1. **Show that if $n+1$ distinct integers are chosen from the set $\{1, 2, \dots, 3n\}$, then there are always two which differ by at most 2.**

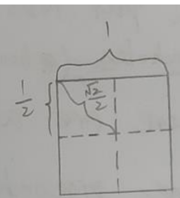
1\. proof:

Assume pigeonholes as: $\{1,2,3\}$, $\{4,5,6\}$, …, $\{3n-2, 3n-1, 3n\}$.

When we take $n+1$ numbers from n pigeonholes, there must be ~~one~~ two numbers be ~~took~~ taken out from the same pigeonhole by the pigeonhole principle and ~~differ 2 at most~~. they differ by at most 2.

2. **Prove that of any five points chosen within a square of side length 1, there are two whose distance apart is at most $\frac{\sqrt{2}}{2}$.**

2\. proof:



As is shown in the picture, we can divide the square ~~of side~~ into four squares of side length 1. When two points are taken from the same small square their distance is at most $\frac{\sqrt{2}}{2}$. According to the pigeonhole principle, there are always two points be taken from the same small square when we take five points.

3. **In a room there are 10 people with integer ages [1, 60]. Prove that we can always find two groups of people (with no common person) the sum of whose ages is the same.** 30

3. proof:

$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}$ represent ten people.

$$\left.\begin{array}{cccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \vdots & & & & & & & & & \vdots \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right\} 2^{10}-1=1023$$

Just like binary number, there are ~~$2^{10}-1=1023$~~ 1023 kinds of age composition. ~~the ten people~~ The sum of 1023 kinds of age composition is integer [10, 600]. We can take [10, 600] as pigeonholes and put 1023 kinds of age composition into 591 pigeonholes. $(600-10+1=591)$. According to the pigeonhole principle, there are always two pigeons be taken into the same pigeonhole, and we can assume the two pigeons ~~from the~~ are

$\{a_{x1}, a_{x2}, \dots, a_{xn}\}, \{a_{y1}, a_{y2}, \dots, a_{ym}\}$

$a_{x1}+a_{x2}+\dots+a_{xn} = a_{y1}, a_{y2}, \dots, a_{ym}$ ①

$x_i, y_j \in \{1, 2, \dots, 10\}$, $i=1,\dots,n$; $j=1,2,\dots,m$

if $x_i \neq y_j$, then $\{a_{x1}, a_{x2}, \dots, a_{xn}\}$, $\{a_{y1}, a_{y2}, \dots, a_{yn}\}$ are two groups that we want get.

if $\exists\, x_i = y_i$, delete x_i and y_j where $x_i = y_j$, then after delete x_i and y_i where $x_i = y_j$, the equation ① still holds, and the rest of the equation are two groups that we want get.