$A = a = (1, 3, 4, 5, 7)$

1\. (1) $q_1 = \frac{a}{\|a\|}$ $\quad \|a\| = \sqrt{1+9+16+25+49} = 10.$

$q_1 = (\frac{1}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{7}{10}) = (\frac{1}{10}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{7}{10})$

$\|b\| = \sqrt{36\times 2 + 64\times 2} = 10\sqrt{2}.$

$B = b - \frac{A^T b}{A^T A} \cdot A = (-7, 3, 4, -5, 1)$ $\quad \|B\| = 10$

$q_2 = \frac{B}{\|B\|} = (-\frac{7}{10[illegible]}, \frac{3}{10[illegible]}, \frac{2}{5[illegible]}, -\frac{1}{2[illegible]}, \frac{1}{10[illegible]}).$

(2) project $\vec{c} = (1, 0, 0, 0, 0)$ to space $A = \begin{pmatrix} 1 & -6 \\ 3 & 6 \\ 4 & 8 \\ 5 & 0 \\ 7 & 8 \end{pmatrix}$

$P = A(A^T A)^{-1} A^T.$

$p = Pb.$

the ~~closest~~ closest vector is $\vec{p} = (0.5, -0.18, -0.24, 0.4, 1.11\times 10^{-16})$

2\. (a) True

Suppose AB is invertible, then $AB \cdot (AB)^{-1} = I.$

$\therefore A^{-1} = B(AB)^{-1}.$

$\therefore A$ is invertible. Contradicted.

(b) False.

$\det P \cdot \det A = \det L \det U.$

$\therefore \det A = \pm (d_1 \cdots d_n)$

(c) False.

Counter example. $A = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$ $\quad B = \begin{pmatrix} 2 & 3 \\ 2 & 2 \end{pmatrix}.$

$\det(A-B) = \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$\det A = -4$ $\quad \det B = -2$. $\quad \therefore \det(A-B) \neq \det A - \det B$

(d) ~~False~~. True: $\det(AB) = \det(A)\det(B) = \det(B)\det(A) = \det(BA)$

~~AB's determinant may not exist.~~

~~If A is m×n, B is n×k. AB is m×k whose determinant does not exist~~

3\. The determinant of A may not exist.

Like in 1 (2).

Actually, we can use $P^2 = P$

$\therefore \det|P|^2 = \det|P| \Rightarrow \det|P| = 0/1/-1.$

4. $\det(A-\lambda I) = \det((A-\lambda I)^T) = \det(A^T - (\lambda I)^T)$

$= \det(A^T - \lambda I)$ $\det(A-\lambda I) = \det(A^T-\lambda I) = 0$

$\therefore$ A and A^T have the same eigenvalues

But the eigvectors may not be the same.

like. $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ $A^T = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$.

the eigenvalue of A is $\lambda_1 = 1$. $\lambda_2 = 3$.

the eigenvectors of A are. $\begin{pmatrix} 1 & 0.70710678 \\ 0 & 0.70710678 \end{pmatrix}$

but the eigenvectors of A^T are. $\begin{pmatrix} 0 & 0.70710678 \\ 1 & -0.70710678 \end{pmatrix}$