Numerical Linear Algebra with Applications S21
HW 4
Due Feb 22 (by noon)

1. 4.25 but replace part (b) with the following:

b) Using either orthogonal iteration, or the QR method, compute the eigenvalues of the matrix from part (a). Do not use MATLAB's `eig(A)` command for this. The stopping condition should be that the iterative error for each eigenvalue satisfies $|v_k - v_{k-1}| \leq 10^{-8}$. Also, both $\sum_{i=1}^{n} \lambda_i$ and $\sum_{i=1}^{n} \lambda_i^2$ are suppose to be integers. How close are you values to satisfying these two conditions? Finally, in your write-up state which method you used to compute the eigenvalues.

2. Let

$$\mathbf{A} = \begin{pmatrix} 3 & 0 & 3 & 0 \\ 0 & 6 & 1 & 5 \\ 3 & 1 & 5 & 0 \\ 0 & 5 & 0 & 5 \end{pmatrix}$$

(a) Construct the directed graph for the matrix, and from this show that it is irreducible.

(b) Use your result from part (a), and Definition 3.1(2), to show that the matrix is positive definite.

3. This problem considers a house with 9 rooms and interconnecting doorways as shown in Figure 1 on the next page. Every hour those in each room will either stay in the room or move into one of the adjacent rooms. Specifically, if the room has m individuals and n doorways, then each hour $m/(n+1)$ will stay, and $m/(n+1)$ will move through each doorway into the corresponding adjacent room.

(a) Express this using a weighted directed graph with 9 vertices.

(b) Based on the given formula, $a_{11} = \frac{1}{3}$, $a_{12} = \frac{1}{4}$, and $a_{25} = \frac{1}{5}$. Determine the other entries, and then write down the matrix $\mathbf{A}$. Explain why it is a probability matrix.

(c) Explain why $\mathbf{A}$ is irreducible.

(d) (`MATLAB`) Suppose there are 1200 people in the house (it's a big house), and at the start they are all in Room 2. Taking $tol = 10^{-8}$, use the power method to determine the eventual distribution of people in the 9 rooms. You should round the values of the eventual distribution to the closest integers. Also, your answer should include the value of k that the iteration stopped at. Is there any connection between the the eventual distribution and the number of doorways to each room?

(e) (`MATLAB`) How many hours does it take to reach the eventual distribution?

(f) (`MATLAB`) Suppose three of the doorways for Room 5 are closed, and the one remaining open connects with Room 6. What is the resulting matrix $\mathbf{A}$? Redo parts (d) and (f) and comment on the differences in your answers.

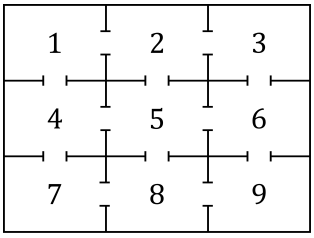

Figure 1: House with 9 rooms interconnected by doorways used in Problem 3.

What to turn in for MATLAB questions 1(b),(c), 3(d),(e),(f): Make a pdf of the results of the MATLAB command window (make sure to include comments, or labels, in the printout indicating the problem being answered). It's OK if there are different pdf's for each problem.