Homework 0

Notes you want the TAs to consider when grading.

Problem 1

1. According to the question:

$$A = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 30 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Because the last number in the last row of matrix A is 0 instead of 1, the condition is not satisfied

$$B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \quad R_B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$R_B R_B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathrm{I}$$

$$R_B^T R_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \quad \det(R_B) = 1$$

$$\mathrm{R}_B R_B^T = R_B^T R_B = I \quad \det(R_B) = 1$$

Therefore,B matrix belongs to the SE(3)

$$C = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad det(R_c) = -1 \neq 1$$

Therefore,C matrix is not belong to the SE(3)

$$D = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad R_D^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_D R_D^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathrm{I}$$

$$R_D^T R_D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \quad det(R_D) = 1$$

$$R_D R_D^T = R_D^T R_D = I \quad \det(R_D) = 1$$

Therefore,D matrix belongs to the SE(3)

2. According to the question:

$$B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad B^{-1} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -5 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$BB^{-1} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -5 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

$$B^{-1}B = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -5 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

$$D = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad D^{-1} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$DD^{-1} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

$$D^{-1}D = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

Problem 2

1.According to the question

$$^1P = {}^1T_2{}^2P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \\ -4 \\ 1 \end{bmatrix}$$

2. According to the question:

$$^0T_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1T_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_2 = {}^0T_1{}^1T_2 \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$Coordinate \quad Transformation \quad Description$:

1. Translate the y-axis by 10 units

2. Translate 3 units on the z-axis and rotate -90 degrees around the x-axis

3. Rotate 180 degrees around the y-axis

4. Translate 3 units along the negative direction of y-axis

5. Translate 10 units along the negative direction of z-axis

6. Rotate -90 degrees around the z-axis

Note:

1. From step 1 to step 5, describe ${}^{0}T_{1}$

2. From step 5 to step 6, describe ${}^{1}T_{2}$

3. From step 1 to step 6, describe ${}^{0}T_{2}$.

Problem 3

1. According to the question

$$ {}^{m}T_{w} = \begin{bmatrix} 0 & 0 & -1 & 4 \\ -1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^{e}T_{m} = \begin{bmatrix} 0 & 0 & 1 & -10 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^{e}T_{w} = \begin{bmatrix} 0 & 1 & 0 & -10 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$For \quad {}^{m}T_{w}$:

Coordinate system transformation: Move four units along the x axis, then move six units along the Y axis. Then we rotate minus 90 degrees about the Z axis, and finally 90 degrees about the X axis

2. Accodrding to the question

$$ {}^{e}T_{m} \; {}^{m}T_{w} = \begin{bmatrix} 0 & 0 & 1 & -10 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 4 \\ -1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -10 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^{e}T_{w} $$

$$ {}^{w}T_{e} = \begin{bmatrix} 0 & -1 & 0 & 6 \\ 1 & 0 & 0 & 10 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{w}T_{e}{}^{-1} = \begin{bmatrix} 0 & 1 & 0 & -10 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^{e}T_{w} $$