MATH 424 Final Project

Verify Delta and Gamma Hedging

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Introduction, simulate data, corresponding part in report and presentation slides - Xinyang Ding
Verify Delta Hedging with simulated data, corresponding part in report and presentation slides - Garrett Rent
Verify Delta and Gamma Hedging with market data, corresponding part in report and presentation slides - Peter Robinson
Verify Gamma Hedging with simulated data, compare simulated data and market data results, conclusion, corresponding part in report and presentation slides - Yueru He

Chapter 1

INTRODUCTION

1.1 Introduction

Firstly, we want to briefly introduce Geometric Brownian motion; Delta, and Gamma hedging. Delta hedging is a way to reduce the directional exposure of a stock or option position. For example, if we have positive position deltas, we can reduce the risk by adding a negative delta that brings the position delta closer to zero. On the other hand, if we have a negative position delta, we can accomplish the same by adding a positive delta. Delta is the first derivative of the function. If we want to compare it with some real-life concept, it can be seen as the velocity in physic. To generally describe delta as an increase or decrease in stock price, how much will change in the option price. For Gamma, it is the second-order derivative of the function. If we want to compare it as we did to Delta, Gamma is like acceleration in Physics. Last, we want to introduce the Brownian motion. Standard Brownian motion is a continuous random process to simulate future stock prices. It has two main characteristics. The first is the movement in the different time periods is independent. It means the movement in different time zones will not affect each other. The second is the increment is a normal distribution. The drifted Brownian motion is (1). In this project, the main purpose of the Geometric Brownian motion is to simulate future stock prices. We denote as the stock price at time t.

$$\mathrm{dS}_t = \mu * \mathrm{S}_t * \Delta \mathrm{t} + \sigma * \mathrm{S}_t * \Delta * \mathrm{w}_t (1)$$

$$\ln(\mathrm{S}_t / S_0) = \sigma * \mathrm{w}_t + (\mu - \sigma^2 / 2) t (2)$$

$$\mathrm{U}_t = ln(S_(i+1)) - ln(S_i) = \sigma * \mathrm{w}_{t(} i+1) - \sigma * \mathrm{w}_{ti} + (\mu - \sigma^2 / 2) dt (3)$$

1.2 Project Summary

In this project, we are using Geometric Brownian motion to first simulate stock price datas. Then, we verify Delta and Gamma hedging using the simulated datas we got in the first part. With this , we can get the proof of Delta and Gamma hedging. Last, We compare simulated results with real-world results to get the conclusion.

Chapter 2

Using GBM to Simulate Stock Price

In the Matlab, we try to use Geometric Brownian Motion to simulate stock price. The Code is to simulate the stock price multiple times using the Geometric Brownian Motion. We first set the time and the stock price for input. Then, we use the formula to generate the data.

2.1 Explanation

We can set the function of Geometric Brownian Motion in the Matlab. Then we can input initial data to generate the result multiple times in order to get a trend graph of the stock price.

We have expected expected drift rate and volatility respectively. It also suggested that the financial meaning of the drift rate is the average rate of return. Equation (3) in the previous page is dtochastic differential equation(SDE). We can first assume the initial stock price which can get equation (3) from (2). Then we can use formula (3) to simulate the stock price in the time period we set. The drift rate and volatility can be calculated from historical stock price. The following is the Matlab code that used to simulate stock price data:

```
% s0 is the initial price of stock
% mu is the drift rate
% sigma it the volatility
% t_all is the total time
% dt is the unit of time
% s_n is the stock price array
% t_n is the time array
function [s_n, t_n] = generate_stock_price(s0, mu, sigma, t_all, dt)
t_n = [0:dt:t_all]'; % get the time array
BM_n = [0 dt0̂.5*cumsum(randn(1,t_all/dt))]'; %construct a standard brownian motion
s_n = s0 * exp((mu-0.5*sigma2̂) * t_n+sigma * BM_n); %construct the stock price plot(t_n,s_n), hold
on
end
```

Chapter 3

Simulated Data Results

3.1 Simulated Data for Delta Hedging

This simulation examined the life of an arbitrary 20-week option that started and ended on arbitrary dates. Given the code that was provided to us, we were to construct a simulation of delta hedging table, listing the Δ, shares purchased (or sold), the cost of purchasing or selling said shares, the interest cost, and the cumulative total cost for each week from the start time of the option to the expiration date.

3.1.1 Constructing Data for Delta Hedging

To properly construct data for the simulated delta hedging, it was important that our values independent of the code we were provided were immutable and the same as the real-life values provided by the market data. Therefore, our risk-free interest rate was 0.25%, which was taken from the fluctuating LIBOR rate that hovered around 0.25% for the market value's term. We decided to keep a flat rate, since an ever-changing rate would've made delta hedging far more difficult than need be for the purposes of this project. We also kept the option price to be 5.6, the spot price to be 278, and the strike price to be 300, to stay in line with the market values. The purpose of maintaining these identical values to the market was to remove any possible discrepancies which would otherwise make comparing the market and simulated data far more difficult.

3.1.2 Simulated Data

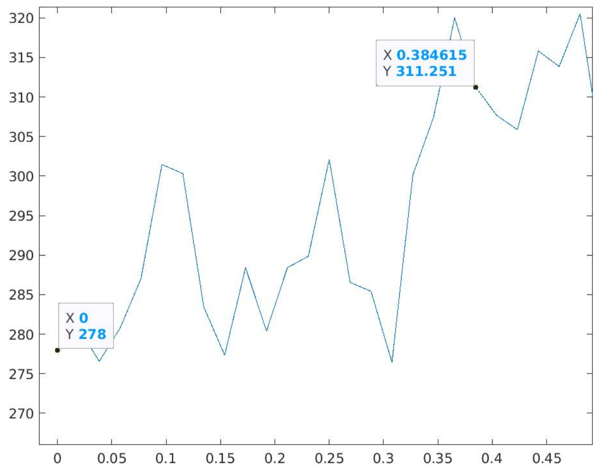

Figure 3.1: Simulated Data for 20-Week Stock Price (y) Over Time (x)

This is the stock price over time graph that was constructed using the code we were provided. Since the shortest length of time we could use was 1 year, we had to divide our sub-lengths of time to weekly intervals, to return the data points that were needed. The data points that are labeled are the start time and expiration date for our option, being 0 and 20 weeks (140 days in 20 weeks/365 days in a year). As we can see from this graph,the stock price was frequently subject to change. Often times the stock price would frequently change places from being in to out of the money, which resulted in many changes in the d1 and delta from week to week.

3.1.3 Verifying Simulated Data for Delta Hedging

Simulation of Delta Hedging

Week	Stock Price	D1	Delta	Shares Purchased	Cost of Shares Purchased (x1000)	Cumulative Cost Excluding Interest Cost (x1000)	Interest Cost
0	278	-0.5443	0.2931	29310	8418.2	8148.2	195.87
0.5	279.39	-0.5112	0.3046	1150	321.3	8469.5	203.59
1	280.78	-0.4797	0.3157	1110	311.7	8781.2	211.09
1.5	278.67	-0.5511	0.2908	-2490	-693.9	8087.3	194.41
2	276.57	-0.6249	0.266	-2480	-685.1	7402.2	177.94
2.5	278.67	-0.5704	0.2842	1820	507.2	7909.4	190.13
3	280.77	-0.515	0.3033	1910	536.3	8445.7	203.02
3.5	283.91	-0.4259	0.3351	3180	902.8	9348.5	224.72
4	287.04	-0.3357	0.3685	3340	958.7	10307.2	247.77
4.5	294.25	-0.1158	0.4539	8540	2486.4	12793.6	307.5
5	301.47	0.1059	0.5542	10030	3023.7	15817.3	380.22
5.5	300.89	0.0875	0.5349	-1930	-580.7	15236.6	366.26
6	300.31	0.0683	0.5272	-770	-231.2	15005.4	360.7
6.5	291.88	-0.2119	0.4161	-11110	-3242.8	11762.6	282.75
7	283.45	-0.5112	0.3046	-11150	-3160.5	8602.1	206.78
7.5	280.4	-0.6339	0.2631	-4150	-1163.7	7438.4	178.81
8	277.35	-0.763	0.2227	-4040	-1120.5	6317.9	151.87
8.5	282.88	-0.5718	0.2837	6100	1725.6	8043.5	193.35
9	288.42	-0.3762	0.3534	6970	2010.3	10053.8	241.68
9.5	284.41	-0.5432	0.2935	-5990	-1703.6	8350.2	200.72
10	280.4	-0.721	0.2355	-5800	-1626.3	6723.9	161.63
10.5	284.41	-0.5762	0.2822	4670	1328.2	8052.1	193.56
11	288.41	-0.4267	0.3348	5260	1517.0	9569.1	230.03
11.5	289.13	-0.4109	0.3406	580	167.7	9736.8	234.06
12	289.85	-0.3946	0.3466	600	173.9	9910.7	238.24
12.5	295.97	-0.1353	0.4462	9960	2943.9	12854.6	309
13	302.1	0.1363	0.5542	10800	3262.7	16117.3	387.44
13.5	294.32	-0.2306	0.4088	-14540	-4279.4	11819.9	284.13
14	286.55	-0.637	0.2621	-14670	-4203.7	7616.2	183.08
14.5	286.08	-0.6939	0.2439	-1820	-520.7	7095.5	170.56
15	285.42	-0.7684	0.2211	-2280	-650.8	6444.7	154.92
15.5	280.93	-1.0827	0.1395	-8160	-2292.4	4152.3	99.81
16	276.44	-1.4433	0.0745	-6500	-1796.9	2355.4	56.62
16.5	288.3	-0.7379	0.2303	15580	4491.7	6847.1	164.59
17	300.16	0.0381	0.5152	28490	8551.6	15398.7	370.16
17.5	303.85	0.3155	0.6223	10710	3254.2	18652.9	448.39
18	307.55	0.6557	0.744	12170	3742.9	22395.8	538.36
18.5	313.79	1.3421	0.9102	16620	5215.2	27611.0	663.72
19	320.03	2.346	0.9905	8030	2569.8	30180.8	725.5
19.5	315.64	2.6023	0.9954	490	154.7	30335.5	729.22
20	311.25	Infinity	1	460	143.2	30478.7	—
Rate	0.25%						
# of Options	100,000			Delta Hedging Total Cost	$30,478,700.00	Payoff from Exercise	$30,000,000.00
Strike Price	300			Interest Cost	$11,062.20	Payoff from Contract	$560,000.00
Option Cost	5.6 per share			Total Cost	$30,489.762.20	Total Payoff	$30,560,000.00
				Total Profit	Payoff — Cost =	$70,237.80	**DELTA HEDGING WORKS**

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Figure 3.2: Simulated Data for 20-Week Delta Hedging

3.1.4 Observations

As seen in the figure above, our option ended up in the money, and as a result our delta eventually converged at 1 at the end of the 20 weeks. The interest costs were also quite low, due to our low risk-free interest rate. Our stock price changing sides of the strike price so frequently that it greatly affected the deltas more than was expected. As a result, the shares purchased was also a greatly changing variable, and as a result our cumulative cost was quite high. A total of 5 times, our stock price would either rise above the strike price, or fall below the strike price. Near the end of the option's lifetime, the stock price made one more rapid rise in value, going from \$276.44 to \$311.25 in the last 5 weeks, with the price peaking in Week 19 where it reached \$320.03. In particular, the strangest week was from Week 16 to week 17, where the price rose from \$276.44 to \$300.16. Previously, the Δ was -1.4433, nearly assuring us that the stock price was going to end up out of the money, and signaling the investor to start selling nearly all of his shares.

However, as the stock price continuously and rapidly rose, the investor was forced to buy large amounts of shares to hedge. While the option did in fact fall in the money, the cumulative cost nearly exceeded the payoff, likely due to the large amounts of shares the investor was forced to buy due to the surprising increase in stock price in the last few weeks of the option's lifetime. Previously, when delta hedging was done every week, it resulted in a loss. However, when delta hedging was done twice a week, we had a profit, showing that the more frequently delta hedging is done, the more profit the investor will yield.

3.2 Simulated Data for Gamma Hedging

Similar to what we did for Delta hedging, we use a different set of simulated data to verify the Gamma hedging. Since the underlying asset has zero gamma value, we need to use induce other derivatives to do gamma hedging. In this project, we construct the Gamma Hedging using a "traded option" and the underlying stock of the portfolio we want to hedge.

3.2.1 Information needed for Gamma Hedging

We used the same code for generating stock price data as in the Delta Hedging part. For Gamma Hedging, we need to simulate two stocks: the underlying stock for the target portfolio and that for the traded option. The underlying stock for the portfolio has the same initial value as the Delta Hedging part, and the underlying stock for the traded option has a different spot price at the starting point. We set S0 to be 300, which is a little bit higher than the S0 for the portfolio. We assume that the institution sells this portfolio for 30,000 dollars. The detailed setup for the portfolio is shown in the following table. For simplicity, we assume the stock is non-dividend paying, and all three options have the same underlying stock. The traded option is a call option with a strike price of 310.

Portfolio Components	Number of options	S0	K	T	σ	dividend
call option	1000	278	300	20 weeks	0.2	0
put option	1000	278	270	20 weeks	0.2	0
call option	1000	278	290	20 weeks	0.2	0

We ran the price generating code several times, with different seeds, since some data varies so widely that it could barely happen in the real world. The following are the plots of the stock prices for the underlying stock for both the portfolio and the traded option. The blue line represents the underlying stock price for the portfolio and the red line represents the underlying stock price for the traded option. Notice that the plots are the simulated data for one year, but for our purpose, we will only use the first 20 weeks (140 days).

3.2.2 Verify Gamma Hedging With Simulated Data

There are mainly four parts of code for verifying Gamma Hedging. The first part is the data generating possess, as discussed above. The second thing we did is to compute the daily total gamma and delta values of the portfolio. We utilized the financial toolbox in Matlab, which has built-in BSM pricing tools. Since we hedge the portfolio once a day, those pricing tools can give us the price of each option in the portfolio in a very short time. The next part is to compute daily Deltas and Gammas for the traded option. The computing process is very similar to the second. The last part is to use the traded option and underlying stock of the portfolio to implement Gamma hedging.

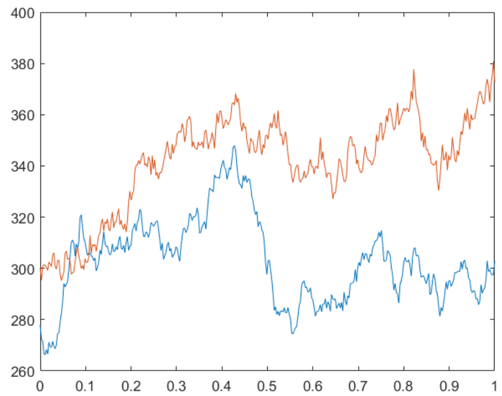

Figure 3.3: Simulated Data for Gamma Hedging

The goal is to keep our holdings Delta-neutral and Gamma-neutral. After obtaining the total Gammas of the portfolio and the traded option, we can use the traded option to offset the Gammas of the portfolio. If the total Gammas of the portfolio at day t is $\Gamma 0$, then we need to get -$\Gamma 0$ from the traded option. If the gamma for one share of traded option is γ, then we need to long Q=-$\Gamma 0/\gamma$ traded options. Then we recalculate Delta at day t and we can hedge Delta using the underlying stock. Suppose the Delta of the portfolio is Δp, and the change in Delta due to the traded option is Δtr. Then we can long -(Δp+Q*Δtr) shares of underlying stocks to make our holdings Delta-neutral.

We need to perform part two to four described in the first paragraph for the Gamma Hedging every day. Therefore, we use a while loop. Except for the first day of hedging, we only need to modify our holdings based on the change of the underlying stocks, so we need variables to keep track of the changes every day. The Deltas, Gammas, the number of stocks we need to buy, and the number of traded options we need to buy are the four variables we need to take care of. Then we use the changes in holdings to compute the cumulative cost of performing Gamma Hedging. The following code is the second and last part of our code. The third part of the code is omitted since it is very similar to the second part.

3.2.3 Results and Observations

The stock price at maturity date is $329.8267, which makes it strongly in the money for both call options in the portfolio, and strongly out of money for the put option in the portfolio. Therefore, the institution receives (1000*300+1000*290)=$590,000 since the buyer will exercise call options in the portfolio. The expenditure we used for Gamma hedging is $28,904. With the initial payment of $30,000, the institution gets a profit of $591,096. This is a big amount of profit, which is rare in the real market since the probability of the stock price to increase by about 20% in 20 weeks is low.

On the other hand, we successfully hedged the portfolio by daily Gamma Hedging. If the simulated stock price is smoother, we can expect the probability of the call options to be exercised to be lower. Besides, we didn't include interest cost in this simulation. Even though the interest cost is not big, the higher the frequency of hedging is, the more important interest cost becomes. If we include the interest cost, the profit should be lower, but still positive. In fact, the portfolio itself hedges loss of the institution, because it consists of both put and call options. No matter the stock increase or decrease, at least one option of the portfolio has the chance to be exercised, which facilitates the Gamma Hedging, makes the cost of hedging lower.

```
%% Step 1: Compute total gamma and delta for portfolio
DataMatrix = [s_n(i)  300  0.38356  0.2   0        % Call
          s_n(i)  270  0.38356  0.2   0        % Put
          s_n(i)  290  0.38356  0.2  0]  ;        % Call
RiskFreeRate = 0.0025; % 0.25%, consistent with real market data

% assign each column of DataMatrix to a column vector whose name reflects
% the type of financial data in the column.
StockPrice   = DataMatrix(:,1);
StrikePrice  = DataMatrix(:,2);
ExpiryTime   = DataMatrix(:,3);
Volatility   = DataMatrix(:,4);
DividendRate = DataMatrix(:,5);

% use BSM price model in the financial toolbox here to compute call
% and put prices, gammas and deltas
[CallPrices, PutPrices] = blsprice(StockPrice, StrikePrice,...
RiskFreeRate, ExpiryTime, Volatility, DividendRate);

[CallDeltas, PutDeltas] = blsdelta(StockPrice,...
StrikePrice, RiskFreeRate, ExpiryTime, Volatility,...
DividendRate);

Gammas = blsgamma(StockPrice, StrikePrice, RiskFreeRate,...
              ExpiryTime, Volatility , DividendRate)';

Prices = [CallPrices(1) PutPrices(2) CallPrices(3)];
Deltas = [CallDeltas(1) PutDeltas(2) CallDeltas(3)];
%total gamma of the portfolio, with 1,000 of each of the three options
Gamma_total=sum(Gammas)*1000;
Delta_total=sum(Deltas)*1000;
%% Step 3: gamma hedging the portfolio using traded option and underlying
%get gamma neutral first
traded_quantity=-Gamma_change/Gamma_T; %quantity we should long in traded
traded_change=traded_quantity-traded_old;
traded_old=traded_quantity;
%then get delta neutral
stock_quantity=-(Delta_change+traded_quantity*Delta_T); %quantity we shoul
stock_change=stock_quantity-stock_old;
stock_old=stock_quantity;
%compute total costs
cost=cost+Price_T*traded_change+stock_change*s_n(i); %s_n(i) is the curren
```

Figure 3.4: Part 2 and Part 4 of the code

Chapter 4

Market Data Results

4.1 Market Data for Delta Hedging

This study examined the life of a 20-week option on the Exchange-Traded Fund (ETF) VOO. VOO is an ETF designed to mimic the S&P 500. The study began on March 29, 2020 and ended with the expiration of the option on October 15, 2020. The scenario represented an institution who had taken a short position in 1000 call options (each option on 100 shares) on VOO. The price of the option was $5.60, the strike price was $300, and the risk-free rate was taken from the LIBOR rate, which hovered around 0.25% for the duration of the term. The delta in the portfolio was rebalanced every week until expiration using calculations from TD Ameritrade. Using the delta-hedging technique, the institution generated $361,448 in profit despite the options rising above their strike. Exact results can be seen in the figure below.

Date	Weeks Till Expi	Stock Price at Open	Delta	Shares Holding	Shares Purchased in Pd	Cost of Shares Purchased	Cumulative Share Cost	Interest Cost	Cumulative Interes	Total Cumulative Cost
5/29/20	20.00	278.03	0.28	28,098.54	28,098.54	7,812,237.08	7,812,237.08	375.59	375.59	7,812,612.66
6/5/20	19.00	291.63	0.43	42,616.12	14,517.58	4,233,761.86	12,045,998.93	579.13	954.72	12,046,953.65
6/12/20	18.00	283.35	0.31	31,401.74	-11,214.38	-3,177,594.57	8,868,404.36	426.37	1,381.09	8,869,785.45
6/19/20	17.00	290.04	0.38	37,848.77	6,447.03	1,869,896.58	10,738,300.94	516.26	1,897.35	10,740,198.29
6/26/20	16.00	282.72	0.27	27,035.34	-10,813.43	-3,057,172.93	7,681,128.01	369.29	2,266.64	7,683,394.65
7/2/20	15.00	288.76	0.35	34,628.80	7,593.46	2,192,687.51	9,873,815.52	474.70	2,741.34	9,876,556.86
7/10/20	14.00	288.72	0.40	39,662.63	5,033.83	1,453,367.40	11,327,182.92	544.58	3,285.92	11,330,468.83
7/17/20	13.00	295.72	0.44	43,662.25	3,999.62	1,182,767.63	12,509,950.54	601.44	3,887.36	12,513,837.90
7/24/20	12.00	294.86	0.42	41,732.74	-1,929.51	-568,935.32	11,941,015.23	574.09	4,461.44	11,945,476.67
7/31/20	11.00	299.45	0.48	48,160.27	6,427.53	1,924,723.86	13,865,739.08	666.62	5,128.07	13,870,867.15
8/7/20	10.00	306.23	0.58	58,074.42	9,914.15	3,036,010.15	16,901,749.24	812.58	5,940.65	16,907,689.89
8/14/20	9.00	309.15	0.61	61,132.00	3,057.58	945,250.86	17,847,000.10	858.03	6,798.68	17,853,798.77
8/21/20	8.00	310.48	0.66	65,845.46	4,713.46	1,463,435.06	19,310,435.16	928.39	7,727.07	19,318,162.22
8/28/20	7.00	321.14	0.77	76,846.07	11,000.61	3,532,735.90	22,843,171.05	1,098.23	8,825.29	22,851,996.35
9/4/20	6.00	317.77	0.68	67,952.24	-8,893.83	-2,826,192.36	20,016,978.69	962.35	9,787.65	20,026,766.34
9/11/20	5.00	308.55	0.60	60,489.23	-7,463.01	-2,302,711.74	17,714,266.96	851.65	10,639.30	17,724,906.25
9/18/20	4.00	309.44	0.58	58,431.21	-2,058.02	-636,833.71	17,077,433.25	821.03	11,460.33	17,088,893.58
9/25/20	3.00	297.58	0.55	55,262.48	-3,168.73	-942,950.67	16,134,482.57	775.70	12,236.02	16,146,718.60
10/2/20	2.00	304.72	0.66	66,282.93	11,020.45	3,358,151.52	19,492,634.10	937.15	13,173.17	19,505,807.27
10/9/20	1.00	317.47	0.93	93,166.29	26,883.36	8,534,660.30	28,027,294.40	1,347.47	14,520.64	28,041,815.03
10/15/20	0.00	315.71	0.96	96,063.30	2,897.01	914,615.03	28,941,909.42	1,391.44	15,912.07	28,957,821.50
Asset	VOO									
Expiration Date	10/16/20									
risk-free rate	0.25%				Total Cost of Delta Hedgin	28,957,821.50	Payoff from Exercise	30,000,000.00		
Cost of Option	5.6 per share				Cost to Complete Option	1,240,729.74	Payoff from Contract	560,000.00		
No. of Options	1000 options at 100 shares each				Total Cost	30,198,551.24	Total Payoff	30,560,000.00		
Strike	300.00						**Profit**	**361,448.76**		
Shares	100,000.00									

Figure 4.1: Delta Hedging Market Data Results

As seen in the figure, the delta was initially very low for an option that was far from its expiration and far below its strike price. It is noticeable that in the week of August 7, the delta rose above .5 when the stock price rose above the strike price for the option. Afterwards, the stock remained firmly above the strike price. As the time until the expiration whittled down, the delta converged to one. By the time that the buyer of the call options was ready to exercise, the investor had most of the shares on hand to fulfill the option. Moreover, the investor had purchased those shares for below market price at the time of the option. This would have been the price the investor had to pay if he or she did not use the delta hedging technique.

4.2 Market Data for Gamma and Delta Hedging

Similar to the delta hedging example, this study examined the effectiveness of simultaneous gamma and delta hedging. This example looked at a 5-week time period from November 11, 2020 to October 16, 2020. The initial investor began with 10 contracts of a call option with strike 310, price 6.45, and expiration on October 16, 2020, which was 5 weeks. The investor re-balanced the portfolio weekly to ensure a gamma-neutral and delta-neutral portfolio.

To do this, the investor would calculate the total gamma of the portfolio. This would be the sum of the gamma of each type of option in the portfolio, $\sum \Gamma_i$, where $\Gamma_i = \Gamma_c * (quantitiyofoptions) * 100$ and $\Gamma_c = \frac{\partial \Delta}{\partial S}$. Every week, as time and stock price changed, the gamma of the portfolio would change as well. To bring the gamma back to zero, the investor would buy or sell a quantity of call options contracts that would offset the gamma in their portfolio and bring it as close to zero as possible. This option would have the same expiration, October 16 but could have varying strike prices. The quantity would be $-\Gamma/\Gamma_c$. The options bought at each re-balancing are shown in the diagram below. The delta of the portfolio would have then changed because time had passed and new options had been added. In response, the investor would buy or sell shares of the asset VOO to satisfy the following delta-hedging requirements. After five weeks of using these techniques, the investor secured a profit of $1827. Exact results can be seen in the section below.

Portfolio at t=0: Institution has 10 contracts of a call option expiring at 10/16											
Strike 310, price 6.45							-6450				
Date	**Weeks Till Expir**	**Stock Price at Open**	Gamma Before Adjustment	Adjustment	Quantity/Strike/Price	Gamma After Adjustment	Cash Flow	New Delta/Shares Holdir	Shares Purchased In Pd	Cost of Shares Purcha	Cumulative Share Cost
9/11/20	5.00	308.55	18.54	Sell Calls	10/315/4.30	0.79	4300	96.39	96.39	29741.1345	29741.13
9/18/20	4.00	309.44	2.42	Sell Calls	1/310/4.20	0.24	420	76.68	-19.71	-6099.0624	23642.07
9/25/20	3.00	297.58	2.34	Sell Calls	1/315/1.275	0.49	127.5	76.15	-0.53	-157.7174	23484.35
10/2/20	2.00	304.72	-3.24	Buy Calls	1/310/4.05	-0.5	-405	124.99	48.84	14882.5248	38366.88
10/9/20	1.00	317.47	-19.36	Buy Calls	4/315/5.15	-0.95	-2060	359.44	234.45	74430.8415	112797.72
10/16/20	0.00	315.71	-47.63								
							-4067.5				

Option Payoffs	Get	Give
t=0	1000 Shares	310000
t=0	315000	1000 Shares
t=1	31000	100 Shares
t=2	31500	100 Shares
t=3	100 Shares	31000
t=4	400 Shares	126000
Cashflows		4067.5
Total	**300 Shares**	**-93567.5**

Note: Delta and Gamma Figures are caluclated from TD Ameritrade software
The delta figure represents the number of shares needed to maintain a delta-neutral portfolio
The gamma figure is weighted, that is, it is (gamma of one option on one share)*100*number of contracts
One options contract is on 100 shares

	Costs	Assets	
	93567	300 shares * $315.71 = $94713	
	112797.72	359.44 shares * $315.71 = 113478.80	
Total	**206364.72**	**208191.8**	**1827.08**

Figure 4.2: Gamma and Delta Hedging Market Data Results

As seen in the figure, the investor began with an initial gamma of 18.54. The investor adjusted by taking a position with negative gamma, and purchased delta shares in order to minimize risk. The gamma did not fluctuate very much at the beginning. Between the first and second period, the gamma only moves from 0.79 to 2.42. However, as expiration drew near, the gamma began to fluctuate wildly. Between the fourth and fifth weeks, the gamma grew from -.95 to -47.63. The investor counteracted this by taking on larger positions in options and in the asset. In the end, his or her strategy paid off. Using the techniques, the investor secured a profit of $1827.

Chapter 5

Comparison Between Market and Simulated Data

Both the simulated data and the market data end up with a higher stock price than the initial price. Also, we can observe the fact that the closer to the maturity time, the larger Deltas are. This observation is consistent with what we've seen in class.

The Delta Hedging fails with simulated data at first. The profit we get from hedging the asset is negative, which means the institution that sells options loses money by hedging the risk. In contrast, market data is Delta hedged successfully. One possible reason for this difference is the difference in volatility.

As mentioned in class, the volatility of a stock partly determines the distribution of the stock price. We can use the implied volatility to predict future stock prices, but we must be careful about the fact that we always use the lognormal assumption to model the distribution of the stock price. The stock price under lognormal distribution has a similar yet different shape to the distribution of the stock price under implied volatility. More precisely, under the implied volatility, the stock price appears to be higher in the center, which means it is more likely to fluctuates around the initial stock price than the lognormal distribution predicts. Also, more extreme price movements might happen under the implied volatility than under the lognormal assumption. These differences explain why the plot of the simulated data is more volatile than real market data.

Another reason might be that we the periods between hedging is too long. In this project, we hedge the portfolio only once a week. Infrequent hedging leads to large hedging errors. This infrequency problem could have been solved by increasing the frequency of hedging. Thus we change our hedging frequency to twice a week. This time the Delta Hedging works. Therefore, we can say that the frequency of hedging plays a significant role in hedging strategies.

By hedging everyday, the Gamma Hedging successes with simulated data. It is hard to perform Gamma Hedging for market data, and we failed to get Gamma-neutral results from the market data. In simulated data, we ignore the interest cost and transaction cost, which increase as the frequency of hedging increases. For the market data, we have the interest cost but still ignore the transaction cost. Despite the failure of Gamma Hedging in market data, we observed a big fluctuation in Gammas when the time to maturity is getting closer, which is also true for the simulated data.

Chapter 6

Conclusion

Although it's compelling to be able to reduce the risk of trading, it is always very complicated and time-consuming to do so. We performed Delta Hedging and Gamma Hedging with both simulated data and real market data. The results are not as ideal as what we have learned in theory. The market data is successfully hedged by both Delta and Gamma Hedging. On the other hand, the simulated data fails delta hedging.

The time we spent on keeping the portfolio both Delta-neutral and Gamma-neutral was also very long. Delta Hedging alone is easy to implement since there's only one letter we need to care about. However, hedging both Delta and Gamma is much more complicated. As mentioned in class, Delta and Gamma change over time. We have to rebalance them regularly. In this project, we hedge the portfolio once a week. We found that it's challenging to keep Delta and Gamma both equal to zero since it involves at least two kinds of options. If we want to hedge the asset more frequently (e.g. once a day, even once an hour), we will need computers that can calculate Delta and Gamma much faster than we did in this project.

In the real world, many limitations prevent people from perfect hedging. For example, one can only buy and sell an integer number of options, which make Delta and Gamma sometimes cannot be exactly equal to zero. The periods between hedging is another limitation. Additionally, the distribution of the stock price does not necessarily follow the Geometric Brownian Motion. Although the simulated data assume the stock price follows GBM, the real stock price might fluctuate differently. We can never model the stock price movement perfectly. Therefore, we cannot hedge the risk perfectly. For both simulated and real market data used in this project, the hedging strategies could not generate the ideal profits one can get from the Black-Scholes-Merton model. Nevertheless, Delta and Gamma Hedging still secure risks to some extent and are widely used together in practice.

Chapter 7

References

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