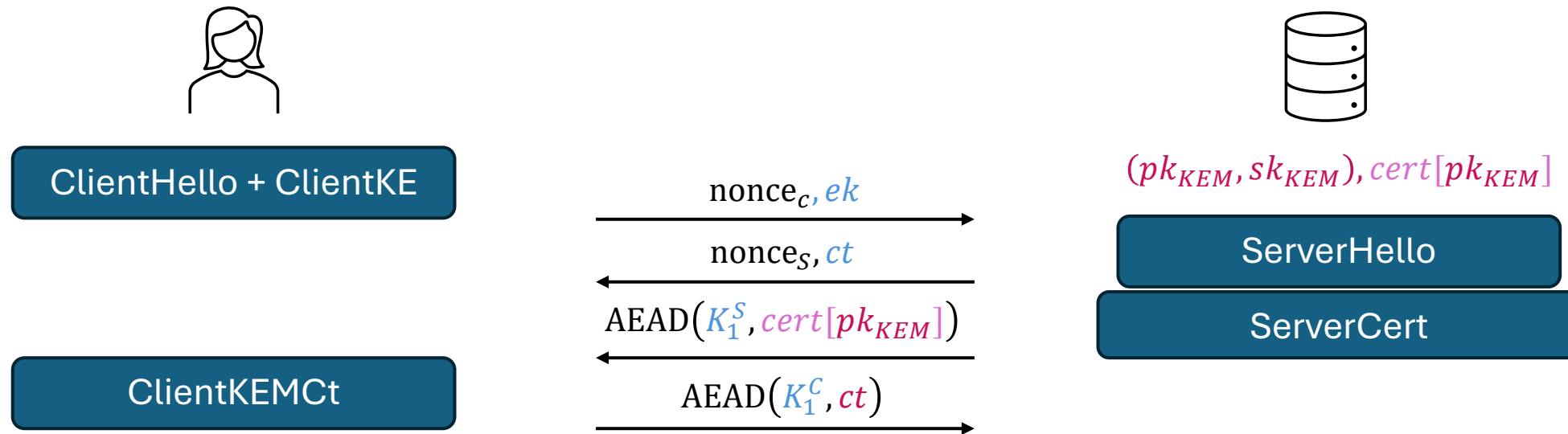
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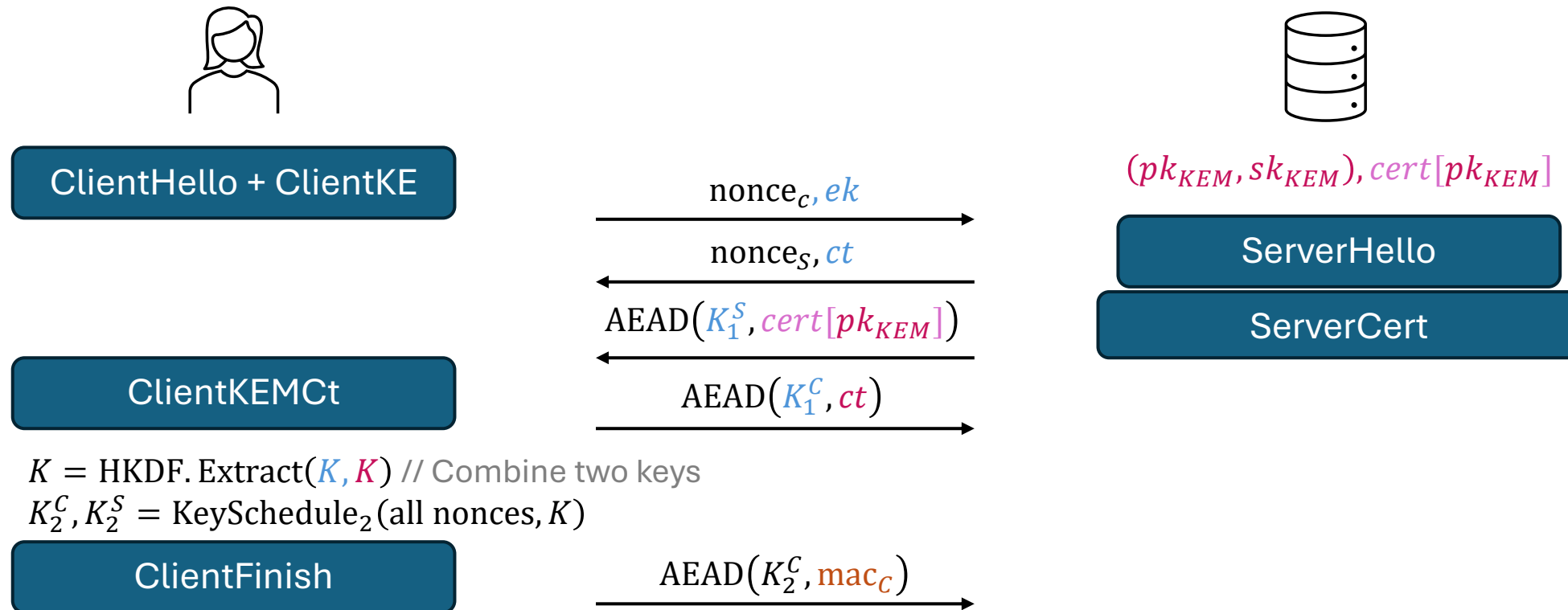


$K = \text{HKDF.Extract}(K, K)$ // Combine two keys

$K_2^C, K_2^S = \text{KeySchedule}_2(\text{all nonces}, K)$

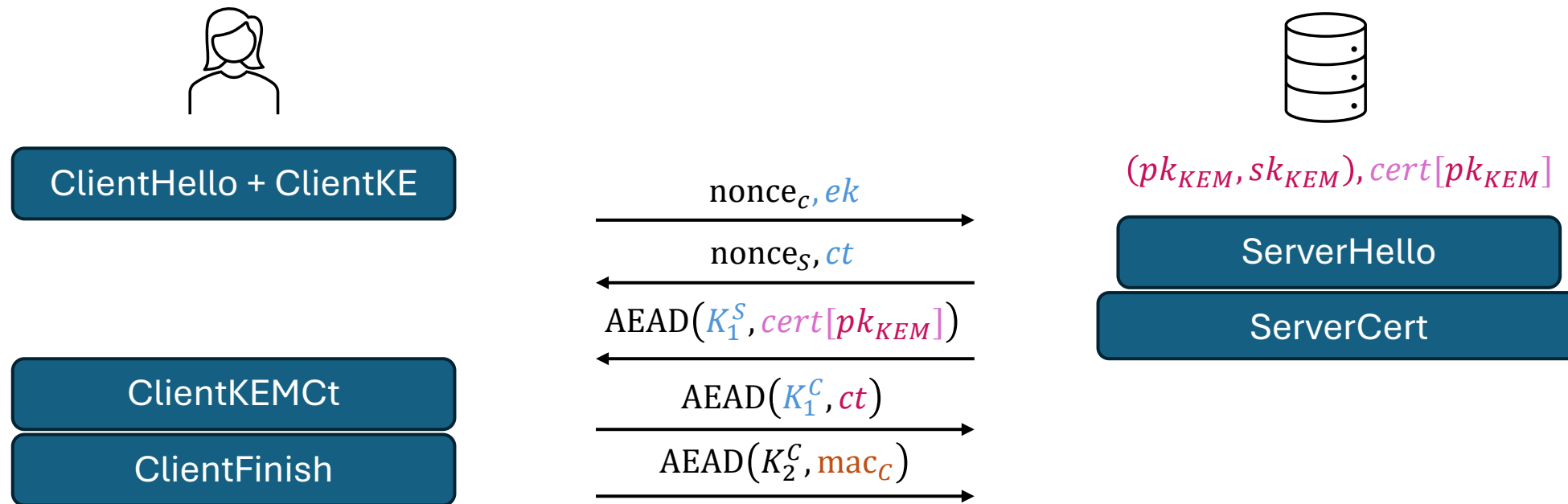
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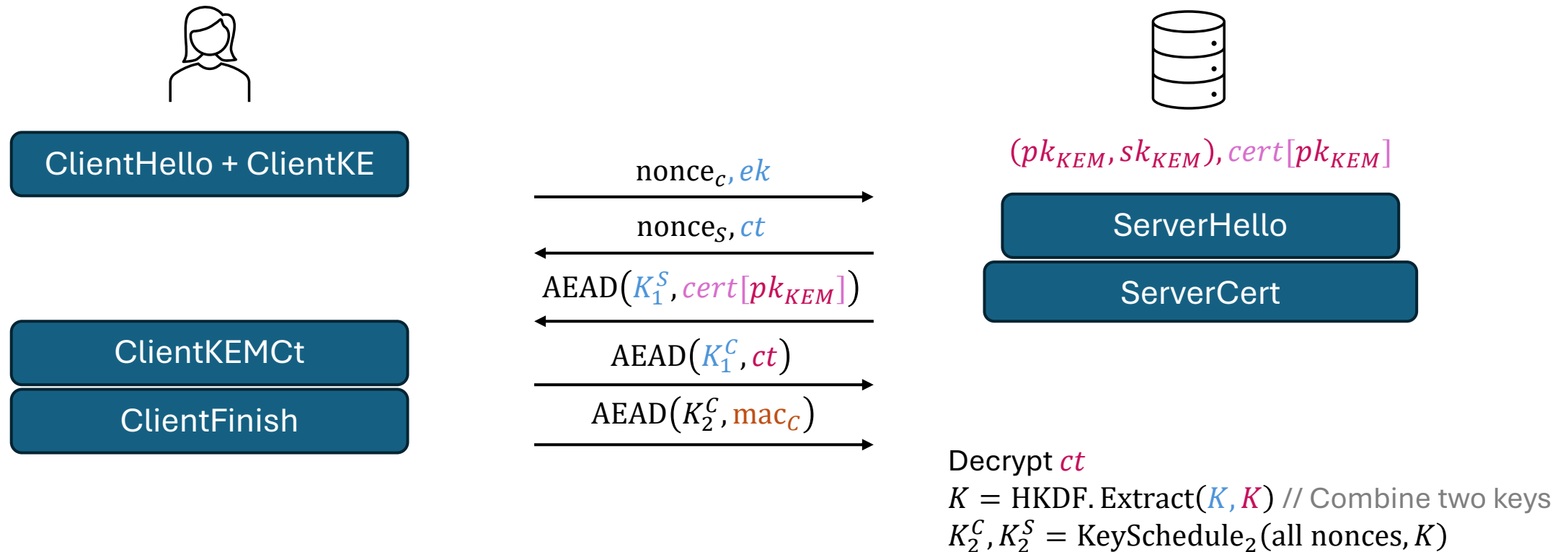
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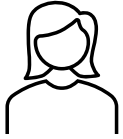


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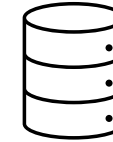
KEM-TLS



ClientHello + ClientKE

ClientKEMCt

ClientFinish

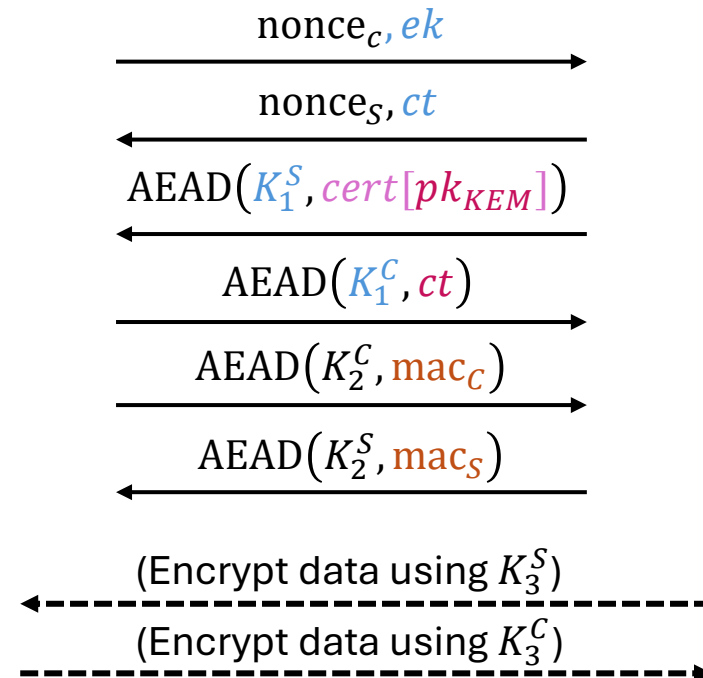


$(pk_{KEM}, sk_{KEM}), cert[pk_{KEM}]$

ServerHello

ServerCert

ServerFinish



Homework

- 1. Find Kyber (or ML-KEM) and Dilithium (or ML-DSA) implementations in your programming language. Then implement the PQ-TLS protocol.
- 2. Find Kyber (or ML-KEM) implementations in your programming language, then implement the KEM-TLS protocol.
- 3. Compare the efficiency between your PQ-TLS and KEM-TLS implementations.
- **DDL: Jan 09th, 2026 at 23:59**
- Rust:
 - https://docs.rs/ml-kem/latest/ml_kem/
 - https://docs.rs/ml-dsa/latest/ml_dsa/
- Python:
 - <https://github.com/GiacomoPope/kyber-py>
 - <https://github.com/GiacomoPope/dilithium-py>

Reference

- PQ-TLS: <https://cic.iacr.org/p/1/2/6>
- KEM-TLS: <https://kemtls.org/>
- Crystal-Kyber and ML-KEM: <https://nvlpubs.nist.gov/nistpubs/FIPS/NIST.FIPS.203.pdf>
- Crystal-Dilithium and ML-DSA: <https://nvlpubs.nist.gov/nistpubs/FIPS/NIST.FIPS.204.pdf>