7.4 Which of the following are correct?

a. False $\models$ True. **Correct**

b. True $\models$ False. **Incorrect**

c. $(A \wedge B) \models (A \Leftrightarrow B)$. **Correct**

d. $A \Leftrightarrow B \models A \vee B$. **Incorrect**

e. $A \Leftrightarrow B \models \neg A \vee B$. **correct**

f. $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$. **Correct**

g. $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$. **Correct**

h. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$. **Correct**

i. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$. **Incorrect**

j. $(A \vee B) \wedge \neg (A \Rightarrow B)$ is satisfiable. **Correct**

k. $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable. **Correct**

l. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C. **Correct**

7.14 According to some political pundits, a person who is radical (R) is electable (E) if he/she is conservative (C), but otherwise is not electable.

a. Which of the following are correct representations of this assertion?
(ii) is right

(i) $(R \wedge E) \Leftarrow\Rightarrow C$

(ii) $R \Rightarrow (E \Leftarrow\Rightarrow C)$

(iii) $R \Rightarrow ((C \Rightarrow E) \vee \neg E)$

b. Which of the sentences in (a) can be expressed in Horn form?

(i),(ii),(iii) all can be expressed in Horn Form.

7.18 Consider the following sentence:

$[(\text{Food} \Rightarrow \text{Party}) \vee (\text{Drinks} \Rightarrow \text{Party})] \Rightarrow [(\text{Food} \wedge \text{Drinks}) \Rightarrow \text{Party}]$.

a. Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable.]

Valid: From the table below. I know that all models satisfy this sentence.

Food	Drink	Party	Food $\wedge$ Drinks	Food $\Rightarrow$ Party	Drink $\Rightarrow$ Party	Left	Right
True	**True**	**True**	**True**	**True**	**True**	**True**	**True**
True	**False**	**True**	**False**	**True**	**True**	**True**	**True**
False	**True**	**True**	**False**	**True**	**True**	**True**	**True**
False	**False**	**True**	**False**	**True**	**True**	**True**	**True**
True	**True**	**False**	**True**	**False**	**False**	**False**	**False**
True	**False**	**False**	**False**	**False**	**True**	**True**	**True**
False	**True**	**False**	**False**	**True**	**False**	**True**	**True**
False	**False**	**False**	**False**	**True**	**True**	**True**	**True**

b. Convert the left-hand and right-hand sides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a).
Left =

$(\neg \boldsymbol{Food} \cup \boldsymbol{Party}) \cup (\neg \boldsymbol{Drink} \cup \boldsymbol{Party}) = \neg \boldsymbol{Food} \cup \neg \boldsymbol{Drink} \cup \boldsymbol{Party}$

Right =

$\neg(\boldsymbol{Food} \cap \boldsymbol{Drink}) \cup \boldsymbol{Party} = (\neg \boldsymbol{Food}) \cup (\neg \boldsymbol{Drink}) \cup \boldsymbol{Party}$

c. Prove your answer to (a) using resolution.

[(Food ⇒ Party) ∨ (Drink ⇒ Party)] ⇒ [(Food ∧ Drink) ⇒ Party]
To prove the negative is empty:

¬[[(Food ⇒ Party) ∨ (Drink⇒ Party)] ⇒ [(Food ∧ Drink) ⇒ Party]]

$= \neg(\neg(\neg \boldsymbol{Food} \cup \boldsymbol{Party}) \cup (\neg \boldsymbol{Drink} \cup \boldsymbol{Party})) \cup (\neg \boldsymbol{Food} \cup \neg \boldsymbol{Drink} \cup \boldsymbol{Party}))$

$= \neg(\neg(\neg \boldsymbol{Food} \cup \neg \boldsymbol{Drink} \cup \boldsymbol{Party}) \cup (\neg \boldsymbol{Food} \cup \neg \boldsymbol{Drink} \cup \boldsymbol{Party}))$

$= \neg((\boldsymbol{Food} \cap \boldsymbol{Drink} \cap \neg \boldsymbol{Party}) \cup (\neg \boldsymbol{Food} \cup \neg \boldsymbol{Drink} \cup \boldsymbol{Party}))$

$= \neg(\boldsymbol{Food} \cap \boldsymbol{Drink} \cap \neg \boldsymbol{Party}) \cap \neg(\neg \boldsymbol{Food} \cup \neg \boldsymbol{Drink} \cup \boldsymbol{Party}))$

$= (\neg \boldsymbol{Food} \cup \neg \boldsymbol{Drink} \cup \boldsymbol{Party}) \cap (\boldsymbol{Food} \cap \boldsymbol{Drink} \cap \neg \boldsymbol{Party}))$

$= \emptyset$

The negative sentence is empty, so the Original sentence is Valid

8.9(Extra) This exercise uses the function MapColor and predicates In(x, y), Borders(x, y), and Country(x), whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3) is syntactically valid but does not express the meaning of the English sentence.
a. Paris and Marseilles are both in France.

(i) In(Paris ∧ Marseilles, France). **2) Use ∧ in side the term.**

(ii) In(Paris, France) ∧ In(Marseilles, France). **1) Correct**

(iii) In(Paris, France) ∨ In(Marseilles, France). **3) ∨ is wrong. Both means ∧**

b. There is a country that borders both Iraq and Pakistan.

(i) ∃ c Country(c) ∧ Border (c, Iraq) ∧ Border (c, Pakistan). **1) Correct**

(ii) ∃ c Country(c) ⇒ [Border (c, Iraq) ∧ Border (c, Pakistan)]. **3) Incorrect**

(iii) [∃ c Country(c)] ⇒ [Border (c, Iraq) ∧ Border (c, Pakistan)]. 2) Invalid ⇒ is wrong

(iv) ∃ c Border (Country(c), Iraq ∧ Pakistan). 2) **Use ∧ in side the term.**

c. All countries that border Ecuador are in South America.

(i) ∀c Country(c) ∧ Border (c,Ecuador) ⇒ In(c, SouthAmerica). **1) Correct**

(ii) ∀ c Country(c) ⇒ [Border (c,Ecuador) ⇒ In(c, SouthAmerica)]. **1) Correct**

(iii) ∀ c [Country(c) ⇒ Border (c,Ecuador)] ⇒ In(c, SouthAmerica). **3) Incorrect RHS is empty**

(iv) ∀c Country(c) ∧ Border (c,Ecuador) ∧ In(c, SouthAmerica).

d. No region in South America borders any region in Europe.

(i) ¬[∃ c, d In(c, SouthAmerica) ∧ In(d, Europe) ∧ Borders(c, d)]. **1) Correct**

(ii) ∀ c, d [In(c, SouthAmerica) ∧ In(d, Europe)] ⇒ ¬Borders(c, d)]. **3) Incorrect It's the negative of the sentence.**

(iii) ¬∀ c In(c, SouthAmerica) ⇒ ∃d In(d, Europe)∧ ¬Borders(c, d). **1) Correct**

(iv) ∀ c In(c, SouthAmerica) ⇒ ∀d In(d, Europe) ⇒ ¬Borders(c, d). **1) Correct**

e. No two adjacent countries have the same map color.

(i) ∀ x, y ¬Country(x) ∨ ¬Country(y)∨ ¬Borders(x, y) ∨

¬(MapColor (x) = MapColor (y)). **1) Correct**

(ii) ∀ x, y (Country(x) ∧ Country(y) ∧ Borders(x, y) ∧ ¬(x = y)) ⇒

¬(MapColor (x) = MapColor (y)). **1) Correct**

(iii) ∀ x, y Country(x) ∧ Country(y) ∧ Borders(x, y) ∧

¬(MapColor (x) = MapColor (y)). **3) Incorrect Use ∧ inside the term**

(iv) ∀ x, y (Country(x) ∧ Country(y) ∧ Borders(x, y)) ⇒ MapColor (x ~_= y). **2)Invalid ≠ inside the term is illegal.**