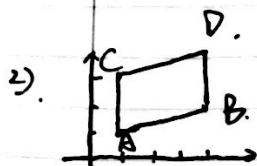


DS-GA 3001. HW1.



$$\vec{AC} = (1,3) - (1,1) = (0,2)$$

$$\therefore \vec{DB} = (0,2) \quad \therefore D(4,0)$$



$$\vec{CA} = (1,1) - (1,3) = (0,-2)$$

$$\therefore \vec{DB} = \vec{CA} = (0,-2) \quad \therefore D(4,4)$$



$$\vec{AB} = (4,1) - (1,1) = (3,0)$$

$$\therefore \vec{DC} = \vec{AB} = (3,0) \quad \therefore D(-2,2)$$

So the three possible answers are $(4,0)$ $(4,4)$ $(-2,2)$

2. $\vec{v} = (1,3)$ $\|\vec{v}\| = \sqrt{1+9} = \sqrt{10}$ $\therefore \vec{u}_1 = (\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}})$

Suppose $\vec{U}_1 = (x_1, y_1)$

$$\|\vec{U}_1\| = 1$$

$$\vec{U}_1 \cdot \vec{v} = 0$$

$$\Rightarrow x_1^2 + y_1^2 = 1$$

$$\Rightarrow x_1 + 3y_1 = 0$$

$$\Rightarrow \begin{cases} x_1 = -\frac{3}{\sqrt{10}}, y_1 = \frac{1}{\sqrt{10}} \\ x_2 = \frac{3}{\sqrt{10}}, y_2 = -\frac{1}{\sqrt{10}} \end{cases}$$

$$\therefore \vec{U}_1 = (-\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}) \text{ or } (\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}})$$

$\vec{w} = (2,1,2)$ $\|\vec{w}\| = \sqrt{4+1+4} = 3$ $\therefore \vec{u}_2 = (\frac{2}{3}, \frac{1}{3}, \frac{2}{3})$

Suppose $\vec{U}_2 = (x_2, y_2, z_2)$

$$\begin{cases} x_2^2 + y_2^2 + z_2^2 = 1 \\ 2x_2 + y_2 + 2z_2 = 0 \end{cases}$$

one solution: $\vec{U}_2 = (\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2})$

3. $(x, y, z) \rightarrow (y, x, z) \rightarrow (y, z, x)$

swap(1,2)

swap(2,3)

$$P_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$



4. ipython notebook attachment.

5. B is the inverse of A^2

$$\therefore A^2 B = I$$

$$\Leftrightarrow A \cdot AB = I$$

$$\Leftrightarrow A(AB) = I$$

$$\Leftrightarrow A = (AB)^{-1}$$

we three possible answers are $(A, 0)$, (A, A) and $(A, -A)$

$$\left(\frac{5}{3}, \frac{1}{3}\right) = \vec{w}$$

$$\|\vec{w}\| = \sqrt{1+1} = \sqrt{2}$$

$$\frac{\partial f}{\partial x} = N, \frac{\partial f}{\partial y} = -1 \Rightarrow X = -\frac{1}{N}$$

$$\begin{cases} 1 = N + X \\ 0 = N + X \end{cases} \Rightarrow \begin{cases} X = 1 - N \\ X = -N \end{cases}$$

$$(N, X) = (1, 0)$$

$$1 = \sqrt{1+0} = 1$$

$$\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = (N, -1) \text{ or } (N, 1)$$

$$\left(\frac{5}{3}, \frac{1}{3}, \frac{5}{3}\right) = \vec{w}$$

$$\|\vec{w}\| = \sqrt{1+1+1} = \sqrt{3}$$

$$(N, X) = (1, 0)$$

$$1 = \sqrt{1+0+0} = 1$$

$$0 = \sqrt{1+0+0} = 1$$

$$\left(\frac{2}{3}, 0, \frac{2}{3}\right) = \vec{w}$$

$$(X, Y, N) \leftarrow (X, Y, N) \leftarrow (X, Y, N)$$

$$(5, 5, 2)$$

$$(5, 5, 2)$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} =$$

