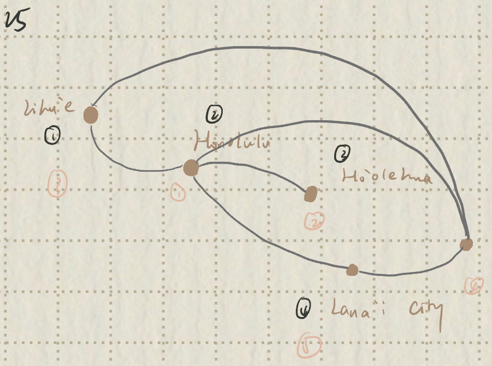
Title HW4 - Ruochen Kong **Date** 2.21

1. 4.25



$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ 4 & 1 & 0 & 1 & 0 & 1 \\ 5\ ⑤ & 1 & 0 & 0 & 1 & 0 \end{array}$$

a)

Let ① Lihu'e, ② Honolulu, ③ Ho'olehua, ④ Lana'i City, and ⑤ Kahului, then the adjacency matrix denodes:

$$A = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \\ ⑤ \end{array} \begin{array}{c} \begin{array}{ccccc} ① & ② & ③ & ④ & ⑤ \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

b) Code and Result Attached in the next page.

I used Orthogonal Iteration.

The result shows:

$\sum_{i=1}^{n} \lambda_i \approx 1.11 \times 10^{-16}$ which is quite close to 0

And $\sum_{i=1}^{n} \lambda_i^2 = 1.2$

which is not quite satisfied to be an integer.

c) change: ① Honolulu, ② Ho'olehua, ③ Lihu'e, ④ Kahului, ⑤ Lana'i City.

Then the adjacency Matrix should be:

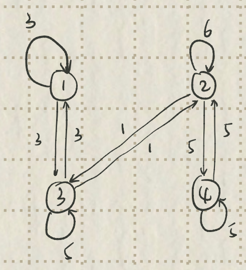
$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The result of e-values is just changed in orders, the sum value changes a little, from 1.11×10^{-16} to 2.78×10^{-16}, and the sum squared values does not change.

2.

$$A = \begin{bmatrix} 3 & 0 & 3 & 0 \\ 0 & 6 & 1 & 5 \\ 3 & 1 & 5 & 0 \\ 0 & 5 & 0 & 5 \end{bmatrix} \qquad \begin{array}{c} [0, 6] \\ [0, 12] \\ [1, 9] \\ [0, 10] \end{array}$$

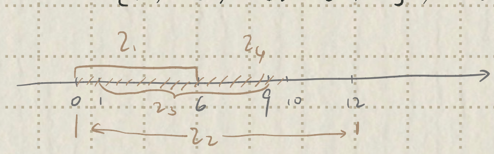
a)



From the graph, we can see that the nodes are connected in a cycle: ①→③→②→④→② →③→①→② ..., and each node has an edge to itself, So every node is connected, the matrix is irreducible.

b) The Gershgorin interval of this matrix should be:

$I_1 = [0, 6]$, $I_2 = [0, 12]$, $I_3 = [1, 9]$, $I_4 = [0, 10]$



From definition 3.1, we need to prove all e-values are positive.

However, from the intervals, we can only conclude

$0 \le \lambda \le 12$,

where the condition $\lambda = 0$ is not excluded.

Recall the theorem ⑤, if we can prove that 1 is an e-value of A, and since A is irreducible, then we can say that all the intervals started at 1.

Assume $\lambda = 1$, then

$$(A - \lambda I)\vec{x} = \begin{bmatrix} 2 & 0 & 3 & 0 \\ 0 & 5 & 1 & 5 \\ 3 & 1 & 4 & 0 \\ 0 & 5 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \vec{0}$$

$$\left[\begin{array}{cccc|c} 2 & 0 & 3 & 0 & 0 \\ 0 & 5 & 1 & 5 & 0 \\ 3 & 1 & 4 & 0 & 0 \\ 0 & 5 & 0 & 4 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 2 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 4 & 0 \end{array}\right]$$

$$\rightarrow \begin{cases} 2x_1 + 3x_3 = 0 \\ x_3 + x_4 = 0 \\ x_1 + x_2 = 0 \\ 5x_2 + 4x_4 = 0 \end{cases} \rightarrow \begin{array}{l} x_2 = -x_1 \\ x_3 = -\frac{3}{2}x_1 \\ x_4 = \frac{3}{2}x_1 \end{array}, \text{ So } \vec{x} = \begin{bmatrix} x_1 \\ -x_1 \\ -\frac{3}{2}x_1 \\ \frac{3}{2}x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -3/2 \\ 3/2 \end{bmatrix} x_1$$

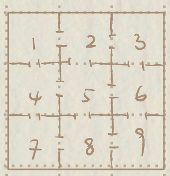
Thus, the e-vector is $\begin{bmatrix} 1 & -1 & -3/2 & 3/2 \end{bmatrix}^T$ which means, $\lambda = 1$ is a true e-value. Therefor the intervals can be rewritten as $I_1 = [1, 6]$, $I_2 = [1, 12]$, $I_3 = [1, 9]$, $I_4 = [1, 10]$.

Thus, $1 \le \lambda \le 12$, λ is always positive. Since A is symmetrical, and all e-values are positive, A is positive definite.

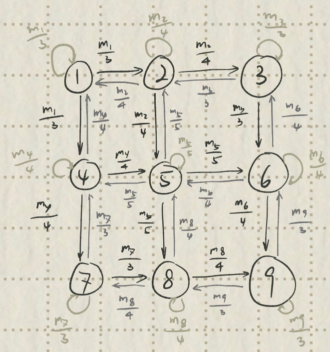
```
>> prob1
-------------part b)-------------
k = 45 iterations

e-value 1: 2.68554393e+00
e-value 2: -1.74911755e+00
e-value 3: -1.27133037e+00
e-value 4: 0.00000000e+00
e-value 5: 3.34903985e-01

Final Q(e-vectors):
    0.4119   -0.4581   -0.2834         0    0.2004
    0.5825    0.6478   -0.4008         0   -0.2835
    0.2169   -0.3704    0.3153         0   -0.8464
    0.4119   -0.4581   -0.2834         0    0.2004
    0.5237    0.1534    0.7611         0    0.3506


Sum of E-valuse        : 1.11022302e-16
Sum of squared E-valuse: 1.20000000e+01

-------------part c)-------------
k = 45 iterations

e-value 1: 2.68554393e+00
e-value 2: -1.74911755e+00
e-value 3: -1.27133037e+00
e-value 4: 3.34903985e-01
e-value 5: 0.00000000e+00

Final Q(e-vectors):
    0.5825   -0.6478    0.4008   -0.2835         0
    0.2169    0.3704   -0.3153   -0.8464         0
    0.4119    0.4581    0.2834    0.2004    0.0000
    0.5237   -0.1534   -0.7611    0.3506         0
    0.4119    0.4581    0.2834    0.2004    0.0000


Sum of E-valuse        : 2.77555756e-16
Sum of squared E-valuse: 1.20000000e+01
>>
```

Title HW 4 - Ruochen Kong **Date** 2.2

3.



a)



b) Adajacency matrix:

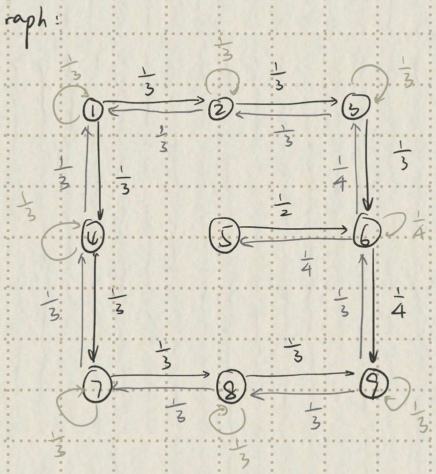
$$A = \begin{array}{c|ccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline 1 & \frac{1}{3} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 2 & \frac{1}{3} & \frac{1}{4} & \frac{1}{3} & 0 & \frac{1}{5} & 0 & 0 & 0 & 0 \\ 3 & 0 & \frac{1}{4} & \frac{1}{3} & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 4 & \frac{1}{3} & 0 & 0 & \frac{1}{4} & \frac{1}{5} & 0 & \frac{1}{3} & 0 & 0 \\ 5 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{5} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 6 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{5} & \frac{1}{4} & 0 & 0 & \frac{1}{3} \\ 7 & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{3} & \frac{1}{4} & 0 \\ 8 & 0 & 0 & 0 & 0 & \frac{1}{5} & 0 & \frac{1}{3} & \frac{1}{4} & \frac{1}{3} \\ 9 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{3} \end{array}$$

Since A is a $n \times n$ matrix, and sum of each row adds up to 1, and clearly $a_{ij} \geq 0 \ \forall i, j$, A is a probability matrix.

Column Sum: 1 1 1 1 1 1 1 1 1 V

c) Similar as for problem 2, we can get to any node of the nine by randomly select a starting node. This means, the matrix is irreducible.

e) Graph:



```
>> prob3
---------------part d)---------------
A =
    0.33    0.25    0.00    0.25    0.00    0.00    0.00    0.00    0.00
    0.33    0.25    0.33    0.00    0.20    0.00    0.00    0.00    0.00
    0.00    0.25    0.33    0.00    0.00    0.25    0.00    0.00    0.00
    0.33    0.00    0.00    0.25    0.20    0.00    0.33    0.00    0.00
    0.00    0.25    0.00    0.25    0.20    0.25    0.00    0.25    0.00
    0.00    0.00    0.33    0.00    0.20    0.25    0.00    0.00    0.33
    0.00    0.00    0.00    0.25    0.00    0.00    0.33    0.25    0.00
    0.00    0.00    0.00    0.00    0.20    0.00    0.33    0.25    0.33
    0.00    0.00    0.00    0.00    0.00    0.25    0.00    0.25    0.33

y0 =
           0
        1200
           0
           0
           0
           0
           0
           0
           0

x1 =
    0.2684
    0.3578
    0.2683
    0.3578
    0.4472
    0.3578
    0.2683
    0.3577
    0.2683

sol =
   109
   145
   109
   145
   182
   145
   109
   145
   109

hours = 21
>>
```

Room 1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9

= 3 : 4 : 3 : 4 : 5 : 4 : 3 : 4 : 3 (Add the room itself as a gate)

3+4+3+4+5+4+3+4+3 = 33

$1200 \times \frac{3}{33} = 109$

$1200 \times \frac{4}{33} = 145$

$1200 \times \frac{5}{33} = 182$

} those are exactly the same number shown in the result.

Thus, assume we have y people at the begining, and the door numbers are represented as $n_1, n_2, \dots, n_9$, then the eventual distribution of room i should be

$$\frac{n_i + 1}{\sum_{m=1}^{9} (n_m + 1)} \cdot y$$

```
>> prob3
---------------part f)---------------
A =
    0.33    0.33    0.00    0.33    0.00    0.00    0.00    0.00    0.00
    0.33    0.33    0.33    0.00    0.00    0.00    0.00    0.00    0.00
    0.00    0.33    0.33    0.00    0.00    0.25    0.00    0.00    0.00
    0.33    0.00    0.00    0.33    0.00    0.00    0.33    0.00    0.00
    0.00    0.00    0.00    0.00    0.50    0.25    0.00    0.00    0.00
    0.00    0.00    0.33    0.00    0.50    0.25    0.00    0.00    0.33
    0.00    0.00    0.00    0.33    0.00    0.00    0.33    0.33    0.00
    0.00    0.00    0.00    0.00    0.00    0.00    0.33    0.33    0.33
    0.00    0.00    0.00    0.00    0.00    0.25    0.00    0.33    0.33

y0 =
           0
        1200
           0
           0
           0
           0
           0
           0
           0

x1 =
    0.3293
    0.3293
    0.3293
    0.3293
    0.2195
    0.4391
    0.3293
    0.3293
    0.3293

sol =
   133
   133
   133
   133
    89
   178
   133
   133
   133

hours = 51
>>
```

doors +1 : 3 3 3 3 2 4 3 3 3

sum = 27

$1200 \times \frac{3}{27} = 133$

$1200 \times \frac{2}{27} = 89$

$1200 \times \frac{4}{27} = 178$

Same as I commented in part d)

But it causes much longer time to getting eventual.