

4. How many distinct positive divisors does each of the following numbers have?

(a) $3^4 \times 5^2 \times 7^6 \times 11$

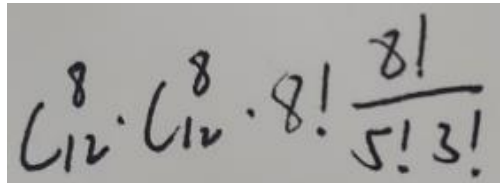
Answer: $5 \times 3 \times 7 \times 2 = 210$

7. In how many ways can four men and eight women be seated at a round table if there are to be two women between consecutive men around the table?

Answer: $7! \times 4! \times 2$

19. We are given eight rooks, five of which are red and three of which are blue.

(b) In how many ways can the eight rooks be placed on a 12-by-12 chessboard so that no two rooks can attack one another?



Handwritten calculation: $C_{12}^8 \cdot C_{12}^8 \cdot 8! \cdot \frac{8!}{5! \cdot 3!}$

Answer:

30. We are to seat five boys, five girls, and one parent in a circular arrangement around a table. In how many ways can this be done if no boy is to sit next to a boy and no girl is to sit next to a girl? What if there are two parents?

Answer:

One parent: $4! \times 5! \times 10$

Two parents: $4! \times 5! \times 10 \times 9$

61. Consider an 9-by-9 board and nine rooks of which five are red and four are blue. Suppose you place the rooks on the board in nonattacking positions at random. What is the probability that the red rooks are in rows 1,3,5,7,9? What is the probability that the red rooks are both in rows 1,2,3,4,5 and in columns 1,2,3,4,5?

Answer:

(1) $9!$

(2) $4!$