$A = a = (1, 3, 4, 5, 7)$

1.(1) $q_1 = \frac{a}{\|a\|}$ $\quad \|a\| = \sqrt{1+9+16+25+49} = 10.$

$q_1 = (\frac{1}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{7}{10}) = (\frac{1}{10}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{7}{10})$.

$\|b\| = \sqrt{36\times2 + 64\times2} = 10\sqrt{2}.$

$B = b - \frac{A^Tb}{A^TA} \cdot A = (-7, 3, 4, -5, 1)$

$q_2 = \frac{B}{\|B\|} = (-\frac{7}{10\sqrt{2}}, \frac{3}{10\sqrt{2}}, \frac{2}{5\sqrt{2}}, -\frac{1}{2\sqrt{2}}, \frac{1}{10\sqrt{2}})$.

(2) project $\vec{c} = (1, 0, 0, 0, 0)$ to space $A = \begin{pmatrix} 1 & -6 \\ 3 & 6 \\ 4 & 8 \\ 5 & 0 \\ 7 & 8 \end{pmatrix}$

$P = A(A^TA)^{-1}A^T.$

$p = Pb.$

the ~~closest~~ closest vector is $\vec{p} = (0.5, -0.18, -0.24, 0.4, 1.11\times10^{-16})$

2. (a) True

Suppose $AB$ is invertible, then $AB \cdot (AB)^{-1} = I$.

$\therefore A^{-1} = B(AB)^{-1}$.

$\therefore A$ is invertible. Contradicted.

(b) False.

$\det P \cdot \det A = \det L \det U.$

$\therefore \det A = \pm(d_1 \cdots d_n)$

(c) False.

Counter example: $A = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$ $\quad B = \begin{pmatrix} 2 & 3 \\ 2 & 2 \end{pmatrix}$.

$\det(A-B) = \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$\det A = -4 \quad \det B = -2. \quad \therefore \det(A-B) \neq \det A - \det B$

(d) False.

$AB$'s determinant may not exist.

If $A$ is $m\times n$, $B$ is $n\times k$, $AB$ is $m\times k$, whose determinant does not exist

3. The determinant of $A$ may not exist.

Like in 1(2).

Actually, we can use $P^2 = P$

$\therefore \det|P|^2 = \det|P| \Rightarrow \det|P| = 0/1/-1.$