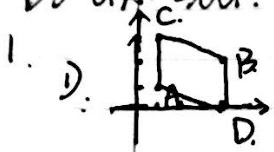
DS-GA 3001. HW1.

1.

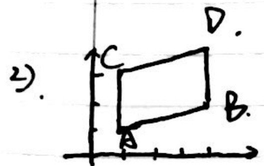
1)



$\vec{AC} = (1,3) - (1,1) = (0,2)$.

$\therefore \vec{DB} = (0,2)$. $\quad \therefore D(4,0)$.

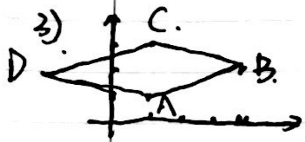
2)



$\vec{CA} = (1,1) - (1,3) = (0,-2)$.

$\therefore \vec{DB} = \vec{CA} = (0,-2)$ $\quad \therefore D(4,4)$.

3)



$\vec{AB} = (4,2) - (1,1) = (3,1)$.

$\therefore \vec{DC} = \vec{AB} = (3,1)$. $\quad \therefore D(-2,2)$.

So the three possible answers are $(4,0)$ $(4,4)$ $(-2,2)$

2. $\vec{v} = (1,3)$. $\quad \|\vec{v}\| = \sqrt{1+9} = \sqrt{10}$. $\quad \therefore \vec{u_1} = (\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}})$.

suppose $\vec{U_1} = (x_1, y_1)$.

$\left.\begin{array}{ll} \|\vec{U_1}\| = 1. & \Rightarrow x_1^2 + y_1^2 = 1 \\ \vec{U_1} \cdot \vec{v} = 0 & \Rightarrow x_1 + 3y_1 = 0. \end{array}\right\} \Rightarrow \begin{array}{l} x_1 = -\frac{3\sqrt{10}}{10},\ y_1 = \frac{\sqrt{10}}{10} \\ x_2 = \frac{3\sqrt{10}}{10},\ y_1 = -\frac{\sqrt{10}}{10} \end{array}$

$\therefore \vec{U_1} = (-\frac{3\sqrt{10}}{10}, \frac{\sqrt{10}}{10})$ or $(\frac{3\sqrt{10}}{10}, -\frac{\sqrt{10}}{10})$

$\vec{w} = (2,1,2)$. $\quad \|\vec{w}\| = \sqrt{4+1+4} = 3$. $\quad \therefore \vec{u_2} = (\frac{2}{3}, \frac{1}{3}, \frac{2}{3})$.

Suppose $\vec{U_2} = (x_2, y_2, z_2)$.

$\begin{cases} x_2^2 + y_2^2 + z_2^2 = 1 \\ 2x_2 + y_2 + 2z_2 = 0. \end{cases}$

one solution: $\vec{U_2} = (\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2})$

3. $(x, y, z) \underset{\text{swap}(1,2)}{\rightarrow} (y, x, z) \underset{\text{swap}(2,3)}{\rightarrow} (y, z, x)$

$P_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

$P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.

$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$.

4. ipython notebook attachment.

5. $\because$ B is the inverse of A^2

$\therefore A^2 \cdot B = I$

$\Leftrightarrow A \cdot AB = I$

$\Leftrightarrow A(AB) = I$

$\Leftrightarrow A = (AB)^{-1}$.