**7.4** Which of the following are correct?

**a**. False |= True. **Correct False is empty and exist no sentence. It entails the True**

**b**. True |= False. **Incorrect. Same reason as above**

**c**. (A ∧ B) |= (A ⇔ B). **Correct Left has one model (A = T, B = T), Right has two Models (A = T,B=T and A= F,B= F ). Right sentence covers the Left.**

**d**. A ⇔ B |= A ∨ B**. Incorrect Same reason as above.**

**e**. A ⇔ B |= ￢A ∨ B.  **correct Right has (A = T, B=T and A= F, B= F and A= F, B=T)**

**f**. (A ∧ B) ⇒ C |= (A ⇒ C) ∨ (B ⇒ C). **Correct, The table belows shows the Left and RHS have same models.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **A** | **B** | **C** | A∧B⇒ C | (A ⇒ C) ∨ (B ⇒ C) |
| **True** | **True** | **True** | **True** | **True** |
| **True** | **False** | **True** | **True** | **True** |
| **False** | **True** | **True** | **True** | **True** |
| **False** | **False** | **True** | **True** | **True** |
| **True** | **True** | **False** | **False** | **False** |
| **True** | **False** | **False** | **True** | **True** |
| **False** | **True** | **False** | **True** | **True** |
| **False** | **False** | **False** | **True** | **True** |

**g**. (C ∨ (￢A ∧ ￢B)) ≡ ((A ⇒ C) ∧ (B ⇒ C)). **Correct, The table below shows the models are same.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **A** | **B** | **C** | ((A⇒ C)∧ (B ⇒ C)) | (C ∨ (￢A ∧ ￢B)) |
| **True** | **True** | **True** | **True** | **True** |
| **True** | **False** | **True** | **True** | **True** |
| **False** | **True** | **True** | **True** | **True** |
| **False** | **False** | **True** | **True** | **True** |
| **True** | **True** | **False** | **False** | **False** |
| **True** | **False** | **False** | **False** | **False** |
| **False** | **True** | **False** | **False** | **False** |
| **False** | **False** | **False** | **True** | **True** |

**h**. (A ∨ B) ∧ (￢C ∨￢D ∨ E) |= (A ∨ B). **Correct, RHS must cover the model in the Left.**

|  |  |  |  |
| --- | --- | --- | --- |
| **A** | **B** | (A ∨ B) | . (A ∨ B) ∧ (￢C ∨￢D ∨ E) |
| **True** | **True** | **True** | **True**∧ (￢C ∨￢D ∨ E) |
| **True** | **False** | **True** | **True**∧ (￢C ∨￢D ∨ E) |
| **False** | **True** | **True** | **True**∧ (￢C ∨￢D ∨ E) |
| **False** | **False** | **True** | **True**∧ (￢C ∨￢D ∨ E) |
| **True** | **True** | **True** | **True**∧ (￢C ∨￢D ∨ E) |
| **True** | **False** | **True** | **True**∧ (￢C ∨￢D ∨ E) |
| **False** | **True** | **False** | **False** |
| **False** | **False** | **False** | **False** |

**i**. (A ∨ B) ∧ (￢C ∨￢D ∨ E) |= (A ∨ B) ∧ (￢D ∨ E). **Incorrect**

Removes a ￢C disjunction will makes the RHS has fewer model.

|  |  |  |  |
| --- | --- | --- | --- |
| **A** | **B** | (A ∨ B) ∧ (￢D ∨ E) | (A ∨ B) ∧ (￢C ∨￢D ∨ E) |
| **True** | **True** | **True**∧ (￢D ∨ E) | **True**∧ (￢C ∨￢D ∨ E) |
| **True** | **False** | **True**∧ (￢D ∨ E) | **True**∧ (￢C ∨￢D ∨ E) |
| **False** | **True** | **True**∧ (￢D ∨ E) | **True**∧ (￢C ∨￢D ∨ E) |
| **False** | **False** | **True**∧ (￢D ∨ E) | **True**∧ (￢C ∨￢D ∨ E) |
| **True** | **True** | **True**∧ (￢D ∨ E) | **True**∧ (￢C ∨￢D ∨ E) |
| **True** | **False** | **True**∧ (￢D ∨ E) | **True**∧ (￢C ∨￢D ∨ E) |
| **False** | **True** | **False** | **False** |
| **False** | **False** | **False** | **False** |

**j**. (A ∨ B)∧ ￢(A ⇒ B) is satisfiable. **Correct**

|  |  |  |  |
| --- | --- | --- | --- |
| **A** | **B** | A ∨ B | ￢(A ⇒ B) |
| **True** | **True** | **True** | **False** |
| **True** | **False** | **True** | **True** |
| **False** | **True** | **True** | **False** |
| **False** | **False** | **False** | **False** |

**k**. (A ⇔ B) ∧ (￢A ∨ B) is satisfiable. **Correct**

|  |  |  |  |
| --- | --- | --- | --- |
| **A** | **B** | ￢A ∨ B | (A ⇔ B) |
| **True** | **True** | **True** | **True** |
| **True** | **False** | **False** | **False** |
| **False** | **True** | **True** | **False** |
| **False** | **False** | **True** | **True** |

**l**. (A ⇔ B) ⇔ C has the same number of models as (A ⇔ B) for any fixed set of

proposition symbols that includes A, B, C. **Correct, They both have 6 models**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **A** | **B** | **C** | A ⇔ B | (A ⇔ B) ⇔ C |
| **True** | **True** | **True** | **True** | **True** |
| **True** | **False** | **True** | **False** | **False** |
| **False** | **True** | **True** | **False** | **False** |
| **False** | **False** | **True** | **True** | **True** |
| **True** | **True** | **False** | **True** | **False** |
| **True** | **False** | **False** | **False** | **True** |
| **False** | **True** | **False** | **False** | **True** |
| **False** | **False** | **False** | **True** | **False** |

**7.14** According to some political pundits, a person who is radical (R) is electable (E) if

he/she is conservative (C), but otherwise is not electable.

1. Which of the following are correct representations of this assertion?

**(ii) is right**

**i) means the person must be R and E if he is C**

**iii) means nothing because** R ⇒ ((C ⇒ E) ∨￢E) is always true

(i) (R ∧ E) ⇐⇒ C

(ii) R ⇒ (E ⇐⇒ C)

(iii) R ⇒ ((C ⇒ E) ∨￢E)

1. Which of the sentences in (a) can be expressed in Horn form?

**(i),(ii),(iii) all can be expressed in Horn Form.**

**i)**

(R ∧ E) ⇐⇒ C ≡ ((R ∧ E) ⇒ C) ∧ (C ⇒ (R ∧ E))

= ((R ∧ E) ⇒ C) ∧ (C ⇒ R) ∧ (C ⇒ E)

**ii)**

R ⇒ (E ⇐⇒ C) ≡ R ⇒ ((E ⇒ C) ∧ (C ⇒ E))

= ¬R ∨ ((¬E ∨ C) ∧ (¬C ∨ E))

= (¬R∨ ¬E ∨ C) ∧ (¬R ∨ ¬C ∨ E)

**iii) true**⇒**true**

**7.18** Consider the following sentence:

[(Food ⇒ Party) ∨ (Drinks ⇒ Party)] ⇒ [(Food ∧ Drinks) ⇒ Party] .

**a**. Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid),

or unsatisfiable.】

**Valid: From the table below. I know that all models satisfy this sentence.**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Food** | **Drink** | **Party** | Food∧Drinks | Food⇒Party | Drink⇒Party | **Left** | **Right** |
| **True** | **True** | **True** | **True** | **True** | **True** | **True** | **True** |
| **True** | **False** | **True** | **False** | **True** | **True** | **True** | **True** |
| **False** | **True** | **True** | **False** | **True** | **True** | **True** | **True** |
| **False** | **False** | **True** | **False** | **True** | **True** | **True** | **True** |
| **True** | **True** | **False** | **True** | **False** | **False** | **False** | **False** |
| **True** | **False** | **False** | **False** | **False** | **True** | **True** | **True** |
| **False** | **True** | **False** | **False** | **True** | **False** | **True** | **True** |
| **False** | **False** | **False** | **False** | **True** | **True** | **True** | **True** |

**b**. Convert the left-hand and right-hand sides of the main implication into CNF, showing

each step, and explain how the results confirm your answer to (a).

Left =

(

Right =

**c**. Prove your answer to (a) using resolution.

[(Food ⇒ Party) ∨ (Drink ⇒ Party)] ⇒ [(Food ∧ Drink) ⇒ Party]

To prove the negative is empty:

¬[[(Food ⇒ Party) ∨ (Drink⇒ Party)] ⇒ [(Food ∧ Drink) ⇒ Party]]

**The negative sentence is empty, so the Original sentence is Valid**

**8.9(Extra)** This exercise uses the function MapColor and predicates In(x, y), Borders(x, y), and

Country(x), whose arguments are geographical regions, along with constant symbols for

various regions. In each of the following we give an English sentence and a number of candidate

logical expressions. For each of the logical expressions, state whether it (1) correctly

expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3)

is syntactically valid but does not express the meaning of the English sentence.

**a**. Paris and Marseilles are both in France.

(i) In(Paris ∧ Marseilles, France ). **2) Use ∧ in side the term.**

(ii) In(Paris, France ) ∧ In(Marseilles, France ). **1) Correct**

(iii) In(Paris, France ) ∨ In(Marseilles, France ). **3)** **∨ is wrong. Both means ∧**

**b**. There is a country that borders both Iraq and Pakistan.

(i) ∃ c Country(c) ∧ Border (c, Iraq) ∧ Border (c, Pakistan). **1) Correct**

(ii) ∃ c Country(c) ⇒ [Border (c, Iraq) ∧ Border (c, Pakistan)]. **3) Incorrect**

(iii) [∃ c Country(c)] ⇒ [Border (c, Iraq) ∧ Border (c, Pakistan)]. 2) Invalid ⇒ is wrong

(iv) ∃ c Border (Country(c), Iraq ∧ Pakistan). 2) **Use ∧ in side the term.**

**c**. All countries that border Ecuador are in South America.

(i) ∀c Country(c) ∧ Border (c,Ecuador ) ⇒ In(c, SouthAmerica). **1) Correct**

(ii) ∀ c Country(c) ⇒ [Border (c,Ecuador ) ⇒ In(c, SouthAmerica)]. **1) Correct**

(iii) ∀ c [Country(c) ⇒ Border (c,Ecuador )] ⇒ In(c, SouthAmerica). **3) Incorrect RHS is empty**

(iv) ∀c Country(c) ∧ Border (c,Ecuador ) ∧ In(c, SouthAmerica).

**d**. No region in South America borders any region in Europe.

(i) ￢[∃ c, d In(c, SouthAmerica) ∧ In(d, Europe) ∧ Borders(c, d)]. **1) Correct**

(ii) ∀ c, d [In(c, SouthAmerica) ∧ In(d, Europe)] ⇒ ￢Borders(c, d)]. **3) Incorrect It’s the negative of the sentence.**

(iii) ￢∀ c In(c, SouthAmerica) ⇒ ∃d In(d, Europe)∧ ￢Borders(c, d). **1) Correct**

(iv) ∀ c In(c, SouthAmerica) ⇒ ∀d In(d, Europe) ⇒ ￢Borders(c, d). **1) Correct**

**e**. No two adjacent countries have the same map color.

(i) ∀ x, y ￢Country(x) ∨ ￢Country(y)∨ ￢Borders(x, y) ∨

￢(MapColor (x) = MapColor (y)). **1) Correct**

(ii) ∀ x, y (Country(x) ∧ Country(y) ∧ Borders(x, y) ∧ ￢(x = y)) ⇒

￢(MapColor (x) = MapColor (y)). **1) Correct**

(iii) ∀ x, y Country(x) ∧ Country(y) ∧ Borders(x, y) ∧

￢(MapColor (x) = MapColor (y)). **3) Incorrect Use ∧ inside the term**

(iv) ∀ x, y (Country(x) ∧ Country(y) ∧ Borders(x, y)) ⇒ MapColor (x ~\_= y). **2)Invalid ≠ inside the term is illegal.**