

Space Debris: A Challenge or An Opportunity

Team 42963

February 2, 2016

Abstract

In this day and age, space debris has become a rather urgent issue. To address such a problem, especially in the Low Earth Orbit(LEO) where the density of the junk reaches its peak, we develop a comprehensive model to meet various scenarios, in the view of being both technologically and economically practical. To be specific, we accomplish the following model establishing path:

- First, we put forward two systems aimed at debris of different sizes - the *Electrodynamic Tether System*(EDT) for larger ones and the *Laser Targeting System* for smaller ones. Mathematical and physical tools have been used to derive them, followed by a detailed sensitivity analysis to determine parameter influences.
- Second, we build a *time-dependent* model to compute the overall efficiency in various situations. Debris with different physical properties(size, shape and etc.), different orbital radius and inclination have been delicately discussed to calculate the efficiency of the two systems.
- Third, *risks and benefits* are carefully compared, which leads to an idea of *model combination* of these two systems. We use *Poisson Stream* to model risks, and benefits are based on efficiency analyzed before. Then a framework of *four practical strategies* is provided to meet various demands.
- Finally, an *economic model* is shown for commercial concern. *Cost and revenue* are quantified, with the consideration of material, energy expense and reduction on loss of spacecrafts due to debris removal. A simulated case is studied, which takes system lifetime and survivability into account, to determine how proposed four strategies could be set up under specific parameter settings.

In conclusion, we provide an efficient and versatile model to combat various space debris problems and an economical analysis to verify its commercial feasibility.

Contents

1	Introduction	3
2	Our Model	3
2.1	Assumptions and Notations	3
2.2	Model One: Electrodynamic Tether System	5
2.2.1	EDT System and Removal Steps	5
2.2.2	Tether Model and Geomagnetic Field	6
2.2.3	Current and Ampere's Force	8
2.2.4	Time Consumed in Orbital Transfer	8
2.3	Laser Targeting System	9
2.3.1	Laser Targeting System and Removal Steps	9
2.3.2	The Laser-induced Momentum Change	10
2.3.3	The Optical Equipment	11
2.3.4	The Orbital Change	11
2.4	Sensitivity Analysis	13
2.4.1	EDT: Ampere Force with Respect to Tether Length and Power	13
2.4.2	EDT: Transfer Time with Respect to Tether Length and Power	14
2.4.3	Laser: Pulse Energy with Respect to D_{eff}, λ, τ	16
2.4.4	Laser: Required Velocity Change with Respect to Zenith Angle	17
3	Simulations for Various Scenarios: Efficiency Analysis	18
3.1	Time dependent EDT System	18
3.1.1	Basic Model: Mono EDT System	18
3.1.2	Orbital Radius Variety: Poly-Mono, Power	20
3.1.3	Mass Variety: Poly-Mono, Power	20
3.1.4	Orbital Inclination Variety: Poly-Mono, Power	21
3.2	Time Dependent Laser Targeting System	21
3.2.1	Large Debris Size	21
3.2.2	Large Debris Orbit	22
3.2.3	Irregular Shaped Debris	23
4	Model Combination - Risk and Benefit Analysis	24
4.1	Risks of the EDT Model	24
4.1.1	Various Risks	24
4.1.2	Collisions Between Tether and Small Debris: Poisson Stream Model	25
4.2	Benefits of the EDT Model	28
4.3	Risks of Laser Targeting Model	28
4.4	Benefits of Laser Targeting Model	29
4.5	Model Combination: EDT and Laser	29
5	Economic Model - Cost and Revenue	30
5.1	Revenue - Efficiency	30
5.2	Cost - Expense and Risk	33
5.3	Net Profit	34
6	Final Remarks	34
6.1	Strengths and Weaknesses	34
6.2	Future Work	35
6.3	Conclusion	36

1 Introduction

The amount of space debris around the earth is ever increasing, which poses a serious threat on space craft safety. The space debris problem is most serious in the low Earth orbit(LEO), where the density of space debris reaches its peak. The fact that most important space crafts are in the LEO makes the debris removal issue in this area even more urgent. Therefore, in our model, we focus on debris removal in the LEO.

Various methods have been proposed to deal with the problem. Among them, the electrodynamic tether system and the laser targeting system are considered to be most practical and efficient by previous research. In our model, we concentrate on these two models and various kinds of combination of them.

In order for a debris removal method to be accepted by a private firm, it must be commercially feasible. Therefore, the costs, risks and benefits of the models must be taken into consideration. In our model, we discuss the feasibility of different methods from both technological and economical perspectives.

2 Our Model

2.1 Assumptions and Notations

Our whole analysis is based on the following necessary assumptions:

- The debris is non-cooperative, which means it has no direct contact with the ground console and can not end its life automatically;
- The tension inside the tether is ignored if not otherwise specified;
- In EDT System, the target debris is large enough to be detected by the ground console(we mainly consider debris with diameter larger than 10cm which can actually be tracked by the Space Surveillance Network according to NASA);
- The debris original orbit is circular
- The mechanical energy merely consists of kinetic energy and gravitational potential energy;
- The orbit plane inclination does not change notably when EDT system is working if not otherwise specified;
- The service satellite will track next task once it has finished one removal job;
- Coriolis force is not taken into consideration;
- In EDT System, debris with small size is to the disadvantage of capture;
- The motion of the tether along the orbit will not be considered;
- The orientation of the tether is ignored;
- The tether direction can be changed automatically to be perpendicular to the horizontal geomagnetic field

- In laser targeting model, we consider two main types of shapes: the spherical debris and the cylindrical debris. The former represents regularly-shaped debris, and the latter represents flat debris like paint flake.

Basic notations and definitions are listed as follows:

Table 1: EDT Notations

sign	meaning	
B	geomagnetic field of Earth	
B_r	radial direction of the magnetic field	
B_h	horizontal direction of the magnetic field	
β	the angle between the point and the magnetic north pole	deg
r	the altitude of a given point	km
B_e	the magnetic field at magnetic equator of the Earth	
R_e	radius of the Earth	km
e_r	radial direction of a given orbit	
e_v	velocity direction of a given orbit	
e_n	orbital plane direction of a given orbit	
\vec{r}	position vector	
\vec{v}	velocity vector	
\vec{n}	orbital plane vector	
v	velocity	m/s
e_l	tether direction	
I	current	A
σ	angle between e_r and e_l	deg
λ	angle between the orbit plane and the geomagnetic equator plane	deg
f_a	Ampere's force	N
f_r	radial direction of Ampere's force	N
f_v	velocity direction of Ampere's force	N
f_n	orbital plane direction of Ampere's force	N
t	time required in orbital transfer	s
E	mechanical energy of the tether	J
G	gravitational constant	Nm^2/kg^2
M_e	Earth mass	kg
m	service satellite mass	kg
V_{bat}	battery load	V
V_{ind}	induced voltage	V
R_t	tether resistance	Ω
R_p	ambient plasma resistance	Ω
P	power load	W
v_f	relative velocity to the geomagnetic field	m/s
ρ	specific resistance	Ωm
S	cross-section area of tether	m^2
γ	angle between the target debris and the service satellite	deg

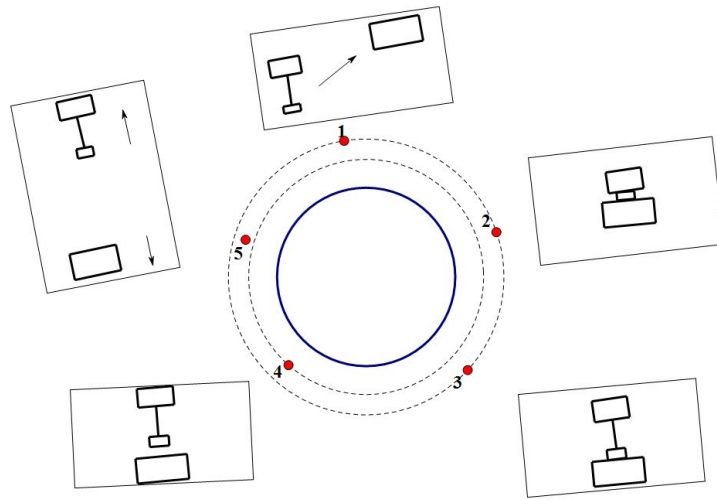
Table 2: Laser Notations

sign	meaning	
I	laser pulse intensity	kWm^{-2}
p	ablation pressure	Pa
C_m	momentum coupling coefficient	NW^{-1}
Φ	laser impulse flux	kJm^{-2}
τ	laser pulse duration	ns
$\Delta\vec{v}$	debris velocity change	ms^{-1}
\vec{k}	unit vector in the direction of the laser beam	
\vec{n}	unit vector normal to the debris surface	
R	debris radius	cm
m	debris mass	kg
H	height of cylindrical-shaped debris	cm
M^2	laser beam quality factor	
λ	laser wavelength	μm
d_s	spot size	cm
z	target distance	km
D_{eff}	illuminated beam diameter	cm
T_{eff}	remaining percentage of laser energy	
W	laser pulse energy	kJ
R_E	radius of the earth	km
M_E	mass of the earth	kg
a	semi-major axis of an ellipse	km
b	semi-minor axis of an ellipse	km
c	focal length of an ellipse	km
G	gravitational constant	Nm^2/kg^2
v_p	debris velocity at perigee	kms^{-1}
v_a	debris velocity at apogee	kms^{-1}
E	mechanical energy of the debris	J
L	angular momentum of the debris	$kgms^{-1}$
r	radius of the debris orbit	km
v_r	radial component of debris velocity	kms^{-1}
v_t	tangential component of debris velocity	kms^{-1}
H_p	minimal perigee height for entering the atmosphere	km
N	required number of periods for removal of a piece of large debris	
t_i	time cost for the i th period	h
t	time cost for removal of a piece of large debris	h

2.2 Model One: Electrodynamic Tether System

2.2.1 EDT System and Removal Steps

We consider a reusable space debris removal system for debris with a size larger than $10cm$ in low Earth orbits (LEO). Electrodynamic tether (EDT) technology is under hot study for its viability to transfer orbits and its relative high efficiency and low cost based on theoretical calculation. An EDT system has a pair of plasma contactors at both ends of the tether, through which the electric



current can flow by processing the circuit via the ambient plasma. The current then interacts with the geomagnetic field of Earth, which induces an Ampere's force to drop the satellite to a lower orbit, carrying the debris. At proper altitude, the debris is released and gradually dragged towards the Earth by the gravity, in which process it could be burned out. Then the EDT System transverse the current, ascends to the previous orbit and make the next debris removal.

The process of debris removal is shown as follows:

1. The satellite changes its orbit to approach the target debris;
2. The satellite retrieves its tether. A propulsion system for rendezvous is utilized for enough thrust;
3. The tether is re-deployed and descends with the target attached to its end;
4. The target is released at a $650km$ altitude circular orbit;
5. The satellite ascends, enabled by a reverse Ampere's force. The free target descends and finally burns.

Figure 2.2.1 provides more intuitive understandings.

2.2.2 Tether Model and Geomagnetic Field

Tether Model In this model, the capture and removal device is referred to as "service satellite". In addition, we call the satellite at the head of the tether the "mother satellite", in contrast, the spacecraft at the tail the "sub satellite" due to its relatively light body. The tether model is shown in figure 1:

Geomagnetic Field The Earth geomagnetic field could be simplified as a magnetic dipole with the axis tilted away from the Earth's spin axis by approximately $\phi = 11.5deg$. The magnetic field vector B can be decomposed along two directions: the component in the radial direction B_r and

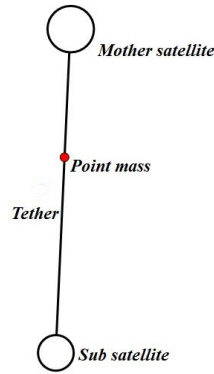
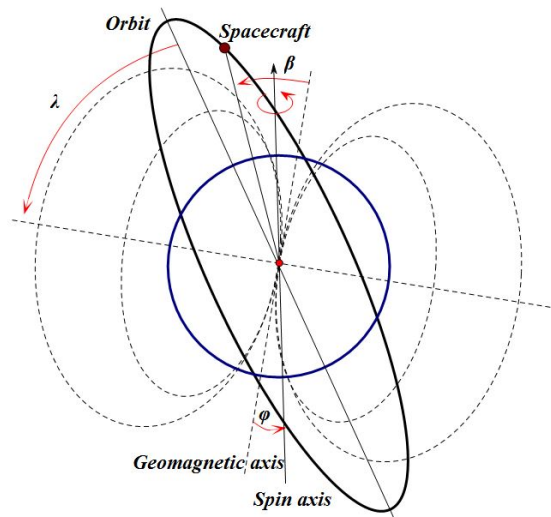


Figure 1: Simple Tether Model



that in the horizontal direction B_h . At any given point, using β to represent the angle between the point and the magnetic north pole, we have the following expressions of B

$$B_r = \frac{B_e R_e^3 \cos \beta}{r^3}, \quad B_h = \frac{B_e R_e^3 \sin \beta}{r^3} \quad (2.1)$$

Since the geomagnetic pole is inconsistent with the Earth rotation axis, we can infer that the geomagnetic axis also rotates around the spin axis, which results in the change in the geomagnetic field distribution. More details can be seen in Figure 2.2.2.

2.2.3 Current and Ampere's Force

We have the following equations:

$$\vec{r} = re_r, \quad \vec{v} = \frac{d\vec{r}}{dt} = ve_v, \quad \vec{n} = r \times v = rve_n, \quad I = Ie_l.$$

By definition, we have $e_l = e_r \cos \sigma + e_v \sin \sigma$. Thus the Ampere's force can be formulated as

$$f_a = LI \times B, \quad (2.2)$$

and

$$\begin{aligned} f_r &= f_l \cdot e_r = BIL \cos \lambda \sin \sigma; \\ f_v &= f_l \cdot e_v = BIL \cos \lambda \cos \sigma; \\ f_n &= f_l \cdot e_n = BIL \sin \lambda \cos u, \end{aligned}$$

where u denotes the argument of latitude.

The voltage consists of the potential drops at the tether, the ambient plasma, anode plasma contactor and cathode plasma contactor. Considering the relative much smaller values of the latter two factors, we merely focus on the former two values. Thus the voltage satisfies the relation

$$|V_{bat} - V_{ind}| = IR_t + IR_p = IR, \quad R = R_t + R_p. \quad (2.3)$$

When $V_{bat} > V_{ind}$, the equation can be written as $V_{bat} - V_{ind} = IR = \frac{P}{I} + B_h Lv_f$, and R can be expressed as $R = \rho \frac{L}{S}$.

Reformulate it, and we can obtain the equation which I satisfies

$$I^2 \rho \frac{L}{S} + B_h ILv_f = P \quad (2.4)$$

Note that $f_a = BIL$, we have the following formula:

$$\rho f_a^2 + SLB_h^2 v_f f_a - PS LB^2 = 0, \quad (2.5)$$

With the given location, we can determine B_h and v_f , and together with given L and P , we can obtain f_a by solving the quadratic equation 2.5.

2.2.4 Time Consumed in Orbital Transfer

In this part, we calculate the time used for a tether to carry one piece of debris to a lower (determined) orbit, specifically, reaching an altitude of $650km$ [8], where the debris will eventually reenter the atmosphere and burn up.

The geomagnetic field is dependent on radius r , so considerable change in altitude will result in the variation of the distribution of magnetic field. To calculate the total transfer time, we divide the whole process into small intervals. Specifically,

$$t = \sum_{i=1}^N t_i, \quad (2.6)$$

where N is an appropriate large integer and in each t_i , the geomagnetic field is considered to be constant, so as the Ampere's force and its velocity direction f_v . The corresponding orbit radius for each interval is $r_0 > r_1 > r_2 > \dots > r_N$, where r_0 is the initial orbit radius of debris, and r_N is the release orbit radius. Now we show how to calculate the small interval t_i .

By definition, the differential time dt can be calculated by

$$dt = \frac{dE}{E'},$$

where dE represents the difference in mechanical energy between the radius r and $r - dr$ and dr denotes the minute change of orbit radius. E' is the derivative of mechanical energy with respect to time, i.e., power. Velocity v can be obtained by:

$$\frac{GM_em}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM_e}{r}}, \quad (2.7)$$

where m is the mass of service satellite

In addition, the mechanical energy can be calculated as the sum of two components: kinetic energy and gravitational potential energy and

$$E_k(r) = \frac{1}{2}mv^2 = \frac{1}{2}\frac{GM_em}{r}, E_p(r) = -\frac{GM_em}{r}.$$

Thus, we can get dE

$$dE = E_k(r - dr) + E_p(r - dr) - (E_k(r) + E_p(r)) = -\frac{GM_emdr}{2r^2}. \quad (2.8)$$

On the other hand, the power can be simply represented as

$$E' = f_v v = f_v \sqrt{\frac{GM_e}{r}}$$

Finally, dt can be derived as follows

$$dt = \frac{dE}{E'} = -\frac{\sqrt{GM_e}mdr}{2f_v r^{\frac{3}{2}}}$$

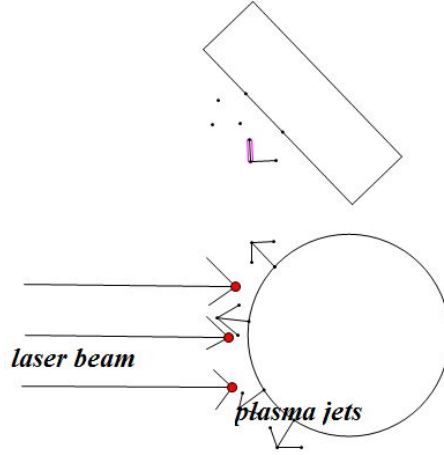
Thus the small interval t_i can be obtained by the following integral:

$$t_i = \int_{r_{i-1}}^{r_i} dt = \frac{m\sqrt{GM_e}}{f_v^{(i)}} \left(\frac{1}{\sqrt{r_i}} - \frac{1}{\sqrt{r_{i-1}}} \right). \quad (2.9)$$

2.3 Laser Targeting System

2.3.1 Laser Targeting System and Removal Steps

To remove space debris of both small size (with diameter between $1cm$ and $10cm$) and large size (with diameter larger than $10cm$), the laser targeting model is a good option. We provide its removal steps as follows:



1. The detector finds the debris and the ground-based laser system targets the debris;
2. The irradiation of the laser on the debris causes plasma jets on the debris and slows down the debris;
3. The orbit of the debris changes due to its momentum change after several laser hits;
4. The debris enters the atmosphere and burns up.

2.3.2 The Laser-induced Momentum Change

When the debris is hit by the laser beam, part of the debris material transforms into plasma regime. The plasma jets give the debris a propulsion, resulting in its momentum change. To measure this energy transformation, we use the momentum coupling coefficient, a concept widely used in laser propulsion. The momentum coupling coefficient C_m is defined as the efficiency of conversion from the laser intensity I to ablation pressure p , that is

$$C_m = \frac{p}{I}. \quad (2.10)$$

According to [11], as laser intensity I increases, C_m rises to maximum and decreases at higher laser intensity. The maximal C_m occurs at the vapor-plasma transition, and the laser pulse flux Φ (laser energy passing through unit area) at this transition is given by the following empirical formula

$$\Phi_{opt} = 4.8 \times 10^8 \sqrt{\tau},$$

where τ is the laser pulse duration. To ensure an efficient use of laser energy, it is natural to require $\Phi = \Phi_{opt}$. The pressure p resulting from laser energy gives a propulsion to change the debris momentum. For different debris sizes and shapes, the resulting debris velocity changes can be different.

The pressure given by plasma jets is perpendicular to the debris surface, which is shown in Figure 2.3.2. Using the theorem of momentum, we can calculate the velocity change $\Delta \vec{v}$ caused by

the laser pulse flux Φ explicitly. We denote by $\vec{k} = (k_1, k_2, k_3)^T$ the unit vector in the direction of the laser beam, and \vec{n} the unit vector normal to the debris surface. We have [5]

$$m\Delta\vec{v} = - \iint C_m \Phi(\vec{k} \cdot \vec{n}) dA \vec{n}.$$

For spherical debris with radius R and mass m ,

$$m\Delta\vec{v} = \frac{2\pi}{3} C_m \Phi R^2 \vec{k},$$

For cylindrical debris with radius R , height H and mass m ,

$$m\Delta\vec{v} = \frac{\pi}{2} C_m \Phi (k_1 RH, k_2 RH, 2k_3 R^2)^T.$$

2.3.3 The Optical Equipment

Since the laser beam travels about $1000km$ to reach the debris, the diffractive spread of the light is significant. To overcome the problem, optical equipment like large mirrors is required [10]. For a laser beam with quality factor M^2 and wavelength λ , the spot size d_s that can be delivered to a target at distance z is given by

$$d_s = \frac{aM^2\lambda z}{D_{eff}},$$

where a is a coefficient equal to $\frac{\pi}{4}$ for a Gaussian beam, or 2.44 for an Airy distribution, and D_{eff} is the illuminated beam diameter inside the aperture D for calculating diffraction. Another important influence that atmosphere has on the laser beam is the transmission loss. We use the coefficient T_{eff} to represent the remaining percentage of laser energy. Thus, the flux Φ generated by a laser pulse with energy W on the earth can be calculated by

$$\Phi = \frac{4WT_{eff}}{\pi d_s^2}. \quad (2.11)$$

Thus, to obtain maximal C_m , the laser pulse energy needed is given by

$$W = \frac{\pi a^2 M^4 \lambda^2 z^2 \Phi_{opt}}{4D_{eff}^2 T_{eff}}. \quad (2.12)$$

2.3.4 The Orbital Change

With the laser-induced momentum change, the debris orbit changes to an ellipse. The new orbit is shown in Figure 2 The semi-major axis, semi-minor axis and focal length of this ellipse is represented by a , b , c . The radius and mass of the earth are represented by R_E and M_E , respectively. The gravitational constant is denoted by G .

For the changed orbit, we suppose that the debris velocity at perigee is v_p , and the debris velocity at apogee is v_a . The mechanical energy and angular momentum of the debris are denoted by E and L , respectively. By the conservation of mechanical energy and angular momentum, we obtain

$$E = \frac{1}{2}mv_p^2 - \frac{GM_E m}{a - c} = \frac{1}{2}mv_a^2 - \frac{GM_E m}{a + c}, \quad L = mv_p(a - c) = mv_a(a + c). \quad (2.13)$$

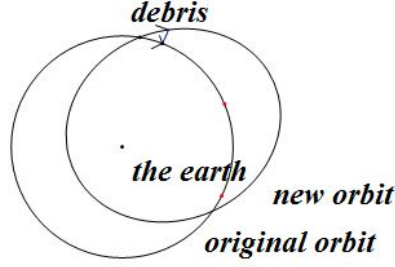


Figure 2:

From the two equations above, we calculate E and L as

$$E = -\frac{GM_E m}{2a}, \quad L = m\sqrt{\frac{GM_E(a^2 - c^2)}{a}}.$$

For debris with initial orbit of radius r , we denote by v_r and v_t the radial component and tangential component of its velocity after laser hit. We have

$$E = \frac{1}{2}m(v_r^2 + v_t^2) - \frac{GM_E m}{r} \quad \text{and} \quad L = mv_t r.$$

From the two equations above, we calculate a and c as

$$a = \frac{GM_E r}{2GM_E - (v_r^2 + v_t^2)r}, \quad c = \sqrt{a^2 - \frac{v_t^2 r^2 a}{GM_E}}. \quad (2.14)$$

In order for the debris to enter the atmosphere and burn up, the perigee height of debris has a minimal height H_p . That is

$$a - c \leq R_E + H_p. \quad (2.15)$$

Therefore,

$$a \geq \frac{GM_E(R_E + H_p)^2}{2(R_E + H_p)GM_E - v_t^2 r^2}, \quad (2.16)$$

i.e.

$$v_r^2 - \left(\frac{r^2}{(R_E + H_p)^2} - 1\right)v_t^2 \geq \frac{2GM_E(R_E + H_p - r)}{(R_E + H_p)r}. \quad (2.17)$$

We denote by ψ the zenith angle. ψ is shown in Figure3.

For illustration, we consider the spherical debris. We have

$$\frac{r}{\sin \psi} = \frac{R_E}{\sin \theta}, \quad v_r = \Delta v \cos \theta, \quad v_t = \sqrt{\frac{GM_E}{r}} - \Delta v \sin \theta. \quad (2.18)$$

Thus, the minimal Δv_0 can be calculated. If we denote by T the average target available time of small debris, the minimal frequency f of the laser beam for single-pass removal is given by

$$f = \frac{\Delta v_0}{\Delta v T}. \quad (2.19)$$

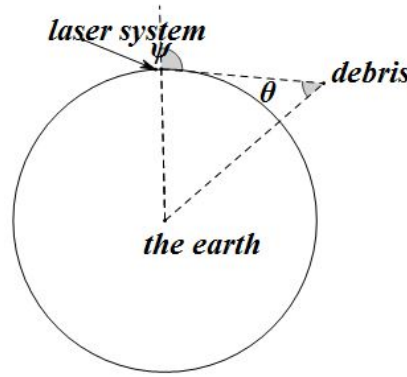


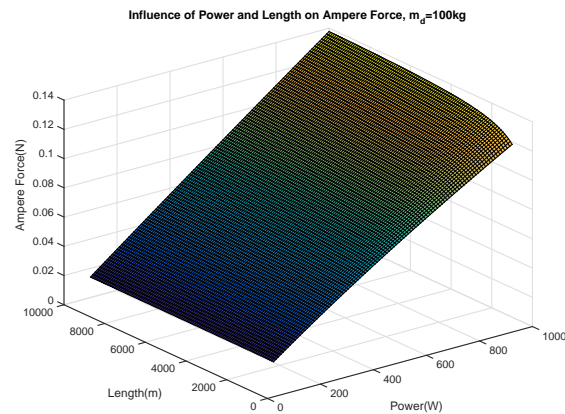
Figure 3:

2.4 Sensitivity Analysis

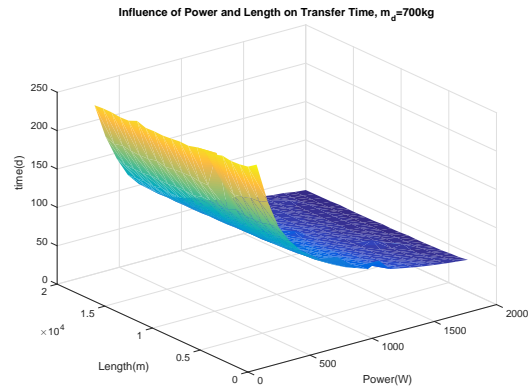
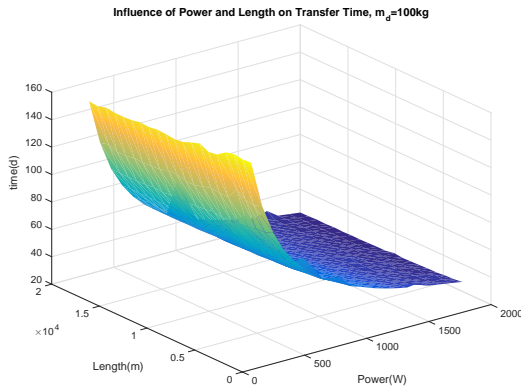
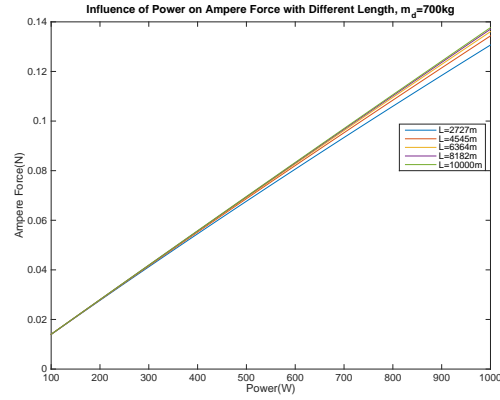
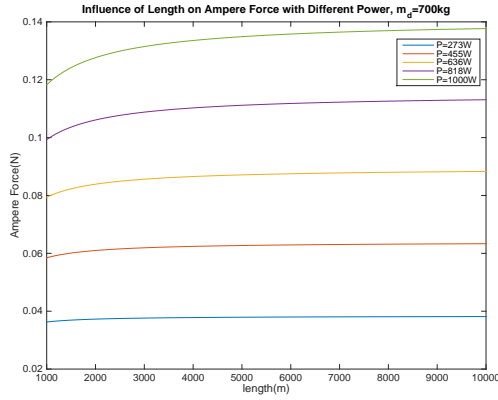
2.4.1 EDT: Ampere Force with Respect to Tether Length and Power

To obtain the most efficient ampere force f_a , we need to determine how tether length L and power P influence the induced force. The relative speed $v_f = v - 2\pi r\omega$, where ω denotes earth's rotation angular velocity, and $v = \sqrt{GM/r}$. Notice that in the LEO setting, $2\pi r\omega$ is small enough compared with the magnitude of v , so we can ignore it and let $v_f \approx v$ for simplicity.

In our experimental setting, we set tether length L to be uniformly distributed in the interval $[3, 10]km$ and power P to be uniformly distributed in the interval $[300, 1000]W$. The numerical results are shown in figure 2.4.1



As it's clear to see from the figure 2.4.1 that ampere force f_a grows with power P . However, the ampere force seems to be insensitive to parameter L . To make an insight into the change of it, we provide the sectional view in figure 2.4.1:



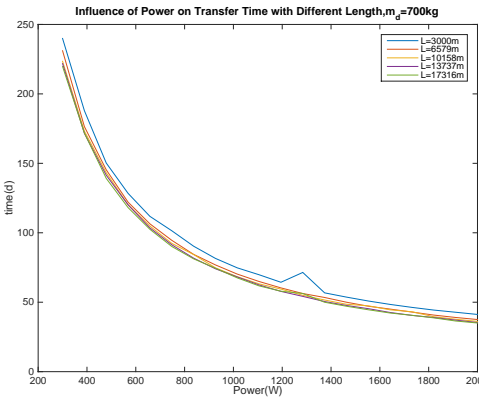
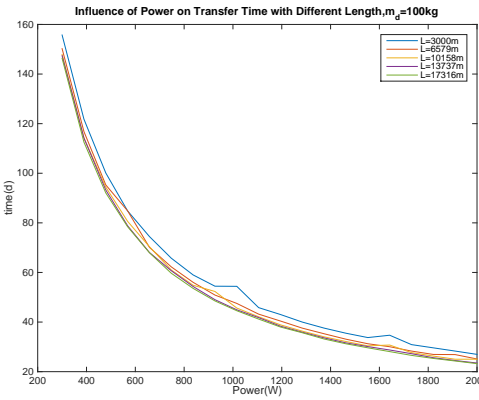
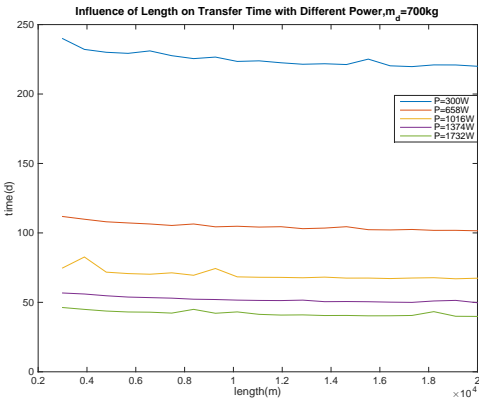
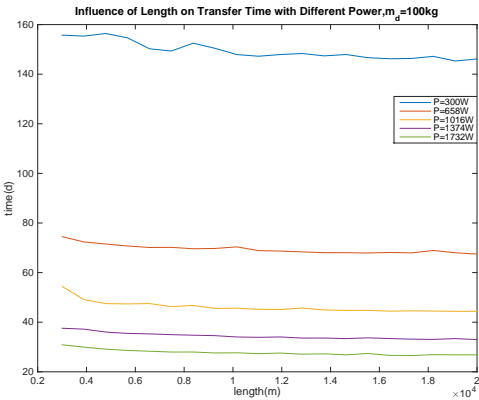
Now we can clearly see that ampere force is not sensitive to L . Besides, the increasing rate of ampere force decreases with power P .

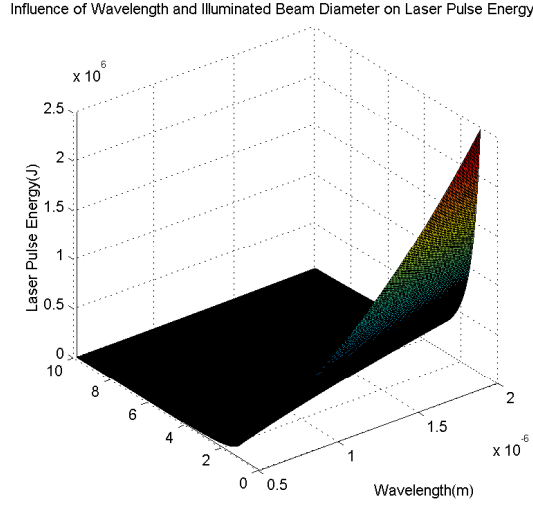
2.4.2 EDT: Transfer Time with Respect to Tether Length and Power

Now we want to verify how tether length and power influence the transfer time. In calculation, we take the angle change into consideration to determine the different magnetic field B_i at different orbit radius r_i . I can be calculated from 2.4, thus we can obtain the force and small interval t_i . Sum up t_i , then total time will be easily worked out.

In the experimental setting, L is chosen to be uniformly distributed in $[1, 10]\text{km}$ and P uniformly distributed in $[100, 1000]\text{W}$. The results are shown as follows in figure 2.4.2, 2.4.2. Similarly, we can see that descending time decreases with power and not sensitive to L , which is reasonable based on the ampere force sensitivity analysis above. To view it more clearly, we provide the sectional views in figure 2.4.2:

It's easy to see that the descending rate of transfer time becomes slow along with power, and





length does not influence notably on transfer time. We have to emphasize that time is not the only consideration when it comes to commercial activity. Larger power and longer tether mean more energy, material consumption and risks, which will definitely lower the company's return.

2.4.3 Laser: Pulse Energy with Respect to D_{eff}, λ, τ

With given optical equipment, the needed laser pulse energy can be calculated by

$$W = \frac{\pi a^2 M^4 \lambda^2 z^2 \Phi_{opt}}{4 D_{eff}^2 T_{eff}}, \quad (2.20)$$

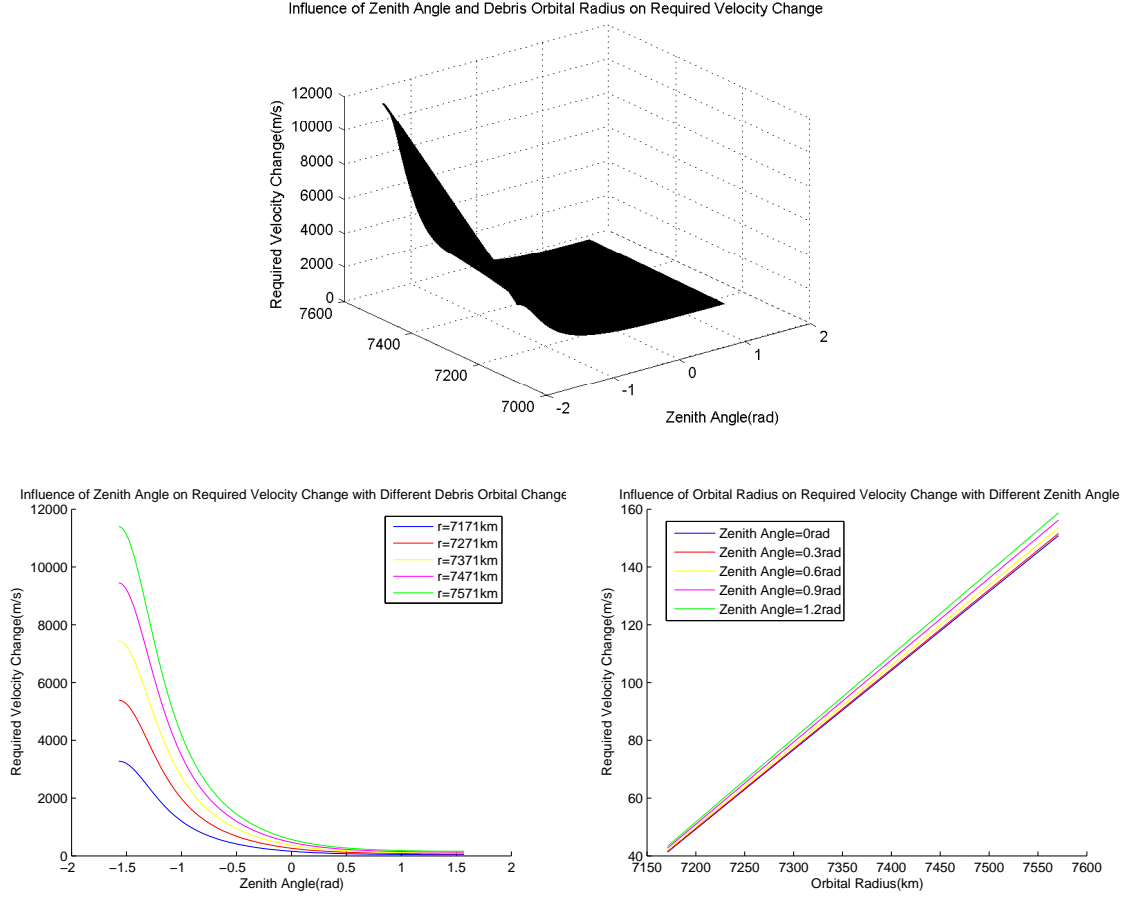
where

$$\Phi_{opt} = 4.8 \times 10^8 \sqrt{\tau}. \quad (2.21)$$

From the formulae above, we observe that W is very sensitive to the optical parameter D_{eff} . If the size of the mirror is enlarged, the required laser pulse energy can be much smaller. Also, the energy efficiency can be improved, for the spot size d_s is smaller, leading to less laser overspill around small debris. Thus, the energy efficiency of the laser targeting system is sensitive to optical equipment parameter D_{eff} .

We also see that W is very sensitive to laser parameters. With a smaller λ and τ , the laser impulse requirement can be lowered greatly. Therefore, the energy cost of the laser targeting system is very sensitive to laser parameters like wavelength and laser impulse duration time.

To illustrate, for $a = 1.7, M^2 = 2.0, z = 1000km, \tau = 5ns, T_{eff} = 0.5$, we provide Figure 2.4.3 showing the influence of λ and D_{eff} on W .



2.4.4 Laser: Required Velocity Change with Respect to Zenith Angle

Now we discuss the energy cost sensitivity with regard to zenith angle ψ . It can be deduced from the model that Δv_0 satisfies the equation

$$\left[1 - \frac{R_E^2 \sin^2 \psi}{(R_E + H_p)^2}\right] \Delta v_0^2 + 2 \left[\frac{r^2}{(R_E + H_p)^2} - 1\right] \sqrt{\frac{GM_E}{r}} \frac{R_E}{r} \sin \psi \Delta v_0 + \frac{GM_E}{r} \left[-\frac{r^2}{(R_E + H_p)^2} + \frac{2r}{R_E + H_p} - 1\right] = 0. \quad (2.22)$$

To test the sensitivity of Δv_0 with regard to the zenith angle ψ , we set $H_p = 650 \text{ km}$, $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$, $R_E = 6371 \text{ km}$ and $M_E = 5.977 \times 10^{24} \text{ kg}$. We further set ψ to be uniformly distributed in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and r to be uniformly distributed in the interval $[7171, 7571] \text{ km}$. The numerical results are shown in figure 2.4.4 and figure 2.4.4.

It can be clearly seen from figure 2.4.4 that the required velocity change Δv_0 decreases as zenith angle ψ increases. From the sectional view in figure 2.4.4, we see that Δv_0 is very sensitive to ψ for fixed r . Therefore, to improve energy efficiency, choosing a larger zenith angle is a good approach.

3 Simulations for Various Scenarios: Efficiency Analysis

3.1 Time dependent EDT System

First, we simulate an LEO environment. Various debris objects are put into the orbit with different sizes and motion states. We still use time to be a factor related to efficiency. Specifically, we will simulate the removal process to determine the number of debris removed during a given time period.

Next, it is obvious that time should be much more delicately handled instead of merely considering the orbital transfer time, as what we have done. The whole process of removing one piece of debris can be separated into four noticeable parts - tracking, capturing, orbit-descending and orbit-ascending.

Orbital transfer time for descending has been discussed before, and the ascending time is almost the same. We will merely consider the differences in mass which includes EDT system mass and debris mass in the descending and EDT system mass only in the ascending.

Tracking time denotes the time for EDT to reach the new target after releasing the last debris. We design the EDT tracks its next target on the lower orbit, i.e., the release orbit. After releasing the debris, the EDT decides its next target on the basis of their size (recall that we have assumed size is proportional to mass) and chases it in the lower orbit. Then the EDT conducts the ascending and reaches its new target. We simply calculate an approximate time for EDT to track debris. Assuming that the debris orbit is at $r_1 = 1000\text{km}$ altitude and the release orbit is at $r_2 = 650\text{km}$ altitude, the maximum angle needed to be caught up with is π . Then the tracking time is:

$$T = \pi / (\omega_{release} - \omega_{debris}) = \pi / (\sqrt{\frac{GM}{r_2^3}} - \sqrt{\frac{GM}{r_1^3}}) \quad (3.1)$$

Recall that in the descending time analysis in the last section, we calculate out that the descending time can be hundreds of days. Thus, we can just ignore this time for tracking.

Capturing time can be far more complicated due to various situations in reality. For instance, the turbulence of the target, such as rotation, will dramatically increase the difficulty to capture it, which results in longer capture time. We use rotation to represent its turbulence. We want to highlight that when the debris is in some extreme revolving condition, the capturer should give up this target in reality and goes for another debris. However, we ignore this possibility in this part and will discuss it in details in the model combination section. Now we estimate the capturing time. Nishida points out in [7] that brush can be used to slow down the rotation rate of the target debris by 10% for 7s. Therefore, this time can also be ignored considering the time consumed for orbital transfer.

In conclusion, the EDT removal algorithm can be extracted as follows

Algorithm 1: Time-Dependent EDT Removal Process

1. Capture the target debris and lower its orbit;
 2. Decide the next target and chase it in the release orbit;
 3. Ascend to the original orbit and back to 1.
-

3.1.1 Basic Model: Mono EDT System

We simulate a case to study the basic model: mono EDT system with $L = 5\text{km}$, $P = 500\text{W}$. The debris orbit altitude is 1000km and we release it at the altitude 650km . Geomagnetic axis intersects

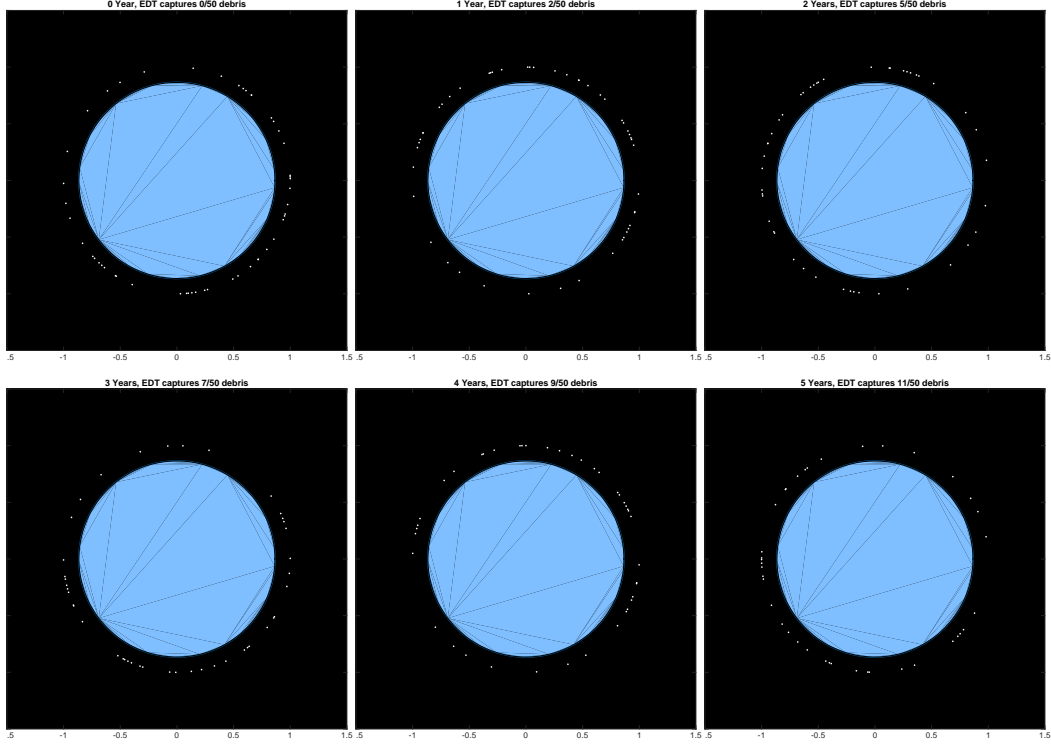


Figure 4: Mono EDT System

the orbit phase with angle $\beta = \pi/3$. The mono EDT system starts at the location with the angle β_n between it and geomagnetic axis to be β . A whole removal time T :

$$T = T_d + T_a, \quad (3.2)$$

where T_d, T_a respectively denotes descent transfer time and ascending transfer time.

We randomly generate 50 debris objects with specified mass and location. The results are shown in figure4

It's clear to see from the figure4 that as time goes by, EDT system can effectively remove debris in the orbit. Such model can be easily applied to poly EDT systems, which we will discuss in the following scenarios. We will only consider the situation when each EDT system is provided with the same power. It's worth noticing that adding only one satellite will tremendously increase the material and technological cost in spite of the same energy cost. In order to determine how many capturers should be launched to the space, we have to develop a new model to quantify the cost, which will be worked out later.

Next, we will discuss different situations where power and the number of EDT systems will be analyzed in various orbital radius, mass and inclination. Tether length will not be taken into consideration since we have known its minor influence from sensitivity analysis shown before.

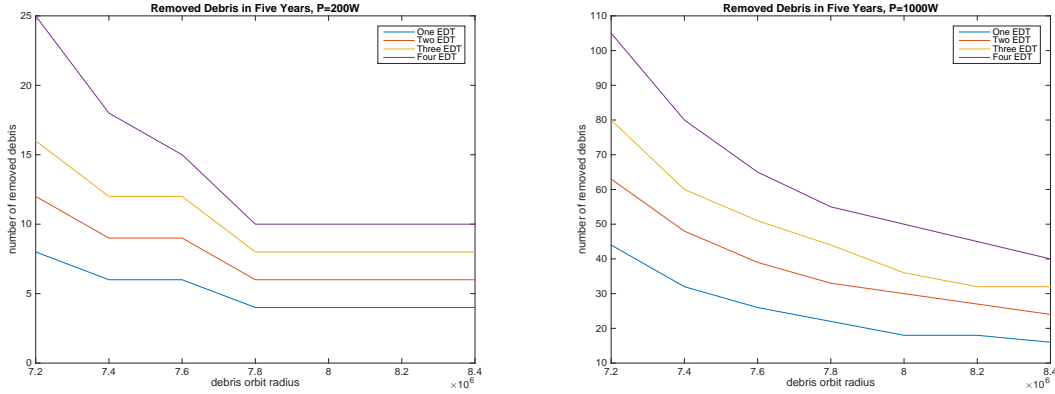


Figure 5:

Case	Heavy				Medium				Light			
EDT#	1	2	3	4	1	2	3	4	1	2	3	4
200W	6	9	12	15	8	12	16	20	10	15	18	25
400W	12	18	24	30	14	21	28	35	18	27	36	45
600W	18	27	36	45	22	33	44	55	28	42	52	65
800W	24	36	48	60	30	45	56	74	36	54	71	90
1000W	30	45	60	75	36	54	72	90	44	66	88	110

3.1.2 Orbital Radius Variety: Poly-Mono, Power

In the simulation, we generate 100 debris objects with mass uniformly distributed in $[500, 1500]kg$. The orbital altitude changes from $800km$ to $2000km$ with inclination $\pi/3$. We test it under power $200W$ and $1000W$ for each EDT. The EDT number is set to be $\{1, 2, 3, 4\}$. We calculate the number of removed debris in five years.

It's clear to know from the figure5 that under low power, orbital radius increasement will not noticeably influence the effect due to the long time for removal. Analyze the radio of different EDT Systems with given radius and power, we can conclude that at low altitude and under low power, adding EDT systems can be most effective.

3.1.3 Mass Variety: Poly-Mono, Power

In the simulation, we generate 100 debris objects at altitude $1000km$ and inclination $\pi/3$. We set three kinds of debris for each case: heavy(mass $1000 \sim 1250kg$), medium(mass $500 \sim 750kg$) and light(mass $0 \sim 250kg$) . Power for each EDT changes from $200W$ to $1000W$. The EDT number is set to be $\{1, 2, 3, 4\}$. We calculate the number of removed debris in five years in table3.1.3.

Calculate the ratio of removed debris objects among $\{1, 2, 3, 4\}$ EDT systems, it's easy to find that increasing the EDT systems can be most effective when dealing with heavy mass debris. The power does not notably influence the effect of EDT systems adding for each case.

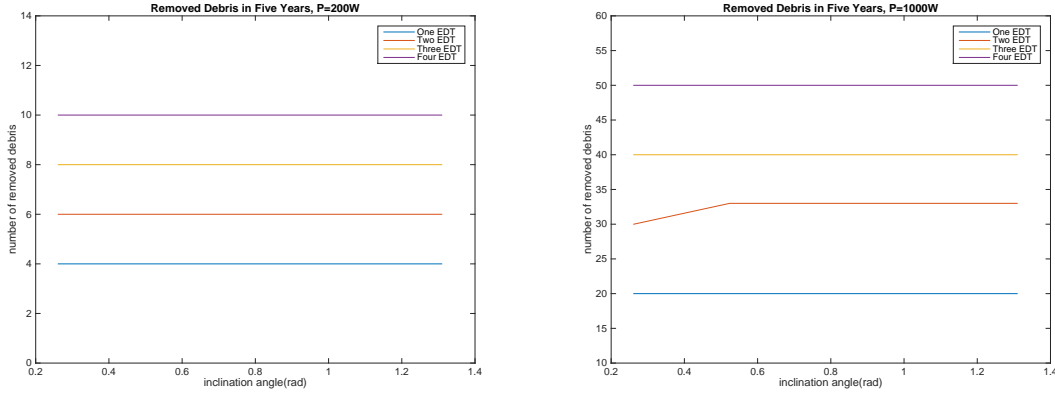


Figure 6:

3.1.4 Orbital Inclination Variety: Poly-Mono, Power

In the simulation, we generate 100 debris objects at altitude 1000km with mass uniformly distributed in $[500, 1500]$ kg. The inclination changes from $\pi/12$ to $5\pi/12$. The EDT number is set to be $\{1, 2, 3, 4\}$. We calculate the number of removed debris in five years.

Clearly, Figure 6 shows that inclination does not influence much on the removal efficiency. In our model, the tether direction can be changed automatically to be perpendicular to the horizontal geomagnetic field. Besides, the orbital transfer time is much longer than the orbital cycle time. Thus, such insensitivity is reasonable.

3.2 Time Dependent Laser Targeting System

3.2.1 Large Debris Size

Large debris (with diameter larger than 10cm) will make the required laser frequency impractical. Therefore, the debris has to go around the earth for many periods to slow down enough. Both the energy cost and the time cost are significant. We perform a simulation for quantification.

We assume that the debris is a spherical one, and denote by N the required number of periods for the debris to enter the atmosphere, i.e.

$$N = \lceil \frac{\Delta v_0}{fT\Delta v} \rceil, \quad (3.3)$$

where $\lceil u \rceil$ denotes the minimal integer larger than u . Thus, the energy cost to remove the debris is

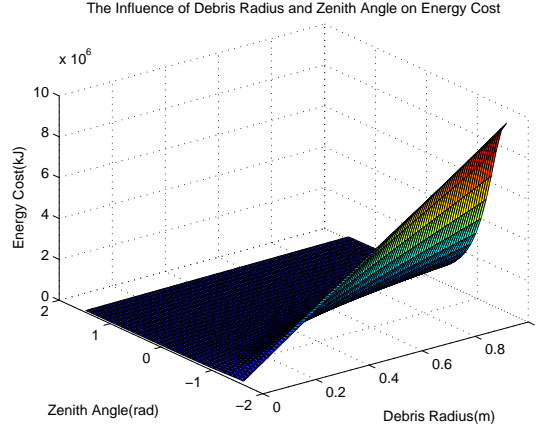
$$E = NW, \quad (3.4)$$

where W is the laser pulse energy.

For the i th period ($1 \leq i \leq N$), the semi-major axis and semi-minor axis of the orbit is given by

$$a = \frac{GM_E r}{GM_E - i^2 f^2 T^2 \Delta v^2 r + 2i f T \Delta v \sin \theta \sqrt{GM_E r}}, b = \sqrt{ar} - i T f \Delta v \sin \theta r \sqrt{\frac{a}{GM_E}}. \quad (3.5)$$

parameter	value	unit
τ	5	ns
λ	1.06	μm
a	1.7	
M^2	2.0	
D_{eff}	10	m
W	7.3	kJ
f	12.5	Hz



Here, θ is determined by

$$\frac{r}{\sin \psi} = \frac{R_E}{\sin \theta}.$$

Thus, the time cost t_i for the i th period is

$$t_i = \frac{2\pi ab}{(\sqrt{GM_E r} - ifTr\Delta v \sin \theta)}$$

The total time cost t for removing the debris is estimated by

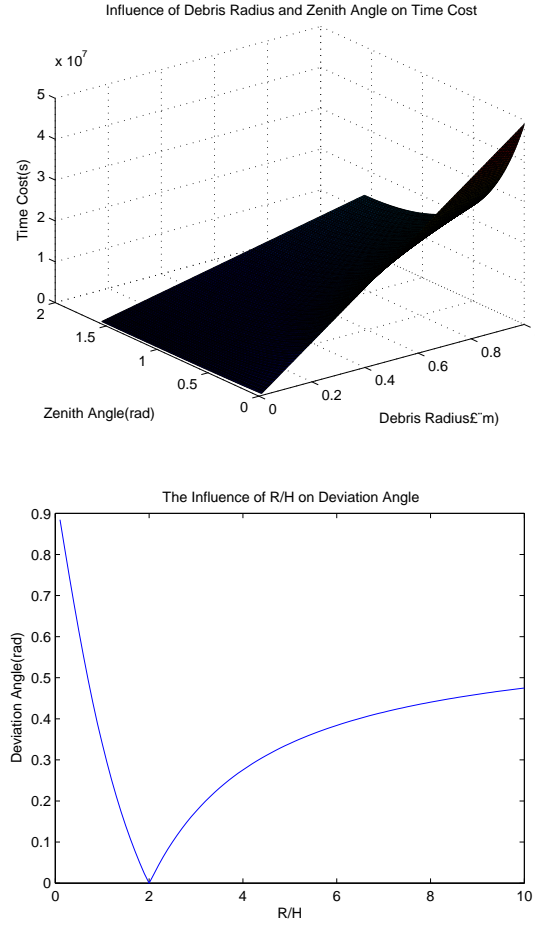
$$t = \sum_{i=1}^N t_i.$$

In our simulation, we assume that the laser system parameters are given in Table 3.2.1.

From the simulation result in Figure 3.2.1 and Figure 3.2.1, we conclude that when debris diameter is very large, the energy cost and time cost is quite significant, posing this system at a disadvantage in practical.

3.2.2 Large Debris Orbit

It can clearly be seen from figure 2.4.4 that the required velocity change Δv_0 increases with larger orbital radius r . From the sectional view in figure 2.4.4, we see that Δv_0 grows nearly linearly with the increase of r .



Therefore, when the debris radius is very large, the required velocity change can be huge, leading to large energy costs for the laser system.

3.2.3 Irregular Shaped Debris

Many pieces of debris results from collision, which leads to their irregular shape. For such debris, the debris velocity change may not be parallel to the laser beam. To deal with it, we have to use the formula

$$m\Delta\vec{v} = - \iint C_m \Phi(\vec{k} \cdot \vec{n}) dA \vec{n},$$

where $\vec{k} = (k_1, k_2, k_3)^T$ is the unit vector in the direction of the laser beam, and \vec{n} is the unit vector normal to the debris surface.

For illustration, we simulate the deviation angle γ between \vec{k} and $\Delta\vec{v}$ for a cylindrical debris with different radius-height ratio $\frac{R}{H}$ when $\vec{k} = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

From the simulation results shown in Figure 3.2.3, we see that γ can be large when $\frac{R}{H}$ is far from 2. For debris like paint flakes, the ratio $\frac{R}{H}$ is so large that the deviation can be significant. Therefore, for some irregularly-shaped debris, the laser system may not work efficiently to remove it. In practice, considering the probability of the appearance of certain kinds of irregularly-shaped debris and adjust the laser targeting system parameters is a reasonable solution.

4 Model Combination - Risk and Benefit Analysis

4.1 Risks of the EDT Model

We first list out additional notations needed in this analysis in table3 :

sign	meaning	
P_s	Survival probability of the tether	
P_c	Probability of the tether to be severed or cut	
d_t	Diameter of the tether	m
d_f	Fatal debris diameter	m
d_d	Diameter of the debris	
$\frac{d_c}{2}$	Critical distance within which the debris passes leading to severing	m
N_f	Number of fatal impacts on a single line	
R_f	Fatal impact rate	
μ	Flux of particles	
S_i	Probability of Status i	
A_e	Effective cross-sectional area of the tether wire	m^2

Table 3: Notations for Risks Analysis of EDT Model

The tether model can be rather complicated in terms of the stability and survivability. In this section we will carefully take account of risks about the EDT system.

4.1.1 Various Risks

There are a variety of factors remained to be considered in practice.

Firstly, EDT's long and thin shape leads to high probability of going wrong, especially at its tether part. The causes includes manufacturing defects, system mal-functions, material degradation, vibrations and contact with other spacecraft elements.

However, generally this risk can be controlled by quality checks and production supervision. Through active efforts this accidental risk can be manipulated to a rather low level.

Secondly, in the space there are risks for collision, which is the main threat on EDT. There are 4 types of collision:

1. The remover collides with the target when capturing it.
2. The removers (tethers) collide with each other.
3. The remover collides with some large spacecrafts moving in a high velocity.
4. The remover collides with rather small pieces of debris.

Tether-Target Debris Collisions While manipulating the tumbled debris, EDT is at great risk due to the violent motion of the target. We have shown the idea of lowering the rotation rate of the debris by a brush on a robot arm. Chances are that the rotation rate was severely underestimated so that the remover would try to capture some extremely turbulent debris. There is high probability of breaking the brush or the robot arm, and of the capturer being driven to rotate by the target, going out of control. However, advanced technologies can reduce such probability through accurate manufacturing and precise monitor and control.

Tether-Tether Collisions Once more than one EDT system are launched to the space, the risk of tether colliding with each other exists. Taking account of the high-risk characteristics of the tether, the collision between themselves will lead to more harm. Numerous studies have been conducted to find the probability. [9] Dutch Delta-Utec Space Research and Consultancy company concluded that if more than four EDT systems work in the same orbit, such probability can not be ignored.

Tether-Spacecraft Collisions In addition to debris, other operating satellites positioned on the orbit can also collide with EDT satellite, which causes most criticism on the tether method. Studies show that if more than ten EDT systems work on the same orbit, this concern could be critical [9]. However, analysis should be done to compare the benefits from the removal with the risks EDT itself poses to the functional satellites.

Tether-Small Debris Collisions Large quantities of small debris harms the EDT in various ways, such as increasing the material degradation speed, misleading the function or even cutting the tether. Due to difficulty of detection, small debris poses greatest risk on EDT system.

In conclusion, our main task to calculate risks of EDT model is to derive specific expressions to quantify the risks from small debris.

4.1.2 Collisions Between Tether and Small Debris: Poisson Stream Model

In this section we will calculate the survivability probability P_s of the tether, which implies the risk between tether and small debris. We notice that one single tether may be easily severed but two or more strands of the tether can be less vulnerable.

Probability Model of Single Line Tether The flux of debris with regard to diameter has been estimated [9] and we will use this statistics directly. The denser the debris flux is, the higher the probability to be severed. Besides, the larger effective cross-sectional area brings about larger P_c . First, we should determine the effective cross-sectional area.

Figure7 below shows the effective cross-sectional area.

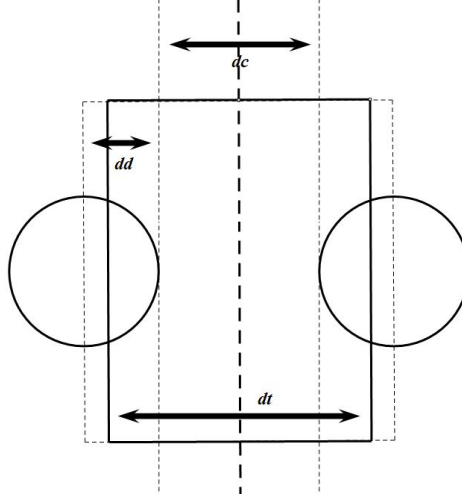


Figure 7:

We have

$$A_e = L(d_c + d_d),$$

The debris with diameters shorter than d_f will not cut the tether. For simplicity, we assume that $d_f = 0.2d_t$, $d_c = 0.7d_t$.

Then we calculate the fatal impacts to be expected. NASAs ORDEM2000 model, coupled with the Grn meteoroid model simulated the flux of debris particles μ . Then the fatal impacts rate will be $A_e\mu$. Both μ and A_e is the function of d_d (debris diameter). Let $d\mu$ denotes the differential flux of particles with respect to d_d , we have

$$R_f = \int_{d_f}^{\infty} A_e(d_d)d\mu(d_d), \quad (4.1)$$

Within a given time interval Δt , for instance, the tether orbit lifetime, we want to calculate the probability that the tether is hit by 0 fatal particle, i.e., the survivability $P_s(\Delta t)$. This model is highly similar to the intuitive perspective of Poisson distribution, with the intensity of $R_f\Delta t$. Thus we have

$$P_s(\Delta t) = e^{-R_f\Delta t}, \quad (4.2)$$

The probability that two or more particles collide with the tether can be ignored, so

$$P_c(\Delta t) = 1 - P_s(\Delta t) = 1 - e^{-R_f\Delta t}, \quad (4.3)$$

L and d_t are parameters whose sensitivity should be analysed. μ can be obtained from the space debris flux model, whose result is shown as a figure in Page 577 of [9]. In order to simulate, we will try to find the most suitable function to fit the curve. Thus the approximate expression of $d\mu$ is determined. In this way, we can obtain $P_c(\Delta t)$, which implies the risk of being severed in terms of the single line tether.

Probability Model of Double Strand Tether To be simplified, we assume that there are only two knots at both ends of the tether, and the severing probability of the knot is ignored.

There are three statuses in space for double strand tether loop:

- Neither of the wires is severed.
- Only one of the wires is severed.
- Both wires are severed.

The tether system survives in the first two situations.

We adopt a stochastic process to illustrate it. S_i denotes the probability of the i_{th} status ($i = 1, 2, 3$) and R_{fij} represents the transition rate from “Status i ” to “Status j ”. More detailed, S_i is the function of t . From 4.2, $S_i(t)$ can be written as

$$S_i(t) = S_i(t_0)e^{-R_{fi}(t-t_0)},$$

And the equivalent differential equation is

$$\frac{dS_i(t)}{dt} = -R_{fi}S_i(t),$$

where $S_i(t_0)$ is the probability of Status i at a given time t_0 and the transition rate

$$R_{fi} = \sum_j R_{fij},$$

Thus specifically

$$\frac{dS_2(t)}{dt} = R_{f02}S_0(t) + R_{f12}S_1(t) - R_{f2}S_2(t) = R_{f02}S_0(t) + R_{f12}S_1(t), \quad (4.4)$$

$$\frac{dS_1(t)}{dt} = R_{f01}S_0(t) + R_{f21}S_2(t) - R_{f1}S_1(t) = R_{f01}S_0(t) - R_{f12}S_1(t), \quad (4.5)$$

$$\frac{dS_0(t)}{dt} = R_{f10}S_1(t) + R_{f20}S_2(t) - R_{f0}S_0(t) = -(R_{f01} + R_{f02})S_0(t), \quad (4.6)$$

In addition,

$$\sum_{i=0}^2 S_i = 1,$$

Our goal is to calculate the survival probability $P_s(t)$

$$P_s(t) = S_0(t) + S_1(t), \quad (4.7)$$

By computing 4.6 and 4.5,

$$S_0(t) = S_0(t_0)e^{-(R_{f01}+R_{f02})t}, \quad (4.8)$$

$$S_1(t) = e^{-R_{f12}t} \left\{ e^{R_{f12}t_0} S_1(t_0) + \frac{R_{f01}S_0(t_0)}{R_{f01} + R_{f02} - R_{f12}} (e^{-(R_{f01}+R_{f02}-R_{f12})t_0} - e^{-(R_{f01}+R_{f02}-R_{f12})t}) \right\} \quad (4.9)$$

,

Then we need to find the explicit solution of the transition rate.

To simplify it, we assume that the distance between two wires of the system is large enough so that one impact is not able to cut two lines at the same time. Thus

$$R_{f02} = 0, R_{f01} = 2R_{f12}, R_{f12} = R_f, \quad (4.10)$$

Then 4.8 and 4.9 can be simplified as

$$\begin{aligned} S_0(t) &= S_0(t_0)e^{-2R_ft}, \\ S_1(t) &= e^{-R_ft}\{e^{R_ft_0}S_1(t_0) + \frac{2R_fS_0(t_0)}{R_f}(e^{-R_ft_0} - e^{-R_ft})\}, \\ P_s(t) &= S_0(t_0)e^{-2R_ft} + e^{-R_ft}\{e^{R_ft_0}S_1(t_0) + 2S_0(t_0)(e^{-R_ft_0} - e^{-R_ft})\}, \end{aligned} \quad (4.11)$$

Where R_f is calculated from 4.1.

The lifetime of the tether system can be approximately calculated as the time when P_s is rather low.

In simulation, we notice that the parameter L , d_t , and t will influence the probability. Specifically, the survivability of the tether with a shorter length and a larger diameter will definitely be enhanced within a shorter period of time. To give an example, we use the typical tether model to simulate the risk, i.e. $L = 5000m$, $d_t = 0.01m$. Assume $t_0 = 0$ and all survivability is equal to 1 at the beginning time. Specifically, we choose one year as the regular time interval. The result will be closely related to our quantization of net profit.

By calculation, we present our results. R_f is around 0.6, thus the survivability within one year will be about 0.79. After three years, P_s has dropped to 0.3, as a result we determine the tether lifetime to be three years.

4.2 Benefits of the EDT Model

In this section we list the benefits of EDT system. Since we have discussed it in detail in the simulation part, we only make a summary on it.

- EDT system is beneficial for reducing the debris orbit lifetime from tens to thousands of years to a few weeks to a few yeas;
- Energy consumed is extremely smaller compared to chemical thrusters. EDT system merely requires an energy mass allocation which is a rather small percent of the total mass, typically, 1 – 5%, while a conventional chemical thruster needs much more (10 – 20%). [9]
- The design of a robot arm with a brush can effectively reduce the rotation rate of the debris, which tremendously increases the success probability of capturing the target.

4.3 Risks of Laser Targeting Model

The ground-based laser system is less likely to be risked than the EDT system. However, delicate spacecrafts with diameter less than 10cm can be mistaken for small debris. After the irradiation, the spacecraft orbit is greatly influenced and will enter the atmosphere, which poses a threat to the spacecraft safety.

4.4 Benefits of Laser Targeting Model

Efficiency The laser targeting system is very efficiency when dealing with small debris. We use the parameters in Table 3.2.1 to illustrate it. The number of small debris pieces is estimated to be $N_1 = 1.9 * 10^5$. We assume the target-laser distance $z = 1000km$, for the density of small debris reaches its peak around the height.

For simplicity, we assume that small debris pieces are uniformly distributed around the earth and it takes the laser system exactly 100s to remove a piece of small debris. Since $\frac{1.9*10^5}{24*3600} = 2.199$ debris pieces passes the system every second, we estimate the time cost t_1 for removing all the small debris by

$$t_1 = N_1 T = 0.602 year$$

Though considering the time for detecting and the generation of small debris from large debris collision, the real removal time can be larger than t_1 , it is sure that all small debris in that orbit can be removed within years by using a laser system.

Energy Consumed When dealing with small debris, the energy cost is not high. The laser energy E for removal of small debris is estimated by

$$E = W f T N_1 = 1.734 * 10^{12} J,$$

which is acceptable considering its return.

4.5 Model Combination: EDT and Laser

We now give four strategies, followed by detailed analysis:

Strategy One Merely employ EDT system to remove large debris.

Strategy Two Merely employ laser targeting system to remove small and large debris

Strategy Three Model Combination:

1. Employ ground-based laser targeting system to remove enough small debris;
2. *Choice A*: Launch EDT system to remove large debris while ground-based laser system is inactive
Choice B: Launch EDT system to remove large debris while ground-based laser system keeps active;

Strategy Four EDT and laser are combined to one system and launched to space.

For simplicity, we categorize the space debris into two groups, the large debris ($> 10cm$) and the small debris ($< 10cm$). EDT system is advantageous for the former, while laser for the latter.

Small debris

Efficiency EDT system is not suitable to remove small pieces of debris for the difficulty to track and precisely capture them, especially ones smaller than $1cm$. However, small debris can be easily eliminated by the laser targeting system for minutes.

Risks Tether faces the risk of being severed by small particles.

Time(d)	EDT 262	laser 1351
Energy Consumed(J)	$1.13e + 10$	$1.75e + 12$

Table 4: Comparison of efficiency and energy between EDT and laser

Large debris

We now give a specific case to show advantages of EDT on efficiency and energy consuming. Suppose a $1000kg$ object at altitude $1500km$ with inclination $\pi/3$, we need to release it at altitude $200km$. The power for EDT is $500W$ and tether length is $5km$. We show the calculated results in table 4. It's clear that EDT model is at great advantage when dealing with large debris.

To determine whether the combination will lead to more profits and to find out the detailed strategy in terms of the combination, we have to develop an economic model which measures the cost and revenue of each strategy.

5 Economic Model - Cost and Revenue

To make the economical analysis for a private commercial company, we first list out notations used in this part in tabel5. As for a private firm, its highlight and only concern is the net profit NP . If $NP > 0$ there is an economically attractive opportunity, otherwise there isn't. So in this section our major task is to quantify NP and maximize it, by which the best working strategy is determined.

Basic Economic Model In economics, net profit is the difference between revenue RV and cost CO ,

$$NP = RV - CO,$$

We will estimate the net profit, which is quantified as money.

5.1 Revenue - Efficiency

We mainly calculate revenue from the efficiency of a working system discussed above. Our idea is that the one who pays the company is the one who benefits from its removal job. As we know, the debris in the space poses great threat to the operating satellites. High efficiency, that is, to remove debris in one year, leads to high revenue.

Since the removal work can be classified as public goods (noncompetitive and non-exclusion), free ride behavior couldn't be eliminated. Thus we cannot expect one country to pay, but some worldwide organizations to take this responsibility. The gross benefits are calculated, which the organizations should pay.

Specifically, we give some steps to compute the benefits the removal mission will bring about.

1. Calculate the loss of each operating spacecraft with regard to the debris flux (the function of diameter), the size, weight, orbit position and value of the spacecraft.
2. Calculate the efficiency of each system or their combinations, i.e., the reduction of debris flux density. By 1 we can obtain the decrease of loss of a typical spacecraft.
3. Find statistics about the number of new spacecrafts required to be launched to the mission orbit per year globally. The gross benefits thus can be determined.

sign	meaning	
NP	Net profit	\$
RV	Revenue	\$
CO	Cost	\$
R_{fsps}	Impact rate of small debris to the spacecraft	
R_{fspm}	Impact rate of medium-sized debris to the spacecraft	
R_{fspl}	Impact rate of large debris to the spacecraft	
A_{esp}	Effective cross sectional area of the spacecraft	m^2
R_{ag}	Aging rate of the spacecraft	
κ	Aging coefficient	
T_{ds}	Designed lifetime for a spacecraft	<i>year</i>
T_{sp}	Real lifetime for a spacecraft	<i>year</i>
c_{sp}	Manufacturing cost of a spacecraft	\$
L_S	Loss due to the impacts of debris	\$
N	Number of debris	
C_{en1}	Energy cost of the EDT system	
C_{ma1}	Material cost of the EDT system	
C_{edt}	Total cost of the EDT system	
C_{en2}	Energy cost of the laser targeting system	
C_{ma2}	Material cost of the laser targeting system	
C_{laser}	Total cost of the laser targeting system	
C_{en3}	Energy cost of the combined model	
C_{ma3}	Material cost of the combined model	
C_{com}	Total cost of the combined model	
τ	Unit energy cost	

Table 5: Notations for Economical Analysis

Simplified Model

For simplicity, we ignore the influence of orbit position on flux of debris and the shape of all concerned spacecrafts. In our model, the average value of a satellite or a spacecraft c_{sp} is estimated as \$300,000,000. We want to calculate the probability of colliding with debris smaller than 1cm, between 1cm and 10cm and larger than 10cm respectively. Let $d_{c1} = 0.01m$, $d_{c2} = 0.1m$

The probability model used in risk analysis of EDT model can be easily applied to calculate impact rate of the small debris(< 1cm) to the spacecraft $R_{f_{sp}}$:

$$R_{f_{sp}} = \int_0^{d_{c1}} A_{esp} d\mu(d_d),$$

Similarly, the impact rate of medium-sized and large debris is

$$R_{f_{spm}} = \int_{d_{c1}}^{d_{c2}} A_{esp} d\mu(d_d), R_{f_{spl}} = \int_{d_{c2}}^{\infty} A_{esp} d\mu(d_d),$$

We can assume an aging rate R_{ag} , which is proportional to $R_{f_{sp}}$, i.e.,

$$R_{ag} = \kappa R_{f_{sp}},$$

where κ is different with size of debris.

Let T_{sp} denote the real lifetime of the spacecraft,

$$T_{sp} = (1 - R_{ag})T_{ds},$$

The loss per year can be expressed as

$$L_s = \frac{R_{ag} c_{sp}}{T_{sp}}, \quad (5.1)$$

In the simulation, we set $A_{esp} = 50m^2$ and $T_{ds} = 10years$.

Simulation

A report in 1992 [11] said that the number of large debris is $NL = 7000$, medium-sized is $NM = 17500$ and small is $NS = 3500000$. NASA gives the statistics of large debris - the number is 20000, 500000, and 10000000 respectively.

In the EDT model, the LEO space is from 200km to 2000km with the volume V .

$$V = \frac{4}{3}\pi(8400^3 - 6600^3),$$

The orbit space is calculated as the space with the radius covered by tether, i.e., 1000m. We calculate the orbit volume V_o .

$$V_o = \frac{4}{3}\pi(7401^3 - 7400^3)$$

Thus the number of debris N_o on the orbit will be

$$N_o = N * \frac{V_o}{V}, \quad (5.2)$$

By calculation, $NL = 100$, $NM = 2500$ and $NS = 50000$.

mother satellite	600	kg	tether diameter	1	cm
sub satellite	400	kg	tether length	5	km

Table 6: EDT System Parameters

From the efficiency part, a tether can remove one piece of large debris within about 3 months. Thus the EDT system will remove about 4 pieces of debris per year, i.e., the change of the large debris flux density is about 4%.

By calculation the initial R_{fspl} is about 10^{-5} . If the density is decreased by 4 percent, the order of R_{fspl} will be about 10^{-6} . While κ is really large, for instance, 1000, due to the destructive consequence of large debris impacts. Finally the reduction of loss per year is calculated as

$$\kappa(10^{-5} - 10^{-6}) * c_{sp}/10 = 2700000$$

Assume that the number of satellites and spacecrafts or any other device launched to space is 300 and the number on the orbital radius is about 200, the total revenue will be

$$2700000 * 200 = 54,000,000.$$

Similarly, the impacts of debris of other sizes can be calculated.

5.2 Cost - Expense and Risk

Expense

We have found some statistics about the expense of our models. We will analyse the cost of the 4 strategies listed in the model combination part.

Strategy One As to a typical tether model, the parameters are shown in table6:

The tether is usually made of aluminum, thus the material cost C_{ma1} is assumed to be \$100,000,000. The energy cost C_{en1} is $C_{en1} = P * t$. When t is one year, the annual energy cost can be determined. We only specify the power to give an example.

The energy consumed when de-orbiting once is calculated as $1.3 \times 10^{10} J$. Thus the annual energy cost is $6.2 \times 10^{10} J$. The expense of unit energy is $\tau = 5\$/kw \cdot h$. Taking into account the expense when launching, designing, as well as staff salaries in the manufacturing process, the total cost of the tether system C_{edt} is estimated to be

$$100000000 + 3000 * 0.5 \approx 100000000,$$

Strategy Two The cost of laser targeting system can be similarly obtained. We assume $C_{ma2} = 120000000$. And the energy consumed is calculated as $5 \times 10^{11} J$.

Strategy Three In the combination model, the material cost could be simply added up to be the total cost C_{ma3} .

$$C_{ma3} = C_{ma1} + C_{ma2}, \quad (5.3)$$

In choice A the combination might have the least effect on efficiency, for they almost don't interact with each other. Thus energy could be added as well.

$$C_{en3} = C_{en1} + C_{en2} \quad (5.4)$$

However, when carefully considered, the usage of laser may be a parameter which affects the energy cost. Through analysis, theoretically we can determine the most suitable period of time of the laser system using. At the specific time point we switch our method to the EDT system, when the risk of the tether is rather low.

In choice B, the energy consumption may be larger, while the removal efficiency will be enlarged. More analyses are needed for trade-off.

Strategy Four The material cost can never be the simple addition of two parts. Further, more materials to attach the two parts are required, as well as finer technology. Thus

$$C_{ma3} = C_{ma1} + C_{ma2} + C_{add}, \quad (5.5)$$

The energy cost is relevant to the utilization of the laser, which is the same consideration in choice B in Strategy Three.

In practice, equipped with more statistics and more time, we can determine which strategy is the best and even give the more detailed method with regard to the laser usage.

5.3 Net Profit

In this section we calculate the net profit. The lifetime of the tether is calculated as 3 years. The survivability within one year is around 0.8. Use the probability model, the return can be expressed as

$$NP = (1-0.8)(RV - C_{ma} - C_{en}) + 0.8(1-0.8)(2RV - C_{ma} - 2C_{en}) + (1-0.2-0.16)(3RV - C_{ma} - 3C_{en}), \quad (5.6)$$

Thus, we determine the net profit for each strategy:

Strategy One We can easily obtain that $NP = \$31,760,000$.

Strategy Two Laser is not common for removing large debris. So intuitively this may not be an economic solution.

Strategy Three Simply calculate C_{en} by direct addition and RV could be up to twice larger because of the combination. Then $NP = \$43,520,000$.

Strategy Four Simply calculate C_{en} by direct addition, $C_{add} = 0.2C_{ma}$ and RV could be 2.5 or more times larger because of the coordination. By calculation, the result is $NP = \$65,400,000$

Through rough comparison, we can see that the combination model 4 may be the most favorable. This is an economically attractive strategy. Annually the return of the company is about $65400000/3 = \$21,800,000$ on average.

6 Final Remarks

6.1 Strengths and Weaknesses

In this paper, we propose EDT model and laser targeting model to remove debris in LEO. Furthermore,

Strengths

- We propose two effective model EDT and laser with different emphasises. The former merely targets at large debris and the latter mainly targets at small debris;
- Different types of debris and orbits can be easily considered;
- Risks are handled delicately for each model and benefits are carefully considered from efficiency analysis;
- Economical analysis has been technically conducted to quantify costs and revenue;
- Strategies of model alternatives and alternative combinations have been put up to handle different situations, satisfying a commercial demand.

Weaknesses

- Only the most urgent environment LEO is considered, some other important environments such as GEO also need delicate design;
- We mainly focus on the circular debris orbit, while in reality elliptical orbit also needs great attention;
- The various physical properties of debris, such as shape, rotation rate and density remain to be handled more delicately;
- We have limited access to realistic commercial prices, which leads to certain deviation in economical analysis.

6.2 Future Work

As we have shown before, there exist many simplifying assumptions in our model. More delicate considerations remain to be done. Our model can be developed by the following ways:

- Apply our model to GEO space and elliptical orbit;
- Shape of debris can be discussed in more details;
- Tension within tether remains to be considered;
- Usages of laser and model combination can be explored further;
- More researches should be done to find information on risks by space debris on spacecrafts, which can lead to a more exact revenue analysis;
- More statistics need to be searched to make a more reasonable economical analysis.

6.3 Conclusion

In this paper, we propose two models to address the space debris problem. The EDT model functions effectively mainly on large debris(diameter $> 10cm$), while laser targeting model works best on small debris(diameter $< 10cm$). We carefully discuss the parameter sensitivity analysis and a lot of “What If” scenarios to determine the efficiency of each model, based on which we obtain the benefits analysis of these two models. Then probability model has been derived to study the risks. Combine all above, we propose four strategies including alternatives and alternative combinations in order to meet different demands. Finally, we give an economical model to quantify the costs and revenue. We further simulate a case to study which strategy put up before is more profitable under specific setting.

References

- [1] Claudio Bombardelli and Jesus Pelaez. Ion beam shepherd for contactless space debris removal. *Journal of Guidance Control & Dynamics*, 34(3):916–920, 2011.
- [2] L. T. Deluca, F. Bernelli, F. Maggi, P. Tadini, C. Pardini, L. Anselmo, M. Grassi, D. Pavarin, A. Francesconi, and F. Branz. Active space debris removal by a hybrid propulsion module. *Acta Astronautica*, 91(10):20–33, 2013.
- [3] Davide Farnocchia, Giacomo Tommei, Andrea Milani, and Alessandro Rossi. Innovative methods of correlation and orbit determination for space debris. *Celestial Mechanics & Dynamical Astronomy*, 107(1-2):169–185, 2010.
- [4] Satomi Kawamoto, Takeshi Makida, Fumiki Sasaki, Yasushi Okawa, and Shin Ichiro Nishida. Precise numerical simulations of electrodynamic tethers for an active debris removal system. *Acta Astronautica*, 59(1):139C148, 2006.
- [5] Duane A. Liedahl, Stephen B. Libby, and Alexander Rubenchik. Momentum transfer by laser ablation of irregularly shaped space debris. In *International Symposium on High Power Laser Ablation*, pages 772–779, 2010.
- [6] Jonathan Missel and Daniele Mortari. Removing space debris through sequential captures and ejections. *Journal of Guidance Control & Dynamics*, 36(3):743–752, 2013.
- [7] Shin Ichiro Nishida and Satomi Kawamoto. Strategy for capturing of a tumbling space debris. *Acta Astronautica*, 68(s 1C2):113–120, 2011.
- [8] Shin Ichiro Nishida, Satomi Kawamoto, Yasushi Okawa, Fuyuto Terui, and Shoji Kitamura. Space debris removal system using a small satellite. *Acta Astronautica*, 65(s 1C2):95–102, 2009.
- [9] Carmen Pardini, Toshiya Hanada, and Paula H. Krisko. Benefits and risks of using electrodynamic tethers to de-orbit spacecraft. *Acta Astronautica*, 64(5-6):571–588, 2009.
- [10] Claude R. Phipps, Kevin L. Baker, Stephen B. Libby, Duane A. Liedahl, Scot S. Olivier, Lyn D. Pleasance, Alexander Rubenchik, James E. Trebes, E. Victor George, and Bogdan Marcovici. Removing orbital debris with lasers. *Advances in Space Research*, 49(9):1283–1300, 2012.
- [11] Lawrence D. Roberts. Addressing the problem of orbital space debris: Combining international regulatory and liability regimes. *B.c.intl & Comp.l.rev*, (1), 1992.
- [12] Wolfgang O. Schall. Removal of small space debris with orbiting lasers. *High-Power Laser Ablation*, pages 564–574, 1998.
- [13] Wolfgang O. Schall. Laser radiation for cleaning space debris from lower earth orbits. *Journal of Spacecraft & Rockets*, 39(39):81–91, 2002.
- [14] G. Tommei, A. Milani, and A. Rossi. Orbit determination of space debris: admissible regions. *Celestial Mechanics & Dynamical Astronomy*, 97(4):289–304, 2007.

Executive Summary

To cope with the serious space debris problem, we have modelled various options and alternatives. In our attempt, we put forth two feasible methods and various combinations of them. We not only analyze the technological efficiency of each option, but emphasize on practical concerns (risks, benefits and costs) as well. We finally come to an option that is considered the most attractive opportunity.

Two promising systems form the basis of our model. The first one is the *electrodynamic tether system* (EDT system), and the second one is the *laser targeting system*. The electrodynamic tether system is a system consisting of a satellite and a tether, which removes a target by capturing it and forcing it to descend. The laser targeting system uses laser to target the debris, forcing it to slow down and burn up in the atmosphere. Several combinations of the two models have been explored to meet various scenarios. The first combination is to employ ground-based laser system to remove small debris, and then launch EDT system to remove large debris while ground-based laser system is inactive. The second combination also uses ground-based laser system to remove small debris, then launch EDT system to remove large debris, but while the EDT system is working, the ground-based laser system keeps active. The third combination is that EDT and laser are combined to one system and launched to space.

As the EDT system is launched into space, its risks are relatively higher, which include the risk of tether going wrong, and the risk of collision. The former one, which possibly results from manufacturing defects, system mal-functions, material degradation, vibrations and contacting with other spacecraft elements, can be controlled by quality checks and production supervision. The latter one serves as the main threat on EDT. There are four main types of collision: the remover may collides with a target, another removal, some large spacecrafts moving in a high velocity or rather small pieces of debris. Among these risks, tether-small debris collision is the most serious due to difficulty of detection of small debris. To reduce this risk, a double strand tether which has a higher probability of survival is advocated.

Although the EDT system has a relatively high risk, its benefits are obvious. The EDT system is efficient in that the debris orbit lifetime can be reduced greatly with the help of it. It is also admired for its low energy consumption compared to chemical thrusters. Also, some special design like a robot arm with a brush makes its efficiency even higher.

Compared to the EDT system, the ground-based laser targeting system has a lower risk, which mainly comes from the concern that delicate spacecrafts may be mistaken for debris. The benefit of the laser targeting system mainly lies in the fact that it can handle both small and large pieces of debris. The EDT system is not suitable to remove small pieces of debris due to the difficulty of precise tracking and

capturing. The laser targeting system, on the contrary, can eliminate small pieces of debris within minutes with low energy consumption. However, as the debris size grows larger, the time cost and energy consumption for laser targeting system increase significantly. Considering this, it is wise to use the EDT system to remove large pieces of debris.

Based on the modelling results of the two original options above, we consider combination models mentioned in the second paragraph to get the best option. To decide on the most economically attractive model to choose, we use economical model to analyze costs and revenue of each option. Then we can decide the net profit, which is quantified as money. If the net value is positive, the option is considered as economically attractive. In this way, different options can be compared according to their net profit.

When we are modelling the revenue of each option, our idea is that the one who pays the company is the one who benefits from its removal job. As the removal work can be classified as public goods, free ride behavior cannot be eliminated. Therefore, we assume that worldwide organizations will take the responsibility. The revenue of the firm can be estimated by the efficiency of the systems, that is, a higher efficiency of a system will lead to a higher revenue. With some simulations, we come to the estimated revenue of different options.

As to the expenses, each options are analyzed specifically. For the EDT system, its costs mainly come from the material cost and energy cost, where material cost is the major cost. For the laser targeting system, the costs can be similarly calculated. For the first combination model, since the two systems almost have no interactions, the expense is simply the addition of that of the two systems. Therefore, the expense is relatively higher. For the second combination model, the energy consumption will be larger than the first one, while the removal efficiency will be enlarged, increasing the revenue. Thus, comparison between the first and the second model needs careful trade-off analysis. For the third combination model, an extra expense is added to the sum of the two systems. This is based on the consideration that more materials and finer technology are needed to combine the EDT system and the laser system and then send them to space.

We simulate a case with specified parameter settings. Under careful calculation and estimation of the revenue and cost of each option, we come to the conclusion that the net profit of the third combination model is the largest. Therefore, in similar situations, we recommend the firm to take this model, that is, combine EDT and laser to one system and send them to space. Other cases can be easily handled by the framework we give in the report.