DS-GA 3001.

HW2. 1. $h(x) = 3f(x) - 4g(x) = 3x^2 - 20x$. $-\infty < x < \infty$

2. A: 2×3 B: 3×2. $AB = I$.

$A \cdot B = I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

$\text{rank}(AB) = 2$.

① $\because AB = I$ $\therefore$ A and B are invertible.

$\therefore$ ~~the C(A)~~ the column vectors of A / B are independent.

$\therefore$ ~~rank(A)~~ A · B are full rank matrix

$\therefore \text{rank}(A) = 2$ $\text{rank}(B) = 2$

② Suppose $BA = I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = [BA_1 \ BA_2 \ BA_3]$.

Since BA is a linear combination of A's column vectors,

$\text{rank}(BA) \le \text{rank}(A) = 2$

Similarly, $\text{rank}(BA) \le \text{rank}(B) = 2$

$\therefore$ Contradicted.

$\therefore BA \ne I$

3 A: $m\times n$. $AX = b$ has no solutions.

(a). ① full row rank. $n-r$ free variables

∞ solutions.

② full column rank $r = n < m$. n pivots. 0 free variables.

$\therefore N(A) = 0$.

particular solution: 0/1.

③ ~~φ~~ square full rank $r = n = m$

always has 1 solution.

④ not full: $r < m$, $r < n$. $0/\infty$ solutions

In summary: $r = n < m$ or $r < m, r < n$

(b) ~~$A^T y = 0$~~.

$\because$ $Ax=b$ has no solution.

$\therefore$ $N(A)=\{0\}$.

In order to find all y satisfy $A^T y = 0$.,

we can try to find ~~$N(A^T)$~~. $\in$

the $C(A^T)$'s orthogonal complement

which means that: $? \perp C(A^T)$.

which is exactly $N(A)=\{0\}$.

$\therefore$ y can only be 0.

4. We should project $b=(2,1,1)$ onto the subspace formed by $(1,2,-1)$, $(1,0,1)$

$$A=\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ -1 & 1 \end{pmatrix}$$

~~$\hat{x} = (A^TA)^{-1}A^Tb$~~

~~$p = A\hat{x} = (2,1,1)$~~.

Suppose the combination is

$x(1,2,-1)+y(1,0,1)$

$=(x+y,\ 2x,\ -x+y)$

The distance^2 $= (x+y-2)^2+(2x-1)^2+(y-x-1)^2 = D$

$$\begin{cases} \dfrac{\partial D}{\partial x} = 2(x+y-2)+2(2x-1)\cdot 2+2(y-x-1)(-1)=0 \\ \dfrac{\partial D}{\partial y} = 2(x+y-2)+2(y-x-1)=0 \end{cases}$$

$$\Rightarrow \begin{cases} x=\frac{1}{2} \\ y=\frac{3}{2} \end{cases}$$

$\therefore$ the combination is $\frac{1}{2}\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \frac{3}{2}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

5. please refer to the attached pnb