

# 1 Experiment

## 1.1 Sparse Linear Regression

Assume that we observe

$$Y = Hx + \sigma\tilde{\zeta},$$

with  $\tilde{\zeta}$  is Gaussian.

The goal is to estimate the optimal  $\hat{x}$  with the relationship  $Y \approx H\hat{x}$ .

## 1.2 ISTA (Iterative Shrinkage-Thresholding Algorithm)

The goal is to find

$$\min_x \frac{1}{2n} \|Y - Hx\|_2^2 + \lambda \|x\|_1.$$

The solution is

$$x_{k+1} = \text{soft}(x_k + \frac{1}{\alpha n} H^T(Y - Hx_k), \lambda).$$

In Ista.py, generate data with  $n = 100, p = 1000$ .  $X \in \mathbb{R}^{p \times 1}$  has 20 nonzero elements.

The optimal  $\lambda = 3\sigma\sqrt{\log(p)} \approx 0.5$ .

## 1.3 Hard Thresholding Algorithm

Loss function  $L : \mathbb{R}^p \rightarrow \mathbb{R}$ ,

$$L = \frac{1}{2n} \|Y - Hx\|_2^2.$$

The solution is

$$\text{hard}(x + \frac{1}{\alpha n} H^T(y - Hx), \lambda).$$