3. Find the number of integers between 1 and 10,000 that are neither perfect squares nor perfect cubes.

```
Answer:

Let S = \{1, 2, -10000\},

AI = \{ai | a \in S \text{ and } a^2 \in S^2\}

Az = \{bi | b \in S \text{ and } b^3 \in S^2\}

because 100^2 = 10000, 10|^2 > 10000 \Rightarrow |AI| = 100

Similarly 2|^3 = 9261 - 22^3 = 10048 > 10000 \Rightarrow |A|^2 = 21

AI \cap Az = \{bi | c \in S, c^{2x} = b^2 \in S^2\}

4b = 4096, 5^6 = 15625 > 1000 \Rightarrow |AI \cap A| = 4

\Rightarrow |AI \cap A| = |S| - |AI| - |A| + |AI \cap A| = 10000 - 100 - 21 + 4 = 9883

Answer: 9883
```

8. Determine the number of solutions of the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 14$ in positive integers x_1, x_2, x_3, x_4 and x_5 not exceeding 5.

Answer:

Let
$$J_1 = X_1 - 1$$
 $\Rightarrow J_1 + J_2 + J_3 + J_4 + J_5 = 9$

Let $S_1 = X_1 - 1$

Let $S_2 = X_1 + J_4 + J_5 = 9$

Let $S_3 = X_1 + J_4 + J_5 = 9$

Let $S_4 = X_1 + J_4 + J_5 = 9$

Let $S_4 = X_1 + J_4 + J_5 = 9$

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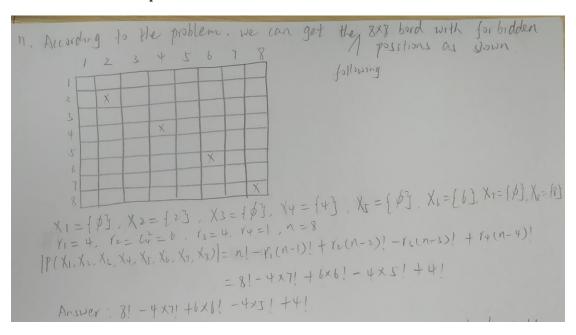
Let $S_4 = X_1 + J_4 + J_5 = 9$

Let $S_4 = X_1 + J_4 + J_5 = 9$

Let $S_4 = X_1 + J_4 + J_5 = 9$

Let $S_4 = X_1 + J_5 = 9$

11. Determine the number of permutations of $\{1, 2, ..., 8\}$ in which no even integer is in its natural position.



25. Count the permutations $i_1i_2i_3i_4i_5i_6$ of $\{1,2,3,4,5,6\}$, where $i_1 \neq 1,5$; $i_3 \neq 2,3,5$; $i_4 \neq 4$; and $i_6 \neq 5,6$.

25. According to the problem, we can get the following bxb	bord with
forbidden positions as shown	
123456	
1 X X X X X X X)
1 = 8 $1 = 6+4+3+3+2+2=20$ $1 = 20$ $1 = 20$	
Answer: 61-8x51+70x41-20x51+7x21	

31. How many circular permutations are there of the multiset

$$\{2 \cdot a, 3 \cdot b, 4 \cdot c, 5 \cdot d\},\$$

where, for each type of letter, all letters of that type do not appear consecutively?

```
Let S is the set that contains all the circular permutations of the multiset \{2:a, 3:b, 4:c, 5:d\};

At its the set that contains all the circular permutations of the multiset \{2:a, 3:b, 4:c, 5:d\}; that "aa" means that two letters of "a" appear consecutively.

Similarly, A2 is \{"bbb", 2:a, 4:c, 5:d\}

A3 is \{"cccc", 2:a, 3:b, 4:c\}

Then, we can get:

|S| = 12 \times \frac{12!}{3!45!} |A|| = \frac{12!}{3!45!} |A| = \frac{11!}{3!45!} |A| = \frac{10!}{2!3!}

|A| = \frac{9!}{2!3!} |A| = \frac{10!}{3!45!} |A| = \frac{10!}{3!45!} |A| = \frac{10!}{3!45!} |A| = \frac{3!}{3!45!} |A| = \frac{3!}{3!45!} |A| = \frac{10!}{3!45!} |A| = \frac{3!}{3!45!} |A| = \frac{3!}{3!45!
```